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Masonry cross vaults: an overview of the historical developments

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Abstract

The cross vault represents one of the most diffused and fascinating structural typologies of the European architectural heritage. Its history began almost two thousand years ago and reached a widespread use during the Middle Ages with the outstanding gothic cathedrals. Without any proper scientific support but only using trial-and-error methods, considering each building as a scaled specimen of a new one to be built, the ancient workmanship achieved a proper competence represented by the so-called rules of thumb. However, despite this long-lasting history, it is only from the eighteenth centuries that scholars have tried to tackle the problem of analytically describing its structural behaviour. In this regard, the first part of the present study is devoted to the evolution of cross vaults from the geometrical and constructive standpoint, whereas the second one describes the historical advancements of its structural behaviour, until the development of modern limit analysis.

Form evolution

The history of cross vaults starts almost two thousand years ago with the Roman construction during the Empire Age, with the so-called groin vault composed by the intersection at right angles of two semi-circular barrel vaults. Basilica of Maxentius and

Baths of Diocletian (Figure 1), both with a span of more than 25m, are remarkable results of the technical skills of that time, when the *opus caementicium* was the undisputed protagonist. However, although Romans conceived the vault as a one-piece structure, the occurrence of cracks led the builders to strengthen the most stressed parts by hidden ribs within the concrete mass, i.e. perimetral arches and internal diagonal brickwork ribs[1].

At the end of the fifth century AD, the decline and the subsequent fall of the Roman Empire brought also a deterioration of the construction techniques and materials, first of all the pozzolana concrete. A new period of economic and cultural decline begun, usually referred to as Middle Ages or Dark Ages because of its impoverishment of civilization. However, looking at the construction that resulted from this period, i.e. the Gothic Architecture, the last designation does not seem appropriate. How can it be possible that the majestic buildings suddenly appeared and spread all over Europe? In which way did the old masons get such a competence, jealously guarded in the secrets of the lodge?

Without doubt, the monuments of that time are a clear sign of the powerful logic of the trial and error methods employed by the medieval builders. Assigning a precise function to each element gave gothic churches a sense of profound elegance with a considerable resources saving: compared with barrel or cloister vaults, the groin vault produces the smallest surface in covering a rectangular space[2]. From a structural point of view, the cross vault represents the optimum in terms of lightness and refinement. This form permitted to direct the self-weight to the four corners, allowing the lateral walls to become non-structural elements, frequently replaced by stained glass windows. Even if this represents the end of the original Roman massive construction, hidden ribs became now of fundamental importance[3].

From the constructive point of view, the intersection of two semi-cylinders produces semi-elliptical diagonals, difficult to be built by the masons of that time who started to adopt the segmental arcs. This practical approach affected inevitably the geometry leading the centre of the vault to be higher than the lateral arches and the webs to lose the cylindrical shape. Besides this first variation, the pointed arch appeared during the 1130s in France and England, representing a geometrical revolution: the height of the lateral arches was no longer constrained and the bay could be rectangular[4]. The geometrical options at disposal of the masons gave way to different cross vaults in terms of overall form and construction variations. Barthel[5] proposed a possible classification of the large variety of quadripartite cross vault. Moving from the groin vault, Figure 2 shows the main types of cross vaults with particular reference to the shape of the generatrix of the webs. Figure 2b s hows the variation of cross vault shapes starting from the same diagonal arch profiles[6].

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[1] Como 2013
[2] Huerta 2004
[3] Como 2013
[4] Tomasoni 2008
[5] Barthel 1993
[6] Strommer 2008

Practical rules

In the antiquity, only Heron of Alexandria (ca. 60 AD) wrote about technical aspects related to vault construction, later cited by Anthemius of Tralles, one of the architects of Hagia Sophia in Constantinople. Unfortunately, both documents were lost[7]. After Vitruvius, who did not provide any practical rule for cross vaults, the next prominent person who wrote about architecture was the polymath Leon Battista Alberti[8]. Moving from the classical architecture and the Vitruvius's three basic principles, Alberti connected indissolubly strength and beauty considering the latter as the most important and noble quality, intimately related to the nature perfection. Traditionally, at the base of this approach there was the Pythagorean concept of beauty founded on numerical proportions as *concinnitas universarum partium*: a beautiful building is well proportioned and, therefore, safe[9].

