*Addendum* to "Weil representation and metaplectic groups over an integral domain"

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In paragraph 4.3 of [CT15] we claim that $\gamma$ takes values in the subgroup of fourth roots of unity in $R^\times$. This is true if the residual characteristic of $F$, denoted by $p$, is different from 2, but not always for $p = 2$. In the latter case, we can actually show that $\gamma$ takes values in the subgroup of eighth roots of unity in $R^\times$.

Formulas $\gamma(x_1^2 - ax_2^2 - bx_3^2 + abx_4^2) = (a, b)$ for every $a, b \in F^\times$ and $\gamma(f)^2 = (D(f), -1)\,\gamma(q_1)^{2m}$ for every non-degenerate quadratic form over $F$ are still valid.

- If $p$ is odd then $-1$ is either a square or a norm from $F(\sqrt{-1})$ to $F$ and so $\gamma(q_4) = (-1, -1) = 1$ and $\gamma(f)^4 = 1$.
- If $p = 2$ then $-1$ can be neither a square nor a norm form $F(\sqrt{-1})$ to $F$ and so we cannot conclude that $\gamma(q_4) = 1$. However, we always have $\gamma(q_4)^2 = (-1, -1)^2 = 1$ and so $\gamma(f)^8 = 1$. Hence, $\gamma$ takes values in the subgroup of eighth roots of unity in $R^\times$ as announced.

Now the question is for which finite extension $F$ of $\mathbb{Q}_2$ we have $(-1, -1) = 1$. Thanks to theorem 7.6 of [Iwa86] the nonzero norms from $F(\sqrt{-1})$ to $F$ are the elements of $F$ whose norms to $\mathbb{Q}_2$ lie in the group of nonzero norms from $\mathbb{Q}_2(\sqrt{-1})$ to $\mathbb{Q}_2$. Since $-1$ is not a norm from $\mathbb{Q}_2(\sqrt{-1})$ to $\mathbb{Q}_2$, we have $(-1, -1) = 1$ if and only if $[F : \mathbb{Q}_2]$ is even, i.e., $(-1, -1) = (-1)^{[F:\mathbb{Q}_2]}$.

Finally, if the residue characteristic of $F$ is odd or if $F$ is a finite extension of $\mathbb{Q}_2$ and $[F : \mathbb{Q}_2]$ is even then $\gamma$ takes values in the subgroup of fourth roots of unity in $R^\times$. Otherwise, if $F$ is a finite extension of $\mathbb{Q}_2$ and $[F : \mathbb{Q}_2]$ is odd then $\gamma$ takes values in the subgroup of eighth roots of unity in $R^\times$.

To conclude, we notice that the results of section 5 do not rely specifically on $\gamma$ taking values in subgroup of fourth roots of unity in $R^\times$, and therefore hold also when $F$ is a 2-adic field. Hence, no further correction is needed.

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## References

[CT15] Gianmarco Chinello and Daniele Turchetti. Weil representation and metaplectic groups over an integral domain. *Communications in Algebra*, 43(6):2388–2419, 2015.

[Iwa86] Kenkichi Iwasawa. *Local class field theory.* Oxford University Press, New York, 1986.