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# **Time Series Clustering and Classification**

1st Edition

Elizabeth Ann Maharaj, Pierpaolo D'Urso, Jorge Caiado

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# Summary

The beginning of the age of artificial intelligence and machine learning has created new challenges and opportunities for data analysts, statisticians, mathematicians, econometricians, computer scientists and many others. At the root of these techniques are algorithms and methods for clustering and classifying different types of large datasets, including time series data.

**Time Series Clustering and Classification** includes relevant developments on observation-based, feature-based and model-based traditional and fuzzy clustering methods, feature-based and model-based classification methods, and machine learning methods. It presents a broad and self-contained overview of techniques for both researchers and students.

#### **Features**

- Provides an overview of the methods and applications of pattern recognition of time series
- Covers a wide range of techniques, including unsupervised and supervised approaches
- Includes a range of real examples from medicine, finance, environmental science, and more
- R and MATLAB code, and relevant data sets are available on a supplementary website

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# Time Series Clustering and Classification

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We dedicate this book to:

Paul and Claudia

Giordana

Dina, Maria, and Martim



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# Preface

The beginning of the age of artificial intelligence and machine learning has created new challenges and opportunities for data analysts, statisticians, mathematicians, econometricians, computer scientists and many others. At the root of these techniques, we find algorithms and methods for clustering and classifying different types of large datasets, including time series data, spatial data, panel data, categorical data, functional data and digital data. The emphasis of this book is on the clustering and classification of time series data, and it can be regarded as a reference manual on this topic.

The subject of clustering and classification of time series with applications in fields such as geology, medicine, environmental science, astronomy, finance and economics, has attracted substantial attention over the last two to three decades. Our goal in publishing this book is to provide research students and other researchers with a broad spectrum of techniques, all of which are located in one place. It provides the relevant developments in observation-based, feature-based and model-based traditional and fuzzy clustering methods, feature-based and model-based classification methods, and machine learning methods, in a concise manner using applied and simulated studies. Presently, these techniques can be found scattered in articles in many different journals and book chapters.

In truth, we have been researching these topics for more than 20 years. Our research has led to numerous publications in scientific journals in several fields, such as economics, business, management, finance, statistics, data analysis, marketing, medicine, physics, biology, hydrology and many others. We have included our work as well as works of several other authors, thus collecting as many methods on the clustering and classifying time series as we could. However, it should be noted that the book contains as many methods as we were aware of at the time of writing, and clearly new methods have since been proposed and published in journals.

We have divided the book into three parts and eleven chapters. Chapter 1 begins with a very brief overview of the contents of the book. Chapter 2 introduces some fundamental concepts in time series, spectral and wavelet analyses that are necessary for understanding the classification and clustering methods discussed in the book. Part I is about unsupervised clustering techniques for time series and consists of five chapters. Chapter 3 outlines the basic concepts of traditional cluster analysis. Chapter 4 discusses fuzzy clustering methods. Chapter 5 considers observation-based clustering methods. Chapter 6 deals with feature-based methods in the time, frequency and

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wavelet domains. Chapter 7 discusses model-based clustering methods, while Chapter 8 discusses other time series clustering approaches. Part II, which deals with supervised classification techniques for time series, consists of two chapters. Chapter 9 discusses discriminant analysis and classification methods based on time series features and models. Chapter 10 explores machine learning methods, such as classification trees, support vector machines and nearest neighbour algorithms. Finally, Part III, which consists of Chapter 11, presents links to computer programs in Matlab and R, data sets and real examples through demonstration.

It would not have been possible to complete this project successfully without the unconditional support of several people. Firstly, we are greatly indebted to our families for their constant support and patience. Secondly, we would like to thank Nuno Crato, Daniel Peña, João Bastos, Andrés Alonso, Livia De Giovanni, José Vilar and the Taylor & Francis's team for their helpful suggestions and contributions. Finally, we would like to thank some of our colleagues in our departments at Monash University, Sapienza - University of Rome and University of Lisbon for their support and encouragement.

A comprehensive webpage providing additional material to support this book can be found at http://www.tsclustering.homepage.pt/

Elizabeth Ann Maharaj, Melbourne, Australia Pierpaolo D'Urso, Rome, Italy Jorge Caiado, Lisbon, Portugal February 2019

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# Introduction

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## 1.1 Overview

Time series clustering and classification has relevance in a diverse range of fields which include geology, medicine, environmental science, finance and economics. Clustering is an unsupervised approach to grouping together similar items of interest and was initially applied to cross-sectional data. However, clustering time series data has become a popular research topic over the past three to four decades and a rich literature exists on this topic. A set of time series can be clustered using conventional hierarchical and non-hierarchical methods, fuzzy clustering methods, machine learning methods and model-based methods.

Actual time series observations can be clustered (e.g., D'Urso, 2000; Coppi and D'Urso, 2001, D'Urso, 2005), or features extracted from the time series can be clustered. Features are extracted in the time, frequency and wavelets domains. Clustering using time domain features such as autocorrelations, partial autocorrelations, and cross-correlations have been proposed by several authors including Goutte et al. (1999), Galeano and Peña (2000), Dose and Cincotti (2005), Singhal and Seborg (2005), Caiado et al. (2006), Basalto et al. (2007), Wang et al. (2007), Takayuki et al. (2006), Ausloos and Lambiotte (2007), Miskiewicz and Ausloos (2008), and D'Urso and Maharaj (2009).

In the frequency domain, features such as the periodogram and spectral and cepstral ordinates are extracted; included in the literature are studies by Kakizawa et al. (1998), Shumway (2003), Caiado et al. (2006), Maharaj and D'Urso (2010, 2011).

The features extracted in the wavelets domain are discreet wavelet transforms (DWT), wavelet variances and wavelet correlations and methods have been proposed by authors such as Zhang et al. (2005), Maharaj et al. (2010), D'Urso and Maharaj (2012) and D'Urso et al. (2014). As well, time series

can be modelled and the parameters estimates used as the clustering variables. Studies on the model-based clustering method include those by Piccolo (1990), Tong and Dabas (1990), Maharaj (1996, 2000), Kalpakis et al. (2001), Ramoni et al. (2002), Xiong and Yeung (2002), Boets (2005), Singhal and Seborg (2005), Savvides et al. (2008), Otranto (2008), Caiado and Crato (2010), D'Urso et al. (2013), Maharaj et al. (2016) and D'Urso et al. (2016).

Classification is a supervised approach to grouping together items of interest and discriminant analysis and machine learning methods are amongst the approaches that have been used. Initially classification was applied to crosssectional data but a large literature now exists on the classification of time series which includes many very useful applications. These time series classification methods include the use of feature-based, model-based and machine learning techniques. The features are extracted in the time domain (Chandler and Polonok, 2006; Maharaj, 2014), the frequency domain (Kakizawa et al., 1998; Maharaj, 2002; Shumway, 2003) and the wavelets domain (Maharaj, 2005; Maharaj and Alonso, 2007, 2014; Fryzlewicz and Omboa, 2012). Model-based approaches for time series classification include ARIMA models, Gaussian mixture models and Bayesian approaches (Maharaj, 1999, 2000; Sykacek and Roberts, 2002; Liu and Maharaj, 2013; Liu et al., 2014; Kotsifakos and Panagiotis, 2014), while machine learning approaches include classification trees, nearest neighbour methods and support vector machines (Douzal-Chouakria and Amblard, 2000; Do et al., 2017; Gudmundsson et al., 2008; Zhang et al., 2010).

It should be noted that clustering and classifying data evolving in time is substantially different from classifying static data. Hence, the volume of work on these topics focuses on extracting time series features or considering specific time series models and also understanding the risks of directly extending the common-use metric for static data to time series data.