Regarding cross vaults, the most famous rule was the so-called *Blondel's rule*, designation due to the popularity and influence of Blondel who reported this rule in his treatise in 1675. However, since Derand in 1643 highlighted the clear gothic origin of the rule, it is also referred as *Fr. Derand's rule*[10]. It consists in the division of the *arc-doubleau* in three equal parts from which it is possible to geometrically obtain the width of the abutments (Figure 3). The principal aspect of this rule is that it addressed positively the problem of larger abutments for larger thrust, from pointed to flat arches[11].

Other two gothic rules are shown in Figure 4. The first one was provided by Hernandez Ruiz el Joven in 1560[12] and it involves the thickness of the arch. The author emphasised the stabilizing importance of the infill at the haunches until half of the height of the arch and prescribed the minimum thickness of the arch equal to 1/10 of the span. The second one regards the German Gothic rules which, starting from the main module (the span of chorus), led up to the smallest details, e.g. the vault ribs cross-section[13].

Almost one century after Alberti, Cataneo[14] wrote his treatise following a completely different approach. He proposed the dimensions of all the parts of five Latin cross plan churches, thus with proportions defined a priori. Figure 5 shows the general plan and the longitudinal cross section of one of the churches: the abutments width of the aisle vaults is one third of the span.

Contemporaneous of Cataneo, Rodrigo Gil de Hontañón focused on the late Spanish gothic cross vaults. He provided detailed rules for the calculation of circular columns and abutments dimensions through analytical formulas (Table 1).

On the other hand, one century later, Friar Lorenzo de San Nicolás addressed general aspects of the vault but not giving any practical rule about their dimensions. However, in case of groin vaults, he pointed out that the stability was guaranteed only thanks to

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[7] Huerta 2004

[8] Alberti 1485

[9] Di Stefano 2000

[10] Benvenuto 1991

[11] Huerta 2004

[12] *Ibidem*

[13] Coenen 1990

[14] Cataneo 1567

the infill weight (until the first third) with no need of abutments[15].
Few decades after, the Age of Enlightenment brought a surprising interest in the analysis of vaulted structures. Several scholars sharpened their intellect to find a method to design and to assess the stability of arches and vaults, having as only reference, naturally, the sound and validated tradition, i.e. geometrical proportions. De la Hire and Belidor (1712 and 1729 respectively) were prominent figures well integrated into the trend of *science after tradition*[16]. They tried to apply a scientific method to the arch stability (based on the wedge theory), but they ended up with a geometrical construction (de la Hire's graphical method is reported in Figure 6a). Although scientifically incorrect, the rule provided a proportion in line with the traditional rules, and it swiftly spread all over the Europe, together with the everlasting Derand/Blondel's rule.
Valadier[17] criticised the last rule, stressing the point that this method did not consider the thickness of the arch and the height of the abutment. He proposed a new graphical method without presenting any demonstration. Although Valadier did not cite de la Hire, he referenced the reader to the memories of *Accademia Reale delle Scienze* of 1712, probably Paris, that is the same year of de la Hire[18]. Figure 6 shows the comparison between the two methods, apparently different. Regarding cross vaults, in order to evaluate the dimensions of the abutments, Valadier considered the two elemental barrel vaults independently, thus the two perpendicular side lengths (Figure 7).

Historical structural analysis methods

The origin of vaulted constructions is lost in the mists of time: vaults are spread all over the world with almost seven thousands years of history[19]. From the structural point of view, the common thread has always been the eternal struggle between the natural laws and the human cunning. But also a sense of deep admiration and surprise affected the scholars who approached this issue. In the words of Tosca: "*the same gravity and weight which should have precipitate them* [the voussoirs] *to the earth, maintain them constantly in the air*"[20]. The only price to pay for this wonder was the thrust which became evident in the overturning moment of the abutments. To overcome this structural challenge, the weight has represented for centuries the only expedient, i.e. *buttressing by loading*[21].
Several authors have tried to understand the behaviour of vaulted structures, feeling the need of giving a scientific foundation to construction. Probably Gautier (1716) was the scholar who better described this feeling of concern and apprehension: "*Only by trial and error, it was possible to build all bridges and vaults of all the buildings. Specific rules which allowed to know the safety capacity of these works have never been followed*"[22].
Without entering into the merits of the history of arch theory, for which the reader is

