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A NEW SEPARATION RESULT FOR EULER-LAGRANGE-LIKE SYSTEMS

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Abstract: This paper presents a separation result for some global stabilization via output feedback of a class of quadratic-like nonlinear systems, under the form of some *stabilizability by state feedback* on the one hand, and *unboundedness observability* on the other hand. They allow to design, for any domain of output initial condition, a dynamic output feedback controller achieving global stability. As an example, these conditions are shown to be satisfied by so-called *Euler-Lagrange* systems, for which a tracking output feedback control law is thus proposed. *Copyright ©2002 IFAC.*

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1. INTRODUCTION

It is well-known that for linear systems, separate possible designs for state feedback and state observer always result in a possible design for output feedback. But it is also known that such a *separation principle* does not systematically hold for *nonlinear* systems, and for this reason various studies towards a generalization of such a result have been proposed. Among them, it has in particular been shown that (semi-)global state feedback stabilizability together with uniform complete observability result in *semi-global* output feedback stabilizability (Khalil and Esfandiari, 1993; Teel and Praly, 1994). Moreover, it has been underlined that beyond basically *quadratic nonlinearities in the unmeasured states*, no *global* output feedback stabilization can be achieved, due to the lack of *unboundedness observability* (UO) (Mazenc *et al.*, 1994): roughly, for any initial condition and bounded input, any finite escape time of the state trajectory can be observed from the output.

For a class of systems which are *affine in the unmeasured states*, a global separation result has been proposed in (Battilotti, 1996), requiring stabilizability both by *state feedback* (SSF) and by *output injection* (SOI): this is not in contradiction with the counterexamples of (Mazenc *et al.*, 1994), since linearity in the unmeasured states plus SOI implies UO. On the other hand, for the considered class of systems, SOI is equivalent by (Sontag and Wang, 1997) to the *Output to State Stability* (OSS): roughly, the state trajectories are bounded whenever the output is bounded and go to zero as the output goes to zero. A first attempt to extend the result of (Battilotti, 1996) to a broader class of systems can be found in (Battilotti *et al.*, 1997), and more recently in (Battilotti, 1999). However, even if these results allow to put in a separation framework various existing contributions on set point control, including for instance the case of Euler–Lagrange systems (EL systems), they are not sufficient to globally solve the output *tracking* problem. This problem for EL systems has been motivating quite a lot of research for the last years (see e.g. (Lefeber and Nijmeijer, 1997; Battilotti *et al.*, 1997; Besançon, 2000; Loria and Panteley, 1999) and references therein). Recently, an interesting solution for rigid robots has been proposed in (Zhang *et al.*, 1997): although the design procedure in this paper is constructive, very little insight is given into the tracking problem in its generality. Moreover, the proposed controller very tightly relies on the *initial value* of the output tracking error, and for this reason is not global in the usual sense. In the present paper we consider the problem of output feedback stabilization for a class of time–varying systems roughly quadratic in the unmeasured states, which in particular includes EL systems. From this, a solution to the tracking problem for such systems is given.

First, we prove that if SSF holds together with OSS, *subject to the constraint that the output stays for all time in some set* Ω (Ω–OSS), then given any set $\mathcal{K}_y$ containing the initial output, stabilization can be achieved *via dynamic output feedback as long as the dynamic controller is suitably chosen w.r.t.* $\mathcal{K}_y$ *and* Ω. From this, our stability result can be said to be *global w.r.t. the unmeasured states* and *semiglobal w.r.t. the measured outputs*. Then we prove that, by restricting the class of systems considered (but still including EL systems), one can still achieve stabilization via output feedback if Ω–OSS is relaxed to some property (Ω–UO) which, by the recent characterization of (Angeli and Sontag, 1999), is equivalent, for the considered systems and for Ω compact, to UO. The basic ingredient to prove these results is an *output constraining* technique (see (Zhang *et al.*, 1997; Besançon *et al.*, 1998; Besançon, 2001) for comparisons).

Section 2 formulates the problem and presents the main results, while section 3 presents an application to the problem of tracking control of Euler-Lagrange systems.

