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# The Choice of the Functional Form in the Consumption Euler Equation: a Critical View

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# Contents

# Abstract

This thesis deals with some analytical questions that arise in the modern theory of consumption based on the intertemporal utility maximization model, also known as the Euler equation approach. It thus builds on the theoretical and empirical literature which has stemmed from the seminal contribution by Hall (1978), who extended the basic life cycle – permanent income model of consumption to the case of uncertainty. In reviewing this literature and the theoretical developments and extensions that the original model has undergone due to the need of accounting for the puzzling empirical evidence, the thesis aims at highlighting the crucial role that the preference parameters play in the specification of the theoretical content of the model and its predictions, and in the definition of the policy implications that may be drawn from it.

Crucial as the role of preference parameters may be both in theory and practice, our analysis reveals, also by making use of some simulation exercises, the substantial inability of the Euler equation approach to give definite content to such parameters, due to the heavy dependence of the results of parameter estimation on the specification of the utility function.

The first paper offers a survey of the literature which, following Hall (1978), has engaged in the task of testing the model against empirical evidence and proposing successive extensions and refinements of the original model ending up in the current enriched versions of it. The paper draws on other surveys but focuses in particular on an overall assessment of the theoretical implications of the extensions which the literature has proposed. The second paper focuses on the literature directly aiming to estimate the parameters characterizing preferences – offering in this respect a specific survey which seems to be lacking in the literature – and highlights the crucial role of the specification of the utility function in such estimates. Drawing on this result, the third paper proposes a number of original simulation exercises aimed at showing that the same time profile of consumption and saving may give rise to the estimation of rather different values of the crucial preference parameters depending on the particular utility function adopted.

# The Euler equation approach: theory and evidence

## 1. Introduction

The modern theory of consumption rests on the idea that individuals maximize lifetime utility subject to an intertemporal budget constraint. The idea is based on the Life Cycle – Permanent Income Hypothesis developed by Modigliani and Brumberg (1954) and Friedman (1957). In his seminal contribution, Hall (1978) extended the model to the case of uncertainty with the introduction of the rational expectations assumption and proposed to use the first order conditions of the intertemporal optimization problem faced by the consumer to derive a set of orthogonality conditions. In the framework considered by Hall, the basic implication of the equilibrium condition of the model – the Euler equation – is that, conditional on current consumption, other current variables, including income, do not help in predicting future consumption. Starting with Hall, the literature focused on testing the model of intertemporal utility maximization relying on the Euler equation and a new approach to consumption, often referred to as the Euler equation approach, has been established.

The early empirical tests of the formulation proposed by Hall found several results that apparently contradicted theoretical predictions. A number of empirical puzzles thus arose. These are known in the literature as: a) excess sensitivity; b) excess smoothness; c) hump in the age profile of consumption; d) retirement puzzle; and d) equity premium puzzle. The subsequent literature tried to provide an interpretation of these empirical puzzles and to progressively modify and enrich the original version of the model so as to render it able to explain the data.

This paper is devoted to the reconstruction of the evolution of the theoretical view of consumer behaviour stemming from the Euler equation approach to consumption. A survey will be proposed of the contributions highlighting the various empirical puzzles and proposing extensions and refinements of the basic model aimed at reconciling theoretical predictions and empirical evidence. In the literature on consumption, the link between empirically-oriented contributions and theoretical developments is in fact quite strong, given the fact that the theoretical evolution of the approach has been strongly influenced by the need to face the empirical puzzles and to find more sophisticated versions of the model that could account for observed facts. This has implied a number of relevant theoretical changes with respect to the original formulation.

Following some well-documented and authoritative surveys, among which Browning and Crossley (2001), Attanasio and Weber (2010), and Jappelli and Pistaferri (2010), we shall here in the first place address the basic question that such reconstructions have addressed, i.e. to assess whether the model, after being so modified and enriched, is now able, in its most sophisticated versions, to adequately replicate the empirical evidence. Attanasio and Weber (2010), on the basis of a very careful and detailed analysis of the various contributions, conclude that this is indeed the case:

> One possible reading of the empirical literature on the life cycle model is that it is possible to construct rich versions of the model that are not inconsistent with available micro data, especially for households headed by prime aged individuals (Attanasio and Weber, 2010, p. 741).

In the present survey, also considering the little updating of the literature that is necessary, we shall find that this result can be substantially confirmed. With respect to Attanasio and Weber (2010), however, the present survey also tries to focus on other two research questions, which, though sparsely hinted at in various contributions, have nonetheless received less systematic attention up to now.

The first of such questions is to evaluate to what extent, and in which directions, the richest versions of the model, in order to achieve the sought-for adherence to empirical evidence, modify the theoretical view on consumption with respect to the original formulation. In particular, the question we shall address is whether the necessary introduction of highly specific assumptions both on preferences and on the budget constraint affects the generality of the theoretical conclusions of the model. As we hope to show, some problematic issues are indeed open in this respect.

The second question has to do with the applicability of the theoretical and empirical analysis of consumption in the Euler equation approach to policy problems. As will be seen below in greater detail, in fact, for all the general theoretical consensus it enjoys, the approach is little used in the design and evaluation of public policy. We shall try to show how this depends, at least in part, on the lack of generality of the policy implications that may be drawn from the various versions of the model, and on the difficulty surrounding the identification of the value of the crucial parameters.

## 2. The consumption Euler equation

The roots of the Euler equation approach lie in the life cycle model proposed by Modigliani and Brumberg (1954) and in the permanent income hypothesis formulated by Friedman

(1957). The basic insight of both these theories is that individuals prefer smooth paths of consumption over the life cycle and thus use saving to prevent income variability from causing large fluctuations in consumption. The theory proposed by Friedman usually applies to a context in which the individual faces an infinite time horizon and is particularly focused on the consumer's attempts to smooth short-run fluctuations in income, whereas the life-cycle model considers a finite horizon and is therefore more oriented to the study of retirement saving. However, the essential implications of the two theories are the same, so that the literature often refers to them as a single theoretical framework, called Life Cycle - Permanent Income Hypothesis (LCPIH).

Modigliani and Friedman contributions nourished the debate on the consumption function originated from the seminal work of Keynes (1936), which took place in the 1940s and 1950s. In fact, when researchers tried to verify empirically the main implications of Keynes's analysis of consumption, several empirical puzzles arose. In particular, the marginal propensity to consume out of income appeared lower when estimated on short run data (cross section) and higher in the long run (in time series). Many scholars thus entered the debate both with empirical contributions and with possible explanations of the observed data. When the LCPIH was proposed, a general consensus was reached and it became the standard theory of consumption. In fact, the LCPIH provides a theory with microeconomic foundations based on individual optimization and is therefore consistent with the methodological premises of modern economic theory and, at the same time, it apparently succeeded in explaining all the main stylized facts about consumption.[1]

Once a consensus was established on the explanation of consumption behaviour, the subsequent theoretical step was an extension of the model that provided a proper treatment of uncertainty. Hall (1978) filled this gap by introducing the rational expectations hypothesis.

The framework considered by Hall is that of an individual endowed with a state and time separable utility function $U(C_1, C_2, \dots)$ which is, as usually assumed, concave increasing in each argument; the consumer has a constant discount factor $1 + \delta$ – with $\delta$ representing the rate of time preference – and faces perfect capital markets. Assuming that the individual maximizes the expected utility of consumption over a certain time horizon subject to an intertemporal budget constraint and a terminal condition on wealth, one obtains

[1] Among the explanations of the empirical evidence proposed in the 1940s, an influential contribution is the Relative Income Hypothesis formulated by Duesenberry (1949). For a survey of the debate on the consumption function and a comparison between Duesenberry's and Friedman's theories, see Trezzini (2012).

the well-known Euler equation for consumption:

$$u'(C_t) = (1+\delta)^{-1}\mathbb{E}_t[(1+r_t)\cdot u'(C_{t+1})] \tag{1}$$

where $u(C_t)$ is the instantaneous utility function and $r_t$ is the stochastic interest rate between $t$ and $t+1$.

The Euler Equation states that in equilibrium there are no intertemporal consumption reallocations that increase the consumer's utility at the margin, i.e. the consumer aims at keeping the marginal (discounted by $1+\delta$) utility of (discounted by $r$) expenditure constant over time.

The optimality condition of the intertemporal maximization problem faced by the consumer thus provides clear theoretical implications: it implies that, *ex ante*, current marginal utility is the best predictor of next period's marginal utility and, *ex post*, marginal utility changes only if expectations are not realized. Therefore, changes in marginal utility should be unpredictable on the basis of past information. For instance, an anticipated income decline should not affect the marginal utility of consumption at the time it occurs because the consumer would have already incorporated the expectation of the income decline in his optimal consumption plan when the information first became known.

The seminal idea of Hall was to derive a set of orthogonality conditions from the consumption Euler equation which allow to both test the validity of the model and to estimate the structural parameters of the utility function without solving the maximization problem of the consumer and finding a closed form solution for consumption.[2] However, in its more general formulation, the Euler equation is consistent with many types of consumption behaviour and has almost no testable implications, since its theoretical predictions concern the *marginal utility* of consumption, which is not observable. Taking the model to the data requires to make some modelling choices and to specify individual preferences, so as to derive predictions about consumption itself rather than its marginal utility. At the same time, once some basic assumptions about insurance and credit markets and a parameterization for the utility function are provided, the Euler equation is a very convenient tool. In fact, it allows to estimate preference parameters by observing consumption only in two different periods, as well as observing interest rates (and possibly the main demographic variables which can affect utility), but does not require to observe wealth or completely model the stochastic environment faced by the consumer. That makes the Euler equation

[2] In order to derive a solution for consumption from the Euler equation, this must be put together with the intertemporal budget constraint, which requires making a number of specific assumptions about the stochastic environment in which the individual acts.

particularly attractive from an empirical point of view and explains the huge flow of econometric research it produced in the last decades. Moreover, since it considers the individual optimisation problem, it allows to overcome the Lucas critique by shifting the focus from the aggregate consumption function to the consumers' attempts to maximize their utility.[3]

In his seminal contribution, Hall found that not only the marginal utility of consumption but also consumption itself is a martingale: *ex ante* current consumption is the best predictor of the next period's consumption; *ex post*, consumption changes only if expectations are not fulfilled. Hall's result, highly influential on subsequent research, lies on several assumptions. First, the consumer is assumed to maximize expected utility, which implies that utility is additive over states of nature. This is the assumption usually adopetd in the literature, but sometimes the Von Neumann-Morgenstern framework is replaced with different axiomatic structures, such as the Kreps-Porteus axiomatization as parametrised by Epstein and Zin (1989). Second, it is assumed that preferences are additively separable over time, which means that in each period, marginal utility is independent from consumption in any other period. This precludes the consideration of durables and habit formation. In addition, utility is considered as a function of consumption as the only argument, so that consumption and labour must be separable, and consumption is considered as a single commodity, which presupposes an aggregation theorem of the type studied by Gorman (1959) or the hypothesis that there are a number of different goods, but utility is additively separable across these goods. Finally, in Hall's contribution it is assumed that preferences are quadratic and, thus, marginal utility is linear.

We shall discuss the implications of the particular framework proposed by Hall in the next sections.

## 3. Early tests of the Euler equation: the emergence of empirical puzzles

After the work of Hall (1978), the literature on consumption focused on the empirical tests of the intertemporal utility maximization model. The first of these tests was the one produced by Hall himself, who tested the implication that, conditional on current consumption, other current variables, including income, do not help in predicting future

[3] Lucas (1976) suggested that the relation between consumption, income, and interest rates depends on the wider macroeconomic framework and may not be stable over time, even though the consumers are always maximizing the same utility function.

consumption, or, equivalently, no variables known in period t−1 (and earlier) should be correlated with changes in consumption between t−1 and t.

Hall (1978) found that on macro U.S. data neither lagged consumption nor lagged income terms are significant, thus corroborating the main prediction of his model.[4] Shortly after Hall's contribution, though, the empirical research found several rejections of Hall's result.

The first empirical failure of the model, which is also the one which received greater attention and generated extensive empirical work, is the excess sensitivity puzzle, highlighted by Flavin (1981), Campbell (1987) and Campbell and Mankiw (1989). In her contribution, Flavin (1981) considers an intertemporal utility maximization model in which income is assumed to follow an autoregressive process and estimates jointly the consumption and income equations, finding evidence of excess sensitivity. In fact, according to her estimates, consumption does depend on predicted income changes, contradicting the prediction that, if the variation of income is anticipated, it should already be incorporated in the behaviour of the consumer and not cause any impact on consumption when it occurs.[5]

Campbell (1987) suggested that, according to Hall's model, saving should encapsulate the superior information of the individual over the econometrician and thus help to forecast income. He specified a model for the joint consumption-income process consistent with this idea but rejected the hypothesis.

Of particular influence, though, is the rejection of the model presented by Campbell and Mankiw (1989). Regressing changes in aggregate U.S. log consumption on interest rates and changes in log disposable income, they found that the latter variable attracted a coefficient of 0.4, statistically different from zero, even after instrumenting current variables with lagged ones to avoid picking up the effects of innovations to the level of permanent income. Thus, they estimated that 45% of U.S. households do not behave as utility maximizing consumers, but, instead, follow a "rule-of-thumb", i.e., they set current consumption equal or proportional to their current income. This clearly constitutes a strong violation of the theory, since it implies that an impressive portion of individuals deviates from rational behaviour. Campbell and Mankiw (1991) replicate the test for a variety of

[4] Hall (1978) employs quarterly, seasonally adjusted U.S. data excluding expenditure on durables, covering the period 1948-1977. In his work, the orthogonality conditions are satisfied, but he does not include both income and interest rates together in the estimation. In addition, he does find that lagged stock market prices have explanatory power.

[5] Flavin's test indicated that the U.S. consumption response to an income innovation was over three times the value predicted by the model.

other countries and confirm the presence of a large number of so-called "Keynesian" consumers.

The evidence of excess sensitivity of consumption was associated with the symmetric but opposite empirical problem: not only consumption appeared to be too *sensitive* to predicted income changes, it also turned out to be too *unsensitive* to unexpected changes in income. According to Hall's model, in fact, consumption should not react to anticipated changes in income, but, when the individual faces a variation of income that is not foreseen, this should affect consumption in the moment it occurs. Empirical evidence suggests instead that consumption does not react enough to unanticipated income changes, leading to excess smoothness. The puzzle was first presented in Deaton (1986), West (1988), Campbell and Deaton (1989) and Flavin (1993).

Deaton (1986) points out that if real disposable income can be adequately represented as a first-order autoregressive process in first differences, as the data seem to suggest, then the life-cycle model implies that changes in consumption should be more variable than innovations in income. Relying on aggregate time-series data from the U.S., he shows that the empirical evidence contradicts this prediction. West (1988), applying a variance bounds test, under the hypothesis adopted by Deaton (1986) that income has a unit root, also finds that consumption is less sensitive to news about income than the theoretical model would predict. Campbell and Deaton (1989) consider the evidence suggesting that the smoothness of consumption cannot be straightforwardly explained by the theory and argue that in postwar U.S. data consumption is smooth because it responds with a lag to changes in income. They also investigate the connection between excess smoothness and excess sensitivity and conclude that the two puzzles reflect the same failure of the model. Flavin (1993) generalizes Campbell and Deaton (1989)'s analysis and argues that the excess sensitivity hypothesis, i.e. a non-zero marginal propensity to consume out of transitory income, implies the finding that consumption is excessively smooth, in the sense that the variance of consumption innovations is smaller than the variance of innovations in permanent income. Her empirical analysis supports the notion that consumption is too smooth: the standard deviation of consumption disturbances is estimated to be about 2.2%, against an estimated standard deviation of innovations in permanent income of about 4.0%.

The most important implication of the standard model is that the time path of income is irrelevant for consumption because individuals use borrowing and saving to smooth fluctuations in income. Carroll and Summers (1991), in a very popular paper, show that life cycle profiles of income and consumption track each other: they both increase during the first part of the life cycle to reach a peak a few years before retirement and decline afterwards. For many countries and different groups of individuals, both income and consumption profiles appear to be "hump shaped", contradicting one of the main predictions of the intertemporal utility maximization model.

Related to this empirical problem is the one referred to as the retirement consumption puzzle, first highlighted by Hamermesh (1984) and then further investigated by Bernheim (1987) and Banks, Blundell, and Tanner (1998). Hamermesh (1984) analysed the relationship between consumption and lifetime wealth and originally suggested that consumers retire with inadequate savings, contradicting the prediction of the utility maximizing model according to which individuals should accumulate wealth during their working life and then start decumulating it to keep a level of consumption consistent to the one afforded before retirement. Bernheim (1987), by following a sample of retired individuals over time, provides evidence indicating that significant dissaving may occur after retirement, particularly among single individuals and early retirees. He also constructs a test of the utility maximization principle using information on the age-wealth profile and concludes that the theory fails to account for savings behaviour after retirement.

Banks, Blundell, and Tanner (1998), analysing data for the UK, find that around two-thirds of the drop in consumption growth at retirement that occurred for those cohorts retiring in the 1970s, 1980s, and early 1990s is consistent with an intertemporal utility maximizing model, but the remaining third suggests that at least some individuals had not saved enough. To the work by Banks, Blundell, and Tanner (1998), was then associated similar evidence found for the U.S. by Bernheim, Skinner, and Weinberg (2001) and for Italy by Miniaci, Monfardini, and Weber (2003 and 2010).

A further empirical problem has arisen in the finance literature. Given the historically high equity premium (the difference between the average return on the stock market and the return on short-term government debt), asset markets equilibrium requires consumers to have very high risk aversion, in contrast with the empirical evidence on the magnitude of the parameter and with equilibrium conditions for the risk-free interest rate. This problem, known as the equity premium puzzle, has been first highlighted by Mehra and Prescott (1985), who employ data for the U.S. from 1889 to 1978 indicating that the average equity premium was 618 basis points. Calibrating an asset pricing model with what they consider reasonable values of the preference parameters (including a coefficient of relative risk aversion below or equal to 10) they are not able to produce more than a 35 basis point equity premium: the high equity premium observed would require consumers to have implausibly high (in the 20-30 range) risk aversion.[6]

Overall, the empirical evidence accumulated in the first decades of the Euler equation approach appears to essentially violate the basic implications of the model. As a consequence, subsequent research focused on interpreting the available empirical evidence, improving the empirical quality of the tests conducted and, at the same time, extending the baseline model in order to reconcile it with the data. The next section deals with those developments.

## 4. Reconciling theory and empirical evidence: extensions of the standard model

Initially, the tests of the intertemporal utility maximization theory, such as Flavin (1981) and Campbell (1987), relied on the peculiar declination of the model proposed by Hall (1978), which includes the key hypothesis of quadratic preferences. When the utility function is quadratic marginal utility is linear, so expected marginal utility of consumption is the same as the marginal utility of expected consumption. This case is known in the literature as the certainty equivalence model, because it implies that the individual consumes the amount he would consume if his future incomes were certain to equal their means. That happens because quadratic utility, exhibiting a null third derivative, rules out precautionary saving.

When preferences are quadratic the Euler equation implies that not only the marginal utility of consumption but consumption itself should be constant over the life cycle, leading to the random walk result first found in Hall (1978). However, the impossibility of capturing the precautionary motive for saving led scholars to consider the model with quadratic preferences very misleading in the presence of uncertainty and, in the 1990s, the certainty equivalence version of the standard model was discarded.[7]

The relevance of the hypothesis of quadratic preferences suggests that some of the rejections originally found in the empirical literature may be regarded as failures of the specific version of the intertemporal utility maximization model proposed by Hall and not as violating the basic insights of the theory. The core of the theory is that individuals maximize lifetime utility subject to an intertemporal budget constraint. This implies that con-

[6] Campbell (2003) and Breeden, Litzenberger, and Jia (2015a) discuss the first stages of development of the literature aiming at explaining the puzzle and Ludvigson (2013) provides a survey of the recent contributions related to consumption-based capital asset pricing.

[7] See Blanchard and Mankiw (1988) and Caballero (1990) as the first works that express discontent with the certainty equivalence model, and Browning and Lusardi (1996) for a more extensive discussion.

sumers should smooth marginal utility, not consumption. As a consequence, the empirical evidence suggesting that individuals have fluctuating paths of consumption does not necessarily contradict the model. For these reasons, in the 1990s the literature started taking account of circumstances likely to affect marginal utility, so that demographic variables such as family composition were introduced in the Euler equation. In particular, Attanasio and Browning (1995) suggest that the excess sensitivity of consumption to income disappears when controlling for demographic variables, whereas Blundell, Browning, and Meghir (1994) and Attanasio *et al.* (1999) show that allowing demographics to affect household preferences and relaxing the assumption of certainty equivalence can generate hump-shaped consumption profiles of the kind found by Carroll and Summers (1991).

A further issue related to the preference specification relates to the impact of labour supply on marginal utility. In this respect, Attanasio and Weber (1995) suggest that if utility is a function of consumption and leisure and the two arguments of the function are nonseparable, then the saving behaviour is affected by anticipated changes in labour supply. This implies that consumption growth is positively correlated with predictable growth in hours of work. Since predicted growth in working hours is very likely to be correlated with predicted income growth, failure to control for labour supply indicators may lead to spurious evidence of excess sensitivity.

The evidence on the retirement consumption puzzle has been partially reconsidered as well. Even though it is undeniable that consumption drops at retirement, this does not necessarily mean that individuals do not save enough during their working life: some part of the drop in consumption maybe planned and related to changes in work status, thus happening without implying an increase in marginal utility. For example, Aguiar and Hurst (2005) suggest that the decrease in food expenditure highlighted in Bernheim, Skinner, and Weinberg (2001) can be explained by a shift in the amount of time spent preparing food and shopping.[8]

Together with a deeper understanding of the implications of the theoretical model and a better interpretation of the empirical tests conducted, a further step concerned the quality of the data and of the econometric techniques used in estimating Euler equations. In particular, most of the early empirical tests of the Euler equation were performed using macroeconomic data, by regressing the aggregate rate of growth of consumption on the rate of growth of income. In the 1990s, though, some papers used individual level data and

[8] For a recent review of the literature on the consumption retirement puzzle see Aguila, Attanasio, and Meghir (2011).

stressed the relevance of the aggregation issues that were at first completely neglected. Attanasio and Weber (1993) show the difference between a consistently aggregated equation, based on the means of the logarithm of consumption, and what is available in macro data (the logarithm of the mean) and suggest that consistently aggregated micro data partially solve rejections of the model that instead appear by making use of macroeconomic data.[9]

As for the econometric issues, some of the tests based on Euler equation estimation encounter difficulties that were initially disregarded or for which proper econometric procedures were not available. In particular, in the excess sensitivity test, it is very hard to find instruments for income growth that are truly exogenous and, at the same time, have good predictive power.[10]

The empirical work conducted in the 1990s, on the whole, led to a considerable reshaping of the evidence contradicting the intertemporal utility maximizing problem. Nevertheless, the main rejections of the model, even to a smaller extent, survived this more mature stage of the empirical research. For this reason, several extensions to the baseline model were proposed, with the purpose of reconciling it with the available data.

A first, important extension of the model consists in the removal of the hypothesis of perfect credit markets. If an individual who would shift resources to the present so as to increase current consumption finds it difficult to borrow, then the Euler equation will not hold as an equality. In that case, current marginal utility will be higher than discounted future expected marginal utility. In the periods in which the constraint is binding, the individual will consume his income (and run down assets completely), acting as a rule of thumb consumer of the kind considered by Campbell and Mankiw (1989).[11]

However, liquidity constraints may help explaining excess sensitivity to anticipated income increases (when current income is low relative to permanent income, if the individual cannot borrow, he consumes current income) but cannot explain why consumption reacts to anticipated income declines (when income is expected to decrease the individual can still

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[9] In addition to Attanasio and Weber (1993), the first influential papers that perform excess sensitivity tests on micro data are Attanasio and Weber (1989 and 1995), Blundell, Browning, and Meghir (1994), and Attanasio and Browning (1995). An example of the much less extensive evidence on micro data concerning excess smoothness is Attanasio and Pavoni (2011).

[10] A thorough discussion of the econometric problems related to the excess sensitivity test, and a detailed presentation of the empirical evidence concerning the puzzle can be found in Jappelli and Pistaferri (2010). For recent contributions investigating the excess sensitivity issue, see Parker (2015) and Islamaj and Kose (2016).

[11] See Zeldes (1989a), Hayashi (1985), Deaton (1991), and Jappelli and Pagano (1994).

save). In a survey of empirical consumption studies, Jappelli and Pistaferri (2010) conclude that liquidity constraints seem to play an important role in explaining why consumption responds to anticipated income changes, because consumption appears much less responsive to anticipated income declines, e.g. after retirement, when liquidity constraints have little bearing.

An excessive rate of growth of consumption may also be explained by precautionary saving. As already mentioned, Hall's random walk result rests on the assumption of quadratic preferences, which exhibit no prudence. When preferences exhibit prudence, income uncertainty reduces current consumption, and thus raises saving: prudence leads individuals to treat future uncertain income cautiously and not to spend as much currently as they would if future income were certain. The saving that results from the knowledge that the future is uncertain is known as precautionary saving.[12]

Precautionary saving interacts with liquidity constraints because the possibility of future binding borrowing constraints provides an additional motive for saving.[13] However, both the versions of the utility maximization models with liquidity constraints and with precautionary saving, need to be associated with a high rate of time preference – i.e., with a large level of impatience – in order to replicate the data.

When a significant degree of uncertainty is associated with a sufficient level of impatience, one obtains the so-called "buffer-stock model", proposed by Deaton (1991) and Carroll (1992 and 1997). In such a model consumers have a target level of liquid assets, above which impatience dominates and assets are run down, and below which the precautionary motive dominates and assets are accumulated. Thus the model predicts a positive correlation between expected income growth and consumption growth.[14]

Among the modifications to the standard model, particularly successful is the one which removes the hypothesis of intertemporal additivity of preferences, according to which marginal utility is, in each period, independent from consumption in any other period. That allows to consider habit formation, either internal (when marginal utility depends on the consumer own past consumption) or external (when marginal utility depends on the society past consumption). Models with habit formation help explain the observed ex-

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[12] A precautionary motive for saving was first introduced in the Euler equation literature by Zeldes (1989b) and Caballero (1990). For more recent analyses, see Kimball and Weil (2009) and Choi, Lugauer, and Mark (2014).

[13] For analyses of the interplay of liquidity constraints and prudence see Carroll and Kimball (2001) and, for a more recent work, Deidda (2014).

[14] See also Ludvigson and Michaelides (2001) and Carroll (2011).

cess smoothness of consumption and have also attracted much attention in the finance literature since they may provide a partial solution to the equity premium puzzle. Indeed, habits increase the disutility associated with large declines in consumption, so they may both explain the high equity premium[15] and a sluggish response of consumption to income changes, because when income increases, an individual accustomed to a low level of consumption finds it optimal to increase saving.[16]

## 5. Conclusions

A general conclusion that emerges from an overall assessment of the literature on consumption based on the Euler equation is that, more than in other fields of economic theory, a very strict link exists between econometric analyses and theoretical developments. The attention given to puzzling empirical results has in fact guided theoretical analysis since its origin, bringing about modifications of the basic theoretical framework aiming at reconciling it with empirical evidence.

