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Twin-waves propagation phenomena in magnetically-coupled structures

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Abstract

The use of magnetic dipoles embedding in an elastic support introduces long-range interaction forces. This is a completely new paradigm in structural mechanics, classically based on local short-range particle interaction. The features of long-range forces produce very new mechanical coupling effects. This paper examines the case in which two identical rod-like structures, each with a dipole distribution embedded, vibrate side by side. Waves generated in one of the rods propagate also in the second and vice versa creating a new effect we name twin-waves. The present investigation unveils the existence of an infinite set of numerable waves even in one-dimensional infinite structures, a new and unusual behaviour in classical waveguides. The physics behind this phenomenon is further investigated also by numerical simulations.

Keywords: metamaterials, elastic metamaterials, wave propagation, long-range interaction,

1. Introduction

Metamaterials are investigated in many different fields of science, such as mechanics, optics, solid state physics, nanoscience, electromagnetism and many others. To better frame this work we intend for metamaterial, materials showing unusual dynamic propagation characteristics [1] and dissipation properties [2–5], due to the microstructure they are composed by and the associated connectivity. More specifically, metamaterials are artificial composites whose properties are not easily found in nature; they *derive their properties not from the properties of the base materials, but from their newly designed structures. Their precise shape, geometry, size, orientation and arrangement gives them their smart properties capable of manipulating electromagnetic* and mechanical *waves.* [...] *The materials are usually arranged in repeating patterns, at scales that are smaller than the wavelengths of the phenomena they influence* [22]. This paper investigates metamaterials in which the introduction of long-range interaction activates new propagation phenomena that the classical dynamics based on local interaction cannot explain without the introduction of unusual mathematical models [6–8].
In this context, an extensive investigation has been carried on in [9–12]. In these papers the static and dynamic behavior of these models is considered, taking advantages of the fractional calculus [13–20]. In [21] it is shown how the

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fractional calculus enables the formulation of fractional generalization of nonlocal elasticity models: *The long-range interactions have been studied in discrete system as well as in their continuous analogous and the continuum equations with derivatives of non-integer orders can be directly connected to lattice models with long-range interactions of power-law type* [6,21].
The present paper considers for the first time a continuous model with long-range interaction of Gaussian-law type, but instead of applying the fractional calculus, the system is modelled by a non-linear integral-differential equation. Two systems are considered: a single waveguide and a system composed by two identical coupled waveguides. This new approach considers the structure of the material as a continuous system made of particles interacting in any possible combination by long-range forces. Their Gaussian-like nature, is aimed at mimiking long-range decay with the distance, that is typical of electro-magnetic forces. The main contribution of this paper is the disclosure of analytical solutions to the integral-differential equations, revealing the existence of propagating modes, a mixing of a wave behavior and modal response. Numerical simulations complete the investigation.

2. Long-range interaction waves: integral-differential equation approach

To explore the waves propagation properties modified by the presence of the long-range interaction, a single dimensional infinite waveguide is considered. We define $F(r) = \mu r e^{-\beta r^2}$ as the interaction force that is borne on the particle at x, because of the particle at ξ, assumed $r = x - \xi + w(x) - w(\xi)$ where $w(x)$ and $w(\xi)$ are the displacements in the elastic medium, μ and β control the intensity and the characteristic interaction length, respectively. The Gaussian-like force considers the decaying long-range effect, an antisymmetric behaviour and a dependency on r only, as typical of magnetic and electrostatic forces. The equation of motion is represented by:

$$\rho \frac{\partial^2 w}{\partial t^2} - E \frac{\partial^2 w}{\partial x^2} - \mu \int_{-\infty}^{+\infty} (1 - 2\beta\xi^2) e^{-\beta\xi^2} w(x - \xi) d\xi = 0 \tag{1}$$

