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Optimization of profits in one-way free-floating car-sharing services, with a user-based relocation strategy that apply dynamic pricing and urban area demand defined gathering real vehicle-sensor data.

Eng. Yuri Maria Chianese

Candidate PhD (cycle XXIX) in Industrial Production Engineering.

Department of Mechanical and Aerospace Engineering

University of Rome "La Sapienza", Italy

Tutor: Prof. Laura Palagi

Department of Computer, Control and Management Engineering "Antonio Ruberti", University of Rome "La Sapienza", Rome, Italy, laura.palagi@uniroma1.it

Co-tutor: Prof. Alessandro Avenali

Department of Computer, Control and Management Engineering "Antonio Ruberti", University of Rome "La Sapienza", Rome, Italy, alessandro.avenali@uniroma1.it

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IV. Notes to the document

This paragraph contains notes to the document that should allow a better comprehension of document organization and notations.

a. References of Papers and other scientific sources: references are listed at the end of the document and are recalled in the document with number at the apex of the recalled concept or citation,

b. Footnotes: footnotes are inserted at the end of the page as roman number at the apex of the recalled text,

1. Abstract

Rapid growing in urbanization and miles driven in the city will triple urban mobility by 2050. This explosion in demand requires switching to Mobility-as-a-Service (MaaS) models, such as Car-sharing. However, a critical issue for Car-sharing one-way free-floating services is the imbalance problem that requires to solve the conflict between the positioning of vehicles "at the right place and time" and the freedom for customers to return vehicles where and when they want.

To better understand the impact of the imbalance problem, we propose to use a grid partition of the served city into zones with different demand potentials. To this aim as first step of the research real data related to vehicle positions of three Car-sharing services have been collected for approximately three months in the cities of Rome, Milan, Turin and Florence (Italy). In the experimental results data of the city of Rome have been used.

This part of the research focuses on analysing user behaviour by using the number of stops in selected city zones (Stop Density) and the duration of any stop (Average Stop Duration); in fact, all the stops of each vehicle belonging to any car-sharing operator, are uniquely associated and mapped to exactly one cell of the city grid representing the Urban Areas, also tracking stop start/end time and trip start/end time. This spatial association is used to calculate Stop Density and Average Stop Duration of each urban area and to map stops to specific time-slots.

Consequently, in each urban area, the Urban Area Value is calculated as a function of Stop Density and Average Stop Duration belonging to the urban area; the results of this research confirm that Urban Area Value is high where high values of Stop Density and low value of Average Stop Duration occurs. Urban Areas are ranked using the Urban Area Value calculated by considering all Car-sharing services operating in the eco-system; a spatial analysis with a thermographic map of Urban Area Value allows to visualize the existence of city zones with crucial different demand potentials.

The analysis derived from such Urban Area Value and from a time-slot dynamic of the Urban Areas Values themselves, that suggested to split the standard operating day in five hourly ranges, is then used to construct a flexible and dynamic pricing mathematical programming model that has been used to derive an optimal setting of tariffs and to perform a validation phase.

In this model the trip fare is defined, based on a trip planning trigger, applying a bonus/malus mechanism to a basic tariff, which considers *vehicle service cost, staff relocation saving* and the *difference of demand value between origin and destination Urban Areas*. If the user desired destination is planned in an urban area which is adjoining urban areas with higher values, alternatives with lower fees are proposed. This approach is applicable, in the reality, to several Car-sharing operators and mobility-sharing aggregators such as Urbi[i].

The model and the outcomes of Urban Area Values have been validated in a study based on real data collected in the city of Rome (Italy) during an observation period of 49 days from April 28th to June 16th, in 2016, and where 287.975 stops observation referring to 1.271 distinct vehicles have been collected. All the stops have been observed in the city of Rome whose grid representation has been partitioned in 636 cells. These results have been presented to the 2017 COMPSAC Conference, July 7th, 2017 in the Workshop "Smart Sharing Mobility in Smart Cities" [1]. These data have been used to

[i] Urbi (https://www.urbi.co/) è una mobile app che aggrega la mobilità urbana e permette di trovare e prenotare la miglior soluzione per raggiungere una determinata destinazione attraverso car/scooter/bike sharing, taxi e ride sharing e trasporto pubblico.

construct an integer linear programming model where only a grid of 25 cells has been considered over the same period of 49 days. The resulting model (which has 84.500 variables and 87.750 constraints) has been solved using AMPL/CPLEX and validated by simulating a trip demand over an observed period. The result of this pricing scheme seems to produce interesting results with a business applicability in urban car–sharing market.

The thesis is organized as follows.
Chapter 1 is focused on the analysis of main challenges of urban mobility, and the role that car-sharing systems can play. *Chapters 2, 3, 4* are devoted to the introduction and a systematic review of the literature. In *Chapter 5* the data collection and cleaning are described and the final Data set is presented. *Chapter 6* includes the grid partition of a city and the procedure to evaluate the Urban Area Value. *Chapter 7* presents a review of the up-to-date pricing models for Car sharing that are used for defining some parameters in the optimization model presented in *Chapter 8*. Finally, in *Chapter 9* the results obtained on the available Data set for the city of Rome are presented.

Keywords: profit optimization, one-way free-floating car-sharing, user-based relocation strategy, dynamic pricing, urban area value, vehicle-sensor data.

2. Importance of sharing-mobility

2.1. Future of Urban Mobility

The Globalization process and the study of global phenomena have long shown that the world's population is becoming increasingly concentrated in cities; in 2014, a UN study showed that 53% of the population lives in urban areas; this percentage is expected to reach 60% by 2030, with urban areas growing at a rate of 1.3 million people every week[2], and 66% in 2050.

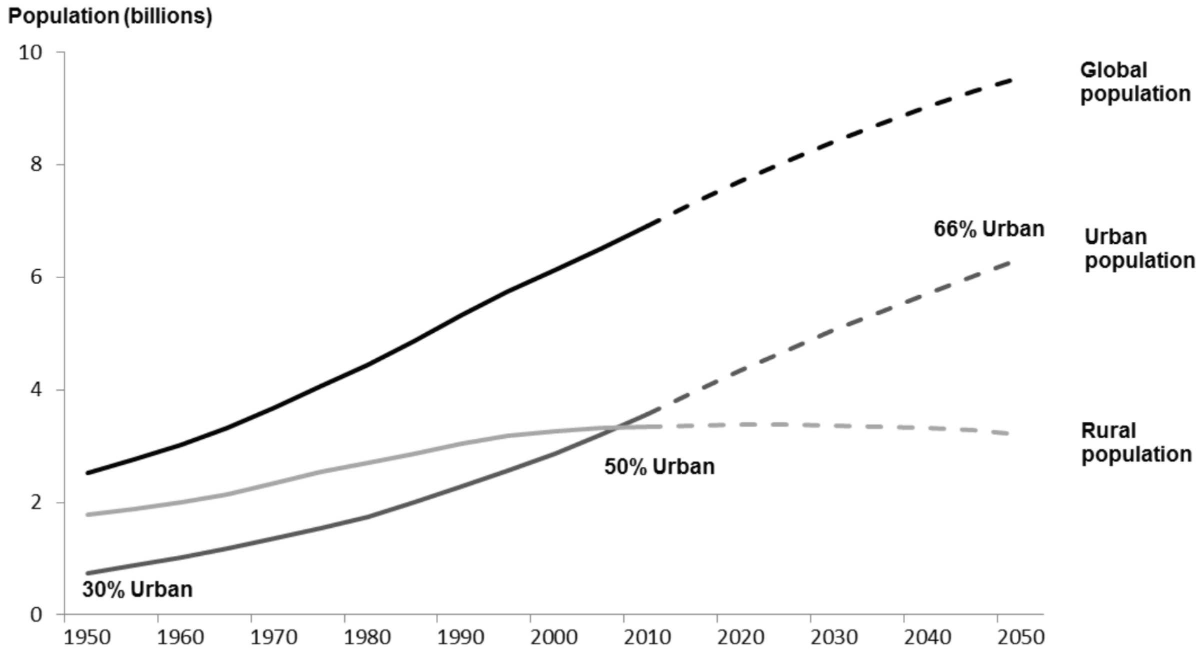

Figure 1 - Global urban and rural population, 1950-2050[3]

From the movement point of view, a recent study has calculated that 64% of miles driven today, covers trips made within the city, and it is estimated that the number of miles driven in the city will triple by 2050[4], growing to 105 billion kilometres globally[5], forcing every citizen to spend in traffic about 106 hours/year (twice today).

This phenomenon will be even more important in European cities, where 74% of the population lives and works in the city, with an increase of the concentration of population in urban areas in 2050 to 82%.

Furthermore, the current trend of urban mobility indicates, globally, a growth of private transport, with an estimated over 6.2 billion daily trips made by private vehicles.

This explosion in demand for urban mobility will be difficult to sustain without a profound change in habits and infrastructure.

For example, it is estimated that by 2025, the urban transport systems will be responsible for an increase of 30% of greenhouse gas emissions compared to 2005.

Particularly in Europe, the urban expansion and the strong dependence on the transport of cars and trucks are carrying more and more congestion in urban areas.

The inefficiency of private travel and the resulting congestion will lead cities in the world to a "dead end" in terms of mobility and urban planning, resulting in increased complexity of shift, rendering inefficient public transport system, for sustainability of citizens (simple mobility, environmentally friendly, reasonable cost, improved safety and lower stress levels).

Such a scenario will require a total rethink of models and systems of urban mobility worldwide with a special focus in Europe.

The priority is therefore to create urban transport systems capable of satisfying the requirements of mobility and social and economic sustainability to ensure the people and goods movements in safety.

One of the most logical answer to this requirement is to enhance and develop existing mobility infrastructure and networks to achieve an integrated mobility platform, which can provide new tools and services to the public and, especially, to provide the tools for strategic planning of urban mobility that can allow involved operators to optimize the distribution and allocation of resources.

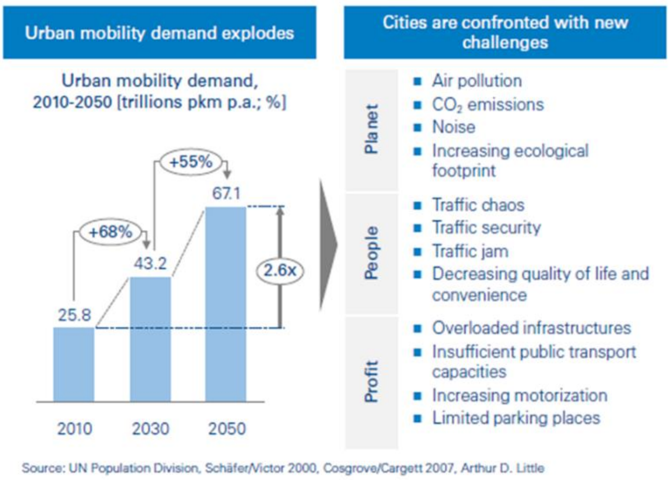

Figure 2 - urban trends in future cities

In fact, doubling the public transport market share Worldwide by 2025, cities will be able to stimulate growth, to help combat climate change and create urban environments more liveable and comfortable.

Doubling the market share of public transport will also allow the stabilization of greenhouse gas emissions in urban transport and energy consumption despite the overall increase in mobility.

In addition to the increasing demand for urban mobility, the mobility requirements themselves are evolving.

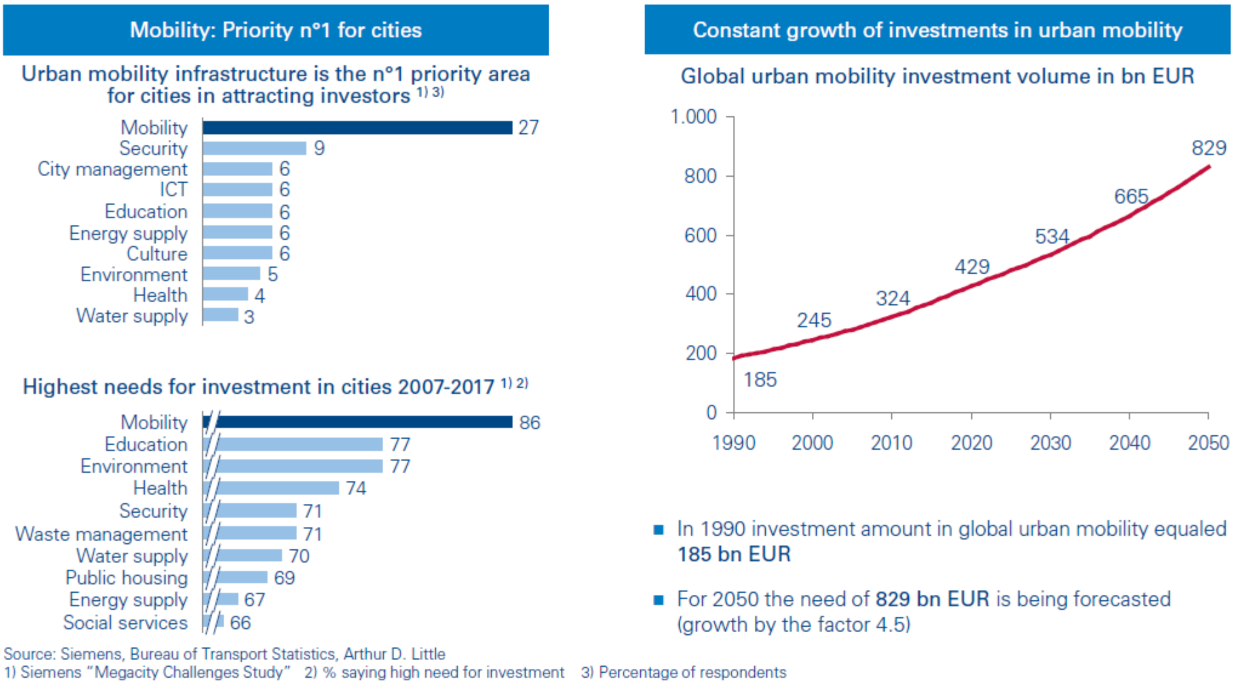

Figure 3 - priority of investing in effective smart mobility models

Changing demand for urban mobility services to meet the needs of citizens - that evolve from one side to the increased convenience, speed and predictability of journey times, the other towards greater personalization, economic and environmental sustainability - will require a significant expansion and evolution of the portfolio of services, with massive investments, strategic and structural, in the very near future, making Mobility a priority for infrastructure development and investments[6].

The challenge then is to find strategies to provide citizens with effective alternatives to private cars that are able to motivate the citizens themselves to adopt more sustainable mobility alternatives. According to research conducted by Arthur D. Little[4], the European systems of urban mobility are now the most mature in terms of mobility performance.

In fact, despite the presence of effective solutions and business models available in theory, few of them have been able to integrate all mobility platforms and express the full potential of the business model.

For this to happen it is necessary in addition to a strategic vision for the ecosystem of urban mobility, collaboration among all actors to pursue innovative business models and integrated offer.

Considering the cities of Western Europe, the strategic imperatives for success in this initiative are:

- **Rethinking the System**: cities in mature economies are characterized by a high percentage of use of private transportation modes, with ineffective use of space (one individual per car) or time (two way travel origin to destination per day); this behaviour will require to **radically redesign last mile urban mobility system** to improve use of public modes instead of private and improve sharing of resources to increase number of passenger per trip and minimize number of vehicles per user to guarantee a better economic and environmental sustainability.
- **Integrate actual Transportation Modes and Networks**: another characteristic of mature cities is the presence large but not integrated public transportation networks; evolution of urban mobility will require to integrate, end to end, the transportation value chain to enable a

multimodal and interoperable city transportation platform, also known as intermodal mobility, to increase attractiveness and use of public transportation systems to citizens (and city users).

The pursuit of these challenges will request Public Administration, City Institutions and Transportation Operators to cooperate at a strategic planning level, using both levers of valorisation of current infrastructures and analysis of insights available from the huge volume of data and information available about mobility, traffic, citizens' behaviours.

To respond to citizens' needs with optimized allocation of transportation modes and hubs is necessary to:

1. analyse and profile urban mobility demand, including mobility behaviours and needs per transport mean, frequency of use, origin to destination trips, trip time and duration, etc.
2. monitor the urban mobility offer through public transportation networks (means, city areas, frequency, capacity by origin-destination etc., …)
3. evaluate action to maximize the optimal distribution of urban transportation capacity within defined goals and constraints.

Thanks to the actual diffusion of data services in people communication data (P2P), process generated data (M2P and P2M) and machine generated data (M2M) citizen access, more and more, with connected mobile devices to digital mobility services.

This explosion in demand for urban mobility will be unsustainable without a profound change in habits and infrastructures, and such a scenario will require a total rethink of models and systems of urban mobility, and of parking lot occupation, globally and particularly in Europe.

An opportunity to rethink the mobility model is represented by Car-sharing services which, allowing more drivers to share the same vehicles during the day, will reduce the number of circulating vehicles, increase the number of daily users per vehicle, reduce traffic and pollution, and increase parking availability, so that citizens' time, stress and costs of urban trips are strongly reduced.

2.2. *Mobility as a Service and Car-sharing*

Car-sharing Market has reached worldwide in 2015[7] more than 86.000 vehicles with over 5.8 million users, 2.5 million of minutes booked and 0.65 billion revenues. Only in Europe Car Sharing services reached 31.000 vehicles and 2.1 million users, covering more than 36% of the vehicles and users of the global market. Additionally, the market is growingly very fast (CAGR of 30%) and expected revenues are 3.7-5.6 Billion Euro in 2020[8].

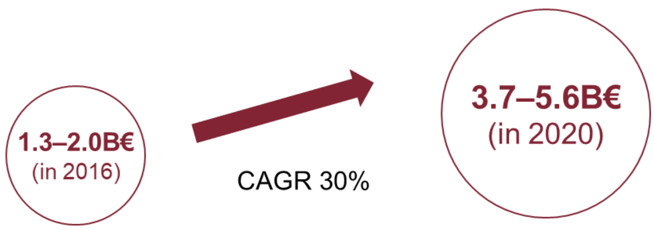

Figure 4 - Car-sharing global trends

2.3. Car-sharing sustainability contribution

The growing penetration of Car-sharing in urban mobility can help to improve environmental and social sustainability. As research studies demonstrate, Car-sharing can generate a significantly positive sustainability impact in:

- **Saving gas and oil consumption** by **reducing pollution and CO_2 emissions** and the **number of kilometres driven by in private trips**. In fact, according to a recent study from University of California, Berkeley, 1 Car-sharing Vehicle can replace from 9 to 13 private vehicles reducing Green-house Gas (GHG) emissions by 34%-41% per household[9] and the Vehicle Miles Travelled (VMT) by 27%-43% per year.

- **Decreasing the need of parking areas**[10] with 36-84 sqm of public spaces freed-up per vehicle.

3. State of the art

3.1. Definition of Car-sharing

Car-sharing is a membership based globally diffused car rental model where people rent cars, for short periods, often by the minute, from Car-sharing companies promoting the service, who generally own a fleet of vehicles that make available to people for sharing[11]. Membership subscription give the user access to the rental service and a usage fee is paid *for time and distance travelled, which already embeds fuel and insurance*. Membership enable users to have immediate access to the available vehicles whenever they want, without needing to fill a rental agreement each time a car is used[12].

Generally, Car-sharing operational models are based on specific characteristics of *location of vehicles and allowed movements*.

Considering pick-up and return of vehicles, the model can be "station based" or "free floating". The "station based" model requires vehicles to be picked-up and returned in predefined locations (stations); the "free floating" model allow the driver to pick-up the vehicle where it is available and return it wherever he wants, respecting a pre-defined service area. Station-based car-sharing services requires that vehicle booking has to be completed before usage. *Free floating system vehicles have on-board GPS equipment to ease management and allow users to locate them by using a smartphone*[13] and allow users to book the vehicle in real-time.

Considering *allowed movements* the model can be generally defined "round trip" or "one way". "Round Trip" models require the users to return the vehicle at the same location of pick-up.

"One Way" models, on the other hand, allow the user to return the vehicle to a different destination from pick-up location, which means that there is no imposition to return the car to any place[14], giving the user a high flexibility in optimizing his movements, since the only constraint is to return the vehicle with legal public access, inside the service area.

In the following thesis two different operating models are recalled: the **station-based model**, which includes both round-trip and one-way movements, and the **free-floating model**, which considers only one-way movements, being round trip an exception to the standard free-floating movement.

The round-trip service model is very simple, requires few staff and can be easily performed with a small number of vehicles; on the other hand, the flexibility for users to adapt the service to their needs, especially if compared with private car usage, is very limited.

The one-way station-based model is a hybrid solution between round trip and private car usage, since it gives the user as many options to trip flexibility as many stations are used to manage the service. In fact, one-way movements give more flexibility to users, being a critical factor to attract new clients to the system[15]. Additionally, it lets a higher utilization of vehicles as they do not need to be idle during the rental period as it happens when clients are forced to a roundtrip.

Between above described business models, the free-floating Car-sharing is the service that can better replace the use of private cars in urban areas. This model is the most flexible and in theory maximizes the opportunity using the vehicle multiple times, between one rental and the other.

3.2. Car-sharing experiences in and extra Europe

Car-sharing systems can be classified, considering their business model in 3 different generations[16], characterized by different level of maturity and features.

3.2.1. *First Generation of Car-sharing systems*

Car-sharing origin can be linked to the *shared use vehicle systems is in 1948, in Zurich, which has been performed by a cooperative called "Sefage"*.

Others shared vehicle models raised in early 1970s and in the 1980s, but all of them were unsuccessful. Their business model required members to return the vehicle at pick-up location (Round Trip), book the vehicle in advance, specifying, while booking, drop-off time (fixed-period reservation). This model generated problem when for unpredictable events, such as incidents or traffic jam the return time could not be respected.

In general, first generation model of car-sharing systems can be seen as tradition Car-rental systems with a higher degree of flexibility consisting in the possibility to book the vehicle per hour instead of per day.

One of most successful initiatives worldwide is ZipCar, founded in 2012. In Italy, most known initiative is the ICS (Car-Sharing Initiative).

ICS[17] is a Convention of Municipalities and other Local Authorities, supported and financed by the Ministry for the Environment and Protection of the Territory and the Sea, whose mission is to promote and support the diffusion of car sharing as a tool for sustainable urban mobility.

The aim is to contribute in designing, in Italian served cities, a more intelligent, more efficient and less impacting mobility system on the environment, where car sharing, and more generally the shared mobility techniques, are integrated with the public transport and other innovative ways of urban transport.

ICS is also connected to the public initiative "Io Guido"[18] still active in 8 Italian cities.

3.2.2. *Second Generation of Car-sharing systems*

Some years later a new generation, called the second generation, of Car-sharing systems was developed providing a different service model to overcome the limits imposed, to users, by the first generation, by the introduction of three new features:

- real-time access to the vehicle, that does not need to be reserved in advance,
- open-ended reservation, allowing the user to close the rent at drop-off time, that can be decided by himself according to his real-time needs
- one-way movement, sine the car could be returned in a different destination from pick-up location.

These new features, if on one side provided greater flexibility to consumers, on the other side create imbalance of vehicles available at the various stations.

This problem, also known as the *imbalance problem*[12], will require relocation strategies to mitigate the impact of having concentration of vehicles in stations with low demand and lack of vehicles in stations with high mobility demand.

The imbalance problem and the relocation strategies will be discussed in detail since their efficient management represents the main goal of this thesis.

3.2.3. Third (new) Generation of Car-sharing systems

After 2010, a third generation of car sharing systems (3G-CS)[ii], also known as *one-way free-floating* systems has been proposed to users by Car-sharing operators. Key features of these models are:

- vehicles can be picked-up along the roads[iii]
- vehicles can be identified using a mobile app with location features
- reservation is immediate and made via mobile app or website
- vehicles pick-up can be done without Car-sharing staff involvement
- vehicles can be returned at any point and any time inside the service area.

This sharing mobility model has been designed to target users who live in residential areas where population is poorly served, in terms of frequency, capacity and destination flexibility by public transport systems.

The main challenge of 3G-CS is the capillarity defined as the *degree of diffusion of vehicles within the application area of the transport system* defined by Ciari et al.[19] and Schwieger [20]

One of more representative case studies of free-floating one-way Car-sharing services is Car2Go, founded in Germany in 2008, with headquarters in Stuttgart, Germany and fully owned by Daimler AG.

Car2Go offers its car-sharing services in 8 countries[iv] in Europe, North America and Asia, and as of July 2017, is the largest Car-sharing company in the world with 2,500,000 registered members, a fleet of 14,115 vehicles in 25 cities, as of November 2017.

In Amsterdam, Madrid and Stuttgart the service is managed using a fleet of 1,400 of full electric vehicle (FEVs), representing about 10% of the fleet.

The business model of Car2Go requires a paid membership to access the service and a rent by the minute with tariffs that may vary from country to country.

In Italy, where the Car2Go covers four cities (Milan, Rome, Turin and Florence/Prato) the tariff scheme requires the registration to the service that may be done online or via mobile App, with a one-shot validation fee of 9 € per user. To complete the subscription process, it is required to validate the driving licence of the new member, to register and validate a credit card and to accept the term of service agreement.

Once the registration is completed a personal account is activated and the vehicle can be rented by the minute with a tariff depending by the vehicle type. The tariff scheme includes the parking cost inside the service area even if additional parking tariff can be applied if the driver uses specific interconnection parking hubs such as airports, railway stations or city hubs.

[ii] Examples of 3G-CS are Car2Go, DriveNow, ReachNOw, Enjoy andSharen'go.
[iii] In urban zone with parking slots shortage Car-sharing allow users to drop-off in affiliated private parking areas by paying an addition fee.
[iv] Car2Go served countries are: Austria, Canada, China, Germany, Italy, Netherlands, Spain, United States

The service steps that a driver has to perform to access the Car-sharing service are:

1. **Search and reserve** a vehicle; in case of Car2Go the first 20 minutes of reservation are included in the rental cost, while additional time is billed by the minute at the same fare of the trip,
2. **Open** the Vehicle, when the driver is next to it; at this point if the reservation time is lower than 20 minutes, starts the fare counter,
3. **Drive** to destination (drive period can be assisted with navigation features to minimize trip time and distance)
4. **Park and close the rent**, which requires to leave the vehicle inside the allowed service area,
5. **Billing and Payment** of the service.

All the process can be performed via mobile App.

3.2.4. ZipCar

Zipcar is an American car-sharing company, founded in 2000 by Antje Danielson and Robin Chase, then acquired for approximately US$500 million by Avis Budget Group[21], providing vehicle reservations to its members, billable by the minute, hour or day; its members pay a monthly or annual membership fee in addition to car reservations charges.

In June 2018, Zipcar[22] claims to have reached *"over million members across 500 cities in offering more than 12,000 vehicles in urban areas on college campuses and at airports"* in ten countries, in Belgium, Canada, Costa Rica, France, Iceland, Spain, Taiwan, Turkey, the United Kingdom and the United States., making it one of the world's leading car rental networks[23].

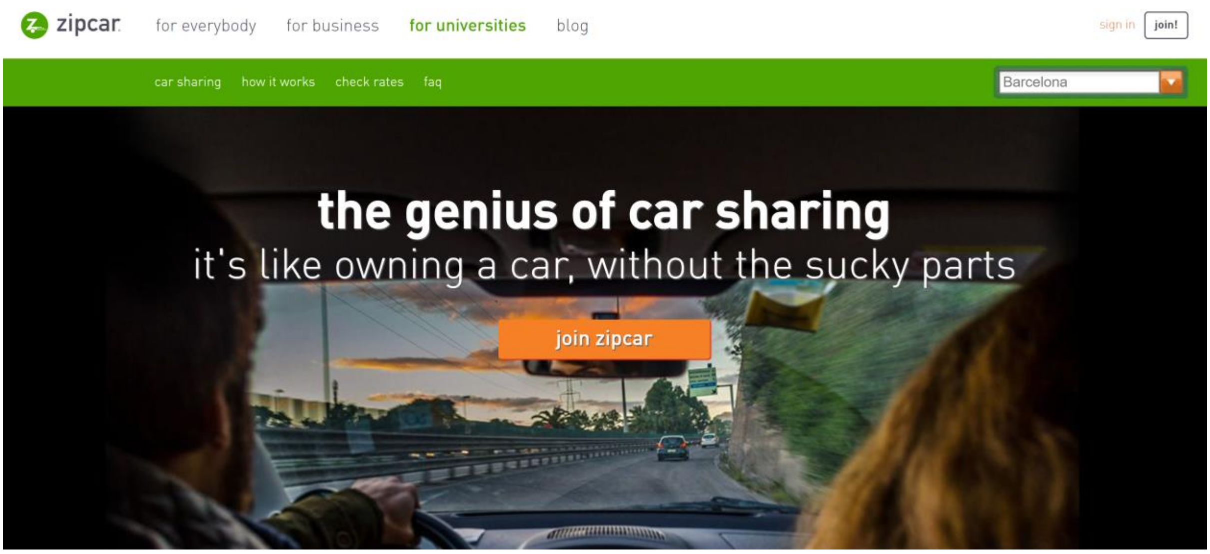

Figure 5 – ZipCar Home page

Zipcar members can reserve vehicles at any time with Zipcar's mobile app or website at any time. Members can use Zipcar's Android or iPhone app to locate a Zipcar.

Vehicle door is unlocked directly by the user; in fact, access to the vehicle is available by using a proprietary access card (Zipcard), or, for mobile users, a mobile app which unlocks the door; the keys are located inside the vehicle. Zipcar charges an annual fee and a rental hourly charge. Fuel, parking, insurance, and maintenance are included in the rental fee.

The user-experience provided by Zipcar can be synthetically descried with:

- JOIN – online application to receive a Zipcard to access vehicles worldwide.
- RESERVE – booking of a vehicle (minimum time: one hour; maximum time: seven days).
- TAP – use of Zipcard to access the vehicle.
- DRIVE – Zipcars can be picked-up and parked in reserved spot.

Zipcar is a first-generation round-trip model, which means the car must be reserved in advance declaring the rental slot and returning the vehicle at the pick-up station.

To improve flexibility of its model Zipcar, allow users to book the car shortly in advance (near real time), and, in case of necessity extend the booking period.

3.2.5. DriveNow

DriveNow[24] car sharing is the mobility concept from BMW and MINI for Europe[v], created as a joint venture between BMW Group and Sixt SE holding with 50% of shares each.

In March 2018, DriveNow became a wholly owned subsidiary of BMW after that in January 2018, BMW announced that Sixt SE will sell its 50% stake for €209 million[25].

With a fleet of the very latest BMW and MINI models, DriveNow offers a third-generation free-floating system not binding the consumer to any station for car pick-up or return.

Figure 6 – DriveNow Home page

In April 2018, BMW Group and Daimler AG agreed to combine their mobility services, including their car sharing devices DriveNow and car2go, to shape sustainable urban mobility for the future.

DriveNow car sharing operates with more than 6.000 vehicles in 13 cities across 8 countries[vi] and has reached more than 1.000.000 users[26].

[v] A similar model is implemented in United States under the brand ReachNow (https://reachnow.com/en/)
[vi] DriveNow is present in Germany (Munich, Berlin, Hamburg, Düsseldorf and Cologne), Austria (Vienna), Belgium (Brussels), Italy(Milan), Sweden(Stockholm), Denmark(Copenhagen), Portugal(Lisbon), Finland(Helsinki) and UK (London)

DriveNow cars are rent, in real-time, per minute rate and services are included in the price: insurance, parking, road taxes and fuel.

The user-experience provided by DriveNow can be synthetically descried with:

- SUBSCRIBE – register and download the app.
- RESERVE – booking of a vehicle real-time.
- DRIVE – pick-up and drive a BMW or MINI.
- RETURN – vehicles can be returned everywhere inside the served area.

3.3. State of the art in Revenue Management and Dynamic Pricing

According to Robert Cross (1997)[27] Revenue Management is "the application of disciplined tactics that predicts consumer behaviour at micro-market level and optimizes availability and price to maximize revenue growth.

Profitability is the main cause of utilization of Revenue Management. Companies need to maximize profitability and need to perform demand forecasts, market segmentation, price setting and inventory management to reach this goal.

Revenue management is born in the airline industry in 1978 with the Deregulation Act in the USA. American Airlines was the first to apply such a strategy to control inventories and maximize revenues[28,29].

Later it has been applied with success in other sectors, such as:

- hotel management[30],
- rail transportation[31],
- car rental[32].

Revenue Management principles derive from the economic theories of "demand and supply theory, equilibrium, invisible hand", firstly introduced by Adam Smith[33], which focus on self-regulation behaviour of systems. The main objective of Revenue Management is to increase companies' productivity and margins. It can be applied whenever decision must be taken about allocation of scarce resources, to match supply and uncertain demand. In fact, "Revenue Management ensures that companies will sell the right product (e.g. seat) to the right customer at the right time"[34] "for the right price"[35].

An extensive overview of Revenue Management can be found at Talluri and Van Ryzin[36] (2006), McGill and Van Ryzin[37] (1999), Chiang et al.[38] (2007) and Walczak et al.[39] (2012).

Before discussing key features of Revenue Management in Car-sharing it is important to illustrate the main aspects of Revenue Management systems

3.3.1. Revenue Management systems in Airline industry

Revenue Management in Airline industry has been crucial, in the last 40 years, for Airline Companies to reach and maintain profitability in a highly competitive market[40].

Since the massification of power-controlled man flight and the rise of low-cost vectors, market share are gained and consumer are attracted mainly with application of lower pricing than competitors and tariff flexibility for the traveller to choose to pay only desired services.

Additionally, airline pricing works on the concept of maximizing flight profit (or minimize flight cost) using seat occupation and urgency to buy so that, in general, lower tariffs are proposed when aircraft occupation is lower and boarding date is far, while higher tariffs are sold with less available seats and close departure date.

Personalization of tariff having the goal to meet both *no-frills*[vii] consumers and business travellers require complex dynamic pricing systems that aim to achieve revenue maximization by leveraging huge amount of data and more complex algorithms.

A new frontier of Revenue Management to capture customer behaviour and to respond to the flexibility need of consumers' in choosing between different alternatives, with respect to airline booking strategy, is to incorporate a Costumer Choice Model into the Revenue Management problem statement[41].

The Customer Choice Model (or CCM) is an individual response model that allows companies to analyse and understand the choices of individual customers in the market. Company can apply this model to marketing and sales analysis and decision making with the goal to better respond with their products and services to customer behaviours. In transportation, *"under a CCM framework, market share of an itinerary is proportional to the attractiveness of that itinerary. In turn, the itinerary's attractiveness depends on multiple factors affecting customer preferences and their relative importance."*[42]

At the state-of-the-art two main CCMs are used in Revenue Management: the Basic Attraction Model[43] (BAM) and the General Attraction Model[44] (GAM). Including CCMs, two main Revenue Management models have been proposed in literature Choice-Based model (CBM) and Sales-Based model (SBM).

One of must known CBMs is the Choice-Based Linear Programming (CBLP) model proposed by Gallego and Phillips[45] (2004) and Liu and Van Ryzin[46] (2008). In CBLP, decision variables represent the period in which a certain offer-set of flight options can be booked. Considering *m* the number of flight options, the model lead to exponential complexity given that the worst case is characterized by are 2^m variables (offer-sets).

Sales-Based models (SBMs) have been proposed more recently by Guillermo et al.[47] (2011) and Gallego et al. (2015)[44]. In SBM[48]s the decision variables are represented by the seats to be allocated. The number of variables is polynomial and if *m* is the number of flight options the model will have *m+1* variables. Gallego et al. (2015) proposed the linear formulation (SBLP) that it is equivalent to CBLP, having the same objective value at the optimum.

Grani et. all[41] (2018) proposed an integer version of the sales based model that can be incorporate within a post departure toolbox Revenue Opportunity Model[viii] (ROM) that uses a metric to evaluate

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[vii] Definition of no-frills: offering or providing only the essentials – not fancy, elaborate, or luxurious a e.g. no-frills airline (Merriam-Webster)
[viii] *ROM is developed by the Sabre Airline Consulting group and it is delivered to Sabre customers*

how the system performs once the full awareness about customers demand and airlines offer has been reached".

3.3.2. Revenue Management systems in Rail transportation industry

Revenue Management in Rail Transportation industry offers a similar approach to revenue management in air transport sector. The analogy is linked, in the first instance, to the fact that the railway industry, like the air transport sector, manages passenger transport and the delivery of goods.

On the other hand, historically, more simplified Revenue Management models have been applied in Rail Transportation, in some cases also for the low competition in the rail transportation sector.

In literature there are two fields of application of Revenue Management principles to the Rail Transportation industry sectors: when operating on passengers' models are classified as Railroad Passenger Revenue Management (also defined RPRM) while when things are transported from one origin to a destination models are classified as Railroad Freight Revenue Management (RFRM).

In RPRM revenues optimization can be done by finding the maximum amount of revenues from tickets that can be sold to passengers by limiting tickets' availability.

On the other hand, the absence of the passenger in the freight rail transportation RFRM requires different optimization approaches that will not be considered for the dynamic pricing of this research.

In passengers' rail transportation "a service is defined as a train travelling from an origin to a destination at a specific time" [49].

In general rail operators, to serve passengers, are forced to leverage divergent challenges[50]:

- Maximize revenues
- Maximizing load factor
- Increasing total ridership
- Meeting needs of different customer segments
- Competing on market share with other transport modes
- Optimizing seat allocation for origin-destination pairs
- Respond to governmental local strategies (e.g. subsidy of regional trips)

There is very little literature about Revenue Management for passengers in Rail Transportation industry (see e.g. Ciancimino et al.[51] (1999)).

Considering the challenges and the lower attention that Revenue Management had in the past it is frequent that RPRM of rail transport are, in some way, similar to (or derived from) Revenue Management Systems applied in Airline industry.

3.3.3. Revenue Management systems in car-rental industry

Car-rental companies have been applying Revenue Management for over 25 years, since National, a US car-rental firm[52], developed the first Revenue Management process including Dynamic Pricing[53,ix].

[ix] **Dynamic pricing** is a pricing strategy in which businesses set flexible prices for product or service based on current market demands. In literature Revenue Management and Dynamic Pricing are sometimes considered as synonyms, while

In early 1990s National Car Rental faced economic crisis, liquidation and the loss of over 7.500 jobs. In 1993 the restructuring process included an extended and comprehensive revenue management program using analytic modelling to manage capacity, pricing, and reservation.

"In July 1994 (National) implemented a state-of-the-art revenue management system, improving revenues by $56 million in the first year."[54]

The problem of efficiently managing car-rental fleets requires taking decisions on various levels (strategic, tactical and operational); main decisions affecting the fleet, as a whole, are the definition of the sizing, its composition, the distribution of cars across rental stations, the definition of reference prices for services, the management of reservations for rentals and the assignment of vehicles to customers.

Compared to airline travelling car-rental requires to consider in addition to itineraries (round/one-way trips) and the resources (cars) also capacity flexibility and rent length that vary for each specific customer. In a real context, these decisions are only linked by tight interactions and time horizons that require high flexibility.

Contrary to expectations linked to over 25 of history of car-rental in the field, as stated by Oliveira B et al.[55] (2016) *literature in the area is scarce and somewhat concentrated in only a few of the problems*. In the Revenue Management field solution to the following problems have been developed:

- *whether to accept or reject a booking request* has been proposed by Guerriero and Olivito[56] (2014), Steinhardt and Gönsch[57] (2012) and by Li and Pang[58] (2016) using two heuristics to decompose a discrete-time stochastic dynamic program over an infinite horizon,
- *pricing decisions* by Oliveira et al.[59] (2015) updating prices for a car rental company in the websites of e-brokers that compare prices in the market

A systematic improvement on use of analytics seems to rise from the companies; in fact, Europecar has been selected in the 2018 INFORMS Franz Edelman Award finalists[60].

