Five Essays on Neoclassical and Neo-Schumpeterian Growth Theory.

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[1] **This paper is co-authored with G.Cozzi and P. Giordani.**

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Introduction

One of the most peculiar features of the early neoclassical growth theory was the exogeneity of the growth rate. The application of the principle of substitution among factors of production to economic growth makes incentives to capital accumulation necessarily vanish as the aggregate production function exhibits decreasing returns to capital. As a result, the economy is doomed to a steady state where the capital-labor ratio is constant and per-capita output is only able to grow by relying on technical progress.

Technical progress, in fact, may be conceived of as a third factor of production of the aggregate production function, but such an assumption is not without cost. The logic of the 'replication argument' requires that the production function is constant returns in capital and labor: given the state of technology, output can always be doubled by reproducing the existing production process. The introduction of a third factor yields increasing returns. If this factor has to be endogenous to the system it must be the outcome of economic agents' decisions and, accordingly, must be rewarded. Such remuneration, though, is at odds with the neoclassical theory of distribution. As, under competitive conditions, factors of productions are paid according to their marginal productivities then the whole product is not sufficient to pay its cost of production; indeed, as is well known, under increasing returns to scale the Euler theorem does not hold. This difficulty forced technical change to be an ‘exogenous’ factor of production for prolonged time. The development of neoclassical growth theory can actually be interpreted as an attempt to make technical change and the steady state growth rate endogenous to the economic system.

Basically two routes have been pursued. The first one derived endogenous growth rates by setting a lower bound to returns to capital; by doing this, the incentive to capital accumulation never ends and the capital-labor ratio never settles down to a steady state value. The so-called ‘AK’ models are the main example of this way out of the exogenous growth. The second route made technical change endogenously determined as the outcome of rewarded R&D activities; along this way perfect competition must be abandoned to make the distributional side of the economy fit. This class of models is usually referred to as ‘Neo-Schumpeterian growth’.

My dissertation mainly focuses on this second stream of research.

The plan of the book is as follows.

In the first essay, ‘Exogenous and Endogenous Growth in a Historical Perspective’, I propose a survey and an interpretation of the structure and of the basic results of neoclassical (exogenous and endogenous) and neo-schumpeterian growth.

The second essay, 'An Uncertainty-Based Explanation of Symmetric Growth in Schumpeterian Growth Models', provides a re-foundation of the symmetric equilibrium characterizing the research sector of most of the R&D driven growth models. This result is usually derived by assuming symmetric expectations on future per-sector R&D expenditure. On the contrary, it is assumed that future per-sector distribution of R&D efforts is characterized by non-probabilistic uncertainty. By adopting the multi-prior expected utility theory axiomatized by Gilboa and Schmeidler (1989) I and my co-authors prove that the symmetric structure of R&D investment is uniquely compatible with uncertainty-averse agents adopting a maxminimizing strategy.

The third paper,'A Quality Ladder Growth Model with Asymmetric Fundamentals', generalizes the standard quality ladder growth model as proposed by Grossman and Helpman (1991) to encompass cases where the economy's fundamentals are asymmetric. In particular, asymmetry is introduced in the research technology and in the utility weights and cost of production of the lines of production. It is shown that the basic results of the symmetric case remain unaffected.

In the fourth paper, 'Uncertainty-Averse Agents in a Quality Ladder Growth Model with Asymmetric Fundamentals', I derived the equilibrium in an economy with asymmetric fundamentals and where the agents are subject to non-probabilistic uncertainty. Agents are assumed to adopt a maxminimizing strategy to face such an uncertainty.

In the fifth essay, 'Time-Varying Elasticity of Substitution and Economic

Growth', I show that in a standard neoclassical growth model framework the path followed by the elasticity between capital and labor must be a non decreasing one; I suggest considering this evolution as a possible source of economic growth.

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Exogenous and Endogenous Growth in a Historical Perspective

0.1 Introduction

The so-called 'marginalist revolution' took place around the 70's of the XIXth century. It produced a theory of the level and distribution of output based on the endowments of production factors, technology and consumer preferences. In such a theory, economic growth had to be conceived of as the result of the increase in the factors' endowments. Early marginalist analyses of economic growth have been provided by Marshall, Cassel and Wicksell. However, we start our survey from Solow's (1956, 1957) formulation of the neoclassical growth model because it later became the basic point of reference of any discussion on neoclassical exogenous and endogenous growth.

In the lecture held for the receiving of the 1987 Nobel Prize for Economics, Robert Solow (1988) reconstructs the development of his research work on growth theory. He remembers how his work started by analyzing the Keynesian growth models by Harrod (1939) and Domar (1946) and by attempting to solve their critical points.

These economists had found the conditions under which the economy can develop along a balanced growth path, when utilizing its productive capacity in a normal way and by employing all the existing labor force. First, these conditions require that investments be equal to savings corresponding to the normal productive capacity utilization, so that:

$I = sY^*$ (Y^* being the normal productive capacity income)

and

$I/K = s/v = g_w$where v is the capital-income ratio and g_w is the so-called warranted growth rate. The second requisite is that the warranted growth rate equals the growth rate of population (the natural growth rate) n. These two growth rates are not related in any definite way, and nothing guarantees that the autonomous investment decisions allow the economy to grow at the warranted growth rate. On the contrary, the growth path corresponding to g_w is, at least in Harrod's opinion, unstable (the so-called 'knife edge') so that the economy cannot grow along it. This kind of instability, which was perceived as highly unsatisfactory since it was not observable in the actual capitalistic economies, has been eliminated by Solow by introducing the neoclassical principle of substitution among factors of production. This principle assures both that investments adjust to the savings corresponding to the full employment of capacity, so that the economy grows at the warranted growth rate, and that the capital-income coefficient v varies until the warranted growth rate adjusts to the natural one so that the economy employs the whole labor supply.

It seems however, that this important advancement in growth theory is responsible for the exogeneity[2] of the long run growth rate, so that the 'new' endogenous growth theories were born to correct this undesired consequence of the principle of substitution and of the related decreasing marginal factors productivity.

0.2 The Exogenous Growth

We briefly illustrate the main characteristics of the exogenous growth model.

0.2.1 The macro model

The represented economy is perfectly competitive. The factors of production, labor and capital, are continuously fully employed and their rewards coincide with their marginal productivity. In this economy there is a single good, which can either be consumed or saved[3]. Population (L) evolves according to the exogenous exponential dynamics:

$L(t) = L(0)e^{nt}$

[2] By exogeneity of the growth rate we mean its independence of agents' preferences. In the opposite case we will speak of endogeneity.

[3] The one-sector hypothesis prevents the model from being affected by the 'capital critique'.

The production function $Y = F(K, L)$[4] exhibits constant returns to scale, is well behaved[5] and satisfies the Inada conditions[6]. By the constant return hypothesis:

$Y/L = F(K/L, 1)$ the product per unit of labor (y) is a function of the sole capital per unit of labor (k), $y = f(k)$.

Savings (S) are assumed to be a constant given share (s) of income, so that $S = sY$

and $S/L = sy = sf(k)$, and by the adjustment hypothesis of investments to saving: $sf(k) = I/L$.

The dynamics of the economy are given by the pace at which capital accumulates:

$K = \dot{I} - \delta k$ (where δ is the depreciation rate of capital), so that:

$sf(k) - \delta k = \dot{K}/L$.

By deriving the identity $k = K/L$ with respect to time, we obtain:

$\dot{K}/L = \dot{k} + nk$

so that

$\dot{k} = sf(k) - (n + \delta)k$.

Capital per worker will grow until $sf(k) > (n + \delta)$ (i.e. $\dot{k} > 0$). The Inada conditions assure that for relatively low levels of k this inequality holds. The

[4] Unless strictly reuired, I drop in what follos the time argment for notational simplicity.

[5] The function is continuous and twice differentiable with $\partial F/\partial K > 0$, $\partial F/\partial L > 0$, $\partial^2 F/\partial K^2 < 0$, $\partial^2 F/\partial L^2 < 0$.

[6] The Inada conditions require that $\lim\limits_{t\to\infty} f'(k) = 0$ and $\lim\limits_{t\to 0} f'(k) = \infty$. These conditions were introduced by Inada (1963) to assure the existence, stability and uniqueness of a stationary growth path. Indeed, in Solow (1956, pp. 77-8) the case of sustained growth is analyzed by assuming production functions which violate these conditions.

same conditions guarantee the existence, uniqueness and stability of the steady-state value k^* . The economy evolves towards $k^*$where $sf(k^*) = (n+\delta)k^*$and $\dot{k} = 0$. In the steady state, the capital K and the labor L grow at the same rate n. Also the product, given the constant return hypothesis, grows at the same rate. Therefore the growth rate of per-capita income is null independently of saving propensity (s), which represents agents' preferences.

This result follows from the variability of the capital-output ratio and the tendency towards full employment of all factors. To catch the underlying intuition, we can look at the growth rate of the economy as the result of the growth rates of capital (s/v) and labour (n). If the economy grows at a rate higher than n, then s/v needs to be higher than n. Capital will be growing faster than labor and the capital-labor ratio is also increasing. Given the decreasing marginal productivity of factors the capital output ratio is also increasing. This means that s/v is decreasing until it adjusts to n.

The only chance to have sustained per capita growth in the long run relies therefore on technical progress. This is represented in the neoclassical theory as a third factor of production (A), so that the production function becomes:

$Y = F(K, L, A)$.

Technical progress can enter the functional form of the production function in various ways, but the only one which allow for the existence of a steady state is the labor augmenting one, which characterizes progress as an increase in the efficiency of the labor factor:

$Y = F(K, AL)$.

The reason why growth is indeed named exogenous is that technical progress is modeled as an original factor of production; analogously to the labor force it is assumed to grow exponentially over time ($A(t) = A(0)e^{\lambda t}$) at a constant rate (λ). It simply occurs in the economy without being produced by any agent whose preferences are therefore irrelevant for its evolution. We discuss later how the neoclassical theory tried to amend this unsatisfactory treatment of technical change.

0.2.2 Convergence

The transitional dynamics of the system show an important implication of the model. Given the rate of technological progress, the growth rate of per-capita capital tends towards the equilibrium value with a speed of convergence inversely proportional to the level of the initial stock of capital. Indeed:

$$\frac{\partial(\dot{k}/k)}{\partial k} = s\left(\frac{f'(k)}{k} - \frac{f(k)}{k^2}\right) = \frac{s}{k^2}\left(f'k)k - f(k)\right) < 0$$

being the quantity in square brackets negative, because it is equal to the opposite of the marginal productivity of labor.

This result is the basis of the convergence hypothesis according to which poorer economies should grow faster than richer ones, and should finally reach them at the steady state value of the per-capita growth rate. However, this conclusion holds only if the different economies have the same steady state. This requires that the parameters which determine the steady state (the propensity to save, the technology, population and depreciation growth rates) must be equal for all the economies. If they differ have different steady state levels of per-capita income and the convergence proposition does not hold. So, the Solow model only implies convergence conditional to the steady state, not absolute convergence (the simple fact that poorer countries grow faster than richer ones).

Thanks to the data set becoming available in the eighties, the convergence hypothesis has been tested both in the absolute and in the conditional versions. Baumol (1986) found evidence to support the absolute convergence hypothesis for the period 1870-1979. The sample he used however, based on Maddison (1982), only considers the sixteen countries which have successfully completed the process of industrialization; it is therefore biased. De Long (1988) in fact showed that the hypothesis is rejected once the sample is completed with those countries which were relatively rich in 1870 but failed to industrialize. Tests on the complete sample of the Summer and Heston (1993) data set for the post-war period have led to the refutation of the absolute convergence hypothesis (cfr. Mankiew, Romer and Weil (MRW, 1992) or Barro and Sala-i-Martin (1995)). Therefore, attention turned to verifying convergence in its conditional form. MRW find evidence of conditional convergence, but they add the saving rate of human capital to the determinants of the steady state. Evidence in favor of conditional convergence also emerges in Barro (1997) and Barro and Sala-i-Martin (1995). Here however, regressions are conditioned to several economic indicators whose role in the growth process remains theoretically unexplained. Therefore, the convergence hypothesis remains doubtful from an empirical point of view[7].

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[7] It can be interesting to note that in Barro (1997, Chap. 1, Tables 1.1 p.13 and 1.3

0.2.3 The micro model

We saw how in the Solow model the propensity to save is taken as a given. Cass (1965) and Koopmans (1965), by refining a seminal paper by Ramsey (1928), provide a micro-foundation of the neoclassical growth model.

The economy is characterized by a finite number of identical infinitely-lived households which supply labor to the firms and receive wages as reward. This reward may be either consumed or saved by purchasing assets on which an income interest is received. Savings define the dynamics of the accumulation of capital. The dynamics of the economy are defined by the maximization problem of the representative household. The household maximizes his discounted intertemporal utility over an infinite time horizon:

$$\underset{c_t}{Max}\left[U = \int_0^\infty e^{-\rho t} u(c_t) e^{nt}\right]$$

where c is consumption, ρ is the rate of time preference and n is the rate of growth of the family size. The maximization is subject to two conditions:

First, it must satisfy the budget constraint:

$$\dot{a} = w + ra - c - na$$

where w is the wage rate, a is the amount of per-capita financial asset and r is the interest rate. Secondly, a no-Ponzi game condition has to be satisfied in order to avoid the possibility that households finance for ever their consumption

pp. 34) conditional convergence is tested by means of two different dependent variables: per-capita growth rate and investment rate. Only the former shows a significant negative relation with the initial level of per-capita income. If this is the case the convergence effect should not be ascribed to the higher capital accumulation of backward countries as implied by the Solow model. Dowrick and Nguyen for example find the convergence effect even when controlling for capital accumulation and labor force participation; they attribute it to catch up in Total Factor Productivity. Analogously Abramovitz (1986) suggests that convergence can be basically driven by the adoption of the technological frontier.

simply by borrowing money:

$$\lim_{t\to\infty}\left(a(t)e^{-\int_0^t (r(u)-n)du}\right) \geq 0$$

which states that the discounted value of assets must be asymptotically non-negative.

The household's maximization problem has to be coupled with the firm's one to determine the economy's path of consumption and accumulation. The representative firm maximizes its profits, that is the difference between the value of its production and the payments to rent labor and capital services:

$\Pi = F(K, L) - wL - (r + \delta)K$[8].

The solution of these two problems defines a system of two differential equations, which describes the optimal path of per-capita capital and consumption. Solving for $\dot{k} = 0$ and $\dot{c} = 0$ provides the steady state pair of values (c^*, k^*), which turns out to be a saddle point. If the economic agents have perfect foresight, this saddle point will be the actual point the economy tends to.

Therefore, in equilibrium the per-capita growth rate of consumption, capital and income are zero. The only possibility for sustained growth derives from the existence of exogenous technical progress. We are back to Solow's conclusions.

To sum up, we can say that the Solow model appeared unsatisfactory both from an empirical and a theoretical point of view. First, apart from the recent debate on the convergence hypothesis, it was soon clear that the model was unable to account for some of the 'stylized fact' regarding growth (Kaldor, 1961). Second, the only way through which it could generate sustained per-capita

[8] We did not consider the intertemporal problem because if there are no adjustment costs maximizing the present value of profits coincides with maximizing profits at any point in time.

growth was to assume exogenous technical progress. The technical progress is described as a factor of production on which the theory has nothing to say. It is not rewarded nor provided by any economic agent[9]. It is a public good, because it is non-rival and non-excludable, but there is no explanation for its production: it appears in the economy like 'manna from heaven'[10]. This seemingly unsatisfactory[11] characteristic of the technical progress is strictly linked to the structure of the neoclassical theory. From a logical point of view, the production function must be linearly homogeneous in labor and capital (the rival inputs), because of the so-called 'replication argument'. The introduction of a third factor (Romer 1990b) engenders increasing returns[12] . If this factor has to be the outcome of economic agents' decisions, it must be rewarded. But the neoclassical theory of distribution cannot account for this remuneration: under increasing return to scale the Euler theorem does not hold and the product is not sufficient to pay the factors of production according to their marginal productivity.

