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Essays on Non-Linearities in Macroeconomics

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Abstract

This dissertation consists of three essays studying different topics in macroeconomics under the common aim of assessing the role of nonlinear dynamics in explaining selected facts of interest.

In Chapter 1, co-authored with Marzio Bassanin and Ester Faia, we explore the linkages between financial crises and debt markets, where collateral constraints and opacity of asset values are the norm. We, therefore, introduce ambiguity attitudes in beliefs formation in a small open economy model where borrowers investing in risky assets face occasionally binding collateral constraints. We estimate the ambiguity attitudes process and derive that borrowers endogenously act optimistically in booms and pessimistically in recessions. Analytically and numerically we show that our ambiguity attitudes coupled with the collateral constraints crucially help explaining asset price and debt cycle facts.

Chapter 2 studies the pass-through of sovereign risk in an environment where latent confidence factors, along with fundamentals, might feed debt crises. A Markov-switching VAR with three variables (private spread, sovereign spread, debt-to-GDP) is estimated on fiscally-leveraged economies (Italy, Spain, Portugal). By allowing fiscal and financial sources of amplification, the model historically identifies: *i*) an *high vulnerability regime*, where sovereign spreads show excessive sensitiveness to fiscal imbalances. Those periods line up mostly with the global financial turmoil and the sovereign European debt crisis; *ii*) an *high synchronization* regime where the sovereign and financial risk measures are strongly tied in a synchronized comovement. Those period identify more the first phases of the two crises.

Finally, Chapter 3, co-authored with Othman Bouabdallah and Pascal Jacquinot, aims to extract an empirical narrative for France on the relationship between fiscal policy and debt sustainability, in the context of fiscal regimes. We build a DSGE model, where Markov-switching dynamics are introduced on the tax revenues response to debt, expenditure and output gap. We then bring the model to the data and show that two distinct fiscal regimes took place over the period 1955-2009: a *sustainable regime* covered 'Les Trente Glorieuses' until 1977 and then re-emerged in 1999 with the euro membership; an *unsustainable regime*, instead, characterized the 1978-1998 period, where a policy mix of disinflation, external and internal balance led to primary deficits and unstable debt-to-GDP accumulation.

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Contents

List of Figures

List of Tables

Chapter 1

Ambiguous Leverage Cycles

1.1 Introduction

Most financial crisis originate in debt markets and asset price as well as leverage cycles have important effects on the real economy. Opacity and collateral constraints are the two most notable features of debt markets and both can be a source of instability (See Holmstrom (2015)). First, collateral constraints expose debt markets to the fluctuations in collateral values and the anticipatory effects associated to their endogenous changes trigger large reversal in debt and asset positions. Second, agents trading in debt markets hold doubts about the fundamental value of the collateral. In this context ambiguity attitudes and endogenous beliefs formation are crucial in determining the dynamic of asset values and debt, also since the latter is tied to the first through the collateral constraint. The surge in asset prices and leverage observed prior to most financial crises and their collapse observed following it have often been linked to a combination of institutional factors, captured by collateral constraints, and endogenous beliefs formation[1]. Optimism in booms, generated by assigning higher subjective beliefs to gains than to losses, can explain the surge in asset demand, prices and, through the collateral channel, in debt. Pessimism in recessions produces the opposite chain of events[2]. Despite the joint relevance of those elements in explaining the unfolding of financial crises, as well as the dynamic of asset prices and leverage over the business cycle, they are absent from the literature.

We fill this gap by assessing the role of ambiguity attitudes in a small open economy model where borrowers, investing in risky assets, are subject to occasionally binding collateral constraints that tie the scarcity or availability of debt to asset valuations. The latter is then affected by ambiguity attitudes, which render beliefs formation endogenous. Indeed the borrower, endowed with a sequence of subjective beliefs upon which he holds different amount of confidence, optimally chooses the degree of entropy, namely the distance between subjective and objective probability distributions, subject to bounds on it. The confidence in subjective beliefs are captured by an ambiguity parameter. Given the optimal entropy or likelihood ratio (LR hereafter), which affects also the value of risky assets through the stochastic discount factor

[1]See also Barberis (2011).

[2]See Barberis (2011) for discussion on the role of over-confidence and under-confidence in particular for asset prices and leverage also at around the 2007-2008 financial crisis.

(SDF here-after), the borrower solves optimal portfolio and leverage decisions.

Importantly we depart from the standard ambiguity aversion framework[3] and consider preferences which combine ambiguity aversion and ambiguity seeking. We model a dynamic extension of the biseparable preferences axiomatized in Ghirardato and Marinacci (2001) and Ghirardato, Maccheroni and Marinacci (2001), which convexity the decision maker problem of finding the optimal beliefs by nesting (depending on the weights) both aversion and seeking behaviour. Extended ambiguity attitudes have also strong support in experimental studies[4]. Specifically we model the decision marker problem using dynamic Lagrangian preferences a' la Hansen and Sargent (2001) and we then convexity them to nest both the entropy minimization problem (ambiguity aversion) and maximization problem (ambiguity seeking). Consistently with Ghirardato, Maccheroni and Marinacci (2001) the weight or the indicator function in the optimal decision problem depends upon expected utilities. To validate our preferences empirically we determine the mapping between the ambiguity attitudes and the expected utility through structural estimation of the model. Specifically, we develop a novel estimation method by adapting the non-linear method of moments to our model-based combined Euler equation, in debt and risky asset[5]. We find that ambiguity aversion prevails when the value function is above its expected value (a case which we often label the loss domain) and viceversa.Those attitudes endogenously result in optimism or right-skewed beliefs in booms and pessimism in recessions[6]. This structure of the beliefs coupled with the anticipatory effects, which are typically associated with occasionally binding collateral constraints[7], have important implications for asset price, debt capacity and leverage dynamic. Consider a boom. Borrowers endogenously tend to act optimistically and increase their demand of risky assets. This boosts asset prices and through anticipatory effects also the demand of debt, which in turn endogenously relaxes the constraint. This is also consistent with the fact that in booms the evaluation of optimistic agents drives the debt capacity. The opposite is true in the loss domain. Ambiguity aversion typically induces persistence, but little volatility. Our preferences which combine the two in a kinked fashion induce the right amount of persistence and volatility needed to match asset price facts and debt dynamic.

With the above model we obtain a series of analytical and numerical results related to asset prices and debt dynamic. Analytically we discuss implications for asset prices and the Sharpe

[3]See pioneering work by Hansen and Sargent (2001), Hansen and Sargent (2007) and Maccheroni, Marinacci and Rustichini (2006).

[4]Ambiguity seeking is strongly supported in experimental evidence. See Dimmock et al. (2015), Dimmock et al. (2016), Baillon et al. (2017) and Trautmann and van de Kuilen (2015) among others.

[5]For this we use the procedure developed in Chen, Favilukis and Ludvigson (2013), where one step involves the estimation of a latent unobservable variable given by the continuation value ratio.

[6]Our macro estimates are well in line with experimental evidence. Abdellaoui et al. (2011) provide foundations for S-shaped preferences with changing ambiguity attitudes and show through experimental evidence that pessimism (left-skewed beliefs) prevails in face of losses, while optimism prevails in face of gains. Further experimental evidence by Boiney (1993) Kraus and Litzenberger (1976) has associated ambiguity seeking (aversion) with right (left) skewed beliefs. On another front, survey evidence by Rozsypal and Schlafmann (2017), shows that low-income households hold pessimistic beliefs about the future, while the opposite is true for high-income households.

[7]Mendoza (2010) shows that the occasionally binding nature of the collateral constraints gives a role to anticipatory effects. As agents expect the constraint to bind in the future, they off-loads risky assets and debt in anticipation.

ratio. For the first, we show that the conditional LR heightens asset price growth in booms and depresses it in recessions. Second, the kink in the stochastic discount factor induced by the shift from optimism to pessimism helps to move the model-based Sharpe ratio closer to the Hansen and Hansen and Jagannathan (1991) bounds.

Next, we solve our model numerically by employing global non-linear methods with occasionally binding constraints[8]. The policy functions and a simulated crisis event, which allow us to discuss the economic intuition behind our model, show that optimism increases the build-up of leverage in booms, while pessimism steepens the recessionary consequence of the crisis. In both cases the comparison is done relatively to a model featuring solely collateral constraints, but no deviations between subjective and objective beliefs. Ambiguity attitudes play a crucial role in this result. In booms optimism boosts collateral values, hence, by relaxing the constraint, it facilitates the build-up of leverage, asset demand and the asset price boom. In recessions pessimism materializes, which drives the transmission channel in the opposite direction. To subject our model and belief formations process to further rounds of empirical validation, we calibrate all parameters by minimizing the distance between some targeted model-based moments and their empirical counterparts using data for the US economy over the sample 1980-2016, namely the sample of both a rapid growth in leverage and then a sudden collapse in debt positions. Under the optimized calibration, the model can match asset price volatilities and equity premia (both the long run and the dynamic pattern), returns, Sharpe ratios, volatilities of debt and its pro-cyclicality [9]. The comparison with the model featuring solely the collateral constraint shows that our model performs better in the data matching. To explain asset price facts borrowers' ambiguity attitudes over the tails are crucial.

The rest of the chapter is structured as follows. Section 1.2 compares the paper to the literature. Section 1.3 describes the model and the ambiguity attitudes specification. Section 1.4 presents the estimation procedure and results. Section 1.5 investigates analytical results. Section 1.6 discusses quantitative findings (the solution method is detailed in the appendix). Section 1.7 concludes.

1.2 Comparison with Past Literature

Following the 2007 financial crisis which was triggered by panics in various debt markets (for structured products, for short-term bank funding and in repo markets, see Gorton and Metrick (2012)) there has been a growing interest in understanding the determinants and the dynamics of the leverage cycle and the role of the underlying externalities (pecuniary and demand) for the real economy. Most recent literature tends to assess the dynamic of debt over the business cycle through models with occasionally binding constraints. Papers on this topic include Geanakoplos (2010), Lorenzoni (2008), Mendoza (2010), which among many others examine both positive and

[8] We employ policy function iterations based on a Tauchen and Hussey (1991) discretization of the state space and by accommodating different regimes (portions of the state space) with binding or non-binding constraints.

[9] It is well documented by Jorda, Schularick and Taylor (2016) at aggregate level and using historical data. But it is also well document for consumer debt, see for instance Fieldhouse, Livshits and MacGee (2016) among others.

normative issues related to the leverage cycle. Papers focusing on the positive aspects show that anticipatory effects produced by occasionally binding constraints are crucial in generating sharp reversals in debt markets and in establishing the link between the tightening of the constraint and the unfolding of financial crisis. None of the past papers however assesses the joint role of financial frictions, in the form of collateral constraints, and belief formation, while both play a crucial role in determining the asset price and leverage cycle in normal times and in explaining endogenously the unfolding of crises even in face of small shocks. One exception is Boz and Mendoza (2014) which introduces learning on asset valuation in a model with occasionally binding collateral constraints. Contrary to them our beliefs are endogenously formed based on ambiguity attitudes toward model mis-specification. Moreover none of the past papers conducts a quantitative analysis aimed at assisting the quantitative relevance of those elements in jointly matching asset price and debt facts and cyclical moments.

The relevance of ambiguity and of the beliefs formation process is crucial in debt markets in which opacity is the norm (see Holmstrom (2015)). Indeed, contrary to equity markets in which buyers of the asset wish to exert monitoring and control on the investment activity, participants in debt markets usually trade under the ignorance of the fundamental value of collateral. For this reason in debt markets a collateral guarantee is part of the contractible set-up. This indeed serves the purpose of overcoming the pervasive asymmetric information. However even if the information asymmetry underlying the specific debt relation is solved through the contracts, doubts remain about the fundamental value of the asset, implying that optimism or pessimism of subjective beliefs affect the agents' saving and investment problem, hence the dynamic of asset prices and leverage. Despite the realism and importance of the connection between ambiguity and debt dynamic, this nexus has not been studied so far.

Since we choose to model endogenous beliefs formation through ambiguity attitudes our model is also connected to the literature on ambiguity aversion (see Hansen and Sargent (2001), Hansen and Sargent (2007) and Maccheroni, Marinacci and Rustichini (2006)). In this context some papers also assess the role of ambiguity aversion for asset prices or for portfolio allocation. For instance Barillas, Hansen and Sargent (2007) show that ambiguity aversion is akin to risk-sensitive preferences a' la Tallarini (2000) and as such it helps the model's Sharpe ratio to get closer to the Hansen and Jagannathan (1991)[10]. Epstein and Schneider (2008) also analyse the properties of asset prices focusing on ambiguity-averse investors. More recently in a production economy Bianchi, Ilut and Schneider (2017) have assessed the role of ambiguity aversion for firms, debt policies and stock prices. We depart from this literature in two important ways. First, we model ambiguity attitudes that encompass both ambiguity aversion and ambiguity seeking behaviour. Ambiguity seeking is well documented in experimental evidence (see Dimmock et al. (2015) and Dimmock et al. (2016), Baillon et al. (2017), Trautmann and van de Kuilen (2015), and Roca, Hogarth and Maule (2006) among others). We introduce the whole span of ambiguity attitudes through the extended multiplier preferences, which have been founded theoretically by Baillon et al. (2017). We confirm the existence and significance

[10]On a different line of research Benigno and Nisticó (2012) show how ambiguity averse preferences can be used to explain the home bias in international portfolio allocations due to the need to hedge against long run risk.

of ambiguity attitudes through time-series estimation or our model. Furthermore, it is only by accounting jointly for ambiguity aversion and ambiguity seeking that our model is able to match numerically the volatilities, the persistence and the cyclical behaviour of asset prices and debt.

We depart from this literature in two important ways. First, we model ambiguity attitudes that encompass both ambiguity aversion and ambiguity seeking behaviour. Our preferences are indeed a dynamic extension of the biseparable preferences axiomatized in a static context by Ghirardato and Marinacci (2001) and Ghirardato, Maccheroni and Marinacci (2001). Both papers show that ambiguity attitudes can be formalized within a general decision model by constructing a biseparable preference, which can nest both ambiguity aversion and ambiguity seeking. Effectively preferences are convexified with respect to the problem of finding the optimal beliefs, so that under a weight of one the decision maker solves a minimization problem (ambiguity aversion) and viceversa. The weights in their formalization depend upon expected utility mapping. In our work we construct a value function, which embed a multiplier on the entropy, that can be convexified, thereby nesting ambiguity aversion (with a positive multiplier on entropy) and ambiguity seeking (negative multiplier). Consistently with Ghirardato, Maccheroni and Marinacci (2001), the indicator function, which non-linearly shifts the preferences from ambiguity averse to its dual, depends upon the deviations of future value functions from a reference level represented by the future expected value. Importantly our state contingent multiplier are estimated as explained below. This not only validates empirically the preferences, but it also allows us to pin down the exact form of the state contingency in the multiplier (negative in the gain domain and positive in the loss domain). Equipped with these preferences we show that beliefs endogenously become pessimistic in the loss domain (when the value function is below its expected value) and optimistic in the gain domain (the opposite case). This has important consequences in our case. Indeed by embedding those preferences into a leverage cycle and risky investment problem we can show that optimism induces price acceleration and excessive leverage, while pessimism induces the opposite. Moreover the combination of ambiguity aversion and seeking delivers the right amount of persistence and volatility needed to explain jointly asset price and debt dynamic. At last the kinked nature of the preferences helps in generating the right volatility in the Sharpe ratios, which governs risk-taking behaviour.

Note that ambiguity seeking as well as the state contingent nature of the ambiguity attitudes also well documented in experimental studies (see Dimmock et al. (2015) and Dimmock et al. (2016), Baillon et al. (2017), Trautmann and van de Kuilen (2015), and Roca, Hogarth and Maule (2006) among others). Multiplier preferences, embedding both gradation of ambiguity, have also been examined by Baillon et al. (2017) through experimental evidence. We confirm the existence and significance of ambiguity attitudes through time-series estimation or our model. Furthermore, it is only by accounting jointly for ambiguity aversion and ambiguity seeking that our model is able to match numerically the volatilities, the persistence and the cyclical behaviour of asset prices and debt.

At last, our paper contributes to the literature on the estimation of SDF with behavioural elements. The closer contribution to ours is Chen, Favilukis and Ludvigson (2013). A series of papers have developed procedures for SDF estimation. We review most of them in the section describing our model estimation. An important aspect we contribute to this literature is the development of an estimation procedure for a model which jointly accounts for collateral constraints and for ambiguity attitudes. Our estimation uncovers the state-contingent nature of ambiguity attitudes, namely optimistic in booms and pessimistic in recessions, while not previously noted in the literature.

1.3 A Model of Ambiguous Leverage Cycle

Our baseline model economy is an otherwise standard framework with borrowers facing occasionally binding collateral constraints. One of the novel ingredients stems form the interaction between ambiguity attitudes and debt capacity. Debt supply is fully elastic with an exogenous debt rate as normally employed in most recent literature on the leverage cycle.[11] Collateral in this economy is provided by the value of the risky asset funded through debt. To this framework we add ambiguity attitudes, which includes both ambiguity aversion and seeking. The latter is modelled through the extended multiplier preferences, for which Baillon et al. (2017) and Abdellaoui et al. (2011) have provided experimental evidence and theoretical foundation. The underlying logic is similar to the one pioneered and proposed by the game-theoretic approach a' la Hansen and Sargent (2007) in which agents are assumed to have fears of model mis-specification and play a two-stage game with a malevolent agent (nature) that amplifies deviations from the true probability model and helps the borrower to explore the fragility of a decision rule with respect to various perturbations of the objective shock distribution. Hansen and Sargent (2007) focus on ambiguity averse attitudes. Under this case the game of interaction between the agents and nature results in the latter inducing more pessimistic beliefs with the goal of testing agents' ability to make robust decisions. Agents therefore optimally attempt to minimize the distortion induced by nature, by enforcing a positive penalty parameter under the case of max-min preferences.

While ambiguity aversion has been the norm in macro and finance models, a crucial departure introduced by our framework is to consider the whole span of ambiguity attitudes, namely ambiguity aversion and ambiguity seeking. Extensive experimental studies (reviewed above) finds support for both. To include the whole span of ambiguity attitudes we employ the extended multiplier preferences discussed in Baillon et al. (2017) and Abdellaoui et al. (2011), as explained above. Moreover through empirical analysis we uncover the state-contingent nature of the ambiguity attitudes, whereby aversion prevails in recessions and ambiguity seeking prevails in booms.

Importantly the contingent reason for considering this extended set-up for ambiguity attitudes is that, as our analysis below shows through several steps, this is crucial for explaining the facts we focus on, namely the patterns observed around the unfolding and development of debt crises as well as the full array of asset price and debt statistics. At this stage it is also useful to mention that within the structure of the zero-sum game the economic interpretation

[11] This model economy corresponds to a limiting case in which lenders are risk-neutral. Alternatively the model can be interpreted as a small open economy with debt supplied from the rest of the world.

of the ambiguity seeking attitudes is just similar to the one described above for the ambiguity averse attitudes. This implies that under positive realizations of income nature induces optimistic beliefs with the intent again of testing robustness of agents' decisional process. Given this interpretation, such beliefs formation process is also akin to the one considered in Brunnermeier and Parker (2005) in which a small optimistic bias in beliefs typically leads to first-order gains in anticipatory utility[12].

Below, we show that ambiguity aversion results endogenously in left-skewed or pessimistic beliefs, relatively to rational expectation, namely relatively to the case in which objective and subjective beliefs coincide. On the other side ambiguity seeking results in right-skewed or optimistic beliefs. Importantly the changing nature of the ambiguity attitudes contributes to the occasionally binding nature of the collateral constraint. As agents become optimist their demand for risky assets contributes to boost collateral values and to expand debt capacity. The opposite is true with pessimism.

1.3.1 Beliefs Formation and Preferences

The source of uncertainty in the model is a shock to aggregate income y_t, which is our exogenous state and follows a finite-space stationary Markov process. We define the state space as S_t, the realization of the state at time t as s_t and its history as $s^t = \{s_0, s_1, \ldots, s_t\}$ with associated probability $\pi(s^t)$. The initial condition of the shock is known and defined with s_{-1}.

Borrowers are endowed with the approximated model $\pi(s^t)$ over the history s^t but they also consider alternative probability measures, indicated by $\tilde{\pi}(s^t)$, which deviate from $\pi(s^t)$.[13] Borrowers can have different degrees of trust in their own subjective beliefs, so that act as ambiguity averse when they fear deviations from the approximated model and they act as ambiguity seeking when they hold high confidence in their beliefs. Following the relevant literature, we introduce the measurable function $M(s^t) = \tilde{\pi}(s^t)/\pi(s^t)$, which we define as the likelihood ratio. We can also define the conditional likelihood ratio as, $m(s_{t+1}|s^t) = \tilde{\pi}(s_{t+1}|s^t)/\pi(s_{t+1}|s^t)$. For ease of notation since now onward we use the following notation convention: $M_t = M(s^t), M_{t+1} = M(s^{t+1})$ and $m_{t+1} = m(s_{t+1}|s^t)$, where the sub-index refers to the next period state. The above definition of M_t allows us to represent the subjective expectation of a random variable x_t in terms of the approximating probability models:

$$\tilde{\mathbb{E}}_t[x_t] = \mathbb{E}_t[M_t x_t] \tag{1.1}$$

where $\mathbb{E}_t$ is the subjective expectation operator conditional to information at time t for the probability $\pi(s^t)$, while $\tilde{\mathbb{E}}_t$ is the expectation operator conditional to information at time t for the probability $\tilde{\pi}(s^t)$. The function M_t follows a martingale process and as such it satisfies the

[12] Some connections between the economic interpretation of ambiguity seeking attitudes under loss domain can also be traced with the news averse preferences introduced by Kőszegi and Rabin (2007) and examined more recently in asset price context by Pagel (2014). In this case as well agents prefer not to receive news fearing the bad ones.

[13] The alternative probability measure $\tilde{\pi}$ is absolutely continuous with respect π. This means that events that receive positive probability under the alternative model, also receive positive probability under the approximating model

following condition $\mathbb{E}[M_{t+1}] = M_t$. We can decompose M_t as follows

$$m_{t+1} \equiv \frac{M_{t+1}}{M_t} \qquad \text{for} \quad M_t > 0 \tag{1.2}$$

and $m_{t+1} = 1$ for $M_t = 0$. These incremental deviations satisfy condition $\mathbb{E}_t[m_{t+1}] = 1$. Moreover, the discrepancy between the approximated and the subjective models is measured by the conditional entropy, defined as follows:

$$\varepsilon(m_{t+1}) = \mathbb{E}_t \{m_{t+1} \log m_{t+1}\} \tag{1.3}$$

where $\varepsilon(m_{t+1})$ is a positive-valued, convex function of $\pi(s^t)$ and is uniquely minimized when $m_{t+1} = 1$, which is the condition characterizing the case with no ambiguity attitudes. Given the probabilistic specifications above, we now introduce the following *kinked multiplier preferences*:

$$V(c_t) = \begin{cases} \min\limits_{\{m_{t+1}, M_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \mathbb{E}_0 \Big\{ \beta^t \pi_t M_t u(c_t) + \beta \theta_t \varepsilon(m_{t+1}) \Big\} & \text{if} \quad \theta_t \geq 0 \\ \\ \max\limits_{\{m_{t+1}, M_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \mathbb{E}_0 \Big\{ \beta^t \pi_t M_t u(c_t) + \beta \theta_t \varepsilon(m_{t+1}) \Big\} & \text{if} \quad \theta_t < 0 \end{cases} \tag{1.4}$$

where $u(c_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma}$. In the above expression, $\theta_t \in \mathbb{R}$ is a process capturing the degree of doubts about the prevailing model, which include ambiguity aversion and ambiguity seeking.

Later on we will characterize this process exactly based on estimated values. For now on it suffices to know that θ_t is a state contingent binary variable which will take positive values for states of the world for which the value function is below its average and negative in the opposite states. Mathematically the value function under θ_t^- is essentially the dual representation of the value function under θ_t^+.

The next session explains more in detail the axiomatic foundations of those preferences. Our preferences can indeed be seen as a multiplier and dynamic extension of the biseparable preferences Ghirardato and Marinacci (2001) and Ghirardato, Maccheroni and Marinacci (2001). Extended multiplier preferences similar to the ones in 4 are suggested also in the experimental work by Baillon et al. (2017), albeit in a static context.

1.3.2 Preferences Formalization

The above preferences can also be derived as a dynamic extension of the biseparable preferences axiomatized in Ghirardato and Marinacci (2001) and Ghirardato, Maccheroni and Marinacci (2001). Both papers show that ambiguity attitudes can be formalized within a general decision model by constructing a biseparable preference, which can nest both ambiguity aversion and ambiguity seeking. Preferences are convexified with respect to the problem of finding the optimal belief. Consider the instantaneous utility function, $u(c_t)$ and the problem of finding the optimal belief. Given again the probabilistic specifications above we can represent preferences as follows:

$$\mathbb{I}_{\theta_t \geq 0} \min_{\{m_{t+1}, M_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \mathbb{E}_0 \Big\{ \beta^t \pi_t M_t u(c_t) + \beta \theta_t \varepsilon(m_{t+1}) \Big\} +$$
$$\mathbb{I}_{\theta_t < 0} \max_{\{m_{t+1}, M_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \mathbb{E}_0 \Big\{ \beta^t \pi_t M_t u(c_t) + \beta \theta_t \varepsilon(m_{t+1}) \Big\} \tag{1.5}$$

As noted in Ghirardato, Maccheroni and Marinacci (2001) the indicator function shall depend only upon expected utility mapping. We design the following expected utility mapping so that $\theta_t > 0$ whenever $V_t \geq EV_t$ (which since now we often refer as the gain domain) and viceversa (in the loss domain). We can therefore re-write our preferences as:

$$\mathbb{I}_{V_t \geq EV_t} \min_{\{m_{t+1}, M_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \mathbb{E}_0 \Big\{ \beta^t \pi_t M_t u(c_t) + \beta \theta_t \varepsilon(m_{t+1}) \Big\} +$$
$$\mathbb{I}_{V_t < EV_t} \max_{\{m_{t+1}, M_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \mathbb{E}_0 \Big\{ \beta^t \pi_t M_t u(c_t) + \beta \theta_t \varepsilon(m_{t+1}) \Big\} \tag{1.6}$$

Three theoretical notes are worth at this point. First, as noted in Ghirardato, Maccheroni and Marinacci (2001) most decision theory models of ambiguity employ those biseparable preferences, but add additional assumptions. For instance ambiguity aversion arises under the assumption of ambiguity hedging, namely the fact that between two indifferent alternatives the ambiguity averse decision marker prefers a convex combination of the two to each one in isolation. Under ambiguity seeking this assumption should be reversed. Second, the dependence of the indicator function upon the expected utility effectively creates a dependence with respect to the state of the economy. Indeed it is only after a sequence of negative shock to wealth that the value function passes its mean and viceversa[14]. Therefore, formally we should condition the indicator function and the θ_t upon the state of the economy. With a slight abuse of notation and for convenience we maintain our notation of a time dependent θ_t as in the context of our model we deal with random shocks in a time series context. Second, note that the general formalization of the decision problem is not explicit about the exact dependence of the indicator function upon the gain or the loss domain. This is effectively an empirical question. Indeed as explained above it has been addressed in the context of experimental studies[15]. For this reason later below we estimate our model and we assign to the Lagrange multiplier state contingent process which is consistent with the data and the evidence that we find. This effectively also serves as an indirect validation of the preferences. Note that for robustness we run two types of estimation. The first is a reduced form through GMM confined to the model-implied Euler equation, the second is a method of moments on the entire model. Both methods give the same

[14] In this respect the preferences are also akin to the news dependent preferences a' la Kõszegi and Rabin (2007). See also recently Pagel (2014). The main difference is that news dependence a' la Kõszegi and Rabin (2007) affects risk aversion, while in our case it affects attitudes toward uncertainty. Second, once again we consider aversion but also its dual.

[15] See Baillon et al. (2017) and Abdellaoui et al. (2011).

consistent answer, albeit understandably they deliver two different values for the estimated parameters[16].

Some additional considerations are worth on the interpretation of our preferences and on their implication for the asset price and the leverage cycle. First, as we show below, when solving the decision maker problem of finding the optimal beliefs, our biseparable or extended (since now on we will use the terms interchangeably) multiplier preferences deliver pessimism (or left-skewed) beliefs in loss domain and optimism (right-skewed) beliefs in gain domain. Framed in the context of the Hansen and Sargent (2007) game with nature, the optimal belief problem has the following interpretation. Under the loss domain nature tests the decision maker by inducing him/her to assign more weights to adverse states, hence the pessimistic beliefs. In a consumption-saving problem this naturally induces more precautionary saving, while in our framework, where financial crises endogenously materialize, pessimistic beliefs are responsible for stronger deleveraging (and fire sales) during the downturn. This effect is well in line with post-crises dynamic. Under the gain domain nature again tests the limit of the decision maker by inducing him/her to assign more weight to the upper tail[17]. This leads to the emergence of risk-taking and excessive leverage. In both cases nature shifts decision makers' behaviour toward the tails. Hence, our preferences are well in line with the prevalent interpretation of model ambiguity. As we show extensively below however considering ambiguity seeking and extended attitudes helps greatly in explaining asset price facts as well as in the context of our leverage model also debt dynamic.

Budget and Collateral Constraint

The rest of the model follows a standard leverage cycle model with risky assets that serve as collateral (see e.g. Mendoza (2010)). The representative agent holds an infinitely lived asset x_t, which pays a stochastic dividend d_t every period and is available in fixed unit supply. The asset can be traded across borrowers at the price q_t. In order to reduce the dimension of the state space, we assume that the dividend is a fraction α of the income realization. Therefore, we indicate with $(1-\alpha)y_t$ the labor income and with $d_t = \alpha y_t$ the financial income. Agents can borrow using one-period non-state-contingent bonds that pay an exogenous real interest rate R. The budget constraint of the representative agents can be expressed as following:

$$c_t + q_t x_t + \frac{b_t}{R} = (1-\alpha)y_t + x_{t-1}[q_t + d_t] + b_{t-1} \tag{1.7}$$

where c_t indicates consumption and b_t the bond holdings. The agents' ability to borrow is restricted to a fraction ϕ of the value of asset holding:

$$-\frac{b_t}{R} \leq \phi q_t x_t \tag{1.8}$$

[16]This is understandable since in one case the estimation uses information from only one part of the model, while in the case it uses information from the entire model.

[17]Given this interpretation, such beliefs formation process is also akin to the one considered in Brunnermeier and Parker (2005) in which a small optimistic bias in beliefs typically leads to first-order gains in anticipatory utility.

The collateral constraint depends on the current period price of the asset in order to reproduce fire-sales driven amplification dynamics, which for this simple model would not be produced with a different formulation of the constraint.[18]

1.3.3 Recursive Formulation

Following Hansen and Sargent (2007), we rely on the recursive formulation of the problem, which allows us to re-write everything only in terms of m_{t+1}. The recursive formulation shall of course be adapted to capture the changing nature of the ambiguity attitudes.

We now partition the state space S_t in the two blocks, given by the endogenous and the exogenous states, $S_t = \{B_t, y_t\}$, where B_t is the aggregate bond holdings and y_t the income realization. Note that the aggregate asset holdings is not a state variable because it is in fixed supply. Moreover, the problem is also characterized by the two individual state variables (b_t, x_t). For the recursive formulation we employ a prime and sub-index to indicate variables at time $t+1$ and no index for variables at time t. The borrowers' recursive optimization problem reads as follows. Conditional on $\theta_t > 0$, the recursive two-stage optimization reads as follows:

$$\begin{aligned} V(b,x,S) = \max_{c,x',b'} \min_{m'} \Big\{ & u(c) + \beta \mathbb{E}_S \left[m' V(b',x',S) + \theta m' \log m' \right] \\ & + \lambda \left[y + q(S)(x+\alpha y) + b - q(S)x' - c - \frac{b'}{R} \right] \\ & + \mu \left[\phi q(S) x' + \frac{b'}{R} \right] + \beta \theta \psi \left[1 - \mathbb{E}_S m' \right] \Big\} \end{aligned} \tag{1.9}$$

Conditional on $\theta_t < 0$, the recursive two-stage optimization reads as follows:

$$\begin{aligned} V(b,x,S) = \max_{c,x',b'} \max_{m'} \Big\{ & u(c) + \beta \mathbb{E}_S \left[m' V(b',x',S) - \theta m' \log m' \right] \\ & + \lambda \left[y + q(S)(x+\alpha y) + b - q(S)x' - c - \frac{b'}{R} \right] \\ & + \mu \left[\phi q(S) x' + \frac{b'}{R} \right] + \beta \theta \psi \left[1 - \mathbb{E}_S m' \right] \Big\} \end{aligned} \tag{1.10}$$

where the aggregate states follow the law of motion $S' = \Gamma(S)$. In the above problem λ and μ are the multipliers associated to the budget and collateral constraints respectively, while the term $\beta\theta\psi$ is the multiplier attached to the constraint $\mathbb{E}_S[m'] = 1$.

