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Electromagnetic Corrections to Hadronic Decays from Lattice QCD

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Abstract. A new method, recently introduced for the lattice calculation of electromagnetic and isospin corrections to weak decays [1], is discussed. Using this method, for the first time, the electromagnetic effects in the leptonic decay rates $\pi^- \to \mu^- \bar{\nu}_\mu$ and $K^- \to \mu^- \bar{\nu}_\mu$ have been evaluated in lattice QCD. Preliminary results for the electromagnetic corrections to charged (neutral) pion and kaon masses and leptonic decay rates are presented.

1. Introduction

In Flavor Physics an accurate determination of the parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix represents a crucial test of the Standard Model (SM). Moreover, improving the accuracy on the CKM parameters is at the heart of many searches for New Physics (NP), where we look for rather small effects. The Unitarity Triangle (UT) analysis, for example, allows a rather precise determination of the Wolfenstein parameters $\bar{\rho}$ and $\bar{\eta}$, obtained from a plethora of flavour measurements. This analysis has been performed by several groups, among which the **UT***fit* collaboration, see for example `http://www.utfit.org` (the results presented here are an update on the "Summer 2016" analysis and they will appear in the web-page as "Winter 2017"). The experimental measurements are mostly taken from `http://www.slac.stanford.edu/xorg/hfag`, while the non-perturbative QCD parameters come from the most recent lattice QCD averages [2]. Using the above inputs a global fit gives $\bar{\rho} = 0.153 \pm 0.013$ and $\bar{\eta} = 0.343 \pm 0.011$, see Figure 1.

For many physical quantities relevant for studies of flavour physics, recent improvements in lattice computations have led to such a precision that electromagnetic effects and isospin breaking contributions cannot be neglected anymore (see e.g. Ref. [2] and references therein). For light-quark flavours, important examples include the calculations of the leptonic decay constants f_K and f_π and of the form factor $f^+(0)$ in semileptonic $K_{\ell 3}$ decays. These are used to determine the CKM matrix element $|V_{us}|$ and the ratio $|V_{us}|/|V_{ud}|$ at high precision. For such quantities, which have been computed with a precision at the sub-percent level, the uncertainty due to the explicit breaking of isospin symmetry (of the order of $(m_u - m_d)/\Lambda_{QCD} \sim 0.01$) and to

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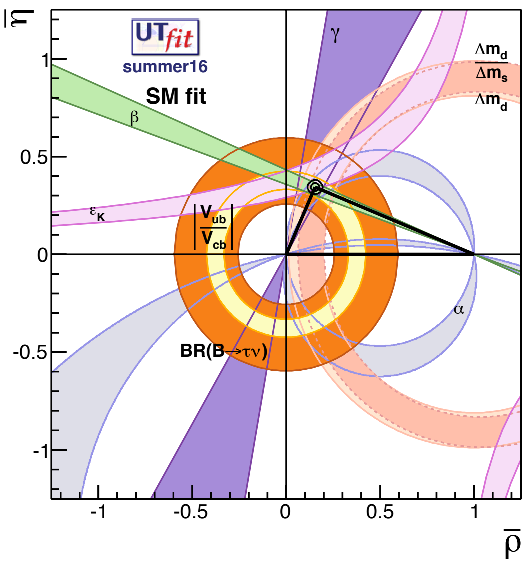

Figure 1: *$\bar{\rho}-\bar{\eta}$ plane showing the result of the SM fit. The black contours display the 68% and 95% probability regions selected by the given global fit. The 95% probability regions selected by the single constraints are also shown.*

electromagnetic corrections (of the order of $\alpha \sim 0.007$) is similar to, or even larger than, the quoted QCD errors [2]. Several collaborations have recently obtained remarkably accurate results in the calculation of the electromagnetic effects in the hadron spectrum and in the determination of quark masses with ab-initio lattice calculations see [3]-[5] for reviews on the subject.

In a recent paper, a new proposal to include electromagnetic and isospin-breaking effects in the non-perturbative calculation of hadronic decays was presented [1] and fully developed in [6]. As an example of the new method, the procedure to compute $O(\alpha)$ corrections to leptonic decays of pseudoscalar mesons was described in detail. This can then be used to determine the corresponding CKM matrix elements.

