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Characterization of defects in plates using shear and Lamb waves

Dimitra V. Achillopoulou[a,*], Annamaria Pau[b]

[a]*Department of Structural and Geotechnical Engineering, La Sapienza University of Rome, via Eudossiana 18, Rome, 00184, Italy*
[b]*Department of Structural and Geotechnical Engineering, La Sapienza University of Rome, via Gramsci 53, Rome, 00197, Italy*

Abstract

This work investigates the interaction of shear and Lamb waves with different kinds of defects in plates, in view of applications to defect characterization purposes. Using a finite element model, the reflection and transmission coefficients of shear and Lamb waves are determined, as a function of size parameters of the defect related to its extension and depth. Notches with elliptical and rectangular profile are examined, together with internal voids, covering both symmetric and asymmetric cases. Low and high frequency times height $(2hf)$ regimes are considered in order to clarify how mode conversion can provide information on the shape of the defect. In this regard, also the role of symmetric and asymmetric waves is elucidated.

Keywords: waveguides, cavities, Shear waves, Lamb waves, damage characterization

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1. Introduction

Damage detection is a matter of fundamental importance in structural health monitoring. In this area, the use of sonic or ultrasonic guided waves has attracted the interest of many researchers for their wide range inspection capability, and for the fact that the scattered field resulting from the interaction of guided waves with defects contains information on its size, location and shape. Although the use of guided waves in detecting the presence and locating a defect is rather consolidated, their application in geometrically sizing and mapping the in-depth profile of the defect is still out of reach, also due to the difficulties in the interpretation of the scattered field [1, 2].

Detailed understanding of wave interaction with defects is useful in the definition of a model-based procedure for defect characterization based on the analysis of the scattered response [3]. Awareness of the variation of scattering coefficients as a function of the geometric characteristics of the discontinuity is fundamental for the solution of the resulting inverse problem, and it can as well help in the selection of modes and frequencies that improve inspection sensitivity to various discontinuities [4, 5].

This study focuses on the numerical investigation of the interaction of shear (SH) and Rayleigh-Lamb waves (RL) with different sizes and profiles of voids in an Aluminum plate. The different profiles investigated are a rectangular and an elliptical discontinuity which have been placed inside or on the surface of the waveguide, either symmetrically

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[*] Corresponding author.
E-mail address: dimitra.achillopoulou@uniroma1.it

or nonsymmetrically with respect to its plane. The analysis is performed using finite element models, and adds useful information to our former research [6, 7], also enabling us to define the most sensitive type of waves to be used to interrogate the structure on the size and shape of its defects.

2. Interaction of guided waves with defects

We consider a plate with thickness $2h$ and waves propagating along the x_1 direction. The system of coordinates is reported in Figure 1. When a guided wave meets discontinuities, such as flaws, voids, cavities, reflected and transmitted waves arise. Let us assume that the incident wave is the n-th wave mode with radian frequency ω. The harmonic response u^l (l is for left) at a point before the discontinuity consists of the superposition of the incident wave mode plus a reflected wave field, represented by the sum of wave modes with reflection coefficients R_{np}, where the subscripts n and p specify both the n-th incident and p-th reflected wave modes:

$$u_2^l = u_{2n}^{inc} + \sum_{p=0}^{N} R_{np} U_{2p}(x_3) e^{i(-k_p x_1 - \omega t)} \tag{1}$$

for shear waves, and

$$u_1^l = u_{1n}^{inc} + \sum_{p=0}^{N} R_{np} U_{1p}(x_3) e^{i(-k_p x_1 - \omega t)} \quad u_3^l = u_{3n}^{inc} + \sum_{p=0}^{N} R_{np} U_{3p}(x_3) e^{i(-k_p x_1 - \omega t)} \tag{2}$$

for Rayleigh-Lamb waves. The summation index extends to the N modes existing at the given frequency, and the coefficients R_{np} in equation 1 and in equations 2, in general, differ, as the shear and flexural problem are not coupled. The transmitted wave field u^r is the superposition of the first N wave modes with transmission coefficients T_{np}:

