

Analytical Method for Next-to-Leading-Order QCD Corrections to Double-Higgs Production

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We propose a new method to calculate analytically higher-order perturbative corrections and we apply it to the calculation of the two-loop virtual corrections to Higgs pair production through gluon fusion. The method is based on the expansion of the amplitudes in terms of a small Higgs transverse momentum. This approach gives a very good approximation (better than per mille) of the partonic cross section in the center-of-mass energy region $\sqrt{\hat{s}} \lesssim 750$ GeV, where $\sim 95\%$ of the total hadronic cross section is concentrated. The presented method is general and can be applied in a straightforward way to the computation of virtual higher-order corrections to other $2 \rightarrow 2$ processes, representing an improvement with respect to calculations based on heavy mass expansions.

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Introduction.—The experimental exploration of the properties of the Higgs boson is one of the major targets of the Large Hadron Collider (LHC). However the self-couplings of the Higgs boson, which in the standard model are fully determined in terms of the mass of the Higgs boson and the Fermi constant, have not been probed yet. While the quartic Higgs self-coupling is not directly accessible at the LHC [1,2], the trilinear self-coupling might be measurable from Higgs pair production processes [3–17].

Those processes, in particular Higgs pair production in gluon fusion, are also sensitive to new physics, that can greatly modify their rates [18–22]. Bounds on $gg \rightarrow HH$ for different final states are reported in Refs. [23–28].

Therefore, a precise prediction of the gluon fusion channel is essential to determine the Higgs trilinear self-coupling and constrain new physics. At leading order (LO) the gluon fusion process has been known since the 1980s [29]. At next-to-leading order (NLO) this process is fully known only numerically [30,31], while analytical results are available in the heavy top mass (m_t) limit [32–34] and partially in the light m_t limit [35]. In Ref. [36] a method was proposed for obtaining an analytical result combining large top mass expansion and a threshold expansion by means of Padé approximants.

The limits of Refs. [32–35] well describe the Higgs pair production in the regions $\sqrt{\hat{s}} < 300$ GeV and $\sqrt{\hat{s}} > 750$ GeV, respectively, where $\sqrt{\hat{s}}$ is the partonic center-of-mass energy, but fail to describe the intermediate region.

We propose a new approach for the analytical calculation of the virtual NLO corrections to the Higgs pair production through gluon fusion. The method is based on the expansion of the amplitudes around a small Higgs transverse momentum p_T and Higgs mass m_h . After properly expanding, the resulting amplitudes are functions of only m_t and $\sqrt{\hat{s}}$ and can be calculated analytically without resorting to further expansions. With this method we are able to correctly describe the Higgs pair production in the region $\sqrt{\hat{s}} \lesssim 750$ GeV, nicely complementing the present literature. It must also be noted that, due to the shape of the gluon parton distribution functions, this region represents 95% of the total hadronic cross section.

Our approach has the virtue of covering larger regions of the phase space with respect to approaches based on heavy mass expansions or high energy expansions and can be easily implemented to other $2 \rightarrow 2$ processes.

In this Letter we describe the basics of the method, and the main results of our calculation, while we will reserve a more detailed discussion of the computation to future works.

Notation and definitions.—In this section we introduce the notation we will use in the rest of the Letter and define a set of kinematical variables. The amplitude $g_a^\mu(p_1)g_b^\nu(p_2) \rightarrow H(p_3)H(p_4)$ can be written as

