# Galactic cluster winds in presence of a dark energy 

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#### Abstract

We obtain a solution for the hydrodynamic outflow of the polytropic gas from the gravitating centre, in the presence of the uniform dark energy (DE). The antigravity of DE is enlightening the outflow and makes the outflow possible at smaller initial temperature, at the same density. The main property of the wind in the presence of DE is its unlimited acceleration after passing the critical point. In application of this solution to the winds from galaxy clusters, we suggest that collision of the strongly accelerated wind with another galaxy cluster, or with another galactic cluster wind, could lead to the formation of a highest energy cosmic rays.


Key words: galaxies: clusters: intracluster medium-dark energy.

## 1 INTRODUCTION

It was shown by Chernin $(2001,2008)$ that outer parts of galaxy clusters (GC) may be under strong influence of the dark energy (DE), discovered by observations of Type Ia supernova at redshift $z \leq 1$ (Riess et al. 1998; Perlmutter et al. 1999), and in the spectrum of fluctuations of the cosmic microwave background radiation (CMB; see e.g. Spergel et al. 2003; Tegmark et al. 2004). Equilibrium solutions for polytropic configurations in the presence of DE have been obtained in papers of Balaguera-Antolínez, Mota \& Nowakowski et al. $(2006,2007)$ and Merafina, Bisnovatyi-Kogan \& Tarasov (2012). The hot gas in the galactic clusters may flow outside due to high thermal pressure, and in the outer parts of the cluster the presence of a DE facilitates the outflow.
Here we obtain a solution of hydrodynamic equations for the winds from galactic clusters in the presence of DE. We generalize the solution for the outflows from the gravitating body, obtained for solar and stellar winds by Stanyukovich (1955) and Parker (1963), to the presence of DE. It implies significant changes in the structure of solutions describing galactic winds.

## 2 NEWTONIAN APPROXIMATION IN DESCRIPTION OF GALACTIC WINDS IN PRESENCE OF DE

A transition to the Newtonian limit, where DE is described by the antigravity force in vacuum, was done by Chernin (2008). In the Newtonian approximation, in the presence of DE, we have the fol-

[^0]lowing hydrodynamic Euler equation for the spherically symmetric outflow in the gravitational field of matter and DE:
\[

$$
\begin{align*}
\rho v \frac{\mathrm{~d} v}{\mathrm{~d} r}+\frac{\mathrm{d} P}{\mathrm{~d} r} & =-\rho\left(\frac{G m_{m}}{r^{2}}-\frac{\Lambda c^{2} r}{3}\right) \\
& =-\rho\left(\frac{G m_{m}}{r^{2}}-\frac{8 \pi G \rho_{\Lambda} r}{3}\right) \tag{1}
\end{align*}
$$
\]

Here, $\rho$ and $P$ are a matter density and pressure, respectively, and $m_{\mathrm{m}}$ is the mass of the matter inside the radius $r$. We use here DE in the form of the Einstein cosmological constant $\Lambda$. Newtonian gravitational potentials produced by matter $\Phi_{\mathrm{g}}$, and $\Phi_{\Lambda}$ by DE , satisfy the Poisson equations

$$
\begin{equation*}
\Delta \Phi_{\Lambda}=-8 \pi G \rho_{\Lambda}, \quad \Delta \Phi_{g}=4 \pi G \rho, \quad \rho_{\Lambda}=\frac{\Lambda c^{2}}{8 \pi G} \tag{2}
\end{equation*}
$$

We consider, for simplicity, the outflow in the field of a constant mass (like in stellar wind) $m_{\mathrm{m}}=M$. Equation (1) in this case is written as

$$
\begin{align*}
\rho v \frac{\mathrm{~d} v}{\mathrm{~d} r}+\frac{\mathrm{d} P}{\mathrm{~d} r} & =-\rho\left(\frac{G M}{r^{2}}-\frac{\Lambda c^{2} r}{3}\right) \\
& =-\rho\left(\frac{G M}{r^{2}}-\frac{8 \pi G \rho_{\Lambda} r}{3}\right) \tag{3}
\end{align*}
$$

