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Book of Short Papers

Giovanni C. Porzio
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Editors

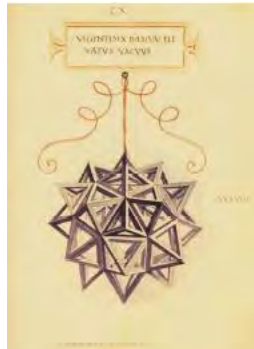


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2019

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Preface

This book collects the short papers presented at CLADAG 2019, the 12th Scientific Meeting of the Classification and Data Analysis Group (CLADAG) of the Italian Statistical Society (SIS).

The meeting has been organized by the Department of Economics and Law of the University of Cassino and Southern Lazio, under the auspices of the SIS and the International Federation of Classification Societies (IFCS). CLADAG is a member of the IFCS, a federation of national, regional, and linguistically-based classification societies. It is a non-profit, non-political scientific organization, whose aims are to further classification research.

Every two years, CLADAG organizes a scientific meeting, devoted to the presentation of theoretical and applied papers on classification and related methods of data analysis in the broad sense. This includes advanced methodological research in multivariate statistics, mathematical and statistical investigations, survey papers on the state of the art, real case studies, papers on numerical and algorithmic aspects, applications in special fields of interest, and the interface between classification and data science. The conference aims at encouraging the interchange of ideas in the above-mentioned fields of research, as well as the dissemination of new findings.

CLADAG conferences, initiated in 1997 in Pescara (Italy), were soon considered as an attractive information exchange market and became a most important meeting point for people interested in classification and data analysis. One reason was

certainly the fact that a selection of the presented papers is regularly published in (post-conference) proceedings, typically by Springer Verlag.

The Scientific Committee of CLADAG2019 conceived the Plenary and Invited Sessions to provide a fresh perspective on the state of the art of knowledge and research in the field. The scientific program of CLADAG 2019 is particularly rich. All in all, it comprises 5 Keynote Lectures, 32 Invited Sessions promoted by the members of the Scientific Program Committee, 16 Contributed Sessions, a Round Table and a Data Competition. We thank all the session organizers for inviting renowned speakers, coming from 28 countries. We are greatly indebted to the referees, for the time spent in a careful review.

The editors would like to express their gratitude to the Rector of the University of Cassino and Southern Lazio and the Director of the Department of Economics and Law for having hosted the meeting. Special thanks are finally due to the members of the Local Organizing Committee and all the people who with their abnegation and enthusiasm have worked for CLADAG 2019.

Special thanks go to Alfiero Klain and Livia Iannucci for the editorial and administrative support.

Last but not least, we thank all the authors and participants, without whom the conference would not have been possible.

Cassino, September 11, 2019

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THE PARAFAC MODEL IN THE MAXIMUM LIKELIHOOD APPROACH

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ABSTRACT: Factor analysis is a well-known model for describing the covariance structure among a set of manifest variables through a limited number of unobserved factors. When the observed variables are collected at various occasions on the same statistical units, the data have a three-way structure and standard factor analysis may fail to discover the interrelations among the variables. To overcome these limitations, three-way models can be adopted. Among them, the so-called Parallel Factor (Parafac) model can be applied. In this article, the structural version of such a model, i.e. as a reparameterization of the covariance matrix, is studied by discussing under what conditions factor uniqueness is preserved.

KEYWORDS: three-way factor analysis, maximum likelihood, factor uniqueness property.

1 Introduction

Factor analysis (FA) (Bartholomew et al., 2011) is a well-known method explaining the relationships among a set of manifest variables, observed on a sample of statistical units, in terms of a limited number of latent variables. In FA data are stored in a matrix, say \mathbf{X} , of order $(I \times J)$ being I and J the number of statistical units and variables, respectively. Thus, FA deals with two-way two-mode data, where the modes are the entities of the data matrix, i.e., statistical units and manifest variables, and the ways are the indexes of the elements of \mathbf{X} , i.e., $i = 1, \dots, I$ and $j = 1, \dots, J$. In many practical situations, it may occur that the scores on the same manifest variables with respect to a sample of statistical units are replicated across K different occasions, e.g. time, locations, conditions etc. In this case, there are three sets of entities (statistical units, manifest variables and occasions), hence three modes and the available information is stored in the so-called array, or tensor, usually denoted by $\underline{\mathbf{X}}$ of order $(I \times J \times K)$. Its generic element is x_{ijk} , $i = 1, \dots, I$, $j = 1, \dots, J$ and $k = 1, \dots, K$, expressing the score of statistical unit i on manifest variable j at occasion k .