In what follows, we analyze how the marginalist theory of growth made the technological progress endogenous while maintaining its theory of distribution. However, it is worth noting here that it is assumed that capital and technical

[9]Shell (1966, 1973) provided models where innovation is a good produced in the economy. However he derives an optimal steady state level of A which in turn is not capable of engendering sustained growth.

[10]Vintage capital models (see for example Solow 1960) do not solve this problem. They require capital accumulation for the existence of technical progress but they do not explain who introduces innovations.

[11]'..exogenous theories of technical change are essentially confession of ignorance..' (Shell, 1973, p.77).

[12]If R is the set of rival inputs, N a non-rival input and $F(R, N)$ the production function, we have: $F(\mu R, N) = \mu F(R, N)$, so that $F(\mu R, \mu N) > \mu F(R, N)$ for $\mu > 1$.

progress are two distinct and independent factors. Kaldor (1957) notes that technical progress is embodied in new capital goods, so that the distinction between movements along the production function due to an increase in the per-capita capital and the shifts of the production function deriving from an increase in the factor 'technical progress' is an illegitimate one[13]. He proposes a 'technical progress function', which relates the growth rate of per-capita capital to that of the per-capita output. This function incorporates both the improvements due to the technical change and those due to the higher intensity of capital.

0.3 The First Generation of Endogenous Growth Models

The basic way through which the new growth theories make the growth rate endogenous is by setting a lower bound to the decrease of the marginal productivity of capital. Indeed, in the Solow model the growth rate necessarily relies on exogenous technical progress because the marginal product of capital tends to zero and the incentive to save necessarily comes to an end. There are basically three ways of stopping the decrease of the marginal product of capital: the introduction of a production externality which offsets the decrease of

[13] Indeed Solow (1957) shows that if the factors of production are paid according to their marginal productivities, it is possible to distinguish changes in per-capita output from those due to technical change to those attributable to a change in the capital labor ratio. However in a following paper Solow (1960, p.90) observes: 'Improvements in technology affect output only to the extent that they are carried into practice either by net capital formation or by replacement of old-fashioned equipment..'. Accordingly he develops a model (vintage model) where technical progress occurs only when investments take place. The conclusions of exogeneity of the rate of growth are however not affected.

the marginal productivity of the private capital, the introduction of the 'human capital' factor which assures that the physical capital never becomes abundant and, eliminating the factor through which the capital would become abundant, i.e. the labor factor.

0.3.1 The externality

Arrow (1962) has been the first to suggest that the technological progress can be due to a process of learning by doing, and that the total stock of capital can be considered as a measure of it. Basically as accumulation goes on, the technological level of the economy increases. The problem of factors reward under increasing returns to scale is solved because firms produce under constant return to scale and they reward the factors of production according to their marginal productivity. At the same time, the accumulation of capital causes an externality on the technological level and therefore increases returns to scale. The production function of the single firm can be represented as:

$Y_i = A(K)F(K_i, L_i)$ where $K = \sum_i K_i$

In this way, the level of technology is taken as a given in the individual choices of production, but it is endogenous because it depends on the level of accumulation and eventually on the propensity to save. However, the Arrow model was developed in the case of a fixed capital-labor ratio and this implied that in the long run the output growth was limited by the growth in population. The output growth was therefore still independent of savings behavior.

The same idea has been made popular by Romer (1986). In an optimal growth framework, Romer introduces an externality into the production function due to the production of 'knowledge', i.e. the technological level. Knowledge (R) is produced by devoting resources to research. The externality arises because of the non-rival nature of the knowledge good. Therefore the single firm production function, when assuming a Cobb-Douglas case, is:

$Y_i = AK_i^{\alpha} L_i^{\beta} R_i^{1-\alpha-\beta} R^{\gamma}$ where $R = R_i$

If $\gamma = \alpha + \beta$

the aggregate production function becomes

$Y = \sum_i Y_i = AK^{\alpha} L^{\beta} R$

The production function exhibits constant return to scale in R[14] . Since in these kinds of models R is commonly thought of as a sort of capital, we have derived a so-called AK model. Its main characteristic is the constant marginal productivity of capital and the rate of growth is given by the saved fraction of the net marginal product of capital, in our case:

$g = s(AK^{\alpha} L^{\beta} - \delta)$.

It must be made clear that the hypothesis $\gamma = \alpha + \beta$, is essential for the result of the endogeneity of the steady state. The mere existence of the externality, $\gamma > 0$, i.e. increasing returns, is not sufficient to generate endogenous growth. From this point of view these kinds of models do not seem robust.

An analogous result had already been put forward by Frankel (1961). He

[14] Anyhow, as Romer pointed out later (1994, p.15), the assumption of a similar production function is not legitimate. Indeed, according to the 'replication argument' the production function need be constant return to scale in the sole rival inputs, labor and capital. Adding a third factor of production should render the production function increasing return to scale. This conclusion has been avoided for the difficulty to handle a similar case as regards the distribution side. The solution suggested by Romer (1987, 1990) will be the introduction of imperfect competition in research growth models.

wanted to combine two different types of aggregate production functions: the Cobb-Douglas to account for the theory of distribution, and a linear production function a la Harrod ($Y = aK$) to re-establish the role of capital accumulation as the engine of growth. The way he achieved this result is exactly the same way as Romer did. He introduced an externality of production (E), which is a function of the capital-labor ratio. The production function becomes $Y = AK^{\alpha}L^{1-\alpha}E$, if $E = (K/L)^{1-\alpha}$ then $Y = AK$. It appears then that the novelty of the 'seminal' 1986 paper by Romer consisted only of focusing on the concept of knowledge instead of capital as the factor of production generating the externality. The scarce hype attributed to Frankel's contribution can probably be attributed to its relying on the 'knife edge' assumption of externality. Apparently this was not sufficient reason to prevent Romer's paper from becoming a revolutionary one.

0.3.2 The human capital.

Human capital (H) constitutes the second line of research along which endogenous growth theories have developed. Lucas (1988) has completed and made popular the Uzawa (1965) human capital growth model. Lucas agrees with Romer on the point of differences in productivity levels among countries being due to differences in knowledge. But the source of knowledge is singled out more in the accumulation of human capital than in research.

The introduction of human capital in the model requires the definition of three different elements: the contribution of human capital to production, the allocation of time between labor and the accumulation of H, the link between the time devoted to accumulation of H and its rate of growth.

The production function is:

$$Y = AK^{\beta}\left[uhL\right]^{1-\beta} h_a^{\gamma}$$

Where u is the share of time devoted to work, h the level of human capital of the individual worker and N the number of workers. The last term h_a^{γ} represents the externality due to the average level of human capital.

The time, $1-u$, devoted to the accumulation of H is related to its growth rate by the differential equation:

$$\dot{h} = \delta h^{\zeta}(1-u)$$

Lucas stresses the necessity to rule out the case $\zeta < 1$, otherwise the model is unable to yield sustained growth. The case $\zeta > 1$ yields explosive growth, therefore he assumes $\zeta = 1$: the production of H is linear in the level of human capital and is homogeneous of degree two in the two factors H and $(1-u)$. In this case the model generates sustained growth. It is worth noting that the existence of the positive externality is unnecessary in order to obtain sustained growth; the key assumption is the 'knife edge' condition $\zeta = 1$[15]. It can be noticed that the human capital factor enters the production function in exactly the same way as the labor augmenting technical progress. It can produce sustained growth because it is assumed to grow exponentially ($\zeta = 1$); it yields endogenous growth because the amount of accumulated human capital is made dependent on the agents' preferences between working and accumulating H.

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[15] It must be stressed however that the model, once the agents are allowed to consume 'leisure', becomes unable to generate sustained growth (Solow, 1992).

Sustained growth with human capital had also been obtained by Uzawa. Lucas (1988, pp. 19-20) acknowledges his contribution but stresses that Uzawa only described a path of optimal accumulation without obtaining it as the outcome equilibrium of the economy.

0.3.3 Constant returns to capital

This class of models is characterized by the same AK production function we have seen in Romer and Frankel. The rationale in obtaining it is slightly different. It is assumed that all the factors of production are reproducible, i.e. they are all capital of some kind. As the factors of production are all producible, the capital never becomes 'abundant' and its marginal productivity and the rate of profit never fall to zero. For example in Rebelo (1991), output is produced by physical and human capital. Both kinds of capital are produced by means of a constant returns technology, which uses the same composite capital as input. Necessarily, the final good is also produced under constant returns and the assumption of a unique method of production makes the profit rate depend only on technology. If

$Y = AK$

then

$r = A$

and the rate of growth is

$g = sA = sr$ (or $g = \dfrac{r-\rho}{\sigma}$ when assuming the maximizing representative agent with ρ being the rate of time preferences and $1/\sigma$ the intertemporal elasticity of substitution of consumption).

0.4 The Imperfect Competition

Up to this point, the analyzed models of endogenous growth rely either on technical progress arising as a by-product of capital accumulation, or on the introduction of a second reproducible factor of production (human capital). The missing point is the analysis of the technical progress as the outcome of intentional choices of profit (or utility) maximizing agents. The prolonged absence of this analysis, which contrasts with the evidence of firms' research and development expenses, can be explained by the difficulty of reconciling increasing returns and perfect competition. It follows from the 'replication argument' that the production function needs to be constant returns in labour and capital. If this is the case, and firms are price-takers, the reward of the two factors exhausts the whole product and there is no scope for the reward of the research factor. It is therefore clear that the introduction of technical progress as the outcome of intentional choices implied the abandonment of perfect competition and the introduction of firms' market power. This is exactly the route followed by Romer in his 87 and 90 models. These models are based on the idea that growth is sustained by the increased specialization of labor across increasing varieties of lines of production. The existence of increasing returns is due to the increasing number of intermediate goods used to produce the final good. The production function is:

$Y = L^{\alpha} \int_0^A x_i^{(1-\alpha)} di$

where x_i is the quantity of the $i-th$ intermediate good and $[0, A]$ is the interval on which the set of intermediate goods is measured. The production set of the intermediate goods is non-convex. Indeed, a fixed cost arising from acquiring or producing a new idea must be paid for producing a new good. The intermediate sector is monopolistic since any firm is the exclusive beneficiary of an idea. Therefore, any firm chooses the value of x by equating its marginal revenue, derived from taking the marginal product of the good as its demand price, to the marginal cost derived from a technology which uses only capital. If any unit of the intermediate good is produced by means of η units of capital (so that $x_i \Longleftarrow \eta x_i$), we find a symmetric equilibrium with $x_i =\underline{\mathrm{x}}$ and $K = A\underline{\mathrm{x}}$. Plugging the expression for K in the production function:

$Y = L^{\alpha} \int_0^A \left(\dfrac{K}{A\eta}\right)^{1-\alpha} di = L^{\alpha} K^{1-\alpha} \eta^{\alpha-1} A^{\alpha}$

So that for given quantities of labor and capital, the increase in the number of intermediate good increases the amount of the final good. Increasing returns are introduced through the differentiation of intermediate good[16]. The dynamics of technological progress are explained by the search for the monopolistic rents arising in the production of ideas, so that for the first time it is the result of intentional and rewarded actions. Then, the introduction of imperfect competition allows the existence of increasing returns to be handled.

Nevertheless, even in this case, assuming linearity in the production of the

[16] More precisely, given the free entry condition it is possible to compute the value of A in terms of K. By plugging the value in the production function we obtain increasing returns in labor and capital.

factor (A) increasing the technological level is essential in order to obtain endogenous growth. Romer himself says: "Linearity in A is what makes unbounded growth possible, and in this sense, unbounded growth is more like an assumption than a result of the model" (Romer, 1990, p. S84). The production function of ideas is $\dot{A} = \delta L_a A$, so that the increase of ideas in the economy is proportional to the stock of existing ideas. And this is a necessary condition: "If A were replaced [..] by some concave function of A-[..]- human capital employed in research would shift out of research and into manufacturing as A becomes larger" (ibid.). Endogenous growth is therefore supported both by the existence of increasing returns and, in an essential way, by the existence of spillovers in research production. The rationale for this assumption is that knowledge is a non-rival good, so that all the researchers can use it freely in their activity. But while the mere existence of this positive spillover seems plausible, the hypothesis that it can assure the linearity in the production of A is doubtful.

In a similar way, Grossman and Helpman (1991, chap.3) derive endogenous growth by means of a private sector engaged in R&D, which produces new ideas and increases the variety of the existing goods. The structure of the economy is analogous to that of Romer 90, but the product differentiation belongs to the consumption sector instead of the intermediate one.

The representative agent instantaneous utility function is:

$C = \left(\int_0^A x_i^\alpha di\right)^{\frac{1}{\alpha}}$ with $0 < \alpha < 1$

This utility function, due to Dixit and Stiglitz (1977), shows a consumer's preference for product variety. It can easily be shown that utility is a rising function of the number of final goods A. This preference anyhow, is not sufficient to assure endogenous growth. Once again, the key role is played by the non-rival nature of knowledge and by the assumption of a linear spill over in its production.

The contributions analyzed so far consider horizontal innovation, which consists of expanding product variety. The appearance of a new good introduces a new sector in the economy, which satisfies new functions and needs. The substitutability between the new product and those already existing is finite. However, innovation can have a different nature. It often consists of improving the quality of already existing goods or developing the production process of a certain good. This is vertical innovation and is the basis of the schumpeterian idea of 'creative destruction', where the destruction consists of rendering a series of goods obsolete.

Like the horizontal case, the vertical innovation can be considered both for the final goods sector (Grossman and Helpman, 1991, chap. 4) and for the intermediate one (Aghion and Howitt, 1992 and 1998, chap. 2,3). In the first case there are N goods each of which can be qualitatively improved an infinite number of times. The producer owning the most updated version of a good becomes monopolist. The profit flow deriving from being monopolist constitutes the incentive to innovate. In the second case however, there is a single final good produced under perfect competition and an intermediate sector in which the owner of the innovation of the last generation is monopolist. There are two basic ways in which vertical innovation differs from horizontal innovation.

First, uncertainty is introduced into the production process of innovations. It is assumed that innovations arrive randomly according to a Poisson distribution of parameter λ. The introduction of uncertainty makes the description of the innovation sector more plausible. Nevertheless, at the aggregate level, the growth rate of ideas is proportional to the arrival rate of the Poisson process λn (n being the number of researchers), so that $\dot{A}/A = \lambda n \ln \gamma$ where γ is a measure of the improvement in technology. The assumptions concerning ideas production are analogous to those of Romer. Second, and more fundamental, the vertical nature of innovation renders any monopoly associated with an innovation transitory. The discovery of a vertically integrated product determines the extinction of the previous monopolist's rents: this is the 'destructive' component of technological progress. The temporary nature of monopolistic rents is taken into account by the firm engaged in R&D and this generates an inverse relation between the amounts of research in two successive periods. This relation requires that agents have perfect foresight on the future amount of research, which will be carried out in their sector.

0.5 The Scale Effect and the Endogenous Growth

When discussing the AK model we saw how the absolute scale of population enters the expression for the growth rate of income. This effect is commonly referred to as 'scale effect' and it characterizes both the AK models and the first generation of R&D models. As capital has constant returns to scale it is capable of yielding a constant growth rate, once population growth is added the growth rate will be ever increasing. Such an explosive trend of output is empirically testable as it implies that larger countries grow faster than the smaller ones. Barro and Sala-i-Martin (1995) analyze two samples: one consisting of 87 countries for the period 1965-75 and one of 97 countries for 1976-85; they find a weak and not significant correlation between the extent of the labor force and the growth rate of output. Nevertheless, they notice, the choice of the single country as the scale variable can be misleading as the variables geographically relevant can be larger (Kremer (1993) finds a positive correlation between world population and per-capita growth) or smaller (e.g. the industrial district).

The first way out of the scale effect appeared as early as the first AK model. We saw how in Frankel (1962) there is no scale effect as the externality is caused not by the total but by the per-capita level of capital stock. By so doing, the labor force disappears from the growth rate of the economy. Analogously, Lucas (1988) attributes the externality to the average level of human capital. While in Arrow (1962) the externality was caused by the total capital stock which represented the cumulated knowledge of producers, it is less clear what is the underlying rationale for focusing on average (physical and human) capital. A shortcoming of this solution is however that the labor factor disappears from the production function so that it does not provide any contribution to production.