Equation (1) should be solved together with the continuity equation in the form
$4 \pi \rho v r^{2}=\dot{M}$,
where $\dot{M}$ is the constant mass flux from the cluster. We consider polytropic equation of state, where pressure $P$ and sound speed $c_{\mathrm{s}}$
are defined as
$P=K \rho^{\gamma}, \quad c_{\mathrm{s}}^{2}=\gamma \frac{P}{\rho}, \quad \rho=\left(\frac{c_{\mathrm{s}}^{2}}{\gamma K}\right)^{\frac{1}{\gamma-1}}$,
$P=\left(\frac{c_{\mathrm{s}}^{2}}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} K^{-\frac{1}{\gamma-1}}$.
Introduce non-dimensional variables as
$\tilde{v}=\frac{v}{v_{*}} \quad \tilde{c}_{\mathrm{s}}=\frac{c_{\mathrm{s}}}{c_{*}}, \quad \tilde{r}=\frac{r}{r_{*}}, \quad r_{*}=\frac{G M}{c_{*}^{2}}, \quad v_{*}=c_{*}$,
$\tilde{\rho}=\frac{\rho}{\rho_{*}}, \quad \tilde{P}=\frac{P}{P_{*}}, \quad \rho_{*}=\left(\frac{c_{*}^{2}}{\gamma K}\right)^{\frac{1}{\gamma-1}}$,
$P_{*}=\left(\frac{c_{*}^{2}}{\gamma}\right)^{\frac{\nu}{\nu-1}} K^{-\frac{1}{\gamma-1}}$.
In non-dimensional variables equation (3) is written as
$\tilde{v} \frac{\mathrm{~d} \tilde{v}}{\mathrm{~d} r}+\frac{2}{\gamma-1} \tilde{c}_{\mathrm{s}} \frac{\mathrm{d} \tilde{c}_{\mathrm{s}}}{\mathrm{d} \tilde{r}}+\frac{1}{\tilde{r}^{2}}-\lambda \tilde{r}=0, \quad \lambda=\frac{\Lambda c^{2} r_{*}^{2}}{3 c_{*}^{2}}$.
The continuity equation (4) in non-dimensional form is written as
$\tilde{\rho} \tilde{v} \tilde{r}^{2}=\dot{m}, \quad \tilde{c}_{\mathrm{s}}^{\frac{2}{\nu-1}} \tilde{v} \tilde{r}^{2}=\dot{m}, \quad \dot{m}=\frac{M}{\dot{M}_{*}}$,
$\dot{M}_{*}=4 \pi \rho_{*} v_{*} r_{*}^{2}$.
It follows from (5), (6) and (8) that
$\frac{d \tilde{\rho}}{\tilde{\rho}}=\frac{2}{\gamma-1} \frac{d \tilde{c}_{\mathrm{s}}}{\tilde{c}_{\mathrm{s}}}, \quad \frac{d \tilde{\rho}}{\tilde{\rho}}+\frac{d \tilde{v}}{\tilde{v}}+2 \frac{d \tilde{r}}{\tilde{r}}=0$.
Using (9) we may write the equation of motion (7) in the form
$\frac{\mathrm{d} \tilde{v}}{\mathrm{~d} \tilde{r}}=\frac{\tilde{v}}{\tilde{r}} \frac{2 \tilde{c}_{\mathrm{s}}^{2}-\frac{1}{\tilde{r}}+\lambda \tilde{r}^{2}}{\tilde{v}^{2}-\tilde{c}_{\mathrm{s}}^{2}}$.
The only physically relevant solutions are those which pass smoothly the sonic point $v=c_{\mathrm{s}}$, being a singular point of equation (10), with
$\tilde{v}=\tilde{c}_{\mathrm{s}}, \quad 2 \tilde{c}_{\mathrm{s}}^{2}-\frac{1}{\tilde{r}}+\lambda \tilde{r}^{2}=0$,
where $\tilde{r}=\tilde{r}_{\mathrm{c}}, \tilde{v}=\tilde{v}_{\mathrm{c}}$ and $\tilde{c}_{\mathrm{s}}=\tilde{c}_{\mathrm{sc}}$. Choosing $c_{*}=c_{\mathrm{sc}}$, we obtain in the critical point
$\tilde{v}_{\mathrm{c}}=\tilde{c}_{\mathrm{sc}}=1, \quad 2-\frac{1}{\tilde{r}_{\mathrm{c}}}+\lambda \tilde{r}_{\mathrm{c}}^{2}=0$.
With this choice of the scaling parameters, we have from (8)
$\dot{m}=\tilde{r}_{\mathrm{c}}^{2}$.
The physical meaning of the parameter $\lambda$ becomes clear after rewriting it, using (2) and (6), in the form
$\lambda=\frac{\Lambda c^{2} r_{*}^{2}}{3 c_{*}^{2}}=\rho_{\Lambda} \frac{8 \pi}{3 M} r_{*}^{3}=\rho_{\Lambda} \frac{8 \pi}{3 M} \frac{r_{\mathrm{c}}^{3}}{\tilde{r}_{\mathrm{c}}^{3}}=\frac{2 \rho_{\Lambda}}{\rho_{M}} \frac{1}{\tilde{r}_{\mathrm{c}}^{3}}$,
where $\rho_{M}=\frac{3 M}{4 \pi r_{\mathrm{c}}^{3}}$ is a density of the matter after smearing the central mass uniformly inside the critical radius $r_{\mathrm{c}}$. The value of $\lambda$ is proportional to the ratio of the DE mass inside the critical radius $M_{\Lambda}=\frac{4 \pi}{3} r_{\mathrm{c}}^{3} \rho_{\Lambda}$ to the mass $M$ of the central body.

