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Mohr Circle-based Graphical Vibration Analysis and Earthquake Response of Asymmetric Systems

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Abstract

The maximum seismic response of torsionally coupled plan asymmetric structures can be rationally visualized and computed through a Mohr Circle Response Spectrum Analysis (MCRSA). This is done combining the graphic modal properties of the torsional dynamic equations of motion with the structural earthquake demand in terms of a displacement spectrum as a function of the modal eigenvalues $S_D(\omega^2)$. A compact representation of the modal properties and of the response envelope is built and visualized in the Mohr plane. The maximum modal responses are then combined using a graphic adaptation of the SRSS and CCQ combination rules based on the elastic response spectrum. This Graphic Dynamic rule proves to be an effective response prediction tool, and is particularly suited to estimate the response of seismic base isolation systems.

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Keywords: Graphic Dynamics; Asymmetric structures; Torsion; Mohr Circle; Earthquake Response Spectrum.

1. Introduction

In-plan irregularity and torsional coupling of structural systems are aspects of primary importance in seismic structural design and assessment. Several seismic codes predict design rules to incorporate torsional behavior that can be due to inherent eccentricity as well as accidental eccentricity, which covers different sources of asymmetric behavior. Torsional effects can often result in unbalanced demand on structural components leading to collapse or poor earthquake performance. This paper presents a graphic computation approach, the Mohr Circle Response Spectrum Analysis (MCRSA), for plan-asymmetric systems based on some new properties of the Mohr Circle. The method uses graphic dynamic properties and the Mohr Circle Modal Analysis (MCMA) to compute the modal system properties, [1, 2], and the Graphical Response Spectrum Analysis (GRSA), [3, 4], to determine the maximum modal response and combinations. Rigid floor diaphragm behavior is assumed to represent the rotational kinematics of the displacements and accelerations through the modal centers of rotation, referred to as the modal rotational pivots, [5, 6, 7].

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The Mohr Circle approach provides useful insight into the dynamic behavior of torsionally coupled systems and a straightforward rule to predict the maximum dynamic torsional response of diaphragm systems. It can be particularly suited to predict the seismic response of base isolated systems, which have an inherent one-way only eccentricity and are characterized by rigid diaphragm behavior.

2. Mohr Circle- Modal Analysis (MCMA) and Response Spectrum Analysis (MCRSA)

Graphic Dynamic methods can be useful to highlight a number of geometric parameters governing the structural behavior and the free vibrations characteristics of torsionally coupled systems, [8 – 12], which can be identified on the Mass Circle of gyration and on the Ellipse of Elasticity, as it is shown in Figure 1a.

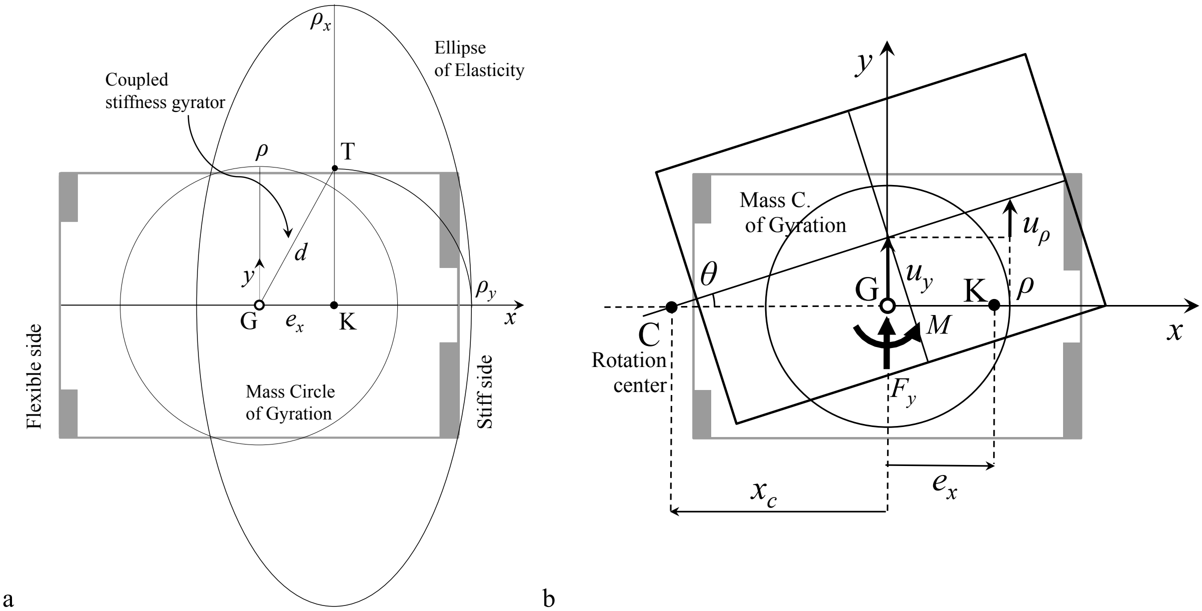

Fig. 1. (a) graphic dynamic properties of plan-eccentric torsional diaphragm systems; (b) rotational deformed shape about a center C, DOFs and equivalent translational DOFs