As already mentioned, in their review of the literature on the approach to consumption based on the Euler equation, Attanasio and Weber (2010) conclude that it is possible to construct rich versions of the intertemporal utility maximization model able to replicate most empirical evidence. A similar consideration can be found in Attanasio and Wakefield (2010):

> A short and definitely subjective and not unbiased summary of these results is that a rich version of the life-cycle model can fit the available data, especially if one focuses on households headed by prime aged individuals (Attanasio and Wakefield, 2010, p. 706).

As a matter of fact, this stream of research has led to a better understanding of the main motives for saving and the circumstances affecting consumption behaviour. At the same time, however, each of the extensions of the model that the literature has proposed ends up being crucial for the ability of the Euler equation framework to replicate the specific empirical evidence taken into consideration. Rather than representing refinements of the basic model that may contribute to form a more general view of consumption behaviour, each of these extensions thus ends up constituting a model in itself. For that matter,

[15] See Abel (1990), Constantinides (1990), and Campbell and Cochrane (1999) as first contributions with habit formation in a capital asset pricing model. For a recent survey see Breeden, Litzenberger, and Jia (2015b).

[16] See Deaton (1986 and 1992) and Alessie and Lusardi (1997).

the intertemporal utility maximization framework, in its most general formulation, is not endowed with specific theoretical content but is rather a basic methodological tool of reasoning, which must be given content through the specification of hypotheses if it has to give indications on consumption behaviour. As suggested by Browning and Lusardi,

> the most general model that allows for capital market imperfections and nonadditive preferences over time does not seem to impose any restrictions on the time path of consumption and asset prices. It is only when we impose restrictions on preferences and budgets that we can derive testable implications. Thus the standard model in its most general form is better thought of as a framework than as a direct source of testable propositions (Browning and Lusardi, 1996, p. 1800).

Attanasio and Weber (2010) stress as well that the modern view of consumption only assumes, in its most basic form, that individuals maximize their own utility subject to the intertemporal constraint, but «this level of generality encompasses many different types of behavior and has almost no testable implications» (Attanasio and Weber, 2010, p. 695).

This very general framework is what allows to formulate the problem of intertemporal choice in a manner that is consistent with the methodological premises of modern economic theory. To this framework various elements may be added, by specifying individual preferences and the stochastic context in which the consumer operates. According to the way in which such hypotheses are specified, the most diverse conclusions on individual consumption choices are obtained. We have in fact that most of the implications of the Euler equation model that were initially emphasized in the literature have been lost due to the subsequent extensions of the model: for example, the conclusion that individuals choose consumption profiles that are rather uniform over time, which was at the basis of Friedman's original insight, falls apart when the hypothesis of quadratic utility is abandoned, or a discrepancy is admitted between the rate of time preference and the rate of interest. Or, to make another example, the peculiar link between short and long period that is implicit in the Euler equation, according to which marginal utilities must be equal not only between two consecutive periods but also between the current period and any other distant period, would be lost once the presence of liquidity constraints is admitted, thus necessarily shortening the time horizon.

While each extension of the basic model of intertemporal utility maximization plays a crucial role for defining the implications of the model, no general consensus has yet emerged on which extensions should be adopted in defining the general theoretical approach to consumption. As Attanasio and Wakefield observe,

> The life-cycle model is an extremely useful framework that can be used to conceptualize the analysis of saving and consumption behaviour. However, if one wants to take the model to the data for serious quantitative prediction, it is necessary to work with relatively complex and sophisticated versions of the model that take into account a number of factors that have been proven to be empirically important. [...] The quantitative implications of more complicated versions of the lifecycle model that involve endogenous labour supply, durables, housing, and so on need to be explored as we still do not have a good idea of what role these phenomena (which are bound to be of first order importance) play (Attanasio and Wakefield, 2010, p. 728).

To better grasp the extreme relevance of the way the model of intertemporal maximization is specified, one may consider, for example, the quantification of the component of savings due to liquidity constraints. This does not depend exclusively on imperfections in the credit market, since liquidity constraints interact with the precautionary motive: thus the intensity of prudence of individuals, the variability and uncertainty of future incomes, all would concur in determining the component of saving due to liquidity constraints. This implies that the choice of "ingredients" which make up the model specification – together with their quantitative definition – play an absolutely essential role.

In this context, particularly crucial is the role played by the specification of individual preferences. The sensitivity of savings to changes in the context in which the consumer operates, for example, does in fact depend on structural parameters: on the way the consumer discounts future utility, on his prudence, on the intensity of his aversion to intertemporal fluctuations of income. The ability of some versions of the basic model to replicate empirical evidence is closely related to the value attributed to the rate of time preference (as for example in the buffer-stock model proposed by Deaton, 1991 and Carroll, 1992).

It is worth noting how the variability of the theoretical implications of the basic model of intertemporal maximization according to the specification of the hypotheses also affects the possibility of using it for policy purposes. As observed by Attanasio and Weber,

> Perhaps as a consequence of this focus on testing, when it came to policy analysis and debates, the model and in particular the empirical evidence that has been accumulated on it have been rarely used (Attanasio and Weber, 2010, p. 694).

It seems in fact that the basic model as enriched by its extensions, for all the theoretical consensus it enjoys, is scarcely suitable as far as design and evaluation of policy measures are concerned. This depends at least on two kinds of reasons. Having fully assimilated Lucas's critique, this literature has naturally focused on individual consumption choices, thus giving up the idea that an aggregate consumption function could be identified and regarding aggregate consumption as merely the result of the behaviour of individual consumers. It is a distinctive character of the Euler equation approach, however, that, even in the microfounded model, it is not capable of determining the *level* of consumption. Thus the approach does not deliver general predictions of the impact of policy measures on the levels of consumption and saving. This is quite clearly illustrated by Attanasio and Weber (2010) and Attanasio and Wakefield (2010):

> One of the reasons for this divorce between the literature on the life cycle model and what should have been its practical use in the design and evaluation of public policy stems from the fact that the Euler equation does not deliver a consumption function. While it can be used to test the model and estimate some of its parameters, it cannot be used to determine the effects of specific policy changes on consumption or saving (Attanasio and Weber, 2010, p. 694-95).

> The price one pays for this [the possibility of not fully specifying the whole stochastic environment in which the individual operates] is that the approach does not deliver a 'consumption function'. It is therefore not possible to establish how consumption or saving will change in reaction to changes in the various variables faced by the individual. This is obviously an important limitation for policy analysis and probably explains the dichotomy mentioned above between the empirical consumption literature and the public finance literature (Attanasio and Wakefield, 2010, p. 707).

This is not, however, the only limitation faced by the Euler equation approach regarding its applicability to policy. If a clear identification of the preference parameters were possible, in fact, this would actually allow to draw from the model some relevant and usable policy indications. For example, understanding the importance of the elasticity of intertemporal substitution in determining the size of the reaction of savings to changes in the interest rates, would allow to assess the effects of taxing asset returns.

However, as shown above, the value of the crucial parameters, and in consequence the policy implications that could be drawn, depend too much on the specification and the particular hypotheses adopted in each version of the model, while at the same time there is no widespread consensus on which formulation of the theory could be regarded as the general one. As we shall show in the second paper, the indeterminacy surrounding the definition of parameters is also related to the difficulties arising from the necessity to specify the utility function.

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# The choice of the functional form in the consumption Euler equation approach: a critical view

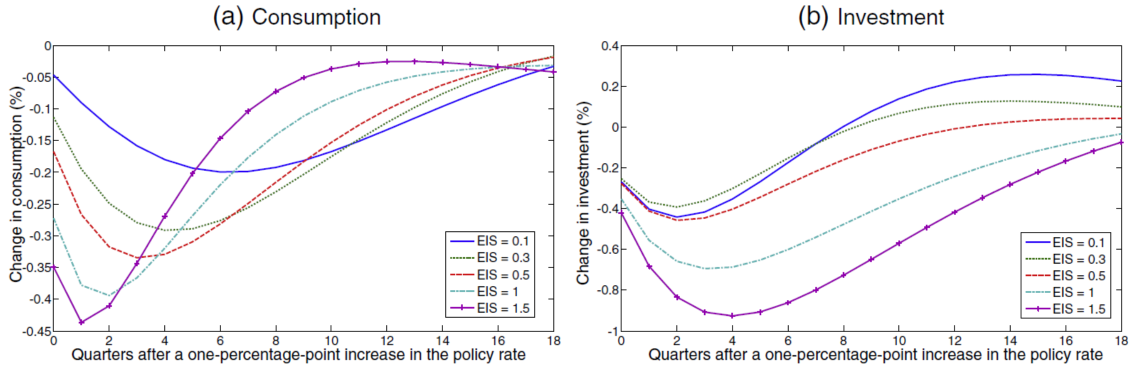
## 1. Introduction

Within the intertemporal utility maximization model that is at the basis of the modern view of consumer behaviour, the main characteristics of individual preferences are represented by a definite set of parameters. Such parameters synthetically describe the crucial features of the observed consumption behaviour, while at the same time representing the properties of individual preferences from which such behaviour derives. Such features are: i) the tendency of individuals to prefer a sum that is certain to an aleatory sum with the same expected value; i.e., consumers are characterized by *risk aversion*; ii) the tendency of individuals to save as a result of the presence of uncertainty over future incomes, i.e., consumers are characterized by *prudence*; iii) the tendency of individuals to save as a result of the incentive represented by interest on saved sums, i.e., consumers have an *intertemporal motive* to save. A specific parameter corresponds to each of these crucial properties of preferences: a coefficient describing (absolute and relative) risk aversion, a coefficient describing (absolute and relative) prudence, and an elasticity of intertemporal substitution.

To give adequate quantitative definition to each of these parameters, and to specify the intensity of each of them through appropriate numerical estimates, is absolutely crucial for a number of reasons. In the first place, the model of intertemporal utility maximization would prove devoid of definite theoretical content without a specification of individual preferences (such specification also including definition of the value of the parameters that characterize preferences). In fact, to state that the choice between consumption and saving derives from the solution of a problem of intertemporal maximization, without any further specification, would only constitute a very general framework of thought. The capability of such framework of providing definite theoretical propositions entirely depends on the specification of the preferences of the consumer. In the second place, in the last decades modern macroeconomics has entirely given up the idea that it is possible to identify an aggregate consumption function. Based on the implications of Lucas critique, it relies instead on an entirely microfounded description of consumption. As a result, the general equilibrium models that underpin modern macroeconomic literature necessarily require the specification of the utility function of the representative agent, including the specification of the structural parameters that characterize it. Such parameters either have to be estimated within the general equilibrium model itself, or have to be inferred from the empirical microeconometric literature. The analysis that such general equilibrium models offer of macroeconomic phenomena and their conclusions depend in no small way on the value attributed to the structural parameters, including the individual preference parameters. In the third place, specification of the utility function and estimation of its parameters are crucial for policy: the parameters being a basic feature both of the microeconomic model of intertemporal utility maximization and of the macroeconomic general equilibrium model, they are crucial for devising and implementing policy measures. For example, within a DSGE model, a different value attributed to the intertemporal elasticity of substitution implies a different reaction to a monetary policy shock. This is very clearly and effectively shown by Havránek *et al.* (2015), who replicate the well-known model of Smets and Wouters (2007) by attributing different values to that parameter (see Figure 1). The experiment shows that the dynamics of consumption and investment changes dramatically if the elasticity of substitution varies between 1.5 (as in Smets and Wouters) and 0.1 (as seems to be widely suggested by empirical evidence, according to which this parameter could well be null or almost null).

To mention just another reason for accurate estimates of the intertemporal elasticity of substitution to be important, they would allow to evaluate the effects on saving of taxing asset returns, which indeed requires quantifying the contribution of interest rates in explaining consumption growth.[1]

Figure 1. Simulated impulse responses to a one-percentage-point increase in the monetary policy rate in the model by Smets and Wouters (2007)




*The figure shows the impact of the elasticity of intertemporal substitution (EIS) on the impulse responses of consumption and investment in the model developed in Smets and Wouters (2007).*
*Source: Havránek et al. (2015).*

[1] See Attanasio and Wakefield (2010), who simulate a life cycle model to understand the importance of the parameter in deterniming the size of the reaction of savings to changes in the interest rate.

The crucial role of the estimation of preference parameters for policy purposes may further be appreciated if we reflect on the role of prudence in evaluating the benefits of a programme of social security. Once the presence of precautionary saving is admitted, it must in fact be concluded that uncertainty over future consumption generates, at least for some individuals, a welfare loss, which may only be quantified once the parameter representing prudence is given. The magnitude of the parameter is therefore essential to determine whether that the introduction of a social welfare scheme aimed at reducing the impact of uncertainty on future incomes would bring about an increase in welfare.[2]

The growing importance of parameters representing preferences, due to the evolution of the economic theory of consumption in the recent decades, shows clearly in the great number of contributions aimed at the estimation of the parameters. Such empirical literature originates within the approach to consumption theory based on the Euler equation. As is well known, the approach predicts that consumers allocate resources over time in order to maximize lifetime utility subject to an intertemporal budget constraint. The first order conditions of the optimisation problem give rise to the following Euler equation for consumption:

$$u'(C_t) = (1+\delta)^{-1}\mathbb{E}_t[(1+r_t)\cdot u'(C_{t+1})] \qquad (1)$$

where $\delta$ is the rate of time preference and $r_t$ is the (stochastic) interest rate between $t$ and $t+1$.

The Euler Equation states that in equilibrium there are no intertemporal consumption reallocations that increase the consumer's utility at the margin, i.e. the consumer aims at keeping the marginal (discounted by $(1+\delta)^{-1}$) utility of (discounted by $r$) expenditure constant over time. The idea that originated the approach, proposed by Hall (1978), consists in deriving a set of orthogonality conditions from equation ( 1 ), which can be verified empirically and allow to both test the validity of the model and to estimate the structural parameters of the utility function.

A very appealing feature of the Euler equation is that, in order to be employed for estimating preference parameters, it requires data on consumption and interest rates in different periods, but it does not require observation of consumers' wealth nor to formulate many hypotheses on the stochastic environment faced by the consumer. Despite being consistent with very different modelling choices, the Euler equation simply requires some basic assumptions about insurance and credit markets as well as the specification of the

[2] Browning and Lusardi (1996) offer both a discussion of this case and a numerical example (cfr. p. 1803).

utility function. That makes the Euler equation particularly attractive from an empirical point of view and explains the huge flow of econometric research it produced in the last decades. However, the results of this literature are controversial, because the estimates are characterised by high variability and there appears to be little consensus regarding the magnitude of the parameters. In fact, while some scholars consider the Euler equation as an effective and convenient tool for estimating preference parameters, others seem rather sceptical. For example, Attanasio and Weber claim that

> Euler equations are remarkably useful because they let researchers estimate important preference parameters in a relatively robust way (Attanasio and Weber, 2010, p. 741).

Conversely, Carroll notices that

> despite scores of careful empirical studies using household data, Euler equation estimation has not fulfilled its early promise to reliably uncover preference parameters (Carroll, 2001, p. 1).

The purpose of this paper is to analyse the most relevant parameters characterizing utility functions, i.e. the coefficients of relative and absolute risk aversion, the intertemporal elasticity of substitution, and the coefficients of relative and absolute prudence. In Section 2, after discussing the meaning of each structural preference parameter, we highlight the relations between them. We thus show that while measuring different aspects of preferences, the main parameters, because of the way they are defined, have close mathematical relationships between each other, which impose strict constraints on the estimates of the parameters themselves. In Section 3 we provide a survey of the most popular utility functions adopted in the literature, emphasizing their main properties in terms of preference parameters. Section 4 reviews the empirical evidence on the parameters collected since Hall's contribution, with a focus on the choice of the utility functions adopted for the estimation. Section 5 concludes.

## 2. Preference Parameters

### *2.1. How to summarize risk attitudes: the coefficients of risk aversion*

An individual is risk averse if he is not willing to accept a fair gamble, i.e. a gamble with an expected return of zero. That is to say that, given the choice between the same amount of consumption achievable through a gamble or through certainty the risk averse person will opt for certainty. Analytically, if we assume two possible states of nature with consumption levels $C_t^1$ and $C_t^2$ and probabilities $\pi$ and $(1-\pi)$, then the utility of the expected value of consumption is:

$$u(\mathbb{E}[C_t]) = u[\pi C_t^1 + (1-\pi)C_t^2]$$

and the expected utility of consumption is:

$$\mathbb{E}[u(C)] = \pi u(C_t^1) + (1-\pi)u(C_t^2).$$

An individual is risk averse if he would prefer the certain level of consumption $\mathbb{E}[C_t]$ rather than the expected utility, or, equivalently, if $u(\mathbb{E}[C_t]) > \mathbb{E}[u(C_t)]$.

The risk attitude is directly related to the curvature of the utility function: if $u(\mathbb{E}[C]) > \mathbb{E}[u(C)]$ then the utility function is concave and so the utility of the expected value of an uncertain amount is greater than the expected utility of that amount. Obviously, the theory requires marginal utility to be decreasing and hence one needs to assume that consumers are risk averse.

The modern literature on risk aversion began with the work of Pratt (1964) and Arrow (1965), who proposed formal measures of risk aversion. Since the degree of risk aversion depends on the curvature of the utility function, they suggested that it could be measured by means of the second derivative of the function. Clearly, preferences – and thus risk attitudes – are unchanged under affine transformations of the utility function, so the second derivative needs to be normalized. Therefore, the Arrow–Pratt measures of absolute and relative risk aversion are, respectively:

$$A(C) = -\frac{u''(C)}{u'(C)} \qquad \text{and} \qquad a(C) = -\frac{u''(C) \cdot C}{u'(C)}$$

The concept of absolute risk aversion is suited to the comparison of attitudes towards risky projects whose outcomes are absolute gains or losses from current consumption, whereas the concept of relative risk aversion is useful to evaluate risky projects whose outcomes are percentage gains or losses of current consumption.[3]

Both Pratt and Arrow suggested in their contributions absolute risk aversion to be decreasing and the subsequent literature has indeed widely agreed that preferences, in order to be plausible, should exhibit decreasing, or at most constant, absolute risk aversion. More controversial appears to be the behaviour of relative risk aversion.

To understand the implications of increasing or decreasing absolute or relative risk aversion, we may consider the case of forming a portfolio with one risky asset and one risk-free asset. If the individual experiences an increase in wealth, he will choose to in-

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[3] Risk is additive in the state variable for the models with income risk and multiplicative for those with capital risk. See Lippman and McCall (2000).

crease the total amount invested in the risky asset if absolute risk aversion is decreasing, and he will choose to increase the fraction of the portfolio held in the risky asset if relative risk aversion is decreasing. It is usually believed that wealthier people are willing to bear more risk than poorer people, which is why a decreasing absolute risk aversion is considered plausible. The hypothesis of decreasing relative risk aversion appears, instead, much stronger, because it implies that individuals become less risk averse with regard to gambles that are proportional to their wealth as their wealth increases. In addition, a risk averse individual with decreasing relative risk aversion will exhibit decreasing absolute risk aversion, but the converse is not necessarily the case. Generally, assuming constant relative risk aversion is thought to be a quite plausible assumption.

### *2.2. Measuring the precautionary motive for saving: relative and absolute prudence*

When preferences exhibit prudence and the consumer faces income uncertainty, he is willing to insure himself against the fall in consumption that would result from a fall in income and so, if insurance markets are not complete, he reduces current consumption: prudence leads individuals to treat future uncertain income cautiously and not to spend as much currently as they would if future income were certain. The saving that results from the knowledge that the future is uncertain is known as precautionary saving.

Leland (1968) demonstrated that precautionary saving requires convex marginal utility in addition to risk aversion. To understand why this happens, following Deaton (1992), we can suppose that, at time $t$, there is a mean-preserving increase in the spread of the distribution of $C_{t+1}$. Such an additional uncertainty about future consumption will affect current consumption as far as it affects the marginal utility of next period's consumption. With a null third derivative of the utility function, and hence a linear marginal utility, there will be no effect, because the mean of marginal utility equals the marginal utility of mean consumption which is unchanged. On the contrary, if marginal utility is convex, a mean preserving increase in risk will increase marginal utility, so that current consumption will have to decrease in order to bring the current marginal utility back into equality. Therefore, whenever the third derivative of the utility function is positive, income uncertainty implies an optimal consumption-age profile that is increasing.

Kimball (1990), in parallel to the Arrow-Pratt coefficients, proposed two measures of the intensity of the precautionary saving motive which rely on the convexity of marginal utility: a measure of absolute prudence and a measure of relative prudence

$$P(C) = -\frac{u'''(C)}{u''(C)} \quad \text{and} \quad p(C) = -\frac{u'''(C) \cdot C}{u''(C)}$$

Since the introduction of those coefficients, the consumption literature has been trying to quantify the importance of the precautionary motive for saving, but, as will be discussed in Section 4, the results are mixed.

### *2.3. The aversion to temporal fluctuations of consumption: the elasticity of intertemporal substitution*

According to the model of intertemporal utility maximization, the consumption profile of the individual should be designed so as to take advantage of the interest rate: if the interest rate between $t$ and $t+1$ is high, then there is an additional incentive to postpone consumption in $t$ in favour of $t+1$. This is immediately clear when we consider the Euler equation in the case of a constant interest rate:

$$u'(C_t) = \frac{1+r}{1+\delta} \mathbb{E}_t[u'(C_{t+1})] \tag{2}$$

The equilibrium condition predicts that consumption increases over time if $r$ exceeds $\delta$ and falls if $r$ is less than $\delta$.

The magnitude of the response of consumption growth to changes in the interest rate is equal to the intertemporal elasticity of substitution

$$\sigma(C) = \frac{d(C_{t+1}/C_t)}{C_{t+1}/C_t} \bigg/ \frac{d(1+r_t)}{1+r_t}$$

which represents the proportional change in consumption growth that must follow to a one percent change in the interest rate in order to keep the marginal utility of expenditure constant. The intertemporal elasticity of substitution depends (inversely) on the curvature of the utility function: if the marginal utility of consumption decreases very quickly as consumption rises, the individual does not need to change his consumption much to take advantage of intertemporal incentives since the profile of marginal utility can be udjusted to the changed interest rate with little change in consumption.

Note that, whereas the effect of the interest rate on consumption depends on the interplay of income, substitution and wealth effects and is therefore ambiguous, its effect on the growth rate of consumption can never be negative ($\sigma \geq 0$) because that would imply non concave utility, that is, non decreasing marginal utility.[4]

In the last decades many contributions have attempted at determining the intertemporal elasticity of substitution relying on Euler equation estimates, but there is still controversy about whether or not that parameter is large enough to make changes in expected interest rates an important factor in fluctuations in consumption growth. Table 5 in the Appendix presents the main empirical evidence on the magnitude of the elasticity of substitution and shows the variability of the results of this line of research.

### *2.4. The relationship between preference parameters*

The degree of risk aversion, the intensity of the precautionary saving motive and the degree of intertemporal substitutability, describe different aspects of individual preferences:

- the measures of risk aversion tell us how much the individual is willing to sacrifice his consumption in the best case scenario in order to achieve a higher level of consumption in the worse;
- the magnitude of prudence tells us how an increase in uncertainty about future income will affect current consumption;
- the intertemporal elasticity of substitution tells us to what extent the individual is willing to substitute future consumption for current consumption in response to the incentive given by interest rates.

However, a close relationship exists between the different preference parameters.

Leland (1968) demonstrated that, under additive utility, for the utility function to exhibit prudence it is necessary that the coefficient of absolute risk aversion be decreasing:

$$P(C) > 0 \Rightarrow A'(C) < 0.$$

Mirman (1971) showed that when risks to wealth are multiplicative (as from uncertain inflation or interest rates), precautionary saving occurs only if:

$$\begin{cases} a(C) > 1 \\ p(C) > 2 \end{cases}$$

Furthermore, Drèze and Modigliani (1972) proved that decreasing absolute risk aversion leads to a precautionary saving motive stronger than risk aversion:

[4] The interest rate represents the price of future consumption relative to current consumption, so when it increases current consumption decreases because of the substitution effect. At the same time, with a higher interest rate, any target level of future consumption is achieved with less saving, which implies an income effect going in the opposite direction of the substitution effect. As for the wealth effect, a higher interest rate implies higher discount factors to be applied to future flows of labour income, and so it leads to a decrease in current consumption, reinforcing the substitution effect.

$$A'(C) < 0 \Rightarrow A(C) < P(C)$$

More generally, the relationship between absolute prudence and absolute risk aversion can be seen by differentiating $A(C)$ (as in Eisenhauer, 2000) to get

$$P(C) = A(C) - \frac{A'(C)}{A(C)} \quad (3)$$

and, multiplying both sides by the consumption level, relative prudence appears as the difference between relative risk aversion and the consumption-elasticity of absolute risk aversion:

$$p(C) = a(C) - \frac{C \cdot A'(C)}{A(C)} \quad (4)$$

From ( 3 ) and ( 4 ) it is easy to see that if the utility function exhibits decreasing absolute risk aversion (DARA), then $A'(C) < 0$ and the individual is more prudent than risk averse. Therefore, when a DARA utility function is chosen, so as to meet the Arrow and Pratt hypothesis, it is automatically implied that the consumer is more inclined to take precautions than to avoid risk. This happens with isoelastic preferences. In contrast, under constant absolute risk aversion (CARA), $A'(C) = 0$ so risk aversion and prudence are exactly the same, as in the exponential utility case. And, finally, with increasing absolute risk aversion (IARA), $A'(C) > 0$ and the individual is more risk averse than prudent, which occurs under quadratic preferences.

| | | |
|---|---|---|
| DARA: | $A'(C) < 0 \Rightarrow P(C) > A(C)$ | (isoelastic preferences) |
| CARA: | $A'(C) = 0 \Rightarrow P(C) = A(C)$ | (exponential preferences) |
| IARA: | $A'(C) > 0 \Rightarrow P(C) < A(C)$ | (quadratic preferences) |

Thus, the magnitude of relative prudence is closely related to the magnitude of relative risk aversion, and yet has important behavioural implications of its own. In fact, a value $p(C) \leq 0$ indicates a lack of precautionary motives, $p(C) > 0$ implies a tendency to undertake precautionary saving in the face of additive risk, while $p(C) > 2$ implies precautionary saving under multiplicative risk.

Such a clear-cut link between the magnitudes of these parameters has long been generally ignored in the literature on the consumption Euler equation and still appears often neglected. For example, Blanchard and Mankiw (1988), talking about the case of CARA, argue that «In general, there need not be any tight relation between the coefficients of risk aversion and the coefficients of prudence» (Blanchard and Mankiw, 1988, p. 174).