The relation (12) determining the dependence $\tilde{r}_{\mathrm{c}}(\lambda)$ in the solution for the galactic wind and accretion, in the presence of DE , is presented in Fig. 1. In the presence of DE the critical radius of the flow is situated closer to the gravitating centre (in non-dimensional


Figure 1. The functions $\tilde{r}_{\mathrm{c}}(\lambda)$ (full curve), according to (12), and $\tilde{r}_{\max }(\lambda)$ (dashed curve) according to (23). It is clear that the critical radius of the flow $r_{\mathrm{c}}$ is always inside the radius of the extremum of the total gravitational potential $r_{\text {max }}$.


Figure 2. The function $h(\lambda)$ for $\gamma=\frac{4}{3}$ (full curve); $\gamma=\frac{3}{2}$ (dashed curve); $\gamma=\frac{5}{3}$ (dash-dot-dot curve), according to relations (12) and (16).
units) with increasing $\lambda$. Equation (7) for the polytropic flow has a Bernoulli integral as
$\frac{\tilde{v}^{2}}{2}+\frac{\tilde{c}_{\mathrm{s}}^{2}}{\gamma-1}-\frac{1}{\tilde{r}}-\frac{\lambda \tilde{r}^{2}}{2}=h$,
$\tilde{c}_{\mathrm{s}}^{2}=\left(\frac{\dot{m}}{\tilde{v} \tilde{r}^{2}}\right)^{\gamma-1}=\left(\frac{\tilde{r}_{\mathrm{c}}^{2}}{\tilde{v} \tilde{r}^{2}}\right)^{\gamma-1}$.
The dimensional Bernoulli integral $H=h c_{\mathrm{sc}}^{2}$. The Bernoulli integral is determined through the parameters of the critical point, with account of (12), as
$h=\frac{\gamma+1}{2(\gamma-1)}-\frac{1}{\tilde{r}_{\mathrm{c}}}-\frac{\lambda \tilde{r}_{\mathrm{c}}^{2}}{2}=\frac{5-3 \gamma}{2(\gamma-1)}-\frac{3}{2}\left(\frac{1}{\tilde{r}_{\mathrm{c}}}-2\right)$.
The dependence $h(\lambda)$ for different polytropic powers $\gamma$ is given in Fig. 2. Note that in the presence of DE the outflow is possible also for negative values of the Bernoulli integral $h$, defined equally. The stationary solution for the wind is determined by two integrals:
constant mass flux $\dot{M}$ and energy (Bernoulli) integral $H$. In the absence of DE, we obtain the known relations
$\tilde{r}_{\mathrm{c}}=\frac{1}{2}, \quad h=\frac{5-3 \gamma}{2(\gamma-1)}$.
At small $\lambda$ we have from (12) and (16)
$\tilde{r}_{\mathrm{c}}=0.5-\frac{\lambda}{16}, \quad h=\frac{5-3 \gamma}{2(\gamma-1)}-\frac{3}{8} \lambda$.
At large $\lambda \rightarrow \infty$ it follows from (12) $\tilde{r}_{\mathrm{c}} \rightarrow \tilde{r}_{c \infty}=\lambda^{-1 / 3}$. Making expansion in (12) around $\tilde{r}_{c \infty}$ in the form
$\frac{1}{\tilde{r}_{\mathrm{c}}}=\lambda^{1 / 3}+\varepsilon$,
we obtain from (12) and (16)

