Treatment of and sensitivity to epistemic uncertainty in seismic risk assessment of infrastructures

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ABSTRACT: Seismic risk assessment of an infrastructure, intended as a system of systems including buildings, lifelines and critical facilities, is typically affected by several sources of uncertainty, classified as aleatory or epistemic. Momentum to this work came from the need not only to properly take into account all these uncertainties, but also to provide the confidence in the estimate and quantify the contribution of the employed models and parameters to the total uncertainty. After a brief overview about treatment of and sensitivity to epistemic uncertainty, this paper focuses on some critical issues, advocates the use of parallel models arranged in a so-called logic tree, and demonstrates the applicability of (modified) ANOVA to evaluate sensitivity of the total variance in the risk to each component of the input epistemic uncertainty, with reference to a "synthetic" city composed of buildings and a water network. Results show how the methodology can give the analyst a clear indication on which models or parameters are the most influential and thus deserve increased "knowledge" in order to reduce the total epistemic uncertainty in the problem.

INTRODUCTION

Modern societies heavily rely on their infrastructure to produce and distribute the continuous flow of essential goods and services they need (PCCIP, 1997). From a system-theoretic point of view, the infrastructure is a system of systems (SOS), a super-system including a number of spatially distributed systems (i.e., buildings, lifelines and critical facilities).

The best practice for the seismic risk assessment of an infrastructure should include not only taking into account all the relevant uncertainties affecting the problem, but also providing the confidence in the estimated output and quantifying the contribution of each input model to the total output uncertainty (i.e., sensitivity).

Uncertainty affects seismic risk to an infrastructure in the following aspects:

- Cause or hazard: Regional seismicity (event magnitude and location, local seismic intensities at vulnerable components' sites).
- Physical damage: Fragility of vulnerable components as a function of local seismic intensities (fragility functions).
- Functional consequences: Network flow analysis.
- Impact (e.g., estimation of injured, fatalities, displaced population, economic loss).

The above uncertainties can be classified as aleatory or epistemic, and both characterize the seismic input as well as the physical system. For instance, the geometry of seismic sources, their activity rate, the maximum magnitude of earthquakes they can generate, are all examples of quantities affected by epistemic uncertainty and entering into the seismic hazard evaluation, while the damage state of a component given a value of the intensity measure is an example of aleatory uncertainty affecting the system.

This paper focuses on epistemic uncertainty, that is, uncertainty in the choice of a model among different candidate models, called Type I in the following, or uncertainty in the parameters of a chosen model, denoted as Type II, or both.

Published literature to date features many works addressing the treatment of and the sensitivity to epistemic uncertainty (of the output) in seismic risk assessment (e.g., Helton and Oberkampf, 2004). However, most of these works give only a partial view of the problem, dealing with either the treatment of uncertainties (e.g., Celic and Ellingwood, 2010, and Rokneddin et al., 2015) or the computation of confidence bounds (e.g., Rubinstein and Kroese, 2016), or the sensitivity (e.g., Celarec et al., 2012). Further, the available works focus on seismic hazard only, or seismic risk assessment of structural systems.

The goals of this paper are i) to summarize relevant approaches in the current literature, discussing some critical issues, and ii) to propose and demonstrate a methodology that overcomes those issues, with reference to the seismic risk assessment of an infrastructure.

The next two sections include a brief overview about treatment of and sensitivity to epistemic uncertainty in seismic risk assessment, while Section 3 presents an application to a "synthetic" city, composed of buildings and a water network, derived from that in Franchin and Cavalieri (2015). Results are presented in terms of distribution of the mean annual frequency (MAF) of exceedance curves of displaced population due to epistemic uncertainty, and contribution of different components of epistemic uncertainty to total output uncertainty.

