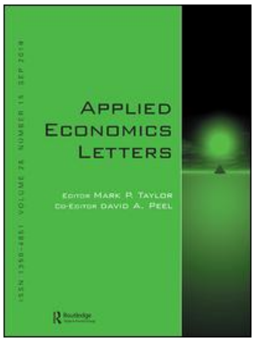

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Combining multi-criteria and directional distances to decompose non-compensatory measures of sustainable banking efficiency

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ABSTRACT
Non compensatory choices are widespread in the economics, strategic management and decision making. Nevertheless, many assessments of productivity still fail to consider non-compensatory preference structures in the measure of the technical inefficiency. This paper proposes a preference elicitation schema, typical of Multi-criteria decision analysis, for the selection of the directional vector in the assessment of a sustainable productivity. The direction choice is based on the weighted aggregation of concordance indexes for each decision criteria on each individual input, such that it represents an index of relative importance according to the decision maker's perspective. The methodology can be used to aid resource allocation and saving, identify benchmarks for efficient practices and more generally for planning environmental policies in many services and industrial organizations. We illustrate the method with an environmental efficiency evaluation of Brazilian Federal Saving Bank branches.

KEYWORDS
Directional distance functions; data envelopment analysis; multi-criteria decision analysis; banks; environmental efficiency

JEL CLASSIFICATION
C67; C44; D81; G21; Q29

I. Introduction

Since the introduction of the first linear formulations to measure the technical inefficiency and productivity of Decision Making Units (DMUs) from Charnes, Cooper, and Rhodes (1978), the so called Data Envelopment Analysis (DEA), countless models, applications and software tools have popularized the field (Daraio et al. 2019a, 2019b). The assessment of the efficiency although is subject to a kind of paradox. On the one hand, we have an objective efficiency measurement which is based on DEA, in which the analyst does not choose a direction along which to gauge the efficiency: the direction is imposed by the DEA linear programme according to which DMUs have to contract inputs (or expand outputs) in a radial proportionate way. On the other hand, we have a Directional Distance framework (Färe and Grosskopf 2004) in which the analyst is free to choose the preferred direction to expand the outputs or to reduce the inputs. The path towards the frontier and the related benchmarking are imposed by this direction. To avoid arbitrary choices, the selected direction must be justified.

This manuscript explores a combination of Directional Distance Functions (DDF) techniques with the outranking Multi-criteria (MCDA) elicitation methods to handle non-compensatory choices. The mathematical approach proposed here has the benefit to offer a detailed representation of the multi-attribute production possibilities to account for indifference, preference and veto thresholds, which may support policy makers to obtain insights on their own preferences and values. The application on the Brazilian Federal Saving Bank highlights the contribution of the proposed approach to classify sustainable units, identify sustainable practices, processes and determine the optimal input-output relationship.

II. Directional distance function

Directional Distances are a very flexible nonparametric way to gauge the performance of Decision Making Units. They allow measuring the technical efficiency of DMUs by the choice of a direction where outputs are expanded and inputs contracted to reach the efficient frontier. The efficiency scores can be estimated by solving the following linear programing model:

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$$\begin{aligned} &\mathrm{D}_t\,(\mathrm{x},\mathrm{y},\mathrm{gx},\mathrm{gy}) = max\,\beta \\ &s.t.\ \sum_{j=1}^{k} z_j y_{jr} \geq y_{or} + \beta g_{y_r} \quad r = 1, 2, \ldots, s \\ &\sum_{j=1}^{k} z_j x_{ji} \leq x_{oi} - \beta g_{x_i} \quad i = 1, 2, \ldots, m \\ &\sum_{j=1}^{k} z_j = 1, z_j \geq 0, \quad j = 1, 2, \ldots, k \end{aligned} \tag{1}$$

Where x_{ji} and y_{jr} are the inputs and outputs for each j unit, and (g_{x_i}, g_{y_r}) are the input vector $g(x_1, x_2, \ldots, x_m)$ and the output vector $g(y_1, y_2, \ldots, y_s)$ defining the direction along which the inputs must be contracted or the production expanded to reach the efficient frontier. Equation (1) represents the overall technical efficiency (see Färe, Grosskopf, and Zelenyuk 2008). The β coefficient measures the absolute technical inefficiency of each decision unit. Efficient units have $\beta = 0$ i.e. they are on the boundary of the production frontier. Note that in the input-oriented case, the first constraint in the proposed model has $g_{y_r} = 0$, providing a cost efficiency direction solely as a function of the input reductions.