The first graphic indicator is the coupled stiffness gyrator d, which is the diagonal built on the eccentricity ex and on the semi-axis ρ_y of the Ellipse of Elasticity. The parameters relevant for the system of equations of dynamics are:

$$\rho_x = \sqrt{\frac{k_\theta}{k_x}},\quad \rho_y = \sqrt{\frac{k_\theta}{k_y}},\quad d = \sqrt{e_x^2 + \rho_y^2},\quad \rho = \sqrt{\frac{I_p}{m}},\quad \Omega_s = \frac{\rho_y}{\rho},\quad q = \frac{d}{\rho},\quad \varepsilon_x = \frac{e_x}{\rho} \tag{1}$$

The parameters ρ_y and Ω_s are commonly used to measure the torsionally stiff vs. torsionally flexible character of the system, while the related parameters d and q account for both the in-plan distribution and magnitude of the stiffness of the lateral load resisting elements and for the additional torsional coupling induced by the eccentricity. It was noted that d figures directly in the rotational diagonal term of the stiffness matrix associated with the degrees of freedom u_y and θ, [2, 8], and represents a coupled stiffness gyrator. The dynamic system's characteristic polynomial equation can be solved in the graphic/nodal form, i.e. in terms of the position $x_{c,1}$ and $x_{c,2}$ of the modal centers of rotation C_1 and C_2 (modal pivots), which fully define the mode shapes. A dimensionless set of equations governing the dynamic torsional problem can be obtained assuming as degrees of freedom the displacement u_y at the center of mass G, and the torsional displacement $u_\rho=\rho\theta$, [13], associated with the dynamic stiffness matrix $\mathbf{k}_\rho$. The system dynamics can be reduced to a classical eigenproblem governed by a geometric-dynamic stiffness matrix with some entries normalized by the radius of gyration and a unity mass matrix, based on the equations of free vibrations (2):

$$\mathbf{I}\ddot{\mathbf{u}}_{\rho}+\frac{1}{m}\mathbf{k}_{\rho}\mathbf{u}_{\rho}=\sum_{n=1}^{N}\left\{\begin{array}{c}-x_{c,n}\\ \rho\end{array}\right\}\ddot{\theta}_{n}(t)+\omega_{y}^{2}\left[\begin{array}{cc}1 & \varepsilon_{x}\\ \varepsilon_{x} & q^{2}\end{array}\right]\sum_{n=1}^{N}\left\{\begin{array}{c}-x_{c,n}\\ \rho\end{array}\right\}\theta_{n}(t)=\mathbf{0} \tag{2}$$

The graphic dynamic indicator consisting in the square $q^2 = (d/\rho)^2$ of the ratio of the coupled stiffness gyrator to the gyrator of mass, describes the translationality of the coupled system. It is important to note that this eigenproblem can be solved graphically through a Mohr Circle construction, which is illustrated in Figure 2a. The MCMA is drawn overlaying the dimensionless plan of the structure per unit-radius of gyration onto the Mohr plane so that the intersection of the mass circle of gyration with the vertical axis through G coincides with the pole P of the circle. Then the modal centers of rotation C_1 and C_2 are found as the intersection of the structure's x-axis with the straight lines passing through the circle's pole P and the normalized eigenvalues. The MCMA performs thus a fully graphic and single-step determination of the position of the modal nodes and of the modal frequencies.

