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Conductor losses calculation in two-dimensional simulations of H-plane rectangular waveguides

G. Castorina [a], G. Torrisi [b], G. Sorbello [b,c], L. Celona [b] and A. Mostacci [a]

[a]Istituto Nazionale di Fisica Nucleare – Laboratori nazionali di Frascati, Frascati, Italy; [b]Istituto Nazionale di Fisica Nucleare – Laboratori Nazionali del Sud, Catania, Italy; [c]Dipartimento di Ingegneria Elettrica, Elettronica e Informatica, Università degli Studi di Catania, Catania, Italy

ABSTRACT
This paper presents a novel numerical approach to simulate H-plane rectangular-waveguide microwave circuits considering a reduced quasi-2D simulation domain with benefits for computational cost and time. With the aim to evaluate the attenuation of the full height 3D component, we propose a modified expression for the waveguide top/bottom wall conductivity. Numerical 2D simulations are validated against results from full wave 3-D commercial electromagnetic simulator. After a benchmark on a simple straight waveguide model, the method has been successfully applied to an asymmetric un-balanced power splitter, where an accurate power loss prediction is mandatory. Simulation time and memory consumption can be reduced by a factor ten and seven respectively, in comparison with complete 3D geometries. Finally, we show that, also for quasi-2D E-bend waveguide, a case where the translational H-plane symmetry is broken, the error on conductor losses computation is mitigated by our approach since the method remains still valid in a first approximation.

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1. Introduction and motivation

Waveguide-based components have been widely investigated and used because of their low insertion loss, wide bandwidth and high-power capability [1].

One approach for the study of waveguide components is the field-theoretical analysis of the whole component by general methods such as finite difference time (FDTD) [2] and frequency domain (FDFD) [3] or finite element method (FEM) [4]. Such general approaches remove any restriction to the shape and symmetry of the studied components. However, full wave simulations are time-consuming and computationally demanding. If we restrict the study to arbitrary H-plane components of height *b*, a number of efficient methods have been proposed for lossless, homogeneous, isotropic, non-dispersive medium [5] that can be extended to lossy components by using a perturbation approach [6].

As a general rule, one way to reduce the simulation time is to consider a reduced computational domain taking advantage of symmetry such as translational/rotational invariance,

CONTACT G. Torrisi giuseppe.torrisi@lns.infn.it, peppetorrisi@lns.infn.it Istituto Nazionale di Fisica Nucleare – Laboratori Nazionali del Sud, Via S. Sofia 62, Catania, Italy

periodicity or even more complex roto-translational periodic symmetry [7]. The design of rectangular waveguide components is completely 2D only if the layout of the waveguide branching network lays entirely in the H-plane. It is well known that H-planar components maintain the matching properties thanks to the translational field invariance in the direction of the electric fields. This circumstance can be exploited to make faster 2D numerical simulations as well as in application where the free parameter height b of the device can be, theoretically, arbitrarily increased to accommodate higher power [8] or reduced to satisfy space constraint as in [9]. Moreover, planar devices can be easily fabricated in planar technologies or from two monolithic pieces [10].

However, a simple two-dimensional model, or a reduced height model would not allow in principle a correct power loss calculation which depends on the height of the guiding structure. In this letter, we present an approach, based on the surface impedance loss model, to correct the loss per unit length in 2D computer-aided design of planar components such as asymmetric H-plane power splitter [11] where accurate power loss prediction is mandatory. Our approach can be easily extended to any H-plane component uniform in the transverse direction y, even complex devices such as substrate integrated waveguides (SIWs) [12].

Since the correction factor we introduced is analytically derived and demonstrated, the method is general and valid also for lossy filling dielectric [13] which can be also inhomogeneous if the filling dielectric constant does not depend on y, i.e. $\epsilon(x, y, z) = \epsilon(x, z)$.