Whereas in the calculation of the hadron spectrum there are no infrared divergences, in the case of electromagnetic corrections to the hadronic amplitudes the infrared divergences are present and only cancel for well defined, measurable physical quantities. This requires the development of a new strategy, different than the usual approaches followed to compute the electromagnetic corrections to the spectrum. We proposed such a strategy in Ref. [1]. There we envisaged that at $O(\alpha)$ the physical observable is the inclusive decay rate of the pseudoscalar meson into a final state consisting of either $\ell^-\bar{\nu}_\ell$ or $\ell^-\bar{\nu}_\ell\gamma$, with the energy of the emitted photon in the rest frame of the pion smaller than an imposed cut-off ΔE. Here ℓ^- is a charged lepton and ν_ℓ the corresponding neutrino. The cut-off ΔE on the energy of the final-state (real) photon should be sufficiently small that for photons with such an energy we can neglect the structure of the meson and treat it as an elementary point-like particle, neglecting the structure-dependent corrections of $O(\alpha\,\Delta E/\Lambda_{\rm QCD})$. At $O(\alpha)$ the inclusive width can be written in the form

$$\Gamma(\Delta E)=\Gamma_0^{\rm tree}+\frac{\alpha}{4\pi}\lim_{L\to\infty}\Big(\Gamma_0(L)+\Gamma_1^{\rm pt}(\Delta E,L)\Big)\ , \tag{1}$$

where the suffix 0 or 1 indicates the number of photons in the final state; $\Gamma_0^{\rm tree}$ is the rate at $O(\alpha^0)$ given in Eq. (9) below; the superscript "pt" on Γ_1 denotes *point-like* and we have exhibited the dependence on L, the spacial extent of the box in which the lattice calculation is to be performed ($V=L^3$). It is now convenient to write

$$\lim_{L\to\infty}\Big(\Gamma_0(L)+\Gamma_1^{\rm pt}(\Delta E,L)\Big)=\lim_{L\to\infty}\Big(\Gamma_0(L)-\Gamma_0^{\rm pt}(L)\Big)+\lim_{L\to\infty}\Big(\Gamma_0^{\rm pt}(L)+\Gamma_1^{\rm pt}(\Delta E,L)\Big)\ . \tag{2}$$

The second term on the right-hand side of Eq. (2),

$$\Gamma^{\rm pt}(\Delta E)=\lim_{L\to\infty}\Big(\Gamma_0^{\rm pt}(L)+\Gamma_1^{\rm pt}(\Delta E,L)\Big) \tag{3}$$

can be evaluated in perturbation theory directly in infinite volume and the result has been presented in Ref. [1]. $\Gamma^{\rm pt}(\Delta E)$ is infrared finite and independent of the scheme used to regulate the divergences which are present separately in $\Gamma_0^{\rm pt}(L)$ and $\Gamma_1^{\rm pt}(\Delta E,L)$; its explicit expression is given by

$$\begin{aligned}\Gamma^{\rm pt}(\Delta E)\ &=\ \Gamma_0^{\rm tree}\times\Bigg(1+\frac{\alpha}{4\pi}\Bigg\{3\log\left(\frac{m_P^2}{M_W^2}\right)+\log\left(r_\ell^2\right)-4\log(r_E^2)\\
&+\frac{2-10r_\ell^2}{1-r_\ell^2}\log(r_\ell^2)-2\frac{1+r_\ell^2}{1-r_\ell^2}\log(r_E^2)\log(r_\ell^2)-4\frac{1+r_\ell^2}{1-r_\ell^2}\,{\rm Li}_2(1-r_\ell^2)-3\\
&+\Bigg[\frac{3+r_E^2-6r_\ell^2+4r_E(-1+r_\ell^2)}{(1-r_\ell^2)^2}\log(1-r_E)+\frac{r_E(4-r_E-4r_\ell^2)}{(1-r_\ell^2)^2}\log(r_\ell^2)\\
&-\frac{r_E(-22+3r_E+28r_\ell^2)}{2(1-r_\ell^2)^2}-4\frac{1+r_\ell^2}{1-r_\ell^2}\,{\rm Li}_2(r_E)\Bigg]\Bigg\}\Bigg)\ ,\end{aligned} \tag{4}$$