$$u_2^r = \sum_{p=0}^{N} T_{np} U_{2p}^r(x_3) e^{i(k_p^r x_1 - \omega t)} \tag{3}$$

for shear waves, and

$$u_1^r = \sum_{p=0}^{N} T_{np} U_{1p}^r(x_3) e^{i(k_p^r x_1 - \omega t)} \quad u_3^r = \sum_{p=0}^{N} T_{np} U_{3p}^r(x_3) e^{i(k_p^r x_1 - \omega t)} \tag{4}$$

for the two components of Rayleigh-Lamb waves, where r (for right) denotes the mechanical characteristics of the right part of the waveguide. Also here, coefficients T_{np} in equation 3 differ from those of equations 4.

The coefficients R_{np} and T_{np} depend on the geometry of the discontinuity, which is described by two size parameters, that are $r = (h - h_d)/h$, the ratio between the depth of the notch $h - h_d$ and the undamaged height h, and $\delta = d/\lambda_w$, where d is the length of the notch and λ_w is the wavelength of the incident wave. The parameter r measures the notch magnitude because $r = 0$ corresponds to a continuous plate, whereas $r = 1$ corresponds to a fully cracked cross section. The different types of voids examined are reported in Figure 1, with labels A, B, C referring to double sharp changes of height and D, E, F to elliptical profiles. The parameter r is varied, while $\delta = 0.16$ is kept constant. This value of δ corresponds to a damage extension that is fit to be detected with the frequency of excitation in use [8].

3. Finite Element Model simulations

Shear and Rayleigh-Lamb waves are simulated with two different finite element models. A plane strain model in the plane x_1, x_3 with in-plane displacements describes Rayleigh-Lamb waves, while an axisymmetric model with very large radius, with out-of-plane displacements (along x_2) describes shear waves. All the simulations were conducted

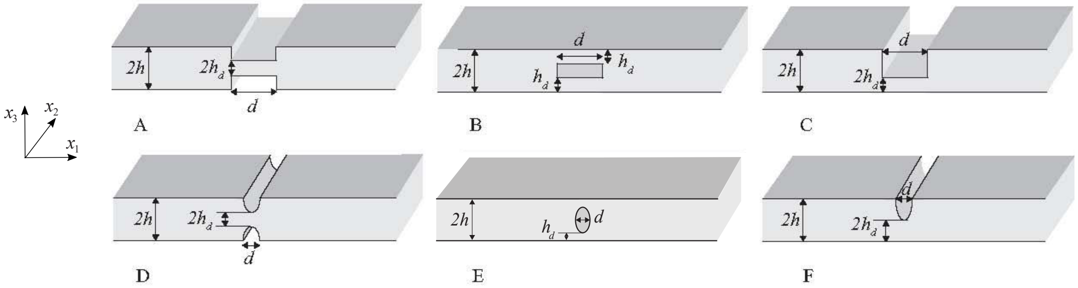

Fig. 1. Profiles of discontinuities and voids under investigation

using the Plane182 2D structural element of the Ansys software library. This element can be used for both plane strain and axisymmetric analyses.

The different kinds of waves investigated are generated by exciting one free-end of the plate with a sine burst so that a narrow frequency band is excited. The sine burst is obtained by enveloping a harmonic wave with a Gaussian window so that around six cycles are included. Such a time-history has a frequency band sufficiently narrow to limit dispersion phenomena. The time-history and Fourier transform of the forcing function are reported respectively in Figure 2 a and b. The spatial distribution of forces is chosen so as to excite different waves that are SH0, S0 and A0, and is depicted in Figure (Fig. 2). In the case of shear waves, the research presented in this work covers two regimes of the product $2hf$ obtained by modifying the plate thickness, that are a low one ($1MHzmm$, only SH0 propagates) and a higher one ($4MHzmm$, SH0, SH1, SH2 propagate), where f is the frequency of the excitation. For Rayleigh Lamb waves only the low value of $2hf = 1MHzmm$ was investigated, with S0 and A0 propagating. The results are obtained by a dynamic transient elastic analysis using Newmark time integration method, with appropriate spatial and time resolutions, equal respectively to one twentieth of the maximum wavelength involved and one twentieth of the central frequency of the sine burst, neglecting damping phenomena. The Aluminum plate examined was simulated using a linear elastic material and had the following parameters: density $\rho = 2810\ kg/m^3$, Lamé coefficients $\mu = 27000$ MPa and $\lambda = 55000$ MPa and longitudinal and shear velocities equal to $c_L = 6200$ m/sec and $c_T = 3071$ m/sec, respectively.

