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Perturbation damage indicators based on complex modes

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Abstract

The papers focusing on dynamic identification of structural damages usually rely on the comparison of two or more responses of the structure; the measure of damage is related to the differences of the vibration signals. Almost all literature methods assume damping proportionality to mass and stiffness; however, this is acceptable for new, undamaged structures, but not for existing, potentially damaged structures, especially when localised damages occur. It is well-known that in non-proportionally damped systems the modes are no longer the same of the undamped system: thus, some authors proposed to use modal complexity as a damage indicator. This contribution presents a perturbation approach that can easily reveal such a modal complexity.

Keywords: Structural damage identification, dynamic identification, non-proportional damping, modal complexity, perturbation approach

1. Introduction

Using dynamic analysis for structural identification is of paramount importance, as it allows to trace the characteristics of existing buildings by structural health monitoring. The problem of structural design is direct, because the structural model and its response to each action is known; the problem of structural dynamic identification is inverse, because it is required to detect the properties of the structural model once the response and the actions are known.

Undamaged structures exhibit proportional damping, yielding real natural modes, as provided by both standard and state-space dynamic analysis [1]. Indeed, the natural modes for discrete systems are described by lists of real numbers according to standard dynamic analysis. According to state-space formulation, the natural modes are described by lists of complex numbers lying on a straight line, hence they can be rigidly rotated [2] to become real, see figure 1.

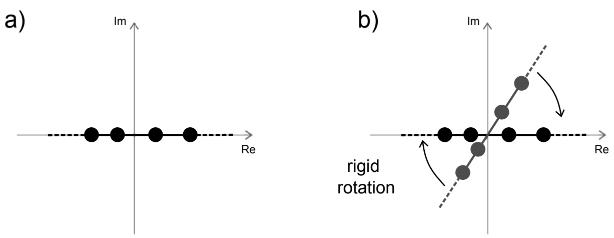

Fig. 1. Undamaged natural modes for a 4-dof discrete system by a) standard, and b) state-space dynamic analysis.

Structural damage, especially localised, implies a loss of stiffness and an increase of dissipated energy during vibration with respect to the undamaged situation: this is associated with non-proportional damping. The natural

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modes are complex and do not form a set of independent vectors; the equations of motion in modal coordinates are coupled. The state-space formulation for dynamic analysis provides the situation in figure 2: the natural modes are lists of complex numbers that do not lie on a line, and remain complex also after a rotation [2]. Thus, a measure of the dispersion of natural modes around the real axis provides the entity of the damage.

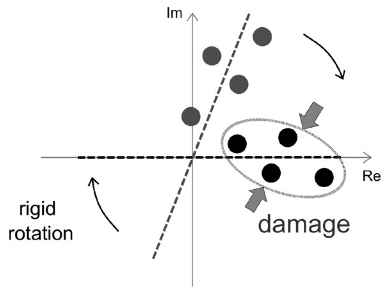

Fig. 2. Natural modes by state-space formulation for dynamic analysis.

In damaged (non-proportionally damped) structures there are modal complexity indexes (that vanish in undamaged structures) measuring the dispersion of the natural modes around the real axis according to a single scale: thus, they can be used for damage identification [3]. These indexes shall be monotonically increasing, pseudo-linear, and damage sensitive. For a discrete system with n dof, the most commonly used complexity indexes are ($i = 1, 2, \ldots n$):

- modal polygons [4]:

$$I_1 = \sum_{i=1}^{n} \frac{A_i}{n \cdot A_{max}}, \qquad A_{max} = n \cos\left(\frac{\pi}{n}\right) \sin\left(\frac{\pi}{n}\right) \tag{1}$$

  where A_{max} is the maximum area of the modal polygons;
- phase differences [4]:

$$I_2 = \sum_{i=1}^{n} \frac{\mid \phi_{i,max} \mid - \mid \phi_{i,min} \mid}{n \cdot \pi} \tag{2}$$