The absolute measure for the technical inefficiency (β) is strongly correlated with the direction the decision maker chooses. A common direction adopted by many practitioners in the productivity and efficiency analysis literature is the unit vector (i.e. $\mathrm{g}_{(\mathrm{x},\mathrm{y})} = (1,1)$). This vector evaluates the contraction in the inputs or the expansion in the outputs equally in the same direction (Färe and Grosskopf 2004). Other feasible choices for the direction proposed in the literature include: the definition of the direction for the expansion/contraction as proportions of the mean (Fukuyama and Weber 2017); infer the direction through a data-driven way (Daraio and Simar 2014, 2016); choosing the distance function according to a reference direction $\mathrm{g}_{(\mathrm{x},\mathrm{y})} = \mathrm{g}_{(\mathrm{x}_\mathrm{R},\ \mathrm{y}_\mathrm{R})}$, where the direction is determined by the proportion of the outputs and inputs from the reference decision unit 'R' (Briec 1997); or to determine the directional vector endogenously, i.e. as a part of the linear problem (Färe, Grosskopf, and Whittaker 2013).

III. Outranking directions

Outranking methods bring outranking relations on the set of available alternatives. They consider the decision marker's preferences for each alternative instead of a single value. The most prominent outranking methods are the ELECTRE (*Elimination and Choice Translating Algorithm*), designed by Benayoun, Roy, and Sussman (1966), and the PROMETHEE (Preference Ranking Organization Method for Enrichment of Evaluations) developed by Brans, Vincke, and Mareschal (1986) and coworkers. Each method differs to the other based on the volume of information required, the decision problem and according to the decision maker's preference structure.

Recent improved developments on non-compensatory outranking methods have been proposed to evaluate job-satisfaction (Peng and Wang 2018), hotel location selection (Ji, Zhang, and Wang 2018A), Resource Allocation in Public Universities (Martins, de Almeida, and Morais 2019), problems with hesitant interval-valued fuzzy sets (Wang et al. 2017) and outsourcing relations (Ji, Zhang, and Wang 2018B; de Carvalho, Poleto, and Seixas 2018). The feasibility from these methods permits exploring pairwise comparison with the definition of concordance and discordance indices that can be applied to practical real-world decision-making problems.

Based on the outranking procedures (see Belton and Stewart 2002) we propose the following adapted model for the preference decomposition into a directional input vector of which the absolute technical inefficiency (slacks) can be derived as input contractions:

$$\begin{aligned} &\mathrm{D}_t\,(\mathrm{x},\mathrm{y},\mathrm{gx},\mathrm{gy}) = max\,\beta \\ &s.t.\ \sum_{j=1}^{k} z_j y_{jr} \geq y_{or}, \quad r = 1, 2, \ldots, s \\ &\sum_{j=1}^{k} z_j x_{ji} \leq (1 - \beta g_{x_i}) x_{oi} \quad i = 1, 2, \ldots, m \\ &\sum_{j=1}^{k} z_j = 1,\ z_j \geq 0, \quad j = 1, 2, \ldots, k \end{aligned} \tag{2}$$

where:

$$g_{(x_i)} = (i-1)^{-1} \sum_{i=1}^{m} \left(\frac{\sum_{a=1}^{A} w_a C_a(i', i)}{\sum_{a=1}^{A} w_i} \right) D_a(i', i),$$
$$\forall (i \neq i') \in I' | \exists f_a(i) : D_a(i', i)\} \tag{3}$$

And:

$$\textit{Concordance index} : C_a(i', i) = \begin{cases} 0 \; \textit{if} \; f_a(i) + q_a \geq f_a(i') \\ 1 \; \textit{if} \; f_a(i) + p_a < f_a(i') \end{cases} \text{for any } i \tag{4}$$

$$\textit{Dcordance index} : D_a(i', i) = \begin{cases} 0 \; \textit{if} \; f_a(i) + t_a < f_a(i') \\ 1 \; \textit{if} \; f_a(i) + t_a \geq f_a(i') \end{cases} \text{for any } i$$