3.3.4. Revenue Management systems in car-sharing industry

The Revenue Management considers different interesting aspects, relevant to car sharing system: customer's willingness to pay or to accept, that is related to price sensitivity, user reservation behaviour and attractivities of markets. Then, Revenue Management tries to allocate nodes' capacities to satisfy the demand and to maximize profits[61].

Reading previous analysis by different authors, a relationship between cars sharing system and revenue management's notions seems to be tricky. Car sharing is close to airlines in terms of Revenue Management when the booking is made in advance because the cars are subjected to depreciation, users can be partitioned and there are differentiations of the service. But, when the booking is made in real time, traditional models in Revenue Management seems to be inefficient for car sharing.

In fact, there are some differences:

- renting by minute despite by day, introduces a change of degree in temporal flexibility;

in other cases Dynamic Pricing is considered as the possibility to flexibly define the price during the Revenue Management process.

- current Car-sharing is typically a one-way service, since according to Morency et al., (2011) only 5% of sold rides are two-way trips[62], so it is in opposition with airline market;
- a car's booking is made in real time, while airplane's reservation is made some days in advance.

Despite all these negative reviews, the goal of this relation isn't to find a relationship between them, but to exploit revenue management's aspects that can be useful in car sharing system.

Examples of these concepts are:

- market segmentation;
- market attractiveness;
- willingness to pay or to accept;
- price sensitivity;
- alternative prices;
- probability of selection;
- large dimension of the network;
- discrete nature of variables;
- randomness of the environment;
- complexity in characterizing real behaviour of users;
- possibility for users to accept or reject a ride;
- a set of alternatives for users;
- possible unavailability of a means in a node;
- possibility to recapture all the refused rides.

3.4. The novel contribution of this thesis

Novelty of this study is mainly based on the following four key research topics that, even if in some cases have been tackled singularly, they have not (or poorly) been developed in a unified framework to generate synergies and market opportunity for Car-sharing companies. They are:

1. **Dynamic Tariff application to relocation movement**. The tariff is determined considering the relocation cost for the Car-sharing company and the access to saving by applying, during peak-hours, discounted fees for movements from colder to hotter spots, to incentive user-based relocation, and application of penalties from hotter to colder spots to disincentive imbalance.
2. **User-based relocation using trip planning techniques**. User-based relocation has been much less investigated than operator-based relocation; additionally, the traditional approach in user-based relocation is focused on convincing the user to change its destination, while in our case the relocation is based on user preference expressed by declaring its destination during reservation process;

3. **Hot and cold spots have been calculated considering the frequency and the latency of movements in specific areas**. The value metric associated to the area has been defined **Urban Area Value**; even if a similar approach has been used sometimes for one-way station-based car-sharing systems no evidence for this approach, have been found **in one-way free-floating car-sharing systems**,
4. **Urban Area Value** has been calculated using data for all one-way free-floating Car-sharing systems operating in the city (car-sharing ecosystem), gathering additional precision and value form data variety and volume.

The following chapters describe in detail the methodology and the model used to reach results and conclusions.

4. The imbalance problem

4.1. Definition of imbalance problem

Unfortunately, one-way Car-sharing model have a strong complexity given by characterization of users demand that leads to *having a surplus of vehicles in stations with high demand as destination, and a lack of vehicles in stations with high demand as origin, unbalancing the demand and supply quotient*[63].

Areas of lower individual mobility demand are defined *cold spots*, while zones of higher demand are defined as *hot spots*[64].

This phenomenon[65], also known as the *imbalance problem*, requires solving the conflict between the positioning of vehicles "at the right place and time", where the same vehicle can be used by different persons[12] and the freedom for customers to return vehicles where and when they want.

The imbalance problem needs to be solved because having vehicles unused for long time in areas of low demand cause a loss of money and impacts on the system diffusion among mobility users.

4.2. Relocation and relocation strategies

Relocation principle consist in moving vehicles from low demand areas (cold spots) to high demand areas (hot spots).

Relocation strategies are criterion used to perform relocation more efficiently and effectively.

A vehicle relocation is always accompanied by costs to perform the movement, so relocation can be done when benefits to the Car-sharing provider compensate relocation cost by additional earnings.

The approach used to solve the imbalance problem is to apply relocation strategies having the goal to reduce the lack of vehicle in hot spots by using the saturation of cold spots.

The relocation focus should especially be on high demand spots caused by a poor public transport connection, because Car Sharing Systems should not substitute existing efficient public transport systems. A good possibility for relocation is by night, when the demand is lower.[64]

Main goals of a relocation strategy are:

- reduce the Car-sharing service management costs
- increase earnings, profits and service quality by providing users higher availability (or lower waiting times) and higher flexibility in vehicle pick-up.

Controlling the system with optimization algorithms can lead to **minimize service management costs** and **maximize earnings and profits**.

According to Cepolina[66], the need for a relocation is triggered by the reach of one of the below defined threshold:

- *high critical threshold*, representing the maximum number of vehicles that should be available in a station or area to support efficient model operability; above the threshold the area has reached its capacity

- *low critical threshold*, representing the minimum number of vehicles that should be available in a station/area; below the threshold the area has a lack of vehicles to properly fulfil users' demand
- *low buffer threshold*[67], representing the minimum number of vehicles that a station/area need to have to be able to send a vehicle to another station
- *high buffer threshold*[67], representing the maximum number of vehicles that a station/area can have to be able to accept a new vehicle from another station.

The relocation can be performed moving vehicles from stations/zones whose number of vehicles is higher than *low buffer threshold* to stations/zones where number of vehicles is closer or equal to the *low critical threshold.*

Relocation criterion are different and in case of staff relocation can be:

- *shortest time* consisting in relocating the vehicle in the fastest station/zone to reach
- *inventory balancing*: consisting in relocating the vehicle in station/zone with fewer vehicle

Relocation strategies are generally classified as:

- **Operator based**, when is the operator that, with the intervention of its staff, manages according to some rules, the relocation of vehicles. The negative effect of this situation is that some trip occurs without users' generating additional cost without any corresponding revenue. The additional cost is generated by vehicle movement cost and staff cost.
- **User-based**, when the user directly carries out to the relocation, following the balancing needs of Car-sharing system. In user-based relocation the user is generally engaged with a bonus, a discount or even a free ride to support relocation strategies proposed by the Car-sharing operator, which generally requires the user to change his original destination to reach the nearest hot-spot identified by the relocation system. This relocation strategy is convenient for the Car-sharing operator from the financial point of view, avoiding the cost of staff to be involved in relocation. In case of free rides, the cost of movement still applies, since for discount based relocation the movement cost is covered by the discounted fee applied to the trip. In user-based relocation strategies also light maintenance cost such as refuel or driving to cleaning centres can be transferred to users.
- **Vehicle-based**[68] which is a new relocation model using AI/self-driving cars paradigm that in the future will allow the cars to relocate themselves based on Car-sharing relocation criterion defined by the Car-sharing operator. One example of vehicle-based relocation is the PICAV[x] Car-sharing[69] where the vehicles are electrically powered, are able to recharge when they are idle at stations, can be available at the station and on the road and can move autonomously so that an automated vehicle-based relocation strategy is proposed.

Most of Car-sharing systems use the *operator-based relocation*, because traditional user-based relocation strategies require the customer to change the desired destination, accepting the alternative proposed by the car-sharing system, or using trip-joining, to gather a discounted ride. It has been demonstrated that in practice it is difficult to engage the customer only suggesting him a different

[x] PICAV: Personal Intelligent City Accessible Vehicles

destination in exchange for a discount, given that, especially in cities, car-sharing users value privacy and convenience over minor transport cost savings[70].

Relocation Strategy	Advantages	Disadvantages
User-based	• staff savings; • no additional vehicle movements • environmentally sustainable	• difficulty to influence the end-user (rejection ratio) • complexity of pricing/ model communication
Operator-based	• reliability of relocation • combination with periodic maintenance	• staff cost of relocation • empty trips without revenues
Vehicle-based	• staff savings; • reliability of relocation; • avoidance of staff for periodic maintenance • environmentally sustainable • no need to influence the end-users	• additional vehicle movements • loss of relocation discounts • model maturity.

Table 1: relocation strategies maintenance.

The performance of a relocation strategy is linked to vehicle availably (or user waiting time) and, according to Shaheen researches, to the capability of car sharing systems to attract users from private transport modes[71].

Car-sharing operators and researchers are strongly focused on analysing company historical data to develop deeper knowledge of their vehicles' usage and the ability to reach an effective vehicle distribution in the city service area, mainly using models that optimize the company staff used to relocate poorly positioned vehicles and to perform vehicle maintenance.

For instance, according to Santos and Correia (2015), a model can be defined to optimize staff activity in real-time, in a rolling horizon planning, managing simultaneously maintenance and relocation operations in a one-way Car-sharing[12].

This model in general does not allow to find solutions that significantly decrease the cost of the service and the actual pricing schemes and fees result not convenient for users of large crowded city to switch from existing transportation modes to car-sharing; this constitutes a barrier strongly affecting mass market scalability for Car-sharing operators.

On the other side, there is a lack of knowledge of the urban market demand in terms of citizens' service needs, urban areas potential, geo-clustering, pricing optimization, operational optimization, interoperability opportunities, which require the integration of data from various sources, with the goal of assembling a single homogeneous database with the information of all Car-sharing services.

In fact, current optimization models proposed in the literature imply that if a car is located for too long in a specific place, or a vehicle is expected to be used more frequently in different place[72], the vehicle is repositioned by company staff, at very high cost[xi]. Consequently, the Total Cost of Ownership per City Fleet, impacted by vehicle productivity and lack of ability to increase the number

[xi] Source: the cost estimated, in Italy, is up to 15 euro/movement; this value has been retrieved in an interview with a leading Car-sharing operator

of Users is a barrier to make the model scalable and cost-attractive especially if compared with Public Transport services.

Therefore, it is critical for Car-sharing operators to act on different users' demand needs to convince them to preferably switch to Car-sharing when the vehicle is poorly used, to reposition the vehicle in a more attractive location. The User-based relocation would in theory enable Car-sharing operators to reduce the cost of vehicle relocation simply discounting the price of the urban trip and saving cost on reducing or avoiding staff involvement dedicated to the task.

In case of *one-way free-floating* systems, the concept of relocation should be considered at area level since the vehicles are distributed in a wider area and not concentrated in a geographically identified station.

In this case, to reduce the impact of the imbalance problem relocation strategies must be developed to move vehicles from *cold area* characterized by an excess of vehicles compared to users' needs, to *hot area* having a shortage of vehicles against booking requests.

There are three main approaches to assist the daily system management in reducing the impact of imbalance problem via vehicle relocation (Jorge and Correia, 2013):

- **operator-based** performing relocations by using operator's staff,
- **user-based** where balancing movements are performed by clients reacting to incentive mechanism based on price discounts, requiring the driver to change destination to fulfil operator's need to relocate the vehicle in a hot area. Generally, users are poorly motivated to change their trip destination to receive a price discount.
- **trip selection** performing demand control to allow only trips matching operator's balancing criteria.

The "MIP model" presented in the work of Santos and Correia[12] (2015) is an operator-based relocation model that is used to optimize the staff activity in real time acting on a rolling horizon where staff uses vehicle to perform simple maintenance (e.g. cleaning) or refuelling procedures and to execute relocation movement. In case of refuelling staff drove to the nearest gas station and left the vehicle inside or next to the gas station.

User-based relocation model generally require the user to accept the optimal destination for the operator and need to be influenced to reach the relocation objective[64]

Research for flexible pricing models has been performed in a station-based model for electric vehicles but not on a free-floating model; in this study pricing is not carefully analysed due to lack of data, an elastic demand formulation is seen as a potential future work[73].

Similarly, a novel technique called FDP (Feedback Dynamic Pricing) for tackling the problem of vehicles balancing in one-way VSSs has been presented for an electric station based operating model[74].

Differently from the largest part of previously described models, our research focuses in solving the flexibility-tariff problem, with the application of flexible pricing schemes, based on Urban Area Demand, where both the Car-sharing end-user and the service provider find a mutual win-win situation, with the service provider reducing cost of vehicle relocation and the city user spending a lower fee to reach his destination.

The most complex operational set up is one way and free floating. This allows individuals to use a vehicle of the system as if it was their own vehicle. However, it doesn't mean complete freedom, since vehicles need to be delivered inside an operating area[75].

The imbalance problems created by one-way movements need to be solved by the operator to minimize the rejected demand and increase vehicle availability levels. This can be done by intervening on the demand side or on the supply side[70]. The amount of vehicle usage in a one-way system, implies the need for daily maintenance operations, such as vehicle cleaning and refuelling. Therefore, the use of staff to perform both maintenance and relocation operations should be regarded.

Car-sharing operators and researchers are strongly focused on analysing company historical data to develop deeper knowledge of their vehicles' usage and the ability to reach an effective vehicle distribution in the city service area, mainly using models that optimize the use of company staff to relocate poorly demanded vehicles and perform vehicle maintenance.

For instance, the model can optimize staff activity in real-time, in a rolling horizon planning approach, managing simultaneously maintenance and relocation operations in a one-way Car-sharing[72].

The approach of using Car-sharing provider employed staff with a multi-skill role covering ordinary maintenance, cleaning and vehicle repositioning represents a way to decrease the cost of the service and it can represent a solution to support sustainability of actual pricing schemas and service fees; unfortunately the actual cost of Car-sharing is considered too expensive for the large crowd of city users and represents a barrier, for a larger use, switching to Car-sharing from private cars. This barrier is consequently affecting mass market scalability of Car-sharing operators preventing a larger coverage of the service due to lack of sustainability.

The definition of a reliable User-based relocation model in one-way free-floating Car-sharing requires a critical quantity of data enabling a very precise knowledge of the urban mobility-sharing demand for each urban area, to be able to assign a demand value to each of the covered urban area; Car-sharing operators tend to investigate only self-generated data which give them a partial view of their demand distribution in served cities.

A more complete view of mobility demand would require the integration of data from diverse sources, with the goal to own a single homogeneous database including the information of all Car-sharing services operating in the city.

4.3. Vehicle Relocation

Ideally, independently from the Car-sharing service model the operator has decided to apply, only the user-based relocation model should apply.

In fact, in case of Station based Car-sharing model both "round-trip" and "one-way" naturally use a user-based relocation model, being the end-user requested to bring back the vehicle to an approved delivery station, to close the vehicle rent.

In one-way free-floating Car-sharing models, the relocation model is ideally similar to the Station-based model, with the difference that the relocation is allowed in defined service area instead of a predefined set of allowed stations. On the other hand, one-way free-floating car-sharing models are affected by the imbalance problem that, as known, concentrates vehicle delivery where there is high destination demand and creates a lack of vehicles where there is a high departure demand.

In such situations Car-sharing operators involve staff to support vehicle relocation movements from high destination demand to high origin demand with an increase of cost including personnel cost, fuel cost and vehicle consumption cost. This relocation approach will be called Staff-based relocation model.

To solve the problem of minimizing Staff-based relocation cost, several methods applied in research focus on creating synergies between vehicle relocation and planned maintenance cost such as vehicle washing or re-fuel; unfortunately, the impact of cost saving risk to be marginal considering the high cost-impact of personnel cost and the relatively minimal impact of planned maintenance events compared to vehicle need for high origin demand for relocation.

4.4. Analysis of Urban Mobility Demand

Several factors impact on the definition of Urban Mobility demand for an Urban Area, which strongly depends on the characteristics of the Urban Area itself. Generally Urban Mobility Demand is empirically analysed using historical data.

For instance, all cities considered in this thesis have distinctive characteristics of Urban Demand distribution.

Example of characteristics of Urban Mobility demand are:

- Hourly range, since end users' urban mobility demand varies in different hours of the day;
- Seasonality, users' demand varies in different period of the year;
- Urban Areas, urban mobility demand changes in different places of the city;
- Week-Day, since different day of the week are affected by diverse urban mobility needs.

4.5. Pricing models in car-sharing

In every business system, one of the most important and significant decisions from a profit's point of view is the definition of a pricing model. Price can not only bring relevant revenues, but it can also influence users' options. So, the challenge is to determine the value that each costumer is disposed to pay for the service, in order to generate high profits. It is necessary to remember that in every business model, the main objective is to satisfy customers.

The price in Transportation modes is strictly connected to several factors. Pricing in Car-sharing mainly depends from:

- travelled distances and rent time,
- car model,
- type and quality of the service;
- other incentive factors defined by the operator.

The choice for a simple pricing policy, in a specific transport mode, isn't often an advantageous selection to reach success in business and consumers satisfaction. For example, is easy to demonstrate that Car Rental pricing model[xii], typically applied to round trip movements, which is based on a daily rental service with a flat day cost, is not convenient for the large consumer segment of city users who

[xii] Generally, Car Rental services provide a flat daily tariff up to 150 kilometres per day with additional cost per kilometres over the threshold. Fuel cost is not included in car rental tariffs.

make few daily movements under 100 kilometres in total, which usually choose public transportation when are cost sensitive or taxi when time sensitive.

A solution to this segment needs is the flexible pricing of a car sharing system, that allows companies to apply a Pricing Model based on time or distance travelled with the option to set a different a price during the day, depending on the current state of the system, and to guarantee flexibility to customers.

From a system's state point of view, types of variable pricing can be:

- *static*, where the unit cost of the service (e.g. flat cost per minute) is independent from the system's state and it is constant during the day, with a pay-per-use model where the price variation depends on the time consumption of the service;
- *dynamic*, when the price derives from system's state and it is variable during the day; it can be considered a balance between supply and demand.

The literature presents an example of dynamic pricing known as *locally dynamic pricing*[76], with a pricing model depending on the states of the station (full parking spots).

Locally dynamic pricing in defined by Waserhole as *"a station state dependent pricing policy can set the price to take a trip from a station A to a station B in function of the current states of stations A and B (parking filling and number of vehicles in transit toward them)."*

For Car-sharing systems, in this research, is applied a variable tariff scheme, that can be called *system's saturation dependent variable pricing*. In this model the price is set to a standard tariff which is applied in a pre-defined range of saturation of the system, generally off-peak and it is variable in peaks during a day.

More precisely, when a positive peak occurs during the day a certain level of addition can be applied to the standard tariff; on the other hand, discounts can be applied to negative peaks to incentivize service usage.

The pricing model can be characterized by:

- continuous pricing, if it is a real number included in a range from a minimum to a maximum value p ∈ {pmin; pmax};
- discrete pricing, if there are some discrete values to choose from p ∈ {p1, p2,…, pn}.

In this study the second pricing characteristics are applied to respect a certain level of pricing simplification that can be easily understood by Car-sharing consumers. Consequently, a pricing package composed by different pricing option is defined. Every price depends from the alternative trip and user can choose the best for his aims.

Generally, main factors influencing the price are:

- time-based pricing, when it depends on the time of booking (there are different prices for long and short time of reservation); this kind of price fits well in rental-car system, but in this relation, it isn't considered because one-way car sharing systems are characterized by short trips;
- distance-based pricing, when price depends on travelled distances (e.g. for long distances there is a tariff rate lower than for short distances); this isn't considered in this thesis because the model describes a limited city and there isn't a relevant difference among travelled distances;

- location-based pricing, if the price depends on the state of stations in terms of requests and available parking lot; this is a kind of pricing closest to our idea of dynamic pricing: if there is a high request in a node, the price is modelled in a way that users are stimulated to locate cars in that station; instead, the thesis doesn't consider the problem of parking congestion: to simplify the analysis, the nodes have infinite parking spots, so there isn't a maximum threshold; only at the end of the day, with relocation activities, it is assumed that in the zones there can't be more than a certain percentage of cars;
- customer segmentation, when there are different prices for users' requirements and needs: for business customers, the price is low because of their utilization frequency, for individual customer, there is a higher price and for family or small group there is an average price;
- quality-based pricing, consisting in setting a higher price for frequent travellers, willing to pay more for a better service and a lower price for a basic service; this kind of differentiation isn't considered here because in one-way car sharing there isn't the possibility to characterize customers' attitude, economic situation and reasons; so, the price is unique for all the users' typologies;
- type of vehicle, if there is a fleet formed by kinds of cars with different performances and an associated price (a family car is less expensive than a sports car); in our model, all types of vehicle are considered as a homogeneous fleet including the same type of vehicles.

The users can access to the service respecting a specific booking protocol scheme:

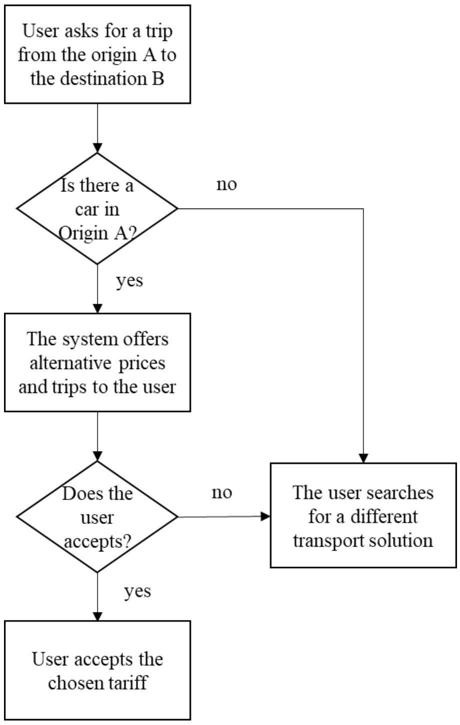

Figure 7 – proposed booking scheme

Actual pricing model in Car-sharing business is similar between one-way free-floating Car-sharing operators in Italy. Analysing[xiii] three mainstream operators in Italy: Car2Go, Enjoy and Sharen'go,

[xiii] Analysis date October 15th, 2017 on three main Car-sharing operators in Italy.

the tariff application resulted similar with small difference in pricing, but substantially applying the same model. In fact, Car-sharing tariff analysis for the Italian market denoted the following pattern described in below paragraph.

4.6. Comparison of operating models

This paragraph compares the operating models of the three operators involved in the research.

- **Car2Go**[77] is the main one-way free-floating Car-sharing operator in Italy covering four cities (Milan, Rome, Turin and Florence) with a fleet of 572[xiv] made up of Fortwo and Forfour produced by Smart.

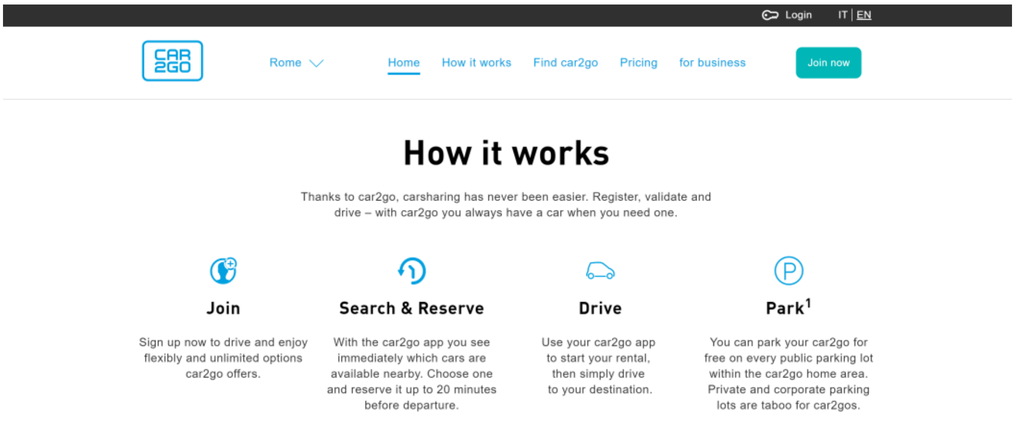

Figure 8 – Car2Go Italy home page (city of Rome selected)

- **Enjoy**[78] is the first challenger one-way free-floating Car-sharing operator in Italy covering five cities (Milan, Rome, Turin, Florence and Catania) with a fleet of 604[79] cars made up of 500 and 500L produced by Fiat Chrysler Automobiles (FCA).

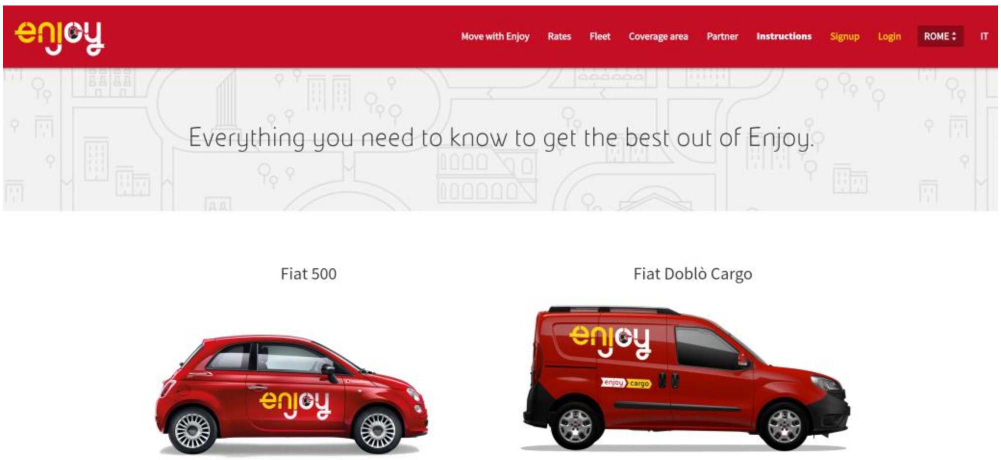

Figure 9 – enjoy home page

- **Share'ngo** is the full-electric one-way free-floating Car-sharing operator in Italy covering four cities[80] (Milan, Rome, Florence and Modena) with a fleet of 400[81] vehicles of type ZD1 produced by Xin Da Yang Electric Vehicles.

[xiv] Number of vehicles calculated on the basis of April 2016 analysis

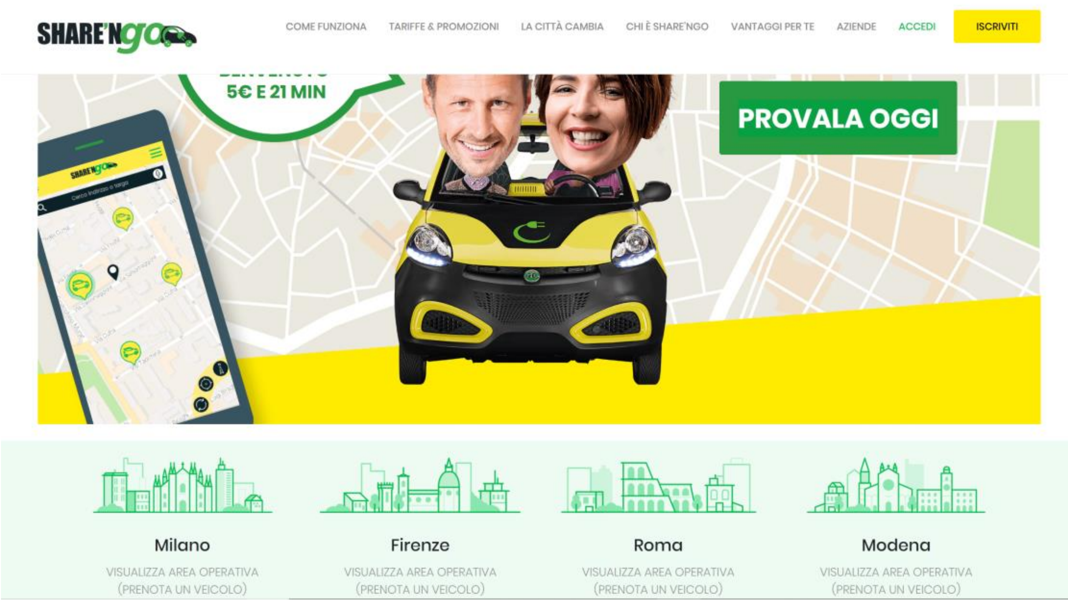

Figure 10 – Sharen'go home page

4.6.1. Car-sharing operating model benchmark

	Car2go	Enjoy	Share'ngo
Subscription fee (€)	9,00	Free	5,00[xv]
Tariff (€/min)	0,24-0,29[xvi]	0,25	0,22-0,28[xvii]
Discounts	Hourly Packages[xviii]	Prepaid	Packages[xix]
Distance included (km)	200[xx]	50[xxi]	Unlimited
Free reservation (min)	20	15[xxii]	20
Full-day rent (€)	60	50	50
User – refuelling	YES[xxiii]	YES[xxiv]	--

Table 2 – car-sharing operators comparison

4.7. Integrating Data of Different Car-sharing Operators

[xv] Includes 15 minutes ride for free
[xvi] The tariff if differentiated on the basis of vehicle model including: smart Fortwo (0,24 €/min), smart Forfour (0,26 €/min), smart fortwo cabrio (0,29 €/min) – smart fortwo cabrio is present only in Rome; minutes are rounded to the upper value.
[xvii] Declared average of 0,24 €/min; minutes rounded to the upper value after 31seconds
[xviii] Hourly packages provide discounted fees for 2/4/6 hours' time-slots.
[xix] Prepaid minute packages and Woman Night Vouchers.
[xx] A long-distance fee of 0,29 €/min is applied to kilometres exceeding the included distance.
[xxi] A long-distance fee of 0,25 €/min is applied to kilometres exceeding the included distance.
[xxii] Extended reservation 0,10 €/min (after the first 15 free minutes, up to 90 minutes).
[xxiii] If fuel level is lower than 25%, a 4 € bonus of car2go credit is given the user for complete refuel.
[xxiv] Only vehicles with a low fuel level can be refilled; user will receive a €5 voucher to use for rentals and Enjoy will pay the whole cost of the fuel

In the experiment data coming from the three different Italian one-way free-floating Car-sharing operators have been collected, used and analysed.

The three operators considered in the analysis had differences in Service Coverage Area, which have been recorded at time of data gathering.

The following map illustrates, in light blue, the Urban Area served by Car2Go during data gathering period.

Figure 11 - Car2Go Coverage Area (date: April 2016)

Similarly, the below map illustrates, in light blue, the Urban Area served by Enjoy vehicle during data gathering period. In this case two different coverage areas can be recognized:

- light blue the Car-sharing coverage;
- light brown the Scooter-sharing coverage (also included in gathered data) which has been closed.

Figure 12 - Enjoy Car & Scooter Coverage Area (date: April 2016)

Finally, the third map illustrates, in light green, the Urban Area served by Share'ngo during data gathering period.

Figure 13 - Share 'ngo Coverage Area (date: April 2016)

The first key issue to be solved in collecting data from different Car-sharing operators' sources is to identify a common data model with homogeneous data to be used for the Data Analysis to identify Urban Area Demand Value Pattern.

Analysing data structure of each of the identified source the immediate evidence is that, even if all data are stored in json format, data structures and storage fields are different between each other and need to be re-organized and/or reprocessed to effectively converge in a common data model enabling a resilient data analysis. Additionally, data are subject to continuous refresh considering that vehicle status changes are highly frequent since end-users continue to book, use and leave the vehicle.

Potentially also the volume of data can be an issue considering the experimental environment. To manage these issues connected to data collection and processing, some big data management techniques such as distributed computing, cloud platforms and switching data processing between in-database and in-memory tasks have been used.

The following paragraphs describe in detail the data structure collected from Car-sharing sources for the services involved in the analysis.

5. Defining and building a Homogeneous Data Model

As mentioned before to reach a homogeneous convergent data structure where to store the raw dataset, all the heterogeneous data formats of involved data sources need to be converted to a target data model. In fact, one of main challenges of “Urban Informatics” is the capability to integrate data belonging to various sources with different data formats and visualization languages, to be able to use data analytics algorithms and techniques to discover hidden patters and receive insights that may support business decisions. This analysis will be possible after data normalization to a Homogeneous Data Model (or HDM), to be reached after a normalization of available data sources of vehicle-sharing transportation modes.

In this case each operator of each Car-sharing one-way free-floating service has a proprietary, independently defined data record, focused on solving the operator data requirements for its proprietary mobility platform, which is obviously different in each case.

To create and fulfil a HDM containing data for all involved Car-sharing operators it is necessary to follow a Data Collection Cycle which is composed by the 7 steps illustrated in the following Figure 14 consisting in:

1. Data Analysis
2. Data Mapping
3. Data Injection Cycle

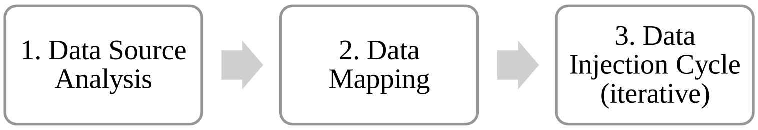

Figure 14 - description of the data collection process

Data Collection Process will be supported by the following Data Collection Architecture described in Figure 15

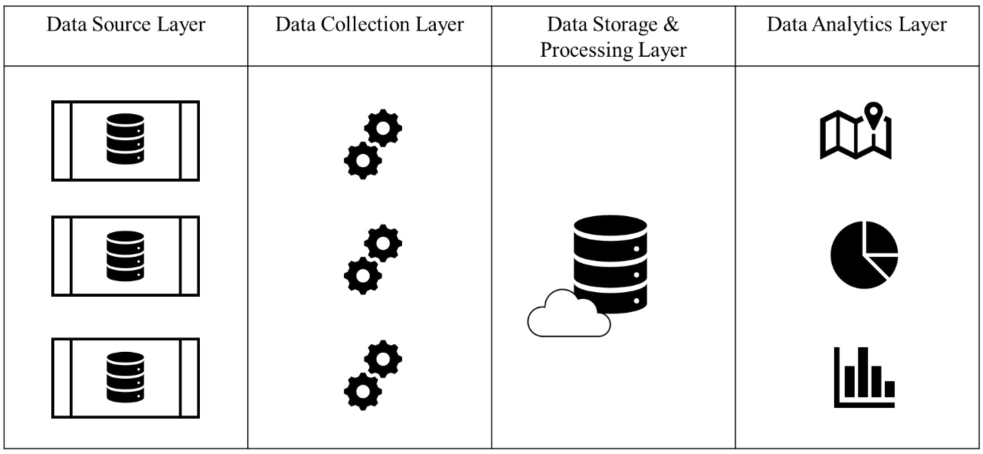

Figure 15 - Data Collection Architecture

Data collection architecture is composed by 4 main layers:

- Data sources, including the various json structured databases of three Car-sharing operators,
- Data collection, including a dedicated collection agent to each source, in order to manage peculiarity of each database
- Data storage and processing layer, where processed and post-process data are maintained; due to large amount of data a cloud database based on Aruba technologies has been used for this research.
- Data analytic layer, where data are analysed.

More in details, the Data collection cycle consisting in data retrieving, data processing, data transformation and data injection in the homogeneous data structure (the cloud-based database) requires to be performed in each iteration to update continuously generated data about Car-sharing vehicles.

Obviously, a Homogeneous Data Model will require flexibility to extend data collection to new or largest smart urban mobility services to insist in single, standardized dataset.

5.1. Data Source Analysis

Main objective of this first step of Data Source Analysis is to identify available sources for data collection, to analyse dataset accessibility by identifying how to connect to the database to retrieve data, to understand how data are stored and what is their structure.

In the case of Car-sharing, Data Structure of the three operators of interest have been analysed and the results are illustrated in the following paragraphs.

As will be explained below there are some similarities between the three data sources, in fact all data sources are accessible via company website and data are displayed using a json key/value pair data structure.

5.1.1. Car2Go Data Structure

The following table describes Car2Go vehicle presence data structure, available in json format, at the source[xxv]. Each vehicle which is available to the end-user has the following data structure:

{

"address":"Via Pietro Capparoni, 3, 00151 Roma",

"coordinates":[12.43952,41.86871,0],

"engineType":"CE",

"exterior":"GOOD",

"fuel":45,

[xxv] Car2go offers an Open API interface https://code.google.com/archive/p/car2go/

"interior":"GOOD",

"name":"194/ET836WT",

"smartPhoneRequired":false,

"vin":"WME4513341K740507"

}

Consequently, Car2Go record data description is:

Field Name	Description	Sample data
Name	Vehicle id containing number and plate	194/ET836WT
Vin	Unique vehicle identification number	WME4513341K740507
Fuel	Level of fuel	45
coordinates.0	Longitude	12.43952
coordinates.1	Latitude	41.86871
coordinates.2	Altitude	0
Address	Address at identified position	Via Pietro Capparoni, 3, 00151 Roma
engineType	Type of engine	CE (combustion engine)
Exterior	External quality of vehicle	GOOD
Interior	Internal quality of vehicle	GOOD
SmartPhoneRequired	--	False

Table 3 - Car2Go data source description

5.1.2. *Enjoy Data Structure*

This paragraph describes Enjoy vehicle data structure, available in json format, with the below data structure described by a sample record:

{

"car_name":"Fiat 500",

"car_plate":"FF852SS",

"fuel_level":30,

"lat":41.81797,

"lon":12.43961,

"address":"Viale C. Sabatini, 138, 00144 Roma RM",

"virtual_rental_type_id":2,

"virtual_rental_id":479677,

"car_category_type_id":1,

"car_category_id":8,

"onClick_disabled":false,

"carModelData":[99,100,141]

}

Consequently, Enjoy’s record data dictionary is:

Field Name	Description	Sample value
car_name	Name of vehicle model	Fiat 500
car_plate	Unique vehicle plate number	FF852SS
fuel_level	Level of fuel in a range 0-100	30
Lat	Latitude coordinates	41.81797
Lon	Longitude coordinates	12.43961
Address	Address at identified position	Viale C. Sabatini, 138, 00144 Roma RM
virtual_rental_type_id	Type of rental	2 [1=MP3, 2=500, 3=500 L]
virtual_rental_id	Rental identification number	479677
car_category_type_id	Type of vehicle	1 [1=Car, 2=Scooter]
car_category_id	Vehicle category	8 [7=MP3, 8=500, 9=500 L]
onClick_disabled	Not available	False

Table 4 - Enjoy data source description

5.1.3. Sharen’go Data Structure

This paragraph describes Share’ngo vehicle data structure, available in json format, with the below structure described by a sample record[xxvi]:

{

"plate":"EH24795",

"manufactures":"Xindayang Ltd.",

"model":"ZD 80",

"active":true,

"intCleanliness":"clean",

"extCleanliness":"clean",

"notes":"LJU70W1Z4GG001473",

"longitude":"12.5144",

"latitude":"41.89145",

"damages":null,

"battery":85,

"busy":false,

"hidden":false,

"rpm":0,

[xxvi] The data record documenting Share’ngo vehicle is very long, in this paragraph only relevant fields have been considered.

"speed":0,

"km":164,

"running":false,

"parking":false,

"status":"operative",

"soc":85,

"charging":false,

"fleet":{---},

…

}

Similarly, to the others' Car-sharing operators, Share'ngo's record data dictionary is:

Field Name	Description	Type of value
Plate	Unique vehicle plate number	EH24795
Manufactures	Name of vehicle brand	Xindayang Ltd.
Model	Name of vehicle model	ZD 80
Active	Vehicle activity	True
intCleanliness	Internal quality of vehicle	Clean
extCleanliness	External quality of vehicle	Clean
Longitude	Longitude coordinates	12.5144
Latitude	Latitude coordinates	41.89145
Damages	Description of damages	Null
Battery	Level of charge in a range 0-100	85
Busy	Vehicle availability	False
Hidden	Vehicle signal presence	False
Rpm	Engine rotation per minute	0
Speed	Vehicle speed	0
Km	Total number of kilometres	164
Running	State of movement	False
Parking	State of parking	False
Status	Operativity status	Operative
Charging	Charging process activated	False

Table 5 - Share'ngo data source description

5.2. *Data Mapping*

Data Mapping phase has the purpose to map multiple data sources to a common target data structure, identifying common fields to all available data sources, by matching exactly each original field to the target field of raw data structure; additionally, all unavailable fields can be analysed to be treated and integrated during this the Data Collection Cycle.

