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A theory of X and Z multiquark resonances

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ABSTRACT

We introduce the hypothesis that diquarks and antidiquarks in tetraquarks are separated by a potential barrier. We show that this notion can answer satisfactorily long standing questions challenging the diquark–antidiquark model of exotic resonances. The tetraquark description of X and Z resonances is shown to be compatible with present limits on the non-observation of charged partners $X^{\pm}$, of the $X(3872)$ and the absence of a hyperfine splitting between two different neutral states. In the same picture, Z_c and Z_b particles are expected to form complete isospin triplets plus singlets. It is also explained why the decay rate into final states including quarkonia are suppressed with respect to those having open charm/beauty states.

1. Introduction

The observed lowest lying X and Z states are found very close or slightly above the meson–meson thresholds with the corresponding quantum numbers. The $X(3872)$, $Z_c(3900)$, $Z_c'(4020)$, $Z_b(10610)$, $Z_b'(10650)$ axial resonances, have central mass values distant by

$$\delta = 0 \pm 0.195,\ +7.8,\ +6.7,\ +2.7,\ +1.8 \text{ MeV} \tag{1}$$

from the closer meson–meson thresholds with 1^+ quantum numbers

$$\bar{D}^0 D^{*0},\ \bar{D}^0 D^{*+},\ \bar{D}^{*0} D^{*+},\ \bar{B}^0 B^{*+},\ \bar{B}^{*0} B^{*+} \tag{2}$$

Some authors believe that, being the δs fairly small, different parametrizations of the lineshapes, combined with updated data analyzes, might eventually show that X, Z states have masses below the aforementioned thresholds (see reviews [1–6] and [7]). In the latter case the hadron molecule interpretation would become tenable, at least from the energetic point of view.

With positive and finite δ values, a reasonable alternative description is in terms of compact tetraquarks, as in the diquark–antidiquark model [8,9]. The model can describe all observed exotic hadrons in a unique scheme, including cases like $Z(4430)$ [10,11], the $J/\psi\,\phi$ resonances [12] and the heavier, positive parity, pentaquark $\mathcal{P}(4570)$ [13,14], which are problematic to fit in the molecular picture. We have to underscore that the existence of exotic charged charmed resonances with decays into $\psi(nS)\,\pi^{\pm}, \rho^{\pm}\cdots$ was a prediction of the diquark–antidiquark model [8] and an unwanted/unnecessary feature for molecular models.

Four quarks produced in high-energy hadron collisions, or in B meson decays, have different alternatives for clustering in color neutral states namely, the diquark–antidiquark alternative

$$\Psi_{\mathcal{D}} = (\epsilon_{ijk}\, Q^{\,j} q^k)\,(\epsilon^{imn}\, \bar{Q}_m \bar{q}'_n) = [Q\,q][\bar{Q}\,\bar{q}'] \tag{3}$$

or the meson–meson alternatives

$$\Psi_{\mathcal{M}} = (Q^{\,i}\bar{q}_i)\,(\bar{Q}_k q'^k) \quad \text{or} \quad (Q^{\,i}\bar{Q}_i)\,(\bar{q}_k q'^k) \tag{4}$$

The $\Psi_{\mathcal{M}}$ component is supposed to be in the continuum spectrum of a shallow potential with no bound states — a residual strong interaction tail at large distances. The $\Psi_{\mathcal{D}}$ component is instead a stationary state in the color binding potential.

The mass of the tetraquark can be slightly higher than the sum of the masses of the two open charm singlets, because strong attraction in color singlet channels is stronger than in color anti-triplet channels. Thus, it is not surprising that the observed tetraquarks appear near to the corresponding meson thresholds, albeit being heavier.