# 1.2 Examples

We discuss three examples to illustrate time series clustering and classification before going into detail about these and other approaches in subsequent chapters. The first example illustrates clustering using time domain features, the second is observation-based and the third illustrates classification using wavelet features.

Example 1.1 D'Urso and Maharaj (2009) illustrate through simulated data, crisp clustering (traditional hierarchical and non-hierarchical) and fuzzy clustering of time series using the time domain features of autocorrelations. The aim here is to bring together series generated from the same process in order to understand the classification success. Fig. 1.1 shows the autocorrelation

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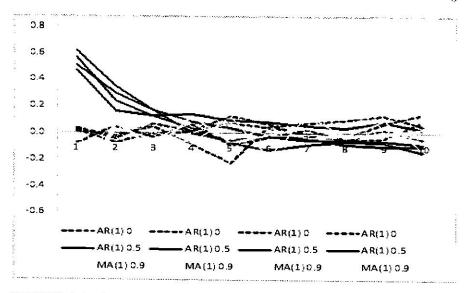


FIGURE 1.1: Autocorrelation function of series generated from three processes.

TABLE 1.1: Percentage of correct classifications using autocorrelation.

	Percentage of correct classifications
k-means	83.5
Single Linkage	85.5
Complete Linkage	93.0
Average Linkage	92.5
Ward's Method	97.8
Fuzzy c-means	87.9 - 99.5

functions (ACFs) over 10 lags for 12 simulated series, 4 of each generated from an AR(1) process with  $\phi=0$  (a white noise process), an AR(1) process with  $\phi=0.5$  and an MA(1) process with  $\theta=0.9$ . The patterns of the ACFs associated with each process are clearly distinguishable at the early lags. Table 1.1 show a summary of results of clustering the 12 series, 4 from each process over 1000 simulations. The fuzzy c-means results are subject to specific choices of parameter values. It is clear from the results in Table 1.1 that the autocorrelations provide good separation features.

**Example 1.2** D'Urso (2005) illustrates the application of a fuzzy clustering model to a set of short synthetic series consisting of three well-separated clusters of time series with 4, 2, and 2 time series each, respectively, and one switching time series (the 7th time series). This illustration is presented in

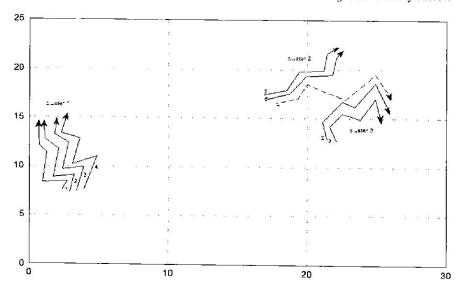


FIGURE 1.2: A set of short time series including a switching time series.

Fig.1.2 from where it can be observed that the switching time series, for the initial time period, presents an instantaneous position and slope similar to the time series belonging to Cluster 2 (series 5 and 6), while at a later time, it has an instantaneous position and slope similar to the time series belonging to Cluster 3 (series 8 and 9). Table 1.2 shows the membership degrees of each time series in each cluster and it is clear that series 1-4, 5-6 and 8-9 have crisp memberships in Clusters 1, 2 and 3 respectively, while series 7 has fuzzy membership in Clusters 2 and 3.

TABLE 1.2: Membership degrees of each time series in each cluster.

	Cluster 1	Cluster 2	Cluster 3
1	0.973	0.013	0.014
2	0.991	0.005	0.004
3	0.995	0.003	0.002
4	0.961	0.024	0.015
5	0.003	0.977	0.002
6	0.001	0.997	0.002
7	0.084	0.497	0.419
8	0.004	0.027	0.969
9	0.001	0.002	0.997

Example 1.3 Maharaj and Alonso (2014) illustrate the classification of multivariate synthetic time series using the wavelet features of variances and correlations with both linear and quadratic discriminant functions. Fig. 1.3 shows

Introduction 5

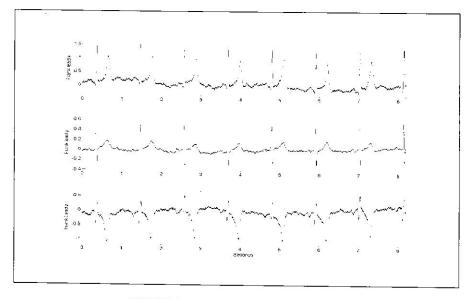


FIGURE 1.3: Synthetic ECG signals.

synthetic electrocardiogram (ECG) signals for three leads based on a three-dimensional formulation of a single dipole of the heart. Refer to Sameni et al. (2007) and Clifford et al. (2010) for more details on the development of these synthetic signals. The signals shown in Fig. 1.3 could represent those of an individual with normal heart beats. One of the parameters,  $\lambda$ , can be varied to simulate signals of an individual with the heart condition, acute myocardial infarction (AMI). This is done by setting  $\lambda > 1$ . Fig. 1.4 shows a single beat of a synthetic ECG that is normal with four scenarios of AMI when  $\lambda$  is varied.

For each population (Normal and AMI), 100 ECGs, each of length T=4096 were simulated and linear and quadratic discriminant analysis applied to the wavelet variances and wavelet correlations extracted from the signals. Fig. 1.5 and 1.6 show the classification rates (from hold-out-one cross-validation) using several wavelet filters with linear and quadratic discriminant functions, respectively. The results reveal with the exception of the scenario where  $\lambda$  was set to the smallest value greater than one, the wavelet variances and the combination of wavelet variance and correlations appear to be reasonably good features for discriminating between normal beats and those associated with AMI.

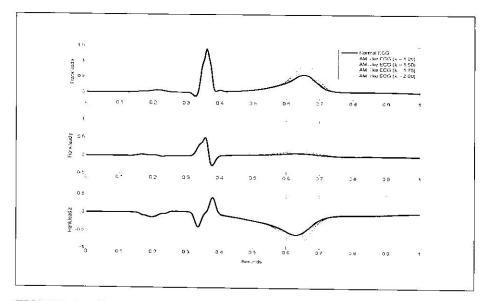


FIGURE 1.4: Single beat of a synthetic ECG signal: normal and acute myocardial infarction.

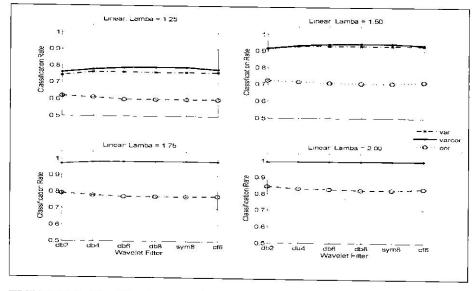


FIGURE 1.5: Classification rates for synthetic ECGs using linear discriminant functions.

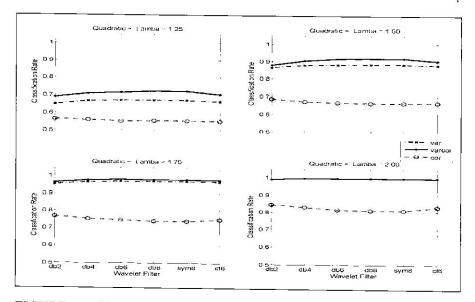


FIGURE 1.6: Classification rates for synthetic ECGs using quadratic discriminant functions.

# 1.3 Structure of the book

After this chapter, time series concepts essential for what is to follow are discussed in Chapter 2. The rest of the book is divided into three parts. Part 1 consisting of Chapters 3 to 8 is on unsupervised approaches to classifying time series, namely, clustering techniques. Traditional cluster analysis and fuzzy clustering are discussed in Chapters 3 and 4, respectively, and this is followed by observation-based, feature-based, model-based clustering, and other time series clustering approaches in Chapters 5 to 8.

