Load identification by operational Statistical Energy Analysis inverse approach

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Abstract

Aim of this paper is to perform a load identification procedure based on Statistical Energy Analysis (SEA) in operative conditions. Three coupled plates laying on a soft support is the test bed chosen to verify the efficiency of this investigated technique. By performing a Power Injection Method (PIM), the SEA parameters, are experimentally identified. These parameters allow to build a SEA model of the structure, that represents the starting point for the load identification procedure in operative conditions. The identification of the power injected in each subsystem is performed by calculating the energy of each plate starting from the dynamic response of the structure and the inversion of the SEA energy balance equation.

A test is presented to identify a “rain-on-the-roof” load. The identified power injected is used together with “rain-on-the-roof” model to evaluate the actual load applied. The identified load is compared with the actual one to evaluate the accuracy of the results.

1 Introduction

In the field of vibroacoustic there are two different kinds of identification problems: the identification of the model by the knowledge of inputs and responses, and the identification of model inputs by the knowledge of responses and model parameters[1]. Both these inverse identification problems are still open issues in structural dynamics. By considering the definition given by Hadamard[2], inverse problems are ill-posed; in fact the solution is not unique and the stability criterion is not satisfied. At a later time, the introduction of the concept of the general solution makes possible to approach this problem that still remains ill-conditioned.

To reduce ill-conditioning, several techniques are developed: most of these are centered on the reduction of condition number by regularization techniques based on singular values decomposition[3]. Due to the nature of dynamic problems, the reduction of ill-conditioning is not always enough to obtain meaningful solution. For example, in the field of force identification in the frequency domain the inversion of the frequency response function (FRF) matrix could not give the expected solution.

In particular conditions, alternative approaches could be used together with regularization techniques. The solution of high frequency dynamic problems, for example, can be performed by energy-based models, as SEA[4-6], that allows to bypass the ill-conditioning of FRF matrix.

This paper will present an identification procedure in operative conditions based on SEA model. The test structure is a three coupled plates laying on a soft support and 14 points are selected on the three plates to perform the tests. Each plate represents a SEA subsystem. First a Single Input/Multi Output test is carried out at the selected points by a white random noise excitation. Then, the FRF matrix is calculated by the obtained measures and the SEA parameters are experimentally identified by the Power Injection Method (PIM) in order to create the SEA model. The validation of this model and of the identification procedure is performed by the identification of the injected power of all considered measurement points of the structure and by comparison with the actual power injected.

Then, two tests are carried out by a *quasi* “rain-on-the-roof” load. The first one, it is a numerical simulation: a random not-correlated load is imposed at the points of the first plate and the response at each point of the system are computed by the calculated FRF matrix. The second test is carried out by loading simultaneously the first plate by multi-impulse forces.

The energy of each subsystem is computed starting from the measured dynamic responses and the power injected is identified through the inversion of the energy balance equations of the system. The identified power injected is used together with the “rain-on-the-roof” model to evaluate the actual load applied

2 Theoretical background

The SEA model is a set of energy balance equations. For M coupled subsystem it can be written as follows[4-6]:

$$P_i = \omega\eta_i E_i + \omega \sum_{j=1,j\neq i}^{M}(\eta_{ij}\, E_i - \eta_{ji}\, E_j) \tag{1}$$

where i and j are the subsystems indices, η_i is the Internal Loss Factor (ILF) of subsystem i and η_{ij} is the Coupling Loss Factor (CLF) of junction between subsystems i and j. P_i is the power injected into subsystem i, ω is the central frequency of considered band and E is the energy stored in each subsystem.

Equation (1) can be written in a synthetic way as follows:

$$\boldsymbol{\eta}\,\mathbf{E} = \mathbf{P} \tag{2}$$

where the matrix $\boldsymbol{\eta}$ is built as shown in the equation (1) by the knowledge of the CLF’s and the ILF’s of subsystems.

