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# Optimum design of the tuned mass-damper-inerter for serviceability limit state performance in wind-excited tall buildings

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## Abstract

Optimally designed tuned mass-damper-inerters (TMDIs) are considered to meet code-prescribed serviceability criteria in typical wind-excited tall buildings subject to vortex shedding effects in a performance-based design context. The TMDI, couples the classical tuned-mass-damper (TMD) with an inerter, a two-terminal device resisting the relative acceleration of its terminals, achieving mass-amplification and higher-modes-damping effects compared to the TMD. A benchmark 74-storey building is considered, where TMDI is added to the structural system assuming ideal linear inerter behavior. The wind action is defined through a non-diagonal power spectral density matrix supporting computationally efficient frequency domain structural analyses. The TMDI is optimally designed for stiffness, damping, and inerter constant parameters via a standard numerical optimization search, for a range of pre-specified attached TMDI mass values. It is shown that the TMDI achieves more lightweight construction in the design of new code-compliant tall buildings against wind.

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*Keywords:* Tuned mass damper, inerter, wind, tall buildings, serviceability, passive vibration control

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## 1. Introduction

Performance-Based Engineering (PBE) is an integrated framework that during last decade received significant attention from researchers and practitioners aiming to optimally design structures achieving pre-specified levels of

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structural performance under different hazards [1]. In this context, PBE has been considered to treat the serviceability performance of tall buildings under wind excitation [2]. In particular, wind-excited slender high-rise buildings with rectangular floor plan are prone to excessive accelerations in the across-wind direction (i.e., within the normal plane to the wind direction) due to vortex shedding effects generated around their edges [3,4]. Ensuring that the across-wind floor accelerations remain below a certain threshold associated with users' comfort becomes a critical performance requirement for serviceability [5]. In general, increasing the stiffness of the building does not lead to suppression of wind-induced peak accelerations [6]. Consequently, supplemental damping systems are often provided to modern tall buildings appropriately designed to meet the occupants' comfort requirements prescribed by building codes and guideline. To this aim, tuned mass-dampers (TMDs), among other devices and configurations for supplemental damping, have been widely used over the past three decades for vibration mitigation in wind-excited tall buildings [7,8]. In its simplest form, the linear passive TMD comprises a mass attached towards the top of the building whose oscillatory motion is to be controlled (primary structure) via linear stiffeners, in conjunction with linear energy dissipation devices (dampers). The effectiveness of the TMD relies on "tuning" its stiffness and damping properties for a given primary structure and attached mass, such that significant kinetic energy is transferred from the dynamically excited primary structure to the TMD mass and eventually dissipated through the dampers.

The two main drawbacks of the TMD regarding the suppression of lateral wind-induced floor accelerations are:

- The TMD is commonly placed at the upper floors of the building and tuned to control the fundamental lateral mode shape of the primary structure (e.g. [9]). Nevertheless, peak floor accelerations are heavily influenced by higher modes of vibrations which the TMD cannot control.
- The effectiveness of the TMD for vibration control depends heavily on the attached mass [8,10]. The latter can rarely exceed 0.5% to 1% of the total building mass in tall buildings as it becomes overly expensive to accommodate its weight and volume due to structural and architectural limitations, respectively.

To address the above issues and concerns in an innovative manner, Giaralis and Petrini [11] explored the potential of incorporating an inerter device to wind-excited TMD-equipped tall buildings, to achieve enhanced floor accelerations suppression in the across-wind direction without increasing the attached TMD mass. The inerter is a line-like two-terminal device introduced by Smith in 2002 [12], having negligible mass/weight resisting relative accelerations between its terminals, and characterized by a scalar variable called "inertance". In [11] the tuned mass-damper-inerter (TMDI) configuration, originally introduced by Marian and Giaralis [13,14] for earthquake engineering applications, was considered. Appreciable gains in reducing peak top floor accelerations in a 74-floor benchmark tall building were achieved compared to the TMD through a parametric study considering non-optimal TMDI stiffness and damping coefficients for fixed attached mass and increasing inertance. These gains are attributed partly to the mass-amplification effect and partly to higher-modes-damping effect endowed to the TMD by the inerter. In the present paper, the same building benchmark structure is used as in [11] to derive optimal TMDI stiffness and damping improving further the TMDI efficiency for floor acceleration control compared to same-attached-mass TMDs.