The above optimization problems are solved sequentially. First an inner optimization and then an outer optimization problem are derived sequentially. In the first stage agents choose the optimal incremental probability distortion for given saving and portfolio choices. In the second stage, for given optimal likelihood ratio, they solve the consumption/saving problem and choose the optimal amount of leverage. Intuitively, the problem is modelled as a game of strategic interactions between the maximizing agents, who face Knightian uncertainty[19], and

[18] Moreover, Bianchi and Mendoza (2015) provide a micro-founded derivation of this constraint, based on a limited enforcement problem.

[19] Knight (1921) advanced the distinction between risk, namely the known probability of tail events, and

a malevolent agent that draws the distribution (see Hansen and Sargent (2007) who proposed this reading).

The Inner Minimization Problem

Through the inner optimization problem the borrowers choose the optimal entropy or conditional likelihood ratio, namely the optimal deviation between his own subjective beliefs and the objective probability distribution. The first order condition with respect to m', which is functionally equivalent under the two cases, is given by:

$$V(b', x', S') + \theta(\log m' + 1) - \theta\psi = 0 \tag{1.11}$$

Rearranging terms, we obtain:

$$\begin{aligned} 1 + \log m' &= -\frac{V(b', x', S')}{\theta} + \psi \\ m' &= \exp\left\{\frac{-V(b', x', S')}{\theta}\right\}\exp\{\psi - 1\} \end{aligned} \tag{1.12}$$

Finally, imposing the constraint over probability deviation m', and defining $\sigma = -\frac{1}{\theta}$ we derive the optimality condition for the conditional likelihood ratio:

$$m' = \frac{\exp\{\sigma V(b', x', S')\}}{\mathbb{E}\left[\exp\{\sigma V(b', x', S')\}\right]} \tag{1.13}$$

Equation (1.13) also defines the state-contingent incremental probability deviation from the rational expectation case. The magnitude and the direction of this deviation depends on the agents' value function and the value for the inverse of σ. We will return on the role of the optimal conditional likelihood ratio later on.

The Outer Maximization Problem

For given optimal LR $m^{'}$ the borrower solves an outer optimization problem in consumption, risky assets and debt. Upon substituting the optimal LR into the value function, the maximization problem reduces to find the optimal allocations of consumption, bond holding and asset holdings. The resulting recursive problem is:

$$\begin{aligned} V(b, x, S) = \max_{c, x', b'} \Bigg\{ & u(c) + \frac{\beta}{\sigma}\log\left[\mathbb{E}_S \exp\left\{\sigma V(b', x', S')\right\}\right] \\ & + \lambda\left[y + q(S)((x + d) + b - q(S)x' - c - \frac{b'}{R}\right] \\ & + \mu\left[\phi q(S)x + \frac{b'}{R}\right]\Bigg\} \end{aligned} \tag{1.14}$$

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uncertainty, namely the case in which such probabilities are not known. Ambiguity usually refers to cases of uncertainty where the state space is well defined, but objective probabilities are not available.

We will now derive and list all the competitive equilibrium conditions. Since now we return to the notation with t and $t+1$ indices as this is needed for our analytical derivations in section 1.5. The borrowers' first order condition with respect to bond holding and risky assets reads as follows:

$$u_c(c_t) = \beta R \mathbb{E}_t \{m_{t+1} u_c(c_{t+1})\} + \mu_t \tag{1.15}$$

$$q_t = \beta \frac{\mathbb{E}_t \{m_{t+1} u_c(c_{t+1})[q_{t+1} + \alpha y_{t+1}]\}}{u_c(c_t) - \phi \mu_t} \tag{1.16}$$

where u_c indicates the derivative of the utility function with respect to consumption. Equation (1.15) is the Euler equation for bonds and displays the typical feature of models with occasionally binding collateral constraint. In particular, when the constraint binds there is a wedge between the current marginal utility of consumption and the expected future marginal utility, given by the shadow value of relaxing the collateral constraint. Equation (1.16) is the asset price condition.

Note that ambiguity attitudes, hence beliefs, affect asset prices since m_{t+1} enters the optimality conditions for risky assets, equation (1.16), and they affect the tightness of the debt limit as m_{t+1} enters the Euler equation 1.15. In other words the optimal m_{t+1} affects the stochastic discount factor and through this it affects the pricing of all assets in the economy. The model characterization is completed with the complementarity slackness condition associated to the collateral constraint:

$$\mu_t \left[\frac{b_{t+1}}{R} + \phi q_t \right] = 0 \tag{1.17}$$

and with the goods and stock markets clearing conditions:

$$c_t + \frac{b_{t+1}}{R} = y_t + b_t \tag{1.18}$$

$$x_t = 1 \tag{1.19}$$

Definition 1.3.1 (Recursive Competitive Equilibrium). *A Recursive Competitive Equilibrium is given by a value function V_t, allocations $(c_t; b_{t+1})$, probability distortions m_{t+1} and prices q_t such that:*

- *given prices and allocations the probability distortions solve the inner minimization problem;*

- *given prices and probability distortions, the allocation and the value function solve the outer maximization problem;*

- *the allocations are feasible, satisfying* (1.18) *and* (1.19)*;*

- *the aggregate states' low of motion is consistent with agents' optimization;*

1.3.4 Pessimism and Optimism

To determine under which states the Lagrange multiplier, θ_t, turns positive or negative we will estimate our model implied Euler equations through GMM in the next section. In the meantime it is useful to discuss how the ambiguity averse or ambiguity seeking attitudes affect the endogenous formation of beliefs, as captured by the optimal likelihood ratio. For simplicity of exposition we report the optimal condition for variable m_{t+1}:

$$m_{t+1} = \frac{\exp\{\sigma_t V(b_{t+1}, x_{t+1}, S_{t+1})\}}{\mathbb{E}_t\left[\exp\{\sigma_t V(b_{t+1}, x_{t+1}, S_{t+1})\}\right]} \tag{1.20}$$

The conditional deviation affects how agents assign different subjective probabilities (with respect to the objective ones) to future events, which can be characterized by high and low utility. In particular, if $m_{t+1} > 1$ agents assign an higher subjective probability, while if $m_{t+1} < 1$ the opposite holds. Given this, the sign of the parameter σ_t affects how these conditions are linked to positive or negative future state realizations.[20] The following lemma summarizes this consideration and defines optimism and pessimism in the agents' attitude.

Lemma 1.3.2. *When $\theta_t < 0$ $m_{t+1} > 1$ in good states and $m_{t+1} < 1$ in bad states. Hence, beliefs endogenously emerge as right-skewed and agents act with optimism. When $\theta_t > 0$ the opposite is true.*

Proof. First we define good states as those in which the current state value function is above its expected value. When $\theta_t < 0$; then $\sigma_t > 0$ in good states $\exp\{\sigma_t V(b_{t+1}, x_{t+1}, S_{t+1})\} > \mathbb{E}_t\left[\exp\{\sigma_t V(b_{t+1}, x_{t+1}, S_{t+1})\}\right]$ namely the risk-adjusted value function for the good states is larger than the average one. Based on the above equation, this implies that $m_{t+1} > 1$. The opposite is true in bad states. When $\theta_t > 0$ then $\sigma_t < 0$ this implies that in good states $\exp\{\sigma_t V(b_{t+1}, x_{t+1}, S_{t+1})\} < \mathbb{E}_t\left[\exp\{\sigma_t V(b_{t+1}, x_{t+1}, S_{t+1})\}\right]$, namely the risk-adjusted value function for the good states is lower than the average one and $m_{t+1} < 1$. The opposite is true in bad states.

Beliefs Formation: A binomial state space example

To gain some intuition we discuss a particular case with only two income states, which we define as high, with a sup-index h, and low, with a sup-index l. We also consider only two periods which we label as $t = 0, 1$. By assumption the high state is high enough that the collateral constraint is slack, while the opposite is true for the low state. This facilitates the computation of the expectation operators. The states have a binomial probability structure such that state h realizes with probability π, while the state l with its complement $1 - \pi$. Equipped with these assumptions we can characterize the dynamic between time 0 and time 1. In this case the

[20] Concerning the size of the distortion, we can say that a large absolute value of θ increases the probability distortion in all future states, meaning that m' is close to unity. At the contrary, a small absolute value of θ, implies that the decisions are far from the rational expectation setting.

likelihood ratio can be specified as follows:

$$m_1 = \frac{\exp\{\sigma_0 V_1\}}{\pi \exp\{\sigma_0 V_1^h\} + (1-\pi)\exp\{\sigma_0 V_1^l\}} \tag{1.21}$$

where $V_1^h > \mathbb{E}_0\{V_1\}$ and $V_1^l < \mathbb{E}_0\{V_1\}$. Note that depending on the time zero realization of the state we have two different values of the inverse of the penalty parameter, σ_0. To fix ideas imagine that the income realization at time zero is the low state, l. Given our Lemma 1.3.2 we have that $\sigma_o^l < 0$. The latter implies that $\exp\{\sigma_0^l V_1^h\} < \mathbb{E}_0\{\exp\{\sigma_0^l V_1\}\}$ and $\exp\{\sigma_0^l V_1^l\} > \mathbb{E}_0\{\exp\{\sigma_0^l V_1\}\}$. Therefore, the marginal likelihood ratio are $m_1^h < 1$ and $m_1^l > 1$. As a consequence, we can define the following subjective probabilities as:

$$\omega^h = \pi m_1^h < \pi \quad \omega^l = (1-\pi)m_1^l > (1-\pi) \tag{1.22}$$

As we can see, agents assign a higher (lower) subjective probability - with respect to the objective probability - to the future negative (positive) events, typical of a pessimistic attitude. The opposite is true when $\sigma_o^l < 0$. In this case $\exp\{\sigma_0^h V_1^h\} > \mathbb{E}_0\{\exp\{\sigma_0^h V_1\}\}$ and $\exp\{\sigma_0^h V_1^l\} < \mathbb{E}_0\{\exp\{\sigma_0^h V_1\}\}$ producing $m_1^h > 1$ and $m_1^l < 1$.

Therefore, agents assign higher (lower) subjective probability to the future positive (negative) events, showing an optimism attitude:

$$\omega^h = \pi m_1^h > \pi \quad \omega^l = (1-\pi)m_1^l < (1-\pi) \tag{1.23}$$

The interesting feature of this state-contingent behaviour concerns its connections with asset prices, the value of collateral and leverage. Further below we explain this in more details through analytical derivations and quantitative analysis. Intuitively, optimism explains why asset price booms and leverage build-ups are steeper in booms and relatively to the model with no beliefs formation. To fix ideas consider the case with a negative θ_0 and that the borrower experiences a good state today and expects a good state tomorrow. Asset price would grow even in the case with no ambiguity attitudes, however under our extended multiplier preferences, borrowers form today subjective beliefs that induce an LR of $m_1^h > 1$. As this makes the borrowers' SDF right-skewed distributed, it induces higher demand for both. This is why we label this case as optimism.

Consider now the opposite case, namely θ_0 lower than zero. According to Lemma 1.3.2 now the optimal LR is left skewed, namely lower than one if associated to future good states and larger than one to bad states. In other words the borrower becomes pessimistic. In this case, if a bad state is expected asset prices will fall according to equation 1.16 and they would do so more sharply than under when $m_1 = 1$ across all states of nature. Hence we shall conclude that pessimism explains why asset price bursts and de-leverages are sharper in recessions and relatively to the case with no ambiguity attitudes. Appendix A.6 considers a more extended version of the three periods model and also shows analytically that our ambiguity attitudes interacting with the collateral constraint induces higher debt levels in booms. Further below we explain through analytical derivations of the full dynamic model and through simulations of it how the ambiguity attitudes contribute to explain asset price and debt dynamics.

1.4 Estimation of the Model Implied SDF

To provide empirical ground to ambiguity attitudes within the context of our model and to uncover how the value of θ_t changes according to the prevailing state we estimate the model implied Euler equations. Once equipped with the process for θ_t we will solve the model analytically to uncover the main economic channels at work and numerically to assess the quantitative relevance.

We devise a novel estimation method apt to a model with collateral constraints and extended multiplier preferences. The method is based on adapting the minimum distance estimation conditional on latent variables to our modelling environment. In a nutshell we derive a moment condition by using the combined non-linear expression for the Euler equations (1.15) and (1.16). As we show in Appendix A.1, the latter depends on the value function. We therefore follow the approach in Chen, Favilukis and Ludvigson (2013), who condition the Euler moment condition to the estimation of the value function. A crucial difference between our method and theirs is that their value function has an unknown functional form, which is estimated semi-nonparametrically, while ours can be derived analytically.

More specifically, the estimation procedure (whose detailed derivations are contained in Appendix A.1) can be described as follows. First, one shall re-write the value function in terms of an ambiguity factor. For this, we adapt the steps used in the recursive preference literature to the case of our extended multiplier preferences (see Appendix A.1.1). Next, the implied SDF is derived (see Appendix A.1.2) and the value function is estimated (see Appendix A.1.3). Next, substituting the derived SDF into the combined Euler equations for debt and risky assets, (1.15) and (1.16), delivers the final moment condition (see Appendix A.1.4). Finally, as it is common for GMM estimation, we condition on a set of instruments, $\mathbf{z_t}$. The resulting moment condition reads as follows:

$$\mathbb{E}_t \left\{ \left[\underbrace{\beta \left(\frac{c_t}{c_{t+1}} \right)^{(1-\sigma_t)} \left(\frac{\exp\left(\frac{V_{t+1}}{t+1}\right)}{\sqrt[\beta]{\exp\left(\frac{V_t}{c_t}\right)}} \right)^{\sigma_t}}_{\Lambda_{t,t+1}} \left(R^s_{t+1} - \phi R_{t+1}\right) + \phi - 1 \right] \mathbf{z_t} \right\} = 0 \tag{1.24}$$

where $R^s_{t+1} = \frac{q_{t+1}+d_{t+1}}{q_t}$ is the cum-dividend return on risky asset and R_{t+1} is the risk-free interest rate, which is time-varying in the data. Note that the expression for the SDF can be decomposed into two factors, $\Lambda^1_{t,t+1} = \beta\left(\frac{c_t}{c_{t+1}}\right)$ and $\Lambda^2_{t,t+1} = \left(\exp\left(\frac{V_{t+1}}{c_{t+1}}\right) / \sqrt[\beta]{\exp\left(\frac{V_t}{c_t}\right)}\right)^{\sigma_t}$, where the second captures the role of ambiguity attitudes. Equation (1.24) is estimated fully non-linearly with GMM methods.[21] Note that tight restrictions are placed on asset returns and

[21] Optimal GMM parameters minimize a quadratic loss function over the weighted distance between population and sample moments, by a two-step GMM.

consumption data since our moment condition embodies both financial and ambiguity attitudes. For the estimation we fix the loan to value ratio at $\phi = 0.6$ and, given that $\theta_t = -\frac{1}{\sigma_t}$, we estimate the preference parameters, β and θ_t.

Regarding the data, we use real per capita expenditures on non-durables and services as a measure of aggregate consumption. For R we use the three-month T-bill rate, while R^s is proxied through the Standard & Poor 500 equity return.[22] The choice of the instruments follows the literature on time-series estimation of the Euler equations.[23] They are grouped into internal variables, namely consumption and interest rates two quarters lagged, and external variables, namely the excess market return, consumption growth, the value and size spreads, the long-short yield spread and the dividend-price ratio (see also Yogo (2006)). A constant is additionally included in order to restrict model errors to have zero mean. Finally, the model's over-identifying restrictions are tested through the J-test (test of over-identifying restrictions, Hansen (1982)). [24]

Table 1.1: Estimation Results

Sample	Estimated parameters				
	β	θ	$\theta(\tilde{v}_t \geq E\tilde{v}_t)$	$\theta(\tilde{v}_t < E\tilde{v}_t)$	$J-test$
1980-2016	0.836		-4.278	4.000	7.014
	(.016)		(.053)	(.068)	(.857)
1985:Q1-2007:Q2	0.814	-4.261			4.51
	(.014)	(.039)			(.985)
2007:Q3-2016:Q4	0.852	5.499			7.318
	(.015)	(.019)			(.885)

Table 1.1 presents the results. The estimated values of θ_t are conditioned to the logarithm of the continuation value ratio, defined as $\tilde{v}_t = \log\left(\frac{V_t}{c_t}\right)$. Consistently we previous definition good states are those for which the latent value function is higher than its mean and vice-versa for bad states. Column 3 shows results conditioned upon the relation $\tilde{v}_t \geq \mathbb{E}\{\tilde{v}_t\}$, while column 4 reports the results for the complementary condition. We find that a negative value (-4.28) prevails over good states, namely those for which $\tilde{v}_t \geq \mathbb{E}\{\tilde{v}_t\}$, and that a positive value (4.00) prevails in bad states, namely those for which $\tilde{v}_t < \mathbb{E}\{\tilde{v}_t\}$. This gives clear indication on the state-contingent nature of the ambiguity attitudes, being averse to entropy deviations in bad states and opportunistic toward them in good states. According to Lemma 1.3.2 above we know that $\theta_t < 0$, which prevails in good states, implies that agents act optimistically. Similarly a

[22] Data sources are NIPA Tables https://www.bea.gov/iTable/index_nipa.cfm, CRSP Indices database http://www.crsp.com/products/research-products/crsp-historical-indexes, and the Shiller database http://www.econ.yale.edu/~shiller/data.htm, respectively

[23] See Stock, Wright and Yogo (2002) for a survey on the relevance of instruments choice in a GMM setting

[24] This is a specification test of the model itself and it verifies whether the moment conditions are enough close to zero at some level of statistical confidence, if the model is true and the population moment restrictions satisfied.

$\theta_t > 0$, which prevails in bad states, speaks in favour of pessimism.

To further test our result above we ran unconditional estimation over two different historical periods. We choose the first to be Great Moderation sample (1985:Q1-2007:Q2), which captures the boom phase preceding the 2007-2008 financial crisis. The sub-sample representing the recessionary states is the period following the crisis, namely the (2007:Q3-2016:Q4). Estimations, reported in the last two rows, confirm the same state-contingent nature uncovered in the conditional estimates. Finally note that for each sample reported the J test fails to reject model in equation (1.24) at conventional significance levels.

Table 1.2: Estimated Moments of the Pricing Kernel

Moments	$\Lambda_{t,t+1}$	$\Lambda^1_{t,t+1}$	$\Lambda^2_{t,t+1}$
Mean SDF	0.806	0.833	0.967
Standard deviation SDF	8.263	0.332	9.874
$Corr(SDF, \Delta c_t)$	-0.105	-0.999	-0.063
$Corr(SDF, R^s_{t+1})$	-0.081	-0.332	-0.067

Next, given the estimated preference parameters we investigate cyclical properties of the pricing kernel, namely the empirical SDF. Among other things this also gives indications on the cyclical properties of the asset price. To this purpose we decompose the SDF in the two components highlighted above, $\Lambda^1_{t,t+1}$ and $\Lambda^2_{t,t+1}$, where the latter captures the role of ambiguity attitudes. For this exercise we use the sample 1980-2016, which among other things is consistent with the one used later on for the data-model moments comparison. The empirical moments of the SDF are listed in table 1.2, which shows that the volatility of $\Lambda_{t,t+1}$ is explained almost in full by the ambiguity factor $\Lambda^2_{t,t+1}$. The latter also accounts for a lower correlation with respect to both consumption and risky returns.

Given the above estimation results the process for θ_t reads as follows:

$$\theta_t = \begin{cases} \theta^- & \text{if} \quad V_t \geq \mathbb{E}V_t \\ \theta^+ & \text{if} \quad V_t < \mathbb{E}V_t \end{cases} \tag{1.25}$$

We will use this process structure since now on.

1.5 Analytical Results

In this section we derive analytical expressions for asset price, premia and Sharpe ratio and show their dependence on the optimal LR and the shadow price of debt, μ_t. The analytical derivations will allow us to gain first economic intuition on the combined role of occasionally binding constraints and ambiguity attitudes for asset prices and leverage.

1.5.1 The Impact of Ambiguity on Asset Prices

Proposition 1.5.1 (Asset Price Recursion)**.** *The recursive formula for the asset price over the infinite horizon in our model reads as follows:*

$$q_t = \lim_{T\to\infty} \mathbb{E}_t \left\{ \sum_{i=1}^{T} d_{t+i} \prod_{j=1}^{i} K_{t+j-1,t+j} \right\} \tag{1.26}$$

where $K_{t,t+1} = \frac{\Lambda_{t,t+1}}{1-\phi\mu_t^{'}}$ *with* $\Lambda_{t,t+1} = \beta\frac{u_c(c_{t+1})}{u_c(c_t)}m_{t+1}$ *and* $\mu_t^{'} = \frac{\mu_t}{u_c(c_t)}$.

Proof is described in Appendix A.2.1. The asset price clearly depends upon the optimal LR, m_{t+1}, and the shadow price of debt, μ_t. Consider first good states. In this case endogenous beliefs are right skewed toward the upper tails according to Lemma 1, hence both $\Lambda_{t,t+1}$ and $K_{t,t+1}$ are higher than when $m_{t+1} = 1$ for all positive states. In good states the asset price grows, due to increase asset demand, but it does so more under optimist beliefs. Similarly in bad states endogenous beliefs are left-skewed toward the lower tails, hence both $\Lambda_{t,t+1}$ and $K_{t,t+1}$ are higher than in the case with no ambiguity for all negative states. Asset price falls, but they do more so with pessimism. This is the sense in which ambiguity attitudes contribute to the heightened dynamic of the asset price boom and bust cycles. The asset price also depends upon the shadow price of debt, which proxies the margin or the down-payment requested to borrowers. When the constraint is binding margins are positive and increasing, in line with empirical observations (see Geanakoplos (2010)). The higher margins paid by borrowers or the higher collateral value of the asset is reflected in higher asset prices. This also contributes to heightened asset price dynamics.

Proposition 1.5.2 (Equity Premium)**.** *The return for the risky asset reads as follows:*

$$\mathbb{E}_t\{R^s_{t+1}\} = \frac{R(1 - cov(\Lambda_{t,t+1}, R^s_{t+1}) - \phi\mu_t^{'})}{1-\mu_t^{'}} \tag{1.27}$$

while the premium of its return over debt return reads as follows:

$$\Psi_t = \frac{1 - cov(\Lambda_{t,t+1}, R^s_{t+1}) - \phi\mu_t^{'}}{1-\mu_t^{'}}. \tag{1.28}$$

where$\Lambda_{t,t+1} = \beta\frac{u_c(c_{t+1})}{u_c(c_t)}m_{t+1}$ *and* $\mu_t^{'} = \frac{\mu_t}{u_c(c_t)}$.

See Appendix A.2.2 for the proof. The above proposition also shows unequivocally the dependence of the premia over the beliefs as captured by m_{t+1} and the shadow price of debt. While the exact dynamic of the equity premium depends on the solution of the full-model and upon its general equilibrium effects, we can draw some general conclusions on the dependence of the equity premium upon the beliefs and the shadow price of debt.

First, a negative covariance between the SDF and the risky asset returns implies that borrowers are less hedged. This results in a higher return required to hold the risky asset. The opposite is true for positive covariances. While we cannot say with certainty whether the $cov(\Lambda_{t,t+1}, R^s_{t+1})$[25], we know by the Cauchy-Schwarz inequality that $cov(\Lambda_{t,t+1}, R^s_{t+1}) \leq \sqrt{Var(\Lambda_{t,t+1})Var(R^s_{t+1})}$. Therefore anything that either increases the variance of $\Lambda_{t,t+1}$ or R^s_{t+1} will increase their covariance, whether in the positive or the negative domain. Endogenous beliefs formation by inducing fluctuations in m_{t+1} tend to increase the variance of the stochastic discount factor which is given by $Var(\Lambda_{t,t+1}) = Var(\beta \frac{u_c(c_{t+1})}{u_c(c_t)} m_{t+1})$.

Second, the premium also depends upon the shadow price of debt. Taking as given again the covariance between the SDF and the risky return, one can compute the following derivative: $\frac{\partial \Psi_t}{\partial \mu_t'} = \frac{(1-\phi)-cov(\Lambda_{t,t+1}, R^s_{t+1})}{(1-\mu_t')^2}$. If the $cov(\Lambda_{t,t+1}, R^s_{t+1})$ is negative the derivative is certainly negative[26]. In other words when there are low hedging opportunities a tightening of the constraint implies that borrowers require higher premia to hold the risky asset. The asset already conveys poor insurance opportunities, a tightening of the constraint by reducing the asset collateral value, reduces its demand. Hence borrowers are willing to hold only at higher premia. Endogenous beliefs also play an indirect role in this dependence. Indeed as explained above fluctuations in beliefs generally raise the absolute value of the covariance. Hence, consider again the case of a negative covariance. In this case fluctuations in beliefs impair even more the hedging abilities of the risky assets and this in turn increases the premium that borrowers ask in face of a tightening of the borrowing limit.

Proposition 1.5.3 (Sharpe Ratio)**.** *The Sharpe ratio in our model reads as follows:*

$$SR = \frac{\mathbb{E}_t\{z_{t+1}\}}{\sigma_z} = \left[\frac{\sigma^2_{\Lambda^*_t}}{\bar{\Lambda}^{*2}} - 2\mu_t' \frac{(\phi-1)\mathbb{E}_t\{z_{t+1}\}}{\sigma^2_z} - \frac{\mu_t'^2}{\bar{\Lambda}^{*2}} \frac{(\phi-1)^2}{\sigma^2_z} \right]^{\frac{1}{2}} \tag{1.29}$$

where $z_{t+1} = R^s_{t+1} - R$ *is the asset excess return* $\bar{\Lambda}$ *is the long run value for the SDF,* $\sigma^2_{\Lambda^*_t}$ *is the volatility of the SDF and* σ^2_z *is the volatility of the excess return.*

Proof is given in Appendix A.2.3. The presence of endogenous beliefs raises the Sharpe ratio and brings it close to the empirical values as we show in Table 1.4. Matching the empirical values of the Sharpe ratios is typically hard for models with asset pricing and/or financial frictions. The reason being that typically an increase in the excess returns of the risky assets is accompanied by an increase in its volatility. Analytically it is easy to see why the Sharpe ratio raises in our model. First fluctuations in m_{t+1} raise fluctuations in the stochastic discount factor, Λ^*_t, hence in its variance. This in turn raises the Sharpe ratio. Second, fluctuations in θ_t enhance fluctuation in beliefs, m_{t+1}. Third, the kinked nature of the value function steepens fluctuations in m_{t+1} and the SDF also since marginal utilities tend to infinity around the kink. In turn any increase in the variance of m_{t+1} raises the variance of Λ^*_t and the Sharpe ratio. Intuitively in presence of *uncertainty* or ambiguity agents require a premium which goes beyond the one needed to cover risk[27] as measured by the volatility of the excess return. If agents knew

[25] This indeed depends on whether $\mathbb{E}_t(\Lambda_{t,t+1}, R^s_{t+1}) > \mathbb{E}_t(\Lambda_{t,t+1})\mathbb{E}_t(R^s_{t+1})$ or $\mathbb{E}_t(\Lambda_{t,t+1}, R^s_{t+1}) < \mathbb{E}_t(\Lambda_{t,t+1})\mathbb{E}_t(R^s_{t+1})$.

[26] If the $cov(\Lambda_{t,t+1}, R^s_{t+1}) > 0$, then whether $\frac{\partial \Psi_t}{\partial \mu_t'}$ is positive or negative depends upon whether the $cov(\Lambda_{t,t+1}, R^s_{t+1}) > (1-\phi)$ or not.

[27] Here we refer to the distinction between uncertainty and *risk* introduce by Knight (1921).

the objective probability distribution, they would need to be compensated only for bearing tail risk. As the tail itself is uncertain, borrowers require a higher premia.

In past literature it was noted that the model implied Sharpe ratio can match the empirical counterpart by assuming implausibly large values for the risk-aversion parameter (see Cochrane (2005), chapter 13). In the numerical simulations below we show that this is not the case for our model.

At last, note also that the Sharpe ratio depends negatively upon the shadow value of debt. When the constraint binds borrowers start to de-leverage and to reduce the demand of risky asset. As a result this reduces the expected excess returns relatively to the return on debt. This is compatible with the pro-cyclical nature of the returns on risky assets observed in the data.

1.6 Quantitative Results

In this section we solve the model numerically employing a global solution method, namely policy function iterations with occasionally binding constraints. We provide details on the solution method in Appendix A.3. We group our results in three. First, we search for the optimal model calibration. To do so we choose some target moments in the data and we search for the set of parameters that minimizes the distance between the targets and the model-implied moments. This gives further empirical validation of our model. Second, under the optimal calibration we verify if the model can match several volatilities and correlations for asset prices, returns, equity premia and leverage. We show that in fact the model does it well. At last, under this optimal calibration we examine policy functions and we conduct a crisis event exercise. Our main result is that with ambiguity the model produces steeper asset prices and leverage increases in booms, which are then followed by sharper de-leverage and crises in recessions.

1.6.1 Calibration Strategy

This section describes the calibration strategy. We divide the set of structural parameters in three groups. The first group includes parameters which are calibrated using external information. Those are the risk free rate, the loan-to-value ratio, the fraction of financial wealth over total wealth. The second group includes parameters calibrated using a matching moments routine. Those are θ, the absolute risk aversion coefficient, the discount factor and the volatility of the income process. The third group includes parameters which are calibrated with the estimation of the income process, more specifically the autocorrelation of the income process. Table 1.3 summarizes the results of the calibration procedure.

In order to calibrate the second group of parameters, we choose to match six empirical moments (the matching is shown in Table 1.4, where also other moments are displayed), namely the volatility of debt σ^b, the autocorrelation of debt ρ^b, the correlation between debt and consumption $Corr(\Delta b^t, \Delta c^t)$, the expected return on risky asset $\mathbb{E}_t(R_t^s)$, the volatility of return on risky asset σ^{R^s}, the correlation between return on risky asset and consumption growth $Corr(R_t^s, \Delta c_t)$. To compute the empirical equivalent we focus on the data sample 1980:Q1-2016:Q4, which captures a period of both of large debt growth and subsequent de-leverage.

More details on the data sources are in Appendix A.4. We do not include the equity premium among our targets because the risk free rate is exogenous in the model, but we show later on that our model can match it well. Note that while the income shock correlation is directly estimated in the data, its volatility is instead calibrated. It is indeed well known from past literature that estimated values exhibit large measurement errors (see Heaton and Lucas (1996) and Deaton (1991)).

Table 1.3: Values for the calibrated parameters

Parameter	**Meaning**	**Strategy**	**Value**
R	*Risk-free rate*	3month T-bill rate	1.0114
ϕ	*Loan-to-value ratio*	Crises Probability	0.15
α	*Share of dividend*	Fraction of financial wealth	0.10
θ	*Penalty parameter*	Matching Moments	-1.35
γ	*Risk aversion*	Matching Moments	2.075
β	*Discount factor*	Matching Moments	0.930
σ_y	*Income Volatility*	Matching Moments	0.0415
ρ_y	*Income Persistence*	Estimation	0.634

The matching moment routine starts from the following grids: $\sigma^y \in [0.02, 0.07]$ for the states of the income shock, $\beta = [0.92, 0.98]$, $\gamma = [1, 2.2]$, and finally $\theta_t \in \{[-5, 5], 100\}$[28]. In the grid for θ_t we introduce the value 100, in order to check if the model with no ambiguity produces theoretical moments which perform better than our model with waves of optimism and pessimism. Moreover, the grid is defined between 5 and -5 because out of these bounds the difference between the model with and without ambiguity becomes negligible.

It is interesting to note that the estimation of the full model through moments matching equally delivers the same type of state-contingent process for the parameter θ_t as the one we uncovered with our GMM estimation above. The estimated values are naturally different between the two estimation methods, since in the GMM case the regression is based on one equation summarizing only borrowers' first order conditions, while in he second case the estimation involves the full set of model equations. But the fact that the two estimations deliver the same type of state-contingent process is important.