2. MAIN RESULTS

Consider a system of the following form:

$$\begin{array}{lll} \dot{y} & = A_{12}(y,t)z; & y,z\in I\!R^n; \\ \dot{z} & = f(y,z,t)+B_2(y,t)u & \\ y & = \text{measured output} & \\ \text{where} & \|A_{12}(y,t)\|\le a_{12}(y),\ \forall y,t, & \\ \text{and} & A_{12},f,B_2 \text{ are smooth functions.} & \end{array} \tag{1}$$

The problem which is studied here is that of **output feedback stabilization**, i.e. finding a dynamic controller only using the measured output:

$$\begin{aligned} \dot{\sigma} &= S(y,\sigma,t) \\ u &= U(y,\sigma,t) \end{aligned} \tag{2}$$

such that the origin of the closed-loop system (1)-(2) is uniformly asymptotically stable.
The stabilization is *global* as soon as the stability of (1)-(2) holds for any initial condition. If for any compact set $\mathcal{K}\subset I\!R^{2n}$ there exists a compact set $\mathcal{K}_\sigma$ and a controller (2) making the closed-loop system asymptotically stable for initial conditions $(\,y(0)^T \quad z(0)^T\,)^T$ in $\mathcal{K}$ and $\sigma(0)\in\mathcal{K}_\sigma$, then the stabilization is *semiglobal* (Teel and Praly, 1995). In fact, in the present note, being given any compact set $\mathcal{K}_y\subset I\!R^n$, we provide a controller achieving asymptotic stability of the closed-loop system for any initial condition $z(0)\in I\!R^n$, any $y(0)\in\mathcal{K}_y$, and any $\sigma(0)\in\mathcal{K}_\sigma$ for some compact $\mathcal{K}_\sigma$. In this sense, the stabilization is *semiglobal w.r.t.* y *and global w.r.t.* z. This extends previously available semiglobal separation results (e.g. as in (Teel and Praly, 1995)), insofar as here no prespecified set for z is any longer required. Notice that the study of output feedback stabilization of the time–varying system (1) is motivated in particular by the fact that, for example, the output tracking of the time invariant system:

$$\dot{y}=z;\ \ \dot{z}=f(y,z)+B_2(y)u,\ \ \text{with } y \text{ measured}\,,$$

can be easily reformulated as the stabilization problem for a system of the form (1).

In the case of linear systems, it is well-known that the problem of output feedback stabilization can be solved from *separate* designs of state feedback and state observer. This separation result has been extended to the case of systems *affine in the unmeasured states* in terms of Lyapunov functions (Battilotti, 1996). Here, we propose *separate conditions* in the same spirit, but now for a class of systems of the form (1), and highlighting the relationship with the notion of ”unboundedness observability”. These conditions are:

(SSF) (i) There exist a $\mathcal{C}^\infty$, proper and positive definite function $V : I\!R^n \to I\!R$ and a $\mathcal{C}^\infty$ function $\mu : I\!R^n\times I\!R\to I\!R^n$ such that $\mu(0,t)=0$ for all t, $\dfrac{\partial\mu}{\partial y}(y,t)$ and $\mu(y,t)$ are bounded w.r.t. t, and :

$$\forall y\neq 0;\ \forall t;\ -\frac{\partial V}{\partial y}(y)A_{12}(y,t)\mu(y,t)\le -a(y),$$

for some positive definite function a;
(ii) $\forall t,y,\ 0<b_1 Id\le B_2(y,t)\le b_2(y)Id$.
(Id denotes the identity matrix).

(Ω–OSS) There exists a $\mathcal{C}^\infty$, proper and positive definite function $W(y,z,t)=\frac{1}{2}z^TP(y,t)z+z^T\zeta(y)+\xi(y)$ such that $0<p_1 Id\le P(y,t)\le p_2 Id$, $\left\|\dfrac{\partial P}{\partial y}(y,t)\right\|\le p_3$ and:

$$\frac{\partial W}{\partial t}+\frac{\partial W}{\partial(y,z)}\begin{pmatrix}A_{12}(y,t)z\\ f(y,z,t)\end{pmatrix}\le -\alpha W(y,z,t)+\Xi(y)$$

$$\text{for some } \Xi,\ \text{for all } t,z \tag{3}$$

$$\text{and } y\in\Omega=\{y\in I\!R^n:\|\zeta(y)\|\le\eta\}, \tag{4}$$

for some η and $\alpha>0$ (independent of η).

Remark *2.1.* The first condition means some global stabilizability by state feedback (use backstepping). On the other hand, it can be noticed that inequality (3) in particular suggests that $f(y,z)$ must in some sense be quadratic in z. Moreover, it is easy to see that, whenever Ω is compact, (3) implies that if y is in Ω and bounded then the state is bounded with exponential decay, and by (Sontag and Wang, 1997) it is equivalent to *output to state stability* (OSS), modulo the special structure required for W. It can also be noticed that (3) is independent of any output injection $(v_1^T, v_2^T)^T$ satisfying $\|v_1\| \leq \frac{p_1\alpha_1}{p_3}$, $\alpha_1 < \alpha$.