## 2. The Tuned Mass-Damper-Inerter (TMDI) for multi-storey building

Conceptually defined by Smith (2002) [12], the ideal inerter is a linear massless two-terminal mechanical element resisting the relative acceleration at its terminals through the so-called inertance coefficient,  $b$ , measured in mass units. In this regard, the inerter element force  $F$  shown schematically as a hatched box in the inlet of Fig.1 reads a  $F = b(\ddot{u}_1 - \ddot{u}_2)$ , where, a dot over a symbol signifies differentiation with respect to time. The ideal inerter can be interpreted as an inertial weightless element whose gain depends on  $b$  and on the relative acceleration observed by its terminals [15].

The above considerations led to the TMDI configuration in [13,14] where an inerter device is used as a mass amplifier contributing additional inertia to the attached mass of the classical TMD without increasing its weight to enhance the TMD vibration suppression effectiveness. Specifically, consider a planar linear  $n$ -storey frame structure modelled as an  $n$ -DOF dynamical system with mass  $m_k$  ( $k=1,2,\dots,n$ ) lumped at the  $k$ -th floor as shown in Fig.1 (a). Treating the above system as the primary structure, the TMDI configuration comprises a mass  $m_{TMDI}$  attached to the top floor via a linear spring of stiffness  $k_{TMDI}$  and a linear dashpot of damping coefficient  $c_{TMDI}$ , and linked to the

penultimate floor by an ideal inerter of inertance  $b$ . The mass  $\mathbf{M}$ , the damping  $\mathbf{C}$ , and the stiffness  $\mathbf{K}$  matrices characterizing the dynamic behaviour of the TMDI equipped system in Fig. 1 are given in Eq. (1) where  $c_{i,j}$  and  $k_{i,j}$  are the  $(i,j)$  elements of the primary structure damping and stiffness matrices, respectively.

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & \dots & \dots & \dots & 0 \\ 0 & m_2 & \dots & \dots & \dots & \vdots \\ \vdots & \vdots & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & 0 & m_{n-1} + b & 0 & -b \\ \vdots & \vdots & 0 & 0 & m_n & 0 \\ 0 & \dots & 0 & -b & 0 & m_{TMDI} + b \end{bmatrix}, \mathbf{C} = \begin{bmatrix} c_{1,1} & c_{1,2} & \dots & \dots & c_{1,n} & 0 \\ c_{2,2} & \dots & \dots & \dots & \vdots & \vdots \\ \vdots & \vdots & \ddots & c_{n-2,n-1} & c_{n-2,n} & 0 \\ \vdots & \vdots & \vdots & c_{n-1,n-1} & c_{n-1,n} & 0 \\ \text{SYM} & & & & c_{n,n} + c_{TMDI} & -c_{TMDI} \\ & & & & & c_{TMDI} \end{bmatrix}, \tag{1}$$

$$\text{and } \mathbf{K} = \begin{bmatrix} k_{1,1} & k_{1,2} & \dots & \dots & k_{1,n} & 0 \\ & k_{2,2} & \dots & \dots & \vdots & \vdots \\ & & \ddots & k_{n-2,n-1} & k_{n-2,n} & 0 \\ & & & k_{n-1,n-1} & k_{n-1,n} & 0 \\ & & & & k_{n,n} + k_{TMDI} & -k_{TMDI} \\ & & & & & k_{TMDI} \end{bmatrix}$$

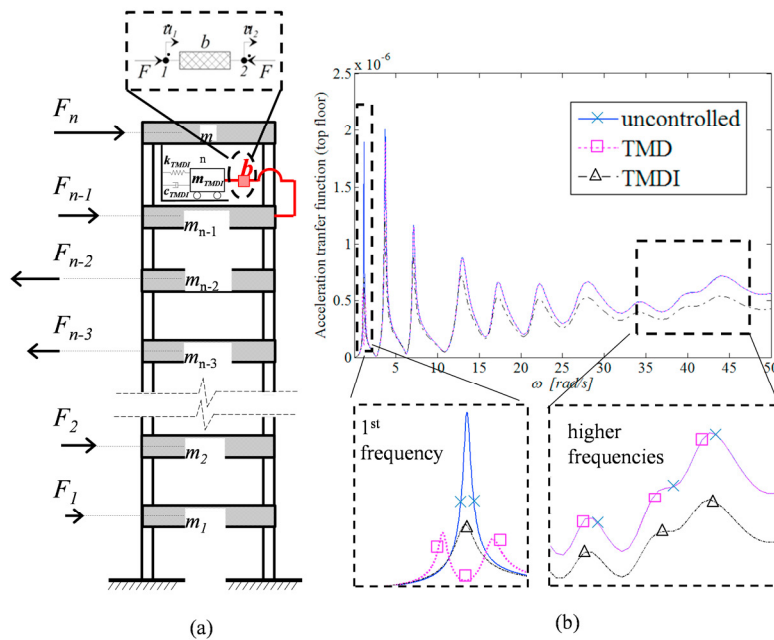


Fig. 1. Lumped-mass linear model of a wind excited n-storey frame building equipped with a TMDI (a); suppression of higher modes of vibration in terms of acceleration transfer function, case  $\beta=0.69, \mu=0.1$  (b)