1.6.2 Empirical Moments Matching

In this section we evaluate the model's ability to match the empirical moments under the optimal calibration determined above. We also compare the theoretical moments of our model with ambiguity attitudes (labelled AA since now on) with those of the equivalent model but with rational expectation (labelled RE since now on). The following Table 1.4 summarizes the main results:

[28]For each parameter we check that the optimal values do not hit the bounds of the grid.

Table 1.4: Empirical and model-based moments

Moments	Mnemonics	Empirical	Model AA[1]	Model RE
	Matched Moments			
Volatility debt	σ^b	12.52	12.37	7.24
Persistence debt	ρ^b	0.846	0.539	0.331
Cyclicality debt	$Corr(\Delta b_t, \Delta c_t)$	0.668	0.378	0.821
Exp risky return	$E_t(R_t^s)$	9.38	8.19	7.38
Volatility risky returns	$\sigma^{R_t^s}$	16.21	17.46	12.40
Cyclicality risky returns	$Corr(\Delta R_t^s, \Delta c_t)$	0.474	0.989	0.989
	Other Relevant Moments			
Equity premium	$E_t(R_t^s - R)$	8.25	7.05	6.24
Sharpe ratio	$\frac{E_t(R_t^s - R)}{\sigma^{R_t^s - R}}$	0.522	0.404	0.503
SDF[2]	$E_t(\Lambda_{t,t+1})$	0.806	0.940	0.939
Volatility SDF	$\sigma^{\Lambda_{t,t+1}}$	8.263	15.10	12.987
Cyclicality SDF	$Corr(\Lambda_{t,t+1}, \Delta c_t)$	-0.105	-0.976	-0.988
Corr SDF with risky returns	$Corr(\Lambda_{t,t+1}, R_t^s)$	-0.081	-0.967	-0.98
Prob(crisis)	-	4.00[3]	3.16	4.51

[1] *Column 2 and 3 compare theoretical moments under ambiguity attitudes versus rational expectation*;
[2] *In the data this refers to the SDF estimated in section* (1.4);
[3] *We do not calculate the empirical frequency of the financial crises but we follow Bianchi and Mendoza (2015), who derive an average of 4 crises every 100 years in the developed countries.*

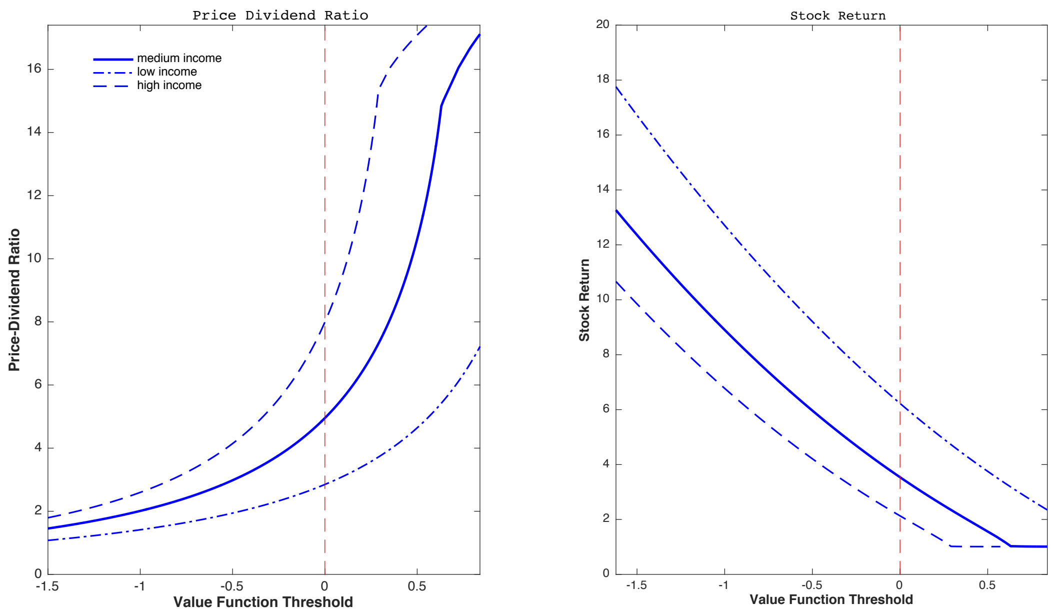
The upper panel of Table 1.4 shows the matched moments (according to the criteria set in the previous section), while in the lower panel other relevant moments are shown. The overall message is that our model fits well the empirical moments. First, it is better capable of matching empirical debt and risky asset return volatilities, relatively to the RE model. This is so despite both models exhibit amplification induced by the occasionally binding collateral constraint. This shows that endogenous beliefs are also needed to explain asset and debt markets dynamics. The equity premium as well as its cyclical properties are also well captured and again the presence of ambiguity attitudes seem to improve even above the benchmark model featuring solely the collateral constraint. As explained in Cochrane (2005) the ability to match contemporaneously the long run equity premia and asset returns and their cyclical properties is related to the agents' attitude toward events on the tails. In our model borrowers are endogenously optimistic, hence risk-takers, on the upper tail, while they are pessimistic, hence risk-sensitive, on the lower tail. This additional effect, stemming from the endogenous waves of optimism, improves the ability of the model to match the equity premium and its cyclical properties. In terms of matching the Sharpe ratio and the empirical SDF, both models seem to perform similarly and with acceptable performance, thereby showing that the kink induced by the occasionally binding collateral constraint contributes alone to this result. At last, both model match the pro-cyclicality of leverage which is well documented in the data. Leverage indeed increases in booms due to a combination of exuberance and lax debt constraints and declines in recessions due to a combination of pessimism and increasing margins, namely borrowers' down-payments. Here neither our model nor the RE model seem to match the empirical value with precision, as the first underestimates, while the second overestimates.

At last note, that the model reasonably matches the empirically probability of the crisis. For the empirical counterpart of such a probability we rely on the value presented in Bianchi and Mendoza (2015).

1.6.3 Excess Returns Predictability

Before turning to the implications of this model for the unfolding of crises, it is instructive to conclude our assessment of its empirical validity by examining also the implied excess returns predictability. In asset pricing this is an important test on whether the model ingredients are able to account for the sources of risk that drive expected returns. A number of empirical observations (Fama and French (1988)) established predictability of risk premia through current or past price-dividend ratios: the pro-cyclical movements in stock prices generate a large countercyclical variation in expected risk premia. In macro so far the introduction of habits in consumption proved successful in providing a theory for return predictability (Campbell and Cochrane (1999)). Here, along the same logic, we evaluate what is the role of our behavioural ingredient, given by ambiguity attitudes, in the debate.

Figure 1.1: Price-consumption ratio and stock returns



Reminding that we first estimate and then define the ambiguity parameter conditioning on the demeaned value function $(\tilde{v}_t - E(\tilde{v}_t))$, we assess in Figure 1.1 the model's implied price-dividend and risk premia determination in terms of our ambiguity attitudes. Note that the plot reports results for 3 realizations of the income process. More importantly when the level of the value function positively departs from its mean we're in an region where agents displays ambiguity-seeking behaviours, while ambiguity aversion prevails for negative deviations. Then, compatibly with our analytical results in Proposition 1.5.1 we conclude that ambiguity attitudes map crucially into the price-dividend ratio generating asset price build-ups and low equity returns under ambiguity seeking; while asset price bursts and high returns under ambiguity aversion.

From these results the intuition that since the type of beliefs deviation depends on the realizations of the demeaned value function and we showed that the latter maps into the price-dividend ratio, then measuring return predictability over the price-dividend ratio (or the dividend yields) would account for the influence of ambiguity attitudes. How they intervene in the ability of the model to forecast future risk premia is evaluated through a return predictability regression (see also Fama and French (1988) and Cochrane (2011)) of future excess returns, at various horizons k, on the dividend yield, $\frac{d_t}{q_t}$, using both historical and model's simulated data. The regression[29] we consider is the following:

$$\mathbb{E}_t\{R^s_{t+k}\} - \mathbb{E}_t\{R_{t+k}\} = a + b\frac{d_t}{q_t} + \varepsilon_{t+k} \tag{1.30}$$

Table1.5 reports the results, comparing empirical evidence and model's performance. For the former, the estimated coefficients are positive: high dividend yields reliably anticipate periods of high returns. Predictability however proves to be poor in the short-run, but increases with the forecasting horizon, as widely stated in the related literature. Model's results, in the second panel of the table, are instead much more significant and informative at all horizons. Moreover, the comparison between the model with ambiguity attitudes (labelled AA as usual) and the model without (labelled RE as usual) highlights the role of ambiguity, which by affecting the price-dividend ratio (as in Figure 1.1) is responsible for substantially higher excess returns predictability.

Table 1.5: Excess return predictability regression

	Historical data: 1960-2016			*AA* Model			*RE* Model		
Horizon (years)	$\frac{d_t}{q_t}$	$t\left(\frac{d_t}{q_t}\right)$	R^2	$\frac{d_t}{q_t}$	$t\left(\frac{d_t}{q_t}\right)$	R^2	$\frac{d_t}{q_t}$	$t\left(\frac{d_t}{q_t}\right)$	R^2
1	1.06	0.60	0.01	18.57	8.42	0.58	7.08	6.20	0.38
5	11.44	1.94	0.06	26.03	11.0	0.54	10.37	6.04	0.37
7	26.04	4.87	0.17	30.66	7.81	0.54	11.88	6.14	0.38
10	54.87	5.90	0.28	35.16	5.27	0.52	11.34	5.46	0.26

1.6.4 Policy Function and Crisis Event

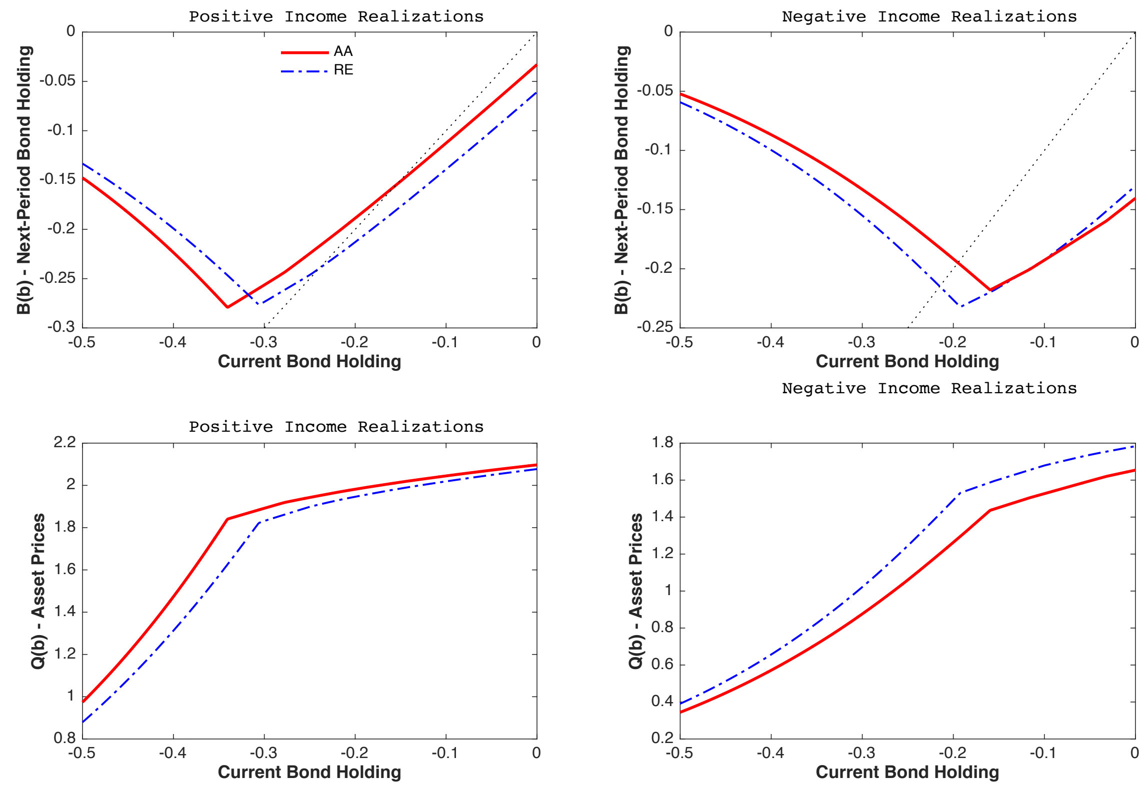
We have argued that our leverage cycle model with state-contingent beliefs' distortions has a sound empirical ground. The estimated SDF implied by our model shows that the role

[29] Estimation is done through overlapping OLS regression, standard errors are computed based on Hansen and Hodrick (1980). Data for this estimation are the price-dividend ratio for the S&P stock returns from the Shiller Database and the 3-months T-bill rate from CRSP Indices Database.

of ambiguity attitudes is significant and sizeable. Under the estimated ambiguity parameter and the empirically optimal calibration, we also showed that our model can account well for several asset price and leverage moments. This second exercise serves a cross-check of the model empirical validity.

Given the above, we proceed describing the dynamic properties of our model in comparison to the RE benchmark and by focusing in particular on the leverage and asset price cycles and on the unfolding of a crisis. We do so in two steps. First, we plot policy functions of debt and asset prices. Next, we simulate a crisis event.

Figure 1.2: Policy functions for debt and asset prices

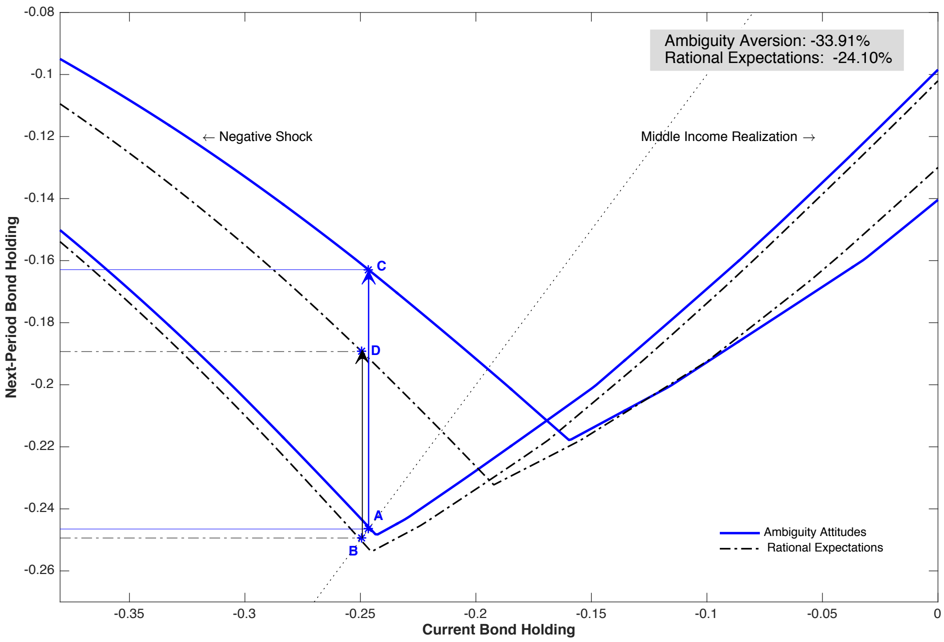


Borrowing and Asset Pricing decisions

Figure 1.2 below shows the decision rules for debt and asset prices with respect to past debt holdings across the model with ambiguity attitudes, labelled AA (red line) and the model with rational expectations, labelled RE (blue line). Note that the full set of policy functions can be found in Appendix A.5. We interpret the results distinguishing between positive (+5% from income trend; left panels) and negative realizations of the shock (−5% from trend; right panels) in order to appreciate the non-linearity arising by the changing ambiguity attitudes over the different states of the economy. Moreover, in each panel the kink separates the constrained from the unconstrained region and it represents the point at which the collateral constraint is marginally binding in each economy. Finally, the intersection between the 45 degree line and the policy function defines the stationary levels of debt. Several considerations emerge.

First, both economies are able to produce the V-shaped bond holdings decision rules, which are a typical feature of models with high deleveraging and financial crises (see e.g. Bianchi (2011) and Bianchi and Mendoza (2015)). To the right of the kink the policy functions are upward-sloping, corresponding to the unconstrained values of debt, while to the left they are downward-sloping identifying the constrained region where next-period bond holdings decrease in current bond holding. The kinked policy functions for asset prices follow accordingly: they increase with wealth and more steeply in the constrained region.

Figure 1.3: Debt Amplification Dynamics



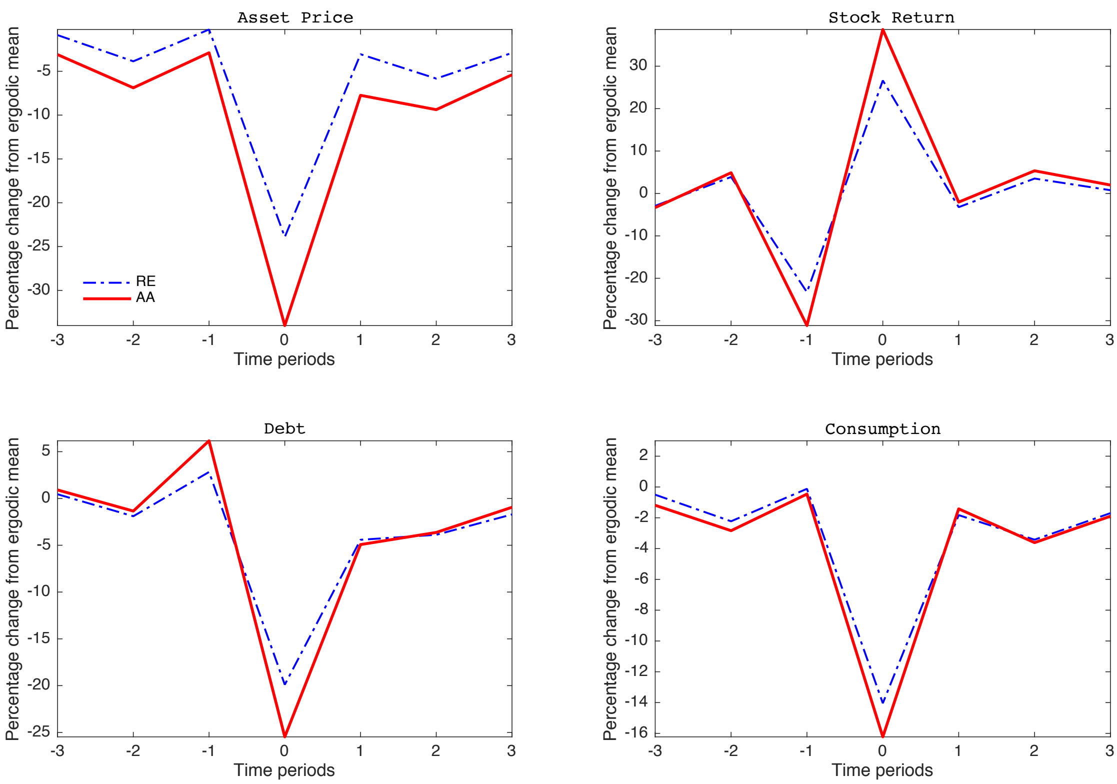
Second, the policy function for the AA model moves away from the one under RE both in the scale of the dynamics in each region and in the position of the kinks. In particular, given a negative state of the economy, higher previous-period debt induces a binding constraint earlier, increasing the probability of lying in the financial amplification region. The opposite holds for booms, where optimism boosts the collateral values, which in turn relaxes the constraint and facilitates the build-up of leverage. Thus, given the shifted location of the binding and slack regions, debt choices under AA, when constrained, associate a sharper or a more damped contraction in debt whether the economy is in booms or in busts. This nonlinearity reflects optimistic and pessimistic attitudes toward future realizations and generates amplification dynamics in the leverage cycles. We will visualize the size of this result below. At last, focusing on the asset price panels the comparison between the two models turns to be quite interesting. Asset prices in the AA model lie always above the RE benchmark in booms and always below in busts, which is coherent with the ability of the AA model to associate to a given initial debt position more debt and less debt, respectively for the two income states. Next we compute how large would be the extent of a de-leverage when the steady state of the economy is perturbed by a one-period 5% fall in income. This exercise offers a clear visualization of the enhanced financial amplification dynamics produced by AA, keeping the parallel with the RE model.

Figure 1.3 reproduces the following experiment. Assume that the two economies lie in equilibrium in A and B, respectively. Then, at the time of the shock the new negative realization of income forces a sharp upward adjustment of the bond decision rules and the temporary equilibria jump to C and D. The arrows define a drop in bond holdings which results to be much more pronounced for the model under the AA model. Interestingly, the AA model generates a drop of -33.9%, which exceeds the RE equivalent by about 10 points. This speaks about the model's quantitative relevance in producing amplified leverage cycles.

Financial Crises

The crisis event displayed in Figure 1.4 proves the model's ability to generate financial crises and studies relevant macro dynamics around it. More in detail, the event analysis is realized using model-simulated data for the two economies, AA and RE, and defining as crises the events in which the collateral constraint binds and the current account is at least two standard deviations above the trend. Then, we construct seven-periods event windows centred on the crisis to analyse pre- and post-crises patterns.

Figure 1.4: Crises Event Study



From the comparison between the two economies lies in the ability of the model with AA to account for stronger build-up of leverage prior to the crisis (around +3%) and sharper de-leveraging at the crisis (around -7%). Again the role of the state contingent distortion is important in understanding this dynamic. In booms optimism boosts collateral values, relaxing the constraint and facilitates the build-up of leverage. In recessions the opposite is true. Pessimism induces assets' fire sales, this generates sharper declines of the collateral values forcing borrowers to de-leverage earlier and more severely. Accordingly, looking at asset prices, consumption and equity returns helps understanding the results around debt decisions. Indeed, all of them display more severe dynamics under ambiguity aversion. The asset price collapses, for instance, playing an important role in explaining the more pronounced decline in debt under the AA model, reflecting a strong Fisherian deflation mechanism. Moreover, consumption falls 2 percentage points more and the risky return results to drive the enhanced pre- and post-crisis debt patterns, falling more sharply in booms and increasing when the crisis occurs.

1.6.5 Intermediation Sector and Intermediation Shocks

Lack of transparency and ambiguity play an important role in crises developments as we showed so far, but by no means instability stemming from the intermediation sector, hence originating in the credit supply, has a major role too. This is particularly true within the context of the 2007-2008 financial crisis. While including all possible sources of intermediation disincentives is beyond the scope of this paper, we nevertheless wish to assess the role of the intermediation channel. This is important as one should test whether the beliefs-related channels described so far persist even when the supply side of credit is inserted in the model. In fact, we find that not only the role of ambiguity attitudes is preserved, but in most cases is amplified and the interaction with the intermediation channels is compelling.

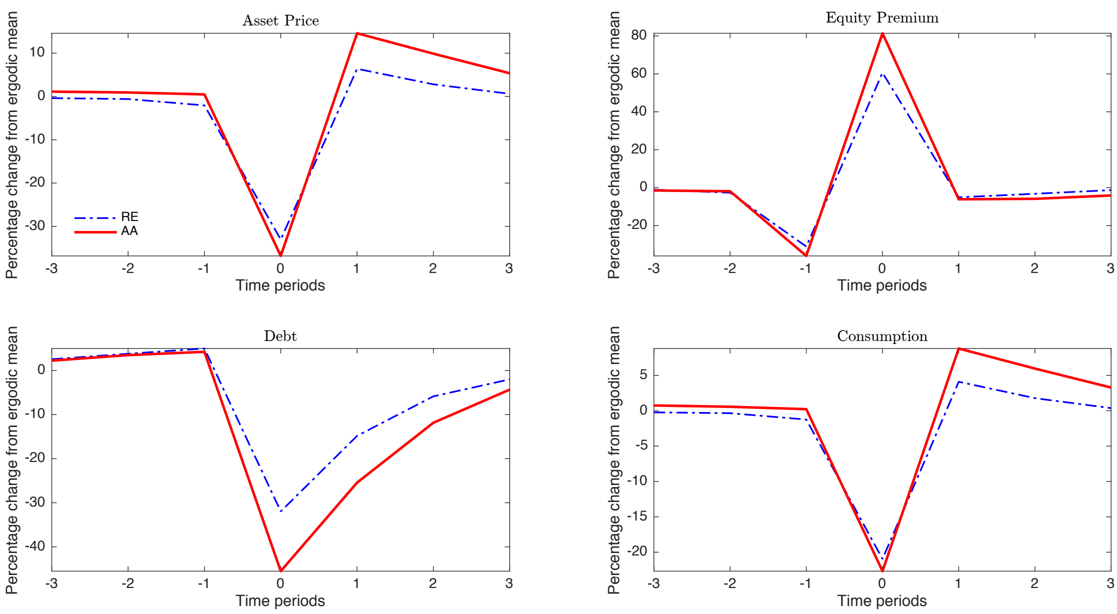
We introduce intermediation by assigning the role of debt monitoring to a bank. This is actually realistic since atomistic lenders do not monitor or screen debtors individually, but largely assign this function to an intermediary. In this context the collateral constraint results from the bank design of a debt contract that is incentive compatible, meaning that it reduces the incentives of the borrower to divert resources and default. We formalize this type of contracts and show how the collateral constraint emerges from such incentive compatibility constraint in Appendix A.7 Within this context an intermediation shock, which suddenly tightens the supply of credit, affects the parameter governing the loan-to-value ratio, ϕ, which itself governs the strength of the incentive problems. Intuitively the shock can be interpreted in two ways, both affecting the contractual agreement in a similar vein. It could capture financial innovation in the form of derivatives and/or asset back securities issuance, which being pervasive prior to the crisis, allowed banks to off-load credit risk and reduced the tightness of the debt contract. A sudden freeze of the asset backed market liquidity due for instance to the sub-prime shock would have then induced a sudden fall in ϕ. A second interpretation, linked to the first, is that higher availability of liquidity[30] prior to the crisis had lessened banks' monitoring incentives, something which resulted in higher loan-to-value ratios, ϕ. After the crisis occurs, the squeeze in liquidity, hence banks' funding, could suddenly tightens the loan-to-value ratio. Both interpretations, which are realistic particularly in the context of the recent financial crisis, have the effect of

[30] This again could be due either to the possibility of raising additional bank liabilities through asset backed securities or through the ample availability of liquidity in interbank and repos markets prior to the 2007-2008 crisis.

producing a sudden tightening of credit supply. Within this context we subject our model to an intermediation shock to ϕ and assess its role as well as its interaction with ambiguity attitudes. We do so by analysing again policy functions, crisis events and second moments of the model.

Before proceeding to the assessment of the quantitative results, a few words are needed regarding the calibration of the shock. We define a high and a low level of the loan-to-value ratio, respectively $\phi_l = 0.22$ and $\phi_h = 0.28$, calibrated in order to match the empirical volatility of debt. The shock then follows a two-state regime-switching Markov process, with a transition matrix calibrated to replicate the empirical probability and duration of the crises events, as in Bianchi and Mendoza Bianchi and Mendoza (2015). More in detail, the probability to remain in a high state, π_{hh} is set equal to 0.955 in order to match a frequency of crises close to 4%, while the transition probability from a low to high state π_{lh} is equal to one, implying a one year duration of the crises. The remaining transition probabilities are set as complements of the previous ones, i.e $\pi_{hl} = 1 - \pi_{hh}$ and $\pi_{ll} = 1 - \pi_{lh}$.

Figure 1.5: Crises Event Study with income and intermediation shock



We start in this case from a crisis event, since this makes immediately visible the role of the credit supply for the crisis development on top of the role of ambiguity attitudes. Figure1.5 compares the crisis event in the model with ambiguity attitudes and with rational expectations. The crisis event is defined as before, but now it is triggered by a combination of income and intermediation shocks. Specifically, we simulate the model in response to both shocks, we then observe that the crisis originates exactly when both shocks turn negative. The Figure shows two interesting facts. First, the role of ambiguity attitudes remains. It is still true that beliefs formation by affecting the value of collateral through endogenous skewed beliefs induce sharper crises than under the case with no ambiguity. Second and interestingly, this time the drop in the crisis is even larger. This is reasonable since now both credit demand side and supply side components are operative. Intuitively the steepness of the crises now depends on two channels.

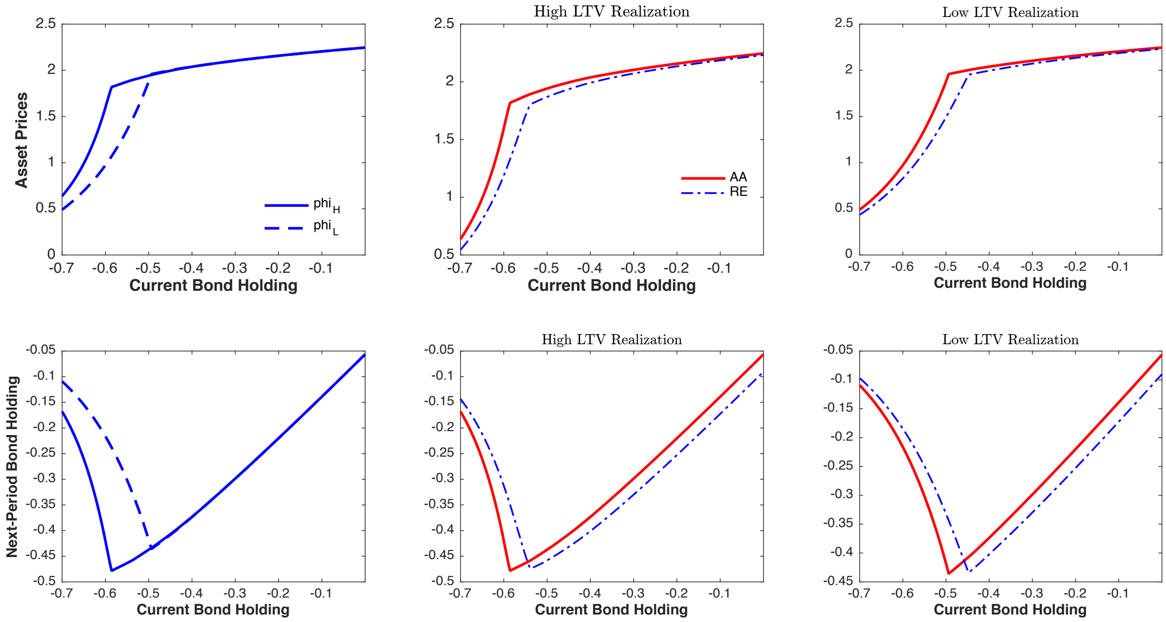
As before the positive skewed beliefs, valid prior to the crisis, induced higher demand for leverage and the negative skewed beliefs, materializing after the crisis, induce de-leveraging. On top of this the progressive reduction of ϕ facilitates debt supply prior to the crisis and produces a credit crunch after the crisis.

To examine more in details the intermediation channel we examine the policy functions for debt and asset prices. Figure 1.6a below shows the policy functions conditional to positive realizations of the income shock for asset prices and debt by comparing various scenarios. In the first column we compare the model with ambiguity attitudes for two values of ϕ. This case allows us to isolate only the contribution of credit supply. As before the kink represents the turn in which the constraint shifts from binding to non-binding. The comparison shows that a low ϕ, namely tight credit due to high monitoring standards or low availability of liquidity, has two effects. On the one side, it enlarges the constrained region. On the other side, it reduces leverage, and this effect can be beneficial in the medium to long run. The second and the third columns compare the models with and without ambiguity attitudes, respectively for low levels of ϕ (second column) and high levels of ϕ (third column). Two interesting observations emerge. First, as before under the model with ambiguity attitudes asset prices are higher and debt displays the previously underlined nonlinear dynamics over constrained and unconstrained regions. This as before is due to the nature of the positive skewed beliefs that emerge under positive income shocks. Second, the comparison between a high and a low level of ϕ shows that the qualitative pattern of the policy functions remains unaltered, albeit the constrained region is expanded under the low loan to value ratio. In other words, the forces operating through the ambiguity channel remain active even when introducing supply side elements. The dominant effect of the latter is more evident in terms of changes in the size of the constrained region. To fully complete the assessment of the policy functions Figure 1.6b shows the results for the policy functions conditional on negative income realizations. The message is largely symmetric to the one described above.