where $r_\ell=m_\ell/m_P$, $r_E=2\Delta E/m_P$ and $0\le r_E\le 1-r_\ell^2$. $\Gamma_0(L)$ is infrared divergent and depends on the infrared regularisation. Since all momentum modes of the virtual photon contribute to Γ_0, it depends on the structure of the meson and is necessarily non-perturbative. It should therefore be computed in a lattice simulation. In Ref. [1] we stressed that the infrared divergence cancels in the difference $\Gamma_0(L)-\Gamma_0^{\rm pt}(L)$. In Ref. [6] we showed that the $1/L$ finite-volume (FV) corrections are also *universal*, that is they are independent of the structure of the pseudoscalar meson and hence cancel in the difference $\Gamma_0(L)-\Gamma_0^{\rm pt}(L)$. This allows us to calculate $\Gamma_0^{\rm pt}(L)$ in perturbation theory with a point-like pseudoscalar meson up to and including the $1/L$ corrections expanded in inverse powers of L

$$\Gamma_0^{\rm pt}(L)=C_0(r_\ell)+\tilde C_0(r_\ell)\log\left(m_PL\right)+\frac{C_1(r_\ell)}{m_PL}+\dots\ , \tag{5}$$

m_P and m_ℓ are the masses of the pseudoscalar meson and the lepton respectively. The explicit expression for $\Gamma_0^{\rm pt}(L)$ has the form

$$\Gamma_0^{\rm pt}(L)=\Gamma_0^{\rm tree}\left\{1+2\frac{\alpha}{4\pi}\,Y(L)\right\}\ , \tag{6}$$

where

$$
\begin{aligned}
Y(L) \;=\; & \left(1+r_\ell^2\right)\left[2(K_{31}+K_{32}) + \frac{\left(\gamma_E + \log\left[\frac{L^2 m_P^2}{4\pi}\right]\right)\log\left[r_\ell^2\right]}{\left(1-r_\ell^2\right)} + \frac{\log^2\left[r_\ell^2\right]}{2\left(1-r_\ell^2\right)}\right] + \\
& + \frac{\left(1-3\,r_\ell^2\right)\log\left[r_\ell^2\right]}{\left(1-r_\ell^2\right)} - \log\left[\frac{M_W^2}{m_P^2}\right] + \log[m_P^2 L^2] - \frac{1}{2}K_P + \frac{1}{12} + \qquad (7) \\
& + \frac{1}{m_P L}\left(\frac{2r_\ell^2}{1-r_\ell^2}\left(K_{21}+K_{22}-2\pi\left(\frac{1}{1+r_\ell^2}+\frac{1}{r_\ell}\right)\right) - \frac{\pi(1+r_\ell^2)}{(1-r_\ell^2)}\left(K_{11}+K_{12}-3\right)\right) .
\end{aligned}
$$

Note that in this expression we did not include the contribution of the muon wave-function renormalisation which is also not computed in $\Gamma_0(L)$ since it cancels exactly in the difference $\Gamma_0(L) - \Gamma_0^{\rm pt}(L)$ [1]. The dimensionless constants K_{ij} only depends on the ratio r_ℓ, and their explicit expression can be found in Ref. [**?**].

The coefficients $C_0(r_\ell)$, $\tilde{C}_0(r_\ell)$ and $C_1(r_\ell)$ are universal, although $C_0(r_\ell)$ and $C_1(r_\ell)$ depend on the infrared regulator. $\tilde{C}_0(r_\ell)$ is universal and does not depend on the regularisation. $C_0(r_\ell)$, $\tilde{C}_0(r_\ell)$ and $C_1(r_\ell)$ cancel the corresponding terms contained in $\Gamma_0(L)$. In this way $\Gamma_0(L)-\Gamma_0^{\rm pt}(L)$ is infrared finite and independent of the infrared regularisation up to terms of $O(1/L^2)$. Higher order FV terms are not universal and thus cannot be corrected with an analytic computation.