- modal collinearity [2]:

$$I_3 = \sum_{i=1}^{n} \frac{\mid Re(\mathbf{Z}_i)^T Im(\mathbf{Z}_i) \mid}{n \sqrt{[Re(\mathbf{Z}_i)^T Re(\mathbf{Z}_i)][Im(\mathbf{Z}_i)^T Im(\mathbf{Z}_i)]}} \tag{3}$$

  where $\mathbf{Z}_i$ is the i-th complex natural mode;
- average of the imaginary parts of the natural modes [2]:

$$I_4 = \sum_{i=1}^{n} \sum_{k=1}^{n} \frac{\mid Im(Z_{ki}) \mid}{n^2} \tag{4}$$

  where Z_{ki} is the k-th component of $\mathbf{Z}_i$;
- weight of the imaginary part of the eigenmode [5]:

$$I_5 = \sum_{i=1}^{n} \frac{||Im(\mathbf{Z}_i)||}{n \, ||\mathbf{Z}_i||} \tag{5}$$

These indexes are able to detect damage from a single observation, without the need to compare various measures. However, the damage identification procedures of literature basing on these indexes allow to detect the presence of damage, but not its quantification or location, and cannot determine the left life of the structure for a given damage, see, for instance, [6]. Here we propose a procedure that uses the indexes in Eqs. (1)–(5) and aims at developing a numerically efficient technique for detecting, quantifying and localising damage for the monitoring of the health status of structures (level 3 identification, see Tab. 1). All indexes but the 2nd are found suitable for damage identification by this procedure.

Table 1. Levels of damage identification.

Level	Presence	Entity	Position	Remaining life
1	√	×	×	×
2	√	√	×	×
3	√	√	√	×
4	√	√	√	√

2. A perturbation approach for identification through complexity indexes

In a discrete system, the search for natural properties leads to an eigenvalue problem in the state space, of the form

$$(\mathbf{B}-\lambda\mathbf{A})\mathbf{Z}=\mathbf{0}, \qquad [\mathbf{A}]=\begin{bmatrix}\mathbf{C} & \mathbf{M}\\ \mathbf{M} & \mathbf{0}\end{bmatrix}, \qquad [\mathbf{B}]=\begin{bmatrix}\mathbf{K} & \mathbf{0}\\ \mathbf{0} & -\mathbf{M}\end{bmatrix}, \qquad [\mathbf{Z}]=\begin{bmatrix}\mathbf{q}\\ \dot{\mathbf{q}}\end{bmatrix} \tag{6}$$

where $\mathbf{M}, \mathbf{C}, \mathbf{K}$ are the mass, damping, and stiffness matrices of the structure; $\mathbf{Z}$ is the vector of the space-state variables of the structure. If damage is modest, the damaged state can be described as a perturbation of the undamaged one [7]. Thus, a formal power series expansion of all the quantities in Eq. (6) can be performed up to the 2nd order of a 'small' quantity ε, seen as the evolution parameter of the modest damage that averts the damaged state from the undamaged one. Thus, if $i = 1, 2, \ldots n$ one gets

$$[(\mathbf{B}_0+\varepsilon\mathbf{B}_1+\varepsilon^2\mathbf{B}_2+\ldots)-(\lambda_{0i}+\varepsilon\lambda_{1i}+\varepsilon^2\lambda_{2i}+\ldots)(\mathbf{A}_0+\varepsilon\mathbf{A}_1+\varepsilon^2\mathbf{A}_2+\ldots)](\mathbf{Z}_{0i}+\varepsilon\mathbf{Z}_{1i}+\varepsilon^2\mathbf{Z}_{2i}+\ldots)=\mathbf{0} \tag{7}$$

The terms of order 0 of the expansion in Eq. (7) represent the unperturbed properties of the system; the higher order terms represent their perturbed counterparts, depending on the damage parameter ε.

Once solved the undamaged problem, the eigenproperties of order 1 turn out to be

$$\lambda_{1i}=\frac{\mathbf{Z}_{0i}^{\top}(\mathbf{B}_1-\lambda_{0i}\mathbf{A}_1)\mathbf{Z}_{0i}}{\mathbf{Z}_{0i}^{\top}\mathbf{A}_0\mathbf{Z}_{0i}}, \qquad \mathbf{Z}_{1i}=\alpha_{ii}\mathbf{Z}_{0i}+\sum_{j=1,\, j\neq i}^{n}\frac{\mathbf{Z}_{0j}^{\top}(\mathbf{B}_1-\lambda_{0i}\mathbf{A}_1)\mathbf{Z}_{0i}}{(\lambda_{0i}-\lambda_{0j})\mathbf{Z}_{0j}^{\top}\mathbf{A}_0\mathbf{Z}_{0j}}\mathbf{Z}_{0j} \tag{8}$$