Our innovative method allows fast and accurate simulation of planar waveguide components with a 2D model, thus leading to a significant reduction of computational requirements. The importance of this reduced model is clearly revealed in a large design where the method allows a significant speed-up. The technique is of general applicability since this can be used in commercially available microwave CAD, and it is of great interest in many time-consuming optimization of complex H-plane y-invariant components including SIW waveguides and structures with lossy filling dielectric.

CAD-assisted design and optimization of microwave devices and systems, especially where losses are of primary concern, could benefit from the method. Appropriate optimization and fine tuning can be easily carried out on many devices including high average-power microwave components, high current particle accelerators which are emerging as critical tools for science and applications [14], planar transmission lines and waveguide devices for instance filters, diplexers, low-loss antenna feeders, delay lines, and phase shifters [15,16]. In particular, loss evaluation is mandatory in specific high-power millimetre-wave applications such as electron cyclotron heating/current drive for fusion plasmas [17].

2. Waveguide loss formulation

The attenuation is the ratio of the power density dissipated per unit length, P_d, to the propagating power P and can be analytically calculated by the power loss method [1, p.82 (2.96)], [18, p.60], [19]:

$$\alpha = \frac{P_d}{2P} \tag{1}$$

where

$$P = \frac{1}{2}\mathrm{Re}\int_S \vec{E} \times \vec{H}^* \cdot \vec{u}_z \,\mathrm{d}S \tag{2}$$

and

$$P_d = \frac{1}{2}\sqrt{\frac{\omega\mu_c}{2\sigma}} \oint_{L\equiv\partial S} \vec{H}_\tau \cdot \vec{H}_\tau^* \,\mathrm{d}L \tag{3}$$

where H_τ is the magnetic field component tangent to the conductor surfaces, ω is the angular frequency, μ_c the waveguide walls magnetic permeability, σ the waveguide walls conductivity, S the waveguide section and $L \equiv \partial S$ is the waveguide contour.

If we consider a rectangular waveguide with cross section $a \times b$, the conductor power loss on the contour $L \equiv 2a + 2b$ can be separated in the sum $P_d = P_{da} + P_{db}$, where

$$\begin{aligned} P_{da} &= \frac{1}{2}\sqrt{\frac{\omega\mu_c}{2\sigma}} \int_{2a} \vec{H}_\tau \cdot \vec{H}_\tau^* \,\mathrm{d}L \\ P_{db} &= \frac{1}{2}\sqrt{\frac{\omega\mu_c}{2\sigma}} \int_{2b} \vec{H}_\tau \cdot \vec{H}_\tau^* \,\mathrm{d}L \end{aligned} \tag{4}$$

with P_{da} the loss on top/bottom sides and P_{db} the one on waveguide sidewalls of height b.

The electromagnetic field for good conductor is well approximated by the solution in a PEC rectangular waveguide. For the planar circuit supporting only the fundamental mode TE_{10} (and eventually higher TE_{p0} modes) in principle all the simulations are independent of the height b thanks to the translational symmetry in the direction of the electric fields (below the direction of y) since the field is well approximated by the TE_{p0} mode below [1,18]:

$$\begin{aligned} \vec{E}_{t\mathrm{TE}_{p0}} &= \vec{e}_{t\mathrm{TE}_{p0}}\, \mathrm{e}^{-j\beta_z z} \\ \vec{H}_{t\mathrm{TE}_{p0}} &= \frac{\beta_z}{\omega\mu}\vec{h}_{t\mathrm{TE}_{p0}}\, \mathrm{e}^{-j\beta_z z} \\ H_{z\mathrm{TE}_{p0}} &= \frac{k_t^2}{j\omega\mu} \cos\frac{p\pi x}{a}\, \mathrm{e}^{-j\beta_z z} \end{aligned} \tag{5}$$

where the non-zero $\vec{e}_{t\mathrm{TE}_{p0}}$ and $\vec{h}_{t\mathrm{TE}_{p0}}$ components are

$$\begin{aligned} e_{y\mathrm{TE}_{p0}} &= -h_{x\mathrm{TE}_{p0}} \\ h_{x\mathrm{TE}_{p0}} &= \frac{p\pi}{a}\sin\frac{p\pi x}{a} \end{aligned} \tag{6}$$

with $k_t^2 = (p\pi/a)^2$ and $\beta_z^2 = \omega^2\epsilon\mu - k_t^2$. It is clear that all fields in (5)–(6) are y-invariant and independent of the height b of the waveguide/H-planar device.