Jones (1995a) showed the empirical weakness of the AK models. He stressed that the trend of post-war per-capita growth rate in the OECD countries is substantially stable. Such stability, if the model is correct, must be coupled with the stability of the variables explaining the long-run growth rate. In the period considered the investement rates have followed an upward trend which should have affected the growth rates; the absence of other variables which could have offset the effect of the investment rates makes Jones reject the empirical soundness of the AK models.

The scale effect characterizes the steady state growth rate of the R&D models. In these models indeed the growth rate basically depends on the amount of labor employed in the research sector. Jones (1995a, b) tested this conclusion by studying the correlation between the number of scientists and engineers and the growth rate. The scale effect is rejected[17].

Moreover Jones (1995a, b) showed how the existence of the scale effect in the R&D models derives from a hypothesis which is both arbitrary and necessary for the models to yields endogenous growth. This is hypothesis is the linearity in the production of the factor which is accumulated endogenously. Jones considered a general production function of ideas:

$$\dot{A} = \delta L_a A^{\varphi}$$

The stock of existing ideas can have different effects on the production of ideas. If the most obvious ideas are discovered first leaving the most difficult ones to be found the spillover deriving from the stock A is negative ($\varphi < 0$); if the increase in knowledge on the contrary spurs the discovery of new ideas, then the spillover is positive. The special case assumed by the R&D models

[17] Jones' empirical analyses, however, have been questioned. Backus et al. (1992) find evidence of scale effect when considering only the manufacturing sector. Moreover it has been argued that his results could be biased because of problems in the measurement of quality improvements of goods and services (see for example Griliches (1994)).

though has no economic explanation ($\varphi = 1$). Jones develop the case $\varphi < 1$: as $\dot{A}/A = \delta L_a / A^{1-\varphi}$, the steady state rate of growth of ideas is $g = n/1 - \varphi$: the growth rate of the economy depends only on technology and population. In order to avoid the scale effect the endogenous factors (namely preferences) disappear from the growth rate. Still, it is not a mere return to exogenous growth as technical change is the outcome of rational choice of profit-maximizing agents: Jones names his model semi-endogenous.

The attempts to eliminate the scale effects from the R&D growth models basically developed along two lines. The first is closer to Jones' solution in that it assumes that as the level of knowledge increases the productivity of the research sector decreases. This idea is developed, among the others, by Kortum (1997) and Segerstrom (1998). Their model, it is shown by Jones (1999), can be subsumed under the case $\varphi < 0$. Analogously to Jones' result the growth rate is exogenous. It is worth noting however that changes in research intensity and population even though do not affect per capita growth rates in fact influence the steady state level of per-capita income.

The second way out of the scale effect instead reassures the endogeneity of the growth rate. The basic argument is that as the number of researchers grows the number of sectors where they are employed increases in a way that leaves per-sector research intensity unaffected. Basically, these models consider both vertical and horizontal innovation. The former is characterized by the usual spillover which enables the model to engender sustained growth (i.e. $\varphi \doteq 1$). As regards the horizontal innovation instead, it is required that the number of sectors in the economy grows proportionally to the population ($A = kL$). Such a condition is necessary to generate endogenous growth without scale effect; if the sectors of the economy grow slower than population the scale effect arises, if they grow faster the models become exogenous. Then, in order to eliminate the scale effect in the endogenous growth literature a further ad hoc condition is required.

0.6 Concluding Remarks

Summing up, it seems that the endogenous growth models are based on the hypothesis of constant returns in the reproducible factor, capital (physical or human) or knowledge. Such a structure implies a permanent influence of changes in the investment rates and in the share of labor devoted to R&D on per-capita growth rate and it opens large possibilities for economic policy and shows that the exogenous- endogenous growth debate is not a simple academic matter.

From a theoretical point of view these models are not robust in that they hinge on 'knife edge' restrictions on technology, which allow them to generate linearity in the production of the reproducible factor of production. This seems to be the reason why many of the results accomplished by the endogenous growth theories, even though already obtained during the 60's, were considered implausible[18]. After all, the Solow model attempted to solve the problem of the

[18] As Stiglitz observes: "We knew how to construct models that 'worked', but felt uneasy making these special assumptions. It was one thing to assume that savings rate were constant-that was a behavioral hypothesis that provided a not bad description of the economy over long periods of time, (■).But it was quite another thing to assume, for instance, that the effects of learning just offset the effects of diminishing returns due to land scarcity! That was

Harrod-Domar instability and the new growth theories share the same feature. As Solow notes: "modern literature is in part just a very complicated way of disguising the fact that it is going back to Domar, and, as with Domar, the rate of growth becomes endogenous" (1992, p.18).

Still, credit must be given to endogenous growth models because they have contributed to the focus on elements like the production of knowledge and human capital that had been underrated until then. The biggest accomplishment of the theories seems however, to be a technical one. By introducing general equilibrium models of imperfect competition they made it possible for the neoclassical theory to cope with both increasing returns and innovation as the outcome of profit-maximizing agents' decisions[19]. From a neoclassical point of view it is a huge result to make innovation and the marginal theory of distribution consistent. Unlike in Schumpeter's Theory of Economic Development (1934) it is no longer necessary to assume that whenever an innovation occurs the marginal theory of distribution breaks down.

a technological assumption, and although we may have agreed with Einstein that God had created a universe of great simplicity, it seemed going to far to assume that he had endowed technology with these special parameters, simply so that we could construct our steady state models" (1990, p.55).

[19]As admitted by Aghion himself: "The main contribution of the new growth theory so far has been predominantly technical in nature. It is now possible to deal with increasing returns and imperfect competition in dynamic general equilibrium models which are simple as those developed in the recent industrial organization literature. This technological breakthrough has in turn made it possible to formalize a number of existing ideas concerning development" (1994, p.7).

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An Uncertainty-Based Explanation of Symmetric Growth in Schumpeterian Growth Models[20]

0.7 Introduction

Most vertical R&D-driven growth models (such as Grossman-Helpman 1991, Segerstrom 1998, Aghion-Howitt 1998, Ch.3) focus on the *symmetric equilibrium* in the research industries, that is, on that path characterized by an equal size of R&D investments per sector. As is well known, in these models the engine of growth is technological progress, which stems from R&D investment decisions

[20]**This paper is co-authored with G.Cozzi and P. Giordani.**

taken by profit-maximizing agents. By means of research each product line can be improved an infinite number of times and the firms manufacturing the most updated version of a product monopolize the relative market and thus earn positive profits. However, these profits have a temporary nature since any monopolistic producer is doomed to be displaced by successive improvements in her product line. The level of expected profits together with their expected duration, as compared with the cost of research, determines the profitability of undertaking R&D in each sector.

Now, the *plausibility* of the symmetric equilibrium requires that each R&D sector be equally profitable, so that the agents happen to be indifferent as to where targeting their investments. The profit-equality requirement implies two different conditions. First, the profit flows deriving from any innovation need be the same for each industry: this is guaranteed by assuming that all the monopolistic industries share the same cost and demand conditions. Second, the monopolistic position acquired by innovating needs be expected to last equally long across sectors: this requires that the agents *expect* the future amount of research to be equally distributed among the different sectors. As is well known to the reader familiar with the neo-Schumpeterian models of growth, future is allowed to affect current (investment) decisions via the forward-looking nature of the Schumpeterian 'creative destruction' effect.

Grossman and Helpman (1991, p.47) recognize the centrality of the assumption of symmetric expected R&D investments in order to justify the selection of the symmetric equilibrium: with the assumption that "the profit flows are the same for all industries [..] an entrepreneur will be indifferent as to the industry in which she devotes her R&D efforts provided that she expects her prospective leadership position to last equally long in each one. We focus hereafter on the symmetric equilibrium in which all products are targeted to the same aggregate extent. In such an equilibrium the individual entrepreneur indeed expects profit flows of equal duration in every industry and so is indifferent as to the choice of industry". Hence in this framework it is crucial to assume that an equal distribution of future R&D efforts across industries is expected.

Although expecting equal future profitability across sectors constitutes a necessary condition for each agent to choose a symmetric allocation of R&D efforts, it is however not sufficient: in fact, equal future profitability makes the investor indifferent as to where targeting research. As a result, the allocation problem of investments across product lines is indeterminate even if symmetric expectations are assumed. Notice also that the way this allocation problem is solved is not always without consequence for this class of models, as recently pointed out by Cozzi (2003). For instance in a Segerstrom's (1998) framework, because of the 'increasing complexity hypothesis', the alternative prevalence of the symmetric or asymmetric equilibrium has powerful effects on the growth rate of the economy: if indifferent agents, for a whatever reason (a 'sunspot'), are induced to allocate their investment only in a small fraction of sectors, the dynamic decreasing returns to R&D investments will imply a lower aggregate growth rate as compared with the one associated with an equal distribution of R&D efforts across all sectors. An equally relevant effect of sunspot-driven asymmetric R&D investments on steady-state growth rates reappears in the Howitt's (1999) extension to an ever expanding set of product lines (see Cozzi (2004)). Hence both solutions to the 'strong scale effect' problem (Jones (2004)) exhibit dependence of growth rates on the intersectoral distribution of R&D.

In this paper we provide an alternative route to make the focus on the symmetric equilibrium compelling. Our basic idea is that the agent's beliefs on the future (per sector) distribution of R&D investments are characterized by uncertainty, in the sense that information about that distribution is too imprecise to be represented by a (single additive) probability measure. The distinction between risk and uncertainty traces back to Frank Knight (1921) and states that risk is associated with ventures in which an objective probability distribution of all possible events is known, while uncertainty characterizes choice settings in which that probability distribution is not available to the decision-maker.

In recent years a number of attempts have been made to extend the expected utility theory in order to embrace the distinction between risk and uncertainty[22]. Here however we will follow the maxmin expected utility (MMEU) decision rule axiomatized by Gilboa and Schmeidler (1989), which formalizes a 'taste for uncertainty'[23]. According to this rule, an uncertainty-averse deci-

[22]For instance Bewley (1986) has developed his theory of the 'status quo', by dropping the axiom of complete preferences inside the Anscombe and Aumann (1963) version of the Savage's theory.

[23]Gilboa and Schmeidler (1989) provide an axiomatic foundation of the maxmin expected utility decision rule in the framework of Anscombe and Aumann (1963). Several applications of this rule have been elaborated over the last few years. We recall, among the others, Epstein and Wang (1994), and Hansen and Sargent (2000). Notice that a 'taste for uncertainty' can alternatively be modeled via the Choquet expected utility (CEU) theory axiomatized by Schmeidler (1989). In it, expected utility is computed according to a capacity (that is, a not necessarily additive probability) via the Choquet integral. MMEU and CEU coincide in a special but important case, namely, when the capacity is convex.

sion maker is provided, not with a unique prior - as in the standard expected utility theory -, but with a *set of priors.* When evaluating the different acts to choose among, this agent will compute the *minimal* expected utility over her set of priors for each of them (acts), and then will single out the one associated with the highest computed value. Consequently, in our framework the decision maker will be assumed to maximize her expected pay-off with respect to the R&D investment decision, while singling out the minimizing choice scenario, that is, the worst probability distribution over the future configuration of R&D investments. Unlike in Epstein and Wang (1994), in this paper the maxmin decision rule eliminates indeterminacy and makes the symmetric - and growth maximizing - allocation of R&D investment emerge as the unique equilibrium.

Importantly, our assumption on the agents' beliefs does not affect any fundamentals of the economy and is to be interpreted as a way of treating sector-specific 'extrinsic uncertainty'. Moreover, since uncertainty does not affect aggregate variables, in order to develop our argument, we need to introduce neither the optimal consumption problem solved by households, nor the profit-maximizing problem solved by firms (for them the reader is referred to Segerstrom (1998)). As the problem is that of distributing a given amount of R&D efforts across product lines, all we need is the description of the R&D sector.

Our result holds for a however small probability that a however small fraction of individual's portfolio be affected by strong uncertainty. Hence a microscopic departure from the standard treatment of extrinsic uncertainty leads to potential macroscopic growth consequences.

The rest of the paper is organized as follows. In Section 2 we briefly describe the basic structure of the R&D sector, with particular reference to the Segerstrom's (1998) formalization. In Section 3 we explain the core of our argument, enunciate and prove the proposition. In Section 4 we conclude with some remarks.

0.8 R&D Sector

In this Section we provide a description of the vertical innovation sector which is basically common to most neo-schumpeterian growth models. This sector is characterized by the efforts of R&D firms to develop better versions of the existing products in order to displace the current monopolists[25]. We assume a continuum of industries indexed by ω over the interval [0,1]. There is free entry and perfect competition in each R&D race. Firms employ labor and produce, through a constant returns technology, a Poisson arrival rate of innovation in the product line they target. Adopting Segerstrom's (1998) notation, any firm hiring l_j units of labor in industry ω at time t acquires the instantaneous probability of innovating $Al_j/X(\omega,t)$, where $X(\omega,t)$ is the R&D difficulty index.

Since independent Poisson processes are additive, the specification of the innovation process implies that the industry-wide instantaneous probability of innovation is $AL_I(\omega,t)/X(\omega,t) \equiv i(\omega,t)$, where $L_I(\omega,t)=\sum_j l_j(\omega,t)$. The parameter $X(\omega,t)$ describes the evolution of technology; as in Segerstrom (1998),

[25] It seems irrelevant to our purpose to distinguish whether the monopolistic sector is that of the final goods - as in Segerstrom (1998) - or that of the intermediate ones - as in Aghion and Howitt (1998, Ch.3) and Howitt (1999).

we assume it to evolve in accordance with

$$\frac{\dot{X}(\omega,t)}{X(\omega,t)} = \mu i(\omega,t)$$

where μ is a constant. However we do not impose any sign restriction on μ, in order to leave the difficulty index increasing, decreasing or remaining constant as research accumulates. In the next section we will return to this problem by specifying the range of values of μ which render our proposition significant.

Whenever a firm succeeds in innovating, she acquires the uncertain profit flow that accrues to a monopolist, that is, the stock market valuation of the firm: let us denote it with $v(\omega,t)$. Thus, the problem faced by an R&D firm is that of choosing the amount of labor input in order to maximize her expected profits:

$$\max_{l_j}[v(\omega,t)A/X(\omega,t)l_j - l_j]$$

which provides a finite, positive solution for l_j only when the arbitrage equation[27] $v(\omega,t)A/X(\omega,t) = 1$ is satisfied. Notice that in this case, though finite, the size of the firm is indeterminate because of the constant return research technology.

Efficient financial markets require that the stock market valuation of the firm yield an expected rate of return equal to the riskless interest rate $r(t)$. The shareholder receives a dividend of $\pi(t)dt$[28] over a time interval of length dt and

[27] We consider the wage rate as the numerarie.

[28] We drop the ω argument from the profit function because, when assuming symmetric cost and demand conditions, the profit flows in each monopolistic industry coincide.

the value of the monopoly appreciates by $\dot{v}(\omega,t)dt$ if no firm innovates in the unit time dt. However, if an innovation occurs, the shareholder suffers a loss of $v(\omega,t)$. It happens with probability $i(\omega,t)dt$, whereas no innovation occurs with probability $[1-i(\omega,t)dt]$. Therefore, the expected rate of return from holding a share of monopolistic firm per unit time is

$$\frac{\pi(t)+\dot{v}(\omega,t)[1-i(\omega,t)]}{v(\omega,t)}-i(\omega,t)$$

which needs be equal to the interest rate $r(t)$. From this equality we can derive the firm's market valuation:

$$v(\omega,t)=\frac{\pi(t)}{r(t)+i(\omega,t)-\dfrac{\dot{v}(\omega,t)}{v(\omega,t)}}$$

so that the R&D equilibrium condition is

$$\frac{\pi(t)A}{X(\omega,t)[r(t)+(1-\mu)i(\omega,t)}=1$$

since

$$\frac{\dot{v}(\omega,t)}{v(\omega,t)}=\frac{\dot{X}(\omega,t)}{X(\omega,t)}=\mu i(\omega,t).$$

The usual focus on the symmetric growth equilibrium is based on the assumption that the R&D intensity $i(\omega,t)$ is the same in all industries ω at time t and strictly positive. The suggestion of a new rationale for this symmetric behavior will be the topic of the next Section.