If the recoil energy E_0 in the center of mass of the color singlets in $\Psi_{\mathcal{M}}$ is high enough [15], a pair of free mesons will be

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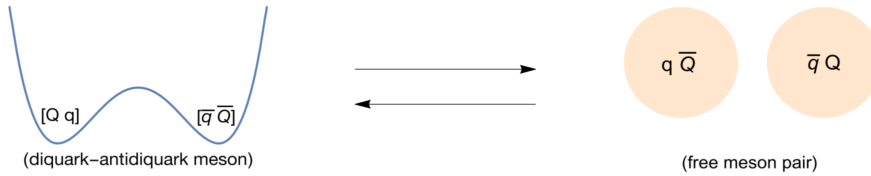

Fig. 1. Tunneling of *light* quarks rearranges the diquark–antidiquark state $\Psi_{\mathcal{D}}$ (left panel) into two color singlets $\Psi_{\mathcal{M}}$ (right panel). The opposite process might proceed if the recoil energy between the color singlets is low enough to keep them in a small volume of configuration space.

detected. If E_0 is sufficiently low (a rare circumstance in prompt production from high energy hadron collisions) the color singlets might rescatter forming a tetraquark state that decays back into a meson–meson pair [2]. The diquark–antidiquark tetraquark can as well be produced promptly.

The fact that E_0 tends to be large in high-p_T events in hadronic collisions at the LHC is compatible with the non-observation of loosely bound molecules, like deuteron, produced promptly in such kinematic conditions [16]. On the other hand, the large prompt production cross section of $X(3872)$ at the LHC appears to be in contradiction with a loosely bound molecule interpretation [15,17,18].

Following an argument of Selem and Wilczek [19], we make the hypothesis that a tetraquark can plausibly be represented by two diquarks in a double well potential separated by a barrier, as in Fig. 1.

The argument can be summarized as follows. At large distances, diquarks see each other as QCD point charges and QCD confining forces are the same as in a quark–antiquark system. At shorter distances, however, forces among different parts that tend to destroy the diquark, *e.g.* attraction between quarks and antiquarks, reduce the binding energy of the diquark. These effects increase at decreasing distance and produce a repulsion among diquark and antidiquark [19], *i.e.* a component in the potential increasing at decreasing distance. If this effect wins against the decrease due to the color attraction, it will produce the barrier depicted in the figure.

It is an hypothesis that we cannot prove, at the moment. However, it has some phenomenological support in the spectrum of $X(3872)$, $Z_c(3900)$, $Z_c'(4020)$. Mass ordering indicates [9] that *i*) spin–spin interactions between constituents located one in the diquark and the other in the antidiquark are definitely smaller than one would guess from the same interactions within mesons and *ii*) the spin–spin interaction inside the diquark is about four times larger than the same interaction in the diquarks inside charmed baryon states. Thus the overlap probability $|\psi_{q\bar{q}'}(0)|^2$ of a quark and an antiquark is suppressed and that of a quark pair $|\psi_{cq}(0)|^2$ is enhanced with respect to what happens in mesons and baryons respectively.

Fig. 1, taken literally, implies the existence of two length scales: the diquark radius, R_{Qq} and the tetraquark radius, R_{4q}, which we assume to be well separated

$$\lambda = R_{4q}/R_{Qq} \geq 3 \tag{5}$$

In principle the diquark radius R_{Qq} can be different if the diquark is in a tetraquark or in a baryon. We will distinguish the latter naming it R_{Qq}^{baryon}.

Using established Constituent Quark Model techniques [20], see also [8,21], we show that this picture can give a novel answer to the present lack of observation, in $B^{0,+}$ decays, of a second neutral state in the vicinity of the $X(3872)$ and of the associated charged state. We find that: *i*) the two neutral states are quasi-degenerate within the mass resolution with which the $X(3872)$ is observed and *ii*) the associated charged state is produced much below the rate expected for a pure isospin $I = 1$ $X(3872)$ multiplet, complying with present limits. For the large charm quark mass, the two-lengths picture leads, in addition, to *iii*) an exponentially suppressed amplitude for $X(3872) \to J/\psi\ \pi\pi$, with respect to $\bar{D}^0 D^{*0}$, qualitatively explaining the large branching fraction of the latter to the former mode, in spite of its much smaller phase space, as observed in the phenomenology [22]. This behavior, as shown in [22], is quite evidently shared by $Z_{c,b}^{(\prime)}$ resonances — the $Z(4430)$, being most likely a radial excitation, may have slightly different features.

An increase of the experimental resolution and statistics are crucial to support or disprove our picture, by searching for a double structure inside the $X(3872)$ line and for $X^{\pm}$ in the decays of B mesons at lower branching fractions than at present.