1. TREATMENT OF EPISTEMIC UNCERTAINTY

1.1. Possible approaches

As already said, uncertainty in the problem is partly aleatory and partly epistemic. It is useful to recall the possible approaches to the treatment of the epistemic component, which vary depending on its type:

1. Epistemic uncertainty of Type I: parallel models Θ are considered in each step of the analysis, arranged in what is often called a logic tree, and distinct simulations are run for each different combination of branches, thus yielding multiple results (e.g., MAF of exceedance curves of a performance metric). Weights, summing up to 1, are attached to branches to reflect subjective degrees of belief of the analyst in each model. This is common practice in probabilistic seismic hazard analysis (PSHA). A typical uncertainty in model form considered in PSHA is represented by the ground motion prediction equation (GMPE). The outcome is usually expressed in terms of mean hazard curve over the logic tree, obtained as a weighted average of curves from each branch (Bommer and Scherbaum, 2008). Often, upper and lower fractile curves or, alternatively, a confidence interval around the mean curve are computed based on the same data (set of curves from the tree) to quantify the effect of epistemic uncertainty on the results.
2. Epistemic uncertainty of Type II: each model parameter θ is modelled with a random variable, whose distribution describes its epistemic uncertainty.
    - a) These variables (e.g., the maximum magnitude M_{max}) can be arranged in a hierarchical model, together with aleatory uncertainty (e.g., the magnitude M). In this case the risk analysis yields a single result, incorporating the effect of both aleatory and epistemic uncertainty (e.g., Franchin and Cavalieri, 2015).
    - b) Alternatively, and with a higher associated computational effort, the risk analysis can be repeated for discrete values of each parameter θ (e.g., 16^{th}, 50^{th} and 84^{th} fractiles). This approach practically leads to arrange parameters in

a logic tree, as done for Type I uncertainty.

3. Epistemic uncertainty of both Types I and II:
   - a) One possibility is to adopt approach 1 for Type I and approach 2a for Type II uncertainties. This comparatively cheaper approach should be followed only as a way to carry out the expectation over all sources of uncertainty, presenting the results as the mean over the logic tree. It should not be used to compute confidence intervals or fractiles, because they would refer only to part of the total epistemic uncertainty.
   - b) The second approach, involving approach 1 for Type I and approach 2b for Type II uncertainties, consists in building an expanded logic tree combining both Type I and II uncertainties. Since they come from a probability distribution, both discrete values of the model parameters and their weights attached to tree branches could be assigned, for instance, according to Miller and Rice (1983).

It should be clear from the above that the best practice for the treatment of Type II epistemic uncertainty would be to adopt approaches 2b or 3b. The problem with these approaches, however, is that, in rigour, they can only be applied when all parameters are statistically independent, otherwise variation in one parameter changes the (conditional) distribution of the others and neglecting this makes the results dependent on the ordering of branches. This occurs in PSHA, for instance, with reference to the parameters of the Gutenberg-Richter recurrence relationship (a and b values, and M_{max}), for which independent sequential branches are adopted, neglecting the correlation between a and b (Bommer and Scherbaum, 2008). To the best knowledge of the authors, this issue remains unsolved, and possible solutions are proposed in Section 2.2.

1.2. Quantification of output uncertainty due to the epistemic component

In a probabilistic framework where a logic tree is used to deal with epistemic uncertainty (approaches 1, 2b and 3b), risk is estimated through a chain of modules, intended as groups of parallel choices (i.e., alternative models or model parameter values). The logic tree results in a number N of MAF of exceedance (λ) curves, denoted λ-curves in the following, for a performance metric of interest. Each λ-curve is related to one out of N simulations (e.g., Monte Carlo), each encompassing multiple runs.

Propagation of epistemic uncertainty through the logic tree results in a distribution $f_X(x)$ of the output X (either λ for a fixed performance metric value or performance metric values for a fixed value of λ). As a minimum, $f_X(x)$ can be summarized through its mean μ_X and variance σ_X^2 . Computing the mean is normally termed harvesting the logic tree in PSHA practice. Finally, variability in X is also often expressed through weighted fractiles, as an alternative to σ_X^2, and the confidence interval around the mean curve is also used (but of course this is related to σ_X^2 but not alternative to it).