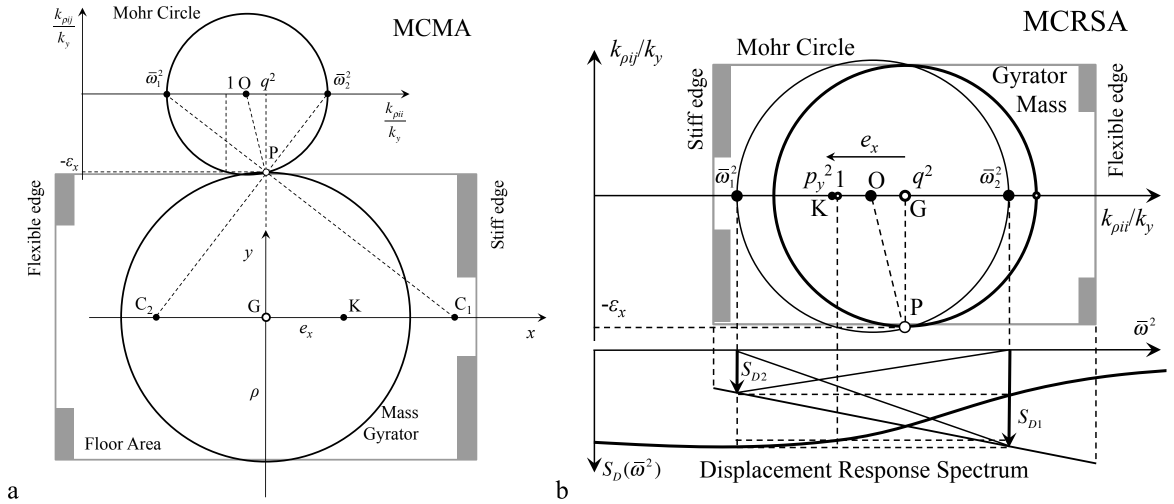

Fig. 2. (a) Mohr Circle Modal Analysis MCMA; (b) Mohr Circle Response Spectrum Analysis MCRSA based on eigenvalue Displacement Spectrum.

The response of a system to earthquake loads is generally computed building on the results from the modal analysis, and expressing the mass-proportional static earthquake forces in terms of their projections onto the modal force vectors, [14]. The intuitive graphic visualization of the modal expansion of the earthquake forces is the basis for the Graphic Response History Analysis (GRHA), and for the related Graphic Response Spectrum Analysis (GRSA) procedures, [3, 4]. These methods can be merged with the MCMA analysis, as it is illustrated in Figure 2b. A useful tool for determining the maximum response, and therefore more suited for drawing design considerations, is the GRSA procedure, which draws from the GRHA response construction adding the simple consideration that, as per definition of response spectrum, the maximum values of the nodal responses $D_n(t)$ are the ordinates of the displacement response spectrum. Therefore it is very straightforward to draw the maximum modal responses $S_D(T_n)$ at the nodes, based on the Displacement Response Spectrum function. By combining the MCMA and the GRHA, it is possible to come up with a comprehensive Mohr Circle based Response Spectrum Analysis (MCRSA). The main concept is to combine the traditional Mohr Circle determination of the modal eigenvalues (frequencies), with the MCMA determination of the modal pivots (mode shapes) and in addition with the seismic input information (response spectrum).

Based on the geometric correspondence drawn in Figure 2a, it is possible to demonstrate that the plan layout of the structure can be superimposed on the Mohr plane so that second diagonal term q^2 of the dimensionless dynamic stiffness matrix corresponds to the center of mass G of the structure, and that the gyration radius is scaled to the normalized eccentricity (off diagonal term). This is illustrated in Figure 2b, plotting the Displacement Spectrum as a function of the modal eigenvalues (squared frequencies). Finally the maximum modal demand can be read on this spectrum in correspondence of the Mohr eigenvalues. The modal deformed shapes can then be drawn accordingly. Finally the linear combination of the maximum absolute modal responses (ABSSUM rule) is obtained connecting the spectral responses at the nodes through straight lines. Moreover the Square Root of the Sums of the Squares (SRSS) rule or through the Complete Quadratic Combination (CQC) rule can be drawn graphically based on simple geometric constructions, [3, 4].

3. Application to earthquake response of base isolated system

The MCRSA with unidirectional earthquake loading is illustrated with an application to a base isolated structure in presence of accidental eccentricity orthogonal to the seismic action, therefore with torsionally coupled behaviour and modes, [15]. The structure has a squared plan layout (L/B = 1), uniformly distributed masses, damping equal to 10% of critical, and an accidental eccentricity e_x = 5%, as it is prescribed by several seismic codes.

The MCRSA is performed with respect to perturbed conditions by fixing the eccentricity and the total stiffness k_y, and changing only the in-plan distribution of stiffness, i.e. chaning the translationality or the torsional amplification factor Ω_s.

