Concept Maps Similarity Measures for Educational Applications

Carla Limongelli[1], Matteo Lombardi[2], Alessandro Marani[2], Filippo Sciarrone[1], and Marco Temperini[3]

[1] Engineering Department, Roma Tre University
Via della Vasca Navale, 79 - 00146 Roma, Italy
{limongel,sciarro}@ing.uniroma3.it
[2] School of Information and Communication Technology, Griffith University,
170 Kessels Road, Nathan, QLD, 4111 Australia
{matteo.lombardi,alessandro.marani}@griffithuni.edu.au
[3] Dept. of Computer, Control and Management Engineering, Sapienza University
Via Ariosto, 25 - 00184 Roma, Italy -
marte@dis.uniroma1.it

Abstract. Concept maps represent a significant tool in education, used to plan and guide learning activities and to help teachers in some endeavors such as analyzing and refining their teaching strategies, retrieving suitable learning material, and supporting the provision of adaptive guidance in adaptive learning environments. Here we propose seven measures of similarity among concept maps, representing course modules. They deal with both structural and didactic aspects of the maps, to find out educational similarities among their associated course modules. The performance of the proposed similarity measures are analyzed and evaluated by means of some significant case studies.

Keywords: Concept map similarity, Learning, Ontology

1 Introduction

A Concept Map (CM) is a well established means for organizing concepts and the relationships among them in a easy and useful visual way. It is used in various fields, such as knowledge management, information systems development [3, 6, 4], collaborative work [10, 9] and industrial fields [5]. A CM can be managed as either an ontology or a graph. In the literature the problem of computing the similarity among ontologies has been already addressed and many approaches have been suggested [11]. On the other hand, very few works consider the particular case of educational CMs. [2] proposes some ways to suggest the user for additional concepts and learning material during the creation of her CM. In [8] is addressed the matching of elements or parts among CMs, based on a similarity flooding algorithm, with the aim to support comparisons and merging of maps. This paper focuses on educational CMs, taking into account both structural aspects of the associated graphs and some didactic aspects, such as the prerequisite relationships and the commonality of concepts, which are of capital importance to state the educational similarity between two CMs. An evaluation of the suggested measures is conducted to check the following research question: *Given two CMs, do the proposed similarity measures capture the didactic aspects of concepts commonality and prerequisite relationships?*

Sec. 2 presents the proposed measures, and Sec. 3 reports a first evaluation of the measures, concluding in Sec. 4 with foreseen future works.

2 Similarity Measures

In the following we present some different and independent ways for comparing two CMs. Let CM be a CM represented by a Direct Acyclic Graph (DAG), where nodes and edges represent, respectively, concepts and "prerequisite" relationships among concepts. We define the set of common nodes CN between two CMs CM_1 and CM_2 as follows: $CN = \{CM_1 \cap CM_2\}$. The *distance between nodes*, $\delta(c_1, c_2, CM)$, given a concept map CM and two nodes $(c_1, c_2) \in CM$, is defined as *the length of the shortest path from* c_1 to c_2 (or ∞, if there is no path). Moreover, the *Predecessor* of a node c in a concept map CM is defined as: $Preds(c, CM) = \{\forall c_i \in CM$ such that there exists a path from c_i to $c\}$.

Overlapping Degree (OD). This measure analyses if there is a significant number of CN (note that $|CN|$ is the *cardinality* of the set) (1) and how such nodes are placed in the maps (2). The following formula expresses how significant is the set of common nodes:

$$a = \frac{|CN|}{min(|CM_1|, |CM_2|)} \in [0 \ldots 1] \tag{1}$$

Then, adjacency matrices of the common nodes are built for both maps.The elements of these matrices are $\delta(c_i, c_j, CM)$ for each pair of nodes $c_i, c_j \in CN$. The formula (2) computes the cosine similarity of the vectors of the two matrices, allowing to determine the similarity of the arrangement of the CN in the two maps.

$$b = CosineSimilarity(\overrightarrow{Adj}_{(CN,CM_1)}, \overrightarrow{Adj}_{(CN,CM_2)}) \tag{2}$$

where $\overrightarrow{Adj}_{(CN,CM_i)}$ is the vector obtained from the linearization of the adjacency matrix of the common nodes in CM_i. The criteria pursued in (1) and (2) are then unified in (3):

$$\text{OD} = \frac{a + b}{2} \cdot \alpha, \quad with\ \alpha = \frac{2|CM_1 \cap CM_2|}{|CM_1| + |CM_2|} \tag{3}$$

Basically, the higher is the OD, the more similar and important is the arrangement of the common nodes in the maps. In other words, the same common nodes could be placed as a common subgraph of the two maps (higher similarity) or just be differently scattered in the maps (lower).