where the α_{ii} are undetermined amplification parameters. The eigenproperties of order 2 turn out to be

$$\begin{aligned}\lambda_{2i}&=\frac{\mathbf{Z}_{0i}^{\top}(\mathbf{B}_1-\lambda_{0i}\mathbf{A}_1-\lambda_{1i}\mathbf{A}_0)\mathbf{Z}_{1i}}{\mathbf{Z}_{0i}^{\top}\mathbf{A}_0\mathbf{Z}_{0i}}+\frac{\mathbf{Z}_{0i}^{\top}(\mathbf{B}_2-\lambda_{0i}\mathbf{A}_2-\lambda_{1i}\mathbf{A}_1)\mathbf{Z}_{0i}}{\mathbf{Z}_{0i}^{\top}\mathbf{A}_0\mathbf{Z}_{0i}}\\ \mathbf{Z}_{2i}&=\beta_{ii}\mathbf{Z}_{0i}+\sum_{j=1,\, j\neq i}^{n}\left(\frac{\mathbf{Z}_{0j}^{\top}(\mathbf{B}_1-\lambda_{0i}\mathbf{A}_1-\lambda_{1i}\mathbf{A}_0)\mathbf{Z}_{1i}}{(\lambda_{0i}-\lambda_{0j})\mathbf{Z}_{0j}^{\top}\mathbf{A}_0\mathbf{Z}_{0j}}+\frac{\mathbf{Z}_{0j}^{\top}(\mathbf{B}_2-\lambda_{0i}\mathbf{A}_2-\lambda_{1i}\mathbf{A}_1)\mathbf{Z}_{0i}}{(\lambda_{0i}-\lambda_{0j})\mathbf{Z}_{0j}^{\top}\mathbf{A}_0\mathbf{Z}_{0j}}\right)\mathbf{Z}_{0j}\end{aligned} \tag{9}$$

where the β_{ii} are undetermined amplification parameters. As usual in static perturbation techniques, higher order quantities (related to the perturbation) are found in closed form in terms of those of the unperturbed problem. Thus, solving the problem of the zeroth order eigenproperties greatly facilitates the computational effort.

The proposed procedure for dynamic identification involves the following steps:

- the experimental modal complexity index is evaluated: the damage parameter providing this index is the unknown of the problem;
- the perturbation approach provides the numerical modal complexity indexes for each value of the damage parameter in the range of interest;
- an objective function represents the trend of the difference of experimental and numerical modal complexity indexes as a function of the damage parameter: its minimum identifies the searched damage parameter.

Only for ideal identification the damage parameter minimising the objective function coincides with that of the experimental data. In the real case, the numerically identified damage parameter deviates from the actual one because of the truncation of the formal series expansion in the perturbation procedure.

3. Application

To validate the proposed identification procedure, a shear-type multi-story frame reduced to a discrete system with 4 degrees of freedom is considered, see Fig. 3. Its motion is described in terms of the absolute displacement of each

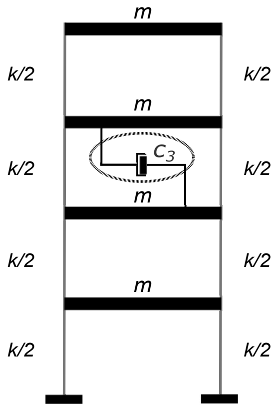

Fig. 3. Structural model

story [8]. The following values of mass and stiffness are considered

$$m = 1\,\text{kg}, \qquad k = 1800\,\text{Nm}^{-1}$$

from which the mass and stiffness matrices are deduced

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & m \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{K} = \begin{bmatrix} 2k & -k & 0 & 0 \\ -k & 2k & -k & 0 \\ 0 & -k & 2k & -k \\ 0 & 0 & -k & k \end{bmatrix} = \begin{bmatrix} 3600 & -1800 & 0 & 0 \\ -1800 & 3600 & -1800 & 0 \\ 0 & -1800 & 3600 & -1800 \\ 0 & 0 & -1800 & 1800 \end{bmatrix}$$

The damping matrix $\mathbf{C}$ of the undamaged system is assumed proportional to mass and stiffness matrices as follows

$$\mathbf{C} = a_0\mathbf{M} + a_1\mathbf{K} \qquad a_0 = 0.3150\,\text{s}, \; a_1 = 0.0003\,\text{s}^{-1}$$

A local damping variation at the third story simulates damage; thus, the damping matrix $\mathbf{C}_d$ of the damaged system is

$$\mathbf{C}_d = \mathbf{C} + \varepsilon \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & c_3 & -c_3 & 0 \\ 0 & -c_3 & c_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where a value of $25\,\text{Nsm}^{-1}$ is adopted for the coefficient c_3. Since the mass and stiffness matrices of the damaged system are the same of the intact case, the damping matrix $\mathbf{C}_d$ is clearly related to non-proportional damping. In what follows, the damage evolution parameter ε is posed to range in the interval $[0, 0.5]$.