This equation is the result of the linearization of the Gaussian-like force about $\varepsilon = w(x) - w(\xi) = 0$, assuming $|\varepsilon| \ll |x - \xi|$. The equation of motion simplifies as:

$$\rho \frac{\partial^2 w}{\partial t^2} - E \frac{\partial^2 w}{\partial x^2} - g(x) * w(x) = 0 \tag{2}$$

where $g(x) = \frac{\partial F(r)}{\partial r}\Big|_{r=x}$. Assuming $w(x, t) = \varphi(x) e^{-J\omega t}$, the equation of motion leads to the dispersion relationship:

$$\rho\, \omega^2 - E\, k^2 + \frac{\mu}{(2\beta)^{\frac{3}{2}}} e^{-\frac{k^2}{4\beta}} k^2 = 0 \tag{3}$$

The final goal of the analysis, to determine an analytical explicit form of the dispersion relationship, is obtained. Because of the transcendent form of equation (3), for any given value of the frequency, the associated wavenumber k assumes an infinite set of numerable values. This fact is quite unusual in standard one-dimensional propagation phenomena, where, for partial differential equations, in general, one obtains a polynomial form of the dispersion relationship, that admits, at most, a number of different roots equal to the degree of the polynomial. If we interpret each root as a wave (propagating or evanescent), this implies that, for each excitation frequency, equation (3) predicts a waveguide response made by the superposition of an infinite set of wavenumbers. In other words, in a standard waveguide (closest neighbour interaction) we expect a sine-shaped waveform translating towards the x axis, when a single-frequency is acting. In the present long-range system, we indeed expect to see that a single-frequency excitation generates a superposition of different sine-shaped waveform, translating at different phase speeds. Thus, even a single frequency excitation produces a complex wave train that is dispersive. Note that each of the wavenumber defines a *propagation mode*, physically different with respect to the classical modes arising in finite size systems. Therefore, a long-range system produces, at each single-frequency, a response that is the superposition of an infinite number of *propagation modes*. Note that the outlined physical phenomenon is new and it has analogues only in two- or three-dimensional systems, but not for pure one-dimensional waveguides. An example is the case of strip-like structures, that are properly two-dimensional systems, with a finite length along one axis and an infinite length along the other. Precisely, along the finite size axis, transverse modes take place, while along the infinite size direction,

a *propagation mode* travels, related to the excited transverse steady mode. Examples of this behaviour are met in optical/electromagnetic waveguides and are known as TE, TM and TEM (transverse electromagnetic modes, or purely electric or magnetic modes), or in mechanical strip-like bending plates, or in laser theory, but needing 2D or 3D systems.
The expression for the displacement associated to each *propagation mode* of wavenumber $k_i(\omega)$ is:

$$w_i(x,t) = \varphi_i(x)e^{-\jmath\omega t} = \int_{-\infty}^{+\infty} \Phi(k_i)e^{\jmath(k_i x-\omega t)}dk_i = \int_{-\infty}^{+\infty} \Phi(k_i(\omega))e^{\jmath(k_i(\omega)x-\omega t)}dk_i(\omega) \tag{4}$$

where $\Phi(k_i) = \mathcal{F}(\varphi_i(x))$.