Note that the inclusion of the inerter changes only the mass  $\mathbf{M}$  of the controlled structure (i.e.,  $\mathbf{C}$  and  $\mathbf{K}$  are the same for the TMD and for the TMDI), and that for  $b=0$  the mass matrix of the TMD-equipped frame building is retrieved (i.e., the TMD is a special case of the TMDI). Furthermore, for  $b \neq 0$ ,  $\mathbf{M}$  in Eq.(1) is not diagonal since the inerter introduces “gyroscopic” inertial cross-terms that couples the DOF of the attached mass, numbered as  $n+1$ , with the DOF of the penultimate floor. These cross-terms alter the dynamics of the primary structure such that higher modes of vibration are damped besides the fundamental mode shape. As a final remark, the fact that the effective inertia corresponding to the DOF of the attached mass is equal to  $(m_{TMDI}+b)$  in Eq.(1) (mass amplification effect of the inerter) motivates the definition of the following frequency ratio  $v_{TMDI}$  and damping ratio  $\zeta_{TMDI}$ .

$$v_{TMDI} = \frac{\sqrt{\frac{k_{TMDI}}{m_{TMDI} + b}}}{\omega_1}, \quad \zeta_{TMDI} = \frac{c_{TMDI}}{2\sqrt{(m_{TMDI} + b)k_{TMDI}}}, \tag{2}$$

to characterize the dynamics of the TMDI given an attached mass  $m_{TMDI}$  and inertance  $b$ .

### 3. Adopted primary structure and wind excitation model

To optimize the TMDI in Fig. 1(a) for suppressing wind induced oscillations in tall buildings, a high-rise building previously considered for the development of a performance-based wind engineering framework [4] is taken as a benchmark structure. The adopted structure is a 74-storey steel frame building of 305m total height with a 50m-by-50m footprint. The building comprises two spatial steel frames, one inner including 12 columns, and one outer formed by 28 columns, the two frames are connected by three outriggers located at 100m, 200m, and 300m in elevation. All columns have hollow square sections, with varying outer dimensions and thickness along the building height ranging in between 1.20m to 0.50m, and 0.06m to 0.025m, respectively. Beams are of various standard double-T steel section profiles and all beam-to-column joints are taken as rigid. The outriggers are braces consisted of double-T beams and hollow-square diagonal struts. The first three natural frequencies of these modes and the corresponding modal participating mass ratios in parentheses are 0.185Hz (0.6233), 0.563Hz (0.1900), and 1.052Hz (0.0745). The modal damping ratios  $\zeta_s$  has been assumed equal to 2% for the first 9 modes [11]. Starting from detailed FE model of the structure, a reduced dynamic system with  $n=74$  DOFs is derived in terms of mass, damping and stiffness matrices to serve as the primary (uncontrolled) structure. The 74 DOFs of the system correspond to the lateral translational DOFs of the FE model. The wind action is considered only in the across-wind direction as this is the critical direction to check for the occupants’ comfort criterion for this particular structure [4]. The wind force components  $F_k$  ( $k=1,2,\dots,74$ ) acting at the slab heights of the primary structure as pictorially shown in Fig.1 are modelled as a zero-mean Gaussian ergodic spatially correlated random field represented in the frequency domain by a  $\mathbf{S}_{FF}^{74}(\omega) \in \mathbb{R}^{74 \times 74}$  PSD matrix.