In fact, while some field can be simply mapped and copied from original data source to the target data source without any, or very low, intervention, other fields require a pre-injection data transformation activity to reliably generate the data for the target table.

During the Data Mapping step, the following operations can be designed to generate the target data table:

- *Copy*: data is collected as it is from the original dataset, and added to the record for the target table, without intervention
- *Load*: data is created from a validated source, external to the original dataset, and is added to the record before it is inserted into target data table,
- *Parse*: data is extracted from a record of the original dataset, and only its part of interest is added to the record before it is inserted into target data table,
- *Process*: data generated during processing operation and is added to the record before the injection into the target table.

In the following table in each of the first three columns, fields potentially covered by the specific dataset are marked in green (plain or light) with the source description inside the cell. In the "Mobility Sharing" case the homogenous data model to merge Car2Go, Enjoy and Share'ngo datasets is the following:

Field Name	Car2Go	Enjoy	Share'ngo	Target data
Plate	Parse	Copy	Copy	Required
Operator	Load	Load	Load	Required
Model	Load	Copy	Copy	Available
Address	Copy	Copy	n/a	Available
Interior	Copy	n/a	Copy	Available
Exterior	Copy	n/a	Copy	Available
Longitude	Copy	Copy	Copy	Required
Latitude	Copy	Copy	Copy	Required
Engine Type	Load	Load	Load	Available
Damages	n/a	n/a	Copy	Unavailable
Fuel/Battery	Copy	Copy	Copy	Required
Date	Process	Process	Process	Required
Time	Process	Process	Process	Required
City	Load	Load	Load	Required
Busy	n/a	n/a	Copy	Unavailable
Hidden	n/a	n/a	Copy	Unavailable
Rpm	n/a	n/a	Copy	Unavailable
Speed	n/a	n/a	Copy	Unavailable
Km	n/a	n/a	Copy	Unavailable
Running	n/a	n/a	Copy	Unavailable
Parking	n/a	n/a	Copy	Unavailable
Status	n/a	n/a	Copy	Unavailable
Charging	n/a	n/a	Copy	Unavailable

Table 6 - data mapping for selected car-sharing services

This mapping designs, for example, that:

- the field “plate” for Car2Go go vehicles can be extracted parsing the key/value “name”: “id/plate” to extract e.g. plate value “ET836WT” parsing the string “194/ET836WT” and saving the characters 5 to 11,
- the fields “date” and “time” can be processed gathering current date and time from a reliable data source (e.g. TIMESTAMP)

Finally, the Data Mapping step is completed by defining in the previous Table 4 which are the field of interest for the target data table; in this case fields have been mapped with:

- Required: where the data are important for the Car-sharing analysis and enough data are available from original sources,
- Available: where the data are available from original sources,
- Unavailable: where few operators own the data and the analysis is inconsistent.

5.3. *Data Collection Cycle*

Data Collection Cycle, described in detail in the following paragraphs, is composed by the five following steps:

1. **Data Extraction**, having the purpose to extract data from original data sources,
2. **Data Filtering**, having the purpose to select only relevant data
3. **Data Transformation**, having the purpose to generate additional data and prepare the appropriate data format
4. **Data integration**, having the purpose to integrate all data belonging to the same vehicle in a single data record
5. **Data injection**, having the purpose to insert data into the homogeneous target table.

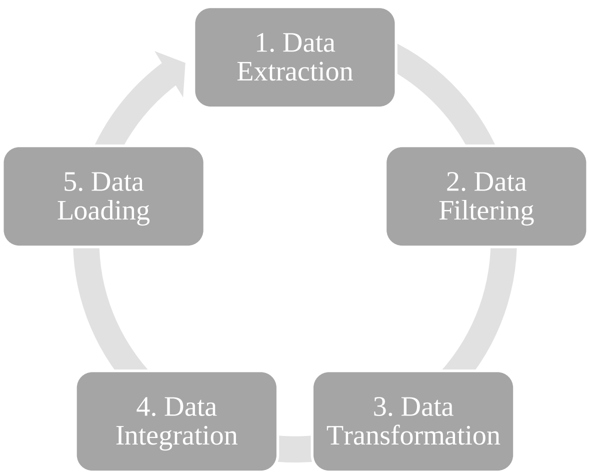

Figure 16 – the Data Collection Cycle

5.3.1. Data Extraction

Data Extraction consist in retrieving data from each of involved source in a synchronized way so that source raw data needed to populate the HDM are available in a common timeframe.

Original datasets had distinct sizes:

- Car2Go, average size is about 110 KB/iteration
- Enjoy, average size is about 145 KB/iteration
- Share'ngo, average size is about 690 KB/iteration

with a total size of about 1MB/iteration considering cycle of 3 minutes, between one iteration and the following.

Considering 1 year of data extraction the total size of extracted data would be only for the city of Rome about 175 TB; about 700 TB of data for Italy.

5.3.2. Data Filtering

To avoid data volume ineffectiveness related to processing time and data overload, a data filtering step is applied so that only valuable fields required to the analysis are selected to be aggregated into a record that is injected in the HDM.

The homogeneous target dataset[xxvii] structure for Car-sharing homogeneous raw data record is the following:

Plate	Lon	Lat	City	Operator	Date	Time	Aut_lev	Vehicle	Model	Engine
R	R	R	R	R	R	R	R	A	A	A
E/C/C	C/C/C	C/C/C	L/L/L	L/L/L	P/P/P	P/P/P	C/C/C	L/C/C	L/L/C	L/L/C

Table 7 - target raw data record

Legenda:

- R: required
- A: available
- E: parsed
- C: copied
- L: loaded
- P: processed

Considering this filtering the target dataset in HDM has a total size per iteration, including newly generated data such as "operator", "city", "vehicle", "date", "time" of about 0,39 MB, versus the size of original data sources of about 1 MB per iteration, with a reduction of data occupancy of about 60%.

5.3.3. Data Processing

During this step the operations of "copy", "parsing", "loading" and "processing" are performed, and all data records are formatted to fit the target homogeneous data row.

[xxvii] In the homogeneous dataset size of each entry is about 0,31 kB, considering about 1271 vehicles (including scooters), 0,39 MB about 60% of original raw data size, which was composed by 380.550 rows

Of course, considering the different nature of source data each source, representing data of vehicles of a specific Car-sharing operator performs dedicated set of transformations.

5.3.4. Data Transformation

Of course, considering the different nature of source data each source, representing data of vehicles of a specific Car-sharing operator performs dedicated set of transformations.

5.3.5. Data Loading

Last phase of the iteration is the Data Injection that inserts the data rows into the Target Data Table of the HDM.

In this case it is critical to opportunely manage injection duration; in this case a frequency during the data acquisition cycle a period of 3 minutes per iteration has been set, while the performance filtering processing and injection cycle that was performed in less than 1 minute.

The chosen trade-off is reliable enough because avoids the loss of vehicle movements, since only the check-in/check-out process requires 3 minutes, additionally the average error value in estimating the stop duration time is 1.5 minutes that considering the average stop duration will introduce an error of less than 1%.

This choice has been made because it represents a good trade-off between the Data Refresh Frequency, Data Extraction Amounts and Data Collection Cycle duration; using less than 1 minutes versus 3 minutes extraction frequency allows an easy and quick data recovery in case of system fault during the Injection Cycle.

6. Defining the value of Urban Areas

To calculate the value of Urban Areas four key steps process must be performed from the Homogeneous Data Table and others sources of information.

1. Organizing the Urban Area in different cells representing the city space
2. Processing vehicle data to extract data about Stops
3. Performing a spatial analysis to join Urban Area and vehicle data
4. Processing vehicle data to extract data about Trips

6.1.1. *Real data case study: City of Rome*

This case study experiments[1] the previously described model applied to the city of Rome (Italy).

The study is based on real data collected during the T observation period of 49 days (T=49) ranging from 2016, April 28th to 2016, June 16th, where the vehicles of three distinct free-floating one-way Car-sharing operators' active in Rome have been monitored.

During the observation period, we collected 287.975 stops observation referring to 1.271 distinct vehicles. All the stops have been observed in a geographical area ($Grid_k$) that has been partitioned in 636 cells (K = 636)

Operator *i*	Number of Vehicles	Number of Stops	Avg ASD (min)
1	95	9.661	577.6
2	572	126.093	258.4
3	604	152.221	134.2

Table 8 - Car-Sharing Vehicle Dataset

All spatial analysis and algorithms have been performed using QGis Desktop 2.18.3[82].

6.2. *Urban Area Organization*

6.2.1. *Urban Area definition*

In this case $Grid_k$ has been defined from existing Taxable Areas defined for the City of Rome by Rome Municipality[83], aggregating them where, as in city centres, the taxable areas are too small, to be at least 1.500 square meters each, and build larger grid cell.

Since cells defined by Taxable Areas can vary in different city zones (for instance we have very small cells of few hundred squared meters in the Centre of City while very large areas of several squared kilometres in more peripheral zones), a minimum surface of at least 1.500 square meters each has been defined to identify a consistent Urban Area that in those case will be built aggregating smaller cells.

Additionally, the cells defined using Taxable Areas are generally irregular and the shape can in some cases be concave; these shapes might have the issue that the position of their centroids is outside the shape. To avoid this problem, each cell was converted in a convex polygon, using the specific QGis function, so to be sure that the centroid of each polygon falls inside the same cell area.

Finally, the centroid of each cell can be calculated using the QGis function "Polygon Centroid"

The Grid for the city of Rome has been defined partitioning the city area in squared cells, using the QGis function “Grid”.

Rome City Urban Area has been defined using the following GPS coordinates to define boundary layers of the map:

- x_{min}: 12.2341551399
- x_{max}: 12.8558382321
- y_{min}: 41.6554062257
- y_{max}: 42.1409693500

The grid partition has been finally performed selecting the destination layer and setting x at 0.005 degrees, corresponding to about 450 meters per cell side, and saving the result in new file.shp.

Figure 17 shows the cell distribution of the Grid used to map the city of Rome.

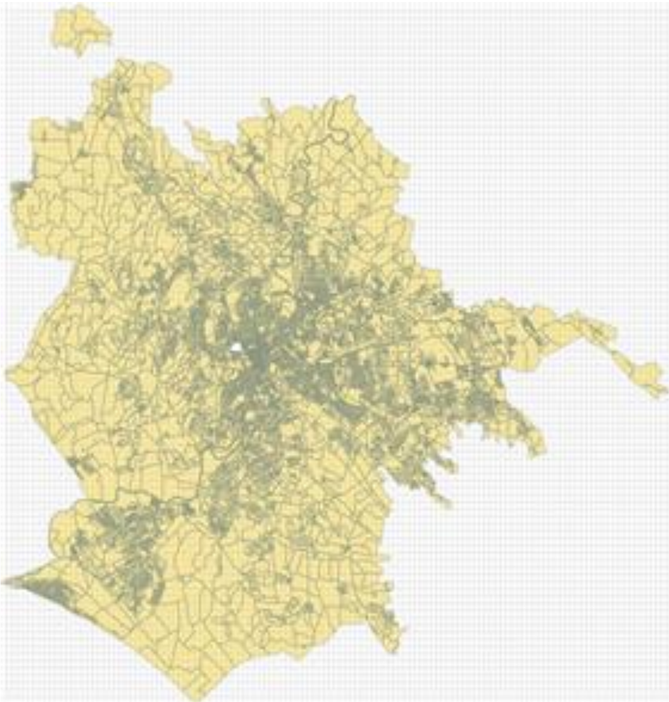

Figure 17 - grid definition for Rome case study

6.3. Generating Stops

6.3.1. Stop definition

Let V be the number of different vehicles observed at list once during the observation window T.

Let K be the number of different cells of the grid partition defined in the Urban Area. For each vehicle define O_j as the number of observations of the *j-th* vehicle during T. Each vehicle observation is performed checking the vehicle position at predefined time intervals. In this study a time interval of 3 minutes has been used.

Let o the number of time slots in which can be portioned the day.

A *stop* is defined as the *i-th* observation of the *j-th* vehicle formalized with the following *tuple*

$$Stop_{ij} = < lat_{ij}, lon_{ij}, aut_{ij}, st_{ij}, et_{ij}, cso_{ij} >$$

where:

- lat_{ij} is the latitude of the *i-th* stop of *j-th* vehicle
- lon_{ij} is the longitude of the *i-th* stop of *j-th* vehicle
- aut_{ij} is the autonomy at the *i-th* stop of *j-th* vehicle
- st_{ij} is the timestamp, date and time, of start of the *i-th* stop of *j-th* vehicle
- et_{ij} is the timestamp, date and time, of the end of the *i-th* stop of *j-th* vehicle
- cso_{ij} is the car-sharing operator id at the *i-th* stop of *j-th* vehicle

We consider the $Stop_{ij}$ belonging to the *k-th* cell of the partition if the point defined by (lat_{ij}, lon_{ij}) is contained in the *k-th* cell area. We refer to *Stop(k)* as the number of tuples $Stop_{ij}$ belonging to the *k-th* cell.

6.3.2. Stop data processing

Vehicles positions are checked using GPS coordinates latitude (*lat*) and longitude (*lon*) of each of the vehicle, since there are several movements per vehicle during the day and positions of the vehicle evolve continuously. In this research the following assumptions have been made to address the right position evaluation.

Problem 1: define the right frequency of data collection from original sources to have accurate evaluation of position changes.

Solution 1: establish a minimal time interval to check if vehicle position has changed, considering that each movement must have a duration higher than *n* minutes to be considered an effectively valid trip. Consequently, extracting data from native sources with a frequency of *n* minutes all position changes are tracked. In this study the minimal trip duration has been set to 3 minutes (n = 3).

Problem 2: avoid errors in position acquisition inducted by GPS precision defects. In fact, as known the GPS has a natural error in accurately defining vehicle positioning between 10 and 100 meters.

Solution 2: establish that each movement must have a distance higher than the maximum error of GPS sensor to be considered a valid trip.

Consequently, extracting data from native sources with a position tolerance to evaluate vehicle position changes inaccuracies of GPS measurement system are avoided.

During stop processing phase only, positions' changes are stored in the STOP table and vehicle data record is updated.

Vehicle position evaluation is conceptually performed using the following logic:

```
RUN PERIOD t
FOR EACH VEHICLE i
        IF (Pt(lat(i), lon(i)) - Pt-1(lat(i), long(i))) < gps_error
                UPDATE (Vehicle Stop Duration)
        ELSE
                UPDATE (Vehicle Trip Start)
END
```

Figure 18 - stop data calculation logic

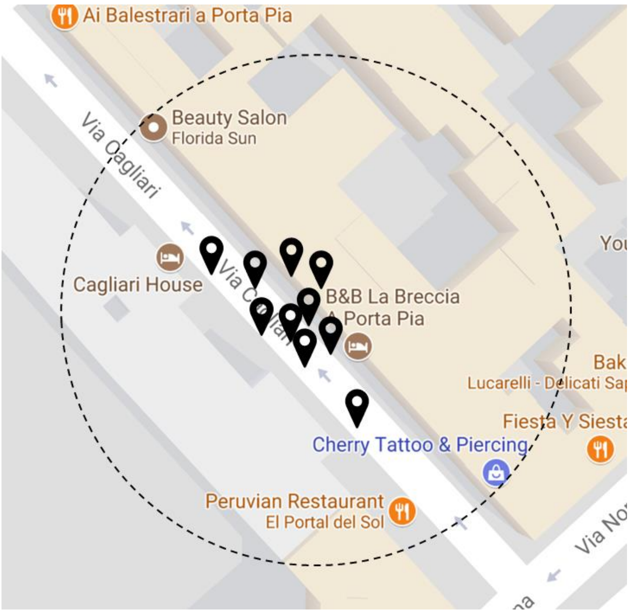

Figure 19 – Example of GPS position tolerance analysis

Considering *n* number of periods in which the vehicle maintains its position, *d* the duration of the interval between a position check and the following p_i the *i-th* period, the Vehicle Stop Duration (vsd) is so calculated.

$$vsd = \sum_{i=1}^{n} p_i * d$$

The following table show an example of the **vehicle-stop** table allowing the analysis of stop characteristics of Car-sharing Operators, Vehicle and Stop Events.

Id	vehicle_id	city	latitude	longitude	area_id	fuel	Soc	status	stop_time	duration	updated_at
115275	EX591DK	roma	41.89859	12.44271	1791	100	0	0	25/04/2016 22:29	130	26/04/2016 10:51
115276	EX592DK	roma	41.90328	12.42898	1512	81	0	0	25/04/2016 22:29	130	26/04/2016 10:51
115278	EX594DK	roma	41.91228	12.54697	1926	100	0	0	25/04/2016 22:29	130	26/04/2016 10:51
115279	EX595DK	roma	41.88136	12.47377	636	100	0	0	25/04/2016 22:29	130	26/04/2016 10:51
115281	EX597DK	roma	41.87847	12.50508	2671	54	0	0	25/04/2016 22:29	130	26/04/2016 10:51
115284	EX601DK	roma	41.89775	12.42997	2337	60	0	0	25/04/2016 22:29	130	26/04/2016 10:51
115285	EX602DK	roma	41.92498	12.48572	2238	81	0	0	25/04/2016 22:29	130	26/04/2016 10:51
115286	EX603DK	roma	41.90584	12.55395	889	39	0	0	25/04/2016 22:29	130	26/04/2016 10:51
115287	EX605DK	roma	41.88522	12.48227	629	100	0	0	25/04/2016 22:29	16	26/04/2016 10:51
115288	EX605DK	roma	41.83280	12.46279	138	100	0	0	25/04/2016 23:22	77	26/04/2016 10:51
115289	EX606DK	roma	41.82003	12.45438	806	100	0	0	25/04/2016 22:29	130	26/04/2016 10:51
115290	EX607DK	roma	41.92270	12.51363	1373	72	0	0	25/04/2016 22:53	73	26/04/2016 10:51
115291	EX607DK	roma	41.92942	12.51855	2244	72	0	0	26/04/2016 00:25	13	26/04/2016 10:51
115292	EX608DK	roma	41.86077	12.44812	1893	51	0	0	25/04/2016 22:29	130	26/04/2016 10:51
115293	EX609DK	roma	41.88124	12.47900	629	27	0	0	25/04/2016 22:29	130	26/04/2016 10:51
115294	EX610DK	roma	41.94473	12.53473	19	100	0	0	25/04/2016 22:29	130	26/04/2016 10:51
115295	EX612DK	roma	41.86347	12.49271	2455	24	0	0	25/04/2016 22:29	120	26/04/2016 10:51
115296	EX613DK	roma	41.86262	12.43965	1885	100	0	0	25/04/2016 22:29	130	26/04/2016 10:51

Table 9 - Stop data sample

Table above has the following structure.

Field Name	Description
Id	Unique stop identification number defined at data platform level
Vehicle_id	Vehicle plate unique identifier
City	Name of the city where the stop occurs
Latitude	Latitude of stop position
Longitude	Longitude of stop position
Area_id	Identifier of the urban area where the vehicle stopped
Fuel	Fuel level in case of combustion engine
Soc	State of charge in case of electric vehicle
Status	Vehicle status
Stop_time	Day and time when stop started
Duration	Stop duration in minutes
Updated_at	Date and time last record updated

Table 10 – Stop data dictionary

6.4. Spatial Analysis

A grid-matching operation now is performed in order to associate one of pre-defined centroids to each cell of the grid. Now that the Urban Grid is defined the Stop Data Table is associated to match each stop position to one grid cell and consequently to a defined urban area. All stops are consequently associated and mapped to a cell, belonging to an Urban Area which uniquely identified by its centroid.

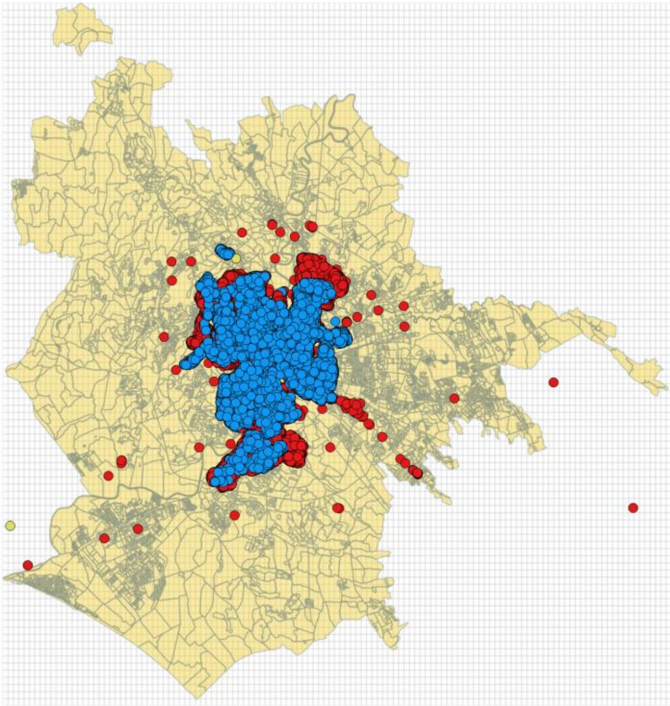

Figure 20 - vehicle position mapping on Rome city grid.

Once the table is associated to the map it will be possible to perform different position analysis with different Car-sharing Operators, or different Analysis in different time slots.

In this case study the above defined metrics SD(k, o), ASD(k, o) and UAV(k, o) have been calculated considering unique time slot, assuming a homogeneous pattern during all day, consequently ignoring daily peak hours in urban mobility demand. Additionally, seasonality has not been considered considering the partial timeframe of the available dataset. Further the study will also be focused in analysing results of the model using different *time slots* during the day and impact of seasonality.

Urban Area Value (also mentioned as UAV) is a function of two spatial key metrics defined for each *k-th* cell of the grid of the city:

a. Stop Density
b. Average Stop Duration

6.4.1. *Defining Stop Density as a spatial key metric*

Stop Density (SD) for the k-th Grid Cell calculated as:

$$SD(k,o) = \frac{Stop(k,o)}{\sum_{k=1}^{K} Stop(k,o)} \qquad k = 1, \dots, K$$

This section describes the results of the research where the above described model has been applied using the collected data in the period T, to the case study of Rome urban area.

Figure 13 shows the results of the vehicle Stop Density (SD) analysis performed on all collected data referring to the three operators together and identifying different level of density with the percentage of total stops belonging to each cell.

The resulting spatial analysis clearly states a different status for $Grid_k$ cells ranging from hot zones (identified with dark red cells) to cold zones (identified with dark blue cells). The dark red area in the middle of the map is Termini railway station.

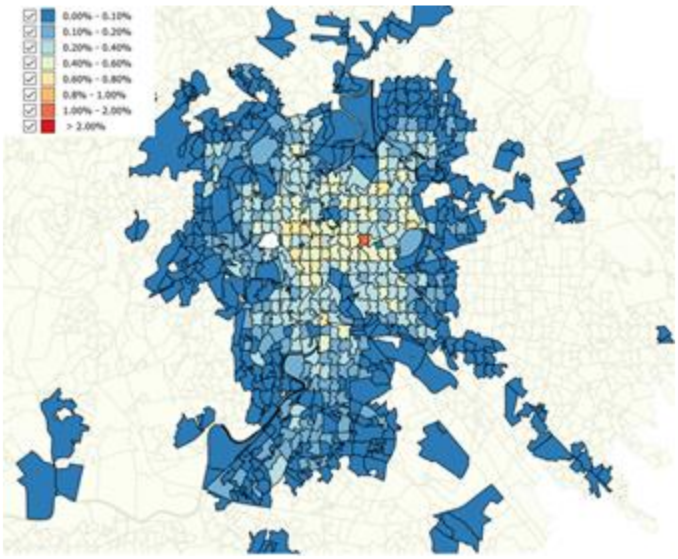

Figure 21 - Stop Density map.

More precisely dark red areas identify zones with more than 4.000 stops during T (more than 50 stops in a day)) and dark blue cells identify zones with less than 1.000 during T (less than 12 stops in a day). The analysis confirms that the SD metric is important in evaluating the UAV of different Urban Areas.

6.4.2. *Defining Average Stop Duration as a spatial key metric*

Average Stop Duration (ASD): is the average duration of stops in the k-th Grid Cell, calculated as:

$$ASD(k,o) = \frac{\sum_{j=1}^{V}\sum_{i=1}^{o_j}\begin{cases}(te_{ij} - ts_{ij}) \; if \; (lat_{ij},\, lon_{ij}) \cap Grid_k \\ 0 \qquad\qquad\qquad\qquad otherwise\end{cases}}{Stop(k,o)}$$

$$k = 1, \dots, K$$

In the model, UAV depends on the two-main metrics defined as *Stop Density* associated to each specific *Urban Area* and *Average Stop Duration* associated to the same *Urban Area*. It very important to properly and effectively define the concept of Grid to accurately perform a proper Spatial Analysis.

Elaborating the second key metric represented by the Average Stop Duration (ASD), calculated from all collected data referring of the three operators together, we obtain the results shown in Figure 22. Also, the spatial analysis on the ASD shows that $Grid_k$ cells have different status value. In fact, dark red areas identify hot zones where the Average Stop Duration is very short and dark blue zones identify cold zones where ASD is very long; all gradient ranging between dark red cells to dark blue cells identify cell status with gradually longer durations.

This second analysis confirms that the ASD is also important in evaluating the UAV of different urban areas.

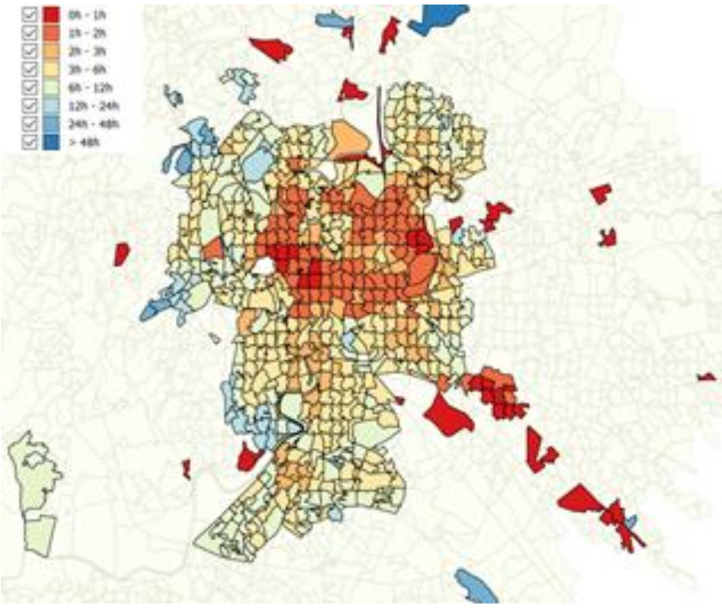

Figure 22 - Average Stop Duration map.

Consequently, results confirm that with aggregated data of the three Car-sharing services it is possible to breakdown the city in uniquely defined cells belonging to a $Grid_k$ to which a UAV can be associated, based on end-user mobility behaviour.

6.4.3. *Defining Urban Area Value*

Consequently, it is possible to define Urban Area Value (UAV) as a *function* of the values that *Stop Density (SD) and Average Stop Duration (ASD)* assume in each k cell of $Grid_k$.

In this case the Urban Area Value (UAV) has been defined as the ratio between Stop Density and Average Stop Duration.

$$UAV(k,o) = \frac{SD(k,o)}{ASD(k,o)}$$

In fact, it is intuitive the UAV increases incrementing SD and decreasing ASD; consequently, high demand urban areas will have high SD and low ASD.

Finally, UAV is calculated for each k cell of $Grid_k$ and classified, using a five-class representation, to distribute the UAV in a discrete scale of High, Medium-High, Medium, Medium-Low and Low value cells. Figure 23 below shows the results of the classified UAV distribution on a spatial thermographic map, obtained with aggregated data of all the three Car-sharing services analysed.

This first result allows us to show the existence of urban zones with crucial different demand potentials. In such a way, it is possible to rank and sort distinct city zones from high to low demand potential areas, classified in five classes.

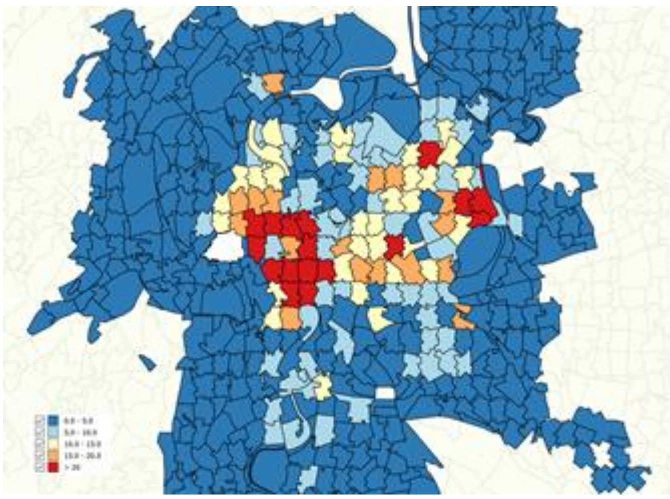

Figure 23 - Five Classes Aggregated UAV thermographic map

In figure 15 we can still recognize among dark red areas Termini railway station, Tiburtina railway station, Vatican City boundaries, Trastevere, Piazza Bologna and some others.

To clarify the difference in using aggregated data versus self-generated data it has also been performed the UAV analysis for the Car-sharing operator 2 (Op2) and the Car-sharing operator 3 (Op3). Data about the third operator are not shown due to the small number of data available and the small coverage of urban area at time of the analysis.

The following Figure 24 shows the five classes UAV analysis applied to SD_Op2 and ASD_Op2 using data belonging only to the Car-sharing operator 2.

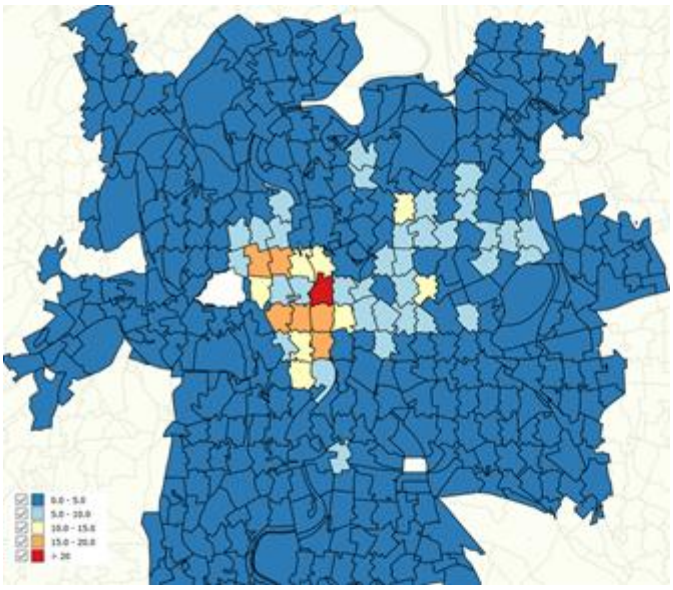

Figure 24 - UAV thermographic map for Operator 2 data

Reading the map above it is clear that the UAV(Op2) describes, for the Car-sharing operator a perceived mapping of Urban Area Value significantly different from the city aggregated UAV map (Figure 23) calculated considering all SD and ASD data. Consequently, it is possible to clearly see the lack of objective data regarding the Urban Area Value can drives to a wrong assignment of UAV to specific cells.

Similarly, in Figure 25, are shown analysis results of the five classes UAV applied to SD_Op3 and ASD_Op3 using data belonging only to the Car-sharing operator 3.

Reading the map, it clear that the UAV_Op3, even if similar to the city UAV, describes a different perception of Urban Area Value both from the city UAV map (Figure 23) and for the UAV_Op2 map (Figure 24). Additionally, comparing city UAV to individual UAV maps, it is also possible, for the Car-sharing operators, to assign a value to urban area not covered by their service, allowing them to evaluate the interest in extending service coverage to specific not-served urban area.

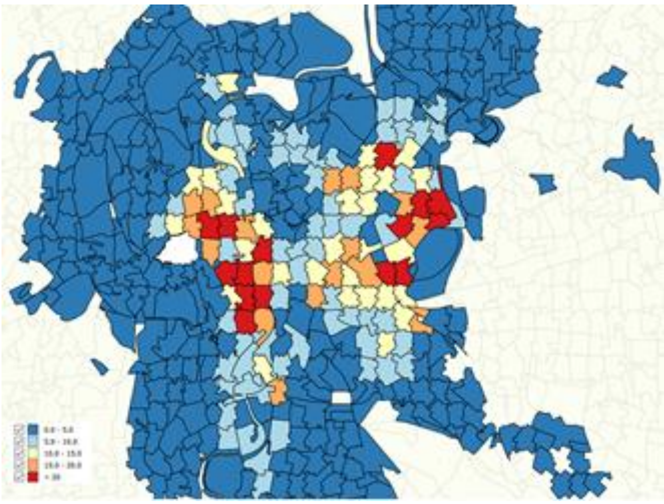

Figure 25 - UAV thermographic map for Operator 3 data

6.5. Generating Trips

6.5.1. Trip definition

A trip is defined, as the *l-th* variation of *j-th* vehicle status between two consecutives stops $(Stop_{lj}, Stop_{(l+1)j})$ by the following tuple:

$$Trip_{lj} = < c_{lj}, o_{lj}, d_{lj}, ts_{lj}, te_{lj}, dist_{lj}, >$$

where:

- c_{lj} is the city where the *i-th* trip of the *j-th* vehicle occurs
- o_{lj} is the identifier of the grid where (lat_{ij}, lon_{ij}) belong (origin)
- d_{lj} is the identifier of the grid where $(lat_{(i+1)j}, lon_{(i+1)j})$ belong (destination)
- ts_{lj} is the trip start corresponding to et_{ij}
- te_{lj} is the trip end corresponding to $st_{(i+1)j}$
- $dist_{lj}$ is the Euclidean distance between (lat_{ij}, lon_{ij}) and $(lat_{(i+1)j}, lon_{(i+1)j})$

6.5.2. Trip data processing

Trips are calculated as the difference between two consequent stops. All trips are stored in the TRIP_TABLE described in the following dataset sample.

Trip_id	Vehicle_id	city	prev_stop_id (ORIGIN)	curr_stop_id (DEST)	start_time	arrival_date	distance	Dur	cons	refuel	updated_at
30383	EW776BJ	roma	459	1931	25/04/2016 22:46	25/04/2016 23:34	5,09300	49	2	F	26/04/2016 10:51
30384	EW776BJ	roma	1931	531	25/04/2016 23:34	26/04/2016 00:06	2,45800	32	1	F	26/04/2016 10:51
30385	EW783BJ	roma	729	1266	25/04/2016 23:30	26/04/2016 00:10	2,35000	16	2	F	26/04/2016 10:51
30386	EW796BJ	roma	2502	1990	25/04/2016 22:30	25/04/2016 23:34	4,29400	37	2	F	26/04/2016 10:51
30387	EW802BJ	roma	908	1931	25/04/2016 22:30	25/04/2016 23:01	2,77700	28	1	F	26/04/2016 10:51
30388	EW802BJ	roma	1931	2243	25/04/2016 23:01	25/04/2016 23:26	1,62600	16	0	F	26/04/2016 10:51
30389	EW804BJ	roma	1792	1938	25/04/2016 22:30	25/04/2016 23:38	3,32800	33	-1	T	26/04/2016 10:51
30390	EW804BJ	roma	1938	785	25/04/2016 23:38	26/04/2016 00:10	2,02100	28	2	F	26/04/2016 10:51
30391	EW810BJ	roma	1089	1100	25/04/2016 22:42	25/04/2016 23:30	0,34300	45	3	F	26/04/2016 10:51
30392	EW810BJ	roma	1100	2459	25/04/2016 23:30	26/04/2016 00:29	6,14600	46	4	F	26/04/2016 10:51

Trip_id	Vehicle_id	city	prev_stop_id (ORIGIN)	curr_stop_id (DEST)	start_time	arrival_date	distance	Dur	cons	refuel	updated_at
30393	EW824BJ	roma	1373	661	25/04/2016 22:34	25/04/2016 23:58	7,67200	28	2	F	26/04/2016 10:51
30394	EW830BJ	roma	2005	2848	25/04/2016 23:18	26/04/2016 00:14	7,91400	44	7	F	26/04/2016 10:51
30395	EW835BJ	roma	320	459	25/04/2016 22:30	26/04/2016 00:06	6,16100	31	3	F	26/04/2016 10:51
30396	EW839BJ	roma	848	1090	25/04/2016 22:34	26/04/2016 00:06	2,94700	31	-1	T	26/04/2016 10:51
30397	EW842BJ	roma	2896	721	25/04/2016 22:30	25/04/2016 23:34	7,53400	45	0	F	26/04/2016 10:51
30398	EW845BJ	roma	2257	2179	25/04/2016 22:30	25/04/2016 22:54	1,00200	20	2	F	26/04/2016 10:51
30399	EW845BJ	roma	2179	1030	25/04/2016 22:54	25/04/2016 23:23	2,95100	25	0	F	26/04/2016 10:51
30400	EW845BJ	roma	1030	161	25/04/2016 23:23	25/04/2016 23:50	1,90100	20	1	F	26/04/2016 10:51
30401	EW862BJ	roma	848	1715	25/04/2016 22:30	25/04/2016 23:30	4,69800	29	1	F	26/04/2016 10:51
30402	EW869BJ	roma	2459	20	25/04/2016 22:30	25/04/2016 23:06	0,89000	32	2	F	26/04/2016 10:51
30403	EW869BJ	roma	20	2625	25/04/2016 23:06	26/04/2016 00:10	7,77000	36	3	F	26/04/2016 10:51
30404	EW873BJ	roma	2671	662	25/04/2016 22:30	25/04/2016 23:14	6,04500	37	1	F	26/04/2016 10:51
30405	EW873BJ	roma	662	2842	25/04/2016 23:14	25/04/2016 23:54	4,01500	20	2	F	26/04/2016 10:51
30406	EW873BJ	roma	2842	2644	25/04/2016 23:54	26/04/2016 00:22	1,51800	16	2	F	26/04/2016 10:51
30407	EW960BH	roma	1373	1523	25/04/2016 22:54	25/04/2016 23:30	3,89700	29	0	F	26/04/2016 10:51

Table 11 - trip sample dataset

Previous table has the following data features:

Field Name	Description
trip_id	Unique trip identification number defined at data platform level
vehicle_id	Vehicle plate unique identifier
City	Name of the city where the stop occurs
prev_stop_id	Origin cell of the trip
curr_stop_id	Destination cell of the trip
start_time	Timestamp of beginning of trip
arrival_date	Timestamp of end pf trip
Distance	Estimate of distance between origin and destination
Dur	Trip duration as difference between start_time and arrival_date
cons	Autonomy consumption
Refuel	Refueling of the vehicle
updated_at	Date of record update

Table 12 - TRIP_TABLE dataset description

The **Origin-Destination Matrix (O-D Matrix)** can be created, filtering from the TRIP_TABLE the columns *vehicle_id*, *prev_stop_id*, *curr_stop_id.*

Each **Vehicle Trip Duration** *(vtd_{ij})* of trip *j* with vehicle *i* can be approximately be calculated as the difference between [arrival_date] and [start_time] with the following formula

$$vtd_{ij} = arrival\_date_{ij} - start\_time_{ij}$$

7. Vehicle Commercial Model in Car-sharing services

According to Revenue Management principles having the goal to maximize profit for the car-sharing service a commercial model for vehicles must be represented.