Part 2 is on supervised classification approaches. This includes feature-based approaches in Chapter 9 and other time series classification approaches in Chapter 10. Throughout the book, many examples of simulated scenarios and real-world applications are provided, and these are mostly drawn from the research of the three authors. Part 3 provides links to software packages, some specific programming scripts used in these applications and simulated scenarios, as well as links to relevant data sets.



- [1] Alagon, J. (1989). Spectral discrimination for two groups of time series, Journal of Time Series Analysis 10:3, 203-214.
- [2] Alcock, R.J., Manolopoulos, Y. (1999). Time-series similarity queries employing a feature-based approach. In: Seventh Hellenic Conference on Informatics, Ioannina, Greece.
- [3] Al-Naima, F., Al-Timemy, A. (2009). In: Peng-Yeng, Y. (Ed.), Neural Network based Classification of Myocardial Infraction: A Comparative Study of Wavelet and Fourier Transforms in Pattern Recognition. In-Tech, 338-352.
- [4] Alonso, A.M., Maharaj, E.A. (2006). Comparison of time series using subsampling, Computational Statistics & Data Analysis 50, 2589-2599.
- [5] Alonso, A.M., Berrendero, J.R., Hernandez, A. and Justel, A. (2006). Time series clustering based on forecast densities. Computational Statistics & Data Analysis 51, 762-776.
- [6] Anderson, G.J., Linton, O.B. and Whang, Y.-J. (2012). Nonparametric estimation and inference about the overlap of two distributions. *Journal* of *Econometrics* 171, 1-23.
- [7] Andrzejak, R.G., Lehnertz, K., Rieke, C., Mormann, F., David, P. and Elger, C.E. (2001). Indications of nonlinear deterministic and finite dimensional structures in time series of brain electrical activity: dependence on recording region and brain state. *Phys. Rev. E* 64, 061907.
- [8] Aach, J., Church, G. (2001). Aligning gene expression time series with time warping algorithms. *Bioinformatics* 17, 495-508.
- [9] Ausloos, M.A., Lambiotte, R. (2007). Clusters or networks of economies? A macroeconomy study through Gross Domestic Product, *Physica A* 382, 16-21.
- [10] Australian Bureau of Statistics (2009). http://www.abs.gov.au/Retail-and-Wholesale-Trade.
- [11] Bahlmann, C., Hassdonk, B. and Burkhardt. (2002). Online handwriting recognition with support vector machines - a kernel approach. Proceedings. 8th International Workshop on Frontiers in Handwriting Recognition (IWFHR), 49-54.

- [12] Banerjee, S., Mitra, M. (2010). ECG feature extraction and classification of anteroseptal myocardial infarction and normal subjects using discrete wavelet transform. In: Proceedings of 2001 International Conference on Systems in Medicine and Biology, 16-18 December 2010, IIT, Kharagpur, India.
- [13] Banfield, J.D., Raftery, A.E. (1993). Model-based Gaussian and non-Gaussian clustering, *Biometrics*, 49:803-821, 345, 354.
- [14] Bastos, J.A., Caiado J. (2014). Clustering financial time series with variance ratio statistics. Quantitative Finance 12, 2121-2133.
- [15] Basalto, N., Bellotti, R., De Carlo, F., Facchi, P., Pantaleo, E. and Pascazio, S. (2007). Hausdorff clustering of financial time series, *Physica A*, 379, 635-644.
- [16] Ben-Hur, D., Horn, H.T., Siegelmann, V. and Vapnik, V. (2001). Support vector clustering. J. Mach. Learn. Res., 2,125-137.
- [17] Berndt, D., Clifford, J. (1994). Using dynamic time series warping to find patterns in time series, AAA1-94 Workshop on Knowledge Discovery in Databases, 229-248.
- [18] Beyen, K., Goldstein, J., Ramakrishnan, R. and Shaft, U. (1999). When is the nearest neighbour meaningful?, Proceedings of the 7th International Conference on Database Theory, 217-235.
- [19] Bezdek, J.C. (1981). Pattern Recognition with Fuzzy Objective Function Algorithms, Kluwer Academic Publishers, Plenum Press, New York.
- [20] Bezdek, J.C. (2001). Pattern Recognition with Fuzzy Objective Function Algorithms, Kluwer Academic Publishers, Plenum Press, New York.
- [21] Biernacki, C., Celeux, G. and Govaert, G. (2000). Assessing a mixture model for clustering with integrated completed likelihood. *IEEE Trans*actions on Pattern Analysis and Machine Intelligence, 22: 719-725, 353.
- [22] Blandford, R.R. (1993). Discrimination of earthquakes and explosions at regional distances using complexity, AFTAC TR 93 044 HQ, Air Force Technical Applications Centre, Patrick Air Force Base, FL.
- [23] Bloomfield, P. (2000). Fourier Analysis of Time Series: An Introduction. Wiley, New York.
- [24] Boeking, B., Stephan, K.C., Detlef, S. and Wong, A.S.W. (2015). Support vector clustering of time series data with alignment kernels. *Pattern Recognition Letters*, 2,125-137.
- [25] Boets, J., De Cock, K., Espinoza, M. and De Moor, B. (2005). Clustering time series, subspace identification and cepstral distances, *Commun. Info. Syst.* 5, 1, 69-96.

[26] Bollerslev, T., Chou, R.Y. and Kroner, K.F. (1992). ARCH modeling in finance. Journal of Econometrics 1-2, 5-59.

- [27] Bousseljot, R., Kreiseler, D. and Schnabel, A. (1995). Nutzung der EKGsignaldatenbank CARDIODAT der PTB ber das internet. Biomedizinische Technik. 40 (S1), S317.
- [28] Bozzola, G., Bortolan, G., Combi, C., Pinciroli, F. and Brohet, C. (1996). A hybrid neuro-fuzzy system for ECG classification of myocardial infraction. *Computers in Cardiology*, 241-244.
- [29] Brandmaier, A.M. (2011). Permutation distribution clustering and structural equation model trees. Ph.D. Thesis Universitat des Saarlandes, Saarbrucken, Germany.
- [30] Brockwell, P.J., Davis, R.A. (1991). Time Series: Theory and Methods. 2nd ed., Springer, New York.
- [31] Box, G.E.P., Jenkins, G.M. and Reinsel, G.C. (1994). Time Series Analysis. Forecasting and Control, 3rd Edition, Prentice Hall. New Jersey.
- [32] Coles, S. (2001). An Introduction to Statistical Modeling of Extreme Values, Springer-Verlag: London.
- [33] Caiado, J., Crato, N. (2010). Identifying common dynamic features in stock returns, Quantitative Finance 10, 797-807.
- [34] Caiado, J. (2006). Distance-Based Methods for Classification and Clustering of Time Series. PhD Thesis, ISEG, University of Lisbon, Portugal.
- [35] Caiado, J., Crato, N. and Peña, D. (2006). A periodogram-based metric for time series classification, Computational Statistics & Data Analysis 50, 2668-2684.
- [36] Caiado, J., Crato, N. and Peña, D. (2009). Comparison of time series with unequal length in the frequency domain, Communications in Statistics-Simulation and Computation 38, 527-540.
- [37] Caiado, J., Crato, N. and Peña, D. (2012). Tests for comparing time series of unequal lengths, Journal of Statistical Computing and Simulation 12, 1715-1725.
- [38] Caiani, E.G., Porta, A., Baselli, G., Turiel, M., Muzzupappa, S., Pieruzzi, F., Crema, C., Malliani, A. and Cerutti, S. (1998). Warped-average template technique to track on a cycle-by-cycle basis the cardiac filling phases on left ventricular volume. *IEEE Computers in Cardiology*. 73-76.

[39] Calinski, T., Harabasz, J. (1974). A dendrite method for cluster analysis. Communications in Statistics, 3(1):1-27.