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[15] Huerta 2004
[16] Benvenuto 1991
[17] Valadier 1832
[18] de la Hire 1712
[19] Como 2013
[20] Huerta 2001, p. 47
[21] Huerta 2006
[22] Huerta 2004, p. 320

referred to other references, such as Heyman 1972 and Benvenuto 1991, Mascheroni[23] was probably the first scholar who addressed the study of compound vaults, that is, those of any form other than the simplest. The objective of his work is well explained in the preface of his treatise: "*But what most required a geometrical research were compound arches and vaults, which are usually executed by architects in various ways and in rather spacious places, and which necessarily imply a large danger*"[24].

Mascheroni dedicated the eleventh chapter of his book to the study of the compound arches and vaults. He approached the study of cross vaults considering independent web strips whose resultant action is applied to the diagonal arch: this is nothing else but a reduction from a three-dimensional problem into a well-known plane one. Considering the diagonal arch and the web as the two main elements of the cross vault (Figure 8), he proposed two dual problems: given the shape of one, calculate the other. With this aim, he extensively used the concept of catenary.

With the contribution by Mascheroni, the end of the eighteenth century marked also the end of the rigid, infinitely resistant voussoirs theory and gave way to new theories, namely elastic beams with curvilinear axis, membranes and shells, gathered together in the framework of the elastic theory. Accordingly, the attention was focused on the true elastic solution of the masonry arch, which is a statically indeterminate problem[25]. Previous scholars have addressed this problem thanks to the intuitive idea of *cracking* the structure up to the collapse mechanism. Even though they were not aware of it, this procedure reduced the degree of indeterminacy and the Archimedean simple machines of wedge and lever are applicable.

Beside the elastic theory, the nineteenth century gave birth also to the concept of line of thrust, as the line connecting the resultant forces in each cross section. Since the problem is statically indeterminate, it cannot be solved only with equilibrium conditions. Several authors tried to address these difficulties by introducing *principles*. Culmann[26], for example, adopted the principle of minimum loading, i.e. the true line of thrust is the one with the smallest deviation from the centre line. However, the most important contribution of Culmann regards the graphical statics, which, at the end of the nineteenth century, gained new vigour and rapidly spread out paving a new way for vaulted structures analysis (Figure 9). In order to apply the slicing technique to complex vaults, Mohrmann suggested following the idea of a ball rolling down the extrados of the webs. The same approach will be adopted by Sabouret[27] and Abraham[28] but, since only the latter provided explicative drawings (Figure 10), the entire credit was given to Abraham[29].

In spite of these last developments with the graphical method and the thrust line, after the rising in popularity of wrought-iron structures starting from 1860s, the supremacy of elastic theory was inevitable. To give a new boost to the masonry vaults analysis, it is necessary to wait until the half of the twentieth century, when the elastic theory

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[23] Mascheroni 1785
[24] *Ibidem*, p. xi
[25] Benvenuto 1991
[26] Culmann 1864
[27] Sabouret 1928
[28] Abraham 1934
[29] Huerta 2009

definitely lost ground to the plastic theory: the ultimate load analysis re-emerged (again, after the developments in the eighteenth century) and took the modern shape. Regarding masonry, Drucker, Kooharian and Prager conducted studies which led to the well-known work by Heyman[30], who collected and arranged all the previous knowledge in the framework of limit analysis.

Conlcusions

The historical developments of cross vaults presented in this paper have shown as the ancient builders achieved such a level of complexity and perfection that only modern and sophisticated structural analysis tools are able to model. The understanding of cross vaults structural behaviour is still a challenging task in the conservation of architectural heritage and more efforts are still needed.