The $X^{\pm}$ charged resonances could also be produced prompt in proton–proton collisions at the LHC. For the time being the prompt production of X^0 is well studied but no signs neither of $X^{\pm}$ nor of $Z_{c,b}^{\pm}$ are found. The experimental situation of Z_c particles in B decays is also unclear.

2. Isospin breaking in tetraquarks

We recall the definitions

$$X_u = \frac{1}{\sqrt{2}}\Big([cu]_0[\bar{c}\bar{u}]_1 + [cu]_1[\bar{c}\bar{u}]_0\Big) \tag{6}$$

$$X_d = \frac{1}{\sqrt{2}}\Big([cd]_0[\bar{c}\bar{d}]_1 + [cd]_1[\bar{c}\bar{d}]_0\Big) \tag{7}$$

in brackets (anti)diquarks with the indicated flavors, in color ($\mathbf{3}$) $\bar{\mathbf{3}}$ and total spin indicated by the subscripts.

In [8,23], we considered the mass difference $\Delta M = M(X_u) - M(X_d)$ to be determined by the *down-up* quark mass difference

$$\Delta M = 2(m_u - m_d) \approx -6\ \text{MeV} \tag{8}$$

A more refined analysis [24,25] introduces the effect of Coulomb and hyperfine electromagnetic interactions and of the $u - d$ mass difference in the strong hyperfine interaction. These effects are parametrized, for baryons, with three phenomenological parameters a, κ, γ defined according to[1]

<u>Electrostatic</u>

$$H_{ij} = Q_i\, Q_j\, a \times \left(\frac{R_{Qq}^{\text{baryon}}}{R_{ij}}\right) \tag{9}$$

<u>Electromagnetic hyperfine</u>

$$H_{q,c} = (Q_u - Q_d)Q_c \frac{\alpha}{\overline{m} m_c} \boldsymbol{S}_q \cdot \boldsymbol{S}_c\, |\psi(0)|^2 = \\ = 2\gamma\,(Q_u - Q_d)Q_c \frac{\overline{m}}{m_c} \frac{|\psi(0)|^2}{|\psi_B(0)|^2}\, 2\boldsymbol{S}_q \cdot \boldsymbol{S}_c \tag{10}$$

[1] In the following equation write $m_u = \overline{m} + (m_u - m_d)/2$ and $m_d = \overline{m} - (m_u - m_d)/2$ where $\overline{m} = (m_u + m_d)/2$. Neglect $(m_u - m_d)^2/4$. The coupling $g_s^2/\overline{m} m_c$ has to be rescaled by $\kappa_{cq}/\kappa_{cq}^{\text{Baryon}}$ (where $\kappa_{cq} \equiv \kappa_{cq}^{\text{diquark}}$) and it is used $\kappa_{cq}^{\text{Baryon}} = g_s^2/\overline{m} m_c\, |\psi_B(0)|^2$.

Table 1
Numerics of mass differences, in MeV, vs λ in Eq. (5).

–	$\lambda = 1$	$\lambda = 3$
$M(X_u) - M(X_d)$	-6.1 ± 0.1	-1.2 ± 0.3
$M(X_u) - M(X^+)$	-5.31 ± 0.05	-1.34 ± 0.12

$$\Delta H_{q,c} = \underbrace{\frac{g_s^2}{m_c}\left(\frac{1}{m_u} - \frac{1}{m_d}\right) \boldsymbol{S}_q \cdot \boldsymbol{S}_c\, |\psi(0)|^2}_{\text{Strong hyperfine}} = = -\kappa_{qc}\frac{m_u - m_d}{\overline{m}}\frac{|\psi(0)|^2}{|\psi_B(0)|^2}\, 2\boldsymbol{S}_q \cdot \boldsymbol{S}_c \quad (11)$$

where we indicate explicitly the dependence from $m_{u/d}$ and $\overline{m}$ denotes the average light quark mass and a sum of the two charge conjugate contributions is understood. R_{ij} can be either R_{Qq} or R_{4q}, Eq. (5), and R_{Qq}^{baryon} is the radius of the diquark in the baryon. $|\psi(0)|^2$ and $|\psi_B(0)|^2$ represent the cq overlap probabilities in tetraquarks and baryons respectively.

