THE GEODESY AND RELATIVITY EXPERIMENTS OF BEPICOLOMBO

Luigi Imperi, Luciano Iess, Mirco J. Mariani

Abstract

BepiColombo is a ESA-JAXA mission aimed to a comprehensive exploration of Mercury, the innermost planet of the solar system. The Mercury Orbiter Radio science Experiment (MORE) will exploit a state of the art microwave tracking system, including an advanced Ka-band transponder, to determine the gravity field and the rotational state of the planet, and to perform extensive tests of relativistic gravity. In this work we analyze all the aspects of the radio science investigation, which include: (i) the solar conjunction experiment in cruise; (ii) the geodesy and rotation experiment; (iii) the fundamental physics test. We report the results of numerical simulations based on the latest mission scenario, with launch in October 2018 and arrival at Mercury in December 2025. We show that the gravity and rotation measurements expected from BepiColombo will allow to better characterise the size of an inner solid core inside the outer liquid core and the properties of the outer mantle and the crust. We discuss how the current knowledge of several parametrized post-Newtonian parameters can be improved by MORE through the determination of the heliocentric motion of Mercury and by measuring the propagation time of radio waves. We also assess in a quantitative way the benefits of an extended mission.

1 Introduction

Mercury is one of the most interesting objects of the solar system. The high density, the peculiar 3:2 spin-orbit resonance, the intrinsic magnetic field and the anomalous perihelion drift of the planet have been one of the most fascinating challenges in planetary physics. So far, only the spacecraft Mariner 10 and MESSENGER (MErcury Surface, Space ENvironment, GEochemistry and Ranging) reached the planet. Mariner 10 surprised the science community with the detection of a relatively strong magnetic field, suggesting that the planet likely has a large molten core. In its four years in orbit around Mercury (from April 2011 to April 2015), MESSENGER carried out an extensive characterisation of the planet, revealing its dynamic past and present, from an early magma ocean, to the extended volcanism spanning over billions of years, and to the present-day, robustly convective, outer core (Johnson and Hauck, 2016).

BepiColombo (Benkhoff et al., 2010) will be the third mission reaching the planet, and the second one entering into hermean orbit after MESSENGER. The mission has been jointly developed by the European Space Agency (ESA) and the Japanese Aerospace eXploration Agency (JAXA). Planned for launch in October 2018, with arrival at Mercury in December 2025, the nominal mission is scheduled to be one year long, with the possibility of an extension by an additional year. The mission entails the release of two spacecraft in hermean orbit. The Mercury Magnetospheric Orbiter (MMO), developed by JAXA, will be placed into an elliptical polar orbit around the planet (590 x 11600 km altitude), and is devoted to explore the exosphere and the magnetosphere. The Mercury Planetary Orbiter (MPO), provided by ESA, will be inserted in a lower, near-circular, polar orbit (480 x 1500 km altitude), to study the surface and the deep interior.

1.1 MORE: The radio science experiment of BepiColombo

The Mercury Orbiter Radio science Experiment (MORE, Iess et al., 2009) is one of the 11 instruments and investigations of the MPO. It exploits advanced radio tracking instrumentation both onboard and on ground, enabling highly accurate range and Doppler measurements. The tracking system architecture (Iess and Boscagli 2001; Simone et al. 2008; Ciarcia et al., 2013} is an evolution of the radio system adopted by the Cassini mission to Saturn. In addition to the deep space transponder used for telemetry, tracking and command functions (TTC) and enabling two downlinks at X and Ka band coherent with a X band uplink, the key onboard instrument is a Ka-band Transponder (KaT), capable of receiving an uplink at 34 GHz and retransmitting it coherently to ground at 32.5 GHz. The multi-frequency radio link in X/X (7.2 GHz uplink / 8.4 GHz downlink), X/Ka (7.2 / 32.5 GHz) and Ka/Ka (34 / 32.5 GHz) allows a nearly complete suppression of plasma noise, the dominant source of noise in S and X bands (Armstrong et al, 1979; Bertotti et al., 1993).

The baseline ground station for MORE is DSS 25, a 34 m dish of NASA's Deep Space Network located in Goldstone (California, USA). DSS 25, previously used for Cassini radio science experiments, is currently the only ground antenna supporting the multifrequency radio link required by MORE with full operational capabilities. The ESA deep space antenna DSA-3 in Malargue (Argentina) has similar capabilities at experimental level. It is expected to reach operational support shortly after the launch of BepiColombo. The expected Allan deviation (ADEV) of the Doppler measurements in the hermean phase is 1-2 x 10^{-14} at 1000 seconds integration time (4-8 x 10^{-14} at 60 s). A remarkable difference between the Cassini and BepiColombo radio system is the multifrequency ranging function. Infact, the KaT is endowed with a novel wide-band ranging system, based upon a high rate (24 Mcps) pseudo-noise ranging code. The expected range accuracy is 20 cm after a few seconds integration time (Iess and Boscagli, 2001). On Cassini, range observable were available only at X band.

The scientific goals of MORE span over three areas:

- *gravimetry*: recovery of the static hermean gravity field and determination of the Love number k_2;
- *rotation*: estimation of Mercury's rotational state (pole direction and librations in longitude);
- *fundamental physics*: test different aspects of General Relativity (GR) through the determination of several parametrized post-Newtonian parameters.

A test of relativistic gravity, exploiting the Shapiro delay (Shapiro, 1964) and the corresponding Doppler shift affecting the propagation of radio-waves (Bertotti et al., 2003) will be performed in the cruise phase to Mercury.

The purpose of this work is twofold. We will give a general review of the MORE investigation, addressing all its aspects (gravity, rotation, and fundamental physics), and we will present the expected accuracies obtained from realistic numerical simulations based on the latest mission scenario with launch in October 2018. The work is organized as follows: Section 2 introduces the orbit determination method and summarizes the simulation scenario. Section 3 describes the relativistic test performed in cruise. The geodesy (gravimetry and rotation) and the fundamental physics experiments are addressed respectively in Section 4 and 5. The results of simulations are presented and discussed in Section 6. In Section 7 we illustrate the prospects and benefits of an extended mission. Section 8 reports on open issues and future works, followed by the conclusions in Section 9.

2 The orbit determination method for MORE

The MORE radio science experiment uses the radio observables (e.g. Doppler and range) as input to the orbit determination process. The observables collected at the ground station are compared with the predictions given by a reference solution. The discrepancies are minimized by correcting the spacecraft state and other model parameters in a least squares fit. The well-known weighted least squares correction with *a priori* information is given by (Tapley, 2004)

$$\delta\hat{\mathbf{x}} = \left(H^T W H + \bar{W}\right)^{-1} \left(H^T W \delta\mathbf{y} + \bar{W}\delta\bar{\mathbf{x}}\right) , \qquad (1)$$

where **x** is the unknown n-dimensional vector of solve-for parameters (differential corrections). The matrix H, called *design matrix,* contains the partial derivatives of the observable quantities with respect to the solve-for parameters, W is a weight matrix and $\delta\bar{\boldsymbol{x}}$, $\bar{W}$ represent respectively the a priori estimate and covariance matrix of **x**. The final estimate is obtained through an iterative procedure based on Equation 1. The inverse of the *normal* matrix (H^TWH + W) can be seen as the covariance matrix of the vector **x.**

In the orbital phase around Mercury, MORE will address use Doppler and range data to (i) determine the Mercury-centric motion of MPO, enabling the gravimetry and rotational experiments; (ii) determine the heliocentric motion of Mercury, thus allowing fundamental physics tests. Furthermore, the integration time at which the errors in integrated Doppler and differenced range become approximately equal is $\sigma_r/\sigma_{rr} \approx 15$ hours. Accordingly, the Mercury-centric orbit of the MPO, characterized by a high frequency dynamics (one revolution around the planet takes about 2.3 hours), is assessed through the Doppler observables. On the contrary, the slower heliocentric motion of Mercury is essentially inferred from range data.

2.1 Non-gravitational accelerations and desaturation maneuvers

Due to the proximity of Mercury to the Sun, the spacecraft dynamics is heavily affected by intense non-gravitational perturbations due to the planetary albedo, infrared emission and thermal thrust, and the solar radiation pressure. To mitigate the problem, the MPO is equipped with a sensitive accelerometer, the Italian Spring Accelerometer (ISA, Iafolla et al., 2010), which will directly measure the vectorial non-gravitational acceleration. The instrument consists of three linear accelerometer elements, arranged along three orthogonal directions, approximately coinciding with the orbital reference frame[1]. The ISA readouts will be sent to ground in the telemetry stream and used to replace non-gravitational accelerations in the dynamical model of MPO, which therefore need not to be modeled. Such readings will be unavoidably affected by some sort of error $\boldsymbol{\varepsilon}(t)$. Especially important are systematic errors driven by temperature variations of the sensitive elements (ISA's sensors are torsional springs). In order to reduce adverse effects on the estimation of the physical quantities of interest, a suitable calibration strategy must be envisaged (see Section 2.3.2).