Prerequisites Constraints Measure (PCM). This measure determines the shared predecessors $Preds$ of CN in the two maps. Given a concept $k \in CN$, let P_1 and P_2 be respectively $Preds(k, CM_1) \cup k$ and $Preds(k, CM_2) \cup k$. The PCM is the sum of the following three elements:

$$a_k = \frac{|P_1 \cap P_2|}{|P_1 \cup P_2|}, \quad b_k = \frac{|CN \cap (P_1 \cup P_2)|}{|P_1 \cup P_2|}, \quad c_k = \frac{min\{|P_1|, |P_2|\}}{max\{|P_1|, |P_2|\}}$$

a_k is the ratio of common predecessors on the total number of predecessors.
b_k is the ratio of the number of predecessors in CN (they may not be common predecessors) on the total number of predecessors.
c_k says the similarity of the amount of knowledge required by k in the two maps.

Given the three aforementioned elements, PCM is stated as follows:

$$\text{PCM} = \frac{1}{|CN|} \sum_{\forall k \in CN} \frac{a_k + b_k + c_k}{3} \tag{4}$$

In summary, this measure analyses the required knowledge for the CN shared in the two maps.
Topological Similarity Measure (TSM). TSM combines the purely structural measure given in (3) with the semantic information given in (4). Concepts might be differently scattered in the maps, so considering only their co-occurrence in the maps might be not enough. On the other hand, the structural information provided by OD can be an improvement to the PCM, so the following definition (5) tries to express a level of integration between the two previous measures:

$$\text{TSM} = \frac{\text{OD} + \text{PCM}}{3} \cdot \alpha \tag{5}$$

where α is given in (3).
Flux-Based Similarity Measure (FBSM). By *flux* we mean a property of a node of the CM representing how much information is passing through it. The higher the *flux* of a node, the more "important" is the associated concept in the map. FBSM computes the similarity of importance of concepts in the two maps, expressed by the accumulated *flux* $\varphi(c, CM)$ of the associated map nodes. The computation of FBSM is based on the spread activation technique [1]. In particular, let $|CM_1| < |CM_2|$ and let $c \in CM_1$, when c is *activated* it receives *flux* equal to 1 in CM_1. If $c \in CM_2$, c is activated in the second map too. In general, when a node receives *flux*, it retains at most an amount T ($= 0.3$ in our case) that is added to its total flux: $\varphi(c, CM) = \varphi(c, CM) + T$. If there is any exceeding flux (which is $flux - T$), such *flux* is spread to the child nodes evenly. So, a concept may receive *flux* from its own activation or from the predecessors. FBSM computes the sum of flux differences of the concepts in CM_1 in the two maps. If $c \notin CM_2$, then $\varphi(c, CM_2)$ is equal to 0.

$$\text{FBSM} = 1 - \frac{\sum_{c \in CM_1} abs(\varphi(c, CM_1) - \varphi(c, CM_2))}{|CM_1|} \tag{6}$$

Flux-Based Similarity Measure on Common Nodes (FBSM-CN). In this case, the same spread activation algorithm of measure (6) is used, but only the flux on CN is considered. This measure results in high scores if common nodes are similarly distributed in the two maps. Given the two vectors of the flux on $c_1, \ldots, c_i \in CN$ in CM_1 and CM_2, $\overrightarrow{V_1} = \langle\varphi(c_1, CM_1), \ldots \varphi(c_i, CM_1)\rangle$, and $\overrightarrow{V_2} = \langle\varphi(c_1, CM_2), \ldots \varphi(c_i, CM_2)\rangle$ respectively, FBSM-CN is computed as follows:

$$\text{FBSM-CN} = CosineSimilarity(\overrightarrow{V_1}, \overrightarrow{V_2}) \tag{7}$$

Comprehensive Flux-Based Similarity Measure (C-FBSM). This measure combines the two previous flux-based measures given in (6) and (7):

$$\text{C-FBSM} = \frac{\text{FBSM} + \text{FBSM-CN}}{2} \tag{8}$$

Comprehensive Similarity Measure (C-SM). This measure is a linear combination of the Topological Similarity Measure (5) and the Flux-Based one (8):

$$\text{C-SM} = \text{TSM} \cdot (1 - \beta) + \text{C-FBSM} \cdot \beta \tag{9}$$