For a tree with N_Θ models and N_θ parameters, μ_X can be estimated as the weighted average of X:

$$\bar{X} = \sum_{n=1}^{N} w_n \cdot x_n \Big/ \sum_{n=1}^{N} w_n = \sum_{n=1}^{N} w_n \cdot x_n \qquad \text{with:}$$
$$N = \prod_{i=1}^{N_\Theta} N_{bi} \prod_{k=1}^{N_\theta} N_{bk}; \quad w_n = \prod_{i=1}^{N_\Theta} w_{i,j(i)} \prod_{k=1}^{N_\theta} w_{k,m(k)} \quad (1)$$
$$x_n = x\left(\Theta_{i,j(i)} \forall i = 1,\ldots,N_\Theta, \theta_{k,m(k)} \forall k = 1,\ldots,N_\theta\right)$$

In (1) N_{bi} and N_{bk} are the numbers of choices for the i-th model and the k-th parameter, respectively, x_n is the MAF or performance metric value corresponding to the n-th branch of the tree, formed from the sequence $j(1), \ldots, j(N_\Theta)$ of models and $m(1), \ldots, m(N_\theta)$ of parameter values, while w_n is the associated weight. It is noted that the final weights w_n, as well as all other weights within each module, are "reliability weights", as opposed to "frequency weights", i.e. they sum up to unity:

$$\sum_{n=1}^{N} w_n = 1; \quad \sum_{j(i)=1}^{N_{bi}} w_{j(i)} = 1; \quad \sum_{m(k)=1}^{N_{bk}} w_{m(k)} = 1 \qquad (2)$$

The variance σ_X^2 can be estimated through the weighted sample variance:

$$s_X^2 = \sum_{n=1}^{N} w_n \left(x_n - \bar{X}\right)^2 \Big/ \left(1 - \sum_{n=1}^{N} w_n^2\right) \qquad (3)$$

which is the unbiased estimator when weights sum up to unity as in (2).

The weighted p-th fractile (with p being a percentage) is defined as the element x_p of X satisfying the condition $\sum_{n=1}^{N} I \cdot w_n = p/100$, where I is equal to 1 if x_n is lower than or equal to x_p, and 0 otherwise. Finally, the confidence interval around the estimate $\bar{X}$ of μ_X, following, e.g., Rubinstein and Kroese (2016), can be obtained recalling that the estimator $\bar{X}$ has approximately a normal distribution, $\mathcal{N}(\mu_X,\sigma_X^2/N)$, where N is the samples size. When σ_X^2 is estimated as in (3), the confidence interval at the confidence level (1-α)100% (e.g., $\alpha = 0.05$ yields a confidence level of 95%), is:

$$\left[\bar{X} - t_{N-1,1-\alpha/2} \frac{s_X}{\sqrt{N}}, \bar{X} + t_{N-1,1-\alpha/2} \frac{s_X}{\sqrt{N}}\right] \qquad (4)$$

where $t_{N-1,1-\alpha/2}$ is the critical value for the Student t distribution with (N-1) degrees of freedom.

2. SENSITIVITY OF OUTPUT UNCERTAINTY TO INPUT EPISTEMIC UNCERTAINTY

2.1. Sensitivity to modules

In order to assess which component of (input) epistemic uncertainty contributes more to the uncertainty in the output, sensitivity analysis should be carried out. Some authors, e.g., Celic and Ellingwood (2010) and Celarec et al. (2012), with reference to seismic risk assessment of structures, rather than infrastructures, perform sensitivity of selected seismic response parameters (drifts, forces, etc.) to the input random variables, which include also Type II epistemic uncertainty. In these works output is commonly computed first with all input variables set to their medians, then, setting one input variable at a time to a lower or upper fractile (typically the 16th and 84th), to represent the resulting variations with tornado diagrams. This procedure can be extended to frameworks including a logic tree. For each choice, the weighted average $\bar{X}$ is computed considering only the logic tree branches involving that choice. The sensitivity of epistemic uncertainty to logic tree choices can be still presented through tornado diagrams (see Figure 3b).