Numerical simulations of planar waveguide device can take advantage of this property to greatly reduce the required computational effort.

A model with reduced height does not introduce any approximation if PEC boundaries are imposed. However, if a finite conductivity boundary condition is taken into account, the calculated loss are overestimated because the attenuation, α, increases as b decreases.

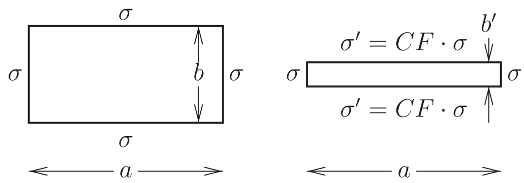

Figure 1. Full height and reduced b'-height models.

It is possible to restore the correct attenuation by introducing a fictitious conductivity σ' for the top/bottom a-wide side of the waveguide:

$$\sigma' = CF \cdot \sigma = \left(\frac{b}{b'}\right)^2 \cdot \sigma \tag{7}$$

where the correction factor $CF = (b/b')^2$, introduced, is given by the ratio between the initial 3D height b and the reduced one b' of the quasi-2D model.

Thanks to the correction factor on top/bottom waveguide walls of the reduced b'-height model, now all terms in (1), P, P_{da} and P_{db}, scale in the same manner with b' restoring the correct value of α (Figure 1) .

The correction introduced above is limited to H-plane devices supporting TE_{p0} modes. However, as far as the waveguide height b is kept low, higher TE_{pq} with $q \neq 0$ are not supported by the structure. Usually standard waveguide devices fulfil this requirement.

3. Numerical results

The simulations carried out rely on the surface impedance concept for both the full wave simulation and the reduced 2D model. This approximation is very well fulfilled for metals up to optical wavelengths. The commercial code Ansys HFSS has been used for the numerical analysis and validation. Simulations have been performed on a workstation equipped with a Xeon E5-1630 v4 3.70 GHz CPU and 128 GB RAM. The method above discussed has been firstly benchmarked with a very simple geometry: a straight waveguide with the standard WR90 dimension. After this first preliminary test, an asymmetric splitter with 1/3 power splitting ratio has been designed. Finally, the method has also been tested for an E-plane device. For all the considered devices the simulations with standard waveguide height, highlighted in bold in the tables of results, are used as reference value for both the S-parameters, memory footprint and computation time. When $CF = 1$, it means that no correction factor has been used for the simulation and the scattering parameters evaluation.

3.1. Straight waveguide

The model has been parametrized respect to the waveguide height b' ranging from the standard WR90 height $b = 10.16$ mm to b' as reported in the first column of Table 1. For each simulation, the value of the S_{21} parameter at 10 GHz frequency has been computed. The maximum length of the mesh elements is $\sim \lambda/6$. An adaptive solution with three passes and an interpolating frequency sweep has been used for the simulation. The waveguide

Table 1. Results of the parametric sweep simulations for the WR90 waveguide.

b' (mm)	CF	σ'	Mesh cells	Mem. (GB)	Time (s)	$\lvert S_{21}\rvert$
b	1	PEC	123174	34.24	680	1
b	**1**	**5.80e7**	**123177**	**29.37**	**1249**	**0.988**
b/2	1	5.80e7	62779	15.17	423	0.979
	4	2.32e8	62775	15.16	418	0.988
b/4	1	5.80e7	31787	7.86	184	0.961
	16	9.28e8	31752	7.85	181	0.988
b/8	1	5.80e7	25540	6.45	154	0.923
	64	3.71e9	25540	6.45	151	0.988

length is 1 m. Results, also in terms of computation time and memory usage, are reported in Table 1.