2.1 Identification of CLF: Power Injection Method

The SEA parameters, CLF’s and ILF’s , can be obtained by theoretical relationships or by the PIM[7-9]. Appropriate experimental tests can be carried out on the studied mechanical system to measure the imposed forces and the dynamics responses (acceleration/velocity); this set of measures supply to determine the total energy of each subsystem and the power injected.

First, the Energy Distribution (ED) model must be considered, then the conditions under which the model can be considered “quasi SEA” or “proper SEA” must be verified[6].

In both cases, the parameters of the systems must satisfy these two hypothesis:

- the sum of columns of n-th CLF’s matrix must be equal to η_i.
- the off- diagonal terms must satisfy the relation: $n_i\eta_{ij} = n_j\eta_{ji}$

The use of PIM requires that a number of independent experiments must be performed. For each experiment, each subsystem is excited and by the measurements of force and response the total energy of each subsystem and the power injected are calculated.

Considering, for example, a system built of M subsystems, the use of PIM implies that M different experiments must be performed. For each experiment, the system of equation (2) becomes:

$$\begin{bmatrix} P_1 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} = \omega\,\boldsymbol{\eta} \begin{bmatrix} E_{11} \\ \vdots \\ \vdots \\ \vdots \\ E_{m1} \end{bmatrix}, \quad \begin{bmatrix} 0 \\ P_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \omega\,\boldsymbol{\eta} \begin{bmatrix} E_{12} \\ \vdots \\ \vdots \\ \vdots \\ E_{m2} \end{bmatrix}, \quad \cdots, \quad \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ \vdots \\ P_m \end{bmatrix} = \omega\,\boldsymbol{\eta} \begin{bmatrix} E_{1m} \\ \vdots \\ \vdots \\ \vdots \\ E_{mm} \end{bmatrix} \tag{3}$$

where E_{mj} indicates the energy of subsystem m when the subsystem j is excited.

So, the CLF’s matrix can be identified by the inversion of the energy matrix:

$$\begin{bmatrix} \eta_{11}+\sum_{j=1,j\neq 1}^{M}\eta_{1j} & -\eta_{21} & \cdots & -\eta_{m1} \\ -\eta_{12} & & & \vdots \\ \vdots & & \ddots & \vdots \\ \vdots & & & \vdots \\ -\eta_{1m} & -\eta_{2m} & \cdots & \eta_{mm}+\sum_{j=1,j\neq m}^{M}\eta_{mj} \end{bmatrix} = \frac{1}{\omega}\begin{bmatrix} P_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & P_m \end{bmatrix}\begin{bmatrix} E_{11} & \dots & E_{1m} \\ \vdots & \dots & \vdots \\ E_{m1} & & E_{mm} \end{bmatrix}^{-1} \quad (4)$$

A similar approach allows writing the identification algorithm in order to calculate directly the vector of CLFs without calculating the $\boldsymbol{\eta}$ matrix. Equation (5) shows the algorithm for three coupled subsystems.

$$\begin{bmatrix} \eta_1 \\ \eta_{12} \\ \eta_{13} \\ \eta_{21} \\ \eta_2 \\ \eta_{23} \\ \eta_{31} \\ \eta_{32} \\ \eta_3 \end{bmatrix} = \begin{bmatrix} E_{11} & E_{11} & E_{11} & -E_{21} & 0 & 0 & -E_{31} & 0 & 0 \\ 0 & -E_{11} & 0 & E_{21} & E_{21} & E_{21} & 0 & -E_{31} & 0 \\ 0 & 0 & -E_{11} & 0 & 0 & -E_{21} & E_{31} & E_{31} & E_{31} \\ E_{12} & E_{12} & E_{12} & -E_{22} & 0 & 0 & -E_{32} & 0 & 0 \\ 0 & -E_{12} & 0 & E_{22} & E_{22} & E_{22} & 0 & -E_{32} & 0 \\ 0 & 0 & -E_{12} & 0 & 0 & -E_{22} & E_{32} & E_{32} & E_{31} \\ E_{13} & E_{13} & E_{13} & -E_{23} & 0 & 0 & -E_{33} & 0 & 0 \\ 0 & -E_{13} & 0 & E_{23} & E_{23} & E_{23} & 0 & -E_{33} & 0 \\ 0 & 0 & -E_{13} & 0 & 0 & -E_{23} & E_{33} & E_{33} & E_{33} \end{bmatrix}^{-1} \begin{bmatrix} P_1 \\ 0 \\ 0 \\ 0 \\ P_2 \\ 0 \\ 0 \\ 0 \\ P_3 \end{bmatrix} \quad (5)$$