The presence of a single solar panel and the ensuing asymmetric shape of the spacecraft generate large torques, which are balanced by reaction wheels. Desaturation maneuvers may occur up to twice per day. The estimated maximum magnitude of the uncompensated Δv is 17 mm/s, 0.2 mm/s and 42 mm/s for respectively the radial, transversal and normal components in the orbital reference frame (Iafolla et al., 2011).

Even using the ISA readings of the non-gravitational accelerations along with a suitable calibration strategy, and including desaturation maneuvers in the set of solve-for parameters, unavoidable residual inaccuracies can still generate significant errors in the state propagation. To cope with this problem, a multi-arc strategy, as described in the following section, is commonly used.

2.2 Multi-arc strategies

The multi-arc strategy allows to absorb unmodeled effects by means of an over-parameterization of the problem. It consists in fragmenting the trajectory of the spacecraft into non-overlapping segments called *arcs*. A new set of spacecraft initial conditions are estimated for each arc, based on the observable quantities (Doppler and range),collected in that arc. The parameters are split into *local* $\mathbf{l}^i$, pertaining to each arc *i* (e.g. the spacecraft state vector and the Δv due to maneuvers) and *global* $\mathbf{g}$, related to the whole trajectory (e.g. the harmonic coefficients of the gravity field). For each set of observables $\mathbf{y}^i$ belonging to the arcs, the design matrix that links the observable residuals to the vector of solved-for parameters correction is expressed separating the local parameters to the global ones as

$$\delta\mathbf{y}^i = \begin{bmatrix} H^i_{l_i} & H^i_g \end{bmatrix} \begin{pmatrix} \delta\mathbf{l}^i \\ \delta\mathbf{g} \end{pmatrix} .$$

If the trajectory is split in *r* arcs, this results in

$$\begin{pmatrix} \delta\mathbf{y}^1 \\ \delta\mathbf{y}^2 \\ \vdots \\ \delta\mathbf{y}^r \end{pmatrix} = \begin{bmatrix} H^1_{l_1} & 0 & \dots & 0 & H^1_g \\ 0 & H^2_{l_2} & \dots & 0 & H^2_g \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & H^r_{l_r} & H^r_g \end{bmatrix} \begin{pmatrix} \delta\mathbf{l}^1 \\ \delta\mathbf{l}^2 \\ \vdots \\ \delta\mathbf{l}^r \\ \delta\mathbf{g} \end{pmatrix} .$$

The technique is properly called *pure* multi-arc strategy when the arcs are handled as completely independent each other, i.e. the corresponding orbits are effectively treated as if they belong to different spacecraft.

The orbit determination problem often shows rank deficiencies, which cause difficulties in the invertion of the normal matrix. For a spacecraft orbiting a planet and tracked from the Earth, there is an approximated rank deficiency if the arc length is negligible

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[1] In this frame, the x-axis points in the opposite direction of nadir (radial direction), the z-axis is normal to the orbital plane in the direction of the angular momentum vector (normal direction); the y-axis completes the right-handed system (transversal direction).

with respect to the orbital motion of the planet (Bonanno & Milani 2002). To mitigate the problem, we adopt a *constrained* multi-arc strategy, where positions and velocities of two subsequent arcs are fixed by given discrepancies at their connection time. This can be obtained including a priori observations in the least squares fit, following the standard procedure illustrated in Milani et al. (1995). Let $\mathbf{r}_T^i$ and $\mathbf{r}_T^{i+1}$ be the spacecraft positions at the connection time T of two subsequent arcs. The continuity condition at the arc boundary can be imposed as

$$r_T^i - r_T^{i+1} = \left[\mathbf{r}_T^i - \bar{\mathbf{r}}_T^i\right] - \left[\mathbf{r}_T^{i+1} - \bar{\mathbf{r}}_T^{i+1}\right] = \mathcal{N}(\sigma_i, 0)\,, \quad (2)$$

where $\bar{\boldsymbol{r}}_T^i$ and $\bar{\boldsymbol{r}}_T^{i+1}$ are a priori values that fulfill the continuity condition. The residual is assumed to have a normal distribution N(σ_i,0) with zero mean and RMS σ_i. Note that the continuity condition is not exact. The selection of σ_i, which requires some experimenting, is a compromise between underconstraining and overconstraining the solution. Making use of the elements of the state transition matrix defined in each arc, the left-hand side of Equation 2 is written as

$$\begin{aligned}\frac{\partial \mathbf{r}_T^i}{\partial \mathbf{x}^i}\delta\bar{\mathbf{x}}^i - \frac{\partial \mathbf{r}_T^{i+1}}{\partial \mathbf{x}^{i+1}}\delta\bar{\mathbf{x}}^{i+1} &= \frac{\partial \mathbf{r}_T^i}{\partial \mathbf{l}^i}\delta\bar{\mathbf{l}}^i + \frac{\partial \mathbf{r}_T^i}{\partial \mathbf{g}}\delta\bar{\mathbf{g}} - \left[\frac{\partial \mathbf{r}_T^{i+1}}{\partial \mathbf{l}^{i+1}}\delta\bar{\mathbf{l}}^{i+1} + \frac{\partial \mathbf{r}_T^{i+1}}{\partial \mathbf{g}}\delta\bar{\mathbf{g}}\right] \\ &= \frac{\partial \mathbf{r}_T^i}{\partial \mathbf{l}^i}\delta\bar{\mathbf{l}}^i - \frac{\partial \mathbf{r}_T^{i+1}}{\partial \mathbf{l}^{i+1}}\delta\bar{\mathbf{l}}^{i+1} + \left[\frac{\partial \mathbf{r}_T^i}{\partial \mathbf{g}} - \frac{\partial \mathbf{r}_T^{i+1}}{\partial \mathbf{g}}\right]\delta\bar{\mathbf{g}}\,.\end{aligned}$$

Similar relations hold for the velocity components.

2.3 Assumptions and simulation setup

The latest mission profile for BepiColombo entails a one year orbital phase around Mercury, starting from 15 March 2026 (Jehn, 2016), with a possible extended mission of one additional year. The initial Mercurycentric orbit of MPO is polar and low eccentricity (480 x 1500 km altitude), with a period of about 2.3 hours and periherm latitude at ≅ 16° N. Two ground antennas are assumed available for tracking, DSS 25 in Goldstone (California), with Ka band uplink capabilities, and DSA 2 in Cebreros, enabling only X band uplink. The visibility conditions are met when the spacecraft elevation above the ground station local horizon is > 15°. Occultations of the orbiter behind Mercury are taken into account as well. As both antennas are in the boreal hemisphere, the observation windows change seasonally from about 6 hours in the boreal winter up to about 11 hours in the boreal summer, when they overlap. The ESA DSA 3 antenna in Malargue (Argentina) has currently experimental Ka band uplink capabilities and will also provide operational support to MORE and BepiColombo starting from 2020. Its availability will allow a better coverage from ground. For simplicity, the analysis is based on multifrequency data from DSS 25 only.

Data collected at DSS 25 (and DSA 3 as well) can exploit the cancellation of plasma noise allowed by the multi-frequency link. Plasma noise cancellation is assumed to be possible till the signal path passes at seven solar radii away from the Sun (Tortora et al., 2004). Data acquired below this threshold (smaller impact parameter) are discarded. The measurement error is considered white and gaussian. Therefore the standard deviation of the measurement noise can be straightforwardly rescaled at any integration time: Doppler and range are simulated respectively every 60 and 300 seconds, with accuracies of 12.25 x 10^{-4} cm/s and 30 cm. These values are equivalent to an ADEV of 10^{-14} at 1000 seconds integration time for Doppler and the acquisition of one range measurement with an accuracy of 20 cm every 10 minutes (a realistic assumption). As DSA 2 only uses X band uplink, an error ten times larger is assumed.

Power constraints impose to switch off some instruments during perihelion transits (mean anomaly ± 30°, corresponding to ≈ 10 days). The current mission guidelines indicate that the KaT can be operated only in one perihelion passage. Therefore we assumed the availability of plasma-free observables only during the first perihelion, taking place on 18 May 2026. This choice allows to fully exploit the superior solar conjunction event occurring on 14 May 2026 for the relativistic gravity test (see Section 4.3). This important restriction will be revisited once the actual performance of the flight system is assessed in cruise.