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$$A^{\mu\nu} = \frac{G_\mu \alpha_s(\mu_R)}{\sqrt{2} 2\pi} \delta_{ab} T_F \hat{s} [A_1^{\mu\nu} F_1 + A_2^{\mu\nu} F_2], \quad (1)$$

where G_μ is the Fermi constant, $\alpha_s(\mu_R)$ is the strong coupling defined at the renormalization scale μ_R , and $T_F = 1/2$ is the normalization factor for the fundamental representation of $SU(N_c)$. In Eq. (1) $A_{1,2}^{\mu\nu}$ are the orthogonal projectors onto the spin-0 and spin-2 states, respectively, while the corresponding form factors $F_{1,2}$ are functions of m_t , m_h , and the partonic Mandelstam variables (all momenta are assumed incoming)

$$\hat{s} = (p_1 + p_2)^2, \quad \hat{t} = (p_1 + p_3)^2, \quad \hat{u} = (p_2 + p_3)^2, \quad (2)$$

via

$$F_1 = F_1(\hat{s}, \hat{u}, m_t^2, m_h^2), \quad F_2 = F_2(\hat{s}, \hat{u}, m_t^2, m_h^2). \quad (3)$$

We defined $A_1^{\mu\nu}$ and $A_2^{\mu\nu}$ as

$$\begin{aligned} A_1^{\mu\nu} &= g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{(p_1 \cdot p_2)}, \\ A_2^{\mu\nu} &= -g^{\mu\nu} + \frac{m_h^2 p_1^\nu p_2^\mu}{p_T^2 (p_1 \cdot p_2)} \\ &\quad - 2 \frac{(p_3 \cdot p_2) p_1^\nu p_3^\mu + (p_3 \cdot p_1) p_3^\nu p_2^\mu - (p_1 \cdot p_2) p_3^\mu p_3^\nu}{p_T^2 (p_1 \cdot p_2)}, \end{aligned} \quad (4)$$

with p_T the transverse momentum of the Higgs particle, which can be expressed in terms of the Mandelstam variables as

$$p_T^2 = \frac{\hat{t} \hat{u} - m_h^4}{\hat{s}}. \quad (5)$$

The Born cross section, then, is

$$\sigma^{(0)}(\hat{s}) = \frac{G_\mu \alpha_s^2(\mu_R)}{512(2\pi)^3} \int_{\hat{t}_-}^{\hat{t}_+} d\hat{t} (|T_F F_1|^2 + |T_F F_2|^2), \quad (6)$$

with $\hat{t}_\pm = -\hat{s}/2(1 - 2m_h^2/\hat{s} \mp \sqrt{1 - 4m_h^2/\hat{s}})$. For our purpose, it is particularly convenient to introduce the prime Mandelstam variables:

$$\begin{aligned} s' &= p_1 \cdot p_2 = \frac{\hat{s}}{2}, & t' &= p_1 \cdot p_3 = \frac{\hat{t} - m_h^2}{2}, \\ u' &= p_2 \cdot p_3 = \frac{\hat{u} - m_h^2}{2}, \end{aligned} \quad (7)$$

for which $s' + t' + u' = 0$. In these variables the Higgs transverse momentum becomes

$$p_T^2 = 2 \frac{t' u'}{s'} - m_h^2. \quad (8)$$

Our ultimate goal is to make an expansion for small $\sqrt{p_T^2 + m_h^2} \sim p_T$. Since the final result is symmetrical in $t' \leftrightarrow u'$, the latter can be achieved expanding for $t' \sim 0$, $u' \sim -s'$ (expanding only in $t' \sim 0$ would not be correct if the final result were not symmetrical in $t' \leftrightarrow u'$). This is going to restrict F_1 and F_2 in Eq. (3) to a forward kinematic, namely to be a function of \hat{s}/m_t^2 only, reducing the computational difficulty from a three scales problem to a single scale one. To perform the expansion we need to express the momenta in terms of the parallel and transverse components with respect to the beam axis. For this purpose we define the combination of momenta:

$$r = p_1 + p_3 \quad \text{and} \quad \bar{r} = p_2 + p_3. \quad (9)$$

It is easy to show that

$$\begin{aligned} r^2 &= \hat{t}, & \bar{r}^2 &= \hat{u}, \\ p_1 \cdot r &= -p_2 \cdot r = t', & p_2 \cdot \bar{r} &= -p_1 \cdot \bar{r} = u', \end{aligned} \quad (10)$$