A first separation result can then be stated as:

Theorem *2.1.* If (SSF) and (Ω–OSS) hold, system (1) is stabilizable by output feedback globally w.r.t. z and semiglobally w.r.t. y, in the sense that for any compact set $\mathcal{K}_y \subset I\!\!R^n$ there exist a controller (2) with a compact $\mathcal{K}_\sigma$, such that for any $z(0) \in I\!\!R^n$, any $y(0) \in \mathcal{K}_y$, and any $\sigma(0) \in \mathcal{K}_\sigma$, the solutions $y(t), z(t), \sigma(t)$ of (1)-(2) satisfy:

$$\lim_{t\to\infty} \|y(t)\| = \lim_{t\to\infty} \|z(t)\| = \lim_{t\to\infty} \|\sigma(t)\| = 0.$$

The proof is given in appendix A: it provides an explicit solution for the output feedback control law, and it is based on an *output constraining* technique, in the sense that the controller is designed such that some output-dependent variable of the closed loop system stays for all times in some set Ω. In particular y remains constrained w.r.t. $\mathcal{K}_y$.

Notice that the proof uses the property that the state cannot escape in finite time as long as it remains in some set, which results from (Ω–OSS). One can actually still achieve stabilization via output feedback if Ω–OSS is relaxed to such an Ω– *unboundedness observability* property (Ω–UO), namely the property that subject to the constraint that $y(t) \in \Omega, \forall t$, *bounded output guarantees a state with no finite escape time*:

(Ω–UO) There exists a $\mathcal{C}^\infty$ proper positive definite $W(y,z,t) = \frac{1}{2}z^T P(y,t)z + z^T\zeta(y) + \xi(y)$ s.t. $p_1 Id \leq P(y,t) \leq p_2 Id$, $\left\| \dfrac{\partial P}{\partial y}(y,t)\right\| \leq p_3$ and:

$$\frac{\partial W}{\partial t} + \frac{\partial W}{\partial (y,z)}\begin{pmatrix} A_{12}(y,t)z \\ f(y,z,t)\end{pmatrix} \leq \alpha W(y,z,t) + \Xi(y) \tag{5}$$

$\forall t, z$ and $y \in \Omega$ as in (4) for some η and $\alpha > 0$

If Ω is compact, by the recent characterization given in (Angeli and Sontag, 1999) one can actually check that this property implies UO, modulo the special structure of W. It can be noticed that (5) is independent of any output injection $(v_1^T, v_2^T)^T$ with $\|v_1\|$ bounded. If in addition $A_{12}(y,t) \geq aId > 0$ for all y and t, it is easy to prove that Ω–UO implies Ω–OSS (just add to W of Ω–UO a term of the form $-z^T K y + H(y)$, for some $K > 0$ and $H(y)$ definite positive, depending on η and α, so as to make (3) satisfied with the new function).

The condition $A_{12}(y,t) \geq aId > 0$ still preserves the state feedback stabilizability property, which can consequently be formulated as:

(RSF) $A_{12}(y,t) \geq aId > 0$ and $0 < b_1 Id \leq B_2(y,t) \leq b_2(y) Id$ for all y and t.

We thus have the following separation result:

Theorem *2.2.* If (RSF) and (Ω–UO) hold, system (1) is stabilizable by output feedback globally w.r.t. z and semiglobally w.r.t. y, in the sense that for any compact set $\mathcal{K}_y \subset I\!\!R^n$ there exist a controller (2) with a compact $\mathcal{K}_\sigma$ such that for any $z(0) \in I\!\!R^n$, any $y(0) \in \mathcal{K}_y$, and any $\sigma(0) \in \mathcal{K}_\sigma$, the solutions $y(t), z(t), \sigma(t)$ of (1)-(2) satisfy:

$$\lim_{t\to\infty} \|y(t)\| = \lim_{t\to\infty} \|z(t)\| = \lim_{t\to\infty} \|\sigma(t)\| = 0.$$

Notice that the restriction $A_{12}(y,t) \geq aId > 0$ actually compensates any lack of detectability, which is not guaranteed by the (Ω–UO) property by itself.

Notice also that (RSF) actually guarantees *usual* semiglobal stabilizability (Teel and Praly, 1995), but here (Ω–UO) further allows to obtain *global* stabilizability w.r.t. unmeasured states.

Finally, notice that similarly, conditions for output feedback stabilization with disturbance attenuation can be formulated (see (Besançon *et al.*, 2000) for a theoretical formulation, and (Besançon *et al.*, 1998) for an example of application).