0.9 The Re-foundation of the Symmetric Equilibrium by Assuming Uncertainty-Averse Agents

We assume that the agent has a fuzzy perception of the future configuration of R&D efforts and formalize her investment strategy as an equilibrium resulting from a 'two-player zero-sum game' characterized by:

· the *minimizing* behavior of a 'malevolent Nature', which selects the prior belief associated with the 'worst possible scenario' inside a pre-specified set of priors and

· the *maximizing* behavior of the agent, whose optimal choice must take into account the worst-case strategy implemented by Nature.

Before proceeding with the analysis, let us clarify two important aspects of the model's structure. In the previous Section we have referred to the R&D firm as the one choosing the size and the distribution among sectors of R&D investments. However, R&D firms are financed by consumers' savings which are channeled to them through the stock market. Thus, since the consumer is allowed to choose the R&D sectors where to employ her savings, she ends up with being our fundamental unit of analysis. The role of the R&D firms merely becomes that of transforming these savings into research activity.

Notice also that in the basic set-up by which our paper is inspired (Grossman and Helpman (1991) and Segerstrom (1998)), the agent is assumed to be risk-averse. In fact, she is assumed to be able to completely diversify her portfolio - by means of the intermediation of costless financial institutions - and then to only care about deterministic mean returns. This assumption is retained in our set-up - which allows for a whatever asymmetric configuration of investments - since, in order to carry out this diversification, it is sufficient to equally allocate investments in a non-zero measure interval of R&D sectors (and not necessarily in the whole of them). The crucial difference with respect to the standard framework is then concerned with the assumption of uncertainty-averse agents, where uncertainty only affects the mean return of the R&D investment and not its volatility, against which the agent has already completely hedged.

Assumption:

$X(\omega, 0) = X_0 \; \forall \omega \in [0, 1]$.

We assume that all industries share the same difficulty index X_0 in order to focus on the role of expectations on the kind of equilibrium that will prevail.

Our problem can be stated as follows: at time $t = 0$ an agent is asked to allocate a given amount of R&D investment among all the existing sectors. As the agent is assumed to be uncertainty-averse, in maximizing her expected pay-off she will take into account the minimizing strategy that a 'malevolent nature' will be carrying out in choosing the composition of future R&D efforts. We denote with $l_m + \alpha(\omega)$ the agent's investment in sector ω, and with $L_I + \varepsilon(\omega)$ the aggregate expected research in sector ω. l_m and L_I are, respectively, the agent's average investment per sector and the average expected research per sector. $\varepsilon(.)$ and $\alpha(.)$ represent deviations from the averages satisfying:

$$\int_0^1 \varepsilon(\omega) d\omega = 0 \qquad \int_0^1 \alpha(\omega) d\omega = 0 \qquad \text{and}$$

$$\varepsilon(\omega) > -L_I \qquad \alpha(\omega) > -l_m.$$

The presence of the two functions $\alpha(.)$ and $\varepsilon(.)$ is intended to allow for asymmetry both in the agent's investment and in expected research[30]. We also

assume the space to be partitioned into two events: symmetric and asymmetric configuration of future R&D efforts. The first is supposed to occur with probability $1-p$ while p stands for the aggregate probability of all possible asymmetric configurations. The interval $[0,p]$ represents the unrestricted set of priors assigned to each of them. As we will see, the minimizing strategy carried out by Nature will end up with assigning probability p to the worst asymmetric configuration (which is function of the agent's choice) and 0 to all the others.

By partitioning the state space 'configuration of future investments' into the events 'asymmetric' and 'symmetric', and by assigning the probability distribution $(p, 1-p)$ to them, we have implicitly assumed that the decision maker has sufficient information to evaluate probabilistically the occurrence of both of them. In fact, what is subject to uncertainty, and then to a 'conservative assessment' through the maxmin strategy, is the particular asymmetric configuration that would possibly take place among all those generated by the deviation ε. Our conclusions do not crucially hinge on this partition[32].

[30] These definitions imply:
$\int_0^1 [L_I + \varepsilon(\omega)]\, d\omega = L_I = L \int_0^1 [l_m + \alpha(\omega)]\, d\omega = Ll_m$
where L denotes the number of agents in the economy.
With reference to Section 2 the following relation between l_j and l_m holds:
$\int_0^1 \sum_j l_j(\omega)\, d\omega = Ll_m$.

[32] For example, it can easily be shown how our result holds for a however small perturbation of the probability distribution which assigns equal probabilities to every possible configuration of future R&D investment across sectors.

We can now enunciate the following:

Proposition 1 *For a however small probability (p) of deviation ($\varepsilon(\omega)$) and for a however small deviation ($\varepsilon(\omega)$) from symmetric expectation on future R&D investment, uncertainty-averse agents who adopt a maxmin strategy to solve their investment allocation problem, choose a symmetric investment strategy, i.e.* $l_m + \alpha(\omega) = l_m$ $\forall \omega \in [0,1]$. *Their optimal investment choice makes them expect a symmetric distribution of future R&D effort among sectors:* $L_I + \varepsilon(\omega) = L_I$ $\forall \omega \in [0,1]$.

Proof. $\max\limits_{\alpha(.)} \left[\min\limits_{\varepsilon(.)} \int\limits_0^1 [l_m + \alpha(\omega)] \dfrac{A}{X_0} v(\omega) d\omega \right]$

$$\text{sub} \qquad \int\limits_0^1 \varepsilon(\omega) d\omega = 0 \qquad ; \qquad \int\limits_0^1 \alpha(\omega) d\omega = 0$$

where $v(\omega) = \dfrac{\pi}{r + \dfrac{A}{X_0}[L_I + \varepsilon(\omega)] - \dfrac{\dot{v}(\omega)}{v(\omega)}}$ with probability $0 \leq p \leq 1$.

Then the problem is equivalent to:

$$\max_{\alpha(.)} \left[\min_{\varepsilon(.)} \int\limits_0^1 [l_m + \alpha(\omega)] \left(p \frac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0}[L_I + \varepsilon(\omega)](1-\mu)} + (1-p) \frac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0} L_I (1-\mu)} \right) d\omega \right] =$$

$$= (1-p) \frac{l_m \frac{A}{X_0}\pi}{r + \frac{A}{X_0} L_I (1-\mu)} + p \max_{\alpha(.)} \left[\min_{\varepsilon(.)} \int\limits_0^1 [l_m + \alpha(\omega)] \frac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0}[L_I + \varepsilon(\omega)](1-\mu)} d\omega \right]$$

which admits the same solution as:

$$\max_{\alpha(.)} \left[\min_{\varepsilon(.)} \int\limits_0^1 [l_m + \alpha(\omega)] \frac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0}[L_I + \varepsilon(\omega)](1-\mu)} d\omega \right].$$

Notice that this is valid for however small probability p.

We restrict our attention to the case: $0 < v(\omega) < +\infty$. As at time $t = 0$,

A, X_0, π, r are assumed to be positive constants, that condition requires $\mu < 1$.

Let us suppose the following (not necessarily minimizing) expected reaction function of Nature: $\varepsilon(\omega) = t\alpha(\omega)$ with $t > 0$. Notice that as t tends to zero the deviation in each sector can be made arbitrarily small independently of population size.

The objective function then becomes: $\int_0^1 [l_m + \alpha(\omega)] \dfrac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0}\left[L_I + t\alpha(\omega)\right](1-\mu)} d\omega.$

Since $[l_m + \alpha(\omega)] \dfrac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0}\left[L_I + t\alpha(\omega)\right](1-\mu)}$ is a strictly concave function of $\alpha(\omega)$, by Jensen's inequality its integral

$\int_0^1 [l_m + \alpha(\omega)] \dfrac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0}\left[L_I + t\alpha(\omega)\right](1-\mu)} d\omega$

is maximized by $\alpha(\omega) = 0$ for all $\omega \in [0,1]$. Therefore, if $\alpha(\omega) \neq 0$ in a non-zero measure subset of $[0,1]$, it follows:

$$\min_{\varepsilon(.)} \int_0^1 [l_m + \alpha(\omega)] \frac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0}\left[L_I + \varepsilon(\omega)\right](1-\mu)} d\omega \leq \int_0^1 [l_m + \alpha(\omega)] \frac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0}\left[L_I + t\alpha(\omega)\right](1-\mu)} d\omega <$$

$$\int_0^1 [l_m + \alpha(\omega)] \frac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0} L_I (1-\mu)} d\omega =$$

$$= \frac{\frac{A}{X_0}\pi l_m}{r + \frac{A}{X_0} L_I (1-\mu)}$$ which is instead attained if $\alpha(\omega) = 0$ for all $\omega \in [0,1]$ (almost everywhere). If indeed $\alpha(\omega) = 0$ for all $\omega \in [0,1]$ (almost everywhere), Jensen's inequality implies:

$$\min_{\varepsilon(.)} \int_0^1 l_m \frac{\frac{A}{X_0}\pi}{r+\frac{A}{X_0}\left[L_I+\varepsilon(\omega)\right](1-\mu)} d\omega = \frac{\frac{A}{X_0}\pi l_m}{r+\frac{A}{X_0}L_I(1-\mu)}$$ which is instead attained if $\varepsilon(\omega)=0$ for all $\omega \in [0,1]$ (almost everywhere).

Therefore the worst harm Nature can inflict to the agent in the case $\alpha(\omega)=0$ is always better for the agent than the worst harm Nature can inflict to the agent in the case $\alpha(\omega) \neq 0$. Hence the symmetric portfolio - and zero measure deviations from it - is the maximinimizing strategy of the agent.

Since this holds for any $t>0$ and $0<p<1$, the statement follows for however small deviations and their probabilities. ■

Then, even if the agent is 'almost sure' ($p \to 0$) of facing a symmetric configuration of future investments (which would leave her in a position of indifference in her current allocation problem), the mere possibility of a slightly different configuration ($\varepsilon \to 0$) makes her strictly prefer to equally allocate her investments across sectors. This occurs because, whenever the agent evaluates an asymmetric allocation of her current investments, she will always be induced to expect the worst configuration of future investments inside the ε-generated set.

0.10 Concluding Remarks

In the neo-schumpeterian growth models the existence of the creative destruction effect implies that expectations on future R&D investments affect the allocation of current ones. Therefore the usual focus on the symmetric equilibrium in the vertical research sector relies on the assumption of a symmetric expected per-sector distribution of R&D expenditure. However, in making the agents indifferent as to where targeting their investments, this assumption is not sufficient to univocally pin down the symmetric structure of R&D efforts: actually symmetric expectation on future R&D leaves the current composition of R&D investments indeterminate, with potentially large effects on growth rates.

We have shown that a possible way out of this indeterminacy is that of assuming uncertainty on the future configuration of R&D investments and maxminimizer agents in the face of this uncertainty. Under this assumption, indeterminacy vanishes and the symmetric allocation of the vertical research expenditures comes out as the unique optimal solution.

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A Quality-Ladder Growth Model with Asymmetric Fundamentals

0.11 Introduction

Since their very appearance horizontal as well as vertical R&D-driven growth models have focused on structurally symmetric economies and on symmetric equilibria in R&D and in the final (or intermediate) goods sector. In particular, Romer (1990) proposed a model where technical progress is due to the expanding variety of final goods, each of which is equally consumed in the economy because of the symmetric structure of the utility function. Grossman and Helpman (1991) elaborated a model of vertical innovation where growth is fostered by the improving quality of the final (or intermediate) goods sector; expenditure is evenly allocated over products and symmetry also characterizes the industry-specific R&D effort.

We develop a quality-ladder growth model with increasing complexity in R&D for an economy with asymmetric fundamentals. In the research sector, asymmetry is introduced both in the quality jumps, i.e. the improvement in utility following each innovation, and in the probabilities of innovating per unit of labor (the arrival rates). Moreover, the unit contribution to the consumer's utility of any final goods is industry-specific and so are their production costs.

As Jones (1995a) convincingly argued against the existence of the so called 'scale effect' in real economies our analysis, in aiming at ruling it out, incorporates the idea of increasing complexity in the research sector in two different specifications respectively formalized by Segerstrom (1998) and Dinopoulos and Segerstrom (1999).

As we will see, the introduction of different quality jumps and utility weights across sectors breaks down the symmetric structure of market demands and of per-sector profits; furthermore, asymmetric arrival rates will cause the profitability of engaging in R&D to vary accordingly. Then, the creative destruction effect resulting from expectations on future research is the sole responsible for equalizing the expected returns in R&D, which is a necessary condition for having positive R&D efforts in each industry. The rational expectations equilibrium requires expectations on research intensities to be equal to their actual values; moreover, as our model assumes increasing complexity, the steady-state analysis makes these intensities be constant over time. It will turn out that the steady-state per-sector research intensities vary according to the specification of the difficulty index adopted.

The paper is organized as follows: in Section 2 we develop the model, analyze its steady-state properties and draw the main comparative statics results. In Section 3 we conclude with some remarks.

0.12 The Model

As usual, we assume a continuum of industries producing final goods indexed by $\omega \in [0,1]$. In each industry firms are distinguished by the quality index j of the goods they supply, with the quality of their goods being increasing in the integer j. At time $t = 0$ in each industry some firms know how to produce a $j = 0$ quality product and no other firms can offer a better one. In order to develop higher quality versions of any product firms engage in R&D races. The winner of a R&D race becomes the sole producer of a good whose quality is one step ahead of the previous quality leader.

0.12.1 Households

We assume a fixed number of dynastic households (normalized to one) whose members grow at the constant rate $n > 0$. Each member shares the same intertemporally additively separable utility $u(t)$ and is endowed with a unit of labor she supplies inelastically. Therefore each household chooses its optimal consumption path by maximizing the discounted utility

$$U \equiv \int_0^\infty L(0)e^{-(\rho-n)t} \log u(t)dt \tag{1}$$

where $L(0) \equiv 1$ is the initial population and $\rho > n$ is the common rate of time preferences.

The instantaneous utility function is a Cobb-Douglas. We introduce asymmetric preferences by allowing the utility weights $(\alpha(\omega))$ to vary across sectors. We also impose $\int_0^1 \alpha(\omega)d\omega = 1$ to represent the homogeneity of degree one of the utility function. With

$\log u(t) \equiv \int_0^1 \alpha(\omega) \log \sum_{j=0}^{j^{\max}(\omega,t)} \lambda^j(\omega)d(j,\omega,t)d\omega,$

the static maximization problem can be represented as:

$$\max \int_0^1 \alpha(\omega) \log \sum_{j=0}^{j^{\max}(\omega,t)} \lambda^j(\omega)d(j,\omega,t)d\omega \tag{2}$$

$$s.t. E(t) = \int_0^1 \left[\sum_{j=0}^{j^{\max}(\omega,t)} p(j,\omega,t)d(j,\omega,t) \right] \tag{3}$$

where $p(j,\omega,t)$ and $d(j,\omega,t)$ denote, respectively, the price and consumption of product ω of quality j at time t. $\lambda(\omega)$ is the size of quality improvements that we assume to be industry-specific to allow for asymmetry in the technical evolution of each line. Vertically differentiated products in a given industry ω are perceived by consumers as perfect substitutes once adjusted for quality differences. $j^{\max}(\omega,t)$ denotes the highest quality reached by product ω at time t and $E(t)$ is the total expenditure at time t.

At each point in time consumers maximize static utility by spreading their expenditure across sectors proportionally to the utility contribution of each product line $(\alpha(\omega))$, and by only purchasing in each product line those products with the lowest price per unity of quality. As we will see in the next subsection, in each product line the $j^{\max}(\omega,t)$ quality product is the only good with the minimum price-quality ratio. Then, the individual static demand functions are:

$$d(j,\omega,t)=\begin{cases}\dfrac{\alpha(\omega)E(t)}{p(j,\omega,t)} & \text{for } j=j^{\max}(\omega,t)\\ \\ 0 & \text{otherwise}\end{cases} \tag{4}$$

Moreover, since the only $j^{\max}(\omega,t)$ quality product is actually purchased, in what follows it will be:

$$\sum_{j=0}^{j^{\max}(\omega,t)}\lambda^j(\omega)=[\lambda(\omega)]^{j^{\max}(\omega,t)}$$

Substituting (4) into (2) and (2) into (1), we get the intertemporal maximum problem

$$\max_E U=\int_0^\infty e^{-(\rho-n)t}\left[\log E(t)+\int_0^1\alpha(\omega)\left[\log\alpha(\omega)+\log[\lambda(\omega)]^{j^{\max}(\omega,t)}-\log p(j,\omega,t)\right]d\omega\right]dt$$

$$\text{s.t. }\int_0^\infty e^{-\int_0^t[r(s)-n]ds}E(t)dt\le A(0),$$

where $r(s)$ is the instantaneous interest rate at time s and $A(0)$ is the present value of the stream of incomes plus the value of initial wealth at time $t=0$.