Without doubt, the shape and the proper geometrical representation of the vault play a central role in its overall stability, i.e. resistant-by-shape structures. Focusing on this feature, confirmed by the *in situ* geometrical survey, could give valuable insight into the load-bearing capacity of the vault without increasing the computational effort of the analysis. On the other hand, the rules of thumb can provide grounds for a geometrical database of the elements related to the cross vault. This investigation has a twofold goal. It may represent the basis of a parametric analysis but, at the same time, it may be a practical reference for practitioners. Apart from the uniqueness of every historical buildings, further work is still requested to validate and to expand the overall database or to delimit it to a particular geographical area.

Finally, the study of the historical methods for the analysis of masonry vaulted structures highlighted the continuous effort of scholars in studying the statics of such a complex element. Even though approximate, they achieved an appreciable understanding of the stability of cross vaults under gravitational loads but no consideration have been ever proposed in case of seismic action. Looking at the high vulnerability of cross vaults within the architectural heritage, this topic still represents an open issue and more research is certainly desirable.

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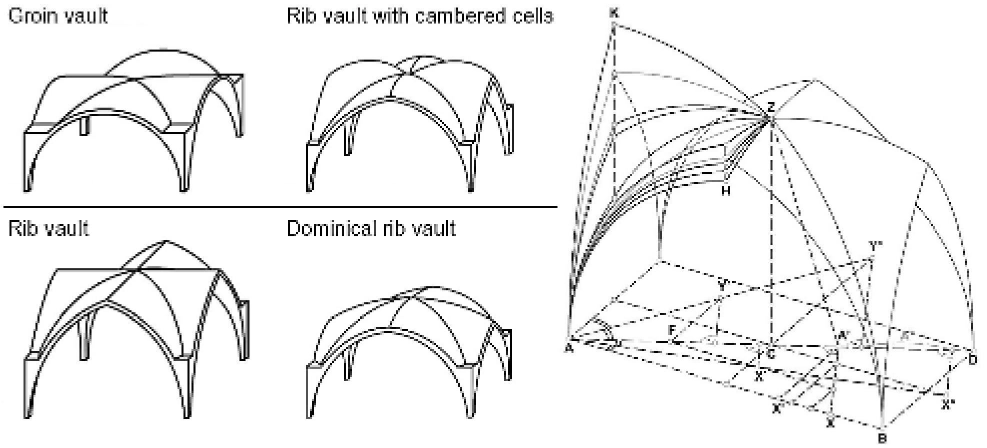
a)

b)

Figure 1. Bath of Diocletian - Rome, AD 298 - 305/6: a) particular from the inner perspective[31]; b) nowadays, Basilica of St. Mary of the Angels and the Martyrs

[31] Paulin 1890



a) b)

Figure 2. Cross vaults geometry: a) analysis according to Barthel[32] and b) different shapes using the same diagonal arches[33]