Since both equations (4) and (5) imply the inversion of the energy matrix and since the energies and the powers are calculated by experimental measurements affected by noise, it is necessary to tackle the ill-conditioning of energy matrix by two different procedures, as those proposed before. Therefore, both these approaches are investigated: even if the systems (4) and (5) are theoretically equivalent, the inverse problems can differ due to the different ill-conditioning of the energy matrices.

The energy of subsystem *i* when the subsystem *k* is excited can be obtained by the knowledge of the dynamic response considering the following equation:

$$E_{i,k} = \frac{1}{n}\sum_{j=1}^{n} \mathrm{m}_j \int_{\omega_1}^{\omega_2} S_{v_{j,i,k}v_{j,i,k}} d\omega \quad (6)$$

where, *n* is the number of measurement point for each subsystem *i*, m_j the mass assigned to each measurement point and $S_{v_{ij,k}v_{ij,k}}$ [10] the velocity of the measurement point *j* of the subsystem *i* when the subsystem *k* is excited. The power injected into subsystem *i* is computed by the knowledge of the cross spectral density between the applied force and the velocity measured at the drive point *j* by the following relationship:

$$P_i = \frac{1}{n}\sum_{j=1}^{n} \mathrm{Re}\left\{\int_{\omega_1}^{\omega_2} S_{F_{j,i}v_{j,i}} d\omega\right\} \quad (7)$$

2.2 Identification of injected power

Since aim of this paper is to perform a force identification technique under operative condition based on SEA equation, after calculating CLF’s and ILF's by PIM, matrix $\boldsymbol{\eta}$ of equation (2) is completely known and the power injected vector can be calculated by the same equation 2:

$$\widehat{\mathbf{P}} = \boldsymbol{\eta}\,\mathbf{E} \quad (8)$$

Matrix **E** is computed by a set of velocity measurements at selected points of each subsystem when an unknown external force acts on the structure. Equation (6) allows to calculate the subsystems energy by neglecting the *k* index:

$$E_i = \frac{1}{n}\sum_{j=1}^{n} \mathrm{m}_j \int_{\omega_1}^{\omega_2} S_{v_{j,i}v_{j,i}} d\omega \quad (9)$$

Vector $\widehat{\mathbf{P}}$ are the identified injected powers.

2.3 Identification of load spectrum

The considered load is a "rain-on-the-roof". This kind of load is a random excitation with delta-correlation spatial resolution and with amplitude proportional to the local mass density[6]. This hypothesis implies that the force power spectral density depends only on frequency and not on space.

Therefore, the mean square force over a frequency band, for this kind of excitation, is given by:

$$\langle F^2 \rangle_\omega = S_{ff}(\omega)\rho\,\Delta\omega \tag{10}$$

Since the injected power averaged on a frequency band is defined as:

$$P_{inj} = \int_{\Delta\omega} \mathrm{S}_{ff} \mathrm{Re}\{M\} d\,\omega \tag{11}$$

where M is the drive point mobility, the identification procedure is performed by inverting the last equation.

3 Experimental setup

Three coupled plates laying on a soft support is the test bed chosen to verify the efficiency of this investigated technique. The material of the plates are steel 1.5 mm thick, connected as shown in figure 1. In table 1 the dimensions of the structure are reported.

