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Against the Fragmentation of Knowledge: The Power of Multidisciplinary Research for the Design of Metamaterials

Francesco dell'Isola, Sara Bucci and Antonio Battista

Abstract The new possibilities arisen in the last years in material manufacturing (3D-printing, electrospinning, roll-to-roll processing, self-assembly, etc.) and the theoretical tools made available by generalized continuum mechanics are still far from achieving their full potential. The main thesis of the present paper is that it is necessary a *multidisciplinary* approach to address the emerging issues in metamaterials' design. Therefore, an improvement in the degree and the depth of the cooperation between scientists from different areas is required. The advancements needed in mechanics and physics of solids and fluids, mathematical and numerical modeling and advanced technology in material construction can be obtained only as a consequence of a synergic effort.

1 Introduction

Technology and hard sciences have always developed in a close and parallel relation. Indeed, a driving force for science to rediscuss the current paradigms, during all History, has been the advancement of new technological possibilities, which allow for new phenomenological evidence to arise. What proves the actual success of the conceptual revolutions connected with the birth of Mechanics, of Thermodynamics

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and Electromagnetism, is that those new theories and models, created with the aim of designing and describing new technological advancements, were in the end point embodied in the what has been called *classical physics*. In the opinion of the authors, a similar change of paradigm is going to be experienced by mechanics in the incoming future. However, the new technological possibilities, which permit to produce and control the properties of material both at the micro- and nanoscales, need substantial progresses from a theoretical point of view. It is evident that the new manufacturing techniques, such as electrospinning, 3D-printing, self-assembly, and so on, which have been developed in the recent years, enforce us to reanalyze some of our ideas concerning theoretical mechanics and above all, on the relation existing between it and technology. Indeed, using such techniques, we are able to produce objects which present an exotic and peculiar behavior from a *classical* point of view.

The key challenge, today, is therefore not only to be able to predict the behavior of already existing (possibly advanced) materials, but also to succeed in prescribing the right constitutive and geometric characteristics at the microscale in order to get a certain (even exotic) behavior at the macroscale. Among many paths we can imagine for future developments, the following ones appear critical with respect to the aforementioned technological innovations:

- the development of an improved theoretical framework for generalized continua based on a systematic assumption of the variational approach;
- the development of suitable homogenization techniques allowing for the determination of reliable macroscopic models based on the given micro-/nanostructure and on the physical properties of the considered materials;
- the development of robust and flexible numerical methods to perform effective simulations of the proposed models;
- a sound basis of experimental evidence, to be developed in close connection with the previously mentioned theoretical knowledge and understanding;
- the concrete realization of proofs of concept, constituted by prototypes of new advanced architectured materials.

Therefore the creation of networks,[1] *which allow the interaction of scientists specialized in each of the aforementioned categories, is an unavoidable step if the scientific community wants to successfully meet the challenge.* This idea is not a novelty,[2] of course, but in the opinion of the authors it has to be reexamined in order to promote actual interaction between different fields and not, as often is the case, mere union of the respective results. Indeed, joining different specializations under a unified line of research having a common goal should exploit the highly specialized knowledge currently existing, avoiding that diversification could become fragmentation. In particular, it should prevent the danger of developing just parallel and independent

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[1] An example is the recently established M&MoCS International Research Center, see http://memocs.univaq.it/?lang=en.

[2] Indeed the establishment of such networks can be traced back to Hellenistic Science (see Russo et al. 2013). In the *Mouseion* at Alexandria, experts from all disciplines (geometers, physicists, mechanicians, physicians, grammars) were all working together, attacking the same problem from all available point of view.

investigations that, in themselves, are not capable to effectively attack complex problems. We remark that in this connection, it is, in our opinion, also critical the involvement of younger generations of researchers, so as to create, since the very beginning of higher level education, a sort of constructive dependence between students coming from different paths of learning and experienced scientists.