Herein it is proposed to perform sensitivity analysis by means of the analysis of variance (ANOVA). ANOVA allows to test the assumption that a sample is divided into groups, by expressing the total variance in the sample, s_X^2, as the sum of a variance "within" each group, $s_{X,W}^2$, and a variance "between" groups, $s_{X,B}^2$. If the ratio $s_{X,B}^2 / s_X^2$ is large, it means that the difference between groups is large. If each module in the tree is taken in turn as a criterion for grouping, and the set of results x_n is divided into N_G sub-groups each corresponding to a different choice in the module, the ratio $s_{X,B}^2 / s_X^2$ can be used to rank modules. The classical ANOVA expression is derived for a sample where all realizations have the same weight and reflect the underlying distribution, which is assumed to be normal. It can be shown that, if the sample is a weighted one as in the case at hand, the following relation holds:

$$s_X^2 \left(1 - \sum_{n=1}^{N} w_n^2\right) = \sum_{n=1}^{N} w_n \left(x_n - \bar{X}\right)^2 =$$
$$\sum_{g=1}^{N_G} \sum_{j=1}^{n_g} w_{gj} \left(x_{gj} - \bar{X}_g\right)^2 + \sum_{g=1}^{N_G} \sum_{j=1}^{n_g} w_{gj} \left(\bar{X}_g - \bar{X}\right)^2 = \qquad (5)$$
$$\left(s_{X,W}^2 + s_{X,B}^2\right)\left(1 - \sum_{n=1}^{N} w_n^2\right)$$

where x_{gj} and w_{gj} are the output value and the associated weight corresponding to the j-th branch within the n_g branches in the g-th group (g-th option in the considered module), and $\bar{X}_g$ is the weighted average within the g-th group. Note that weights within a group do not sum up to unity.

2.2. Case of correlated parameter values

As pointed out above, in the presence of correlation between parameters in logic trees (approaches 2b and 3b), sequential modules can still be used, but care should be taken in conditioning the values of the subsequent parameter to those of the preceding one, in order for the results to be independent on the ordering of modules. One possibility is to change the weights, obtaining them from the distribution of each parameter conditional on the value of the correlated parameter that precedes it. This approach may not work in cases where there are other constraints involved. One such case that arises in the context of seismic risk analysis is related to the use of fragility functions, which are commonly postulated as lognormal and are thus defined by two parameters, the log-mean and the log-standard deviation. Epistemic uncertainty on these parameters is sometimes available. The two parameters are obviously correlated. As long as the component state is defined as binary, i.e. a single fragility function is employed, to determine whether the component is intact or damaged, the modification of the weights can be used. The problem arises when the component is not treated as binary but multiple states of damage are considered. In this case two or more fragilities are used. These fragility functions must be such that they never intersect.

To overcome this issue, herein it is proposed to avoid the use of sequential modules corresponding to correlated parameters, but instead to lump them all together into the same module, accounting for their dependence within the module itself. Needed input data are the marginal distributions (e.g., normal with mean μ and standard deviation σ) of the fragility curve parameters and their correlation matrix. One parameter is fixed to a certain fractile and the same fractile of the remaining parameters is obtained using the conditional mean and standard deviation. For the formulation to be exhaustive, all combinations of conditioning and conditioned parameters are included. Indicating with ρ the correlation coefficient, with x_1 the conditioning parameter (fixed at a fractile) and with $\mathbf{x}_2$ the vector of the conditioned parameters, it is possible to write:

$$\mathbf{D}_{\mathbf{xx}} = \begin{bmatrix} \sigma_{x_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{x_n} \end{bmatrix}; \ \mathbf{R}_{\mathbf{xx}} = \begin{bmatrix} 1 & \cdots & \rho_{x_1 x_n} \\ \vdots & \ddots & \vdots \\ \rho_{x_n x_1} & \cdots & 1 \end{bmatrix} \quad (6)$$
$$\mathbf{C}_{\mathbf{xx}} = \begin{bmatrix} \sigma^2_{x_1} & \mathbf{C}_{x_1\mathbf{x}_2} \\ \mathbf{C}_{\mathbf{x}_2 x_1} & \mathbf{C}_{\mathbf{x}_2\mathbf{x}_2} \end{bmatrix} = \mathbf{D}_{\mathbf{xx}}\mathbf{R}_{\mathbf{xx}}\mathbf{D}_{\mathbf{xx}}$$

The conditional mean vector and covariance matrix are obtained as:

$$\begin{aligned} \mu_{\mathbf{x}_2|x_1} &= \mu_{\mathbf{x}_2} + \mathbf{C}_{\mathbf{x}_2 x_1}\left(x_1 - \mu_{x_1}\right)/\sigma^2_{x_1} \\ \mathbf{C}_{\mathbf{x}_2\mathbf{x}_2|x_1} &= \mathbf{C}_{\mathbf{x}_2\mathbf{x}_2} - \left(\mathbf{C}_{\mathbf{x}_2 x_1}\mathbf{C}_{x_1\mathbf{x}_2}\right)/\sigma^2_{x_1} \end{aligned} \quad (7)$$

The conditional standard deviations are simply the square root of the diagonal elements of $\mathbf{C}_{\mathbf{x}_2\mathbf{x}_2|x_1}$. Starting from the conditional means and standard deviations one can retrieve any fractile of the conditioned parameters.

3. APPLICATION

For demonstration purposes, a seismic risk assessment was carried out on the "synthetic" city shown in Figure 1, simplified from that in Franchin and Cavalieri (2015). The OOFIMS software for quantitative probabilistic seismic risk analysis, namely Object-Oriented Framework for Infrastructure Modeling and Simulation, developed (Franchin and Cavalieri, n.d.) within SYNER-G (2012), was used for the computations.

Figure 1 shows that the city (infrastructure) is composed of two layers: buildings (BDG) with

15 by 15 km square footprint, and a water supply system (WSS), whose size, properties, and topology mimic those of real systems. Five seismogenic areas affect the city.

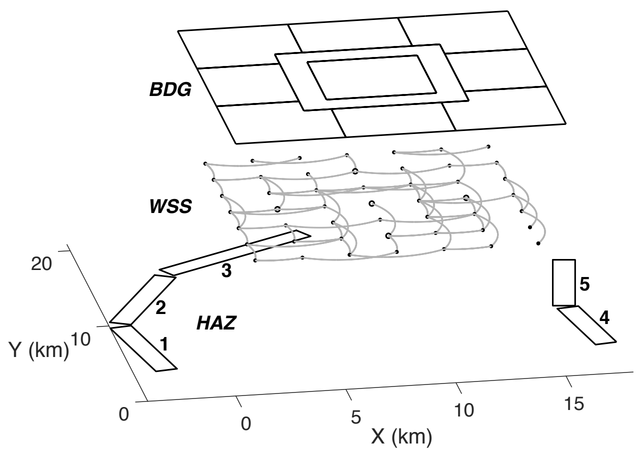

Figure 1: The synthetic city and seismic environment. Black dots and empty circles in the WSS layer denote demand nodes and sources, respectively.

For simplicity, only reinforced concrete (RC) buildings are present. A set of two lognormal fragility curves is provided, whose parameters are themselves characterized as joint normal variables (epistemic uncertainty of Type II), with a 4x4 correlation matrix. Further details can be found in Franchin and Cavalieri (2015).