The solution to this problem obeys the differential equation:

$$\frac{\dot{E}(t)}{E(t)}=r(t)-\rho. \tag{5}$$

0.12.2 Manufacture

Each good is produced by only employing labor through a constant return to scale technology. We modify the standard framework by introducing different production costs across sectors: in order to produce one unit of good ω firms hire l_ω units of labor regardless of quality.

In each industry the $j^{\max}(\omega,t)$ quality product can only be manufactured by the firm which has discovered it. Since firms engage in Bertrand competition, the quality leader monopolizes her relative market until a new innovation is introduced in her product line. Indeed, having a quality advantage over her competitors she can charge a price higher than her unit cost, with the adjusted-quality price being still ε-lower than those of her followers. Moreover, because of the so-called Arrow effect the quality leader does not engage in R&D races. Hence she is always one step ahead of her immediate follower and the limit price that still monopolizes the market is $\lambda(\omega)wl_\omega$. With $D(\omega,t) = \dfrac{\alpha(\omega)E(t)L(t)}{p\left[j^{\max}(\omega,t),\omega,t\right]}$ being the market demand of good ω at time t, its unit elastic structure makes the quality leader exactly set the limit price. Then:

$$p\left[j^{\max}(\omega,t),\omega,t\right] = \lambda(\omega)wl_\omega.$$

We can now calculate the profit flow in each sector:

$$\pi(\omega,t) = \frac{\lambda(\omega)-1}{\lambda(\omega)}\alpha(\omega)E(t)L(t).$$

0.12.3 R&D races

This sector is characterized by the efforts of R&D firms to develop better versions of the existing products in order to displace the current monopolists. We assume free entry and perfect competition in each R&D race. Firms employ labor and produce, through a constant returns technology, a Poisson arrival rate of innovation in the product line they target. The asymmetry in the research sector consists of different arrival rates across sectors. Any firm hiring l_k units of labor in industry ω at time t acquires the instantaneous probability of innovating $A(\omega)l_k/X(\omega,t)$, where $X(\omega,t)$ is the R&D difficulty index.

Since independent Poisson processes are additive, the specification of the innovation process implies that the industry-wide instantaneous probability of innovation (or research intensity) is

$$\frac{A(\omega)L_I(\omega,t)}{X(\omega,t)} \equiv i(\omega,t) \tag{6}$$

where $L_I(\omega,t)=\sum_k l_k(\omega,t)$. The R&D difficulty index $X(\omega,t)$ describes the evolution of technology; it has been assumed to increase over time in order to rule out the 'scale effect', that is, to allow for constant growth rates even with a growing population. In what follows we will suppose it to evolve in accordance with two alternative specifications, identified with the acronyms TEG and PEG by Dinopoulos and Segerstrom (1999)[36]. The first formalizes the idea that, in each sector, easier inventions are earlier discovered:

$$\frac{\dot{X}(\omega,t)}{X(\omega,t)} = \mu i(\omega,t) \tag{TEG}$$

where the difficulty parameter μ is a positive constant. Actually, in analyzing the properties of the steady-state under TEG in the next subsection, we will assume μ to be industry-specific. The second formulation has been developed to give reason for the increasing difficulty of introducing new products in more crowded markets:

$$X(\omega,t) = kL(t), \tag{PEG}$$

where k is a positive constant.

Whenever a firm succeeds in innovating, she acquires the uncertain profit

[36] TEG stands for 'Temporary effects on growth' of policy measures such as subsidies and taxes. The first TEG formulation has been proposed by Jones (1995b) and has been subsequently developed, among the others, by Kortum (1997). Here we discuss Segerstrom's (1998) version. PEG stands for 'Permanent effects on growth' of policy measures such as subsidies and taxes. It has been independently developed by Young (1998), Dinopoulos and Thompson (1998), Peretto (1998) and Howitt (1999). We adopt the formalization suggested by Dinopoulos and Segerstrom (1999). Useful surveys on the scale effect problem and the way it has been solved are Dinopoulos and Thompson (1999) and Jones (1999 and 2003).

flow that accrues to a monopolist, that is, the stock market valuation of the firm: let us denote it with $v(\omega, t)$. Thus, the problem faced by a R&D firm is that of choosing the amount of labor input in order to maximize her expected profits:

$$\max_{l_k} \left[\frac{v(\omega,t)A(\omega)}{X(\omega,t)} l_k - l_k \right]$$

which provides a finite, positive solution for l_k only when the arbitrage equation[37]:

$$\frac{v(\omega,t)A(\omega)}{X(\omega,t)} = 1$$

is satisfied. Notice that in this case, though finite, the size of the firm is indeterminate because of the constant return research technology.

Efficient financial markets require that the stock market valuation of the firm yields an expected rate of return equal to the riskless interest rate $r(t)$. The shareholder receives a dividend of $\pi(\omega,t)dt$ over a time interval of length dt and the value of the monopoly appreciates by $\dot{v}(\omega,t)dt$ if no firm innovates in the unit time dt. However, if an innovation occurs, the shareholder suffers a loss of $v(\omega,t)$. It happens with probability $i(\omega,t)dt$, whereas no innovation occurs with probability $[1 - i(\omega,t)]dt$. Therefore, the expected rate of return from holding a share of monopolistic firm per unit time is

$$\frac{\pi(\omega,t) + \dot{v}(\omega,t)[1 - i(\omega,t)]}{v(\omega,t)} - i(\omega,t)$$

which needs be equal to the interest rate $r(t)$. From this equality we can derive the firm's market valuation:

[37] We consider the wage rate as the numerarie.

$$v(\omega,t)=\frac{\pi(\omega,t)}{r(t)+i(\omega,t)-\dfrac{\dot{v}(\omega,t)}{v(\omega,t)}}$$

so that the R&D equilibrium condition is

$$\frac{\pi(\omega,t)A(\omega)}{X(\omega,t)\left[r(t)+i(\omega,t)-\dfrac{\dot{v}(\omega,t)}{v(\omega,t)}\right]}=1. \tag{7}$$

0.12.4 The labor market

Since in each industry the market demand $D(\omega,t)=\dfrac{\alpha(\omega)E(t)L(t)}{\lambda(\omega)l_\omega}$ requires $D(\omega,t)l_\omega$ units of labor in order to be produced, the total employment in the manufacturing sector is:

$$\int_0^1 \frac{\alpha(\omega)E(t)L(t)}{\lambda(\omega)}d\omega.$$

Then, the labor market-clearing condition implies:

$$L(t)=\int_0^1 \frac{\alpha(\omega)E(t)L(t)}{\lambda(\omega)}d\omega+\int_0^1 L_I(\omega,t)d\omega. \tag{8}$$

where $\int_0^1 L_I(\omega,t)d\omega$ is the total employment in the research sector.

0.12.5 Balanced growth paths

We now focus on the steady-state growth path where the endogenous variables all grow at constant rates. We develop the analysis by distinguishing between the two specifications of the difficulty index.

Steady-State under TEG Specification

In addition to the previous hypotheses on the asymmetric structure of our economy, let us also assume the difficulty parameter μ to be ω-specific. As a result, the TEG specification can now be written as:

$$\frac{\dot{X}(\omega,t)}{X(\omega,t)} = \mu(\omega)i(\omega,t).$$

(TEG) implies that in steady-state $i(\omega,t)$ must be a constant over time ($i(\omega,t) = i(\omega)$). Then, differentiating (6) with respect to time:

$$\frac{\dot{i}(\omega)}{i(\omega)} = 0 = \frac{\dot{L}_I(\omega)}{L_I(\omega)} - \frac{\dot{X}(\omega)}{X(\omega)}$$

and using (TEG), it follows:

$$\frac{\dot{L}_I(\omega)}{L_I(\omega)} - \mu(\omega)i(\omega) = 0. \tag{9}$$

Since in steady-state $\frac{\dot{E}(t)}{E(t)} = 0$, the market-clearing condition implies that employment in both manufacturing and R&D sector is growing at the population growth rate (n). Moreover, a constant allocation of R&D employment among sectors is required by a steady-state analysis:

$$\frac{\dot{L}_I(\omega)}{L_I(\omega)} = \frac{\dot{L}_I}{L_I} = n$$

and from (9) we get:

$$i(\omega) = \frac{n}{\mu(\omega)}.$$

The expression above states that, per each sector, the instantaneous probability of innovation is inversely proportional to the parameter μ characterizing the law of motion of X.

As (4) implies the steady-state interest rate to be $r(t) = \rho$, and as differentiating the arbitrage equation $A(\omega)v(\omega,t)/X(\omega,t) = 1$ with respect to time yields $\frac{\dot{v}(\omega,t)}{v(\omega,t)} = n$, the balanced-growth research arbitrage condition is:

$$\frac{\pi(\omega,t)A(\omega)}{X(\omega,t)\left[\rho + \frac{n}{\mu(\omega)} - n\right]} = 1. \tag{10}$$

Then the equilibrium function for $X(\omega,t)$ is

$$X(\omega,t)=\frac{\pi(\omega,t)A(\omega)}{\left[\rho+\dfrac{n}{\mu(\omega)}-n\right]}.$$

Given (6), the equilibrium per-sector research employment is

$$L_I(\omega,t)=\frac{nX(\omega,t)}{\mu(\omega)A(\omega)}=\frac{\pi(\omega,t)}{\mu(\omega)\left[\dfrac{\rho}{n}+\dfrac{1-\mu(\omega)}{\mu(\omega)}\right]}$$

and its population-adjusted value is given by

$$l_I(\omega)\equiv\frac{L_I(\omega,t)}{L(t)}=\frac{\dfrac{\lambda(\omega)-1}{\lambda(\omega)}\alpha(\omega)E}{\mu(\omega)\left[\dfrac{\rho}{n}+\dfrac{1-\mu(\omega)}{\mu(\omega)}\right]}. \tag{11}$$

As in steady-state the per-capita version of the resource condition (8) becomes:

$$1=E\int_0^1\frac{\alpha(\omega)}{\lambda(\omega)}d\omega+\int_0^1 l_I(\omega)d\omega, \tag{12}$$

then the system made up of (11) and (12) defines the steady-state values of per-capita consumption E and of the population-adjusted research employment in each sector $l_I(\omega)$. Using (11) to substitute for $l_I(\omega)$ into (12), we get:

$$E=\left[\int_0^1\left(\frac{\alpha(\omega)}{\lambda(\omega)}+\frac{\dfrac{\lambda(\omega)-1}{\lambda(\omega)}\alpha(\omega)}{\mu(\omega)\left[\dfrac{\rho}{n}+\dfrac{1-\mu(\omega)}{\mu(\omega)}\right]}\right)d\omega\right]^{-1}=\bar{E}.$$

Plugging $\bar{E}$ into (11), we finally obtain:

$$l_I(\omega)=\frac{\dfrac{\lambda(\omega)-1}{\lambda(\omega)}\alpha(\omega)\bar{E}}{\mu(\omega)\left[\dfrac{\rho}{n}+\dfrac{1-\mu(\omega)}{\mu(\omega)}\right]}.$$

We now turn to comparative statics analysis and state the following:

Proposition 2 .

-In each sector the population-adjusted research effort is an increasing function of the quality jump and utility weight and a decreasing function of the difficulty parameter.

-the population-adjusted value of the aggregate research effort is negatively correlated with the rate of time preferences and positively correlated with the population growth rate.

Proof. See the appendix. ■

Since the growth rate of individual utility can be thought of as the measure of the economy's growth rate, we can now solve for its steady-state value. Substituting for $p(j,\omega,t) = \lambda(\omega)l_\omega$ and $E(t) = E$, the balanced growth value of the utility implies:

$$\log u(t) = \log E + \int_0^1 \alpha(\omega)\left[\log\alpha(\omega) + \log\left[\lambda(\omega)\right]^{j^{\max}(\omega,t)} - \log\left[\lambda(\omega)l_\omega\right]\right]d\omega =$$

$$= \log E + \int_0^1 \alpha(\omega)\log\alpha(\omega)d\omega + \int_0^1 \alpha(\omega)\log\left[\lambda(\omega)\right]^{j^{\max}(\omega,t)}d\omega - \int_0^1 \alpha(\omega)\log\left[\lambda(\omega)l_\omega\right]d\omega.$$

Since $\int_0^1 \alpha(\omega)\log\left[\lambda(\omega)\right]^{j^{\max}(\omega,t)}d\omega = \int_0^t\int_0^1\left[i(\omega,\tau)\alpha(\omega)\log\lambda(\omega)d\omega\right]d\tau$ (where $\int_0^t i(\omega,\tau)d\tau$ represents the expected number of successes in industry ω up to time t), differentiating $\log u(t)$ with respect to time yields:

$\frac{\dot{u}}{u} = n\int_0^1 \frac{\alpha(\omega)}{\mu(\omega)}\log\lambda(\omega)d\omega \equiv n\log\hat{\lambda}$ where $i(\omega,\tau)$ has been substituted for its steady-state value.

Steady-State under PEG Specification

In balanced growth the arbitrage equation (7) becomes:

$$\frac{\pi(\omega,t)A(\omega)}{X(\omega,t)\left[\rho + i(\omega,t) - n\right]} = 1$$

where again $\rho = r(t)$ and $\frac{\dot{v}(\omega)}{v(\omega)} = \frac{\dot{X}(\omega)}{X(\omega)}$ with (PEG) requiring $\frac{\dot{X}(\omega)}{X(\omega)} = n$.

Then $i(\omega) = \frac{\pi(\omega,t)A(\omega)}{X(\omega,t)} - \rho + n = \frac{\lambda(\omega)-1}{\lambda(\omega)k}\alpha(\omega)A(\omega)E - \rho + n.$

Notice that the probabilities of innovations $i(\omega)$ in steady-state are now proportional to industry-specific profits and research technologies.

As $l_I(\omega) = \dfrac{i(\omega)X(\omega,t)}{L(t)A(\omega)}$, from the expression above we get:

$$l_I(\omega) = \frac{\lambda(\omega)-1}{\lambda(\omega)}\alpha(\omega)E - \frac{k(\rho-n)}{A(\omega)} \qquad (13)$$

The steady-state resource condition is given by:

$$1 = \int_0^1 \frac{\alpha(\omega)E}{\lambda(\omega)}d\omega + \int_0^1 l_I(\omega)d\omega. \qquad (14)$$

The two equations (13) and (14) define the steady-state values of per-capita consumption E and of the population-adjusted research employment in each sector $l_I(\omega)$.

Using (13) to substitute for $l_I(\omega)$ into (14), we obtain the steady-state value for E:

$$\bar{E} = 1 + k(\rho-n)\int_0^1 \frac{1}{A(\omega)}d\omega.$$

Plugging the expression above into (13) we get the steady-state value for $l_I(\omega)$:

$$l_I(\omega) = \frac{\lambda(\omega)-1}{\lambda(\omega)}\alpha(\omega) + k(\rho-n)\left[\frac{\lambda(\omega)-1}{\lambda(\omega)}\alpha(\omega)\int_0^1 \frac{1}{A(\omega)}d\omega - \frac{1}{A(\omega)}\right].$$

As before, we proceed with the comparative statics analysis by stating the following:

Proposition 3 .

-In each sector the population-adjusted research effort is an increasing function of the quality jump and utility weight.

-the population-adjusted value of the aggregate research effort is negatively correlated with the rate of time preferences and positively correlated with the population growth rate.

