Analysis of echocardiographic movies by variational methods

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Introduction

- Biomedical Movie Processing
  - A posteriori non-invasive medical parameter determination
  - Graphic application developed for Patient's signal synchronization
- Images segmentation and enhancing
  - Level-set method
    - Curve evolution by eikonal equation
    - Speed term choice
  - Image Pre-Processing
    - Energy-based-Functional minimization
    - Dynamic approach for time series image
- Applicability of methods
  - Discrete and continuos consideration
- Numerical algorithm
  - Parallel and sequential computation
- Simulation results

Ventricular AREA recognition of Echographic frame

We focus ours attention to a ventricular cavity:

Figure: Echocardiographic frame and its recognized Area

Ventricular AREA trends in the movie

Results of the final elaboration on entire clinical movies

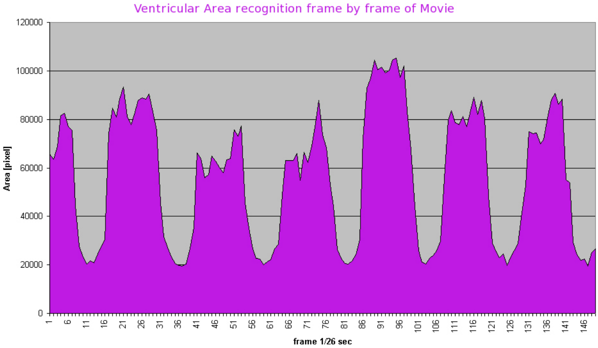
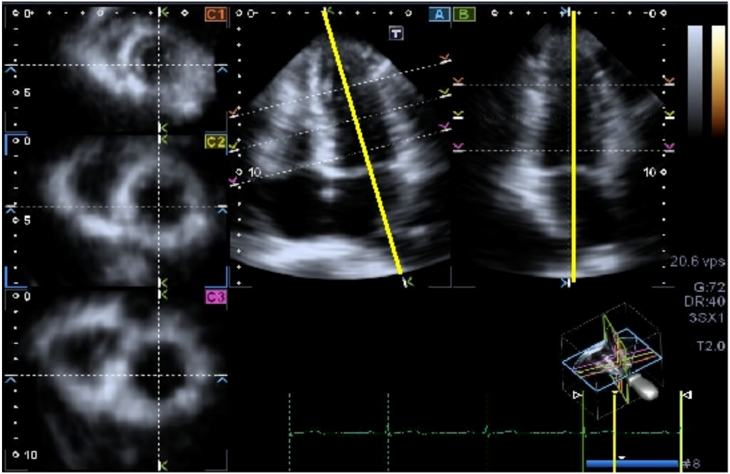


Figure: Biomedical parameters of cardiological efficiency are determinated on this area trends.

Future Project with 3D echographic sampling

Sapienza Math dep. Policlinico Umberto I and Toshiba collaboration

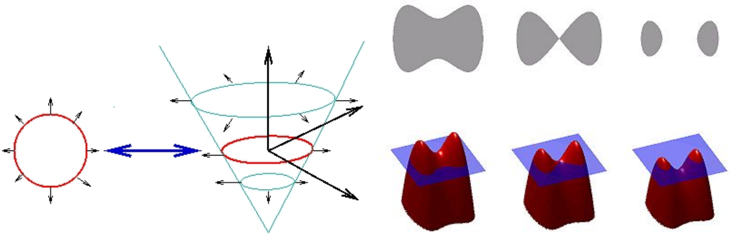


Presentation OUTLINE

1. Model Problem and Numerical Approximation
2. Protocol building and Simulation results
3. Developed Software for Clinical Application

Curve as the level-set of a surface in $\mathbb{R}^3$.

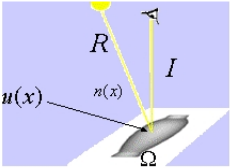
Area recognition as the level-set of a surface evolving by outward normal direction:



Adoption of a model where the speed of the front is related to the presence of an object in the image.