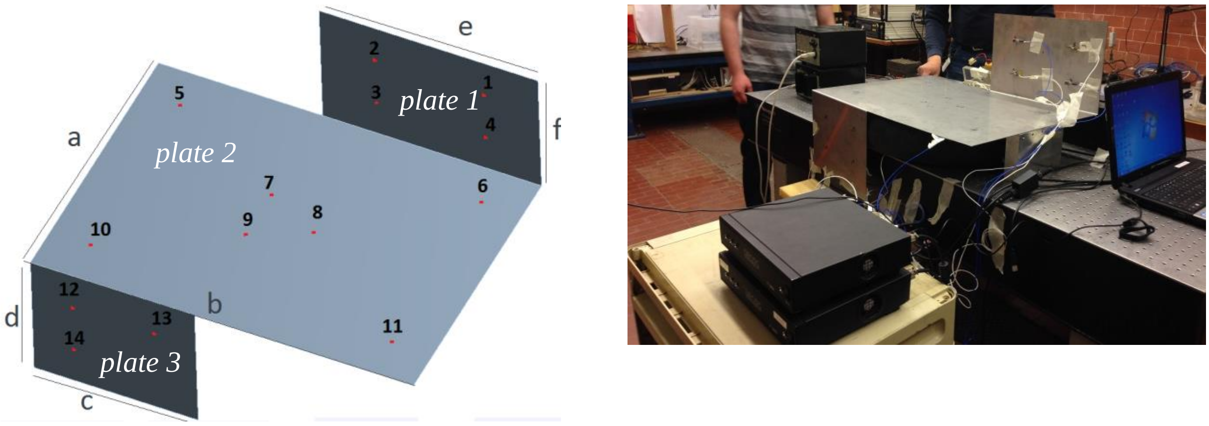

Figure 1: Experimental setup

Fourteen measurement points on the three plates are considered: 4 on first plate, 7 on the second plate and 3 on the third one. The points are not uniformly distributed; in fact, usually in operative condition the uniform distribution of acquiring points is not applicable.

a[mm]	b[mm]	c[mm]	d[mm]	e[mm]	f[mm]
500	700	300	250	400	250

Table 1: Structure dimensions

Whit the purpose of perform SEA parameter identification by using PIM, the system is excited by white random noise obtained by hammer multi impulse at all the 14 points. The acceleration is acquired for 20 seconds at 5000 Hz sample frequency.

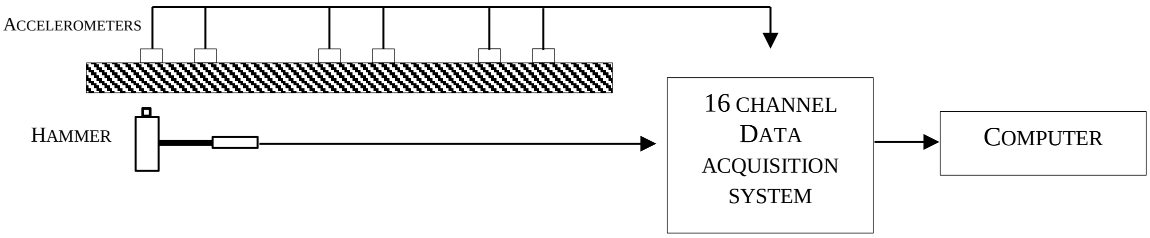

Figure 2: Measurement chain

By this first test, the SEA model and the FRF matrix are calculated.

In order to perform load identification the system is excited by by a *quasi* “rain-on-the-roof” load on the first plate and the acceleration is acquired at all the considered points.

4 Results

4.1 CLF’s and ILF’s identification

The CLF’s and ILF’s are identified by considering each point of each subsystem as drive point and the results are averaged as shown in figure 3.