In brief a commercial model, which is the evaluation of revenues, costs and profits of a vehicle during its operational period, can be based the Vehicle Lifetime Value (VLV).

Vehicle Lifetime Value, in case of Sharing Mobility services such as Car-sharing, can be defined as the difference of Total Lifetime Incomes (TLI) and Total Cost of Ownership (TCO).

$$VLV = TLI - TCO$$

Where:

- TLI is defined as the sum of all service incomes generated during the operational period,
- TCO is defined as the evaluation of the total cost to own a vehicle for a predefined operational period, typically ranging from 2 to 5 years, which includes all type the expenses e.g. in fuel, insurance, maintenance, repairs, service, interest on loan payments as well as the losses incurred due to depreciation of the car at the end of the same period.

7.1. Total Cost of Ownership in Car-sharing services

TCO can be spilt in five main cost categories:

- Acquisition Costs, which is the cost that the Car-sharing Operator pays the vehicle manufacturer for vehicles property after adjusting for discounts, incentives, closing costs and other necessary expenditures but before sales taxes[84].
- Recurring Fixed Costs, which are the costs non-dependent from the user behaviours, and are mainly related to assurance, taxes, fixed parking costs, traffic limited zones access fees, weekly cleaning and programmed maintenance; these costs can be calculated as an average cost per kilometre as average of fleet statistics (standard fixed costs)
- Extraordinary Variable Costs, which are costs that cannot be programmed, and which partially depend from user behaviour (e.g. accidents); these costs can be, also, calculated as an average cost per kilometre based on the fleet population (standard extraordinary costs)
- Variable Operational Costs, that are related to usage intensity and are impacted by users' behaviour such as fuel
- Relocation Cost, cost related to optimize vehicle demand/offer fitting Urban Areas, acting to reduce the imbalance problem.

A vehicle commercial model with some similarities to the one introduced in the following chapter has been used by Boyac et al. (2014) for different goals.

TCO can be calculated using the following cost variables

$$TCO = VIC(km, VRV(v, t, km)) + VMC(km, t) + VFC(km) + VLC(t) + VTC(t) + VPC(t, p) + Avg(VRC) + VCC(t) + VRLC(d, t, ve, fc, hsc) + VDC(v, t, km)$$

with

- VIC: Vehicle Insurance Cost, is the recurring cost of annual insurance price that must be paid per each vehicle depending on the yearly distance performed in km, and the VRV; VIC is paid annually and will generally be the same for all fleet vehicles,
- VRV: Vehicle Residual Value, which is the residual asset value when the vehicle is brought out of production; VRV depends on vehicle brand, model and duration of the operating period,
- VMC: Vehicle Maintenance Cost, is a recurring cost based on distance (km) and age of vehicle,
- VTRC: Vehicle Traction Cost, is a recurring cost based on the distance depending on vehicle efficiency based on fuel cost (FC), kilometres per fuel unit (kfu) and distance (d) (fuel can be electric charge or oil refill),
- VTC: Vehicle Taxation Cost, recurring cost of annual taxes that must be paid per each vehicle depending on vehicle brand and model (n*tax), where n is the number of years and tax is the yearly tax amount,
- VPA: Vehicle Purchase Amount, representing the purchase cost of the vehicle including the car-sharing setup and configuration. This is a one-time cost occurring when the vehicle is bought by the Car-sharing service provider and depends on the vehicle brand and model,
- VPC: Vehicle Parking Cost, is the parking fee that annually must be paid to municipalities and/or parking hubs for usage of parking spaces,
- VRC: Vehicle Repair Cost, is an extraordinary cost based on vehicle repair needs that can be estimated in average considering average fleet repair cost,
- VCC: Vehicle Cleaning Cost, is the vehicle cleaning cost that is generally calculated on the basis of a total of 52 cleaning operations estimated with a unit cost of 7 €
- VDC: Vehicle Depreciation Cost, is the loss of value of the vehicle during car-sharing service period
- VRSC: Vehicle Relocation Staff Cost, is the cost that the Car-sharing operator must sustain to move the vehicle from a cold to hot spots and can be estimated by the product of the average number of relocation and the cost of a relocation movement (e.g in Rome about15 €/each).

Supposing that a Car-sharing service fleet has N vehicles, with Y being the average number of vehicle operating years, for *i-th* vehicle the total cost of ownership can be calculated as

$$TCO_i = VDC_i + \sum_{y=1}^{Y}(VTC_{iy} + VIC_{iy} + VPC_{iy} + VCC_{iy}) + Avg(VTRC_i) * kfu * D + Avg(VRC_i) + Avg(VMC_i) + Avg(VRSC_i)$$

with

$$VDC_i = VPA_i - VRV_i$$

Consequently, the Average Vehicle Cost per minute for the Car-sharing operator can be calculated with following equation.

$$AVCpm = \frac{\sum_{i=1}^{N} TCO_i}{N * Y * T}$$

being T the yearly average operating minutes[xxviii].

$$T = \frac{\sum_{i=1}^{N} \sum_{j=1}^{365} m_{ij}}{N}$$

so with:

$$AVCpm = \frac{\sum_{i=1}^{N} TCO_i}{N * Y * \frac{\sum_{i=1}^{N} \sum_{j=1}^{365} m_{ij}}{N}} = \frac{1}{Y} * \frac{\sum_{i=1}^{N} TCO_i}{\sum_{i=1}^{N} \sum_{j=1}^{365} m_{ij}}$$

Considering that *vehicle fixed operating costs* (VFOC) are calculated as:

$$VFOC_i = \sum_{y=1}^{Y} (VTC_{iy} + VIC_{iy} + VPC_{iy} + VCC_{iy}) + \frac{1}{N} \sum_{j=1}^{N} \sum_{y=1}^{Y} VRC_{ijy}$$

with

$$Avg(VRC_i) = \frac{1}{N * Y} \sum_{j=1}^{N} \sum_{y=1}^{Y} VRC_{ijy}$$

TCO can also be expressed as

$$TCO_i = VDC_i + VFOC_i + Avg(VTRC_i) * kfu * D + Avg(VMC_i) + Avg(VRSC_i)$$

7.2. Focus on the Relocation Cost (VRLC).

It is also known that in case of staff relocation, vehicle movement is unproductive, so the formula based on variable cost can be split to evidence productive (*user*) and unproductive (*staff*) cost components, based on trip distance performed by the user and distance performed by staff for relocation.

Considering the objective of this research, the positive contribution that the User-based Relocation Model can give to One-way Free-floating Car-sharing Operators (OFCOs) is focused in reducing the relocation cost and maximising revenues, active users and productive trips.

Consequently, it is necessary to directly link VRLC to an optimization tariff model.

The first step in achieving this goal requires to define a generalized formula to **calculate the VRLC.**

As already expressed, VRLC is a function of:

- *distance* (D_{Staff}) driven during the relocation event, measured in kilometres,

[xxviii] If the average cost per minute is calculated using every minute of the year an “Utilization Rate” *k* must be considered in determining the tariff per minute. Average Vehicle Utilization is the percentage of minutes of fleet usage in a defined period. In car-sharing fleet utilization is generally low, between 8-12%. 525.600 is the theoretical maximum number of minutes of vehicle operability.

- *traction unit cost* (Tuc) which is the cost per unit of traction source used to power the vehicle such as fuel or gas in case of endothermic vehicles or electric power in case of electric vehicles,
- *vehicle efficiency* (Ve) calculated as the unit of traction per kilometre (tuc/km), which is the amount of traction source necessary to cover in average the distance of 1 kilometre; considering that vehicle used in OFCOs models by Car-sharing operators can have multiple engines types, even in the same fleet (e.g. Car2Go in Munich) this definition applies, for example, to l/km in case of petrol, m^3/km in case of gas and uoc/km in case of electric vehicles,
- *service relocation cost (Src)*, which the cost per minute of personnel dedicated to relocation movements
- and time (t) calculated in minutes.

Considering below definitions and assumptions the *i-th* vehicle TCO formula can also be expressed as

$$TCO_i = VDC_i + VFOC_i + Avg(VMC_i) + Avg(VTRC_i) * Ve_i * D_i + \ Avg(VRSC_i)$$

with the total distance (D_i) performed by the i-th vehicle considered as the sum of total distance performed by users (D_i^u) and the total distance performed by staff (D_i^s).

$$D_i = D_i^u + D_i^s$$

the TCO of the *i-th* vehicle can be expressed as

$$\begin{aligned} TCO_i = VDC_i + VFOC_i + Avg(VMC_i) + Avg(VTRC_i) * Ve_i * D_i^u + \ Avg(VTRC_i) * Ve_i * D_i^s \\ + Avg(VRSC_i) \end{aligned}$$

If the i-th vehicle is considered, VMC_i can be expressed as:

$$VMC_i = \ MC_{km} * \ \frac{D_i^u + D_i^s}{D_i} = \frac{MC_{km}}{D_i} * (D_i^u + \ D_i^s) = \frac{MC_{km}}{D_i} * D_i^u + \ \frac{MC_{km}}{D_i} \ * \ D_i^s$$

considering the VMC_{km}^u associated to the user and VMC_{km}^s associated to the staff

$$VMC_{km}^u = \frac{MC_{km}}{D_i} \ D_i^u$$

$$VMC_{km}^{s} = \frac{MC_{km}}{D_i}\ D_i^s$$

and the relocation cost of the *i-th* vehicle is:

$$VRLC_i = ((Avg(VTRC_i) * Ve_i) + \frac{MC_{km}}{D_i}) * D_i^s + Avg(VRSC_i)$$

This formula can be more explicit in evaluating Vehicle Relocation Service Cost, $VRSC_i$ if staff relocation time is tracked at vehicle level. In this case, considering the:

- SC_m as the minute cost of Staff in charge of relocation
- t_i as the time spent in relocation activities by dedicated staff

$$VRSC_i = SC_m * t_i$$

$$VRLC_i = ((Avg(VTRC_i) * Ve_i) + \frac{MC_{km}}{D_i}) * D_i^s + SC_m * t_i$$

In user-based relocation models the staff involvement can be considered as cost saving while there might be an additional distance cost considering that the user might bring the vehicle to a farer place than the one planned by staff to relocate the vehicle.

Considering the *j-th* trip the $VRLC_{i,j}^{u}$ formula can be expressed as

$$VRLC_{ij}^{u} = ((Avg(VTRC_i) * Ve_i) + \frac{MC_{km}}{D_i}) * (D_i^s + \left(D_{ij}^u - D_{ij}^s\right))$$

7.3. Base Tariff Calculation

As known in main Car-sharing systems the business model is frequently based on Trip Revenue Amount (*R*) calculation as the product of vehicle rent time rt (which, in one-way free-floating services, is generally measured in minutes) and of a standard tariff (t_{STD}), with the following formula:

$$R = t_{STD} * rt$$

In this research, the first approximation of t_{STD} will be calculated starting from the Vehicle Cost, considering that revenues for the Car-sharing operator will also include a standard margin on the average cost of the service; in this case the formula can be written as follows:

$$R = (m_{STD} + c_{STD}) * rt$$

or

$$R = m_{STD} * rt + c_{STD} * rt = M + C$$

witht

$$C = c_{STD} * rt$$

which in this case is equal to

$$C = AVC_{pm} * rt$$

In case of Relocation Movements performed by staff $R_{staff} = 0$ and the margin M of the trip is negative

$$0 = M_{staff} + C_{staff}$$

$$M_{staff} = -C_{staff} = -VRLC_i = -(((Avg(VTRC_i) * Ve_i) + \frac{MC_{km}}{D_i}) * D_i^s + SC_m * t_i)$$

$$M_{staff} = -C_{staff} = -\left(\left(AVC_{pm} * rt\right) + (SC_m * t_i)\right)$$
$$= -(((Avg(VTRC_i) * Ve_i) + \frac{MC_{km}}{D_i}) * D_i^s + SC_m * t_i)$$

7.4. The user-based relocation tariff strategy.

The tariff definition could be based, as generally happening in car-sharing on time and/or distance travelled and potentially on daily hourly-range and seasonality.

Defining *h* as the origin of a car-sharing trip starting in a so-called Urban Area[xxix] (U_1) of departure of a vehicle starting and *k* as the destination to a Urban Area (U_2) of arrival when finishing the movement, a trip will be identified as $T_{h,k}$ represented as:

$$T_{h,k} = T(U_1, U_2)$$

[xxix] Representing an Urban Area, the location variable will be called U_i

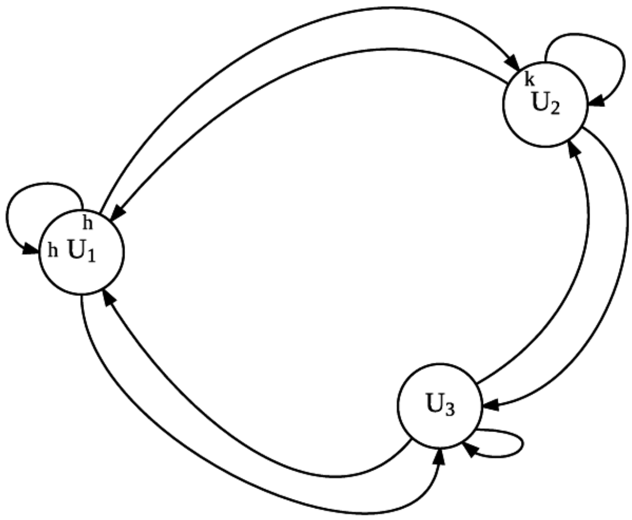

Figure 26 - sample of origin to destination movements between Urban Areas

In the case in which the departure and destination of the vehicle refer to the same Urban Area h = k.

$$T_{h,h} = T(U_1, U_1)$$

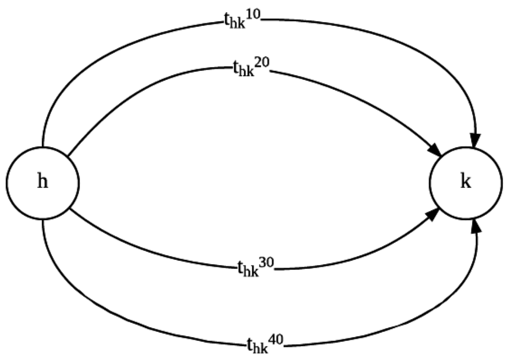

Figure 27 - sample movements considering time slots and tariffs

Considering that, as mentioned before, Rome is divided in taxable areas defined from Rome Municipality aggregated in 636 cells, and imagining the city like a graph, **each area can be represented as a node** with destinations represented by arcs linking bidirectionally each node to another.

In other words, we should see the map of Rome made up of spots with different dimension and a dense network of lines which connect the spots.

Remember that to simplify and make the experiment more flexible each zone has been collapsed to its centroid which can be considered as "station" which cars reach or leave.

Even if the similarity with a station-based system seems to be in contrast with one-way free-floating systems this hypothesises makes the analysis computationally more tolerable to manage and to control, without losing the assumption that the user is free to drop the car whenever he wants inside the service area.

In fact, the high number of cells (636) give the experiment a very good approximation with a real free-floating system.

To reduce the number of our grid's cells and simplify the computation in the following experiment 25 spots representing different type of Urban Area Value have been considered.

The study made on Urban Areas permitted to identify five different groups of cells based on Urban Area Value:

- Low identified by Dark Blue color (DB)
- Medium-Low identified by Light Blue (LB);
- Medium identified by Yellow (Y);
- Medium – High identified by Orange (O);
- High identified by Red (R).

This groups of cells correspond to a segmentation of demand rate during the day.

The so called Low zones are spots of the city in which there is a low percentage of users: this means that there are few bookings and the cars remains in their stationary position for a long time (the combined blue colour represents a kind of cold zone, namely it specifies that in those zones there is an abundance of means of transport and users can always find available cars).

Instead, the High zones are places in which there is much demand from users and the cars' stationarity time is low (the red color underlines "hot areas" where there is an under-capacity of vehicles and a user could not find an available car).

The Medium zones are a middle way between the previous two. It can be considered the "normal" state of the system, because there are neither an under-utilization of the service nor a lack of cars in the nodes.

7.4.1. Urban zones dependent discounts

The problem of establishing a correct price and its discount thresholds, to optimize margins and incentives of the user-based relocation model, depends to the desired level of demand. We know that lower tariffs generate higher requests, but it is difficult to find a scientific correlation between them.

Tariff discounts (bonus) and increases (malus) are treated as commercial correction coefficients (K), where the final tariff, proposed to the end user, is calculated by multiplying the standard tariff by these coefficients.

Some systems consider an additive formula or a mixed form. In this study, the system offers to users' different commercial coefficients based on the following considerations.

The analysis of Car-sharing systems in Italy enables the validation of following assumptions:

- service tariff is per minute;
- the standard price is set to 0,25 €/min[xxx].

[xxx] This is choice is legitimated because current tariff per minute of main Car-sharing operators, which together cover about 90% of market share are: 0,24/0,26 for Car2Go; 0,25 for Enjoy. The pricing model doesn't consider the subscription

- discounts directly depend on different value of Urban Areas as expressed in table 13, considering the following 5-level value scale.

Urban Area Value	Symbol	UAV
High	R	1,20
Medium-High	O	1,08
Medium	Y	1,00
Medium-Low	LB	0,92
Low	DB	0,80

Table 13 - Urban Area Value

$$valueOfTariff_{h,k} = (1 - (UAV_h - UAV_k)) * \ t_{std} = (1 - \ K_{h,k}) * \ t_{std}$$

The prices reflect a specular symmetry around the standard value in terms of multiplicative factor (K-matrix) and an increasing linear trend in terms of tariffs' values.

K Matrix (h,k)	R	O	Y	LB	DB
R	0,00	0,08	0,20	0,32	0,40
O	-0,08	0,00	0,08	0,20	0,32
Y	-0,20	-0,08	0,00	0,08	0,20
LB	-0,28	-0,20	-0,08	0,00	0,08
DB	-0,40	-0,28	-0,20	-0,08	0,00

Table 14 - discount matrix (K-matrix)

As an example, a vehicle moving from a DB to a R area, will have a 40% discount while a vehicle moving from R to DB area will have a 40% tariff increase based on the standard tariff.

Discounts are applied when a user performs an attractive trip, moving from a non-attractive to a more attractive node. In the opposite way, increments are applied when user goes from an attractive to less-attractive areas. Moving in the same class area is indifferent to discount or penalty application, so that standard tariff is applied to users going from a station to another with the same degree of attraction.

Considering the attractiveness as a staircase with different steps, each corresponding to a level of attraction, the user can access the applied tariff as a "climber" who can go up or down the stairs. This means that to go from an attractive zone to a non-attractive one, price is higher.

The choice of these tariffs can be motivated by the unit cost per minute of a ride, namely knowing the cost, the lowest discount is set equal to the cost, so that there is 0-margin for a user relocation

Remember that discount has the purpose to stimulate relocation by user, then fixing a price equals to cost brings 0-profit, but it balances the system, avoiding further staff relocation costs (when there is a staff relocation, the companies must pay not only the movements, but also manpower).

Therefore, the expectation is to reduce or avoid staff relocation and to increase user-relocation, varying tariffs during the day. The above-mentioned cost per minute includes both fixed costs (e.g.

fee of each customer, but only the sold rides; considering this approximation doesn't impact the standard pricing because Car2Go requires a una-tantum cost of 10 €/subscriber and Enjoy subscription is free.

insurance, taxes, reserved parking, cleaning) and costs depending on time (e.g. depreciation, reparation, maintenance, fuel).

To simplify the analysis this cost comprehends all these expenses in a simple constant value per minute: 0,17 €/min[xxxi]. Now, it is clearer the choice of the lowest price: we expect that with a "near-0-margin" strategy for user-based relocation, the customers are involved to select the best alternative to balance the system.

A danger could be the possibility that users do not accept any variable tariff: this causes not only unbalancing, but also a reduction of the margin. As clarified later, this danger seems to be avoided according to model's results.

As mentioned in the next chapters, the day is divided into five time slots: two peaks and three off-peaks, i.e. respectively parts of the day with high demand and parts of the day with few requests.

During the off-peaks the system offers the classical standard tariff because there is a homogeneous distribution of the demand in the city. It is rare that the system can be unbalanced during off-peaks because there isn't a tendency of customers, then a so-called flow's semi-conservation constraint is always respected (in a node, all the entering rides are almost equal to all the outgoing rides). During the peaks, the situation is opposite. There are particular patterns in users' behaviour and needs: during the morning there is the so-called house-work wave and during the evening there is the opposite flow (work-house).

In our first analysis, we haven't considered other peaks such as lunch or dinner because they aren't as relevant and distinct as the first cited ones.

The expectation is that during peak hours, there is a higher price to discourage the demand, but during our peaks the systems offers different alternatives to user:

- for users who cause imbalance, from attractive to less attractive zones, the alternatives are the desired trip at a higher price or closer destinations at lower prices;
- for users who cause spontaneous balancing, the alternatives are the lowest prices;
- for users who do not change the system equilibrium[xxxii], the tariff is standard.

As explained later, the presence of four discounts and four increments thresholds depend on the configuration of the demand during the day. During peaks, different trips are distinguished, from a Car-sharing operator point of view with different levels of attractiveness or not attractiveness.

The idea is that the higher the unbalancing caused from the user, higher the price offered to him.

After this digression the choice of only 25 zones can be justified. Observing the colored partition of the city, corresponding to a thermographic analysis, which takes a bearing of the demand rate during a day, it can be noticed that there is a symmetric distribution of the demand, that can be reduced to this "incorporated" nodes.

[xxxi] It is a coherent average value.
[xxxii] Because they go from attractive zones to attractive zones or from non-attractive zones to non-attractive zones.

Consequently, according to Urban Area Value definition where demand rate depends on frequency of travels and stop time of cars, the 636 areas have been aggregated in 25 cells, assigning to each cell a UAV representing the average of the aggregated cells.

To better understand, in the picture below describes the “transformation”.

As a first step a grid of 25 squared cells with a side wideness of 1 kilometre has been defined and has been superimposed on the thermographic map defining the UAV using data from all 3 Car-sharing operators.

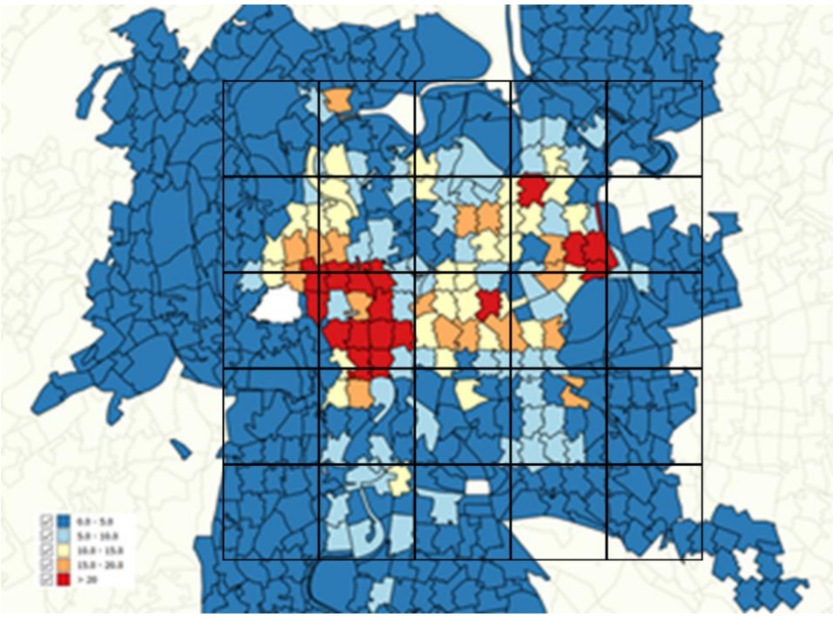

Figure 28 – Grip superimposed to the thermographic map

Than a color had been assigned to each squared area, based on the average UAV of the original map. The following grid represents the reduced graph.

DB1	LB1	LB2	LB3	DB2
Y1	O1	Y2	R1	DB3
Y3	R2	O2	O3	DB4
DB5	Y4	DB6	LB4	DB7
DB8	LB5	DB9	DB10	DB11

Figure 29 – reduced graph

This mode of detection of service areas is very faithful to reality, precisely because it is obtained by aggregating the original cells, reflecting, with the due, obvious, approximation, the distribution of the real cells, according to their value.

In fact, the red zones are in the central part of the city and they are separated from different medium nodes. In the peripheric area there are the blue and light blue spots and the medium cells are scattered among red and blue cells. In other words, there is a concentration of all the city's areas in 25 cells. This means that the total demand level is the same of the real situation, but in each merged zone the requests' frequency is higher, namely there are more rides.

Note that this characteristic doesn't affect the computation. To confirm this last sentence, we have built a model which reflects the real current condition, concentrating the analysis in these 25 reduced areas and the results show a strict correspondence between the two patterns' solutions.

This ideal model also assumes that travellers don't pick up and leave cars at specified locations (like in free floating system) but in a restricted subarea, which is simply called area or zone or cell or node. This notation can cause confusion: what happens if a user asks for a ride from a station to the same one? In this case, it is assumed that he needs a car for a short trip inside the same area or, if you prefer, that he moves towards another station internal to the subarea. So, it is accepted a ride from a station to the same one because there isn't the constraint that in each area there is only one station.

Furthermore, a coherent and realistic percentage of cells' groups can be respected with 25 nodes:

- 11 dark blue spots for a percentage of 44%: this means that about half graph is composed from dark blue areas;
- 5 light blue spots for a percentage of 20%,
- 4 yellow spots for a percentage of 16%,
- 3 orange spots for a percentage of 12%,
- 2 red spots for a percentage of 8%.

This distribution also highlights that about 2/3 of the spots (64%) are below the average UAV.

	# Spots	% Spots
Dark Blue	11	44%
Light Blue	5	20%
Yellow	4	16%
Orange	3	12%
Red	2	8%
Total	25	100%

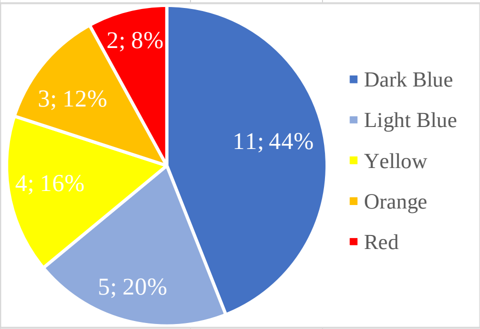

Figure 30 - Pie Chart of the nodes distribution

An important assumption is that all the areas have the same geometric form: every cell is a square with side of 1 km. This means that each node occupies 1 km^2 of space and the squares' diagonal measures (for Pythagorean theorem) $\sqrt{2}$ km, that is approximately 1,4 km.

Considering that the total surface of Rome is approximately 1.300 km^2, the previous shrinking's concept is confirmed: there is a scale reduction of 1:5

A hypothesis of the model is that the distance from an origin *h* to a destination *k* is equal to the diagonal multiplied by the number of crossed zones between *h* and *k*. Mathematically speaking: $d_{AB} = \sqrt{2} \times n_{AB}$, where d is the distance, square of 2 is the diagonal and n is the number of zones between A and B. The choice of the diagonal as distance is made to adapt model to real case. For example, the maximum distance between the furthest cells (see the graph, Spot1 and Spot6) is: $\sqrt{2} \times 5 \cong 7{,}1$ km. If origin and destination are the same station, the number of crossed zone is set to 1, then the distance is equal to $\sqrt{2} \cong 1{,}4$ km.

Thinking the city of Rome as a simple geometrical figure, it can be approximated to a square of side 36 km ($\cong \sqrt{1300}$), that is about equal to 7,1 x 5 km. Considering that car sharing companies have a more restricted area of service than the total surface and that Rome can't be replaced with a geometrical figure, it legitimate to consider 36 km as the maximum distance. So, we establish that the multiplicative factor, that establishes a correspondence between real length and the model's one, is the integer number 5.

Another assumption is that users drive crossing the minimum number of cells. This is reasonable because customers tend to choose the fastest and shortest ride. Each time a user stops the car, turns off and gets out of it, a trip end. If the same user reuses the same car, the model considers a new ride.

This is acceptable and coherent with car sharing principles: vehicle sharing is a public transport and it must guarantee the service to every member. In other words, there isn't exclusivity.

As a coin with two opposite sides, in this model a trip's distance can be considered at the same time the worst and the best case: best case because it is assumed that user always choose the shortest route; the worst one because unit movement is fixed equal to the diagonal, that is the longest unit shift.

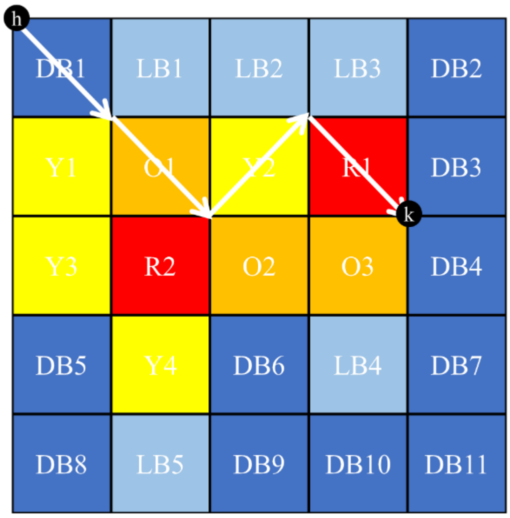

Table 15 – representation fo a sample trip from h to k.

Then, the pessimistic unit distance ($\sqrt{2}$) is compensated by the optimistic minimum number of passed nodes.

Once identified and justified the spatial configuration of the problem, we can consider other characteristics. From a capacity's point of view, the actual car sharing platform has a homogeneous fleet of 1248 vehicles, which must serve a lot of rides.

The total capacity of the system is sum of all the cars. This total capacity is distributed on the origin nodes in a certain manner (initially there is an equal distribution among the five colored zones, but other different options have been considered for a comparison). Each station has a finite number of cars and there isn't the problem of parking's lack.

From a schedule's point of view, the thesis is focused on a single day. The day is divided into 5 different time-slots or hourly ranges: two peaks and three off-peaks. As defined before, a peak is the part of the day with a high demand rate, while an off-peak refers to a "semi-static" condition of the day with a low level of requests.

Name of time slot	Time range	Day-time
1st Off-Peak	00:00-06:59	night
1st Peak	07:00-09:59	early morning
2nd Off-Peak	10:00-16:59	late morning - early afternoon
2nd Peak	17:00-19:59	evening
3rd Off-Peak	20:00-23:50	night

Table 16 – hourly-ranges for 5 time-slots

Note that durations of the slots are different: the first peak's length (3 hours) equals the second peak's one, but the first off-peak (7 hours) and the second one differs from the third one (4 hours).

Although the initial goal was to maintain a model's symmetry in terms of data's representation, observing and analysing real data, this temporal distribution is coherent with current information. If the durations are different, there is constancy in the distribution of users willing to pay.

This means that there isn't a mathematical proportion between demand and slot's duration, but during these slots some percentages of demand are respected (during peak one the demand is similar but not the same one of peak two, and so on). This explains the selection of 5 slots.

We assume that all rides, which start in an hourly range, terminate in the same one or, if you prefer, that a ride, which crosses two adjacent slots, is split in two different trips, each for every temporal range. In other words, we don't consider the possibility that a trip finishes in the following time slot. This confirm the complexity of modelling a real car sharing system. Although the dimension of temporal slots is excessively big, in this way we can avoid or decrease overlaps of cars. In an "old" version of the model there is an estimation of capacities taking into account cars in transit from a slot and the following one (trans-capacity).

Another important hypothesis is that all cars can do only a trip for time slot. To make the analysis coherent with real case, the number of cars is quintupled. The choice of a 5-factor derives from the fact that in a real context, each car are expected to make about 5 runs during the day. Multiplying the capacity for 5, is not intended to say that there are 1248 x 5 = 6240 cars, but that each car makes 5 runs, one per time-slot. Summarizing, for the model it's like there are 6240 cars, but practically speaking there are 1248 cars that can make 5 trips each: this is an optimization problem and not a simulation that focuses on each single movement of all cars.

Considering driving speed, we confirm the separation between peak periods and off-peak periods:

- during peaks, the average speed is 10 km/h;
- during off-peaks, the average speed is doubled to 20 km/h.

This separation can justify also the presence of traffic congestion: during demand's peak there is a high number of cars that involve congestion and the driving speed is lower; vice versa, during demand's off-peak the lesser number of vehicles is linked to higher driving speed. The speeds' choice isn't random: they are average values deriving from a characterization of real data[85, 86].

From a simple physical formula, we deduce the average driving time or rent time to go from a station h to a station k in the o hourly-range:

$$RentTime_{hk}^{o} = Distance_{hk} * Speed^{o}$$

Then, the rent time is a parameter depending on distance, speed and hourly-range and it's measured in hours (hrs) because Km/Km/hrs = hrs. To obtain the measure in minutes, RentTime is multiplied by 60 because 1 h = 60 min. Remember that unit price and cost depend on minutes of driving time.

According to Revenue Management's notions, car sharing is a system that can be divided in different markets. Each market corresponds to a trip from an origin A to a destination B for a total of 25 x 25 = 625 markets and it has different rent time, tariff, demand and, above all, different alternatives.

As mentioned before, in the peak periods, when there is a high probability of unbalancing on cars' distribution, and in zones with many requests, the system raises the price of that desired trip and proposes to the user some alternatives to stimulate user-based relocation and to balance the system: an alternative is a closer less attractive station with a lower value of tariff.

When a user access to protocol reservation e.g. from a smartphone application, the system provides alternatives and user can choose to accept or to reject a ride. This doesn't mean that the system imposes to choose another trip, but simply that it directs user to select the alternative, which is useful to the system.

The expectation is that the attractiveness of user for the alternatives increases when the tariff decreases, and he is discouraged by a higher price.

All the markets and relative alternatives have a different attractivity, that in the model is called also *willingness to pay* or *willingness to accept*; it is a real number between 0 and 1 depending on tariff, market, alternative trips and alternate urban transport, quality of the service and specific hourly range.

It represents the attractiveness of a trip or the probability that a user is willing to pay or to accept the ride.

For a better explanation, we consider two ideal opposite situations as examples:

1. in case there aren't other public or private means of transport, the user hasn't a private car and he must arrive to destination, that is far, the tariff is low, there isn't traffic congestion, namely when all conditions are favourable to accept, we can assume that probability to pay a ride is equal to 1;
2. in case there are public or private efficient transport, or the user has a private car, there isn't traffic or there is the possibility to walk, the tariff is high, namely when all conditions are unfavourable to accept, then probability to pay is equal to 0.

Obviously, these situations never happen, but they can be considered as two ideal borders: the optimal condition and the worst condition. All the middle ways between these two ideal cases provide a probability to accept between 0 and 1.

From the demand's point of view, initially we can say that the daily sold trips are around 6.240. Contrary to some researchers[87], the general assumption is that 6.240, namely the sold trips, doesn't represent the effective users' demand during a day. In other words, the historical data can partially represent demand, in fact there is a part of demand that is obscured.

Users who want to go from A to B comprehend not only the members, but all the travellers between the two spots. It is important to consider the possibility of a service's expansion and to face the consequent demand's increment and a possible fleet's increase. So, we can say that it is known only the number of sold trips, that is simply a part of the total possible demand: there are some traveller that actually prefer different kind of transport because of tariffs convenience or traffic congestion.

To keep this aspect into account, it has been estimated that demand is equal to 1,6 x sold trips, which means that all the sold rides are 60% of the total demand. We know that this percentage can be an overestimation, but it is a computational choice to simplify the analysis and it is coherent with successive choices.

Once defined demand and attractiveness, according to a study of Grani et al.[30], we define two parameters: *not-demand* and *not-willingness*. As explained later in the model and in the solutions, these two terms are important to allow users to accept or reject a ride.

Not-demand includes not only travellers who choose other means of transport, but also all the users who refuse a ride and its alternatives. Not-willingness refers to the lack of attractiveness of a ride and, compared with the willingness, it defines the number of people that accept, and decline offers.

So, we distinguish the sold trips from the refused or unsold rides.

Providing the alternatives, the system offers to users the following tariff scheme

Price Matrix	R	O	Y	LB	DB
R	0.25	0.27	0.30	0.33	0.35
O	0.23	0.25	0.27	0.30	0.33
Y	0.20	0.23	0.25	0.27	0.30
LB	0.18	0.20	0.23	0.25	0.27
DB	0.15	0.18	0.20	0.23	0.25

Table 17 – Price Matrix

The lowest tariff is equal to the fixed unit cost per minute. This allows to balance system without staff's costs, satisfying the customers, but obtaining near-0-profit (with the lowest tariff the margin slightly negative but very close to 0).

7.4.2. *Time dependent discounts*

The following results are developed on the hypothesis that discounts are time dependent, so that different discounts can be applied to the same origin-to-destination movement if performed in a different time-slot where origin and/or destination Urban Area Value changes during the day.

Trip offering lets define C_h the demand capacity in the Origin Urban Area *h*, and C_k the offer capacity in the Destination Urban Area

$$max \sum_{j} \sum_{o} \sum_{hk} t_{hk}^{oj} x_{hk}^{oj} - \sum_{hk} C_{hk}^{R} y_{hk}^{oj}$$

defining:

- o the hourly range
- j the applicable tariff
- t_{hk}^{oj} as the j^{th} tariff applied to the trip $O_h \rightarrow D_k$ at time o
- x_{hk}^{oj} the number of trips that can sold per origin/destination $O_h \rightarrow D_k$ at the j^{th} tariff during the timeslot o
- y_{hk} the number of relocation trips per origin/destination $O_h \rightarrow D_k$

The following formula should also include demand, flow and offer constraints.

$$x_{hk}^{oj} \in Z$$

Figure below, shows an example of movement flow dynamics considering user movements *x* and staff movements *y*.

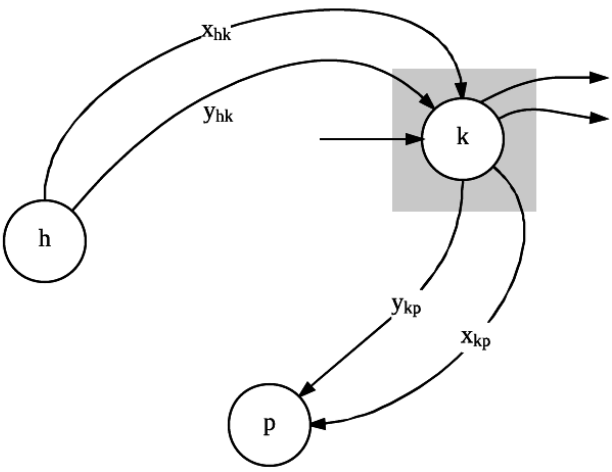

Figure 31 - movement flow dynamic at k-node with user and staff movements

Hourly tariffs can be calculated as discounted tariffs starting from a base standard tariff t_{std} with the following formula.

$$valueOfTariff_{hk}^{o} = \left(1 - (UAV_h^o - UAV_k^o)\right) * t_{std} = (1 - K_{h,k}^{o}) * t_{std}$$

7.4.3. *Applied relocation model*

The goal of the analysis is to maximize profit, considering the possibility of a partial user-based relocation. According to the price and the quality of service (distance from desired destination, traffic congestion, rent time), user can choose the preferred ride.

There will be travellers who accept high tariff for a more comfortable service, travellers willing to pay low tariff and to go to destination by walk, and travellers who prefer other means of public transport such as bus and/or underground. The system provides a solution and let users decide.

At the end of the day, if there is the imbalance, the staff can shift cars' position to re-establish a correct distribution. The system provides only two operations: renting and relocating, ignoring refuelling and other activities generally performed by Car-sharing system staff that are are marginal to the results.