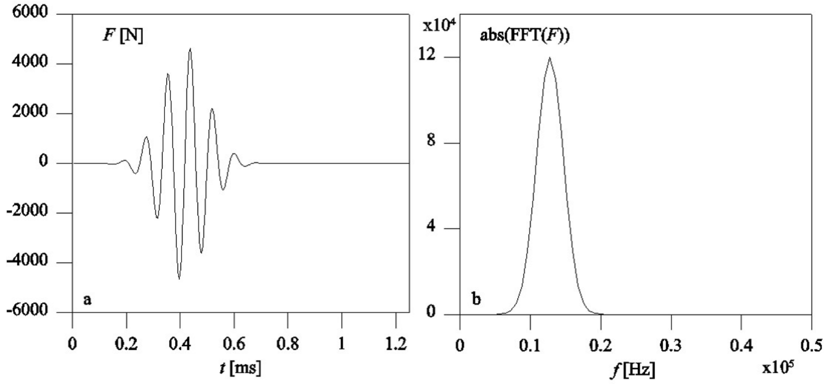

Fig. 2. Time history (a) and Fourier transform (b) of the forcing function

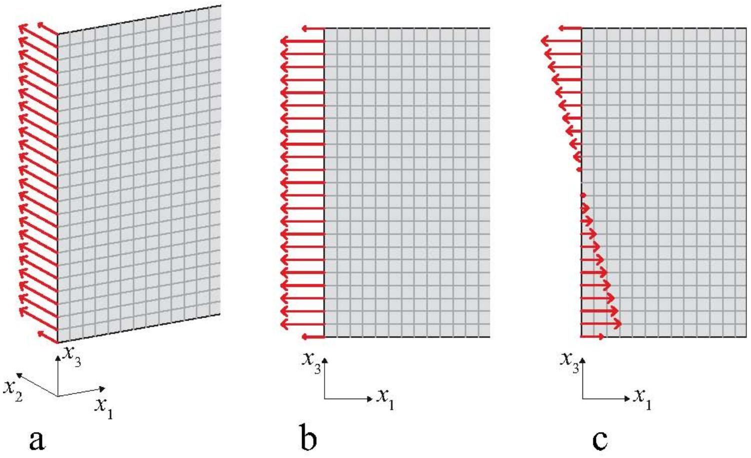

Fig. 3. Different spatial distributions of the forcing function used to generate different waves (a) SH0 (b) S0 RL (c) A0 RL

4. Results and Discussion

4.1. Shear waves

For low $2hf = 1MHzmm$, Figure 3(a) reports the coefficients of reflection and transmission of the mode SH0, for SH0 incident, that are R_{00} and T_{00}. Rectangular profiles A-C are depicted in Figure 3 (a), and elliptical D-F in Figure 3(b). The response is exactly the same for cases A and B, and for cases D and E, pointing out that for such vale of $2hf$, provided that the notch is symmetric, the response is insensitive to the in-depth location of the notch, that is to say whether it is internal or external. Moreover, the differences between the couples A-B and D-E are practically negligible, which denounces a feeble dependence of the response on the profile of the notch. Only asymmetric notches present slight differences (cases C and F), due to the arising of local, nonpropagating, nonsymmetric modes, that are not detectable in the far-field.