The response displacement and acceleration PSD matrices of the TMDI-equipped primary structure are obtained using the frequency domain input-output relationships

$$\mathbf{S}_{xx}(\omega) = \mathbf{B}(\omega)^* \mathbf{S}_{FF}(\omega) \mathbf{B}(\omega) \quad \text{and} \quad \mathbf{S}_{\ddot{x}\ddot{x}}(\omega) = \omega^4 \mathbf{S}_{xx}(\omega) \tag{3}$$

respectively. In Eq. (3),  $\mathbf{S}_{FF}$  is the PSD wind force matrix  $\mathbf{S}_{FF}^{74}$  augmented by a zero row and a zero column corresponding to the DOF of the TMDI which is not subjected to any wind load and the “\*” superscript denotes complex matrix conjugation, and the transfer matrix  $\mathbf{B}$  is given as  $\mathbf{B}(\omega) = [\mathbf{K} - \omega^2 \mathbf{M} + i\omega \mathbf{C}]^{-1}$ , being  $i = \sqrt{-1}$ . Finally, peak  $k$ -th floor accelerations are estimated by the expression  $\text{peak}\{\ddot{x}_k\} = g\sqrt{\sigma_{\ddot{x}_k}^2}$ , and the peak factor  $g$  is estimated by the widely used empirical formula  $g = \sqrt{2\ln(\eta T_{wind})} + 0.577/\sqrt{2\ln(\eta T_{wind})}$ .

### 4. Performance-Based optimization of the TMDI

The performance of the building in term of occupants comfort are evaluated by comparing the hourly peak accelerations of the floors as experimented under a design wind having an annual return period (gradient wind velocity of the atmospheric boundary layer  $V_{ref}=35$  m/s taken at an height of 810m in urban congested area), with code-prescribed threshold values depending on the first natural frequency of the building in across-wind direction.

Then, considering the configuration in Fig. 1(a), optimization of the TMDI parameters (Design Variables - DVs)  $v_{TMDI}$  and  $\zeta_{TMDI}$  in Eq. (2) is conducted for fixed values of the TMD mass ratio  $\mu = m_{TMDI}/M_{building}$  and inertance ratio  $\beta = b/M_{building}$  by using the pattern search algorithm [16]. Since the maximum peak acceleration is always attained by the top floor of the building, the goal of the optimization problem is to minimize the hourly peak top floor acceleration  $\text{peak}\{\ddot{x}_{74}\}$  induced by the considered wind loads, subjected to the constrains of meeting the structural performances in terms of peak top floor accelerations ( $\text{peak}\{\ddot{x}_{74}\} \leq \ddot{x}_{threshold}$  for building occupant comfort) and maximum peak drift along the height of the building ( $\max_{1 \leq j \leq 74} [\text{peak}\{x_j - x_{j-1}\}] \leq \theta_{threshold}$  structural and non-structural

damage limitation). The details of the structural optimization problem are summarized in Table 1, showing the threshold values considered for the constrains and the value ranges considered for the DVs.

Table 1. Parameters' values in the TMDI optimization

Design variables	min	Max	Note: the objective is to minimize the peak top floor acceleration at the top of the building
$v_{TMDI}$	0.2	1.2	
$\zeta_{TMDI}$	$10^{-5}$	0.8	
Fixed parameters		Threshold response value	
$\mu$	01%-0.9%		$\ddot{x}_{threshold}$ 102.9 mm/s <sup>2</sup>
$\beta$	0-0.4		$\theta_{threshold}$ 0.004

The resulting values of the optimization are shown in Figure 2, where the performance of the optimized configurations in terms of peak  $\{\dot{x}_{74}\}$  are also compared to the classical TMD (case  $\beta=0$ ). Moreover, the peak inerter force, obtained by multiplying the inertance  $b$  by the peak relative acceleration between the two inerter terminals, is shown in Fig. 2(d). It is seen that this force takes on reasonable values that can well be accommodated by the structure.