With the definitions in Eqs. (9) to (11) and defining $\Delta_m = m_u - m_d$, one finds the mass differences

$$M(X_u) - M(X_d) = = 2\Delta_m + \frac{4}{3}a' - \frac{5}{3}\frac{a'}{\lambda} + \kappa'_{cq}\frac{\Delta_m}{\overline{m}} - \frac{4}{3}\gamma'\frac{\overline{m}}{m_c} \quad (12)$$

$$M(X_u) - M(X^+) = = \Delta_m + \frac{2}{3}a' - \frac{4}{3}\frac{a'}{\lambda} + \kappa'_{cq}\frac{\Delta_m}{2\overline{m}} - \frac{2}{3}\gamma'\frac{\overline{m}}{m_c} \quad (13)$$

Primed quantities refer to (anti)diquarks in tetraquarks and have to be scaled using the ratio of the hyperfine strong couplings, κ_{cq} and κ'_{cq} in baryons and tetraquarks. The term a'/λ, representing the electrostatic attraction between diquark and antidiquark, has been further rescaled to the tetraquark radius. We find $\kappa'_{cq} = 67$ MeV, from the mass difference of $Z(4020)$ and $Z(3900)$ [8] and $\kappa_{cq} = 15$ MeV, from the hyperfine mass differences of single charm baryons [26]. Ref. [25] finds $\kappa_{cq} = 19$ MeV. We take $\kappa_{cq} = 17 \pm 2$ MeV as an indication of the error. Accordingly,

$$r = \frac{\kappa'_{cq}}{\kappa_{cq}} = 3.94 \pm 0.45, \quad \frac{R_{cq}^{\text{baryon}}}{R_{cq}} = r^{1/3} = 1.58 \pm 0.06 \quad (14)$$

From a fit to the isospin violating mass differences of light baryons, Ref. [25] obtains: $2\Delta_m = -4.96$; $a = 2.83$; $\gamma = -1.30$, $\overline{m} = 308$, $m_c = 1665$. Thus we obtain: $a' = 4.47$; $\gamma'\overline{m}/m_c = -0.95$ (all in MeV). Numerical results are shown in Table 1.

The separation of the two scales makes a big effect. For $\lambda = 1$, the electrostatic repulsion in the (anti)diquark is almost compensated by the electrostatic attraction between the diquark and the antidiquark, and the mass difference is dominated by Δ_m. As we get to $\lambda = 3$, the electrostatic repulsion dominates and the mass difference is greatly reduced, to the extent that $X_{u,d}$ may be considered to be quasi-degenerate, within the present experimental resolution of about 1 MeV. The result justifies why only one line is seen in the $D^0\bar{D}^{*0}$ channel and none in $D^+D^{*-} + D^-D^{*+}$. X^+ is expected to be below threshold for the decay into $D^0D^{*+} + D^+D^{*0}$ but it should be found among the products of charmonium decays of B mesons, however within the bounds we shall consider now.

3. Charmonium decays of *B* mesons

Starting from the overall weak process with one $q\bar{q}$ pair from the sea:

$$[\bar{b}d]_{B^0} \to \bar{c}\, c\bar{s} + (d\bar{d}, \text{or } u\bar{u}) + d$$

one can describe the decays $B \to X\, K$ with two amplitudes, corresponding to the kaon being formed from the $\bar{s}$ with the spectator d quark, A_1, or with a d or u quark from the sea, A_2.