Where

$$\beta = \left(\frac{\sum_{c \in CM_1} outgoingArcs(C, CM_1)}{|CM_1| - |sinks(CM_1)|} + \frac{\sum_{c \in CM_2} outgoingArcs(C, CM_2)}{|CM_2| - |sinks(CM_2)|} \right) \cdot \frac{1}{2 \cdot N}$$

here, $N \in [7..10]$ is a parameter of the algorithm, and *sinks* denotes the nodes having no successors. In practice, β is expected to express the significance of the flux-based measures according to the structure of the concept maps: the more the concept maps are linear or sequential, the less flux-based measures are expressive.

3 Evaluation

This section presents an evaluation of the similarity measures presented in Sec. 2 and Fig. 1 shows the sample of CMs used for this goal. The sample is composed by a set of five CMs which includes the seed ontology CM_0 and its progressive variations; CM_0 will be compared Vs. all the others, including itself. The rationale is to show the behavior of the proposed measures for different variations of the seed ontology CM_0, as suggested by ontology matching literature [11].
Here we discuss the five comparison cases, whose results are reported in Tab. 1:
Evaluation I: CM_0 Vs. CM_0. This is the comparison between two identical maps, so all the similarity measures must be equal to 1, (cfr. Tab. 1).
Evaluation II: CM_0 Vs. CM_2. This is the case where two CMs differ for a concept only, namely concept C. Not surprisingly, all the measures report a lower similarity than the previous case (refer to Tab. 1) but with different trends. FBSM-CN falls very slightly from 1 to 0.998, whereas TSM is the most sensible falling to 0.853. The other measures are in between.

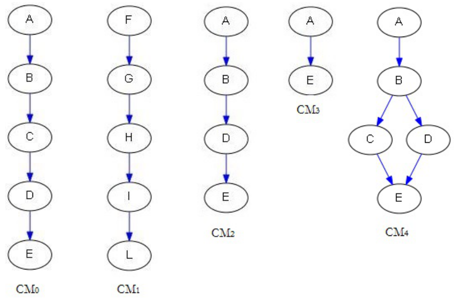

Fig. 1. The sample of CMs. CM_0 represents the seed CM.

Evaluation III. CM_0 Vs. CM_3. As expected, the similarity measures still decrease because CM_3 is a very small subset of CM_0; it consists of only the source concept A and the target concept E of CM_0. All the similarity measures capture such situation, especially the flux based measures with the highest similarity values. This happens because A has the same amount of flux and E is a sink in both maps.
Evaluation IV. CM_0 Vs. CM_4. This is the case where the FBSM-CN similarity presents the highest value with respect to the other measures: almost 1. This is because the Flux-Based measure captures a similar knowledge dissemination on concepts C and D in both maps; all the other measures increase report a more didactic similarity.
Evaluation V: CM_0 Vs. CM_1. The two CMs are formed by all different concepts. Consequently, all the measures return a similarity score equal to 0.

In all the evaluation cases, we notice that the similarity measures were able to capture both topological and educational aspects (common concepts and prerequisites relationships) shared by a pair of CMs.

Eval.	CMs	OD	PCM	TSM	FBSM	FBSM-CN	C-FBSM	C-SM
I	(CM_0, CM_0)	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**
II	(CM_0, CM_2)	0.889	0.888	0.853	0.975	0.998	0.986	0.899
III	(CM_0, CM_3)	0.571	0.700	0.514	0.750	0.886	0.818	0.615
IV	(CM_0, CM_4)	0.976	0.967	0.973	0.680	0.958	0.819	0.890
V	(CM_0, CM_1)	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**

Table 1. Results of Evaluations I-V using the similarity measures presented in Sec. 2.

4 Conclusions

In this paper we addressed the problem of measuring the similarity among educational CMs. Seven similarity measures have been presented and evaluated in order to test their capability to capture both topological and educational differences between two concept maps. The evaluation shows that the research question is strengthened: all the measures are able to capture both topological and educational aspects. As a future work we plan to strengthen the evaluations of all the measures with a larger set of CMs involving teachers to asses their validity. Finally, the proposed measures would significantly benefit of tools for domain-based retrieval of synonyms, like SynFinder [7] or Word2Vec[4] for a more appropriate identification of common nodes of two CMs.

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