2 Multidisciplinary Nature of Metamaterial Research

When trying to graphically represent the properties of (natural and artificial) materials, one finds that all possible charts matching relevant mechanical and physical properties are made by zones occupied by actually existing materials, as well as by very large empty areas. Some of these areas will remain empty for long because, as far as we understand, the materials there lying would be impossible for fundamental reasons, but others are simply empty even though, in principle, they could be filled by means of a suitable exploitation of the technological possibilities (Ashby 2013). Compared even to few years ago, today the situation has changed so abruptly that the more important and challenging question is related to what can be seen as an *inverse problem*: given a continuum model, which are those mechanical systems that, at a certain length scale, behave as specified by the chosen continuum model? In setting the question in this way, the key point is to understand the microscopic properties of such systems to obtain information on how to realize them (dell'Isola et al. (2015f), Bouaziz et al. (2008), Gibson and Ashby (1997), Brechet and Embury (2013), Fleck et al. (2010)). If the challenge of a multidisciplinary approach is accepted, it will be capable to promote a quantum leap in the exploitation of the technological possibilities already potentially existing, and to drive the advanced manufacturing technology toward the most promising further developments. The new level of ambition in the requirements of peculiar multifunctional properties often create a bottleneck which cannot be overcome by means of traditional materials. Manufacturing materials with the degree of freedom and precision allowed by techniques like 3D printing and electrospinning, associating them, and suitably selecting their geometries, represent an innovative strategy to meet the newly arisen (and yet to come) engineering challenges.

Summarizing the following two points are essential in order to achieve further progresses:

- to establish long-term links among research groups from different areas;
- to avoid the fragmentation of skills and knowledge, i.e., a situation in which the team work is performed without much exchange of information between different specialists.

In other words, multidisciplinarity has to play, in our context, a major role, even more relevant than in the generality of technological and scientific research. Indeed, in the opinion of the authors, all the research directions have to systematically interact:

mathematicians need, in order to supply the rigorous results necessary for numerical computation, to be informed about the conjectures formulated by modelers; physicists and engineers must interact to integrate the capability of understanding phenomena in order to be able to solve more efficiently practical problems; experimentalists must be guided by theoretical knowledge if very unusual phenomena must be discovered and exploited.

Specifically, the networks we have in mind should favour:

- the exchange between physicists and engineers in order to improve the state-of-the-art capability to design application-tailored materials;
- the interaction between mathematicians and more applied scientists in order to direct the theoretical investigation toward more applicable problems and in particular toward the formulation of highly predictive numerical tools;
- the joint work of theoretical mechanicians, physicists, and mathematicians in order to formulate mathematical models capable to drive the design and realization of newly conceived architectured materials, based on complex microstructures and multiphysics/multiscale phenomena;
- the interaction of mathematicians with experienced numerical analysts in order to improve the capability of tailoring macroscopic homogenized models to the description of microscopic complexity (up to nanolevel);
- the collaboration for the study of scientific and technological problems involved in mechanics of natural and artificial tissues and in electromagnetic action on architectured materials together with its possible application for health protection.

We believe that, in this interaction, a special role has to be played by the coordinating power of experienced components and by the capabilities and the willingness of junior components. This will, in our opinion, maximize the chances of developing sharp and ground-breaking solutions in the direction of the concrete realization of new technological application of architectured materials.

3 Some Aspects of the Implementation of a Multidisciplinary Research Work

A long way has been covered by material technology since its beginning (dating back to non-Sapiens hominids). The basic steps of this adventurous travel can be summarized as Brechet and Embury (2013):

1. using the materials available on site (e.g., native metals, bone, or wood);
2. gradual evolution toward the optimization of specific classes of materials on an empirical basis (e.g., development of empirical metallurgy techniques);
3. science-based approaches (e.g., scientific metallurgy and later polymer science etc.);
4. what can be called *hyperchoice of materials*, i.e., the development of scientific methods and tools for comparing and selecting materials coming from different

classes which, individually considered, were already optimized for a specific set of engineering applications;

5. search for multifunctionality of materials, with increasingly ambitious requirements for materials capable of fulfilling conflicting needs.

The importance of theoretical modeling has of course grown very much passing from (1) to (4), and today the demand for multifunctionality is such that the already available theoretical models are not suitable anymore for the full exploitation of the technical possibilities, nor for meeting the needs coming from industry (Bréchet 2000; Fleck et al. 2010). Material scientists have developed an impressive and very specialized body of knowledge, while theoretical mechanicians have pushed the study of classical mechanical models close to the theoretical limit of their potential, and sound nonclassical theoretical frameworks have been developed concerning microstructured/micromorphic media (Green and Naghdi 1995; Masiani et al. 1995; Neff and Forest 2007; Altenbach and Eremeyev 2009; Carcaterra et al. 2015; Federico and Grillo 2012), some of which can by now be considered as *classics* (Eringen 1968; Germain 1973). The developments in numerical analysis in case of micro- and even nanosystems are by now remarkable. However, the interaction between the two aforementioned areas is still not as intensive and fruitful as it can be. The proposed point of view about the modeling of architectured materials and metamaterials by means of suitably reformulated generalized continuum theories is capable, in our opinion, to make it seem even obsolete a clear-cut distinction between the two fields.