The system can be considered a hybrid model between user-based and operator-based relocation, in which operators' activities remain a necessity because of the demand's stochasticity. The last statement means that the current service is characterized from randomicity and not that the model is stochastic, in fact we assume to know demand, requests' rate, tariffs and other model's parameters.

The future expectation of the car sharing companies is that operator-based relocation can be completely substituted by user-based relocation setting to zero the relocation costs and creating a self-regulating system. It is important to underline that this shouldn't cause, with actual car-sharing systems, a strong impact on job loss for staff members because they will still be required for cleaning, maintenance and other tasks such as customer service.

8. The Optimization Model

The optimization model is the core of the dynamic pricing application in this research thesis; it is defined by the *mathematical model* and has been applied with the *implemented model* using AMPL; both models are described in the detail the paragraph below.

The mathematical model has been developed integrating spatial and time dynamics of the analysed car-sharing eco-system. In fact, the origin (h) and destination (k) have been defined in a spatial grid and cells have been obtained by aggregating a more atomic view of the detailed spatial analysis developed to define attractiveness of zones using Urban Area Value.

On the other hand, also a time-grid of five time-slots has been defined to differentiate mobility behaviours during the operating day.

Finally, user behaviour mainly focused on the *willingness to pay* typical of a Costumer Choice Models described in the Revenue Management chapter. In defining the *willingness to accept* of both discounted rates for desired destination or for close alternatives to planned destination have been of inspiration the work on the General Attraction Model proposed in literature as Sales-Based Model (SBM) and of Gallego et al.[44] and Grani et al[48].

This approach has been already experimented in a simpler way in the MSc graduation thesis of Eng. Graziano Ciucciarelli *"Applying Mathematical Programming Approaches to User-Relocation in a Station-based Car Sharing System with variable Tariffs"*[88] in which the Author of this thesis has contributed as Co-tutor.

8.1. The Mathematical Model

In this paragraph is described the mathematical model used for the optimization of profit by applying a user-relocation strategy.

8.1.1. Sets and indices:

- c ∈ Cell: set of all the cells and relative indices;
- o ∈ HourlyRange: set of time slots and relative indices; it is a set of 5 hourly ranges HourlyRange ≔ {1, 2, 3, 4, 5};
- m ∈ Market: set of markets and relative indices; Market is the Cartesian product between Cell sets (Market = Cell x Cell); it is the set of all the pairs *a* and *b*, where *a* ∈ Cell and *b* ∈ Cell;
- h ∈ Origin ⊆ Cell: set of origins and relative indices; it is the set of all the origin points, that can be seen as a subset of Cell (in this case we assume that they are coincident because all the cells are both origins and destinations); considering the Market set as a two-column set, one for Origin and another one for Destination, the Origin set can be extracted from the first column;
- k ∈ Destination ⊆ Cell: set of destinations and relative indices;
- db ∈ DB ⊆ Cell: auxiliary[xxxiii] set of cells and relative indices representing all the zone less attractive areas (dark blue nodes); it is a subset of Cell;

[xxxiii] This sets have been used to validate the model or to control computation behavior or during starting test models.

- lb ∈ LB ⊆ Cell: auxiliary set of cells and relative indices representing all the medium-low or light blue areas; it is a subset of Cell;
- y ∈ Y ⊆ Cell: auxiliary set of cells and relative indices representing all the medium or yellow areas; it is a subset of Cell;
- o ∈ O ⊆ Cell: auxiliary set of cells and relative indices representing all the medium-high or light blue areas; it is a subset of Cell;
- r ∈ R ⊆ Cell: auxiliary set of cells and relative indices representing the areas with higher demand rate (red nodes); it is a subset of Cell.

8.1.2. Model Parameters:

The following parameters must be defined for the mathematical model:

- $nodes_{hk}$: determines the distance between nodes;
- $close_{hk}$: determines neighbour nodes including the node itself
- $closepotential^{o}_{hk}$: determines neighbour nodes with higher potential; it is managed using a matrix with cell value of {0,1} where value 1 identifies the neighbour nodes with higher potential.
- $alter^{0}_{hk}$: determines the neighbour nodes to destination with potential to be proposed as alternatives, and it is represented by the transpose of *closepotential*
- $demand^{o}_{hk}$ (or $demand^{o}_{m}$): estimated demand from origin h to destination k, or demand in market m, during the hourly range o; it is an integer value, greater than or equal to 0;
- $notDemand^{o}_{hk}$ (or $notDemand^{o}_{m}$): not-demand from origin h to destination k, or not-demand in market *m*, during the hourly range *o*; it is an integer value, greater than or equal to 0; it is an estimation of all the unsatisfied requests, or in other words an estimation of the number of drivers who doesn't take a ride;
- $willingnessQ^{o}_{hk}$ (or $willingnessQ^{o}_{m}$): attractiveness or probability to accept/pay a run; it depends on origin *h* and destination *k*, or market *m*, slot time *o*; it is a real value between 0 and 1;
- $willingnessA^{oa}_{hk}$ (or $willingnessA^{oa}_{m}$): probability to accept an alternative destination; it depends on origin *h* and destination *k*, or market *m*, slot time *o* and alternative *a*; it is a real value between 0 and 1;
- $notWillingness^{o}_{hk}$ (or $notWillingness^{o}_{m}$): unattractiveness of destination or probability to reject the trip and its alternatives; it depends on origin *h* and destination *k*, or market *m*, and slot time *o*; it is a real value between 0 and 1;
- $valueOfTariff^{o}_{hk}$ (or $valueOfTariff^{o}_{m}$): it is the tariff's value associated to a run *hk* during an hourly range *o*; it is a real value greater than 0; it is measured in €/min (euro per minute);
- $speed$: is the average speed of vehicle depending on the Hourly Range; speed has a value of 20 km/hr during off-peaks and a value of 10 km/hr during peaks
- $distance$: the unitary distance between two cells; distance is measured as the diagonal of the cells and has a value of sqrt(2), considering the cell width of 1 km.

- $rentTime^{o}_{hk}$ (or $rentTime^{o}_{m}$): it is the driving time to go from origin h to destination k during the time slot *o*; it depends on the level of traffic congestion and it is calculated as the ratio between distance and driving speed; it is a real number, greater than 0 and it is measured in minutes because the tariff depends on minute;
- $vehicles$**:** indicates the number of vehicles composing the fleet;
- $unitCost$: fixed unit cost (in the experiments 0,17 €/min); it includes fixed costs (purchase, insurance, taxes, reserved parking, cleaning) and costs depending on time (depreciation, reparation, maintenance, fuel);
- c^{1}_{h}: starting (i.e. at the beginning of temporal slot number 1) capacity of node or origin h; it is an integer value greater than or equal to 0; considering all the cells, it represents the starting distribution of the model.

8.1.3. Decision variables:

- q^{o}_{hk} (or q^{o}_{m}) $\in \mathbb{N}$: number of sold or accepted trips from origin h to destination k during hourly range o; it is an integer value;
- a^{oa}_{hk} (or a^{oa}_{m}) $\in \mathbb{N}$: number of sold or accepted trips, for alternative a, from origin h to destination k during hourly range; it is an integer value;
- $notQ^{o}_{hk}$ (or $notQ^{o}_{m}$) $\in \mathbb{N}$: total quantity of lost rides from origin h to destination k during slot *o*, because of rejection by users or lack of demand and capacity (in case there is capacity, but there isn't enough demand and vice versa);
- $capacity^{o}_{h}$ $\in \mathbb{N}$: capacity of node *h* at the beginning of time slot *o*.

8.1.4. Objective function:

The objective function is the **maximization of profit**:

$$Profit = \sum_{h\in O}\sum_{k\in D}\sum_{o\in H}\{[q^{o}_{hk}\cdot\ rentTime^{o}_{hk}\cdot(valueOfTariff^{o}_{hk}-UnitCost)] + \sum_{a\in D}[a^{oa}_{hk}\cdot alter^{o}_{ha}\cdot rentTime^{o}_{ha}\cdot(valueOfTariff^{o}_{ha}-UnitCost)]\}$$

Abbreviations:

- O is the set of Origins,
- D is the set of Destinations,
- H is the set of Hourly Ranges,
- q^{o}_{hk} for quantity of sold trip from origin h to destination k,
- $a^{oa}_{hk}$for alternative of sold trip from origin h to destination a.

The objective function is a linear function.

8.1.5. Constraints:

- ***Availability***:

$$(\sum_{k\in D} \mathrm{q}_{hk}^{o} + \sum_{a\in \mathrm{D}} a_{hk}^{oa} \cdot alter_{ha}^{o}) \leq \mathrm{capacity}_{h}^{o} \quad \forall\ \mathrm{h} \in \mathrm{O}, \mathrm{o} \in \mathrm{H}$$

For each origin *h* and time slot *o*, the sold trips from *h* to every destination *k* and its alternatives must be lower than or equal to the capacity in the origin during the slot time *o*. It is an availability constraint or a capacity constraint (it is a linear inequality) meaning that the system cannot sell more trips than the number of cars in the origin area (not exceeding capacity).

- ***Total Market Demand***:

$$\mathrm{q}_{hk}^{o} + \sum_{a\in \mathrm{D}} a_{hk}^{oa} \cdot alter_{ha}^{o} + \mathrm{notQ}_{hk}^{o} = \mathrm{demand}_{hk}^{o} \quad \forall\ \mathrm{h} \in \mathrm{O}, \mathrm{k} \in \mathrm{D}, \mathrm{o} \in \mathrm{H}$$

It is a total market demand constraint and it is a linear equality constraint. For each market *m* (from origin *h* to destination *k*) and time slot *o*, the sum of sold trips from *h* to every destination and alternative and lost quantity is equal to the demand for that market and during that time slot. This means that demand is formed from sold quantities and lost quantities or that not-quantity is equal to the difference between demand and sold quantities.

- ***Lower Bound:***

$$\mathrm{notQ}_{hk}^{o} \geq \mathrm{notDemand}_{hk}^{o} \forall\ \mathrm{h} \in \mathrm{O}, \mathrm{k} \in \mathrm{D}, \mathrm{o} \in \mathrm{H}$$

It is the lower bound (linear inequality) of not-quantity that considers demand recapture by not-demand. This means that not-demand is always available for sale and there is the possibility to recapture refused trips. To better understand, we can consider *notDemand* as a pessimistic value of lost trips which indicates a minimum number of rides that will be not sold (it is estimated from real data).

Combining constraints of "Total Market Demand" and "Lower Bound", we obtain:

$$\mathrm{q}_{hk}^{o} + \sum_{a\in \mathrm{D}} a_{hk}^{oa} \cdot alter_{ha}^{o} + \mathrm{notQ}_{hk}^{o} = \mathrm{demand}_{hk}^{o}$$

$$\text{notQ}_{hk}^{o} \geq \text{notDemand}_{hk}^{o}$$

then

$$\text{notQ}_{hk}^{o} = \text{demand}_{hk}^{o} - (\text{q}_{hk}^{o} + \sum_{a \in \text{D}} a_{hk}^{oa} \cdot alter_{ha}^{o}) \geq \text{notDemand}_{hk}^{o}$$

$$\text{demand}_{hk}^{o} - \text{notDemand}_{hk}^{o} \geq \left(\text{q}_{hk}^{o} + \sum_{a \in \text{D}} a_{hk}^{oa} \cdot alter_{ha}^{o}\right) \geq 0$$

consequently

$$\text{demand}_{hk}^{o} - \text{notDemand}_{hk}^{o} \geq 0$$

then,

$$\text{demand}_{hk}^{o} \geq \text{notDemand}_{hk}^{o}$$

and the sum of q and alternatives must be lower than or equal to the difference between demand and not-demand, that includes the number of users willing to accept that run. If the capacity of the node is 0, there could be people willing to pay who aren't satisfied by the system and this is a missed opportunity to make profit.

- ***WillingnessQ:***

$$\text{notWillingness}_{hk}^{o} \cdot \text{q}_{hk}^{o} - \text{notQ}_{hk}^{o} \cdot \text{willingnessQ}_{hk}^{o} \leq 0 \qquad \forall\, h \in O, k \in D, o \in H$$

This is an acceptation or willingness constraint (linear inequality). Remember that user can choose to accept or reject a proposal of the system.

We can simply see that:

$$\text{q}_{hk}^{o} \leq \frac{\text{willingnessQ}_{hk}^{o}}{\text{notWillingness}_{hk}^{o}} \cdot \text{notQ}_{hk}^{o} \leq \frac{\text{willingnessQ}_{hk}^{o}}{\text{notWillingness}_{hk}^{o}} \cdot \text{demand}_{hk}^{o}$$

the meaning of the acceptation constraint is easily understood: if the probability to accept is greater than the probability to reject a ride, the sold quantity is lower than or equals to a value, greater than unsold trips (in fact, the ratio between *willingness* and *notWillingness* is greater than one); in the opposite situation, the sold quantity is lower than or equal to a number lower than unsold rides.

In fact, the constraint states that in case of available capacity, if the probability to accept is high then the sold quantity can be high.

- ***WillingnessA:***

$$\text{notWillingness}_{ha}^{o} \cdot a_{hk}^{oa} \cdot alter_{ha}^{o} - \text{notQ}_{ha}^{o} \cdot \text{willingnessA}_{hk}^{oa} \leq 0 \quad \forall\, h \in O, k \in D, o \in H, a \in D$$

This is an acceptation or willingness constraint (linear inequality). Remember that user can choose to accept or reject a proposal of the system.

We can simply see that:

$$a_{hk}^{oa} \cdot alter_{ha}^{o} \leq \frac{\text{willingnessA}_{hk}^{oa}}{\text{notWillingness}_{ha}^{o}} \cdot \text{notQ}_{ha}^{o} \leq \frac{\text{willingnessA}_{hk}^{oa}}{\text{notWillingness}_{ha}^{o}} \cdot \text{demand}_{hk}^{o}$$

- ***Capacity Update:***

$$\frac{capacity_{h}^{o+1}}{5} = \frac{capacity_{h}^{o}}{5} - \sum_{\text{k}\in\text{D}} (\text{q}_{hk}^{o} + \sum_{a\in\text{D}} a_{hk}^{oa} \cdot alter_{ha}^{o}) + \sum_{\text{k}\in\text{O}} (\text{q}_{kh}^{o} + \sum_{a\in\text{D}} a_{ka}^{oh} \cdot alter_{ah}^{o}) \ \forall\, h \in \text{O}, \text{o} \in \text{H}: \text{o} \leq 4$$

Where by $a \in \text{A}(\text{j}, \text{o}): a = h$ we denote that vehicle destination is h, and not every other alternative.

This constraint represents a kind of flow conservation constraint. Considering a slot time, the capacity of the successive slot is equal to the current capacity minus every exiting run plus every entering car. This constraint allows to evaluate the distribution of cars at the end of a period as well as the distribution of cars at the beginning of the successive period. For example, capacity at the beginning of the second temporal slot is equal to the capacity at the end of the first range.

In sustainable Car-sharing systems, as already mentioned, it is estimated that a vehicle performs between 4 and 5 trips per day, approximately 1 for each HourlyRange; consequently in the model it is assumed that the number of cars is quintuplicated because it is complex to model a single car's state to reply real-time users' behaviour in large hourly ranges and we are obliged to renounce some details, preferring a simple characterization.

Then, multiplying by 5 we want to underline that each car can make 5 runs during an operational day. This is the cause of the division by 5 in the Capacity constraint.

In the end, observe that the hourly range must be lower than or equal to 4 because the HourlyRange set is made of 5 elements, then the capacity of a sixth period[xxxiv] doesn't exist and the model would suggest the presence of an error.

To overcome this drawback, the following constraint is introduced:

- ***Upper Bound:***

$$\frac{capacity_{h}^{5}}{5} - \sum_{\text{k}\in\text{D}} (\text{q}_{hk}^{5} + \sum_{a\in\text{D}} a_{hk}^{5a} \cdot alter_{ha}^{5}) + \sum_{\text{k}\in\text{O}} (\text{q}_{kh}^{5} + \sum_{a\in\text{D}} a_{ka}^{5h} \cdot alter_{ah}^{5}) \geq 0 \ \forall\, h \in \text{O}$$

This upper bound constraint requires that the capacity of each node at the end of the fifth period is greater than or equal to 0.

[xxxiv] The analysis is focused on an operational day, considering it independent from previous days and following ones. The model is like a frame of a long film, which can be an operational week, month or year.

- ***Non-negativity:***

1. $q_{h,k}^{o} \geq 0 \ \forall \ h \in O, k \in \mathrm{D}, o \in \mathrm{H}$
2. $notQ_{h,k}^{o} \geq 0 \ \forall \ h \in \mathrm{O}, k \in \mathrm{D}, o \in \mathrm{H}$
3. $q_{h}^{o} \geq 0 \ \forall \ h \in \mathrm{O}, o \in \mathrm{H}$
4. $a_{h,k}^{o,a} \geq 0 \ \forall \ \mathrm{h} \in \mathrm{O}, \mathrm{k} \in \mathrm{D}, \mathrm{o} \in \mathrm{H}, \ \mathrm{a} \in \mathrm{D}$

The non-negativity constraint shows that all the variables are greater than or equal to 0.

- ***Vehicles' Number:***

$$\sum_{\mathrm{h} \in \mathrm{O}} capacity_{h}^{o} = vehicles \quad \forall \ o \in \mathrm{H}$$

It is an auxiliary constraint to control that the sum of capacities is equal to the number of vehicles.

- ***Probability:***

$$willingnessQ_{hk}^{o} + notWillingness_{hk}^{o} = 1 \quad \forall \ h \in \mathrm{O}, k \in D$$

$$willingnessA_{hk}^{oa} + notWillingness_{ha}^{o} = 1 \quad \forall \ h \in \mathrm{O}, k \in D, a \in D$$

It is an auxiliary constraint to assert that willingness and not-willingness are probabilities.

The model is a variation of a transport model: there are origins and destinations, customers and sold goods, unit costs and prices, the objective to maximize revenue and to satisfy customers, but there isn't the constraint that availability must be equal to requests. This model represents an integer linear programming problem because of the linear nature of functions (objective and constraints) and the integer nature of variables.

In the model there are some simplifications, that are expressed in the following assumptions or hypothesis:

- it is focused on a single day;
- an operating day is divided in 5 time-slots and each user completes trip before the end of the range;
- there are relocations not during the day, but at the end of it;
- the relocations are instantaneous;
- each car can make 5 runs per day;
- the initial distribution and the location of the zones is known a priori;
- the desired final distribution is chosen a priori;
- number of stations, cars, value of cost, price, demand and rent time are known;
- user can take and leave cars only in the cells;
- all cells are linked;

- all the areas have the same geometrical shape and dimension;
- users always prefer the shortest route;
- all the rides are completed in a specific hourly range;
- infinite availability of parking spots;
- the utility[xxxv] of user depends from distance, tariff, alternative urban transport, alternatives proposed by system, operating day, time slot;
- other concepts already expressed above.

8.2. *Implemented Model*

8.2.1. *AMPL Modelling and Optimization tool*

This chapter describes in detail the optimization model implemented using the AMPL software, IDE Version: 3.1.0.201510231950.

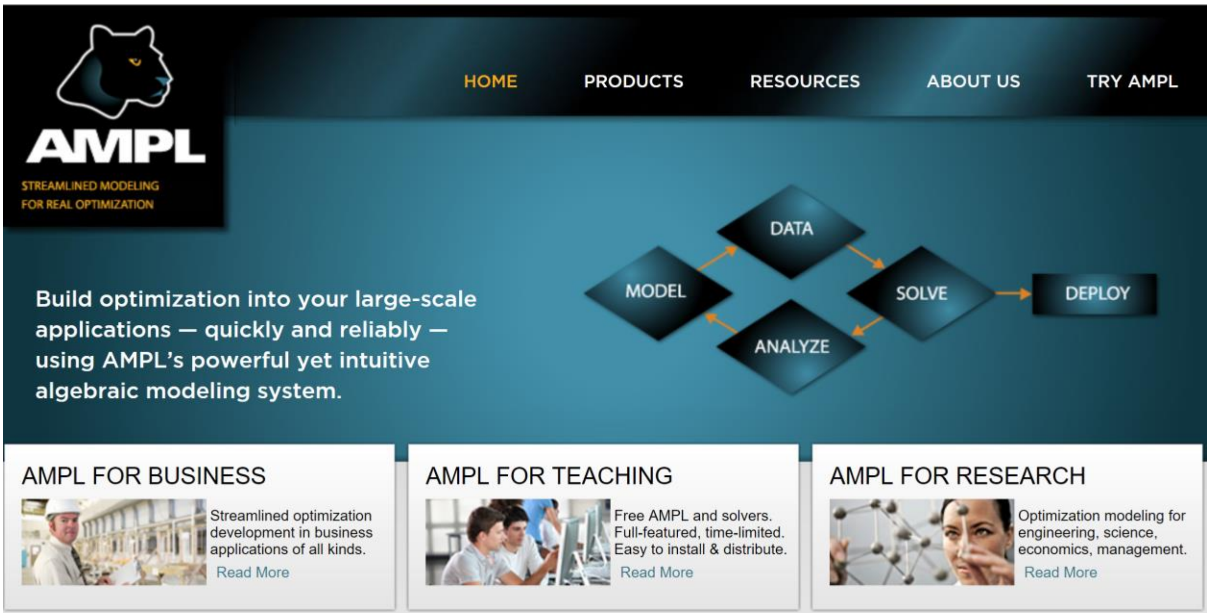

Figure 32 – AMPL Home Page

AMPL[89] is a sophisticated and user-friendly modelling tool that by using high-level representation promotes rapid development and reliable results, supporting end-to-end the optimization lifecycle:

- development,
- testing,
- deployment,
- maintenance.

AMPL allows to write model in parametric form using the same concepts and syntax for streamlined application-building, integrating:

- a modelling language for describing optimization data, variables, objectives, and constraints,
- a command language for browsing models and analysing results,

[xxxv] It is difficult to find a link between utility and all these influencing factors

- a scripting language for gathering and manipulating data and for implementing iterative optimization schemes,

separating declaration (model file), assignment (data file) and resolution (run file).

8.2.2. The Model file

The model file, which has the extension *.mod*, loads the model. From AMPL's console the model can be loaded with the instruction: *model file.mod*.

In this paragraph the structure and the content of the model file is described

```
### Definitions ###
set Cell;

set HourlyRange;

set Market:= Cell cross Cell;

set Origin := setof{(h,k) in Market} h;

set Destination := setof{(h,k) in Market} k;


## Urban Area Value definition ##
set DB within Origin;      #Dark Blue    → low potential

set LB within Origin;      #Light Blue   → medium-low potential

set Y within Origin;       #Yellow       → medium potential

set O within Origin;       #Orange       → medium-high potential

set R within Origin;       #Red          → high potential


### parameters' definition ###
param nodes{h in Origin, k in Destination}

param close{h in Origin, k in Destination}

param closepotential{h in Origin, k in Destination, o in HourlyRange}

param alter{h in Origin, k in Destination,o in HourlyRange}
```

```
param willingnessQ{(h,k) in Market, o in HourlyRange}

param willingnessA{h in Origin, k in Destination, a in Destination, o in HourlyRange}

param demand{(h,k) in Market, o in HourlyRange}

param notDemand{(h,k) in Market, o in HourlyRange}

param notWillingness{(h,k) in Market, o in HourlyRange}

param speed{o in HourlyRange}

param distance = sqrt(2)

param valueOfTariff{h in Origin, k in Destination, o in HourlyRange}

param rentTime{(h,k) in Market, o in HourlyRange}

param c{h in Origin}


### decision variables' definition ###
var quantity{(h,k) in Market, o in HourlyRange} integer, >=0

var alternative{h in Origin, k in Destination, a in Destination, o in HourlyRange}
integer, >= 0

var notQ{(h,k) in Market, o in HourlyRange} integer, >= 0

var capacity{h in Origin, o in HourlyRange} integer, >= 0


### objective function's definition ###
maximize profit: sum{h in Origin, k in Destination, o in HourlyRange} (quantity[h,k,o]
+sum {a in Destination}
(alternative[h,k,a,o]*alter[k,a,o])*5*rentTime[h,k,o]*(valueOfTariff[h,k,o]-0.17));


### constraint's definition ###
subject to availability_constraint{h in Origin, o in HourlyRange}: sum{k in
Destination} (quantity[h,k,o]+sum {a in Destination}
(alternative[h,k,a,o]*alter[k,a,o])) <= capacity[h,o]
```

```
subject to totalDemand_constraint{h in Origin, k in Destination,o in HourlyRange}:
quantity[h,k,o] + sum {a in Destination} (alternative[h,a,k,o]*alter[a,k,o]) +
notQ[h,k,o] = demand[h,k,o]

subject to lb_constraint{h in Origin,k in Destination, o in HourlyRange}: notQ[h,k,o]>=
notDemand[h,k,o];

subject to willingness_constraint{(h,k) in Market,o in HourlyRange}:
notWillingness[h,k,o]*quantity[h,k,o]- willingnessQ[h,k,o]*notQ[h,k,o] <= 0

subject to willingness_constraint_alt{(h,k) in Market, a in Destination, o in
HourlyRange}: notWillingness[h,a,o]*(alternative[h,k,a,o]*alter[k,a,o]) -
willingnessA[h,k,a,o]*notQ[h,a,o] <= 0

subject to capacityUpdate_constraint{h in Origin, o in HourlyRange: o <= 4}:
capacity[h,o+1]/5 = capacity[h,o]/5 -sum{k in Destination} (quantity[h,k,o]+sum {a in
Destination} (alternative[h,a,k,o]*alter[a,k,o])) + sum{k in Origin}
(quantity[k,h,o]+sum {a in Destination} (alternative[k,a,h,o]*alter[a,h,o]))

subject to upper_bound{h in Origin}: capacity[h,5]/5 - sum{k in Destination}
(quantity[h,k,5]+sum {a in Destination} (alternative[h,a,k,5]*alter[a,k,5])) +sum{k in
Origin} (quantity[k,h,5]+sum {a in Destination} (alternative[k,a,h,5]*alter[a,h,5])) >=
0
```

8.2.3. *Data file*

The data file has the extension *.dat* and it groups all the data. To load the file from AMPL's prompt the instruction is: *data file.dat*.

```
set Cell:= DB1 DB2 DB3 DB4 DB5 DB6 DB7 DB8 DB9 DB10 DB11 LB1 LB2 LB3 LB4 LB5 Y1 Y2 Y3
Y4 O1 O2 O3 R1 R2;
```

There are 25 cells of which 11 Dark Blue (DB1 – DB11), 5 Light Blue (LB1-LB5), 4 Yellow (Y1-Y4), 3 Orange (O1-O3), 2 Red (R1, R2).

```
set DB:= DB1 DB2 DB3 DB4 DB5 DB6 DB7 DB8 DB9 DB10 DB11;
set LB:= LB1 LB2 LB3 LB4 LB5;
set Y:=  Y1 Y2 Y3 Y4;
set O:=  O1 O2 O3;
set R:=  R1 R2;
```

The HourlyRange is definedby 5 elements:

- 1 for off-peak1,
- 2 for peak1,
- 3 for off-peak2,
- 4 for peak2
- 5 for off-peak3.

```
set HourlyRange:= 1 2 3 4 5;
```

Close Matrix

This matrix determines neighbour nodes including the node itself.

	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	Y1	Y2	Y3	Y4	O1	O2	O3	R1	R2
DB1	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0
DB2	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
DB3	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0
DB4	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
DB5	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
DB6	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
DB7	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0
DB8	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
DB9	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0
DB10	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
DB11	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0
LB1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1	0	0	1	0	0	0	0
LB2	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	1	0	0	1	0
LB3	0	1	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	1	0
LB4	0	0	0	1	0	1	1	0	1	1	1	0	0	0	1	0	0	0	0	0	0	1	1	0	0
LB5	0	0	0	0	1	1	0	1	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
Y1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	1	0	0	0	1
Y2	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	1	1	1	1	1
Y3	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	0	0	0	1
Y4	0	0	0	0	1	1	0	1	1	0	0	0	0	0	0	1	0	0	1	1	0	1	0	0	1
O1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1	1	0	1	1	0	0	1
O2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	0	1	1	1	1	1	1
O3	0	0	1	1	0	1	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	1	1	0
R1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
R2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Table 18 – Close Matrix representation

Closepotential matrix

These matrixes determine neighbour nodes with higher potential using cell value of {0,1} where value 1 identifies the neighbour nodes with higher potential.

In first, second and third hourly ranges, corresponding to first, second and third off-peaks, each destination hasn't alternatives. This means that during off-peaks, the system provides user only the destination with standard tariff.

To not make heavy and redundant the data representation, we omit the parameter *closepotential* for *o* equal to 3 and 5 that are analogous to the parameter for *o* equal to 1.

	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	Y1	Y2	Y3	Y4	O1	O2	O3	R1	R2
DB1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LB1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LB2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LB3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LB4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LB5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
O1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
O2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
O3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 19 – Closepotential matrix for time-slots 1,3,5

In the second time-slot, that corresponds to the first peak (morning), there is a general tendency to shift car from low zones to red ones. It is the so-called house-work trend, with a concentration of demand in the focus zones. In other words, there is the shift from suburbs to city's centre. Coherently with the trend, the system proposes to user different alternatives for more requested zones.

	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	Y1	Y2	Y3	Y4	O1	O2	O3	R1	R2
DB1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0
DB2	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
DB3	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0
DB4	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
DB5	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
DB6	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
DB7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0
DB8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
DB9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0
DB10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
DB11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
LB1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0
LB2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0
LB3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0
LB4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
LB5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Y1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
Y2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
Y3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
Y4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
O1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
O2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
O3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
R1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 20– Closepotential matrix for time-slots 2 (first peak)

In the second peak (evening) there is the opposite trend of morning peak. Demand goes from the central zones to the suburbs.

	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	Y1	Y2	Y3	Y4	O1	O2	O3	R1	R2
DB1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LB1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LB2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LB3	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LB4	0	0	0	1	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LB5	0	0	0	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Y2	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0
Y3	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y4	0	0	0	0	1	1	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
O1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1	1	0	0	0	0	0	0
O2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0
O3	0	0	1	1	0	1	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0
R1	0	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	1	1	0	0
R2	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0

Table 21 – Closepotential matrix for time-slots 4 (second peak)

Capacity distribution at the beginning of operating day (parameter c)

DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	Y1	Y2	Y3	Y4	O1	O2	O3	R1	R2
23	23	24	23	24	24	23	24	25	24	23	52	52	52	52	52	52	52	52	52	86	86	88	130	130

Table 22 – Vehicle distribution initialization

The Nodes distribution is represented by the matrix below.

	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	Y1	Y2	Y3	Y4	O1	O2	O3	R1	R2
DB1	1	5	5	5	4	4	5	5	5	5	5	2	3	4	4	5	2	3	3	4	2	3	4	4	3
DB2	5	1	2	3	4	4	4	5	5	5	5	4	3	2	4	5	5	3	5	4	4	3	3	2	4
DB3	5	2	1	2	5	3	3	5	4	4	4	4	3	2	3	4	5	3	5	4	4	3	2	2	4
DB4	5	3	2	1	5	3	2	5	3	3	3	4	3	3	2	4	5	3	5	4	4	3	2	2	4
DB5	4	4	5	5	1	3	5	2	3	4	5	4	4	4	4	2	3	3	2	2	3	3	4	4	2
DB6	4	4	3	3	3	1	3	3	2	2	3	4	4	4	2	2	3	3	3	2	3	2	2	3	2
DB7	5	4	3	2	5	3	1	5	3	2	2	4	4	4	2	4	5	3	5	4	4	3	2	3	4
DB8	5	5	5	5	2	3	5	1	3	4	5	5	5	5	4	2	4	4	3	2	4	3	4	4	3
DB9	5	5	4	3	3	2	3	3	1	2	3	5	5	5	2	2	4	4	3	2	4	3	3	4	3
DB10	5	5	4	3	4	2	2	4	2	1	2	5	5	5	2	3	4	4	4	3	4	3	3	4	3
DB11	5	5	4	3	5	3	2	5	3	2	1	5	5	5	2	4	5	4	5	4	4	3	3	4	4
LB1	2	4	4	4	4	4	4	5	5	5	5	1	2	3	4	5	2	2	3	4	2	3	4	4	3
LB2	3	3	3	3	4	4	4	5	5	5	5	2	1	2	4	5	3	2	4	4	2	3	3	2	3
LB3	4	2	2	3	4	4	4	5	5	5	5	3	2	1	4	5	4	2	4	4	3	3	3	2	3
LB4	4	4	3	2	4	2	2	4	2	2	2	4	4	4	1	3	4	3	4	3	3	2	2	3	3
LB5	5	5	4	4	2	2	4	2	2	3	4	5	5	5	3	1	4	4	3	2	4	3	3	4	3
Y1	2	5	5	5	3	3	5	4	4	4	5	2	3	4	4	4	1	3	2	3	2	3	4	4	2
Y2	3	3	3	3	3	3	3	4	4	4	4	2	2	2	3	4	3	1	3	3	2	2	2	2	2
Y3	3	5	5	5	2	3	5	3	3	4	5	3	4	4	4	3	2	3	1	2	2	3	4	4	2
Y4	4	4	4	4	2	2	4	2	2	3	4	4	4	4	3	2	3	3	2	1	3	2	3	3	2
O1	2	4	4	4	3	3	4	4	4	4	4	2	2	3	3	4	2	2	2	3	1	2	3	3	2
O2	3	3	3	3	3	2	3	3	3	3	3	3	3	3	2	3	3	2	3	2	2	1	2	2	2
O3	4	3	2	2	4	2	2	4	3	3	3	4	3	3	2	3	4	2	4	3	3	2	1	2	3
R1	4	2	2	2	4	3	3	4	4	4	4	4	2	2	3	4	4	2	4	3	3	2	2	1	3
R2	3	4	4	4	2	2	4	3	3	3	4	3	3	3	3	3	2	2	2	2	2	2	3	3	1

Table 23 – Nodes distribution

The starting distribution is a homogeneous-like split of cars in each area. This means that approximately 1248/5 ≅ 250 cars are in each colored area: high (20%), medium-high (20%), medium (20%), medium-low (20%), low (20%).

In reality the applied distribution is Red: 260 (≈21%); Orange: 260 (≈21%); Yellow: 208 (≈16%); Light Blue: 260 (≈21%); Light Blue: 260 (≈21%).

The choice isn’t random: we suppose that, before the beginning of operating day, staff have relocated cars to obtain a symmetric distribution. The starting capacity is a vector of 25 elements, one for each cell.

Tariff matrixes.

The tariff of all the off-peaks is the same one. For all the trips, there is the standard tariff of 0,25€/min, that is the current pricing scheme. The tariff parameter is formed by 5 (one for each time slot) squared matrixes of 25 rows and 25 columns (625 elements) for a total of 625 x 5 = 3125 elements. For period 1, 3, 5 there is the same pricing pattern.

	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	Y1	Y2	Y3	Y4	O1	O2	O3	R1	R2
DB1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
DB2	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
DB3	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
DB4	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
DB5	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
DB6	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
DB7	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
DB8	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
DB9	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
DB10	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
DB11	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
LB1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
LB2	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
LB3	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
LB4	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
LB5	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
Y1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
Y2	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
Y3	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
Y4	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
O1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
O2	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
O3	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
R1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
R2	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25

Table 24 – Price-matrix during off-peaks

	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	Y1	Y2	Y3	Y4	O1	O2	O3	R1	R2
DB1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.30	0.33	0.33	0.33	0.35	0.35
DB2	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.30	0.33	0.33	0.33	0.35	0.35
DB3	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.30	0.33	0.33	0.33	0.35	0.35
DB4	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.30	0.33	0.33	0.33	0.35	0.35
DB5	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.30	0.33	0.33	0.33	0.35	0.35
DB6	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.30	0.33	0.33	0.33	0.35	0.35
DB7	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.30	0.33	0.33	0.33	0.35	0.35
DB8	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.30	0.33	0.33	0.33	0.35	0.35
DB9	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.30	0.33	0.33	0.33	0.35	0.35
DB10	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.30	0.33	0.33	0.33	0.35	0.35
DB11	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.30	0.33	0.33	0.33	0.35	0.35
LB1	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.33	0.33
LB2	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.33	0.33
LB3	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.33	0.33
LB4	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.33	0.33
LB5	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.25	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.27	0.30	0.30	0.30	0.33	0.33
Y1	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.23	0.23	0.23	0.23	0.23	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.30	0.30
Y2	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.23	0.23	0.23	0.23	0.23	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.30	0.30
Y3	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.23	0.23	0.23	0.23	0.23	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.30	0.30
Y4	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.23	0.23	0.23	0.23	0.23	0.25	0.25	0.25	0.25	0.27	0.27	0.27	0.30	0.30
O1	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.20	0.20	0.20	0.20	0.20	0.23	0.23	0.23	0.23	0.25	0.25	0.25	0.27	0.27
O2	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.20	0.20	0.20	0.20	0.20	0.23	0.23	0.23	0.23	0.25	0.25	0.25	0.27	0.27
O3	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.20	0.20	0.20	0.20	0.20	0.23	0.23	0.23	0.23	0.25	0.25	0.25	0.27	0.27
R1	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.18	0.18	0.18	0.18	0.18	0.20	0.20	0.20	0.20	0.23	0.23	0.23	0.25	0.25
R2	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.18	0.18	0.18	0.18	0.18	0.20	0.20	0.20	0.20	0.23	0.23	0.23	0.25	0.25

Table 25 – tariff matrix during first peak in 2nd time-slot

The tariff of the first peak (morning) reflects the demand’s level. Where there is a high request, there is a higher price. To go from a zone to another one with the same color, the tariff is standard. From an attractive to an unattractive zone there are two possible discounts, while in the opposite case there are two possible increments.