However, the link between risk aversion and intertemporal substitution is even closer. The standard version of the intertemporal maximization model, indeed, implies a double additivity induced by the simultaneous assumptions of intertemporal separability and expected utility (additivity over periods and over states of nature), which entails a negative relation between risk aversion and intertemporal substitution: individuals who are risk averse will be unresponsive to intertemporal incentives, while those who are willing to reallocate their consumption in response to changes in the interest rate will display relatively little aversion towards risk. That happens because both parameters are governed by the concavity of the utility function:

$$\sigma(C) = -\frac{u'(C)}{c \cdot u''(C)} = \frac{1}{a(C)}$$

Many scholars believe that risk aversion and intertemporal substitution are independent aspects of consumer preferences, so that any formulation that mixes them up is incorrect. Hall is one of them:

> it would be desirable to eliminate this automatic connection between intertemporal substitution and risk aversion. The empirical finding that intertemporal substitution is weak or absent does not contradict any widely held beliefs about consumer behavior. But the corresponding conclusion that the coefficient of relative risk aversion is close to infinity is incompatible with observed willingness of consumers to take risk (Hall, 1988, p. 343).

More recently, Kimball and Weil argue that

> Because it does not distinguish between aversion to risk and aversion to intertemporal substitution, the traditional theory of precautionary saving based on intertemporal expected utility maximization is a framework within which one cannot ask questions that are fundamental to the understanding of consumption in the face of labor income risk (Kimball and Weil, 2009, p. 245).

Other economists do not feel so uncomfortable in accepting the link between the two parameters, such as Deaton (1992), according to whom,

> time and uncertainty are so intimately connected that (at least to this writer) there is strong intuitive support for a relationship between attitudes towards risk and attitudes towards substitution. In an uncertain world, the substitution of future consumption for current consumption inevitably increases exposure to risk, and those who are willing to contemplate the former must be willing to face the latter (Deaton, 1992, p. 20).

Although it may be the case that a negative correlation exists between risk aversion and intertemporal substitution, it appears quite restrictive to constrain the response to the incentive given by interest rates to be, *a priori*, exactly the reciprocal of the coefficient summarizing the attitude towards risk.

In order to disentangle the two parameters, one needs to give up one of the two sources of additivity, which means to either reject the axioms of expected utility theory or to abandon intertemporal additivity. As for the former route, Epstein and Zin (1989) and Weil (1990) have pursued it by developing the non-expected utility model of choice proposed by Kreps and Porteus (1978). The resulting functional form, often called Epstein-Zin-Weil (EZW) recursive utility, has been increasingly adopted in the Euler equation literature on consumption in recent years. In fact, it has been employed in the literature aiming at explaining the empirical evidence concerning the consumption puzzles arisen in the 1980s, playing an important role in the research on the equity premium puzzle[5]. Moreover, because of the flexibility which it seems to provide in terms of the magnitude of preference parameters, EZW utility has also been employed in the empirical work aiming at evaluating risk attitudes and the degree of intertemporal substitutability. For these reasons, this functional form is analysed in Section 3, when illustrating the main preference specifications adopted in the consumption literature, while Section 4 reviews the empirical evidence on the parameters relying on it. As will be discussed, EZW preferences, while keeping intertemporal substitution and risk aversion separated, imply a degree of prudence implicit in the definition of the other two parameters and, moreover, require the introduction of an additional aspect of preferences, the so-called timing premium, which relates to the preference for the timing of resolution of uncertainty.

As for the alternative route toward a separation of the coefficient of relative risk aversion and the elasticity of intertemporal substitution, that is to say the hypothesis of intertemporal dependence of preferences, this has been investigated extensively as well. Indeed, the assumption of additivity over periods is considered very strong, as it prevents from capturing important phenomena such as habit formation and durability. Furthermore, intertemporal dependent preferences may explain several empirical puzzles. In particular, habits help to explain the observed "excess smoothness" of consumption[6] and have attracted much attention in the finance literature because they may provide a partial solution to the equity premium puzzle. Indeed, habits increase the disutility associated with large de-

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[5] The puzzle was first highlighted by Mehra and Prescott (1985). They analyse the difference between the average return on the stock market and the return on short-term government debt in the U.S. from 1889 to 1978 and find that the average equity premium was 618 basis points. Their asset pricing model is able to generate such an high equity premium only by assuming an implausibly high risk aversion.

[6] According to the model presented in Hall (1978), when the individual faces a variation of income which is not foreseen, this should affect consumption in the moment it occurs, instead the empirical evidence suggests that consumption does not react enough to unanticipated income changes, leading to excess smoothness. The puzzle was highlighted by Deaton (1986).

clines in consumption, so they may explain the high equity premium[7] and they may imply a sluggish response of consumption to income changes, because when income increases, an individual accustomed to a low level of consumption finds it optimal to increase saving.[8]

However, most studies relying on Euler equation estimation have found little or no evidence of habits, such as Meghir and Weber (1996) and Dynan (2000).[9]

## 3. Utility Functions

This section provides a survey of the most popular utility functions adopted in the literature on the Euler equation, i.e. quadratic, exponential, isoelastic, and recursive Epstein-Zin-Weil preferences. The functional forms are illustrated in Table 1 in the Appendix. Here, we discuss the Euler equations associated with each preference specification and describe the implications of the functional forms in terms of parameters' values and behaviour. For a presentation of the quantitative definition of the parameters of each utility function see Table 2 in the Appendix.

### *3.1. Quadratic preferences: certainty equivalence*

Quadratic preferences imply linear marginal utility: expected marginal utility of consumption is the same as the marginal utility of expected consumption. This entails that the third derivative of the function is zero, that is to say, quadratic preferences exhibit no prudence, so uncertainty about future income has no impact on consumption.

Earlier attempts at testing the utility maximization model of consumption relied on the special case of quadratic preferences. This case is known in the literature as the Permanent Income model with certainty equivalence or, simply, certainty equivalence model (examples can be found in Hall, 1978; Flavin, 1981; Campbell, 1987).

Under the assumption of quadratic preferences, the Euler equation ( 2 ) rewrites as:

$$a - bC_t = \left(\frac{1+r}{1+\delta}\right) \mathbb{E}_t[(a - bC_{t+1})]. \quad (5)$$

It is common practice, when illustrating the implications of quadratic utility, to assume, for simplicity, that both the interest rate and the rate of time preference are zero

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[7] See Abel (1990), Constantinides (1990) and Campbell and Cochrane (1999).

[8] See Deaton (1986).

[9] For a review of the empirical evidence on habit formation see the meta-analysis in Havránek, Rusnák, and Sokolova (2015).

(see, for example, Blanchard and Mankiw, 1988 and Romer, 2012). Introducing this hypothesis in equation ( 5 ) yields:

$$a - bC_t = a - b\mathbb{E}_t[C_{t+1}]$$
$$\mathbb{E}_t[C_{t+1}] = C_t. \qquad (6)$$

Hence, in the special case of quadratic preferences, consumption behaves as a martingale:

$$C_t = C_{t-1} + \varepsilon_t \qquad (7)$$

where $\varepsilon_t = C_t - \mathbb{E}_{t-1}[C_t]$ is a consumption innovation, i.e., the effect on consumption of all new information about the sources of uncertainty faced by the consumer.

The Euler equation ( 6 ) implies that, *ex ante*, current consumption is the best predictor of next period's consumption; *ex post*, consumption changes only if expectations are not fulfilled.

In the case of quadratic preferences, it is possible to derive the solution of the maximization problem by combining equation ( 6 ) with the intertemporal budget constraint. If we assume a finite horizon $T$, an initial non-human wealth $A_0$, and labour income $Y_t$, the constraint takes the form

$$\sum_{t=1}^{T} C_t \leq A_0 + \sum_{t=1}^{T} Y_t,$$

and the optimal behaviour is such that

$$C_1 = \frac{1}{T}\left(A_0 + \sum_{t=1}^{T} \mathbb{E}_1[Y_t]\right), \qquad (8)$$ [10]

so that consumption is a linear function of initial wealth and the present value of expected future income: consumption does not depend on higher moments.

From ( 8 ) it is apparent that quadratic utility implies certainty equivalence: the consumption function is the same as under certainty once expectations are replaced by realizations; the individual consumes the amount he would if his future incomes were certain to equal their means.

The impossibility of capturing the precautionary motive for saving is the reason why the model with quadratic preferences is universally considered to be potentially very misleading in the presence of uncertainty.[11] However, the lack of prudence is not the only reason why quadratic utility is unappealing. In fact, it also implies increasing absolute risk aversion, which means that the amount of consumption that individuals are willing to give

[10] Clearly, one needs to assume that the individual's wealth is such that consumption is always in the range where marginal utility is positive.

[11] Among the first works stressing this point, see Blanchard and Mankiw (1988) and Caballero (1990). For a thorough discussion and a numerical example illustrating how misleading the certainty equivalence model can be, see Browning and Lusardi (1996).

up to avoid a given amount of uncertainty about the level of consumption rises as they become wealthier. This property is considered «unappealing on theoretical grounds and strongly counterfactual (riskier portfolios are normally held by wealthier households)» (Attanasio and Weber, 2010, p. 707).[12]

Quadratic preferences also imply that the elasticity of intertemporal substitution is decreasing in consumption, which is another feature that makes its use unappealing: when the elasticity of intertemporal substitution is decreasing, poor consumers are willing to substitute consumption over time in response to interest rate changes more than rich consumers, which does not seem to be the case.[13]

### *3.2. Exponential preferences: constant absolute risk aversion*

Exponential utility has played an important role in the literature on the consumption Euler equation, because it has proven analytically tractable (see Kimball and Mankiw, 1989 and Caballero, 1990).

For a consumer with exponential instantaneous utility function, Euler equation ( 2 ) becomes:

$$\exp[-\alpha C_t] = \left(\frac{1+r}{1+\delta}\right) \mathbb{E}_t\big[\exp[-\alpha C_{t+1}]\big]$$

$$1 = \left(\frac{1+r}{1+\delta}\right) \mathbb{E}_t\big[\exp[-\alpha(C_{t+1}-C_t)]\big]$$

$$\alpha(\mathbb{E}_t[C_{t+1}]-C_t) = \ln\left(\frac{1+r}{1+\delta}\right)$$

$$\mathbb{E}_t[C_{t+1}] = C_t + \ln\left(\frac{1+r}{1+\delta}\right)^{1/\alpha}. \qquad (9)$$

The term $\ln\left(\frac{1+r}{1+\delta}\right)^{1/\alpha}$ in equation ( 9 ) represents the intertemporal substitution motive for saving. As the Euler equation shows, under exponential utility intertemporal substitution does not take a multiplicative form that implies changes affecting the rate of growth of consumption, but entails additive changes in the level of consumption.

This functional form is unique in exhibiting constant absolute risk aversion (CARA), which implies that the elasticity of risky investments with respect to wealth is zero.

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[12] Talking about the increasing absolute risk aversion hypothesis underlying quadratic utility, Blanchard and Mankiw state that «Introspection and casual evidence suggest that this is a poor description of behavior under uncertainty» (Blanchard and Mankiw, 1988, p. 173–174).

[13] In fact, as will be shown in Section 4, several empirical studies, such as Blundell, Browning, and Meghir (1994), Attanasio and Browning (1995), and Atkeson and Ogaki (1996), suggest that the elasticity of intertemporal substitution increases with consumption.

Exponential preferences also exhibit constant absolute prudence. This is considered quite unattractive a feature, since it implies that the increase in consumption required to keep the same level of expected marginal utility in the face of an increase in uncertainty is independent of the initial level of consumption. That means that poor people and rich people reduce their consumption by exactly the same amount in reaction to a given risk.

On the other hand, relative risk aversion is, in the exponential case, increasing in consumption. That appears to be quite unrealistic an assumption too. In addition, as in the quadratic preferences case, intertemporal elasticity of substitution is decreasing.

These features, together with the fact that exponential utility does not rule out negative consumption, make this functional form an implausible representation of individual preferences. Nonetheless, it has played an important role in the Euler equation literature, since it allows deriving a closed form solution for consumption. In particular, it was employed in the first works investigating the relevance of a precautionary motive for saving, because it allows to go beyond certainty equivalence by retaining much of its analytical tractability.

### *3.3. Isoelastic preferences: constant relative risk aversion*

Isoelastic (or CRRA, or power) utility, a very popular preference specification, has been used in the consumption literature since the papers by Hansen and Singleton (1982 and 1983) and it has played a preeminent role in many theoretical studies of asset pricing and in the empirical tests of the validity of the consumption model based on the Euler equation. In fact, the assumptions of CRRA utility and lognormality of the joint distribution of consumption and stock returns together lead to an empirically tractable, closed-form characterization of the restrictions implied by the model.

This functional form is also the usual choice in empirical works aiming at estimating the coefficient of relative risk aversion, the intensity of the precautionary motive for saving, or the response to intertemporal incentives: CRRA utility leads to an approximation of the Euler equation which is linear in parameters, which implies high econometric tractability.

If we assume that the instantaneous utility function takes the isoelastic form, then the Euler equation becomes:

$$\mathbb{E}_t\left[\frac{C_{t+1}}{C_t}\right] = \left(\frac{1+r_t}{1+\delta}\right)^{\frac{1}{\gamma}}. \quad (10)$$

One can derive a testable relationship from equation ( 10 ) by taking logs and assuming that consumption is approximately log-normally distributed:

$$\Delta \ln C_{t+1} = \tilde{\beta} + \frac{1}{\gamma} r_t + \frac{\gamma}{2} \mathrm{Var}(\Delta \ln C_{t+1}) + u_{t+1} \quad (11)$$

with $\mathbb{E}_t[u_{t+1}] = 0$ and where $\tilde{\beta} = \frac{1}{\gamma} \ln\left(\frac{1}{1+\delta}\right)$ is the discount factor (a lower $\tilde{\beta}$ implies an increase in impatience, which leads to higher current consumption and hence lower consumption growth); $\frac{1}{\gamma}$ measures the response to the interest rate so it represents the intertemporal elasticity of substitution; $\mathrm{Var}(\Delta \ln C_{t+1})$ is the variance of future consumption $C_{t+1}$ conditional on information available at time $t$, so $\frac{\gamma}{2}$ captures the intensity of the precautionary motive for saving.

As it appears clear from equation ( 11 ), the CRRA specification imposes strong restrictions on preferences: the elasticity of intertemporal substitution of consumption is, in this context, constant and equal to the reciprocal of the degree of risk aversion. The fact that the intertemporal elasticity of substitution is constrained, *a priori*, to be independent of consumption (it is the same for rich and poor), can be considered quite unappealing in the light of the empirical evidence suggesting an increasing willing to substitute over time. Furthermore, the curvature parameter $\gamma$ plays a double role. On the one hand it is equal to the coefficient of relative risk aversion and therefore summarizes the consumer's attitude towards risk. On the other, its reciprocal is equal to the elasticity of intertemporal substitution and therefore measures the way consumption growth changes when the relative price of present and future consumption changes.

In the CRRA case there is also a direct link between risk aversion (or intertemporal substitution) and prudence: specifying the degree of risk aversion also pins down the degree of prudence. This follows from the fact that we have only one parameter in the utility function, which has to control all aspects of preferences. Table 3 in the Appendix reports possible values assigned to the coefficient of the isoelastic function, chosen accordingly to the range considered reasonable by the literature, and shows the implications in terms of preference parameters. For example, logarithmic utility implies the coefficient of relative risk aversion and the elasticity of intertemporal substitution to be both equal to unity and the coefficient of relative prudence to be 2;[14] a coefficient of relative risk aversion of 5 would imply an elasticity of intertemporal substitution equal to 0.2 and a coefficient of relative prudence equal to 6.

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[14] Note that logarithmic utility is an isoelastic function in which $\gamma = 1$ since $\lim_{\gamma \to 1} \frac{C_t^{1-\gamma} - 1}{1-\gamma} = \ln(C_t)$.

*3.4. Nonexpected utility: recursive Epstein-Zin-Weil preferences*

A functional form that implies a higher degree of flexibility is the one inspired by the work of Kreps and Porteus (1978) and developed by Epstein and Zin (1989) and Weil (1990), where utility is recursively defined over current consumption and a certainty equivalent of future random utility.

With EZW preferences there are two parameters in the utility function so the degree of risk aversion (equal to $\gamma$) is disentangled from the elasticity of intertemporal substitution ($\sigma$). As in the isoelastic case, they are both constrained to be constant, but, unlike the other functional forms, it is possible to admit a high aversion to risk and a high aversion to intertemporal substitution to coexist. In fact, EZW preferences can be conceived as a generalization of the standard time additive expected utility function, which collapses to the isoelastic case when the coefficient of relative risk aversion equals the reciprocal of the elasticity of intertemporal substitution, i.e. when $\sigma\gamma = 1$. However, the separation between the preference parameters governing attitudes towards *time* and *state* fluctuations of consumption comes at the cost of accepting that the individual has a preference for the timing of resolution of uncertainty: whenever the individual has greater (smaller) aversion to risk than to intertemporal substitution, early (late) resolution of uncertainty is preferred. Epstein and Zin (1989) show that recursive utility implies that the temporal resolution of risk matters and Epstein, Farhi, and Strzalecki (2014) provide a quantitative assessment of timing premia. The timing premium is defined as the fraction of consumption stream that the individual would be willing to give up in order for all risk to be resolved, and depends on both the preference parameters and on the nature of the endowment process: the farther the elasticity of intertemporal substitution from the reciprocal of relative risk aversion and the more persistent the consumption process, the greater the timing premium.

Basically, if the isoelastic specification entails that a single parameter governs three aspects of preferences, with recursive EZW utility two parameters control four aspects of preferences. In fact, once the elasticity of intertemporal substitution and the degree of risk aversion are set, prudence follows accordingly. Kimball and Weil (2009) provide a thorough discussion of how the links between preference parameters take form in the case of recursive preferences. In particular, in the most popular version of recursive preferences, where both the time aggregating function and the function which synthetises attitudes towards risk are of the isoelastic form, relative prudence is $p(C) = \gamma(1 + \sigma)$.

Table 4 in the Appendix reports possible values assigned to the coefficients of the EZW function – $\gamma$ and $\sigma$ – and shows the implications in terms of preference parameters.

Being the isoelastic function a special case of the recursive specification, the elaborations can be seen as an extension of those in Table 3 referred to isoelastic utility.

### 4. The empirical evidence on the structural parameters of the utility function

One difficulty that arises when reviewing the empirical literature on preference parameters is that, because of the links between them, most empirical estimates are conducted in such a way that, while investigating a specific aspect of preferences, they also have implications for the size of the parameters that do not represent the focus of the research. For example, if an empirical contribution relying on isoelastic utility (as most papers) provides an estimated value for the elasticity of intertemporal substitution equal to 0,5, it implicitly conveys an estimation for relative risk aversion of 2.[15]

Therefore, after a brief discussion of some influential empirical contributions related to each parameter, we choose to focus on a single one of them – the elasticity of intertemporal substitution – assuming that the implications of every estimate in terms of the other two should be clear from the arguments presented so far.

The first estimates of preference parameters appeared in the finance literature and concerned the degree of risk aversion. Breeden (1979) developed the intertemporal consumption model in the form that has been widely employed in subsequent research, but the finance paper that has had the greater influence is probably that by Hansen and Singleton (1983). Their estimates of the coefficient of relative risk aversion were between 0 and 2 with five out of six estimates being smaller than unity. Hansen and Singleton's framework has been adopted by several empirical papers, with extremely different estimates of the average value of relative risk aversion, so that there appears to be little consensus regarding the magnitude of this parameter or the direction in which it changes as wealth increases.

The attempts to quantify the importance of the precautionary motive for saving have been similarly extensive. Most of the early empirical works include some measure of risk in a linear approximation to a consumption Euler equation and test for its significance. These tests typically find that the estimated effects of consumption uncertainty on consumption growth are small, indicating that precautionary motives are weak or nonexistent.[16] A very popular contribution is that by Dynan (1993), who examines lifetime saving behaviour in the U.S. and obtains extremely small and statistically insignificant estimates

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[15] As discussed in Section 2, Table 3 in the Appendix provides further examples.

[16] Browning and Lusardi (1996) provide a detailed survey of the early contributions.

of relative prudence. Kuehlwein (1991), Grossberg (1991), and Parker (1999) each fails to obtain evidence of precautionary saving as well. Guiso, Jappelli, and Terlizzese (1992) find some evidence of prudence in Italy, but identified by a very low rate of precautionary saving, explaining about 2 percent of wealth accumulation. Later, Merrigan and Normandin (1996), using data for the UK, obtain estimates of relative prudence ranging from 1.78 to 2.33, and Kazarosian (1997) estimates that a 10 percent increase in uncertainty, as measured by the standard deviation of transitory income, would increase the ratio of wealth to permanent income by just 2.9 percent.

Overall, there has been a wide discrepancy between the values of prudence assumed in simulations and those – very small – implied by estimations performed on actual savings data, which suggests a widespread belief in prudent attitudes and precautionary motives and leads to suppose that the empirical tests relying on Euler equation estimation failed to capture the true extent of prudence.

As for the intertemporal elasticity of substitution, the most common approach to estimate it is through the Euler equation for consumption applied to aggregate time series or household level data. An influential and incessantly cited contribution is that by Hall (1988), who claims that, at least for the U.S., the intertemporal elasticity of substitution is close to zero. Many subsequent studies find similar results. Then, Attanasio and Weber (1995) suggest that the intertemporal elasticity of substitution is a function of several variables, including the level of consumption, and obtain positive estimates of the parameter, but small in absolute value (between 0.2 and 0.4) and not significantly different from zero. Blundell, Browning, and Meghir (1994), Atkeson and Ogaki (1996) and Attanasio and Browning (1995) also suggest that the elasticity increases with consumption.

Starting with Epstein and Zin (1991), many scholars have tried to employ a recursive specification for utility in estimating the elasticity of intertemporal substitution and have sometimes obtained greater estimated values.

In a recent paper, Jappelli and Padula (2013) estimate an Euler equation for consumption augmented by indicators of financial sophistication finding that the expected growth rate of consumption increases with financial literacy and that the intertemporal elasticity of substitution ranges between 0.1 and 0.4.

In the last decades, the contributions attempting at determining the intertemporal elasticity of substitution relying on Euler equation estimates are countless, but there is still controversy about whether or not that parameter is large enough to make changes in expected interest rates an important factor in fluctuations in consumption growth. For that matter, Attanasio and Wakefield (2010) argue that

> Despite the clear relevance of the EIS, a remarkable feature of the sizeable recent literature on the effect of the preferential taxation of retirement wealth on personal and national saving is that it never refers to the literature that has studied the life-cycle model and estimated preference parameters, including the EIS (Attanasio and Wakefield, 2010, p. 682).

The main empirical evidence on the elasticity of intertemporal substitution is presented in Table 5 in the Appendix, where the values of the parameter estimated in 28 empirical works are reported, specifying, for each piece of evidence, the data source and the utility function adopted.

## 5. Conclusions

The preceding sections have considered the most relevant parameters characterizing individual preferences and discussed the main properties of the utility functions commonly used in the consumption literature. The analysis suggests that the line of research devoted to estimating preference parameters has been deeply affected by the constraints imposed by the quantitative definition of the parameters themselves and by the utility functions adopted.

In the literature on the consumption Euler equation there has been a thorough discussion of the various econometric issues affecting the empirical results. Nevertheless, the implications of adopting a specific utility function appear to have received little attention.

It could be argued that the choice of the functional form is often made regardless of the features which make that function a reasonable representation of individual preferences: unappealing implications in terms of the direction in which preference parameters change when consumption increases seem to be usually ruled out from the criteria of choice of the utility function. In addition, the impact of the functional form on the results of the empirical work appears to be to some extent neglected. For example, the use of CARA preferences has been perhaps due, at least partially, to the fact that this utility function allows the derivation of a closed form solution for consumption. However, as has been argued, this functional form can be considered unappealing on theoretical grounds. As for CRRA utility, which is definitely the most popular function in the consumption literature, its main advantage is analytic convenience, as it yields first order conditions that are log-linear in consumption. However, such a specification imposes strong restrictions on preferences because it has only one parameter which must control prudence, risk aversion and intertemporal substitution and, in addition, it implies that relative risk aversion is the reciprocal of the elasticity of intertemporal substitution.

In the light of the essential role of the values assigned to preference parameters for the implications of the intertemporal utility maximization theory and, especially, for devising and implementing policy measures, it seems important to conduct a survey of the empirical evidence accumulated on the magnitude of the parameters and to make a general assessment of the results of this line of research. This paper provides such a contribution. The analysis conducted shows that the mathematical definition of preference parameters implies quantitative constraints linking them to one another which make the specification of the utility function crucial in assessing their values.