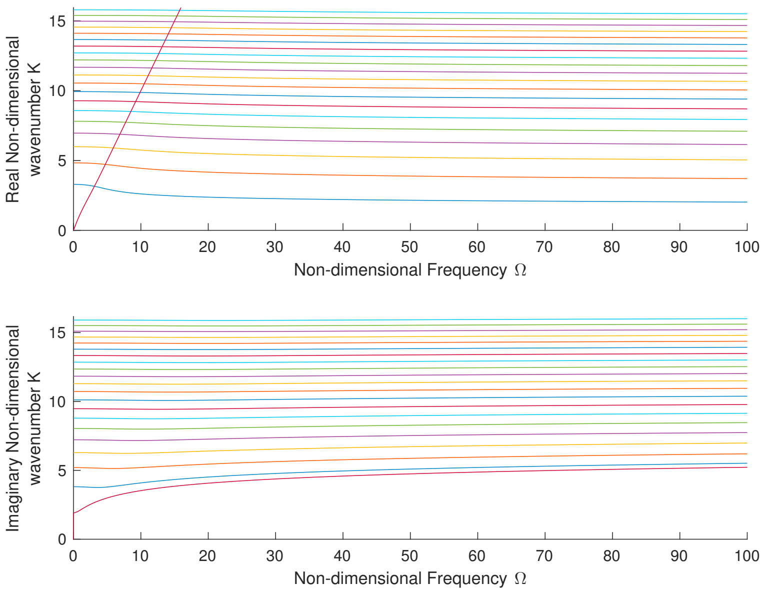

Fig. 1. Propagation modes wavenumbers for the Gaussian-like force.

The upper plot of Figure 1 shows the real part of the first twenty wavenumbers related to the propagation modes, together with the purely real standard propagation wavenumber, related to the standard wave (sloping curve). The lower plot replicates the same, but for imaginary parts and the lowest curve represents a purely imaginary wavenumber. Therefore, the general solution to the equation of motion is:

$$w(x,t) = \sum_{i=1}^{+\infty} \int_{-\infty}^{+\infty} \Phi(k_i)e^{\jmath(k_i x-\omega t)}dk_i = \sum_{i=1}^{+\infty} \int_{-\infty}^{+\infty} \left[\Phi_+(k_i)e^{\jmath(k_i x-\omega t)} + \Phi_-(k_i)e^{\jmath(k_i x-\omega t)}\right] d\omega \tag{5}$$

This expression is self-explaining: it matches the double nature of the solution, as a mixing of propagation and mode response, through the presence of an integral (normally related to the response representation for infinite-size systems) and summation (normally related to the modal response in finite-size systems). This is the main feature of the long-range structure here investigated: the borne of 1D *propagation modes*.

3. Twin waves

The long-range interaction produces the chance of coupling different waveguides. Therefore, here two waveguides are considered, each characterised by long-range interaction of Gaussian-like type in its own motion, but further coupled due to their long-range ability. This implies that on each of the point of each waveguide, we see the effect related to the forces generated by the points of the same waveguide and those of the forces generated by the points of

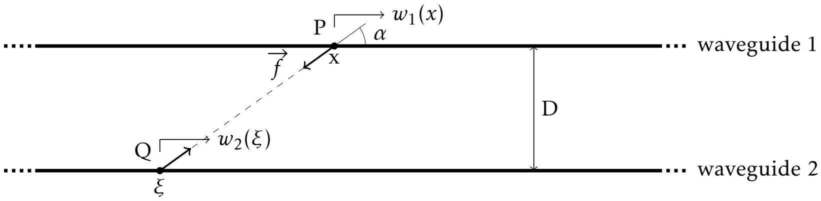

Fig. 2. Twin waves coupling model

the second.
Let us model the cross long-range interaction between the waveguides, following Figure 2.

We assume the interaction force $\mathbf{f}$ depends on the $\mathbf{PQ}$ distance, $r = |\mathbf{PQ}| = \left[D^2 + (w_1 + x - w_2 - \xi)^2\right]^{\frac{1}{2}}$. Moreover, we assume only longitudinal motion of the particles, that implies only the axial component $\mathbf{f}_x(\mathbf{r})$ is active. This leads to $f_x(\mathbf{r}) = |\mathbf{f}(\mathbf{r})| \cos\alpha$, where $\cos\alpha = \frac{x-\xi+w_1-w_2}{r}$. The projection of the interaction force on x produces:

$$f_x(\mathbf{r}) = -\frac{|\mathbf{f}(\mathbf{r})|}{r}(x - \xi + w_1 - w_2) \tag{6}$$

In particular, selecting $f(r) = \mu r e^{-\beta r^2}$, the expression of the force $F_x(x) = -\int_{-\infty}^{+\infty} f_x(r) d\xi$ becomes:

$$F_x(x) \approx \int_{-\infty}^{+\infty} h\, w_2\, d\xi = h * w_2 \tag{7}$$

where $h(x)$ is a term deriving from the linearization of $f_x(r)$.