Mathematical modeling of materials has been developed in the nineteenth century on the basis of some reasonable and well-grounded assumptions, which are verified by the majority of natural materials and by the great majority of the materials used up to now in engineering. Some natural materials which show sophisticated and often unexpected behaviors are those living tissues produced by natural Darwinian selection whose microstructure:

- is very complex;
- exhibits multiple characteristic length scales;
- involves coupled multiphysics phenomena;
- shows strongly inhomogeneous physical properties at every characteristic length (Dunlop and Fratzl 2013; Tomic et al. 2014).

It is clear that the assumptions used in classical physics for describing mechanical behavior are not anymore suitable when one wants to model living tissues or when one wants to design and build exotic artificial materials tailored to high-performance technological (possibly biomedical) applications, e.g., for the replacement of natural tissues and for providing protection against externally induced damage for living organisms.

It is clear that some well-established *classical* concepts may have to be rethought. For instance, the same concept of stress, strain, local and contact interaction, deformation energy, balance equations, constitutive equations, yield and damage criteria need to be reformulated in some respects, in a context never explored before. To this aim the competences of mathematicians, physicists, numerical analysts, theoretical and applied mechanicians need to be joint to obtain a more performing and predictive Weltanschauung, i.e., a vision of the physical world by means of which physicists and engineers can shape materials and their behaviors for the upcoming future. In other words we need to emancipate from well-established simplification assumptions and modeling hypotheses so as to lead to the development of new design procedures and solutions, as well as, of homogenization techniques under new microstructure assumptions, like high contrast and multiscale implying indeed higher gradient and/or microstructured continuum modeling. Classical mechanics, and in particular continuum mechanics, assumes usually (and rightly!) very simple hypotheses about the mechanical and physical behavior of materials, from which it is naive to expect a universal predictive power. However, a theoretical framework that in our opinion can still have an effective unifying power is the variational one, possibly improved by means of the Hamilton–Reyleigh dissipation mechanism.

4 Short-Term and Long-Term Scientific, Technological, and/or Socioeconomic Impacts

Up to now, one of the strongest features of European science has always been its capability of integrating toward one specific challenge, different competences and capabilities. Notwithstanding the increasing difficulties in supporting multidisciplinary research Europe seems to be able to keep its scientific leadership continuing this long-lasting tradition (which could be tracked up to the achievements of the Hellenistic scientists and in particular to Archimedes: a geometer whose theoretical knowledge produced impressive technological applications). Technology cannot advance without the nourishment supplied by fundamental science, fundamental science finds its ultimate motivation and justification in technological applications. This is why, in our opinion, it is necessary to embody the just-stated principle while confronting a specific challenge: the development of theoretical and experimental tools needed to conceive, optimize, and build novel highly performing architectured materials.

Standard methods, optimization routines, and already existing finite-element analyses could be used to effectively refine a given architectured material once its general constitutive and geometrical characteristics have been chosen. However, what the standard methods are not so good at is a reasonably quick scan of alternative combinations (Ashby 2013), which is of course crucial as the possibilities entailed by the new computer-guided manufacturing techniques are virtually infinite and cannot be tried extensively. As an example, we can consider what are called hybrid materials. The equivalent properties of hybrid materials lie on a trajectory (in the space of possible materials) with end points at the materials that are combined to make them.

A suitable theoretical model should provide a good prevision about the shape of the trajectories. A particular care has to be paid in order to specify the physical meaning of the portions of these curves where small changes of one parameter are associated to great variations of the other parameters: the singularity phenomena arising in these circumstances can indeed be source of very interesting effects, as well as instabilities of even “difficult” types (Luongo 2010; Di Egidio et al. 2007; Luongo 2015). Among the short-term targets, there should be also the concrete realization of prototypes of architectured materials manufactured by means of the technological possibilities displayed by the experimental researchers involved, which should be capable to perform advanced 3D-printing and electrospinning. The focus of the networks should be even more strongly directed toward long-term targets. In this respect, the goal should be to replace the *incremental* character that the advancements in material technology have experienced in the last years with a quicker and sharper, step-like evolution function. We believe that the synergistic effort by theoretical mechanicians, applied mathematicians, numerical analysts, and experimentalists can indeed produce the aforementioned conceptual revolution in continuum mechanics and material science. As for the socioeconomic impact, the achievement of the aforementioned objectives will have an inestimable value for industry and also potentially for environmental issues, as the realization of lighter and not overdimensioned objects will relevantly affect power consumption in manufacturing and shipping. Last, but not least, the materials which we have in mind may play a relevant role in biomechanics and medicine. This feature is to us so valuable that an upper estimate of its potential socioeconomic impact looks simply impossible.