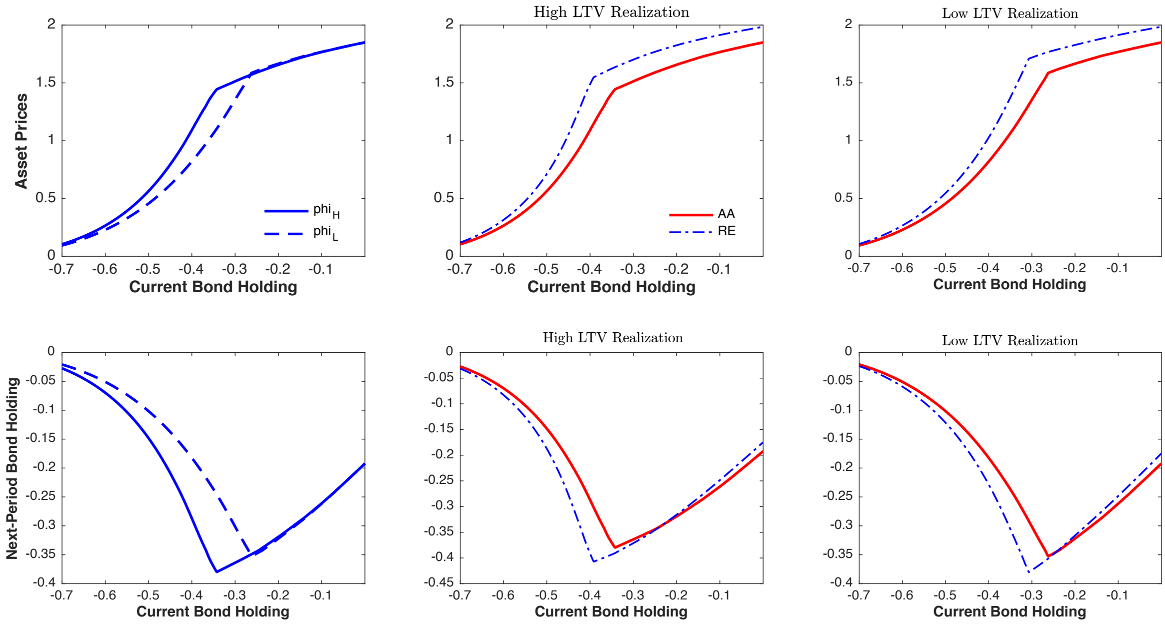
At last, we ask whether the introduction of the intermediation shock can improve upon the moment matching and if so along which dimension. Table1.6 below shows again the comparison of a selected numbers of second moments between the data, the model with and without ambiguity attitudes. This time the comparison is done by simulating the model also in response to the intermediation shock. The addition of the intermediation shock preserves most of the previous moments and improves in terms of data matching along other dimensions. The Table highlights primarily moments that change with the introduction of the intermediation shock. The most noteworthy result is that the introduction of credit supply fluctuations increases debt pro-cyclicality, which as discussed before, is an important stylized fact. The reason is intuitive. The double occurrence of the negative income and credit supply shock tightens leverage much more sharply. Equally the double-coincidence of the positive income and credit supply realizations heightens the build-up of leverage. Those movements on the tails help to increase average pro-cyclicality. The volatility of debt is also somewhat higher, mostly so in the model with ambiguity attitudes, and is closer to the data value. This again might be due to the contribution of the tails. On the other side, it shall be mentioned that the introduction of the intermediation

Figure 1.6: Policy Functions for the model with intermediation

(a) Positive intermediary shock realization



(b) Negative intermediary shock realization



shock worsens the volatility of risky returns, which now goes above the one detected in the data. This effect is possibly due to the fact that our model does not account for loss absorption capacity of the intermediation sector in terms of equity capital and/or liquidity buffers. Those elements would indeed limit the extent of fire sales in risky assets when credit supply tightens, hence they would reduce fluctuations in asset prices.

Table 1.6: Selected empirical and model-based moments

Moments	Mnemonics	Empirical	**Model AA**	**Model RE**
Matched Moments				
Volatility debt	σ^b	12.52	11.55	9.78
Persistence debt	ρ^b	0.846	0.432	0.385
Cyclicality debt	$Corr(\Delta b_t, \Delta c_t)$	0.668	0.792	0.795
Exp risky return	$E_t(R_t^s)$	9.38	8.67	7.88
Volatility risky returns	$\sigma^{R_t^s}$	16.21	23.45	19.40
Cyclicality risky returns	$Corr(\Delta R_t^s, \Delta c_t)$	0.474	0.983	0.992
Equity premium	$E_t(R_t^s - R)$	8.255	7.013	7.050
Prob(crisis)	-	4.0	4.06	5.53

To sum up the main contribution of the intermediation channel in our model is that of modifying the size of the constrained versus the unconstrained region, that of contributing to explain the severity of a financial crisis and that of contributing to explain debt pro-cyclicality.

1.7 Conclusions

Financial crisis are most often triggered by endogenous instability in debt markets. The latter are typically characterized by collateral constraints and opacity in asset values. Under lack of transparency the beliefs formation process acquires an important role since eventually it affects the value of collateral and with it the debt capacity. The narrative of most crises depict sharp increases in debt and asset prices prior to them and sharp reversal afterwards.

We therefore introduce in a model in which borrowers fund risky assets through debt and are subject to occasionally binding collateral constraints, beliefs formation, driven by ambiguity attitudes that endogenously induce optimism in booms and pessimism in recessions. In booms optimistic borrowers demand more risky assets, which results in higher asset price growth (compared to the case with only collateral constraints), and lever up more. In recessions pessimistic borrowers de-leverage sharply and off load risky assets. This beliefs formation process coupled with the occasionally binding nature of the collateral constraint is a crucial element in explaining the combined amplified dynamic of asset prices and leverage as well as the whole span of their long run and short run statistics. Importantly we assess the empirical validation of our model both through GMM estimation of the Euler equation and through data-model moment matching.

Chapter 2

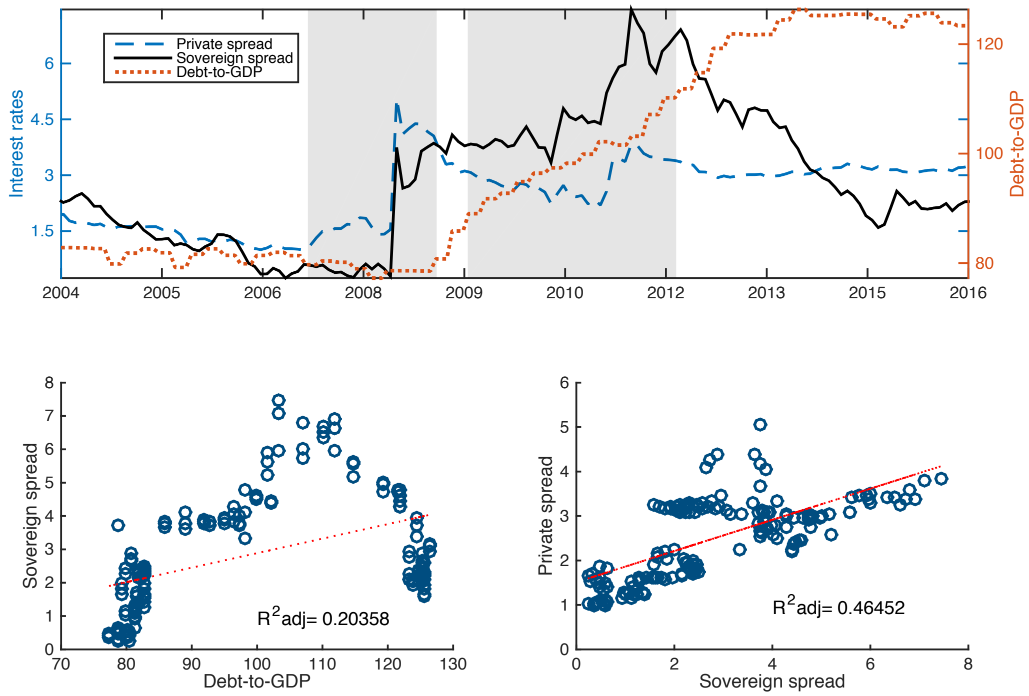
Synchronization in Sovereign and Financial Vulnerability

2.1 Introduction

Following the global financial crisis, a number of European countries experienced strong increases in both sovereign and private debt interest rate spreads. The common view about the surge in spreads is that they reflect a vicious spiral of fiscal and financial distress (Diebold and Yilmaz (2009)): on the one hand, sovereign risk feeds back into banks' fragility through their balance sheet exposure to government imbalances; on the other hand, banks' risk spills to sovereign instability when it calls for government guarantees and for its recessionary implications throughout credit contractions. In parallel, moreover, a growing attention is devoted to the determinants of rising sovereign risk premia, with the ultimate goal to discuss policy implications. One view addresses the deterioration of the fiscal outlook priced by markets as entirely driven by the worsening of macro fundamentals, where the debt-to-GDP ratio is generally conceived as the most significant variable at the roots of the sovereign capacity to service debt obligations (see Yeyati and Panizza (2011) and Mendoza and Yue (2012)). Under this perspective, reforms of fiscal consolidation are justified. A more critical view (Calvo (1988)), instead, believes that expectations-driven factors, weakly related to debt dynamics, generate confidence crises, which severely inject financial panic in sovereign and financial markets. This theory calls for interventions on markets' miss-pricing, as at least complements of fiscal corrections. We perform an empirical analysis to intervene jointly on the two debates. By using Markov-switching methods, we design a small-scale framework which captures, given our identification strategy, endogenous feedback dynamics between corporate and sovereign spreads, and a measure of macro risk, given by the debt-to-GDP ratio. We then evaluate the properties of the model under the realizations of regimes, identified on two independent sources of macro-financial instability: one addressing the link between debt sustainability and sovereign yields; the other interesting the private-sovereign risk nexus.

In order to grasp a first intuition on the mechanisms under study, Figure 2.1 shows some data properties for Italy, Spain and Portugal, selected as reference countries having recently experienced both fiscal strain and increased spreads. We report here the evolution, since 2003 up to the end of 2016, of sovereign and private spreads, along with that of the macroeconomic fundamentals, summarized by the general government debt-to-GDP ratio. For the sovereign and private rate we take the 10-year government bond and the lending rate for non-financial corporate debt (over 5 years maturity), respectively; while the German 6-month Zero-Coupon Bonds (ZCB) is assumed to be the risk-free asset. The grey areas identify the financial and sovereign debt crises, respectively.

Figure 2.1: Sovereign Debt and Spreads. Italy, Spain, Portugal



The dynamics displayed in Figure 2.1 signal that strong nonlinearities characterize the sovereign risk channel. Both the linkage between the debt ratio and the sovereign bond spread and that between the latter and the non-financial corporate spread can be interestingly discussed decoupling the sample between the crises period and the pre and post crisis. Until the unfolding of the 2008 burst, fiscal balances, with an average debt-to-GDP ratio of 81%, were not perceived as a looming concern, as signalled by low spreads on sovereign debt (on average 1.5%) and private credit (1.55%). However, that was a period in which good growth performance and a benign financial environment masked the accumulation of an array of macroeconomic and financial vulnerabilities (Lane (2012)). Since 2008, indeed, the severe global financial crisis injected abruptly high risk on the markets. With the collapse of Lehman Brother, private spreads raised at 5.81%, and public spreads aligned at 3.75%. Financial instability shook the area-wide system and, while the main focus of the political agenda was on the banking system (calming private spreads), markets' concerns moved to address country-specific fiscal risks. Indeed, fiscal imbalances were gradually worsening, increasing at an 8% annual rate between 2008 and the end of 2012. In parallel, sovereign spreads never decreased since 2008 and started to diverge from private spreads when the sovereign debt crisis hit. For Italy the public spread against Germany reached 7.16% in 2011, without reflecting an apparent fundamental macro change. Markets' fear, investors' pessimism or policy uncertainty were contributing to the surge in spreads. This has been even clearer with Draghi's speech[1], which curbed agents' expectations and, with them, private and public spreads, inducing a new scenario of stability around an average level of debt at 124% of GDP.

The analysis of the linkages between fiscal imbalances, sovereign spread and private spreads allows us to capture two main interesting facts. First of all, what clearly stems from this evidence is that both relatively low and high government imbalances are consistent with low-risk economic environments. This pattern is clearly displayed in the scatter plot on the bottom left of Figure 2.1, where both low and high levels of debt are associated with low sovereign spreads, whereas for intermediate debt realizations the relationship describes a more defined tendency. Overall, we find that the debt-to-GDP alone accounts for only 20% of sovereign risk variability, suggesting that, as long as fundamentals are the ultimate cause of risk premia, markets proved unable to provide a correct pricing of risk (De Grauwe and Ji (2013)). These results can be interpreted stating that the dynamics in place during the crises might be determined or might be interacting with additional latent factors (expectation-driven or non-fundamental factors) feeding sources of financial instability. Secondly, the joint analysis of the two spreads (bottom-right panel) reveals high linear correlation (the sovereign spreads explains a good 46% of private spreads' variability). However, also this linkage suggests complex dynamics: the high degree of synchronization between the two risk proxies decreased after the first crisis and broke down with the second one, when private spreads reacted more timidly but more persistently. Since then, the two spreads followed different trajectories: private spreads remained high, while sovereign spreads faced a sharp drop. We can again attribute the change in the degree of coordination between sovereign and private risks to latent (potentially different) factors.

Based on these puzzling evidences, the paper aims to empirically identify latent sources of risk underlying both the transmission of debt surges to sovereign spreads and the channel linking sovereign to private risk pricing. By estimating a small-scale Markov-Switching Vector Autoregression (MS-VAR) for a pooled sample of European countries (Italy, Spain and Portugal) in three variables (private spreads, sovereign spreads, debt-to-GDP ratio), we derive macro evidence about the emergence of substantial regime changes affecting independently two main structural relationships in the model. On one hand, we efficiently extract regimes of different degrees of sensitivity of sovereign risk pricing to debt dynamics. We address them as driven by a latent factor feeding uncertainty, fear or pessimism in the determination of sovereign spreads. We will, therefore, refer to them as *vulnerability regimes*. On the other hand, the channel connecting sovereign to private spreads fluctuations displays regularities of strong and loose degrees of tightness. The implied regimes are defined as *synchronization regimes*. Two main considerations emerge from the analysis: *i*) from an historical assessment of the identified regimes, high sovereign vulnerability to debt seems to broadly characterize the peak of the global financial

[1] Speech at the Global Investment Conference in London 26 July 2012, `https://www.ecb.europa.eu/press/key/date/2012/html/sp120726.en.html`

crisis and the different phases of the sovereign debt crisis, while high risk synchronization marks mainly the first phases of both crises; *ii* for a quantification of the regime-specific dynamics, we simulate a debt-to-GDP shock, which induces a more than two times larger surge in sovereign spreads under the high vulnerability regime. This nonlinear effect spills over private spreads, which are in turn directly interested by an additional source of risk amplification under the high synchronization regime.

The Chapter is organized as follows. Section 2.2 presents the related literature. Section 2.3 describes the MS-VAR estimation procedure, while its results are presented and discussed in Section 2.4. Section 2.5 concludes.

2.2 Related Literature

This chapter relates to two distinct strands of the macro literature. First, it contributes to the literature on the nexus between sovereign and bank risk. Robust empirical evidence uncovering the strong co-movements between sovereign and financial crises can be found in Diebold and Yilmaz (2009), in Reinhart and Rogoff (2011) with aggregate data, and in Gennaioli, Martin and Rossi (2014) with cross-country panel data on banks. Theoretically, numerous papers address the different channels feeding the 'doom loop'. Bocola (2016) studies the impact of sovereign risk on banks' balance sheets, credit provision and output losses. Faia (2017) intervenes in the debate modelling a more comprehensive set of sovereign risk mechanisms, featuring a balance sheet, a collateral and a liquidity channel. Acharya, Drechsler and Schnabl (2014), Cooper and Nikolov (2017), and Farhi and Tirole (2016) analyse the nexus under the lens of banks' bailouts incentives and costs. Finally, Konig, Kartik and Heinemann (2014) employ a global game approach to show that the credibility and effectiveness of the guarantees are intertwined with the sovereign funding risk. Exploiting the properties of a MS-VAR, we contribute to the literature identifying regimes where a sizeable sovereign-banking nexus recurrently alternates with a normal scenario, where the related linkage is negligible. We analyse the financial amplification coming from the implied risk loop in a setup where fiscal imbalances carry themselves a relevant source of macro risk. To the best of our knowledge this is the first contribution applying MS techniques to this topical area.

We also relate to the literature on self-fulfilling debt crises, which aims to account for the role of expectations-driven factors in explaining sovereign risk fluctuations. Bocola and Dovis (2017), using the modelling framework of self-fulfilling rollover crises á la Cole and Kehoe (2000), provides the first quantitative measurement of the fundamental versus non-fundamental description of sovereign spreads during the Eurozone crisis. The literature extensively investigates on the occurrence of multiple equilibria at the origin of non-fundamental sovereign risk sources. Indeed, descriptions of alternative mechanisms date back to Calvo (1988), but are also recently studied by Lorenzoni and Werning (2014), Ayres et al. (2016) and Broner et al. (2014). We do not explicitly model non-fundamental equilibria but we empirically interpret them using the concept of latent factors. They are assumed to be induced by uncertainty, lack of trust or policy credibility, which alter the degree of vulnerability of markets' pricing to fiscal fluctuations. Our main contribution concerns the analysis of the role of latent (non-fundamental) factors, not only in terms of transmission of fiscal imbalances to sovereign risk, but also in terms of implied co-movement between sovereign and private spreads.

2.3 MS -VAR evidence

We evaluate the low-frequency nonlinearities in the structural relations linking sovereign, private credit spreads and macroeconomic fundamentals by estimating a Markov-switching VAR model (MS-VAR) on monthly data spanning the period 2003:M1-2016:M12. We run the analysis on a pool of three countries: Italy, Spain and Portugal. Aggregation is realized with HICP weights. Three variables are considered in the MS-VAR: the private lending spread, the sovereign spread and the government debt-to-GDP ratio. Sovereign spreads are measured by yields differentials between long-term debt rates and a German ZCB with a residual maturity of 6 months; private spreads are, instead, given by the yield differential between the lending rate for the non-financial corporate debt and the German ZCB. Finally, the nominal value of general government debt is scaled over GDP. A detailed description of variables' definitions and data sources is available in Appendix B.1.

Regime-switching dynamics are introduced by adding two channels of parameters' instability in the systematic component of the VAR: *i*) one affects the equation for sovereign spreads, according to the latent variable ξ^{vul}, which defines the *vulnerability regimes*; *ii*) the other controls the equation for private spreads, according to the latent variable ξ^{syn}, which defines the *synchronization regimes*. An independent Markov chain ξ^{vol} in the stochastic component of the VAR, i.e. governing the variance-covariance matrix, captures shocks' heteroskedasticity. Therefore, in this nonlinear setting the exogenous determination of low-frequency switches in model's coefficients defines stochastically-generated regimes, which have to be interpreted conditionally to the shocks' size.

Collecting the two independent Markov chains affecting the model's structural parameters, under the composite process $\xi_{sp} = \{\xi_{vul}, \xi_{syn}\}$, the following Bayesian MS-VAR model is considered:

$$\mathbf{y}_t^{'}\mathbf{A}_0(\xi_t^{sp}) = \mathbf{c}(\xi_t^{sp}) + \sum_{i=1}^{\rho} \mathbf{y}_{t-i}^{'}\mathbf{A}_i(\xi_t^{sp}) + \epsilon_t^{'}\mathbf{\Sigma}(\xi_t^{vol})^{-1} \quad (2.1)$$

where $\mathbf{y}_t$ is the three-dimensional vector of endogenous variables, $\mathbf{c}(\xi_t^{sp})$ the vector of constants, $\mathbf{A}_0(\xi_t^{sp})$ the invertible matrix of the contemporaneous correlations, ρ the lag length, $\mathbf{A}_i(\xi_t^{sp})$ the autoregressive dynamic cross-correlations. We fix $\rho = 13$, as suggested by AIC information criteria[2]. A conditional multivariate normal distribution for the orthogonal structural shocks ϵ_t is assumed:[3]

[2] Ivanov and Kilian (2005) implement MCMC simulations to conclude that for monthly VAR models the Akaike Information Criterion (AIC) tends to produce the most accurate structural and semi-structural impulse response estimates for realistic sample sizes

[3] Conditional normality over the reduced-form residuals space opens up a much wider class of distributions for the error terms than the unconditional normality, meeting non-Gaussian evidence in applied works.

$$P(\epsilon_t|\mathbf{Y}^{t-1}, \mathbf{\Xi}_t, \theta, q) = \mathcal{N}(\epsilon_t|\mathbf{0}_n, \mathbf{I}_n) \tag{2.2}$$

where the structural shocks' standard deviations are given by the diagonal elements of the matrix $\mathbf{\Sigma}^{-1}(\xi_t^{vol})$, θ denotes the vector of model's structural parameters, $\mathbf{\Xi}_t$ and $\mathbf{Y}^{t-1}$ collect past information on the latent processes and data, respectively. Regimes' dynamics are driven by the composite process of two independent chains $\xi_t = \{\xi_t^{sp}, \xi_t^{vol}\}$, otherwise interpretable in Bayesian analysis as a vector of nuisance parameters, which obeys the first-order Markovian property $p(\xi_t|Y_{t-1}, \theta, q, \Xi_{t-1}) = q_{\xi_t,\xi_{t-1}}$. The transition probabilities $q_{i,j}$ to go from state i to state j are collected in the composite transition matrix $Q = (q_{i,j})_{(i,j)\in H\times H)} \in \Re^{h^2}$, where $H = \{1...h\}$ is the set of possible regimes for ξ_t, and Q is nonlinearly restricted to the tensor product form $Q = Q^{sp} \otimes Q^{vol}$. We allow for two regimes per Markov chain and estimate the MS-VAR model by using Bayesian methods. Appendix B.2 provides the estimation details.

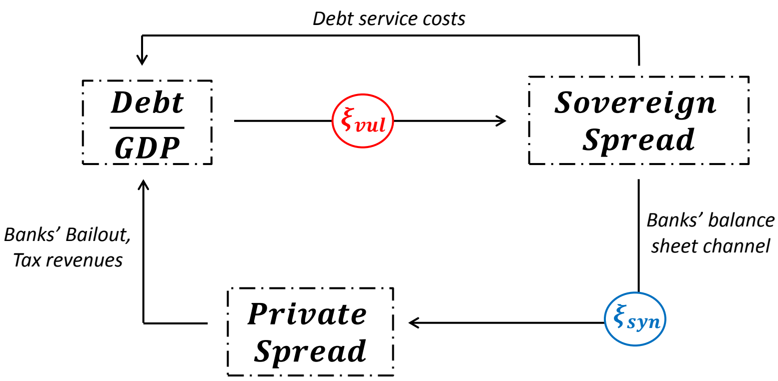
2.3.1 Model Identification

Identification is achieved by means of exclusion restrictions on both the contemporaneous and the dynamic structure of the VAR (Sims, Waggoner and Zha (2008), Waggoner and Zha (2003)). As Figure 2.2 illustrates, we employ the following scheme of assumptions regarding the interactions between model's variables. The two-sided link between debt-to-GDP and sovereign spread is left free to display its contemporaneous and dynamic effects, both in terms of higher fiscal burden induced by sovereign spreads changes on the debt-to-GDP ratio, and in terms of higher risk pricing coming from high debt levels. The channel between sovereign and private spreads is, instead, allowed to hold only in the direction from sovereign to private risk. We, therefore, account for the balance-sheet effects operating through the banks' exposure to government bond holdings, ruling out the reverse direction of causation, contemporaneously and dynamically[4]. The latter operates only indirectly through the linkages connecting banks' credit contractions to drops in production and rise of fiscal guarantees supporting banks. Debt-to-GDP does not directly determine private spreads neither on impact nor at dynamically. Sovereign spread is assumed to react with a one-month delay to fiscal unexpected shocks. Identification (global and local) is tested and verified through the methods of Rubio-Ramirez, Waggoner and Zha (2010), designed for restrictions on Markov-switching models.

The peculiar feature of our analysis intervenes in two nodes: *i*) the determination of sovereign spreads (red node), which - given our identifying assumptions - depend only on the fiscal variable; *ii*) the determination of private spreads (blue node), which, instead, depend only on sovereign spreads. Two independent latent drivers are responsible for significantly divergent nonlinear transmission channels featuring the origins and the pass-through of sovereign risk.

[4] Moody's (2004), in assessing how credit risk linkages vary over time, documents a chain of spillover effects operating mainly in the direction from sovereign to financial stress, for the cases of Italy and Greece.

Figure 2.2: VAR Identification Scheme



2.3.2 Model Selection

Here we evaluate our switching structure in terms of model fit. Table 2.1 reports the log Marginal Data Density (MDD) of four differently specified models. The latter are compared under three measures, which differently specify the weighting function for the MDD's numerical approximation: the new modified harmonic mean method (MHM) proposed by Sims, Waggoner and Zha (2008); the bridge sampling method of Meng and Wong (1996); the Müller method (Liu, Waggoner and Zha (2011)). We consider two regimes per chain in every model.

Table 2.1: Marginal Data Densities

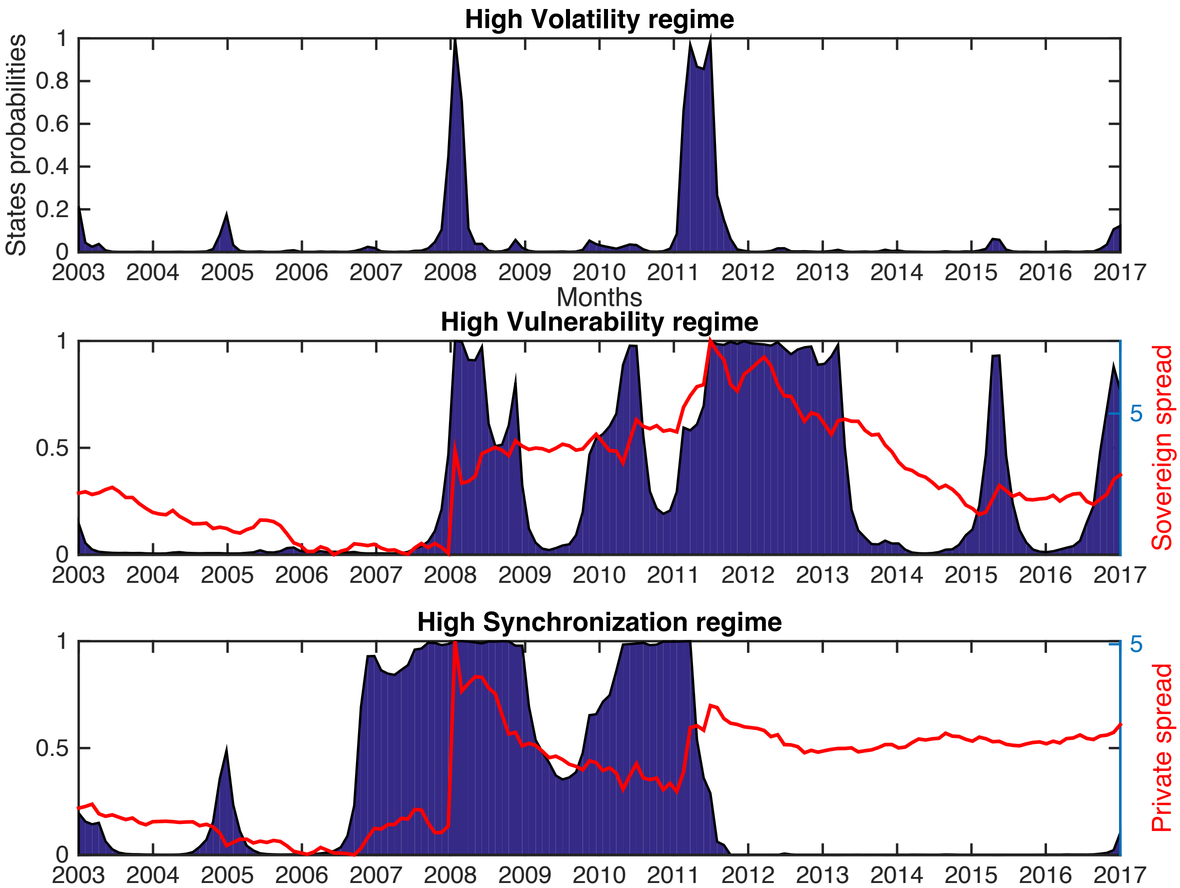
Methods	Regimes			
	ξ^{vol}	$\{\xi^{vol}, \xi^{vul}\}$	$\{\xi^{vol}, \xi^{syn}\}$	$\{\xi^{vol}, \xi^{vul}, \xi^{syn}\}$
SWZ(08)'s MHM	529.7685	550.3061	580.9696	592.4230
Bridge sampling	526.7853	546.1993	580.0308	589.0128
Müller's	526.5279	546.1991	580.1027	587.7441

A set of results are derived. First, the model with only stochastic volatilities (ξ^{vol}), keeping a time-invariant structure on structural parameters, is clearly outperformed by all other models. Indeed, by including an independent source of discrete variability on the determination of sovereign spreads in terms of debt, ($\{\xi^{vol}, \xi^{vul}\}$), or, alternatively, on the sovereign-private risk pass-through, ($\{\xi^{vol}, \xi^{syn}\}$), we always improve in terms of model fit over the stochastic volatility case. Secondly, the latter performs better than the former. Finally and more importantly, the best-fit model, ($\{\xi^{vol}, \xi^{vul}, \xi^{syn}\}$), entails the inclusion of both risk channels, providing robust statistical support to our analysis.

2.4 Financial and Fiscal Regimes

Figure 2.3 reports the smoothed probabilities at the posterior mode for the three Markov chains, obtained by conditioning to the shifts in the covariance matrix (top panel), to the shifts in the sovereign spread equation (middle panel), and to the shifts in the private spread equation (bottom panel). Note that our identification scheme implies that sovereign risk depends only on debt-to-GDP, as well as private risk depends only on sovereign risk. Only these relationships are allowed to switch over Markov states. The top panel displays states' probabilities for the regime capturing higher shocks' size. Therefore, we label it the *high volatility regime*.[5] The middle panel, instead, displays probabilities for the regime that we consider as driven by exogenous factors intervening in rising the degree of vulnerability of sovereign spread to debt-to-GDP. Thus, we label it the *high vulnerability regime*. Finally, the bottom panel reports the regime featuring strong co-movements between sovereign and private spreads. Therefore, we label it the *high synchronization regime*. In order to visually facilitate regimes' interpretation, the last two plots show also the dynamics of sovereign and private spreads.

Figure 2.3: Posterior States' Probabilities



Some considerations are worth noting. The high volatility regime captures two sharp short-lived events observed with the two abrupt surges in sovereign spreads in 2008 and 2011, lining up with the dates of the financial and the sovereign debt crises. Differently, the high vulnerability

[5] On this regard, a large literature preceding the crisis interpreted monetary policy regimes considering heteroskedastic error terms only (see Sims and Zha (2006) and Primiceri (2005)). In the last decade, however, many contributions arose finding strong empirical confirmation of structural deviations from regularities (Bianchi (2013), Bianchi and Melosi (2017), Bianchi and Ilut (2017), and Hubrich and Tetlow (2015).

and synchronization regimes are estimated to cover wider entire time windows, mainly around the two crises, rather than only spikes. This is line with the interpretation of the structural regimes as covering periods where low-frequency fluctuations prevail and suggest that, albeit the increase in spreads had its origin in big shocks, the following developments are mainly explained by structural factors, for which we can provide an economic interpretation. Indeed, we observe that the high vulnerability regime captures the period when the debt-to-GDP ratio started its gradual surge and markets were aggressively pricing sovereign and financial risk. These features describe both crises, but more broadly the sovereign debt turmoil. The high risk synchronization regime, instead, seem to characterize more the first phases of both the global and debt crises. See Table B.3 in Appendix for the associated conditional moments.

2.4.1 Impulse Response Analysis

Once derived how the data properties are captured by regime changes, we interpret the regime-specific economic dynamics by simulating an unexpected worsening in macroeconomic fundamentals resulting from a debt-to-GDP surge. The crucial question here is whether, and under which conditions, fiscal imbalances spill over sovereign and private risk.