The time required for the reference simulation with a finite conductivity is 1249 s. This value can be reduced by a factor three when the height of the waveguide is halved, and a correction factor of $CF = 4$ is employed. When a quarter value of the height is used the time required for simulation is reduced by $\sim$6.5 factor. Further reductions of the height dimension do not entail to a substantial speed-up of the simulation. However, for larger geometries, the b' dimension could be further decreased with computational time benefits. Thanks to the correction factor, the evaluated $|S_{21}|$ is equal to the $b' = b$ reference simulation also for the case $b' = b/8 = 1.27$ mm where the initial evaluated $|S_{21}|$ (without CF) has an appreciable error respect to the $b' = b$ reference simulation Also the memory requirements are greatly reduced when this method is used: for the full 3D-simulation more than 34 GB of RAM are required while for the quasi-2D simulation, with $b' = b/8 = 1.27$ mm, only 6.5 GB of RAM are needed.

If the operating frequency is close to cut-off frequency f_c, the waveguide loss becomes very high. However, the presented reduced model is still valid since it relies on a local property based on the surface impedance concept. Further insight is given by an analytical derivation and numerical experiments are shown in Appendix.

3.2. Unbalanced power splitter

This microwave device (see Figure 2) is designed to obtain a power ratio of one-third between two ports arranged with a 90° degree angle. From the $|S_{21}|$ and $|S_{31}|$ values of the preliminary reference simulation with a standard dimension of the waveguide (see Table 2, first row), we can verify that the power division obtained is satisfactory ($|S_{21}|^2 = 0.8673^2 = 0.752 \approx 0.75$ and $|S_{31}|^2 = 0.244 \approx 0.25$).

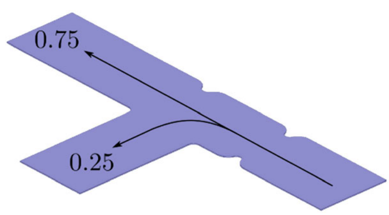

Figure 2. CAD drawing of the H-plane splitter designed to obtain an un-balanced power division.

Table 2. Results for the asymmetric power splitter at 11.424 GHz.

b' (mm)	CF	σ'	Mem. (GB)	Time (s)	$\lvert S_{21}\rvert$	$\lvert S_{31}\rvert$
b	**1**	**5.80e7**	**10.35**	**545**	**0.8673**	**0.4939**
b/32	1	5.80e7	1.59	54	0.8382	0.5204
b/32	1024	5.94e10	1.50	53	0.8673	0.4939

Table 3. Results for the waveguide with a bend in the E-plane at 11.424 GHz.

b' (mm)	CF	σ'	Mem. (GB)	Time (s)	$\lvert S_{11}\rvert$	$\lvert S_{21}\rvert$
b	**1**	**5.80e7**	**48.85**	**946**	**2.7e−3**	**0.9878**
$b/32$	1	5.80e7	3.99	125	5.5e−7	0.7372
$b/32$	1024	5.94e10	3.99	124	6.5e−7	0.9878

The same device has been simulated with a reduced height without ($CF = 1$) and with the correction factor ($CF = 1024$) on conductivity. In both cases, the time required has been reduced by an order of magnitude and the memory employed by a factor of 7. Thanks to the correction factor, the same value of the reference simulation for the S-parameters is again obtained, therefore successfully validating the method proposed here.

3.3. Non-planar devices

The electric field of the TE_{p0} modes is modified when they propagate inside an E-bend. In this case, Equations (6) are no more strictly valid nor y invariant. However, if the bend radius R is at least some wavelength long, the method here proposed can be used with a limited error. A sufficiently large radius of curvature (greater than 1.5 wavelength) is also needed to avoid a large insertion loss of the bend. In almost all designs of microwave waveguide devices, this condition is naturally satisfied. An E-bend with a radius of 40 mm ($\sim 1.5\lambda$) has been simulated at the working frequency of 11.424 GHz. The results of the simulations are reported in Table 3.