3. EXAMPLE

In this section we show how theorem 2.1 provides for so-called Euler-Lagrange systems an output feedback tracking controller similar to those of (Zhang *et al.*, 1997; Besançon *et al.*, 1998), but not depending any more on the exact value of $y(0)$. Such systems are classically described by:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau,$$

where q denotes the generalized positions in $I\!\!R^n$ assumed to be measured, D the inertia matrix, $C(q,\dot{q})\dot{q}$ Coriolis and centrifugal forces, $G(q)$ the gravitation, and τ the control forces. Moreover we assume as usual that $\forall q$: $\dot{D} = C(q,\dot{q}) + C^T(q,\dot{q})$, $0 < d_1 Id \leq D(q) \leq d_2 Id$, $\|C(q,\dot{q})\| \leq \kappa\|\dot{q}\|$ and $C(q,q_1)q_2 = C(q,q_2)q_1$ (Spong and Vidyasagar, 1989).

Let us now consider some twice differentiable trajectory $q_d(t)$ to be tracked, with $\|\dot{q}_d(t)\| \leq Q_d$, and $u = \tau - \tau_d$ with $\tau_d = D(q)\ddot{q}_d + C(q,\dot{q}_d)\dot{q}_d + G(q)$. Then $\tilde{q} := q - q_d$ then satisfies:

$$D(q)\ddot{\tilde{q}} + C(q,\dot{q})\dot{\tilde{q}} + C(q,\dot{q}_d)\dot{\tilde{q}} = u. \tag{6}$$

The problem of output feedback control for this new system turns to be a problem of **tracking control without velocity measurement** for the original Euler Lagrange system. As mentioned in the introduction, this problem has motivated a lot of recent work. Here, showing that assumptions (SSF) and (Ω–OSS) of previous section are satisfied by the error system (6) provides us with a *global-like* solution (as in theorem 2.1) to this problem. With $y := \tilde{q}$ measured and $z := \dot{\tilde{q}}$, system (6) indeed takes the form (1):

$$\dot{y} = z;\ \dot{z} = -D^{-1}(y+q_d)[C(y+q_d,\dot{q}_d)z \\ +C(y+q_d,z+\dot{q}_d)z-u] \\ = f(y,z,t)+B_2(y,t)u.$$

Let us sketch how conditions (SSF) and (Ω–OSS) are satisfied:

(SSF) One can simply take $V = \frac{l}{2}y^Ty$, $l > 0$ and any function $\mu(y)$ making $y^T\mu(y)$ positive definite (for instance $\mu = y$ or, as in (Besançon *et al.*, 1998), $\mu = tanh$) on the one hand, and notice that here $B_2 = D^{-1}$ classically satisfies (SSF)-(ii) on the other hand.

(Ω–OSS) Taking for W:

$$W := \frac{1}{2}\dot{\tilde{q}}^T D(q)\dot{\tilde{q}} - k\dot{\tilde{q}}^T\tilde{q} + \frac{l}{2}\tilde{q}^T\tilde{q} = \frac{1}{2}z^TD(y+q_d)z \\ -kz^Ty + \frac{l}{2}y^Ty$$

with $k > 0$, and l large enough so that W be positive definite, one can check by direct computations that for $k > \kappa Q_d + 1$, inequality (3) indeed holds with $\eta = \frac{d_1}{2\kappa}$ and $\alpha = \frac{1}{d_2}$.
From this, theorem 2.1 applies, and thus one can obtain an output feedback controller for the tracking error system (6), as in the proof of theorem 2.1. Taking for instance:

$$V_{y,z,\sigma} = \frac{1}{2}(z+D^{-1}\hat{\sigma}+\varepsilon\mu(y))^TD(z+D^{-1}\hat{\sigma}+\varepsilon\mu(y)) \\ -\frac{l}{2}Log(1-\frac{y^Ty}{c+\varepsilon}) - \frac{1}{2}\sum_{i=1}^{n}Log(1-\frac{n\hat{\sigma}_i^2}{\varepsilon^2})$$

for any μ as in (SSF), and any c s.t. $\|y(0)\|^2 \leq c$, one obtains (with Ψ as in appendix A):