[32] Barthel 1993

[33] Strommer 2008

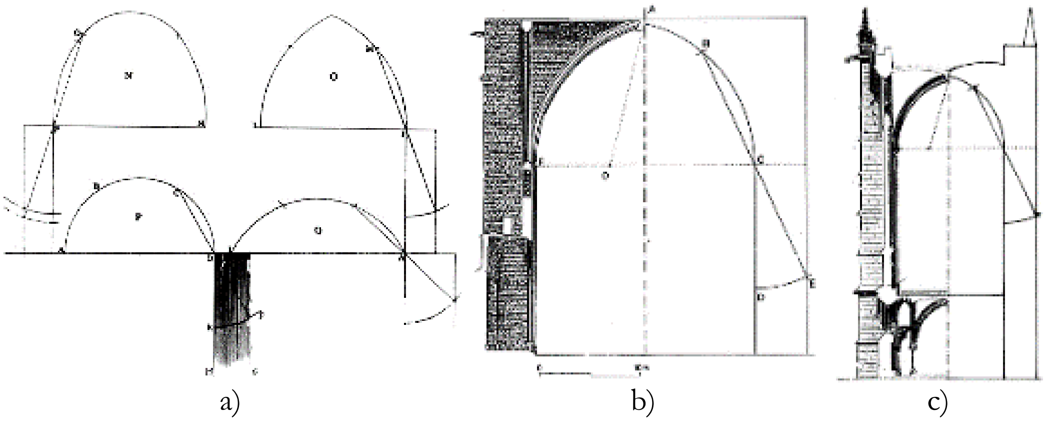
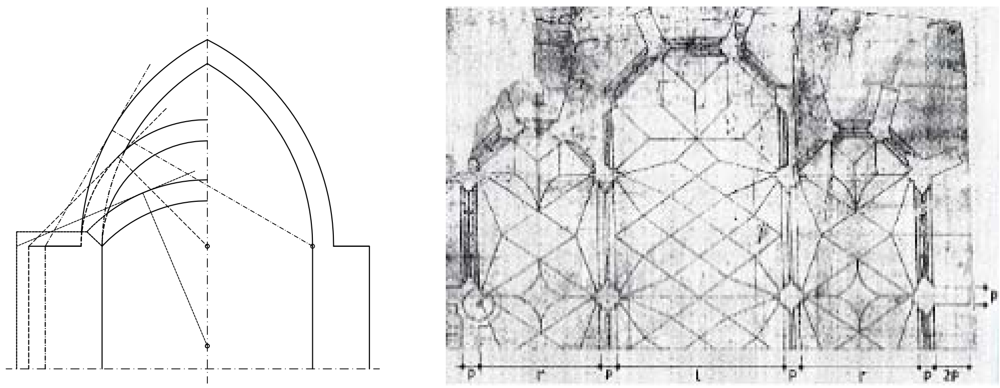


Figure 3. Fr. Derand's rule: a) application to different type of arches[34] and to b) the Cathedral of Gerona and c) the Sainte Chapelle of Paris[35]

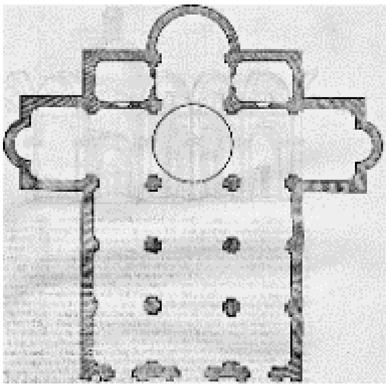
[34] Derand 1743, p. 2, plate 1
[35] Huerta 2004



a) b)

Figure 4. Abutment width calculation: a) Hernandez Ruiz el Joven's rule considering the arch thickness; b) German gothic proportion where *l* represents the chorus span[36]

[36] Ivi



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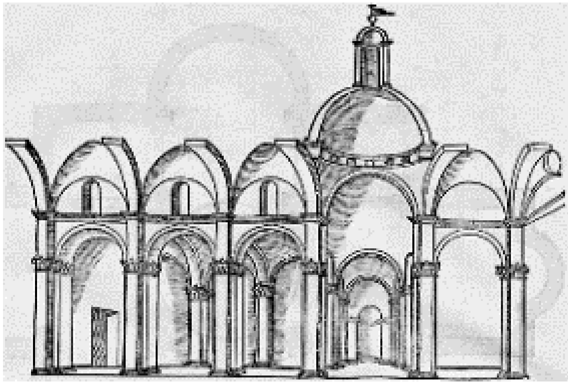

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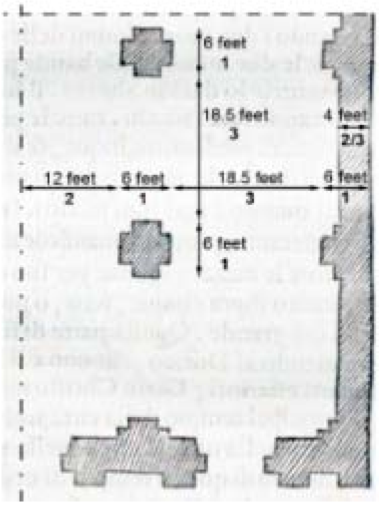

