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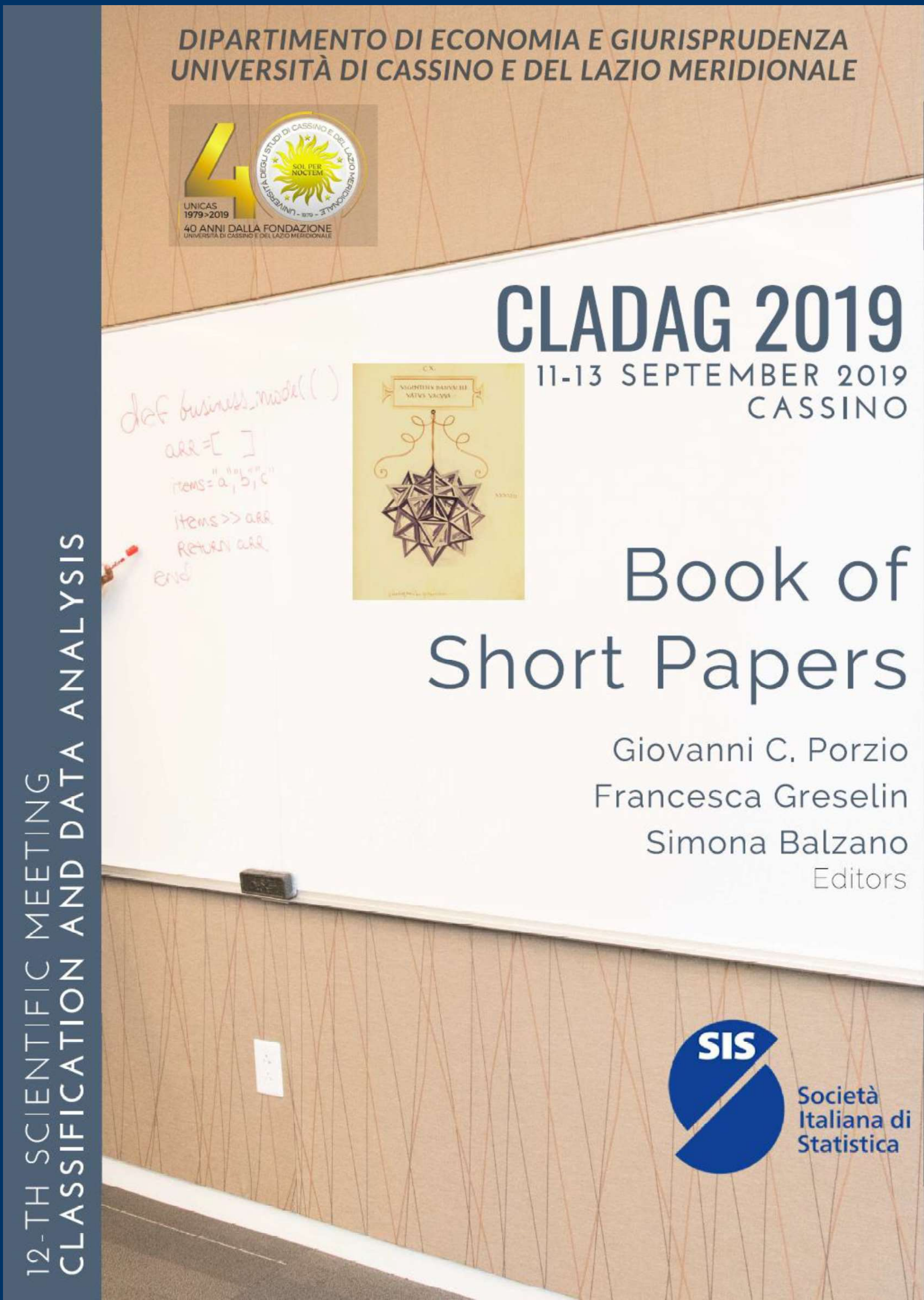