We can proceed similarly to the previous examined cases with $w_1 = \varphi_1(x)e^{-\jmath\omega t}$ and $w_2 = \varphi_2(x)e^{-\jmath\omega t}$. Thus, one obtains with obvious meaning of the symbols:

$$\begin{cases} -\rho_1\omega^2\Phi_1 + E_1k^2\Phi_1 - G_1\Phi_1 + H\Phi_2 = 0 \\ -\rho_2\omega^2\Phi_2 + E_2k^2\Phi_2 - G_2\Phi_2 - H\Phi_1 = 0 \end{cases} \tag{8}$$

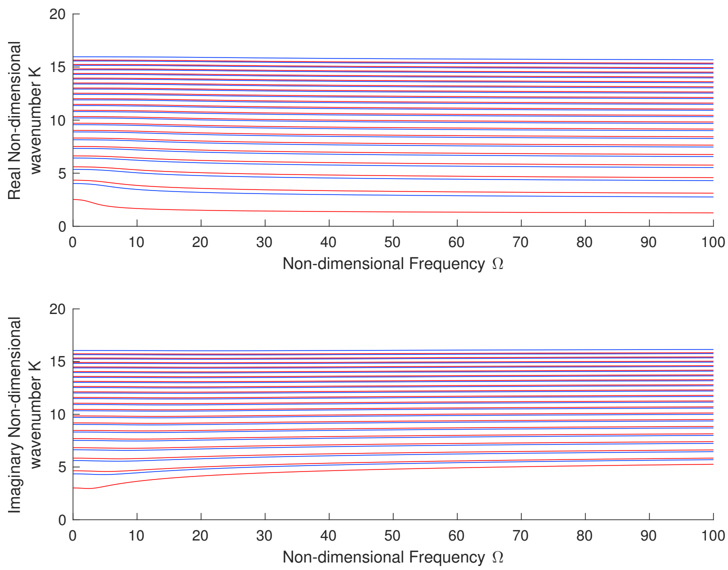

Fig. 3. Twin waves dispersion relationships

Assuming the two waveguides are twins, and thanks to the contribution $H = -\frac{\gamma}{(2\beta)^{\frac{3}{2}}} e^{-\frac{k^2}{4\beta}} k^2$ and $G = -\frac{\mu}{(2\beta)^{\frac{3}{2}}} e^{-\frac{k^2}{4\beta}} k^2$, the dispersion relationship becomes:

$$\left(-\rho\omega^2 + E\,k^2 + \frac{\mu}{(2\beta)^{\frac{3}{2}}} e^{-\frac{k^2}{4\beta}} k^2\right)^2 + \left(-\frac{\gamma}{(2\beta)^{\frac{3}{2}}} e^{-\frac{k^2}{4\beta}} k^2\right)^2 = 0 \tag{9}$$

The effect due to the long-range interaction appears for twin waveguides analogous to the single waveguide. In fact, the dispersion relationship (9) shows an infinite numerable set of wavenumbers associated to the same frequency ω. Figure 3 shows the first twenty solutions for k, when varying the frequency ω.

4. Numerical Simulations

The behaviour of the twin waveguides has been numerically analysed by considering its discrete counterpart. The system consists of a set of two discrete subsystems of uniform masses, each one connected with springs to its first neighbours. The long-range interaction not only links masses of the same waveguide, but also the two waveguides together. The distance D between the waveguides is considered in terms of the typical long-range interaction length $\delta_0 = \sqrt{\frac{1}{\beta}}$. Numerical simulations are performed for a distance of the same order of δ_0, $D \sim \delta_0$.