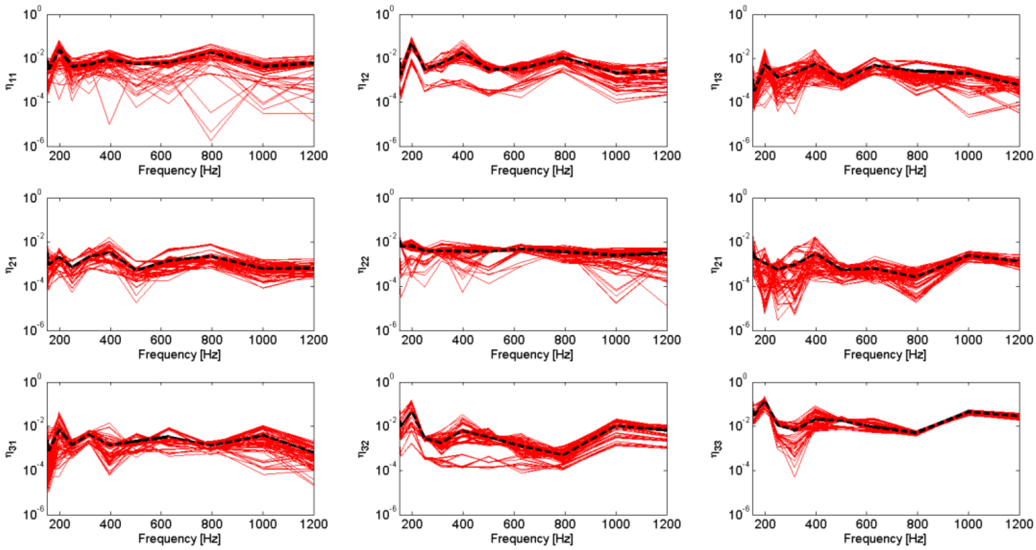

Figure 3: Comparison between CLF’s computed for each measurement (-)and the average and (--)

Identification of CLFs involves the inversion of the energy matrix, and the results obtained are different depending on the chosen procedure (equation (4) and (5)). Figure 4 displays the comparison between ILF's and CLF's computed by equation (4) and (5). As shown the CLF's identified by the two methods are almost equal, though the ILF's show larger differences; this entails to evaluate what results are better to perform power identification.

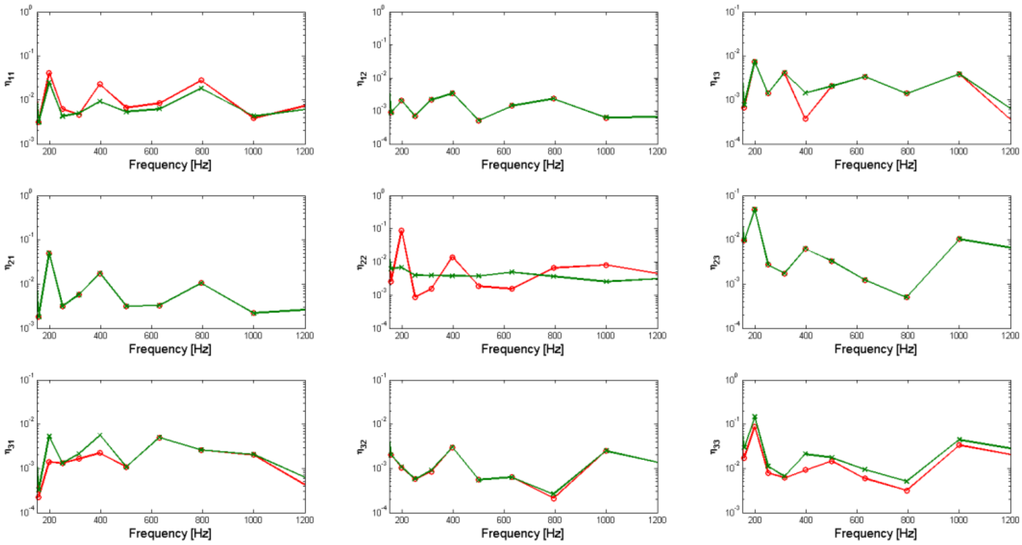

Figure 4: Comparison between ILF's and CLF's computed by equation 4 (**-o-**)and equation 5(**-x-**)

4.2 Power identification

4.2.1 Single Input Multi Output: experimental test

With the purpose to validate the results obtained from PIM and the identification procedure, the identified injected power at all the considered measurement points of the structure is compared with the actual injected power. Figure 5 and 6 show the comparison between the powers identified at two points by the two different sets of the SEA model.