- [40] Campello, R.J.G.B., Hruschka, E.R. (2006). A fuzzy extension of the silhouette width criterion for cluster analysis. Fuzzy Sets and Systems, 157(21):2858-2875.
- [41] cardioPATTERN, Telemedical ECG-Evaluation and Follow up (2009). http://radib.dyndns.org.
- [42] Chandler, G., Polonik, W. (2006). Discrimination of locally stationary time series based on the excess mass functional. *Journal of the American Statistical Association*, 101:473, 240-253.
- [43] Chatfield, C. (2004). The Analysis of Time Series, An Introduction, 3rd Edition. Chapman and Hall/CRC, New York.
- [44] Chernoff, H. (1952). A measure of asymptotic efficiency for tests of a hypothesis based on the sum of observations. The Annals of Mathematical Statistics 23:4, 493-507.
- [45] Chen, Y., Keogh, E., Hu, B., Begum, N., Bagnall, A., Mucen, A. and Batista, G. (2015). The UCR time series classification archive. URL www.cs.ucr.edu/~eamonn/time\_series\_data.
- [46] Chinganda, E.F., Subrahaniam, K. (1979). Robustness of the linear discriminant function to nonnormality: Johnson's system. *Journal of Statistical Planning and Inference* 3, 69-77.
- [47] Chinipardaz, R., Cox, T.F. (2004). Nonparametric discrimination of time series, *Metrika* 59:1, 13-20.
- [48] Chow, K., Denning, K.C. (1993). A simple multiple variance ratio test, Journal of Econometrics 58:3, 385-401.
- [49] Chu, S., Keogh, E., Hart, D. and Pazzani, M. (2002). Iterative deepening dynamic time warping for time series. *Proc 2nd SIAM International Conference on Data Mining*.
- [50] Cilibrasi, R., Vitányi, P.M.B. (2005). Clustering by compression. IEEE Tranactions on Information Theory. 51, 1523-1545.
- [51] Clifford, G., Nemati, S. and Sameni, R. (2010). An artificial vector model for generating abnormal electrocardiographic rhythms, *Journal of Phys*iological Measurements 31 (5), 595-609.
- [52] Coates, D.S., Diggle, P.J. (1986). Test for comparing two estimated spectral densities. J. TimeSer. Anal. 7, 7-20.
- [53] Cochrane, J. (1991). A critique of the application of unit root tests, Journal of Economic Dynamics and Control 15:2, 275-284.

[54] Coles, S. (2001). An Introduction to Statistical Modeling of Extreme Values. Springer-Verlag: London (2001).

- [55] Coppi, R., D'Urso, P. (2000). Fuzzy time arrays and dissimilarity measures for fuzzy time trajectories, in *Data Analysis, Classification, and Related Methods* (eds. H.A.L Kiers, J.P. Rasson, P.J.F. Groenen, M. Schader), 273-278, Springer-Verlag, Berlin.
- [56] Coppi, R., D'Urso, P. (2001). The geometric approach to the comparison of multivariate time trajectories, in Advances in Data Science and Classification (eds. S. Borra, R. Rocci, M. Vichi, M. Schader), 93-100, Springer-Verlag, Heidelberg.
- [57] Coppi, R., D'Urso, P. (2002). Fuzzy K-Means Clustering models for triangular fuzzy time trajectories, Statistical Methods and Applications, 11, 21-40.
- [58] Coppi, R., D'Urso, P. (2003). Three-Way Fuzzy Clustering models for LR fuzzy time trajectories, Computational Statistics & Data Analysis, 43, 149-177.
- [59] Coppi, R., D'Urso, P. (2006). Fuzzy unsupervised classification of multivariate time trajectories with the Shannon Entropy Regularization, Computational Statistics & Data Analysis, 50 (6), 1452-1477.
- [60] Coppi, R., D'Urso, P. and Giordani, P. (2006). Fuzzy c-medoids clustering models for time-varying data. In Bouchon-Meunier, B., Coletti, G. and Yager, R.R. (eds), Modern Information Processing: From Theory to Applications, Elsevier, Perugia.
- [61] Coppi, R., D'Urso, P. and Giordani, P. (2010). A Fuzzy clustering model for multivariate spatial time series, *Journal of Classification*, 27, 54-88.
- [62] Corduas, M., Piccolo, D. (2008). Time series clustering and classification by the autoregressive metric, *Computational Statistics & Data Analysis* 52, 1860-1872.
- [63] Cornish, C.R., Bretherton, C.S. and Percival, D.B. (2004). Maximal overlap wavelet statistical analysis with applications to atmospheric turbulence, *Boundary-Layer Meteorology*, 119 (2), 339-374.
- [64] Crato, N., Taylor, H. (1996). Stationary persistent time series misspecified as nonstationary ARIMA. Statistische Hefte/Statistical Papers 37, 215-223.
- [65] Cuturi, M., Vert, J.-P., Birkenes, O. and Matsui, T. (2007). A kernel for time series based on global alignments. *Proceedings of ICASSP*, volume II, 413-416.

[66] Dahlhaus, R. (1997). Fitting time series models to nonstationary processes, The Annals of Statistics 25:1 1-37.

- [67] Dargahi-Noubary, G.R. and Laycock, P.J. (1981). Spectral ratio discriminants and information theory. *Journal of Time Series Analysis* 2:2 71-86.
- [68] Davé, R.N. (1991). Characterization and detection of noise in clustering. Pattern Recognition Letters, 12(11):657-664.
- [69] Davé, R.N., Sen, S. (1997). Noise clustering algorithm revisited. In 1997 Annual Meeting of the North American Fuzzy Information Processing Society. NAFIPS-97, pages 199-204. IEEE.
- [70] Davé, R.N., Sen, S. (2002). Robust fuzzy clustering of relational data. IEEE Transactions on Fuzzy Systems, 10(6):713-727.
- [71] Davies, R.B., Harte, D.S. (1987). Tests for Hurst effect, Biometrika 74: 1, 95-101.
- [72] Davis R.A., Mikosch, T. (2009). The extreme ogram: a correlogram for extreme events, *Bernoulli*, 15(4), 977-1009.
- [73] Dahlhaus, R. (1997). Fitting time series models to nonstationary processes, The Annals of Statistics 25:1 1-37.
- [74] De Chazal, P., Reilly, R.B. (2000). A comparison of the ECG classification performance of different feature sets. Computers in Cardiology 27, 327-330.
- [75] Department of Climate Change, Commonwealth of Australia (2009). www.climatechange.gov.au , ISBN: 978-1-921298-71-4 .
- [76] DeJong, D.N., Nankervis, J.C., Savin, N.E. and Whiteman, C.H. (1992). Integration versus trend stationary in time series, *Econometrica* 60:2, 423-433.
- [77] Dette, H., Hallin, M., Kley, T. and Volgushev, S. (2014). Of copulas, quantiles, ranks and spectra: an  $l_1$ -approach to spectral analysis, ArXiv e-prints, arXiv:1111.7205v2.
- [78] Diebold, F.X., Rudebusch, G.D. (1991). On the power of Dickey-Fuller tests against fractional alternatives, *Economics Letters* 25:2, 155-160.
- [79] Diggle, P.J., Fisher, N.I. (1991). Nonparametric comparison of cumulative periodograms, Appl. Statist. 40, 423-434.
- [80] Disegna, M., D'Urso, P. and Durante, F. (2017). Copula-based fuzzy clustering of spatial time series, Spatial Statistics 21, 209-225.

[81] Do, C., Douzal-Chouakria, A., Marié, S. and Rombaut, M. (2017). Multi-modal and multi-scale temporal metric learning for a robust time series nearest neighbours classification. *Information Sciences* 418-419, 272-285.