2. Calculation of the relevant amplitudes

The inclusive decay rate for $P^- \to \ell^- \bar{\nu}_\ell[\gamma]$ of Eq. (2) can be written as

$$\Gamma(\Delta E) = \Gamma_0^{(tree)} \times \left[1 + \delta R_P\right] \ , \qquad (8)$$

where $\Gamma_0^{(tree)}$ is the tree-level decay rate given by

$$\Gamma_0^{(tree)} = \frac{G_F^2}{8\pi}|V_{q_1 q_2}|^2 m_\ell^2 \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2 \left[f_P^{(0)}\right]^2 m_P \ , \qquad (9)$$

and

$$\delta R_P = \alpha_{em}\frac{2}{\pi}\log\left(\frac{M_Z}{M_W}\right) + 2\frac{\delta A_P}{A_P^{(0)}} - 2\frac{\delta m_P}{m_P^{(0)}} + \Gamma^{(pt)}(\Delta E) \ . \qquad (10)$$

The quantity $\Gamma^{(pt)}(\Delta E)$ can be read off from Eq. (4), while the term containing $\log(M_Z/M_W)$ comes from short-distance electroweak corrections, m_P is the physical pseudo scalar meson mass including both e.m. and strong Isospin Breaking (IB) corrections, and $A_P^{(0)}$ is the QCD axial amplitude

$$A_P^{(0)} \equiv Z_A \langle 0|\bar{q}_2\gamma_0\gamma_5 q_1|P^{(0)}\rangle \equiv f_P^{(0)} m_P^{(0)} \qquad (11)$$

where $f_P^{(0)}$ defines the pseudo scalar meson decay constant known in pure QCD, and Z_A is the (lattice) renormalization constant of the axial current. Throughout this contribution the superscript $^{(0)}$ indicates quantities that do not contain e.m. and strong IB corrections.

The evaluation of δA_P and δm_P is obtained from the diagrams shown in Figs. 5 and 6 of Ref. [1]. It is in δA_P that the subtraction procedure described in Eq. (2), see also (7), is applied. We adopt the quenched QED approximation, which neglects the sea-quark electric charges and corresponds to consider only the connected diagrams shown in Fig. 2. In addition, with Wilson like fermions, one should include also the contributions coming from the tadpole operator, the e.m. shift in the critical mass and the insertion of the isovector scalar density (see Refs. [7, 8]).

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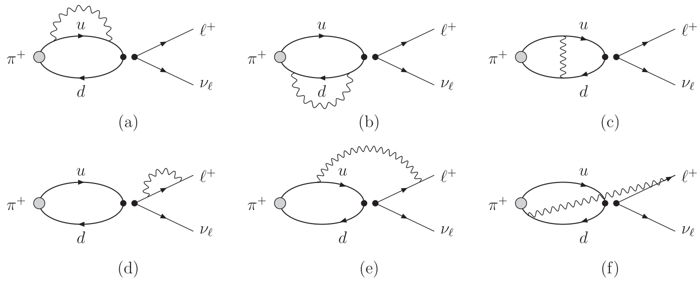

Figure 2: *Connected diagrams contributing at $O(\alpha_{em})$ to the amplitude for the decay $\pi^+ \to \ell^+\nu$. The labels (a)-(f) identify the individual diagrams.*

3. Results for charged pion and kaon decays

We now insert the various ingredients described in the previous Sections in the master formula (10) in the case of the decays $\pi^- \to \mu^-\bar{\nu}_\mu[\gamma]$ and $K^- \to \mu^-\bar{\nu}_\mu[\gamma]$. It is understood that m_P represents the charged pseudo scalar mass, which includes e.m. and strong IB corrections. The latter are calculated as in Refs. [7, 8] and are generated by the mass difference $\delta m = m_d - m_u$ previously determined.

Preliminary results for the corrections δR_π and $\delta R_{K\pi} \equiv \delta R_K - \delta R_\pi$ are shown in Fig. 3, with the caveat that all photon energies (i.e. $\Delta E = \Delta E^{max} = m_P(1 - m_\mu^2/m_P^2)/2$) are included, since the experimental data on $\pi_{\ell 2}$ and $K_{\ell 2}$ decays are fully inclusive. The "universal" finite volume corrections up to order $O(1/L)$ are subtracted from the lattice data and the combined chiral, continuum and infinite volume extrapolations are performed using the fitting function

$$\delta R_P = A_0 + \frac{3}{16\pi^2}\log(\xi) + A_1\xi + A_2\xi^2 + Da^2 + \frac{K_2}{m_P^2 L^2} + \frac{K_2^\ell}{E_\ell^2 L^2} + \Gamma^{pt}(\Delta E^{max}) \ , \tag{12}$$

where $\xi \equiv m_P^2/(4\pi f_0)^2$, E_ℓ is the lepton energy in the meson rest frame, $A_{0,1,2}$, D, K_2 and K_2^ℓ are free parameters, and the coefficient of the chiral log is taken from Ref. [9].