and that

$$r^\mu = \frac{t'}{s'} (-p_1^\mu + p_2^\mu) + r_\perp^\mu, \quad \bar{r}^\mu = \frac{u'}{s'} (p_1^\mu - p_2^\mu) + \bar{r}_\perp^\mu, \quad (11)$$

where $r_\perp^\mu = \bar{r}_\perp^\mu$ is perpendicular to p_1 and p_2 and, as expected,

$$r_\perp^2 = m_h^2 + 2t' + 2 \frac{t'^2}{s'} = -p_T^2. \quad (12)$$

Finally, in this reparametrization, $A_{1,2}^\mu$ assume particularly simple forms:

$$A_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{s'}, \quad A_2^{\mu\nu} = A_1^{\mu\nu} + 2 \frac{r_\perp^\mu r_\perp^\nu}{p_T^2}. \quad (13)$$

Expansion.—From Eq. (5), assuming real valued \hat{t} and \hat{u} , we obtain the condition

$$p_T^2 + m_h^2 \leq \frac{\hat{s}}{4}, \quad (14)$$

which allows us to expand for $p_T^2/s' \ll 1$ and $m_h^2/s' \ll 1$.

Although our program is clear, it is hindered by the fact that p_T does not appear directly at the amplitude level. However, it is possible to show that an expansion for $r^\mu \sim 0^\mu$ is equivalent to an expansion in $p_T^2 \sim 0$. Using Eqs. (11) and (12), and noticing that r_\perp is purely spacelike, we can exchange the expansion in $p_T^2 \sim 0$ with an expansion in $r^\mu \sim 0^\mu$ or, equivalently, $p_3^\mu \sim -p_1^\mu$.

This observation is one of the main results of this Letter, and it allows us to proceed. We can then rewrite the form factors in Eq. (1) as

$$F_{1,2} = F_{1,2}|_{p_3=-p_1} + r^\mu \frac{\partial F_{1,2}}{\partial p_3^\mu} \Big|_{p_3=-p_1} + \frac{1}{2} r^\mu r^\nu \frac{\partial^2 F_{1,2}}{\partial p_3^\mu \partial p_3^\nu} \Big|_{p_3=-p_1} + \dots \quad (15)$$

Although Eq. (14) is always valid, the expansion proposed in Eq. (15) requires a hierarchy between r^2 and m_t^2 . We are going to estimate the range of validity of the small p_T expansion by comparing at the LO the result obtained via Eq. (15) with the exact LO result (see next section).

We conclude this section with an important remark on how to correctly truncate the series in Eq. (15). Since the final result should be symmetrical for $p_1 \leftrightarrow p_2$ and $\partial F_{1,2}/\partial p_3^\mu$ is a rank 1 tensor, the second term in the rhs of Eq. (15) can be rewritten as $r^\mu \mathcal{F}_{1,2}(p_1^\mu + p_2^\mu) = \mathcal{F}_{1,2}(\hat{i} + \hat{u})$, with $\mathcal{F}_{1,2}$ a function of s' and m_t^2 . For similar arguments, the third term should instead be proportional to $r^\mu r^\nu (g_{\mu\nu} + \dots) = p_T^2 + \dots$. It is clear, then, that to expand to the first order in p_T^2 , one has to expand to the second order in p_3^μ , or, more in generally, an order n expansion in p_T^2 needs the order $2n$ expansion in p_3^μ .

Computation and results.—We generated the relevant amplitudes for the virtual NLO corrections to $gg \rightarrow HH$ with FEYNARTS [37]. The amplitudes were contracted with the two orthogonal projectors in Eq. (4), using FEYNCALC [38], and reduced to a combination of scalar integrals. The integrals were then Taylor expanded, as described in the previous section. Subsequently, the resulting integrals were reduced in terms of a basis of master integrals using FIRE [39] and LITERED [40]. All of the master integrals, of which nearly the totality can be expressed in terms of multiple polylogarithms, were already known in the literature [41–47]. However, we evaluated them again directly in the phase space region of interest. We cross-checked our results using SECDEC [48]. The details of the calculation presented here, as well as a detailed study of the validity of our approximation at the hadronic level, will be the topic of a second paper on this argument [49], while in this Letter we will focus on the final result.