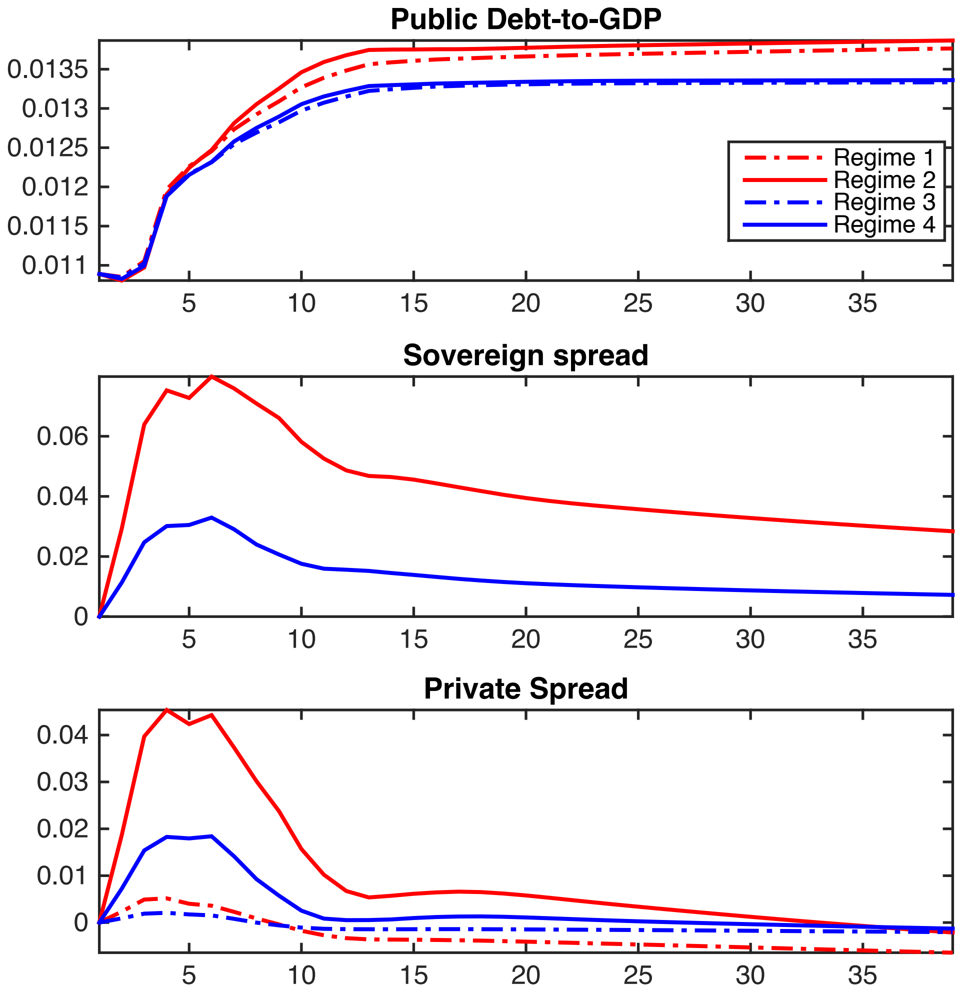
Figure 2.4 shows the impulse response functions to a 1% increase in government debt-to-GDP conditional[6] on the different regimes identified in the previous section. Several results are worth noticing. First of all, both spreads respond positively to the worsening of the fundamental, highlighting a significant risk's transmission across the two sectors. Moreover, focusing on the response of the sovereign spread (the middle panel) we can notice how the degree of transmission crucially depends on the structural regime in place. Indeed, a strong amplification of the effects, in terms of size and persistence, is observed for the sovereign spread under what we call the *high vulnerability regime* (the red line), while a smaller and a relatively short-lived response characterizes the complementary regime. More in detail, the chart shows that the peak of sovereign spread response moves from 3% in the *low vulnerability* regime to 8% in the *high vulnerability regime*.

Finally, the response of the private spread reveals an additional source of nonlinearity characterizing the comovement between private and sovereign spreads, independently on the direct impact of the debt-to-GDP shock on the sovereign spread. The bottom panel of Figure 2.4 displays the impulse response functions produced in the four regimes identified by different levels of vulnerability and synchronization, highlighting how the size of the private spread response is mainly affected by the degree of risk synchronization. Indeed, when a *low synchronization regime* realizes, the response of private spread is negligible, independently on the degree of vulnerability. Instead, when the *high synchronization* regimes materializes, the *high vulnerability* realization amplifies an already sizeable response of the private spread. More in detail, we can see how in the "low vulnerability-high synchronization" regime the private spread response reaches a peak of 2% while in the "high vulnerability-high synchronization" it raises up to 5%.

To sum-up, the interaction between the two sources of non-linearity, identified by the

[6] In a Markov-switching model the conditional impulse responses are computed assuming that over the relevant horizon a specific regime will prevail.

Figure 2.4: Impulse Responses: Debt-to-GDP shock



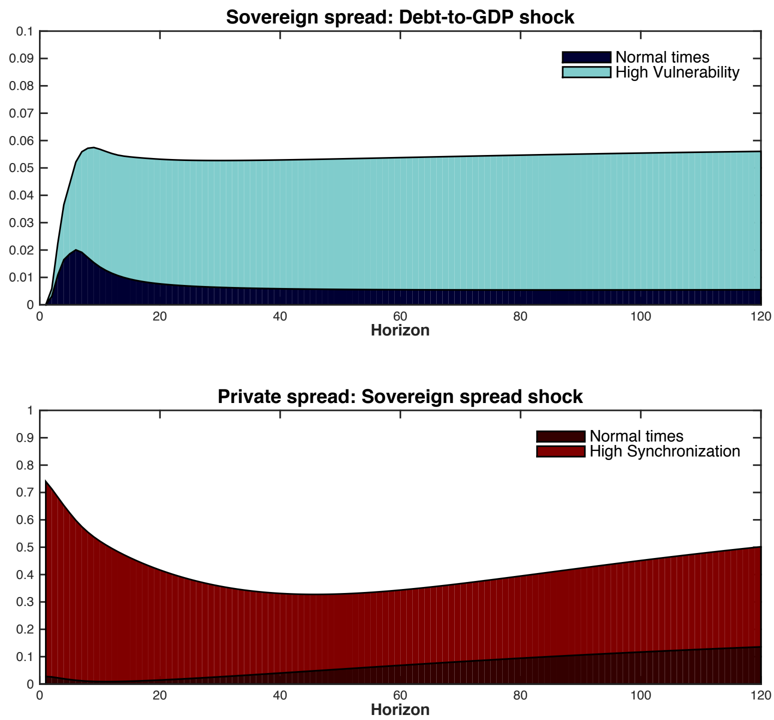
Regime 1: High Vulnerability-Low Synchronization; Regime 2: High Vulnerability-High Synchronization; Regime 3: Low Vulnerability-Low Synchronization; Regime 4: Low Vulnerability-High Synchronization.

Markov-switching structure, controls the extent to which a worsening in fiscal imbalances is transmitted to sovereign spreads and, trough this, to the private sector. The realization of a "high vulnerability-high synchronization" regime defines a situation with the highest risk pricing in both markets and the most aligned comovement in private and public spreads.

2.4.2 Variance Decomposition Analysis

So far, in Section 2.4 we derived the structural regimes affecting the determination of sovereign and private risk measures, whose properties are shown to be clearly interpretable in terms of transmission channels. Indeed, we evaluated the response of our model's variables by simulating a shock to the debt-to-GDP ratio. In order to add some evidence on the regimes' interpretation, we here quantify the shocks' relative contributions to the two spreads' variability, and we analyse them under the identified regimes. To this purpose, Figure 2.5 selects some results obtained by a dynamic variance decomposition analysis, performed conditioning on a particular regime path.

Figure 2.5: Forecast Error Variance Decomposition



In particular, the Figure illustrates the contribution of the debt-to-GDP shock to the variance of sovereign spreads (top panel); as well as the contribution of the sovereign spread shock to the variance of private spreads (bottom panel). Both are evaluated conditioning on two polar regimes: a *normal times* regime, which corresponds to the states' combination where simultaneous low vulnerability and low synchronization realize; the latter is compared to a regime of *high vulnerability* and *high synchronization*, respectively. Both are selected under the low volatility regime, but no remarkable differences emerge under the high volatility regime. This implies that, given our regimes' identification, the shocks' size do not seem to matter for this analysis. Results show that debt shocks explain a 6% and a 2% variation of sovereign spreads at short horizons, under respectively the high vulnerability and the normal times regimes. The difference across the two is more remarkable if evaluated at longer horizons, where in normal times debt shocks seem uninformative, while the high vulnerability regime still captures a persistent degree of contribution. A stronger evidence is, then, derived when assessing the private spreads' variance in terms of sovereign spreads' shocks, under the two scenarios. Indeed, the second panel of the Figure clearly shows that at short horizons the private spreads' variance is mostly explained by sovereign spreads' shocks, with a 70% share of contribution under the high synchronization regime. This result holds even at longer horizons; while in normal times some small significant effects arise only in the long run.

The above evidence is in line with the regimes' interpretation emerging from the impulse response analysis and suggests that sources of fiscal instability induce a nonlinear degree of variation in sovereign interest rates; the latter, in turn, inject nonlinear financial amplification, through private spreads.

2.5 Conclusions

The nexus between government and banking system played a key role in explaining the euro-area sovereign debt crisis: indicators of sovereign and bank credit risk for the periphery countries spiked together both with the global financial crisis and right after the Greek bailout in 2010. Financial panic spread in the two markets with apparently few filters. Although this evidence is well established in the literature, a key open question addresses the sources of the risk transmission. How much of this synchronized risk surge comes from macro risk factors? and how the emerging fragility in both markets can be reconciled with the structural regularities driving periods of low financial stress?

We address these questions by estimating a MS-VAR on the southern EZ countries (Italy, Portugal and Spain). We historically identify periods of high financial amplification, as opposed to normal times. By using the information of both scenarios, the model extracts two distinct latent sources of risk: one featuring the determination of sovereign spreads in terms of debt-to-GDP; the other interesting the nexus between private and sovereign spreads. The former generates regimes of high and low vulnerability of sovereign spread to fiscal imbalances. High sovereign risk sensitiveness to debt is then channelled to the private financial markets under the high synchronization regime, which, instead, is generated by the latter source of instability.

Chapter 3

Euro Area Fiscal Regimes: The case of France[1]

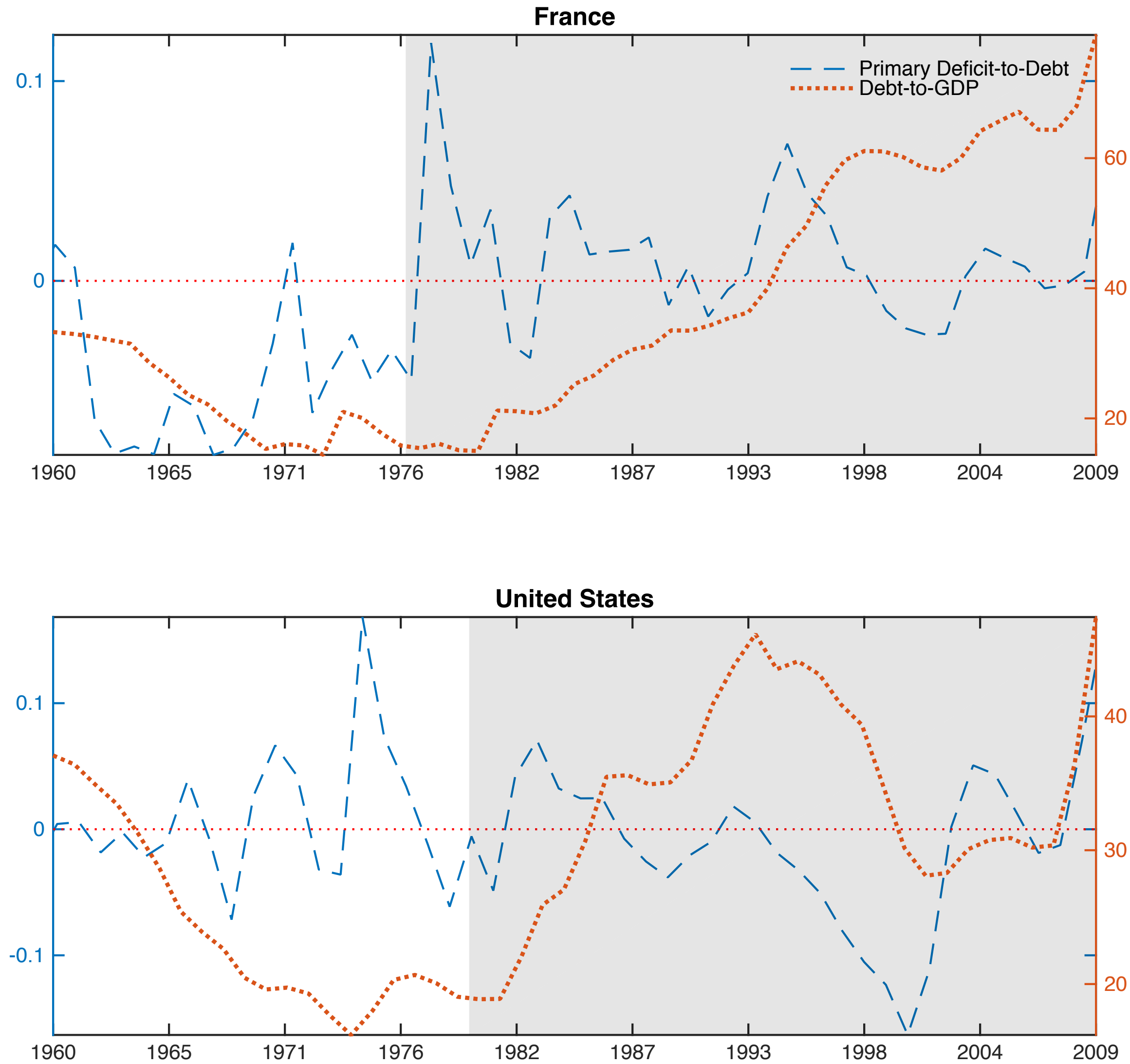
3.1 Introduction

The global financial and the sovereign debt crises severely worsened the Euro area (EA) fiscal imbalances putting renewed emphasis on whether the outstanding debt and its projected path are consistent with short-run primary balance dynamics. Despite it's core policy implications, the question of short-run stabilization versus long-run sustainability is still an open debate, both at the national and the EA level. Our aim is twofold: *i*) first, we evaluate the extent to which government's fiscal attitudes toward debt adjustments (fiscal stance) are compatible with debt accumulation dynamics; *ii*) in light of the above relationship, we identify regimes of fiscal sustainability, which occur recurrently replacing unsustainable regimes. We perform this analysis using data evidence for France.

The degree to which fiscal policy is consistent with intertemporal solvency has been for years the focus of a well established empirical literature, which builds on the seminal contribution of Bohn (1998). His approach consists in defining sustainability conditions on the response of the primary balance to debt changes: a positive and significant adjustment signals a sustainable fiscal stance (passive fiscal policy); while a weak relationship identifies unsustainable fiscal imbalances (active fiscal policy). Applications of Bohn's test have initially concerned US data (Bohn (2008)), but there are also extensions to panels of emerging and developed economies due to Mendoza and Ostry (2008), while EU countries are treated in Melitz (2000), among others. Results hardly find evidence of active fiscal behaviours. By estimating the rule for the primary balance, fiscal solvency seems to be credibly guaranteed. However, as emphasized in Leeper and Leith (2016), such analyses might produce misleading inferences about fiscal behavior, when they do not embody the bond valuation equation. The latter is an equilibrium condition which bring in the forward-looking nature of nominal government debt and, therefore, calls for a fully specified DSGE framework.

[1]This chapter should not be reported as representing the views of the European Central Bank (ECB). The views expressed are those of the authors and do not necessarily reflect those of the ECB.

Figure 3.1: Primary balance and debt-to-GDP dynamics. France and US



We use the modelling environment proposed by the empirical literature on monetary and fiscal policy regimes, by Bianchi and Ilut (2017), and we adapt it to study debt sustainability in France. Indeed, while their model provides a successful analytical tool for the US case, the application to the EA turns to be not trivial. Two main issues arise in the latter case: *i*) a proper analysis of the monetary policy at the EA level cannot be abstracted from the consideration of a currency union setup, where an independent central bank and a fragmented collection of multiple fiscal authorities are crucially tied; *ii*) while for US debt and price dynamics seem to be jointly determined[2], independently on the fiscal stance (primary budget balance), for France, instead, data evidence shows that we can explain debt facts already by studying fiscal regimes on the relationship between short-run stabilization and long-run sustainability (fiscal policy and solvency), independently on inflation dynamics. Moreover, Sims (2013) tells us that

[2]The US post-World War II inflation and debt facts can be interpreted through the lens of the fiscal theory; while the post-Walker era is consistent with the traditional monetary view.

the EA strategic environment alters and substantially weakens the degree of policy interactions, inducing a looser link between national fiscal stances and inflationary pressures.

With the aim of studying debt determination through the lens of fiscal regimes, Figure 3.1 reports some evidence on France (top panel) and US (bottom panel), covering the period 1959-2009. It shows the evolution of the primary deficit-to-debt ratio, taken as a measure of the country's fiscal stance, observed jointly with the debt-to-GDP ratio, as an index of sustainability. Some stylized facts can be already identified. First, over the first 10 years of the sample, France was running primary surpluses compatibly with a sustainable debt; whereas, since then, a series of primary deficits were feeding a fast and steady surge in the debt-to-GDP ratio. The comparison with US data is rather informative and suggests a crucial difference between the two countries' fiscal events. In US, indeed, under the pre-Volker period a low debt-to-GDP ratio was guaranteed by high inflation and low real interest rates, even though the government was running primary deficits, while during the post-Volker high debt arose with low inflation and primary surpluses. The two fiscal phenomena suggest different analyses, and for France debt sustainability concerns seem to be closely tied to fiscal variables.

We, therefore, use a general equilibrium model, in the tradition of Lubick and Shorfeide (2004) and Clarida, Galí and Gertler (2010), to extract an empirical narrative of debt determination and fiscal stance, using a newly-built French dataset from 1955 to 2009. Fiscal policy regimes are introduced on the response of primary surplus to outstanding debt, thus on the rule for tax revenues, whose parameters are controlled by a latent exogenous policy factor. Specifically, we allow them to undertake two regimes of debt sustainability, given a monetary policy actively targeting inflation. As a result, we focus exclusively on Ricardian equilibria, for which the government budget constraint must be satisfied, for any path of the price level. This assumption seems consistent with the constraints imposed by the European Monetary System in 1979 and the EMU framework since 1999. The model is solved with Markov-switching perturbation methods by Maih (2015). In this framework, agents form expectations taking into account the probabilistic distribution over future regime changes (Bianchi (2013)). Model's performance is assessed on the ability to recover stylized facts in line with the historical narratives. Our empirical results for France identify two regions of debt sustainability, where fiscal data evidence is the outcome of two differently specified processes for the tax rule. Specifically, a *sustainable regime* covered 'Les Trente Glorieuses' until 1977 and then re-emerged in 1999 with the euro membership. This is consistent with an estimated positive response of the primary balance to outstanding debt. An *unsustainable regime*, instead, characterized the 1978-1998 period, years of a costly transition, where a prudent policy mix aiming at disinflation, external balance and nominal exchange rate stability lead to primary deficits and increasing debt-to-GDP accumulation. Over this period, taxes were approximately insensitive to long-term debt concerns.

The Chapter is organized as follows. Section 3.2 reviews the literature. Section 3.3 presents the model. Section 3.4 estimates the structural model, extracts the fiscal regimes and sets the direction for future progresses. Section 3.5 concludes.

3.2 Literature Review

This paper relates to three strands of the literature. First, we borrow from the empirical literature, which provides reduced-form and structural tests for fiscal sustainability (see D'Erasmo, Mendoza and Zhang (2016) for an overview). Regarding the non-structural interpretations, Canzoneri, Cumby and Diba (2011) define regimes of policy interactions, based on the response of future surpluses and the real value of government debt to a positive shock to surpluses. A second branch of the correlation-based testing literature follows Bohn (1998)'s seminal work, which defines fiscal regimes on the interpretation of the fiscal rule coefficients. If primary balances weakly correct debt changes, fiscal behaviour is active and delivers unstable debt dynamics; if, instead, the government is taking actions to counteract changes in debt, fiscal policy is passive and sustainable. As discussed in Leeper and Leith (2016), however, surplus-debt estimates which do not take into account the bond evaluation equation are subject to simultaneity bias, producing misleading inferences about fiscal behaviour. Biases arise from the failure to model the general equilibrium relationships between government debt and surpluses, conditions that bring in the forward-looking nature of nominal debt valuation and the role of monetary policy. A close inspection on this is provided by Leeper and Li (2017). Regarding the structural approach, instead, a unified theory for US inflation historical dynamics is developed by Bianchi and Ilut (2017) and Bianchi and Melosi (2017) in a DSGE framework with a fully specified fiscal sector, but no contributions are so far interesting the identification of Euro Area fiscal regimes. We fill this gap with the case of France.

Our paper is also partially related to the large literature on the macroeconomic role of fiscal and monetary policy interactions in the determination of inflation and debt dynamics. Following the seminal contribution of Stiglitz and Weiss (1981), who developed the analysis in a deterministic environment, various studies (Sims (1994), Woodford (1994, 1995, 2011)) focused on the determinacy properties of model's equilibrium, placing fiscal policy in a coordination game with the monetary authority. Leeper (1991) defines conditions for uniqueness and existence of model's solution under different combinations of policy regimes. His paper led the way towards a vast literature studying inflation as a fiscal phenomenon (Sims (2016)), as well as monetary. Since then, regimes of fiscal dominance, supporting the Fiscal Theory of the Price Level (FTPL), are studied both in isolation (Cochrane (2005, 2001)) and inside a unified framework[3] of power imbalances between the two policy authorities, which studies: *i*) the implications of fiscal imbalances on the price level; *ii*) how the conduct of the monetary policy affects debt sustainability concerns. Modelling setups including price rigidity, a maturity struc-

[3]Two main equilibrium outcomes are generally identified. The first considers agents forming expectations compatibly with a fiscal authority able to take adequate corrective fiscal measures to stabilize debt dynamics, while the central bank is credibly committed to inflation. This case is generally called the monetary-led regime (Regime M), where fiscal shocks have little effects on inflation and real activity. The second, instead, considers a scenario where agents don't believe in future fiscal backing, inflation expectations tend to rise, the monetary policy accommodates. Changes in inflation and bond prices induce nominal government debt revaluations, causing a temporary economic boom and reduction in the fiscal burden. The literature refers to this case as the fiscally-led regime (Regime F). In both scenarios it is assumed that the monetary and fiscal authorities act in coordination, keeping debt on a stable path and inflation controlled. However, cases of policy conflicts Bianchi and Melosi (2017) proved to be able to explain some historically and currently relevant puzzles.

ture for government debt, distortionary taxes and fiscal rules for government spending widen the scope of policy interactions, as discussed in Leeper and Leith (2016). We inherit the general equilibrium approach from this strand of the literature, but we depart from it considering only Ricardian equilibria, evaluating regimes of fiscal sustainability.

Finally, our paper borrows extensively also from the insights of the literature on the role of agents' expectations over policy regimes in generating crucial equilibrium dynamics within the context of forward looking behaviours. Agents' current decision rules are, indeed, affected by the degree of credibility and enforceability of the fiscal behaviour (Sims (1982)), through what Leeper and Zha (2003) call the 'expectations formation effects'. Their relevance proves to be significant both within regime and as a source of shifts over them, as various counterfactual simulations testify. On this ground, Markov-switching DSGE models proved to provide a good understanding of recent sensitive dynamics. For monetary policy shifts Schorfheide (2005), Fernández-Villaverde, Guerron-Quintana and Rubio-Ramiírez (2010), Liu, Waggoner and Zha (2011) and Foerster (2015, 2016); for the fiscal and monetary policy mix Davig and Leeper (2006, 2007), Bianchi and Ilut (2017) and policy uncertainty at the ZLB Bianchi and Melosi (2017). Applications in different settings produce extensive accounting for new and past prevailing stylized facts, which emerge nonlinearly from historical events. Moreover, since regime-switching models account for the role of expectations over regimes, they count on expanded regions of equilibrium determinacy (Davig and Leeper (2007), Farmer, Waggoner and Zha (2009), Ascari, Florio and Gobbi (2017)). This feature allows the interpretation of cases of policy conflicts, which proved to explain crucial historical puzzles. We borrow this framework to characterize agents expectations around policymakers' behaviour, and assess their implications in terms of debt sustainability.

3.3 The model

Our framework builds on a basic monetary DSGE model in the tradition of Lubick and Shorfeide (2004) and Clarida, Galí and Gertler (2010), augmented with a fiscal block, whose details are borrowed by Bianchi and Ilut (2017).

We consider a production economy subject to nominal rigidities à la Rotemberg (1982). Sticky prices, indeed, provide a channel for policy interaction since they account for monetary policy effects on debt dynamics, through real debt service costs. We introduce a maturity structure for government debt and persistence through external habits in consumption and inflation indexation. Fiscal policy is described by rules on tax revenues and government expenditure[4]. We add Markov-switching properties on the elasticities characterizing the tax rule, which are therefore modelled as the outcome of a two-state discrete exogenous latent process. Importantly, the model is designed to capture the role of agents beliefs over regimes, property which widens the model's determinacy regions, creating room for understanding conditions of lack of policy coordination. As mentioned above, indeed, the paper studies fiscal regimes of debt

[4]The assumption of explicit rules for fiscal instruments reflects more realistically the last stabilization measures privileging government expenditure adjustments (Leeper, Traum and Walker (2017)).

sustainability, given a monetary policy committed to control inflation.

3.3.1 Agents and Their Decision Problems

The structure of the economy is as follows. A representative household consumes, saves, and supplies labor. The final output is assembled by a competitive final good producer who uses as inputs a continuum of intermediate goods manufactured by monopolistic competitors. The intermediate goods' producers rent labor from the household and set prices à la Rotemberg (1982). Finally, the government fixes the one-period nominal interest rate, sets taxes and decides over government expenditure. The economy is hit by seven shocks: a preference shock on the households' side; a technology and a cost-push shock on the supply side; and four policy shocks (one monetary and three fiscally-driven).

Households

The representative household receives utility from consumption C_t, and disutility from labor supply h_t. Preferences are described by the following discounted flow of period utilities:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t e^{d_t} \left[\log(C_t - \Phi \tilde{C}_{t-1}) - h_t \right] \right\} \tag{3.1}$$

Preferences, logarithmic in consumption and linear in labor, show external habits, where $\tilde{C}_t$ is a measure of the aggregate lagged level of consumption, while Φ is the degree of habits' formation. The preference shock has an AR(1) representation: $d_t = \rho_d d_{t-1} + \sigma_d \varepsilon_t^d$. The budget constraint is given by:

$$P_t C_t + P_t^m B_t^m + P_t^s B_t = P_t W_t h_t + B_{t-1} + (1 + \rho P_t^m) B_{t-1} + P_t D_t - T_t + TR_t \tag{3.2}$$

where P_t is the aggregate price level, W_t the real wage, D_t are the real dividends paid by firms, T_t denotes lump-sum taxes, and finally TR_t represents government transfers. Agents have access to two kinds of government bonds, differing in maturity: B_t indicates a one-period bond in zero net supply, while B_t^m is the long period bond in non-zero net supply.[5] Bonds' prices are given by $P_t^s = \frac{1}{R_t}$ and P_t^m, respectively. The price recursion for long-term bonds can be defined as $P_{t+j}^{m-j} = \rho^j P_{t+j}^m$, where the parameter $\rho \in [0, 1]$ controls the average maturity of debt. The latter is defined as $(\beta\rho - 1)^{-1}$.

The bond-pricing equations for short and long term securities are :

$$1 = \beta \mathbb{E}_t \left\{ e^{d_{t+1} - d_t} \frac{u_c(t+1)}{u_c(t)} \frac{1 + \rho P_{t+1}^m}{P_t^m} \Pi_{t+1}^{-1} \right\} \tag{3.3}$$

$$1 = \beta \mathbb{E}_t \left\{ e^{d_{t+1} - d_t} \frac{u_c(t+1)}{u_c(t)} \frac{R_t}{\Pi_{t+1}} \right\} \tag{3.4}$$

[5] The long-term bonds are modelled as a portfolio of infinitely many bonds, with weight along the maturity structure given by $\rho^{T-(t+1)}$ for $T > t$ (see Eusepi and Preston (2013) and Woodford (2011) for a detailed explanation).

where $u_c(t) = C_t - \Phi\tilde{C}_{t-1}$ is the marginal utility of consumption and $\Pi_t = \frac{P_t}{P_{t-1}}$ stands for inflation. In equation (3.3) the realized return of the maturity bond is $R^m_{t+1} = \frac{1+\rho P^m_{t+1}}{P^m_t}$, while combining the two conditions we derive the no-arbitrage condition:

$$R_t = \mathbb{E}_t\left\{R^m_{t+1}\right\} \tag{3.5}$$

Finally, the labor supply condition is characterized by:

$$W_t = C_t - \Phi\tilde{C}_{t-1} \tag{3.6}$$

Firms

The supply side of the economy is populated by a continuum of monopolistically competitive firms producing differentiated intermediate goods, used as inputs by a perfectly competitive firm, which combines them in a final output and sells it to households. The intermediate firms are price takers in factors' markets and price makers in goods' market. They use the following CES production function:

$$Y_t = \left(\int_0^1 Y_t(j)^{1-\nu_t} dj\right)^{\frac{1}{1-\nu_t}} \tag{3.7}$$

where $Y_t(j)$ is the intermediate good produced by the $j \in [0,1]$ firm and $\frac{1}{\nu_t}$ is the time-varying elasticity of substitution between two differentiated goods. The latter translates in a price markup shock $\mu_t = \frac{\kappa}{1+\iota\beta}\log\left(\frac{1-\nu}{\nu_t}\right)$, where $\kappa = \frac{1-\nu}{\nu\psi\Pi^2}$; ν stands for the steady state level of v_t and Π is the steady state level of inflation. Finally, we assume that the mark-up shock evolves according to $\mu_t = \rho_\mu\mu_{t-1} + \sigma_\mu\varepsilon^\mu_t$.