5 A Closer Look at How to Face the Problems Involved

In the present section, we propose a possible way to rationalize the team work. We selected eight main fields of research (FOR), each of which should be involved both for specific tasks and for interacting with the others:

FOR1: Theoretical Continuum Mechanics and Variational Approach

The construction of the general theoretical framework for the description and the prevision of the behavior of advanced architectured materials is probably, as mentioned, the soundest possible ground for exploring the exotic phenomena we have in mind. In particular, they have to extend and generalize the already existing higher gradient and micromorphic models; moreover they have to drive the work of numerical analysts. The researchers should provide assessment about the realization of theoretical models for complex metamaterials showing a good match with both numerical simulations and experimental results. A detailed coverage of the scientific novelties introduced in the modeling have to be clear, offering a general understanding and a unifying perspective relating the proposed theoretical findings with their technological potential and their contribution to the general progress of science.

In order to model architectured materials (i.e., metamaterials in the sense specified in Del Vescovo and Giorgio 2014; a rational collection of recent results is Placidi et al. 2015b), the variational approach is very effective and allows us to obtain well-posed problems with the minimum constitutive assumptions possible (see, e.g., Auffray et al. 2015; dell'Isola et al. 2015a, e, 2016a; dell'Isola and Placidi 2012; Cuomo et al. 2014). Architectured materials can be classified and interpreted as microstructured materials. Particular examples of these materials are for instance micropolar materials (see, e.g., Altenbach and Eremeyev 2009, 2014; Pietraszkiewicz and Eremeyev 2009a, b) but also higher gradient materials (see, e.g., Placidi et al. 2015a; Auffray et al. 2015; dell'Isola et al. 2015a, 2016a) can be seen as a particular case of micromorphic models.

Sometimes the complexity of the medium can be modeled in the framework of mixture theory when different phases of some constituents coexist (see, e.g., Placidi and Hutter 2005, 2006; Andreaus et al. 2014; Giorgio et al. 2015).

A very delicate issue in the study of complex materials is, finally, the damage detection and the characterization of the evolution of cracks inside them (see, e.g., Andreaus and Baragatti 2011, 2009; Placidi 2015, 2014; Thiagarajan and Misra 2004; Rinaldi and Placidi 2014).

FOR2: Advanced Homogenization Techniques

The most suitable continuous (i.e., macroscopic) limit, for a wide class of discrete mechanical system characterized by a high degree of complexity in the microstructure, have to be provided if one wants to fasten the numerical investigation and, as a consequence, the prototyping process. Looking for the correct micro-macro identification, for every conceived microstructured metamaterial, and driving the numerical work toward the most promising models of architectured materials, would be the main concern of researchers in this field. Their objectives have to focus on the description of a suitable continuous limit for:

- (micro) lattices characterized by multiple length scales;
- multilayered materials obtained combining in 3D structures the previous lattices

'Ad hoc' homogenization techniques should be conceived to deal with these particulars micro-macro identifications. Some results in such a direction can be found in Assidi et al. (2011), Carcaterra et al. (2015), dell'Isola et al. (2016b), Dos Reis and Ganghoffer (2010, 2012), Goda et al. (2012), Rahali et al. (2015).

FOR3: Mathematics of Nonlinearity

It is also essential to deepen the understanding of the complex nonlinear phenomena occurring when considering complex multiphysics systems (see, e.g., Javili et al. 2013; Piccardo et al. 2014). This can be done through the study of the instability resulting from the behavior of the considered systems. The ultimate goal should be to assess the degree of accuracy required in order to realistically expect a good fit between models, numerical analyses, and experimental results.

FOR4: Numerical Investigations

New numerical techniques should be developed for the investigation of novel architectured materials. Refined FEM schemes flexible enough to be able to consider even complex geometries and robust enough to work with high degrees of complexity in the microstructure should be also constructed.