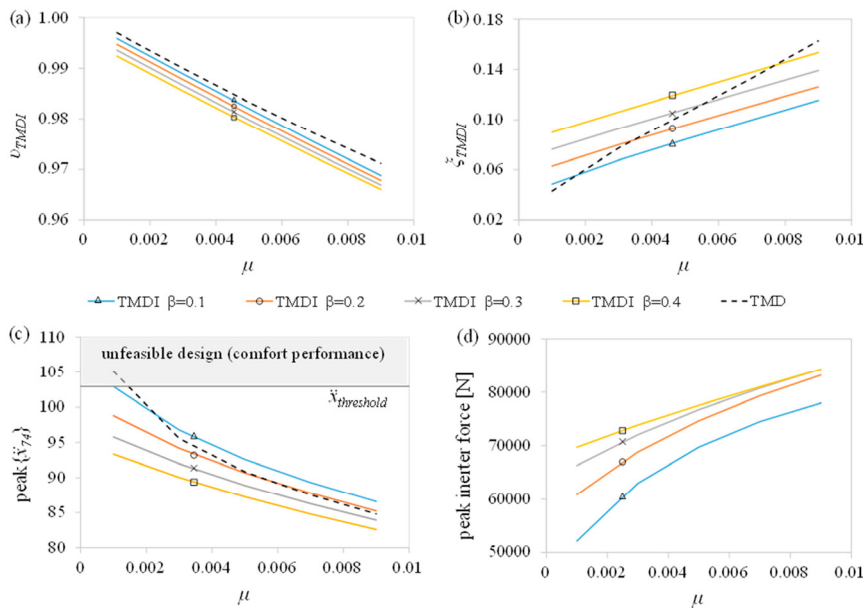


Fig. 2. Results of the TMDI optimization in at fixed values of  $\mu$  and  $\beta$  : (a) optimal  $v_{TMDI}$  values; (b) optimal  $\zeta_{TMDI}$  values; (c) structural performances of the optimal configurations ; (d) peak inerter force for the optimal configurations

In order to express the optimal values of  $v_{TMDI}$  and  $\zeta_{TMDI}$  in closed form, a linear regression of the values in figures 2(a) and 2(b) with  $\mu$ , i.e.  $v_{TMDI}=a_1\mu+c_1$  and  $\zeta_{TMDI}=a_2\mu+c_2$  is first undertaken for different values of  $\beta$ . Next, the obtained values of the  $a_i$  and  $c_i$  ( $i=1,2$ ) parameters are fitted with polynomial linear and/or quadratic laws with respect to  $\beta$  and/or  $\sqrt{\beta}$ . Finally, the obtained polynomial coefficients are expressed by fractions of two integer numbers. At the end of the fitting process, the optimal values are written as

$$v_{TMDI} = \mu \left( \frac{26}{84} \beta - \frac{288}{84} \right) - \frac{1}{84} \beta + 1, \quad \zeta_{TMDI} = 11\mu(\beta - \sqrt{\beta} + 1) + \frac{11}{65} \beta + \xi_{s1} \tag{6}$$

where  $\xi_{s1}$  is the modal damping ratio of the first structural mode of the primary structure equal to 2% in the examined case. The matching between the numerical values obtained by the optimization algorithm (dots of various shapes) and the values from Eq. (6) (dashed lines) are shown in Figure 3, where the agreement between the two is shown to be satisfactory, with a maximum error (absolute difference between the true and the fitted values divided

by the true value) equal to 0.06% for  $v_{TMDI}$ , and 7% for  $\zeta_{TMDI}$ , in both cases occurring for the largest considered  $\beta$  value.

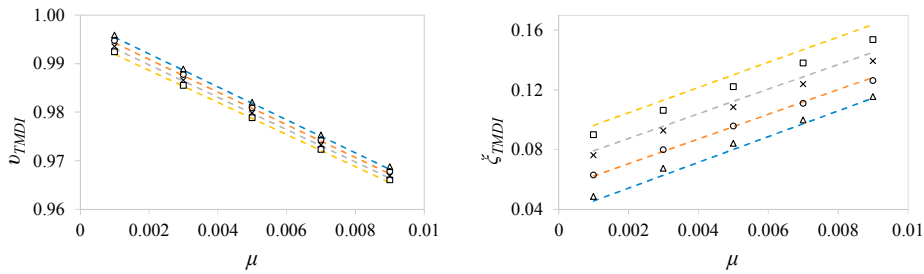


Fig. 3. Comparison between fitted (Eq. (6)- dashed lines), and optimum values ( $\beta=0.1$  ( $\Delta$ ),  $0.2$  ( $\circ$ ),  $0.3$  ( $\times$ ),  $0.4$  ( $\square$ )) for  $v_{TMDI}$  and  $\zeta_{TMDI}$

## 5. Conclusions

Optimal TMDI damping and frequency ratio parameters ( $\zeta_{TMDI}$  and  $v_{TMDI}$  respectively) are found to follow the same trend of classical TMD parameters at increasing *mass* for fixed *inertance* values. Structural response results (Fig. 2(c)) confirm that for small attached mass the TMDI performs significantly better than the TMD. Peak inerter force is found to be at manageable levels for connecting the inerter terminals with the primary structure. Lastly, closed form expression of  $v_{TMDI}$  and  $\zeta_{TMDI}$  parameters as a function of *mass* and *inertance* have been derived by means of ordinary polynomial fitting techniques in the range of values that are relevant to tall buildings.

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