- [82] Dose, C., Cincotti, S. (2005). Clustering of financial time series with application to index and enhanced-index tracking portfolio, *Physica A*, 355, 145-151.
- [83] Dunn, C. (1974). Well separated clusters and optimal fuzzy partitions J. Cybern 4, 1, 95-104.
- [84] Dunn, C. (1977). Indices of partition fuzziness and detection of clusters in large data sets, in: Fuzzy Automata and Decision Processes, M. Gupta, G. Saridis (eds.) Elsevier, New York.
- [85] Douzal-Chouakria, A., Amblard, C. (2012). Classification trees for time series. *Pattern Recognition* 45, 1076-1091.
- [86] D'Urso, P., Vichi, M. (1998). Dissimilarities between trajectories of a three-way longitudinal data set, in *Advances in Data Science and Clas*sification, (eds. A. Rizzi, M. Vichi, H.H. Bock), 585-592, Springer-Verlag, Berlin.
- [87] D'Urso, P. (2000). Dissimilarity measures for time trajectories, Statistical Methods and Applications, 1-3, 53-83.
- [88] D'Urso, P. (2004). Fuzzy C-Means Clustering models for multivariate time-varying data: different approaches, *International Journal of Un*certainty, Fuzziness and Knowledge-Based Systems, 12 (3), 287-326.
- [89] D'Urso, P. (2005). Fuzzy clustering for data time arrays with inlier and outlier time trajectories, *IEEE Transactions on Fuzzy Systems*, 13, 5, 583-604.
- [90] D'Urso, P., De Giovanni, L. (2008). Temporal self-organizing maps for telecommunications market segmentation, *Neurocomputing*, 71, 2880-2892.
- [91] D'Urso, P., Maharaj, E.A. (2009). Autocorrelation-based fuzzy clustering of time series, Fuzzy Sets and Systems 160, 3565-3589.
- [92] D'Urso, P., Maharaj, E.A. (2012). Wavelets-based clustering of multivariate time series, Fuzzy Sets and Systems 196, 33-61.
- [93] D'Urso, P., Di Lallo, D. and Maharaj, E.A. (2013a). Autoregressive model-based fuzzy clustering and its application for detecting information redundancy in air pollution monitoring networks, *Soft Computing* 17, 83-131.

[94] D'Urso, P., Cappelli, C., Di Lallo, D. and Massari, R. (2013b). Clustering of financial time series, Physica A: Statistical Mechanics and its Applications, 392(9): 2114-2129.

- [95] D'Urso, P., De Giovanni, L., Massari, R. and Di Lallo, D. (2013c). Noise fuzzy clustering of time series by the autoregressive metric, *Metron* 71, 217-243.
- [96] D'Urso, P., De Giovanni, L., Maharaj, E.A. and Massari, R. (2014). Wavelet-based Self-Organizing Maps for classifying multivariate time series. *Journal of Chemometrics* 28, 1, 28-51.
- [97] D'Urso, P. (2015). Fuzzy Clustering. In Hennig, C., Meila, M., Murtagh, F. and Rocci, R., editors, *Handbook of Cluster Analysis*, 545-573. Chapman and Hall.
- [98] D'Urso, P., De Giovanni, L. and Massari, R. (2015). Time series clustering by a robust autoregressive metric with application to air pollution, Chemometrics and Intelligent Laboratory Systems 141(15), 107-124.
- [99] D'Urso, P., De Giovanni, L. and Massari, R. (2016). GARCH-based robust clustering of time series, Fuzzy Sets and System, 305: 1-28.
- [100] D'Urso, P., Maharaj, E.A. and Alonso, A.M. (2017a). Fuzzy clustering of time series using extremes, Fuzzy Sets and Systems 318, 56-79.
- [101] D'Urso, P., Massari, R., Cappelli, C. and De Giovanni, L. (2017b). Autoregressive metric-based trimmed fuzzy clustering with an application to P M10 time series. Chemometrics and Intelligent Laboratory Systems, 161:15-26.
- [102] D'Urso, P., De Giovanni, L. and Massari, R. (2018). Robust fuzzy clustering of multivariate time trajectories, *International Journal of Approximate Reasoning*, 99, 12-38.
- [103] Dudoit, S., Fridlyand, J. and Speed, T.P. (2002). Comparison of discrimination methods for the classification of tumors using gene expression data. *Journal of the American Statistical Association* 97:457, 77-87.
- [104] Enders, W. (1995). Applied Econometric Time Series, John Wiley & Sons, New York.
- [105] Everitt, B.S., Landau, S., Leese, M. and Stahl, D. (2011). Cluster Analysis, 5th Edition. Wiley, United Kingdom.
- [106] Fatti, L.P., Hawkins, D.M. and Raath, E.L. (1982). Discriminant analysis. In: Hawkins, D.M. (Ed.), Topics in Applied Multivariate Analysis. Cambridge University Press, Cambridge, pp. 1-71.

[107] Fruhwirth-Schnatter, S., Kaufmann, S. (2008). Model-based clustering of multiple time series, *Journal of Business and Economic Statistics*, 26: 1, 78-89.

- [108] Fryzlewicz, P., Ombao, H. (2012). Consistent classification of non-stationary time series using stochastic wavelet representations, *Journal of the American Statistical Association* 104:485, 299-312.
- [109] Fu, K.S. (1982). Syntactic Pattern Recognition and Applications. Academic Press, San Diego.
- [110] Galeano, P., Peña, D. (2000). Multivariate analysis in vector time series, Resenhas 4, 383-404.
- [111] Gan, G., Wu, J. (2008). A convergence theorem for the fuzzy subspace clustering (FSC) algorithm. Pattern Recognition, 41(6):1939-1947.
- [112] García-Escudero, L.A., Gordaliza, A. (1999). Robustness properties of k-means and trimmed k-means. Journal of the American Statistical Association, 94(447):956-969.
- [113] García-Escudero, L.A., Gordaliza, A. (2005). A proposal for robust curve clustering. *Journal of Classification*, 22(2):185-201.
- [114] García-Escudero, L.A., Gordaliza, A. and Matrán, C. (2003). Trimming tools in exploratory data analysis. *Journal of Computational and Graph*ical Statistics, 12:434-449.
- [115] García-Escudero, L.A., Gordaliza, A., Matrán, C. and Mayo-Iscar, A. (2010). A review of robust clustering methods. Advances in Data Analysis and Classification, 4:89-109.
- [116] Ghassempour, S., Girosi, F. and Maeder A. (2014). Clustering multivariate time series using hidden Markov models. Int. J. Environ. Res. Public Health. 11:3 2741-2763.
- [117] Goldberger, A.L., Amaral, L.A.N., Glass, L., Hausdorff, J.M., Ivanov, P.Ch., Mark, R.G., Mietus, J.E., Moody, G.B., Peng, C.-K. and Stanley, H.E. (2000). PhysioBank, physiotoolkit, and physionet: components of a new research resource for complex physiologic signals. Circulation 101 (23), e215-e220. Circulation Electronic Pages: http://circ.ahajournals.org/egi/content/full/101/23/e215. The PTB Diagnostic ECG Database: http://www.physionet.org/physiobank/database/ptbd.
- [118] Glosten, L.R., Jagannathan, R. and Runkle, D.E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks, *Journal of Finance* 48:5, 1779-1801.

[119] Goutte, C., Toft, P., Rostrup, E., Nielsen, F. and Hansen, L.K. (1999). On clustering fMRI time series, NeoroImage 9 (3), 298-310.