Before proceeding with the inverse problem, the sensitivity of the solution of the direct problem to the perturbation approach is studied first. Fig. 4 shows the damage indices in Eqs. (1), (3)–(5) obtained using the mode shapes provided by the exact solution (Eq. (6): the relevant curves bring the label ES), and according to the perturbation approach (Eqs. (8)–(9): the relevant curves bring the label PA with the specification 1^o and 2^o, standing for the first- and second-order formal series expansion, respectively). As usual in perturbation approaches, the PA curves are initially close to the exact ones, then they diverge from the latter as long as the damage parameter increases, due to the truncation of the higher order terms in the formal power series expansion in Eq. (7). Under reasonable regularity hypotheses, higher order terms in the formal expansion would imply that the solution according to the perturbation approach deviates from the exact one for larger values of the evolution parameter. We remark that the

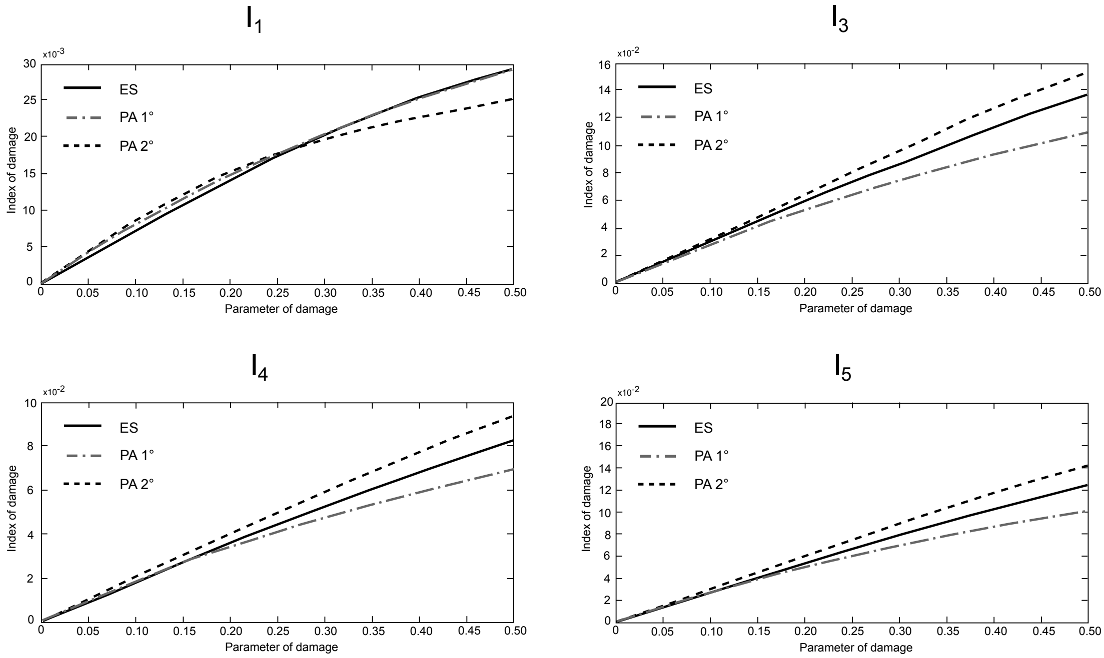

Fig. 4. Damage indexes for $\varepsilon \in [0, 0.5]$.

analysis performed by the modal complexity indexes 3 to 5 is so that the actual curve is always enclosed between the two approximating ones provided by the perturbation approach. This monotonicity condition could be of some importance in the applications. In addition, it is not surprising that the first (second) order curve in Fig. 4 does not represent the best linear (quadratic) approximation of the actual curve, since the formal series expansion refer to the eigensolutions and not to the damage indexes.