2.3.1 Arcs configuration

The subdivision of the trajectory in arcs is done by attributing to each arc a duration of 24 h and by including in each arc a daily tracking session of both ground stations. For the constrained multi-arc strategy, we set the discrepancies δr and $\delta v = n\delta r$ for each of the position and velocity components in the inertial frame (the Earth mean ecliptic J2000 in our setup), where $n \approx 7.3 \times 10^{-4}$ rad/s is the mean motion of MPO. The tighter the constraints, the smaller the formal uncertainties are with respect to those attainable with a pure multi-arc solution. The constraints become effective when δr is set below a few tens of meters. The main advantage of the constrained multi-arc is the improvement of the spacecraft positioning in the transverse and normal directions of the orbital reference frame. The improvements in the radial positioning, hence in the geodesy results, are much less appreciable. How tight the continuity condition can be results from a compromise between the effectiveness of the constraints, the stability of the solution, and the adverse effect on the estimate (bias). In this work we have assumed a conservative value $\delta r = 1$ m, therefore $\delta v = 7.3 \times 10^{-4}$ m/s.

Figure 1 summarizes the typical arc configuration: the state vector is determined at the central time of each arc, exploiting a forward/backward integration. Following the expectations of the mission planners, a first desaturation maneuver is assumed during the X band tracking period, and a second one after the multi-frequency tracking pass. Both maneuvers are modeled as simple impulsive burns. In a pure multi-arc strategy there is no sensitivity to the second maneuver, taking place when tracking from ground is not available. However, the Δv from this maneuver can be estimated in a constrained multi-arc approach. In this case, the estimate will be controlled by the tolerance selected for the velocity discrepancy among subsequent arcs (see Section 6.4).

2.3.2 Accelerometer calibration

The calibration of the error $\boldsymbol{\varepsilon}(t)$ introduced by the ISA readouts in the reconstructed dynamics of the MPO is based on the inclusion of an additional acceleration $\mathbf{c}(t,\psi_i)$. This is a function of a certain number of unknown parameters ψ_i, which are hence added to the solution. The proper choice of $\mathbf{c}(t,\psi_i)$ guarantees that the estimate of the parameters ψ_i is such that $\mathbf{c}(t) \approx -\boldsymbol{\varepsilon}(t)$, canceling out the error at the penalty of some degradations of the formal solution. We envisaged two strategies, aimed at absorbing respectively the low and high frequency components of the error. The typical time scales of the two components are respectively 24 h and 2.3 h (the orbital period). For the low frequency components, we consider in each arc an acceleration $\mathbf{a}_{LF}=\mathbf{b}_0+ \mathbf{b}_1 t$. The bias $\mathbf{b}_0$ and the bias rate $\mathbf{b}_1$ are included in the solution as local, solve-for, parameters. A rough calibration of the high frequency errors is obtained by including a periodic acceleration $\mathbf{a}_{HF} = \boldsymbol{\alpha}_s \sin[\nu(t)] + \boldsymbol{\alpha}_c \cos[\nu(t)]$, where $\nu(t)$ is the true anomaly of MPO. We estimate the parameters $\boldsymbol{\alpha}_s$ and $\boldsymbol{\alpha}_c$ as global parameters.

Since ground tests provide only a crude assessment of the ISA low frequency noise, we set $\varepsilon(t) = 0$. Therefore, in our analysis we cannot address the impact in the solution of some residual errors not absorbed by calibration. We discuss this point in Section 8.2.

3 The Solar Conjunction Experiment in cruise

According to GR, the presence of any mass induces a curvature in the space-time. In a parametrized post-Newtonian expansion of the Minkowsky metric, the space-time curvature induced by a mass is controlled by the parameter γ, unity in GR. As a consequence, the propagation of radio waves undergoes a deflection, a delay (known as Shapiro delay), and a frequency shift. These phenomena are magnified when the signal path is close to the curvature-generating body, e.g. when the signal is exchanged between a ground antenna and a spececraft close to a superior solar conjunction (SSC). In this situation, the delay and frequency shift affecting the single-leg propagation can be approximated as (Will, 2006; Bertotti et al, 2003)

$$\Delta t = \frac{(1+\gamma)GM_{\odot}}{c^3}\ln\left(\frac{4r_1r_2}{b^2}\right); \quad \frac{d\Delta\nu}{\nu} = \frac{d\Delta t}{dt} = -2\frac{(1+\gamma)GM_{\odot}}{c^3 b}\frac{db}{dt};$$

where G is the gravitational constant, M_0 the mass of the Sun, the velocity of light and b is called *impact parameter*, defined as the distance from the center of mass of the Sun and the light path. The lower the value of b and the higher that of db/dt, the higher the delay and frequency shift are. The baseline trajectory with launch in October 2018 entails a 7.2 years cruise and eleven superior solar conjunctions before arrival at Mercury (Jehn, 2016). Each SSC offers a chance to measure the parameter γ and to carry out a test of GR.

A comprehensive analysis of the BepiColombo SCE in cruise is given in Imperi and Iess (2017). The parameter γ can be determined with an accuracy of $\cong 6 \times 10^{-6}$ exploiting data of just one favourable conjunction in July 2022, thus potentially improving by a factor of four the estimate provided by Cassini (Bertotti et al., 2003). Data from different conjunctions may be combined in order to increase the accuracy of the result. However, the solar-electric propulsion system of the spacecraft shall be active during many SSC, which therefore cannot be exploited for the test because of the large dynamical noise induced on the spacecraft. Nevertheless, an accuracy of $\cong 3.5 \times 10^{-6}$ can be attained by exploiting only thrust-free SSC. Moreover, provided that small changes in the thrust profile are considered by the project, an accuracy below 3×10^{-6} could be attained if solar electric propulsion is deactivated for two weeks around each conjunction.

It should be stressed that the SCE of BepiColombo may suffer of a significant dynamical noise due to oscillations of the solar irradiance over time scales of days/weeks, which cannot be neither predicted nor measured. The degradation of the results is currently under investigation.

4 The fundamental physics tests

4.1 The role of Mercury in testing general relativity

Urbain Le Verrier (LeVerrier, 1859) was the first to point out in 1859 that Mercury's orbit was precessing at a rate (38 arcsec/cy) that could not be explained by perturbations due to known solar system bodies. The anomalous precession, slightly adjusted to 43 arcsec/cy by Simon Newcomb (Newcomb, 1895), was matter of speculations for many years. Different explanations were proposed, such as the presence of an additional mass, still hidden to observation, distributed in a belt of many small bodies (the *Vulcanoids*), or by a Mercury's moon. A solar equatorial bulge, as well as modifications of the classical $1/r^2$ law of gravity, or velocity dependent corrections to Newtonian gravity, were also suggested. All proposed solutions failed the observational tests and did not give a coherent solution to the problem. This puzzling finding was finally and brilliantly solved only by GR and the new law of gravity (Einstein, 1915). Among many other predictions, GR implies deviations from a pure Newtonian motion: the higher the velocity of the body and the gravity field in which it is moving, the higher the deviations will be. For Mercury, the innermost planet of the solar system and the one laying deeper in the gravity field of the Sun, GR predicts an extra, non-Newtonian, perihelion drift of exactly 43 arcsec/century.

A precise determination of Mercury's dynamics can be used to put limits to the validity of GR. Starting from 1959, interplanetary radar observations showed that GR agreed with the measured echo delays at a level of 0.3% (Shapiro, 1990), the main limitation being the poor knowledge in Mercury's topography (Pitjeva, 1993). The inclusion of MESSENGER data in a global planetary fit has allowed to improve the knowledge of Mercury motion (Mazarico et al., 2014, Fienga et al., 2014, Folkner et al., 2014) and to perform GR tests (Verma et al., 2014, Fienga et al., 2015, Park et al., 2017). The accuracy of MESSENGER data is however limited however, they are limited by the accuracy of radio observables. A major advance in the accuracy of GR tests is expected from BepiColombo, whose full complement of onboard instruments and state of the art ground tracking system allow a strong reducion of the measurement and dynamical noise.

4.2 Dynamical relativistic models

To test the predictions of GR, the simplest and more frequently adopted approach uses the slow-motion, weak-field limit (Will, 1993). This approach is sufficiently accurate to encompass most solar system tests, including MORE. In this approximation, known as the Post-Newtonian (PN) limit, the space-time metric can be written as an expansion in terms of dimensionless gravitational potentials about the Minkowsky metric. An arbitrarily, multiplicative, parameter is introduced in front of each PN term of the metric. The formalism using parameters to describe the PN limit of metric theories of gravity is called the *Parameterized* PN formalism (PPN). In particular, each PPN parameter can be chosen such as to have a special physical significance. The goal of the MORE investigation is to measure the contribution of each PPN parameter, i.e. of each property of the metric, to the dynamics of Mercury. Determining the value of each PPN parameters with a given experimental accuracy provides a test of the validity of GR predictions, as well as the predictions of competing theories of gravity (see Will 2014 for an exhaustive discussion).

