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Encyclopedia of Continuum Mechanics

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| Editors: Holm Altenbach, Andreas Öchsner

Variational Methods in Continuum Damage and Fracture Mechanics

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Synonyms

[Variational approach to damage and fracture mechanics](#); [Variational formulation of damage and fracture mechanics](#)

Definitions

Damage is defined as the loss of material stiffness under loading conditions. This process is intrinsically irreversible and, therefore, dissipative. When the stiffness vanishes, fracture is achieved. In order to derive governing equations, variational methods have been employed. Standard variational methods for non-dissipative systems are here formulated in order to contemplate dissipative systems as the ones considered in continuum damage mechanics.

Principle of Least Action for Dissipative Systems

Variational principles and calculus of variations have always been important tools for formulating mathematical models of physical phenomena (dell’Isola and Placidi, [2011](#)). Indeed, they are the main tool for the axiomatization of physical theories because they provide an efficient and elegant way to formulate and solve mathematical problems which are of...

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