- [120] Granger, C.W.J., Joyeux, R. (1980). An introduction to long memory time series and fractional differencing, *Journal of Time Series Analysis* 1:1, 15-29.
- [121] Guedes Soares, C., Scotto, M.G. (2004). Application of the r-order statistics for long-term predictions of significant wave heights. *Coast. Eng.* 51, 387-394.
- [122] Gudmundsson, S., Runarsson, T.P. and Sigurdsson, S. (2008). Support vector machines and dynamic time warping for time series, Neural Networks, 2008. IJCNN 2008. (IEEE World Congress on Computational Intelligence). IEEE International Joint Conference, 2772-2776.
- [123] Hagemann, A. (2013). Robust spectral analysis, ArXiv and e-prints, arXiv:1111.1965v1.
- [124] Hassler, U., Wolters, J. (1994). On the power of unit root tests against fractional alternatives, *Economics Letters* 45:1, 1-5.
- [125] Hastie, T., Tibshirani, R., Friedman. J. (2009). The Elements of Statistical Learning. Springer, New York.
- [126] Heden, B., Ohlin, H., Rittner, R. and Edenbrandt, L. (1997). Acute myocardial infarction detected in 12-lead ECG by artificial neural networks. Circulation: American Heart Association Inc. 96, 1798-1802.
- [127] Heiser, W.J., Groenen, P.J.F. (1997). Cluster differences scaling with a within-clusters loss component and a fuzzy successive approximation strategy to avoid local minima. *Psychometrika*, 62(1):63-83.
- [128] Hettich, S., Bay, S.D. (1999). The UCI KDD Archive. Irvine, CA: University of California, Department of Information and Computer Science, Available from: http://kdd.ics.uci.edu.
- [129] Hosking, J.R.M. (1981). Fractional differencing, Biometrika 68:1, 165-176.
- [130] Huang, H., Ombao, H. and Stoffer, D.S. (2004). Discrimination and classification of nonstationary time series using the SLEX model, *Journal of the American Statistical Association* 99:467, 763-774.
- [131] Huang, X., Ye, Y., Xiong, L., Lau, R.Y.K., Jiang, N. and Wang, S. (2016). Time series k-means: a new k-means type smooth subspace clustering for time series data, *Information Sciences*, 367-368, 1-13.
- [132] Izakian, H., Pedrycz, W. and Jamal, I. (2015). Fuzzy clustering of time series data using dynamic time warping distance, Engineering Applications of Artificial Intelligence, 39, 235-244.

[133] Jebara, T., Song, Y. and Thadani, K. (2007). Spectral clustering and embedding with hidden Markov models. *ECML*, 164-175.

- [134] Jeong, Y.-S., Jeong, M.K. and Omitaomu, O.A. (2011). Weighted dynamic time warping for time series classification, *Pattern Recognition*, 44, 2231-2240.
- [135] Johnson, R.A., Wichern, D.W. (1992). Applied Multivariate Statistical Analysis. 3rd edition., Englewood Cliffs, Prentice-Hall.
- [136] Kakizawa, Y., Shumway, R.H. and Taniguchi, M. (1998). Discrimination and clustering for multivariate time series, *Journal of the American Sta*tistical Association 93, 328-340.
- [137] Kalpakis, K., Gada, D. and Puttagunta, V. (2001). Distance measures for the effective clustering of ARIMA time-series. Proceedings of the IEEE International Conference on Data Mining, San Jose, 273-280.
- [138] Kamdar, T., Joshi, A. (2000). On Creating Adaptive Web Servers using Weblog Mining. Technical report TR-CS- 00-05, Department of Computer Science and Electrical Engineering, University of Maryland, Baltimore County.
- [139] Kannathal, N., Choo, M.L., Acharya, U.R. and Sadasivan, P.K. (2005). Entropies in the detection of epilepsy in EEG. Comput. Methods Programs Biomedicine 80:3, 187-194.
- [140] Kaufman, L., Rousseeuw, P.J. (1987). Clustering by Means of Medoids, Statistics Data Analysis based on the L1-Norm and Related Methods (Ed. Y. Dodge). Elsevier, North-Holland, Amsterdam.
- [141] Kaufman, L., Rousseeuw, P.J. (1990). Finding Groups in Data: An Introduction to Cluster Analysis. New York: J. Wiley and Sons.
- [142] Kaufman, L., Rousseeuw, P.J. (2009). Finding Groups in Data: an Introduction to Cluster Analysis, Volume 344. John Wiley & Sons.
- [143] Keogh, E., Lonardi, S., Ratanamahatana, C.A., Wei, L., Lee, S.H. and Handley, J. (2007). Compression-based data mining of sequential data. Data Mining and Knowledge Discovery, 14, 1, 99-129.
- [144] Keogh, E., Pazzani, S. (2000). Scaling up dynamic time warping for data mining applications, 6th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining. Boston, 16-22.
- [145] Keogh, E., Pazzani, S. (2001). Derivative dynamic time warping, Proceedings of the 2001 SIAM International Conference on Data Mining.
- [146] Kiefer, J. (1959). K-Sample analogues of the Kolmogorov-Smirnov and Cramer-V. misses tests, The Annals of Mathematical Statistics 30: 2 420-447.

[147] Kannathal, N., Choo, M.L., Acharya, U.R. and Sadasivan, P.K. (2005). Entropies in the detection of epilepsy in EEG. Computer Methods and Programs in Biomedicine 80, 3, 187-194.

- [148] Kopiec, D., Martyna, J. (2011). A hybrid approach for ECG classification based on particle swarm optimization and support vector machine. In: Corchado, E., Kurzynski, M., Wozniak, M. (Eds.), HAIS2011, Part 1, LNAI, vol. 6678. pp. 329-337.
- [149] Kotsifakos, A., Panagiotis, P. (2014). Model-based time series classification. Advances in Intelligent Data Analysis XIII in Lecture Notes in Computer Science, Springer International Publishing, 8819, 179-191.
- [150] Košmelj, K., Batagelj, V. (1990). Cross-sectional approach for clustering time varying data, *Journal of Classification*, 7, 99-109.
- [151] Krishnapuram, R., Joshi, A., Nasraoui, O. and Yi, L. (2001). Low-complexity fuzzy relational clustering algorithms for web mining. *IEEE Transactions on Fuzzy Systems*, 9(4):595-607.
- [152] Krishnapuram, R., Joshi, A. and Yi, L. (1999). A fuzzy relative of the k-medoids algorithm with application to web document and snippet clustering. In Fuzzy Systems Conference Proceedings, 1999. FUZZ-IEEE-99. 1999 IEEE International, volume 3, pages 1281-1286. IEEE.
- [153] Kruse, R., Doring, C. and Lesot, M.-J. (2007). Fundamentals of fuzzy clustering. In De Oliveira, J.V., Pedrycz, W., editors, Advances in Fuzzy Clustering and its Applications, pages 3-30. Wiley.
- [154] Kullback, S., Leibler, R.A. (1951). On information and sufficiency, The Annals of Mathematical Statistics 22:1 79-86.
- [155] Lachenbruch, P.A., Mickey, M.R. (1968). Estimation of error rates in discriminant analysis. *Technometries* 10, 1-10.
- [156] Lafuente-Rego, B., D'Urso, P. and Vilar, J.A. (2017). Robust fuzzy clustering of time series based on the quantile autocovariances, submitted.
- [157] Lafuente-Rego, B., Vilar, J.A. (2015). Clustering of time series using quantile autocovariances. Advances in Data Analysis and Classification, 10, 391-415.
- [158] Lee, J., Rao, S. (2012). The quantile spectral density and comparison based tests for nonlinear time series, *Unpublished manuscript*, Department of Statistics, Texas A&M University, College Station, USA, arXiv:1112.2759v2,
- [159] Liao, T.W. (2005). Clustering of time series data a survey. Pattern Recognition 38, 1857-1874.

[160] Li, M., Chen, X., Li, X., Ma, B. and Vitányi, P.M.B. (2004). The similarity metric. IEEE Transactions on Information Theory. 50, 12, 3250-3264.