As expected, the obtained results are not in agreement with the 3D-full wave reference simulation. While the $|S_{21}|$ can be correctly estimated, the $|S_{11}|$ is greatly underestimated. However, the proposed method can be used for a rough preliminary estimation.

4. Conclusions

A method to drastically reduce the computational effort when simulating H-planar waveguide device has been presented. This method exploits the field configuration of the TE_{p0} modes which are independent respect to the height of the waveguide. A full 3D simulation can be reduced to a quasi-2D simulation where the conduction losses can be evaluated by introducing a fictitious conductivity which restores the exact attenuation constant. This general and accurate approach can be successfully used in the design of power divider, directional coupler and power distribution network. Respect to the cases here analysed, the simulation time has been reduced by an order of magnitude and the memory usage by a factor of about seven. Also in the case of a waveguide with E-bends, i.e. non-H-planar devices, the error on the transmitted power is negligible compared to full 3D simulation.

Such a drastic time reduction represents a great advantage for a large design as well as for automated design and time-consuming optimization.

Disclosure statement

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Notes on contributors

G. Castorina was born in Catania, Italy, in 1985. He received the Ph.D. in telecommunications engineering from the University of Catania, Catania, in 2018. He received a fellowship from INFN – LNF and Università degli Studi di Roma, "La Sapienza". Currently he is employed as Senior RF Engineer in AVO-ADAM, a CERN spin-off company developing proton LINACs for cancer treatment.

G. Torrisi was born in Catania, Italy, in 1987. He received the M.S. Laurea degree (summa cum laude) in telecommunication engineering from the University of Catania (Catania, Italy) in 2011 and the Ph.D degree in information engineering from Università Mediterranea of Reggio Calabria (Reggio Calabria, Italy) in 2016. He is currently a temporary staff researcher of the Istituto Nazionale di Fisica Nucleare (INFN) at Laboratori Nazionali del Sud (LNS), Catania. His scientific activity is concerned with electromagnetic propagation in microwave generated plasmas produced by ion sources for particle accelerators.

G. Sorbello received the degree in Electronics Engineering cum laude at the University of Catania, Italy, in 1996, and the Ph.D. degree in Electronics and Communications Engineering at the Polytechnic Institute of Milan in 2000. In 2002 he became Assistant Professor of Electromagnetic Fields at University of Catania. Since 2014, he is Associate Professor of Electromagnetic fields at the Dipartimento di Ingegneria Elettrica, Elettronica e Informatica, University of Catania. His research interests include the study of single-mode solid-state waveguide lasers and amplifiers, integrated optics, the development of planar antennas and ultra-wideband compact antennas and antenna arrays, the study of microwave devices and computational electromagnetism with a special interest in RF-plasma interactions and particle accelerators.

L. Celona received the degree in Electronic Engineering at the University of Catania, in 1995 and he joined the Istituto Nazionale di Fisica Nucleare in 1996, at the Laboratori Nazionali del Sud, becoming Principal Technological Engineer in 2006 and Technology Research Director in 2019. His main field of activity covers all the aspects of the production of singly and highly charged beams together with their acceleration to increase the performances of Particles Accelerators for Nuclear and Applied Physics. Experienced in all the design stages of an ion source: from mechanical design and manufacturing through the installation and final commissioning, he is also active in research and development, proposing different innovative concepts concerning the role of microwaves in the development of ECR and microwave ion sources.