	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	Y1	Y2	Y3	Y4	O1	O2	O3	R1	R2
DB1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.20	0.18	0.18	0.18	0.15	0.15
DB2	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.20	0.18	0.18	0.18	0.15	0.15
DB3	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.20	0.18	0.18	0.18	0.15	0.15
DB4	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.20	0.18	0.18	0.18	0.15	0.15
DB5	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.20	0.18	0.18	0.18	0.15	0.15
DB6	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.20	0.18	0.18	0.18	0.15	0.15
DB7	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.20	0.18	0.18	0.18	0.15	0.15
DB8	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.20	0.18	0.18	0.18	0.15	0.15
DB9	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.20	0.18	0.18	0.18	0.15	0.15
DB10	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.20	0.18	0.18	0.18	0.15	0.15
DB11	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.20	0.18	0.18	0.18	0.15	0.15
LB1	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.18	0.18
LB2	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.18	0.18
LB3	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.18	0.18
LB4	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.18	0.18
LB5	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.25	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.23	0.20	0.20	0.20	0.18	0.18
Y1	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.27	0.27	0.27	0.27	0.27	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.20	0.20
Y2	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.27	0.27	0.27	0.27	0.27	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.20	0.20
Y3	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.27	0.27	0.27	0.27	0.27	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.20	0.20
Y4	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.27	0.27	0.27	0.27	0.27	0.25	0.25	0.25	0.25	0.23	0.23	0.23	0.20	0.20
O1	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.30	0.30	0.30	0.30	0.30	0.27	0.27	0.27	0.27	0.25	0.25	0.25	0.23	0.23
O2	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.30	0.30	0.30	0.30	0.30	0.27	0.27	0.27	0.27	0.25	0.25	0.25	0.23	0.23
O3	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.30	0.30	0.30	0.30	0.30	0.27	0.27	0.27	0.27	0.25	0.25	0.25	0.23	0.23
R1	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.33	0.33	0.33	0.33	0.33	0.30	0.30	0.30	0.30	0.27	0.27	0.27	0.25	0.25
R2	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.33	0.33	0.33	0.33	0.33	0.30	0.30	0.30	0.30	0.27	0.27	0.27	0.25	0.25

Table 26 – tariff matrix during second peak in 4th time-slot

The pricing scheme of the second peak is opposite to the first peak's one because of the inversion of demand's tendency during evening. There is the so-called work-house trend.

rentTime matrixes

[*,*,1]	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	4,2	21,2	21,2	21,2	17,0	17,0	21,2	21,2	21,2	21,2	21,2	8,5	12,7	17,0	17,0	21,2	8,5	12,7	17,0	17,0	12,7	8,5	12,7	12,7	17,0
DB2	21,2	4,2	8,5	12,7	17,0	17,0	17,0	21,2	21,2	21,2	21,2	17,0	12,7	8,5	17,0	21,2	17,0	12,7	12,7	8,5	17,0	21,2	12,7	21,2	17,0
DB3	21,2	8,5	4,2	8,5	21,2	12,7	12,7	21,2	17,0	17,0	17,0	17,0	12,7	8,5	12,7	17,0	17,0	12,7	8,5	8,5	17,0	21,2	12,7	21,2	17,0
DB4	21,2	12,7	8,5	4,2	21,2	12,7	8,5	21,2	12,7	12,7	12,7	17,0	12,7	12,7	8,5	17,0	17,0	12,7	8,5	8,5	17,0	21,2	12,7	21,2	17,0
DB5	17,0	17,0	21,2	21,2	4,2	12,7	21,2	8,5	12,7	17,0	21,2	17,0	17,0	17,0	17,0	8,5	12,7	12,7	17,0	17,0	8,5	12,7	12,7	8,5	8,5
DB6	17,0	17,0	12,7	12,7	12,7	4,2	12,7	12,7	8,5	8,5	12,7	17,0	17,0	17,0	8,5	8,5	12,7	8,5	8,5	12,7	8,5	12,7	12,7	12,7	8,5
DB7	21,2	17,0	12,7	8,5	21,2	12,7	4,2	21,2	12,7	8,5	8,5	17,0	17,0	17,0	8,5	17,0	17,0	12,7	8,5	12,7	17,0	21,2	12,7	21,2	17,0
DB8	21,2	21,2	21,2	21,2	8,5	12,7	21,2	4,2	12,7	17,0	21,2	21,2	21,2	21,2	17,0	8,5	17,0	12,7	17,0	17,0	12,7	17,0	17,0	12,7	8,5
DB9	21,2	21,2	17,0	12,7	12,7	8,5	12,7	12,7	4,2	8,5	12,7	21,2	21,2	21,2	8,5	8,5	17,0	12,7	12,7	17,0	12,7	17,0	17,0	12,7	8,5
DB10	21,2	21,2	17,0	12,7	17,0	8,5	8,5	17,0	8,5	4,2	8,5	21,2	21,2	21,2	8,5	12,7	17,0	12,7	12,7	17,0	12,7	17,0	17,0	17,0	12,7
DB11	21,2	21,2	17,0	12,7	21,2	12,7	8,5	21,2	12,7	8,5	4,2	21,2	21,2	21,2	8,5	17,0	17,0	12,7	12,7	17,0	17,0	21,2	17,0	21,2	17,0
LB1	8,5	17,0	17,0	17,0	17,0	17,0	17,0	21,2	21,2	21,2	21,2	4,2	8,5	12,7	17,0	21,2	8,5	12,7	17,0	17,0	12,7	8,5	8,5	12,7	17,0
LB2	12,7	12,7	12,7	12,7	17,0	17,0	17,0	21,2	21,2	21,2	21,2	8,5	4,2	8,5	17,0	21,2	8,5	12,7	12,7	8,5	12,7	12,7	8,5	17,0	17,0
LB3	17,0	8,5	8,5	12,7	17,0	17,0	17,0	21,2	21,2	21,2	21,2	12,7	8,5	4,2	17,0	21,2	12,7	12,7	12,7	8,5	12,7	17,0	8,5	17,0	17,0
LB4	17,0	17,0	12,7	8,5	17,0	8,5	8,5	17,0	8,5	8,5	8,5	17,0	17,0	17,0	4,2	12,7	12,7	8,5	8,5	12,7	12,7	17,0	12,7	17,0	12,7
LB5	21,2	21,2	17,0	17,0	8,5	8,5	17,0	8,5	8,5	12,7	17,0	21,2	21,2	21,2	12,7	4,2	17,0	12,7	12,7	17,0	12,7	17,0	17,0	12,7	8,5
O1	8,5	17,0	17,0	17,0	12,7	12,7	17,0	17,0	17,0	17,0	17,0	8,5	8,5	12,7	12,7	17,0	4,2	8,5	12,7	12,7	8,5	8,5	8,5	8,5	12,7
O2	12,7	12,7	12,7	12,7	12,7	8,5	12,7	12,7	12,7	12,7	12,7	12,7	12,7	12,7	8,5	12,7	8,5	4,2	8,5	8,5	8,5	12,7	8,5	12,7	8,5
O3	17,0	12,7	8,5	8,5	17,0	8,5	8,5	17,0	12,7	12,7	12,7	17,0	12,7	12,7	8,5	12,7	12,7	8,5	4,2	8,5	12,7	17,0	8,5	17,0	12,7
R1	17,0	8,5	8,5	8,5	17,0	12,7	12,7	17,0	17,0	17,0	17,0	17,0	8,5	8,5	12,7	17,0	12,7	8,5	8,5	4,2	12,7	17,0	8,5	17,0	12,7
R2	12,7	17,0	17,0	17,0	8,5	8,5	17,0	12,7	12,7	12,7	17,0	12,7	12,7	12,7	12,7	12,7	8,5	8,5	12,7	12,7	4,2	8,5	8,5	8,5	8,5
Y1	8,5	21,2	21,2	21,2	12,7	12,7	21,2	17,0	17,0	17,0	21,2	8,5	12,7	17,0	17,0	17,0	8,5	12,7	17,0	17,0	8,5	4,2	12,7	8,5	12,7
Y2	12,7	12,7	12,7	12,7	12,7	12,7	12,7	17,0	17,0	17,0	17,0	8,5	8,5	8,5	12,7	17,0	8,5	8,5	8,5	8,5	8,5	12,7	4,2	12,7	12,7
Y3	12,7	21,2	21,2	21,2	8,5	12,7	21,2	12,7	12,7	17,0	21,2	12,7	17,0	17,0	17,0	12,7	8,5	12,7	17,0	17,0	8,5	8,5	12,7	4,2	8,5
Y4	17,0	17,0	17,0	17,0	8,5	8,5	17,0	8,5	8,5	12,7	17,0	17,0	17,0	17,0	12,7	8,5	12,7	8,5	12,7	12,7	8,5	12,7	12,7	8,5	4,2

Table 27 – origin-destination rentTime example during off-peaks

[*,*,2]	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	8,5	42,4	42,4	42,4	33,9	33,9	42,4	42,4	42,4	42,4	42,4	17,0	25,5	33,9	33,9	42,4	17,0	25,5	33,9	33,9	25,5	17,0	25,5	25,5	33,9
DB2	42,4	8,5	17,0	25,5	33,9	33,9	33,9	42,4	42,4	42,4	42,4	33,9	25,5	17,0	33,9	42,4	33,9	25,5	25,5	17,0	33,9	42,4	25,5	42,4	33,9
DB3	42,4	17,0	8,5	17,0	42,4	25,5	25,5	42,4	33,9	33,9	33,9	33,9	25,5	17,0	25,5	33,9	33,9	25,5	17,0	17,0	33,9	42,4	25,5	42,4	33,9
DB4	42,4	25,5	17,0	8,5	42,4	25,5	17,0	42,4	25,5	25,5	25,5	33,9	25,5	25,5	17,0	33,9	33,9	25,5	17,0	17,0	33,9	42,4	25,5	42,4	33,9
DB5	33,9	33,9	42,4	42,4	8,5	25,5	42,4	17,0	25,5	33,9	42,4	33,9	33,9	33,9	33,9	17,0	25,5	25,5	33,9	33,9	17,0	25,5	25,5	17,0	17,0
DB6	33,9	33,9	25,5	25,5	25,5	8,5	25,5	25,5	17,0	17,0	25,5	33,9	33,9	33,9	17,0	17,0	25,5	17,0	17,0	25,5	17,0	25,5	25,5	25,5	17,0
DB7	42,4	33,9	25,5	17,0	42,4	25,5	8,5	42,4	25,5	17,0	17,0	33,9	33,9	33,9	17,0	33,9	33,9	25,5	17,0	25,5	33,9	42,4	25,5	42,4	33,9
DB8	42,4	42,4	42,4	42,4	17,0	25,5	42,4	8,5	25,5	33,9	42,4	42,4	42,4	42,4	33,9	17,0	33,9	25,5	33,9	33,9	25,5	33,9	33,9	25,5	17,0
DB9	42,4	42,4	33,9	25,5	25,5	17,0	25,5	25,5	8,5	17,0	25,5	42,4	42,4	42,4	17,0	17,0	33,9	25,5	25,5	33,9	25,5	33,9	33,9	25,5	17,0
DB10	42,4	42,4	33,9	25,5	33,9	17,0	17,0	33,9	17,0	8,5	17,0	42,4	42,4	42,4	17,0	25,5	33,9	25,5	25,5	33,9	25,5	33,9	33,9	33,9	25,5
DB11	42,4	42,4	33,9	25,5	42,4	25,5	17,0	42,4	25,5	17,0	8,5	42,4	42,4	42,4	17,0	33,9	33,9	25,5	25,5	33,9	33,9	42,4	33,9	42,4	33,9
LB1	17,0	33,9	33,9	33,9	33,9	33,9	33,9	42,4	42,4	42,4	42,4	8,5	17,0	25,5	33,9	42,4	17,0	25,5	33,9	33,9	25,5	17,0	17,0	25,5	33,9
LB2	25,5	25,5	25,5	25,5	33,9	33,9	33,9	42,4	42,4	42,4	42,4	17,0	8,5	17,0	33,9	42,4	17,0	25,5	25,5	17,0	25,5	25,5	17,0	33,9	33,9
LB3	33,9	17,0	17,0	25,5	33,9	33,9	33,9	42,4	42,4	42,4	42,4	25,5	17,0	8,5	33,9	42,4	25,5	25,5	25,5	17,0	25,5	33,9	17,0	33,9	33,9
LB4	33,9	33,9	25,5	17,0	33,9	17,0	17,0	33,9	17,0	17,0	17,0	33,9	33,9	33,9	8,5	25,5	25,5	17,0	17,0	25,5	25,5	33,9	25,5	33,9	25,5
LB5	42,4	42,4	33,9	33,9	17,0	17,0	33,9	17,0	17,0	25,5	33,9	42,4	42,4	42,4	25,5	8,5	33,9	25,5	25,5	33,9	25,5	33,9	33,9	25,5	17,0
O1	17,0	33,9	33,9	33,9	25,5	25,5	33,9	33,9	33,9	33,9	33,9	17,0	17,0	25,5	25,5	33,9	8,5	17,0	25,5	25,5	17,0	17,0	17,0	17,0	25,5
O2	25,5	25,5	25,5	25,5	25,5	17,0	25,5	25,5	25,5	25,5	25,5	25,5	25,5	25,5	17,0	25,5	17,0	8,5	17,0	17,0	17,0	25,5	17,0	25,5	17,0
O3	33,9	25,5	17,0	17,0	33,9	17,0	17,0	33,9	25,5	25,5	25,5	33,9	25,5	25,5	17,0	25,5	25,5	17,0	8,5	17,0	25,5	33,9	17,0	33,9	25,5
R1	33,9	17,0	17,0	17,0	33,9	25,5	25,5	33,9	33,9	33,9	33,9	33,9	17,0	17,0	25,5	33,9	25,5	17,0	17,0	8,5	25,5	33,9	17,0	33,9	25,5
R2	25,5	33,9	33,9	33,9	17,0	17,0	33,9	25,5	25,5	25,5	33,9	25,5	25,5	25,5	25,5	25,5	17,0	17,0	25,5	25,5	8,5	17,0	17,0	17,0	17,0
Y1	17,0	42,4	42,4	42,4	25,5	25,5	42,4	33,9	33,9	33,9	42,4	17,0	25,5	33,9	33,9	33,9	17,0	25,5	33,9	33,9	17,0	8,5	25,5	17,0	25,5
Y2	25,5	25,5	25,5	25,5	25,5	25,5	25,5	33,9	33,9	33,9	33,9	17,0	17,0	17,0	25,5	33,9	17,0	17,0	17,0	17,0	17,0	25,5	8,5	25,5	25,5
Y3	25,5	42,4	42,4	42,4	17,0	25,5	42,4	25,5	25,5	33,9	42,4	25,5	33,9	33,9	33,9	25,5	17,0	25,5	33,9	33,9	17,0	17,0	25,5	8,5	17,0
Y4	33,9	33,9	33,9	33,9	17,0	17,0	33,9	17,0	17,0	25,5	33,9	33,9	33,9	33,9	25,5	17,0	25,5	17,0	25,5	25,5	17,0	25,5	25,5	17,0	8,5

Table 28 - origin-destination rentTime example during peaks

The *rent time* or *driving time* is a parameter with the same characteristics of *tariff* parameter: it is formed by 5 squared matrixes because it depends from market (origin row and destination column) and the hourly range (one matrix for each slot). We distinguish two kinds of rent time: **slow** and **fast**. **Slow** rent time is the average driving time characterizing all peak periods because of traffic congestion which leads slowdowns in circulation, while **fast** rent time is the average driving time during off-peaks hours.

In this case that *rent time* is equal to the ratio between *distance* and *speed*. For example, a rent time's value of 42 minutes during a peak corresponds to the ratio between 5 x √2 (distance) and 10 km/h (speed), multiplied by 60 (for the equivalence in minutes). To avoid heavy text these matrixes are not represented, but this choice drive to the evidence that all the peaks have the same *slow rent time* and all the off-peaks have the same *fast rent time*.

Demand matrixes

[*,*,1]	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	7	2	1	2	1	0	1	1	0	1	1	8	1	2	1	2	7	1	0	0	0	8	1	2	1
DB2	0	7	0	1	1	2	2	0	1	0	0	1	0	7	2	0	1	1	1	8	1	1	1	1	0
DB3	0	2	6	1	0	1	0	2	2	1	1	1	2	8	0	1	0	1	5	5	1	1	0	0	1
DB4	0	1	1	7	0	1	2	0	2	0	1	0	1	7	2	0	1	0	0	6	1	2	0	0	2
DB5	2	1	2	2	5	2	0	1	1	1	0	1	0	7	0	2	0	0	0	8	1	2	2	0	0
DB6	1	2	2	1	1	5	0	2	1	0	2	2	1	7	2	0	1	1	1	7	2	0	2	1	2
DB7	2	0	2	0	1	0	5	0	0	2	2	2	1	1	7	1	0	2	7	2	2	2	1	1	2
DB8	1	2	0	0	2	1	1	5	0	1	1	2	0	1	2	6	1	2	1	0	2	0	1	1	8
DB9	2	0	1	1	2	2	1	0	7	2	2	2	1	1	8	7	1	0	0	1	2	1	1	2	7
DB10	0	0	2	1	2	1	0	2	0	6	2	0	2	1	7	2	1	0	2	0	0	1	1	1	2
DB11	2	1	2	0	1	0	2	2	2	2	6	2	2	2	8	1	2	2	1	0	1	0	2	1	1
LB1	8	0	1	2	2	1	0	1	0	2	2	8	6	0	0	2	8	0	1	2	1	6	8	0	1
LB2	1	0	0	2	0	1	2	0	1	2	0	8	5	7	0	0	7	0	0	5	0	0	5	0	1
LB3	2	8	5	1	0	1	0	2	2	2	0	0	8	5	1	2	1	1	0	8	0	2	8	0	2
LB4	2	0	2	5	1	6	8	0	7	6	5	1	2	2	7	2	0	6	7	0	2	0	1	1	0
LB5	2	2	0	1	6	6	0	7	8	0	0	1	1	1	0	6	0	2	1	1	2	0	2	1	8
O1	6	2	1	1	0	2	1	1	2	1	2	7	8	2	2	1	6	8	2	1	5	8	8	5	2
O2	1	1	2	2	0	6	2	2	2	0	2	1	1	0	8	1	7	8	8	8	7	2	6	1	5
O3	2	1	7	8	0	5	7	1	0	0	1	1	2	0	6	0	2	8	8	8	2	1	6	0	2
R1	0	2	0	2	1	0	0	1	2	1	1	2	0	2	2	1	2	0	2	6	2	2	2	0	0
R2	1	0	0	1	2	2	1	2	0	2	0	2	0	1	1	2	1	0	0	0	6	1	0	0	2
Y1	6	1	0	0	1	0	0	1	1	1	0	7	1	2	2	1	7	2	0	2	8	7	1	7	1
Y2	1	2	1	1	0	1	2	0	0	2	1	8	6	5	0	1	8	6	6	8	8	0	7	0	0
Y3	2	1	2	0	8	2	2	1	1	2	0	2	2	1	2	2	6	1	2	1	6	8	1	6	7
Y4	2	0	2	1	7	7	1	8	5	0	1	2	2	0	0	6	2	7	1	2	5	0	0	5	5

Table 29 – thermographic map of origin-destination demand matrix during first off-peak hours

The situation is different during peaks

[*,*,2]	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	0	2	2	2	1	1	0	1	1	1	2	4	2	4	2	5	16	15	16	26	28	11	11	11	12
DB2	1	3	0	4	4	2	4	1	1	0	1	5	3	4	6	3	18	16	19	30	28	9	11	10	12
DB3	2	1	4	4	1	1	3	3	3	3	3	2	8	7	4	5	17	17	14	28	28	12	9	11	10
DB4	0	2	0	4	2	4	1	3	0	4	0	5	5	3	3	4	18	14	14	26	28	9	11	11	10
DB5	1	1	2	4	2	1	2	2	3	0	4	2	6	8	2	7	17	15	15	30	27	10	10	11	11
DB6	2	1	3	2	1	1	2	1	3	0	2	8	3	7	4	8	15	17	18	23	26	12	10	9	9
DB7	4	2	2	3	1	1	3	1	1	1	4	6	3	5	8	7	18	16	18	24	28	10	9	12	10
DB8	0	2	4	3	0	3	3	4	3	1	0	4	3	4	2	3	13	19	13	25	27	12	11	11	9
DB9	0	1	2	2	0	4	0	3	0	2	0	2	5	3	4	4	17	13	16	29	27	12	11	10	10
DB10	3	0	0	1	4	3	3	4	3	3	3	3	8	2	7	3	18	17	14	23	24	12	10	10	12
DB11	2	3	1	0	4	4	0	0	1	4	3	6	8	4	3	2	13	18	18	25	27	9	9	12	12
LB1	2	2	1	1	0	3	1	1	3	2	3	0	1	4	2	0	9	9	12	15	18	6	5	3	2
LB2	1	0	0	3	3	1	0	3	3	2	1	0	0	0	2	2	11	9	11	19	19	5	7	3	3
LB3	1	1	3	0	0	2	3	1	2	3	1	1	0	3	0	2	10	10	11	14	14	7	5	6	4
LB4	3	0	2	3	1	2	1	1	3	2	3	4	4	1	2	3	10	12	11	14	17	2	6	4	6
LB5	3	0	2	2	0	0	0	0	0	0	0	2	4	0	0	1	9	11	12	16	16	8	6	5	7
O1	0	1	1	1	0	0	1	1	1	1	0	1	0	0	2	2	2	3	3	8	8	0	2	3	3
O2	1	1	0	1	1	0	1	1	1	0	1	0	1	1	2	2	3	3	2	3	6	2	1	1	0
O3	1	1	0	0	1	0	1	1	1	1	0	1	1	0	0	2	2	2	3	5	5	0	1	1	2
R1	0	0	1	0	1	0	0	0	1	0	0	1	0	1	0	0	3	2	3	1	1	1	0	2	0
R2	0	1	0	0	0	0	0	0	1	0	0	1	1	1	0	0	1	0	2	3	2	1	2	1	2
Y1	2	2	0	0	1	2	2	0	0	2	0	2	3	3	0	0	5	3	2	9	12	3	0	2	2
Y2	2	1	2	2	0	0	2	2	2	2	0	0	3	1	3	1	5	8	5	12	12	2	4	1	2
Y3	0	1	0	1	2	2	1	2	1	0	2	3	2	0	2	3	3	4	7	9	11	1	3	3	3
Y4	1	0	0	2	0	0	2	2	0	2	2	1	2	3	1	1	3	7	8	9	12	0	4	3	0

Table 30 – thermographic map of origin-destination demand matrix during first peak hours

Table below describes, during the first peak, demand characteristics from origins (rows) to destinations (columns); to avoid long text, only some examples are commented.
Main characteristics highlight, for instance, that *demand* is:

- high (range: 24-30) from low (DB_y) to high demand areas (R_x),
- medium-high (range: 13-19) from low (DB_y) to medium-high (O_x) demand areas and from medium-low (LB_y) to high-demand areas (R_x),
- very-low (range: 0-4) from low (DB_y) to low (DB_x).

Demand changes again during the 4th time-slot characterising the second traffic peak.

[*,*,4]	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	4	1	1	3	2	3	2	2	1	3	4	3	2	1	0	0	0	1	1	0	1	0	1	1	2
DB2	2	4	3	4	4	4	2	0	0	2	0	2	2	0	1	3	1	0	1	1	0	2	1	2	2
DB3	2	3	2	1	1	0	0	0	4	0	4	3	2	0	0	1	1	1	1	1	0	2	1	0	0
DB4	1	0	1	4	1	2	2	0	1	4	4	3	2	1	3	2	1	1	1	1	1	2	0	0	2
DB5	2	1	4	1	3	2	4	1	2	4	2	1	3	1	3	0	0	0	0	0	0	0	0	2	0
DB6	2	0	1	3	1	3	4	4	3	2	1	1	2	2	2	1	1	1	0	0	0	0	2	0	2
DB7	4	0	0	0	2	2	3	4	0	0	0	2	2	3	0	1	0	1	0	1	0	0	2	1	1
DB8	4	4	3	1	2	2	3	3	0	4	1	2	1	3	1	1	1	1	0	0	0	1	1	0	0
DB9	0	4	0	0	4	1	1	4	2	0	3	3	2	2	0	0	0	1	1	0	0	2	2	0	1
DB10	2	1	4	4	4	4	1	4	4	2	1	1	0	0	0	1	1	0	1	1	0	2	1	2	2
DB11	4	1	2	2	2	4	4	3	1	0	0	3	1	3	3	1	0	0	1	0	0	2	0	1	2
LB1	4	5	8	3	6	5	3	6	3	5	3	4	2	3	2	2	1	0	2	0	0	0	2	0	1
LB2	5	5	6	2	8	8	5	4	6	6	8	0	4	1	1	3	2	0	1	0	0	0	1	0	0
LB3	3	3	7	5	4	8	4	2	8	7	7	1	3	1	3	4	0	2	0	1	0	0	2	3	1
LB4	8	5	4	2	6	2	4	6	2	7	6	1	2	2	4	3	0	1	0	0	1	0	2	1	3
LB5	8	4	6	4	2	4	3	7	2	3	6	3	3	2	3	4	1	0	1	0	0	1	2	1	2
O1	19	15	13	13	13	14	17	19	13	14	15	11	11	10	10	12	4	0	4	2	3	6	8	6	6
O2	18	18	14	13	19	17	19	18	15	15	14	9	9	12	11	9	3	3	2	0	0	7	8	8	4
O3	14	16	17	15	14	16	15	13	17	13	14	10	11	11	12	12	0	0	1	1	3	3	5	8	4
R1	28	27	29	24	29	25	30	29	23	26	30	13	17	19	15	13	6	5	4	4	1	11	10	12	9
R2	25	27	29	24	26	27	30	26	26	30	23	17	19	18	16	13	4	8	7	4	3	12	10	12	9
Y1	9	12	12	11	11	9	12	12	12	9	11	6	5	3	3	6	3	2	0	2	1	0	3	1	0
Y2	9	9	12	10	12	9	11	12	12	9	10	3	6	2	7	8	1	0	1	1	2	2	2	3	1
Y3	12	12	9	9	10	12	9	12	11	11	12	3	4	8	5	4	2	0	1	1	2	4	4	0	2
Y4	12	10	11	9	11	12	9	12	9	12	11	4	5	6	2	3	2	3	1	0	0	1	2	3	1

Table 31 – thermographic map of origin-destination demand matrix during second peak hours

During the second peak there is the known inversion of trend. This means that all that areas that was attractive during the first peak become unattractive.

notDemand matrixes

[*,*,1]	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	2	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	2	0	0	0	0	3	0	0	0
DB2	0	2	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	3	0	0	0	0	0
DB3	0	0	2	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	2	2	0	0	0	0	0
DB4	0	0	0	2	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	2	0	0	0	0	0
DB5	0	0	0	0	2	0	0	0	0	0	0	0	0	2	0	0	0	0	0	3	0	0	0	0	0
DB6	0	0	0	0	0	2	0	0	0	0	0	0	0	2	0	0	0	0	0	2	0	0	0	0	0
DB7	0	0	0	0	0	0	2	0	0	0	0	0	0	0	2	0	0	0	2	0	0	0	0	0	0
DB8	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	3
DB9	0	0	0	0	0	0	0	0	2	0	0	0	0	0	3	2	0	0	0	0	0	0	0	0	2
DB10	0	0	0	0	0	0	0	0	0	2	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0
DB11	0	0	0	0	0	0	0	0	0	0	2	0	0	0	3	0	0	0	0	0	0	0	0	0	0
LB1	3	0	0	0	0	0	0	0	0	0	0	3	2	0	0	0	3	0	0	0	0	2	3	0	0
LB2	0	0	0	0	0	0	0	0	0	0	0	3	2	2	0	0	2	0	0	2	0	0	2	0	0
LB3	0	3	2	0	0	0	0	0	0	0	0	0	3	2	0	0	0	0	0	3	0	0	3	0	0
LB4	0	0	0	2	0	2	3	0	2	2	2	0	0	0	2	0	0	2	2	0	0	0	0	0	0
LB5	0	0	0	0	2	2	0	2	3	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	3
O1	2	0	0	0	0	0	0	0	0	0	0	2	3	0	0	0	2	3	0	0	2	3	3	2	0
O2	0	0	0	0	0	2	0	0	0	0	0	0	0	0	3	0	2	3	3	3	2	0	2	0	2
O3	0	0	2	3	0	2	2	0	0	0	0	0	0	0	2	0	0	3	3	3	0	0	2	0	0
R1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0
R2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0
Y1	2	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	2	0	0	0	3	2	0	2	0
Y2	0	0	0	0	0	0	0	0	0	0	0	3	2	2	0	0	3	2	2	3	3	0	2	0	0
Y3	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	2	3	0	2	2
Y4	0	0	0	0	2	2	0	3	2	0	0	0	0	0	0	2	0	2	0	0	2	0	0	2	2

Table 32 – thermographic map of origin-destination notDemand matrix during second peak hours

[*,*,2]	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	2	7	7	7	13	14	4	4	4	5
DB2	0	1	0	1	1	0	1	0	0	0	0	2	1	1	2	1	8	7	9	15	14	3	4	4	5
DB3	0	0	1	1	0	0	1	1	1	1	1	0	4	2	1	2	8	8	6	14	14	5	3	4	4
DB4	0	0	0	1	0	1	0	1	0	1	0	2	2	1	1	1	8	6	6	13	14	3	4	4	4
DB5	0	0	0	1	0	0	0	0	1	0	1	0	3	3	0	2	8	7	7	15	14	4	4	4	4
DB6	0	0	1	0	0	0	0	0	1	0	0	3	1	2	1	3	7	8	8	11	13	5	4	3	3
DB7	1	0	0	1	0	0	1	0	0	0	1	2	1	2	3	2	8	7	8	12	14	4	3	5	4
DB8	0	0	1	1	0	1	1	1	1	0	0	1	1	1	0	1	6	9	6	13	14	5	4	4	3
DB9	0	0	0	0	0	1	0	1	0	0	0	0	2	1	1	1	8	6	7	15	14	5	4	4	4
DB10	1	0	0	0	1	1	1	1	1	1	1	1	4	0	2	1	8	8	6	11	12	5	4	4	5
DB11	0	1	0	0	1	1	0	0	0	1	1	2	4	1	1	0	6	8	8	13	14	3	3	5	5
LB1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	3	3	5	7	8	2	2	1	0
LB2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	3	4	9	9	2	2	1	1
LB3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	6	6	2	2	2	1
LB4	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	4	5	4	6	8	0	2	1	2
LB5	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	3	4	5	7	7	3	2	2	2
O1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	0	0	0	0
O2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0	0	0	0
O3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0
R1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	0	3	5	0	0	0	0
Y2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	3	2	5	5	0	1	0	0
Y3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	3	4	0	0	0	0
Y4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	3	5	0	1	0	0

Table 33 – thermographic map of origin-destination notDemand matrix during first peak hours

[*,*,4]	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB2	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB3	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB4	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB5	0	0	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB6	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB7	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB8	1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB9	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB10	0	0	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB11	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LB1	1	2	3	1	2	2	1	2	1	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
LB2	2	2	2	0	3	3	2	1	2	2	3	0	1	0	0	0	0	0	0	0	0	0	0	0	0
LB3	1	1	2	2	1	3	1	0	3	2	2	0	0	0	0	1	0	0	0	0	0	0	0	0	0
LB4	3	2	1	0	2	0	1	2	0	2	2	0	0	0	1	0	0	0	0	0	0	0	0	0	0
LB5	3	1	2	1	0	1	1	2	0	1	2	0	0	0	0	1	0	0	0	0	0	0	0	0	0
O1	9	7	6	6	6	6	8	9	6	6	7	4	4	4	4	5	1	0	1	0	0	2	3	2	2
O2	8	8	6	6	9	8	9	8	7	7	6	3	3	5	4	3	0	0	0	0	0	2	3	3	1
O3	6	7	8	7	6	7	7	6	8	6	6	4	4	4	5	5	0	0	0	0	0	1	2	3	1
R1	14	14	15	12	15	13	15	15	11	13	15	6	8	9	7	6	2	2	1	1	0	4	4	5	3
R2	13	14	15	12	13	14	15	13	13	15	11	8	9	8	7	6	1	3	2	1	1	5	4	5	3
Y1	3	5	5	4	4	3	5	5	5	3	4	2	2	1	1	2	0	0	0	0	0	0	0	0	0
Y2	3	3	5	4	5	3	4	5	5	3	4	1	2	0	2	3	0	0	0	0	0	0	0	0	0
Y3	5	5	3	3	4	5	3	5	4	4	5	1	1	3	2	1	0	0	0	0	0	1	1	0	0
Y4	5	4	4	3	4	5	3	5	3	5	4	1	2	2	0	1	0	0	0	0	0	0	0	0	0

Table 34 – thermographic map of origin-destination notDemand matrix during second peak hours

The *notDemand* parameter has the same structure of *demand* and is defined by 5 matrixes. It represents the minimum number of people that aren't willing to pay the offered tariff. Each term of the matrixes is determined by the lower rounding of the product between *not-willingness* and *demand*. In other words, it is the difference between demand and the minimum number of people willing to accept.

Observing the three sample matrixes; it is important to note that in all off-peaks slots 40% of users isn't willing to accept the tariff, because it has been supposed that nowadays 60% of the total demand is disposed to pay a standard tariff.

Consequently, with a standard tariff, the estimated and expected unsold trips are equal to 40% of total demand. During peaks, there is a higher probability to reject rides with higher tariffs, and a lower probability to reject runs where the price is low.

Then, from an attractive zone to an unattractive one, there is a high number of people willing to pay, despite the demand is low.

Considering the proportion of users willing to accept compared to demand, we expect that attractive markets have a proportion lower than unattractive markets' one.

The parameter *willingness* depends from origin, destination, hourly range and alternative. The *attractiveness* depends on different factor, such tariff, distance from the desired destination, alternative urban transport, etc. and it represents the probability to accept or to pay a trip. In this simplified model, focused on variable prices, it is licit to think this parameter depending only on tariff.

AMPL requires a specific notation to represent it: for each destination and hourly range, there is a matrix whose rows are the origin points and the columns are the alternatives.

During off-peaks hourly ranges, the *willingnessQ* is equal to 0,60. This denotes that the probability to accept and pay the offered tariff for the proposed run is equal to 60% of the demand or, if you prefer, that a trip has a 60% of attractiveness.

Note that this doesn't mean that the sold trips are a 60% of the total demand, but that users willing to pay forms a 60% of demand and the system must satisfy the higher number of requests with the available capacity. Without this parameter, the solver would choose all the trips with the longest distance and the maximum margin, neglecting users' behaviour. We omit other data of the off-peaks to not make redundant the text.
During the first peak, a low (or dark-blue) origin, characterized by few requests, has one of the following willingness' value:

[*,*,2]	DB(k)	Tariff
DB(h)	0,60	Standard
LB(h)	0,70	Discount (1)
Y(h)	0,75	Discount (2)
O(h)	0,80	Discount (3)
R(h)	0,85	Discount (4)

Table 35 –willingnessQ to accept from DARK BLUE origin h (to destination k)

First of all, it is important to comment the difference among values: 0,6 indicates a 60% probability to accept a ride; it is assumed that for a 20% unit discount (from 0,25 to 0,20) on the unit standard tariff, there is around a 30% increment (from 60% to 80%) of attractivity and that for a 32% discount (from 0,25 to 0,17), there is a 50% (from 60% to 90%) increment of attractiveness. Although increments of willingness seem to be overestimated, we must consider not only that there is a coherent difference (10%) between the twos, but also that this excessive difference from the "standard" (initial) willingness is compensated by its decreases (see later to better understand).
To avoid excessive information about willingness only yellow and red destination will be commented considering that for additional details the full matrixes can be consulted below.

[*,*,2]	Y(k)	Tariff
DB(h)	0,45	Penalty (2)
LB(h)	0,50	Penalty (1)
Y(h)	0,60	Standard
O(h)	0,70	Discount (1)
R(h)	0,75	Discount (2)

Table 36 – willingness to accept from YELLOW origin h (to destination k)

[*,*,2]	R(k)	Tariff
DB(h)	0,35	Penalty (4)
LB(h)	0,40	Penalty (3)
Y(h)	0,45	Penalty (2)
O(h)	0,50	Penalty (1)
R(h)	0,60	Standard

Table 37 – willingness to accept from RED origin h (to destination k)

The full willingness matrix is displayed below for the first peak (second time-slot) is illustrated below.

[*,*,2]	DB1	DB10	DB11	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,50	0,50	0,50	0,50	0,50	0,40	0,40	0,40	0,35	0,35	0,45	0,45	0,45	0,45
DB10	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,50	0,50	0,50	0,50	0,50	0,40	0,40	0,40	0,35	0,35	0,45	0,45	0,45	0,45
DB11	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,50	0,50	0,50	0,50	0,50	0,40	0,40	0,40	0,35	0,35	0,45	0,45	0,45	0,45
DB2	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,50	0,50	0,50	0,50	0,50	0,40	0,40	0,40	0,35	0,35	0,45	0,45	0,45	0,45
DB3	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,50	0,50	0,50	0,50	0,50	0,40	0,40	0,40	0,35	0,35	0,45	0,45	0,45	0,45
DB4	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,50	0,50	0,50	0,50	0,50	0,40	0,40	0,40	0,35	0,35	0,45	0,45	0,45	0,45
DB5	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,50	0,50	0,50	0,50	0,50	0,40	0,40	0,40	0,35	0,35	0,45	0,45	0,45	0,45
DB6	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,50	0,50	0,50	0,50	0,50	0,40	0,40	0,40	0,35	0,35	0,45	0,45	0,45	0,45
DB7	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,50	0,50	0,50	0,50	0,50	0,40	0,40	0,40	0,35	0,35	0,45	0,45	0,45	0,45
DB8	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,50	0,50	0,50	0,50	0,50	0,40	0,40	0,40	0,35	0,35	0,45	0,45	0,45	0,45
DB9	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,50	0,50	0,50	0,50	0,50	0,40	0,40	0,40	0,35	0,35	0,45	0,45	0,45	0,45
LB1	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,60	0,60	0,60	0,60	0,60	0,45	0,45	0,45	0,40	0,40	0,50	0,50	0,50	0,50
LB2	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,60	0,60	0,60	0,60	0,60	0,45	0,45	0,45	0,40	0,40	0,50	0,50	0,50	0,50
LB3	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,60	0,60	0,60	0,60	0,60	0,45	0,45	0,45	0,40	0,40	0,50	0,50	0,50	0,50
LB4	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,60	0,60	0,60	0,60	0,60	0,45	0,45	0,45	0,40	0,40	0,50	0,50	0,50	0,50
LB5	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,70	0,60	0,60	0,60	0,60	0,60	0,45	0,45	0,45	0,40	0,40	0,50	0,50	0,50	0,50
O1	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,75	0,75	0,75	0,75	0,75	0,60	0,60	0,60	0,50	0,50	0,70	0,70	0,70	0,70
O2	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,75	0,75	0,75	0,75	0,75	0,60	0,60	0,60	0,50	0,50	0,70	0,70	0,70	0,70
O3	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,75	0,75	0,75	0,75	0,75	0,60	0,60	0,60	0,50	0,50	0,70	0,70	0,70	0,70
R1	0,85	0,85	0,85	0,85	0,85	0,85	0,85	0,85	0,85	0,85	0,85	0,80	0,80	0,80	0,80	0,80	0,70	0,70	0,70	0,60	0,60	0,75	0,75	0,75	0,75
R2	0,85	0,85	0,85	0,85	0,85	0,85	0,85	0,85	0,85	0,85	0,85	0,80	0,80	0,80	0,80	0,80	0,70	0,70	0,70	0,60	0,60	0,75	0,75	0,75	0,75
Y1	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,70	0,70	0,70	0,70	0,70	0,50	0,50	0,50	0,45	0,45	0,60	0,60	0,60	0,60
Y2	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,70	0,70	0,70	0,70	0,70	0,50	0,50	0,50	0,45	0,45	0,60	0,60	0,60	0,60
Y3	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,70	0,70	0,70	0,70	0,70	0,50	0,50	0,50	0,45	0,45	0,60	0,60	0,60	0,60
Y4	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,75	0,70	0,70	0,70	0,70	0,70	0,50	0,50	0,50	0,45	0,45	0,60	0,60	0,60	0,60

Table 38 – full matrix of willingnessQ in first peak slot (2nd time-slot)

During peak one, the medium and high destinations are more attractive than low areas. For this, the system provides different alternatives for yellow, orange and red destinations. As mentioned in previous chapters, user who wants to go to an unattractive cell must decide if he want to pay an increased tariff to reach the desired destination or to choose an alternative with a discounted tariff, contributing to relocation.

It is supposed that the probability to accept or to pay depends only on the price, according to this scheme, for example:

- if a user goes from a red zone to a dark blue zone, the willingness is 0,35 (reduction of more than 41% from the standard attractiveness) because there is a 32% increment on the standard tariff (from 0,25 to 0,33);
- if a user goes from a cell to another with the same color, willingness is still 0,6 because the tariff is standard;
- if a user goes from dark blue to red area, the willingness increases to 85% thanks to the 20% discount.

Once defined parameters of desired destination, we define the alternatives' willingness.

Considering *n* as the number of alternatives, if there at least one alternative (n > 0) the willingness to accept the alternative is so defined:

$$willingnessA_{hk}^{oa} = \frac{0{,}2 \times \left(1 - willingnessQ_{hk}^{ok}\right)}{n} \qquad \forall\ \mathrm{o, h, k, a} \neq \mathrm{k};$$

Of course, if n = 0 then there aren't alternatives and the willingness to accept the alternative is 0.