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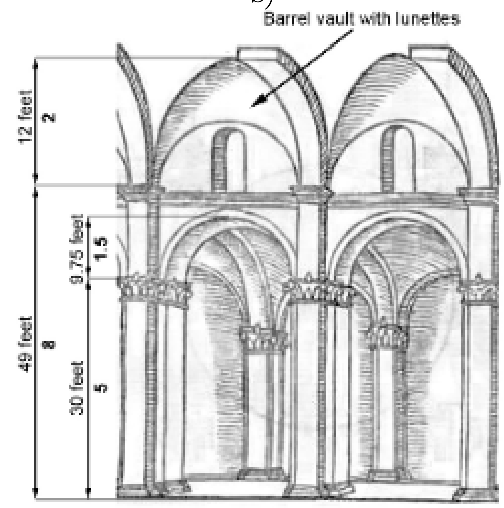
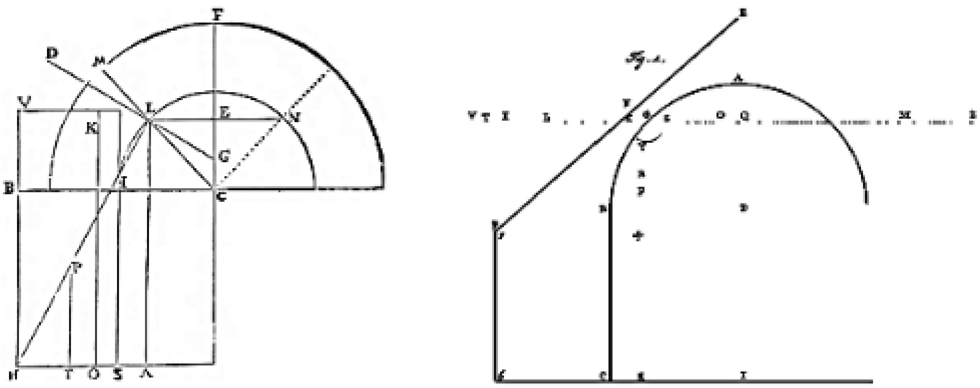


d)

Figure 5. Latin cross plan church according to Cataneo[37]: a) general plan scheme and b) longitudinal cross-section; the relative main dimensions are reported in c) and d).

[37] Cataneo 1567



a) b)

Figure 6. Graphical construction for the abutment width of an arch according to a) de la Hire[38] and b) Valadier[39]

[38] de la Hire 1712

[39] Valadier 1832

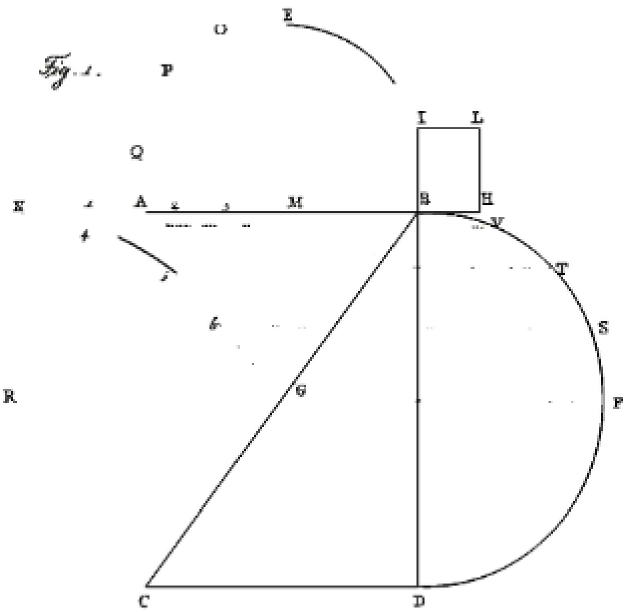


Figure 7. Valadier's geometrical construction[40] for calculating the abutment's dimensions of a cross vault