$$
\begin{align*}
\varepsilon & =\frac{2}{3}, \quad \tilde{r}_{\mathrm{c}}=\frac{1}{\lambda^{1 / 3}+\frac{2}{3}}, \quad h=\frac{5-3 \gamma}{2(\gamma-1)}-\frac{3}{2} \lambda^{1 / 3}+2 \\
& =\frac{\gamma+1}{2(\gamma-1)}-\frac{3}{2} \lambda^{1 / 3} \quad \text { at } \quad \lambda \rightarrow \infty . \tag{19}
\end{align*}
$$

In the outflow from the physically relevant quasi-stationary object, the antigravity force from DE should be less than the gravitational force from the matter on the outer boundary, which we define at $r=r_{*}$. Therefore, the value of $\Lambda$ is restricted by the relation (see e.g. Bisnovatyi-Kogan \& Chernin 2012)
$2 \rho_{\Lambda}=\frac{\Lambda c^{2}}{4 \pi G}<\bar{\rho}=\frac{4 \pi M}{3 r_{*}^{3}}$.
In non-dimensional variables this restriction, with account of (6), (7) is written as
$\lambda<\frac{16 \pi^{2}}{9}=17.55=\lambda_{\lim }$.
It is reasonable to consider only the values of $\lambda$ smaller than $\lambda_{\lim }$. It follows from (12) that $\tilde{r}_{\mathrm{c}}$ is monotonically decreasing with increasing $\lambda$. For $\lambda=\lambda_{\lim }=17.55$ we obtain $\tilde{r}_{\mathrm{c}}=\tilde{r}_{\mathrm{c}, \lim } \approx 0.29$. The effective gravitational potential $\tilde{\Phi}$ is formed by the gravity of the central body, and antigravity of DE
$\tilde{\Phi}=-\frac{1}{\tilde{r}}-\frac{\lambda \tilde{r}^{2}}{2}$.
To overcome the gravity of the central body, the value of $h$ should exceed the maximum value of the gravitational potential, defined by the extremum of $\tilde{\Phi}$
$h \geq \tilde{\Phi}_{\max }\left(\tilde{r}_{\max }\right)=-\frac{3}{2} \lambda^{1 / 3}, \quad \tilde{r}_{\max }=\lambda^{-1 / 3}$.
The function $\tilde{r}_{\max }(\lambda)$ is represented in Fig. 1, it is always $\tilde{r}_{\max }>\tilde{r}_{\mathrm{c}}$. Therefore, in the presence of DE the outflow of the gas from the cluster to the infinity is possible even at the negative values of $h$. In the absence of DE the non-negative value of $h$ and the outflow are possible only at $\gamma \leq \frac{5}{3}$.

## 3 SOLUTIONS OF THE GALACTIC WIND EQUATION IN THE PRESENCE OF DE

### 3.1 Analytical solutions

Analytical solutions exist only in the absence of $\mathrm{DE}, \Lambda=0$. In this case we have from (15)-(17)
$\tilde{r}_{\mathrm{c}}=\frac{1}{2}, \quad h=\frac{\tilde{v}^{2}}{2}+\frac{\tilde{c}_{\mathrm{s}}^{2}}{\gamma-1}-\frac{1}{\tilde{r}}=\frac{5-3 \gamma}{2(\gamma-1)}$.