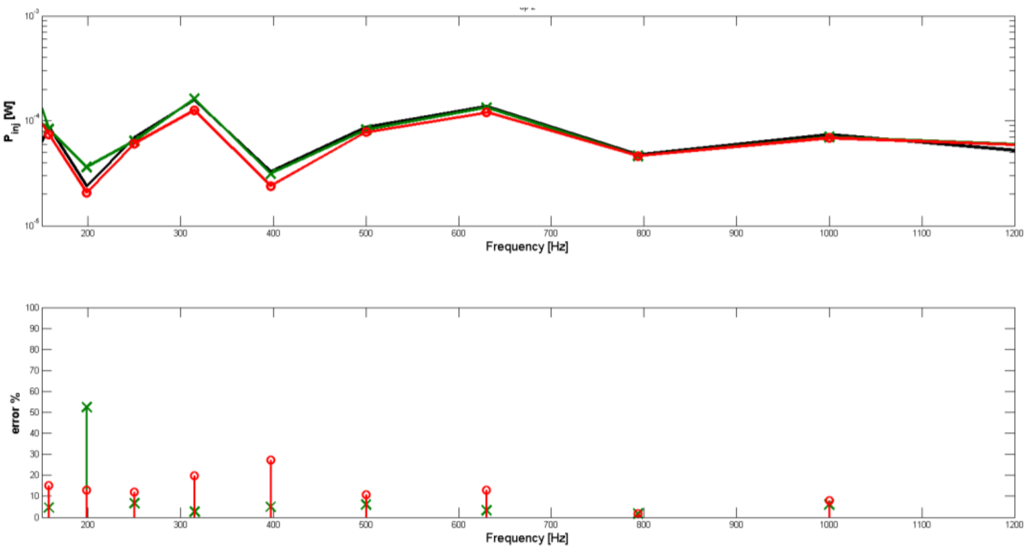

Figure 5: Comparison between injected power: actual power (-), power identified by ILF's and CLF's computed by equation 4 (**-o-**) and equation 5(**-x-**) for dp 2

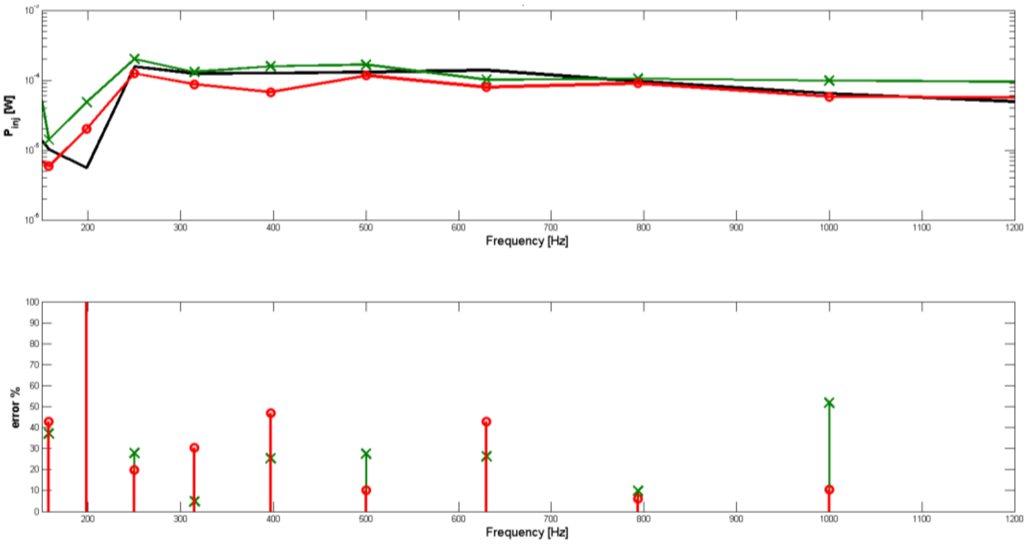

Figure 6: Comparison between injected power: actual power (-), power identified by ILF's and CLF's computed by equation 4 (**-o-**) and equation 5(**-x-**) for dp 14

As expected, the power identified is close to the actual power. As shown, the results obtained by using the two sets of SEA parameters are quite similar.