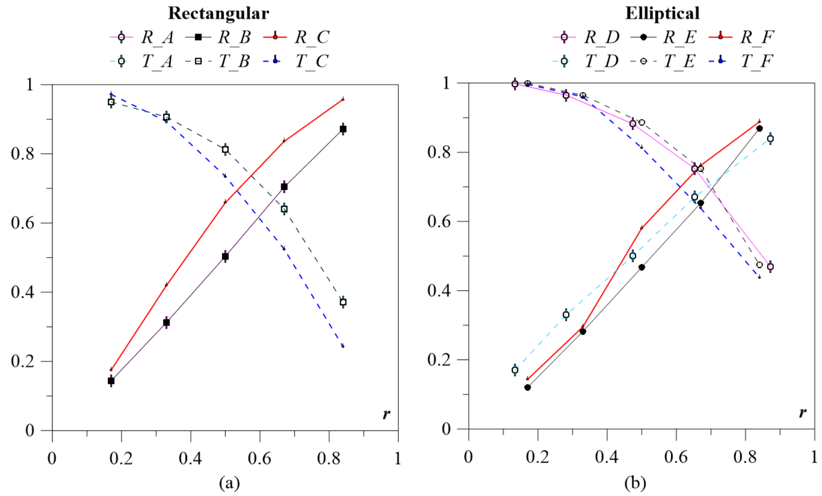

Fig. 4. R_{00} and T_{00} as a function of r (a) rectangular profiles (b) elliptical profiles