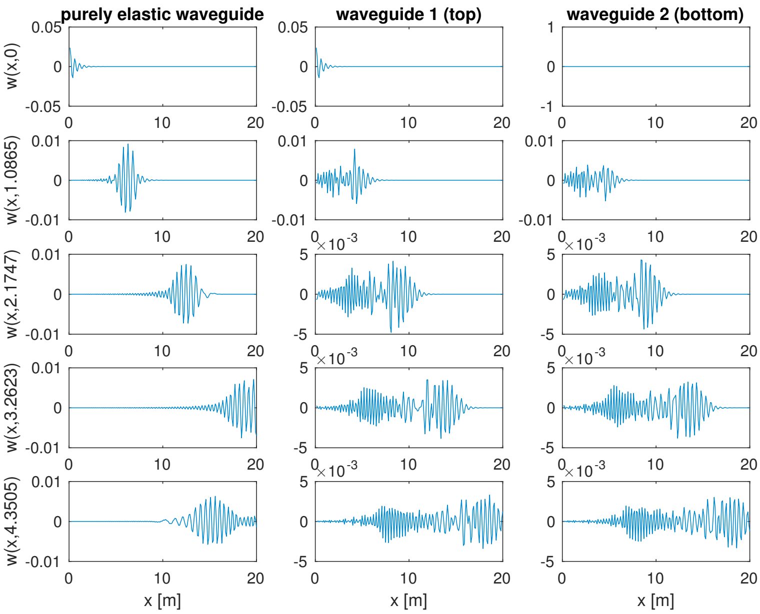

Fig. 4. Twin waves vs purely elastical simulations

To perform numerical simulations, an initial non-zero displacement has been applied only to the top waveguide (*waveguide 1*), leaving at rest the second (*waveguide 2*). The main noticeable effect is an induced wave propagation in the bottom waveguide, clearly only due to the propagation on the top one. This confirms what analytically observed: the pair of waveguides is intrinsically coupled, and propagating phenomena occurring in one of them will be transmitted to the other by the mean of long-range forces. A second remarkable effect is the different wavenumber content of the purely elastic waveguide compared with the twin ones. While the initial wave train packet travels, along the classical waveguide, some dispersion is observed, since the theoretical system represented by the standard waveguide $E\frac{\partial^2 w}{\partial x^2} - \rho\frac{\partial^2 w}{\partial t^2} = 0$ is non-dispersive, the weak dispersion observed in Figure 4 (left column) is due to the numerical discretization (a finite difference scheme is used) as it is known from the theory of finite difference equations. In fact, the dispersion relationship is in this case non-linear, but represented by a branch of sine, implying higher frequencies propagate slightly slower. The wavenumber content of the twin waveguides is clearly richer with respect to the classical waveguide, and the initial wave packet spreads over the waveguide length. This effect is explained by equation (5), as a consequence of the infinite solutions of the dispersion relationship (3), or equivalently (9), predicting the appearance of a superposition of propagation modes, the evidence of which is in the second and third column of Figure 4. Moreover, it appears a slow down of the wavefront speed in the presence of long-range interaction.

5. Conclusions

In this paper, a continuous system with long-range interaction of Gaussian-like type has been investigated. The dynamic behaviour is ruled by a non-linear integral-differential equation. Two systems are considered: a single waveguide and a system composed by two identical rod-like structures. In both cases, the remarkable result consists of the generation of a singular mixing of waves and modes. This discloses a form of displacement that matches the double nature of the solution, through the presence of an integral (normally related to the response representation for infinite-size systems) and summation (normally related to the modal response in finite-size systems). This is the main feature of the long-range structure here investigated: the borne of 1D *propagation modes*. Waves are related to a set of natural frequencies, the mutual distance of which goes to zero when the length of the structure goes to infinite. This collapses the modal summation to an integral. In the present case equation (5) can be interpreted considering that even for an infinite length it exists a cluster of waves each with modal index i generating a complex motion composed by a system of waves.