The choice of the PPN parameters to be estimated by MORE is dictated by the actual possibility of improving their determination. Following the first assessment by Milani et al. (2002), we choose:

- The *Eddington* parameters γ and β. These parameters are unity in GR. In the standard PPN gauge, β is related to the non-linearity in the superposition law for gravity (Will, 1993), while γ measures the space curvature produced by a unit rest mass and parameterizes the velocity-dependent modification of the 2-body interaction;
- The *Nordtvedt* parameter η, which describes violations of the strong equivalence principle. In GR, the gravitational self-energy of a body does not contribute to the

gravitational mass (η=0). This principle states the equivalence between the gravitational and inertial mass.

- The *preferred frame effect* parameters α_1 and α_2. They describe phenomenologically the effects due to the presence of a gravitationally preferred frame. We follow the standard assumption and identify the preferred frame as the frame where the cosmic microwave background has no dipole component (Durrer, 2015).

Additional physical quantities are taken into account and determined. They are the gravitational parameter of the Sun μ_o=GM_o, its time derivative $\zeta = 1/\mu_o \, (d\mu_o/dt)$ and the solar oblateness factor J_2. These parameters have both a physical relevance and a non-negligible uncertainty for the purpose of the experiment. In particular, the effects of the solar equatorial bulge on the motion of Mercury plays a crucial role because they are very similar to those due to β (see Section 4.4). Concerning ζ, its interest lies in the possibility to detect, or set upper limits to, temporal variations of Newton's gravitational constant *G*, envisaged by many alternative theories of gravity. While a desirable goal would be a determination of *dG/dt*, the choice of ζ is driven by the fact that in practice MORE could not discriminate between time changes of *G* and M_o. The estimated value of $(dM_o/dt)/M_o$ due to electromagnetic radiation and the solar wind is about 10^{-13} (Withbroe and Noyes, 1977). While the radiative losses are well known, the particle mass loss due to solar wind is more uncertain. Anyway, starting from a precise estimate of ζ, information on *dG/dt* can be retrieved Under reasonable assumptions on the mass rate of the Sun, we show that MORE can provide an interesting estimate of *dG/dt*.

4.3 Superior conjunctions during the orbital phase

Among PPN parameters, γ affects both the dynamics of Mercury and the propagation of radio waves. Hence, its determination can take advantage from both effects. As the synodic period between Mercury and the Earth is about four months, three SSC will take place during the nominal mission, while three more could be exploited in the extended phase, as summarized in Table 1. Note that during the first SSC in May 2026 Mercury will be occulted behind the solar disk ($b_{min} < R_o$).

4.4 Metric theories of gravity

A solar equatorial bulge was one of the hypotheses proposed in order to explain the anomalous perihelion drift of Mercury before the breakthrough of GR. In fact, the main orbital effect predicted by GR is the precession of the argument of perihelion, whose rate

$$\dot{\omega} = \frac{3GMn}{a(1-e^2)c^2}\left[\frac{1}{3}\left(2+2\gamma-\beta\right)+\frac{1}{6}\left(2\alpha_1-\alpha_2\right)\frac{m}{M}\right]+\frac{3nJ_2R^2}{2a^2(1-e^2)^2}$$

depends on the quantity $2\gamma - \beta$. In the above formula (valid in the approximation m<<M, therefore the contributions of alpha1 and alpha2 is negligible) M is the mass of the Sun, m the mass of Mercury, n is Mercury's mean motion, a the semimajor axis, e the eccentricity, R the equatorial radius of the sun and J2 zonal quadrupole coefficient if the sun. The solar gravitational oblateness (J_2) causes also a precession of the longitude of the ascending node. However, given the small angular distance between the solar equatorial plane and the orbital plane of Mercury (about 3.3°), this precession is not well bserved. The net effect of the solar oblateness is hence very similar to that induced by GR. As γ is independently measured by means of the Shapiro time delay and Doppler shift of photons, separating the effect of β and J_2

cannot be carried out with good accuracy by looking only at the motion of Mercury. This correlation is broken in metric theories of gravity by imposing the Nordtvedt relation (Nordtvedt, 1970):

$$\eta = 4(\beta - 1) - (\gamma - 1) - \alpha_1 - \frac{2}{3}\alpha_2 \,. \qquad (3)$$

4.5 Rank deficiencies in the planetary orbit determination problem

The radio observables are sensitive to variations in the orbits of Mercury and the Earth. Therefore, even if only Mercury is subjected to relevant relativistic effects, one needs to solve also for the ephemeris of the Earth. However, it turns out that the Mercury-Earth dynamics configuration necessarily implies four rank deficiencies (see Milani et al., 2002 for an exhaustive discussion). Indeed, in a pure three-body problem with Mercury, the Sun and the Earth (and J_2 = 0), there would be an exact symmetry with respect to the full rotation group applied to the orbits of Mercury and the Earth. Moreover, if all the lengths are rescaled by a factor λ, all masses by a factor μ and all time intervals by a factor τ, provided that the scaling factors are related by $\lambda^3 = \tau^2\mu$ (Kepler's third law), the equations of motions would not change.

The coupling of the Earth-Mercury system with other bodies breaks the exact rank deficiencies. However, approximate rank deficiencies are still present, making a determination of all 12 initial conditions (position and velocity) of Mercury and the Earth impossible without a substantial deterioration of the global solution. To overcome this problem, we fix the Earth position (x,y,z) and its velocity component (v_z) orthogonal to the ecliptic orbital plane, thus making rescaling and rotation impossible. Therefore, only 8 of the 12 state vector parameters are determined, i.e. the entire state vector of Mercury and the velocity components of the Earth in the ecliptic plane.

4.6 Current status of GR tests in the solar system

The most accurate estimate of the Eddington parameter γ - 1 = (2.1 ± 2.3) x 10^{-5} was provided by measurements carried out during a solar conjunction in the cruise phase of the Cassini mission to Saturn (Bertotti et al., 2003).

Tests of GR are performed by means of global fits of optical and radio data, such as with the Developments Ephemeris (DE), the Integrateur Numerique Planetarie de l'Observatorie de Paris (INPOP) and the Ephemerides of Planets and Moon (EPM). The inclusion of some MESSENGER range data in the INPOP fit lead to a determination of the parameter β - 1 at level of ≈ 7 x 10^{-5} and of J_2 at level of 1.2 10^{-8} (Fienga et al., 2015). From helioseismology are instead inferred the values J_2 = (2.2 ± 0.1) x 10^{-7} (Mecheri et al., 2004) and J_2 = (2.18 ± 0.06) x 10^{-7} (PiJpers 1998). Partk et al. 2017, exploited range data of MESSENGER inferring the estimates β - 1 = (-2.7 ± 3.9) x 10^{-5} and J_2 = (2.25 ± 0.09) x 10^{-7}. A bound on α_1, at level of 4 x 10^{-5} is attained from the orbit of the pulsar-white dwarf system J1738+033333 (Shao & Wex 2012), while α_2 is determined up to an accuracy of 1.6 x 10^{-9} by combining data from two solitary milliseconds pulsars, PSRs B1937+21 and J1744-1134 (Shao et al. 2013). Alternative estimates are attained by Iorio (2014) who used the INPOP ephemeris to obtain the estimates α_1 = (-1 ± 6) x 10^{-6} and α_2 = (-0.9 ± 3.5) x 10^{-5} from solar system planetary precession. A different global planetary fit with the EPM dynamical model, provided the value of ζ = (-6.3 ± 4.3) x 10^{-14} (Pitjeva and Pitjev, 2013). The more conservative estimate $\zeta = (0.1 \pm 1.6) \times 10^{-13}$ was instead obtained studying the ephemeris of Mars by means of Mars Reconnaissance Orbiter (MRO) range data (Konopliv et al., 2011). For the gravitational parameter of the Sun, an uncertainty of 10 km^3/s^2 is obtained in Pitjeva, 2015, the same level of accuracy is quoted in the NASA Horizon system (Link in una nota apposita).

The best bound on the Nordtvedt parameter η, is obtained by accurately measuring the geocentric motion of the Moon with the Lunar Laser Ranging (LLR). Indeed η causes a polarization of the Moon's orbit about the Earth (Nordtvedt, 1968). LLR investigation provided the estimates $\beta - 1 = (1.2 \pm 1.1) \times 10^{-4}$ and $\eta = (4.4 \pm 4.5) \times 10^{-4}$ (Williams et al., 2009), under the hypothesis of metric theories of gravity, i.e. imposing the Nordtvedt relation in Equation 3.

The current limits available in literature are summarized in Table 2.