In particular

$$\begin{aligned} &\text{Amp}(B^0 \to X_d\, K^0) \sim A_1 + A_2 \\ &\text{Amp}(B^0 \to X_u\, K^0) \sim A_1 \\ &\text{Amp}(B^0 \to X^-\, K^+) \sim A_2 \end{aligned} \quad (15)$$

and

$$\begin{aligned} &\text{Amp}(B^+ \to X_d\, K^+) \sim A_1 \\ &\text{Amp}(B^+ \to X_u\, K^+) \sim A_1 + A_2 \\ &\text{Amp}(B^+ \to X^+\, K^0) \sim A_2 \end{aligned} \quad (16)$$

With near degeneracy of $X_{u,d}$, even a small $q\bar{q}$ annihilation amplitude inside the tetraquark could produce sizeable mixing. We consider the mass eigenstates in the isospin basis, namely

$$\begin{aligned} X_1 &= \cos\phi\, \frac{X_u + X_d}{\sqrt{2}} + \sin\phi\, \frac{X_u - X_d}{\sqrt{2}} \\ X_2 &= -\sin\phi\, \frac{X_u + X_d}{\sqrt{2}} + \cos\phi\, \frac{X_u - X_d}{\sqrt{2}} \end{aligned} \quad (17)$$

It is straightforward to compute the rate for B going to $X(3872)$, the sum of two unresolved, almost degenerate lines, followed by decay into $J/\psi + 2\pi/3\pi$, as function of ϕ and of the ratio of the isospin zero and one amplitudes, $2\alpha = 2A_1 + A_2$, $2\beta = A_2$, respectively. Note that, when going from B^0 to B^+ in the 3π to 2π ratio, $\alpha \to \alpha,\ \beta \to -\beta$.

From PDG [28] we find close values of the two ratios within errors

$$\begin{aligned} R(B^0) &= \frac{\Gamma(B^0 \to K^0\, X(3872) \to K^0\, J/\psi\, 3\pi)}{\Gamma(B^0 \to K^0\, X(3872) \to K^0\, J/\psi\, 2\pi)} \\ &= 1.4 \pm 0.6 = \frac{p_\rho}{p_\omega}\, F^0\left(\phi, \frac{\beta}{\alpha}\right) \end{aligned} \quad (18)$$

$$\begin{aligned} R(B^+) &= \frac{\Gamma(B^+ \to K^+\, X(3872) \to K^+\, J/\psi\, 3\pi)}{\Gamma(B^+ \to K^+\, X(3872) \to K^+\, J/\psi\, 2\pi)} \\ &= 0.7 \pm 0.4 = \frac{p_\rho}{p_\omega}\, F^+\left(\phi, \frac{\beta}{\alpha}\right) \end{aligned} \quad (19)$$

where $p_{\rho,\omega}$ are decay momenta (averaged over Breit–Wigner distributions, see [8]). Fig. 2 reports the contour plots of the two experimental ratios $R(B^{+,0})$. We also define

$$\begin{aligned} R^-(B^0) &= \frac{\Gamma(B^0 \to K^+X^- \to K^+\, J/\psi\, \rho^-)}{\Gamma(B^0 \to K^0X(3872) \to K^0\, J/\psi\, \rho^0)} \\ &= G^-\left(\phi, \frac{\beta}{\alpha}\right) \end{aligned} \quad (20)$$

$$R^+(B^+) = G^+\left(\phi, \frac{\beta}{\alpha}\right) = G^-\left(\phi, -\frac{\beta}{\alpha}\right) \quad (21)$$

The two allowed regions with $\phi \sim \pm 20^0$ are compatible with the present limits $R^-(B^0), R^+(B^+) < 1$, see [28]. The center of the allowed region corresponds to $R^-(B^0) = 0.3$ and $R^+(B^+) = 0.2$.

