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## Numerical and experimental analysis of the leaning Tower of Pisa under earthquake

Gabriele Fiorentino<sup>a\*</sup>, Davide Lavorato<sup>a</sup>, Giuseppe Quaranta<sup>b</sup>, Alessandro Pagliaroli<sup>c</sup>,  
Giorgia Carlucci<sup>d</sup>, Camillo Nuti<sup>a,e</sup>, Fabio Sabetta<sup>f</sup>, Giuseppe Della Monica<sup>d</sup>, Maurizio  
Piersanti<sup>d</sup>, Giuseppe Lanzo<sup>g</sup>, Giuseppe Carlo Marano<sup>h,e</sup>, Giorgio Monti<sup>g,i</sup>, Nunziante  
Squeglia<sup>j</sup>, Raffaello Bartelletti<sup>j</sup>

<sup>a</sup>Department of Architecture, Roma Tre University, Via Giovanni Battista Marzi 10, Rome 00152, Italy

<sup>b</sup>Department of Structural and Geotechnical Engineering, Sapienza University of Rome, Via Eudossiana 18, Rome 00184, Italy

<sup>c</sup>Department of Engineering and Geology, University of Chieti – Pescara, viale Pindaro 42, Pescara 65129, Italy

<sup>d</sup>Department of Science, Roma Tre University, Via della Vasca navale 84, Rome 00146, Italy

<sup>e</sup>College of Civil Engineering Fuzhou University, 2 Xue Yuan Road, Fuzhou 350108, China

<sup>f</sup>Department of Civil Protection, via Vitorchiano 4, Rome 00189, Italy

<sup>g</sup>Department of Structural and Geotechnical Engineering, Sapienza University of Rome, Via Gramsci 53, Rome 00197, Italy

<sup>h</sup>Department of Science of Civil Engineering and Architecture, Technical University of Bari, Via Orabona 4, Bari 70125, Italy

<sup>i</sup>College of Civil Engineering, Nanjing Tech University, 30 South Puzhu Rd., Nanjing 211816, China

<sup>j</sup>University of Pisa, Via Diotallevi 2, Pisa 56126, Italy

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### Abstract

Twenty years have passed from the most recent studies about the dynamic behavior of the leaning Tower of Pisa. Significant changes have occurred in the meantime, the most important ones concerning the soil-structure interaction. From 1999 to 2001, the foundation of the monument was consolidated through under-excavation, and the "Catino" at the basement was rigidly connected to the foundation. Moreover, in light of the recent advances in the field of earthquake engineering, past studies about the Tower must be revised. Therefore, the present research aims at providing new data and results about the structural response of the Tower under earthquake. As regards the experimental assessment of the Tower, the dynamic response of the structure recorded during some earthquakes has been analyzed in the time- and frequency-domain. An Array 2D test has been performed in the Square of Miracles to identify a soil profile suitable for site response analyses, thus allowing the definition of the free-field seismic inputs at the base of the Tower. On the other hand, a synthetic evaluation of the seismic input in terms of response spectra has been done by means of a hybrid approach that combines Probabilistic and Deterministic Seismic Hazard Assessment methods. Furthermore, natural accelerograms have been selected and scaled properly. A finite element model that takes into

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\* Corresponding author. Tel.: +39-06-57332908; fax: +39-06-57332940.

E-mail address: [gabriele.fiorentino@uniroma3.it](mailto:gabriele.fiorentino@uniroma3.it)

account the inclination of the structure has been elaborated, and it has been updated taking into account the available experimental results. Finally, current numerical and experimental efforts for enhancing the seismic characterization of the Tower have been illustrated.

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## 1. Introduction

The leaning Tower of Pisa is one of the most famous monuments in the world, especially because of its peculiar inclination. The total height of the Tower is equal to 58.4 m from the base foundation and its cross-section is ring-shaped, with an external diameter equal to 19.6 meters at the base. The current tilt of the structure is about  $5.5^\circ$  in the N-S direction. The inclination was reduced by around  $0.5^\circ$  as consequence of stabilization works made on the Tower between 1999 and 2001. The estimated weight of the Tower is 14.453 tons and the height of the center of mass from the base foundation is 22.6 m. The response of the Tower under earthquake was first studied by Grandori and Faccioli [1], who presented the results of dynamic analyses performed on a simplified finite element (FE) model of the Tower in which the seismic input was defined in terms of response spectrum. As regards the experimental assessment of the monument, the modal parameters were identified by ISMES [2] from forced vibrations due to a vibrodyne whereas there are no recent data about the seismic response of the Tower. Therefore, the current state of knowledge about the seismic behavior of the Tower has to be updated and improved.

