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Effective filtering of modal curvatures for damage identification in beams

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Abstract

In this work, we investigate the effectiveness of a damage identification technique recently proposed in [1] and assess how it is affected by the number and position of the sensors used. Mode shapes and curvatures have been claimed to contain local information on damage and to be less sensitive to environmental variables than natural frequencies. It is known that notch-type damage produces a localized and sharp change in the curvature that unfortunately could be difficult to detect experimentally without the use of an adequate number of sensors. However, we have recently shown that even a coarse description of the modal curvature can still be employed to identify the damage, provided that it is used in combination with other modal quantities. Here, by exploiting the perturbative solution of the Euler-Bernoulli equation, we consider the inverse problem of damage localisation based on modal curvatures only and we ascertain the feasibility of their sole use for recostructing the damage shape. To do so, we set up a filtering procedure acting on modal curvatures which are expressed in a discrete form enabling further investigation on the effect of using a reduced number of measurement points. The sensitivity of the procedure to damage extension is further assessed.

Keywords: damage detection; inverse problems; modal curvatures

1. Introduction

Structural health monitoring techniques based on the measurement of modal response have attracted the interest of many researchers in the last decades [2]. Recent technological advances have made available several kinds of low-cost, reliable sensors suitable to monitor the state of large civil constructions including buildings, bridges and aqueducts [3, 4, 7, 5, 6], and have stimulated countless applications to real structures.

The change of natural frequencies has been one of the first approaches used in damage detection [8, 9, 10, 11], thanks to the easiness and robustness of their measurement in comparison to other modal quantities. The intrinsic drawback in the use of modal frequencies is though their well-known low sensitivity to local variations of the mechanical characteristics [12], which might lead to significant errors in the identified parameters. What is more, the inverse problem obtained by damage detection techniques based on eigenfrequencies is often ill-conditioned and sometimes undetermined.

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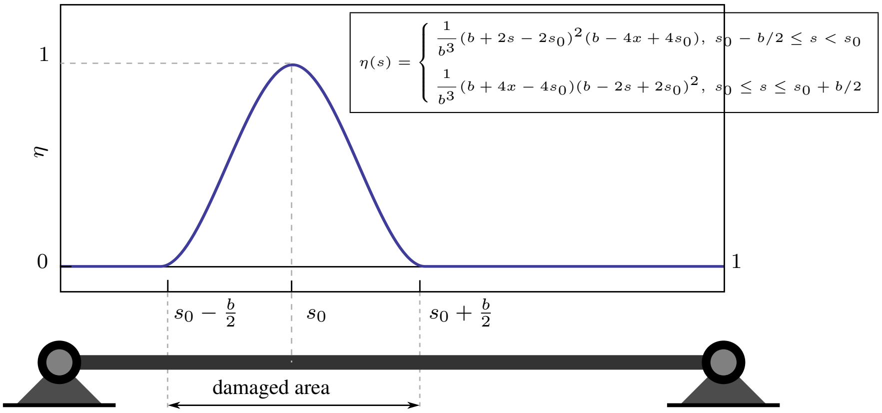

Fig. 1. Simply supported beam and damage shape function used in the examples of Section 3.

Some of these limitations can be overcome by using the changes of modal shapes and curvatures [13, 1, 14]. Such an approach was explored in the last decade of last century by Pandey [15] and, since then, has given rise to quite a number of follower studies investigating the use of modal curvatures as to solve the problem of damage assessment and localization in beams, among which, [16, 17, 18, 19, 20, 13, 21].

Theoretically, modal curvatures would be the most effective quantities to be observed as, for narrow damage such as a notch, their increase localizes in the neighborhood of damage. Moreover, they should be less sensitive to environmental variables than natural frequencies. Non-localized damage causes the change in modal curvature to be a more complex function which could result in an erroneous localization [1]. Effective damage identification can be obtained by various filtering techniques including spline interpolation, wavelet transforms [22], modified Laplace operator [23] or space-wavenumber Fourier transforms [24]. All these techniques require a high number of measurement points to obtain reliable values of modal curvatures.

