MODELS FOR REMODELING IN POROUS BONE RECONSTRUCTED TISSUES SATURATED WITH INTERSTITIAL FLUID

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Outline

Objectives

- Bone is a tissue containing a fluid phase, a solid matrix, and cells.
- In order to consider interactions between bone tissue and bio-resorbable material used for bone grafts, we consider a 2D sample made of a mixture composed of three phases, two of them constituted by a binary solid matrix of bone and bio-resorbable material and the third by a fluid that fills the connected pores of the solid matrix.
- the theory of porous materials saturated with fluid developed by Biot (1) can be employed for the mechanical behavior of such a mixture; we generalize this model with a fully non local term in the internal energy accounting for the compressibility of the fluid and its mass conservation.
- the evolution model proposed by the references (2) and (3) is used to describe biological phenomena associated to the remodeling processes

References I

(1) M.A. Biot (1941)
General theory of three-dimensional consolidation.

(2) A. Madeo, T. Lekszycki, F. dell'Isola (2011)
A continuum model for the bio-mechanical interactions between living tissue and bio-resorbable graft after bone reconstructive surgery.

(3) A. Madeo, D. George, T. Lekszycki, M. Nierenberger, and Y. Rémond (2012)
A second gradient continuum model accounting for some effects of micro-structure on reconstructed bone remodeling.

Model and Materials

2D Numerical simulations of the following solid mixture of bone tissue and bio-resorbable material saturated with interstitial fluid are performed

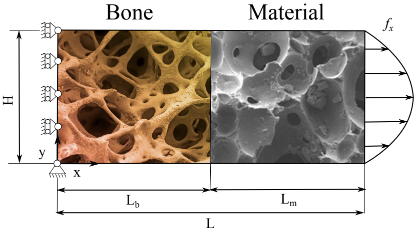


Figure : Initial configuration

The solid material under investigation is constituted by a mixture of bio-resorbable material (used in bone reconstructive surgery) on the right-hand side and of a living bone-tissue on the left-hand side, both porous materials are saturated with an interstitial fluid. In our model the total mass of such an interstitial fluid is assumed to be constant

Preliminary Assumptions I

Porous Materials saturated with interstitial fluid

1. we consider a mixture composed of three phases, a binary solid matrix constituted by bone and bio-resorbable material with connected pores which are filled with a fluid. Let ρ_b, ρ_m and ρ_f be the apparent mass densities in the mixture;
2. the mixture density is given by

$$\rho = \sum_{i=\mathrm{b,m,f}} \rho_i = \sum_{i=\mathrm{b,m,f}} \hat{\rho}_i \varsigma_i = \hat{\rho}_\mathrm{b} \varsigma_\mathrm{b} + \hat{\rho}_\mathrm{m} \varsigma_\mathrm{m} + \hat{\rho}_\mathrm{f} (1 - \varsigma_\mathrm{b} - \varsigma_\mathrm{m}) \quad (1)$$

in which ς_b and ς_m are the volume fraction respectively of bone and bio-material, while $\varsigma_\mathrm{f} = 1 - (\varsigma_\mathrm{b} + \varsigma_\mathrm{m})$ is the porosity; the saturation condition has been used stating that the pore-fluid fills the whole pore space, and $\hat{\rho}_i$ with $\{i = \mathrm{b,m,f}\}$ are the true mass densities of the mixture phases.

Modeling

Stored Energy Density

1. It is introduced a stored energy density ψ associated with the Green-Saint Venant strain tensor G, and with the change of porosity from the reference configuration ϑ:

$$\psi(G,\vartheta) = \frac{1}{2}\left[\frac{Y(\rho_{\mathrm{b}}^*,\rho_{\mathrm{m}}^*)\nu}{(1-2\nu)(1+\nu)} + \alpha^2 Q\right]\mathrm{tr}(G)^2 + \frac{1}{2}\frac{Y(\rho_{\mathrm{b}}^*,\rho_{\mathrm{m}}^*)}{(1+\nu)}\mathrm{tr}(G^2) + \\ + \frac{1}{2}Q\vartheta^2 - \alpha\, Q\, \vartheta\, \mathrm{tr}(G) + \beta M^2 \frac{\varsigma_f\,(1+\mathrm{tr}(G))}{\left[\int_{\mathcal{B}}\varsigma_f\,(1+\mathrm{tr}(G))\right]^2}$$

where $Y(\rho_{\mathrm{b}}^*,\rho_{\mathrm{m}}^*)$ and ν are the elastic modulus and Poisson's ratio, respectively; α and Q are Biot's parameters and then we have the last term.