Proof. See the appendix. ■

Analogously to the TEG case, we can derive the steady-state growth rate of utility. Differentiating $\log u(t)$ with respect to time yields:

$$\frac{\dot{u}}{u} = \int_0^1 i(\omega)\alpha(\omega)\log\lambda(\omega)d\omega.$$

0.13 Concluding Remarks

In the previous pages we have generalized a standard quality-ladder model with increasing complexity in order to encompass economies with asymmetric fundamentals. We now sum up the basic differences with respect to the symmetric case both in the hypotheses and in the results.

We have assumed a R&D sector characterized by industry-specific quality jumps ($\lambda(\omega)$) and arrival rates ($A(\omega)$). While the first assumption makes the mark-up charged by each monopolist (and then profits) vary across industries, the second alters the per-sector profitability of engaging in R&D. Furthermore, as in the standard case, the consumer's utility is represented by a Cobb-Douglas function; however asymmetric utility contributions ($\alpha(\omega)$) of each good are now assumed. Accordingly, sector-specific market demands and profits are derived. Finally in manufacture we have assumed different production costs across sectors (l_ω). This hypothesis does not affect equilibrium quantities.

While these features are independent of the kind of increasing complexity adopted, we now discuss the specific results relative to TEG and PEG hypotheses. The steady-state under TEG specification is characterized by industry-specific R&D efforts ($L_I(\omega)$) and research intensities ($i(\omega)$): these intensities are negatively correlated with the industry-specific difficulty parameters $\mu(\omega)$. Actually, with sector-specific arrival rates ($A(\omega)$) and profits ($\pi(\omega)$) (due to asymmetric fundamentals), returns in R&D are in fact equalized by the industry-specific equilibrium values of $X(\omega)$. On the other hand, the steady-state values of research intensity under PEG specification are industry-specific and proportional to the different arrival rates, utility weights and the mark-ups charged in each sector. In this case, as the law of motion of $X(\omega)$ is ω-independent, the per-sector research employment ($L_I(\omega)$) is the variable which makes the engaging in each R&D sector equally profitable. The different configuration of the research intensities under TEG and PEG specification is finally reflected in two distinct steady-state growth rates of utility. The comparative statics analysis on the steady-state values of E and $l_I(\omega)$ yields results analogous to those of the standard symmetric case.

Appendix

Proof of proposition 1.

Since the zero measure of each industry makes negligible the contribution of the variation of a ω-specific λ to $\bar{E}$, then $\dfrac{d\bar{E}}{d\lambda} = 0$. Analogously it will be $\dfrac{d\bar{E}}{d\alpha} = 0$ and $\dfrac{d\bar{E}}{d\mu} = 0$. Then, for any given ω:

$$\frac{dl_I}{d\lambda} = \frac{\frac{1}{\lambda^2}\alpha\bar{E}}{\mu\left(\frac{\rho}{n} + \frac{1-\mu}{\mu}\right)} > 0 \text{ as, by assumption, } \mu > 0$$

$$\frac{dl_I}{d\alpha} = \frac{\frac{\lambda-1}{\lambda}\bar{E}}{\mu\left(\frac{\rho}{n} + \frac{1-\mu}{\mu}\right)} > 0 \text{ as, by assumption, } \mu > 0$$

$$\frac{dl_I}{d\mu} = -\frac{\frac{\rho - n}{n}\frac{\lambda - 1}{\lambda}\alpha\bar{E}}{\left[\mu\left(\frac{\rho}{n} + \frac{1-\mu}{\mu}\right)\right]^2} < 0.$$

As $L_I \equiv \int_0^1 l_I(\omega)d\omega = 1 - \bar{E}\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega$, then $\frac{dL_I}{d\rho} = -\frac{d\bar{E}}{d\rho}\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega$ and $\frac{dL_I}{dn} = -\frac{d\bar{E}}{dn}\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega$.

Therefore, since $\frac{d\bar{E}}{d\rho} = -\int_0^1 \left(\frac{\alpha(\omega)}{\lambda(\omega)}\frac{[\mu(\omega)(1-\lambda(\omega))]}{\left[\left(\frac{\rho}{n}-1\right)\mu(\omega)+1\right]^2}\right)d\omega\bar{E}^2 > 0$ and

$$\frac{d\bar{E}}{dn} = -\int_0^1 \frac{\alpha(\omega)\mu(\omega)\rho\lambda(\omega)\left[\lambda(\omega)-1\right]}{n^2\left[\lambda(\omega)\left[\left(\frac{\rho}{n}-1\right)\mu(\omega)+1\right]\right]^2}\bar{E}^2 < 0 \text{ (as } \lambda(\omega) > 1\text{)},$$

we can finally state:

$$\frac{dL_I}{d\rho} < 0 \text{ and } \frac{dL_I}{dn} > 0. \blacksquare$$

Proof of proposition 2.

$$\frac{dl_I}{d\lambda} = \frac{1}{\lambda^2}\alpha\left[1 + k(\rho - n)\int_0^1 \frac{1}{A(\omega)}d\omega\right] > 0$$

$$\frac{dl_I}{d\alpha} = \frac{\lambda(\omega)-1}{\lambda(\omega)}\left[1 + k(\rho - n)\int_0^1 \frac{1}{A(\omega)}d\omega\right] > 0.$$

As $L_I \equiv \int_0^1 l_I(\omega)d\omega = 1 - \bar{E}\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega$, then $\frac{dL_I}{d\rho} = -\frac{d\bar{E}}{d\rho}\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega$ and $\frac{dL_I}{dn} = -\frac{d\bar{E}}{dn}\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega$.

Therefore, since $\frac{d\bar{E}}{d\rho} = k\int_0^1 \frac{1}{A(\omega)}d\omega > 0$ and $\frac{d\bar{E}}{dn} = -k\int_0^1 \frac{1}{A(\omega)}d\omega < 0$,

we can finally state:

$$\frac{dL_I}{d\rho} < 0 \text{ and } \frac{dL_I}{dn} > 0.$$

$$\frac{dE}{dk} = (\rho - n)\int_0^1 \frac{1}{A(\omega)}d\omega > 0. \blacksquare$$

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Uncertainty-Averse Agents in a Quality-Ladder Growth Model with Asymmetric Fundamentals

Quality ladders growth models (such as Grossman-Helpman 1991, Segerstrom 1998) focus on the role of technical progress as the main source of economic growth. In this class of models technical change is the outcome of R&D investment decisions taken by profit maximizing firms. Any product line can be improved an infinite number of times by means of research and the firms manufacturing the most updated version of a product monopolize the relative market and thus earn positive profits. However, these profits have a temporary nature as they only last until the next improvement in the same product line occurs;

in the neo-Schumpeterian literature this effect is commonly referred to as 'creative destruction'. The profitability of undertaking R&D in each sector depends on three sets of conditions: the magnitude of the profit flows associated with any monopolistic position, the difficulty of acquiring such a position (i.e. the probability of innovating) and its expected duration. We can refer to the first two sets of conditions as the fundamentals of the economy, as they respectively depend on the costs and demand conditions of the commodity sector and on the technology of the research industry. On the other hand the expected duration of the profit flows in a particular R&D sector depends on the agents' expectations on the future amount of research which will be carried out in that sector; that is, future affects current investment decisions to the extent that agents anticipate the 'creative destruction' effect.

The standard literature assumes that the economy has symmetric fundamentals: any sector shares the same profit flows and probabilities of innovating. In order to equalize the overall profitability across the R&D sectors, symmetric expectations on future R&D investments are also assumed. With these assumptions, the focus on the symmetric equilibrium in R&D investments is made plausible.

In chapter 3 I have generalized the standard model (in particular I referred to Segerstrom 1998) by assuming asymmetric fundamentals. In this case, the creative destruction effect due to the expectations on future research is the sole responsible for equalizing the expected returns in R&D; such equalization is required if we want to derive positive R&D efforts in each industry. Moreover, as rational expectations equilibrium requires the equality between expectations on research investments and their actual values, the actual investments in R&D will equalize the returns across sectors. Still, it does not seem that this equilibrium is uniquely pinned down; equal future profitability makes the investor indifferent as to where targeting research. As a result, when the expectations on future research equalize the expected returns in R&D, the allocation problem of investments across product lines is indeterminate.

In this paper I assume that expectations on the future distribution of research investment are uncertain. The basic idea is that, while the agents perfectly foresee the fundamentals of the economy, their beliefs on the future (per sector) distribution of R&D investments are characterized by uncertainty, in the sense that information about that distribution is too imprecise to be represented by a (single additive) probability measure. The distinction between risk and uncertainty traces back to Frank Knight (1921) and states that risk is associated with ventures in which an objective probability distribution of all possible events is known, while uncertainty characterizes choice settings in which that probability distribution is not available to the decision-maker. In order to formalize the decision making under uncertainty I adopt the maxmin expected utility decision rule axiomatized by Gilboa and Schmeidler (1989). According to this rule, an uncertainty-averse decision maker is provided, not with a unique prior - as in the standard expected utility theory -, but with a *set of priors*. When evaluating the different acts to choose among, this agent will compute the *minimal* expected utility over her set of priors for each of them (acts), and then will single out the one associated with the highest computed value. Accordingly to this framework I will assume the decision maker to maximize her expected pay-off with respect to the R&D investment decision, while singling out the minimizing choice scenario, that is, the worst probability distribution over the future configuration of R&D investments. In this paper the maxmin decision rule eliminates indeterminacy by selecting a unique equilibrium. As we will see below the emerging equilibrium contradicts the conclusion of the standard model as it does not equalize returns across sectors.

The rest of the paper is organized as follows. In Section 2 I recall the structure of the R&D sector. In Section 3 I explain the core of the argument, enunciate and prove the propositions. In Section 3 I conclude with some remarks.

0.14 The R&D Sector

The structure of the model is assumed to be the same as the one outlined in chapter 3 under PEG specification of the increasing complexity index. As the problem I focus on consists of the agent's choice on how to allocate her saving among the different research industries I briefly recall the structure of the R&D sector. The research sector is made up of firms engaging in R&D to develop better versions of the existing products (indexed by ω) in order to displace the current monopolists. Free entry and perfect competition are assumed in each R&D race. Firms employ labor and produce, through a constant returns technology, a Poisson arrival rate of innovation in the product line they target. Any firm hiring l_k units of labor in industry ω at time t acquires the instantaneous probability of innovating $A(\omega)l_k/X(\omega,t)$, where $X(\omega,t)$ is the R&D difficulty index introduced to rule out the 'scale effect'. Since independent Poisson processes are additive, the specification of the innovation process implies that the industry-wide instantaneous probability of innovation (or research intensity) is

$$\frac{A(\omega)L_I(\omega,t)}{X(\omega,t)} \equiv i(\omega,t)$$

where $L_I(\omega,t)=\sum_k l_k(\omega,t)$. As R&D proceeds, its difficulty index $X(\omega,t)$ is supposed to increase over time in order to rule out the scale effect. With reference to Dinopoulos and Segerstrom (1999), we model the increasing complexity hypothesis according to the 'PEG specification':

$$X(\omega,t) = \mu L(\omega,t) \qquad \text{(PEG)}$$

where μ is a positive constant.

Whenever a firm succeeds in innovating, she acquires the uncertain profit flow that accrues to a monopolist, that is, the stock market valuation of the firm: $v(\omega,t)$. Thus, the problem faced by a R&D firm is that of choosing the amount of labor input in order to maximize her expected profits:

$$\max_{l_k}\left[\frac{v(\omega,t)A(\omega)}{X(\omega,t)}l_k - l_k\right]$$

which provides a finite, positive solution for l_k only when the arbitrage equation[38]:

$$\frac{v(\omega,t)A(\omega)}{X(\omega,t)} = 1$$

[38] We consider the wage rate as the numerarie.

is satisfied. Efficient financial markets require that the stock market valuation of the firm yields an expected rate of return equal to the riskless interest rate $r(t)$. Then, the firm's market valuation is:

$$v(\omega,t) = \frac{\pi(\omega,t)}{r(t) + \frac{A(\omega)L_I(\omega,t)}{X(\omega,t)} - \frac{\dot{v}(\omega,t)}{v(\omega,t)}}$$

where $\pi(\omega,t)$ is the profit flow in industry ω.

Then R&D equilibrium condition is

$$\frac{\pi(\omega,t)A(\omega)}{X(\omega,t)\left[r(t) + \frac{A(\omega)L_I(\omega,t)}{X(\omega,t)} - \frac{\dot{v}(\omega,t)}{v(\omega,t)}\right]} = 1.$$

0.15 The Equilibrium with Uncertainty-Averse Agents

In deriving the result I assume that the agent's beliefs on the future composition of R&D efforts are characterized by Knightian uncertainty. This key-assumption essentially incorporates the idea that the agent ignores the future composition of R&D investments across sectors as well as a probability distribution over any of them.

In the decision-making process, a preference for certainty (or 'uncertainty-aversion') is introduced and basically formalized as in Gilboa-Schmeidler (1989). The choice of an uncertainty-averse agent is thus represented as the result of a two-player zero-sum game in which, say, player 1 selects the minimizing scenario with respect to the variable subject to strong uncertainty, while player 2 maximizes her expected pay-off with respect to her decision variable by taking into account the 'worst-case strategy' carried out by player 1. In particular, our agent is asked to allocate a certain amount of R&D investment among all the existing sectors. As the agent is assumed to be uncertatinty-averse, and his set of priors is unbounded, in maximizing her expected pay-off she will assign probability one to the worst possible configuration that a 'malevolent nature'will be choosing. We denote with $l_m + \gamma(\omega, t)$ the agent's investment in sector ω, and with $L_I^e(t) + \varepsilon(\omega, t)$ the aggregate expected research in sector ω. l_m and $L_I^e(t)$ are, respectively, the average agent's investment per sector and the average expected research per sector. $\varepsilon(.)$ and $\gamma(.)$ represent deviations from the averages satisfying:

$$\int_0^1 \varepsilon(\omega, t) d\omega = 0 \qquad \int_0^1 \gamma(\omega, t) d\omega = 0 \qquad \text{and}$$

$$\varepsilon(\omega, t) > -L_I^e(t) \qquad \gamma(\omega, t) > -l_m.$$

The presence of the two functions $\gamma(.)$ and $\varepsilon(.)$ is intended to allow for asymmetry both in the agent's investment and in expected research[39]. It is worth noting that unlike in chapter 3 the expected amount of research in any sector $L_I^e(\omega, t)$, on which the 'creative destruction effect' depends, is allowed to differ from the actual value ($L_I(\omega, t)$). In fact, the assumption that beliefs on the future per-sector composition of research effort are characterized by strong

[39] These definitions imply:
$\int_0^1 [L_I + \varepsilon(\omega)] \, d\omega = L_I = L \int_0^1 [l_m + \alpha(\omega)] \, d\omega = Ll_m$
where L denotes the number of agents in the economy.
With reference to Section 2 the following relation between l_k and l_m holds:
$\int_0^1 \left[\sum_k l_k(\omega)\right] d\omega = Ll_m.$

uncertainty will break down the identity between the actual and expected R&D efforts. As we will see below in the steady state analysis the result of this procedure is a non rational expectation equilibrium where expectations on future R&D turn out to be continuously inconsistent with their actual values.

With reference to the quality-ladder model with asymmetric fundamentals sketched out in the previous chapter and given PEG specification, we can enunciate the following

Proposition 4 *Uncertainty-averse agents, who adopt a maxmin strategy to face strong uncertainty on future R&D per-sector distribution, choose an asymmetric investment strategy:* $l_m + \gamma(\omega,t) = l_m \pi(\omega,t) / \int_0^1 \pi(\omega,t) d\omega$, $\forall \omega \in [0,1]$. *Their optimal investment choice makes them expect an asymmetric composition of future R&D effort among sectors:*

$$L_I^e(t)+\varepsilon(\omega,t) = L_I^e(t)\pi(\omega,t)/\int_0^1 \pi(\omega,t)d\omega+[\rho-n]\,\mu L(T)\left[\pi(\omega,t)/\int_0^1 \pi(\omega,t)d\omega \int_0^1 1/A(\omega)d\omega - 1/A(\right.$$

$\forall \omega \in [0,1]$.

