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Perspectives in Lie Theory

 Springer

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Preface

The present volume contains contributions related to the INdAM intensive research period which took place in the Centro De Giorgi in Pisa in the period December 2014–February 2015.

The volume is divided in two parts. Part I contains the lectures notes of the four minicourses delivered during the trimester. Part II contains papers contributed by participants.

Here is a brief description of the contents of the lecture notes. The first one by Victor Kac is devoted to giving an introduction to the theory of vertex algebras and Poisson vertex algebras. After a quick review of the basic notions related to vertex algebras, the notes give special emphasis to their classical limit, Poisson vertex algebras, and their applications to the classification of integrable Hamiltonian PDE's and their conservation laws. This starts with the famous paper of Drinfeld and Sokolov from 1985 and the notes contain an up to date report of the state of the art in this subject illustrating results by Kac himself, De Sole and others.

The second set of notes, due to Fyodor Malikov, gives an introduction to the theory of chiral differential operators. In the notes one starts with the, by now classical, localization result of Beilinson and Bernstein which is illustrated in the case of the group SL_2 . Inspired by this, via the notion of Chiral Algebroid, the author gives the general notion of algebra of chiral differential operators and illustrates it with a wealth of interesting examples.

The third set of notes is by Vera Serganova. It gives a comprehensive introduction, with plenty of examples, of the theory of finite dimensional representations of basic Lie superalgebras. The notion of a basic Lie superalgebras is recalled and, among other things the author introduces analogues of many of the typical constructions which one performs in the finite dimensional representation theory of semisimple Lie algebras.

Finally, the last set of notes is by Tomoyuki Arakawa. The notes give an introduction to the theory of finite and affine W -algebras and their representation theory. In particular, they provide an outline of the proof of the conjecture of Frenkel, Kac and Wakimoto on the existence and construction of the so called minimal models of W -algebras, which gives rise to rational conformal field theories

as in the case of the integrable representations of affine Kac-Moody algebras and the minimal models of the Virasoro algebra.

Part II contains 13 papers which cover a variety of subjects. Some of them are related to the theory of Lie algebras and their representations, affine algebras and vertex algebras. Others relate to the study of braid groups, the topology of hyperplane arrangements, and various applications.

The INdAM intensive period has been organized with the contribution of INdAM, Sapienza Università di Roma, CRM “Ennio de Giorgi”, Foundation Compositio Mathematica, Università di Pisa, Università di Bologna, FIRB “Perspectives in Lie Theory”, PRIN “Spazi di Moduli e Teoria di Lie” Dipartimento di Matematica di Roma Tor Vergata, Università di Padova, NSF, EMS.

The organizers acknowledge the generous support of INdAM and warmly thanks the CRM De Giorgi for their hospitality.

Pisa, Italy
Padova, Italy
Bologna, Italy
Roma, Italy
Roma, Italy
February 2017

Filippo Callegaro
Giovanna Carnovale
Fabrizio Caselli
Corrado De Concini
Alberto De Sole

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Filippo Callegaro received his PhD in Mathematics from the Scuola Normale Superiore of Pisa in 2007 under the direction of Mario Salvetti. In 2008 he took on a teaching position at the Scuola Normale, before moving to the University of Pisa in 2013. His work focuses on the topology of configuration spaces, cohomology of braid groups and generalizations, and hyperplane and toric arrangements.

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Corrado De Concini received his PhD in Mathematics from the University of Warwick in 1975 under the direction of George Lusztig. He has been a professor of algebra since 1981. Since 1996 he has been a professor at the University of Roma, La Sapienza. He has been a visiting researcher at numerous institutions, including Brandeis University, Mittag Leffler Institute, Tata Institute of Fundamental Research, Harvard University, and MIT. In addition, he has made valuable contributions in several areas of algebra and algebraic geometry, including invariant theory, commutative algebra, algebraic and quantum group theory, the Schottky problem, and hyperplane arrangements.

Alberto De Sole received his PhD in Mathematics from the Massachusetts Institute of Technology in 2003 under the supervision of Victor Kac. He has been a B.P. assistant professor at Harvard University's Department of Mathematics, a researcher

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