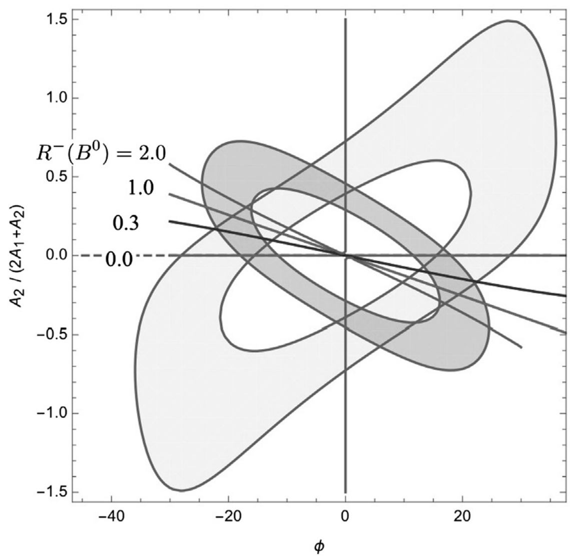

Fig. 2. Contour regions of $F^0(\phi, \frac{A_2}{2A_1+A_2})$, light shaded, and $F^+(\phi, \frac{A_2}{2A_1+A_2})$, shaded, see text. Four overlap areas correspond to regions of parameters which reproduce the experimental values of both F^+ and F^0. Solutions close to $\phi = 0$ correspond to $R^-(B^0) \sim 2$ and are not acceptable. Solutions close to $\phi \sim \pm 20^0$ correspond to $R^-(B^0) \leq 2$. As indicated by level curves reported in the figure, a good fraction of the allowed region is compatible with the present limit $R^-(B^0) < 1$, see [28], and with $R^+(B^+) < 0.5$ (not reported in the figure). The center of the allowed region corresponds to $R^-(B^0) = 0.3$ and $R^+(B^+) = 0.2$.

4. Tunneling

The diquark–antidiquark system can rearrange itself into a color singlet pair of the type $\Psi_{\mathcal{M}}$ by exchanging quarks through a tunneling transition.

The small overlap between the constituent quarks in different wells suppresses quark–antiquark direct annihilation even in neutral tetraquarks and it leaves us with a two stage process: *i*) switch of a quark and an antiquark among the two wells *ii*) evolution of the quark–antiquark pairs (in their color singlet component) into the corresponding mesons.

To illustrate the structure of decay amplitudes, we consider the state made by a diquark localized at x and an antidiquark localized at y, with u and $\bar{u}$ light quarks as in

$$\Psi_{\mathcal{D}} = [cu](x)[\bar{c}\bar{u}](y) \tag{22}$$

We can cluster quarks and antiquarks together by a Fierz rearrangement on color indices, which leads to, *e.g.*

$$\Psi_{\mathcal{D}} \sim (c(x)\bar{u}(y))\,(\bar{c}(y)u(x)) \tag{23}$$

(round brackets indicate that we have to take the projections over color singlets). However this is not enough, since we still need to bring the light quark and the antiquark in the respective positions of $\bar{c}$ and c ($y \leftrightarrow x$). This involves tunneling below the barrier between the two wells, Fig. 1. The amplitude for a heavy quark tunneling is exponentially suppressed with the mass of the heavy quark $\sim \exp(-\sqrt{m_c E}\ \ell)$, where E and ℓ are height and the extension of the barrier, so that: *compact tetraquark couplings are expected to favor the open charm/beauty modes with respect to charmonium/bottomonium ones.*

In addition, tunneling may provide dynamical factors in front of the various components of the Fierz rearranged expression. Including the diquark spins (subscripts), consider the states

$$\Psi_{\mathcal{D}}^{(1)} = [cu]_0[\bar{c}\bar{u}]_1$$
$$\Psi_{\mathcal{D}}^{(2)} = \mathcal{C}\Psi_{\mathcal{D}}^{(1)} = [cu]_1[\bar{c}\bar{u}]_0 \tag{24}$$

with $\mathcal{C}$ the charge conjugation operation. We start by performing a Fierz rearrangement on color indices of $\Psi_{\mathcal{D}}^{(1)}$ and focus on the first (leading) term

$$\Psi_{\mathcal{D}}^{(1)} \sim [c^\alpha \sigma_2 u^\beta](x)[\bar{c}_\beta \sigma_2 \boldsymbol{\sigma}\, \bar{u}_\alpha](y) \tag{25}$$

which encodes the $c\bar{u}$ and $u\bar{c}$ color singlets (and singles out $c\bar{c}$ terms). After a Fierz rearrangement of spin indices we get

$$\begin{aligned}\Psi_{\mathcal{D}}^{(1)} &= A[c^\alpha(x)\sigma_2\bar{u}_\alpha(x)][\bar{c}_\beta(y)\sigma_2\boldsymbol{\sigma}\,u^\beta(y)] \\ &\quad - B[c^\alpha(x)\sigma_2\boldsymbol{\sigma}\,\bar{u}_\alpha(x)][\bar{c}_\beta(y)\sigma_2 u^\beta(y)] + \\ &\quad + iC[c^\alpha(x)\sigma_2\boldsymbol{\sigma}\,\bar{u}_\alpha(x)]\times[\bar{c}_\beta(y)\sigma_2\boldsymbol{\sigma}\,u^\beta(y)]\end{aligned} \tag{26}$$