3.1. Modules, models and parameters

A logic tree according to approach 3b in Section 1.1 is used to estimate the effect of epistemic uncertainty on the seismic risk analysis results, and its sensitivity to models and parameters. The four employed modules, containing different choices of models or parameter values, concern (weights are indicated in brackets):

1. M_{max} for all the seismic sources

   a) 6.5 [0.4]; b) 7.0 [0.6]

2. GMPE

   a) Akkar and Bommer (2010) [0.7]
   b) Boore and Atkinson (2008) [0.3]

3. Fractiles of $\varepsilon_{\mathrm{RR}}$ in WSS fragility function

   a) 8.5^{th} [0.25]; b) 50^{th} [0.5]; c) 91.5^{th} [0.25]

4. Fractiles of parameters of lognormal fragility curves for RC buildings

   a) 8.5^{th} [0.25]; b) 50^{th} [0.5]; c) 91.5^{th} [0.25]

The fractile values and corresponding weights in modules #3 and #4 were set according to the work by Miller and Rice (1983). A Gaussian quadrature procedure and a selected weighting function are employed to approximate a continuous cumulative distribution with a user-defined number of pairs of random variable values and cumulative probability. Each value is paired with a probability, so to obtain the probability mass function of the discretized random variable. Such probabilities are used as branch weights in a logic tree module containing several parameter values. For the case at hand, the parameter distributions in modules #3 and #4 were discretized with three points, namely the 8.5^{th}, 50^{th} and 91.5^{th} fractiles with weights 0.25, 0.5 and 0.25, respectively.

As already pointed out, the four parameters of the two lognormal fragility curves for RC buildings are correlated. Following the approach proposed in Section 2.2 to take into account parameter correlation, the RC fragility module (#4 above) in the logic tree includes four branches for each fractile value. When the 50^{th} fractile is considered, all branches provide the same set of fragility curves (i.e., the mean curves), so that only one branch has to be considered in this case. To summarize, the proposed formulation requires four combinations for both 8.5^{th} and 91.5^{th} fractiles and one combination for the 50^{th} fractile, for a total of nine branches to include in module #4, thus resulting to be computationally affordable.

3.2. Results

The performance metrics of interest are the System Serviceability Index (SSI) of the water network and the displaced population (P_d). SSI is defined as the ratio of the sum of delivered flows in the post-earthquake damaged conditions to the sum of the water demands. In the implemented model, population can be displaced from their homes either because of direct physical damage (building usability) or because of lack of basic services/utilities (building habitability, function of utility loss and hence SSI) resulting from damage to interdependent utility systems (only water system in this case). Details can be found in Franchin and Cavalieri (2015).

The employed logic tree required a total of 2×2×3×9=108 Monte Carlo simulations with 1,000 runs each, which yield stable estimates of the considered performance metrics.

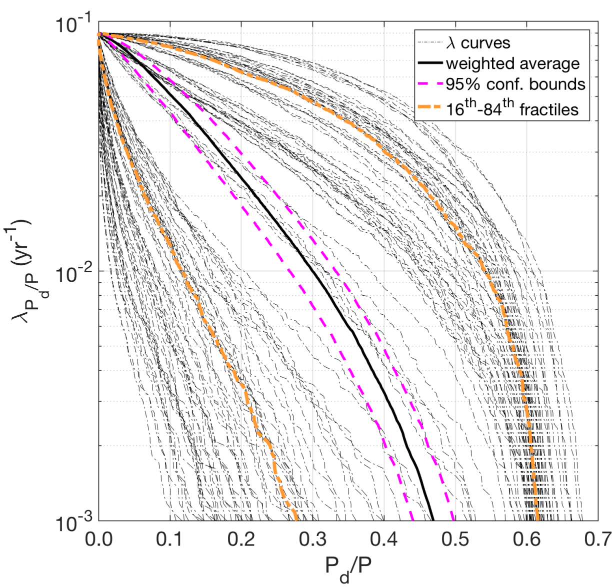

Figure 2: MAF curves of P_d/P, with weighted average curve, 95% confidence bounds and lower and upper fractile curves.