A. Mostacci received the degree in Electronics Engineering cum laude (1997) and the Ph.D. degree in Applied Electromagnetism and Electro-physical Sciences (2001) in Sapienza, University of Rome, Italy. Since 2018, he is Associate Professor of Experimental Physics at the Department of Basic and Applied Sciences for Engineering and he is managing the Accelerator Laboratory of his department. His research interest deal with electromagnetism applied to science and technology of particle accelerators: wakefield effects, design of beam manipulation RF devices (from 3 GHz to 12 GHz), microwave measurements. He is also involved in R&D of high brightness electron accelerators and novel plasma-based compact accelerators.

ORCID

G. Castorina http://orcid.org/0000-0003-2967-0393
G. Torrisi http://orcid.org/0000-0002-2819-8800
G. Sorbello http://orcid.org/0000-0002-5081-3486
L. Celona http://orcid.org/0000-0002-6328-5926
A. Mostacci http://orcid.org/0000-0002-4407-7312

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Appendix

We show hereinafter that the reduced model is valid also close to cut-off, i.e. in a working point where it is well known that the standard variational power loss method fails [1, p.82], [20].

For a TE_{p0} mode from (1) we have

$$\alpha = \frac{P_d}{2P} = \frac{P_{da} + P_{db}}{2P} \tag{A1}$$

where (4)

$$P_{da} = \frac{1}{2}\sqrt{\frac{\omega\mu_c}{2\sigma}} \int_{2a} |\vec{H}_\tau|^2 \, dL = \frac{1}{2}\sqrt{\frac{\omega\mu_c}{2\sigma}}\, 2\int_0^a (\underbrace{|H_z|^2}_{\text{trasv.}} + \underbrace{|H_x|^2}_{\text{longit.}})\, dx,$$

$$P_{db} = \frac{1}{2}\sqrt{\frac{\omega\mu_c}{2\sigma}} \int_{2b} |\vec{H}_\tau|^2 \, dL = \frac{1}{2}\sqrt{\frac{\omega\mu_c}{2\sigma}}\, 2\int_0^b (\underbrace{|H_z|^2}_{\text{trasv.}} + 0)\, dy, \tag{A2}$$

i.e. in the presence of a TE mode, we have longitudinal and transverse currents both giving contributions to the total attenuation

$$\alpha = \alpha_{\text{trasv.}} + \alpha_{\text{longit.}} \tag{A3}$$

Recalling the expression of fields (5) and (6), $k_t^2 = (p\pi/a)^2$ and $\beta_z = \omega\sqrt{\epsilon\mu}\sqrt{1-(f_c/f)^2}$ we have

$$2\int_0^a \left(|H_z|^2 + |H_x|^2\right) dx = 2\int_0^a \left(\left|\frac{k_t^2}{\omega\mu}\cos\frac{p\pi x}{a}\right|^2 + \left|\frac{\beta_z}{\omega\mu}\frac{p\pi}{a}\sin\frac{p\pi x}{a}\right|^2 \right) dx = a\left(\left|\frac{k_t^2}{\omega\mu}\right|^2 + \left|\frac{\beta_z}{\omega\mu}k_t\right|^2 \right)$$

$$2\int_0^b \left(|H_z|^2 + 0\right) dy = 2\int_0^b \left(\left|\frac{k_t^2}{\omega\mu}\cos\frac{p\pi 0}{a}\right|^2 + 0 \right) dy = 2b\left|\frac{k_t^2}{\omega\mu}\right|^2 \tag{A4}$$

and (2):

$$P = \frac{1}{2}\mathrm{Re}\int_S \vec{E}\times\vec{H}^*\cdot\vec{u}_z\, dS = \frac{1}{2}\int_0^a\int_0^b \left|\frac{\beta_z}{\omega\mu}\right| \left|\frac{p\pi}{a}\sin\frac{p\pi x}{a}\right|^2 dx\, dy = \frac{1}{2}\left|\frac{\beta_z}{\omega\mu}\right| k_t^2 \frac{ab}{2}; \tag{A5}$$