In fact, it is known that notch-type damage produces a localized and sharp change in the curvature that could be difficult to detect experimentally without the use of an adequate number of sensors. However, we have recently shown [14] that even a coarse description of the modal curvature can improve the localization of damage when used in combination with other modal quantities. Here, by exploiting the perturbative solution of the Euler-Bernoulli equation described in [1], we consider the inverse problem of damage identification based on modal curvatures only and we ascertain the feasibility of their sole use in a damage identification procedure. The results obtained are expressed in discrete form, providing a tool to further investigate the effect of a limited number of measurement points. The sensitivity of the results is assessed in terms of damage extension and position of measurement points.

2. Modal curvatures as observable quantities

The transverse motion of a cracked beam can be studied by exploiting a perturbative solution of the dynamic Euler-Bernoulli equation. The procedure is fully described in [1] and shortly recalled here.

The dimensionless equation governing the i-th transverse mode of a damaged beam is:

$$\frac{d^4 v_i^*(s)}{ds^4} - \varepsilon \frac{d^2}{ds^2}\left[\eta(s)\,\frac{d^2 v_i^*(s)}{ds^2}\right] - \lambda_i^* v_i^*(s) = 0 \tag{1}$$

where $s \in [0,1]$ is the dimensionless abscissa, $v^*(s)$ is the transverse displacement and $\eta(s)$ the damage shape function depicted in Fig. 1, such that $\|\eta(s)\| = 1$. The eigenfunctions v_i^* and eigenvalues λ_i^* can be expanded as a power series in terms of the damage intensity ε ,i.e.,

$$v_i^*(x) = v_i^0(x) - \varepsilon\, v_i^1(x) + \mathrm{O}(\varepsilon^2), \qquad \lambda_i^* = \lambda_i^0 - \varepsilon\, \lambda_i^1 + \mathrm{O}(\varepsilon^2) \tag{2}$$

where v_i^0 and λ_i^0 are respectively the ith eigenfunction and eigenvalue of the undamaged beam, and v_i^1 and λ_i^1 are their first order variations. By taking into account only the contributions up to the first order in ε, the following system of ordinary differential equations is obtained

$$\text{0-}th\text{ order:} \qquad \frac{d^4 v_i^0(s)}{dx^4} - \lambda_i^0 v_i^0(s) = 0 \tag{3}$$

$$\text{1-}st\text{ order:} \qquad \frac{d^4 v_i^1(s)}{ds^4} - \lambda_i^0 v_i^1(s) = \lambda_i^1 v_i^0(s) - \frac{d^2\eta_i(s)}{ds^2} \tag{4}$$

where the function $\eta_i(s)$ is the damage shape weighted through the ith modal curvature, *i.e.* $\eta_i(s) := \eta(s)\ d^2v_i^0/ds^2$. Eq.(3) is simply the governing equation of the undamaged system.

The difference between the modal curvature of the damaged and undamaged beams, $\Delta_i''(s)$, can be expressed in terms of the modal quantities of the undamaged system and the damage shape function $\eta(s)$ as in [1], i.e.,

$$\Delta_i''(s) = -\eta_i(s) + \frac{\lambda_i^1}{\lambda_i^0}\ w_i^0(s) + \sum_{\substack{k=1\\k\neq i}}^{+\infty} \frac{\lambda_i^0}{\lambda_k^0\ (\lambda_i^0 - \lambda_k^0)} < \eta_i, w_k^0 >\ w_k^0(s) \tag{5}$$

where the $< \cdot, \cdot >:= \int_0^1 \cdot\ \cdot\ \mathrm{d}s$ indicates the scalar product and the notation $w_i^0(s)$ stands for the second derivative, that is the curvature, of the normalised mode shape, i.e., $w_i^0(s) = \|v_i^0\|^{-1}(d^2v_i^0/ds^2)$. It is noted that (5) is the sum of three contributions: the ith modal damage shape, the ith modal curvature and a term taking into account the contribution of the other modal curvatures. A sensitivity analysis of Eq. (5) in terms of damage position and width, carried out in [1], shown that broad damage, that is damage with non-localised extension along the axis, causes the modal curvature difference to have multiple peaks outside the damage region giving false indication of the damage position.