Proof. See Appendix A1. ■

We now solve the model for its steady-state values. In steady-state all endogenous variables grow at constant rates and $r(t) = \rho$ (as $\frac{\dot{E}(t)}{E(t)} = 0$). Hence, by substituting for $L_I^e + \varepsilon(\omega,t)$ as derived in proposition 1 into the research arbitrage condition (7), we get:

$$\frac{\pi(\omega,t)A(\omega)}{\mu L(t)} = \rho - n + \frac{A(\omega)}{\mu L(t)}\left[L_I^e(t)\frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega} + \mu L(t)[\rho-n]\left[\frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega}\int_0^1 \frac{1}{A(\omega)}d\omega - \frac{1}{A(\omega)}\right]\right]$$

whence:

$$L_I^e(t) = \int_0^1 \pi(\omega,t)d\omega - \mu L(t)[\rho - n]\int_0^1 \frac{1}{A(\omega)}d\omega \qquad (9)$$

Notice that, while strong uncertainty hits the expectations on per-sector composition of future R&D, it does not affect expectations on the total amount of R&D which will be carried on. Then, even when allowing for $[l_m + \gamma(\omega,t)]\, L(t) \neq L_I^e(t) + \varepsilon(\omega,t)$, $\forall\omega \in [0,1]$, it is however $L_I^e(t) = l_m L(t) \equiv L_I(t)$.

The market-clearing condition is:

$$L(t) = \int_0^1 \frac{\alpha(\omega) E L(t)}{\lambda(\omega)} d\omega + L(t)\int_0^1 [l_m + \gamma(\omega,t)]\, d\omega$$

Manipulating, it can be written as:

$$1 = E\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)} d\omega + l_m \qquad (10)$$

Provided that $\pi(\omega,t) = \dfrac{\lambda(\omega)-1}{\lambda(\omega)}\alpha(\omega)E(t)L(t)$, the steady-state resource (10) and arbitrage (9) equations allow us to find the equilibrium values of l_m and E.

$$E = 1 + \mu(\rho - n)\int_0^1 \frac{1}{A(\omega)}d\omega$$

$$l_m = 1 - \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega - \mu(\rho - n)\int_0^1 \frac{1}{A(\omega)}d\omega \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega$$

It is easy to show that the usual comparative statics results hold:

$\dfrac{dl_m}{d\rho} < 0$ and $\dfrac{dl_m}{dn} > 0$

$\dfrac{dE}{d\rho} > 0$ and $\dfrac{dE}{dn} < 0$.

As the per-sector investment strategy is given by:

$$L(t)\,[l_m + \gamma(\omega)] = L(t) l_m \frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega},$$

we can now complete the model by solving for the steady-state values of per-capita investment in each research sector:

$$[l_m + \gamma(\omega)] = \left[1 - \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)} d\omega - \mu(\rho - n) \int_0^1 \frac{1}{A(\omega)} d\omega \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)} d\omega\right] \frac{\pi(\omega, t)}{\int_0^1 \pi(\omega, t) d\omega}$$

$$[l_m + \gamma(\omega)] = \alpha(\omega) - \frac{\alpha(\omega)}{\lambda(\omega)} - \mu(\rho - n) \int_0^1 \frac{1}{A(\omega)} d\omega \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)} d\omega \frac{\alpha(\omega) - \frac{\alpha(\omega)}{\lambda(\omega)}}{1 - \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)} d\omega}.$$

In order to understand the effect on uncertainty on the steady state R&D per sector investment, we need to compare this result with what we derived in the previous chapter when agents were assumed to have perfect foresight on the future distribution of R&D investment. In that case we had:

$$l_I(\omega) = \alpha(\omega) - \frac{\alpha(\omega)}{\lambda(\omega)} + \mu(\rho - n) \left[\left(\alpha(\omega) - \frac{\alpha(\omega)}{\lambda(\omega)}\right) \int_0^1 \frac{1}{A(\omega)} d\omega - \frac{1}{A(\omega)}\right]$$

It is straightforward to notice that R&D investments are different if we assume certainty or uncertainty conditions as in the latter case they do not depend on the industry specific arrival rate ($A(\omega)$). As in the first case investments were derived in such a way that returns across sectors were equalized, it turns out that when agents are uncertainty averse the steady state returns cross sectors differ systematically. This conclusion is of course surprising as we would expect intersectoral competition to erase differences in the overall profitability across sectors. However, as the agents are assumed to always expect the worst possible scenario as regards the future distribution of research, the allocation which maximizes their payoff requires that returns are not equalized.

0.16 Conclusions

One of the basic features of neo-schumpeterian growth models is the forward looking nature of the creative destruction effect; that is expectations on future

R&D investments affect the current allocation of R&D resources. In Chapter 3 I have derived the R&D investments which equalize returns across sectors of an economy with asymmetric fundamentals. However, analogously to the symmetric case of Chapter 1, it can be argued that equalization of expected returns is not sufficient to make the agent choose the investments configuration which actually equalizes returns; in fact agents become indifferent as their investments strategy, so that the returns-equalizing equilibrium is not univocally determined.

In order to rule out indeterminacy I have assumed that beliefs on the future distribution of R&D efforts are uncertain and that agents adopt a max-minimizing strategy to face such an uncertainty. While this assumption allows solving the indeterminacy problem it does not refound the returns equalizing equilibrium.

Appendix

Proof. Proof. $\max\limits_{\gamma(\cdot)}\left[\min\limits_{\varepsilon(\cdot)}\int\limits_0^1 \left[l_m+\gamma(\omega,t)\right]v(\omega,t)\frac{A(\omega)}{X(\omega,t)}d\omega\right]$ ■

s.t.$\int\limits_0^1 \gamma(\omega,t)d\omega=\int\limits_0^1 \varepsilon(\omega,t)d\omega=0$

$\varepsilon(\omega,t)>-L_I^e; \qquad \gamma(\omega,t)>-l_m.$

where $v(\omega,t)\equiv\dfrac{\pi(\omega,t)}{r(t)-\dfrac{\dot{v}(\omega,t)}{v(\omega,t)}+\dfrac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right]}.$

Under PEG specification $X(\omega,t)=\mu L(t)$, whence $\dfrac{\dot{X}(\omega,t)}{X(\omega,t)}=n$. Then, by differentiating (7) with respect to time, we obtain: $\dfrac{\dot{v}(\omega,t)}{v(\omega,t)}=\dfrac{\dot{X}(\omega,t)}{X(\omega,t)}=n.$

Given these conditions, we first solve for the minimization problem:

$$\min_{\varepsilon(\cdot)} \int_0^1 \frac{[l_m + \gamma(\omega,t)]\,\pi(\omega,t)}{\frac{\mu L(t)}{A(\omega)}(r(t)-n) + L_I^e(t) + \varepsilon(\omega,t)} d\omega$$

s.t. $\int_0^1 \varepsilon(\omega,t)d\omega = 0$

We set $e(\omega,t) = \int_0^\omega \varepsilon(s,t)ds$; then $e'(\omega,t) = \varepsilon(\omega,t)\ \forall\omega \in [0,1]$

and the minimization problem ($\mathrm{P_{min}}$) can be expressed as:

$$\min_{e'(\cdot)} \int_0^1 G(e')d\omega$$

s.t. $e(0) = 0$; $e(1) = 0$

where $G(e') = \dfrac{[l_m + \gamma(\omega,t)]\,\pi(\omega,t)}{\frac{\mu L(t)}{A(\omega)}(r(t)-n) + L_I^e(t) + e'(\omega,t)}$

This is the simplest problem of calculus of variations.

Since under the conditions specified above $G(e') \in C^2$, we can apply the Euler theorem stating that:

if $G(e,e',\omega) \in C^2$ and e^* is optimal and C^1, then e^* must necessarily solve:

$$G_e - \frac{d}{d\omega}G_{e'} = 0 \qquad \text{(E-E)}$$

As in our case G does not depend on e, $G_e = 0$; hence E-E becomes:

$\dfrac{d}{d\omega}G_{e'} = 0$, implying that $G_{e'} \equiv G_\varepsilon = -\dfrac{\pi(\omega,t)\,[l_m + \gamma(\omega,t)]}{\left[\frac{\mu L(t)}{A(\omega)}\,[r(t)-n] + L_I^e(t) + \varepsilon(\omega,t)\right]^2}$

be constant with respect to ω.

Thus, $\dfrac{\pi(\omega,t)\,[l_m + \gamma(\omega,t)]}{\left[\frac{\mu L(t)}{A(\omega)}\,[r(t)-n] + L_I^e(t) + \varepsilon(\omega,t)\right]^2} = k_1$ where k_1 is a real constant.

Now we solve the expression above for $\varepsilon(\omega)$:

$$\sqrt{\frac{\pi(\omega,t)[l_m + \gamma(\omega,t)]}{k_1}} = \frac{\mu L(t)}{A(\omega)}\,[r(t)-n] + L_I^e(t) + \varepsilon(\omega,t)$$

$$\varepsilon(\omega) = \sqrt{\frac{\pi(\omega,t)\left[l_m+\gamma(\omega,t)\right]}{k_1}} - \frac{\mu L(t)}{A(\omega)}[r(t)-n] - L_I^e(t) \qquad \text{(R-F)}$$

This function can easily be interpreted as the *reaction function* (R-F) of the 'nature' to the agent's decision. We can now plug it into the maximization problem ($P_{\max}$) and solve for γ:

$$\max_{\gamma(.)} \int_0^1 [l_m+\gamma(\omega,t)] \frac{\pi(\omega,t)}{\frac{\mu L(t)}{A(\omega)}[r(t)-n] + L_I^e(t) + \sqrt{\frac{\pi(\omega,t)\left[l_m+\gamma(\omega,t)\right]}{k_1}} - \frac{\mu L(t)}{A(\omega)}[r(t)-n] - L_I^e(t)} d\omega$$

$$\text{sub} \int_0^1 \gamma(\omega,t)d\omega = 0$$

Rearranging, this problem becomes:

$$\max_{\gamma(.)} \int_0^1 [l_m+\gamma(\omega,t)]^{\frac{1}{2}} \left(\pi(\omega,t)k_1\right)^{\frac{1}{2}} d\omega$$

$$\text{sub} \int_0^1 \gamma(\omega,t)d\omega = 0.$$

Again, we solve $P_{\max}$ as a problem of calculus of variations.

By setting:

$c(\omega,t) = \int_0^\omega \gamma(s,t)ds$, so that $c'(\omega,t) = \gamma(\omega,t)$,

$P_{\max}$ becomes:

$$\max_{c'} \int_0^1 F(c')d\omega$$

sub $c(0)=0$; $c(1)=0$

where $F(c') \equiv F(\gamma) = [l_m+\gamma(\omega,t)]^{\frac{1}{2}} \left[\pi(\omega,t)k_1\right]^{\frac{1}{2}}$

With the same reasoning as before, the Euler Equation $F_c - \frac{d}{d\omega} F_{c'} = 0$ implies:

$$F_{c'} \equiv F_\gamma = -\frac{(\pi(\omega,t)k_1)^{\frac{1}{2}}}{2[l_m + \gamma(\omega,t)]^{\frac{1}{2}}} = -k_2 \quad \text{where } k_2 \in R_+. \qquad [F_\gamma]$$

From F_γ we can derive the expression for $\gamma(\omega,t)$:

$$\gamma(\omega,t) = \frac{\pi(\omega,t)k_1}{4k_2^2} - l_m \qquad (1)$$

Plugging it into the (R-F), we get:

$$\varepsilon(\omega) = \sqrt{\frac{\pi(\omega,t)\left[l_m + \frac{\pi(\omega,t)k_1}{4k_2^2} - l_m\right]}{k_1}} - \frac{\mu L(t)}{A(\omega)}[r(t)-n] - L_I^e(t)$$

$$= \frac{\pi(\omega,t)}{2k_2} - \frac{\mu L(t)}{A(\omega)}[r(t)-n] - L_I^e(t) \qquad (2)$$

Now we can use the two conditions imposed by the constraints in order to find the constants k_1, k_2:

$$\int_0^1 \gamma(\omega,t)d\omega = 0 \iff \int_0^1 \left[\frac{\pi(\omega,t)k_1}{4k_2^2} - l_m\right] d\omega = 0$$

Hence:

$$k_1 = \frac{4k_2^2 l_m}{\int_0^1 \pi(\omega,t)d\omega} \qquad (3)$$

$$\int_0^1 \varepsilon(\omega,t)d\omega = 0 \iff \int_0^1 \left[\frac{\pi(\omega,t)}{2k_2} - \frac{\mu L(t)}{A(\omega)}[r(t)-n] - L_I^e(t)\right] d\omega = 0,$$

whence:

$$k_2 = \frac{\int_0^1 \pi(\omega,t)d\omega}{2L_I^e(t) + 2\mu L(t)[r(t)-n]\int_0^1 \frac{1}{A(\omega)}d\omega} \qquad (4)$$

Substituting (4) into (3), we obtain:

$$k_1 = \frac{l_m \int_0^1 \pi(\omega,t)d\omega}{\left[L_I^e(t) + \mu L(t)[r(t)-n]\int_0^1 \frac{1}{A(\omega)}d\omega\right]^2} \qquad (5)$$

Finally we can plug (4) and (5) into (1) and (2) in order to get the optimal pair $\gamma^*(\omega,t)$, $\varepsilon^*(\omega,t)$:

$$\gamma^*(\omega) = \frac{\pi(\omega,t)k_1}{4k_2^2} - l_m = \frac{\pi(\omega,t)\dfrac{l_m\int\limits_0^1 \pi(\omega,t)d\omega}{\left[L_I^e(t)+\mu L(t)[r(t)-n]\int\limits_0^1 \dfrac{1}{A(\omega)}d\omega\right]^2}}{4\left[\dfrac{\int\limits_0^1 \pi(\omega,t)d\omega}{2L_I^e(t)+2\mu L(t)[r(t)-n]\int\limits_0^1 \dfrac{1}{A(\omega)}d\omega}\right]^2} - l_m$$

$$= l_m\left[\frac{\pi(\omega,t)}{\int\limits_0^1 \pi(\omega,t)d\omega} - 1\right]$$

$$\varepsilon^*(\omega) = \frac{\pi(\omega,t)}{\int\limits_0^1 \pi(\omega,t)d\omega}\left[L_I^e(t)+\mu L(t)[r(t)-n]\int\limits_0^1 \frac{1}{A(\omega)}d\omega\right] - \frac{\mu L(t)}{A(\omega)}[r(t)-n] - L_I^e(t) =$$

$$= L_I^e(t)\left[\frac{\pi(\omega,t)}{\int\limits_0^1 \pi(\omega,t)d\omega} - 1\right] + \mu L(t)[r(t)-n]\left[\frac{\pi(\omega,t)}{\int\limits_0^1 \pi(\omega,t)d\omega}\int\limits_0^1 \frac{1}{A(\omega)}d\omega - \frac{1}{A(\omega)}\right]$$

■

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Time-Varying Elasticity of Substitution and Economic Growth

0.17 Introduction

Since the very beginning of neoclassical growth theory it has been apparent that the elasticity of substitution between factors of production has a relevant role in determining the growth path of potential output. In his seminal paper 'A Contribution to the Theory of Economic Growth' Solow (1956, p.77) produced an example where a production function with a sufficiently high elasticity of substitution yields sustained growth and no steady states. That function constituted the first example of the constant elasticity of substitution (CES) production function; Solow himself, together with Arrow *et alii* (1961), later developed and made popular its general form and its basic properties[40]. The general form in per capita terms is well known:

$y = f(k) = \gamma\left[\delta k^{\rho} + (1-\delta)\right]^{\frac{1}{\rho}}$ where y is per capita output, k the capital-labor ratio, γ is an efficiency parameter, $\delta \in [0,1]$ a distribution parameter and $\rho \in (-\infty, 1]$ a substitution parameter. In particular, ρ is a transform of the elasticity of substitution (σ), with $\sigma = 1/(1-\rho)$. When $\sigma > 1$ the marginal product of capital is bounded from below; this is a necessary condition for the incentive to capital accumulation to never vanish and then for sustained growth to be possible. In the example we mentioned above Solow was actually assuming $\sigma = 2$.[41] . Recent literature has assessed the relationship between the elasticity of substitution of a CES production function and per capita output growth. de La Grandville (1989) derived the threshold value of σ above which per capita accumulation never comes to an end; such a threshold turns out to be an inverse function of the population growth rate and of capital depreciation rate, and it is an increasing function of the saving rate[42]. Moreover, he showed by a graphical argument that, even if σ is not high enough to yield sustained growth, the growth rate of per-capita income during the transition to the steady state and the steady state values of per-capita income are both increasing function of σ. Klump and de La Grandville (2000) and Klump and Preissler (2000) basically proved the

[40] This function had already been introduced by Pitchford (1960). Brown and De Cani (1963) also developed it independently.