Edge-detector as function of the Brightness Intensity

A detector of contours is a positive real coefficient, which is dependent of $I(x)$'s gradient



represented by a filter function decreasing with z

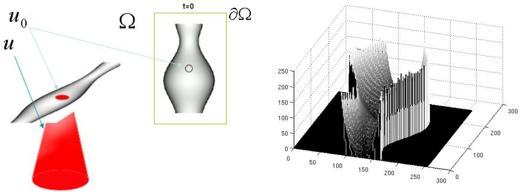
$$g : \mathbb{R}^+ \longrightarrow \mathbb{R}^+, \quad g(z) = \frac{1}{1+z}, \quad \lim_{z\to\infty} g(z) = 0$$

such that $0 \leqslant g$ and $g(0) = 1$. We use this kind of filter to represent the speed along the normal outward direction of the parametrized curve in each of its points. $g_I(x) \;\; \forall x \in \Omega$.

The *level* − *set* standard model (Sethian, [8], [6]et al.).

The segmentation problems:

$$\begin{cases} u_t(x,t) - g_I(|\nabla I(x)|)|\nabla u(x,t)| = 0 & x \in \Omega \subset \mathbb{R}^2 \times [0,T] \\ u(x,0) = u_0(x) & x \in \Omega \subset \mathbb{R}^2 \end{cases} \quad (1)$$



where $\nabla I = \nabla_x I = (I_{x1}, I_{x2})^T$ is the spatial gradient.
A threshold parameter assure the border detection $g_I(x) \leqslant th \Rightarrow g_I(x) = 0$.
The filter function involving the dipendence of $I(x)$. For a continuous problem, we have to control that is possible to calculate the Gradient and it remains bounded.

Eikonal equation solution M. Falcone '97[4], HJPACK M.Rorro (CASPUR)

We refer to an open library package developed for the solution of the Hamilton-Jacoby equation.

$$\begin{cases} u_t(x,t) + H(\nabla u(x,t)) = 0 & x \in \Omega,\ t \in [0,T] \\ u(x,0) = u_0(x) & x \in \Omega. \end{cases}$$

Description of the library use:

- u_T=HJ1D$(H, L(\Delta x, \Delta y, \Delta t), I(x), u_0, th, T)$
- Retrive the OpenMP calculation time and u_T at level 0
- Matlab Area calculation of the closed curve is performed with a pixel measure unit.

It require $u_0(x)$ such that it is a Lipschitz continuous function. where

$$lip(f) := \sup_{x,y\in\Omega; x\neq y} \frac{|f(x)-f(y)|}{|x-y|} < \infty$$

Consideration about continuos approach

In the continuous model problem the image has treathed as a surface, the speed term $V(x, t, \nabla(I(x)) = g_I$ in the equation depends of its gradient.
If exists a fracture the gradient jumps to infinity value.
Then a regularization is needed!

Regularization choice.

- **Convolution with Mollifier:** *level* − *set* standard model

$$g_I(x) = g(|\nabla(G_\sigma * I(x))|) = \frac{1}{1 + |\nabla(G_\sigma * I(x))|},$$

(Osher, Sethian et al.[8, 6], '95-2000, P. Master thesis, 2003).

- **Regularization and Edge Enhancing** by functional minimization

$g_I(x) = \frac{1}{1 + |\nabla u_{k_\epsilon}(x)|}$, u_{k_ϵ} minimum of an energy functional

Mollifying by heat eq. solution

With null source function and Dirichlet boundary conditions:

$$\begin{cases} u_t - \Delta u(x,t) = 0 & (x,t) \in \Omega \times [0,T] \\ u(x,0) = u_0(x) & x \in \Omega \end{cases} \tag{2}$$

Solving (2) is equivalent to carrying out a Gaussian linear filtering.

The explicit solution is:

$$u(x,t) = \int_\Omega G_{\sqrt{2t}}(x-y)u_0(y)dy = (G_{\sqrt{2t}} * u_0)(x), \quad G_\sigma(x) = \frac{e^{-\frac{|x|^2}{2\sigma^2}}}{2\pi\sigma^2}$$

$G_\sigma(x)$ denotes the two-dimensional Gaussian Kernel. It corresponds to low-pass filtering.