$A,\ B,\ C$ are non-perturbative coefficients associated to different barrier penetration amplitudes for *different light quark spin configurations*. Using an evident meson field notation we can write

$$\Psi_{\mathcal{D}}^{(1)} = A\, D^0 \bar{\boldsymbol{D}}^{*0} - B\, \boldsymbol{D}^{*0}\bar{D}^0 + iC\, \boldsymbol{D}^{*0} \times \bar{\boldsymbol{D}}^{*0} \tag{27}$$

Similarly

$$\Psi_{\mathcal{D}}^{(2)} = B\, D^0 \bar{\boldsymbol{D}}^{*0} - A\, \boldsymbol{D}^{*0}\bar{D}^0 - iC\, \boldsymbol{D}^{*0} \times \bar{\boldsymbol{D}}^{*0} \tag{28}$$

5. X_u, X_d and $X^\pm$

Following Eqs. (6) and (7), X_u can be casted in the form

$$X_u \sim \frac{\Psi_{\mathcal{D}}^{(1)} + \Psi_{\mathcal{D}}^{(2)}}{\sqrt{2}} = \frac{A+B}{\sqrt{2}}\,(D^0\bar{\boldsymbol{D}}^{*0} - \boldsymbol{D}^{*0}\bar{D}^0) \tag{29}$$

whereas

$$X_d \sim \frac{A+B}{\sqrt{2}}\,(D^+\boldsymbol{D}^{*-} - \boldsymbol{D}^{*+}D^-) \tag{30}$$

Similar considerations apply to $X^\pm$, described by

$$X^\pm \sim \frac{A+B}{\sqrt{2}}\,(D^\pm\bar{\boldsymbol{D}}^{*0} - \boldsymbol{D}^{*\pm}\bar{D}^0) \tag{31}$$

With the results of Table 1, X_d is below threshold for the decay suggested by (30). Both mass eigenstates in (17) decay in $D^0\ \bar{D}^{0*}$ via mixing. Charged partners are also lighter than the corresponding meson thresholds in (31) and their decay occurs via the subleading charmonium decays considered below.

6. $Z_c^{(\prime)}$ and $Z_b^{(\prime)}$

In the case of Z_c and Z_b resonances, charged and neutral states are observed. Two neutral tetraquarks are expected in this case too, although potentially quasi-degenerate.

Consider the neutral, $u\bar{u}$ component of the Z_c multiplet

$$\begin{aligned}Z_c &= \frac{1}{\sqrt{2}}\Big([cu]_0[\bar{c}\bar{u}]_1 - [cu]_1[\bar{c}\bar{u}]_0\Big) = \\ &= \frac{A-B}{\sqrt{2}}\,(D^0\bar{\boldsymbol{D}}^{0*} + \boldsymbol{D}^{0*}\bar{D}^0) + i\sqrt{2}C\,\boldsymbol{D}^{*0}\times\bar{\boldsymbol{D}}^{*0}\end{aligned} \tag{32}$$

The non-trivial dependence of tunneling factors from the light quark spin (*i.e.* $A \neq B$ unlike in the naive Fierz transformation), allows Z_c to decay in $D^0\bar{\boldsymbol{D}}^{*0}$, the decay in $\boldsymbol{D}^{*0}\bar{\boldsymbol{D}}^{*0}$ being forbidden by phase space. The $d\bar{d}$ component would be coupled to the neutral combination of charged charmed mesons. The two decay channels for the mass eigenstates might get mixed.

The expression for charged states follows naturally from (32), but this time, (see (1)), there is enough phase space to decay into charged open charm components.