5 The geodesy experiment

5.1 The rotational state of Mercury

Mercury's orbital and rotational periods are respectively 87.96 and 58.65 days (Stark 2015. The capture in the 3:2 spin-orbit resonance is a consequence of the chaotic dynamics of the planet (Correia and Laskar, 2004), although how exactly and when the capture took place is still an open question (see e.g. Noyelles et al., 2014, for a detailed discussion).

Colombo and Shapiro (1966), and Peale (1969) proposed that Mercury occupies a Cassini state of type 1[2], as it has been shown by radar measurements (Margot et al., 2007). Mercury exhibits longitude librations around the 3:2 spin-orbit secular equilibrium, due to the non-spherical mass distribution of the planet, which triggers a gravitational torque from the Sun. The torque is variable along the orbit because of the eccentricity and the difference between orbital and rotational periods. The main libration at the orbital period of ≅ 88 days was measured as ≅ 39 arcsec (Margot et al., 2007, 2012, Stark et al., 2015). This value corresponds to a maximum displacement of ≅ 460 meters at the equator.

Changes in the heliocentric motion of Mercury due to planetary perturbations give a rise to further forced librations. They have been predicted by several authors, who pointed out how different interior models of a differentiated Mercury lead to librations of different periods and amplitude (Dufey et al., 2008; Peale et al., 2009; Yseboodt et al., 2010; Dumberry, 2011; Dumberry et al., 2013; Yseboodt et al., 2013). These librations are expected to have small amplitudes unless the frequency of the forcing torque is close to a free libration frequency of Mercury (a situation that would trigger a resonance).

5.2 From gravimetry and rotation to the internal structure of Mercury

Optical, radar and Mariner 10 investigations yielded the mean density of Mercury as ≅ 5440 kg/m^3. This large value have been taken as evidence that iron is the most abundant contributor to bulk composition (Ness, 1978). The intrinsic magnetic field and the preference for an internal dynamo as the source of this field (Ness, 1979) are the primary motivations for believing the existence of a conducting molten core inside the planet. Peale (1976) argued that the measurement of the second degree gravitational harmonic coefficients, obliquity, and

[2] A Cassini state implies that the spin axis, the orbit normal, and the normal to the Laplace plane, are coplanar. The obliquity θ, i.e. the angle among the spin and the orbital normal, remains constant during the precession around the pole of the Laplace plane. The Cassini state 1 is the case when the orbit normal lies in between the others two. The Laplace plane is the plane about which variations in orbital inclination are minimized, ideally zero. The Laplace pole of Mercury is located at about 66.6° longitude and 86.725° latitude with respect to the ecliptic frame of J2000. The precessional period is ≈ 300000 years (Yseboodt and Margot, 2006).

amplitude of the physical libration in longitude would be sufficient to determine whether or not Mercury has a molten core.

The method proposed by Peale is described in detail in several works (Peale 1988; Peale et al., 2002; Peale 2005, among others). It is based on the assumption of a multi-layer model of Mercury's interior structure, made up by a core and a mantle (plus crust). The principal moments of the inertia of the entire planet and of the mantle alone are respectively $A < B < C$ and $A_m < B_m < C_m$. The Peale's procedure provides the ratio $C_m/C \leq 1$, which would be unity for a rigid body, and significantly less (about 0.5) for a body with a liquid layer or core. The ratio can be written as

$$\frac{C_m}{C} = \left(\frac{C_m}{B-A}\right) \times \left(\frac{MR^2}{C}\right) \times \left(\frac{B-A}{MR^2}\right), \tag{4}$$

where M and R are the mass and radius of Mercury. The procedure relies on two main assumptions: (i) the core must not follow the 88 days physical libration of the mantle; (ii) the core must follow the mantle on the timescale of the ≈ 300000 years precession of the spin in Cassini state 1. Under these assumptions, the factors in Equation 4 can be found from the following relations (Peale 2005)

$$\phi = \frac{3}{2}\left(\frac{B-A}{C_m}\right)\left(1 - 11e^2 + \frac{959}{48}e^4 + O(e^6)\right); \quad \frac{B-A}{MR^2} = 4C_{22};$$

$$\frac{C}{MR^2} = \frac{n\left[\frac{J_2}{(1-e^2)^{\frac{3}{2}}} + 2C_{22}\left(\frac{7}{2}e - \frac{123}{16}e^3 + O(e^5)\right)\right]}{\frac{\omega_p \sin i_L}{\vartheta_C} + \omega_p \cos i_L};$$

where ϕ is the amplitude of the 88 days librations in longitude, θ_C is the obliquity of the Cassini state 1 which Mercury is expected to occupy, i_L is the inclination of the orbit plane to the Laplace plane on which Mercury precesses at the constant rate $-\omega_p$, n is the mean orbital motion and e the orbital eccentricity. The Peale's procedure requires therefore the measurement of just three quantities, namely the amplitude of the physical libtrations in longitude, the obliquity, and the quadrupole gravity coefficients J2 and C22. The quantity C/MR^2, known as the concentrator factor, is 0.4 for an homogenous body, and is substantially lower for a body like Mercury.

5.2.1 Mantle-core interaction and inner core

Mercury is thought to host a solid inner core inside its liquid core. The formalism presented in Section 5.2 does not take into account distortions at the core-mantle boundary (Gao and Stevenson, 2012), and, if present, at the inner core boundary. Many efforts have been done in order to evaluate the core contribution to the rotational state (Dumberry, 2011; Dumberry et al., 2013; Van Hoolst et al., 2012; Yseboodt et al., 2013; Peale et al., 2014, 2016). If the inner solid core is large enough (> 1000 km), it affects substantially both the short-term and long-term rotational behaviour. The amplitude of the 88 days libration may change by up to 1 arcsec (or 10 m on the equator) if the inner core radius is 1500 km (Van Hoolst et al., 2012). This value is at the threshold of current accuracy, but is potentially well measurable by MORE. The planetary-induced librations would change as well (see Section 5.5). Furthermore, (Peale et al., (2016) showed also how a large solid inner core would require a revision of the concentrator factor estimate from the obliquity.

5.3 Geodetic models

Using spherical coordinates (r, φ, λ) in a body-fixed reference frame and a normalized representation, the potential *V* generated outside by an isolated body of mass *M* and mean radius *R*, is expressed as (Kaula, 1966)

$$V(r,\varphi,\lambda) = -\frac{GM}{r}\left[1+\sum_{\ell=2}^{\infty}\sum_{m=0}^{\ell}\left(\frac{R}{r}\right)^{\ell}\bar{P}_{\ell m}(\sin\varphi)\left[\bar{C}_{\ell m}\cos(m\lambda)+\bar{S}_{\ell m}\sin(m\lambda)\right]\right], \qquad (5)$$

where *l* and *m* are the spherical harmonics's degree and order, $\bar{P}_{lm}$ are the normalized Legendre polynomials and $\bar{C}_{lm}$, $\bar{S}_{lm}$ are normalized harmonics coefficients. Generally, the coefficients of the same degree have the same order of magnitude. The power associated with degree *l* in a spherical harmonics expansion is introduced as $P_l = [\frac{1}{2l+1}\sum_m \overline{(C}^2{}_{lm} + \bar{S}^2{}_{lm})]^{1/2}$, analogous to the energy spectrum (apart of the square root) for a two-dimensional scalar field.

Equation 5 describes the gravity field in a body-fixed frame. A rotation matrix provides the transformation to the inertial frame in which the satellite's equations of motion are solved. The elements of the matrix are functions of time-dependent angles, such as the right ascension, declination and prime meridian of the planet-fixed frame with respect to the inertial frame. These angles are in turn functions of the obliquity, amplitude of physical librations, and mean rotation rate. Hence, the determination of the MPO orbit provides information on the rotational state of Mercury from the rotation of its gravity field.

The Love numbers k_l, controlling Mercury's gravitational response to the tidal field caused by the Sun, are important quantities in building models of the interior structure of the planet (VanHoolst and Jacobs, 2003). As the ratio between the radius of Mercury and the Sun-Mercury distance is small (≈ 3.5×10^{-3}), only the quadrupole Love number k_2 is accessible to BepiColombo. (A determination of k_3, although possible, is marginal.) A precise measurement of the Love number k_2 could rule out a range of interior models that are otherwise compatible with observations of the rotation and gravity of Mercury. Indeed, k_2 is particularly sensitive to core size and outer solid shell thickness. It also varies with the rigidity and temperature of the mantle layer (Padovan et al., 2014).