If we consider the unit vector, the model becomes a version of Shephard's input distance function (Färe and Grosskopf 2004). The function $f_a(i')$represents the score of the input i being evaluated for the decision criteria a, compared to all the other inputs $f_a(i)$. The veto threshold t works to constraint the degree of compensation among the different inputs so that the gain in contribution from one input must not be sufficient to offset a significant lack of contribution from the other. After the aggregation procedure, the compensation among the inputs is reduced (given a small veto) or abolished (given a high veto threshold). The indifference threshold q is defined by the decision maker or can be postulated as the weighting standard deviation:

$$q_i = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} (w_i - \bar{w}_i)^2} \tag{5}$$

This is important to define a complete elicitation on the preference structure of the policy maker. The manager freely defines weights according to the degree of the global importance he/she attributes to the reduction of each i inputs according to some decision criteria in a regular production process. This procedure produces *concordance* and *discordance* matrices by the pairwise comparisons in (4), which implies the inexistence of *trade-offs* between scaling factors, leaving the decision maker free to choose any quantitative measure of any scale. Lastly, the (aggregate) concordance and discordance matrices are multiplied (see Equation (3)) to produce a value between 0 and 1 that represents the non-compensatory contraction on each individual input.

IV. Application on the sustainable banking

The flexibility of Directional Distance Functions (DDF) in the efficiency assessment provides a tangible way to include the concerns of sustainability into the evaluation of the financial institutions technical performance. As an application in the banking industry, we have collected data on 26 units of the Brazilian Federal Saving Bank (*Caixa Econômica Federal*), which is the largest state-owned bank in the Latin America. A weighting scheme that defines the preference for sustainable banking was defined by the manager from one of the decision units with the elicitation of 4 decision criteria (Cost, Environmental Impact, Availability and Accessibility to the inputs) to compare the inputs utilized to produce business transactions.

The thresholds for preference, indifference and veto (based on the profit contribution) were defined by the analyst. The inputs considered are the electricity consumption, printing services and employees. Table 1 presents the scores for each decision criteria, the information on the thresholds and the defined direction from the pairwise comparison (4) and aggregation (3).

Table 2 brings the optimal level of contraction from the slacks in the input distance function, which represents the absolute measure of technical inefficiency. The benchmark units are those with zero slacks (A, G, H, O, R, S, X, Y, and Z). They serve

Table 1. Input criteria matrix and directions.

Thresholds/Inputs	Cost	Environment Impact	Availability	Accessibility	Profit Contribution	Directions $g_{(x_i)}$
Weights	10	8	6	6	Contribution Threshold = 5	-
Indifference	20,000	0.000000001	0.3	0.3		-
Preference	35,000	0.000000002	0.5	0.5		-
Electricity (x1)	194,676.96	0.000000009	5	5	2	0.7
Printing (x2)	39,597.11	0.000000724	4	4	2	0.466,667
Employees (x3)	2,239,581.84	0.000000000	3	1	10	0

Table 2. Results of the assessments.