In order to show that our method correctly describes the partonic cross section for $\sqrt{\hat{s}} < 750$ GeV, we will start applying it to the LO.

In Fig. 1 we report our calculation for the partonic cross section using Eq. (15). As discussed in the Introduction, while the heavy m_t expansion describes well only the range $\sqrt{\hat{s}} < 2m_t$, with our method we are able to correctly describe a wider range. It is also interesting to note that an expansion up to order $\mathcal{O}(p_T^4)$ is already sufficient to describe the complete result with enough precision.

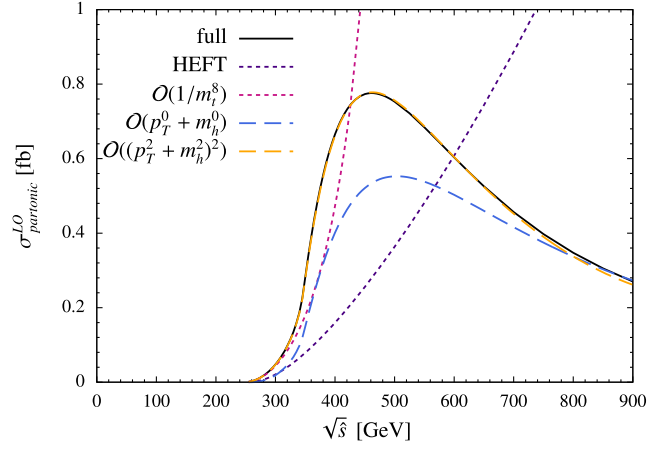


FIG. 1. Partonic cross section of $gg \rightarrow HH$ as a function of the partonic center-of-mass energy. The black continuous line is the full result [29]. The dotted lines represent the line of Higgs effective theory (HEFT) corresponding to the zero order in the heavy m_t limit and the 4th order in the heavy m_t limit. The dashed lines are the result of the small p_T^2 approximation presented in this Letter.

The range of validity of the small p_T expansion can be estimated comparing the partonic cross section calculated with our method with the one from the full LO calculation.

In Table I, we show

$$\Delta\sigma = \left| \frac{2(\sigma_{\text{full}} - \sigma_{\text{approx}})}{(\sigma_{\text{full}} + \sigma_{\text{approx}})} \right|, \quad (16)$$

where σ_{full} is the cross section calculated without expansion, and σ_{approx} is the one calculated in this Letter. Table I indicates that $\Delta\sigma$ is small and very well under control up to values of the partonic c.m. energy of about ~ 750 GeV. Moreover, in the region of interest, the approximation rapidly improves as one considers higher order in the expansion in p_T^2 and m_h^2 . The range of validity of our formulas is complementary to the one present in the literature, and represents 95% of the total hadronic cross section.

This behavior is confirmed (and even improved) in the comparison with the full numerical result at NLO. It is well known that the NLO virtual corrections are IR divergent and these divergences cancel against the ones that come

TABLE I. Relative difference between the approximated and the exact LO cross sections, for different orders of expansion, at various \hat{s} .

$\Delta\sigma - \hat{s}$	$4m_t^2$	$6m_t^2$	$8m_t^2$	$12m_t^2$	$16m_t^2$	$32m_t^2$
$p_T^0 \times 10^{-1}$	6.2	4.4	3.2	1.8	1.0	0.3
$p_T^2 \times 10^{-2}$	8.5	4.4	1.1	2.4	5.1	33.2
$p_T^4 \times 10^{-2}$	1.3	0.1	0.4	0.2	0.9	2.8
$p_T^6 \times 10^{-3}$	2.3	0.9	1.0	0.1	3.5	450

