Parametric frequency conversion of short optical pulses controlled by a CW background

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Abstract: We predict that parametric sum-frequency generation of an ultra-short pulse may result from the mixing of an ultra-short optical pulse with a quasi-continuous wave control. We analytically show that the intensity, time duration and group velocity of the generated idler pulse may be controlled in a stable manner by adjusting the intensity level of the background pump.

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References and links

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1. Introduction

Optical parametric amplification in quadratic nonlinear crystals has been studied since the invention of the laser, as it provides a versatile means of achieving widely tunable frequency conversion [1]. In parametric processes, the effective interaction length of short optical pulses is limited by temporal walk-off owing to chromatic dispersion, or group velocity mismatch (GVM) [2, 3]. Compression and amplification of ultra-short laser pulses in second harmonic and sum-frequency (SF) generation in the presence of GVM was theoretically predicted [4, 5] and observed in several experiments [6, 7]. The conversion efficiency of generated SF pulses may be optimised [8, 9, 10, 11, 12] by operating in the soliton regime [13, 14]. In fact, the temporal collision of two short soliton pulses in a quadratic nonlinear crystal may efficiently generate a short, time-compressed SF pulse [8]. However this SF pulse is unstable: its energy decays back into the two incident pulses after a relatively short distance.

In this Paper we consider the parametric SF conversion from the mixing of an ultra-short signal pulse with a quasi-continuous wave (CW) or background pump, in the presence of GVM. Quite surprisingly we find that parametric mixing of these waves may lead to highly efficient generation of stable and ultra-short idler pulses. Indeed, we predict that the interaction of an ultra-short signal with a CW pump may generate a stable three-wave resonant interaction simulton (TWRIS) [15, 16], consisting of a locked bright-bright-dark triplet (signal-idler-pump) that propagates with a single nonlinear velocity [17, 18]. We analytically show that the intensity, time duration and group velocity of the generated idler pulse may be controlled in a stable manner [19] by means of simply adjusting the intensity level of the CW background. Although we shall restrict our attention in this work to a travelling-wave interaction geometry, we may anticipate that our results will have important ramifications in the optimization of the efficiency of ultrashort pulse optical parametric oscillators [20, 21].

2. Three-wave-interaction equations

The equations describing the quadratic resonant interaction of three waves in a nonlinear medium read as:

$$
\begin{aligned}
\frac{\partial A_1}{\partial \xi} + \delta_1 \frac{\partial A_1}{\partial \tau} &= iA_2^* A_3, \\
\frac{\partial A_2}{\partial \xi} + \delta_2 \frac{\partial A_2}{\partial \tau} &= iA_1^* A_3, \\
\frac{\partial A_3}{\partial \xi} + \delta_3 \frac{\partial A_3}{\partial \tau} &= iA_1 A_2,
\end{aligned}
\tag{1}
$$

with

$$A_j = \pi \chi^{(2)} \sqrt{\frac{n_j \omega_1 \omega_2 \omega_3}{n_1 n_2 n_3 \omega_j}} \, E_j. \tag{2}$$

Here $\tau = t/t_0$, t_0 is an arbitrary time parameter; $\xi = z/z_0$, z_0 is an unit space-propagation parameter. E_j are the slowly varying electric field envelopes of the waves at frequencies ω_j, n_j are the refractive indexes, $\chi^{(2)}$ is the quadratic nonlinear susceptibility, $\delta_j = z_0/(v_j t_0)$ with v_j the linear group velocities, and $j = 1,2,3$. We assume that the group velocity v_3 of the wave with the highest frequency ($\omega_3 = \omega_1 + \omega_2$) lies between the group velocities of the other waves, i.e. $v_1 > v_3 > v_2$. With no loss of generality, we shall write the Eqs. (1) in a coordinate system such that $\delta_1 = 0$, which implies $0 < \delta_3 < \delta_2$. Eqs. (1) exhibit the conserved quantities

$$U_{13} = U_1 + U_3 = \frac{1}{2} \int_{-\infty}^{+\infty} (|A_1|^2 + |A_3|^2) d\tau, \tag{3}$$

$$U_{23} = U_2 + U_3 = \frac{1}{2} \int_{-\infty}^{+\infty} (|A_2|^2 + |A_3|^2) d\tau, \tag{4}$$

$$U = U_1 + U_2 + 2U_3 = \frac{1}{2} \int_{-\infty}^{+\infty} (|A_1|^2 + |A_2|^2 + 2|A_3|^2) d\tau. \tag{5}$$

where U_1, U_2 and $2U_3$ represent the energies at the frequencies ω_1, ω_2 and ω_3.