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## 7. Appendix

Table 1. Functional form and marginal utility for the main preferences specifications adopted in the Euler equation literature

| Expected utility HARA preferences | | |
|---|---|---|
| | Utility function | Marginal utility |
| Quadratic preferences | $u(C) = aC - \frac{b}{2}C^2$ | $u'(C) = a - bC$ |
| Exponential preferences | $u(C) = -\frac{e^{-\alpha C}}{\alpha}$ | $u'(C) = e^{-\alpha C}$ |
| Isoelastic preferences | $u(C) = \frac{C^{1-\gamma}}{1-\gamma}$ | $u'(C) = C^{-\gamma}$ |
| Logarithmic preferences | $u(C) = \ln C$ | $u'(C) = \frac{1}{C}$ |
| **Nonexpected utility recursive preferences** | | |
| | Utility function | Marginal utility |
| Isoelastic EZW preferences | $V_t = \left\{C_t^{1-\frac{1}{\sigma}} + \beta\left[\mathbb{E}_t\left(V_{t+1}^{1-\gamma}\right)\right]^{\frac{1-\frac{1}{\sigma}}{1-\gamma}}\right\}^{\frac{\sigma}{\sigma-1}}$ | $\frac{\partial V_t}{\partial C_t} = \left(\frac{V_t}{C_t}\right)^{\frac{1}{\sigma}}$ |

Table 2. Preference parameters of the main utility functions adopted in the Euler equation literature

| | Quadratic preferences | Exponential preferences | Isoelastic preferences | Logarithmic preferences | Recursive EZW preferences |
|---|---|---|---|---|---|
| $A(C)$ | $A(C) = \frac{b}{a-bC}$ | $A(C) = \alpha$ | $A(C) = \frac{\gamma}{C}$ | $A(C) = \frac{1}{C}$ | $A(C) = \frac{\gamma}{C}$ |
| $a(C)$ | $a(C) = \frac{b}{a-bC}C$ | $a(C) = \alpha C$ | $a(C) = \gamma$ | $a(C) = 1$ | $a(C) = \gamma$ |
| $\sigma(C)$ | $\sigma(C) = \frac{a-bC}{bC}$ | $\sigma(C) = \frac{1}{\alpha C}$ | $\sigma(C) = \frac{1}{\gamma}$ | $\sigma(C) = 1$ | $\sigma(C) = \sigma$ |
| $P(C)$ | $P(C) = 0$ | $P(C) = \alpha$ | $P(C) = \frac{1+\gamma}{C}$ | $P(C) = \frac{2}{C}$ | $P(C) = \frac{\gamma(1+\sigma)}{C}$ |
| $p(C)$ | $p(C) = 0$ | $p(C) = \alpha C$ | $p(C) = 1+\gamma$ | $p(C) = 2$ | $p(C) = \gamma(1+\sigma)$ |

Table 3. Preference parameters for isoelastic preferences

| | | | | | | |
|---|---|---|---|---|---|---|
| γ | 0.5 | 1 | 2 | 5 | 10 | 18 |
| σ | 2 | 1 | 0.5 | 0.2 | 0.1 | 0.06 |
| p | 1.5 | 2 | 3 | 6 | 11 | 19 |

Table 4. Preference parameters for recursive EZW preferences: relative prudence for different values of γ and σ

| | γ = 1 | γ = 2 | γ = 5 | γ = 8 | γ = 10 | γ =18 |
|---|---|---|---|---|---|---|
| σ = 1 | 2 | 4 | 10 | 16 | 20 | 36 |
| σ = 0.5 | 1.5 | 3 | 7.5 | 12 | 15 | 27 |
| σ = 0.2 | 1.2 | 2.4 | 6 | 9.6 | 12 | 21.6 |
| σ = 0.13 | 1.13 | 2.25 | 5.63 | 9 | 11.25 | 20.25 |
| σ = 0.1 | 1.1 | 2.2 | 5.5 | 8.8 | 11 | 19.8 |
| σ = 0.06 | 1.06 | 2.11 | 5.28 | 8.44 | 10.56 | 19 |

*Source: elaborations on Kimball and Weil (2009).*

Table 5. Empirical evidence on the elasticity of intertemporal substitution

| Authors | Data set | Sample period | Preferences | Estimates of the EIS | Other implications |
|---|---|---|---|---|---|
| Summers (1982) | Macro USA | 1950-1978 | CRRA | $\sigma \approx 1$ | |
| Muellbauer (1983) | Macro UK | 1955-1979 | CRRA | $\sigma \approx 1$ | |
| Wickens and Molana (1984) | Macro UK | 1963-1980 | CRRA | $\sigma = 0.74$ | |
| Hall (1988) | Macro USA | 1921-1983 | CRRA | $0 < \sigma < 0.2$ | |
| Attanasio and Weber (1989) | FES (UK) | 1970-1984 | EZW | $\sigma = 2$ | Average cohort data provides higher and better estimates |
| Browning (1989) | Macro USA | 1959-1982 | CRRA | $\sigma = 0.44$ (impr. est.) | EIS is not constant over the year |
| Zeldes (1989) | PSID (USA) | 1968-1982 | CRRA | $\sigma = 0.4$ (impr. est.) | |
| Campbell and Mankiw (1989) | Macro USA | 1953-1986 | CRRA | $\sigma = 0.2$ | One half of households are rule of thumb consumers |
| Epstein and Zin (1991) | Macro USA | 1959-1986 | EZW | $0.05 < \sigma < 1$ | |
| Attanasio and Weber (1993) | FES (UK) | 1970-1986 | CRRA | $\sigma = 0.3$ (ag. data)<br>$\sigma = 0.8$ (c. data) | Importance of demographics and labour supply |
| Blundell *et al.* (1994) | FES (UK) | 1970-1987 | Gen. CRRA | $0.64 < \sigma < 1.17$ | EIS increases with consumption |
| Attanasio and Weber (1995) | CEX (USA) | 1980-1990 | CRRA | $0.2 < \sigma < 0.4$ | Importance of nonseparability between food and other consumption |
| Attanasio and Browning (1995) | FES (UK) | 1970-1986 | Gen. CRRA | $\sigma = 0.2$ (impr. est.) | Importance of demographics and labour supply; EIS increases with consumption |
| Atkeson and Ogaki (1996) | ICRISAT; Macro India and USA | 1960-1987 | Gen. CRRA | $0.5 < \sigma < 0.8$ (panel)<br>$\sigma = 0.40$ (USA ag. data)<br>$\sigma = 0.27$ (India ag. data) | EIS increases with consumption<br>Little effect of subsistence levels |
| Ogaki and Reinhart (1998) | Macro USA | 1951-1983 | Gen. CRRA | $0.32 < \sigma < 0.45$ | Importance of nonseparability between nondurables and durables |
| Attanasio *et al.* (1999) | CEX (USA) | 1982-1990 | CRRA | $\sigma = 0.73$ | Importance of demographics |
| Mulligan (2002) | Macro USA | 1947-1997 | EZW | $\sigma > 1$ | |

Table 5. (*cont.*)

| Authors | Data set | Sample period | Preferences | EIS | Other implications |
|---|---|---|---|---|---|
| Vissing-Jørgensen (2002) | CEX (USA) | 1980-1996 | EZW | $0.3 < \sigma < 0.4$ (stocks)<br>$0.8 < \sigma < 1$ (bonds)<br>$\sigma = 0.2$ (no assets) | Importance of asset market participation |
| Yogo (2004) | Macro* | 1970-1998* | EZW | $\sigma \approx 0$ | Weak instruments can explain low estimates |
| Gruber (2006) | CEX (USA) | 1979-2002 | CRRA | $\sigma = 2$ | |
| Alan, Attanasio, and Browning (2009) | PSID (USA) | 1974-1987 | CRRA | $\sigma = 1.45$ | Importance of dealing with measurement error |
| Khorunzhina and Gayle (2011) | PSID (USA) | 1974-1987 | CRRA with habits | $0.083 < \sigma < 0.193$ | EIS variable across households and over time |
| Jappelli and Padula (2013) | SHIW (Italy) | 2006-2010 | CRRA | $0.2 < \sigma < 0.4$ | Importance of financial literacy |
| Chen, Favilukis, and Ludvigson (2013) | Macro USA | 1952-2005 | EZW | $\sigma > 1$ | |
| Khvostova, Larin, and Novak (2014) | RLMS-HSE (Russia) | 2000-2013 | CRRA with habits | $\sigma = 4.167$ | The effects of habit formation are not significant |
| Choi, Lugauer, and Mark (2014) | URHS (China) and CEX (USA) | 2007 (China)<br>1992-2007 (US) | EZW | $\sigma = 1.44$ (USA)<br>$\sigma = 2.16$ (China) | China and USA are more similar in their $\gamma$ than in their $\sigma$ |
| Gomes and Ribeiro (2015) | Macro USA | 1952-2000 | EZW | $0.4 < \sigma < 1.8$ | Evidence against CRRA utility function |
| Best *et al.* (2015) | PSD (UK) | 2008-2014 | EZW | $0.05 < \sigma < 0.25$ | |

*FES: Family Expenditure Survey (UK); PSID: Panel Study of Income Dynamics (USA); CEX: Consumer Expenditure Survey (USA); ICRISAT: Indian panel; SHIW: Survey of Household Income and Wealth (Italy); RLMS-HSE: Russian Longitudinal Monitoring Survey of the Higher School of Economics (Russia); PSD: Product Sales Database (UK); impr. est.: imprecisely estimated; ag. data: aggregate data; c. data: cohort data.*

**Australia 1970-1998; Canada 1970-1999; France 1970-1998; Germany 1979-1998; Italy 1971-1998; Japan 1970-1998; Netherlands 1977-1988; Sweden 1970-1999; Switzerland 1976-1998; UK 1970-1999; USA 1970-1998.*

# The choice of the functional form in the consumption Euler equation approach: a simulation exercise

## 1. Introduction

In their thorough review of the early literature on the consumption Euler equation, Browning and Lusardi (1996) present a simple two period model in which an individual maximizes expected utility endowed with a certain current income and a future uncertain one. The purpose of this example is to show how misleading the certainty-equivalence model can be in the presence of uncertainty. By certainty-equivalence model, they mean the theoretical framework proposed by Hall (1978) in his influential contribution that extended the life cycle – permanent income hypothesis developed in the fifties by Modigliani and Friedman[1] to the case of uncertainty about life cycle income and interest rates. Hall introduced the rational expectations hypothesis and derived a set of orthogonality conditions from the intertemporal optimisation problem faced by the consumer, giving birth to the consumption Euler equation approach.

The certainty-equivalence model proposed by Hall, widely used in the subsequent empirical research,[2] is built on the hypothesis that agents have intertemporally additive quadratic utility functions with a constant discount factor, face perfect capital markets, and maximize expected utility forming rational expectations. This model implies that the time pattern of income is irrelevant for consumption because the individual uses borrowing and saving to smooth the path of consumption. In particular, the famous result found by Hall is that consumption follows a martingale: in each period, expected next period consumption equals current consumption. This means that the marginal propensities to consume out of current and future expected income are the same and the marginal propensity to consume out of future income is independent of the riskiness of that income: the individual consumes the amount he would consume if his future income were certain to equal its mean. In other words, uncertainty about future income has no impact on consumption.

As Browning and Lusardi point out, most of the conclusions reached by Hall derive from the hypothesis that preferences are quadratic: without assuming a specific functional form for the utility function, the model only leads to the result that agents keep the ex-

[1] Modigliani and Brumberg (1954) and Friedman (1957).

[2] See, for example, Flavin (1981), Wickens and Molana (1984), and Campbell and Mankiw (1989).

pected (discounted) marginal utility of expenditure constant over time; it is because of the linearity of marginal utility typical of quadratic preferences that Hall claims that consumption itself, and not only marginal utility, follows a martingale.[3]

Analytically, the consumption Euler equation is

$$u'(C_t) = \beta \mathbb{E}_t[(1 + r_t) \cdot u'(C_{t+1})] \quad (1)$$

where $C_t$ is consumption in period $t$, $\beta = (1 + \delta)^{-1}$ is the discount factor, $\delta$ is the rate of time preference and $r_t$ is the stochastic interest rate. In the case of quadratic preferences, the Euler equation takes the form

$$C_t = \beta \mathbb{E}_t[(1 + r_t)C_{t+1}] \quad (2)$$

To illustrate the importance of uncertainty in determining the shape of the lifetime path of consumption, Browning and Lusardi consider a theoretical framework analogous to the one found in the certainty-equivalence model, with the decisive difference that preferences are logarithmic rather than quadratic. Through a simple but effective numerical example, they show how crucial the hypothesis of quadratic preferences is for the conclusions drawn in Hall (1978), which happened to deeply influence the early stages of development of the Euler equation approach.[4]

The purpose of this paper is to analyse the impact that the functional form of the utility function had on the literature on the consumption Euler equation. As well known, the Euler equation allows to both test the validity of the underlying model and to estimate the structural parameters of the utility function, whose magnitude has become increasingly important for the calibration of modern dynamic general equilibrium models. We shall argue that the empirical works devoted to the estimation of preference parameters have been substantially affected by the necessity of specifying a functional form for preferences, since the particular utility functions chosen played a crucial role in determining the results obtained. In fact, it appears that the impact of the functional form on the empirical tests has been – at least partially – neglected in the debate and may deserve closer attention.

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[3] Actually, as Browning and Lusardi (1996) stress, there are supplementary implicit assumptions which are required to reach the martingale result. In particular, one needs to assume that consumption and labour supply are additively separable and, more generally, that the marginal utility of expenditure does not depend on (predictable) changes in demographic variables. For a discussion see, in addition to Browning and Lusardi (1996), Attanasio and Weber (2010).

[4] In fact, the martingale result has given rise to a stream of research, beginning with the work of Flavin (1981), which aims at explaining why empirical data suggest that consumption growth is mostly predictable. This evidence, conflicting with Hall's result, has come to be known as the "excess sensitivity puzzle". For a detailed survey, see Jappelli and Pistaferri (2010).

In order to highlight the importance of the functional form adopted, the numerical example proposed by Browning and Lusardi (1996) will be extended to the case of different utility functions and slightly more complicated hypotheses about the stochastic framework faced by the consumer. This simple simulation exercise is meant to show that the same saving behaviour can be associated with pretty different attitudes towards risk and uncertainty – i.e. to different magnitudes of the preference parameters – depending on the utility function adopted. Obviously, such a simple two period model like the one which is going to be presented in the next sections does not allow to derive general indications on the magnitude and the direction of the influence of the functional form on the outcome of an intertemporal optimising problem. Nevertheless, it seems a straightforward way to suggest that the issue may deserve further investigation. Moreover, the simulation exercise will be associated with a more formal and general analysis of the impact of parameters values on the saving behaviour of the consumer, conducted by means of a decomposition of the rate of growth of consumption derived by the consumer's Euler equation. Such an analysis is inspired by the work of Choi, Lugauer, and Mark (2014), who employ a model of precautionary saving to explain saving rate differences between the U.S. and China and, thanks to a recursive specification of utility, present a decomposition of saving into precautionary and non-precautionary components.

The paper is organised as follows. In the next section, the numerical example proposed by Browning and Lusardi (1996) is presented and discussed; Section 3 provides a review of the preference parameters and the utility functions that have been the focus of the empirical work on the consumption Euler equation; Section 4 presents a decomposition of the rate of growth of consumption on the lines of Choi, Lugauer, and Mark (2014); Section 5 extends the numerical example proposed by Browning and Lusardi (1996); Section 6 concludes.

## 2. The misleading role of quadratic preferences: a simple numerical example

In the two period example proposed by Browning and Lusardi (1996), the agent maximizes his expected utility $\mathbb{E}[U] = \mathbb{E}[\sum u(C_t)]$ with a logarithmic instantaneous utility function $u(C_t) = \ln C_t$ where $C_t$ is consumption in period t. In the first period the agent has a certain income $Y_1$ and in the second period earnings are stochastic: they are zero with probability $\varepsilon$ and $Y_2/(1-\varepsilon)$ with probability $(1-\varepsilon)$. Thus, the expected value of second period income is $Y_2$ and an increase in $\varepsilon$ represents a mean preserving spread in future income risk. Finally, they assume that the real rate of interest, as well as the rate of time preference, is zero.

Browning and Lusardi imagine two possible cases, which imply, respectively, low or high first period income relative to second period expected income. They first show that in the case of perfect certainty ($\varepsilon = 0$), regardless of the time profile of income, the agent acts as if he had quadratic preferences, i.e. he consumes half of lifetime (expected) resources in each period. That happens because, in the case of certainty and in the case of quadratic utility function, the only motive for saving is the life cycle motive, i.e. the individual saves in order to smooth the lifetime path of consumption. On the contrary, for $\varepsilon > 0$ (in their example $\varepsilon = 0{,}01$) the time profile of income becomes crucial in determining that of consumption. In the latter case, indeed, in addition to the life cycle motive for saving, the individual has a precautionary motive triggered by uncertainty.

Assuming first $Y_1 = 2$ and $Y_2 = 1$ (high first period income scenario) and then $Y_1 = 1$ and $Y_2 = 2$ (low first period income scenario), they calculate first period consumption, the saving rate from first period income, the marginal propensities to consume (MPC) out of first and second period income, the effective discount rate used to discount future expected earnings (the discount rate for future expected earnings which would give the associated first period consumption if the agent had quadratic preferences), and the variance of second period log consumption (see Table 1).

The first conclusion they stress is that the degree to which the certainty-equivalence model approximates the model with logarithmic preferences depends on the time path of expected income. In particular, if first period cash-on-hand is low relative to second period expected earnings, then there can be wide divergences between the certainty-equivalence model and a model with a different specification of preferences.

Table 1: Simulations by Browning and Lusardi (1996)

| | $\varepsilon = 0$<br>$Y_1 = 1$ $Y_2 = 2$ | $\varepsilon = 0.01$<br>$Y_1 = 2$ $Y_2 = 1$ | $\varepsilon = 0.01$<br>$Y_1 = 1$ $Y_2 = 2$ |
|---|---|---|---|
| First period consumption | 1.5 | 1.49 | 0.98 |
| Saving rate | -0.5 | 0.255 | 0.02 |
| MPC from first period income | 0.5 | 0.51 | 0.97 |
| MPC from second period income | 0.5 | 0.47 | 0.01 |

The example also shows that, with low first period income, the MPC out of cash-on-hand can be close to unity even though there is a very small amount of uncertainty. Furthermore, with low first period income the MPC out of the future expected income can be close to zero. Thus, the certainty-equivalence model prediction that the MPCs out of the current and future income are the same is wildly wrong for some agents. Finally, even a small amount of uncertainty may be sufficient to stop the agent from borrowing in the first period.

Overall, this numerical example, though very simple, provides a clear insight on the crucial role that an inappropriate utility function can play. The explanation for such misleading results, in the case of quadratic preferences considered by Browning and Lusardi, lies in the linearity of marginal utility: when marginal utility is linear, the expected marginal utility of consumption is the same as the marginal utility of expected consumption. It is only when marginal utility is convex, rather than linear, that uncertainty has an impact on current consumption. In fact, the positive third derivative of the utility function increases the consumer's desire to insure himself against the fall in consumption that would result from a fall in income and so increases savings.

The explicative power of the numerical example just discussed suggests extending this simple model to the case of alternative utility functions in order to evaluate the impact of the functional form and the values of preference parameters on saving behaviour.

## 3. Preference parameters and utility functions[5]

With the purpose of evaluating the impact of the utility function chosen in the estimation of preference parameters through the Euler equation, Section 5 replicates the simulations proposed by Browning and Lusardi (1996) with different utility functions: isoelastic (also called power or CRRA – constant relative risk aversion) preferences, exponential (or CARA – constant absolute risk aversion) preferences, and recursive Epstein-Zin-Weil (EZW) preferences. The literature aiming at estimating the preference parameters rests on these functional forms and, as we shall see, the simplest case of logarithmic utility found in Browning and Lusardi (1996) is a special case of isoelastic preferences.

The preference parameters whose magnitude has been estimated by means of the Euler equation are the coefficient of relative risk aversion, the coefficient of relative prudence and the elasticity of intertemporal substitution. Each of these parameters describes a specific aspect of individual preferences: the measure of risk aversion indicates how much the individual is willing to sacrifice his consumption in the best case scenario in order to achieve a

[5] For a more detailed presentation of the preference parameters and the main utility functions see, respectively, Section 2 and Section 3 of the previous paper, of which is here presented a brief summary.

higher level of consumption in the worse; the magnitude of prudence tells how an increase in uncertainty about future income affects current consumption; the intertemporal elasticity of substitution measures to what extent the individual is willing to substitute future consumption for current consumption in response to the incentive given by the interest rate.

The risk attitude of the individual is summarised through the formal measures proposed by Pratt (1964) and Arrow (1965), which rest on the second derivative of the utility function normalised by the first. The Arrow–Pratt measure of absolute risk aversion, suited to additive risks, is $A(C) = -u''(C)/u'(C)$ and the corresponding measure of relative risk aversion, for multiplicative risks, is $a(C) = -u''(C) \cdot C/u'(C)$.

In parallel to the Arrow-Pratt coefficients, Kimball (1990) proposed two measures of the intensity of the precautionary saving motive which rely on the convexity of marginal utility: a measure of absolute prudence $P(C) = -u'''(C)/u''(C)$ and a measure of relative prudence $p(C) = -u'''(C) \cdot C/u''(C)$.

As for the aversion to intertemporal fluctuations in consumption, that is measured through the elasticity of intertemporal substitution $\sigma(C) = -u'(C)/u''(C) \cdot C$, which represents the proportional change in consumption growth that must follow to a one percent change in the interest rate in order to keep the marginal utility of consumption constant. The magnitude of the response of consumption growth to changes in the interest rate depends (inversely) on the curvature of the utility function: if the marginal utility of consumption is very sensitive to changes in consumption, then the individual can adjust the profile of marginal utility to the new interest rate with little change in consumption and the intertemporal motive for saving will be weak.

The coefficients just listed, despite measuring different aspects of preferences, have close mathematical relationships between each other because of the way they are defined, which impose strict constraints on the estimates of the parameters values.[6] In addition to that, the choice of the functional form adopted to represent individual preferences, although inescapable for the estimation of preference parameters through Euler equations, adds further constraints on the range of values the coefficients can take.

It therefore appears useful to summarise the main features of the utility functions employed in the consumption Euler equation literature, beginning with the simplest and first adopted, which is the quadratic form. Table A.1 in Appendix A presents the functional

[6] For a discussion of the links between the parameters of preference see Kimball and Weil (2009) or the paragraph 2.4 of the previous paper.

forms here discussed with the corresponding Euler equations.

The quadratic utility function $u(C_t) = aC_t - \frac{b}{2}C_t^2$ implies linear marginal utility and therefore exhibits no prudence. Furthermore, it implies increasing absolute risk aversion, which means that the amount of consumption that individuals are willing to give up in order to avoid a given amount of uncertainty about the level of consumption rises as they become wealthier. This property is unappealing on theoretical grounds and strongly counterfactual (riskier portfolios are normally held by wealthier households). Quadratic preferences also imply that the willingness to substitute over time is a decreasing function of consumption, which is another feature that makes its use unappealing: according to this functional form, poor consumers should react much more to interest rate changes than rich consumers, which is not the case. All these issues make the quadratic utility function unattractive and caused this functional form to disappear pretty soon from the consumption Euler equation literature. However, with quadratic utility preference parameters are not identified since the Euler equation reduces to $\mathbb{E}_t[C_{t+1}] = C_t$, so this function is just unsuitable for the estimation of preference parameters.

Another functional form which played an important role in the early stages of the Euler equation approach is the exponential utility function $u(C_t) = -\frac{e^{-\alpha C_t}}{\alpha}$, as it is the only functional specification, besides quadratic preferences, which allows deriving a closed-form solution for consumption.[7] This utility function exhibits constant absolute risk aversion $\alpha$, as well as constant (equal) absolute prudence. This implies that poor people and rich people reduce their consumption by exactly the same amount in reaction to a given risk: the increase in consumption required to keep the same level of expected marginal utility in the face of an increase in risk is independent of the initial level of consumption. On the other hand, relative risk aversion is, in the exponential case, increasing in consumption. These appear to be not very realistic assumptions and led, together with the possibility of negative solutions for consumption, to a dismissal of exponential preferences.[8]

As for the isoelastic utility function $u(C_t)=\frac{{C_t}^{1-\gamma}}{1-\gamma}$, it has been used in the consumption literature since the papers by Hansen and Singleton (1982 and 1983) and it has played a preeminent role in many theoretical studies of asset pricing and in the empirical tests of the validity of the standard consumption model. In fact, the assumptions of CRRA utility

[7] For a derivation of an expression for consumption under exponential preferences see Merton (1971) and Caballero (1990).

[8] For a discussion see Blanchard and Mankiw (1988).

and lognormality of the joint distribution of consumption and stock returns together lead to an empirically tractable, closed-form characterization of the restrictions implied by the model. That functional form is also the usual choice in empirical works aiming at estimating preference parameters because it leads to an approximation of the Euler equation that is linear in parameters, which implies high econometric tractability.

In the special case of isoelastic preferences the coefficient of relative risk aversion, the intertemporal elasticity of substitution and the coefficient of relative prudence, respectively equal to $\gamma$, $1/\gamma$, and $\gamma + 1$, are all constant. Therefore, the elasticity of substitution is equal to the reciprocal of the degree of risk aversion and specifying the degree of risk aversion also pins down the degree of prudence. This follows because there is only one parameter in the utility function so this must control all the aspects of preferences. Thus, though very tractable from an analytical point of view, this preference specification imposes strong restrictions on the parameters to be estimated and lacks the flexibility which would be desirable to take a utility function to the data.[9]

A functional form that implies a higher degree of flexibility is the one inspired by the work of Kreps and Porteus (1978) and developed by Epstein and Zin (1989) and Weil (1990), where utility is recursively defined over current consumption and a certainty equivalent of future random utility.

With Epstein-Zin-Weil (EZW) preferences $V_t = \left\{C_t^{1-\frac{1}{\sigma}} + \beta\left[\mathbb{E}_t\left(V_{t+1}^{1-\gamma}\right)\right]^{\frac{1-\frac{1}{\sigma}}{1-\gamma}}\right\}^{\frac{\sigma}{\sigma-1}}$ there are two parameters in the utility function so the degree of risk aversion (equal to $\gamma$) is disentangled from the elasticity of intertemporal substitution ($\sigma$). As in the isoelastic case, they are both constrained to be constant, but, unlike the other functional forms, it is possible to admit a high aversion to risk and a high aversion to intertemporal substitution to coexist. In fact, EZW preferences can be conceived as a generalization of the standard time additive expected utility function, which collapses to the isoelastic case when the coefficient of relative risk aversion equals the reciprocal of the elasticity of intertemporal substitution, i.e. when $\sigma\gamma = 1$. However, the separation between the preference parameters governing attitudes towards *time* and *state* fluctuations of consumption comes at the cost of accepting

[9] According to Browning and Lusardi, «Among the criteria that should guide the choice of functional form are: congruence with theory (we should be able to impose theoretical restrictions in a simple way); flexibility (for example, important elasticities should not be constrained to be constant); and econometric tractability (for example, linearity in parameters is desirable)» (Browning and Lusardi, 1996, p. 1826).

that the individual has a preference for the timing of resolution of uncertainty: whenever the individual has greater (smaller) aversion to risk than to intertemporal substitution, early (late) resolution of uncertainty is preferred. Epstein and Zin (1989) show that recursive utility implies that the temporal resolution of risk matters and Epstein, Farhi, and Strzalecki (2014) provide a quantitative assessment of timing premia. The timing premium is defined as the fraction of consumption stream that the individual would be willing to give up in order for all risk to be resolved, and depends on both the preference parameters and on the nature of the endowment process: the farther the elasticity of intertemporal substitution from the reciprocal of relative risk aversion and the more persistent the consumption process, the greater the timing premium. Basically, if the isoelastic specification entails that a single parameter governs two aspects of preferences, with recursive EZW utility two parameters control three aspects of preferences.

As for the intensity of the precautionary motive for saving, EZW preferences imply $p(C) = \gamma + \gamma\sigma$, that is to say that once aversion to risk and aversion to intertemporal substitution are set, prudence follows accordingly.

This brief review of the main utility functions suggests that every functional form entails strong – and sometimes unappealing – assumptions on preferences and, in particular, implies some restrictions on the magnitude of the preference parameters and on their direction of variation following a change in consumption. To further investigate the role of the parameters in each preference specification, the next section provides an analysis of the sensitivity of the optimal saving rate to a change of the coefficients of the utility function.

## 4. A decomposition of the saving rate

Once we leave the quadratic preferences assumption, closed-form solutions for consumption or the saving rate are not available, but we can deduce some of the properties of the optimal behaviour of the consumer from an analysis of the Euler equation. The purpose is to infer how a change in the parameters of the utility function affects the saving rate in each of the functional forms employed in the simulations presented in the next section.

In the case of exponential preferences, if a non stochastic interest rate $r = 1 - R$ is assumed, the Euler equation becomes:

$$\exp[-\alpha C_t] = \beta R \mathbb{E}_t\big[\exp[-\alpha C_{t+1}]\big]$$

or, rearranging:

$$\mathbb{E}_t[C_{t+1}] = C_t + \ln \beta R^{1/\alpha}.$$

The term $\ln \beta R^{1/\alpha}$ represents the intertemporal substitution motive for saving. As the Euler equation shows, under exponential utility intertemporal substitution does not take a multiplicative form that implies changes affecting the rate of growth of consumption, but entails additive changes in the level of consumption.

Under suitable hypotheses, Caballero (1990) derives a closed-form solution for consumption in the case of CARA utility and shows that this functional form implies a consumption function that, when $\beta R = 1$, may be decomposed, additively, in a term analogous to that of the certainty equivalence case and a precautionary saving component. That means that for any level of wealth, consumption is equal to consumption under quadratic preferences minus a term related to precautionary saving behaviour. If $\beta R \neq 1$, it is still possible to decompose the consumption function into a precautionary saving term and a second term but the latter is no longer identical to consumption under certainty equivalence.

Caballero also shows that the slope of the consumption path when $\beta R = 1$ is positive.