$$\begin{aligned}
\dot{\sigma}_I &= -(\sigma_{II}+1)k[D^{-1}\hat{\sigma}+\varepsilon\mu] + \dot{\sigma}_{II}ky \\
&\quad -\Psi^{-1}[\beta\hat{\sigma} - \frac{l}{c+\varepsilon-y^Ty}D^{-1}y]; \\
\dot{\sigma}_{II} &= -\frac{\lambda}{(c+\varepsilon-\frac{l}{2}\|y\|^2)\prod_{i=1}^{n}(\frac{\varepsilon^2}{n}-\hat{\sigma}_i^2)}\sigma_{II}; \\
\hat{\sigma} &= \sigma_I - (\sigma_{II}+1)ky;\ \sigma_I(0)=0;\ \sigma_{II}(0)=0; \\
u &= -\varepsilon D\frac{\partial D^{-1}}{\partial y}(D^{-1}\hat{\sigma}+\varepsilon\mu-\dot{q}_d)D\mu \qquad (7) \\
&\quad +\Psi^{-1}[\beta\hat{\sigma} - \frac{l}{c+\varepsilon-y^Ty}D^{-1}y] + \\
&\quad \varepsilon D\frac{\partial\mu}{\partial y}[D^{-1}\hat{\sigma}+\varepsilon\mu] - \frac{ly}{c+\varepsilon-y^Ty} \\
&\quad +(\sigma_{II}+1)k\Psi\hat{\sigma} - C(q,\dot{q}_d)(D^{-1}\hat{\sigma}+\varepsilon\mu) \\
&\quad +C^T(q,\dot{q}_d-D^{-1}\hat{\sigma}-\varepsilon\mu)(D^{-1}\hat{\sigma}+\varepsilon\mu),
\end{aligned}$$

for $k > \kappa Q_d + 1$, $\beta > 0$, $\varepsilon\|\frac{\partial\mu}{\partial y} - \frac{\partial D^{-1}}{\partial y}D\mu\| < \frac{1}{2d_2}$,

$$\varepsilon + \varepsilon\|D\mu\| \leq \frac{d_1}{2\kappa} \text{ when } \|y\|^2 \leq c.$$

Remark *3.1.* Notice that in the present example, stronger properties than (SSF) and (Ω–OSS) are satisfied, namely the state feedback and, respectively the "output to state stability", can be made "arbitrarily fast". Thus in view of appendix A, one can increase k instead of reducing ε and canceling (A.7) by u so as to make $\dot{V}$ definite negative. Notice also that if μ is bounded one can choose $\sigma_{II}(0) = 0$ and $\sigma_I(0)$ s.t. $|\sigma_I(0) - ky(0)| < \varepsilon$, so as to still make $\dot{V} < 0$. In that case the achieved stabilization is somehow only *local w.r.t.* y, and coincides with that of (Besançon *et al.*, 1998) (see also (Zhang *et al.*, 1997)).

As an example, consider the two-link robot manipulator of (Zhang *et al.*, 1997) defined by:

$$D(q) = \begin{pmatrix} p_1+2p_3cos(q_2) & p_2+p_3cos(q_2) \\ p_2+p_3cos(q_2) & p_2 \end{pmatrix};$$
$$C(q,\dot{q}) = \begin{pmatrix} -p_3sin(q_2)\dot{q}_2 & -p_3sin(q_2)(\dot{q}_1+\dot{q}_2) \\ p_3sin(q_2)\dot{q}_1 & 0 \end{pmatrix}$$

with $p_1 = 3.473$, $p_2 = 0.193$, $p_3 = 0.242$, and reference trajectories satisfying:

$$q_{1d}(t) = 1.57sin(2t)(1-e^{-0.05t^3}); \\ q_{2d}(t) = 1.2sin(3t)(1-e^{-0.05t^3}).$$

Simulations results, obtained with controller (7) with $\varepsilon = 1, k = 5$ (according to remark 3.1), $l = \lambda = \beta = 5, c = 100, \mu = y$, and initial conditions $q(0) = (-5,2)^T$, $\dot{q}(0) = (2,10)^T$, are illustrated by figure 1. The results reflect the expected asymptotic stability, even for quite large initial errors on $\dot{q}$, and for fair values of the controls (in $[-50,30]$).

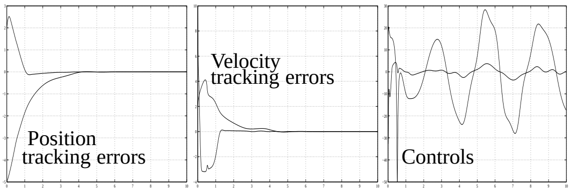

Fig. 1. Simulation results.

4. CONCLUSIONS

In this paper, a problem of output feedback control design for a class of nonlinear systems has been considered. It has been addressed via the formulation of a *separation* result, in the sense that the output feedback controller is based on some *separate* properties related to *state feedback*, on the one hand, and *output to state stability* (OSS) on the other hand. In particular, the OSS condition has been discussed at the light of the "unboundedness observability" property (UO). As an illustration, the conditions have been shown to be satisfied by Euler-Lagrange systems, resulting in a control design which actually improves a technique of (Zhang *et al.*, 1997). In particular it gives some "methodological" interpretation of such an output feedback controller. Moreover in (Zhang *et al.*, 1997) the controller requires the exact value $y(0)$, while in our approach for any compact set containing $y(0)$ a controller can be designed. From this, the proposed controller achieves *semiglobal* stabilization *only w.r.t. the measured state* y (and globally w.r.t. the unmeasured state z), extending previous results on semiglobal stabilization (e.g. (Teel and Praly, 1995)).