The Z'_c resonances are constructed in a very similar way, with different non-perturbative coefficients in (32), *e.g.*

$$Z'_c = \left([cu]_1\,[\bar{c}\bar{u}]_1\right)_{J=1} = \\ = E\,(D^0\bar{\boldsymbol{D}}^{0*} + \boldsymbol{D}^{0*}\bar{D}^0) + iF\,\boldsymbol{D}^{*0}\times\bar{\boldsymbol{D}}^{*0} \tag{33}$$

An interesting experimental check is that of studying the mass difference between the charged and neutral components of the Z_c resonance, which we would expect to be almost degenerate, as is the case for the X.

There are no qualitative differences in the description of the Z_b and Z'_b resonances except the fact that thresholds are closer, as indicated in (1) — this could be due to the reduced chromomagnetic couplings by the large b quark mass. As a consequence, the analog of the $X(3872)$ in the beauty sector could be pushed below threshold by spin interactions and forced to decay in the subleading bottomonium modes.

7. Sub-leading decays

Heavy quark tunnelings amplitudes do not vanish for finite heavy quark masses. In particular it is found

$$X_u \sim a\,i\,\boldsymbol{J}/\boldsymbol{\psi}\times(\boldsymbol{\omega}^0+\boldsymbol{\rho}^0) \tag{34}$$

$$Z_u \sim b\,\eta_c(\boldsymbol{\omega}^0+\boldsymbol{\rho}^0) - c\,\boldsymbol{J}/\boldsymbol{\psi}(\eta_q+\pi^0) \tag{35}$$

while

$$Z'_u \sim d\,\eta_c(\boldsymbol{\omega}^0+\boldsymbol{\rho}^0) + e\,\boldsymbol{J}/\boldsymbol{\psi}(\eta_q+\pi^0) \tag{36}$$

where the non-perturbative coefficients $a, b, ..., e$ are all equal in the limit of naive Fierz couplings. The formulae for X_d, Z_d, Z'_d are obtained by letting $\boldsymbol{\rho}^0 \to -\boldsymbol{\rho}^0$ and $\pi^0 \to -\pi^0$.

For an orientative estimate, we may take the leading semiclassical approximation of tunneling amplitudes (see [27])

$$\mathcal{A}_M \sim e^{-\sqrt{2ME}\,\ell} \tag{37}$$

We use the quark masses, m_q and m_c, quoted before from Ref. [25], the orientative values: $E = 100$ MeV and $\ell = 2$ fm to obtain, neglecting factors of $\mathcal{O}(1)$

$$R = \left(\frac{a}{A+B}\right)^2 \sim \left(\frac{\mathcal{A}_{m_c}}{\mathcal{A}_{m_q}}\right)^2 \sim 10^{-3} \tag{38}$$

With decay momenta (in MeV): $p_\rho \sim 124$ [8], $p_{DD^*} \sim 2$ [28], one would find

$$\frac{\Gamma(X(3872)\to J/\psi\,\rho)}{\Gamma(X(3872)\to D\bar{D}^*)} = \frac{p_\rho}{p_{DD^*}}\,R \sim 0.1 \tag{39}$$

compatible with: $B(X(3872)\to J/\psi\ \rho) \sim 2.6\times 10^{-2}$, $B(X(3872)\to D\bar{D}^*) \sim 24\times 10^{-2}$ [28].

8. Conclusions

In this paper we have analyzed the typical objections raised against the tetraquark model in the diquark–antidiquark realization. The replies we provide are based on a picture of the diquark correlations in hadrons, that we have advocated several times in the past, and examined now in all of its consequences. On this basis we show that the neutral and charged components of X could be quasi-degenerate. As a consequence, the $X^\pm$ should not be observed in open charm decays but only in final states containing charmonia. However the charged X may have much smaller branching fractions in B meson decays than expected and this requires some dedicated experimental effort to go beyond the bounds which have been set years ago. The decay modes of the $Z^{(\prime)}$ particles are also explained and their occurrence in isospin triplets is understood. A number of questions on the $Z_{c,b}$ particles are left open by the experiment — all of them have a crucial role to the assessment of the considerations made here. In particular all X, Z resonances should be produced in prompt pp collisions, whereas there are no hints yet on Z particles in these production channels. Also, Zs should be seen in B decays too and a similar hyperfine structure of neutral Z could eventually be resolved.

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