On standard mesh: $\Delta x = (b-a)/M, \Delta y = (d-c)/N$

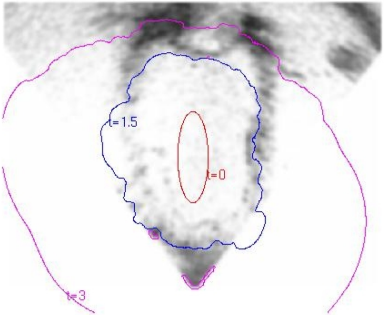
five-point approximation scheme become:

$$u^{n+1} = u^n + \Delta t \frac{u^n_{i+1} - 2u^n_i + u^n_{i-1}}{\Delta x^2} + \Delta t \frac{u^n_{j+1} - 2u^n_j + u^n_{j-1}}{\Delta y^2} \tag{3}$$

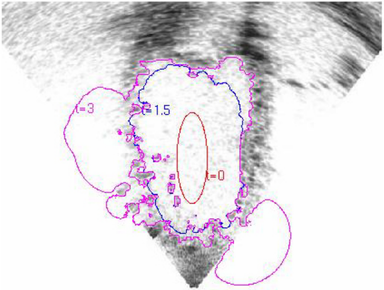
where T is the time horizon with a time step Δt.

Convolution with Mollifier:
Curve evolution on smoothed frame

threshold th=0.125, time horizon=3.



Smooth-Iterations=10, $\sigma = 10\sqrt{2}$



Smooth-Iteration=1, $\sigma = \sqrt{2}$

Ambrosio-Tortorelli(A-T) (I. Birindelli, S. Finzi Vita, '98) [3].

Approximated discrete minimization by iterative solve two systems:

$$F_\epsilon(u,S) = \mu \int_\Omega (u-g)^2\, dx + \int_\Omega S^2 \mid \nabla u \mid^2 dx + \alpha \int_\Omega \left(\epsilon \mid \nabla S \mid^2 + \frac{1}{4\epsilon}(1-S)^2 \right) dx$$

we implement a discrete minimization algorithm:

-$S^{(0)} = 1$; $u^{(0)} = g := I(x)$ for fixed *Nit* and a tolerance ϵ

-for $n = 1, 2, ..., Nit$ find $u^{(n+1)}$ for $S := S^{(n)}$ fixed, by solving:

$$\begin{cases} \widehat{u}_{i,j}^{(n)} = (u_{i+1,j}^{(n)} + u_{i-1,j}^{(n)} + u_{i,j+1}^{(n)} + u_{i,j-1}^{(n)}) \\ \frac{u_{i,j}^{(n+1)}}{\mu h^2 g_{i,j}} = \frac{\widehat{u}_{i,j}^{(n)}(S_{i,j}^2+K_\epsilon)+(S_{i+1,j}^2+K_\epsilon)(u_{i+1,j}^{(n)})+(S_{i-1,j}^2+K_\epsilon)(u_{i-1,j}^{(n)})+(S_{i,j+1}^2+K_\epsilon)(u_{i,j+1}^{(n)})+(S_{i,j-1}^2+K_\epsilon)(u_{i,j-1}^{(n)})}{(\mu h^2+4S_{i,j}^2+S_{i+1,j}^2+S_{i-1,j}^2+S_{i,j+1}^2+S_{i,j-1}^2+8K_\epsilon)} \\ \frac{\partial u_n}{\partial \mathbf{n}} = 0, \quad \text{in } \partial\Omega. \end{cases}$$

and S^{n+1} for $u := u^{(n+1)}$ *fixed* by solving:

$$\begin{cases} S_{i,j}^{(n+1)} = \frac{\alpha\epsilon\,(S_{i+1,j}^{(n)}+S_{i-1,j}^{(n)}+S_{i,j+1}^{(n)}+S_{i,j-1}^{(n)})+\alpha\,\frac{h^2}{4\epsilon}}{4\alpha\epsilon+h^2|\nabla u|_{i,j}^2+\alpha\,\frac{h^2}{4\epsilon}} \\ \frac{\partial S_n}{\partial \mathbf{n}} = 0, \quad \text{in } \partial\Omega. \end{cases}$$

-stop for $n+1 = Nit$. To obtain $F_{k_\epsilon}^{(n+1)}(u_{k_\epsilon}^{(n+1)}, S_{k_\epsilon}^{(n+1)})$.

Regularization: Conjecture for continuos model

Bounded gradient at given iteration, Pedone '08.