Therefore, the elements have three indexes and the array three ways. For all of these reasons data are three-way three-mode (see, e.g., Kroonenberg, 2008).

The basic FA model is not adequate to handle three-way three-mode data. It has been extended in order to take into account and exploit the increasing complexity of three-way three-mode data. The most famous three-way three-mode extensions of FA are the Tucker3 (Tucker, 1966) and Parafac (Harshman, 1970) models, where the latter can be seen as a particular case of the former with a useful property of parameter uniqueness (Kruskal, 1977). Such extensions were born as suitable generalizations of Principal Component Analysis (PCA) and are mainly devoted to fit the model to the data according to a certain criterion. Some authors revised these proposals as structural models for the covariance structure of the manifest variables (e.g., Bentler et al., 1988). In this paper, after recalling the main features of the Parafac model following the above-mentioned two approaches, a structural extension of Parafac is considered and its uniqueness property is analysed when some specific factors are correlated across occasions, or variables.

2 The Parafac model

The Parafac model (Harshman, 1970) summarizes the three-way three-mode tensor $\underline{\mathbf{X}}$ by looking for a limited number of components for the modes. Let \mathbf{X}_A be the matrix of order $(I \times JK)$ obtained by juxtaposing next to each other the frontal slabs of $\underline{\mathbf{X}}$, i.e. the standard two-way two-mode matrices \mathbf{X}_k ($k = 1, \dots, K$) of order $(I \times J)$ collected at the different occasions. The Parafac model can be formulated as

$$\mathbf{X}_A = \mathbf{A}(\mathbf{C} \bullet \mathbf{B})' + \mathbf{E}_A, \quad (1)$$

where the symbol ' \bullet ' denotes the Khatri-Rao product of matrices, i.e., it is $\mathbf{C} \bullet \mathbf{B} = [\mathbf{c}_1 \otimes \mathbf{b}_1, \dots, \mathbf{c}_S \otimes \mathbf{b}_S]$, where \mathbf{b}_s and \mathbf{c}_s are the s -th columns of \mathbf{B} and \mathbf{C} , respectively ($s = 1, \dots, S$), being S the number of components for the modes, and the symbol ' \otimes ' denotes the Kronecker product of matrices. The matrices \mathbf{A} , \mathbf{B} , \mathbf{C} have order $(I \times S)$, $(J \times S)$, $(K \times S)$, respectively, and give the scores of the entities of the various modes on the components. Like Principal Component Analysis, the parameter estimates are found in the ordinary least squares (OLS) sense by minimizing the sum of squares of the error term \mathbf{E}_A . For this purpose, alternating least squares (ALS) algorithms can be applied.

The most interesting feature of Parafac is that under mild conditions the factors are essentially unique. This point has been deeply investigated by Kruskal (1977), who has found the following result. Let us denote by $k\text{-rank}(\mathbf{Z})$ the so-called k -rank of a matrix \mathbf{Z} . It is defined as the largest number k such that every subset of k columns of \mathbf{Z} is linearly independent. Moreover, let $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ and $(\mathbf{A}_T, \mathbf{B}_T, \mathbf{C}_T)$ be two optimal Parafac solutions. Kruskal (1977) has shown that if

$$k\text{-rank}(\mathbf{A}) + k\text{-rank}(\mathbf{B}) + k\text{-rank}(\mathbf{C}) \geq 2S + 2 \quad (2)$$

then, by considering (1),

$$\mathbf{A}(\mathbf{C}\cdot\mathbf{B})' = \mathbf{A}_T(\mathbf{C}_T\cdot\mathbf{B}_T)' \quad (3)$$

implies that there exists a permutation matrix \mathbf{P} and three diagonal matrices \mathbf{D}_A , \mathbf{D}_B and \mathbf{D}_C , for which $\mathbf{D}_A\mathbf{D}_B\mathbf{D}_C = \mathbf{I}$, such that

$$\mathbf{A}_T = \mathbf{A}\mathbf{P}\mathbf{D}_A, \mathbf{B}_T = \mathbf{B}\mathbf{P}\mathbf{D}_B, \mathbf{C}_T = \mathbf{C}\mathbf{P}\mathbf{D}_C. \quad (4)$$