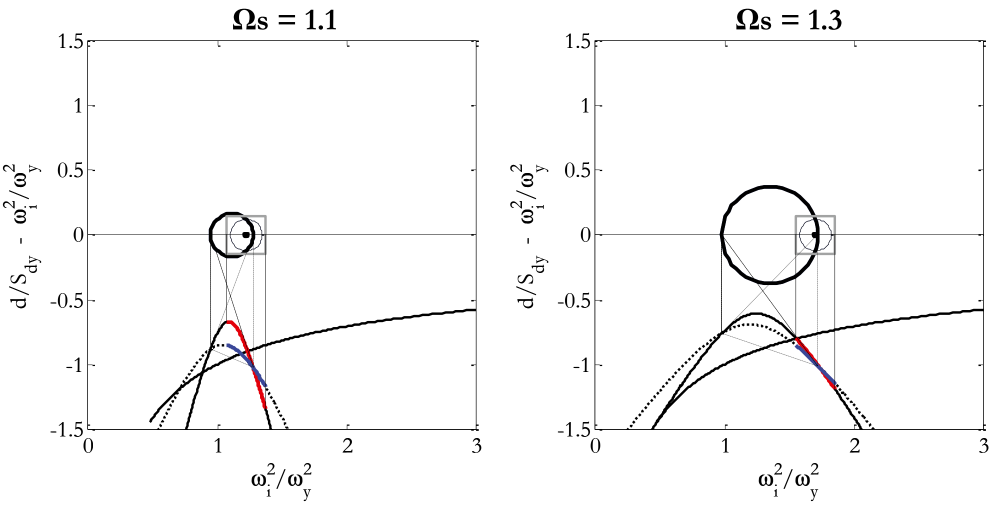

Fig. 3. application of MCRSA to perturbation analysis of a torsionally coupled base isolated structure due to code-based 5% accidental eccentricity and deformed shape obtained through SRSS (red line) and CQC (blue line) combination rules, for changing torsional amplification factor: (a) Ωs = 1.1, (b) Ωs =1.3.

The deformed shape in plan is represented through the final combinations via the SRSS and CQC rules. In Figure 3 it can be easily visualized that structures that are torsionally stiff ($\Omega_s > 1$) have a higher amplification on the flexible side, and that by increasing the torsional amplification factor the maximum displacement is reduced, since the translational behaviour becomes dominant. Also the differences between SRSS and CQC are reduced since the statistical correlation decreases as the modes become more uncoupled. Figure 4 depicts the torsionally stiff configurations ($\Omega_s < 1$) where the deformed shape becomes inverted, i.e. the maximum displacement is attained on the stiff side, since the rotational mode becomes predominant with respect to the more translational mode.

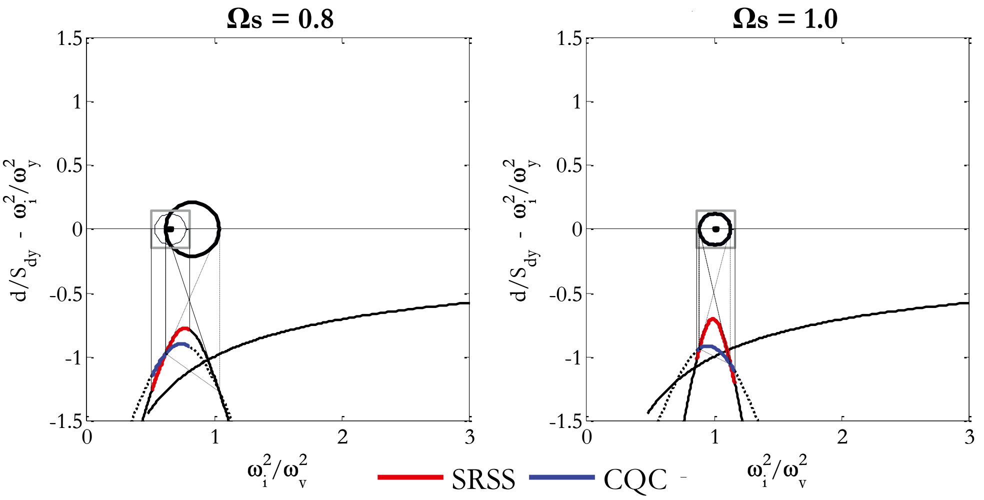

Fig. 4. application of MCRSA to perturbation analysis of a torsionally coupled base isolated structure due to code-based 5% accidental eccentricity and deformed shape obtained through SRSS (red line) and CQC (blue line) combination rules, for changing torsional amplification factor: (a) Ωs = 0.8, (b) Ωs = 1.0.

4. Conclusions

This paper presented a synthetic graphic method for computing the seismic displacement response of a single-story one-way eccentric rigid-diaphragm structure subjected to unidirectional earthquake forces orthogonal to the axis of symmetry, and an application of the method to a plan-asymmetric base isolated structure due to code accidental eccentricity of 5%. Based on the results of the Mohr Circle Modal Analysis MCMA and of the Graphic Response Spectrum Analysis GRSA, it is possible to define the Mohr Circle Response Spectrum Analysis MCRSA, which combines under a single graphic representation the MCMA, the GRSA, and the eigenvalue input Displacement Spectrum.

An application example of perturbation analysis of a base isolated structure shows that the MCRSA allows an effective determination of the modal properties of the eccentric torsional system, and to compute/visualize the maximum torsionally coupled seismic demand. In torsionally stiff configurations amplification is higher on the flexible side, and increasing the torsional amplification factor the maximum displacement is reduced. The differences between SRSS and CQC are reduced as the statistical correlation decreases due to torsional uncoupling. In torsionally stiff configurations the deformed shape is inverted, and displacement is maximum on the stiff side.

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