In the case of isoelastic or recursive EZW preferences, from the log-linearized version of the Euler equation it is possible to infer the variation of the (expected) rate of growth of consumption $\mathbb{E}(\Delta \ln C_{t+1})$ in response to a change of the parameters of the utility function. An increase in the consumption growth rate can be interpreted as an increase in the saving rate, so as to drive some considerations on the effects of changes in preference parameters.

Under the assumption of isoelastic preferences, the Euler equation takes the form

$$1 = \beta(1 + r)\mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right]. \qquad (3)$$

By assuming that consumption is log-normally distributed, we can derive the log-linearized version of the Euler equation usually employed in the empirical work:

$$\mathbb{E}(\Delta \ln C_{t+1}) = \frac{r-\delta}{\gamma} + \frac{\gamma}{2} \mathrm{Var}(\Delta \ln C_{t+1}). \qquad (4)$$

As appears from (4), under isoelastic utility the saving rate depends on a mean-variance relationship for consumption growth: an increase in the variance of consumption implies a higher precautionary saving and therefore higher consumption growth.

The first term in (4) summarises the incentive given by the interest rate, in fact the coefficient on $(r - \delta)$ is the elasticity of intertemporal substitution $\frac{1}{\gamma}$. The second term in (4) is the effect of the precautionary motive on mean consumption growth, which depends on the variability of future consumption $\mathrm{Var}(\Delta \ln C_{t+1})$, so $\frac{\gamma}{2}$ captures the intensity of the precautionary motive for saving. As already stressed, the same parameter $\gamma$ regulates both the attitude towards uncertainty and the attitude towards intertemporal substitution.

Differentiating ( 4 ) with respect to $\gamma$ gives

$$\frac{\partial \mathbb{E}(\Delta \ln C_{t+1})}{\partial \gamma} = \left(\frac{\delta - r}{\gamma^2}\right) + \frac{1}{2} \mathrm{Var}(\Delta \ln C_{t+1}) + \frac{\gamma}{2} \frac{\partial \mathrm{Var}(\Delta \ln C_{t+1})}{\partial \gamma}. \qquad (5)$$

Equation ( 5 ) shows that an increase in risk aversion (or, equivalently, a decrease of the elasticity of intertemporal substitution) has a direct effect on the saving rate, given in the first two terms of ( 5 ), and an indirect effect through the impact on the variance of consumption. If $\gamma$ raises, the elasticity of intertemporal substitution decreases, which means that the consumer is more averse to intertemporal fluctuations of consumption. That leads to lower consumption volatility, i.e. $\frac{\partial \mathrm{Var}(\Delta \ln C_{t+1})}{\partial \gamma} < 0$ which implies that the indirect effect is negative. The direct effect, on the contrary, is positive for "impatient" individuals, i.e. when $\delta > r$. The total effect of an increase in $\gamma$ on the saving rate of the consumer, under isoelastic utility, should therefore be positive as long as the indirect effect is not too high.

The analysis of the impact of a change in preference parameters becomes much more rich and complex under the hypothesis of recursive EZW utility, since it implies a distinction between risk aversion and intertemporal substitution.

With EZW preferences the Euler equation takes the form

$$1 = \beta(1+r)\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}} \left\{\frac{V_{t+1}}{\left[\mathbb{E}_t\left(V_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right\}^{\frac{1-\gamma\sigma}{\sigma}}\right]. \qquad (6)$$

Following Parker and Preston (2005) and Choi, Lugauer, and Mark (2014), we let $Z_{t+1}$ be next period utility relative to its certainty-equivalent:

$$Z_{t+1} \equiv \frac{V_{t+1}}{\left[\mathbb{E}_t\left(V_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}$$

and refer to $Z_{t+1}$ as a preference shifter. With this notation, we can write the Euler equation as

$$1 = \beta(1+r)\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}} Z_{t+1}^{\frac{1-\gamma\sigma}{\sigma}}\right]. \qquad (7)$$

Assuming that consumption and utility are log-normally distributed gives the log-linearized version of the Euler equation

$$\mathbb{E}[\Delta \ln C_{t+1} - (1-\gamma\sigma)\ln Z_{t+1}] = \sigma(r-\delta) + \frac{1}{2\sigma}\mathrm{Var}[\Delta \ln C_{t+1} - (1-\gamma\sigma)\ln Z_{t+1}] \qquad (8)$$

From a comparison between ( 8 ) and ( 5 ), we see that under EZW preferences the mean-variance relationship between consumption growth includes the preference shifter as well, generalizing to $\Delta \ln C_{t+1} - (1-\gamma\sigma)\ln Z_{t+1}$.[10]

In order to better assess the impact of variations in the parameter values on the saving rate, it is useful to rearrange ( 8 ) and express the expected rate of consumption growth as

$$\mathbb{E}(\Delta \ln C_{t+1}) = \theta + \phi + \psi \tag{9}$$

where

$$\theta \equiv \sigma(r-\delta) \tag{10}$$

$$\phi \equiv (1-\gamma\sigma)\mathbb{E}(\ln Z_{t+1}) \tag{11}$$

$$\psi \equiv \frac{1}{2\sigma}\mathrm{Var}[\Delta \ln C_{t+1} - (1-\gamma\sigma)\ln Z_{t+1}]. \tag{12}$$

Following Choi, Lugauer, and Mark (2014), we shall refer to $\theta$ as the intertemporal substitution effect, to $\phi$ as the preference shifter, and to $\psi$ as the precautionary effect.

The intertemporal substitution effect, as obvious, depends on the gap between the interest rate and the subjective rate of time preference and on the elasticity of intertemporal substitution $\sigma$.

The preference shifter represents the preference for the timing of resolution of uncertainty. The term $\mathbb{E}(\ln Z_{t+1})$ can be interpreted as the cost of carrying uncertainty to the future and $\gamma\sigma$ as the risk adjusted elasticity of substitution for uncertainty resolution.[11] If $\gamma < 1/\sigma$ then $\phi$ is positive and the consumer prefers later resolution of uncertainty and raises consumption growth by consuming less today. Under the log-normality assumption the cost of carrying uncertainty to the future is $\mathbb{E}(\ln Z_{t+1}) = (\gamma-1)\mathrm{Var}(\ln Z_{t+1})/2$. Substituting into Equation ( 11 ) gives

$$\phi = \frac{(1-\gamma\sigma)(\gamma-1)\mathrm{Var}(\ln Z_{t+1})}{2} \tag{13}$$

As for the precautionary effect, $\psi$ raises with volatility and its intensity depends on both risk aversion and the elasticity of intertemporal substitution (under EZW preferences the coefficient of relative prudence is $p(C) = \gamma + \gamma\sigma$).

Having clarified the channels through which a variation in the parameter values affect the saving rate, let us analyse both the impact of an increase in $\sigma$ and in $\gamma$.

As for the elasticity of intertemporal substitution, it does not have a monotonic relationship with the saving rate. If the consumer is impatient ($\delta > r$), then increasing $\sigma$ lowers expected consumption growth (and hence saving) directly by depressing the

[10] Notice that equation ( 8 ) reduces to ( 5 ), the log-linearized Euler equation under isoelastic preferences, by setting $\gamma = 1/\sigma$.

[11] Notice that when $\gamma\sigma = 1$ EZW preferences collapse into isoelastic utility: in that case, the timing of resolution of uncertainty becomes irrelevant and, consistently, the preference shifter goes to zero.

intertemporal substitution effect: $\frac{\partial\theta}{\partial\sigma} < 0$. When shifting consumption across time periods is easy for an impatient individual, he will shift consumption towards the present.

The effect of increasing the elasticity of intertemporal substitution on the preference shifter is

$$\frac{\partial\phi}{\partial\sigma} = -\frac{\gamma(\gamma-1)\mathrm{Var}(\ln Z_{t+1})}{2} + \frac{(1-\gamma\sigma)(\gamma-1)}{2}\frac{\partial\mathrm{Var}(\ln Z_{t+1})}{\partial\sigma} \tag{14}$$

The first term in ( 14 ) is a direct effect and is negative when risk aversion is greater than one. In this case, raising $\sigma$ results in a stronger preference for the early resolution of uncertainty and lowers the preference shifter: $\frac{\partial\phi}{\partial\sigma} < 0$. Thus, the consumption growth (and hence saving) falls with $\sigma$. The second term works through the variance of utility and is an indirect effect. Making it easier for people to move consumption across time periods with higher $\sigma$ results in higher volatility of consumption and utility (hence $Z_{t+1}$). Therefore, the indirect effect is negative when the risk aversion and intertemporal substitution are high ($\gamma > 1$ and $\gamma\sigma > 1$).

The effect of increasing the elasticity of intertemporal substitution on the precautionary component is

$$\begin{aligned}\frac{\partial\psi}{\partial\sigma} = & -\frac{1}{2\sigma^2}\mathrm{Var}[\Delta\ln C_{t+1} - (1-\gamma\sigma)\ln Z_{t+1}] \\ & +\frac{\gamma}{\sigma}[(\gamma\sigma-1)\mathrm{Var}(\ln Z_{t+1}) + \mathrm{Cov}(\Delta\ln C_{t+1}, \ln Z_{t+1})] \\ & +\frac{1}{2\sigma}\left[\frac{\partial\mathrm{Var}(\Delta\ln C_{t+1})}{\partial\sigma} + 2(\gamma\sigma-1)\frac{\partial\mathrm{Cov}(\Delta\ln C_{t+1}, \ln Z_{t+1})}{\partial\sigma}\right]\end{aligned} \tag{15}$$

The first two terms in ( 15 ) are the direct effect. The first term is clearly negative. Given volatility, $\mathrm{Var}[\Delta\ln C_{t+1} - (1-\gamma\sigma)\ln Z_{t+1}]$, raising $\sigma$ directly lowers the precautionary saving motive as people can easily substitute consumption across time. The second term represents the change in the contribution of variation in the preference shifter $\ln Z_{t+1}$ on volatility. When $\gamma\sigma > 1$, the variation of the preference shifter contributes positively to the overall volatility. In this case the precautionary saving motive rises with $\sigma$. For the overall direct effect, when $\gamma\sigma > 1$, the two terms have opposite signs. Thus, the direct effect of increasing $\sigma$ on the precautionary component is ambiguous. Combining the first and second terms gives

$$\frac{1}{2\sigma^2}[(\gamma\sigma)^2\mathrm{Var}(\ln Z_{t+1}) - \mathrm{Var}(\ln Z_{t+1} - \Delta\ln C_{t+1})] \tag{16}$$

which is increasing in $\gamma$ and $\sigma$. Thus, for the overall direct effect, increasing $\sigma$ lowers the precautionary saving motive when risk aversion is low and raises $\psi$ when risk aversion is high. The last term in ( 15 ) is the indirect effect working through the changes in the variations of consumption growth and utility. As the variability of consumption and utility rise with substitutability, the indirect effect is positive when substitutability is not too low, $\gamma\sigma > 1$.

The overall relationship between the intertemporal elasticity of substitution and the saving rate cannot be unambiguously signed, but Choi, Lugauer, and Mark (2014) consider reasonable to suppose that increasing $\sigma$ lowers the saving when risk aversion is low and raises the saving rate when risk aversion is high. Combining all effects, we have

$$\begin{aligned}\frac{\partial \mathbb{E}(\Delta \ln C_{t+1})}{\partial \sigma} &= (r-\delta) + \frac{\gamma}{2}\mathrm{Var}(\ln Z_{t+1}) - \frac{1}{2\sigma^2}\mathrm{Var}(\ln Z_{t+1} - \Delta \ln C_{t+1}) \\ &+ \frac{1}{2\sigma}\left\{\frac{\partial \mathrm{Var}(\Delta \ln C_{t+1})}{\partial \sigma} + (\gamma\sigma - 1)(\sigma - 1)\frac{\partial \mathrm{Var}(\Delta \ln Z_{t+1})}{\partial \sigma}\right. \\ &\left. + 2(\gamma\sigma - 1)\frac{\partial \mathrm{Cov}(\Delta \ln C_{t+1}, \ln Z_{t+1})}{\partial \sigma}\right\}\end{aligned} \quad (17)$$

The first three terms are the direct effects and the last term is the indirect effect. The overall indirect effect, the last term, is positive when $\gamma\sigma > 1$ and $\sigma > 1$. The overall direct effect is increasing in $\gamma$ and $\sigma$ given volatility. For the direct effect, increasing $\sigma$ lowers (raises) the saving rate when $\gamma$ is relatively low (high).

For a moderate value of $\gamma$, the saving rate profile has a U shape with respect to $\sigma$. The desire to accumulate a buffer-stock of assets to hedge against adverse income shocks intensifies with greater risk aversion. Raising $\sigma$ makes moving consumption around across time easier and leads to higher saving if $\gamma$ is high enough (for there to be buffer stock asset accumulation). On the other hand, if risk aversion is low, people do not build up a buffer stock. There is less desire to sacrifice current consumption, and when it is easy for people to move consumption across time periods, they will, due to their impatience, move it to the present.

The overall effect of increasing risk aversion on the saving rate is also ambiguous. In terms of Choi, Lugauer, and Mark (2014) decomposition, risk aversion has no effect on the intertemporal substitution effect $\theta$. Increasing risk aversion affects the preference shifter:

$$\begin{aligned}\frac{\partial \phi}{\partial \gamma} &= \frac{(1-\gamma\sigma)\mathrm{Var}(\ln Z_{t+1})}{2} - \frac{\sigma(\gamma - 1)\mathrm{Var}(\ln Z_{t+1})}{2} \\ &+ \frac{(1-\gamma\sigma)(\gamma - 1)}{2}\frac{\partial \mathrm{Var}(\ln Z_{t+1})}{\partial \gamma}\end{aligned} \quad (18)$$

The first term is the effect of change in uncertainty cost, $\mathbb{E}(\ln Z_{t+1})$ which is increasing in risk aversion. If the substitutability is large enough, $\gamma\sigma > 1$, an increase in the uncertainty cost lowers $\phi$ and hence saving. The second term is the direct effect of change in the risk adjusted elasticity of substitution. If $\gamma > 1$, the uncertainty cost is positive. Then, raising risk adjusted substitutability increases the desire for early resolution of uncertainty. This channel lowers $\phi$ and hence saving. Combining these two effects, increasing risk aversion has a negative impact (lowering the saving rate) when risk aversion is relatively high, $\gamma > \frac{\sigma+1}{2\sigma}$. When risk aversion is low (high), increasing $\gamma$ raises (lowers) the preference shifter and contributes towards higher (lower) saving. Thus, for the direct effects, the preference shifter profile has a hump shape with respect to the risk aversion coefficient. The third term is the indirect effect. If consumption and utility volatility declines with higher risk aversion, then the last term has a positive impact when $\gamma\sigma > 1$ and $\gamma > 1$.

The effect of increasing risk aversion on the precautionary component is

$$\begin{aligned}\frac{\partial\psi}{\partial\gamma} &= [(\gamma\sigma-1)\mathrm{Var}(\ln Z_{t+1}) + \mathrm{Cov}(\Delta\, C_{t+1}, \ln Z_{t+1})] \\ &\quad + \frac{1}{2\sigma}\left[\frac{\partial \mathrm{Var}(\Delta\ln C_{t+1})}{\partial\gamma} + (\gamma\sigma-1)^2\frac{\partial \mathrm{Var}(\ln Z_{t+1})}{\partial\gamma}\right. \\ &\quad \left. + 2(\gamma\sigma-1)\frac{\partial \mathrm{Cov}(\Delta\, C_{t+1}, \ln Z_{t+1})}{\partial\gamma}\right]\end{aligned} \tag{19}$$

The first term in (19) is also the second term in equation (15). Hence, the sign of this term is ambiguous at low levels of risk aversion but clearly positive if $\gamma\sigma > 1$. The predicted profile of the saving rate with respect to the direct effect of risk aversion is either that the saving rate rises with $\gamma$ or that it displays a U shape. The second term is negative when $\gamma\sigma > 1$ since the variability of consumption and utility is expected to fall with $\gamma$.

Combining all effects, we have

$$\begin{aligned}\frac{\partial\mathbb{E}(\Delta\ln C_{t+1})}{\partial\gamma} &= \left(\frac{\sigma-1}{2}\right)\mathrm{Var}(\ln Z_{t+1}) + \mathrm{Cov}(\Delta\ln C_{t+1}, \ln Z_{t+1}) \\ &\quad + \frac{1}{2\sigma}\left\{\frac{\partial \mathrm{Var}(\Delta\ln C_{t+1})}{\partial\gamma} + (\gamma\sigma-1)(\sigma-1)\frac{\partial \mathrm{Var}(\ln Z_{t+1})}{\partial\gamma}\right. \\ &\quad \left. + 2(\gamma\sigma-1)\frac{\partial \mathrm{Cov}(\Delta\ln C_{t+1}, \ln Z_{t+1})}{\partial\gamma}\right\}\end{aligned} \tag{20}$$

The overall indirect effect, the last term, is negative when $\gamma\sigma > 1$ and $\sigma > 1$, but is ambiguous for other values. The first two terms are the direct effects. The overall direct effect is positive for $\sigma > 1$.

When $\sigma > 1$, increasing $\gamma$ raises (lowers) the saving rate when $\gamma$ is relatively low (high) so that the indirect effect is relatively small or positive (high or negative). Thus the saving rate profile should exhibit a 'hump shape' with respect to $\gamma$, and the peak occurs earlier with lower $\sigma$. Note that, if the volatility of consumption (hence utility) is relatively high so that the positive direct effect always dominates, the saving rate will monotonically rise with $\gamma$.

## 5. The choice of the functional form: a simulation exercise

Simulations of intertemporal maximization models have played an important role in the consumption literature of the last decades. Among the first works are those by Deaton (1991) and Carroll (1992), who solve buffer-stock models with prudent and impatient consumers. Also influential are the contributions of Attanasio *et al.* (1999) and Gourinchas and Parker (2002), who rest on simulations to analyse the role of demographics in explaining the evidence of consumption tracking income. More recently, Attanasio and Wakefield (2010) simulated a life cycle model to assess the importance of the elasticity of intertemporal substitution in determining the size of the reaction of savings to changes in the interest rate.[12]

The importance of simulation techniques stems from the fact that, except for very special cases, the Euler equation for consumption does not provide closed-form solutions and so does not allow to establish how a change in the framework faced by the consumer affects his level of consumption.

However, simulation results should be considered with caution. In fact, to simulate a model, one needs to make very specific modelling choices and specify the utility function by assigning a value to each of its parameters. As stressed by Attanasio and Wakefield,

> The big difficulty of this approach, if it wants to be realistic and of policy relevance, is that one needs to specify all of the details of the stochastic environment in which the consumer lives. And, [...] some of these details are quantitatively and qualitatively important for the results one obtains (Attanasio and Wakefield, 2010, p. 707).

As realistic as the model simulated can be, some aspects are bound to remain stylized, and, at the same time, the sensitivity to assumptions suggests that such results should be interpreted with prudence.

The simulation exercise here presented, though, does not seem to be affected by the

[12] Attanasio and Weber (2010) survey the main results of the literature relying on simulations.

general limitations of the approach. In fact, the purpose is not that of replicating actual paths of consumption across the life cycle of an individual, nor do we aim at giving a quantitative assessment of the impact of the functional form on preference parameters. On the contrary, the simulation exercise is supposed to show that, *coeteris paribus*, a different utility function leads to associate to the same consumption behaviour very different values of the structural parameters.

For this reason, the consideration of a very simple model, in which all other circumstances affecting the values of preference parameters are ruled out, seems particularly effective for our purposes.

In this section the results derived from the different models of intertemporal optimisation are presented. The first and simplest model consists of a replication of the numerical example proposed by Browning and Lusardi (1996) in which, rather than logarithmic utility, exponential and isoelastic preferences are assumed. When utility is logarithmic the coefficient of relative risk aversion and the elasticity of intertemporal substitution are equal to one and the coefficient of relative prudence is two. That stems from the fact that, as already mentioned, logarithmic preferences are a special case of isoelastic utility in which the parameter of the function, $\gamma$, is set equal to one. Once we leave the logarithmic specification, we are free to assign any other value to the parameter of the function and so we can actually consider as many numerical examples as the number of values we choose to include in our analysis. Obviously, the range of values selected is consistent with the existing empirical literature. However, this first, basic extension of the model already allows us to consider a wide range of cases which are interesting to compare and bring about some general considerations.

In the example proposed by Browning and Lusardi (1996) there is a positive (though very little) chance that second period income may be null. This is not a problem for the purpose of their argument but in the context here considered such a strong hypothesis could crucially affect the results. In fact, when the utility function implies very high disutility of very low consumption levels, i.e., when marginal utility goes to infinity when consumption goes to zero, then if null income is a non-zero probability event, the individual would never want to borrow. As a consequence, a first, simple extension of the baseline model will consist in assuming a positive second period income also in the worst case scenario.

Evidently, as long as the assumptions of a zero rate of time preference and a zero interest rate are retained, we can evaluate the interplay of life-cycle and precautionary motives in determining the saving behaviour of the individual, but the intertemporal substitution motive is ruled out. For this reason, it seems interesting to extend the model to the case of a positive interest rate.

As is well known, the standard consumption theory predicts that an individual with a rate of time preference $\delta$ greater than the market interest rate $r_t$ will have an incentive to anticipate consumption from time $t+1$ to time $t$. Consequently, the simulations presented will consider the case of an impatient individual in the sense of Carroll (1997), which implies that the subjective rate of time preference is greater than the rate of interest ($\delta > r$).

Moreover, to further enrich the analysis, in this more sophisticated exercise the analysis will be applied to the more general recursive EZW functional form in addition to the exponential and isoelastic functions.

### *5.1 A numerical example with different preference specifications*

The first simulation we are going to discuss is a replication of the two period model example proposed by Browning and Lusardi (1996) in which we adopt isoelastic and exponential preferences rather than logarithmic.

In the first period the agent has a certain income $Y_1$ and in the second period earnings are stochastic: they are zero with probability $\varepsilon$ and $Y_2/(1-\varepsilon)$ with probability $(1-\varepsilon)$ (thus, the expected value of second period income is $Y_2$). Finally, the real rate of interest, as well as the rate of time preference, is zero. Two different cases are considered: in the first case, $Y_1 = 2$ and $Y_2 = 1$ (high first period income scenario) while in the second $Y_1 = 1$ and $Y_2 = 2$ (low first period income scenario).

As obvious, the level of period 1 consumption chosen by the agent depends on his degree of prudence, which is determined by the coefficient $\alpha$ in the exponential preferences case and by $\gamma$ when dealing with the isoelastic function. Since both these functional forms only have one parameter, once we set the degree of prudence, the level of risk aversion is also established.

What the results show, as displayed in Figure 1, is that the same saving rate is associated with pretty different attitudes towards risk and uncertainty depending on the utility function adopted. Consider for example the case with low first period income and imagine we observe an agent consuming 0.855 in the first period, which implies a saving rate equal to 0.145. If we assume an isoelastic utility function, we shall conclude that the agent displays a degree of relative prudence equal to 3.54, while, if we adopt exponential preferences, we shall deduce that relative prudence is equal to 5.55. That also means that the corresponding coefficient of relative risk aversion ranges from 2.54 (isoelastic utility) to 5.55 (exponential preferences). Such an ambiguity shows that the role of the functional form adopted in the estimation of preference parameters through the Euler equation is crucial.

Figure 1. Relative risk aversion and relative prudence as functions of the saving rate

Figure 1a. Simulations with low first period income

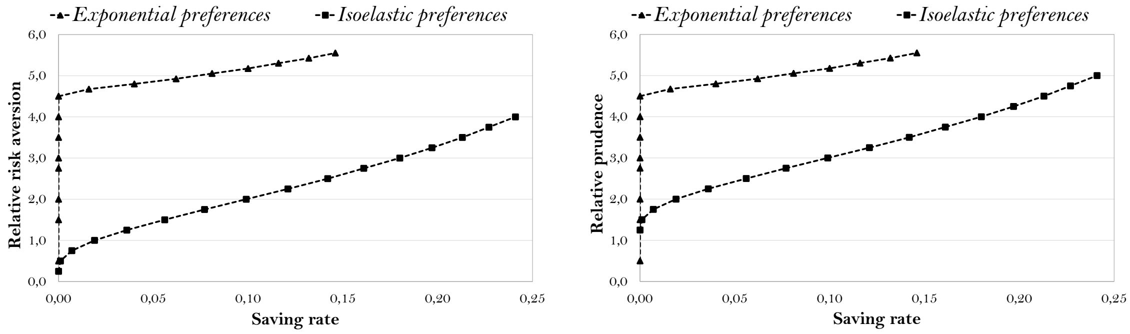
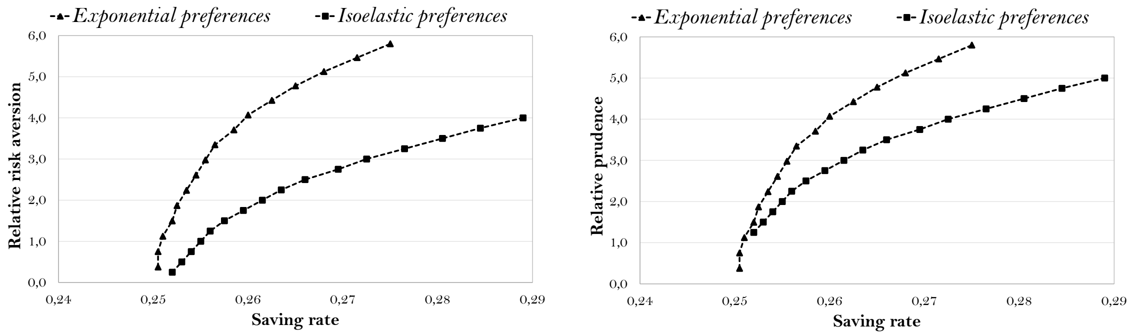



Figure 1b. Simulations with high first period income




*5.2 The interplay of precautionary and intertemporal motives in a two period model*

In this section the results of a slightly more sophisticated model are presented.

First of all, the possibility is ruled out of a zero second period income. It is supposed that in the second period earnings are $Y_2/2$ with probability $\varepsilon$ and $\frac{(2-\varepsilon)Y_2}{2(1-\varepsilon)}$ with probability $1-\varepsilon$, thus expected second period income is still equal to $Y_2$ as in the previous model. In addition, we extend the model by considering a positive rate of interest in order to capture the intertemporal motive for saving. The values assigned to $\beta$ and R are, respectively, 0.96 and 1.02, which correspond to a rate of time preference $\delta = 4.2\%$ and a rate of interest $r = 2\%$. Thus $\beta R = 0.98$, which means that the individual is impatient. These values seem to be consistent with those encountered in the literature.[13] Finally, a higher degree of uncertainty seems more appropriate for the analysis of the relations between consumption profiles and preference parameters, hence $\varepsilon$ is set at 5%.

The model is simulated for exponential, isoelastic and recursive EZW preferences. The preference specification and the equilibrium condition for the two period EZW utility is presented in Table A.2 of Appendix A. In the case of EZW preferences, both the coefficient of relative risk aversion and the elasticity of intertemporal substitution can change. The simulations are performed for the case of $\gamma = 1$ and a variable elasticity of intertemporal substitution and for the case in which $\sigma = 1$ and relative risk aversion changes. Clearly, both cases collapse to the logarithmic scenario when $\gamma = \frac{1}{\sigma}$.