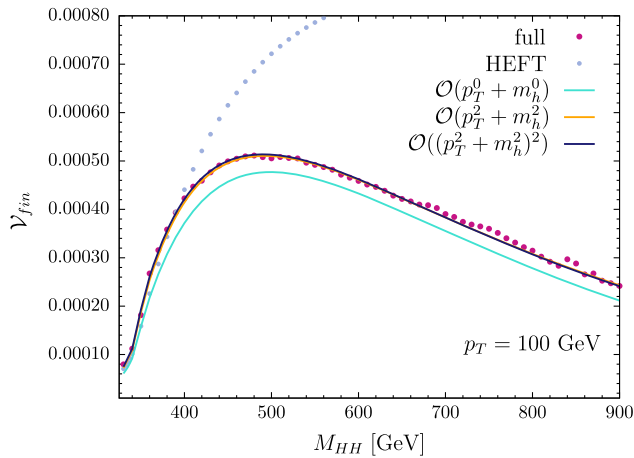


FIG. 2. Finite part of the virtual corrections as a function of the invariant mass of the two Higgs system. The pink points are extracted with the interpolation function from Ref. [50]. The dotted light blue points correspond to reweighted HEFT [51]. The solid lines are the respective orders in our calculation. We do not show $\mathcal{O}((p_T^2 + m_h^2)^3)$ as the line lies perfectly on top of the one of $\mathcal{O}((p_T^2 + m_h^2)^2)$.

from real corrections [32–34]. Following Ref. [34], we cancel the IR divergences by adding the counterterm $1/(2\epsilon^2)F_{1,2}^{\text{LO}}(\epsilon^2)(\hat{s})^{-\epsilon}$, where $F_{1,2}^{\text{LO}}(\epsilon)$ are the LO form factors with the inclusion of the $\mathcal{O}(\epsilon, \epsilon^2)$ terms. In Fig. 2 we compare our result to the numerical results from Ref. [30], at the partonic level, using the grid and the interpolation function for the finite part of the virtual corrections \mathcal{V}_{fin} provided in Ref. [50]. As can be inferred from the figure, our expansion perfectly agrees with the full result when the first correction in p_T and m_h is included. It can clearly be seen that our lines smooth out the error on the full result stemming from the interpolation. Furthermore, we compare the numerical results of the authors of Ref. [30] with ours in several points of the grid provided by the same authors. For $p_T \lesssim 200$ GeV, we find agreement between the two computations within the error quoted for each point of the grid from the numerical integration. For larger p_T the agreement is still quite good (for $p_T \lesssim 300$ GeV is within twice the numerical error quoted), showing a degradation with the increase of p_T .

Conclusion.—In this Letter we have proposed a novel approach for the analytical computation of the NLO virtual corrections to Higgs pair production through gluon fusion. This method, based on an expansion for small p_T^2 , allows us to describe accurately the region $\hat{s} \lesssim 750$ GeV that until now has been explored only numerically. In particular, we showed that a few terms in the expansion already reproduce the full LO within 10^{-3} , in the region of interest. At NLO we find excellent agreement already at $\mathcal{O}(p_T^2 + m_h^2)$ comparing to the full result of Ref. [30]. To judge the usefulness of our analytic method, we compare the CPU time needed to produce a phase-space point in our approach with that

needed in the numerical calculation of Ref. [30]. In order to compute one single phase-space point, Ref. [30] quotes an average of 2 h per node using 16 Dual NVIDIA TESLA K20X GPU nodes, while in our approach the computation of one single phase-space point took ~ 4 sec on a MacBook Air. We remark that this method is general and can be useful for the analytic computation of radiative corrections to other fundamental processes for the physics program of the LHC. In particular, the application of this method to the computation of the NLO virtual corrections to the top contribution in the HZ , ZZ , and $\gamma\gamma$ gluon fusion production processes is expected to be straightforward, while processes where the top and bottom contribution cannot be separate, like, e.g., in the WW gluon fusion production, deserve a more detailed investigation.

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