References

[1] A. Madeo, G. Barbagallo, M.V. D'Agostino. *First evidence of non-locality in real band-gap metamaterials: determining parameters in the relaxed micromorphic model*. Proceedings of the Royal Society. A (2016)
[2] N. Roveri, A. Carcaterra, A. Akay. *Vibration absorption using non-dissipative complex attachments with impacts and parametric stiffness*, Journal of the Acoustical Society of America, Vol. 126 (5), (2009) pp. 2306-2314
[3] A. Carcaterra, A. Akay, I.M. Koc. *Near-irreversibility in a conservative linear structure with singularity points in its modal density*, The Journal of the Acoustical Society of America Vol. 119 (4), (2006) 2141-2149
[4] A. Carcaterra, A. Akay. *Fluctuation-dissipation and energy properties of a finite bath*. (2016) Physical Review E, 93(3), 032142.
[5] A. Carcaterra, A. Akay. *Dissipation in a finite-size bath*. (2011) Physical Review E, 84(1), 011121.
[6] A. Carcaterra, F. Coppo, F. Mezzani, S. Pensalfini. *Metamaterials: wave propagation control*. Conference Paper. ISMA 2016 Proceedings, KU Leuven, 2016
[7] A. Carcaterra, R. Esposito, M. Pulvirenti. *Macroscopic description of microscopically strongly inhomogenous systems: A mathematical basis for the synthesis of higher gradients metamaterials*. (2015) arXiv preprint arXiv:1504.08015.
[8] A. Carcaterra, A. Akay, C. Bernardini. *Trapping of vibration energy into a set of resonators: Theory and application to aerospace structures* (2012). Mechanical Systems and Signal Processing, 26, 1-14.
[9] V. E. Tarasov, George M. Zaslavsky. *Fractional dynamics of coupled oscillators with long range interaction*, American Institute of Physics DOI 10.1063/1.2197167
[10] V. E. Tarasov. *Lattice model of Fractional gradient and integral elasticity: long-range interaction of Grünwaldletnikov-riesz type*, Mechanics of metamaterial vol.70 no. 1 (2014) 106-114
[11] V. E. Tarasov, G. Zaslavsky. *Fractional dynamics of systems with long-range interaction*, Communication in nonlinear science and numerical simulation 11 (2006) 885-898
[12] V. E. Tarasov. *Lattice with long-range interaction of power-law type for fractional on local elasticity*, International journal of solids and structures (2014)
[13] A. Carpinteri, F. Mainardi (Eds.). *Fractals and Fractional calculus in continuum mechanics*, (Springer, New York, 1997)
[14] A. C. J. Luo, V. S. Afraimovich (Eds.). *long-range interaction, stochasticity and fractional dynamics* (Springer, Berlin, 2010)
[15] J. Klafter, S. C. Lim, R. Metzler (Eds.). *Fractional Dynamics. Recent Advances* (World Scientific, Singapore, 2011).
[16] F. Mainardi. *Fractional Calculus and waves in linear viscoelasticity: An introduction to Mathematical models*, (World scientific, Singapore, 2010)
[17] V. E. Tarasov. *Theoretical Physics models with Integro-differentiation of fractional order*, (IKI, RCD, 2011) in Russian.
[18] V. V. Uchaikin. *Fractional derivatives for Physicists and engineers*, Vol.1 Background and Theory. Vol.2 Application. (Springer, 2013)
[19] V. E. Tarasov. *Review of some promising fractional physical models*, International Journal of modern Physics B. Vol.27. No.9 (2013) 1330005. (32 pages)
[20] V. E. Tarasov. *Fractional dynamics: application of fractional calculus to dynamics of particles fields and media* (Springer, New York,2011)
[21] V. E. Tarasov. *Lattice with long-range interaction of power-law type of fractional non-local elasticity*, International Journal of Solids and Structures, (2014)
[22] Metamaterial general description. Available online: https://en.wikipedia.org/wiki/Metamaterial