Given technology, the demand for intermediate inputs is therefore the following:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{\frac{1}{\nu_t}} Y_t \tag{3.8}$$

where $P_t(j)$ is the price of the intermediate good produced by firm j. Intermediate inputs are produced by a continuum of firms operating under monopolistic competition and endowed with the following production technology:

$$Y_t(j) = A_t h_t(j)^{1-\alpha} \tag{3.9}$$

where labor is the only input, $\alpha \in (0,1)$ and A_t is the total factor productivity evolving according to a random walk with drift process $\ln(A_t/A_{t-1}) = \gamma + a_t$ with $a_t = \rho_a a_{t-1} + \sigma_a\varepsilon^a_t$. The parameter γ is the growth rate of the economy[6]. The labor demand is given by:

$$W_t = \varphi_t(j)(1-\alpha)A_t h_t(j)^{-\alpha} \tag{3.10}$$

[6] The presence of the growth rate γ makes the economy not stationary, and then the aggregate variable must be de-trended in order to guarantee stationarity. Given the variable, X_t we define the respective detrended variable $\hat{X}_t = \frac{X_t}{A_t}$.

where $\varphi_t(j)$ is the real marginal cost. Each firm j has monopolistic power in the production of its variety and therefore solves a pricing setting problem. To allow for a real effect of monetary policy, we introduce nominal rigidities à la Rotemberg (1982). Firms face a quadratic cost in adjusting their price in the form of an output loss:

$$AC_t(j) = \frac{\psi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \frac{\Pi^j_{t-1}}{\Pi^{\iota-1}}\right)^2 Y_t(j)\frac{P_t(j)}{P_t} \tag{3.11}$$

where ψ determines the degree of nominal price rigidity. Moreover, indexation to lagged inflation is controlled by the parameter ι. Each firm sets its price $P_t(j)$ to maximize the present value of future profits:

$$\max_{P_t(j)} \mathbb{E}_t\left\{\sum_{t=0}^{\infty} Q_t\left[\left(\frac{P_t(j)}{P_t}\right)Y_t(j) - W_t h_t(j) - AC_t(j)\right]\right\} \tag{3.12}$$

subject to equations (3.9), (3.10), (3.11) and (3.8)

where Q_t is the marginal value of a unit of consumption good. Firms face all the same problem and therefore choose the same price ($P_t(j) = P_t$) and produce the same quantity ($Y_t(j) = Y_t$ and $\varphi(j) = \varphi_t$). After imposing the symmetric equilibrium, the optimal price setting rule is given by:

$$\begin{aligned}\frac{1}{\nu_t}[1-(1-\alpha)\varphi_t] = 1 - \psi\left(\Pi_t - \frac{\Pi^\iota_{t-1}}{\Pi^{\iota-1}}\right)\Pi_t - \frac{\psi}{2}\left(\Pi_t - \frac{\Pi^\iota_{t-1}}{\Pi^{\iota-1}}\right)^2\frac{\nu_t - 1}{\nu_t} \\ +\mathbb{E}_t\left\{\beta\frac{C_t - \Phi\tilde{C}_{t-1}}{C_{t+1} - \Phi\tilde{C}_t}\psi\left(\Pi_{t+1} - \frac{\Pi^\iota_t}{\Pi^{\iota-1}}\right)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right\}\end{aligned} \tag{3.13}$$

Finally, substituting the labor supply schedule (equation (3.6)) into the labor demand curve (equation (3.10)) we derive the real marginal cost equation:

$$\varphi_t = (C_t - \Phi\tilde{C}_{t-1})\frac{h_t}{(1-\alpha)Y_t} \tag{3.14}$$

Government

Imposing the restriction that one-period debt is in zero net supply, policy choices must be consistent with the government's flow budget constraint $E_t + B^m_{t-1}(1+\rho P^m_t) + TP_t = P^m_t B^m_t + T_t$, where $P^m_t B^m_t$ is the market value of debt, E_t represents the government expenditure, T_t stands for tax revenues and TP_t is a shock capturing residual features, such as changes in the maturity structure and the term premium. Government expenditure is the sum of good purchases and transfers: $E_t = P_t G_t + TR_t$. Rewriting the government budget constraint in terms of GDP ratios yields:

$$e_t + b^m_{t-1}\frac{R^m_t}{\Pi_t}\frac{Y_{t-1}}{Y_t} + tp_t = b^m_t + \tau_t$$

where $b_t^m = \frac{P_t^m B_t^m}{P_t Y_t}$, e_t, tp_t and τ_t are expressed as a fraction of GDP. The term-premia process follows: $tp_t = \rho_{tp} tp_{t-1} + \sigma_{tp}\epsilon_t^{tp}$. We, then, assume that government expenditure has a short-term and a long-term component: $e_t = e_t^s + e_t^L$. The latter is assumed exogenous, $e_t^L = \rho_{e^L} e_{t-1}^L + \sigma_{e^L}\varepsilon_t^{e^L}$, capturing large programs following political processes. The short-term component, instead, accounts for cyclical fiscal measures and follows:

$$e_t^S = \rho_{e^S} e_{t-1}^S + (1-\rho_{e^S})\Big[e^S + \phi_{y,\xi_t^{pol}}\big(\hat{y}_t - \hat{y}_t^n\big)\Big]\sigma_{e^S}\varepsilon_t^{e^S} \tag{3.15}$$

where the term $\big(\hat{y}_t - \hat{y}_t^n\big)$ indicates the output gap, i.e the deviation of the actual output from its *natural* level prevailing in the absence of nominal rigidities. We link the parameter ϕ_y to the same Markov chain controlling switches in the tax rule. Moreover, we define the fraction of public expenditure devoted to government goods' purchases with $\chi_t = \frac{P_t G_t}{E_t}$, which in terms of GDP ratios becomes:

$$\chi_t = \left(\frac{G_t}{Y_t}\right) e_t^{-1} \tag{3.16}$$

where $\chi_t = \rho_\chi \chi_{t-1} + \sigma_\chi \epsilon_t^\chi$.

Monetary Policy

Monetary policy is governed by a rule on the short-term nominal interest rate R_t. The central bank obeys a Taylor-type rule, for which the nominal interest rate adjusts to deviations of both inflation from its target and output from its natural level:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R}\left[\left(\frac{\Pi_t}{\Pi}\right)^{\psi_\pi}\left(\frac{Y_t}{Y_t^n}\right)^{\psi_y}\right]^{(1-\rho_R)} e^{\sigma_R \varepsilon_t^R} \tag{3.17}$$

where R is the steady-state for the gross nominal interest rate[7] and ε_t^R the exogenous monetary policy shock with persistence ρ_R. Note that we assume that the Taylor principle ($\psi_\pi > 1$) is always satisfied.

Market Clearing

The model closes with the aggregate resource constraint given by $Y_t = C_t + G_t$. This specification for the market clearing condition assumes that the losses from the quadratic adjustment costs are paid by the intermediate goods producers and transmitted to the households trough the distributed profits. Expressing the market clearing condition in terms of $g_t = \frac{1}{1-\frac{G_t}{Y_t}}$[8] gives:

$$Y_t = C_t + Y_t\left(1 - \frac{1}{g_t}\right) \tag{3.18}$$

[7] We denote all the variables in steady state without the time sub-index: for example, given the general variable X_t the respective steady state level is indicated by X.

[8] This specification of the market clearing conditions simplified the log-linearized version of the model that become $\tilde{y}_t = \tilde{c}_t + \tilde{g}_t$

3.3.2 Fiscal Policy Regimes

The fiscal authority sets tax revenues, τ_t, according to the following rule:

$$\tau_t = \rho_\tau \tau_{t-1} + \rho_\tau \Big[\tau + \delta_{b,\xi_t^{pol}} (b_t - b) + \delta_{e,\xi_t^{pol}} (e_t - e) + \delta_{y,\xi_t^{pol}} (y_t - y_t^n) \Big] + \sigma_\tau \varepsilon_t^\tau \tag{3.19}$$

where τ denotes the steady state of the tax-to-GDP ratio and ε_t^τ the exogenous tax shock. The latent state variable ξ_t^{pol} captures Markov-switching regimes in fiscal policy behaviour, as described by tax changes. The same Markov chain defines the fiscal authority's attitude toward intertemporal debt stabilization, measured by $\delta_{b,\xi_t^{pol}}$, budget balancing, measured by the response to the expenditure-to-GDP ratio, $\delta_{e,\xi_t^{pol}}$, and cyclical fluctuations, measured by the taxes' response to output gap, $\delta_{y,\xi_t^{pol}}$. The unobserved variable ξ_t^{pol} takes on two possible states and follows a Markov chain evolving according to the transition matrix $H^{pol} = \{h_{i,j}\}_{i,j \in \{0,1\}}$, where $h_{i,j}$ indicates the probability to switch from state i to state j.

We, therefore, consider two regimes for fiscal policy making, given that the Taylor principle for monetary policy is always satisfied in our model. We follow the convention labelling as *sustainable regime* (S) the fiscal behaviour according to which taxes are set to guarantee debt stability, namely the coefficient on debt-to-GDP is strictly greater than the real interest rate: $\delta_{b,S} > \beta^{-1} - 1$. The *unsustainable regime* (U) reflects, instead, lack of fiscal commitment and, thus, a weak reaction of the primary surplus to debt: $\delta_{b,U} < \beta^{-1} - 1$. We, therefore, define $\xi_t^{pol} = \{S, U\}$. Furthermore, although we set the regimes' labelling only according to the primary adjustment to debt, the latter is defined together with switches in the other fiscal rule's parameters, namely the one on expenditure, $\delta_{e,\xi_t^{pol}}$, and the output gap, $\delta_{y,\xi_t^{pol}}$. It is worth clarifying that we can solve the model under both the sustainable and the unsustainable regime, given an active monetary policy, thanks to the extended determinacy properties of the equilibrium, guaranteed by the Markov-switching framework. Indeed, this modelling setup assumes that agents form expectations taking into account the possibility of future changes in policymakers' behaviour. This feature widens the equilibrium determinacy regions.

3.3.3 Solution and Estimation methods

Since the technology process A_t exhibits trend growth, the model is rescaled before being linearized around the unique deterministic steady state. Indeed, the policy regimes enter only on variables expressed as deviations from steady states. We solve the model using the efficient perturbation methods applied to Markov-switching models elaborated by Maih (2015), and differently proposed also by Foerster et al. (2016)[9]. A detailed description of the solution method is reported in Appendix C.3. The model's first-order approximated solution can be written in

[9] They differ in the fixed point around which they perform the Taylor series expansion. Foerster et al. (2016) expand around the steady state associated to the ergodic mean of the parameters. The approach pioneered by Maih (2015), instead, expands around the steady state associated to each regime taken in isolation. In our case, the regimes do not alter the steady state. Therefore, the two approaches would not differ.

the following form:

$$\Upsilon_t = T_{\xi_t^{pol}}(\Upsilon_{t-1}, \sigma, \epsilon_t) \tag{3.20}$$
$$T_{\xi_t^{pol}} = T_{\xi_t^{pol}}(z) + DT_{\xi_t^{pol}}(z)(z_t - z)$$

where Υ_t is the vector of model's variables, $T_{\xi_t^{pol}}$ the Taylor first-order expansion, σ defines the perturbation parameter, ϵ_t the vector of structural shocks and DT the matrix of first-order derivatives. The expansion point is $z_{\xi_t^{pol}} = (\Upsilon, 0, 0)$, where $z_t = (\Upsilon_{t-1}, \sigma, \epsilon_t)$ and Υ identifies the variables' steady states. The law of motion (3.3.3) is combined with a system of observation equations to build a state space system. The model is estimated with Bayesian methods. The likelihood is computed with a variant of the Kim filter[10], originally proposed by Kim and Nelson (1999). The states' probabilities are, instead, extracted using the Hamilton (1994) filter. The obtained likelihood is, then, combined with a prior distribution for the parameters, thereby forming the posterior kernel. We first find the posterior modes, and then use them as starting points to initialize the Metropolis-Hastings algorithm[11] to sample from the posterior distribution. After running the MH algorithm, we perform diagnostics to ensure convergence of the MCMC chain.

3.4 Debt Sustainability Regimes

The model is estimated using seven data series for France, collected at annual frequency, spanning the sample 1955-2009. They include: the real GDP growth, GDP deflator inflation, the short term interest rate and four fiscal variables, expressed as GDP ratios: government debt, current tax revenues, government expenditure and a transformation of government purchases. Appendix C.4 provides a detailed description of the data used and their mapping in the model.

We calibrate the discount factor β to 0.9832, which implies an annual 2% percent real interest rate. We set the labor share at 66%, $\alpha = 0.33$, and the average maturity to 5 years, implying $\rho = 0.8137$. Moreover, in order to separate the short from the long term component of government expenditure, we fix $\rho_{e^L} = 0.9606$, annual equivalent of a quarterly 0.99, and $\sigma_{e^L} = .1\%$. The Taylor rule is entirely calibrated, given the substantial misspecification coming from using country level data to describe a monetary policy lead by different types of external factors (ERM since 1979 and EMU since 1999). We, therefore, fix $\psi_\pi = 1.5$, $\psi_y = 0.5$ and $\rho_R = 0.7$, coherently with the estimates by Jondeau and Sahuc (2008) for France. In accordance with them, we also fix the price indexation at $\iota = 0.35$, because poorly identified. Weak identification induces to calibrate also the preference and tax shocks' persistence parameters at $\rho_\tau = 0.7$ and $\rho_d = 0.7$.

[10]In the regime switching case, the Kalman filter cannot be used because the shocks' distributions are no longer Gaussian, being a mixture of Gaussians. Furthermore, they depend on the entire history of regimes. The exact distributions cannot be recovered, but by truncating the mixture to contain only the Gaussians associated to few regimes, the likelihood can be approximated. This is the Kim filter.

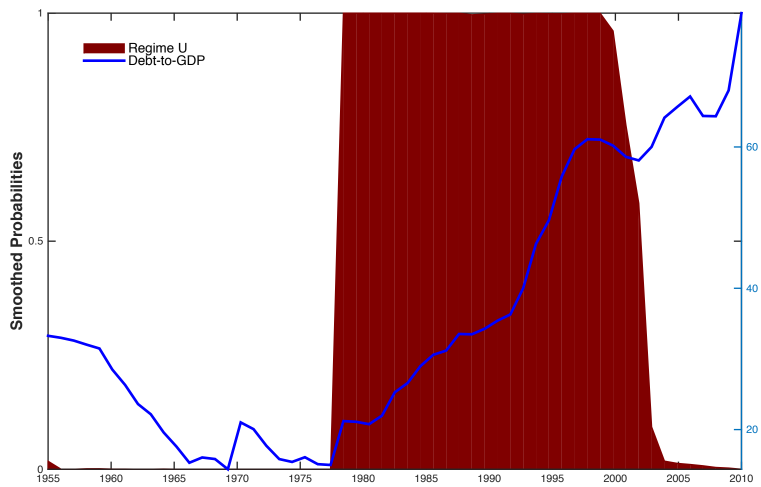
[11]We generate 2 chains, each consisting of 100,000 draws (1 every 10 draws is saved), with a burn-in period of 15% of them. The scale parameter is set to obtain an acceptance rate of around 35%.

Table 3.1: Posterior Modes, Means, 90% Error Bands and Priors of the Model Parameters

	Posterior				Prior		
	Mode	Mean	5%	95%	Type	Mean	Std. Dev
$\delta_b(S)$	0.04462	0.04507	0.03508	0.05432	N	0	0.5
$\delta_b(U)$	0.01226	0.00739	-0.00609	0.02079	N	0	0.5
$\delta_e(S)$	0.84994	0.80286	0.35805	1.24552	N	1	0.5
$\delta_e(U)$	1.37539	1.61890	1.09759	2.12823	N	1	0.5
$\delta_y(S)$	0.33789	0.35438	0.01920	0.69052	N	0	0.5
$\delta_y(U)$	-0.01237	0.09734	-0.32669	0.52676	N	0	0.5
$\phi_y(S)$	-0.73815	-0.80143	-0.99314	-0.64234	N	-0.5	0.5
$\phi_y(U)$	-0.75982	-0.80142	-1.02243	-0.61425	N	-0.5	0.5
H_{SU}^{pol}	0.02638	0.03635	0.00710	0.07979	Dir	0.3	0.2
H_{US}^{pol}	0.08144	0.09615	0.03487	0.17605	Dir	0.3	0.2
ρ_χ	0.98301	0.98458	0.97122	0.99458	B	0.5	0.2
ρ_{e^S}	0.41307	0.44134	0.33740	0.54536	B	0.4	0.1
$\rho_t p$	0.56841	0.53987	0.38636	0.68847	B	0.5	0.2
ρ_μ	0.75150	0.72426	0.61539	0.82700	B	0.5	0.2
ρ_a	0.32519	0.34957	0.11537	0.59417	B	0.4	0.2
Φ	0.75313	0.72375	0.64224	0.79804	B	0.5	0.1
κ	0.01372	0.01607	0.00186	0.03407	G	0.2	0.2
$\exp(\gamma)$	1.03429	1.03145	1.02382	1.03884	N	1.03	0.3
B	0.68295	0.75170	0.54646	0.96328	N	0.6	0.5
G	1.14811	1.15001	1.13799	1.16225	N	1.15	0.01
T	0.19182	0.20062	0.18475	0.21581	N	0.27	0.1
$\ln(P)$	0.03688	0.04030	0.01940	0.05922	N	0.064	0.1
σ_R	0.01861	0.01967	0.01717	0.02246	IG	0.01	5
σ_χ	0.03634	0.03442	0.02951	0.03990	IG	0.01	5
σ_τ	0.00554	0.00580	0.00499	0.00670	IG	0.005	5
σ_{e^S}	0.00329	0.00360	0.00266	0.00457	IG	0.005	5
σ_{tp}	0.08096	0.08675	0.07576	0.09901	IG	0.01	5
σ_a	0.02735	0.02891	0.01950	0.03962	IG	0.05	5
σ_d	0.22102	0.20474	0.16145	0.25342	IG	0.1	5
σ_μ	0.00345	0.00391	0.00257	0.00544	IG	0.01	5

Priors' specification and posterior estimates are reported in Table 3.1. The priors for the constant parameters are in line with previous contributions in the literature; while regarding those interesting the regime-switching parameters, they are symmetric across regimes and assumed very loose. In addition, the priors for the steady state ratios of government spending, debt and taxes to output are centred at their respective first data moments. The first panel of the Table shows results for the parameters interested by regime-switching properties. Indeed, each parameter of the fiscal rule assumes two distinct values, whether in regime S or in U. Under the sustainable regime, as defined in Section 3.4, taxes react to debt according to a positive coefficient, which turns to be larger than the threshold value used to identify a passive fiscal behaviour ($\beta^{-1} - 1 = 0.0171$); while under the unsustainable regime the response is very weak, approximately close to zero. At the same time, however, the parameters controlling how taxes are set over expenditure and output gap changes display distinct dynamics over regimes: the U regime is associated with lower prociclicality (δ_y) and higher elasticity within the budget balance (δ_e). Both regimes are quite persistent, but when the system is in U the probability to switch back to S is higher, implying that S is more recurrent and persistent than U. The estimated transition matrix is not only informative regarding the realized regime sequence, but also on how model dynamics induce changes across regimes.

Figure 3.2: Unsustainable Fiscal Regime. Probabilities at the posterior mode



With the aim of linking regime changes to historical accounts of the French fiscal policy, Figure 3.2 plots the smoothed probabilities assigned to the unsustainable regime U, as defined in Section 3.4 and whose parameters are estimated in Table 3.1. The complementary area characterizes the sustainable regime S, which, as a first phase, marks *Les Trente Glorieuses* (from 1945 to 1975), as coined by Fourastié (1979). Those were years of steady economic and industrial development, policies of economic dirigisme were supporting entire key sectors, several devaluations (1958,1969) were boosting growth favouring competitiveness, and high inflation was overheating the economy. At the same time, primary surpluses and low levels of the debt-to-GDP ratio provided the fiscal space for sustainability. However, since 1977 policies of disinflation, external balance and nominal exchange rate stability (Plan Barre (1976), compliance to ERM (1979) and Mitterand's *Tournant de la rigeur*) set the bases of major rethinking of French economic policy, which lead to the generation of primary deficits, together with a debt-to-GDP ratio at the first steps of a steady surge, as also shown in Figure 3.1. Consistently, under regime U, the red area in Figure 3.2, our estimates report no fiscal commitment towards debt, but high sensitivity to expenditure. Evidently, this fiscal behaviour turned in negative budget balances and uncontrolled debt accounts. This motivates even more the definition of the resulting regime as unsustainable for French public finances. Moreover, on this regard, even if our analysis is not informative on the monetary-fiscal interactions, thinking of nominal debt revaluations driven by inflation is quite hard for those years. Indeed, with the exception of the last 1970s, since 1983, inflation in France experienced a clear-cut decline, which evidently rules out the hypothesis of a passive monetary policy sustaining debt. Finally, the model predicts also the occurrence of a subsequent gradual switch back to the sustainable regime, transition which started in 1999 with the euro membership. The latter evidence is in line with the findings of Gomes and Seoane (2018), while in contrast with Welchenrleder and Zimmer (2014). However, a full investigation on the dynamics in place at the euro area aggregate level would require a different framework.

3.4.1 Way forward

Our results rely on a framework where fiscal regimes are extracted on the tax rule of a standard New Keynesian DSGE model. It is worth remarking here that the literature is pretty exhaustive on the theoretical side, but empirically structural investigations covered mainly the US narrative. At the Euro Area level, instead, the field is still unexplored, given the set of additional challenges which the EMU scenario and country-level heterogeneities would entail. Our contribution represents a first attempt to extract stylized facts by looking at the fiscal side of the French historical dynamics. Although this is already a contribution per sè, our approach is to bring our results forward to further layers of exploration, and we are willing to do so based on the limitations that our analysis encounters from different perspectives. First, among our results we don't report policy simulations because our assumption of lamp-sum taxes implies no differences on the real effects of fiscal shocks across regimes. In order to solve this point in our single-country framework, the introduction of distortionary taxation would be a compelling addition to interpret how model dynamics are affected by fiscal regimes. Furthermore and more importantly, a critical issue of our approach arises from the assumption of no regimes' interactions between the fiscal and the monetary policy authorities. We, indeed, work with a model where the Taylor principle is always satisfied and this implies that: *i*) inflation is always under the control of the monetary policy; *ii*) any inflationary-induced debt revaluations are not taken into account; *iii*) fiscal backing is always guaranteed; *iv*) fiscal regimes do not translate into regime-specific real effects, under lamp-sum taxation. Although we argue that this setup could approximate the macro framework starting by 1979 with the ERM pegged system and then by 1999 with the EMU, a full narrative on EA fiscal regimes would require building a multi-country model, where monetary policy is conducted by a central authority, the ECB. Once the monetary policy is appropriately described, a further extension, then, could account for the ELB on the more recent macro evidence.

3.5 Conclusions

Fiscal imbalances at the EA level are broadly perceived as a major source of aggregate risk and uncertainty. Within this environment, we address the question of French debt sustainability using an historical approach, which maps short-run primary fiscal measures to long-run solvency over the period 1955-2009. The analysis is conducted estimating a DSGE model with Markov-switching dynamics driving the specification of two different fiscal rules for tax revenues. The latter are considered as interacting with each other thorough their effects on agents' expectations, since agents are assumed to know the probability distribution of future policy changes. Two fiscal regimes are identified on the tax rule estimates. A *sustainable regime* describes the fiscal evidence of low debt and primary surpluses prevailing up to 1977, when a sudden switch reverts the system to an *unsustainable regime* accounting for primary deficits and unstable debt dynamics. In 1999, then, with the euro membership, a gradual transition to the sustainable region is observed.

Our analysis intervenes on a field, which is still largely unexplored in the macro literature. The EA policy debate still cannot rely on a structural assessment of fiscal events under an empirically founded historical account. Our contribution provides evidence at a country level, based only on fiscal policy regimes. However, a proper accounting of the EA monetary framework would be necessary to evaluate policy mix interactions and their role in price and debt determination.

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Appendix A

Appendix to Chapter 1

A.1 GMM Estimation of the Ambiguity Parameter

In this section we detail the derivations needed to achieve the moment condition that is the object of our estimation. Further below we also provide a description of the dataset used in the estimation.

A.1.1 General Approach

We use a GMM estimation procedure based on the moment condition obtained from the combined Euler equation for debt and risky assets and is a variant of the techniques developed for asset pricing models with recursive preferences, pioneered by Epstein and Zin (1989) and Kreps and Porteus (1978). Hence the starting point is to reformulate our value function, capturing multiplier preferences, in terms of an ambiguity term. The latter is achieved by mapping the multiplier preferences to a special case of the recursive preferences. This can be done by assuming a logarithmic continuation value, a logarithmic utility function and an exponential ambiguity adjustment factor, Q which accounts for waves of optimism. Indeed we depart from the well-known equivalence between multiplier and recursive preferences by embedding state-contingent ambiguity attitudes. We start by reporting the value function derived after substituting the solution of the inner problem, presented in Section 1.3.3:

$$V_t = u(c_t) - \beta\theta_t log\left[\mathbb{E}_t\left\{\exp\left(-\frac{V_{t+1}}{\theta_t}\right)\right\}\right] \tag{A.1}$$

The above equation embeds a logarithmic ambiguity-adjusted component $Q_t(V_{t+1})$, which maps future continuation values into current realizations. Indeed we can re-write (A.2) as follows:

$$\begin{aligned} V_t =& u(c_t) + \beta h^{-1}\mathbb{E}_t\left\{h(V_{t+1})\right\} \\ & u(c_t) + \beta Q_t(V_{t+1}) \end{aligned} \tag{A.2}$$

The equivalence between specifications under recursive and multiplier preferences is achieved by assuming the following functional form $h(V)$ (Hansen et al. (2007)): $h(V_{t+1}) = \left(-\frac{V_{t+1}}{\theta_t}\right)$. The latter implies that the exponential ambiguity adjustment component reads as follows:

$$Q_t(V_{t+1}) = h^{-1}\mathbb{E}_t\left\{h(V_{t+1})\right\} = -\theta_t \log\left[\mathbb{E}_t\left\{\exp\left(-\frac{V_{t+1}}{\theta_t}\right)\right\}\right] \tag{A.3}$$

A.1.2 Pricing Kernel-SDF

The next step to obtain our moment condition is to derive an expression for the stochastic discount factor as function of the $Q_t(V_{t+1})$. To this purpose, we shall derive expressions for the marginal utilities in period t and $t+1$. Given the needed functional forms detailed above, namely a logarithmic utility function $u(c_t) = log(c_t)$, the marginal utility of consumption simplifies to $MC_t = c_t^{-1}$. The marginal utility of next-period continuation value is instead derived as follows:

$$\begin{aligned} MV_{t+1} &= \frac{\partial V_t}{\partial Q_t(V_{t+1})}\frac{\partial Q_t(V_{t+1})}{\partial V_{t+1}} = \beta\frac{\exp(-\frac{V_{t+1}}{\theta_t})}{\mathbb{E}_t\left\{\exp(-\frac{V_{t+1}}{\theta_t})\right\}} \\ &= \beta\exp\Big(-\frac{1}{\theta_t}(V_{t+1} - Q_t(V_{t+1}))\Big) \end{aligned} \tag{A.4}$$

Using the above expressions for the marginal utility we can derive the SDF as function of the Q_t factor:

$$\Lambda_{t,t+1} = \frac{MV_{t+1}MC_{t+1}}{MC_t} = \beta\frac{c_{t+1}}{c_t}^{-1}\underbrace{\exp\left(-\frac{1}{\theta_t}(V_{t+1} - Q_t(V_{t+1}))\right)}_{m_{t+1}} \tag{A.5}$$

where $m_{t+1} = \exp\left(-\frac{1}{\theta_t}(V_{t+1} - Q_t(V_{t+1}))\right)$ is the optimal likelihood ratio. Equation (A.5) shows that the SDF has a two-factor structure. The first factor is the standard fundamental consumption growth, while the second is the added ambiguity factor, which is conditioned to the distance between the future value function and its certainty equivalent (the future insurance premium). Under no uncertainty this premium vanishes[1].

A.1.3 Estimation of the Continuation Value Ratio

Since estimation requires strictly stationary variables, we shall re-scale the value function (A.2) by consumption (see Hansen, Heaton and Li (2008) (HHL henceforth). We take log deviations from the log of consumption, $\tilde{c}_t = log(c_t)$, where the tilde indicates logarithms:

$$\tilde{v}_t = \beta Q_t(\tilde{v}_{t+1} + \Delta\tilde{c}_{t+1}) \tag{A.6}$$

We define $\tilde{v}_t$ as the log value of *continuation value ratio*, $\frac{V_t}{c_t}$. Next using (A.3) into (A.6) we

[1]Indeed the continuation value would be perfectly predictable $\left(\exp\left(-\frac{V_{t+1}}{\theta}\right) = \mathbb{E}_t\exp\left(-\frac{V_{t+1}}{\theta}\right), m^*_{t+1} = 1\right)$ with zero adjustment ($Q_t(V_{t+1}) = V_{t+1}$).

obtain:

$$\tilde{v}_t = -\beta\theta_t \log(\mathbb{E}_t \{\exp[\sigma_t(\tilde{v}_{t+1} + \Delta\tilde{c}_{t+1})]\}) \tag{A.7}$$

where $\sigma_t = -1/\theta_t$, and it is negative when $\theta_t > 0$ and positive when $\theta_t < 0$. We rely on this expression when we guess a process for $\tilde{v}_t$, which we then estimate. Indeed, since the continuation value ratio is a function of state variables governing the dynamic behaviour of consumption growth, we start by assuming that the latter is a function of state, ξ_t, which in turn evolves according to the following first-order Markov process:

$$g^c_{t+1} = \tilde{c}_{t+1} - \tilde{c}_t = \mu_c + H\xi_t + \mathbf{A}\epsilon_{\mathbf{t+1}} \tag{A.8}$$

$$\xi_{t+1} = F\xi_t + \mathbf{B}\epsilon_{\mathbf{t+1}} \tag{A.9}$$

where ϵ_{t+1} is a $(2x1)$ i.i.d. vector with zero mean and covariance matrix I. A and B are $(2x1)$ vectors. The exogenous states, ϵ_{t+1}, which could capture income shocks, have both a direct impact on consumption and an indirect one through the endogenous state, ξ_t. The latter can indeed capture endogenous movements in wealth which affect consumption one period later. The estimated value of the endogenous states, $\hat{\xi}_t$, is obtained through Kalman filtering consumption data. The value function depends upon the estimated endogenous states, $\hat{\xi}_t$, and consumption growth, g^c_{t+1}. Since the latter also depends upon the endogenous states, we can guess the continuation value ratio as follows:

$$\tilde{v}_t = \mu_v + U_v\hat{\xi}_t \tag{A.10}$$

where $U_v\hat{\xi}_t$ is the discounted sum of expected future growth rates of consumption. After some derivations we can write U_v and μ_v as follows:

$$U_v \equiv \beta(I - \beta F)^{-1}H \tag{A.11}$$

$$\mu_v \equiv \frac{\beta}{1-\beta}\left(\mu_c + \frac{\sigma_t}{2}|A + U_vB|^2\right)$$

where the term $A + U_vB$ captures the dependence between the the continuation value and the exogenous shocks.

A.1.4 SDF and the Euler Equation

Next, given the estimated $\tilde{v}_t$ from (A.10). Substituting (A.7) into (A.5) delivers:

$$\Lambda_{t,t+1} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{-1}\left(\frac{\exp(\frac{V_{t+1}}{c_{t+1}})\frac{c_{t+1}}{c_t}}{\exp(Q_t\left(\frac{V_{t+1}}{c_{t+1}} + \Delta\tilde{c}_{t+1}\right))}\right)^{\sigma} \tag{A.12}$$

Note that equation (A.12) is equivalent to the SDF obtained under Epstein and Zin (1989) preferences and given the assumption of unitary EIS. At last, we substitute the above SDF into the combined Euler for debt and risky asset (1.15) and (1.16), which results in:

$$\mathbb{E}_t \left\{ \underbrace{\beta \left(\frac{c_{t+1}}{c_t}\right)^{(1-\sigma)} \left(\frac{\exp \frac{V_{t+1}}{c_{t+1}}}{\sqrt[\beta]{\exp \frac{V_t}{c_t}}}\right)^{\sigma}}_{\Lambda_{t,t+1}} \left(R^s_{t+1} - \phi R_{t+1}\right) + \phi - 1 \right\} = 0 \tag{A.13}$$

where $R^s_{t+1} = \frac{d_{t+1}+q_{t+1}}{q_t}$ and for the estimation we shall write the debt rate as time-varying.

A.2 Analytical Derivations

This appendix derives analytical expressions for asset prices and returns.

A.2.1 Asset Price

From the borrowers' optimality condition on risky assets:

$$\begin{aligned} q_t =& \beta \mathbb{E}_t \left\{ \frac{u_c(c_{t+1})}{u_c(c_t)} m_{t+1}(q_{t+1} + d_{t+1}) \right\} + \phi \mu_t^{'} q_t \\ =& \beta \mathbb{E}_t \{\Lambda_{t,t+1}(d_{t+1} + q_{t+1})\} + \phi \mu_t^{'} q_t \end{aligned} \tag{A.14}$$

using the definitions for $\Lambda_{t,t+1} = \beta \frac{u_c(c_{t+1})}{u_c(c_t)} m_{t+1}$ and $\mu_t^{'} = \frac{\mu_t}{u_c(c_t)}$. Then denoting $K_{t,t+1} = \frac{\Lambda_{t,t+1}}{1-\phi\mu_t^{'}}$, we derive the following expression for the asset price:

$$q_t = \mathbb{E}_t\{K_{t,t+1}(d_{t+1} + q_{t+1})\} \tag{A.15}$$

Proceeding by forward recursion:

$$\begin{aligned} q_t =& \mathbb{E}_t\{K_{t,t+1}(d_{t+1} + K_{t+1,t+2}(d_{t+2} + q_{t+2}))\} \\ =& \mathbb{E}_t\{K_{t,t+1}(d_{t+1} + K_{t+1,t+2}d_{t+2})\} + \mathbb{E}_t\{K_{t,t+1}K_{t+1,t+2}K_{t+2,t+3}(d_{t+3} + q_{t+3})\} \\ =& \mathbb{E}_t\{K_{t,t+1}(d_{t+1} + K_{t+1,t+2}d_{t+2} + K_{t+1,t+2}K_{t+2,t+3}d_{t+3})\} + \\ & + \mathbb{E}_t\{K_{t,t+1}K_{t+1,t+2}K_{t+2,t+3}K_{t+3,t+4}(d_{t+4} + q_{t+4})\} \\ =& \mathbb{E}_t\{K_{t,t+1}(d_{t+1} + K_{t+1,t+2}d_{t+2} + \\ & + K_{t+1,t+2}K_{t+2,t+3}d_{t+3} + K_{t+1,t+2}K_{t+2,t+3}K_{t+3,t+4}d_{t+4})\} + \\ & + \mathbb{E}_t\{K_{t,t+1}K_{t+1,t+2}K_{t+2,t+3}K_{t+3,t+4}q_{t+4}\} \end{aligned} \tag{A.16}$$

At the final recursion step, the solution for the asset price:

$$q_t = \mathbb{E}_t \left\{ \sum_{i=1}^{T} d_{t+i} \prod_{j=1}^{i} K_{t+j-1,t+j} \right\} + \mathbb{E}_t \left\{ \prod_{i=0}^{T} K_{t+i,t+i+1} q_{t+T} \right\} \tag{A.17}$$

Taking the limit for $T \to \infty$ of the above condition delivers equation (1.26).