4.2.2 Multi input Multi Output: numerical test

For a further validation of the identified SEA parameters and of load identification procedure, 4 different loads are numerically generated and applied at the 4 points of the first plate, the numerical dynamic response is computed by the experimental FRF matrix and the identification of injected power is performed.

Figure 7 displays the comparison between actual and identified injected power. This results is useful to validate the results obtained by PIM. Indeed in this case the SEA parameters obtained from equation (4) and (5) give very similar and good results.

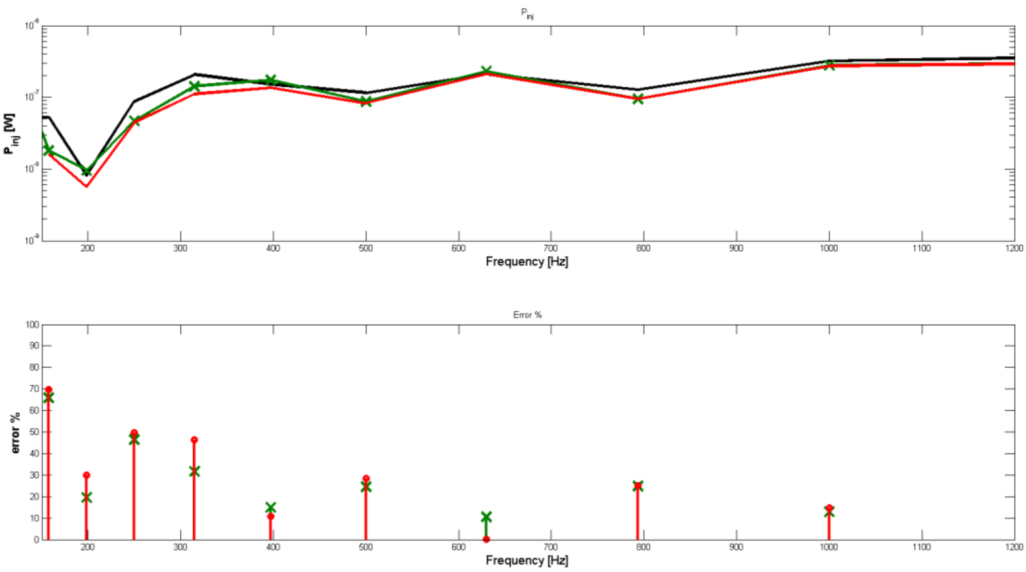

Figure 7: Comparison between injected power: actual power (-), power identified by ILF's and CLF's computed by equation 4 (**-o-**) and equation 5(**-x-**)

Starting from the identified injected power the load identification is performed by inverting equation (11). The results are shown in figure 8.