This means that each exact alternative has a willingness that is equal to $\frac{1}{\mathrm{n}}$ -th of a certain percentage of the probability to reject the desired destination. This percentage is equal to 20%. The choice of 20% derives from the supposition that only a 20% of users who don't accept the increased tariff is disposed to pay a lower tariff for a less adaptable alternative ride. The other 80% prefer to reject the ride and all its alternatives and to take another mean of transport. The number n has the goal to make uniform alternatives' willingness.

Note that if there are zones with more than a color, the percentages for the two colored areas are less than 20%, but in a way that their sum is 20.

For example, if there are 3 alternatives then willingness is equal to:

1/3 x 0,2 x (1-willingnessQ).

In case of [DB4, *, *,2] the willingnessA to accept alternatives[xxxvi] is

	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	Y1	Y2	Y3	Y4
LB1	0,100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LB3	0	0,050	0,050	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LB4	0	0	0	0,017	0	0,017	0,017	0	0,017	0,017	0,017	0	0	0	0	0	0	0	0	0	0	0	0
LB5	0	0	0	0	0,025	0,025	0	0,025	0,025	0	0	0	0	0	0	0	0	0	0	0	0	0	0
O1	0,020	0	0	0	0	0	0	0	0	0	0	0,020	0,020	0	0	0	0	0	0	0,020	0,020	0,020	0
O2	0	0	0	0	0	0,030	0	0	0	0	0	0	0	0	0,030	0	0	0	0	0	0,030	0	0,030
O3	0	0	0,020	0,020	0	0,020	0,020	0	0	0	0	0	0	0	0,020	0	0	0	0	0	0,020	0	0
R1	0	0,016	0,016	0,016	0	0	0	0	0	0	0	0	0,016	0,016	0	0	0	0,016	0,016	0	0,016	0	0
R2	0	0	0	0	0,016	0,016	0	0	0	0	0	0	0	0	0	0	0,016	0,016	0	0,016	0,016	0,016	0,016
Y1	0,055	0	0	0	0	0	0	0	0	0	0	0,055	0	0	0	0	0	0	0	0	0	0	0
Y2	0	0	0	0	0	0	0	0	0	0	0	0,037	0,037	0,037	0	0	0	0	0	0	0	0	0
Y3	0	0	0	0	0,110	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y4	0	0	0	0	0,022	0,022	0	0,022	0,022	0	0	0	0	0	0	0,022	0	0	0	0	0	0	0

Table 39 – example of willingnessA to accept alternative destinations for lower tariffs

In the second peak (evening), the same considerations of the first peak are valid, but it is important to remember that the most attractive areas are now the suburban nodes.

During the second peak (fourth time-slot) the trend in inverted so that *willingnessQ* is the transposed of the matrix of the time-slot.

[*,*,4]	DB1	DB10	DB11	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,70	0,70	0,70	0,70	0,70	0,80	0,80	0,80	0,85	0,85	0,75	0,75	0,75	0,75
DB10	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,70	0,70	0,70	0,70	0,70	0,80	0,80	0,80	0,85	0,85	0,75	0,75	0,75	0,75
DB11	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,70	0,70	0,70	0,70	0,70	0,80	0,80	0,80	0,85	0,85	0,75	0,75	0,75	0,75
DB2	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,70	0,70	0,70	0,70	0,70	0,80	0,80	0,80	0,85	0,85	0,75	0,75	0,75	0,75
DB3	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,70	0,70	0,70	0,70	0,70	0,80	0,80	0,80	0,85	0,85	0,75	0,75	0,75	0,75
DB4	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,70	0,70	0,70	0,70	0,70	0,80	0,80	0,80	0,85	0,85	0,75	0,75	0,75	0,75
DB5	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,70	0,70	0,70	0,70	0,70	0,80	0,80	0,80	0,85	0,85	0,75	0,75	0,75	0,75
DB6	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,70	0,70	0,70	0,70	0,70	0,80	0,80	0,80	0,85	0,85	0,75	0,75	0,75	0,75
DB7	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,70	0,70	0,70	0,70	0,70	0,80	0,80	0,80	0,85	0,85	0,75	0,75	0,75	0,75
DB8	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,70	0,70	0,70	0,70	0,70	0,80	0,80	0,80	0,85	0,85	0,75	0,75	0,75	0,75
DB9	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,60	0,70	0,70	0,70	0,70	0,70	0,80	0,80	0,80	0,85	0,85	0,75	0,75	0,75	0,75
LB1	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,60	0,60	0,60	0,60	0,60	0,75	0,75	0,75	0,80	0,80	0,70	0,70	0,70	0,70
LB2	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,60	0,60	0,60	0,60	0,60	0,75	0,75	0,75	0,80	0,80	0,70	0,70	0,70	0,70
LB3	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,60	0,60	0,60	0,60	0,60	0,75	0,75	0,75	0,80	0,80	0,70	0,70	0,70	0,70
LB4	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,60	0,60	0,60	0,60	0,60	0,75	0,75	0,75	0,80	0,80	0,70	0,70	0,70	0,70
LB5	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,60	0,60	0,60	0,60	0,60	0,75	0,75	0,75	0,80	0,80	0,70	0,70	0,70	0,70
O1	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,45	0,45	0,45	0,45	0,45	0,60	0,60	0,60	0,70	0,70	0,50	0,50	0,50	0,50
O2	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,45	0,45	0,45	0,45	0,45	0,60	0,60	0,60	0,70	0,70	0,50	0,50	0,50	0,50
O3	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,45	0,45	0,45	0,45	0,45	0,60	0,60	0,60	0,70	0,70	0,50	0,50	0,50	0,50
R1	0,35	0,35	0,35	0,35	0,35	0,35	0,35	0,35	0,35	0,35	0,35	0,40	0,40	0,40	0,40	0,40	0,50	0,50	0,50	0,60	0,60	0,45	0,45	0,45	0,45
R2	0,35	0,35	0,35	0,35	0,35	0,35	0,35	0,35	0,35	0,35	0,35	0,40	0,40	0,40	0,40	0,40	0,50	0,50	0,50	0,60	0,60	0,45	0,45	0,45	0,45
Y1	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,50	0,50	0,50	0,50	0,50	0,70	0,70	0,70	0,75	0,75	0,60	0,60	0,60	0,60
Y2	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,50	0,50	0,50	0,50	0,50	0,70	0,70	0,70	0,75	0,75	0,60	0,60	0,60	0,60
Y3	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,50	0,50	0,50	0,50	0,50	0,70	0,70	0,70	0,75	0,75	0,60	0,60	0,60	0,60
Y4	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,45	0,50	0,50	0,50	0,50	0,50	0,70	0,70	0,70	0,75	0,75	0,60	0,60	0,60	0,60

Table 40 – full matrix of willingnessQ in second peak slot (4th time-slot)

<u>**notWillingness** matrixes</u>

[xxxvi] Non displayed rows and columns have a willingness to accept the alternative equal to 0.

notWillingness is a parameter representing, for each time slot, the probability to reject a ride (or the unattractiveness) of a destination; in this case it defined by 5 matrixes.

It is defined as the 80% of users who refuse an increased tariff to go to desired destinations, so it is so calculated:

$$notWillingness_{hk}^{o} = 0{,}8 \times (1 - willingnessQ_{hk}^{o}) \quad \forall\, h, k, o,$$

where $willingnessQ_{hk}^{o}$ is the attractivity of the desired destination (k).

Value of notWillingness are illustrated in the tables below.

[*,*,1]	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
DB2	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
DB3	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
DB4	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
DB5	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
DB6	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
DB7	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
DB8	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
DB9	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
DB10	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
DB11	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
LB1	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
LB2	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
LB3	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
LB4	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
LB5	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
O1	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
O2	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
O3	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
R1	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
R2	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
Y1	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
Y2	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
Y3	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40
Y4	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40

Table 41 – notWillingness origin-destination for off-peaks (time slots 1, 3 and 5).

[*,*,2]	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,50	0,40	0,40	0,40	0,48	0,48	0,48	0,52	0,52	0,44	0,44	0,44	0,44
DB10	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,50	0,40	0,40	0,40	0,48	0,48	0,48	0,52	0,52	0,44	0,44	0,44	0,44
DB11	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,50	0,40	0,40	0,40	0,48	0,48	0,48	0,52	0,52	0,44	0,44	0,44	0,44
DB2	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,50	0,40	0,40	0,40	0,48	0,48	0,48	0,52	0,52	0,44	0,44	0,44	0,44
DB3	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,50	0,40	0,40	0,40	0,48	0,48	0,48	0,52	0,52	0,44	0,44	0,44	0,44
DB4	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,50	0,40	0,40	0,40	0,48	0,48	0,48	0,52	0,52	0,44	0,44	0,44	0,44
DB5	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,50	0,40	0,40	0,40	0,48	0,48	0,48	0,52	0,52	0,44	0,44	0,44	0,44
DB6	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,50	0,40	0,40	0,40	0,48	0,48	0,48	0,52	0,52	0,44	0,44	0,44	0,44
DB7	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,50	0,40	0,40	0,40	0,48	0,48	0,48	0,52	0,52	0,44	0,44	0,44	0,44
DB8	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,50	0,40	0,40	0,40	0,48	0,48	0,48	0,52	0,52	0,44	0,44	0,44	0,44
DB9	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,50	0,40	0,40	0,40	0,48	0,48	0,48	0,52	0,52	0,44	0,44	0,44	0,44
LB1	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,32	0,40	0,32	0,32	0,32	0,44	0,44	0,44	0,48	0,48	0,40	0,40	0,40	0,40
LB2	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,32	0,40	0,32	0,32	0,32	0,44	0,44	0,44	0,48	0,48	0,40	0,40	0,40	0,40
LB3	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,32	0,40	0,32	0,32	0,32	0,44	0,44	0,44	0,48	0,48	0,40	0,40	0,40	0,40
LB4	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,32	0,40	0,32	0,32	0,32	0,44	0,44	0,44	0,48	0,48	0,40	0,40	0,40	0,40
LB5	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,30	0,32	0,40	0,32	0,32	0,32	0,44	0,44	0,44	0,48	0,48	0,40	0,40	0,40	0,40
O1	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,25	0,20	0,20	0,20	0,32	0,32	0,32	0,40	0,40	0,24	0,24	0,24	0,24
O2	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,25	0,20	0,20	0,20	0,32	0,32	0,32	0,40	0,40	0,24	0,24	0,24	0,24
O3	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,25	0,20	0,20	0,20	0,32	0,32	0,32	0,40	0,40	0,24	0,24	0,24	0,24
R1	0,15	0,15	0,15	0,15	0,15	0,15	0,15	0,15	0,15	0,15	0,15	0,16	0,20	0,16	0,16	0,16	0,24	0,24	0,24	0,32	0,32	0,20	0,20	0,20	0,20
R2	0,15	0,15	0,15	0,15	0,15	0,15	0,15	0,15	0,15	0,15	0,15	0,16	0,20	0,16	0,16	0,16	0,24	0,24	0,24	0,32	0,32	0,20	0,20	0,20	0,20
Y1	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,24	0,30	0,24	0,24	0,24	0,40	0,40	0,40	0,44	0,44	0,32	0,32	0,32	0,32
Y2	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,24	0,30	0,24	0,24	0,24	0,40	0,40	0,40	0,44	0,44	0,32	0,32	0,32	0,32
Y3	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,24	0,30	0,24	0,24	0,24	0,40	0,40	0,40	0,44	0,44	0,32	0,32	0,32	0,32
Y4	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,24	0,30	0,24	0,24	0,24	0,40	0,40	0,40	0,44	0,44	0,32	0,32	0,32	0,32

Table 42 – notWillingness origin-destination for 1st peak (time slot 2)

[*,*,4]	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,24	0,24	0,24	0,24	0,24	0,16	0,16	0,16	0,15	0,15	0,20	0,20	0,20	0,20
DB10	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,24	0,24	0,24	0,24	0,24	0,16	0,16	0,16	0,15	0,15	0,20	0,20	0,20	0,20
DB11	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,24	0,24	0,24	0,24	0,24	0,16	0,16	0,16	0,15	0,15	0,20	0,20	0,20	0,20
DB2	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,24	0,24	0,24	0,24	0,24	0,16	0,16	0,16	0,15	0,15	0,20	0,20	0,20	0,20
DB3	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,24	0,24	0,24	0,24	0,24	0,16	0,16	0,16	0,15	0,15	0,20	0,20	0,20	0,20
DB4	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,24	0,24	0,24	0,24	0,24	0,16	0,16	0,16	0,15	0,15	0,20	0,20	0,20	0,20
DB5	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,24	0,24	0,24	0,24	0,24	0,16	0,16	0,16	0,15	0,15	0,20	0,20	0,20	0,20
DB6	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,24	0,24	0,24	0,24	0,24	0,16	0,16	0,16	0,15	0,15	0,20	0,20	0,20	0,20
DB7	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,24	0,24	0,24	0,24	0,24	0,16	0,16	0,16	0,15	0,15	0,20	0,20	0,20	0,20
DB8	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,24	0,24	0,24	0,24	0,24	0,16	0,16	0,16	0,15	0,15	0,20	0,20	0,20	0,20
DB9	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,32	0,24	0,24	0,24	0,24	0,24	0,16	0,16	0,16	0,15	0,15	0,20	0,20	0,20	0,20
LB1	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,32	0,32	0,32	0,32	0,32	0,20	0,20	0,20	0,20	0,20	0,24	0,24	0,24	0,24
LB2	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,32	0,32	0,32	0,32	0,32	0,20	0,20	0,20	0,20	0,20	0,24	0,24	0,24	0,24
LB3	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,32	0,32	0,32	0,32	0,32	0,20	0,20	0,20	0,20	0,20	0,24	0,24	0,24	0,24
LB4	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,32	0,32	0,32	0,32	0,32	0,20	0,20	0,20	0,20	0,20	0,24	0,24	0,24	0,24
LB5	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,40	0,32	0,32	0,32	0,32	0,32	0,20	0,20	0,20	0,20	0,20	0,24	0,24	0,24	0,24
O1	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,44	0,44	0,44	0,44	0,44	0,32	0,32	0,32	0,30	0,30	0,40	0,40	0,40	0,40
O2	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,44	0,44	0,44	0,44	0,44	0,32	0,32	0,32	0,30	0,30	0,40	0,40	0,40	0,40
O3	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,44	0,44	0,44	0,44	0,44	0,32	0,32	0,32	0,30	0,30	0,40	0,40	0,40	0,40
R1	0,52	0,52	0,52	0,52	0,52	0,52	0,52	0,52	0,52	0,52	0,52	0,48	0,48	0,48	0,48	0,48	0,40	0,40	0,40	0,40	0,40	0,44	0,44	0,44	0,44
R2	0,52	0,52	0,52	0,52	0,52	0,52	0,52	0,52	0,52	0,52	0,52	0,48	0,48	0,48	0,48	0,48	0,40	0,40	0,40	0,40	0,40	0,44	0,44	0,44	0,44
Y1	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,40	0,40	0,40	0,40	0,40	0,24	0,24	0,24	0,25	0,25	0,32	0,32	0,32	0,32
Y2	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,40	0,40	0,40	0,40	0,40	0,24	0,24	0,24	0,25	0,25	0,32	0,32	0,32	0,32
Y3	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,40	0,40	0,40	0,40	0,40	0,24	0,24	0,24	0,25	0,25	0,32	0,32	0,32	0,32
Y4	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,40	0,40	0,40	0,40	0,40	0,24	0,24	0,24	0,25	0,25	0,32	0,32	0,32	0,32

Table 43 – notWillingness origin-destination for 2nd peak (time slot 4)

8.2.4. Run file

The run file permits to write all the instructions in a simple script. It is launched by AMPL with the command: *include filename.run*, so its extension is *.run*. The scripts provide a series of memorized instruction, executed in sequence, among which: loading model and data, specifying solver and options, launching solver and stamping results. The run file can be viewed in Appendix A.

Any code reference for additional coding information can be find in the “The AMPL Book” on AMPL website[90].

9. Results

In this chapter are illustrated all the results obtained by the model application and its evidenced characteristics.

Model characteristics	Quantity
Number of Vehicles	1.248
Number of Origins	25
Number of Destinations	25
Number of Time-slots	5
Number of Cells	25

Model characteristics	Quantity
Number of Markets	625
Number of Variables	84.500
Number of Constraints	87.750
Number of Objective Functions	1
Number of Tariffs	5[xxxvii]

Table 44 – Characteristics of the model

As shown in the above table, there is only one function to be optimized and a lot of variables and constraints.

The *number of variables* is calculated using the formula:

$$\#variables = \#accepted\ trips + \#accepted\ alteratives + \ \#lost\ trips + \#capacities^{xxxviii},$$

where:

$\#accepted\ trips = \#origins * \#destinations * \#slots$

$\#lost\ trips = \#origins * \#destinations * \#slots$

$\#capacities = \#cells * \#slots$

The *number of constraints* is so computed:

$$\#constraints = \#availability + \#totalMarketDemand + \#lowerBound$$
$$+\#willingnessQ + \#willingnessA + \#capacityUpdate + \#upperBound$$

where

$\#availability = \#cells \times \#slots$

$\#totalMarketDemand = \#origins * \#destinations * \#slots$

$\#lowerBound = \#origins * \#destinations * \#slots$

$\#willingnessQ = \#origins * \#destinations * \#slots$

$\#willingnessA = \#origins * \#destinations * \#destinations * \#slots$

$\#capacityUpdate = \#origins * \#destinations * (\# - 1)slots^{xxxix}$

$\#upperBound = \#cells$

Outputs:

1) *Time slot 1 - Off-peak 1*
   - Sold trips

[xxxvii] The five tariffs are: standard, discount1, discount2, increment1, increment2
[xxxviii] Note that we know the starting distribution of the operating day, but not the final distribution.
[xxxix] 4 and not 5 because there isn't a sixth slot.

Slot_1	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	4	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB2	0	4	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB3	0	1	3	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB4	0	0	0	4	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB5	1	0	1	1	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB6	0	1	1	0	0	3	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB7	1	0	1	0	0	0	3	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB8	0	1	0	0	1	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB9	1	0	0	0	1	0	0	0	4	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB10	0	0	1	0	1	0	0	1	0	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DB11	0	0	1	0	0	0	1	1	1	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LB1	4	0	0	1	1	0	0	0	0	1	1	4	3	0	0	1	4	0	0	1	0	3	4	0	0
LB2	0	0	0	1	0	0	1	0	0	1	0	4	3	4	0	0	4	0	0	3	0	0	3	0	0
LB3	1	4	3	0	0	0	0	1	1	1	0	0	4	3	0	1	0	0	0	0	0	1	4	0	1
LB4	1	0	1	3	0	3	4	0	4	3	3	0	0	0	4	0	0	0	0	0	0	0	0	0	0
LB5	1	1	0	0	3	3	0	4	4	0	0	0	0	0	0	3	0	1	0	0	0	0	1	0	4
O1	3	1	0	0	0	1	0	0	1	0	1	4	4	1	1	0	3	4	1	0	3	4	4	3	1
O2	0	0	1	1	0	3	1	1	1	0	1	0	0	0	4	0	4	4	4	4	4	1	3	0	3
O3	1	0	4	4	0	3	4	0	0	0	0	0	1	0	3	0	1	4	4	4	1	0	3	0	1
R1	0	1	0	1	0	0	0	0	1	0	0	1	0	1	1	0	1	0	1	3	1	1	1	0	0
R2	0	0	0	0	1	1	0	1	0	1	0	1	0	0	0	1	0	0	0	0	3	0	0	0	1
Y1	3	0	0	0	0	0	0	0	0	0	0	4	0	1	1	0	4	1	0	1	4	4	0	4	0
Y2	0	1	0	0	0	0	1	0	0	1	0	4	3	3	0	0	4	3	3	4	4	0	4	0	0
Y3	1	0	1	0	4	1	1	0	0	1	0	1	1	0	1	1	3	0	1	0	3	4	0	3	4
Y4	1	0	1	0	4	4	0	4	3	0	0	1	1	0	0	3	0	4	0	1	0	0	0	1	3

Table 45 – sold trips during the 1st time-slot

- Lost Trips

Slot_1	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	3	1	1	1	1	0	1	1	0	1	1	8	1	2	1	2	7	1	0	0	0	8	1	2	1
DB2	0	3	0	1	1	2	1	0	1	0	0	1	0	7	2	0	1	1	1	8	1	1	1	1	0
DB3	0	1	3	1	0	1	0	1	1	1	1	1	2	8	0	1	0	1	5	5	1	1	0	0	1
DB4	0	1	1	3	0	1	1	0	1	0	1	0	1	7	2	0	1	0	0	6	1	2	0	0	2
DB5	1	1	1	1	2	1	0	1	1	1	0	1	0	7	0	2	0	0	0	8	1	2	2	0	0
DB6	1	1	1	1	1	2	0	1	1	0	1	2	1	7	2	0	1	1	1	7	2	0	2	1	2
DB7	1	0	1	0	1	0	2	0	0	1	1	2	1	1	7	1	0	2	7	2	2	2	1	1	2
DB8	1	1	0	0	1	1	1	2	0	1	1	2	0	1	2	6	1	2	1	0	2	0	1	1	8
DB9	1	0	1	1	1	2	1	0	3	1	1	2	1	1	8	7	1	0	0	1	2	1	1	2	7
DB10	0	0	1	1	1	1	0	1	0	3	1	0	2	1	7	2	1	0	2	0	0	1	1	1	2
DB11	2	1	1	0	1	0	1	1	1	2	3	2	2	2	8	1	2	2	1	0	1	0	2	1	1
LB1	4	0	1	1	1	1	0	1	0	1	1	4	3	0	0	1	4	0	1	1	1	3	4	0	1
LB2	1	0	0	1	0	1	1	0	1	1	0	4	2	3	0	0	3	0	0	2	0	0	2	0	1
LB3	1	4	2	1	0	1	0	1	1	1	0	0	4	2	1	1	1	1	0	8	0	1	4	0	1
LB4	1	0	1	2	1	3	4	0	3	3	2	1	2	2	3	2	0	6	7	0	2	0	1	1	0
LB5	1	1	0	1	3	3	0	3	4	0	0	1	1	1	0	3	0	1	1	1	2	0	1	1	4
O1	3	1	1	1	0	1	1	1	1	1	1	3	4	1	1	1	3	4	1	1	2	4	4	2	1
O2	1	1	1	1	0	3	1	1	1	0	1	1	1	0	4	1	3	4	4	4	3	1	3	1	2
O3	1	1	3	4	0	2	3	1	0	0	1	1	1	0	3	0	1	4	4	4	1	1	3	0	1
R1	0	1	0	1	1	0	0	1	1	1	1	1	0	1	1	1	1	0	1	3	1	1	1	0	0
R2	1	0	0	1	1	1	1	1	0	1	0	1	0	1	1	1	1	0	0	0	3	1	0	0	1
Y1	3	1	0	0	1	0	0	1	1	1	0	3	1	1	1	1	3	1	0	1	4	3	1	3	1
Y2	1	1	1	1	0	1	1	0	0	1	1	4	3	2	0	1	4	3	3	4	4	0	3	0	0
Y3	1	1	1	0	4	1	1	1	1	1	0	1	1	1	1	1	3	1	1	1	3	4	1	3	3
Y4	1	0	1	1	3	3	1	4	2	0	1	1	1	0	0	3	2	3	1	1	5	0	0	4	2

Table 46 – lost trips during the 1st time-slot

Results for the 1st slot are:

Variable	**Value**
Number of Sold Trips:	465
Number of Lost Trips:	889
Total Demand:	1.354
Profit	1.737,79 €

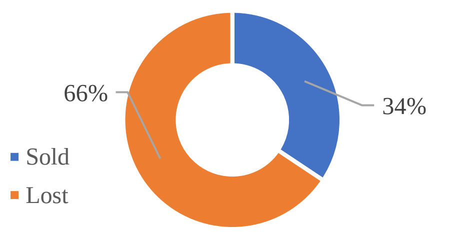

Table 47– Summary results for first time-slot

2) *Time slot 2 – 1st peak*

- Sold trips

Slot_2	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	0	1	1	1	0	0	0	0	0	0	1	0	0	2	1	2	0	6	7	10	11	0	1	0	6
DB2	0	1	0	2	2	1	2	0	0	0	0	0	0	0	3	1	7	0	0	0	11	4	0	5	0
DB3	1	0	2	2	0	0	1	1	1	1	1	0	0	0	2	2	7	0	0	0	11	6	0	5	5
DB4	0	1	0	2	1	2	0	1	0	2	0	0	0	0	1	2	8	6	0	0	11	4	0	5	5
DB5	0	0	1	2	1	0	1	1	1	0	2	0	0	2	1	0	7	6	6	12	0	0	0	0	0
DB6	1	0	1	1	0	0	1	0	1	0	1	4	1	3	2	0	6	0	0	9	0	6	5	4	1
DB7	2	1	1	1	0	0	1	0	0	0	2	0	0	0	0	1	8	0	0	0	11	5	0	6	5
DB8	0	1	2	1	0	1	1	2	1	0	0	0	0	2	1	0	5	0	5	10	9	0	5	0	0
DB9	0	0	1	1	0	2	0	1	0	1	0	0	0	1	0	0	7	5	3	11	10	0	5	0	0
DB10	1	0	0	0	2	0	1	2	1	1	1	0	0	1	0	0	8	6	0	9	0	0	5	5	0
DB11	1	1	0	0	2	0	0	0	0	2	1	0	0	0	0	0	4	0	0	10	10	4	0	6	0
LB1	1	1	0	0	0	2	0	0	2	1	2	0	0	2	1	0	4	4	6	6	8	3	2	1	1
LB2	0	0	0	2	2	0	0	2	2	1	0	0	0	0	1	1	5	4	5	8	8	2	3	1	1
LB3	0	0	2	0	0	1	2	0	1	2	0	0	0	1	0	1	5	5	5	6	6	3	2	3	2
LB4	2	0	1	2	0	1	0	0	2	1	2	2	2	0	1	1	5	6	5	6	7	1	3	2	3
LB5	2	0	1	1	0	0	0	0	0	0	0	1	2	0	0	0	4	5	6	7	7	4	3	2	3
O1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	4	4	0	1	2	2
O2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	3	1	0	0	0
O3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	2	2	0	0	0	1
R1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	2	0	0	0	0	1	0
R2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	1
Y1	1	1	0	0	0	1	1	0	0	1	0	1	2	2	0	0	2	1	1	4	6	1	0	1	1
Y2	1	0	1	1	0	0	1	1	1	1	0	0	2	0	2	0	2	4	2	6	6	1	2	0	1
Y3	0	0	0	0	1	1	0	1	0	0	1	2	1	0	1	2	1	2	3	4	5	0	1	1	1
Y4	0	0	0	1	0	0	1	1	0	1	1	0	1	2	0	0	1	3	4	4	6	0	2	1	0

Table 48– sold trips during the 1st peak (2nt time-slot)

- Lost trips

Slot_2	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	0	1	1	1	1	1	0	1	1	1	1	4	2	2	1	3	16	9	9	16	17	11	10	11	6
DB2	1	2	0	2	2	1	2	1	1	0	1	5	3	4	3	2	11	16	19	30	17	5	11	5	12
DB3	1	1	2	2	1	1	2	2	2	2	2	2	8	7	2	3	10	17	14	28	17	6	9	6	5
DB4	0	1	0	2	1	2	1	2	0	2	0	5	5	3	2	2	10	8	14	26	17	5	11	6	5
DB5	1	1	1	2	1	1	1	1	2	0	2	2	6	6	1	7	10	9	9	18	27	10	10	11	11
DB6	1	1	2	1	1	1	1	1	2	0	1	4	2	4	2	8	9	17	18	14	26	6	5	5	8
DB7	2	1	1	2	1	1	2	1	1	1	2	6	3	5	8	6	10	16	18	24	17	5	9	6	5
DB8	0	1	2	2	0	2	2	2	2	1	0	4	3	2	1	3	8	19	8	15	18	12	6	11	9
DB9	0	1	1	1	0	2	0	2	0	1	0	2	5	2	4	4	10	8	13	18	17	12	6	10	10
DB10	2	0	0	1	2	3	2	2	2	2	2	3	8	1	7	3	10	11	14	14	24	12	5	5	12
DB11	1	2	1	0	2	4	0	0	1	2	2	6	8	4	3	2	9	18	18	15	17	5	9	6	12
LB1	1	1	1	1	0	1	1	1	1	1	1	0	1	2	1	0	5	5	6	9	10	3	3	2	1
LB2	1	0	0	1	1	1	0	1	1	1	1	0	0	0	1	1	6	5	6	11	11	3	4	2	2
LB3	1	1	1	0	0	1	1	1	1	1	1	1	0	2	0	1	5	5	6	8	8	4	3	3	2
LB4	1	0	1	1	1	1	1	1	1	1	1	2	2	1	1	2	5	6	6	8	10	1	3	2	3
LB5	1	0	1	1	0	0	0	0	0	0	0	1	2	0	0	1	5	6	6	9	9	4	3	3	4
O1	0	1	1	1	0	0	1	1	1	1	0	1	0	0	1	1	1	2	2	4	4	0	1	1	1
O2	1	1	0	1	1	0	1	1	1	0	1	0	1	1	1	1	2	2	1	2	3	1	1	1	0
O3	1	1	0	0	1	0	1	1	1	1	0	1	1	0	0	1	1	1	2	3	3	0	1	1	1
R1	0	0	1	0	1	0	0	0	1	0	0	1	0	1	0	0	1	1	1	1	1	1	0	1	0
R2	0	1	0	0	0	0	0	0	1	0	0	1	1	1	0	0	1	0	1	2	1	1	1	1	1
Y1	1	1	0	0	1	1	1	0	0	1	0	1	1	1	0	0	3	2	1	5	6	2	0	1	1
Y2	1	1	1	1	0	0	1	1	1	1	0	0	1	1	1	1	3	4	3	6	6	1	2	1	1
Y3	0	1	0	1	1	1	1	1	1	0	1	1	1	0	1	1	2	2	4	5	6	1	2	2	2
Y4	1	0	0	1	0	0	1	1	0	1	1	1	1	1	1	1	2	4	4	5	6	0	2	2	0

Table 49 – lost trips during the 1st peak (2nd time-slot)

Results for the 2nd slot are:

Variable	Value
Number of Sold Trips:	912
Number of Lost Trips:	2.099
Total Demand:	3.011
Profit	16.727 €

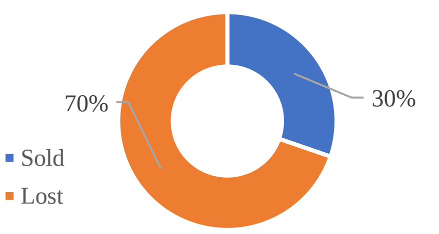

Table 50 – Summary results for second time-slot

The result is an undistributed shift of cars from suburbs (low zones) to center (medium and high zones) during the first peak because of the already mentioned house-work trend, but this tendency is alleviated by the tariffs' increments, which stimulate user-relocation, discouraging an uncontrolled "exodus".

3) Off-peak2

- Sold quantities

Slot_3	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB11	0	0	1	0	0	1	1	1	0	1	0	0	0	1	1	1	0	1	1	0	1	2	1	1	0
DB2	1	0	0	0	1	0	0	1	1	0	1	1	0	2	0	0	1	1	1	0	1	1	1	0	1
DB3	1	0	0	1	0	0	0	0	1	0	1	0	0	3	1	0	1	1	3	0	0	0	0	1	1
DB4	1	0	0	0	0	0	0	1	0	1	1	0	1	4	1	0	0	1	0	0	1	1	0	1	1
DB5	0	1	1	1	0	0	1	1	0	1	0	0	1	4	1	0	1	0	1	4	0	0	1	0	1
DB6	1	1	1	1	1	0	1	1	1	0	1	1	1	4	1	1	0	1	0	4	0	1	1	0	1
DB7	1	1	0	0	1	1	0	0	1	0	0	1	1	0	3	1	1	1	0	0	1	0	1	0	0
DB8	0	1	0	1	0	0	0	0	1	0	1	1	1	1	1	4	0	1	1	0	0	0	1	1	4
DB9	1	0	0	1	1	0	0	0	0	0	1	1	1	1	0	4	0	1	1	0	0	1	1	1	4
DB1	0	1	1	0	1	0	0	0	0	0	1	0	0	1	4	0	1	0	1	0	1	1	1	1	0
DB10	0	0	1	1	1	1	0	1	1	0	0	0	1	1	0	1	0	1	1	1	1	0	1	1	0
LB1	3	1	0	0	0	0	0	0	1	1	1	4	4	1	1	1	5	1	0	0	1	5	4	0	0
LB2	1	1	0	1	0	0	0	1	0	1	0	5	3	3	1	1	4	1	1	4	1	1	5	1	1
LB3	1	5	5	1	1	1	0	1	1	1	1	1	3	4	1	1	1	1	1	3	1	1	5	1	1
LB4	0	0	0	0	0	3	2	1	3	4	1	1	1	0	0	1	0	4	1	0	0	0	1	1	1
LB5	1	1	0	0	1	1	1	3	0	1	1	1	1	1	1	3	0	1	1	1	0	0	0	1	4
O1	4	1	1	1	0	0	1	1	0	1	1	4	5	0	0	0	5	4	0	0	4	4	5	3	1
O2	1	1	1	0	1	4	1	1	1	1	1	0	1	0	4	1	4	4	5	3	5	1	4	1	4
O3	0	1	3	3	1	3	5	0	1	0	0	0	1	1	3	1	1	4	4	4	1	0	4	0	1
R1	1	1	1	0	0	1	0	0	0	1	0	1	1	1	1	0	1	1	1	4	1	0	1	1	1
R2	1	0	1	1	0	1	0	1	0	0	0	1	0	1	0	1	1	1	0	1	4	1	1	1	1
Y1	4	1	0	1	1	1	0	0	1	1	0	5	1	1	0	1	4	1	0	0	3	4	0	4	0
Y2	0	1	0	1	1	0	0	0	1	1	1	3	4	4	1	1	4	3	4	4	3	0	5	0	1
Y3	1	1	1	0	3	0	1	0	1	0	1	1	1	0	1	0	4	1	0	1	3	4	1	4	4
Y4	0	1	0	1	5	5	1	4	3	0	1	0	0	0	0	3	1	4	0	0	0	0	0	4	4

Table 51 – sold trips during the 2^{nd} off-peak (3^{rd} time-slot)

- Lost trips

Slot_3	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	9	1	2	1	1	2	1	2	1	2	1	8	1	2	2	2	7	2	2	1	2	5	2	1	1
DB2	1	6	2	1	2	1	1	2	2	1	1	1	2	5	1	1	2	1	2	9	1	1	2	1	1
DB3	1	1	7	2	1	1	1	1	2	1	1	1	1	3	1	1	2	2	4	9	1	1	1	1	2
DB4	2	3	3	7	1	1	2	1	1	2	1	1	1	3	2	1	1	1	1	8	2	1	1	2	1
DB5	1	2	2	1	6	1	2	2	1	1	1	1	1	3	2	1	1	1	1	4	3	2	1	2	1
DB6	2	1	2	1	2	7	2	1	1	3	1	2	1	4	2	1	1	1	1	3	3	1	1	1	1
DB7	1	1	1	1	2	2	8	1	2	1	3	2	2	1	6	2	2	1	8	3	2	1	2	1	1
DB8	1	2	1	2	1	1	1	9	1	1	2	1	1	2	2	3	1	2	2	2	2	2	1	2	4
DB9	1	1	3	2	2	1	1	1	7	1	1	1	2	1	7	3	1	2	1	2	1	1	2	1	3
DB10	1	1	2	1	1	1	3	1	1	8	2	1	1	1	4	1	2	1	2	1	2	2	2	1	1
DB11	1	1	2	2	1	1	3	2	2	3	8	1	2	1	6	1	1	1	2	1	1	1	2	1	1
LB1	3	1	1	1	1	1	1	1	2	2	2	3	4	2	2	1	4	2	1	1	2	4	3	1	1
LB2	2	2	1	2	1	1	1	1	1	2	1	4	3	3	1	2	3	2	1	4	2	2	4	1	2
LB3	1	4	4	2	2	2	1	1	1	1	1	1	3	4	2	1	1	2	1	3	2	2	4	2	1
LB4	1	1	3	7	1	3	7	2	3	5	5	2	1	1	8	2	3	3	5	1	2	1	1	1	1
LB5	2	1	1	1	8	8	1	5	9	2	2	2	1	1	2	3	2	1	1	2	1	3	1	1	3
O1	3	1	1	2	1	1	1	1	1	1	2	3	4	1	1	1	4	4	1	1	3	4	4	3	1
O2	2	1	2	1	2	4	1	1	1	2	2	1	1	1	3	2	4	3	4	3	4	2	3	2	3
O3	1	1	3	3	2	3	4	1	2	1	1	1	2	1	3	1	1	4	3	4	2	1	4	1	1
R1	1	2	1	1	1	1	1	1	1	2	1	2	1	1	2	1	1	2	2	4	2	1	1	1	1
R2	2	1	2	1	1	1	1	2	1	1	1	1	1	1	1	1	2	2	1	1	3	2	1	1	1
Y1	3	1	1	1	1	2	1	1	2	2	1	4	1	1	1	2	3	2	1	1	3	3	1	3	1
Y2	1	2	1	2	1	1	1	1	2	1	2	3	3	4	2	1	4	3	4	3	3	1	4	1	1
Y3	2	2	2	1	3	1	1	1	2	1	2	2	1	1	2	1	4	1	1	1	3	3	1	4	4
Y4	1	1	1	1	4	4	2	3	5	1	1	1	1	1	1	3	1	3	1	3	6	1	1	4	3

Table 52 – lost trips during the 2^{nd} off-peak (3^{rd} time-slot)

Results for the 3^{rd} slot are:

Note that users' behavior reflects the situation of the first off-peak with the only difference that there is a higher amount of demand and requests. In fact, the middle off-peak goes from 10:00 to 17:00, a range that includes also lunch, that can be considered a little peak because it is less relevant than other peak moments.