A.2.2 The Risk Premium

Expanding the borrower's FOC for the risky asset and plugging in it the derivation for $\mathbb{E}_t\{\Lambda_{t,t+1}\}$ and the definition $R^s_{t+1} = \frac{q_{t+1}+d_{t+1}}{q_t}$ we get:

$$\begin{aligned} 1 &= \mathbb{E}_t\{\Lambda_{t,t+1}\frac{q_{t+1}+d_{t+1}}{q_t}\} + \phi\mu'_t \\ &= \mathbb{E}_t\{\Lambda_{t,t+1}\}\mathbb{E}_t\Big\{\frac{q_{t+1}+d_{t+1}}{q_t}\Big\} + \text{Cov}\Big(\Lambda_{t,t+1}, \frac{q_{t+1}+d_{t+1}}{q_t}\Big) + \phi\mu'_t \\ &= \Big(\frac{1-\mu'_t}{R}\Big)\mathbb{E}_t\{R^s_{t+1}\} + \text{Cov}(\Lambda_{t,t+1}, R^s_{t+1}) + \phi\mu'_t \end{aligned} \tag{A.18}$$

The return on risky assets is obtained:

$$\mathbb{E}_t\{R^s_{t+1}\} = \frac{R(1 - cov(\Lambda_{t,t+1}, R^s_{t+1}) - \phi\mu'_t)}{1-\mu'_t} \tag{A.19}$$

Dividing by the risk-free return rate, the premium between the return on the risky asset and the risk-free rate can be derived:

$$\Psi_t = \frac{1 - cov(\Lambda_{t,t+1}, R^s_{t+1}) - \phi\mu'_t}{1-\mu'_t}. \tag{A.20}$$

A.2.3 The Sharpe Ratio and the Hansen and Jagannathan (1991) Bounds

Writing down the two borrowers' optimal conditions for the risk-free and risky assets, respectively:

$$1 = \mathbb{E}_t\{\Lambda_{t,t+1}R\} + \mu'_t \tag{A.21}$$
$$1 = \mathbb{E}_t\{\Lambda_{t,t+1}R^s_{t+1}\} + \phi\mu'_t \tag{A.22}$$

where $\mu'_t = \frac{\mu_t}{u_c(c_t)}$, $\Lambda_{t,t+1} = \beta\frac{u_c(c_{t+1})}{u_c(c_t)}m_{t+1}$ and $R^s_{t+1} = \frac{q_{t+1}+d_{t+1}}{q_t}$. In order to derive the excess return between the risky asset and the risk-free asset, we subtract (A.21) from (A.22), obtaining:

$$0 = \mathbb{E}_t\{\Lambda_{t,t+1}(R^s_{t+1} - R)\} + \mu'_t(\phi - 1). \tag{A.23}$$

Then, we define the excess return as $z_{t+1} = R^s_{t+1} - R$. Assuming a linear general form for the stochastic discount factor $\Lambda_{t.,t+1}$:

$$\Lambda^*_{t.,t+1} = \bar{\Lambda}^* + \tilde{\beta^m}(z_{t+1} - E_t z_{t+1}) \tag{A.24}$$

it must satisfy the following condition:

$$0 = \mathbb{E}_t\{\Lambda^*_{t,t+1}z_{t+1}\} + \mu'_t(\phi - 1)\}, \tag{A.25}$$

which, once expanded, gives:

$$
\begin{aligned}
0 &= \mathbb{E}_t\{\Lambda^*_{t,t+1}\}\mathbb{E}_t\{z_{t+1}\} + cov(\Lambda^*_{t,t+1}, z_{t+1}) + \mu_t^{'}(\phi - 1) \\
&= \mathbb{E}_t\{\Lambda^*_{t,t+1}\}\mathbb{E}_t\{z_{t+1}\} + \mathbb{E}_t\{(z_{t+1} - \bar{z})(\Lambda^*_{t,t+1} - \bar{\Lambda}^*)\} + \mu_t^{'}(\phi - 1) \\
&= \mathbb{E}_t\{\Lambda^*_{t,t+1}\}\mathbb{E}_t\{z_{t+1}\} + \mathbb{E}_t\{(z_{t+1} - \bar{z})(z_{t+1} - \bar{z})\tilde{\beta^m}\} + \mu_t^{'}(\phi - 1) \\
&= \mathbb{E}_t\{\Lambda^*_{t,t+1}\}\mathbb{E}_t\{z_{t+1}\} + \sigma_z^2\tilde{\beta^m} + \mu_t^{'}(\phi - 1).
\end{aligned} \tag{A.26}
$$

Hence:

$$
\tilde{\beta^m} = -(\sigma_z^2)^{-1}\mathbb{E}_t\{\Lambda^*_{t,t+1}\}\mathbb{E}_t\{z_{t+1}\} - (\sigma_z^2)^{-1}\mu_t^{'}(\phi - 1) \tag{A.27}
$$

The variance of the stochastic discount factor is then obtained:

$$
\begin{aligned}
Var(\Lambda^*_{t,t+1}) &= Var((z_{t+1} - \mathbb{E}_t\{z_{t+1}\})'\tilde{\beta^m}) \\
&= \tilde{\beta^{'m}}\sigma_z^2\tilde{\beta^m} \\
&= (-(\sigma_z^2)^{-1}\bar{\Lambda}_t^*\mathbb{E}\{z_{t+1}\} - (\sigma_z^2)^{-1}\mu_t^{'}(\phi - 1))^{'}\sigma_z^2 \\
&(-(\sigma_z^2)^{-1}\bar{\Lambda}_t^*\mathbb{E}\{z_{t+1}\} - (\sigma_z^2)^{-1}\mu_t^{'}(\phi - 1)) \\
&= (\sigma_z^2)^{-1}(\bar{\Lambda}^*)^2(\mathbb{E}_t\{z_{t+1}\})^2 + \\
&+ 2\mu_t^{'}(\phi - 1)((\sigma_z^2)^{-1}\bar{\Lambda}_t^*\mathbb{E}\{z_{t+1}\} + ((\sigma_z^2)^{-1}(\mu_t^{'})^2(\phi - 1)^2.
\end{aligned} \tag{A.28}
$$

Hence:

$$
\frac{\sigma^2_{\Lambda_t^*}}{\bar{\Lambda}^{*2}} = \frac{(\mathbb{E}_t\{z_{t+1}\})^2}{\sigma_z^2} + 2\mu_t^{'}\frac{(\phi - 1)\mathbb{E}_t\{z_{t+1}\}}{\sigma_z^2} + \frac{\mu_t^{'2}}{\Lambda^{*2}}\frac{(\phi - 1)^2}{\sigma_z^2}. \tag{A.29}
$$

The Sharpe Ratio (SR hereafter) on stock asset returns over bonds results to be:

$$
SR = \frac{(\mathbb{E}_t\{z_{t+1}\})^2}{\sigma_z^2} = \frac{\sigma^2_{\Lambda_t^*}}{\bar{\Lambda}^{*2}} - 2\mu_t^{'}\frac{(\phi - 1)\mathbb{E}_t\{z_{t+1}\}}{\sigma_z^2} - \frac{\mu_t^{'2}}{\Lambda^{*2}}\frac{(\phi - 1)^2}{\sigma_z^2} \tag{A.30}
$$

Thus, the SR depends on the variance of the adjusted for distorted beliefs stochastic discount factor and on $\mu_t^{'}$.

A.3 Numerical Method

Our numerical method extends the algorithm of Jeanne and Korinek (2010) to persistent shocks and state-contingent ambiguity attitudes. The method, following the endogenous grid points approach of Carroll (2006), performs backwards time iteration on the agent's optimality conditions. We derive the set of policy functions $\{c(b, s),\ b'(b, s),\ q(b, s),\ \mu(b, s),\ V(b, s)\}$ that solve competitive equilibrium characterized by the system:

$$c(b,s)^{-\gamma} = \beta R\,\mathbb{E}\left\{m(b',s')c(b',s'^{-\gamma})\right\} + \mu(b,s) \tag{A.31}$$

$$q(b,s) = \beta\,\frac{\mathbb{E}\left\{m(b',s')c(b',s'^{-\gamma}[q(b',s') + \alpha y']\right\}}{c(b,s)^{-\gamma} - \phi\mu(b,s)} \tag{A.32}$$

$$\mu(b,s)\left[\frac{b'(b,s)}{R} + \phi q(b,s)\right] = 0 \tag{A.33}$$

$$c(b,s) + \frac{b'(b,s)}{R} = y + b \tag{A.34}$$

$$V(b,s) = \frac{c(b,s)^{1-\gamma} - 1}{1-\gamma} + \frac{\beta}{\sigma}\ln\mathbb{E}\left\{\exp\{\sigma V(b',y')\}\right\} \tag{A.35}$$

where $m(b,s)$ is the expectation distortion increment. The solution method works over the following steps:

1. We set a grid $G_b = \{b_1, b_2, \ldots, b_H\}$ for the next-period bond holding b'; and a grid $G_s = \{s_1, s_2, \ldots, s_N\}$ for the shock state space $s = \{y, \sigma\}$. The income process y, is discretized with Tauchen and Hussey (1991) method, while the grid for the *inverse* of the penalty parameter σ (recall that θ is the inverse of σ) follows a simple two-state rule:[2]

$$\sigma = \begin{cases} \sigma^- & \text{if} \quad V < \mathbb{E}\{V\} \\ \sigma^+ & \text{if} \quad V \geq \mathbb{E}\{V\} \end{cases} \tag{A.36}$$

2. In iteration step k, we start with a set of policy functions $c_k(b,s)$, $q_k(b,s)$, $\mu_k(b,s)$ and $V_k(b,s)$. For each $b' \in G_b$ and $s' \in G_s$:

   a) we derive the expectation distortion increment:

   $$m_k(b',s') = \frac{\exp\{\sigma V_k(b',s')\}}{\mathbb{E}\left[\exp\{\sigma V_k(b',s')\}\right]} \tag{A.37}$$

   and then, the distorted expectations in the Euler equation for bonds and for the risky assets (equations (1) and (2)).

   b) we solve the system of optimality conditions under the assumption that the collateral constraint is slack:

   $$\mu^u(b',s) = 0 \tag{A.38}$$

   As a result, $c^u(b',s)$, $q^u(b',s)$, $\mu^u(b',s)$, $V^u(b',s)$ and $b^u(b',s)$ are the policy functions for the unconstrained region;

   c) in the same way, we solve the system for the constrained region of the state space, where the following condition holds:

   $$q^c(b',s) = -\frac{b'/R}{\phi} \tag{A.39}$$

[2] We use 800 grids point for bonds and 45 grid points for the exogenous shocks; we implement linea interpolation in order to approximate the policy functions outside the grids

$c^c(b', s)$, $q^c(b', s)$, $\mu^c(b', s)$, $V^c(b', s)$ and $b^c(b', s)$ are the respective policy functions.

d) we derive the next period bond holding threshold $\bar{b}'$ such that the borrowing constraint is marginally binding. For each $s \in G_s$ it satisfies the following condition:

$$\bar{b}'^c(\bar{b}', s) + \frac{\bar{b}'(s)}{R} = 0 \tag{A.40}$$

When this point is out of the grid we use linear interpolation. Given this value, we can derive for each policy function the frontier between the binding and non-binding region: $x^u(\bar{b}'^c(\bar{b}', s)$ for $x = \{c, b, q, \mu, V\}$.

3. In order to construct the step k+1 policy function, $x_{k+1}(b, s)$, we interpolate on the pairs $(x^c(b'^c(b', s))$ in the constraint region, and on the pairs $(x^u(b'^u(b', s))$ in the unconstrained region. As a result we find: $c_{k+1}(b, s)$, $q_{k+1}(b, s)$, $\mu_{k+1}(b, s)$ and $V_{k+1}(b, s)$

4. We evaluate convergence. If

$$\sup ||x_{k+1} - x_k|| < \epsilon \qquad \text{for} \qquad x = c, q, \mu, V \tag{A.41}$$

we find the competitive equilibrium. Otherwise, we set $x_k(b, s) = (1 - \delta)x_{k+1}(b, s) + \delta x_k(b, s)$ and continue the iterations from point 2. We use a value of δ close to 1.

A.4 Data Description for Empirical Moments

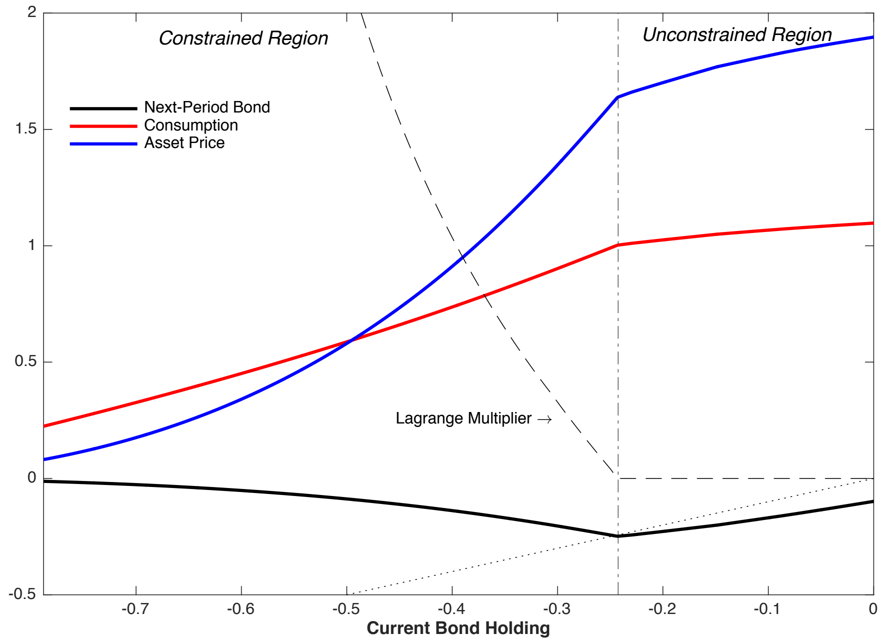
In this section we describe the data employed for the computation of the empirical moments used for model matching. We compute several moments for asset prices, returns and debt data. Data are from the US. The used sample spans 1980:Q1 to 2016:Q4, since this corresponds to the period of rapid debt growth The dataset is composed as follows: debt is given by private non-financial sector, by all sectors (BIS: http://www.bis.org/publ/qtrpdf/r_qt1403g.pdf); consumption is given by Personal Consumption Expenditure (NIPA Tables[3]), GDP (NIPA Tables); the risk-free rate is the 3month T-bill rate (CRSP Indices database[4]); risky returns are proxied by the $S\&P500$ equity return with dividends (Shiller Database[5]). All variables are deflated by CPI index. Note that HP-filtered series are computed as deviations from a long-term trend. Therefore, we work with a much larger smoothing parameter ($\lambda = 400,000$) than the one employed in the business cycle literature, to pick up the higher expected duration of the credit cycle (see http://www.bis.org/publ/bcbs187.pdf).

A.5 Policy Functions

Figure A.1 shows the policy functions $c_t(b, y)$, $q_t(b, y)$, $b_{t+1}(b, y)$ and $\mu_t(b, y)$ for a medium income shock realization. It proves that our model, even with state contingent ambiguity attitudes, is

[3]See https://www.bea.gov/iTable/index_nipa.cfm.
[4]See http://www.crsp.com/products/research-products/crsp-historical-indexes.
[5]See http://www.econ.yale.edu/~shiller/data.htm

Figure A.1: Policy Functions



able to reproduce all the salient characteristics of the financial crises models (see Jeanne and Korinek (2010)). Indeed, in the binding region the next period bond holding is downward sloping and the policy functions for consumption and asset price display a higher inclination than in the unconstrained region. The latter feature implies that in the constrained region variables respond very strongly to changes in the current wealth, as the financial amplification theory states.

A.6 Three Period Model

In this section we outline an extended version of the three period model with occasionally binding collateral constraints and with ambiguity attitudes. The goal is to show the combined effect of those two elements on debt growth. The economy we consider is populated by a continuum of agents, who live for three periods: $t = 0, 1, 2$. Preferences are given by the following specification:

$$U = u(c_0) + \mathbb{E}_S \left[\beta u(c_1) + \beta^2 u(c_2)\right] \tag{A.42}$$

where $u(c) = \frac{1}{2}[\bar{c} - c]^2$. In period 0 we can assume a linear utility function $u(c_0) = c_0$ in order to simplify the analysis. We also assume that $\beta R = 1$. The endowment structure is characterized as follows. Agents receive endowment income in period 1 and 2, but none in period 0. In period 1 the endowment is stochastic depending on the realization of the state $s \in S$. We assume that $S = \{s_1, s_2, ...s_N\}$ is a monotone increasing sequence. The realization of the endowment are affected monotonically from the realization of s, so that for example $y^{s_n} > y^{s_{n-1}}$. The probability that a state s occurs is given by π_s. Similarly to the main text we assume that the dividend is lead by the same source of volatility. This allows us to simplify the state space. Therefore, in each period a fraction $(1-\alpha)y_t$ is the labor income, and the fraction $d_t = \alpha y_t$ is the dividends' income. The budget constraints for each period reads as follows:

$$c_0 + q_0 x_0 + \frac{b_0}{R} = 0 \tag{A.43}$$

$$c_1^s + q_1^s x_1^s + \frac{b_1^s}{R} = (1-\alpha)y_1^s + x_0(q_1^s + \alpha y_1^s) + b_0 \tag{A.44}$$

$$c_2^s = (1-\alpha)y_2 + x_1 \alpha y_2 + b_1^s \tag{A.45}$$

Note that the sup-index s in period 1 indicates that uncertainty materializes in this period. We have assumed that $b_{-1} = b_2 = 0$, $x_{-1} = x_2 = 0$, $q_2 = 0$ and $d_{-1} = 0$. In period 1 the collateral constraint limits the amount of debt:

$$-\frac{b_1^s}{R} \leq \phi q_1^s x_1^s \tag{A.46}$$

The agents expectation formation process is derived as in the main text. Since uncertainty refers to period 1 income, agents form expectation in period 0. Their optimal likelihood ratio in period 0 is given by:

$$m_1^s = \frac{\exp\{\sigma_0 V_1^s\}}{\mathbb{E}_0\left\{\exp\{\sigma_0 V_1^s\}\right\}} \tag{A.47}$$

where the value function recursion is defined as following[6]: $V_1^s = u(c_1^s) + \beta u(c_2^s)$. The relation that links the level of m_1^s to the state of the economy is:

$$\text{if} \qquad V_1^s < \mathbb{E}_0\left\{V_1^s\right\} \qquad \text{then} \qquad m_1^s > 1 \tag{A.48}$$

Given the above optimization problems the *decentralized equilibrium* is characterized as follows. The bonds' Euler equations between periods 0 and 1 and between periods 1 and 2, read as follows:

$$1 = \beta R \mathbb{E}_0\left\{m_1^s u_c(c_1^s)\right\} \tag{A.49}$$

$$u_c(c_1^s) = \beta R u_c(c_2^s) + \mu_1^s \tag{A.50}$$

The Euler conditions on the risky asset between periods 0 and 1 and between periods 1 and 2 read as follows:

$$q_0 = \beta \mathbb{E}_0\left\{m_1^s u_c(c_1^s)[q_1^s + \alpha y_1^s]\right\} \tag{A.51}$$

$$q_1^s = \beta \frac{u_c(c_2^s)\alpha y_2}{u_c(c_1^s) - \phi \mu_1^s} \tag{A.52}$$

The complementarity slackness condition is:

$$\mu_1^s \left[\frac{b_1^s}{R} + \phi q_1^s\right] = 0 \tag{A.53}$$

[6]This simplified representation is obtained under the assumption that there is no uncertainty in period 2.

Finally, the decentralized equilibrium is closed with a condition on expectations, equation (1.7), and the following market clearing conditions:

$$c_0 + q_0 + \frac{b_0}{R} = 0 \tag{A.54}$$

$$c_1^s + \frac{b_1^s}{R} = y_1^s + b_0 \tag{A.55}$$

$$c_2^s = y_2 + b_1^s \tag{A.56}$$

where we have imposed the stock market clearing condition $x_t = 1$.

A.6.1 Time 1 Continuation Equilibrium

We now proceed to the model solution by backward induction. We start from period the last period and since there is no uncertainty between time 1 and time 2 we can solve for the two periods simultaneously. We start from characterizing the continuation value under the *unconstrained region.* The system of equilibrium conditions for the unconstrained region (the sup-index U will be used since now on to indicate the solution for this region) is (we can use $\beta = R^{-1}$ and $\mu_1 = 0$):

$$u_c(c_1^s) = u_c(c_2^s) \qquad c_1^s = c_2^s = c^{U,s} \tag{A.57}$$

$$q_1^s = \beta \frac{u_c(c_2^s)}{u_c(c_1^s)} \alpha y_2 \tag{A.58}$$

$$c_1^s + \frac{b_1^s}{R} = y_1^s + b_0 \tag{A.59}$$

$$c_2^s = y_2 + b_1^s \tag{A.60}$$

Given the above the consumption function depends on lifetime wealth and reads as follows:

$$c^{U,s} = \frac{1}{1+\beta}\left(y_1^s + b_0 + \frac{y_2}{R}\right) \tag{A.61}$$

Using the budget constraint and the consumption function one can derive the optimal level of debt:

$$b_1^U(s) = \frac{\beta}{1+\beta}\left(y_1(s) + b_0 - \frac{y_2}{R}\right) \tag{A.62}$$

Finally, the equilibrium asset price condition, which depends on the value of the dividend in the last period, reads as follows:

$$q_1 = \beta \alpha y_2 \tag{A.63}$$

In the *constrained region* ($\mu_t > 0$, the sup-index C is used since now onward to indicate equilibrium values for this region)), the system of equilibrium conditions reads as follows:

$$\mu_1^s = u_c(c_1^s) - u_c(c_2^s) \qquad c_1^s < c_2^s \tag{A.64}$$

$$q_1^s = \beta \frac{u_c(c_2^s)}{u_c(c_1^s) - \phi\mu_1^s}\alpha y_2 \tag{A.65}$$

$$c_1^s + \frac{b_1^s}{R} = y_1^s + b_0 \tag{A.66}$$

$$c_2^s = y_2 + b_1^s \tag{A.67}$$

$$\frac{b_1^s}{R} = -\phi q_1^s \tag{A.68}$$

A.6.2 Time Zero Equilibrium

To characterize the time 0 equilibrium we first partition the state space into two blocks, S^C and S^U, where the constraint is binding and slack respectively. Assuming that the $u(c_0) = c_0$ we have:

$$1 = \sum_{s\in S^U} \pi_s m_1^{U,s} u_c^{U,s}(b_0; y_1, y_2)) + \sum_{s\in S^C} \pi_s m_1^{C,s} u_c^{C,s}(b_0; y_1, y_2)) \tag{A.69}$$

$$q_0 = \beta \left\{ \begin{array}{c} \sum_{s\in S^U} \pi_s m_1^{U,s} u_c^{U,s}(b_0; y_1, y_2))[q_1^U + y_1^s] \\ + \sum_{s\in S^C} \pi_s m_1^{C,s} u_c^{C,s}(b_0; y_1, y_2))[q_1^{C,s}(b_0; y_1, y_2) + y_1^s] \end{array} \right\} \tag{A.70}$$

$$c_0 = -\frac{b_0}{R} - q_0 \tag{A.71}$$

where $c_1^{i,s}, b_1^{i,s}, q_1^{i,s}$ are the solutions of the time 1 continuation equilibrium.

A.6.3 The Expectation Distortion under a Binomial State Space

Our goal is to assess the role of ambiguity attitudes on debt growth. To this purpose we shall derive a closed form solution for policy functions. To do that we assume a simple binomial structure for the state space. Hence we assume that the state space is comprised of two states, which we label high, with sup-index h, occurring with probability π, and low, with sup-index l, occurring with probability $(1-\pi)$. The exogenous state space therefore reads as follows $S = \{h, l\}$. We assume that the in state h the income realization is high enough that the collateral constraint is slack. Similarly we assume that in state l, the income realization is low enough that the collateral constraint binds. Given this structure for the objective probability, the expectation distortions are given by:

$$m_1^s = \frac{\exp\{\sigma_0 V_1^s\}}{\pi \exp\{\sigma_0 V_1^h\} + (1-\pi)\exp\{\sigma_0 V_1^l\}} \tag{A.72}$$

where the value function has the following form, $V_1^s = u(c_1^s) + \beta u(c_2^s)$. Given the assumptions on the state space, it follows that:

$$V_1^h > \mathbb{E}_0\{V_1^s\} \quad \text{and} \quad V_1^l < \mathbb{E}_0\{V_1^s\} \tag{A.73}$$

Equation (A.72) and jointly imply that, if $\theta_0 > 0$, hence $\sigma_0 = -\frac{1}{\theta_0} < 0$, the following holds:

$$\exp \sigma_0 V_1^h < \mathbb{E}_0 \left\{ \exp \sigma_0 V_1^s | \right\} \quad \Rightarrow \quad m_1^h < 1 \tag{A.74}$$

$$\exp \sigma_0 V_1^l > \mathbb{E}_0 \left\{ \exp \sigma_0 V_1^s \right\} \quad \Rightarrow \quad m_1^l > 1 \tag{A.75}$$

Intuitively the above implies that agents assign an higher subjective probability (with respect to the objective) to the bad history and a lower probability to the good history. We can call this behaviour *pessimism*. Similarly if $\theta_0 < 0$, then $\sigma_0 = -\frac{1}{\theta_0} > 0$, we have that:

$$\exp \sigma^0 V_1^h > \mathbb{E}_0 \left\{ \exp \sigma_0 V_1^s \right\} \quad \Rightarrow \quad m_1^h > 1 \tag{A.76}$$

$$\exp \sigma_0 V_1^l < \mathbb{E}_0 \left\{ \exp \sigma_0 V_1^s \right\} \quad \Rightarrow \quad m_1^l < 1 \tag{A.77}$$

Note that in this second case agents assign an higher subjective probability to the good history and a lower probability to the bad history, depicting borrowers' *optimistic* behaviour. We shall now solve the equilibrium and derive the implied debt policy functions under the above beliefs' structure. We start by characterizing the equilibrium at time zero, given by the optimal decisions (b_0, c_0, q_0). We also compare the two solutions to the case with rational expectations. The debt policy function is best characterized by the following relation:

$$b_0 = -R[c_0 + q_0] \tag{A.78}$$

Next to characterize the time 0 policy function for consumption we rely on the Euler equation between period 0 and period 1:

$$u_c(c_0) = \pi m_1^h u_c(c_1^h) + (1 - \pi) m_1^l u_c(c_1^l) \tag{A.79}$$

We can reformulate the above equation in terms of the subjective weights of the ambiguity averse agent:

$$u_c(c_0) = \psi^h u_c(c_1^h) + (1 - \pi) \psi^l u_c(c_1^l) \tag{A.80}$$

where $\psi^h = \pi m_1^h$ and $\psi^l = (1 - \pi) m_1^l$. Given the model structure (incomplete financial markets, hence lack of insurance to equalize consumption), the events structure and the condition on the collateral constraint, we can conclude that:

$$c_1^h > c_1^l \quad \Rightarrow \quad u_c(c_1^h) < u_c(c_1^l) \tag{A.81}$$

Next, recall that in the *optimism* case beliefs imply:

$$\psi^h = \pi m_1^h > \pi \tag{A.82}$$

$$\psi^l = (1 - \pi) m_1^l < (1 - \pi) \tag{A.83}$$

This implies that agents assign a higher weight, with respect to the RE case, to the component $u_c(c_1^h)$. Hence, the marginal utility of consumption in $t = 0$ is lower (than under rational expectations) and the consumption is higher:

$$c_0^o > c_0^{RE} \tag{A.84}$$

where c_0^o indicates consumption under optimism behaviour, while c_0^{RE} indicates consumption under no ambiguity. Intuitively agents assign higher weight to good future states, hence they prefer to postpone consumption and to invest in the risky asset. This in turn will raise asset price, since the demand of asset has increased. As investment takes place through leverage, they will also leverage more. In the *pessimism* case the borrower assigns the following weights:

$$\psi^h = \pi m_1^h < \pi \tag{A.85}$$

$$\psi^l = (1-\pi) m_1^l > (1-\pi) \tag{A.86}$$

This implies:

$$c_0^u < c_0^{RE} \tag{A.87}$$

where c_0^u indicates consumption under pessimistic behaviour. In this case the agent expects more likely the bad state to take place in the future. The agent will then anticipate consumption and invest less in the risky asset. They will in turn leverage less. We can generalize this relation with the following condition:

$$c_0^o > c_0^{RE} > c_0^u \tag{A.88}$$

A.7 Intermediation Channel

In this section we provide micro-foundations for a delegated monitoring problem in which the collateral constraint emerges as resulting from an incentive-compatible debt contract enforced through a bank. The micro-foundations follows Bianchi and Mendoza (2015). Debt contract are signed by a bank that must enforce debtor incentives. Between periods borrowers can divert revenues for an amount $\tilde{d}$. At the end of the period the diversion is no longer possible and payment is enforced. Banks can monitor financial diversion due to special relationship lending abilities[7]. If the bank detects the diversion asset can be seized up to a percentage ϕ. As common in dynamic economies we assume that the contract is done under no memory, so that in the next periods borrowers can re-enter debt agreement even if they defaulted in the previous period. This assumption allows us to preserve the Markov structure of the contracting/intermediation problem.

We shall show that the collateral constraint can emerge as resulting from an incentive compatibility constraint imposed by the bank through the debt design. Specifically the collateral constraint can be derived as an implication of incentive-compatibility constraints on borrowers if limited enforcement prevents banks from redeploying more than a fraction ϕ of the value of the assets owned by a defaulting borrower. Define V^R and V^D respectively the value of repayment

[7]We assume zero monitoring costs for simplicity. Extending it to the case with positive monitoring costs is rather straightforward.

and default and define as V the continuation value.

If the borrower defaults the diverted resources enter his budget constraint and the recursive problem reads as follows (for notational convenience we skip the beliefs constraints for the purpose of this derivation):

$$
\begin{aligned}
V^D(b, x, S) = \max_{c, x', b'} \{ & u(c) + \beta \mathbb{E}_S + \\
& + \lambda \left[y + q(S)(x + \alpha y) + \tilde{d} + b - q(S)x' - c - \frac{b'}{R} \right] + \\
& + \mu \left[\phi q(S)x' + \frac{b'}{R} \right]
\end{aligned}
\tag{A.89}
$$

On the other side if the borrower repays his value function reads as:

$$
\begin{aligned}
V^D(b, x, S) = \max_{c, x', b'} \{ & u(c) + \beta \mathbb{E}_S + \\
& + \lambda \left[y + q(S)(x + \alpha y) + b - q(S)x' - c - \frac{b'}{R} \right] + \\
& + \mu \left[\phi q(S)x' + \frac{b'}{R} \right]
\end{aligned}
\tag{A.90}
$$

The comparison of the two easily shows that the households repay if and only if $\tilde{d}' < \phi q(S)x'$.

Appendix B

Appendix to Chapter 2

B.1 MS-VAR Data

Countries: Italy, Spain, Portugal

- *Private Interest rate*: Lending rate to non-financial corporations -over 5years maturity. Source: SDW European Central Bank. Database: MFI Interest Rate Statistics. Webpage: http://sdw.ecb.europa.eu
- *Sovereign Interest rate*: Long-term (10 years maturity) government bonds interest rates. Source: OECD. Database: Monthly Monetary and Financial Statistics (MEI). Webpage: http://stats.oecd.org
- *Risk-free Interest rate*: Yields on zero-coupon German government securities with a residual maturity of 6 months (Nelson-Siegel-Svensson method). Source: Deutsche Bundesbank. Webpage: https://www.bundesbank.de/Navigation/EN/Statistics/Time_series_databases/time_series_databases.html
- *Government Debt-to-GDP*: General Government Maastricht debt. Source: SDW European Central Bank. Database: Government Finance Statistics. Webpage: http://sdw.ecb.europa.eu
- *HICP Country weights*: Harmonized Indices of Consumer Prices weights for the euro area. Source: Eurostat. Webpage: http://ec.europa.eu/eurostat/data/database

B.2 MS-VAR Estimation

For the empirical section on MS-BVAR estimation we closely follow Sims, Waggoner and Zha (2008). A detailed description of the Bayesian inference is reported below.