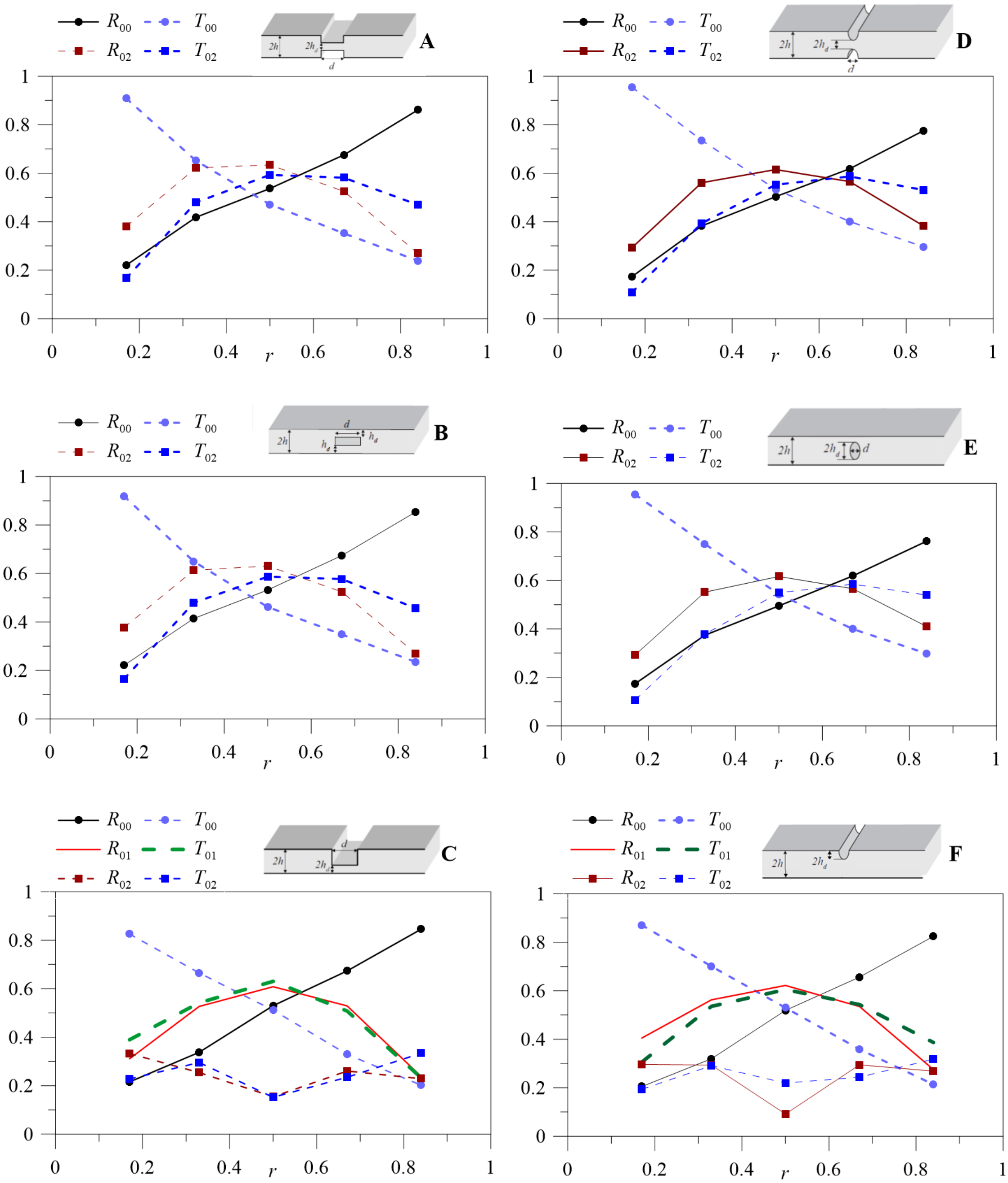

Fig. 5. Scattering coefficients R_{00}, R_{01}, R_{02}, T_{00}, T_{01} and T_{02} as a function of r for rectangular (A, B, C) and elliptical profiles (D, E, F)

In the high ($2hf = 4MHzmm$) frequency-height regime, SH0, SH1 and SH2 can, in principle, propagate. In this case, strong differences can be observed among symmetric and non-symmetric discontinuities. Figure 5 reports the reflection and transmission coefficients of modes SH0, SH1 and SH2, for SH0 mode incident. It can be seen that when the discontinuity is symmetric, only symmetric modes are retrieved in the response. On the contrary, when the discontinuity is nonsymmetric, also the mode SH1 appears. Similarly to what happens for $2hf = 1MHzmm$, the scattered field is the same for cases A-B and D-E, that is, its dependence on the notch profile is limited.

4.2. Rayleigh- Lamb waves

The coefficients of reflection and transmission of Rayleigh-Lamb waves are reported in Figure 6 for incident S0 and A0 modes. It can be observed that, if the discontinuity is symmetric (cases A, B, D, E), the scattered field contains only the incident mode. Among these four cases, and for S0 incident, some differences can be observed in the coefficients, but the same regular trend holds. Instead, when A0 is incident, internal and external notches cause a totally different response, with high sensitivity to r for internal defects, and very limited sensitivity for external defects. This is due to the mechanism of transmission of the wave: below a certain length of the discontinuity, the transmission through transverse shear prevails, and the sensitivity to the depth of the notch is limited. If the discontinuity is asymmetric (C, F), mode conversion occurs, that means that when S0 (A0) is incident, both S0 and A0 are scattered, with S0 (A0) prevailing over A0 (S0).