Appendix A. PROOF OF THEOREM 2.1

Let us consider V, μ as in (SSF), W as in (Ω–OSS), and some compact set $\mathcal{K}_y$ for y. For simplicity, we will only present computations for time–independent functions. Moreover, when there is no ambiguity, the arguments of the functions will be omitted, and for any vector r, r_i will denote its ith component.

Now for some $\varepsilon > 0$ to be specified later, and given any $x = (x_1, \dots x_n)^T \in I\!R^n$, let us define $\Psi(x)$ a diagonal matrix with $\frac{1}{\frac{\varepsilon^2}{n} - x_i^2}$ for $i = 1$ to n as diagonal entries.

Then take c s.t. $\{y : V(y) \leq c\} \supset \mathcal{K}_y$ and set $\hat{\sigma} := \sigma_I + (\sigma_{II}+1)\zeta(y)$ with $\sigma_I \in I\!R^n, \sigma_{II} \in I\!R$ and:

$$\dot{\sigma}_{II}(t) = -\nu(t)\sigma_{II}(t) \;\; where \;\; \nu := \frac{\lambda}{D}, \;\; with$$
$$\lambda > 0; \;\; D := (c + \varepsilon - V(y)) \prod_{i=1}^{n} (\frac{\varepsilon^2}{n} - \hat{\sigma}_i^2) \tag{A.1}$$

$\dot{\sigma}_I(t)$ to be chosen later, $(\sigma_I(0), \sigma_{II}(0)) \in \mathcal{K}_\sigma$

for some compact subset $\mathcal{K}_\sigma$ of $\{(\sigma_I, \sigma_{II}) \in I\!R^{n+1} : \forall y \in \mathcal{K}_y, |\hat{\sigma}_i| \leq \frac{\varepsilon}{2\sqrt{n}}; i = 1, \dots n\}$. [1]

Finally, set $\Omega_\varepsilon := \{y, z, \sigma_I \in I\!R^n, \sigma_{II} \in I\!R : V(y) < c + \varepsilon \; and \; |\hat{\sigma}_i| < \frac{\varepsilon}{\sqrt{n}}; i = 1, ...n\}$. Clearly from the choice of $\sigma_I(0)$ and $\sigma_{II}(0)$, for any $y(0) \in \mathcal{K}_y$ and any $z(0) \in I\!R^n$, the extended state $x_e := (\, y^T \;\; z^T \;\; \sigma_I^T \;\; \sigma_{II}\,)^T$ starts in Ω_ε. Then let us choose:

$$V_{OF}(y, z, \sigma_I, \sigma_{II}) = V_{y,z,\sigma_I} + V_{\sigma_{II}}$$

with $V_{\sigma_{II}} = \frac{1}{2}\sigma_{II}^2$ and

$$V_{y,z,\sigma_I} = \frac{1}{2}(z + P^{-1}\hat{\sigma} + \varepsilon\mu(y))^T P (z + P^{-1}\hat{\sigma} + \varepsilon\mu(y))$$
$$-Log(1 - \frac{V(y)}{c+\varepsilon}) - \frac{1}{2}\sum_{i=1}^{n} Log(1 - \frac{n\hat{\sigma}_i^2}{\varepsilon^2}). \tag{A.2}$$

Notice that V_{OF} is positive definite and radially unbounded on Ω_ε, and $\|\hat{\sigma}\| < \varepsilon$ on Ω_ε.

Let us now show that for appropriate choices of $\varepsilon, \dot{\sigma}_I$ and u, the time derivative $\dot{V}_{OF}$ along the trajectories of the closed loop system can be made negative definite on Ω_ε before the state leaves Ω_ε (and thus the state always remains in Ω_ε with our initial conditions). To that end, notice first that μ and P^{-1} being bounded on Ω_ε, the smaller ε is chosen, the smaller $\|P^{-1}\hat{\sigma} + \varepsilon\mu(y)\|$ is on Ω_ε. Let us then first compute $\dot{V}_{y,z,\sigma_I}$ for the closed loop system (1)-(2) with $u, \dot{\sigma}_I$ to be found:

$$\dot{V}_{y,z,\sigma_I} =$$
$$(z + P^{-1}\hat{\sigma} + \varepsilon\mu)^T P \big[f(y,z) + B_2 u + \frac{\partial P^{-1}}{\partial y}(A_{12}z)\hat{\sigma}$$
$$+ \; P^{-1}(\dot{\sigma}_I + (\sigma_{II}+1)\frac{\partial \zeta}{\partial y}A_{12}z + \dot{\sigma}_{II}\zeta) + \varepsilon\frac{\partial \mu}{\partial y}A_{12}z\big]$$
$$+ \; (z + P^{-1}\hat{\sigma} + \varepsilon\mu)^T \frac{\partial P}{2\partial y}(A_{12}z)(z + P^{-1}\hat{\sigma} + \varepsilon\mu)$$
$$+ \; \frac{1}{c + \varepsilon - V}\frac{\partial V}{\partial y}A_{12}z + \hat{\sigma}^T\Psi(\hat{\sigma})(\dot{\sigma}_I$$
$$+(\sigma_{II}+1)\frac{\partial \zeta}{\partial y}A_{12}z + \dot{\sigma}_{II}\zeta).$$

[1] One can e.g. simply take $\mathcal{K}_\sigma = \{\sigma_I = 0, \sigma_{II} = -1\}$.

Gathering all terms in $\bar{z} := z + P^{-1}\hat{\sigma} + \varepsilon\mu$ gives:

$$\dot{V}_{y,z,\sigma_I} = \sigma_{II}\bar{z}^T[\frac{\partial \zeta}{\partial y}A_{12}]\bar{z} \tag{A.3}$$
$$+ \; \bar{z}^T[\frac{\partial \zeta}{\partial y}A_{12}]\bar{z} + \bar{z}^T\frac{\partial P}{2\partial y}(A_{12}z)\bar{z} \tag{A.4}$$
$$+ \; \bar{z}^T P f(y,z)\bar{z}^T P\frac{\partial P^{-1}}{\partial y}(A_{12}z)(\hat{\sigma} + \varepsilon P\mu) \tag{A.5}$$
$$+ \; \bar{z}^T P\Big[B_2 u + \varepsilon\frac{\partial P^{-1}}{\partial y}A_{12}[P^{-1}\hat{\sigma} + \varepsilon\mu]P\mu$$
$$-\varepsilon\frac{\partial \mu}{\partial y}A_{12}[P^{-1}\hat{\sigma} + \varepsilon\mu] \tag{A.6}$$
$$+P^{-1}(\dot{\sigma}_I - (\sigma_{II}+1)\frac{\partial \zeta}{\partial y}A_{12}[P^{-1}\hat{\sigma} + \varepsilon\mu] + \dot{\sigma}_{II}\zeta)$$
$$+P^{-1}A_{12}^T\left(\frac{1}{c+\varepsilon-V}\frac{\partial V}{\partial y} + (\sigma_{II}+1)\hat{\sigma}^T\Psi\frac{\partial \zeta}{\partial y}\right)^T\Big] \tag{A.7}$$
$$+\hat{\sigma}^T\Psi[\dot{\sigma}_I - (\sigma_{II}+1)\frac{\partial \zeta}{\partial y}A_{12}[P^{-1}\hat{\sigma} + \varepsilon\mu] + \dot{\sigma}_{II}\zeta] \tag{A.8}$$
$$-\frac{1}{c+\varepsilon-V}\left(\varepsilon\frac{\partial V}{\partial y}A_{12}\mu + \frac{\partial V}{\partial y}A_{12}P^{-1}\hat{\sigma}\right) \tag{A.9}$$
$$+\bar{z}^T P[\varepsilon\frac{\partial \mu}{\partial y}A_{12}\bar{z} - \varepsilon\frac{\partial P^{-1}}{\partial y}A_{12}\bar{z}P\mu]. \tag{A.10}$$

At this point, one needs to notice that from (Ω–OSS), if $\|\hat{\sigma} + \varepsilon P\mu(y)\| \leq \eta$, terms (A.4)-(A.5) can be upper bounded by:

$$-\alpha\bar{z}^T P\bar{z} \tag{A.11}$$
$$+\bar{z}^T\Gamma(y, \hat{\sigma} + \varepsilon P\mu) \text{ for some } \Gamma. \tag{A.12}$$

(just reorder (3) w.r.t. $\bar{z} := z + P^{-1}\zeta$ and notice that here $\hat{\sigma} + \varepsilon P\mu$ plays the role of ζ).

Thus, we choose ε small enough so as to ensure $\|\hat{\sigma} + \varepsilon P\mu(y)\| \leq \eta$ on Ω_ε, and consequently we get a first upper bounding term (A.11) which is negative definite in $\bar{z} = z + P^{-1}\hat{\sigma} + \varepsilon\mu(y)$. The second term (A.12) can be gathered with (A.6).