[41] Such necessary condition has been singled out by Pitchford (1960) for the CES production function; Ferguson (1965a, p.469) showed that it holds even for the case of a general constant returns to scale production function.

[42] Solow (1956,p.77) derives the threshold for sustained growth in terms of the saving rate (s). The production function he adopts is : $Y = (a\sqrt{K} + \sqrt{L})^2$,the threshold he obtains is $s > n/a^2$ where n is the population growth rate.

analogous results algebraically in the general framework of the normalized CES production function[43].

It is then clear that in the neoclassical theory the elasticity of substitution is an important source of growth. de La Grandville (1989) suggested that a high elasticity of substitution could have a played a key role in the development of the South Asian countries; his hypothesis has been tested and confirmed by Yuhn (1991). However, while the importance of the *level* of the elasticity of substitution is widely recognized in many growth accounting studies and it is viewed as a determinant of growth alternative to technical change[44], an *increasing* elasticity as a source of growth has been seldom applied to a growth model. Arrow et alii (1961,) envisioned the possibility that capital deepening might shift the elasticity of substitution; this suggestion led to the elaboration of the variable elasticity of substitution (VES) production functions where σ is a function of capital-labor ratio (see for example Sato and Hoffman (1968), Lu and Fletcher (1968) and Revankar (1971)). These functions simply posited a relation between σ and k; to my knowledge, the first attempt to make such a relation endogenous and to incorporate it in a standard neoclassical growth model has been provided by Miyagiwa and Papageorgiou (2004).

My basic idea is that the evolution of the elasticity of substitution is to be

[43] The normalization procedure adopted in de La Grandville (1989) consists in choosing arbitrarily baseline values for $\bar{k}$, $\bar{y}{=}f(\bar{k})$ and $\bar{m} = [f(\bar{k}) - \bar{k}f'(\bar{k})]/f'(\bar{k})$ (the marginal rate of substitution between capital and labor) and then solving for the parameters γ and δ as function of the sole baseline values and the elasticity of substitution.

[44] At this point it seems relevant to mention a fundamental 'impossibility theorem'. Diamond, McFadden and Rodriguez (1978) proved the impossibility of measuring both the elasticity of substitution and the bias of technical change given the time series of the relevant market variables (see also Nerlove (1967) for an accessible expostion). The literature we refer to avoids this impossibility by assuming some particular structure of the technical progress.

related to the mere passing of time. A similar hypothesis has been suggested by Sato and Hoffman (1968); they argued that as time passes and new technologies become available the opportunities for factor substitution are increased. I intend to micro-found this intuition by showing that profit-maximizing firms will be always willing to adopt substitution enhancing combinations of factors of production; then, the actual path followed by the elasticity of substitution has necessarily to be a non-decreasing one.

0.18 Anecdotal Empirical Evidence

The idea that the passage of time is factors' substitution enhancing came to my mind when studying the empirical literature on the estimates of σ. A complete survey of the huge existing literature on the subject is out of the scope of the analysis, but it is possible to quote some of the basic results to support the intuition.

We should first notice that comparing results deriving from different studies is usually misleading. Even when narrowing our attention to the U.S. economy and to estimates based on CES production functions, several sources can account for differences in the estimated σ. Among these, differences in the data sets, in the econometric techniques and in the assumptions on technical progress are worth mentioning. A fundamental distinction has to be made between time-series and cross-sectional estimates; the latter tend to be significantly higher than the former. Berndt (1976) reconciled the results obtained with the two different methodologies. However, recently Antras (2004) showed that Berndt's results could be due to the assumption of neutral technical progress; such an assumption necessarily biases upward the estimates.

Time-series estimates for the period 1899 up to 1960 never surpass $\sigma = .6$. Arrow et al. (1961) for the period 1909-49 obtain .57; Kendrick and Sato (1963) for 1919-60 get .58; Wilkinson (1968) for 1899-1953 provides .54; David and van de Klundert (1965) for 1899-1960 have .32; Sato (1970) for 1909-60 has .52. Of all these estimates the most reliable are probably Wilkinson's and Sato's ones as they both allow for factor augmenting non-neutral technical progress and they do not suffer of simultaneous equations biases.

When the starting year of the sample is moved forward to 1929 we get higher estimates. Ferguson (1965b) for the period 1929-63 obtains .67 ; Panik (1976) for years 1929-66 gets .76; Kalt (1978) for 1929-67 has .76. Berndt (1976) for 1929-1968 obtains estimates not significantly different from one but we have already mentioned why they could be biased upward. Even though this evidence is not compelling at all, it appears plausible to guess that the elasticity of substitution has increased from the period 1899-1929 to 1930-1968. Such trend seems to be confirmed by Antràs (2004) estimates for the period 1948-1998. When he allows for biased technical change his estimates range from .681 to .891 when computed with the econometric technique he considers the most efficient.

Stronger support in favor of our hypothesis comes from the studies of Brown and De Cani (1963) and again Ferguson (1965b). The former split their sample into 1919-37 and 1938-58 and the estimates for σ rises from .08 to .11; the latter finds that when his sample is restricted to the period 1948-63 σ reaches 1.16 (from .67). It is clear that these increases are to be trusted more than the previous evidence as they are obtained by using homogeneous methodologies and data, whatever they be. It then seems that the famous quotation by Nerlove (1967, p.58) : "even slight variations in the period [..] tend to produce drastically different estimates of the elasticity" is confirmed in the sense that later periods show a higher elasticity. Two important results however question our idea. Zarembka (1970, p.47) claims that 'changes in period do not produce significantly different estimates of the elasticity'; he supports his claim by showing that the elasticities for 1957 and 1958 do not significantly differ. It seems however that two years do not constitute a period long enough to assess the issue. Sato and Hoffman (1968) directly considers the possibility of a time trend in the elasticity of substitution by estimating the equation $\sigma = a + bt$ (where a and b are parameters and t is a measure of time). Their findings reject our conjecture for the U.S.1909-60 ($b < 0$) while they confirm it for Japan 1930-60 ($b > 0$). Still, this result should be considered carefully as it is based on the assumption of a linear time trend. I will show below that a change in such assumtion changes the result. No clear evidence of such positive time trend can instead be derived by cross-section estimates of the elasticity of substitution. Many of the studies provide disaggregated estimates for two-digit industries of the manufacturing sector; then the overall factors' elasticity of substitution of the economy is not readily available and comparisons across time become difficult. All these estimates are closer to one than those obtained in time series analysis. Still, some support to our hypothesis can be found. Griliches (1967) and Zarembka (1970), respectively for year 1958 and 1957-58, found that the elasticity of substitution does not significantly depart from one. Recent estimates instead tend to suggest that σ has become significantly higher than one. Bentolila and Saint Paul (2003) find $\sigma = 1.06$ for OECD countries (1972-93). Duffy and Papageorgiou (2000) find $\sigma > 1$ for a sample of 82 countries from 1960 to 1987 both with and without adjusting the labor force for human capital; the same result is confirmed by Masanjala and Papageorgiou (2003) for a 98 non-oil countries sample.

0.19 The model

The model consists of two parts[45]. In the first one I aim at showing that an increase in the elasticity of substitution increases profits so firms will always be willing to adopt substitution-enhancing changes in technology. In the second one I propose a way to model the evolution of the elasticity of substitution; as only increases in the elasticity will be actually implemented by firms, the resulting outcome of the actual technology in the economy should provide a non-decreasing path for the elasticity of substitution. According to the results derived by de La Grandville (1989) and Klump and de La Grandville (2000) such time-increasing path could be interpreted as an alternative explanation of

[45] The model relies on the assumption of a standard neoclassical production function with a single capital good homogeneous to final output. As it is well known from the Cambridge capital controversy, once heterogeneous capital goods are introduced, the basic properties connecting production and distribution in the neoclassical theory cannot be proved. Accordingly, there is no claim that our results can be extended to the case of heterogeneous capital goods.

economic growth. Moreover endogenous growth emerges as a possible outcome in case the process for σ converges to a level of σ higher than the threshold level singled out by de La Grandville (1989).

0.19.1 Firm

Let us consider perfectly competitive firms which maximize profits subject to a CES production function. We want to show that an increase in σ raises their profits for any given value of capital-labor ratio. The result follows by applying results derived by Klump and de La Grandville (2000) to the firm maximizing problem.

As it is well known, in a competitive equilibrium firms earn zero-profit and the scale of production is indeterminate. In equilibrium we have:

$\pi = pf(k) - rk - w = 0$

where π are profits per-worker, p is the output price and r and w are, respectively, the rental rate of capital and labor. As we want to study the comparative statics of production functions which differ only for the values of the elasticity of substitution, we normalize the CES function as in de La Grandville (1989). The normalization procedure consists in choosing arbitrarly baseline values for $\bar{k}$, $\bar{y}$=$f(\bar{k})$ and $\bar{m} = [f(\bar{k}) - \bar{k}f'(\bar{k})]/f'(\bar{k})$ (the marginal rate of substitution between capital and labor) and then solving for the parameters γ and δ as functions of the sole baseline values and the elasticity of substitution. Klump and de La Grandville (2000) derive the normalized CES function as:

$$y = f_\sigma(k) = \bar{y}\left(\frac{\bar{k}^{1-\rho}k^\rho + \bar{m}}{\bar{k}\ + \bar{m}}\right)^{\frac{1}{\rho}}$$

they also compute

$\dfrac{\partial f_\sigma(k)}{\partial \sigma} = -\dfrac{1}{\sigma^2}\dfrac{1}{\rho^2}y\left[\Pi\ln(\dfrac{\bar{\Pi}}{\Pi}) + (1-\Pi)\ln(\dfrac{1-\bar{\Pi}}{1-\Pi})\right]$ where Π and $\bar{\Pi}$ are the actual and the baseline profit share.

It turns out that $\dfrac{\partial f_\sigma(k)}{\partial \sigma} > 0\ \forall k \neq \bar{k}$ (see Klump and de La Grandville (2000, p.285)).

Then, no matter what the optimal capital-labor ratio any firm chooses, it is $\dfrac{\partial \pi}{\partial \sigma} > 0$.

0.19.2 The Evolution of the Elasticity of Substitution

In the previous section I have shown that the actual technology implemented by firms has to yield a non decreasing path of the elasticity of substitution. I propose here a possible process which approximates such path. A restriction I want to impose is bounding the elasticity of substitution from infinity to derive a finite value to which the process converges.

A process fitting these requirements is a first order difference equation:

$\sigma_t = \sigma_{t-1} + \varepsilon_t$ where $\varepsilon_t = \psi\varepsilon_{t-1}\tilde{u}_t$, $\psi \in (0,1)$, and $\sigma_0 \geq 0$, $\varepsilon_0 > 0$.

$$\text{where } \tilde{u}_t = \begin{cases} u_t \text{ if } u_t \geq 0 \\ \\ 0 \text{ if } u_t < 0. \end{cases}$$

and u_t is a zero-mean random variable rectangularly distributed which takes values in $[-\beta, +\beta]$ with $\beta < 1-\psi$. Notice that while u_t is a random variable representing the innovation in the elasticity of substitution, only substitution enhancing innovations will be actually implemented if firms are profit-maximizing.

The autoregressive component of the process, ε_t, captures a decreasing trend in the first difference of the elasticity of substitution; the underlying intuition being that the most obvious possibilities for factor substitution are introduced first and they vanish over time. The random shock u_t represents the innovation in the process: it is not implemented if negative and its domain is restricted in such a way that the process for σ is not explosive. That is to say

$$\lim_{t\to\infty} \sigma_t = \sigma_0 + \varepsilon_0 \tilde{u}_1 + \psi \varepsilon_0 \tilde{u}_2 + .. + \psi^{t-1} \varepsilon_0 \tilde{u}_t + .. = \sigma_0 + \varepsilon_0 \sum_{i=0}^{\infty} \psi_i^{i-1} \tilde{u}_i < +\infty.$$

Indeed, our restriction assures that $\lim_{t\to\infty} \sigma_t < \sigma_0 + \dfrac{\varepsilon_0}{\psi} \dfrac{1}{1-\beta\psi}$, and

$\lim_{t\to\infty} E_0[\sigma_t] = \sigma_0 + \dfrac{\varepsilon_0}{\psi} E_0 \sum_{i=0}^{\infty} \psi^i \tilde{u}_i = \sigma_0 + \dfrac{\varepsilon_0}{\psi 2} \dfrac{1}{1-\beta\psi}$.

If the elasticity of substitution does actually follow a similar process, the steady state per capita income will be increasing over time; moreover it is possible to compare the limit value of σ_t with the threshold value $\hat{\sigma}$ derived by de La Grandville (1989) $\hat{\sigma} = 1/[1 - \log(\delta(\hat{\sigma})\gamma^{\frac{\hat{\sigma}-1}{\hat{\sigma}}}(\hat{\sigma}))/\log(\frac{n}{s})]$. If it turns out that $\sigma_\infty > \hat{\sigma}$ the model generates endogenous growth. In determining this relation a key role is played by the parameters ψ and β. As $\beta\psi \to 1$ $\sigma_\infty \to \infty$, that is, the more persisent is the process for the elasticity increases and the larger is the domain for innovations, the more likely is the possibility for endogenous growth.

0.20 Testing the Hypothesis

Arrow *et al.* (1961, pp. 229-30) derived the CES production function from the relation:

$\log y = \log a + b \log w \quad (i)$

where a and b are constants and y and w are per-capita income and real wage. They also proved that the following relation holds: $b = \sigma$. Accordingly, our hypothesis of an increasing elasticity of substitution over time can be tested from (i) when assuming $b = b(t)$. The time trend which seems to best approximate the process described in the previous section is the logarithmic one, then I assume $b = c + d \log t$. The equation to be estimated is:

$\log y = \log a + c \log w + d \log t \log w.$

We are interested in assessing the sign and the significance of the parameter d to determine wheter the elasticity of susbstitution has a logatithmic time-trend. The first step has been to build up the series for $\{\log y\}_t$ and $\{\log w\}_t$. The period considered is 1947-2000 for the U.S. Data for GDP, price indexes, wages and employment are taken from the National Income and Product Account (Tables 1.1.5, 1.1.4, 2.2, 2.4.4, 6.4). Data series are all quarterly but for the employment which is annual; I have computed the quarters by imputing one fourth of the annual change in employment to any of the quarters. Although the Dickey-Fuller test for both series rejects the null hypothesis of a unit root, I thought detrending the series would be appropriate to single out only their cyclical component. To this purpose I have applied the band pass filter with the standard band of eight to thirty-two quarters; such procedures makes us lose twelve observation at the beginning and at the end of the period. Once the series are thus manipulated a least square estimation confirm our hypothesis as $d = 1.39$ with p-value $= 0.000$; also the overall fit of the estimation looks good: R-squared =0.999 and p-value(F-statistic) $= 0.000$.

0.21 Conclusions

Standard neoclassical growth theory has always relied on the increase (be that exogenous or endogenous) in total factors productivity in order to explain growth in per-capita income. I suggest here that a possible alternative explanation of economic growth is to be found in the evolution of the elasticity of substitution over time. I have shown that profit-maximizing firms will make the actual path followed by the elasticity of substitution a non-decreasing one. Such path according to the results derived by Klump and de La Grandville (2000) and Klump and Preissler (2000) determines a positive trend in the steady state per-capita income; this process is also capable of producing endogenous growth in case the elasticity of substitution converges to a sufficiently high value. Empirical evidence seems to confirm the existence of a positive logarithmic time trend.

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