	Traditional Analysis				Sustainable Analysis			
Unit	Inefficiency	slack.x1	slack.x2	slack.x3	Inefficiency	slack.x1	slack.x2	slack.x3
A	0	0	0	0	0	0	0	0
B	6.32	3.29	0	3.97	6.32	9.66	31.43	0
C	7.86	9.23	77.93	0.65	7.86	81.32	240.5	0
D	11.36	0	59.3	5.84	11.36	11.53	119.98	0
E	21.9	249.37	600.24	16.93	21.9	277.41	681.07	0
F	6.72	0	129.89	2.84	6.92	0	159.59	0
G	0	0	0	0	0	0	0	0
H	0	0	0	0	0	0	0	0
I	13.48	0	50.88	10.28	13.48	1.99	69.72	0
J	13.11	6.78	0	8.51	13.11	34.66	67.83	0
K	10.55	0	162.53	5.22	10.55	30.56	246.08	0
L	14.92	36.28	126.01	10.71	14.92	57.71	184.85	0
M	11.47	0	79.62	8.31	11.48	0.18	94.64	0
N	20.17	0	252.19	17.01	20.18	0	285.86	0
O	0	0	0	0	0	0	0	0
P	5.37	0	1.09	2.81	5.37	2.58	8.53	0
Q	17.47	9.51	248.69	13.46	17.47	26.04	299.5	0
R	0	0	0	0	0	0	0	0
S	0	0	0	0	0	0	0	0
T	5.68	0	101.03	3.32	5.68	0	139.99	0
U	1.35	0	3.73	0	1.35	0	23.32	0
V	7.38	11.06	12.92	4.82	7.38	12.49	19.74	0
W	5.25	0	6.87	2.54	5.25	0	28.87	0
X	0	0	0	0	0	0	0	0
Y	0	0	0	0	0	0	0	0
Z	0	0	0	0	0	0	0	0
Sum	180.36	325.52	1912.93	117.21	180.58	546.13	2701.5	0
Environmental Impact Reduction (Kilowatt | Reams | Employee):						220,610.00	788.57	-
Environmental Impact Reduction (Litters of Water | Trees):						1,270,713,600.00	39.43	-

Table 3. Comparison among efficiency models.

Models	Electricity (x1)*	Printing (x2)**	Employees (x3)
CCR-DEA	12,916.64	1038.39	11.89
Non-Discretionary	509,111.00	2636.37	0.00
Traditional DDF	325,520.00	1912.93	117.21
Non-Compensatory DDF	546,130.00	2701.50	0.00
Saving Potential ***	(213,229,440 to 3,071,308,963.4)	(3.26 to 83.16)	-

*Kilowatt; ** Reams; *** Litters of Water (Electricity) and Trees (Printing)

as reference for the linear combination from which efficient practices might be inferred. The saving potential from the sustainable analysis is compared to the traditional DDF analysis. The slacks from x1 to x3 are defined by the number of kilowatts, paper reams and employees that can be reduced, respectively, in order to achieve a sustainable-efficient production of business transactions by each of the 26 decision units based on the predefined preference structure.

The gains to the ecosystem are translated in the last row. Especially concerning the potential water saving by the hydroelectric provision of electricity, more than one billion liters of water can be saved with the reduction on electricity consumption by the 26 bank branches during one year, instead of reducing employees in the first assessment. Some units have become slightly more inefficient than before. This is due the different targets on the frontier that different directions aim to achieve.

Lastly, in the Table 3 we compare our results with those obtained by other classical and most used methods to measure the technical inefficiency. Three models are considered in the comparison analysis: The traditional Constant Return to Scale CCR model from Charnes, Cooper, and Rhodes (1978); the Non-Discretionary DEA model from Charnes et al. (1985), considering the employees as the variable beyond the managerial control of units; and the traditional Färe and Grosskopf (2004) DDF model. Both the inputs for electricity and printing have the greater environmental impact reduction in the sustainable non-compensatory analysis. The impact in the environment in terms of water saving and trees are significant. Applying a sustainable direction in the efficiency targets for the branches can save from 213 million (compared to the Non-Discretionary method) to 3 billion (compared to the traditional CCR model) litters of water. Likewise, it has a saving potential from 3 (compared to the Non-Discretionary method) to 83 (compared to the traditional CCR model) trees.

V. Conclusion

In this paper we propose a representation of the multi-attribute production possibility which includes the decision maker's preferences and value judgments over acceptable targets. By combining Multi-criteria methods (that allows us to elicitate scores for the global importance of the resources, limiting or abolishing the compensation among the inputs) with the flexibility of Directional Distance Functions we propose a representation of the production process in the efficiency analysis considering complex trade-offs. The efficiency projection from the preference schema in this evaluation compels managers to impose narrower constraints in the usage of some (environmental-related) resources than traditional frontier assessments. In return, it allows a sustainable gain with the identification of processes, policies and actions that benchmark units adopt to minimize the environmental impact.

Disclosure statement

No potential conflict of interest was reported by the authors.

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