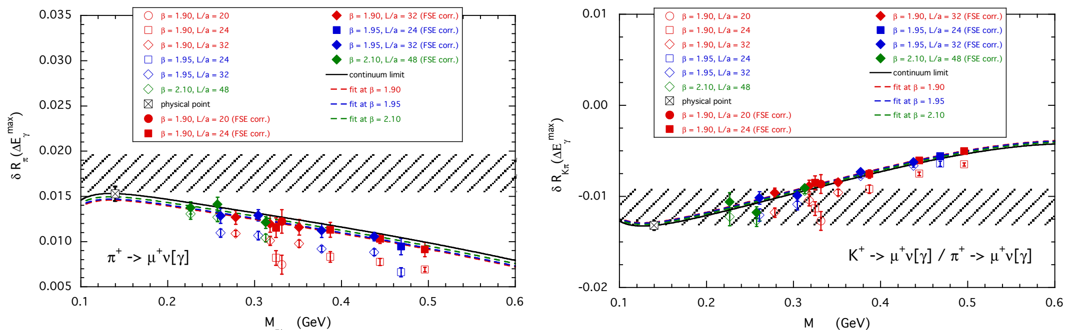

Figure 3: *Results for the corrections δR_π (left panel) and $\delta R_{K\pi} \equiv \delta R_K - \delta R_\pi$ (right panel) obtained after the subtraction of the "universal" FV terms computed in perturbation theory (7). The full markers correspond to the lattice data corrected by the residual FV corrections obtained using the fitting function (12) including the chiral log. The dashed lines represent the results in the infinite volume limit at each value of the lattice spacing, while the solid lines are the results in the continuum limit. The crosses represent the values δR_π^{phys} and $\delta R_{K\pi}^{phys}$ at the physical point. The shaded areas correspond respectively to the values* 0.0176(21) *and* −0.0112(21) *at 1-sigma level, obtained using ChPT (see Refs. [10, 11]).*

Adopting different fitting functions and FSE subtractions in order to estimate systematic errors and averaging the corresponding results, we finally get at the physical point

$$\delta R_\pi^{phys} = +0.0169\ (8)_{stat+fit}\ (11)_{chiral}\ (7)_{FSE}\ (2)_{a^2} = +0.0169\ (15)\ , \tag{13}$$

$$\delta R_{K\pi}^{phys} = -0.0137\ (11)_{stat+fit}\ (6)_{chiral}\ (1)_{FSE}\ (1)_{a^2} = -0.0137\ (13)\ , \tag{14}$$

where the errors do not include the QED quenching effects. Our findings (13)-(14) can be compared with the corresponding ChPT predictions 0.0176(21) and $-0.0112(21)$ [10, 11].

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[1] N. Carrasco, V. Lubicz, G. Martinelli, C. T. Sachrajda, N. Tantalo, C. Tarantino and M. Testa, Phys. Rev. D **91** (2015) no.7, 074506 [arXiv:1502.00257 [hep-lat]].
[2] S. Aoki *et al.*, arXiv:1607.00299 [hep-lat].
[3] N. Tantalo, PoS LATTICE **2013** (2014) 007 [arXiv:1311.2797 [hep-lat]].
[4] A. Portelli, PoS LATTICE **2014** (2015) 013 [arXiv:1505.07057 [hep-lat]].
[5] A. Patella at Lattice 2016, https://conference.ippp.dur.ac.uk/event/470/session/1/contribution/400.
[6] V. Lubicz, G. Martinelli, C. T. Sachrajda, F. Sanfilippo, S. Simula and N. Tantalo, arXiv:1611.08497 [hep-lat].
[7] G. M. de Divitiis *et al.*, Phys. Rev. D **87** (2013) no.11, 114505 [arXiv:1303.4896 [hep-lat]].
[8] G.M. de Divitiis *et al.*, JHEP **1204** (2012) 124 [arXiv:1110.6294 [hep-lat]].
[9] M. Knecht *et al.*, Eur. Phys. J. C **12** (2000) 469 [hep-ph/9909284].
[10] J. L. Rosner *et al.*, Submitted to: Particle Data Book [arXiv:1509.02220 [hep-ph]].
[11] V. Cirigliano and H. Neufeld, Phys. Lett. B **700** (2011) 7 [arXiv:1102.0563 [hep-ph]].