The newly introduced energy term

Stored energy density ψ

$$\psi = \psi_{Biot} + \psi_{NL}$$

$$\psi_{NL} = \beta M^2 \frac{\varsigma_f \left(1 + \mathrm{tr}(G)\right)}{\left[\int_{\mathcal{B}} \varsigma_f \left(1 + \mathrm{tr}(G)\right)\right]^2}$$

where M is the total mass of the interstitial fluid that is assumed to be constant and β is a newly introduced constitutive parameter. It has also to be remarked that this new energy term can be expressed in the following form

$$\psi_{NL} = \beta \hat{\rho}_f^2 \, \varsigma_f (1 + \mathrm{tr}(G))$$

where we appreciate a quadratic representation of the internal energy with respect to the true mass density of the interstitial fluid

Evolution rules of growth and resorption

$$Y = Y_{\mathrm{b\,Max}} (\varsigma_{\mathrm{b}}{}^{*})^{\beta_{\mathrm{b}}} + Y_{\mathrm{m\,Max}} (\varsigma_{\mathrm{m}}{}^{*})^{\beta_{\mathrm{m}}}$$

Governing equations for the mass densities of the two phases

$$\begin{cases} \dot{\rho}^{*}_{\mathrm{b}} & = A_{\mathrm{b}}\left(S\right)\ H\left(\varsigma_{f}\right) \\ \dot{\rho}^{*}_{\mathrm{m}} & = A_{\mathrm{m}}\left(S\right)\ H\left(\varsigma_{f}\right) \end{cases}$$

$$H = k\,\varsigma_{f}\left(1 - \varsigma_{f}\right)$$

$$A_{\mathrm{b}}\left(S\right) = \begin{cases} s_{\mathrm{b}} S & \text{for } S \geq 0 \\ r_{\mathrm{b}} S & \text{for } S < 0 \end{cases}$$

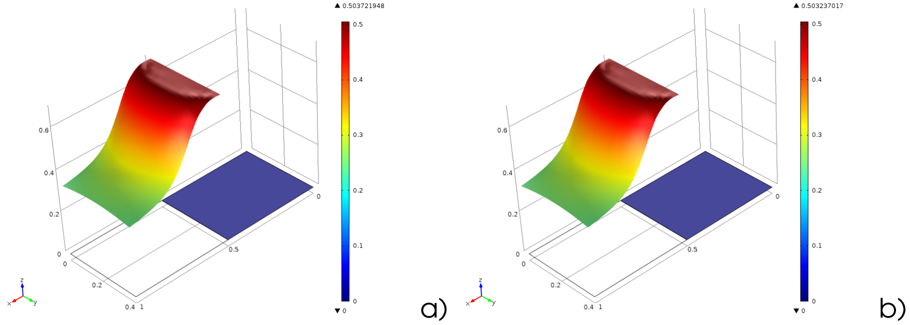
$$A_{\mathrm{m}}\left(S\right) = \begin{cases} 0 & \text{for } S \geq 0 \\ r_{\mathrm{m}} S & \text{for } S < 0 \end{cases}$$

Stimulus

$$S\left(\boldsymbol{X}, t\right) = \int_{\Omega} \psi\left(\boldsymbol{X}_{0}, t\right)\, d\left[\rho_{\mathrm{b}}\left(\boldsymbol{X}_{0}, t\right)\right]\ e^{-f\left(\boldsymbol{X} - \boldsymbol{X}_{0}\right)} d\boldsymbol{X}_{0} - P_{\mathrm{ref}} = P\left(\boldsymbol{X}, t\right) - P_{\mathrm{ref}}$$

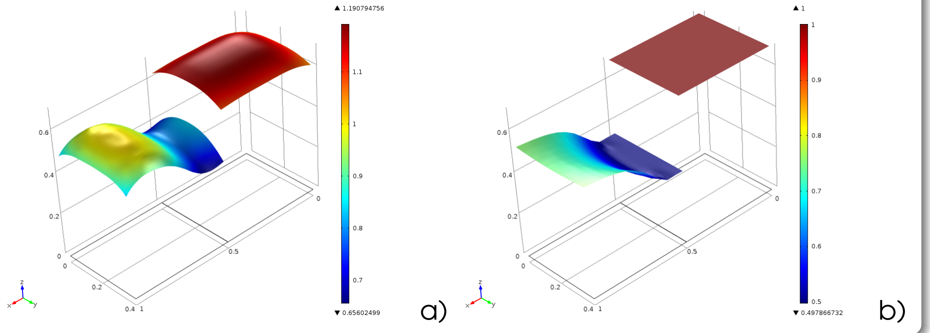
Results

Final mass densities of bio-material ρ_m^* with ψ_{NL} **a)** and without ψ_{NL} **b)**