## 2. Seismic input and soil characterization

### 2.1. Seismic input

Seismic hazard assessment has been performed by combining a probabilistic approach (PSHA) and a deterministic one (DSHA). SP96 [3] and AB10 [4] Ground Motion Predictive Equations (GMPE) have been selected. Uniform Hazard Spectra (UHS) on rock were computed for return periods (RPs) equal to 130 years and 500 years. These values are based on the correlations between MCS intensity and RP, already used by Grandori and Faccioli [1]. Disaggregation results were used to look for controlling earthquakes. Based on the Italian seismic catalogue CPTI15 [5], it was possible to find two controlling earthquakes: a M 5.15 seismic event with  $R_{epi}=19$  km has been selected for 130 years RP (Livorno 1742) whereas a M 5.71 earthquake with  $R_{epi}=21$  km (Orciano Pisano 1846) has been considered for 500 years RP, and they are related to MCS intensities VI and VII, respectively. The target response spectrum for EC8 class B site was evaluated by means of the Akkar and Bommer GMPE, including the subsoil term of the equation depending on the  $V_{s,30}$  value of the site. Eight accelerograms were selected for each RP from the European Strong Motion Database [6], considering  $5 < M < 5.5$  for 130 years RP and  $5.3 < M < 6.2$  for 500 years RP. The selected components of the horizontal accelerograms have been scaled in such a way that the average spectrum of each set of accelerograms well approximates the target spectrum for Soil B. This task has been accomplished using In-Spector software [7]. The scaling was made in the range of the fundamental periods 0.3 s-1.1 s in order to take into account the periods of the first two bending modes (about 1 s) and that of the third mode (about 0.3 s), thus obtaining the proper scale factor SF for each record (see Fig.1). In order to obtain the vertical time histories on Soil B, each original vertical record taken from ESMD database was scaled with the corresponding SF (see Fig. 1).

### 2.2. Geophysical tests

A 2D array test was designed to obtain a shear-wave velocity profile of the soil underlying the Square of Miracles. This kind of test can reach a depth of approximately 100 m, much greater than that obtained with the existing Down-Hole and Cross-Hole tests made in the past years in the Square. The measurement was performed

using nine REFTEK130 stations equipped with Lennartz 3D 5s velocimeters and placed according to a triple equilateral triangle geometry. The central station was located near the Baptistery.

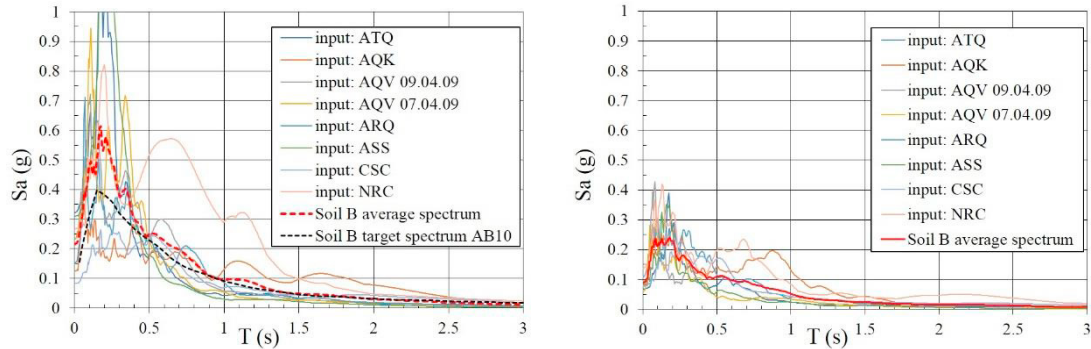


Fig. 1 Spectrum-compatible horizontal acceleration time histories for EC8 Soil B with 500 years RP: horizontal (left) and vertical (right) components

The software GEOPSY was used for the analysis [8]. The elaboration of the recordings has revealed the presence of a rigid layer ( $V_S \approx 500$  m/s) at a depth of about 100 m (Fig. 2). A single station analysis was performed within the same test in order to evaluate H/V spectral ratios, from which a resonance peak at 1.3 Hz related to the interface at 40 m depth has been found. Another peak, due to a deeper layer, was identified at 0.3 Hz. Moreover, the H/V curve with the identified resonance peaks is reported in Fig. 2.