3. Soliton-based parametric sum-frequency conversion

Figure 1 illustrates a typical example of the efficient SF parametric interaction of two short optical pulses in the soliton regime [8].
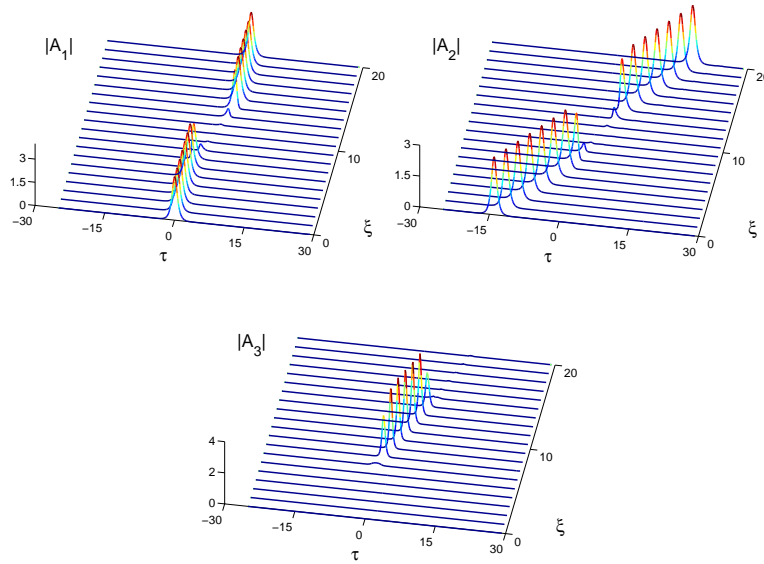

Fig. 1. Sum-frequency parametric interaction of two short optical signals at ω_1 and ω_2. The characteristic delays are $\delta_1 = 0, \delta_2 = 2, \delta_3 = 1$.

At the crystal input, two isolated pulses A_1 and A_2 with frequencies ω_1 and ω_2 propagate with speeds v_1 and v_2. Whenever the faster pulse overtakes the slower one, an idler pulse A_3
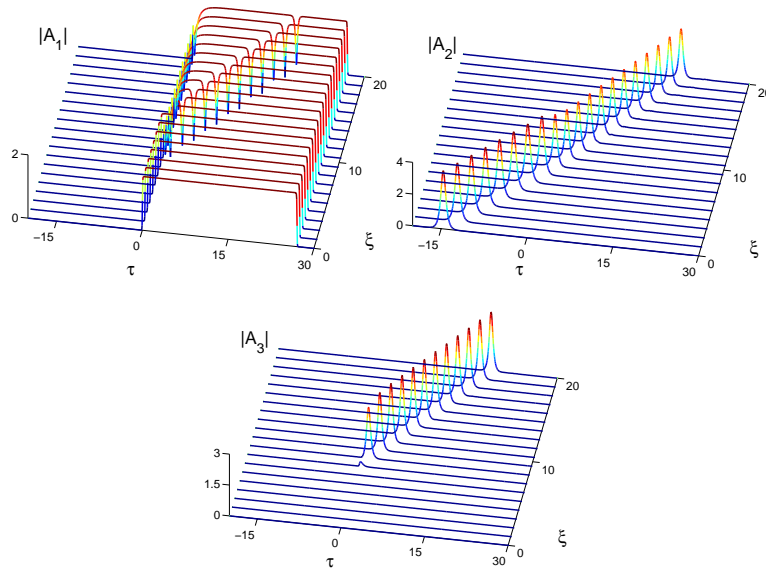
Fig. 2. Sum-frequency parametric interaction of a short pulse at ω_2 and a quasi-CW control at ω_1. The characteristic delays are $\delta_1 = 0, \delta_2 = 2, \delta_3 = 1$.

at the SF $\omega_1 + \omega_2$ is generated and propagates with the linear speed v_3. Depending on the time widths and intensities of the input pulses, the duration of the SF pulse is reduced with respect to the input pulse widths. Correspondingly, the SF pulse peak intensity grows larger than the input pulse intensities. Figure 1 shows that, eventually, the SF idler pulse decays back into the two original isolated pulses at frequencies ω_1 and ω_2. Note that the shapes, intensities and widths of the input pulses are left unchanged in spite of their interaction. As shown in Ref. [8], the above discussed SF pulse generation process may be analytically described in terms of soliton solutions of Eqs. (1) [13, 14]. The decay of the SF pulse which is shown in Fig. 1 may be a significant drawback in practical applications, since it implies that a given nonlinear crystal length yields efficient conversion for a limited range of input pulse intensities and time widths only.