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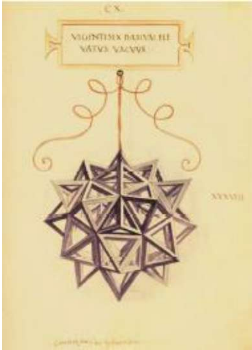

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Book of Short Papers

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COMPOSITE LIKELIHOOD INFERENCE FOR SIMULTANEOUS CLUSTERING AND DIMENSIONALITY REDUCTION OF MIXED-TYPE LONGITUDINAL DATA

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ABSTRACT: We introduce a multivariate hidden Markov model (HMM) for mixed-type (continuous and ordinal) variables. As some of the considered variables may not contribute to the clustering structure, we built a hidden Markov-based model such that we are able to recognize discriminative and noise dimensions. The variables are considered to be linear combinations of two independent sets of latent factors where one contains the information about the cluster structure, following an HMM, and the other one contains noise dimensions distributed as a multivariate normal (and it does not change over time). The resulting model is parsimonious, but its computational burden may be cumbersome. To overcome any computational issue, a composite likelihood approach is introduced to estimate model parameters.

KEYWORDS: mixed-type data, data reduction, HMM, composite likelihood, EM algorithm.

1 Introduction

In this work we focus our attention on longitudinal multivariate-mixed type data (continuous and ordinal variables). This means there are three major dependency structures: correlation between multivariate variables, temporal dependence and heterogeneity. Furthermore, to be realistic, we assume the presence of dimensions (named noise) that are uninformative for capturing the heterogeneity over time and could obscure the true data structure. To simplify, the aim of the proposal is to recover the cluster structure underlying the data that varies over time through some discriminative factors. Following the the Underlying Response Variable (URV) (see e.g. Jöreskog, 1990, Lee *et al.* ,

1990) approach, both the continuous and the categorical ordinal variables follow a Gaussian mixture model (Mclachlan & Peel, 2000), where the ordinal variables are only partially observed through their ordinal counterparts. To take into account the temporal dependence, we assume that the Gaussian mixture changes over time according to the realizations of an homogeneous first order Markov chain. In other words we are assuming a partially observed hidden Markov model (HMM). This extends the mixture model for mixed-type data (Everitt, 1988; Ranalli & Rocci, b 2017) over time. As regards the presence of noise variables, in literature there are approaches based on a family of mixture models which fits the data into a common discriminative subspace (see e.g. Bouveyron & Brunet, 2012; Kumar & Andreou, 1998; Ranalli & Rocci, 2017). The key idea is to assume a common latent subspace to all latent states that is the most discriminative. This allows to project the data into a lower dimensional space preserving the clustering characteristics over time, leading to a better and more parsimonious visualization and interpretation of the underlying structure of the data. The model can be formulated as a HMM with a particular set of constraints on the latent state parameters. The parameter estimates is based on a composite likelihood approach (Lindsay, 1988). The material is organized as follows. In section 2, we present the model specification. In section 3, we outline the model parameter estimation. The EM-like algorithm and an example of application on real data showing the effectiveness of the proposal will be presented elsewhere for lack of space.