5.4 Current status of knowledge

With the analysis of three years of MESSENGER radio data, a global determination of Mercury's gravity field to degree and order 50 (named *HgM005*), was provided by Mazarico et al. (2014), extending earlier gravity solutions (Smith et al. 2012; Genova et al. 2013). The orbit of MESSENGER was highly eccentric (200x15000 km altitude), with periherm located at ≅ 60° N. Because of the large altitude variations, Kaula's rule ($P_l=A_K/l^2$) with scale factor A_K = 1.25×10^{-5} was used to constrain the gravity harmonics coefficients for degrees $l > 7$. Gravity perturbations further reduced the periherm altitude down to few tens of km in the last phase of the mission, allowing an excellent gravity mapping of the northern hemisphere. Gravity and topography data collected over the northern regions were used to infer the thickness of Mercury's crust as (35 ± 18) km (Padovan et al. 2015). The uncertainty of the gravity anomalies are at the level of few mGal in the northern hemisphere, growing up to more than 100 mGal in the southern regions.

Plausible estimates for the tidal Love number k_2 are in the range 0.43 - 0.5, with a preferred value of 0.451. A different analysis of MESSENGER radio data, covering almost the same time span, was recently provided by Verma and Margot (2016). The solution for the gravity field is fully consistent with HgM005, with a value of k_2 in the range 0.420 - 0.465 and a preferential estimate of 0.464. According to Padovan et al. (2014), this larger value would favor interior models with a hotter and weaker mantle than the estimate from Mazarico et al. (2014). The HgM005 formal solution and the estimated errors in the quadrupole coefficients, along with the estimates of the Love number k_2 are summarized in Table 3.

The obliquity θ and the amplitude of the 88 day librations in longitude ϕ obtained with radar observations allowed Margot et al. (2007) to confirm that Mercury is in or near to a Cassini state and that the mantle is decoupled from a large, at least partially molten, core. The results were further refined later on with additional radar data, ending in a final estimate of ϕ=38.5 ± 1.6 arcsec and θ = 2.04 ± 0.08 arcmin. The use of the procedure in Section 5.2, provided a concentration factor C/MR^2 = 0.346 ± 0.014 and a ratio C_m/C = 0.431 ± 0.025 (Margot et al. 2012). Stark et al. (2015) measured the physical librations and the obliquity using a completely different method based on laser altimeter and imaging camera data. This new analysis confirmed previous estimates. In addition, Stark et al. also estimated the mean rotation rate of the planet, which was found higher by 14.72 arcsec/year than the value expected from a 3:2 spin-orbit resonance. A possible explanation is a long-period forced libration due to Jupiter perturbations on the Mercury's heliocentric motion (see Section 5.5).

The current estimates are summarized in Table 4.

Gravity and rotation inputs were used in Hauck et al. (2013) to estimate that the outer limit of the liquid core is found at a radius of (2020 ± 30) km. The mean density above and below this boundary is respectively (3380 ± 200 kg/m^3) and (6980 ± 280 kg/m^3). Similar results are found in Rivoldini and Van Hoolst (2013). A discussion on the possible inner core size is given in Dumberry and Rivoldini (2015). Current data cannot exclude an inner core larger than 1000 km. However, an inner core smaller than 1000 km is preferred according to data and models. Likewise, dynamo models able of reproducing Mercury's magnetic field favor an inner core with radius < 1000 km (Cao et al. 2014).

5.5 Planet-induced librations

Librations induced by planetary perturbations on the heliocentric motion of Mercury can be enhanced if the period of the perturbation is close to a free libration period of the planet. A two layer model of Mercury predicts a unique free libration mode whose angular frequency is (Peale et al. 2009; Yseboodt et al. 2010):

$$\omega_0 = n\sqrt{3\left(\frac{7}{2}e + \frac{123}{16}e^3 + O(e^5)\right)\frac{B-A}{C_m}},$$

which corresponds to ≅ 11.6 years accounting for the value of $(B-A)/C_m$ computed with the available estimates of ϕ. As the perturbation of Jupiter has a periodicity of 11.86 years, the Jupiter-induced librations near the free libration frequency of Mercury could be magnified to several tens of arcsec (see Section 6.5). Such a situation is consistent, and could explain, the measured mean rotation rate detected by Stark et al. (2015), as suggested by the authors.

However, a contribution to the librations from the inner core is an equally plausible scenario. Indeed, Yseboodt et al. (2013) showed that the inclusion of an inner core inside the outer liquid core in Mercury's rotational model generates a second free libration mode. For a small inner core, the first free period is close to Jupiter one, but a large inner core would led the free period close to a perturbation driven by Saturn ($\approx$ 14 years, half Saturn's orbital period). Depending on the size of the inner core, the second free period may be instead resonant with several orbit perturbations, even the 11.86 year perturbation due to Jupiter.

6 Numerical simulations

Thanks to its full set of dedicated geodesy instrumentation, BepiColombo is expected to improve significantly MESSENGER's results. We have quantified this potential improvement by means of numerical simulations based on the reference scenario and the assumptions given in Section 2. The simulations include the relativity, geodesy, and rotation experiment in a single setup.

The solve-for parameters related to the fundamental physics test are:

- the state vector of Mercury and the velocity components of the Earth in the ecliptic plane, determined at the epoch of November 1, 2025 (see Section 6.2);
- the PPN parameters β, γ, η, α_1, α_2, and $\mu_\odot$, $J_{2\odot}$ and ζ.

First guess values for $J_{2\odot}$ and $\mu_\odot$, as well as masses and orbits of all planetary bodies (including the Moon), are taken from the JPL ephemeris DE430 (Folkner et al. 2014). The first guess for the PPN parameters is the prediction of GR (i.e. $\beta = \gamma = 1$; $\eta = \alpha_1 = \alpha_2 = \zeta = 0$).

The estimated parameters in the gravimetry and rotational experiments are:

- the state vector of MPO in each arc;
- the gravity field harmonics coefficients up to degree and order 50;
- the (complex) Love number k_2;
- right ascension α and declination δ of Mercury's spin axis in the ICRF at J2000;
- the amplitude of the libration in longitude at 88 days ϕ;

The reference values for Mercury physical parameters and gravity field (mass, radius, gravity harmonics, and Love number k_2) are those provided in Mazarico et al. (2014). The rotation model is based on Margot (2009). The simulate dvalue of the obliquity θ is 2.1 arcmin. Its formal accuracy is computed with the classical uncertainty propagation formula from righ ascension and declination estimates.

The results that we present are obtained with a global fit addressing the three experiments (gravimetry, rotation and relativity) simultaneously. However, we verified with dedicated simulations that the fundamental physics tests and the geodesy experiment are *de facto* decoupled.

6.1 PPN and related parameters

The results for the PPN and related parameters are reported in the top panel of Table 5. Taking as a reference the current accuracies on each parameter reported in Table 2, with our assumptions, an improvement of two orders of magnitude is expected about β η and $\mu_\odot$. Improvements at a level of one order of magnitude can be attained for the parameters α_1, γ

and $J_{2\odot}$. It is important to remark that the accuracy of 1.1 x 10^{-6} obtained for γ is six times smaller than the estimate attainable from the SCE in cruise (exploiting a sinble favourable SSC, Imperi and Iess, 2017). A smaller improvement is attained for ζ, whose estimate is only about 1.5 times better than the lower boundary existing in literature, obtained from a EPM planetary fit. As the determination of ζ depends essentially on the perturbation in the hermean longitude (an effect that is quadratic in time, Damour & Fraiese 1994), a significant improvement is expected if the mission will be extended by an additional year (see Section 7.1).

As discussed in Sect. 4.4, the determination of β and $J_{2\odot}$ is strongly correlated. The correlation coefficient among these parameters is 0.987 if no constraints are provided. The Nordtvedt relation (Eq. 3) breaks the correlation. Indeed, the estimate of \eta, weakly correlated with solar J2, constrains \beta, which in turn becomes disentangled from J2 in the Mercury's nodal precession. When the Nordtvedt relation is used, the accuracy in β and J2 is respectively 1 x 10^{-6} and 5.5 x 10^{-10}, a factor of 30 and 5 better than the unconstrained estimate.
For what concerns the parameter alpha_2, the estimate attainable with MORE could not challenge what obtained by looking for the precession of milliseconds pulsars, that is almost two orders of magnitude more accured. On the other hand, if we assume alpha2 known to the state of the art accuracy (2 x 10^{-9}) in our simulations, this implies about a 15% accuracy improvement in the estimates of J2, beta, alpha1 and eta, and 5% about mu. The determination of zeta and gamma, not related to alpha2 (neither physically nor through the Nordtvedt relation), remain instead unchanged.