[40] Ivi, chart 256

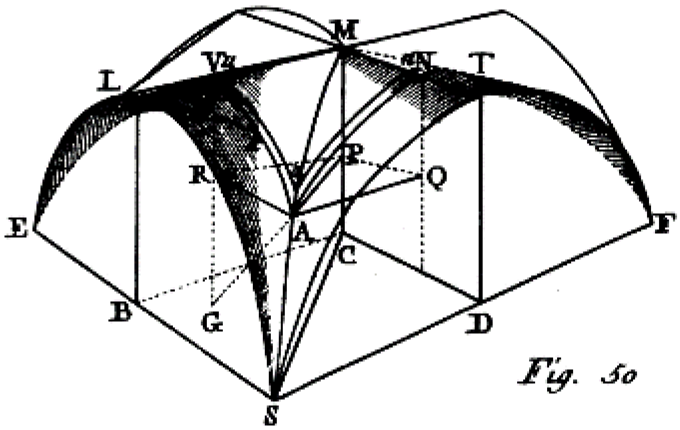


Figure 8. Mascheroni’s analysis of cross vault[41]

[41] Mascheroni 1785, chart XII

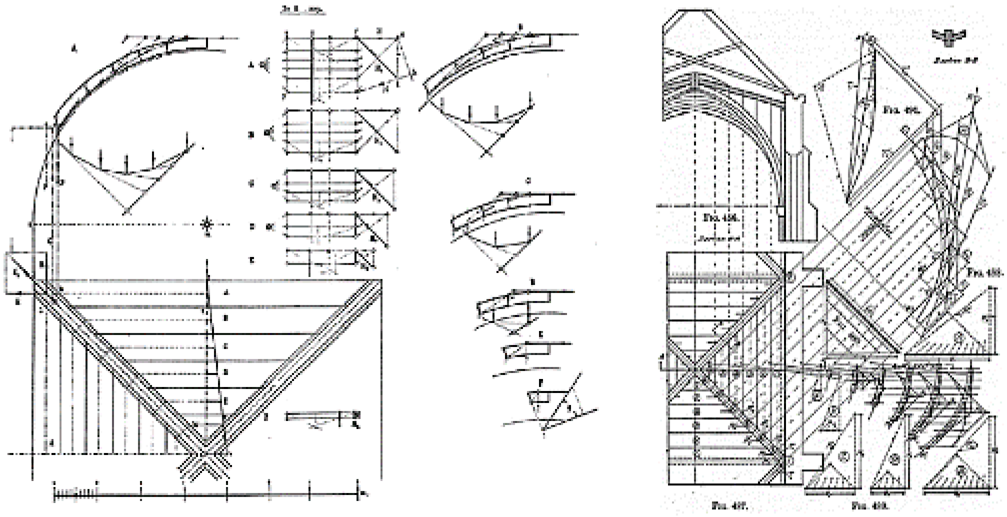
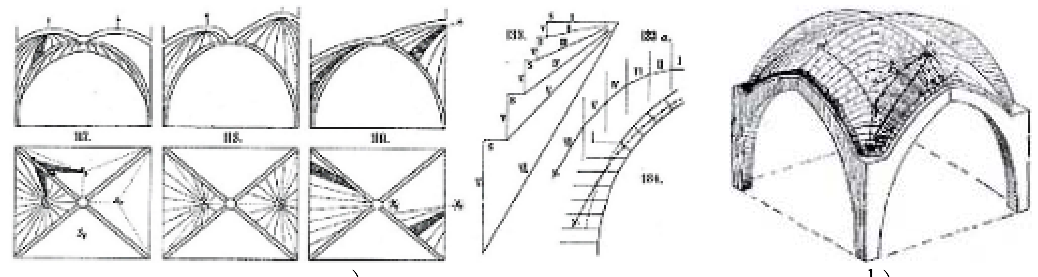


Figure 9. Graphical statics applied to cross vaults at the beginning of nineteenth century[42]

42 Körner 1901, Wolfe 1921



a) b)

Figure 10. Slicing technique: a) patterns of slicing[43] and b) “ball principle”[44]

[43] Ungewitter 1890

[44] Abraham 1934

Column diameter at the base	$d = \frac{1}{2}\sqrt{h_n + s + a}$	h_n s a	nave height nave span bay length
Abutment width at the base	$l = \frac{2}{3}\sqrt{h_a + \frac{2}{3}\sum n_i}$	h_a n_i	abutment height semi-length of all the ribs connected to the abutment (except for the formeret arches)

Table 1. Rodrigo Gil de Hontañón’s suggestions regarding columns and abutments dimensions [feet]