Here we demonstrate that the parametric sum-frequency conversion of an ultra-short signal and a quasi-CW background pump-control may be exploited as a means to reduce or even eliminate the decay of the generated idler wave. In the presence of GVM, the parametric SF conversion of an ultra-short optical signal and a quasi-CW pump typically leads to the generation of a low-intensity and relatively long idler pulse, whose duration is associated with the interaction distance in the crystal. This scenario changes dramatically in the soliton regime. Figure 2 illustrates the efficient generation of a stable, ultra-short SF idler pulse from the parametric SF conversion of a properly prepared ultra-short signal and an arbitrary intensity level CW background control.

In Fig. 2 we injected in the quadratic nonlinear crystal the short signal at frequency ω_2, along with a delayed and relatively long pump-control pulse at frequency ω_1. Initially, the two pulses propagate uncoupled; as soon as the faster pulse starts to overlap in time with the slower quasi-CW control, their nonlinear mixing generates a short SF idler pulse. The sum-frequency process displayed in Fig. 2 can be analytically explained and explored in terms of stable TWRIS solutions [18]. In the notation of Eqs. (1), the TWRIS solution reads as

$$A_1 = \{1 + \frac{2pb^*}{|b|^2 + a^2}[1 - \tanh[B(-\tau + \delta\xi)]]\} \frac{ia\,g_1 \exp(iq_1\tau_1)}{g(\delta_2 - \delta_3)}$$

$$A_2 = \frac{2pa}{\sqrt{|b|^2 + a^2}} \frac{g_2}{g(\delta_2 - \delta_3)} \frac{\exp[i(q_2\tau_2 + \chi\tau + \omega\xi)]}{\cosh[B(-\tau + \delta\xi)]},$$

$$A_3 = \frac{-2pb^*}{\sqrt{|b|^2 + a^2}} \frac{g_3}{g(\delta_2 - \delta_3)} \frac{\exp[-i(q_3\tau_3 - \chi\tau - \omega\xi)]}{\cosh[B(-\tau + \delta\xi)]}, \tag{6}$$

where

$$b = (Q-1)(p + ik/Q), \quad r = p^2 - k^2 - |a|^2,$$

$$Q = \frac{1}{p}\sqrt{\frac{1}{2}[r + \sqrt{r^2 + 4k^2p^2}]},$$

$$B = p[\delta_2 + \delta_3 - Q(\delta_2 - \delta_3)]/(\delta_2 - \delta_3),$$

$$\delta = 2\delta_2\delta_3/[\delta_2 + \delta_3 - Q(\delta_2 - \delta_3)],$$

$$\chi = k[\delta_2 + \delta_3 - (\delta_2 - \delta_3)/Q]/(\delta_2 - \delta_3),$$

$$\omega = -2k\delta_2\delta_3/(\delta_2 - \delta_3), \tau_n = -\tau + \delta_n\xi$$

$$q_n = q(\delta_{n+1} - \delta_{n+2}), g_n = |(\delta_n - \delta_{n+1})(\delta_n - \delta_{n+2})|^{-1/2}$$

$$g = g_1 g_2 g_3, \quad n = 1,2,3 \ mod\,(3). \tag{7}$$

For a given choice of the characteristic linear group velocities, we are left with the four real independent parameters p, a, k, q. The parameter p is associated with the re-scaling of the wave amplitudes, and of the coordinates τ and ξ. Whereas a measures the amplitude of the CW background in wave A_1 (namely $a\sqrt{\delta_2\delta_3}$). The value of k is related to the soliton wavenumber. The parameter q simply adds a phase shift which is linear in both τ and ξ (see [18] for parameter details).

At the input, the properly prepared short pulse at frequency ω_2 and with a speed v_2 is a stable single component TWRIS (6) with parameters $p > 0, k, q, a = 0$. The background control in the interaction region can be modeled with $A_1(\tau) = Ce^{-i\gamma\tau}$. When this faster pulse, pre-delayed with respect to the slower quasi-CW pump at frequency ω_1, overtakes the background (at $\tau = 0$, in Fig. 2), their collision leads to the generation of a short idler pulse at the SF ω_3. Additionally, a dip appears in the quasi CW-control; whereas the intensity, duration and propagation speed of the input wave at frequency ω_2 are modified. Indeed, the signal-pump interaction generates a new stable TWRIS (6), with parameters $\overline{p}, \overline{k}, \overline{q}, \overline{a}$, moving with the locked nonlinear velocity $\overline{v} = z_0/(t_0\overline{\delta})$, where $\overline{\delta}$ is given in (7). It is remarkable that we may analytically predict the parameters $\overline{p}, \overline{k}, \overline{q}, \overline{a}$ of the generated TWRIS from the corresponding parameters of the input single wave TWRIS and the complex amplitude of the pump control. This can be achieved by supposing that the input TWRIS adiabatically (i.e., without emission of radiation) reshapes into a new TWRI simulton after its collision with the quasi-CW pump at a given point in time (say, at $\tau = 0$). Under this basic hypothesis, the conservative nature of the three-wave interaction permits us to suppose that: i) the energy U_{23} (4) of the input TWRI soliton is conserved in the generated TWRI simulton; ii) the phase of the ω_2 frequency components of the input TWRI soliton and of the generated TWRI simulton is continuous across their time interface (i.e., at $\tau = 0$); iii) the amplitude and phase of the control pump coincide with the corresponding values of the asymptotic plateau of the generated TWRI simulton component at frequency ω_1. By imposing the above three conditions, after some straightforward calculations we obtain the following relations that relate the parameters of the incident and of the transmitted TWRIS