The results of the simulations are shown in Appendix B, where tables are presented for each utility function[14] and each scenario (*low* or *high* first period income). For reasonable values of preference parameters, the corresponding level of first period consumption, saving rate, expected value of second period consumption and variance of second period log consumption are reported.

As a benchmark case, let us first consider how the individual would behave in the case of quadratic preferences that rule out the effect of uncertainty. In such a scenario, independently of the path of income, consumption exhibits a decreasing profile because of the hypothesis concerning the rate of time preference: since the individual is impatient, he allocates a larger portion of lifetime resources to the current relative to the future period. This happens regardless of the degree of uncertainty on future income and of the ratio between current and future expected earnings, because of the certainty equivalent character of the model with quadratic preferences.

In particular, the consumer would set first period consumption such that

$$C_1 = \frac{Y_1}{1+\beta} + \frac{Y_1}{R(1+\beta)}.$$

Therefore, the individual would consume 1.51 in the first period, which implies an expected value of future consumption equal to 1.48.

Once we leave the special case of quadratic preferences, first period consumption depends upon the values of preference parameters. Let us consider how consumption behaviour changes with changes in the intertemporal elasticity of substitution. Figure 2 plots

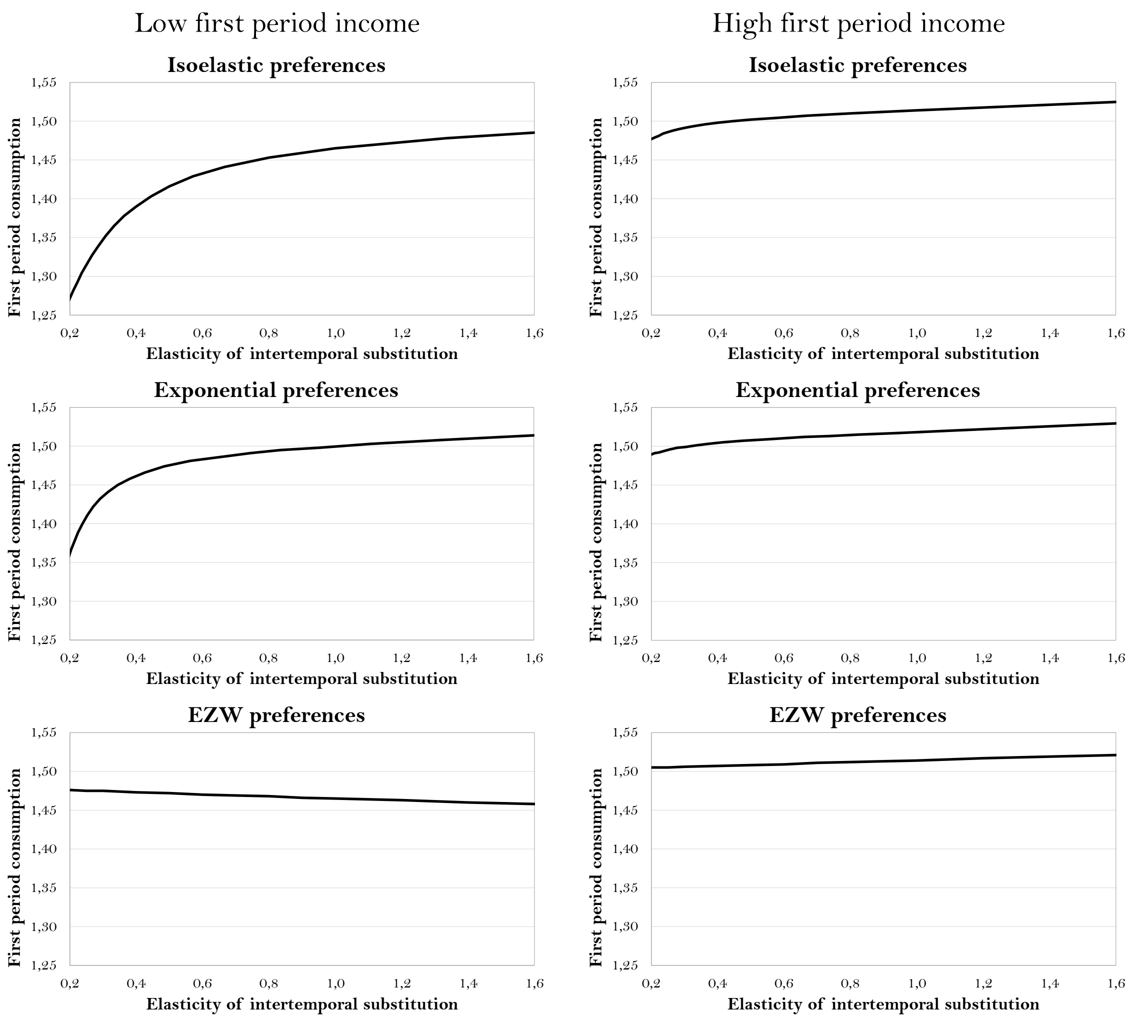
[13] For example, Attanasio and Wakefield (2010) set, in the baseline model, the rate of time preference at 2.5% and the interest rate at 2%.

[14] The simulation results for exponential utility are reported in Tables 1 and 2, for isoelastic utility in Tables 3 and 4, for recursive EZW utlity in Tables 5 to 12.

the values of $\sigma(C)$ and $C_1$ when the elasticity varies between 0.2 and 1.5, which is the range in which most empirical estimates fall.

First of all, Figure 2 shows that the variability of current consumption in response to changes in the elasticity of intertemporal substitution depends on the ratio between first period income and second period expected income. This happens because in the case of high first period income the individual is a net saver.

Figure 2. Elasticity of intertemporal substitution and first period consumption




When $\sigma(C)$ goes up, the individual is less adverse to intertemporal fluctuations of consumption and, at the same time, in the exponential and isoelastic case, aversion to risk goes down. For CRRA utility, the decomposition of the saving rate illustrated in Section 4 shows that a rise in the elasticity of intertemporal substitution has a negative direct effect on the saving rate when the individual is impatient, and a positive indirect effect through the variance of second period consumption. The overall effect on saving is negative as long as the effect operating through consumption variability is not too high, and thus causes current consumption to rise.[15]

In the case of EZW preferences, varying the elasticity of intertemporal substitution does not entail any changes in the degree of risk aversion, so that we can observe how consumption behaviour changes by keeping $\gamma$ constant and equal to one. In this case, when the aversion to intertemporal fluctuations in income decreases, there are an intertemporal substitution effect $\phi$ and a precautionary saving effect $\psi$ on the rate of growth of consumption. In our model, consumption declines when the elasticity of intertemporal substitution goes up in the case of low first period income and vice versa.

On the whole, the simulations performed show that the impact of the value of the preference parameters on consumption behaviour depends crucially upon the specification adopted for the utility function.

## 6. Final remarks

The purpose of this paper has been to analyse the impact that the functional form of the utility function had on the literature devoted to estimating preference parameters through the consumption Euler equation. The estimation of the structural parameters of the utility function has engaged researchers over the last thirty years and has produced a great amount of empirical evidence, often rather conflicting. On the other hand, reaching an agreement on the values which are considered reasonable for preference parameters has become increasingly important for the theoretical and practical implications of the intertemporal utility maximization model and for the calibration of modern macroeconomic models.

Throughout the paper, it is argued that the empirical works devoted to the estimation of preference parameters have been substantially affected by the necessity of specifying a functional form for preferences, and that the particular utility functions chosen may have played a fundamental role in determining the results obtained.

In order to highlight the importance of the functional form adopted, a simulation exercise has been presented and discussed in the light of a decomposition of the rate of growth of consumption derived by the consumer's Euler equation. The analysis suggests that the same saving behaviour can be associated to pretty different attitudes towards uncertainty

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[15] The simulations are conducted for values of the elasticity up to 4 and first period consumption is monotonically increasing in $\sigma$.

and intertemporal incentives – i.e. to different values of the preference parameters – depending on the utility function adopted. Obviously, such a simple two period model like the one here presented does not allow deriving general indications on the magnitude or the direction of the influence of the functional form on the outcome of an intertemporal optimising problem. Nevertheless, it seems a straightforward way to suggest that the issue, that seems to have been at least partially neglected in the empirical and theoretical literature on the consumption Euler equation, may deserve further investigation.

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## 8. Appendix A: Preference specifications

Table A.1. Functional form and Euler equation of HARA and recursive EZW preferences

| Expected utility HARA preferences | | |
|---|---|---|
| | Instantaneous utility function | Euler equation |
| Quadratic preferences | $u(C_t) = aC_t - \frac{b}{2}{C_t}^2$ | $a - bC_t = \beta R\mathbb{E}_t[(a - bC_{t+1})]$ |
| Exponential preferences | $u(C_t) = -\frac{e^{-\alpha C_t}}{\alpha}$ | $\mathbb{E}_t[C_{t+1}] = C_t + \ln \beta R^{1/\alpha}$ |
| Isoelastic preferences | $u(C_t) = \frac{{C_t}^{1-\gamma}}{1-\gamma}$ | $1 = \beta R\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]$ |
| Logarithmic preferences | $u(C_t) = \ln C_t$ | $1 = \beta R\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-1}\right]$ |

| Nonexpected utility recursive EZW preferences | |
|---|---|
| Utility function | Euler equation |
| $V_t = \left\{C_t^{1-\frac{1}{\sigma}} + \beta\left[\mathbb{E}_t\left(V_{t+1}^{1-\gamma}\right)\right]^{\frac{1-\frac{1}{\sigma}}{1-\gamma}}\right\}^{\frac{\sigma}{\sigma-1}}$ | $1 = \beta R\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}\left\{\frac{V_{t+1}}{\left[\mathbb{E}_t\left(V_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right\}^{\frac{1-\gamma\sigma}{\sigma}}\right]$ |

Table A.2. Recursive EZW preferences in the two period case

| $V_1 = u(C_1) + \beta u\left[V^{-1}\mathbb{E}_1\left(V(C_2)\right)\right]$ where $u(C) = \frac{C^{1-\rho}}{1-\rho}$ and $V(C) = \frac{C^{1-\gamma}}{1-\gamma}$ | | |
|---|---|---|
| | Utility function | Euler equation |
| $\sigma \neq 1 \wedge \gamma \neq 1$ | $V_1 = \frac{{C_1}^{1-\rho}}{1-\rho} + \beta\frac{1}{1-\rho}\left[\mathbb{E}_1\left({C_2}^{1-\gamma}\right)\right]^{\frac{1-\rho}{1-\gamma}}$ | $1 = \beta R\mathbb{E}_1\left[\left(\frac{C_2}{C_1}\right)^{-\rho}\left\{\frac{C_2}{\left[\mathbb{E}_t\left(C_2^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right\}^{\rho-\gamma}\right]$ |
| $\sigma = 1 \wedge \gamma \neq 1$ ($u(C) = \ln C$) | $V_1 = \ln C_1 + \beta \ln\left[\left[\mathbb{E}_1\left({C_2}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}\right]$ | $1 = \beta R\mathbb{E}_1\left[\left(\frac{C_2}{C_1}\right)^{-1}\left\{\frac{C_2}{\left[\mathbb{E}_t\left(C_2^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right\}^{1-\gamma}\right]$ |
| $\sigma \neq 1 \wedge \gamma = 1$ ($V(C) = \ln C$) | $V_1 = \frac{{C_1}^{1-\rho}}{1-\rho} + \beta\frac{1}{1-\rho}\left[e^{\mathbb{E}_1(\ln C_2)}\right]^{1-\rho}$ | $1 = \beta R\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left\{\frac{C_2}{e^{\mathbb{E}_1(\ln C_2)}}\right\}^{\rho-1}\right]$ |
| $\sigma = 1 \wedge \gamma = 1$ (Logarithmic) | $V_1 = \ln C_1 + \beta\mathbb{E}_1(\ln C_2)$ | $1 = \beta R\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-1}\right]$ |

## 9. Appendix B: Simulation results

Table B.1

| Exponential preferences low Y1 ε=0,05 βR = 0,98 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A(C1) | a(C1) | σ(C1) | p(C1) | P(C1) | C1 | Saving rate | E(C2) | Var(lnC2) |
| 0.10 | 0.16 | 6.26 | 0.16 | 0.100 | 1.598 | -59.8% | 1.390 | 0.081 |
| 0.20 | 0.31 | 3.24 | 0.31 | 0.200 | 1.544 | -54.4% | 1.445 | 0.070 |
| 0.30 | 0.46 | 2.19 | 0.46 | 0.300 | 1.525 | -52.5% | 1.465 | 0.067 |
| 0.40 | 0.61 | 1.65 | 0.61 | 0.400 | 1.515 | -51.5% | 1.475 | 0.065 |
| 0.50 | 0.75 | 1.33 | 0.75 | 0.500 | 1.508 | -50.8% | 1.482 | 0.064 |
| 0.60 | 0.90 | 1.11 | 0.90 | 0.600 | 1.503 | -50.3% | 1.487 | 0.063 |
| 0.70 | 1.05 | 0.95 | 1.05 | 0.700 | 1.498 | -49.8% | 1.492 | 0.062 |
| 0.80 | 1.20 | 0.84 | 1.20 | 0.800 | 1.495 | -49.5% | 1.495 | 0.062 |
| 0.90 | 1.34 | 0.75 | 1.34 | 0.900 | 1.491 | -49.1% | 1.499 | 0.061 |
| 1.00 | 1.49 | 0.67 | 1.49 | 1.000 | 1.487 | -48.7% | 1.503 | 0.061 |
| 1.20 | 1.78 | 0.56 | 1.78 | 1.200 | 1.481 | -48.1% | 1.509 | 0.060 |
| 1.40 | 2.06 | 0.48 | 2.06 | 1.400 | 1.474 | -47.4% | 1.517 | 0.059 |
| 1.60 | 2.35 | 0.43 | 2.35 | 1.600 | 1.466 | -46.6% | 1.525 | 0.058 |
| 1.80 | 2.62 | 0.38 | 2.62 | 1.800 | 1.458 | -45.8% | 1.533 | 0.056 |
| 2.00 | 2.90 | 0.34 | 2.90 | 2.000 | 1.450 | -45.0% | 1.541 | 0.055 |
| 2.20 | 3.17 | 0.32 | 3.17 | 2.200 | 1.441 | -44.1% | 1.550 | 0.054 |
| 2.40 | 3.44 | 0.29 | 3.44 | 2.400 | 1.432 | -43.2% | 1.559 | 0.053 |
| 2.60 | 3.70 | 0.27 | 3.70 | 2.600 | 1.422 | -42.2% | 1.570 | 0.052 |
| 2.80 | 3.95 | 0.25 | 3.95 | 2.800 | 1.411 | -41.1% | 1.581 | 0.051 |
| 3.00 | 4.20 | 0.24 | 4.20 | 3.000 | 1.400 | -40.0% | 1.592 | 0.050 |
| 3.20 | 4.44 | 0.22 | 4.44 | 3.200 | 1.389 | -38.9% | 1.603 | 0.048 |
| 3.40 | 4.68 | 0.21 | 4.68 | 3.400 | 1.377 | -37.7% | 1.615 | 0.047 |
| 3.60 | 4.92 | 0.20 | 4.92 | 3.600 | 1.366 | -36.6% | 1.627 | 0.046 |
| 3.80 | 5.15 | 0.19 | 5.15 | 3.800 | 1.354 | -35.4% | 1.639 | 0.045 |
| 4.00 | 5.37 | 0.19 | 5.37 | 4.000 | 1.343 | -34.3% | 1.650 | 0.044 |
| 4.25 | 5.65 | 0.18 | 5.65 | 4.250 | 1.329 | -32.9% | 1.664 | 0.043 |
| 4.50 | 5.92 | 0.17 | 5.92 | 4.500 | 1.315 | -31.5% | 1.679 | 0.042 |
| 4.75 | 6.18 | 0.16 | 6.18 | 4.750 | 1.302 | -30.2% | 1.692 | 0.041 |
| 5.00 | 6.44 | 0.16 | 6.44 | 5.000 | 1.289 | -28.9% | 1.705 | 0.040 |
| 5.25 | 6.71 | 0.15 | 6.71 | 5.250 | 1.278 | -27.8% | 1.716 | 0.039 |
| 5.50 | 6.96 | 0.14 | 6.96 | 5.500 | 1.266 | -26.6% | 1.729 | 0.038 |
| 5.75 | 7.22 | 0.14 | 7.22 | 5.750 | 1.256 | -25.6% | 1.739 | 0.037 |
| 6.00 | 7.48 | 0.13 | 7.48 | 6.000 | 1.246 | -24.6% | 1.749 | 0.037 |
| 6.25 | 7.73 | 0.13 | 7.73 | 6.250 | 1.237 | -23.7% | 1.758 | 0.036 |
| 6.50 | 7.98 | 0.13 | 7.98 | 6.500 | 1.228 | -22.8% | 1.767 | 0.035 |
| 6.75 | 8.23 | 0.12 | 8.23 | 6.750 | 1.220 | -22.0% | 1.776 | 0.035 |
| 7.00 | 8.49 | 0.12 | 8.49 | 7.000 | 1.213 | -21.3% | 1.783 | 0.034 |
| 7.25 | 8.74 | 0.11 | 8.74 | 7.250 | 1.205 | -20.5% | 1.791 | 0.034 |
| 7.50 | 8.99 | 0.11 | 8.99 | 7.500 | 1.199 | -19.9% | 1.797 | 0.034 |
| 7.75 | 9.24 | 0.11 | 9.24 | 7.750 | 1.192 | -19.2% | 1.804 | 0.033 |

Table B.2

| Exponential preferences high Y1 ε=0,05 βR = 0,98 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A(C1) | a(C1) | σ(C1) | p(C1) | P(C1) | C1 | Saving rate | E(C2) | Var(lnC2) |
| 0.10 | 0.16 | 6.22 | 0.16 | 0.100 | 1.609 | 19.6% | 1.399 | 0.010 |
| 0.20 | 0.31 | 3.21 | 0.31 | 0.200 | 1.556 | 22.2% | 1.453 | 0.009 |
| 0.30 | 0.46 | 2.17 | 0.46 | 0.300 | 1.539 | 23.1% | 1.470 | 0.009 |
| 0.40 | 0.61 | 1.63 | 0.61 | 0.400 | 1.530 | 23.5% | 1.479 | 0.009 |
| 0.50 | 0.76 | 1.31 | 0.76 | 0.500 | 1.524 | 23.8% | 1.486 | 0.009 |
| 0.60 | 0.91 | 1.10 | 0.91 | 0.600 | 1.520 | 24.0% | 1.490 | 0.009 |
| 0.70 | 1.06 | 0.94 | 1.06 | 0.700 | 1.517 | 24.2% | 1.493 | 0.009 |
| 0.80 | 1.21 | 0.83 | 1.21 | 0.800 | 1.515 | 24.3% | 1.495 | 0.009 |
| 0.90 | 1.36 | 0.73 | 1.36 | 0.900 | 1.513 | 24.4% | 1.497 | 0.009 |
| 1.00 | 1.51 | 0.66 | 1.51 | 1.000 | 1.512 | 24.4% | 1.498 | 0.009 |
| 1.20 | 1.81 | 0.55 | 1.81 | 1.200 | 1.509 | 24.6% | 1.501 | 0.008 |
| 1.40 | 2.11 | 0.47 | 2.11 | 1.400 | 1.507 | 24.7% | 1.503 | 0.008 |
| 1.60 | 2.41 | 0.42 | 2.41 | 1.600 | 1.505 | 24.8% | 1.505 | 0.008 |
| 1.80 | 2.71 | 0.37 | 2.71 | 1.800 | 1.503 | 24.9% | 1.507 | 0.008 |
| 2.00 | 3.00 | 0.33 | 3.00 | 2.000 | 1.501 | 25.0% | 1.509 | 0.008 |
| 2.20 | 3.30 | 0.30 | 3.30 | 2.200 | 1.499 | 25.1% | 1.511 | 0.008 |
| 2.40 | 3.60 | 0.28 | 3.60 | 2.400 | 1.498 | 25.1% | 1.512 | 0.008 |
| 2.60 | 3.89 | 0.26 | 3.89 | 2.600 | 1.496 | 25.2% | 1.514 | 0.008 |
| 2.80 | 4.18 | 0.24 | 4.18 | 2.800 | 1.494 | 25.3% | 1.516 | 0.008 |
| 3.00 | 4.48 | 0.22 | 4.48 | 3.000 | 1.492 | 25.4% | 1.518 | 0.008 |
| 3.20 | 4.77 | 0.21 | 4.77 | 3.200 | 1.491 | 25.5% | 1.519 | 0.008 |
| 3.40 | 5.06 | 0.20 | 5.06 | 3.400 | 1.489 | 25.6% | 1.521 | 0.008 |
| 3.60 | 5.35 | 0.19 | 5.35 | 3.600 | 1.487 | 25.7% | 1.523 | 0.008 |
| 3.80 | 5.64 | 0.18 | 5.64 | 3.800 | 1.485 | 25.8% | 1.525 | 0.008 |
| 4.00 | 5.93 | 0.17 | 5.93 | 4.000 | 1.482 | 25.9% | 1.528 | 0.008 |
| 4.20 | 6.22 | 0.16 | 6.22 | 4.200 | 1.480 | 26.0% | 1.530 | 0.008 |
| 4.40 | 6.50 | 0.15 | 6.50 | 4.400 | 1.478 | 26.1% | 1.532 | 0.008 |
| 4.60 | 6.79 | 0.15 | 6.79 | 4.600 | 1.476 | 26.2% | 1.534 | 0.008 |
| 4.80 | 7.07 | 0.14 | 7.07 | 4.800 | 1.473 | 26.4% | 1.538 | 0.008 |
| 5.00 | 7.35 | 0.14 | 7.35 | 5.000 | 1.471 | 26.5% | 1.540 | 0.008 |
| 5.20 | 7.63 | 0.13 | 7.63 | 5.200 | 1.468 | 26.6% | 1.543 | 0.008 |
| 5.40 | 7.92 | 0.13 | 7.92 | 5.400 | 1.466 | 26.7% | 1.545 | 0.008 |
| 5.60 | 8.19 | 0.12 | 8.19 | 5.600 | 1.463 | 26.9% | 1.548 | 0.008 |
| 5.80 | 8.47 | 0.12 | 8.47 | 5.800 | 1.460 | 27.0% | 1.551 | 0.008 |
| 6.00 | 8.74 | 0.11 | 8.74 | 6.000 | 1.457 | 27.2% | 1.554 | 0.008 |
| 6.20 | 9.02 | 0.11 | 9.02 | 6.200 | 1.455 | 27.3% | 1.556 | 0.008 |
| 6.40 | 9.29 | 0.11 | 9.29 | 6.400 | 1.452 | 27.4% | 1.559 | 0.008 |
| 6.60 | 9.56 | 0.10 | 9.56 | 6.600 | 1.449 | 27.6% | 1.562 | 0.008 |
| 6.80 | 9.83 | 0.10 | 9.83 | 6.800 | 1.446 | 27.7% | 1.565 | 0.008 |
| 7.00 | 10.10 | 0.10 | 10.10 | 7.000 | 1.443 | 27.9% | 1.568 | 0.008 |

Table B.3

| Isoelastic preferences low Y1 ε=0,05 βR = 0,98 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A(C1) | a(C1) | σ(C1) | p(C1) | P(C1) | C1 | Saving rate | E(C2) | Var(lnC2) |
| 0.16 | 0.25 | 4.00 | 1.25 | 0.816 | 1.532 | -53.2% | 1.457 | 0.068 |
| 0.27 | 0.40 | 2.50 | 1.40 | 0.930 | 1.506 | -50.6% | 1.484 | 0.063 |
| 0.33 | 0.50 | 2.00 | 1.50 | 1.003 | 1.496 | -49.6% | 1.494 | 0.062 |
| 0.51 | 0.75 | 1.33 | 1.75 | 1.184 | 1.478 | -47.8% | 1.512 | 0.059 |
| 0.68 | 1.00 | 1.00 | 2.00 | 1.365 | 1.465 | -46.5% | 1.526 | 0.057 |
| 0.86 | 1.25 | 0.80 | 2.25 | 1.549 | 1.453 | -45.3% | 1.538 | 0.056 |
| 1.04 | 1.50 | 0.67 | 2.50 | 1.735 | 1.441 | -44.1% | 1.550 | 0.054 |
| 1.22 | 1.75 | 0.57 | 2.75 | 1.924 | 1.429 | -42.9% | 1.562 | 0.053 |
| 1.41 | 2.00 | 0.50 | 3.00 | 2.119 | 1.416 | -41.6% | 1.576 | 0.051 |
| 1.60 | 2.25 | 0.44 | 3.25 | 2.316 | 1.403 | -40.3% | 1.589 | 0.050 |
| 1.80 | 2.50 | 0.40 | 3.50 | 2.518 | 1.390 | -39.0% | 1.602 | 0.049 |
| 2.00 | 2.75 | 0.36 | 3.75 | 2.721 | 1.378 | -37.8% | 1.614 | 0.047 |
| 2.20 | 3.00 | 0.33 | 4.00 | 2.930 | 1.365 | -36.5% | 1.628 | 0.046 |
| 2.40 | 3.25 | 0.31 | 4.25 | 3.143 | 1.352 | -35.2% | 1.641 | 0.045 |
| 2.61 | 3.50 | 0.29 | 4.50 | 3.361 | 1.339 | -33.9% | 1.654 | 0.044 |
| 2.83 | 3.75 | 0.27 | 4.75 | 3.580 | 1.327 | -32.7% | 1.666 | 0.043 |
| 3.04 | 4.00 | 0.25 | 5.00 | 3.802 | 1.315 | -31.5% | 1.679 | 0.042 |
| 3.26 | 4.25 | 0.24 | 5.25 | 4.026 | 1.304 | -30.4% | 1.690 | 0.041 |
| 3.48 | 4.50 | 0.22 | 5.50 | 4.257 | 1.292 | -29.2% | 1.702 | 0.040 |
| 3.71 | 4.75 | 0.21 | 5.75 | 4.485 | 1.282 | -28.2% | 1.712 | 0.039 |
| 3.93 | 5.00 | 0.20 | 6.00 | 4.717 | 1.272 | -27.2% | 1.723 | 0.038 |
| 4.16 | 5.25 | 0.19 | 6.25 | 4.952 | 1.262 | -26.2% | 1.733 | 0.038 |
| 4.39 | 5.50 | 0.18 | 6.50 | 5.188 | 1.253 | -25.3% | 1.742 | 0.037 |
| 4.62 | 5.75 | 0.17 | 6.75 | 5.426 | 1.244 | -24.4% | 1.751 | 0.036 |
| 4.85 | 6.00 | 0.17 | 7.00 | 5.663 | 1.236 | -23.6% | 1.759 | 0.036 |
| 5.09 | 6.25 | 0.16 | 7.25 | 5.904 | 1.228 | -22.8% | 1.767 | 0.035 |
| 5.33 | 6.50 | 0.15 | 7.50 | 6.148 | 1.220 | -22.0% | 1.776 | 0.035 |
| 5.56 | 6.75 | 0.15 | 7.75 | 6.389 | 1.213 | -21.3% | 1.783 | 0.034 |
| 5.80 | 7.00 | 0.14 | 8.00 | 6.633 | 1.206 | -20.6% | 1.790 | 0.034 |
| 6.04 | 7.25 | 0.14 | 8.25 | 6.875 | 1.200 | -20.0% | 1.796 | 0.034 |
| 6.28 | 7.50 | 0.13 | 8.50 | 7.119 | 1.194 | -19.4% | 1.802 | 0.033 |
| 6.52 | 7.75 | 0.13 | 8.75 | 7.365 | 1.188 | -18.8% | 1.808 | 0.033 |
| 6.76 | 8.00 | 0.13 | 9.00 | 7.608 | 1.183 | -18.3% | 1.813 | 0.033 |
| 7.00 | 8.25 | 0.12 | 9.25 | 7.852 | 1.178 | -17.8% | 1.818 | 0.032 |
| 7.25 | 8.50 | 0.12 | 9.50 | 8.099 | 1.173 | -17.3% | 1.824 | 0.032 |
| 7.49 | 8.75 | 0.11 | 9.75 | 8.348 | 1.168 | -16.8% | 1.829 | 0.032 |
| 7.74 | 9.00 | 0.11 | 10.00 | 8.598 | 1.163 | -16.3% | 1.834 | 0.032 |
| 7.98 | 9.25 | 0.11 | 10.25 | 8.844 | 1.159 | -15.9% | 1.838 | 0.031 |
| 8.23 | 9.50 | 0.11 | 10.50 | 9.091 | 1.155 | -15.5% | 1.842 | 0.031 |
| 8.47 | 9.75 | 0.10 | 10.75 | 9.340 | 1.151 | -15.1% | 1.846 | 0.031 |
| 8.71 | 10.00 | 0.10 | 11.00 | 9.582 | 1.148 | -14.8% | 1.849 | 0.031 |