For $\gamma=\frac{5}{3}$ the constant $h=0$ for the critical solution. At $h=0$ and $\gamma=\frac{5}{3}$ there is a whole family of solutions with arbitrary constant Mach number $\mathrm{Ma}=\frac{v}{c_{\mathrm{s}}}$ in all space. It is more convenient to write this solution in dimensional variables, with Bernoulli integral
$H=\frac{v^{2}}{2}+\frac{3 c_{\mathrm{s}}^{2}}{2}-\frac{G M}{r}=0$.
There is an exact solution in the form of
$v^{2}=\frac{2 G M}{r} \frac{\mathrm{Ma}^{2}}{3+\mathrm{Ma}^{2}}, \quad c_{\mathrm{s}}^{2}=\frac{2 G M}{r} \frac{1}{3+\mathrm{Ma}^{2}}$.
At $\mathrm{Ma}=1$ this solution corresponds to the critical solutions for $\gamma=$ $\frac{5}{3}$. The relations (26) describe both wind and accretion solutions, which have different signs of the velocity, positive for the wind and negative for the accretion.
Another analytic solution takes place at $\gamma=1.5$, with the nondimensional Bernoulli integral $h=0.5$. Using from (15), for $\gamma=1.5$, $r_{\mathrm{c}}=0.5$,
$\tilde{c}_{\mathrm{s}}^{2}=\left(\frac{\tilde{r}_{\mathrm{c}}^{2}}{\tilde{\tilde{v} \tilde{r}^{2}}}\right)^{\gamma-1}=\sqrt{\frac{1}{4 \tilde{v} \tilde{r}^{2}}}$,
we obtain
$h=\frac{\tilde{v}^{2}}{2}+\frac{1}{\tilde{r} \sqrt{\tilde{v}}}-\frac{1}{\tilde{r}}=\frac{1}{2}$.
This non-linear equation has two solutions
$\begin{array}{ll}\text { 1. } \tilde{v}=1 ; & \text { 2. } \tilde{r}=\frac{2}{\sqrt{\tilde{v}}(1+\sqrt{\tilde{v}})(1+\tilde{v})} .\end{array}$
Therefore, the first solution corresponds to the wind and the second one corresponds to the accretion case, with negative velocity; $\tilde{v}$ in (27) represents the absolute value of this variable.

### 3.2 Numerical solution

Equation (10) has a sound critical point of the saddle type, and two physical (critical) solutions going through this critical point. One of this solutions describes a wind outflow and has a positive $\tilde{v}$. Another solution corresponds to an accretion (inflow) and has a negative $\tilde{v}$ (Stanyukovich 1955; Parker 1963). To obtain a physically relevant critical solution of (10), with $\tilde{c}_{\mathrm{s}}^{2}$ from (15), we obtain expansion in the critical point with $\tilde{v}^{2}=\tilde{c}_{\mathrm{s}}^{2}=1$ in the form

$$
\begin{align*}
\tilde{v}= & 1+\alpha\left(\tilde{r}-\tilde{r}_{\mathrm{c}}\right), \quad \alpha_{1}=-\frac{2}{\tilde{r}_{\mathrm{c}}} \frac{\gamma-1}{\gamma+1}, \\
& +\frac{1}{\tilde{r}_{\mathrm{c}}} \frac{2}{\gamma+1} \sqrt{2+\frac{1}{4 \tilde{r}_{\mathrm{c}}}+\frac{\lambda \tilde{r}_{\mathrm{c}}^{2}}{2}-\gamma\left(2-\frac{1}{4 \tilde{r}_{\mathrm{c}}}-\frac{\lambda \tilde{r}_{\mathrm{c}}^{2}}{2}\right)}, \\
\alpha_{2}= & -\frac{2}{\tilde{r}_{\mathrm{c}}} \frac{\gamma-1}{\gamma+1} \\
& -\frac{1}{\tilde{r}_{\mathrm{c}}} \frac{2}{\gamma+1} \sqrt{2+\frac{1}{4 \tilde{r}_{\mathrm{c}}}+\frac{\lambda \tilde{r}_{\mathrm{c}}^{2}}{2}-\gamma\left(2-\frac{1}{4 \tilde{r}_{\mathrm{c}}}-\frac{\lambda \tilde{r}_{\mathrm{c}}^{2}}{2}\right)} . \tag{28}
\end{align*}
$$

Here, $\alpha_{1}$ corresponds to the wind solution and $\alpha_{2}$ is related to the case of accretion, where $\tilde{v}$ defines the absolute value. At $\lambda=0$ we have a well-known expansion with
$\alpha_{1}=\frac{4}{\gamma+1}\left[\sqrt{\frac{5-3 \gamma}{2}}-(\gamma-1)\right]$,
$\alpha_{2}=-\frac{4}{\gamma+1}\left[\sqrt{\frac{5-3 \gamma}{2}}+(\gamma-1)\right]$.


Figure 3. The integral curves of equations (10) and (15), for $\gamma=4 / 3$ and $\lambda=0, r_{\mathrm{c}}=0.5$ (dashed curves); $\lambda=1.10, r_{\mathrm{c}}=0.45$ (dash-dot-dot curves); and $\lambda=5.13, r_{\mathrm{c}}=0.37$ (full curves). Wind solutions correspond to curves with increasing velocity at large radius. The curves with decreasing velocities correspond to the accretion solution with negative $v$, so that its absolute value is presented.