$$\overline{p} = p, \ \ \overline{a} = |C|/\sqrt{\delta_2\delta_3}, \ \ \overline{q} = \gamma, \ \ \overline{k} = k + (\overline{q} - q)(\delta_2 - \delta_3)/2. \tag{8}$$

As an example, in Fig. 2 the input TWRI soliton at frequency ω_2 is described by Eqs. (6) with $p = 1.3, k = 0, q = 0, a = 0$, and the background control amplitude with $C = 1.7, \gamma = 0$. After the collision with the CW background, the above equations predict that the generated TWRIS is again described by Eqs. (6), with $\overline{p} = 1.3, \overline{k} = 0, \overline{q} = 0$, and $\overline{a} = 1.2$. The accuracy of this prediction is well confirmed by its comparison with the numerical solutions of the TWRI Eqs. (1). Indeed, Fig. 3 compares the numerical with the analytical evolutions (along the crystal length ξ) of the energy, the pulse duration and the velocity of the idler and signal pulses which correspond to the case shown in Fig. 2. We performed further extensive numerical simulations, which confirmed the general validity of the above described adiabatic transition model for TWRIS generation upon collision with a CW background.
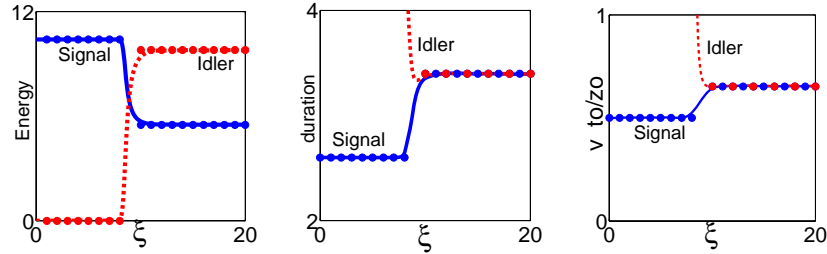

Fig. 3. Numerical evolution (lines) and theoretical predictions (circles) of energy, pulse duration and velocity of idler (red) and signal (blue) waves reported in Fig.2.

Indeed, by increasing or decreasing the CW background amplitude $|C|$ in the range $[0, p\sqrt{\delta_2\delta_3}]$, we observed that stable TWRISs with different velocity, duration and energy distributions may be adiabatically shaped. The important consequence of this result is that, by means of Eqs. (6)–(8), we may analytically predict and control the characteristics of the generated idler pulse (namely, its velocity, time duration and energy) simply as a function of the intensity level of the CW pump. Moreover, we would like to emphasize that the stability of the whole conversion process is ensured by the underlying stability of the generated TWRIS [19].

We would like to point out that the observation of the above described sum–frequency short pulse generation phenomena appears to be readily achievable in nonlinear optical experiments. For instance, let us consider the *eee* interaction of three-waves with carrier wavelengths of $\lambda_1 = 1.55\mu m, \lambda_2 = 3.4\mu m, \lambda_3 = 1.064\mu m$ in a $2cm$ ($8cm$) long periodically poled bulk Lithium Niobate crystal with $28\mu m$ periodicity. In this case, the parametric mixing of a $100fs$ ($1ps$) incident pulse with a quasi–CW (say, with a $3ps$ ($30ps$) time duration) control pulse leads to the generation of an ultrashort sum-frequency pulse of approximately the same time width of the incident short pulse whenever the field intensities of the two input pulses are of the order of a few hundreds of MW/cm^2 (or a few MW/cm^2, respectively).

4. Conclusions

In conclusion, we demonstrated the parametric SF conversion of an ultra-short pulse from the mixing of an ultra-short optical pulse with a quasi-continuous wave control in quadratic nonlinear crystals in the presence of GVM. We analytically showed that the intensity, time duration and group velocity of the generated pulses may be controlled in a stable manner by simply adjusting the intensity level of the background pump.

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