If u_{k_ϵ} represents a solution at k^{th} iteration of the Ambrosio-Tortorelli sequence ([1]) given from alternate solution of the elliptic system for fixed number of iteration, then u_{k_ϵ} is enough smooth to calculate $|\nabla u_{k_\epsilon}(x)|$ i.e. $\|\nabla u_{k_\epsilon}(x)\|_\infty < \infty \quad \forall x \in \Omega^\circ$ □

In the internal points of the domain Ω the amplitude of the fracture is ϵ's proportional, then we can found, at every step of the iterative solution of AT$_\epsilon$ algorithm, a costant C_{Nit} such that

$$|\nabla u_{k_\epsilon}(x)| \leq \frac{C_{Nit}}{\epsilon} \quad \text{then} \quad v(x) = \frac{1}{1 + |\nabla u_{k_\epsilon}(x)|} \geq \frac{1}{1 + \frac{C_{Nit}}{\epsilon}} > 0.$$

It is then possible to calculate the speed term in the eikonal equation.

Regularization: Critical points

We can observe that the function u_{k_ϵ} has in every direction passing to the internal point $x \in \Omega$, a profile regularized by a C^2 arcs in the fracture. (A. Francfort et al. [5], 2009.)

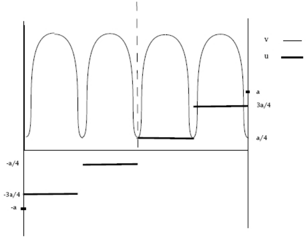


Figure: Approximation of the one dimensional fracture

Image Enhancing: Discrete approach, time dependence

From a computational point of view the discretized image has bounded intensity profile, so we can assume as lipschitz costant the difference beetween the maximum and the minimum of the u_{k_ϵ}'s points around the jump set given by the AT sequence at the Nit^{th} iteration.

Image Enhancing: Gradient calculus, time series

Different approach in Gradient calculus:

- Standard spatial way:
- Following the **optical flow** model (see page 184 [2]), we introduce dependence on time in the gradient term ($|\nabla u|^2$).

$$(|\nabla u|_{i,j}^{(f)})^2 = \frac{1}{4h^2}\left((u_{i+1,j} - u_{i-1,j})^2 + (u_{i,j+1} - u_{i,j-1})^2 + (u_{i,j}^{(f-1)} - u_{i,j}^{(f+1)})^2\right)$$

$$(|\nabla u|_{i,j}^{(f)})^2 = \frac{1}{4h^2}\left(\frac{(u_{i+1,j} - u_{i,j})^2 + (u_{i,j} - u_{i-1,j})^2}{2} + \frac{(u_{i,j+1} - u_{i,j})^2 + (u_{i,j} - u_{i,j-1})^2}{2}\right) + \frac{1}{4h^2}\left(\frac{(u_{i,j}^{(f-1)} - u_{i,j}^{(f)})^2 + (u_{i,j}^{(f)} - u_{i,j}^{(f+1)})^2}{2}\right). \quad (4)$$

Represents a new formulation of the model with a dynamic "mean" between frame.

Time dependence: Open problem R. March, G. Riey

Complete dynamical formulation of Time series functional: where $\varphi_{\eta,L}$ is a cut function.
$g : \Omega \times [0, T]$, $\Omega \in \mathbb{R}^2$
$u \in \mathbb{R}^2 \times [0, T]$,

$$F(u) = \int_0^T dt \int_\Omega \left| u - g \cdot \varphi_{\eta,L} \left(\left| \frac{\partial g}{\partial t} \right| \right) \right|^2 dx + \int_0^T dt \int_\Omega \left| \tilde{\nabla}_x u \right|^2 dx + \int_0^T dt \cdot \mathcal{H}^1 \left(S_{u(t)} \right) +$$

$$+ \int_0^T dt \left(\frac{\partial u}{\partial t} \right)^2 dx + \int_0^T dt \int_\Omega \left| \tilde{\nabla}_x \left(\frac{\partial u}{\partial t} \right) \right|^2 dx + \int_0^T dt \cdot \mathcal{H}^1 \left(S_{\frac{\partial u}{\partial t}} \right)$$

is the time regularization parts.