In this work, we are interested in assessing the effects of the sampling rate of the modal curvatures and, in particular, the minimum number of points that allows the damage shape to be accurately identified. This problem has indeed a great relevance for real world structures where a limited number of sensors is used for monitoring purposes. To this end we focus on the discrete form of (5), i.e.,

$$\Delta_i''^{\,(m)} := \Delta_i''(s_m) = -\eta_i^{(m)} + \frac{\lambda_i^1}{\lambda_i^0}\ w_i^{0\ (m)} + \sum_{\substack{k=1\\k\neq i}}^{K} \frac{\lambda_i^0}{\lambda_k^0\ (\lambda_i^0 - \lambda_k^0)} \sum_{p=1}^{M} \left(\frac{1}{M}\eta_i^{(p)} w_k^{0\ (p)}\right)\ w_k^{0\ (m)} \tag{6}$$

that is the results of sampling the modal curvatures in M equally spaced points, i.e., $0 < s_1 < ... < s_M < 1$ and using K modes in the series (5). It is apparent from (6) that, if the sampling points $s_1, s_2, ..., s_M$ lie outside the support of the function $\eta_i(s)$, $\eta_i^{(m)} = 0$ and the only contribution in $\Delta_i''^{\,(m)}$ is $\lambda_i^1/\lambda_i^0\ w_i^{0\ (m)}$, i.e. the i-th modal curvature. As a consequence, the reconstruction of the damage shape in this situation becomes cumbersome and a large number of modes in the series are needed.

Further details can be gained by inverting (5) to derive $\eta_i(s)$. Equation (6) can be recast in vector form as

$$\widetilde{\boldsymbol{\Delta}}_i'' = -\left[\boldsymbol{I} - \sum_{\substack{k=1\\k\neq i}}^{K} \frac{\lambda_i^0/\lambda_k^0}{M(\lambda_i^0 - \lambda_k^0)}(\boldsymbol{\omega}_k^0 \otimes \boldsymbol{\omega}_k^0)\right]\boldsymbol{\eta}_i \tag{7}$$

where $\widetilde{\Delta}_i''^{\,(m)} = \Delta_i''^{\,(m)} - (\lambda_i^1)/(\lambda_i^0) w_i^{0\ (m)}$ and $\otimes$ indicates the dyadic product between vectors, i.e., $(a\otimes b)_{i\,j} = a_i b_j$. $\boldsymbol{\Delta}_i''$, $\boldsymbol{w}_i^0$ and $\boldsymbol{\eta}_i$ are the M-row vectors containing the corresponding quantities at each sampling point; as such

$$\mathcal{K} = \boldsymbol{I} - \sum_{\substack{k=1\\k\neq i}}^{K} \frac{\lambda_i^0/\lambda_k^0}{M(\lambda_i^0 - \lambda_k^0)}(\boldsymbol{\omega}_k^0 \otimes \boldsymbol{\omega}_k^0) \tag{8}$$

is a $M \times M$ matrix and $\boldsymbol{I}$ is the $M \times M$ identity matrix. The properties of the matrix $\mathcal{K}$ that strongly affect the solution of (7) are studied in the next paragraph through numerical examples.