6.2 planetary ephemeris

The position of Mercury can be determined with an accuracy of few centimeters with respect to the solar system barycenter. Its velocity and the two velocity components of the Earth orthogonal to the Ecliptic plane can be assessed at the level of few cm/s x 10^{-6} . We noticed that the estimate of the planet ephemeris, along with those of eta and beta, are very sensitive to the reference epoch chosen for the planet state vector estimation, this is due to the method we used to avoid planetary rank deficiencies (see Section 4.5). Infact, determining the full state vector of Mercury and the Earth, which is possible by means of suitable apriori constraints among parameters, turned out to solve the problem, making the solution to be almost insensitive from the reference epoch. The results presented here are in line with this second solution, which is still under investigation: they are obtained by selecting the November 1, 2025 as a reference epoch for the estimate of planetary state vector (8 components).

6.2 Gravity field

The global solution for the gravity field is plotted in Figure 2. MORE can bring a substantial improvement to the MESSENGER HgM005 gravity field, whose solution largely relies on the Kaula's constraint. Simulations performed without assuming any regularization show how the MORE gravity harmonics are substantially constrained by Kaula's rule only for degrees l > 30. The gravity anomaly uncertainties, plotted in the top panel of Figure 3, are below 5 mGal on the entire planet surface, with a better definition in the equatorial regions. This would result in an improvement of at least a factor 20 to the gravity field representation in the southern hemisphere, potentially enabling a comprehensive correlation with topography data. The better coverage of the equatorial and Southern mid-latitude regions is essentially due to the pericenter location, which precesses toward the South pole. Due to perturbations induced by the odd harmonics, the eccentricity increases, driving the pericenter to lower altitudes (Genova et al., 2013).

The uncertainties for the static quadrupole coefficients and the tidal Love number k_2 are reported in Table 6. The expected, excellent determination of k_2 (at the level of 2 and 3 x 10^{-4} for respectively the real and imaginary part) can be a powerful tool to discriminate the interior of Mercury among the models consistent with the rotation measurements (Padovan et al. 2014). The orientation of the principal axes in longitude with respect to the reference frame $\delta\psi = \arctan(S_{22}/C_{22})$ can be attained with an accuracy of about 1 arcsec, or about 10 meters on the equator.

6.3 Rotational parameters

The results for the estimate of rotational parameters are reported in Table 7. The obliquity can be detected at the level of 1 x 10^{-3} arcmin. We find an accuracy of 1.3 x 10^{-1} arcsec (or 1.5 m on the equator) for the 88 days libration in longitude ϕ. This value, one order of magnitude better than the current estimates, may either confirm or exclude the presence of a large inner core inside the outer liquid core (Van Hoolst et al. 2012). Cicalo et al. (2016) noted a strong correlation between ϕ and the high frequency error of the ISA accelerometer, simulated as a pure sinusoid at the orbital period. While our simulations effectively confirm this correlation under the same assumptions, we noticed that the correlation is strongly reduced when the argument of the sinusoid is more realistically assumed as the true anomaly of MPO, as we did consider (see Section 2.3.2).

6.4 Spacecraft initial conditions and desaturation maneuvers

The formal solution for the ephemeris of MPO are reported in Figure 4. The radial positioning of the spacecraft in a Mercury-centric frame can be attained at the centimeter level, a quite significant result for the proper referencing of the altimetric observables of BELA, the BepiColombo Laser Altimeter (Thomas et al. 2007). The position in the along-track and transverse directions is much more poorly determined, still with accuracies below ten meters.

The solution for the \delta-V from the wheel desaturation maneuvers (see Sect. 2.1) are plotted in Figure 5. The first maneuver, carried out in the X-band tracking period, is essentially estimated from radio tracking in that pass. The second maneuver, carried out when no tracking is available, is instead constrained only by imposing matching conditions among subsequent arcs (δr, δv), as described in Sect. 2.3.1.We have chosen the value of 0.73 mm/s as a threshold for the velocity discrepancies among subsequent arcs for each of the Cartesian components. Therefore, this value represents a limit for the absolute sensitivity of the estimate to the maneuver. This effectively arises from our results, where, apart the radial component of the maneuver, the transversal and normal ones are detected with an accuracy higher than the selected δv. This is an important indication that the constrained multi-arc strategy is effective. Note that this level of accuracy is comparable to the knowledge of the uncompensated Δv; an even better estimate may be possible by means of the ISA accelerometer (Iafolla et al. 2011). In any case, the inclusion of the out of tracking maneuver in in the solve-for list allows to better account for uncertainties in state propagation during the dark period.

6.5 Long-term forced librations

In Figure 6 we used the model described in(Peale et al., 2009; Yseboodt et al. 2010) to compute the amplitude and phase of the Jupiter-induced librations as a function of the parameter (B-A)/C_m. According to these predictions and to the latest estimates (Table 4), we included a libration at 11.86 years with an amplitude of 40 arcsec[3] in our model. We also added the amplitude of this forced libration in the set of solve-for parameters, finding an accuracy of 8.5 x 10^{-1} arcsec. The penalty is just a small degradation (less than a factor of 2) in the estimate of S_{22}. This accuracy is quite good, substantially lower than the expected possible amplitudes of the forced libration (see left side of Figure 6). The estimate is facilitated by the epoch of the BepiColombo injection into Mercury's orbit, taking place when the amplitude rate is close to its maximum (see Figure 7).

Although this result could be of a paramount importance for constraining the internal structure of the planet, a number of issues shall be carefully taken in consideration. When the current uncertainties on the parameter (B-A)/C_m, shown in Table 4, are compared with the right panel in Fig. 6, it is apparent that the phase of the Jupiter-induced libration cannot be inferred. Actually, the situation is far more complicated, as the possible presence of a (large) inner core admits the enhancement of different planetary-induced librations (Yseboodt et al. 2013). Given these considerations, including in the set of solve-for parameters the amplitude of a libration with fixed frequency and phase may led to incorrect results.

We performed a simulation including the mean rotation rate of Mercury rather than a fixed libration in the least squares fit. We find that the mean rate can be measured with an accuracy of 8.5 x 10^{-10} deg/day. The drawback is a strong deterioration of the estimate of S_{22}, whose formal error grows up by a factor of about 20, resulting in an error in positioning of the prime meridian as large as 200 meters on the equator. Nevertheless, this penalty may be justified. Indeed, such a determination of the mean rotation rate, combined with the mean rotation rate detected by MESSENGER (and the prcecise estimate of the 88 days libration in longitude), allows to constrain the rotation of Mercury over a time scale of more than 10 years. This would strongly reduce the range of the admitted long-term librations, therefore driving the estimation strategy.

7 Benefits from an extended mission

The mission BepiColombo has been approved and funded for one year operations at Mercury. However there are excellent chances that the mission could be further extended for at least an additional year before running out of hydrazine to carry out the wheel desaturation maneuvers and loosing attitude control. In this section we assess the benefits of an extended mission for the science goals of the MORE investigation.

7.1 Fundamental physics

All relativistic effects producing secular perturbations to the orbit of Mercury are better estimated by extending the time basis of the observations. It is therefore apparent that the estimation of the PPN parameters \beta, \eta, \alpha1, \alpha2 (and ζ and J2 of the sun as well) is substantially improved in a two year mission. Of course, including some Mercury normal points from the MESSENGER mission would potentially provide an even better determination of those parameters.

Concerning ζ, we found an accuracy of 2.8 x 10^{-14} y^{-1} from the nominal mission, a value just 1.5 times smaller than the lower boundary found in the literature (although one may

[3] For our purposes, the amplitude is not important, although this value is realistic.

consider also the more conservative estimate provided by MRO, at level of 1.6 x 10^{-13} y^{-1}). As the main effect of ζ is on the mean longitude of Mercury, which grows quadratically with time (Damour and Esposito-Farese, 1994), its determination may greatly take advantage from an extended mission. This is clearly shown in Figure 8 where we report results of simulations assuming a mission duration of 15, 18 and 24 months. The results are given in terms of a gain factor, defined as the ratio between the formal accuracy obtained from the nominal and an extended mission. In a two years mission scenario, the determination of ζ can benefit of an improvement of almost one order of magnitude. Likewise, a significant gain is also found for α_1, with a factor ≅ 5 improvement, and by μ_o and J_{2o}, improved by almost a factor of 4. The benefits for the PPN parameters beta, eta and alpha2 are less significant, with gains below a factor of three. Note that even with an extended phase, the MORE estimate of alpha2 cannot reach the accuracy of 2 x 10^{-9} that is its the current best estimate.

7.2 Geodesy

The Keplerian elements of MPO hermean orbit at the beginning of the mission are written in Table 8. Thanks to the gravity field provided by MESSENGER, the evolution of the orbit's geometry due to gravity perturbations can be predicted very accurately (Genova et al., 2013). As the orbital inclination is ≅ 90°, the longitude of the ascending node and the inclination itself will not change, while substantial changes will occur on the argument of the pericenter and the eccentricity. The combination of the effects results in a drift of the pericenter toward the southern hemisphere and a reduction of its altitude on the surface. The expected behaviour of pericenter and argument of pericenter are plotted in the left side of Figure 9, while the right side gives the overview of the hermean orbit at the beginning of the mission and after two years.