2 Model specification

Let $\mathbf{x}_t = [x_1, \ldots, x_O]'$ and $\mathbf{y}_t^{\bar{O}} = [y_{O+1}, \ldots, y_P]'$ be O ordinal and $\bar{O} = P - O$ continuous variables, respectively, with $t = 1, \ldots, T$. The associated categories for each ordinal variable are denoted by $c_i = 1, 2, \ldots, C_i$ with $i = 1, 2, \ldots, O$. Following the URV approach, the ordinal variables $\mathbf{x}$ are considered as a categorization of a continuous multivariate latent variable $\mathbf{y}_t^O = [y_1, \ldots, y_O]'$. We assume that the temporal evolution of these data is driven by a multinomial process in discrete time $\boldsymbol{\xi}_{1:T} = (\boldsymbol{\xi}_t, t = 1, \ldots, T)$, where $\boldsymbol{\xi}_t = (\xi_{t1}, \ldots, \xi_{tK})$ is a multinomial random variable with K classes. We specifically assume that such process is distributed as a homogeneous Markov chain, whose distribution, say $p(\boldsymbol{\xi}_{1:T}; \boldsymbol{p})$, is known up to a vector of parameters $\mathbf{p}$ that includes the initial probabilities and the transition probabilities of the chain. Conditionally on the value assumed each time by the Markov chain, the distribution of the data at time t depends on the specific component parameters of a partially observed multivariate normal. Formally, let define K initial probabilities as $p_k = P(\xi_{1k} = 1)$ with $\sum_{k=1}^{K} p_k = 1$ and K^2 transition probabilities as $p_{hk} = P(\xi_{tk} = 1 \mid \xi_{(t-1)h} = 1)$ with $h, k = 1, \ldots, K$ and $\sum_{h=1}^{K} p_{hk} = 1$. It follows that the Markov chain process is $p(\boldsymbol{\xi}_{1:T},\mathbf{p}) = \prod_{k=1}^{K} p_k^{\xi_{1k}} \prod_{t=1}^{T}\prod_{h=1}^{K}\prod_{k=1}^{K} p_{hk}^{\xi_{(t-1)h}\xi_{tk}}$. According to the URV, the joint distribution of $\mathbf{x}$ and $\mathbf{y}^{\bar{O}}$ can be constructed as follows. The latent relationship between $\mathbf{x}$ and $\mathbf{y}^{O}$ is explained by the threshold model, $x_i = c_i \Leftrightarrow \gamma_{c_i-1}^{(i)} \leq y_i < \gamma_{c_i}^{(i)}$, with $c_i = 1,\ldots,C_i$ and where $-\infty = \gamma_0^{(i)} < \gamma_1^{(i)} < \ldots < \gamma_{C_i-1}^{(i)} < \gamma_{C_i}^{(i)} = +\infty$ are the thresholds defining the C_i categories collected in a set $\boldsymbol{\Gamma}$ whose elements are given by the vectors $\boldsymbol{\gamma}^{(i)}$. To accommodate both cluster structure and dependence within the groups, we assume that the distribution $\mathbf{y}_t = [\mathbf{y}_t^{O\prime},\mathbf{y}_t^{\bar{O}\prime}]'$ given a particular point in time, say t and conditioning on $\boldsymbol{\xi}_t$, follows a partially observed multivariate normal, $f(\mathbf{y}_{nt} \mid \boldsymbol{\xi}_t) = \prod_{k=1}^{K} \phi_P(\mathbf{y}_{nt} \mid \boldsymbol{\mu}_k,\boldsymbol{\Sigma}_k)^{\xi_{nkt}}$, where the ξ_{nkt} is a Bernoulli variable that assumes value 1 if the $n-$th observation is classified in state k at time t, $\phi_P(\mathbf{y};\boldsymbol{\mu}_k,\boldsymbol{\Sigma}_k)$ is the density of a P-variate normal distribution with mean vector $\boldsymbol{\mu}_k$ and covariance matrix $\boldsymbol{\Sigma}_k$.

Let us set $\boldsymbol{\psi} = \{\mathbf{p},\boldsymbol{\mu}_1,\ldots,\boldsymbol{\mu}_K,\boldsymbol{\Sigma}_1,\ldots,\boldsymbol{\Sigma}_K,\boldsymbol{\Gamma}\} \in \boldsymbol{\Psi}$, where $\boldsymbol{\Psi}$ is the parameter space. For a random i.i.d. sample of size N, $(\mathbf{x}_1,\mathbf{y}_1^{\bar{Q}}),\ldots,(\mathbf{x}_N,\mathbf{y}_N^{\bar{Q}})$, the log-likelihood is