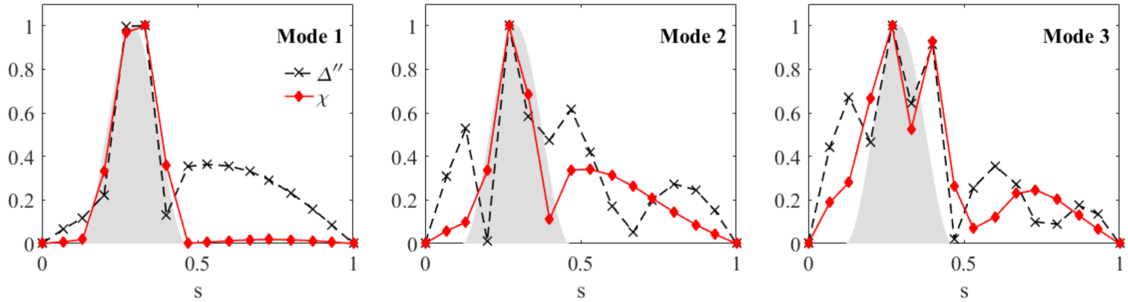

Fig. 2. Curvature variations Δ''_i and modal damage shape η_i obtained by Eq. (7) for the first three modes ($i = 1, 2, 3$) in the case of a broad damage located at $s_0 = 0.29$ with width $b = 0.35$ and $M = 16$ sampling points.

3. Numerical Results

The direct problem of evaluating the curvature for the simply supported beam shown in Fig. 1 was studied by the finite element method. To have an accurate estimate of the modal curvature throughout the beam length a very fine discretisation was used corresponding to 100 standard beam elements. The damage shape function is assumed to be a piecewise cubic function of the normalised abscissa s and to depend on two damage parameters, i.e., position s_0 and width b as indicated in Fig. 1.

The inverse problem was studied by sampling the modal curvature, obtained from the modal displacement of the direct problem, in M equally spaced points. The damage shape function η_i evaluated by inversion of (7) by using $K = 3$ modes is shown in Figs. 2 and 3 for two damage cases together with the curvature variations Δ''_i. In the former, a broad damage ($s_0 = 0.29$ and $b = 0.35$) is considered and the results show that the filtering procedure drastically reduces the peaks in the modal curvature differences outside the damaged area and an accurate estimate of the modal damage shape η_i is achieved although with different accuracy for each mode. When very sharp damage, typical of crack, is considered, different situations may occur. In the first case (left) in Fig. 3 the damage is located between two sampling points of the modal curvature, meaning that in Eq. (6) all the contributions of the damage shape η_1 vanish and, in fact, the modal curvature difference gives the shape of the (first) modal curvature without any significant information on the actual damage position or shape. On the contrary, the inversion of (7) with $K = 3$ modes still gives a clear indication of the damage. The situation is completely different when at least one sampling point lies in the damaged region. In this case, as already pointed out in [14], both the modal curvature difference and the filtered quantity convey very accurate information on the damage region. Indeed, on this particular case are based the satisfactory results presented in [15].

For given location and extension of damage, and using the first two modes, Figure 4 illustrates the consequences of reducing the number of sampling points of the modal curvatures, mimicking the use of a reduced number of sensors. The results indicate that in all cases the unfiltered modal curvatures fail to provide a clear indication of the damage zone, even with 13 sampling points. On the opposite, the peak in the filtered quantities is always in close proximity of the damaged region independently on the number of sampling points. As expected, the error of the identification procedure increases by reducing the number of sampling points.

4. Conclusions

Curvature variation is a quantity that has gained interest in damage detection procedures for its local information content and its limited sensitivity to environmental effects. To clarify some aspects of damage identification based solely on the use of curvature change, we report here selected results obtained from a perturbative solution of the damaged Euler-Bernoulli beam equation of motion that relates in closed form the curvature difference before and after the damage and the position and shape of the damage itself. The solution reported expresses the modal curvature variations, as well as the weighted modal curvature shape, as a function of the measured discrete mode shapes. This would enable to investigate the effect of a coarse description of mode shapes, that is, the actual possibility to use modal curvature as observed quantity in experimental tests on real structures, when a limited number of measurement

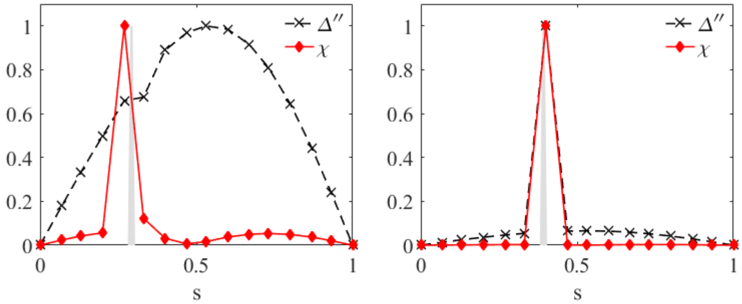

Fig. 3. Curvature variations Δ_1'' and modal damage shape η_1 obtained by Eq. (7) in the case of a sharp damage with width $b = 0.03$ located at $s_0 = 0.29$ (left) and $s_0 = 0.11$ (right). In both cases $M = 16$ sampling points were used.