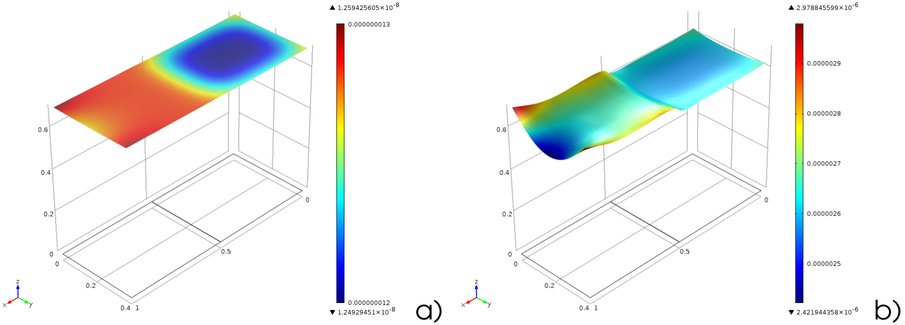
Results

Final mass densities of bone tissue ρ_b^* with ψ_{NL} **a)** and without ψ_{NL} **b)**



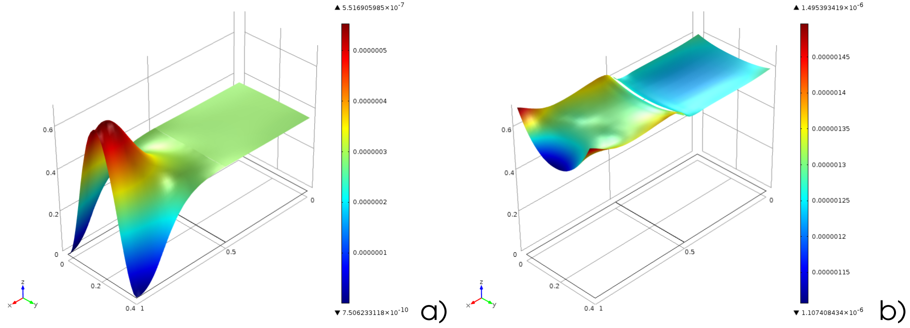
Results

Stored energy density ψ_{NL} at initial time **a**) and at the end **b**)



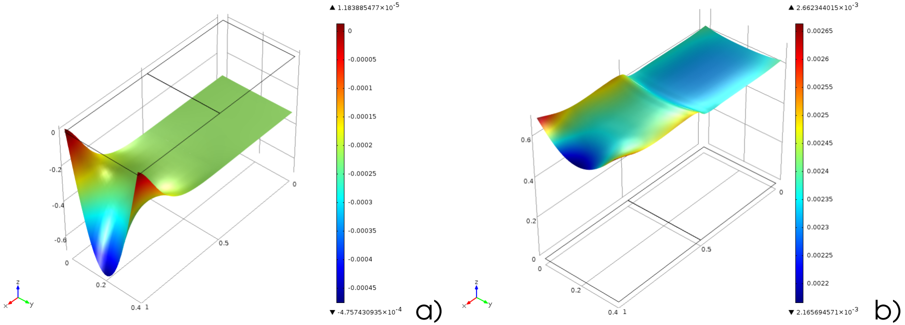
Results

Stored energy density ψ_{Biot} at initial time **a**) and at the end **b**)



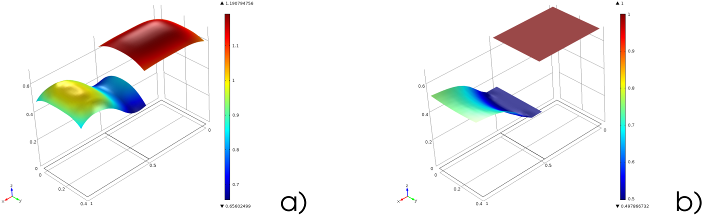
Results

The change of porosity θ at initial time **a**) and at the end **b**)



Conclutions

1. The Biot model for a mixture of bone tissue and bio-resorbable material with interstitial fluid is numerical investigated;
2. a new fully non local term in the internal energy is introduced and analyzed;
3. the new term produces an important growth also at the interface between the bone tissue and the bio-resorbable material;



4. The usual anisotropic characteristic of the remodeling process due to the application of load is also observed;

Thanks

THANK YOU VERY MUCH FOR YOUR KIND ATTENTION