$$\ell(\boldsymbol{\psi}) = \sum_{n=1}^{N} \log\left[\sum_{\boldsymbol{\xi}_{1:T}} p(\boldsymbol{\xi}_t,\mathbf{p})\phi_{\bar{O}}(\mathbf{y}_{nt}^{\bar{O}} \mid \boldsymbol{\xi}_t,\boldsymbol{\mu}_k^{\bar{O}},\boldsymbol{\Sigma}_k^{\bar{O}})\pi_{nt}\left(\boldsymbol{\mu}_{nt;k}^{O|\bar{O}},\boldsymbol{\Sigma}_k^{O|\bar{O}},\boldsymbol{\Gamma},\boldsymbol{\xi}_t\right)\right], \quad (1)$$

where, with obvious notation

$$\pi_{nt}\left(\boldsymbol{\mu}_{n;k}^{O|\bar{O}},\boldsymbol{\Sigma}_k^{O|\bar{O}},\boldsymbol{\Gamma},\boldsymbol{\xi}_t,\right) = \int_{\gamma_{c_1-1}^{(1)}}^{\gamma_{c_1}^{(1)}} \cdots \int_{\gamma_{c_O-1}^{(O)}}^{\gamma_{c_O}^{(O)}} \phi_O(\mathbf{u}_{nt};\boldsymbol{\mu}_{nt;k}^{O|\bar{O}},\boldsymbol{\Sigma}_k^{O|\bar{O}})d\mathbf{u}_{nt},$$

where $\pi_n\left(\boldsymbol{\mu}_{nt;k}^{O|\bar{O}},\boldsymbol{\Sigma}_k^{O|\bar{O}},\boldsymbol{\gamma}\right)$ is the conditional joint probability of response pattern $\mathbf{x}_{nt} = (c_1^{(1)},\ldots,c_O^{(O)})$ given the cluster k and the continuous variables $\mathbf{y}_{nt}^{\bar{O}}$. In order to identify the discriminative dimensions, it is assumed that there is a set of P latent factors $\tilde{\mathbf{y}}_t$, formed of two independent subsets.

In the first one, there are Q (with $Q \leq P$) factors that have some clustering information distributed as a mixture of Gaussians with class conditional means and variances equal to $E(\tilde{\mathbf{y}}^{Q} \mid k) = \boldsymbol{\eta}_k$ and $\mathrm{Cov}(\tilde{\mathbf{y}}^{Q} \mid k) = \boldsymbol{\Omega}_k$, respectively. In the second set there are $\bar{Q} = P - Q$ noise factors defining the so-called noise dimensions, that are independent of $\tilde{\mathbf{y}}^{Q}$ and their distribution does not vary from one class to another: $E(\tilde{\mathbf{y}}^{\bar{Q}} \mid k) = \boldsymbol{\eta}_0$ and $\mathrm{Cov}(\tilde{\mathbf{y}}^{\bar{Q}} \mid k) = \boldsymbol{\Omega}_0$. The link between $\tilde{\mathbf{y}}$ and $\mathbf{y}$ is given by a non-singular $P \times P$ matrix $\mathbf{A}$, as $\mathbf{y} = \mathbf{A}\tilde{\mathbf{y}}$. The final step is to identify the variables that could be considered as noise. Intuitively y_p is a noise variable if it is well explained by $\tilde{\mathbf{y}}^{\bar{Q}}$. Exploiting the independence between $\tilde{\mathbf{y}}^{Q}$ and $\tilde{\mathbf{y}}^{\bar{Q}}$, it is possible to compute proportions of each variable's variance that can be explained by the noise factors, and by one's complement, the proportions of each variable's variance that can be explained by the discriminative factors at each time point.

3 Construction of surrogate functions

The corresponding complete-data log likelihood involves multidimensional integrals that makes the maximum likelihood estimation computationally demanding and infeasible. To overcome this, we adopt a composite likelihood approach (Lindsay, 1988) based on $O(O-1)/2$ marginal distributions each of them composed of two ordinal variables and $\bar{O}$ continuous variables. The parameter estimates are carried out through an EM-like algorithm along with Baum-Welch recursion, that works in the same manner as the standard EM for HMMs.

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