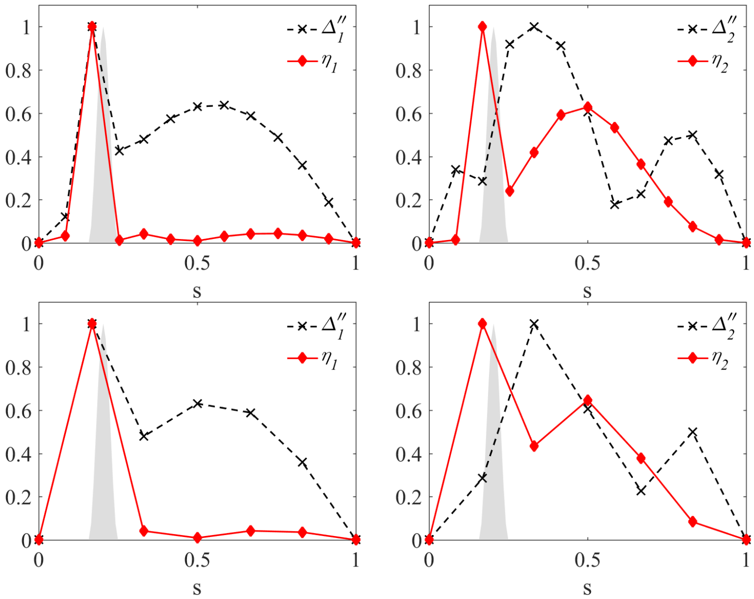

Fig. 4. Sensitivity on the number of sampling points of Δ_i'' and η_i applied to the first two modes ($i = 1, 2$) for a damage located at $s_0 = 0.2$ with width $b = 0.1$. (a)-(b) $M = 13$ sampling points; (c)-(d) $M = 7$ sampling points.

points is often available. A numerical example is reported to show that, when damage is broad, several peaks in the modal curvature outside the damage region may occur, also in the absence of numerical or experimental error, due to the contribution of higher modes in Eq. 5. This means that differently from the common perception, the curvature variations can not always give a reliable information on damage location and shape. We also show that, when damage is narrow, in the absence of a measurement point within the damage region, the modal curvature variation is again unable to provide meaningful results. On the contrary, the filtered modal curvature appears to be a fitting quantity for the localization of damage. It is further shown that the procedure presented is able to provide meaningful results also when using a limited number of sensors.

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References

[1] J. Ciambella, F. Vestroni, The use of modal curvatures for damage localization in beam-type structures, J. Sound Vib. 340 (2015) 126–137.