Table B.4

| Isoelastic preferences high Y1 ε=0,05 βR = 0,98 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A(C1) | a(C1) | σ(C1) | p(C1) | P(C1) | C1 | Saving rate | E(C2) | Var(lnC2) |
| 0.16 | 0.25 | 4.00 | 1.25 | 0.799 | 1.564 | 21.8% | 1.445 | 0.009 |
| 0.26 | 0.40 | 2.50 | 1.40 | 0.909 | 1.540 | 23.0% | 1.469 | 0.009 |
| 0.33 | 0.50 | 2.00 | 1.50 | 0.979 | 1.532 | 23.4% | 1.477 | 0.009 |
| 0.49 | 0.75 | 1.33 | 1.75 | 1.151 | 1.520 | 24.0% | 1.490 | 0.009 |
| 0.66 | 1.00 | 1.00 | 2.00 | 1.321 | 1.514 | 24.3% | 1.496 | 0.009 |
| 0.83 | 1.25 | 0.80 | 2.25 | 1.490 | 1.510 | 24.5% | 1.500 | 0.008 |
| 1.00 | 1.50 | 0.67 | 2.50 | 1.659 | 1.507 | 24.7% | 1.503 | 0.008 |
| 1.16 | 1.75 | 0.57 | 2.75 | 1.828 | 1.504 | 24.8% | 1.506 | 0.008 |
| 1.33 | 2.00 | 0.50 | 3.00 | 1.997 | 1.502 | 24.9% | 1.508 | 0.008 |
| 1.50 | 2.25 | 0.44 | 3.25 | 2.167 | 1.500 | 25.0% | 1.510 | 0.008 |
| 1.67 | 2.50 | 0.40 | 3.50 | 2.336 | 1.498 | 25.1% | 1.512 | 0.008 |
| 1.84 | 2.75 | 0.36 | 3.75 | 2.507 | 1.496 | 25.2% | 1.514 | 0.008 |
| 2.01 | 3.00 | 0.33 | 4.00 | 2.677 | 1.494 | 25.3% | 1.516 | 0.008 |
| 2.18 | 3.25 | 0.31 | 4.25 | 2.849 | 1.492 | 25.4% | 1.518 | 0.008 |
| 2.35 | 3.50 | 0.29 | 4.50 | 3.020 | 1.490 | 25.5% | 1.520 | 0.008 |
| 2.52 | 3.75 | 0.27 | 4.75 | 3.192 | 1.488 | 25.6% | 1.522 | 0.008 |
| 2.69 | 4.00 | 0.25 | 5.00 | 3.365 | 1.486 | 25.7% | 1.524 | 0.008 |
| 2.86 | 4.25 | 0.24 | 5.25 | 3.538 | 1.484 | 25.8% | 1.526 | 0.008 |
| 3.04 | 4.50 | 0.22 | 5.50 | 3.714 | 1.481 | 26.0% | 1.529 | 0.008 |
| 3.21 | 4.75 | 0.21 | 5.75 | 3.888 | 1.479 | 26.1% | 1.531 | 0.008 |
| 3.39 | 5.00 | 0.20 | 6.00 | 4.062 | 1.477 | 26.2% | 1.533 | 0.008 |
| 3.56 | 5.25 | 0.19 | 6.25 | 4.237 | 1.475 | 26.3% | 1.536 | 0.008 |
| 3.74 | 5.50 | 0.18 | 6.50 | 4.416 | 1.472 | 26.4% | 1.539 | 0.008 |
| 3.91 | 5.75 | 0.17 | 6.75 | 4.592 | 1.470 | 26.5% | 1.541 | 0.008 |
| 4.09 | 6.00 | 0.17 | 7.00 | 4.772 | 1.467 | 26.7% | 1.544 | 0.008 |
| 4.27 | 6.25 | 0.16 | 7.25 | 4.949 | 1.465 | 26.8% | 1.546 | 0.008 |
| 4.45 | 6.50 | 0.15 | 7.50 | 5.130 | 1.462 | 26.9% | 1.549 | 0.008 |
| 4.62 | 6.75 | 0.15 | 7.75 | 5.308 | 1.460 | 27.0% | 1.551 | 0.008 |
| 4.80 | 7.00 | 0.14 | 8.00 | 5.491 | 1.457 | 27.2% | 1.554 | 0.008 |
| 4.99 | 7.25 | 0.14 | 8.25 | 5.674 | 1.454 | 27.3% | 1.557 | 0.008 |
| 5.17 | 7.50 | 0.13 | 8.50 | 5.854 | 1.452 | 27.4% | 1.559 | 0.008 |
| 5.35 | 7.75 | 0.13 | 8.75 | 6.039 | 1.449 | 27.6% | 1.562 | 0.008 |
| 5.53 | 8.00 | 0.13 | 9.00 | 6.224 | 1.446 | 27.7% | 1.565 | 0.008 |
| 5.72 | 8.25 | 0.12 | 9.25 | 6.410 | 1.443 | 27.9% | 1.568 | 0.008 |
| 5.90 | 8.50 | 0.12 | 9.50 | 6.593 | 1.441 | 28.0% | 1.570 | 0.008 |
| 6.08 | 8.75 | 0.11 | 9.75 | 6.780 | 1.438 | 28.1% | 1.573 | 0.008 |
| 6.27 | 9.00 | 0.11 | 10.00 | 6.969 | 1.435 | 28.3% | 1.576 | 0.008 |
| 6.45 | 9.25 | 0.11 | 10.25 | 7.153 | 1.433 | 28.4% | 1.578 | 0.008 |
| 6.64 | 9.50 | 0.11 | 10.50 | 7.343 | 1.430 | 28.5% | 1.581 | 0.007 |
| 6.83 | 9.75 | 0.10 | 10.75 | 7.533 | 1.427 | 28.7% | 1.584 | 0.007 |
| 7.02 | 10.00 | 0.10 | 11.00 | 7.719 | 1.425 | 28.8% | 1.587 | 0.007 |

Table B.5

| Recursive preferences RRA = 1 and variable EIS low Y1 ε=0,05 βR = 0,98 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A(C1) | a(C1) | σ(C1) | p(C1) | P(C1) | C1 | Saving rate | E(C2) | Var(lnC2) |
| 0.68 | 1.00 | 0.10 | 1.10 | 0.744 | 1.478 | -47.8% | 1.512 | 0.059 |
| 0.68 | 1.00 | 0.20 | 1.20 | 0.813 | 1.476 | -47.6% | 1.514 | 0.059 |
| 0.68 | 1.00 | 0.25 | 1.25 | 0.847 | 1.475 | -47.5% | 1.516 | 0.059 |
| 0.68 | 1.00 | 0.30 | 1.30 | 0.881 | 1.475 | -47.5% | 1.516 | 0.059 |
| 0.68 | 1.00 | 0.40 | 1.40 | 0.950 | 1.473 | -47.3% | 1.518 | 0.059 |
| 0.68 | 1.00 | 0.50 | 1.50 | 1.019 | 1.472 | -47.2% | 1.519 | 0.058 |
| 0.68 | 1.00 | 0.60 | 1.60 | 1.088 | 1.470 | -47.0% | 1.521 | 0.058 |
| 0.68 | 1.00 | 0.70 | 1.70 | 1.157 | 1.469 | -46.9% | 1.522 | 0.058 |
| 0.68 | 1.00 | 0.80 | 1.80 | 1.226 | 1.468 | -46.8% | 1.523 | 0.058 |
| 0.68 | 1.00 | 0.90 | 1.90 | 1.296 | 1.466 | -46.6% | 1.525 | 0.058 |
| 0.68 | 1.00 | 1.00 | 2.00 | 1.365 | 1.465 | -46.5% | 1.526 | 0.057 |
| 0.68 | 1.00 | 1.20 | 2.20 | 1.504 | 1.463 | -46.3% | 1.528 | 0.057 |
| 0.68 | 1.00 | 1.40 | 2.40 | 1.644 | 1.460 | -46.0% | 1.531 | 0.057 |
| 0.69 | 1.00 | 1.60 | 2.60 | 1.783 | 1.458 | -45.8% | 1.533 | 0.056 |
| 0.69 | 1.00 | 1.80 | 2.80 | 1.924 | 1.455 | -45.5% | 1.536 | 0.056 |
| 0.69 | 1.00 | 2.00 | 3.00 | 2.065 | 1.453 | -45.3% | 1.538 | 0.056 |
| 0.69 | 1.00 | 2.20 | 3.20 | 2.205 | 1.451 | -45.1% | 1.540 | 0.056 |
| 0.69 | 1.00 | 2.40 | 3.40 | 2.346 | 1.449 | -44.9% | 1.542 | 0.055 |
| 0.69 | 1.00 | 2.60 | 3.60 | 2.490 | 1.446 | -44.6% | 1.545 | 0.055 |
| 0.69 | 1.00 | 2.80 | 3.80 | 2.632 | 1.444 | -44.4% | 1.547 | 0.055 |
| 0.69 | 1.00 | 3.00 | 4.00 | 2.774 | 1.442 | -44.2% | 1.549 | 0.054 |
| 0.69 | 1.00 | 3.20 | 4.20 | 2.917 | 1.440 | -44.0% | 1.551 | 0.054 |
| 0.70 | 1.00 | 3.40 | 4.40 | 3.060 | 1.438 | -43.8% | 1.553 | 0.054 |
| 0.70 | 1.00 | 3.60 | 4.60 | 3.201 | 1.437 | -43.7% | 1.554 | 0.054 |
| 0.70 | 1.00 | 3.80 | 4.80 | 3.345 | 1.435 | -43.5% | 1.556 | 0.054 |
| 0.70 | 1.00 | 4.00 | 5.00 | 3.489 | 1.433 | -43.3% | 1.558 | 0.053 |
| 0.70 | 1.00 | 4.25 | 5.25 | 3.669 | 1.431 | -43.1% | 1.560 | 0.053 |
| 0.70 | 1.00 | 4.50 | 5.50 | 3.852 | 1.428 | -42.8% | 1.563 | 0.053 |
| 0.70 | 1.00 | 4.75 | 5.75 | 4.032 | 1.426 | -42.6% | 1.565 | 0.053 |
| 0.70 | 1.00 | 5.00 | 6.00 | 4.213 | 1.424 | -42.4% | 1.568 | 0.052 |
| 0.70 | 1.00 | 5.25 | 6.25 | 4.395 | 1.422 | -42.2% | 1.570 | 0.052 |
| 0.70 | 1.00 | 5.50 | 6.50 | 4.577 | 1.420 | -42.0% | 1.572 | 0.052 |
| 0.71 | 1.00 | 5.75 | 6.75 | 4.760 | 1.418 | -41.8% | 1.574 | 0.052 |
| 0.71 | 1.00 | 6.00 | 7.00 | 4.944 | 1.416 | -41.6% | 1.576 | 0.051 |
| 0.71 | 1.00 | 6.25 | 7.25 | 5.124 | 1.415 | -41.5% | 1.577 | 0.051 |
| 0.71 | 1.00 | 6.50 | 7.50 | 5.308 | 1.413 | -41.3% | 1.579 | 0.051 |
| 0.71 | 1.00 | 6.75 | 7.75 | 5.493 | 1.411 | -41.1% | 1.581 | 0.051 |
| 0.71 | 1.00 | 7.00 | 8.00 | 5.678 | 1.409 | -40.9% | 1.583 | 0.051 |
| 0.71 | 1.00 | 7.25 | 8.25 | 5.859 | 1.408 | -40.8% | 1.584 | 0.050 |
| 0.71 | 1.00 | 7.50 | 8.50 | 6.046 | 1.406 | -40.6% | 1.586 | 0.050 |
| 0.71 | 1.00 | 7.75 | 8.75 | 6.232 | 1.404 | -40.4% | 1.588 | 0.050 |

Table B.6

| Recursive preferences RRA = 1 and variable EIS high Y1 ε=0,05 βR = 0,98 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A(C1) | a(C1) | σ(C1) | p(C1) | P(C1) | C1 | Saving rate | E(C2) | Var(lnC2) |
| 0.67 | 1.00 | 0.10 | 1.10 | 0.732 | 1.503 | 24.9% | 1.507 | 0.008 |
| 0.66 | 1.00 | 0.20 | 1.20 | 0.797 | 1.505 | 24.8% | 1.505 | 0.008 |
| 0.66 | 1.00 | 0.25 | 1.25 | 0.831 | 1.505 | 24.8% | 1.505 | 0.008 |
| 0.66 | 1.00 | 0.30 | 1.30 | 0.863 | 1.506 | 24.7% | 1.504 | 0.008 |
| 0.66 | 1.00 | 0.40 | 1.40 | 0.929 | 1.507 | 24.7% | 1.503 | 0.008 |
| 0.66 | 1.00 | 0.50 | 1.50 | 0.995 | 1.508 | 24.6% | 1.502 | 0.008 |
| 0.66 | 1.00 | 0.60 | 1.60 | 1.060 | 1.509 | 24.6% | 1.501 | 0.008 |
| 0.66 | 1.00 | 0.70 | 1.70 | 1.125 | 1.511 | 24.5% | 1.499 | 0.009 |
| 0.66 | 1.00 | 0.80 | 1.80 | 1.190 | 1.512 | 24.4% | 1.498 | 0.009 |
| 0.66 | 1.00 | 0.90 | 1.90 | 1.256 | 1.513 | 24.4% | 1.497 | 0.009 |
| 0.66 | 1.00 | 1.00 | 2.00 | 1.321 | 1.514 | 24.3% | 1.496 | 0.009 |
| 0.66 | 1.00 | 1.20 | 2.20 | 1.450 | 1.517 | 24.2% | 1.493 | 0.009 |
| 0.66 | 1.00 | 1.40 | 2.40 | 1.580 | 1.519 | 24.1% | 1.491 | 0.009 |
| 0.66 | 1.00 | 1.60 | 2.60 | 1.709 | 1.521 | 24.0% | 1.489 | 0.009 |
| 0.66 | 1.00 | 1.80 | 2.80 | 1.837 | 1.524 | 23.8% | 1.486 | 0.009 |
| 0.66 | 1.00 | 2.00 | 3.00 | 1.966 | 1.526 | 23.7% | 1.483 | 0.009 |
| 0.65 | 1.00 | 2.20 | 3.20 | 2.094 | 1.528 | 23.6% | 1.481 | 0.009 |
| 0.65 | 1.00 | 2.40 | 3.40 | 2.221 | 1.531 | 23.5% | 1.478 | 0.009 |
| 0.65 | 1.00 | 2.60 | 3.60 | 2.348 | 1.533 | 23.4% | 1.476 | 0.009 |
| 0.65 | 1.00 | 2.80 | 3.80 | 2.476 | 1.535 | 23.3% | 1.474 | 0.009 |
| 0.65 | 1.00 | 3.00 | 4.00 | 2.601 | 1.538 | 23.1% | 1.471 | 0.009 |
| 0.65 | 1.00 | 3.20 | 4.20 | 2.727 | 1.540 | 23.0% | 1.469 | 0.009 |
| 0.65 | 1.00 | 3.40 | 4.40 | 2.853 | 1.542 | 22.9% | 1.467 | 0.009 |
| 0.65 | 1.00 | 3.60 | 4.60 | 2.977 | 1.545 | 22.8% | 1.464 | 0.009 |
| 0.65 | 1.00 | 3.80 | 4.80 | 3.103 | 1.547 | 22.7% | 1.462 | 0.009 |
| 0.65 | 1.00 | 4.00 | 5.00 | 3.228 | 1.549 | 22.6% | 1.460 | 0.009 |
| 0.64 | 1.00 | 4.25 | 5.25 | 3.383 | 1.552 | 22.4% | 1.457 | 0.009 |
| 0.64 | 1.00 | 4.50 | 5.50 | 3.537 | 1.555 | 22.3% | 1.454 | 0.009 |
| 0.64 | 1.00 | 4.75 | 5.75 | 3.691 | 1.558 | 22.1% | 1.451 | 0.009 |
| 0.64 | 1.00 | 5.00 | 6.00 | 3.846 | 1.560 | 22.0% | 1.449 | 0.009 |
| 0.64 | 1.00 | 5.25 | 6.25 | 3.999 | 1.563 | 21.9% | 1.446 | 0.009 |
| 0.64 | 1.00 | 5.50 | 6.50 | 4.151 | 1.566 | 21.7% | 1.443 | 0.009 |
| 0.64 | 1.00 | 5.75 | 6.75 | 4.302 | 1.569 | 21.6% | 1.440 | 0.009 |
| 0.64 | 1.00 | 6.00 | 7.00 | 4.453 | 1.572 | 21.4% | 1.437 | 0.009 |
| 0.64 | 1.00 | 6.25 | 7.25 | 4.606 | 1.574 | 21.3% | 1.435 | 0.009 |
| 0.63 | 1.00 | 6.50 | 7.50 | 4.756 | 1.577 | 21.2% | 1.431 | 0.010 |
| 0.63 | 1.00 | 6.75 | 7.75 | 4.905 | 1.580 | 21.0% | 1.428 | 0.010 |
| 0.63 | 1.00 | 7.00 | 8.00 | 5.054 | 1.583 | 20.9% | 1.425 | 0.010 |
| 0.63 | 1.00 | 7.25 | 8.25 | 5.205 | 1.585 | 20.8% | 1.423 | 0.010 |
| 0.63 | 1.00 | 7.50 | 8.50 | 5.353 | 1.588 | 20.6% | 1.420 | 0.010 |
| 0.63 | 1.00 | 7.75 | 8.75 | 5.500 | 1.591 | 20.5% | 1.417 | 0.010 |

Table B.7

| Recursive preferences EIS = 1 and variable RRA low Y1 ε=0,05 βR = 0,98 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A(C1) | a(C1) | σ(C1) | p(C1) | P(C1) | C1 | Saving rate | E(C2) | Var(lnC2) |
| 0.21 | 0.25 | 1.00 | 0.50 | 0.416 | 1.203 | -20.3% | 1.793 | 0.034 |
| 0.31 | 0.40 | 1.00 | 0.80 | 0.625 | 1.280 | -28.0% | 1.714 | 0.039 |
| 0.38 | 0.50 | 1.00 | 1.00 | 0.755 | 1.324 | -32.4% | 1.670 | 0.042 |
| 0.53 | 0.75 | 1.00 | 1.50 | 1.064 | 1.410 | -41.0% | 1.582 | 0.051 |
| 0.68 | 1.00 | 1.00 | 2.00 | 1.365 | 1.465 | -46.5% | 1.526 | 0.057 |
| 0.84 | 1.25 | 1.00 | 2.50 | 1.671 | 1.496 | -49.6% | 1.494 | 0.062 |
| 1.00 | 1.50 | 1.00 | 3.00 | 1.991 | 1.507 | -50.7% | 1.483 | 0.064 |
| 1.16 | 1.75 | 1.00 | 3.50 | 2.324 | 1.506 | -50.6% | 1.484 | 0.063 |
| 1.34 | 2.00 | 1.00 | 4.00 | 2.674 | 1.496 | -49.6% | 1.494 | 0.062 |
| 1.52 | 2.25 | 1.00 | 4.50 | 3.043 | 1.479 | -47.9% | 1.511 | 0.059 |
| 1.71 | 2.50 | 1.00 | 5.00 | 3.429 | 1.458 | -45.8% | 1.533 | 0.056 |
| 1.92 | 2.75 | 1.00 | 5.50 | 3.833 | 1.435 | -43.5% | 1.556 | 0.054 |
| 2.13 | 3.00 | 1.00 | 6.00 | 4.252 | 1.411 | -41.1% | 1.581 | 0.051 |
| 2.34 | 3.25 | 1.00 | 6.50 | 4.690 | 1.386 | -38.6% | 1.606 | 0.048 |
| 2.57 | 3.50 | 1.00 | 7.00 | 5.147 | 1.360 | -36.0% | 1.633 | 0.046 |
| 2.81 | 3.75 | 1.00 | 7.50 | 5.622 | 1.334 | -33.4% | 1.659 | 0.043 |
| 3.06 | 4.00 | 1.00 | 8.00 | 6.112 | 1.309 | -30.9% | 1.685 | 0.041 |
| 3.31 | 4.25 | 1.00 | 8.50 | 6.615 | 1.285 | -28.5% | 1.709 | 0.039 |
| 3.57 | 4.50 | 1.00 | 9.00 | 7.137 | 1.261 | -26.1% | 1.734 | 0.038 |
| 3.83 | 4.75 | 1.00 | 9.50 | 7.667 | 1.239 | -23.9% | 1.756 | 0.036 |
| 4.11 | 5.00 | 1.00 | 10.00 | 8.217 | 1.217 | -21.7% | 1.779 | 0.035 |
| 4.39 | 5.25 | 1.00 | 10.50 | 8.772 | 1.197 | -19.7% | 1.799 | 0.034 |
| 4.67 | 5.50 | 1.00 | 11.00 | 9.338 | 1.178 | -17.8% | 1.818 | 0.032 |
| 4.96 | 5.75 | 1.00 | 11.50 | 9.914 | 1.160 | -16.0% | 1.837 | 0.032 |
| 5.24 | 6.00 | 1.00 | 12.00 | 10.490 | 1.144 | -14.4% | 1.853 | 0.031 |
| 5.54 | 6.25 | 1.00 | 12.50 | 11.082 | 1.128 | -12.8% | 1.869 | 0.030 |
| 5.83 | 6.50 | 1.00 | 13.00 | 11.659 | 1.115 | -11.5% | 1.883 | 0.029 |
| 6.13 | 6.75 | 1.00 | 13.50 | 12.250 | 1.102 | -10.2% | 1.896 | 0.029 |
| 6.42 | 7.00 | 1.00 | 14.00 | 12.832 | 1.091 | -9.1% | 1.907 | 0.028 |
| 6.71 | 7.25 | 1.00 | 14.50 | 13.426 | 1.080 | -8.0% | 1.918 | 0.028 |
| 7.00 | 7.50 | 1.00 | 15.00 | 14.006 | 1.071 | -7.1% | 1.928 | 0.027 |
| 7.29 | 7.75 | 1.00 | 15.50 | 14.581 | 1.063 | -6.3% | 1.936 | 0.027 |
| 7.58 | 8.00 | 1.00 | 16.00 | 15.152 | 1.056 | -5.6% | 1.943 | 0.027 |
| 7.86 | 8.25 | 1.00 | 16.50 | 15.729 | 1.049 | -4.9% | 1.950 | 0.026 |
| 8.14 | 8.50 | 1.00 | 17.00 | 16.284 | 1.044 | -4.4% | 1.955 | 0.026 |
| 8.42 | 8.75 | 1.00 | 17.50 | 16.843 | 1.039 | -3.9% | 1.960 | 0.026 |
| 8.70 | 9.00 | 1.00 | 18.00 | 17.391 | 1.035 | -3.5% | 1.964 | 0.026 |
| 8.97 | 9.25 | 1.00 | 18.50 | 17.944 | 1.031 | -3.1% | 1.968 | 0.026 |
| 9.24 | 9.50 | 1.00 | 19.00 | 18.482 | 1.028 | -2.8% | 1.971 | 0.026 |
| 9.51 | 9.75 | 1.00 | 19.50 | 19.024 | 1.025 | -2.5% | 1.975 | 0.025 |
| 9.78 | 10.00 | 1.00 | 20.00 | 19.550 | 1.023 | -2.3% | 1.977 | 0.025 |