- [161] Li, R.P., Mukaidono, M. (1995). A maximum-entropy approach to fuzzy clustering. In *Proceedings of the 4th IEEE Conference on Fuzzy Systems* (FUZZ-IEEE/IFES-95), volume 4, pages 2227-2232, Yokohama. IEEE.
- [162] Li, R.P., Mukaidono, M. (1999). Gaussian clustering method based on maximum-fuzzy-entropy interpretation. Fuzzy Sets and Systems, 102(2):253-258.
- [163] Li, S., Lu, Y.L., Yang, K. and Li, S. (2012). ECG Analysis using multiple instance learning for myocardial infarction detection. *IEEE Transac*tions on Biomedical Engineering 59 (12), 3348-3356.
- [164] Liu, S., Maharaj, E.A. (2013). A hypothesis test using bias-adjusted AR estimators for classifying time series, Computational Statistics and Data Analysis 60, 32-49.
- [165] Liu, S., Maharaj, E.A. and Inder, B.A. (2014). Polarization of forecast densities: a new approach to time series classification, *Computational Statistics & Data Analysis* 70, 245-361.
- [166] Lo, A.W., MacKinlay, A.C. (1988). Stock market prices do not follow random walks: evidence from a simple specification test, *Review of Fi*nancial Studies 1:1, 41-66.
- [167] Longford, N.T., D'Urso, P. (2012). Mixtures of autoregressions with an improper component for panel data, *Journal of Classification* 29, 341-362.
- [168] MacQueen, J. (1967). Some methods for classification and analysis of multivariate observations. In Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability. volume 1, pages 281-297. California, USA.
- [169] Maharaj, E.A. (1996). A significance test for classifying ARMA models, Journal of Statistical Computation & Simulation 54, 305-331.
- [170] Maharaj, E.A. (1997). Pattern recognition techniques for time series, PhD Thesis, Monash University, Australia.
- [171] Maharaj, E.A. (1999). The comparison and classification of stationary multivariate time series, *Pattern Recognition* 32, 7, 1129-1138.
- [172] Maharaj, E.A. (2000). Clusters of time series, Journal of Classification 17, 297-314.
- [173] Maharaj, E.A. (2002). Comparison of non-stationary time series in the frequency domain, Computational Statistics & Data Analysis 40, 131-141.

- [174] Maharaj, E.A. (2005). Using wavelets to compare time series patterns, International Journal of Wavelets, Multiresolution & Information Processing 3, 4, 511-521.
- [175] Maharaj, E.A., Alonso, A.M. (2014). Discrimination of multivariate time series: application to diagnosis based on ECG signals, Computational Statistics & Data Analysis 70, 67-87.
- [176] Maharaj, E.A., D'Urso, P. (2010). A coherence-based approach for the pattern recognition of time series, *Physica A: Statistical Mechanics* 389, 17, 3516-3537.
- [177] Maharaj, E.A., D'Urso, P. and Galagedera, D.U.A. (2010). Wavelets-based fuzzy clustering of time series, *Journal of Classification* 27, 2, 231-275.
- [178] Maharaj, E.A., D'Urso, P. (2011). Fuzzy clustering of time series in the frequency domain, *Information Sciences*, 181, 1187-1211.
- [179] Maharaj, E.A. (2014). Classification of cyclical time series using complex demodulation. Statistics and Computing 24,6,1031-1046.
- [180] Maharaj, E.A., Alonso, A.M. (2007). Discriminant analysis of locally stationary time series using wavelets, Computational Statistics & Data Analysis 52, 879-895.
- [181] Maharaj, E.A., Alonso, A.M. and D'Urso, P. (2016). Clustering seasonal time series using extreme value analysis: an application to Spanish temperature time series, Communications in Statistics: Case Studies and Data Analysis, 1, 175-191.
- [182] Makridakis, S., Wheelwright, C., Hyndman, R.J. (1998). Forecasting: Methods and Applications, 3rd Edition. Wiley, New York.
- [183] Montero, P., Villar, J. (2014). TClust: an R package for time series clustering, Journal of Statistical Software 62, 1, 1-43.
- [184] McLachlan, G.J. (1992). Discriminant Analysis and Statistical Pattern Recognition. Wiley, New York.
- [185] McBratney, A.B., Moore, A.W. (1985). Application of fuzzy sets to climatic classification. Agricultural and Forest Meteorology, 35(1):165-185.
- [186] Mikosch, T., Stărică, C. (2000). Limit theory of the sample autocorrelation and extreme of a GARCH(1,1) process, Annals of Statistics, 28(5), 1427-1451.
- [187] Milligan, G.W., Cooper, M.C. (1985). An examination of procedures for determining the number of clusters in a data set. *Psychometrika*, 50, 159-179.

[188] Miskiewicz, J., Ausloos, M. (2008). Correlation measure to detect time series distances, whence economy globalization, *Physica A* 387, 6584-6594.

- [189] Miyamoto, S., Mukaidono, M. (1997). Fuzzy c-means as a regularization and maximum entropy approach. In *IFSA 97 Prague: Proceedings of the Seventh International Fuzzy Systems Association World Congress*, pages 86-92.
- [190] Nasraoui, O., Krishnapuram, R., Joshi, A. and Kamdar, T. (2002). Automatic web user profiling and personalization using robust fuzzy relational clustering. In *E-commerce and Intelligent Methods*, 233-261. Springer.
- [191] Nigam, V.P., Graupe, D. (2004). A neural-network-based detection of epilepsy. Neurological Research 26, 1, 55-60.
- [192] Oates, T., Firoiu, L., Cohen, P.R. (1999). Clustering time series with Hidden Markov Models and Dynamic Time Warping. Proceedings of the IJCAI
- [193] Ohashi, Y. (1984). Fuzzy clustering and robust estimation. In Ninth Meeting of SAS Users Group Int.
- [194] Otranto, E. (2008). Clustering heteroskedastic time series by model-based procedures, Computational Statistics & Data Analysis 52, 4685-4698.
- [195] Otranto, E. (2010). Identifying financial time series with similar dynamic conditional correlation, Computational Statistics & Data Analysis 54, 1, 1-15.
- [196] Ord, K., Fildes, F. (2013). Principles of Business Forecasting. Southwestern Cenage Learning.
- [197] Pamminger, C., Fruhwirth-Schnatter, S. (2010). Model-based clustering of categorical time series, *Bayesian Analysis*, 2, 345-368.
- [198] Percival, D.B., Walden, A.T. (2000). Wavelet Methods for Time Series Analysis. Cambridge University Press, Cambridge.
- [199] Pham, D.T., Chan, A.B. (1998). Control chart pattern recognition using a new type of self-organizing neural network. Proc. Inst. Mech. Eng. 212: 2, 115-127.
- [200] Piccolo, D. (1990). A distance measure for classifying ARIMA models, Journal of Time Series Analysis 11, 2, 153-164.

[201] Povinelli, R.J., Johnson, M.T. and Lindgren, A.C. (2004). Time series classification using Gaussian mixture models of reconstructed phase spaces. *IEEE Tranactions on Knowledge and Data Engineering* 16: 6, 779-783.