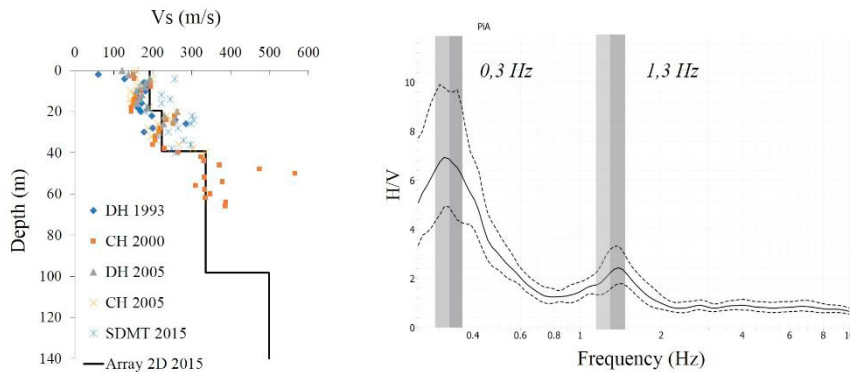


Fig. 2 Comparison between soil profile from array 2d test and old DH/CH tests (left) and H/V spectral curve for a single station (right)

### 2.3. Subsoil model and site response analysis

The subsoil model adopted for site response analyses is reported in Table 1. The subsoil of the Tower area was proposed by Viggiani and Pepe [9] and it takes into account the geotechnical investigations carried from 1907 to 1993. According to this study, three distinct horizons can be distinguished: A (sandy and clayey silt), B (marine clays) and C (dense sand), which can be further subdivided into the lithotypes described in Table 1. Thickness and unit weight for each lithotype were assumed according to the data reported in Ref. [9]. The assumed shear wave velocity profile is based on the outcomes of the 2D seismic array analysis carried out in the present work. However, it should be pointed out that this profile satisfactorily matches the  $V_S$  values measured by other geophysical tests in the upper part (SDMT tests carried out in 2015 up to 40 m and a 65 m deep cross-hole test in 1999). According to the  $V_S$  profile, the seismic bedrock ( $V_S > 800$  m/s) is not localized in the explored depth range because  $V_S = 500$  m/s is reached in the lower sand (horizon C). For this reason, considering the uncertainties in the  $V_S$  profile at higher depth, the input motion for site response analyses was defined at EC8 class B soil (instead of rock conditions), as

described in Section 2.1. Regarding the nonlinear properties, most of lithotypes were characterized based on the resonant column (RC) tests [10].

Table 1. Subsoil model adopted for site response analyses. Legend: LT=Lithotype, ΔH=layer thickness, γ=unit weight, V<sub>s</sub>=shear wave velocity, NL=Nonlinear Characterization, BR=Bedrock

LT	ΔH (m)	γ (kN/m <sup>3</sup> )	V <sub>s</sub> (m/s)	NL	LT	ΔH (m)	γ (kN/m <sup>3</sup> )	V <sub>s</sub> (m/s)	NL
MG	3.0	18.50	180	[13] average	B7	4.6	18.62	230	RC tests
A1	5.4	18.94		DSDSS test S4-C2 σ <sup>'</sup> <sub>v</sub> =65 kPa	B8	1.4	18.41		RC tests
A2	2.0	18.07		[12] PI=30 σ <sup>'</sup> <sub>v</sub> =55 kPa	B9	4.0	19.01		RC tests
B1	3.5	17.00		RC tests	B10	2.6	19.38		RC tests
B2	2.0	17.49		RC tests	C1	27.6	20.52	340	[12] PI=0 σ <sup>'</sup> <sub>v</sub> =350 kPa
B3	4.9	16.67		RC tests	C2	11.1	20.52		[12] PI=15 σ <sup>'</sup> <sub>v</sub> =500 kPa
B4	1.2	19.48		RC tests	C3	16.3	20.52		[12] PI=0 σ <sup>'</sup> <sub>v</sub> =600 kPa
B5	3.0	19.76	230	RC tests	BR (C3)	-	21.00	500	-
B6	2.4	19.11		[12] PI=8 σ <sup>'</sup> <sub>v</sub> =200 kPa					