Figure 2 shows the MAF curves of the displaced population normalized to the total population, P_d/P, as obtained from the sequence of 108 simulations. A single curve, corresponding to an individual branch of the logic tree, quantifies the aleatory uncertainty contained in the employed models and parameters, while the spread of the curves around the average gives a clear indication of the epistemic uncertainty. Therefore, the distribution of P_d/P corresponding to the full suite of curves captures both aleatory and epistemic uncertainties (Bommer and Scherbaum, 2008). Together with the weighted average curve of P_d/P (for fixed MAF values), both the 95% confidence bounds and 16th and 84th weighted fractile curves are also reported, since they represent two different aspects, as already said. In particular, the confidence in the estimate increases (and thus the confidence interval is reduced) with the number of simulations, N, while the weighted fractiles are only function of the sampled values.

Figure 3(a) presents the module importance ranking obtained from ANOVA, for return periods of 100 and 500 years. The two displayed bars for each module indicate the total variance for the two return periods, while the hatched portion inside the bars indicates the variance between groups and thus the importance of the module in terms of contribution to total epistemic uncertainty. The building fragility model has the highest importance, while the contribution of M_{max} is negligible. This plot gives the analyst a clear indication of which modules are the most influential and thus deserve increased "knowledge" (enhanced modelling, more data for calibration, etc.), in order to reduce the total epistemic uncertainty in the problem.

Through the tornado diagram displayed in Figure 3(b), the sensitivity of the weighted average of P_d/P to the different choices in the considered modules is presented, for the same return periods used for ANOVA. The plot partly reflects the information gained from Figure 3(a), namely the importance ranking of modules, since the ones characterized by larger bounds between the extreme choices clearly give higher contribution to uncertainty. Further insight from Figure 3(b) concerns the only module containing models (i.e., #2 above): to yield higher P_d/P weighted average, the Akkar and Bommer (2010) GMPE must provide higher shaking intensities, compared to the Boore and Atkinson (2008) model. Finally, for all three modules containing parameter values, the sensitivity results are consistent with the input settings, in the sense that higher magnitude, upper fractile for $\boldsymbol{\varepsilon}_{RR}$ and lower fractile for building fragility parameters yield higher values of displaced population, and vice versa, as expected.

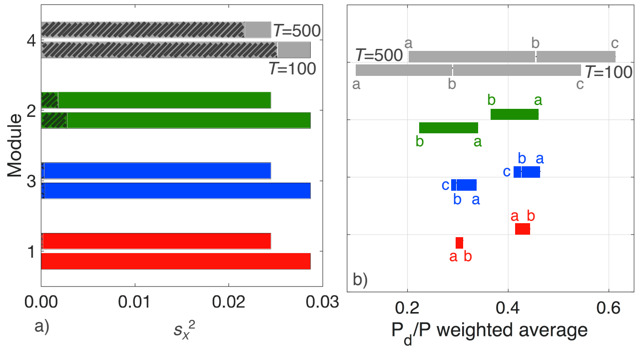

Figure 3: Ranking of modules (importance in contributing to the total output epistemic uncertainty) according to the proposed ANOVA method (a) and based on tornado diagrams (b).

4. CONCLUSIONS

The paper discusses treatment of and sensitivity to epistemic uncertainty within the context of seismic risk assessment of infrastructures. The latter include a number of spatially distributed systems, namely buildings, lifelines and critical facilities. The use of a logic tree approach is advocated. Such a methodology includes the treatment of the relevant uncertainties (both aleatory and epistemic) in the problem, as well as the sensitivity of uncertainty in the output to the components of epistemic uncertainty in the input. The latter is carried out by means of ANOVA. The formulation is general, and thanks to the proposed solution for handling parameter correlation, it can be applied to a range of risk assessment problems for structural and infrastructural systems.

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