Table B.8

| Recursive preferences EIS = 1 and variable RRA high Y1 ε=0,05 βR = 0,98 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A(C1) | a(C1) | σ(C1) | p(C1) | P(C1) | C1 | Saving rate | E(C2) | Var(lnC2) |
| 0.21 | 0.25 | 1.00 | 0.50 | 0.414 | 1.207 | 39.7% | 1.809 | 0.005 |
| 0.31 | 0.40 | 1.00 | 0.80 | 0.621 | 1.289 | 35.6% | 1.725 | 0.006 |
| 0.37 | 0.50 | 1.00 | 1.00 | 0.748 | 1.337 | 33.2% | 1.676 | 0.006 |
| 0.52 | 0.75 | 1.00 | 1.50 | 1.043 | 1.438 | 28.1% | 1.573 | 0.008 |
| 0.66 | 1.00 | 1.00 | 2.00 | 1.321 | 1.514 | 24.3% | 1.496 | 0.009 |
| 0.80 | 1.25 | 1.00 | 2.50 | 1.590 | 1.572 | 21.4% | 1.437 | 0.009 |
| 0.93 | 1.50 | 1.00 | 3.00 | 1.856 | 1.616 | 19.2% | 1.392 | 0.010 |
| 1.06 | 1.75 | 1.00 | 3.50 | 2.122 | 1.649 | 17.6% | 1.358 | 0.011 |
| 1.19 | 2.00 | 1.00 | 4.00 | 2.389 | 1.674 | 16.3% | 1.333 | 0.011 |
| 1.33 | 2.25 | 1.00 | 4.50 | 2.656 | 1.694 | 15.3% | 1.312 | 0.012 |
| 1.46 | 2.50 | 1.00 | 5.00 | 2.927 | 1.708 | 14.6% | 1.298 | 0.012 |
| 1.60 | 2.75 | 1.00 | 5.50 | 3.200 | 1.719 | 14.1% | 1.287 | 0.012 |
| 1.74 | 3.00 | 1.00 | 6.00 | 3.476 | 1.726 | 13.7% | 1.279 | 0.013 |
| 1.88 | 3.25 | 1.00 | 6.50 | 3.757 | 1.730 | 13.5% | 1.275 | 0.013 |
| 2.02 | 3.50 | 1.00 | 7.00 | 4.039 | 1.733 | 13.4% | 1.272 | 0.013 |
| 2.16 | 3.75 | 1.00 | 7.50 | 4.328 | 1.733 | 13.4% | 1.272 | 0.013 |
| 2.31 | 4.00 | 1.00 | 8.00 | 4.622 | 1.731 | 13.5% | 1.274 | 0.013 |
| 2.46 | 4.25 | 1.00 | 8.50 | 4.919 | 1.728 | 13.6% | 1.277 | 0.013 |
| 2.61 | 4.50 | 1.00 | 9.00 | 5.223 | 1.723 | 13.9% | 1.283 | 0.013 |
| 2.76 | 4.75 | 1.00 | 9.50 | 5.530 | 1.718 | 14.1% | 1.288 | 0.012 |
| 2.92 | 5.00 | 1.00 | 10.00 | 5.845 | 1.711 | 14.5% | 1.295 | 0.012 |
| 3.08 | 5.25 | 1.00 | 10.50 | 6.166 | 1.703 | 14.9% | 1.303 | 0.012 |
| 3.24 | 5.50 | 1.00 | 11.00 | 6.490 | 1.695 | 15.3% | 1.311 | 0.012 |
| 3.41 | 5.75 | 1.00 | 11.50 | 6.821 | 1.686 | 15.7% | 1.320 | 0.012 |
| 3.58 | 6.00 | 1.00 | 12.00 | 7.160 | 1.676 | 16.2% | 1.330 | 0.011 |
| 3.75 | 6.25 | 1.00 | 12.50 | 7.508 | 1.665 | 16.8% | 1.342 | 0.011 |
| 3.93 | 6.50 | 1.00 | 13.00 | 7.855 | 1.655 | 17.3% | 1.352 | 0.011 |
| 4.11 | 6.75 | 1.00 | 13.50 | 8.217 | 1.643 | 17.9% | 1.364 | 0.011 |
| 4.29 | 7.00 | 1.00 | 14.00 | 8.578 | 1.632 | 18.4% | 1.375 | 0.011 |
| 4.48 | 7.25 | 1.00 | 14.50 | 8.951 | 1.620 | 19.0% | 1.388 | 0.010 |
| 4.66 | 7.50 | 1.00 | 15.00 | 9.328 | 1.608 | 19.6% | 1.400 | 0.010 |
| 4.86 | 7.75 | 1.00 | 15.50 | 9.712 | 1.596 | 20.2% | 1.412 | 0.010 |
| 5.05 | 8.00 | 1.00 | 16.00 | 10.107 | 1.583 | 20.9% | 1.425 | 0.010 |
| 5.25 | 8.25 | 1.00 | 16.50 | 10.503 | 1.571 | 21.5% | 1.438 | 0.009 |
| 5.45 | 8.50 | 1.00 | 17.00 | 10.904 | 1.559 | 22.1% | 1.450 | 0.009 |
| 5.66 | 8.75 | 1.00 | 17.50 | 11.312 | 1.547 | 22.7% | 1.462 | 0.009 |
| 5.86 | 9.00 | 1.00 | 18.00 | 11.726 | 1.535 | 23.3% | 1.474 | 0.009 |
| 6.07 | 9.25 | 1.00 | 18.50 | 12.147 | 1.523 | 23.9% | 1.487 | 0.009 |
| 6.29 | 9.50 | 1.00 | 19.00 | 12.574 | 1.511 | 24.5% | 1.499 | 0.009 |
| 6.50 | 9.75 | 1.00 | 19.50 | 13.000 | 1.500 | 25.0% | 1.510 | 0.008 |
| 6.72 | 10.00 | 1.00 | 20.00 | 13.441 | 1.488 | 25.6% | 1.522 | 0.008 |

Table B.9

| Recursive preferences RRA = 1 and variable EIS low Y1 ε=0,01 βR = 0,98 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A(C1) | a(C1) | σ(C1) | p(C1) | P(C1) | C1 | Saving rate | E(C2) | Var(lnC2) |
| 0.67 | 1.00 | 0.10 | 1.10 | 0.737 | 1.493 | -49.3% | 1.497 | 0.012 |
| 0.67 | 1.00 | 0.20 | 1.20 | 0.803 | 1.494 | -49.4% | 1.496 | 0.012 |
| 0.67 | 1.00 | 0.25 | 1.25 | 0.837 | 1.494 | -49.4% | 1.496 | 0.012 |
| 0.67 | 1.00 | 0.30 | 1.30 | 0.870 | 1.494 | -49.4% | 1.496 | 0.012 |
| 0.67 | 1.00 | 0.40 | 1.40 | 0.936 | 1.495 | -49.5% | 1.495 | 0.012 |
| 0.67 | 1.00 | 0.50 | 1.50 | 1.003 | 1.496 | -49.6% | 1.494 | 0.012 |
| 0.67 | 1.00 | 0.60 | 1.60 | 1.069 | 1.497 | -49.7% | 1.493 | 0.012 |
| 0.67 | 1.00 | 0.70 | 1.70 | 1.135 | 1.498 | -49.8% | 1.492 | 0.012 |
| 0.67 | 1.00 | 0.80 | 1.80 | 1.201 | 1.499 | -49.9% | 1.491 | 0.012 |
| 0.67 | 1.00 | 0.90 | 1.90 | 1.267 | 1.500 | -50.0% | 1.490 | 0.012 |
| 0.67 | 1.00 | 1.00 | 2.00 | 1.333 | 1.500 | -50.0% | 1.490 | 0.012 |
| 0.67 | 1.00 | 1.20 | 2.20 | 1.465 | 1.502 | -50.2% | 1.488 | 0.012 |
| 0.66 | 1.00 | 1.40 | 2.40 | 1.596 | 1.504 | -50.4% | 1.486 | 0.013 |
| 0.66 | 1.00 | 1.60 | 2.60 | 1.726 | 1.506 | -50.6% | 1.484 | 0.013 |
| 0.66 | 1.00 | 1.80 | 2.80 | 1.858 | 1.507 | -50.7% | 1.483 | 0.013 |
| 0.66 | 1.00 | 2.00 | 3.00 | 1.988 | 1.509 | -50.9% | 1.481 | 0.013 |
| 0.66 | 1.00 | 2.20 | 3.20 | 2.119 | 1.510 | -51.0% | 1.480 | 0.013 |
| 0.66 | 1.00 | 2.40 | 3.40 | 2.249 | 1.512 | -51.2% | 1.478 | 0.013 |
| 0.66 | 1.00 | 2.60 | 3.60 | 2.378 | 1.514 | -51.4% | 1.476 | 0.013 |
| 0.66 | 1.00 | 2.80 | 3.80 | 2.508 | 1.515 | -51.5% | 1.475 | 0.013 |
| 0.66 | 1.00 | 3.00 | 4.00 | 2.637 | 1.517 | -51.7% | 1.473 | 0.013 |
| 0.66 | 1.00 | 3.20 | 4.20 | 2.767 | 1.518 | -51.8% | 1.472 | 0.013 |
| 0.66 | 1.00 | 3.40 | 4.40 | 2.895 | 1.520 | -52.0% | 1.470 | 0.013 |
| 0.66 | 1.00 | 3.60 | 4.60 | 3.024 | 1.521 | -52.1% | 1.469 | 0.013 |
| 0.66 | 1.00 | 3.80 | 4.80 | 3.152 | 1.523 | -52.3% | 1.467 | 0.013 |
| 0.66 | 1.00 | 4.00 | 5.00 | 3.281 | 1.524 | -52.4% | 1.466 | 0.013 |
| 0.66 | 1.00 | 4.25 | 5.25 | 3.440 | 1.526 | -52.6% | 1.463 | 0.013 |
| 0.65 | 1.00 | 4.50 | 5.50 | 3.599 | 1.528 | -52.8% | 1.461 | 0.013 |
| 0.65 | 1.00 | 4.75 | 5.75 | 3.758 | 1.530 | -53.0% | 1.459 | 0.013 |
| 0.65 | 1.00 | 5.00 | 6.00 | 3.916 | 1.532 | -53.2% | 1.457 | 0.013 |
| 0.65 | 1.00 | 5.25 | 6.25 | 4.077 | 1.533 | -53.3% | 1.456 | 0.013 |
| 0.65 | 1.00 | 5.50 | 6.50 | 4.235 | 1.535 | -53.5% | 1.454 | 0.014 |
| 0.65 | 1.00 | 5.75 | 6.75 | 4.392 | 1.537 | -53.7% | 1.452 | 0.014 |
| 0.65 | 1.00 | 6.00 | 7.00 | 4.548 | 1.539 | -53.9% | 1.450 | 0.014 |
| 0.65 | 1.00 | 6.25 | 7.25 | 4.708 | 1.540 | -54.0% | 1.449 | 0.014 |
| 0.65 | 1.00 | 6.50 | 7.50 | 4.864 | 1.542 | -54.2% | 1.447 | 0.014 |
| 0.65 | 1.00 | 6.75 | 7.75 | 5.019 | 1.544 | -54.4% | 1.445 | 0.014 |
| 0.65 | 1.00 | 7.00 | 8.00 | 5.178 | 1.545 | -54.5% | 1.444 | 0.014 |
| 0.65 | 1.00 | 7.25 | 8.25 | 5.333 | 1.547 | -54.7% | 1.442 | 0.014 |
| 0.65 | 1.00 | 7.50 | 8.50 | 5.491 | 1.548 | -54.8% | 1.441 | 0.014 |
| 0.65 | 1.00 | 7.75 | 8.75 | 5.645 | 1.550 | -55.0% | 1.439 | 0.014 |

Table B.10

| Recursive preferences RRA = 1 and variable EIS high Y1 ε=0,01 βR = 0,98 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A(C1) | a(C1) | σ(C1) | p(C1) | P(C1) | C1 | Saving rate | E(C2) | Var(lnC2) |
| 0.66 | 1.00 | 0.10 | 1.10 | 0.730 | 1.506 | 24.7% | 1.504 | 0.002 |
| 0.66 | 1.00 | 0.20 | 1.20 | 0.796 | 1.507 | 24.7% | 1.503 | 0.002 |
| 0.66 | 1.00 | 0.25 | 1.25 | 0.829 | 1.508 | 24.6% | 1.502 | 0.002 |
| 0.66 | 1.00 | 0.30 | 1.30 | 0.861 | 1.509 | 24.6% | 1.501 | 0.002 |
| 0.66 | 1.00 | 0.40 | 1.40 | 0.927 | 1.510 | 24.5% | 1.500 | 0.002 |
| 0.66 | 1.00 | 0.50 | 1.50 | 0.992 | 1.512 | 24.4% | 1.498 | 0.002 |
| 0.66 | 1.00 | 0.60 | 1.60 | 1.058 | 1.513 | 24.4% | 1.497 | 0.002 |
| 0.66 | 1.00 | 0.70 | 1.70 | 1.122 | 1.515 | 24.3% | 1.495 | 0.002 |
| 0.66 | 1.00 | 0.80 | 1.80 | 1.187 | 1.516 | 24.2% | 1.494 | 0.002 |
| 0.66 | 1.00 | 0.90 | 1.90 | 1.252 | 1.518 | 24.1% | 1.492 | 0.002 |
| 0.66 | 1.00 | 1.00 | 2.00 | 1.317 | 1.519 | 24.1% | 1.491 | 0.002 |
| 0.66 | 1.00 | 1.20 | 2.20 | 1.445 | 1.522 | 23.9% | 1.488 | 0.002 |
| 0.66 | 1.00 | 1.40 | 2.40 | 1.574 | 1.525 | 23.8% | 1.485 | 0.002 |
| 0.65 | 1.00 | 1.60 | 2.60 | 1.702 | 1.528 | 23.6% | 1.481 | 0.002 |
| 0.65 | 1.00 | 1.80 | 2.80 | 1.829 | 1.531 | 23.5% | 1.478 | 0.002 |
| 0.65 | 1.00 | 2.00 | 3.00 | 1.956 | 1.534 | 23.3% | 1.475 | 0.002 |
| 0.65 | 1.00 | 2.20 | 3.20 | 2.082 | 1.537 | 23.2% | 1.472 | 0.002 |
| 0.65 | 1.00 | 2.40 | 3.40 | 2.208 | 1.540 | 23.0% | 1.469 | 0.002 |
| 0.65 | 1.00 | 2.60 | 3.60 | 2.333 | 1.543 | 22.9% | 1.466 | 0.002 |
| 0.65 | 1.00 | 2.80 | 3.80 | 2.458 | 1.546 | 22.7% | 1.463 | 0.002 |
| 0.65 | 1.00 | 3.00 | 4.00 | 2.582 | 1.549 | 22.6% | 1.460 | 0.002 |
| 0.64 | 1.00 | 3.20 | 4.20 | 2.706 | 1.552 | 22.4% | 1.457 | 0.002 |
| 0.64 | 1.00 | 3.40 | 4.40 | 2.830 | 1.555 | 22.3% | 1.454 | 0.002 |
| 0.64 | 1.00 | 3.60 | 4.60 | 2.953 | 1.558 | 22.1% | 1.451 | 0.002 |
| 0.64 | 1.00 | 3.80 | 4.80 | 3.075 | 1.561 | 22.0% | 1.448 | 0.002 |
| 0.64 | 1.00 | 4.00 | 5.00 | 3.197 | 1.564 | 21.8% | 1.445 | 0.002 |
| 0.64 | 1.00 | 4.25 | 5.25 | 3.348 | 1.568 | 21.6% | 1.441 | 0.002 |
| 0.64 | 1.00 | 4.50 | 5.50 | 3.501 | 1.571 | 21.5% | 1.438 | 0.002 |
| 0.63 | 1.00 | 4.75 | 5.75 | 3.651 | 1.575 | 21.3% | 1.434 | 0.002 |
| 0.63 | 1.00 | 5.00 | 6.00 | 3.800 | 1.579 | 21.1% | 1.429 | 0.002 |
| 0.63 | 1.00 | 5.25 | 6.25 | 3.951 | 1.582 | 20.9% | 1.426 | 0.002 |
| 0.63 | 1.00 | 5.50 | 6.50 | 4.098 | 1.586 | 20.7% | 1.422 | 0.002 |
| 0.63 | 1.00 | 5.75 | 6.75 | 4.245 | 1.590 | 20.5% | 1.418 | 0.002 |
| 0.63 | 1.00 | 6.00 | 7.00 | 4.394 | 1.593 | 20.4% | 1.415 | 0.002 |
| 0.63 | 1.00 | 6.25 | 7.25 | 4.540 | 1.597 | 20.2% | 1.411 | 0.002 |
| 0.63 | 1.00 | 6.50 | 7.50 | 4.688 | 1.600 | 20.0% | 1.408 | 0.002 |
| 0.62 | 1.00 | 6.75 | 7.75 | 4.832 | 1.604 | 19.8% | 1.404 | 0.002 |
| 0.62 | 1.00 | 7.00 | 8.00 | 4.975 | 1.608 | 19.6% | 1.400 | 0.002 |
| 0.62 | 1.00 | 7.25 | 8.25 | 5.121 | 1.611 | 19.5% | 1.397 | 0.002 |
| 0.62 | 1.00 | 7.50 | 8.50 | 5.263 | 1.615 | 19.3% | 1.393 | 0.002 |
| 0.62 | 1.00 | 7.75 | 8.75 | 5.405 | 1.619 | 19.1% | 1.389 | 0.002 |

Table B.11

| Recursive preferences EIS = 1 and variable RRA low Y1 ε=0,01 βR = 0,98 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| A(C1) | a(C1) | σ(C1) | p(C1) | P(C1) | C1 | Saving rate | E(C2) | Var(lnC2) |
| 0.21 | 0.25 | 1.00 | 0.50 | 0.421 | 1.188 | -18.8% | 1.808 | 0.007 |
| 0.31 | 0.40 | 1.00 | 0.80 | 0.629 | 1.272 | -27.2% | 1.723 | 0.008 |
| 0.38 | 0.50 | 1.00 | 1.00 | 0.756 | 1.322 | -32.2% | 1.672 | 0.008 |
| 0.53 | 0.75 | 1.00 | 1.50 | 1.053 | 1.424 | -42.4% | 1.568 | 0.010 |
| 0.67 | 1.00 | 1.00 | 2.00 | 1.333 | 1.500 | -50.0% | 1.490 | 0.012 |
| 0.80 | 1.25 | 1.00 | 2.50 | 1.608 | 1.555 | -55.5% | 1.434 | 0.014 |
| 0.94 | 1.50 | 1.00 | 3.00 | 1.883 | 1.593 | -59.3% | 1.395 | 0.016 |
| 1.08 | 1.75 | 1.00 | 3.50 | 2.167 | 1.615 | -61.5% | 1.373 | 0.017 |
| 1.23 | 2.00 | 1.00 | 4.00 | 2.460 | 1.626 | -62.6% | 1.361 | 0.018 |
| 1.38 | 2.25 | 1.00 | 4.50 | 2.766 | 1.627 | -62.7% | 1.360 | 0.018 |
| 1.54 | 2.50 | 1.00 | 5.00 | 3.086 | 1.620 | -62.0% | 1.368 | 0.017 |
| 1.71 | 2.75 | 1.00 | 5.50 | 3.420 | 1.608 | -60.8% | 1.380 | 0.017 |
| 1.88 | 3.00 | 1.00 | 6.00 | 3.769 | 1.592 | -59.2% | 1.396 | 0.016 |
| 2.07 | 3.25 | 1.00 | 6.50 | 4.132 | 1.573 | -57.3% | 1.416 | 0.015 |
| 2.25 | 3.50 | 1.00 | 7.00 | 4.507 | 1.553 | -55.3% | 1.436 | 0.014 |
| 2.45 | 3.75 | 1.00 | 7.50 | 4.899 | 1.531 | -53.1% | 1.458 | 0.013 |
| 2.65 | 4.00 | 1.00 | 8.00 | 5.305 | 1.508 | -50.8% | 1.482 | 0.013 |
| 2.86 | 4.25 | 1.00 | 8.50 | 5.728 | 1.484 | -48.4% | 1.506 | 0.012 |
| 3.08 | 4.50 | 1.00 | 9.00 | 6.164 | 1.460 | -46.0% | 1.531 | 0.011 |
| 3.31 | 4.75 | 1.00 | 9.50 | 6.616 | 1.436 | -43.6% | 1.555 | 0.011 |
| 3.54 | 5.00 | 1.00 | 10.00 | 7.082 | 1.412 | -41.2% | 1.580 | 0.010 |
| 3.78 | 5.25 | 1.00 | 10.50 | 7.565 | 1.388 | -38.8% | 1.604 | 0.010 |
| 4.03 | 5.50 | 1.00 | 11.00 | 8.065 | 1.364 | -36.4% | 1.629 | 0.009 |
| 4.29 | 5.75 | 1.00 | 11.50 | 8.576 | 1.341 | -34.1% | 1.652 | 0.009 |
| 4.55 | 6.00 | 1.00 | 12.00 | 9.105 | 1.318 | -31.8% | 1.676 | 0.008 |
| 4.82 | 6.25 | 1.00 | 12.50 | 9.645 | 1.296 | -29.6% | 1.698 | 0.008 |
| 5.10 | 6.50 | 1.00 | 13.00 | 10.196 | 1.275 | -27.5% | 1.720 | 0.008 |
| 5.38 | 6.75 | 1.00 | 13.50 | 10.766 | 1.254 | -25.4% | 1.741 | 0.007 |
| 5.67 | 7.00 | 1.00 | 14.00 | 11.345 | 1.234 | -23.4% | 1.761 | 0.007 |
| 5.97 | 7.25 | 1.00 | 14.50 | 11.934 | 1.215 | -21.5% | 1.781 | 0.007 |
| 6.27 | 7.50 | 1.00 | 15.00 | 12.531 | 1.197 | -19.7% | 1.799 | 0.007 |
| 6.57 | 7.75 | 1.00 | 15.50 | 13.136 | 1.180 | -18.0% | 1.816 | 0.006 |
| 6.87 | 8.00 | 1.00 | 16.00 | 13.746 | 1.164 | -16.4% | 1.833 | 0.006 |
| 7.18 | 8.25 | 1.00 | 16.50 | 14.360 | 1.149 | -14.9% | 1.848 | 0.006 |
| 7.49 | 8.50 | 1.00 | 17.00 | 14.978 | 1.135 | -13.5% | 1.862 | 0.006 |
| 7.80 | 8.75 | 1.00 | 17.50 | 15.597 | 1.122 | -12.2% | 1.876 | 0.006 |
| 8.11 | 9.00 | 1.00 | 18.00 | 16.216 | 1.110 | -11.0% | 1.888 | 0.006 |
| 8.42 | 9.25 | 1.00 | 18.50 | 16.849 | 1.098 | -9.8% | 1.900 | 0.006 |
| 8.73 | 9.50 | 1.00 | 19.00 | 17.463 | 1.088 | -8.8% | 1.910 | 0.006 |
| 9.04 | 9.75 | 1.00 | 19.50 | 18.072 | 1.079 | -7.9% | 1.919 | 0.005 |
| 9.34 | 10.00 | 1.00 | 20.00 | 18.674 | 1.071 | -7.1% | 1.928 | 0.005 |

Table B.12

| Recursive preferences EIS = 1 and variable RRA high Y1 ε=0,01 βR = 0,98 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A(C1) | a(C1) | σ(C1) | p(C1) | P(C1) | C1 | Saving rate | E(C2) | Var(lnC2) |
| 0.21 | 0.25 | 1.00 | 0.50 | 0.420 | 1.191 | 40.5% | 1.825 | 0.001 |
| 0.31 | 0.40 | 1.00 | 0.80 | 0.626 | 1.277 | 36.2% | 1.737 | 0.001 |
| 0.38 | 0.50 | 1.00 | 1.00 | 0.752 | 1.329 | 33.6% | 1.684 | 0.001 |
| 0.52 | 0.75 | 1.00 | 1.50 | 1.045 | 1.436 | 28.2% | 1.575 | 0.001 |
| 0.66 | 1.00 | 1.00 | 2.00 | 1.317 | 1.519 | 24.1% | 1.491 | 0.002 |
| 0.79 | 1.25 | 1.00 | 2.50 | 1.579 | 1.583 | 20.9% | 1.425 | 0.002 |
| 0.92 | 1.50 | 1.00 | 3.00 | 1.837 | 1.633 | 18.4% | 1.374 | 0.002 |
| 1.05 | 1.75 | 1.00 | 3.50 | 2.093 | 1.672 | 16.4% | 1.335 | 0.002 |
| 1.17 | 2.00 | 1.00 | 4.00 | 2.347 | 1.704 | 14.8% | 1.302 | 0.002 |
| 1.30 | 2.25 | 1.00 | 4.50 | 2.603 | 1.729 | 13.6% | 1.276 | 0.002 |
| 1.43 | 2.50 | 1.00 | 5.00 | 2.857 | 1.750 | 12.5% | 1.255 | 0.003 |
| 1.56 | 2.75 | 1.00 | 5.50 | 3.113 | 1.767 | 11.7% | 1.238 | 0.003 |
| 1.68 | 3.00 | 1.00 | 6.00 | 3.369 | 1.781 | 11.0% | 1.223 | 0.003 |
| 1.81 | 3.25 | 1.00 | 6.50 | 3.625 | 1.793 | 10.4% | 1.211 | 0.003 |
| 1.94 | 3.50 | 1.00 | 7.00 | 3.885 | 1.802 | 9.9% | 1.202 | 0.003 |
| 2.07 | 3.75 | 1.00 | 7.50 | 4.144 | 1.810 | 9.5% | 1.194 | 0.003 |
| 2.20 | 4.00 | 1.00 | 8.00 | 4.405 | 1.816 | 9.2% | 1.188 | 0.003 |
| 2.33 | 4.25 | 1.00 | 8.50 | 4.668 | 1.821 | 9.0% | 1.183 | 0.003 |
| 2.47 | 4.50 | 1.00 | 9.00 | 4.934 | 1.824 | 8.8% | 1.180 | 0.003 |
| 2.60 | 4.75 | 1.00 | 9.50 | 5.200 | 1.827 | 8.7% | 1.176 | 0.003 |
| 2.74 | 5.00 | 1.00 | 10.00 | 5.470 | 1.828 | 8.6% | 1.175 | 0.003 |
| 2.87 | 5.25 | 1.00 | 10.50 | 5.744 | 1.828 | 8.6% | 1.175 | 0.003 |
| 3.01 | 5.50 | 1.00 | 11.00 | 6.021 | 1.827 | 8.7% | 1.176 | 0.003 |
| 3.15 | 5.75 | 1.00 | 11.50 | 6.301 | 1.825 | 8.8% | 1.179 | 0.003 |
| 3.29 | 6.00 | 1.00 | 12.00 | 6.583 | 1.823 | 8.9% | 1.181 | 0.003 |
| 3.44 | 6.25 | 1.00 | 12.50 | 6.872 | 1.819 | 9.1% | 1.185 | 0.003 |
| 3.58 | 6.50 | 1.00 | 13.00 | 7.163 | 1.815 | 9.3% | 1.189 | 0.003 |
| 3.73 | 6.75 | 1.00 | 13.50 | 7.459 | 1.810 | 9.5% | 1.194 | 0.003 |
| 3.88 | 7.00 | 1.00 | 14.00 | 7.761 | 1.804 | 9.8% | 1.200 | 0.003 |
| 4.03 | 7.25 | 1.00 | 14.50 | 8.065 | 1.798 | 10.1% | 1.206 | 0.003 |
| 4.19 | 7.50 | 1.00 | 15.00 | 8.375 | 1.791 | 10.5% | 1.213 | 0.003 |
| 4.35 | 7.75 | 1.00 | 15.50 | 8.693 | 1.783 | 10.9% | 1.221 | 0.003 |
| 4.51 | 8.00 | 1.00 | 16.00 | 9.014 | 1.775 | 11.3% | 1.230 | 0.003 |
| 4.67 | 8.25 | 1.00 | 16.50 | 9.343 | 1.766 | 11.7% | 1.239 | 0.003 |
| 4.84 | 8.50 | 1.00 | 17.00 | 9.676 | 1.757 | 12.2% | 1.248 | 0.003 |
| 5.01 | 8.75 | 1.00 | 17.50 | 10.017 | 1.747 | 12.7% | 1.258 | 0.003 |
| 5.18 | 9.00 | 1.00 | 18.00 | 10.363 | 1.737 | 13.2% | 1.268 | 0.003 |
| 5.36 | 9.25 | 1.00 | 18.50 | 10.712 | 1.727 | 13.7% | 1.278 | 0.002 |
| 5.54 | 9.50 | 1.00 | 19.00 | 11.072 | 1.716 | 14.2% | 1.290 | 0.002 |
| 5.72 | 9.75 | 1.00 | 19.50 | 11.437 | 1.705 | 14.8% | 1.301 | 0.002 |