Approximated functional

As in A-T way:

$$F_\epsilon(u) = \int_0^T dt \int_\Omega \left| u - g \cdot \varphi_{\eta,L}\left(\left|\frac{\partial g}{\partial t}\right|\right)\right|^2 dx +$$

$$\int_0^T dt \int_\Omega S^2 |\nabla_x u|^2 dx + \int_0^T dt \int_\Omega \left\{\epsilon |\nabla_x S|^2 + \frac{1}{4\epsilon}(1-S)^2\right\} dx + \int_0^T dt \left(\frac{\partial u}{\partial t}\right)^2 dx +$$

$$\int_0^T dt \int_\Omega Z^2 \left|\nabla_x \left(\frac{\partial u}{\partial t}\right)\right|^2 dx + \int_0^T dt \int_\Omega \left\{\epsilon |\nabla_x Z|^2 + \frac{1}{4\epsilon}(1-Z)^2\right\} dx.$$

Then we obtain the system:

$$\begin{cases} u - g\varphi_{\eta,L}\left(\left|\frac{\partial g}{\partial t}\right|\right) - div\left(S^2 \nabla_x u\right) - \frac{\partial^2 u}{\partial t^2} + \frac{\partial}{\partial t}\left[div\left(Z^2 \nabla_x \frac{\partial u}{\partial t}\right)\right] = 0 \\ S \mid \nabla_x u \mid^2 - \epsilon \Delta_x S - \frac{1}{2\epsilon}(1-S) = 0 \\ Z \mid \nabla_x \left(\frac{\partial u}{\partial t}\right) \mid^2 - \epsilon \Delta_x Z - \frac{1}{2\epsilon}(1-Z) = 0. \end{cases}$$

The existence of minimum and the approximation are in course of development.

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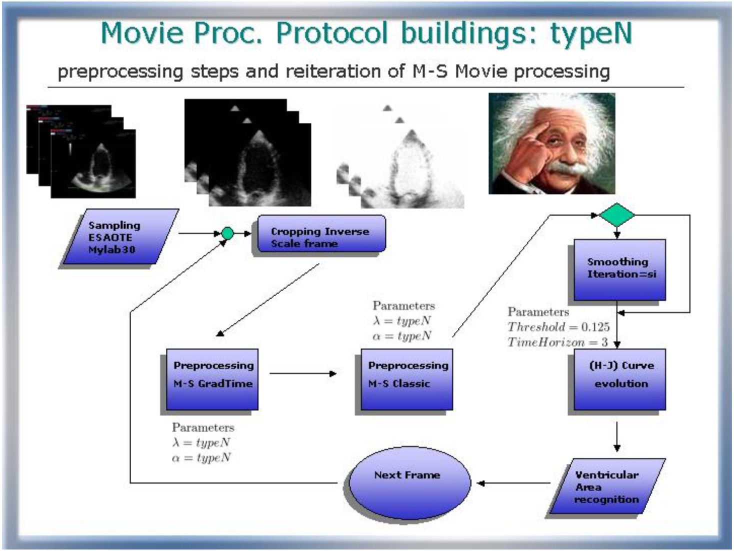
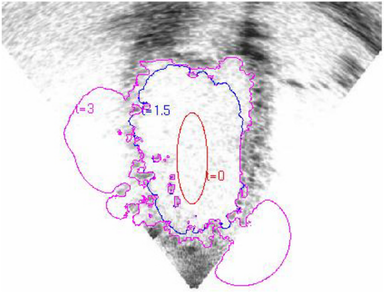

Movie Proc. Protocol buildings: typeN
preprocessing steps and reiteration of M-S Movie processing
Sampling ESAOTE Mylab30
Cropping Inverse Scale frame
Smoothing Iteration=si
Parameters
λ = typeN
α = typeN
Parameters
Threshold = 0.125
TimeHorizon = 3
Preprocessing M-S GradTime
Preprocessing M-S Classic
(H-J) Curve evolution
Parameters
λ = typeN
α = typeN
Next Frame
Ventricular Area recognition

Type 1: Original image

Protocol of elaboration:

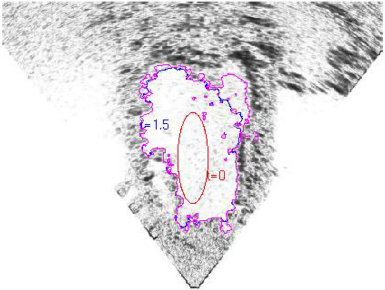
- Cropping: Frame[800,652] $\rightsquigarrow$ Frame[501,411] [pixel]
- Preprocessing: n/a
- Processing:
    - Smoothing: eq. Heat 1 iteration
    - H-J: Threshold (th=0.125)
      Time-Horizon (3, step 0.03)
      Spatial mesh (0 ..5.0,0 ..4.1) Step (0.01,0.01)



Type 2: Time gradient

Protocol of elaboration:

- Cropping: Frame[800,652] $\rightsquigarrow$ Frame[501,411] [pixel]
- Preprocessing:
  - M-S Grad Temp ($\lambda = 0.02, \alpha = 0.0001$) $(UOrig(f-1, f, f+1)) \rightsquigarrow (Ureg_1(f), S_1(f))$
- Processing:
  - H-J: Threshold (th=0.125) Time-Horizon (3, step 0.03) Spatial mesh (0 ..5.0,0 ..4.1) Step (0.01,0.01)



Type 3: M-S algorithm iterated on image U

Protocol of elaboration:

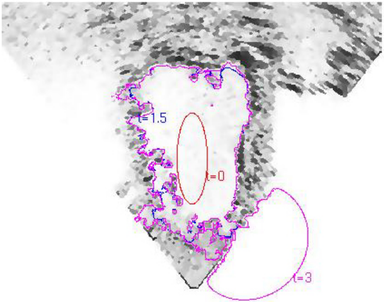
- Cropping: Frame[800,652] $\rightsquigarrow$ Frame[501,411] [pixel]
- Preprocessing:
  - Step1, M-S Time Grad $(\lambda = 0.02, \alpha = 0.0001)$ $(UOrig(f-1, f, f+1)) \rightsquigarrow (Ureg_1(f), S_1(f))$
  - Step2, Classic M-S $(\lambda = 50, \alpha = 0.02)$

    $(Ureg_1(f), S_1(f)) \rightsquigarrow$

    $(Ureg_2(f), S_2(f))$
- Processing:
  - H-J($Ureg_2(f)$): Threshold (th=0.125) Time-Horizon (3, step 0.03)

    Spatial mesh(0..5.0,0..4.1)

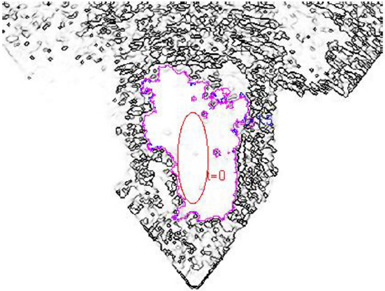
    Step(0.01,0.01)



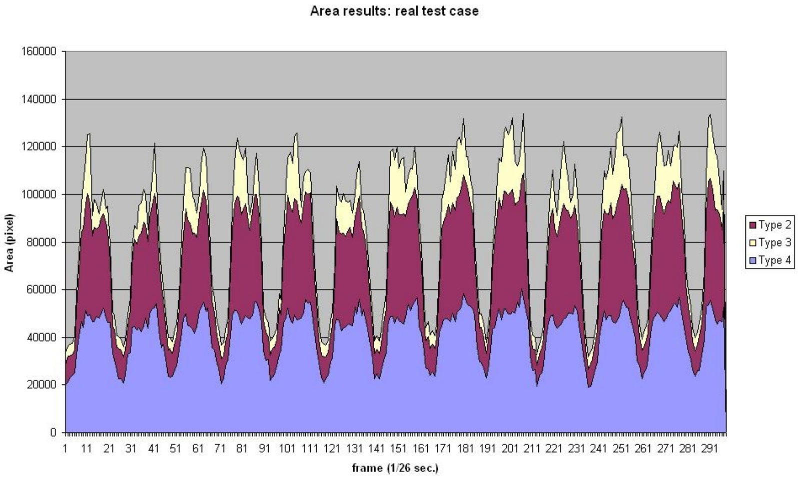
Type 4: M-S algorithm iterated on the jump set S

Protocol of elaboration:

- Cropping: Frame[800,652] $\rightsquigarrow$ Frame[501,411] [pixel]
- Preprocessing: Step1, M-S Time Grad ($\lambda = 0.02, \alpha = 0.0001$) $(UOrig(f-1, f, f+1)) \rightsquigarrow (Ureg_1(f), S_1(f))$
- Step2, Classic M-S ($\lambda = 50, \alpha = 0.02$) $(Ureg_1(f), S_1(f)) \rightsquigarrow (Ureg_2(f), S_2(f))$
- Processing:
- H-J($S_2(f)$): Threshold(th=0.125) Time-Horizon(3, step 0.03) Spatial mesh(0 ..5.0,0 ..4.1) Step(0.01,0.01)



Ventricular area trend in the frames



CPU time and OpenMP parallel computing

- H-J curve evolution F90 code:
  - Parallel OpenMP:CASPUR Cluster Power5,8 CPU, 1.9 GHz, 4x32 GB Ram, AIX Fortran90.
  - Serial PC: PIV Linux RedHat Fedora core 2.
- M-S Pre-Processing Frame enhancing
  - Matlab R14 PIII 800 Mhz, 768 Mb Ram, Windows 2000 Server.
  - Matlab R14b PIV 2000 Mhz, 1GByte Ram, Linux RedHat Fedora core 2.