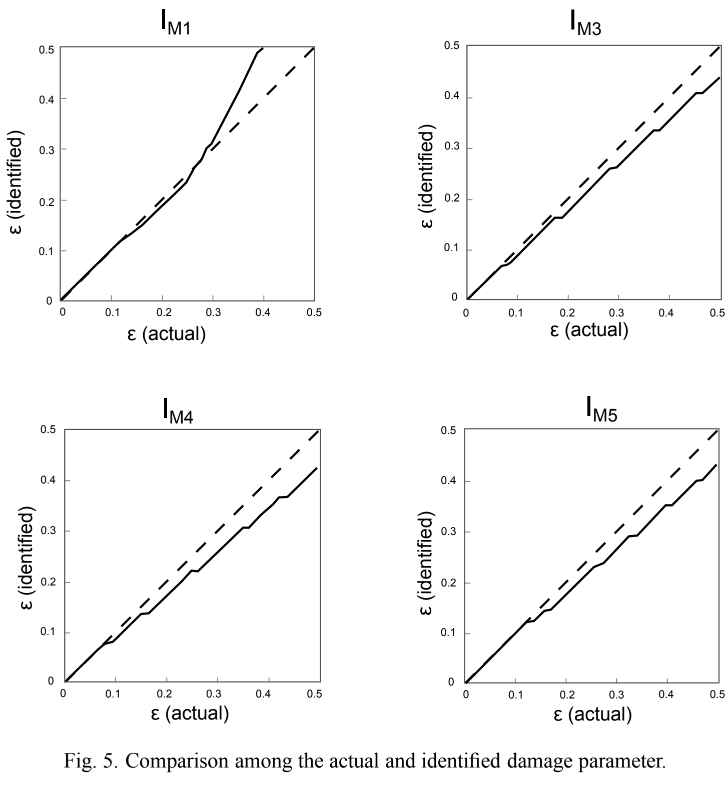

Fig. 5. Comparison among the actual and identified damage parameter.

The inverse problem is coped with by adopting the second-order formal series expansion. Fig. 5 shows plots of the identified damage evolution parameter versus the actual one provided by the assumption on the modified damping; if identification were perfect, the two values should coincide, hence yielding the bisecating dashed line. In reality, Fig. 5 shows that the identified damage evolution parameter deviates from the actual one in a similar way to what happens in the direct problem. In detail, until a value $\varepsilon \approx 0.3$ (i.e., for a small evolution of damage from the intact shape), the identification scheme is effective and accurate, regardless of the definition of the damage index. Indeed, as we already remarked, with the exception of the modal complexity index I_2, which was not found suitable for this analysis and was dropped either in the direct and in the inverse problem, all the other indexes provide good information on the damage evolution. In addition, also for the inverse problem we observe the different behaviour of the identification provided by the indexes I_3–I_5 with respect to I_1: the former always underestimate the actual damage, the latter crosses the line of perfect identification and may overestimate the actual damage - this may be interesting in the applications. For values of the damage evolution parameter $\varepsilon > 0.3$, the discrepancies between the two lines cannot in any case be neglected. It has to be remarked, however, that these results are confined to the knowledge of only the first two perturbation terms and use of higher order terms could improve the identification capability, without affecting the identification scheme. Thus, it seems that the proposed procedure may actually be effective.

4. Conclusions

A perturbation approach for the dynamic identification of damaged structures has been proposed. The fundamental hypothesis is that localised damages can be related to non-proportional structural damping, and, hence, to complexification of the natural modes, which are real in undamaged structures. In the literature we already find indexes of such a complexification.

In detail, this contribution focuses on the development of a numerically effective procedure able to detect, locate and quantify damage. The starting point is the assumption of localised, ‘small’ damages, that is, from an analytical standpoint, damages that can be viewed as ‘small’ perturbations of the initial (healthy) model.

The proposed method, based on the minimisation of an objective function (which was not reported for the sake of brevity, but which is ineffective for the generality of the results provided), is independent of the truncation of the formal series expansion of the perturbation approach, and inherits the advantages of the latter, ensuring a low computational effort.

Lastly, the effectiveness of the method has been proved through the application to a shear-type multi-story frame reduced to a discrete system with 4 dof. The structural damage was simulated as an increase of the dissipative capacity localised at the third floor, and both the direct and the inverse problem were solved. The results show that the direct problem is well described by the adopted perturbation approach, and that the identification technique in the inverse problem looks actually effective.

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