It follows from expansion (28) that physically relevant solutions exist only with positive value under the square root. It gives the restriction for the value of $\gamma$ as a function of $\lambda$ in the form
$\gamma \leq \gamma_{\max }=\frac{2+\frac{1}{4 \tilde{r}_{\mathrm{c}}}+\frac{\lambda \vec{r}_{c}^{2}}{2}}{2-\frac{1}{4 \overrightarrow{r_{c}}}-\frac{\lambda \vec{r}_{c}^{2}}{2}}$.
At $\lambda=32, \tilde{r}_{\mathrm{c}}=0.25$, the limiting value $\gamma_{\text {max }}$ goes to $\infty$, so that at $\lambda \geq 32$ the wind solutions exist formally for all polytropic powers $\gamma$.

The numerical solution of (10) was obtained using predictorcorrector Runge-Kutta method of fourth order, with a fixed relative precision, written in Fortran 77, see Press et al. (1992) The integration started from the critical point with $\tilde{v}=\tilde{c}_{\mathrm{s}}=1$, using expansion (27), both inside and outside the critical point, for two types of the flow: the wind flow, corresponding to the coefficient $\alpha_{1}$ in (27), and accretion flow, corresponding to $\alpha_{2}$ in (27). The critical solutions of equation (10), with account of (15), are presented in Figs 3-5 for different values of $\gamma$ and $\lambda$. Both wind and accretion solutions are presented.

The wind and accretion solutions are plotted in the same Figs 35, but the positive velocities correspond only to the wind solutions. The outflow solutions have increasing velocities in the presence of DE with $\lambda>1$, but at $\lambda=0$ the behaviour at large radius $\tilde{r}$ depends on the adiabatic power $\gamma$. The velocity is increasing in the wind solution at $\gamma=\frac{4}{3}$ (Fig. 3). At $\gamma=\frac{3}{2}$ the wind solution has a constant outflow velocity, according to the analytic solution (26), see also Fig. 4. At $\gamma=\frac{5}{3}$ the wind solution has a decreasing outflow velocity with a constant Mach number, according to the analytic solution (25), see also Fig. 5.

The accretion solutions in Figs 3-5 are represented by the absolute values of the inflow velocity $|\tilde{v}|$, and the inflow velocity during accretion has a negative sign. The inflow velocity inside the critical point at $\gamma=\frac{4}{3}, \frac{3}{2}$ and all $\lambda$ converges to the same free-fall velocity $\tilde{v} \rightarrow-\sqrt{\frac{2}{\tilde{r}}}$, according to the Bernoulli integral (15), with $\tilde{r} \ll 1$, in the supersonic flow with $\tilde{v} \gg \tilde{c}_{\mathrm{s}}$. At $\gamma=\frac{5}{3}$ the inflow solution at $\tilde{r} \ll 1$ is approaching to the constant Mach number analytical solution (25). The inflow solutions given in Fig. 5 correspond to $\mathrm{Ma}=1$. Equation (10) is invariant to the transformation $\tilde{v} \rightarrow-\tilde{v}$;


Figure 4. The integral curves of equations (10) and (15), for $\gamma=3 / 2$ and $\lambda=0, r_{\mathrm{c}}=0.5$ (dashed curves); $\lambda=1.10, r_{\mathrm{c}}=0.45$ (dash-dot-dot curves); and $\lambda=5.13, r_{\mathrm{c}}=0.37$ (full curves). Wind solutions correspond to curves with increasing (or constant) velocity at large radius. The curves with decreasing velocities correspond to the accretion solution with negative $v$, so that its absolute value is presented.


Figure 5. The integral curves of equations (10) and (15), for $\gamma=5 / 3$ and $\lambda=0, r_{\mathrm{c}}=0.5$ (dashed curve); $\lambda=1.10, r_{\mathrm{c}}=0.45$ (dash-dotdot curves); and $\lambda=5.13, r_{\mathrm{c}}=0.37$ (full curves). For non-zero $\lambda$, wind solutions correspond to curves with increasing velocity at large radius. The curves with decreasing velocities correspond to the accretion solution with negative $v$, so that its absolute values are presented. At $\lambda=0$ both wind and accretion solutions are presented by the same curve, which corresponds to the wind for positive $v$ and to the accretion for negative $v$.
therefore, the accretion solution was possible to obtain numerically for the absolute values of the velocity.