CPU Table	Power5	PIV	PIII 800 Mhz
H-J F90 OpenMP	239.8 sec.	288.0 sec.	312.0 sec.
M-S Matlab		1.06 sec.	
M-S gradt(2 point) 3 frame		1.34 sec.	
M-S gradt(3 point) 3 frame		1.61 sec.	

Cardiovascular physiology parameter

Ejection Fraction (E_f) is the fraction of blood pumped out of a ventricle at each heart beat.
We identify the volume of blood within a ventricle:

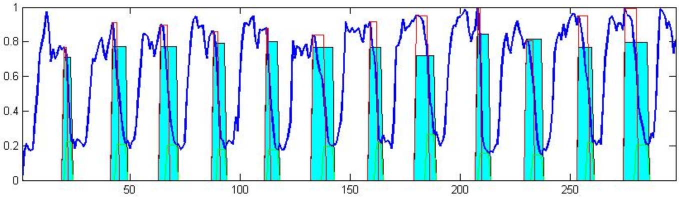
- immediately before a contraction: end-diastolic volume(EDV).
- at the end of contraction: end-systolic volume (ESV).

$$E_f = \frac{EDV - ESV}{EDV}$$

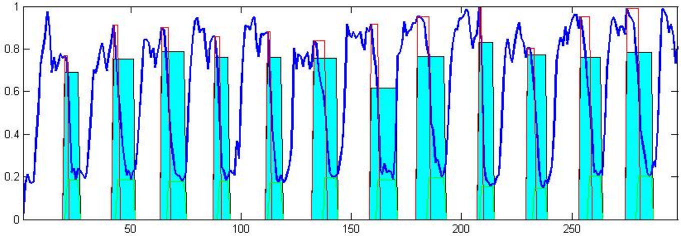
By calculating the mean value of the desired profile of ventricular area, we can perform a zero value detection, with a standard method. Than for every decreasing front determine the two interesting point: (ESV), (EDV)

Ejection Fraction

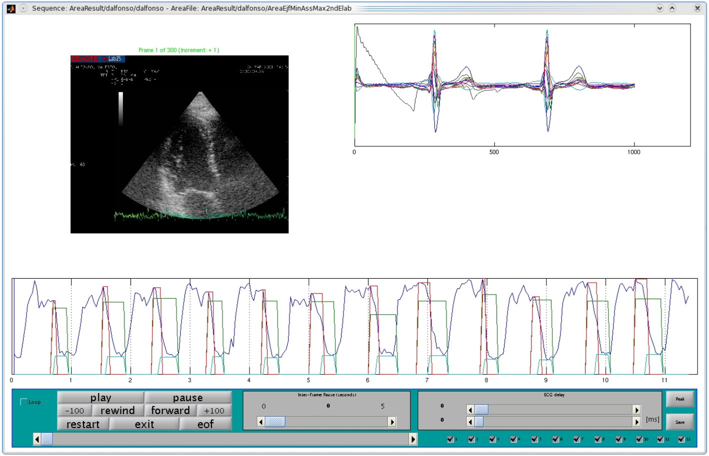
First minimal point of decreasing front for end-systolic phase (ESV):



Absolute minimal point of end-systolic phase (ESV):



Application for signal synchronization



Conclusions

Possible technical refinement in analysis of Echocardiographic image sequences for non-invasive and a-posteriori medical diagnostics of heart left-ventricle diseases:

- Determination of ventricular local pressure.
- Volume recognition by 3D Echo Images.

Mathematical framework upgrading

- Fast marching methods.
- Newer approach to time series problem.
  - Time dependent functional(Open Problem).
- Finite-element analysis to optimize echographic “cone”

Essential references

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Thanks

Thanks for your Attention!

Refer to the last section of the paper document for a list of persons that I wish to thank for the collaboration.
http://pedoneweb.phys.uniroma1.it/max/phd/tesi