Now choose $\dot{\sigma}_I$ so as to cancel (A.8) plus a term $-\Psi^{-1}[\beta\hat{\sigma} - \frac{1}{c+\varepsilon-V}P^{-1}A_{12}^T(\frac{\partial V}{\partial y})^T]$ which will provide $-\beta\|\hat{\sigma}\|^2 + \frac{1}{c+\varepsilon-V}\frac{\partial V}{\partial y}A_{12}P^{-1}\hat{\sigma}$. Together with (A.9) and (SSF)-(i) we thus obtain definite negative terms in $\hat{\sigma}$ and y: $-\beta\|\hat{\sigma}\|^2 - \frac{\varepsilon}{c+\varepsilon-V(y)}a(y)$. Then by (SSF)-(ii), one can choose u so as to cancel (A.6)-(A.12). Moreover, (A.10) clearly being quadratic in $\bar{z}$, one can choose ε small enough so that this term is dominated by (A.11), i.e. (A.10)+ (A.11) $\leq -\bar{\alpha}\bar{z}^T P\bar{z}$ for some $\bar{\alpha} > 0$. Finally, with (A.3), we get:

$$\dot{V}_{y,z,\sigma_I} \leq -(z + P^{-1}\hat{\sigma} + \varepsilon\mu)^T(\bar{\alpha}P -$$
$$|\sigma_{II}|\,\|\frac{\partial \zeta}{\partial y}A_{12}\|)(z + P^{-1}\hat{\sigma} + \varepsilon\mu) \tag{A.13}$$
$$-\beta\|\hat{\sigma}\|^2 - \frac{\varepsilon}{c+\varepsilon-V(y)}a(y).$$

Here let us notice that from (Ω–OSS), z does not have finite escape time as long as y and u are bounded. In view of our candidate for u and our working set Ω_ε, this in particular means that z cannot escape in finite time as long as the extended state x_e remains in Ω_ε.

Let us now show that σ_{II} becomes small enough so as to make the right-hand side of (A.13) definite negative before x_e leaves Ω_ε. To that end, notice that $\dot{V}_{\sigma_{II}} = -\nu(t)\sigma_{II}^2$, where $\nu(t)$ tends to infinity when either y or σ_I approaches the boundary of Ω_ε. Moreover $\nu(t) \geq \frac{\lambda}{(c+\varepsilon)\varepsilon^2} =: \nu_0$, which implies $\dot{V}_{\sigma_{II}} \leq -\nu_0\sigma_{II}^2$, and thus $|\sigma_{II}| \leq \lambda_0 e^{-\nu_0 t}$.

Then if δ is such that $\|\frac{\partial\varsigma}{\partial y}A_{12}\| \leq \delta$ on Ω_ε, we get from (A.13), that the state might tend to the boundary of Ω_ε as long as $\bar{\alpha}p_1 < e^{-\nu_0 t}\delta\lambda_0$. Obviously, since $e^{-\nu_0 t}$ is decreasing to zero, if neither y nor σ_I approaches the boundary of Ω_ε in finite time, there exists a finite time t_0 such that for all $t \geq t_0$, $\bar{\alpha}p_1 > e^{-\nu_0 t}\delta\lambda_0$, i.e. the right-hand side of (A.13) becomes uniformly definite negative, while z still is finite, and thus the conclusion follows.

Now considering the case when $V(y)$ (resp. $\hat{\sigma}_i$) would tend to $c+\varepsilon$ (resp. $\frac{\varepsilon}{\sqrt{n}}$) in finite time t_1, we obtain that in this case: $lim_{t\to t_1} t\nu(t) = lim_{t\to t_1}\nu(t) = +\infty$. In particular, there exists $t_2 < t_1$ such that for any $t \geq t_2$, $t\nu(t) \geq t_0\nu_0$, and there exists $t_3 < t_1$ such that for all $t \geq t_3$, $\nu(t) \geq \frac{t_0\nu_0}{t_2}$, i.e. $|\sigma_{II}| \leq \lambda_0 e^{-\frac{t_0\nu_0}{t_2}t}$. Hence for all $t \geq max(t_2, t_3)$, we have $|\sigma_{II}| \leq \lambda_0 e^{-t_0\nu_0}$ and we have seen above that the right-hand side of (A.13) is then uniformly definite negative.

Finally, from this analysis, V_{OF} remains bounded at any time, and after some time, it decreases to zero, which gives the final result.

Appendix B. REFERENCES

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