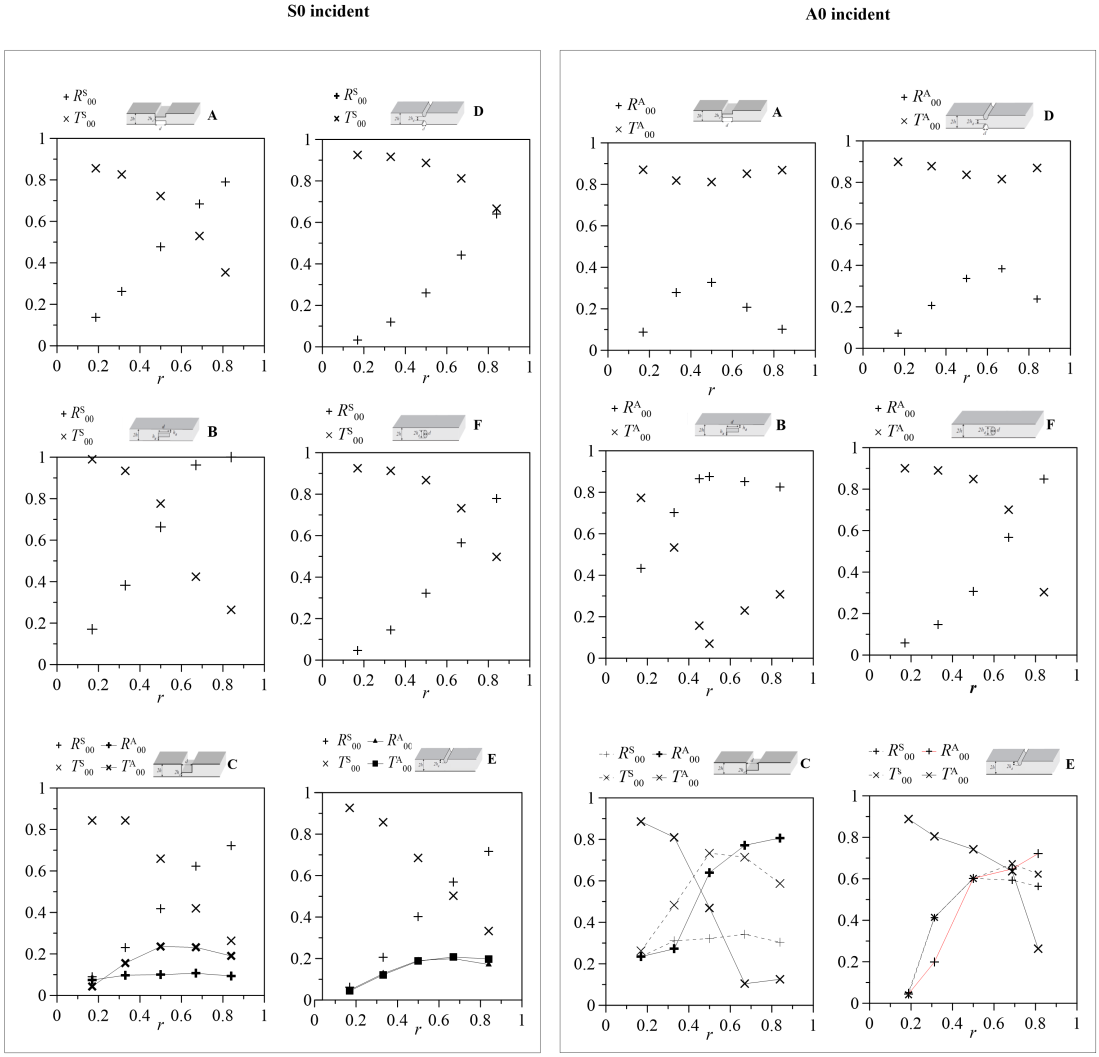

Fig. 6. R^S_{00}, R^A_{00}, T^S_{00}, and T^A_{00} as a function of r for rectangular (A, B, C) and elliptical profiles (D, E, F), for S0 incident

5. Conclusions

This paper describes the interaction of shear and Rayleigh-Lamb waves with voids of different profiles and depth, with given extension. Symmetric and nonsymmetric notches are investigated, where symmetry is meant with respect to the plane of the plate. The wave phenomena are described through a finite element model. Different regimes, arising by varying the product $2hf$, are investigated. These regimes differ for the presence of single or multiple propagating waves. It is found that when the product $2hf$ is low and only one mode can propagate, the scattered field is insensitive to the in-depth symmetry of the discontinuity and depends weakly on the profile of the void. Instead, when multiple propagating modes can exist, if the void is symmetric, the scattered response will contain only symmetric or nonsymmetric modes, respectively, for symmetric or nonsymmetric incident wave modes. If the void is nonsymmetric, the scattered field will contain both symmetric and nonsymmetric wave modes, both for symmetric and nonsymmetric incident wave modes.

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