Starting from the original formulation in (1) we can derive what is the corresponding covariance structure. We limit our attention to the i -th row of \mathbf{X}_A , say \mathbf{x}_{Ai}' , pertaining to the i -th statistical unit. \mathbf{x}_{Ai}' is the vector of length JK containing the scores of statistical unit i on the J manifest variables during the K occasions. By explicitly considering a vector of intercepts and rewriting the model in terms of column vectors, we get

$$\mathbf{x}_{Ai} = \boldsymbol{\mu} + (\mathbf{C}\cdot\mathbf{B})\mathbf{a}_i + \mathbf{e}_{Ai}. \quad (5)$$

As usual in standard FA, we assume that the common factors \mathbf{a}_i and the specific factors \mathbf{e}_{Ai} are random with $E(\mathbf{a}_i) = \mathbf{0}$ and $E(\mathbf{e}_{Ai}) = \mathbf{0}$, without loss of generality because of $\boldsymbol{\mu}$, and $E(\mathbf{a}_i\mathbf{a}_i') = \mathbf{0}$. If $E(\mathbf{a}_i\mathbf{a}_i') = \boldsymbol{\Phi}$ and $E(\mathbf{e}_{Ai}\mathbf{e}_{Ai}') = \boldsymbol{\Psi}$ are positive definite, then the covariance matrix of \mathbf{x}_{Ai} is given by

$$\boldsymbol{\Sigma} = E[(\mathbf{x}_{Ai} - \boldsymbol{\mu})(\mathbf{x}_{Ai} - \boldsymbol{\mu})'] = (\mathbf{C}\cdot\mathbf{B})\boldsymbol{\Phi}(\mathbf{C}\cdot\mathbf{B})' + \boldsymbol{\Psi}. \quad (6)$$

The generic element of the matrix $\boldsymbol{\Sigma}$ (of order $JK \times JK$), $\sigma_{jk,j'k'}$, holds the covariance between manifest variable j at occasion k and manifest variable j' at occasion k' ($j, j' = 1, \dots, J; k, k' = 1, \dots, K$). Bearing in mind the standard FA model, it should be clear that the Parafac model is a constrained version of standard FA. If we set $\boldsymbol{\Lambda} = (\mathbf{C}\cdot\mathbf{B})$, then (6) coincides with the oblique FA model where $\boldsymbol{\Lambda}$ is the matrix of factor loadings having a particular form depending on the three-way three-mode structure of the data. Maximum likelihood theory is used for estimating the parameters of the structural Parafac model assuming that the vectors \mathbf{x}_{Ai} , $i = 1, \dots, I$, are independent and identically distributed as a multivariate normal.

3 Results

In this work we analyzed whether the constraints $\boldsymbol{\Lambda} = (\mathbf{C}\cdot\mathbf{B})$ affect the parameter identifiability under different covariance structure of the specific factors. In particular, we proved that the Parafac model in the structural formulation maintains the uniqueness property when, as in the standard FA model, the specific factors are assumed to be uncorrelated, i.e. the matrix $\boldsymbol{\Psi}$ is diagonal, and when the specific factors of the different variables are correlated within the same occasion, i.e. the matrix $\boldsymbol{\Psi}$ is block-diagonal, i.e.,

$$\Psi = \text{diag}(\Psi_{11}, \dots, \Psi_{kk}, \dots, \Psi_{KK}), \quad (7)$$

where Ψ_{kk} denotes the covariance matrix of order $(J \times J)$ for the specific factors at occasion k , $k = 1, \dots, K$. The Parafac covariance model in (6) with the correlation structure of the specific factors given in (7) represents a more realistic model able to fit reasonably well in many practical three-way three-mode studies. It is important to note that what follows can be extended to the case where the specific factors of the same variable are correlated across the different occasions. Such an extension can be easily obtained by exploiting the symmetry of the model with respect to variables and occasions. When Ψ is diagonal, the proof is based on Theorem 5.1 of Anderson & Rubin (1956). When Ψ is block-diagonal, the conditions of Anderson & Rubin (1956) cannot be longer applied. To prove the uniqueness, the results of Browne (1980), formulated in the context of the FA model for multiple batteries of tests, is considered. For further details, see Giordani et al. (2019). During the meeting, we show how the factor uniqueness property hold in the above described cases. Moreover, we illustrate the effectiveness of the proposal by means of a real-life example in the multitrait-multimethod analysis framework.

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