The inflow solutions for the accretion starts at large radiuses by a slow motion to the gravitating centre. The velocity increases in a subsonic regime, and after crossing the critical point the supersonic infall to the gravitating centre starts. Note that the accretion solutions have a physical sense only for small $\lambda$, when the region with a attractive gravitational force is sufficiently large. In the regions with repulsing force due to DE antigravity, the critical accretion solutions of equation (10) formally exist, but they correspond to anomalous density distribution increasing with radius, what cannot be expected in reality.


Figure 6. The integral curves of equations (10) and (15) for the wind solution, at $\gamma=5 / 3$ and $\lambda=0, r_{\mathrm{c}}=0.5$ (dashed curve); $\lambda=1.10, r_{\mathrm{c}}=0.45$ (dash-dot-dot curve); and $\lambda=0.58, r_{\mathrm{c}}=0.47$ (full curve).

## 4 DISCUSSION

It is clear that the presence of DE tends to help the outflow of the hot gas from the gravitating object, as well as to the escape of rapidly moving galaxies (Chernin et al. 2013). Here we have obtained the solution for outflow in the presence of DE, which generalize the well-known solution for the polytropic solar (stellar) wind. Presently, the DE density exceeds the average density of the dark matter together with the barionic matter. The clusters which outer radius is approaching the zero gravity radius may not only loose galaxies, which join the process of Hubble expansion, but also may loose the hot gas from the outer parts of the cluster. Let us consider outer parts of the Coma cluster at radius $R_{\mathrm{C}}=$ 15 Mpc , with the mass inside $M_{\mathrm{C}}=5 \times 10^{15} \mathrm{M}_{\odot}$, from Chernin et al. (2013). For the present value of $\rho_{\Lambda}=0.71 \times 10^{-29} \mathrm{~g} \mathrm{~cm}^{-3}$, supposing that $R_{\mathrm{C}}=r_{*}$ is the critical radius of the wind, we obtain from (2) and (7) the non-dimensional constant $\lambda$ as
$\lambda=\frac{\Lambda c^{2} r_{*}^{2}}{3 c_{*}^{2}}=\frac{8 \pi}{3} \frac{\rho_{\Lambda} r_{*}^{3}}{M} \approx 0.59$,
$c_{*}=\sqrt{\frac{G M_{\mathrm{C}}}{R_{\mathrm{C}}}} \approx 1200 \mathrm{~km} / \mathrm{c}$.
It corresponds to the temperature about $T \approx 6 \times 10^{7} \mathrm{~K}, k T \approx 5 \mathrm{keV}$. Observations of the hot gas distribution in the Coma cluster (Watanabe et al. 1999) on ASCA satellite have shown the presence of hot region with $k T=11-14 \mathrm{keV}$, and more extended cool region with $k T=5 \pm 1 \mathrm{keV}$, what is in good accordance with our choice of parameters.

Wind solutions for $\lambda=0 ; 0.58 ; 1.1$ are presented in Fig. 6. The solution with $\lambda=0.58$ is the closest to the description of the outflow from Coma cluster. The density of the gas in the vicinity of $r=r_{\mathrm{c}}$ is very small, so the flow may be considered as adiabatic (polytropic) with the power $\gamma=5 / 3$. Without DE such gas flow is inefficient,
its velocity is decreasing $\sim 1 / \sqrt{r}$, see equation (25). In the presence of DE , the wind velocity is increasing two times at the distance of $\sim 5 r_{\mathrm{c}} \sim 75 \mathrm{Mpc}$ from Coma.

After quitting the cluster the gas is moving with acceleration, acting as a snowplough for the intergalactic gas. The shell of matter, forming in such a way, may reach a high velocity, exceeding considerably the speed of galaxies in cluster. If the shell meets another cluster, or another shell moving towards, the collision of such flows may induce a particle acceleration. Due to high speed, large sizes and low density such collisions may create cosmic rays of the highest possible energy (EHECR). We may expect the largest effect when two clusters move to each other. The influence of DE is decreasing with a redshift; therefore, the acceleration of EHECR in this model should take place in the periphery, or between the closest rich GCs.

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