[2] C.R. Farrar, N.J. Lieven NJ, Damage prognosis: the future of structural health monitoring, Philos. Trans. R. Soc. A 365 (2007) 623–632.
[3] Y.F. Fan, J. Zhou, Z.Q. Hu, T. Zhu, Study on mechanical response of an old reinforced concrete arch bridge, Struct. Control Health. Monit. 14(6) (2007) 876–894.
[4] S. Jiang, F. Xu, C. Fu, Intelligent damage identification model of an arch bridge based on box-counting dimension and probabilistic neural network, J. Comput. Inf. Syst. 6(4) (2010) 1185–1192.
[5] J. F. Unger, A. Teughels, G. De Roeck, System Identification and Damage Detection of a Prestressed Concrete Beam, Journal of Structural Engineering, 132(11), 1691.
[6] M. Abdel Wahab, G. De Roeck, Damage Detection in Bridges Using Modal Curvatures: Application To a Real Damage Scenario. Journal of Sound and Vibration, 226(2), 217–235.
[7] F. Magalhães, A. Cunha, E. Caetano, Vibration based structural health monitoring of an arch bridge: from automated OMA to damage detection, Mech. Syst. Signal Process. 28 (2012) 212–228.
[8] F. Vestroni, D. Capecchi, Damage detection in beam structures based on frequency measurements, J. Eng. Mech. 126(7) (2000) 761–768.
[9] D. Montalvao, A Review of vibration-based structural health monitoring with special emphasis on composite materials, Shock. Vib. Dig. 38(4) (2006) 295–324.
[10] C. Papadimitriou, E. Ntotsios, D. Giagopoulos, S. Natsiavas, Variability of updated finite element models and their predictions consistent with vibration measurements, Struct. Control. Health Monit. 19(5) (2012) 630–654.
[11] V.V. Nguyen, U. Dackermann, J. Li, M.M. Alamdari, S. Mustapha, P. Runcie, L. Ye, Damage identification of a concrete arch beam based on frequency response functions and artificial neural networks, Electron. J. Struct. Eng. 14(1) (2015) 75–84.
[12] F. Vestroni, A. Pau, Dynamic characterization and damage identification, in: G.M.R. Gladwell G.M.R., A. Morassi (Eds.), Dynamic inverse problems: theory and application, CISM Courses and Lectures, n. 529, Springer-Verlag, Wien, 2011, pp. 151–178.
[13] D. Dessi, G. Camerlengo, Damage identification techniques via modal curvature analysis: overview and comparison, Mech. Syst. Signal Process. 52–53 (2015) 181–205.
[14] D. Capecchi, J. Ciambella, A. Pau, F. Vestroni, Damage identification in a parabolic arch by means of natural frequencies, modal shapes and curvatures, 51(11) Meccanica (2016) 2847–2859.
[15] A.K. Pandey, M. Biswas, M.M. Samman, Damage detection from changes in curvature mode shapes, J. Sound Vib. 145(2) (1991) 321–332.
[16] J. Ciambella, F. Vestroni, S. Vidoli, Damage observability, localization and assessment based on eigenfrequencies and eigenvectors curvatures, Smart. Struct. Syst. 8(2) (2011) 191–204.
[17] M. Chandrashekhar, R. Ganguli, Damage assessment of structures with uncertainty by using mode-shape curvatures and fuzzy logic, J. Sound Vib. 326(3–5) (2009) 939–957.
[18] M. Dilena, A. Morassi, Dynamic testing of a damaged bridge, Mech. Syst. Signal Process. 25(5) (2011) 1485–1507.
[19] S. He, L.R.F. Rose, C.H. Wang, A numerical study to quantify delamination damage of composite structures using inverse the method, Aust. J. MultiDiscip. Eng. 10(2) (2013) 145–153.
[20] M. Cao, M. Radzieński, W. Xu, W. Ostachowicz, Identification of multiple damage in beams based on robust curvature mode shapes, Mech. Syst. Signal Process. 46(2) (2014) 468–480.
[21] S. Rucevskis, R. Janeliukstis, P. Akishin, A. Chate, Mode shape-based damage detection in plate structure without baseline data, Struct. Control Health. Monit. 23 (2017) 1180–1193.
[22] M. Cao, W. Xu, W. Ostachowicz, Z. Su, Damage identification for beams in noisy conditions based on Teager energy operator-wavelet transform modal curvature, J. Sound Vib. 333(6) (2014) 1543–1553.
[23] M. Cao, P. Qiao, Novel Laplacian scheme and multiresolution modal curvatures for structural damage identification, Mech. Syst. Signal Process. 23(4) (2009) 1223–1242.
[24] Z.B. Yang, M. Radzieński, P. Kudela, W. Ostachowicz, Fourier spectral-based modal curvature analysis and its application to damage detection in beams, Mech. Syst. Signal Process. 84 (2017) 763–781.