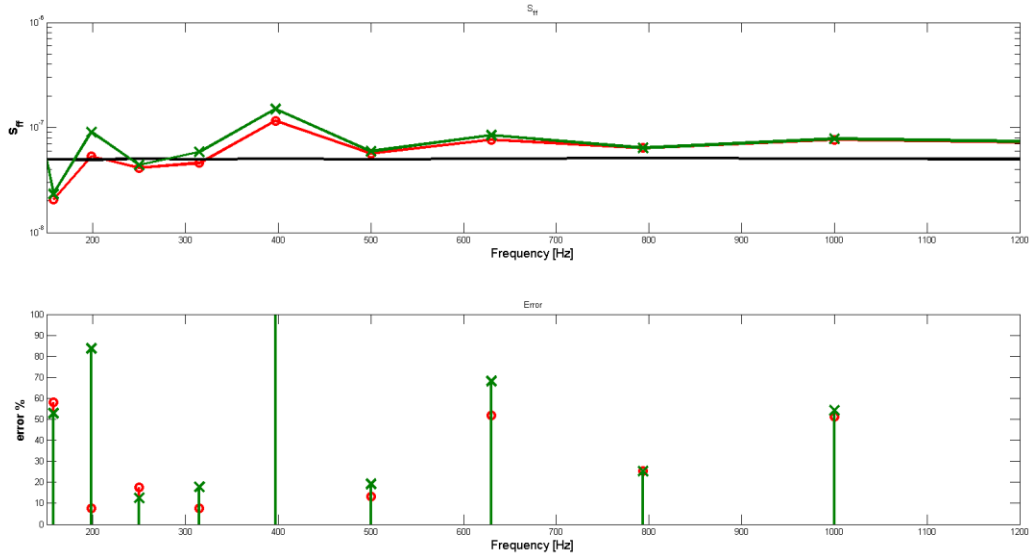

Figure 8: Comparison between force power spectral density: actual S_{ff} (-), S_{ff} identified by ILF's and CLF's computed by equation 4 (**-o-**) and equation 5(**-x-**)

As expected the identification of the power spectral density of the force is good almost over all the considered frequency range, except for the fifth band where the error is more than 100%.

4.2.3 "Rain-on-the-roof": experimental test

The last test is carried out by exciting the first plate by a *quasi* "rain-on-the-roof" load. The structure is excited by spatial uncorrelated hammer multi impulse excitations.

Figure 9 shows the comparison between the actual and identified injected power. By inverting equation 11, the power spectral density of the correspondent force is calculated and drawn in figure 10.

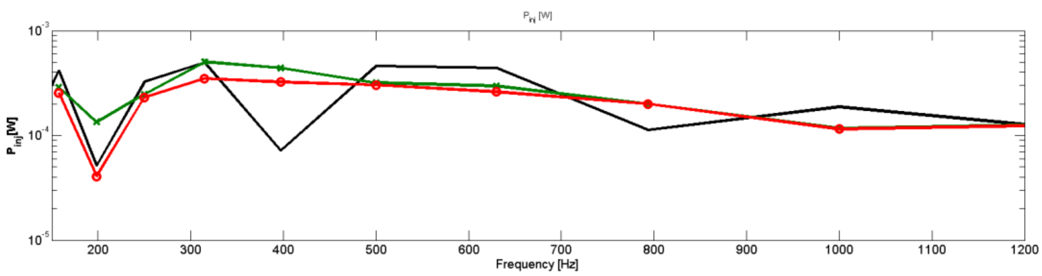

Figure 9: Comparison between injected power: actual power (-), power identified by ILF's and CLF's computed by equation 4 (**-o-**) and equation 5(**-x-**)

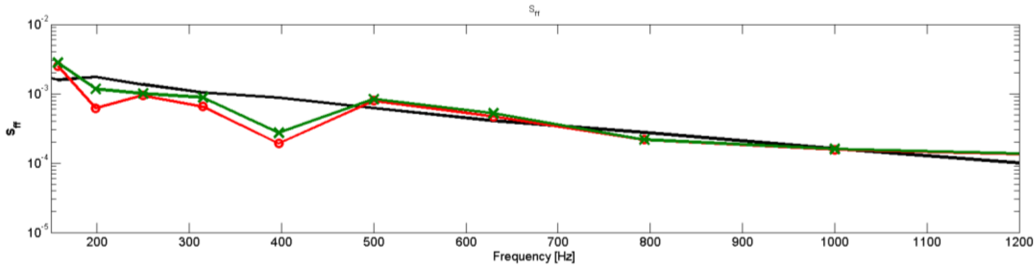

Figure 10: Comparison between force power spectral density: actual S_{ff} (-), S_{ff} identified by ILF's and CLF's computed by equation 4 (**-o-**) equation 5(**-x-**)

5 Conclusion

In this paper, an operative identification procedure based on Statistical Energy Analysis is presented.

The CLF's and the ILF's are identified by PIM and are validated by different tests. Three different tests are performed to identify the injected power by inverting SEA equation. The comparison between these identified power and actual injected power shows a good agreement.

The last two tests are carried out by exciting with a *quasi* "rain-on-the-roof" load. The agreement between the imposed actual force and the identified one is good. As expected, since a SEA model gives better results when the modal overlap factor is high, the best results are obtained at highest frequency bands.

Acknowledgements

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