After the one year nominal mission, the periherm will drift in a specular position ≅ 15° S across the equator. Since its altitude is simultaneously decreased down to ≅ 330 km, the gravity field in the southern regions is better resolved, as it can be noted in the top panel of Figure 3. During an extended phase, the pericenter will further drift toward the southern regions, while its altitude keeps decreasing. After two years the periherm will be at ≅ 43° S, at an altitude of ≅ 255 km. As a consequence the gravity field in the southern hemisphere could be estimated with much improved accuracy, while the gravity determination in the northern regions is essentially unchanged. This result is shown in Figure 3, where the gravity uncertainties attained with a mission duration of one and two years are compared. While there are no differences in the northern hemisphere, an accuracy at level of one mGal can be attained over extended regions of the southern hemisphere. The corresponding gravity field estimation in terms of harmonics cefficients is reported in Figure 2.

An extended mission would bring significant improvements also to the determination of the the rotational parameters and the Love number k_2. For a one year extension, the numerical simulations indicate gains of ≈ 1.5 - 2 with respect to the results shown in Tables 6 and 7. Even larger gains (about a factor of 3) are found for the Jupiter-induced librations.

8 Final remarks and future work

The DE430 solar system model (Folkner et al. 2014), taken as a reference, shall be slightly refined for the purpose of MORE. The indirect figure effect between Mercury and the

Sun (i.e. mainly the effect on Mercury itself due to the interaction of the hermean oblateness with the gravity field of the Sun) causes deviation on the motion of the planet above 1 m over the mission lifetime. Moreover, the Lense-Thirring acceleration due to the Sun's angular momentum can generate signatures of few meters in range data (Iorio et al. 2011). As the knowledge in the Sun's angular momentum is at the level of ≅ 1% thanks to helioseismology (Pitiev 1998), the (deterministic) inclusion of the effect should prevent any biased estimate.

Except for a small number of parameters, we assumed that the reference dynamical model of the solar system (masses and orbits of planets, satellites, and asteroids) is perfectly known. In reality, errors in the reference model may generate spurious signatures in range data, difficult to disentangle from actual deviations of GR. De Marchi et al. (2016) showed that a more realistic accuracy for the parameters η and β may be up to one order of magnitude worse than quoted in this paper. Using different assumptions, Ashby and Bender (20xx) also found much less encouraging results. (The discrepancy between previous estimates of the BepiColombo sensitivity to GR violations is currently under investigation.) Our preliminary attempts to quantify this problem indicate that this level of degradation may occur in all parameters but γ.(Indeed, γ is essentially determined from the propagation delay and Doppler shift of photons, which produce a time-localized signature in the data.)

A major issue in the determination of the parameters η and β (strictly correlated when the Nordtvedt relation is used) could be caused by errors in positioning the Earth. These errors were not considered in De Marchi et al. (2016). In Section 6.1 we stated that initial conditions for Mercury can be determined with centimeter level accuracy, provided that the Earth position is fixed (i.e. perfectly known). Aactually, real errors in positioning the Earth are expected to be at least some tens of cm in the radial heliocentric position[4], releasing our original assumption would invariably produce a degradation. This issue will be addressed in future work by implementing an estimate of the full state vector of Mercury and the Earth, using constraints to remove planetary rank deficiencies. This approach is encouraging, as it leads to some advantages in the estimate of η and β, and removes the dependence of the results from the reference epoch selected for the planetary state vector estimation (see Section 6.2)

Errors expected from an imperfect calibration of ISA readouts error, as well as systematic errors affecting the end-to-end ranging accuracy (expected not to exceed few cm), are less relevant in the relativistic gravity experiment (Schettino et al. 2016).

There are a number of assumptions which may affect the gravimetry and rotation results. In principle, we may use a tighter δr for constraining subsequent arcs, or an even longer time span for each arc, e.g. some days. This approach would result in better accuracies. Degradation of the gravity and orbit estimates are found when additional parameters are included in the solution to absorb unmodeled effects, such as the error introduced by the accelerometer readouts. Once real data are available, the selection of the calibration parameters, as well as the delta r and arc length, will be driven by the capability to fit radio tracking data to the noise level. The set of calibration parameters primary depends on the performance of the ISA measurements of the non-gravitational accelerations. Any systematic effects in the accelerometer (related for example to thermal drifts) will inject errors the reconstruction of the spacecraft dynamics.

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[4] This value can be roughly assessed by comparing different ephemerides releases. Different ephemerides provide different errors, up to some meters when comparing DE and INPOP with EPM. It is nearly impossible to evaluate the level of accuracy expected at the epoch of BepiColombo insertion into Mercury's orbit.

Although ground tests have met the design specifications, the behavior of ISA in the hermean environment may differ from the expectations. A better insight in the actual performances of the accelerometer will be gained from dedicated tests in the cruise phase. Radio tracking during the solar conjunction experiments, when the spacecraft dynamical and thermal environment is very stable, will assess the ISA performances at low frequencies by comparing the readouts of the instrument with the estimate of non-gravitational accelerations gathered from the least squares fit during cruise SCE (Imperi and Iess, 2017). Nevertheless, in order to have more realistic indications on the actual MORE sensitivities, we are investigating the suitable calibration strategy starting from different accelerometer error models. To absorb un-modeled effects, we are also studying the possible use of stocastic dynamical models, not considered so far.

Finally, we recall that the rotational parameters can be independently estimated from optical tracking of surface landmarks (Pfyffer et al. 2011). Note also that, if optical imaging and laser altimetry will provide accuracies similar to those attainable from gravity measurements, one could verify that the crust and the deep interior share the same pole and spin rate. Provided that this is the case, one can attain a combined solution. We have not attempted yet suach a combination. Cicalo et al. (2016) have shown that including optical observations in a global fit together with radio data tracking data, increases the accuracy in the estimate of the rotational parameters if the camera pointing errors (mostly due to thermal deformations) are at the level of 2-5 arcsec.

9 Conclusions

In this paper we analysed the radio science experiment of the mission BepiColombo to Mercury, taking into account the current mission profile and the instrument performances resulting from the final tests. . We described the physical background of the experiments, and used an up to date simulation scenario to perform numerical simulations covering all investigations to be carried out by MORE, both in fundamental physics and geodesy

In an optimal simulation scenario, and assuming that gravitation is described by a metric theory, we find that MORE will attain quite significant improvements in the determination of most PPN parameters. The parameters β and η can be improved by about two orders of magnitude over the current knowledge. Improvements at the level of one order of magnitude can be attained for α_1, γ. The precise determination of the hermean orbit brings allows also better estimates of other parameters relevant to solar system dynamics, such as the gravitational parameter and oblateness of the Sun (μ_o and J_{2o}). If the mission will be extended for an additional year, significant improvements will be attained also in the determination of the rate of change ζ of μ_o. On the contrary, the MORE estimate of alpha2 could not challenge the precise determination attained in Shao et al. 2013.

Systematic effects in range and non-gravitational acceleration measurements are not a limiting factor for the relativity investigation. The main limitation to tests of relativistic gravity comes instead from the imperfect knowledge of the solar system properties.

The global numerical simulations have updated and refined the expected accuracies of the gravity and rotation experiments at Mercury. We have shown that precise gravity measurements enabled by Ka band and multilink radio tracking of BepiColombo, complemented by measurements of the non-gravitational acceleration with the ISA accelerometer, may improve by a factor of 10 the estimate of the 88 day libration amplitude, thus suggesting or disproving the presence of a large inner core inner core inside the outer liquid core. Moreover, MORE could detect, or set upper limits to, Jupiter-induced librations, or different long-term planetary induced librations, therefore putting new, strong, constraints to the presence and the size of an inner solid core

The retrieval of the gravity anomalies is possible to accuracies below 5 mGal on the entire planet, while the gravity field can be determined to a full degree and order 30. The precise determination of the (complex) Love number k_2 will provide an independent constraint to further discriminate among models of the interior of Mercury. Thanks to the pericenter drift toward the south pole and the increasing eccentricity, we have shown that much improved results are attainable locally in the southern hemisphere if the mission will be extended by one year. High accuracies in the gravity determinations are important for correlative analysis with the topography measured by the BepiColombo laser altimeter.

As a final remark, we point out that BepiColombo and MESSENGER have complementary coverages of Mercury's gravity, with better accuracies respectively in the southern and northern hemispheres. For this reason, a combined solution from BepiColombo and MESSENGER radio tracking data would provide a more uniform map of the hermean gravity. Similar sinergies are possible also for the determination of some PPN parameters.

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