The A1 lithotype, for which no cyclic data were available, was characterized within this study through DSDSS (Double Specimen Direct Simple Shear) tests conducted on a soil sample extracted at 6.3 m depth (S4-C2) [11]. The cyclic tests were conducted at various vertical effective consolidation stresses σ<sup>'</sup><sub>vc</sub> (65-131-262 kPa) and the corresponding results are reported in Fig. 3 in terms of normalized secant shear modulus and damping ratio variation as function of the shear strain amplitude (G/G<sub>0</sub>-γ and D-γ curves).

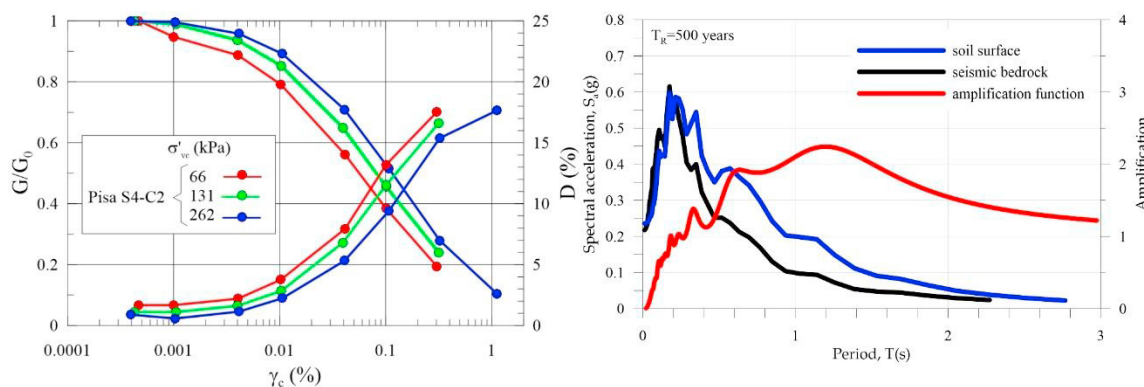


Fig. 3. G/G<sub>0</sub> and D curves obtained through DSDSS test on S4-C2 sample for A1 lithotype (left); input and output average response spectra obtained from equivalent linear site response analyses carried out for 500 years RP; average amplification function between seismic bedrock and soil surface is also reported (right).

Because of the lack of experimental data, literature curves obtained for similar soils were employed for remaining lithotypes [12-13] (see Table 1). Site response analyses were carried out with the 1D frequency domain equivalent linear STRATA code [14]. The results for a return period of 500 years are reported in Fig. 2 in terms of horizontal acceleration response spectra computed at ground surface (averaged over all input motions applied). Average input spectrum at seismic bedrock and nonlinear amplification function between seismic bedrock and soil surface are also reported for comparison. Moderate amplification phenomena take place in the medium-to-long periods with a maximum amplification ratio slightly higher than 2 around 1.2 s, where the corresponding average spectral accelerations are about 0.2g. Spectral accelerations as high as 0.5-0.6g (as average) are achieved at the ground surface in the 0.2-0.4 s period range.

### 3. Experimental seismic response

The Tower is equipped with a network of accelerometers designated for continuous seismic monitoring. The available seismic records were analyzed using different tools, such as Fast Fourier Transform (FFT), Continuous Wavelet Transform (CWS) and Wavelet Cross Spectrum (WCS). The CWT of the seismic response of the Tower recorded at S2 during a 2012 seismic event is shown in Fig. 4. The analyses have allowed the identification of the frequencies of the first four modes of the Tower. The first two are bending modes in N-S and E-W direction, respectively, both with a frequency close to 1 Hz. The third is a vertical mode with a frequency about 3 Hz. It is interesting to remark that the only evidence in the literature about the vertical mode of the monument is given by Nakamura [15]. The results of the seismic monitoring are discussed in Ref. [16].