- [202] Ramoni, M., Sebastiani, P. and Cohen, P. (2002). Bayesian clustering by dynamics, *Mach. Learning* 47, 1, 91-121.
- [203] Rényi, A. (1961). On measures of entropy and information. Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, 1, Berkeley: University of California Press, 547-561.
- [204] Rousseeuw, P.J. (1987). Silhouettes: a graphical aid to the interpretation and validation of cluster analysis. *Journal of Computational and Applied Mathematics*, 20, 53-65.
- [205] Runkler, T.A., Bezdek, J.C. (1999). Alternating cluster estimation: a new tool for clustering and function approximation. *IEEE Transactions* on Fuzzy Systems, 7(4):377-393.
- [206] Sakiyama, K., Taniguchi, M. (2004). Discriminant analysis for locally stationary processes, *Journal of Multivariate Analysis*, 90:2, 282-300.
- [207] Sameni, R., Clifford, G.D., Jutten, C. and Shamsollahi, M.B. (2007). Multi-channel ECG and noise modeling: application to maternal and fetal ECG signals. *IEURASIP Journal on Advances in Signal Processing*, 14 pages. http://dx.doi.org/10.1155/2007/13407. Article ID 43407.
- [208] Sarafidis, V., Weber, N. (2015). A partially heterogeneous framework for analyzing panel data, Oxford Bulletin of Economics and Statistics, 77, 2, 274-296.
- [209] Savvides, A., Promponas, V.J. and Fokianos, K. (2008). Clustering of biological time series by cepstral coefficients based distances, *Pattern Recognition* 41, 2398-2412.
- [210] Serroukh, A., Walden, A.T. and Percival, D.B. (2000). Statistical properties and uses of the wavelet variance estimator for the scale analysis of time series. *Journal of the American Statistical Association* 95: 450, 184-196.
- [211] Serroukh, A., Walden, A.T. (2000). Wavelet scale analysis of bivariate time series II: statistical properties for linear processes. *Journal of Non-parametric Statistics* 13, 37-56.
- [212] Scotto, M.G., Barbosa, S.M. and Alonso, A.M. (2010). Clustering time series of sea levels: extreme value approach, J. Waterway, Port, Coastal, Ocean Eng. 136, 2793-2804.

[213] Shumway, R.H. (2003). Time-frequency clustering and discriminant analysis. Statist. Probab. Let. 63, 3, 307-314.

- [214] Shumway, H., Stoffer, D.S. (2016). Time Series Analysis and Its Applications, Springer, New York.
- [215] Shumway, H., Unger, A.N. (1974). Linear discriminant functions for stationary time series, *Journal of the American Statistical Association* 69:348-948-956.
- [216] Singhal, A., Seborg, D. (2005). Clustering multivariate time series data, J. Chemometrics 19, 427-438.
- [217] Silverman, B.W. (1986). Density Estimation for Statistics and Data Analysis. Chapman & Hall, London.
- [218] Sykacek, P., Roberts, S. (2002). Bayesian time series classification. Advances in Neural Information Processing Systems 14, T. G. Dietterich,
   S. Becker and Z. Ghahramani (eds.), 937-944, MIT Press, Boston.
- [219] Takayuki, M., Takayasu, H. and Takayasu, M. (2006). Correlation networks among currencies, *Physica A*, 364, 336-342.
- [220] Taniguchi, M. (1991). Third-order asymptomic properties of a class of test statistics under a local alternative, *Journal of Multivariate Analysis* 37:2, 223-238.
- [221] Tarpey, T., Kinateder, K.K.J. (2003). Clustering functional data, Tarpey, Thaddeus; Kinateder, Kimberly K J., eds. *Journal of Classi-fication*, 20.1 93-114.
- [222] Taylor (2007). Modelling Financial Time Series. World Scientific Publishing, London.
- [223] Tong, H., Dabas, P. (1990). Clusters of time series models: An example, Journal of Applied Statistics 17, 187-198.
- [224] Tsay, R.S. (2010). Analysis of Financial Time Series, 3rd Edition, South-western Cenage Learning. John Wiley and Sons, Canada.
- [225] University of Hawaii Sea Level Centre UHSLC, <br/> http://ulislc.soest.hawaii.edu/data/download/rq
- [226] Vapnik, V.N. (1995). The Nature of Statistical Learning Theory, Springer-Verlag, New York, Inc., New York, NY, USA.
- [227] Vilar, J.A., Pertega, S. (2004). Discriminant and cluster analysis for Gaussian stationary processes: local linear fitting approach, Nonparametric Statistics 16:3-4, 443-462.

[228] Vilar, J.A., Alonso, A.M., Vilar, J.M. (2010). Non-linear time series clustering based on non-parametric forecast densities. *Computational Statistics & Data Analysis*, 54, 2850-2865.

- [229] Vilar, J.A., Lafuente-Rego, B., D'Urso, P. (2018). Quantile autocovariances: a powerful tool for hard and soft partitional clustering of time series, Fuzzy Sets and Systems, 340, 38-72.
- [230] Vilar, J.A., Vilar J.M. (2013). Time series clustering based on nonparametric multidimensional forecast densities. *Electronic Journal of Statistics*, 7, 1019-1046.
- [231] Vullings, H.J.L.M., Verhaegen, M.H.G., Verbruggen, H.B. (1998). Automated ECG segmentation with dynamic time warping. Proceedings of the 20th Annual International Conference of the IEEE.
- [232] University of Hawaii Sea Level Centre UHSLC, http://uhslc.soest.hawaii.edu/data[download/rq.
- [233] Wang, N., Bolstein, S. (2004). Adaptive zero-padding OFDM over frequency-selective multipath channels, *Journal of Applied Signal Pro*cessing, 10, 1478-1488.
- [234] Wang, X., Smith, R. and Hyndman, R. (2006). Characteristic-based clustering of time series data. *Data Mining and Knowledge Discovery*, 13, 335-364.
- [235] Wang X., Wirth, A. and Wang, L. (2007). Structure-based statistical features and multivariate time series clustering. Seventh IEEE International Conference on Data Mining, 351-360.
- [236] Wang, W., Zhang, Y. (2007). On fuzzy cluster validity indices. Fuzzy Sets and Systems, 158(19):2095-2117.
- [237] Wedel, M., Kamakura, W.A. (2012). Market Segmentation: Conceptual and Methodological Foundations, Volume 8. Springer Science & Business Media.
- [238] Wichern, D.W. (1973). The behaviour of the sample autocorrelation function for an integrated moving average process, *Biometrika* 60:2, 235– 239.
- [239] Wright, J.H. (2000). Alternative variance-ratio tests using ranks and signs. Journal of Business and Economic Statistics 18:1, 1-9.
- [240] Wu, K.-L., Yang, M.-S. (2002). Alternative c-means clustering algorithms. Pattern Recognition, 35(10):2267-2278.
- [241] Xie, X.L., Beni, G. (1991). A validity measure for fuzzy clustering. IEEE Transactions on Pattern Analysis and Machine Intelligence, 13(8):841-847.

[242] Xiong, Y., Yeung, D.Y. (2002). Mixtures of ARMA models for model-based time series clustering, Proceedings of the IEEE International Conference on Data Mining, Maebaghi City, Japan, 9-12 December, 2002.

- [243] Xu, Y., Brereton, R.G. (2005). A comparative study of cluster validation indices applied to genotyping data. Chemometrics and Intelligent Laboratory Systems, 78(1):30-40.
- [244] Xu, R., Wunsch, D.C. (2009). Clustering. Hoboken, New Jersey: IEEE Press, Wiley & Sons, Inc.
- [245] Yang, M.-S., Wu, K.-L. (2006). Unsupervised possibilistic clustering. Pattern Recognition, 39(1):5-21.
- [246] Young, P.C., Pedregal, D.J. and Tych, W. (1999). Dynamic harmonic regression, *Journal of Forecasting*, 18, 369-394.
- [247] Zellner, A. (1962). Estimators for seemly unrelated regression equations and test of aggregation bias. *Journal of the American Statistical Asso*ciation, 57, 500-509.
- [248] Zhang, D.Q., Chen, S.-C. (2004). A comment on "Alternative c-means clustering algorithms", *Pattern Recognition*, 37(2): 173-174.
- [249] Zhang, D., Zuo, W. and Zhang. D. (2010). Time series classification using support vector machine with Gaussian elastic metric kernel. Proceedings 2010 International Conference on Pattern Recognition 29-32.
- [250] Zhang, H., Ho, T.B., Zhang, Y. and Lin, M. (2005). Unsupervised feature extraction for time series clustering using orthogonal wavelet transform, *Informatica* 30, 305-319.
- [251] Zakoian, J. (1994). Threshold heteroskedastic models, Journal of Economic Dynamics and Control 18:5, 931-955.