Variable	Value
Number of Sold Trips:	679
Number of Lost Trips:	1.246
Total Demand:	1.925
Profit	3.095 €

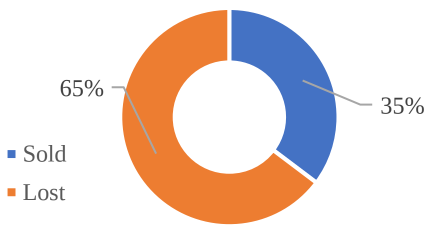

Table 53 – Summary results for third time-slot

4) *Peak2*

- Sold quantities

Slot_4	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	2	0	0	1	1	1	1	1	0	1	2	2	1	0	0	0	0	0	0	0	0	0	0	0	1
DB2	1	2	1	2	2	2	1	0	0	1	0	1	1	0	0	2	0	0	0	0	0	1	0	1	1
DB3	1	1	1	0	0	0	0	0	2	0	2	2	1	0	0	0	0	0	0	0	0	1	0	0	0
DB4	0	0	0	2	0	1	1	0	0	2	2	2	1	0	2	1	0	0	0	0	0	1	0	0	1
DB5	1	0	2	0	1	1	2	0	1	2	1	0	2	0	2	0	0	0	0	0	0	0	0	1	0
DB6	1	0	0	1	0	1	2	2	1	1	0	0	1	1	1	0	0	0	0	0	0	0	1	0	1
DB7	2	0	0	0	1	1	1	2	0	0	0	1	1	2	0	0	0	0	0	0	0	0	1	0	0
DB8	2	2	1	0	1	1	1	1	0	2	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0
DB9	0	2	0	0	2	0	0	2	1	0	1	2	1	1	0	0	0	0	0	0	0	1	1	0	0
DB10	1	0	2	2	2	2	0	2	2	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1
DB11	2	0	1	1	1	2	2	1	0	0	0	2	0	2	2	0	0	0	0	0	0	1	0	0	1
LB1	2	2	4	1	3	2	1	3	1	2	1	2	1	1	1	1	0	0	1	0	0	0	1	0	0
LB2	2	2	3	1	4	4	2	2	3	3	4	0	2	0	0	1	1	0	0	0	0	0	0	0	0
LB3	1	1	3	2	2	4	2	1	4	3	3	0	1	0	1	2	0	1	0	0	0	0	1	2	0
LB4	4	2	2	1	3	1	2	3	1	3	3	0	1	1	2	1	0	0	0	0	0	0	1	0	2
LB5	4	2	3	2	1	2	1	3	1	1	3	1	1	1	1	2	0	0	0	0	0	0	1	0	1
O1	8	6	5	5	5	6	7	8	5	6	6	5	5	5	6	6	2	0	2	1	2	3	4	3	3
O2	8	8	6	5	8	7	8	8	6	6	6	4	4	6	6	4	1	1	1	0	0	3	4	4	2
O3	6	7	7	6	6	7	6	5	7	5	6	5	5	5	7	6	0	0	0	0	2	1	2	4	2
R1	11	10	11	9	11	10	12	11	9	10	12	5	7	10	7	5	3	2	2	2	0	5	5	6	5
R2	10	10	11	9	10	10	12	10	10	12	9	7	8	10	8	5	2	4	3	2	1	6	5	6	5
Y1	4	6	6	5	5	4	6	6	6	4	5	3	2	1	1	3	2	1	0	1	0	0	1	0	0
Y2	4	4	6	5	6	4	5	6	6	4	5	1	3	1	3	4	0	0	0	0	1	1	1	1	0
Y3	6	6	4	4	5	6	4	6	5	5	6	1	2	4	2	2	1	0	0	0	1	2	2	0	1
Y4	6	5	5	4	5	6	4	6	4	6	5	2	2	3	1	1	1	2	0	0	0	0	1	1	0

Table 54 – sold trips during the 2nd peak (4th time-slot)

- Lost trips

Lost_4	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	2	1	1	2	1	2	1	1	1	2	2	1	1	1	0	0	0	1	1	0	1	0	1	1	1
DB10	1	2	2	2	2	2	1	0	0	1	0	1	1	0	1	1	1	0	1	1	0	1	1	1	1
DB11	1	2	1	1	1	0	0	0	2	0	2	1	1	0	0	1	1	1	1	1	0	1	1	0	0
DB2	1	0	1	2	1	1	1	0	1	2	2	1	1	1	1	1	1	1	1	1	1	1	0	0	1
DB3	1	1	2	1	2	1	2	1	1	2	1	1	1	1	1	0	0	0	0	0	0	0	0	1	0
DB4	1	0	1	2	1	2	2	2	2	1	1	1	1	1	1	1	1	1	0	0	0	0	1	0	1
DB5	2	0	0	0	1	1	2	2	0	0	0	1	1	1	0	1	0	1	0	1	0	0	1	1	1
DB6	2	2	2	1	1	1	2	2	0	2	1	1	1	1	1	1	1	1	0	0	0	1	1	0	0
DB7	0	2	0	0	2	1	1	2	1	0	2	1	1	1	0	0	0	1	1	0	0	1	1	0	1
DB8	1	1	2	2	2	2	1	2	2	1	1	1	0	0	0	1	1	0	1	1	0	1	1	1	1
DB9	2	1	1	1	1	2	2	2	1	0	0	1	1	1	1	1	0	0	1	0	0	1	0	1	1
LB1	2	3	4	2	3	3	2	3	2	3	2	2	1	2	1	1	1	0	1	0	0	0	1	0	1
LB2	3	3	3	1	4	4	3	2	3	3	4	0	2	1	1	2	1	0	1	0	0	0	1	0	0
LB3	2	2	4	3	2	4	2	1	4	4	4	1	2	1	2	2	0	1	0	1	0	0	1	1	1
LB4	4	3	2	1	3	1	2	3	1	4	3	1	1	1	2	2	0	1	0	0	1	0	1	1	1
LB5	4	2	3	2	1	2	2	4	1	2	3	2	2	1	2	2	1	0	1	0	0	1	1	1	1
O1	11	9	8	8	8	8	10	11	8	8	9	6	6	5	4	6	2	0	2	1	1	3	4	3	3
O2	10	10	8	8	11	10	11	10	9	9	8	5	5	6	5	5	2	2	1	0	0	4	4	4	2
O3	8	9	10	9	8	9	9	8	10	8	8	5	6	6	5	6	0	0	1	1	1	2	3	4	2
R1	17	17	18	15	18	15	18	18	14	16	18	8	10	9	8	8	3	3	2	2	1	6	5	6	4
R2	15	17	18	15	16	17	18	16	16	18	14	10	11	8	8	8	2	4	4	2	2	6	5	6	4
Y1	5	6	6	6	6	5	6	6	6	5	6	3	3	2	2	3	1	1	0	1	1	0	2	1	0
Y2	5	5	6	5	6	5	6	6	6	5	5	2	3	1	4	4	1	0	1	1	1	1	1	2	1
Y3	6	6	5	5	5	6	5	6	6	6	6	2	2	4	3	2	1	0	1	1	1	2	2	0	1
Y4	6	5	6	5	6	6	5	6	5	6	6	2	3	3	1	2	1	1	1	0	0	1	1	2	1

Table 55 – lost trips during the 2nd off-peak (3rd time-slot)

Results for the 4th slot are:

Note that users' behavior is opposite to the first peak' behavior: there is the already mentioned work-house trend. Note also that this tendency's inversion reflects the colors' change, depending on attractiveness.

Variable	Value
Number of Sold Trips:	1.317
Number of Lost Trips:	1.733
Total Demand:	3.050
Profit	23.508 €

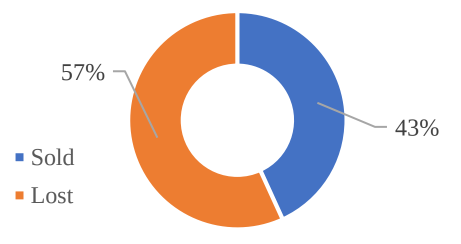

Table 56 – Summary results for fourth time-slot

5) *Off-Peak3*

- Sold quantities

Slot_5	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	2	0	0	1	0	1	0	1	1	0	1	4	1	0	0	0	2	1	1	0	0	3	1	0	1
DB10	0	4	1	1	0	1	1	1	0	0	1	0	0	3	0	0	1	0	0	2	0	1	1	0	0
DB11	0	1	3	1	0	1	0	1	1	1	1	1	0	2	1	1	0	0	4	3	0	1	1	0	0
DB2	1	1	1	4	0	1	0	1	0	1	0	0	0	4	0	0	0	0	1	2	1	0	1	1	1
DB3	0	1	1	1	3	0	1	0	1	1	1	0	0	4	1	0	0	1	1	3	1	1	1	0	0
DB4	1	1	0	1	1	2	1	0	1	0	1	0	1	3	1	0	0	0	1	4	0	1	0	0	0
DB5	0	1	0	1	0	0	4	1	0	0	0	0	1	1	2	0	0	0	4	0	0	0	0	0	0
DB6	0	0	1	1	0	1	1	3	0	0	0	1	1	1	1	4	1	1	1	1	1	1	1	1	4
DB7	1	1	1	1	0	1	0	1	4	0	0	1	0	1	2	4	0	1	1	0	1	0	0	1	2
DB8	1	1	1	1	1	0	0	0	0	2	1	1	0	0	3	0	0	0	1	0	0	1	1	0	1
DB9	0	0	0	1	0	1	1	0	1	0	3	0	1	1	3	1	0	0	1	0	1	0	0	1	1
LB1	4	1	0	1	1	1	0	1	1	0	1	4	4	0	1	0	3	0	1	1	1	4	3	1	0
LB2	0	1	1	0	0	0	0	0	1	0	1	3	3	3	0	1	3	0	1	4	0	0	3	1	1
LB3	0	3	2	0	0	0	0	1	0	0	1	0	2	2	0	0	1	0	1	3	1	0	3	0	0
LB4	0	0	0	3	0	4	2	0	3	2	4	0	0	1	4	0	1	3	3	0	1	0	0	1	0
LB5	0	1	0	1	4	3	0	4	3	1	1	0	1	1	0	3	0	1	0	0	1	1	0	1	3
O1	2	1	0	1	1	1	0	0	0	0	1	3	4	0	0	0	2	4	1	0	3	3	3	4	0
O2	1	1	0	1	0	3	0	1	1	0	1	1	1	0	4	1	3	3	4	2	2	1	4	0	3
O3	0	0	4	3	0	3	3	1	0	0	1	0	0	0	4	1	0	4	3	3	0	1	3	0	0
R1	0	1	1	1	1	1	0	1	1	0	0	0	1	0	1	1	0	1	0	3	0	0	0	1	0
R2	1	0	1	0	1	1	1	0	0	0	1	1	1	0	1	1	1	0	1	1	3	1	1	0	0
Y1	3	0	0	1	1	0	0	0	1	1	1	4	1	1	0	0	3	1	1	1	4	4	0	3	0
Y2	1	0	0	0	1	1	1	1	1	0	0	3	4	4	1	1	4	3	4	4	4	1	3	1	0
Y3	0	0	0	0	4	1	0	0	0	1	1	0	0	0	1	0	2	1	0	0	3	2	0	4	3
Y4	1	1	0	0	4	3	1	0	0	1	0	0	1	0	1	3	1	4	1	1	2	1	0	0	4

Table 57 – sold trips during the 3rd off-peak (5th time-slot)

- Lost trips

Lost_5	DB1	DB2	DB3	DB4	DB5	DB6	DB7	DB8	DB9	DB10	DB11	LB1	LB2	LB3	LB4	LB5	O1	O2	O3	R1	R2	Y1	Y2	Y3	Y4
DB1	2	1	0	2	0	2	1	2	2	1	1	4	2	0	0	1	2	2	1	1	1	3	1	0	2
DB2	0	4	1	2	1	2	2	1	1	1	1	1	0	3	0	0	2	0	1	2	0	2	1	0	1
DB3	0	2	3	1	1	1	0	2	2	1	2	2	0	2	1	1	0	0	3	3	0	1	1	0	0
DB4	1	1	2	4	1	2	1	1	0	2	1	1	1	4	0	1	0	1	2	2	2	1	2	2	1
DB5	0	2	1	2	2	1	1	0	1	2	2	1	0	4	2	0	1	1	2	3	1	2	2	0	1
DB6	2	1	0	2	2	2	2	1	2	0	2	0	2	2	1	1	1	0	2	3	1	1	1	0	1
DB7	0	2	0	2	0	1	3	2	0	1	0	1	1	2	2	1	1	1	4	0	0	0	1	1	0
DB8	0	1	2	1	0	1	1	2	0	1	0	1	1	2	1	4	1	2	1	1	1	2	2	2	4
DB9	2	1	2	2	0	1	1	1	4	0	0	1	0	2	2	3	1	1	2	1	1	1	0	1	2
DB10	2	2	2	2	2	1	0	1	1	2	1	1	1	1	3	1	1	1	2	1	1	1	1	0	1
DB11	1	0	0	2	0	2	2	1	1	0	2	0	1	1	2	2	1	0	1	1	2	0	1	2	1
LB1	3	2	1	2	1	1	1	2	1	0	2	3	3	0	2	1	2	1	1	1	1	4	3	1	1
LB2	1	2	1	0	1	0	0	0	2	0	2	2	2	3	1	1	3	1	2	3	0	0	2	1	1
LB3	1	2	2	1	0	0	0	2	1	1	2	0	2	2	0	0	2	0	2	2	2	0	3	0	1
LB4	1	0	0	3	0	3	2	1	2	2	3	0	0	2	3	0	1	3	2	1	2	1	0	2	1
LB5	1	1	0	2	4	3	0	4	2	1	1	1	1	1	0	3	0	1	1	0	1	2	1	2	3
O1	2	2	0	1	1	1	0	0	0	1	1	3	4	1	0	1	2	4	1	1	2	2	2	4	1
O2	1	2	1	2	0	3	0	1	2	0	1	2	1	1	3	2	2	2	4	2	2	2	4	0	2
O3	1	1	4	2	0	2	2	2	1	1	1	0	1	1	4	1	1	4	2	2	0	1	2	0	1
R1	1	1	1	1	1	2	0	1	2	1	0	0	2	1	1	2	0	1	1	3	1	0	1	1	1
R2	1	0	2	1	2	2	1	0	1	0	1	1	1	0	1	2	2	0	1	1	3	1	1	1	1
Y1	3	1	1	1	2	0	0	0	2	1	2	3	2	2	0	0	3	1	1	1	4	3	1	3	1
Y2	2	1	1	1	1	1	2	2	2	0	0	2	4	4	2	1	3	3	4	4	4	2	3	1	1
Y3	1	1	0	0	4	2	0	1	0	1	1	1	0	0	2	0	2	2	1	1	2	2	1	3	2
Y4	1	2	1	0	4	3	2	8	4	1	1	1	1	1	2	3	1	3	1	2	2	2	0	6	3

Table 58 – lost trips during the 3rd off-peak (5th time-slot)

Results for the 5th slot are:

Variable	Value
Number of Sold Trips:	635
Number of Lost Trips:	857
Total Demand:	1.492
Profit	2.732 €

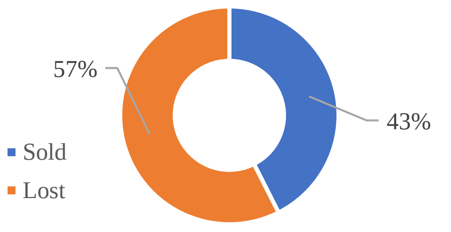

Table 59 – Summary results for fifth time-slot

6) Vehicle distribution evolution during time-slots.

Distribution	Sta1	End1:Sta2	End2:Sta3	End3:Sta4	End4:Sta5	End5
DB1	23	40	3	12	87	85
DB2	23	34	3	9	68	71
DB3	24	37	3	6	79	73
DB4	23	34	3	4	56	61
DB5	24	36	4	4	73	73
DB6	24	40	5	3	74	85
DB7	23	34	3	3	74	76
DB8	24	37	4	3	78	70
DB9	25	39	4	3	64	61
DB10	24	31	3	4	67	62
DB11	23	29	3	4	68	74
LB1	52	48	11	9	28	21
LB2	52	48	11	7	26	27
LB3	52	40	11	7	29	41
LB4	52	41	5	7	27	27
LB5	52	37	5	7	22	14
Y1	52	43	61	55	10	7
Y2	52	44	50	52	13	0
Y3	52	32	55	50	5	3
Y4	52	39	49	49	6	0
O1	86	74	157	151	50	44
O2	86	67	124	114	9	0
O3	88	64	119	105	7	10
R1	130	137	261	275	101	124
R2	130	143	291	305	127	139

Table 60 – changes in vehicle capacity between time-slots

9.1. Aggregate data

The model results in a *total profit* obtained by user-relocations in a standard week/day of 48.000 euro, demonstrating a positive impact of the model.
Main demand and economics data are:

Variable	Value
Total profit:	47.799 €
Total Sold Trips:	4.008
Number of Lost Trips:	6.824
Total Demand:	10.832

Table 61 – Summary results for all time-slots

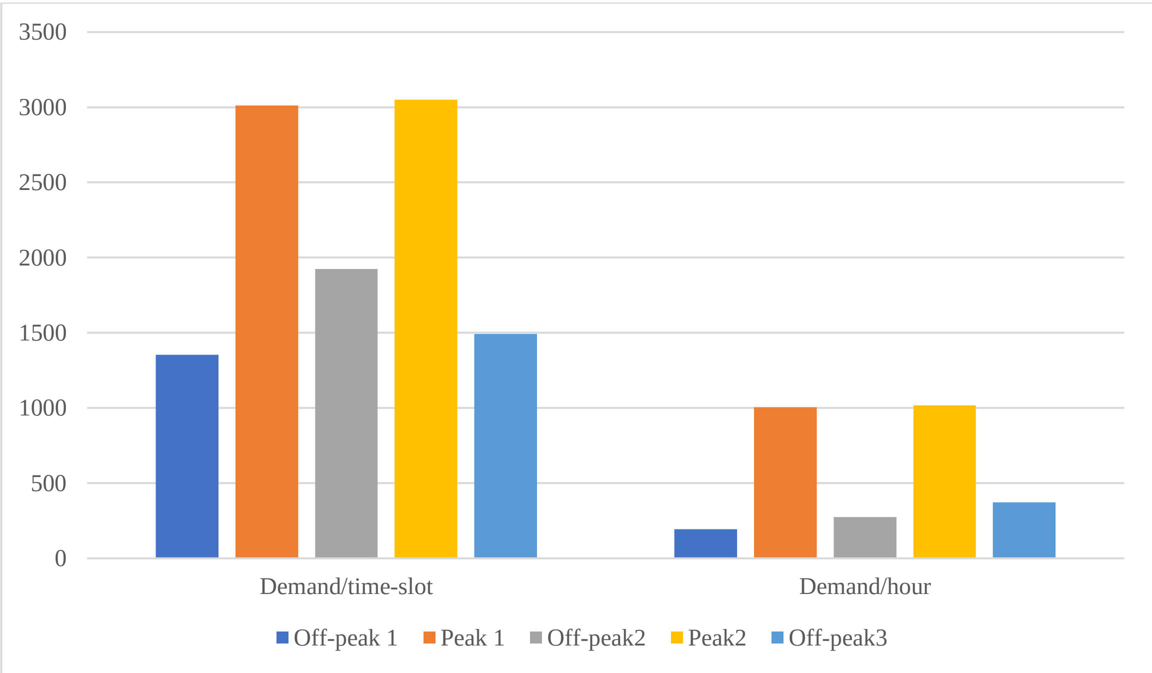

Figure 33 – Demand evolution per time-slot and per-hour

Daily *demand per time-slot* is illustrated in left histogram; there are two peaks during the second and fourth period. In the second period (off-peak2), the demand is greater than other off-peaks' ones because it includes the lunch's little peak.

The right histogram represents the demand per hour, which has been obtained as ratio of demand and relative range's hours.

This means for example that peak 2, which duration is 3 hours, has a demand of 3011 and a average hourly demand (demand/h) of ≅ 1004[xl]. Note that during the last off-peak the demand/h (that we can call demand's rate) is greater than the other off-peaks' ones because it is relative to only 4 hours, instead of 7.

[xl] Demand per hour: 3001:3 ≅ 1104

This justifies that during the night there is a higher demand rate, probably deriving from after dinner requests to go to places of interest and leisure.

Considering the Sold Trips results the two graphs below show different perspectives about sold quantities.

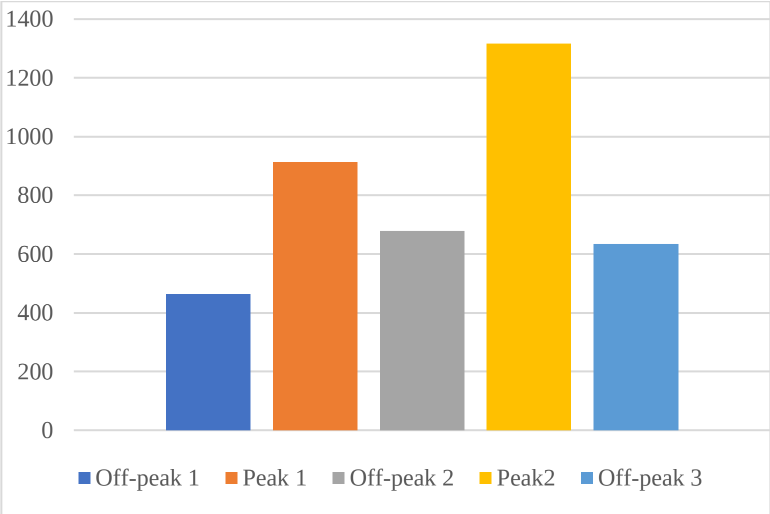

Figure 34 – Sold trips evolution per time-slot

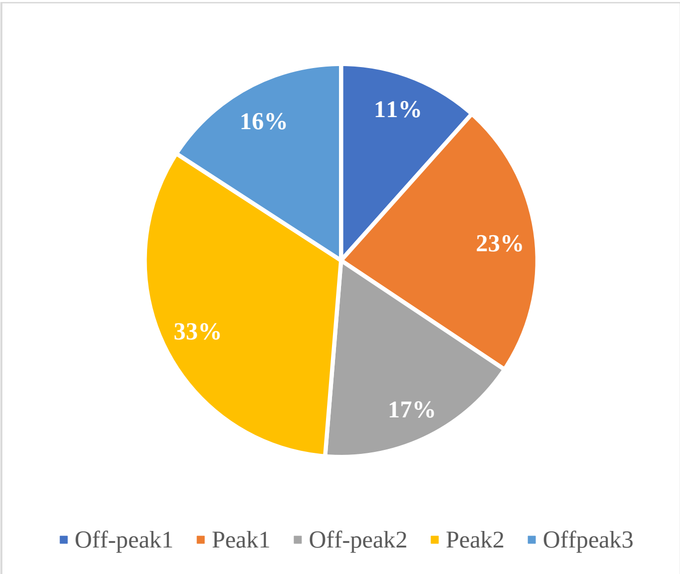

Figure 35 – Sold trips distribution across daily time-slots

Although there is a substantial difference about demand's levels among the 5 time slots (the 2 daily peaks cover 56% of sold trips daily), this difference is mitigated by tariffs' increments and discounts. This is the key of the model: discouraging users with a higher price and encouraging them with a lower price. The following graphs confirms this behaviour.

Some valuable information can be extracted by the Lost Trip analysis.

For example, analysing lost quantities of the 5 periods, and the ratio between lost trips and relative demand. As we can observe there is not only a higher level of losses during peaks, but also a higher percentage of losses with respect to demand. This is caused by the difference between accepted and rejected trips on the base of tariff's variation.

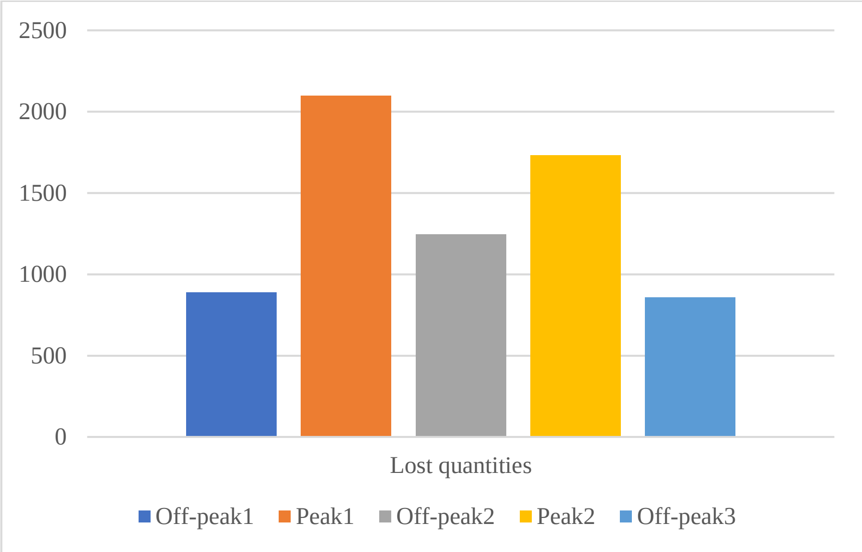

Figure 36 – Lost trips evolution per time-slot

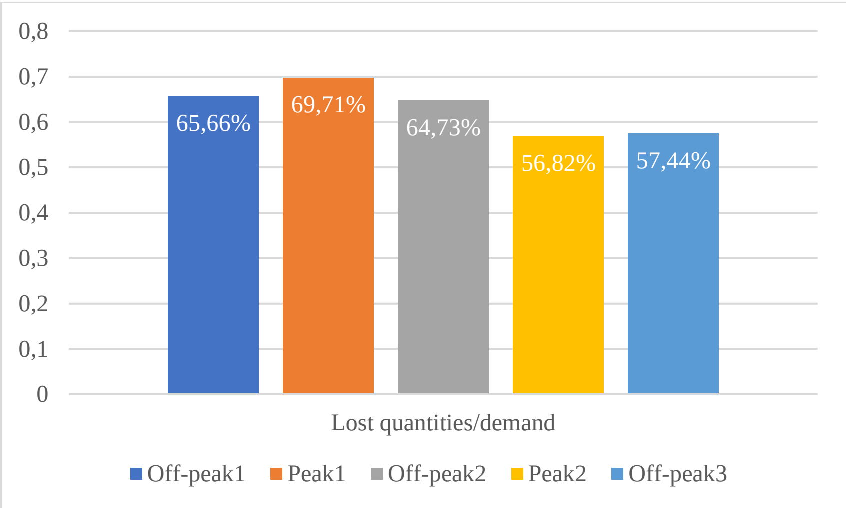

Figure 37 – Lost trips ration of relative demand

According to results of the model the profit is concentrated during peak hours

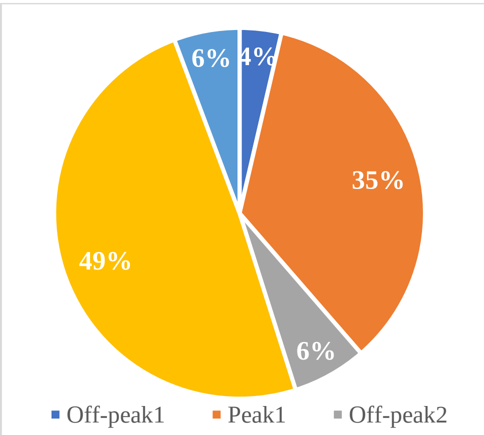

Figure 38 – Distribution of Profits per time-slot

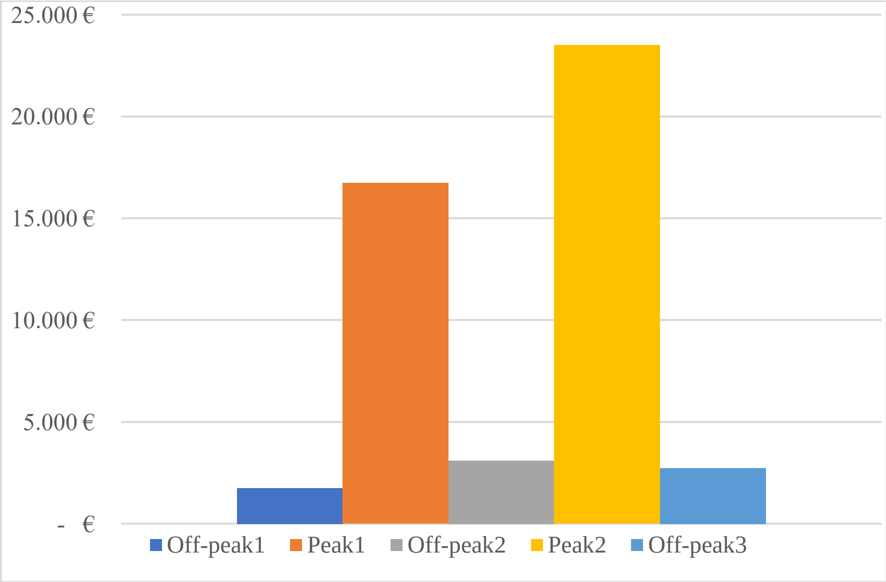

Figure 39 – value of Profits per time-slot

Linking all the ranges' profits, there is the typical sawtooth trend with high values during peaks (because of a high number of sold trips, traffic congestion, variable tariffs), but during a real operating day, the behaviour isn't so regular as in the above graph.

9.2. End of operating day

At the end of "operating day" (typically during the 5th slot) the staff relocation starts, if its required, to recover existing imbalance problems. As mentioned before, the assumption is that relocation is made instantaneously or during an ideal time in which there aren't requests. There are different relocation strategies and we consider the following ones:

- *Uniform*: uniform distribution of cars in the five macro-classes, namely low, medium-low, medium, medium-high and high; this means that approximately 20% of the cars must be in each macro-area;
- *Equal*: distribution is performed to reach the starting distribution status of the 1st time-slot;
- *Threshold*: distribution must respect two thresholds (minimum of 10 cars and maximum[xli] of 90[xlii]); this is a choice that is in contrast with a hypothesis of the model (infinite capacity of destinations), so it is the less accurate because it is like we consider the problem of parking availability only at the end of the day.

The unit cost of relocation equals to 15€[xliii] and the already defined starting distribution (there is a percentage of around 20% in each colored macro-group).

These strategies are compared between each-other and with the *current relocation strategy* that is full operator-based.

[xli] It is a threshold that consider an additive constraint: maximum number of parking slots.

[xlii] Supposing that 1248/25≈50 is the average number of cars in each node and the minimum threshold is fixed to 10, the maximum threshold is equal to 50+(50-10)=90; the threshold are seen as two values with the same distance from the medium value.

[xliii] Real data provided by Car2Go for Rome Municipality (2016).

The *current model* is the model using the actual pricing scheme (fixed unit tariff of 0,25 €/min) Car-sharing operators currently use in Italy.

9.2.1. Business comparison

This subchapter illustrates the comparison between different analysed model, considering the above described model as a benchmark.

Relocation results depend on relocation model and final distributions that in case of the chosen user-based relocation strategy compared to the current relocation strategy present the following distributions at the end of the day.

The *equal* model has the following result

EQUAL			
End	**Target**	**Relocation**	**Cost**
791	260	-531	- €
130	260	130	1.950 €
10	208	198	2.970 €
54	260	206	3.090 €
263	260	-3	- €
		Cost of relocation	**8.010 €**

Table 62 – EQUAL model relocation cost for end-of-day operator-based relocation

similarly, the *uniform* model considering that initial distribution is similar to the uniform

UNIFORM			
End	**Target**	**Relocation**	**Cost**
791	250	-541	- €
130	249	119	1.785 €
10	249	239	3.585 €
54	250	196	2.934 €
263	250	-13	- €
		Cost of relocation	**8.304 €**

Table 63 – UNIFORM model relocation cost for end-of-day operator-based relocation

On the other hand, the *threshold* model seems much more effective in terms of relocation results as stated by the table below.

THRESHOLD			
End	**Target**	**Relocation**	**Cost**
791	791	0	- €
130	130	0	- €
10	62	30	450 €
54	86	10	150 €
263	180	83	1.245 €
		Cost of relocation	**1.845 €**

Table 64 – THRESHOLD model relocation cost for end-of-day operator-based relocation

Finally, the *current* model, which is not using the user-based relocation strategy daytime has given the following results.

CURRENT			
End	**Target**	**Relocation**	**Cost**
1039	260	-779	- €
142	260	118	1.770 €
16	208	192	2.880 €
11	260	249	3.735 €
40	260	220	3.300 €
		Cost of relocation	**11.685 €**

Table 65 – CURRENT model relocation cost for end-of-day operator-based relocation

Results of the user-based relocation used during the day and the operator-based relocation applied at the end of the day by staff are described in the table below.

	Equal	**Uniform**	**Threshold**	**Current**
Gross Profit	47.799 €	47.799 €	47.799 €	36.103 €
Operator-based Relocation cost	8.010 €	8.304 €	1.845 €	11.685 €
Relocation Net Profit	***39.789 €***	***39.495 €***	***45.954 €***	***24.418 €***
%Profit	83%	83%	96%	68%
Number of relocations	528	538	123	779

Table 66 –Profit comparison of different relocation model with current.

As we can observe user-based relocation model used to relocate vehicles at the end of the day has different impact on profit percentage (up to 13% gap) between threshold model and Uniform/Equal, apparently representing a better situation. All user-based relocation models are much more convenient that current relocation model.

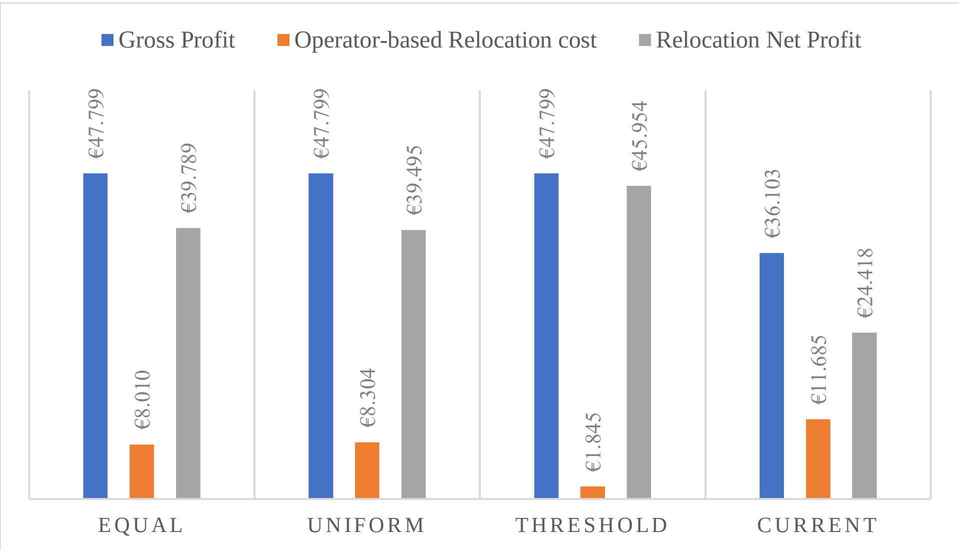

Figure 40 – Profit comparison for different relocation models

Results of the business case demonstrate the effectiveness to implement a user-based relocation strategy daytime, since all operator-based relocation model applied at the end of the day have higher

Gross Profits and lower *Operator-based relocation cost* that *current* model which is not applying the user-based relocation.

In comparing the *equal* and the *current* relocation model, which have to reach the same target distribution at the end of the day, the results show a higher margin (+11.696€/day) and a lower cost (-3.675€/day) for a final net margin of 15.371€/day, projecting an economic result if +5,6 million euro yearly.

10. Conclusions

The goal of this research work was to demonstrate that, the proposed user-based relocation strategy can be profitable for one-way free-floating car-sharing models.

In this case the user-based relocation strategy proposes to the user, when a trip planning inquiry is done, a flexible pricing scheme based the desired destination and a set of alternatives in boundary areas.

Flexible pricing is based on the difference of Urban Area Value (UAV), associated to the origin, the destination and the alternatives.

Results clearly demonstrate that the novel dynamic pricing applied to the city of Rome is profitable for the Car-sharing Operator, providing at the same time a significant increase in service profit and good reduction of staff involvement to perform vehicle relocation.

Additionally, other tangible results of this research are:

1. UAV metric enables Car-sharing operator to classify different Cells of the Urban Area, on the basis of a mobility demand value, distinguishing between areas with higher and lower demand potential[1].
2. Reliability of UAV calculation, of specific urban area, depends on the volume of data used for the analysis and using data from other sources of mobility-sharing services operating in the same city. The higher the volume, the higher the reliability; it is always better to use UAV calculated with data from different and complementary data sources, if available[1,91].
3. Aggregate calculation of UAV also gives valuable information of potential urban demand in non-served areas, enabling the Car-sharing operator to evaluate its interest in extending the service area to new catchment areas[1]. This approach may give a very high added value to Car-sharing operators entering in market with strong competitor presence.
4. Enabling potential users in performing origin-to-destination trip planning, Car-sharing operator can understand interest in destination of end-users even for Lost Trips.

There are different issues in Car Sharing system. Among them there is the imbalance problem. The relocation is an effective way to mitigate it, but it is inefficient when there is traffic congestion and it is too expensive, particularly in a city like Rome. The cost of a relocation often doesn't justify the unit profit, namely it is an investment that often hasn't an earning. To solve this situation, the thesis proposes an optimized model based on the user-relocation policy. The results show that there is a significative reduction of costs when the tariff is variable during an operating day and this is an optimal starting point to study in deep user-based relocation as a solution of unbalancing and a way to make more profits. The confirm arrives from the comparison between the current model and the proposed one: there is not only a decrease of relocation costs, but also a higher earning. The suggested system involves users to relocate cars indirectly, offering different alternatives with variable prices leaving the customers free to choose the best alternative for his needs.

11. Appendixes

11.1. Appendix A – Run file

```
reset;
reset;
model modelloyuri.mod;
data modelloyuri.dat;

param q{h in Origin, k in Destination};
param nq{h in Origin, k in Destination};
param p{o in HourlyRange};
param cf{h in Origin};

fix {h in Origin} capacity[h,1]:= 5*c[h];

option solver cplex;
printf "\n\n ### GRAPH WITH %d NODES (2 High, 3 Medium-High, 4 Medium, 5 Medium-Low, 11 
Low) ### \n\n", card(Origin);

param vehicles:=sum{h in Origin} c[h];
printf "Total number of cars: %d \n", vehicles;
printf "Number of variables: %d \n", _nvars;
printf "Number of constraints: %d \n", _ncons;
printf "Number and name of the objective function: %d %s \n", _nobjs, "Profit";
printf"\n\n";
solve;

for {o in HourlyRange} {

printf "\n\n### SLOT TEMPORALE NUMERO: %d ### \n\n", o;

printf "\nTariffs: ";
for {k in Destination} {
printf "%s ",k;
}
printf"\n";
for {h in Origin} {
printf "%s ",h;
 for {k in Destination} {
printf "%f ",valueOfTariff[h,k,o];
}
printf"\n";
}


for{h in Origin, k in Destination} {
 let q[h,k]:= quantity[h,k,o]+sum {j in Destination} alternative[h,j,k,o];
}

for{h in Origin, k in Destination} {
 let nq[h,k]:= notQ[h,k,o];
}

let p[o]:= sum{h in Origin, k in Destination}
(quantity[h,k,o]*5*rentTime[h,k,o]*(valueOfTariff[h,k,o]-0.17)+sum {a in Destination}
(alternative[h,k,a,o]*alter[k,a,o])*5*rentTime[h,a,o]*(valueOfTariff[h,a,o]-0.17));

printf "Sold quantity for each trip:\n";
```

```
display q;
printf "Lost Trips:\n";
display nq;
printf "Profit = %f \n\n", p[o];

printf "\nStarting Distribution: \n";
for{h in Origin} {
 printf "%s %d \n",h,capacity[h,o]/5;
 }


let{h in Origin} cf[h]:= capacity[h,o]/5 -sum{k in Destination} (quantity[h,k,o]+sum {a
in Destination} (alternative[h,a,k,o]*alter[a,k,o])) + sum{k in Origin}
(quantity[k,h,o]+sum {a in Destination} (alternative[k,a,h,o]*alter[a,h,o]));

printf "\nFinal Distribution: \n";
for{h in Origin} {
if o<5 then printf "%s %d \n",h, capacity[h,o+1]/5;
if o=5 then printf "%s %d \n",h, cf[h];
 }

}


printf "\nTotal profit: %f", profit;
printf "\nTotal sold quantity: %d %s %d %s", sum{h in Origin, k in Destination, o in
HourlyRange} (quantity[h,k,o] +sum {a in Destination}
(alternative[h,k,a,o]*alter[k,a,o])), "of which ",sum{h in Origin, k in Destination, o
in HourlyRange, a in Destination} alternative[h,k,a,o]," alternatives";
printf "\nTotal lost quantity: %d", sum{h in Origin, k in Destination, o in
HourlyRange} notQ[h,k,o];
printf "\nTotal demand: %d", sum{h in Origin, k in Destination, o in HourlyRange}
demand[h,k,o];

printf "\n";
```

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