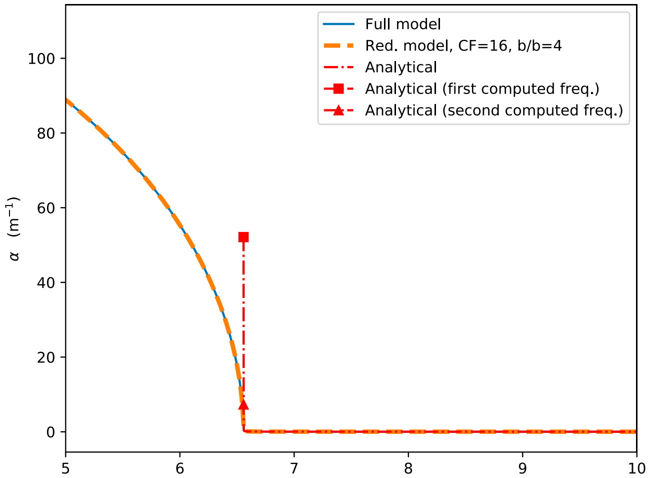
substituting in (A1) we have

$$\alpha = \frac{P_{da}+P_{db}}{2P_d} = \frac{1}{2}\sqrt{\frac{\omega\mu_c}{2\sigma}}\,\frac{a(k_t^2+\beta_z^2) + 2bk_t^2}{\omega\mu\ \beta_z \frac{ab}{2}} = \frac{1}{2}\sqrt{\frac{\omega\mu_c}{2\sigma}}\,\frac{a(\omega^2\epsilon\mu) + 2b(\omega_c^2\epsilon\mu)}{\omega\mu\ \omega\sqrt{\epsilon\mu}\sqrt{1-\frac{f_c}{f}}\frac{ab}{2}} \tag{A6}$$

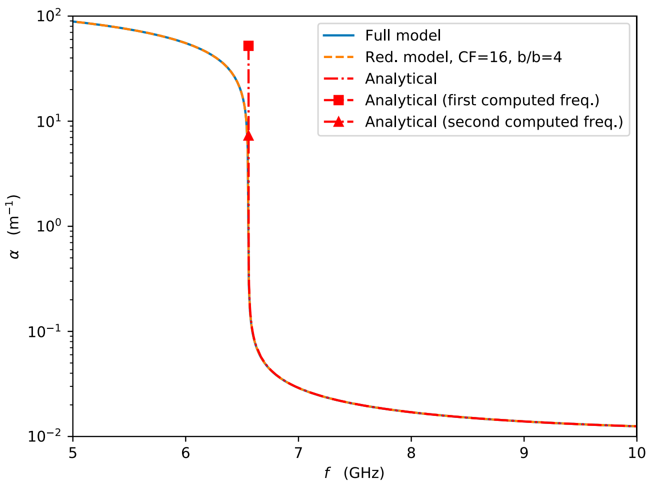
and

$$\alpha = \alpha(f) = \sqrt{\frac{\pi f \mu_c}{\sigma}}\sqrt{\frac{\epsilon}{\mu}}\,\frac{\frac{1}{b} + \frac{2}{a}\left(\frac{f_c}{f}\right)^2}{\sqrt{1-\frac{f_c}{f}}}. \tag{A7}$$

In Figure A1, we report results for full, reduced and the power loss perturbation method (A7) (the latter method is valid for $f > f_c$). As it can clearly be seen in Figure A1, the attenuation constant, α, computed for full height and reduced models are in a very good agreement, below and above cutoff as well as when the frequency approaches f_c and the power loss method (A7) fails. In the simulation carried out to generate Figure A1, we used a discrete sweep with the ratio $b/b' = 4$ (and $CF = 16$) obtaining, also in this very simple configuration, a speedup factor 2.14 and less than one-third of the mesh element.



(a)



(b)

Figure A1. Attenuation evaluated with full and reduced model. The variational semi-analytical formulation (A7) is valid above cut-off and fails close to cut-off. (a) Linear and (b) semilog y-axis.