B.2.1 The posterior

The notation: θ are the model's parameters; while $q = (q_{i,j}) \in \mathbb{R}^{h^2}$ are the regimes' transition probabilities; $Y_t = (y_1, .., y_t) \in (\mathbb{R}^n)^t$ are observed data, with n denoting the number of endogenous variables; $S_t = (s_0, .., s_t) \in H^{t+1}$ is the latent process, with $H \in 1, .., h$.

The log-likelihood, $p(Y_T|\theta, q)$, is combined with the prior density, $p(\theta, q)$ to obtain the posterior density, $p(\theta, q|Y_T) = p(\theta, q)p(Y_T|\theta, q)$.

The likelihood

Following Hamilton (1994), Sims and Zha (2006), ans Sims, Waggoner and Zha (2008), we employ a class of Markov-switching VAR with the following compact form:

$$y_t' A_0(s_t) = x_t' F(s_t) + \epsilon_t' \Sigma^{-1}(s_t)$$

with $x_t' = [y_{t-1}' ... y_{t-\rho}' \ 1]$ and $F(s_t) = [A_1(s_t) ... A_\rho(s_t) \ C(s_t)]'$. Let $a_j(k)$ be the j^{th} column of $A_0(k)$, $f_j(k)$ be the jth column of $F(k)$ and $\xi_j(k)$ be the jth diagonal element of $\Sigma(k)$. The conditional likelihood is as follows:

$$p(y_t|s_t, Y_{t-1}) = |A_0(s_t)| \prod_{j=1}^{n} |\xi_j(s_t)| exp\left(-\frac{\xi^2(s_t)}{2}(y_t' a_j(s_t) - x_t' f_j(s_t))^2\right)$$

Formalizing the model's identifying restrictions in the following form: $a_j(s_t) = U_j b_j(k)$ and $f_j(s_t) = V_j g_j - W_j U_j b_j(k)$, where U_j and V_j are matrices with orthonormal columns, we re-write it in terms of the free parameters of $A(s_t)$ and $F(s_t)$. Then,

$$|A_0(s_t)| \prod_{j=1}^{n} |\xi_j(s_t)| exp\left(-\frac{\xi^2(s_t)}{2}((y_t' + x_t' W_j) U_j b_j(s_t) - x_t' V_j g_j(s_t))^2\right)$$

The log-likelihood function is given by:

$$p(Y_T|\theta, q) = \sum_{t}^{T} ln\left\{\sum_{s_t=1}^{h} p(y_t|s_t, Y_{t-1}) Pr[s_t|Y_{t-1}]\right\}$$

The overall likelihood function is computed using the modified Kalman filter described in Kim and Nelson (1999). It is, therefore, obtained by integrating over unobserved states the conditional likelihood at each time t and by recursively multiplying these conditional likelihood functions forward.

The prior

Following Sims and Zha (1998), we exploit the idea of a Litterman's random-walk prior on the BVAR coefficients. Dummy observations are introduced as a component of the prior in order to allow for unit roots and co-integration relationships. Priors are assumed to be symmetric across regimes. Applying the model's identifying restrictions, the overall prior, $p(\theta, q)$ is given by:

$$p(b_j(k)) = \mathcal{N}(b_j(k)|0, \Sigma_{b_j})$$
$$p(g_j(k)) = \mathcal{N}(g_j(k)|0, \Sigma_{g_j})$$
$$p(\xi_j^2(k)) = \mathcal{G}(\xi_j^2(k)|\bar{\alpha_j}, \bar{\beta_j})$$
$$p(q_j) = \mathcal{D}(q_{i,j}|\alpha_{i,j}, \alpha_{k,j})$$

where k is the generic regime and Σ_{b_j}, Σ_{g_j} denote the prior covariance matrices for the BVAR coefficients. The hyperparameters controlling the tightness of the prior are set to the standard values for monthly data, as suggested by Sims and Zha (2006): $\mu_1 = 0.57$, $\mu_2 = 0.13$, $\mu_3 = 0.1$, $\mu_4 = 1.2$, $\mu_5 = 10$, $\mu_6 = 10$. A Gamma prior is, instead, applied on the structural shocks' variances, with $\bar{\alpha_j}$ and $\bar{\beta_j}$ both set to 1. Finally, the prior of the transition matrix takes a Dirichlet form, as suggested by **?**, with $\alpha_{i,j} = 1$ and $\alpha_{k,j} = 5.7$. The latter implies a prior belief that the average duration of staying in the same regime is 21 months.

B.2.2 MCMC simulation: Gibbs Sampling

Following Sims, Waggoner and Zha (2008), a MCMC simulation method is used to approximate the joint posterior density, $p(\theta, q, S_T|Y_T)$. When working with models whose posterior distribution is very complicated in shape it is very important to initialize the MCMC simulation at the peak of the posterior density. The latter is obtained with the blockwise optimization algorithm (BFGS algorithm conditional on blocks) developed by Sims, Waggoner and Zha (2008).

We follow them and use the Gibbs sampler to obtain the joint posterior distribution. The Gibbs sampler involves sampling alternatively from the following conditional posterior distributions: $p(S_T|Y_T, \theta, q)$, $p(q|Y_T, S_T, \theta)$ and $p(\theta|Y_T, q, S_T)$. The simulation performs 500,000 draws, with a 10% burn-in sample.

Conditional posterior density: $p(\theta|Y_T, q, S_T)$: In order to simulate draws of $\theta \in \{b_j(k), g_j(k), \xi_j^2(k)\}$ from $p(\theta|Y_T, q, S_T)$, we start by using the Metropolis-Hastings (MH) algorithm to sample from:

$$p(b_j(k)|y_t, S_t, b_t(k)) = \\ exp\left(-\frac{1}{2}b_j'(k)\Sigma_{b_j}^{-1}b_j(k)\right) \\ x\prod_j\left[|A_0(k)|exp\left(-\frac{\xi^2(s_t)}{2}(y_t'a_j(k) - x_t'f_j(k))^2\right)\right]$$

Then, a multivariate Normal distribution is employed to draw $g_j(k)$:

$$p(g_j(k)|y_t, S_t) = \mathcal{N}(g_j(k)|\mu_{g_j(k)}^{-}, \Sigma_{g_j(k)}^{-})$$

Finally, shocks' variances ξ_j^2 are simulated from a gamma distribution:

$$p(\xi_j^2(k)|y_t, S_t) = \mathcal{G}(\xi_j^2(k)|\alpha_j\bar{(}k), \beta_j\bar{(}k))$$

Conditional posterior density: $p(S_T|Y_T,\theta,q)$: A multi-move Gibbs-sampling is employed to simulate $S_t, t=1,2...T$. First, we draw s_t according to:

$$p(s_t|y_t,S_t)=\sum_{s_{t+1}\in H}p(s_t|Y_T,\theta,q,s_{t+1})p(s_{t+1}|Y_T,\theta,q)$$

where

$$p(s_t|Y_T,\theta,q,s_{t+1})=\frac{q_{s_{t+1},s_t}p(s_{t+1|Y_t,\theta,q})}{p(s_{t+1}|Y_t,\theta,q)}$$

Then, in order to generate s_t, a uniform distribution is used: if the generated number is less than or equal to the calculated value of $p(s_t|y_t,S_t)$, we set $s_t=1$; otherwise $s_t=0$.

Conditional posterior density: $p(q|Y_T,S_T,\theta)$: The conditional posterior distribution of q_j is:

$$p(q_j|Y_t,S_t)=\prod_{i=1}^{h}(q_{i,j})^{n_{i,j}+\beta_{i,j}-1}$$

where $n_{i,j}$ is the number of transitions from s_{t-1} to $s_t=i$.

B.3 MS-VAR. Some results

Table B.1: Relative structural shocks' standard deviations, by regime $(\xi^{vol})^{-1}$)

Regimes($\xi_t^{vol} = 1, 0$)	Private Spread	Sovereign Spread	Debt-to-GDP
High Volatility (Hvol)	1	1	1
Low Volatility (Lvol)	0.37925	0.38502	1.00940

Table B.2: Transition matrix

	Volatility Regimes		Vulnerability Regimes		Synchronization Regimes	
Regimes	High	Low	High	Low	High	Low
High	0.78418	0.019721				
Low	0.21582	0.98028				
High			0.89567	0.05312		
Low			0.10433	0.94688		
High					0.94204	0.02341
Low					0.057962	0.97659
Duration	4.6335	50.7099	9.5850	18.8253	17.2533	42.7168

Table B.3: Conditional moments, by regime

	Conditional means			Share
Regimes	Private Spread	Sovereign Spread	Debt-to-GDP	%
H volatility	3.685	5.221	95.492	4.487
L volatility	2.484	2.772	97.67	95.513
H vulnerability	3.296	4.689	104.62	29.487
L vulnerability	2.221	2.125	94.765	70.513
H synchronization	2.623	2.897	88.146	27.564
L synchronization	2.506	2.876	101.42	72.436
Sample	2.5008	2.822	96.261	100

Appendix C

Appendix to Chapter 3

C.1 Set of Model Conditions

The model is characterized by the following set of equation. Variables defined with the symbol $\hat{X}_t$ are detrended variables (normalized for the level of aggregate TFP).

$$1 = \beta \mathbb{E}_t \left\{ e^{d_{t+1}-d_t} \frac{\hat{c}_t - \Phi \hat{c}_{t-1} e^{-(\gamma+a_t)}}{\hat{c}_{t+1} - \Phi \hat{c}_t e^{-(\gamma+a_{t+1})}} e^{-(\gamma+a_{t+1})} \frac{R^m_{t+1}}{\Pi_{t+1}} \right\} \tag{C.1}$$

$$R^m_t = \frac{1+\rho P^m_t}{P^m_{t-1}}, \quad R_t = \mathbb{E}_t [R^m_{t+1}] \tag{C.2}$$

$$\varphi_t = \left[\hat{c}_t - \Phi \hat{c}_{t-1} e^{-(\gamma+a_t)} \right] \frac{h^\alpha_t}{1-\alpha} \tag{C.3}$$

$$\hat{y}_t = h^{1-\alpha}_t \tag{C.4}$$

$$\frac{1}{\nu_t}\Big(1 - \varphi_t(1-\alpha)\Big) = 1 - \psi \left(\Pi_t - \frac{\Pi^\iota_{t-1}}{\Pi^{\iota-1}} \right) \Pi_t - \frac{\psi}{2} \left(\Pi_t - \frac{\Pi^\iota_{t-1}}{\Pi^{\iota-1}} \right)^2 \frac{\nu_t - 1}{\nu_t} \tag{C.5}$$

$$+\beta \mathbb{E}_t \left\{ \frac{\hat{c}_t - \Phi \hat{c}_{t-1} e^{-(\gamma+a_t)}}{\hat{c}_{t+1} - \Phi \hat{c}_t e^{-(\gamma+a_{t+1})}} \psi \left(\Pi_{t+1} - \frac{\Pi^\iota_t}{\Pi^{\iota-1}} \right) \Pi_{t+1} \frac{\hat{y}_{t+1}}{\hat{y}_t} \right\} \tag{C.6}$$

$$\frac{\kappa}{1+\iota\beta} \log \left(\frac{1-\nu}{1-\nu_t} \right) = \mu_t \tag{C.7}$$

$$b_t = b_{t-1} \frac{R^m_t}{\Pi_t} \frac{\hat{y}_{t-1}}{\hat{y}_t} e^{-(\gamma+a_t)} - \tau_t + e_t \tag{C.8}$$

$$\frac{\chi_t}{\chi} = \left(\frac{\chi_{t-1}}{\chi} \right)^{\rho_\chi} \left(\frac{\hat{y}_t}{\hat{y}^n_t} \right)^{\iota_y (1-\rho_\chi)} e^{\sigma_\chi \varepsilon^\chi_t} \tag{C.9}$$

$$\chi_t = \left(1 - \frac{1}{g_t} \right) e^{-1}_t \tag{C.10}$$

$$e_t = e^S_t + e^L_t \tag{C.11}$$

$$e^L_t = \rho_{e^L} e^L_{t-1} + \sigma_{e^L} \varepsilon^{e^L}_t \tag{C.12}$$

$$e^S_t = \rho_{e^S} e^S_{t-1} + (1-\rho_{e^S}) \Big[e^S + \phi_y \Big(\hat{y}_t - \hat{y}^n_t \Big) \Big] + \sigma_{e^S} \varepsilon^{e^S}_t \tag{C.13}$$

$$\tau_t = \rho_\tau \tau_{t-1} + (1-\rho_\tau)\left[\tau + \delta_b(b_{t-1} - b) + \delta_e(e_t - e) + \delta_y\left(\hat{y}_t - \hat{y}_t^n\right)\right] + \sigma_\tau \varepsilon_t^\tau \tag{C.14}$$

$$g_t \equiv \frac{1}{1 - \frac{\hat{G}_t}{\hat{y}_t}}; \qquad 1 = \frac{\hat{c}_t}{\hat{y}_t} + \frac{g_t - 1}{g_t} \tag{C.15}$$

$$\hat{y}_t^n = \left[\frac{1-\nu_t}{\frac{\hat{y}_t^n}{g_t} - \Phi \frac{\hat{y}_{t-1}^n}{g_{t-1}} e^{-(\gamma + a_t)}}\right]^{\frac{1-\alpha}{\alpha}} \tag{C.16}$$

C.2 Steady State System

The steady state of the model is characterized by the following set of equations:

$$R^m = \frac{\Pi}{\beta} e^\gamma \tag{C.17}$$

$$P^m = \frac{1}{R^m - \rho} \tag{C.18}$$

$$R = R^m \tag{C.19}$$

$$\varphi = \frac{1-\nu}{1-\alpha} \tag{C.20}$$

$$e = \left(1 - \frac{1}{\beta}\right) b^* + \tau^* \tag{C.21}$$

$$e^S = e \tag{C.22}$$

$$\chi = \left(1 - \frac{1}{g^*}\right) e^{-1} \tag{C.23}$$

$$h = \varphi(1-\alpha)\frac{g^*}{1 - \Phi e^{-\gamma}} \tag{C.24}$$

$$y = h^{1-\alpha} \tag{C.25}$$

$$c = \frac{y}{g^*} \tag{C.26}$$

$$y^n = y \tag{C.27}$$

C.3 Solution Methods

The Markov-switching DSGE model is solved using the perturbation method of Foerster et al. (2016). They develop an iterative procedure that approximate the model's solution by guessing a set of approximations under each regime; given a guess, each regime's approximation follows from standard perturbation techniques, and the iterative algorithm stops when obtained approximations equal the guesses. This perturbation approach has two major advantages. First, it provides a flexible environment for models, like ours, in which switching dynamics affect the steady state of the economy. This is a feature that perturbation handles easily. In addition, perturbation allows for second and higher-order approximations, which improve, on one hand, the solution accuracy and, on the other hand, the ability to capture the role of agents' beliefs over regimes.

In order to technically describe the solution method, it's convenient to stack our variables into a group of exogenous and endogenous predetermined variables, $\mathbf{x}_t \in \mathbb{R}^{n_x}$, and a group of non-predetermined (control) variables, $\mathbf{y}_t \in \mathbb{R}^{n_y}$. Then, we define the vector of structural shocks as $\epsilon_t \in \mathbb{R}^{n_\epsilon}$ and the switching parameters' vector as $\theta(\xi_t^{sp}) \in \mathbb{R}^{n_\theta}$. Given the vector of state variables ($\mathbf{x}_{t-1}$, ϵ_t, ξ_t^{sp}), the equilibrium conditions for our model have the following general form:

$$\mathbb{E}_t \mathbf{f}\Big(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{x}_t, \mathbf{x}_{t-1}, \chi\epsilon_{t+1}, \epsilon_t, \theta(\xi_{t+1}^{sp}, \chi), \theta(\xi_t^{sp}, \chi)\Big) = 0_{n_y+n_y}$$

where $\mathbf{f}$ is a nonlinear function. Then, the algorithm works as an extension of conventional perturbation methods (**?**, **?**), where not only ϵ_{t+1} is perturbed, but also the switching parameters, $\theta(\xi_{t+1}^{sp})$, $\theta(\xi_t^{sp})$. Since in our model the steady state is affected by the policy regime in place, the perturbation function for $\theta(\xi_t^{sp})$ is: $\theta(k, \chi) = \chi\theta(k) + (1-\chi)\bar{\theta}$, where $\chi \in \mathbb{R}$ is the perturbation parameter, k indicates a generic regime and $\bar{\theta} = [\theta(1)...\theta(n_s)]\bar{p}$ is the ergodic mean of $\theta(\xi_t^{sp})$.

Stacking the regime-dependent solutions for $\mathbf{y}_t$ and $\mathbf{x}_t$, the algorithm assumes the following functional forms for $\mathbf{Y}_t = y_t(\mathbf{e}_{s_t}^T \otimes \mathbf{I}_{n_y})^{-1}$ and $\mathbf{X}_t = x_t(\mathbf{e}_{s_t}^T \otimes \mathbf{I}_{n_x})^{-1}$:

$$\mathbf{Y}_t = \mathbf{G}(\mathbf{x}_{t-1}, \epsilon_t, \chi) = \begin{bmatrix} g_{\xi_t^{sp}=1}(\mathbf{x}_{t-1}, \epsilon_t, \chi) \\ \vdots \\ g_{\xi_t^{sp}=n_s}(\mathbf{x}_{t-1}, \epsilon_t, \chi) \end{bmatrix}$$

$$\mathbf{X}_t = \mathbf{H}(\mathbf{x}_{t-1}, \epsilon_t, \chi) = \begin{bmatrix} h_{\xi_t^{sp}=1}(\mathbf{x}_{t-1}, \epsilon_t, \chi) \\ \vdots \\ h_{\xi_t^{sp}=n_s}(\mathbf{x}_{t-1}, \epsilon_t, \chi) \end{bmatrix}$$

where $\mathbf{g}_{\xi_t^{sp}} : \mathbb{R}^{n_x+n_\epsilon+1} \to \mathbb{R}^{n_y}$ and $\mathbf{h}_{\xi_t^{sp}} : \mathbb{R}^{n_x+n_\epsilon+1} \to \mathbb{R}^{n_x}$ are continuously differentiable regime-dependent solutions. Second-order perturbation around the point $(\mathbf{x}_{ss}, \mathbf{0}_{ss}, 0)$ is represented by:

$$\mathbf{G}(z_t) \approx \mathbf{Y}_{ss} + D\mathbf{G}(\mathbf{z}_{ss})(\mathbf{z}_t - \mathbf{z}_{ss}) + \frac{1}{2}\sum_{l_1}^{n_z}\sum_{l_2}^{n_z} D_{l_2}D_{l_1}\mathbf{G}(\mathbf{z}_{ss})(z_{t,l_1} - z_{ss,l_1})(z_{t,l_2} - z_{ss,l_2})$$

$$\mathbf{H}(z_t) \approx \mathbf{X}_{ss} + D\mathbf{H}(\mathbf{z}_{ss})(\mathbf{z}_t - \mathbf{z}_{ss}) + \frac{1}{2}\sum_{l_1}^{n_z}\sum_{l_2}^{n_z} D_{l_2}D_{l_1}\mathbf{H}(\mathbf{z}_{ss})(z_{t,l_1} - z_{ss,l_1})(z_{t,l_2} - z_{ss,l_2})$$

where $\mathbf{z}_t = [\mathbf{x}_{t-1}, \epsilon_t, \chi]$, $\mathbf{z}_{ss} = [\mathbf{x}_{ss}, 0_{n_\epsilon}, 0]$, and $z_{t,l}$ and $z_{ss,l}$ are the l^{th} components of $\mathbf{z}_t$ and $\mathbf{z}_{ss}$.

C.4 Observation Equations

The model's law of motion for the variables S_t is combined with the following system of observation equations:

$$X_t = D + ZS_T \tag{C.28}$$

where $X_t = [\Delta\log(Y_t^{\text{obs}}),\ \Pi_t^{\text{obs}},\ R_t^{\text{obs}},\ b_t^{\text{obs}},\ \log(g_t^{\text{obs}}),\ t_t^{\text{obs}},\ e_t^{\text{obs}}]'$ containes the observables, D the vector of constants and Z_t provides the mapping between the model's solution and the data. The system of equations reads as follows:

$$
\begin{aligned}
\Delta\log(Y_t^{\text{obs}}) &= \gamma + a_t + y_t - y_{t-1} \\
\Pi_t^{\text{obs}} &= \log\frac{P_t}{P_{t-1}} = \log(\Pi) + \Pi_t \\
R_t^{\text{obs}} &= 4\left((\Pi - 1) + \frac{\gamma}{\beta} - 1\right) + 4r_t \\
b_t^{\text{obs}} &= b + b_t \\
\log(g_t^{\text{obs}}) &= \log(\mathrm{g}) + g_t \\
t_t^{\text{obs}} &= t + t_t \\
e_t^{\text{obs}} &= e + e_t
\end{aligned}
$$

where the percentage deviation of the detrended output from its steady state is $y_t = \log\left(\frac{Y_t^{\text{obs}}}{A_t}\frac{Y}{A}\right)$ and the percentage deviations for inflation, government purchases and interest rates are $\Pi_t = \log\left(\frac{\Pi_t^{\text{obs}}}{\Pi}\right)$, $g_t = \log\left(\frac{g_t^{\text{obs}}}{g}\right)$ with $g_t^{\text{obs}} = \frac{1}{1-G_t^{\text{obs}}/Y_t^{\text{obs}}}$, and $R_t = \log\left(\frac{R_t^{\text{obs}}}{R}\right)$. For the variables normalized with respect to GDP the linear deviations are $b_t = b_t^{\text{obs}} - b$, $t_t = t_t^{\text{obs}} - t$ and $e_t = e_t^{\text{obs}} - e$.

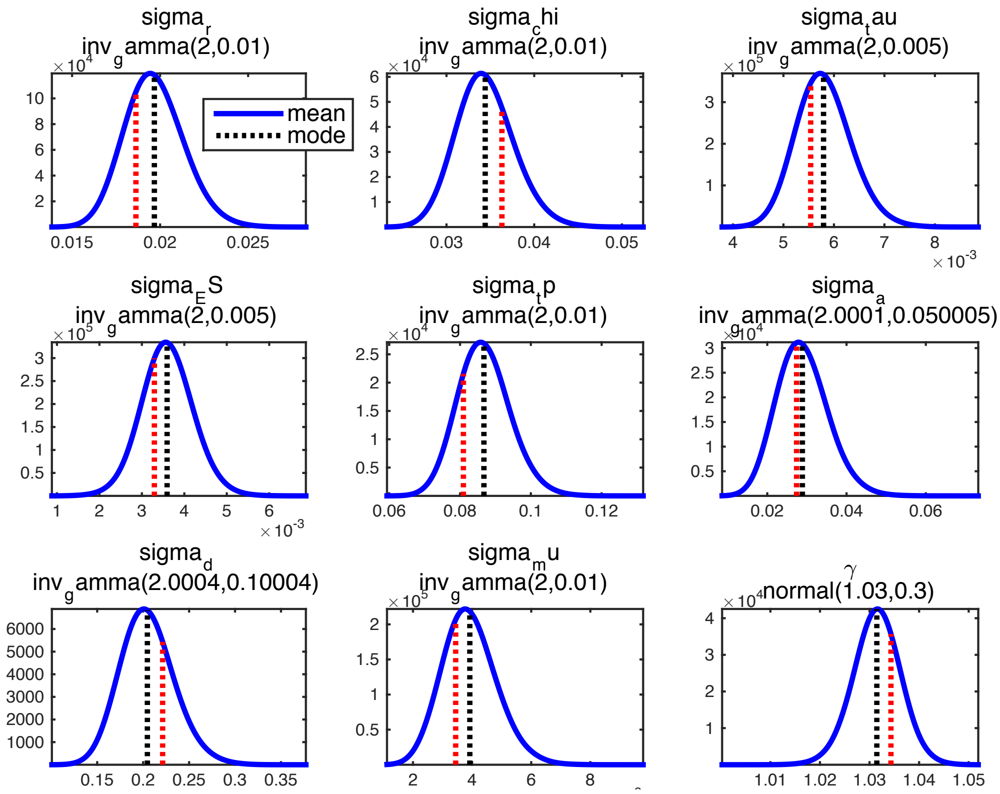
C.5 Data

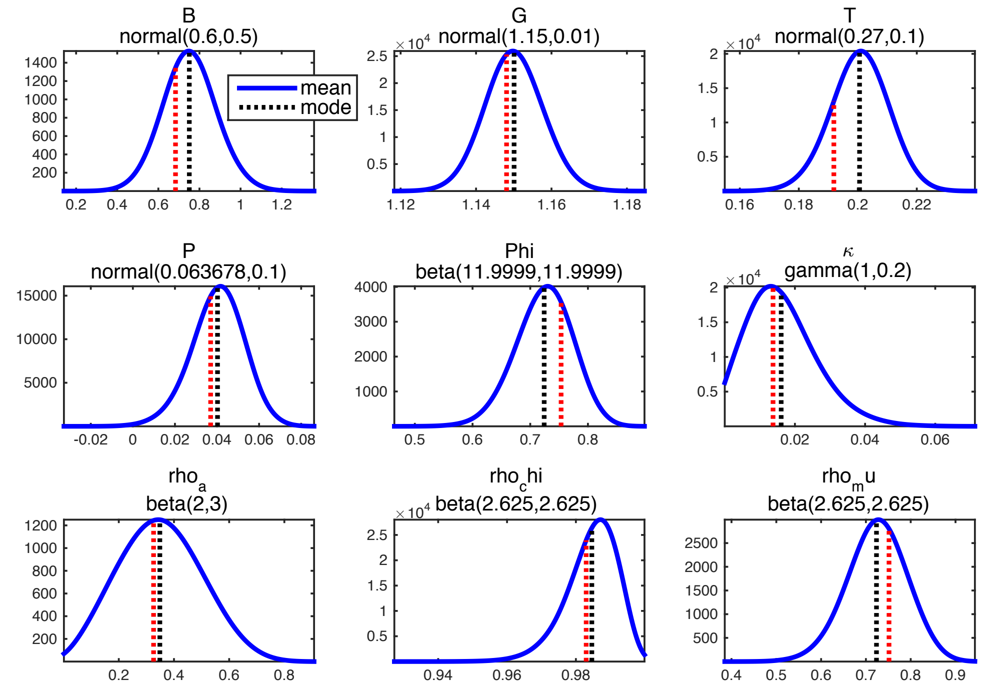
Country: France

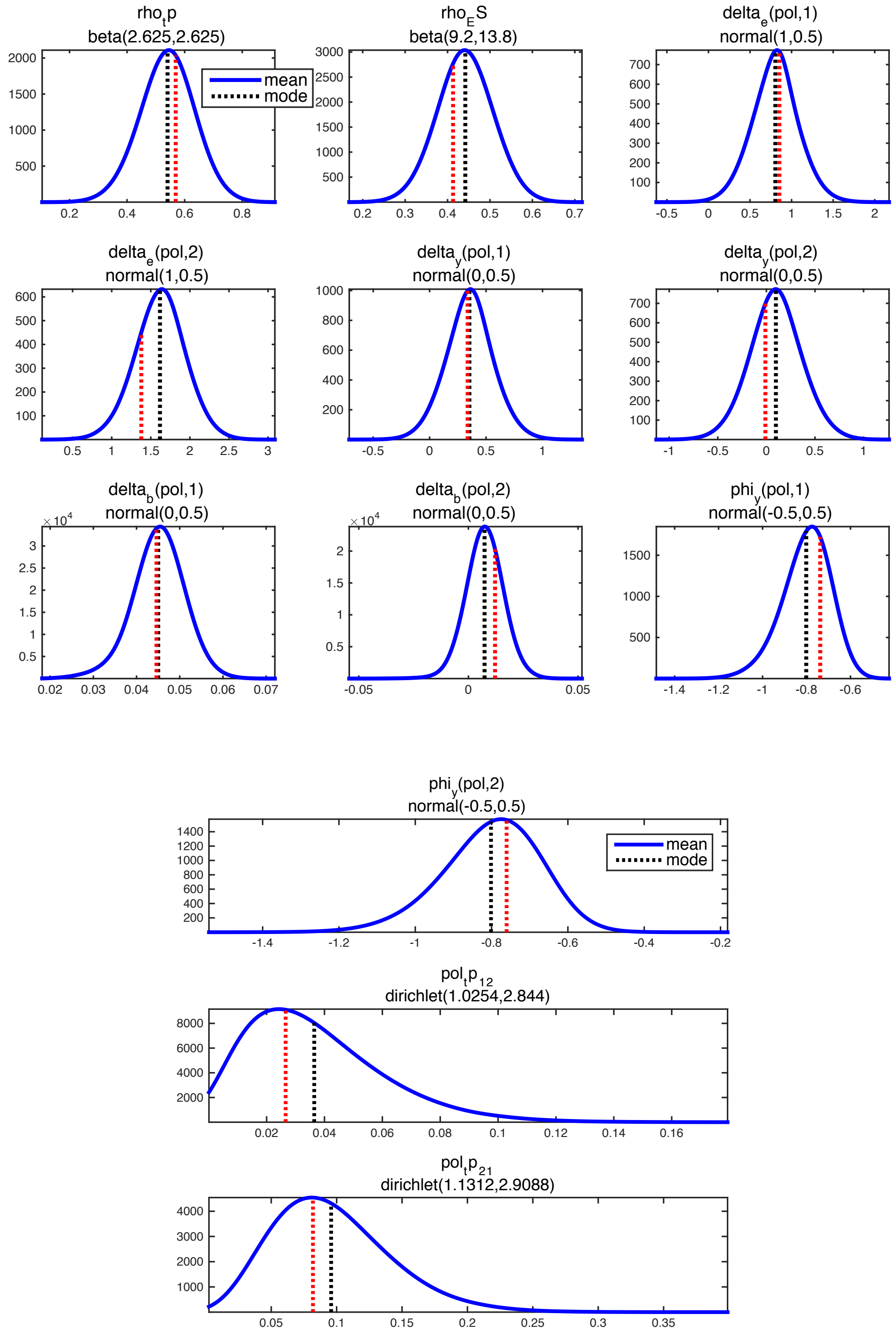
- Y_t^{obs}: Real GDP. Source: INSEE. Database: National accounts. Webpage: https://insee.fr/en/information/2868584#titre-bloc-1
- Π_t^{obs}: GDP deflator inflation. Source: INSEE. Database: National accounts.
- R_t^{obs}: short-term nominal interest rate. Source: OECD. Database: Main Economic Indicators. Webpage: http://stats.oecd.org
- b_t^{obs}: Gov. debt-to-GDP ratio, Maastricht debt. Source: INSEE, IMF Historical Public Debt Database. Webpage: https://www.imf.org/en/Publications/WP/Issues/2016/12/31/A-Historical-Public-Debt-Database-24332
- G_t^{obs}: Government purchases=consumption expenditure+gross government investment+net purchases of non-produced assets-consumption of fixed capital. Source: INSEE. Database: National accounts.
- t_t^{obs}: Tax revenues-to-GDP ratio, where Taxes=current receipts-current tranfer receipts. Source: INSEE. Database: National accounts.

- e_t^{obs}: Government Expenditure-to-GDP, where Gov. Expenditure=Gov.Purchases+Transfers, and Transfers=net current transfer payments+subsidies+net capital transfers. Source: INSEE. Database: National accounts.

C.6 MCMC Simulation: Posterior densities






rho_t p
beta(2.625,2.625)
mean
mode
rho_E S
beta(9.2,13.8)
delta_e(pol,1)
normal(1,0.5)
delta_e(pol,2)
normal(1,0.5)
delta_y(pol,1)
normal(0,0.5)
delta_y(pol,2)
normal(0,0.5)
delta_b(pol,1)
normal(0,0.5)
delta_b(pol,2)
normal(0,0.5)
phi_y(pol,1)
normal(-0.5,0.5)
phi_y(pol,2)
normal(-0.5,0.5)
mean
mode
pol_t p_12
dirichlet(1.0254,2.844)
pol_t p_21
dirichlet(1.1312,2.9088)