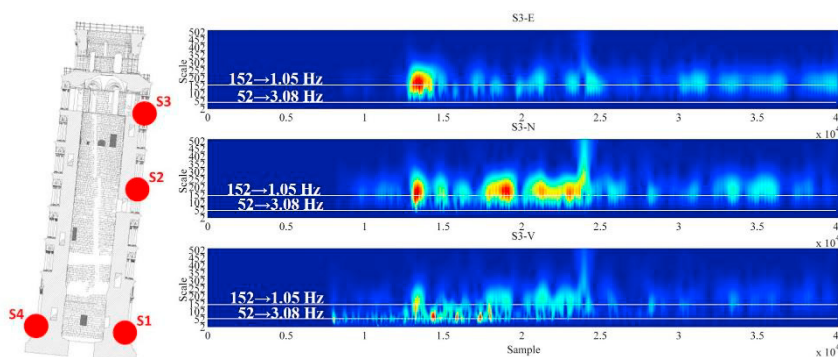


Figure 4. Location of sensors on the Tower (left) and CWT of the response recorded at S3 (right)

### 4. Modal analysis of the Tower including Soil-Structure Interaction

A simplified FE stick model was built considering the inclination of the Tower in the N-S direction [17]. It consists of 14 elements and 16 nodes, with 6 degrees of freedom per node. For each story of the Tower, the coordinates of the centroid were defined according to the work by Macchi and Ghelfi [18]. For each centroid, 3 translational masses and 3 rotational masses were defined. Geometric parameters were taken from the work by Grandori and Faccioli [1]. Three translational springs and rotational springs were assigned at the base of the model. Table 2 shows the comparison between the results of the modal analysis and the frequencies obtained experimentally. It is possible to observe that, for a nominal value of  $G=77326 \text{ kN/m}^2$ , the frequencies obtained by considering the foundation alone and the foundation with the "Catino" are 0.873 Hz and 0.884 Hz, respectively.

Table 2. Comparison between numerical and experimental frequencies before and after model updating

Exp. mode	Exp. frequency (Hz)	Foundation alone		Foundation with "Catino"		Model updating
		$G=77326 \text{ kN/m}^2$	$G=95000 \text{ kN/m}^2$	$G=77326 \text{ kN/m}^2$	$G=95000 \text{ kN/m}^2$	
Bending N-S	0.958 Hz	0.873 Hz	0.958 Hz	0.884 Hz	0.971 Hz	0.950 Hz
Bending E-W	1.025 Hz	0.873 Hz	0.958 Hz	0.885 Hz	0.971 Hz	1.025 Hz
Vertical	2.98 Hz	2.822 Hz	3.12 Hz	2.829 Hz	3.128 Hz	2.964 Hz
Torsional	6.29 Hz	4.309 Hz	4.729 Hz	5.925 Hz	6.432 Hz	6.294 Hz

A sensitivity analysis based on the modulus  $G$  has been performed to look for a better agreement with the experimental evidences. A satisfactory result was found for  $G=95.000 \text{ kN/m}^2$ , which leads to a natural frequency equal to 0.958 Hz for the first and the second (bending) mode whereas the frequency calculated for the vertical mode was found equal to 3.12 Hz. The model updating has been performed on the elements of the impedance matrix

of the foundations in order to obtain an improved agreement between the natural frequencies estimated experimentally and those obtained from the modal analysis.

## 5. Conclusions

Recent advances on the numerical and experimental characterization of the dynamic behavior of the leaning Tower of Pisa have been presented in this study, together with an updated definition of the seismic input. The definition of the seismic input on EC8 Soil class B was presented in terms of accelerograms obtained by means of a hybrid approach which combines PSHA and DSHA. New geophysical and geotechnical tests were carried out in the Square of Miracles, including array 2d and H/V tests, which allowed to identify a soil B layer ( $V_s \approx 500$  m/s) at a depth equal to 100 m. Cyclic tests on the A1 lithotype was also carried out. These results were used to build a subsoil model that was employed to perform a site response analysis, thus obtaining the free-field input ground motion. The numerical model built on the base of the experimental analyses will be used to obtain the dynamic response of the Tower including soil-structure interaction.

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