The complexity of these newly conceived architectured material gives rise to new numerical issues related to different sources, as for instance the presence of strong heterogeneities resulting from the features of microstructure considered, viscous or independent-rate dissipation, internal resonances, and so forth (see, e.g., Aristodemo and Turco 1994; Turco 2001; Bilotta and Turco 2014; Della Corte et al. 2015; Misra et al. 2007 for some numerical tools on this regard). Besides, in many cases (when the standard Cauchy theory seems to be inadequate) the use of microstructured or higher gradient models may be appropriate to describe a greater variety of unusual behaviors. Recently, very powerful tools in numerical analysis have been developed in the framework of isogeometric analysis (see, e.g., Cazzani et al. 2014b, a, 2015; Greco and Cuomo 2013, 2014, 2016 for more details) in order to deal with the difficulties arisen from this type of models.

The complexity and heterogeneities of the microstructure could be, as already said, source of lack of stability in the materials considered. Typically examples of these phenomena can be observed, e.g., in cellular materials as ceramic or metallic foams as well as honeycomb composites. Usual tools, as perturbation analysis, may encounter problems because they require analytical solution as a reference solution that is often not available. Then in many cases these problems can only be faced with numerical methods (see, e.g., Di Egidio et al. 2007; Luongo and Piccardo 2005; Rizzi et al. 2013; Gabriele et al. 2012).

FOR5: Manufacturing

This research team should be aimed in building prototypes of architectured materials exhibiting strong multifunctionality.

They should extent the technological possibilities in building complex multi-physics systems involving a certain desired coupling between the components and compare the experimental results with the theoretical and numerical previsions in order to refine the models and to adjust the relative parametrization. At the end they should be able to describe the experimental results obtained in addition to the concrete realization of working prototypes and the application of patent for the produced prototypes.

With the spirit of synthesizing the material on the basis of a suitable constitutive assumption on the stored strain energy, some examples of pantographic structures (dell'Isola et al. 2015d, c, 2016b) can be interpreted as a particular carrying out of the fiber sheets described in dell'Isola and Steigmann (2015), Steigmann and Dell'Isola (2015), D'Agostino et al. (2015). Other possibilities that can be explored are those based on a proper modification of an existing material in order to modify its mechanical properties. For instance the addition of some microfillers in a concrete matrix in order to improve the dynamic performances as done in Giorgio and Scerrato (2016), Scerrato et al. (2015, 2014).

FOR6: Mechanics of Discrete Systems

These researchers should try to characterize the discrete mechanical systems which will be selected as the most promising in view of obtaining the desired macroscopic properties and behaviors. They should therefore provide a detailed mechanical description of the micro-level of a large class of architectured materials, including for instance (micro) lattices characterized by multiple length scales, multilayered materials obtained combining in 3D structures the previous lattices and other structures displaying even more complex geometry at the microscale.

FOR7: Coupled Phenomena

The potentiality entailed by the conceivable coupling of the different constitutive elements of the considered multiphysics systems is still far from exhausted. A suitable description of the coupling in a certain class of specific cases is indeed still incomplete. Among the most relevant cases, in the opinion of the authors, there are piezo- and flexo-electromechanical systems thought as elementary components of complex structures, and in general nonlinear electroelastostatics. They will be the main object of the work of FOR5 and will allow for the realization of suitable prototypes of multiphysics architectured materials, exhibiting the desired coupling properties. Materials in which there is a coupling between mechanical and electric states (Lagrangian variables) are also of interest especially in the field of vibration control and noise attenuation. Piezo- or flexoelectric materials can be profitably employed for this purpose (see for some examples Andreaus et al. 2004; Giorgio et al. 2009; Enakoutsa et al. 2015).

FOR8: Engineering and Biomechanical Applications

As already said, finding solutions to already existing problems from engineering and biomedical areas by means of a suitable exploitation of the properties of the developed architectured materials is, in a sense, the main challenge in the field of metamaterials for both its difficulty and usefulness. The involved researchers should start from gathering the requirements from the industrial and biomedical world that can best fit with the results expected and obtained by the network, and communicate efficiently with the industrial sector in order to maximize the possibility of fruitful interaction. We will provide some specific example. The study of wind-excited structures (see, e.g., Pagnini 2010; Piccardo et al. 2015; Luongo and Piccardo 2005) has proven that a high strength–weight ratio under certain shape constraints, as well as targeted anisotropic behaviors and piezo- and flexoelectric induced damping, are of great importance for improving the reliability. It is clear, therefore, that metamaterials can have a great potential impact in this regard. In bone reconstructive surgery, designing (from both mechanical and biological points of view) of suitable implants made of bioresorbable artificial material is an attractive challenge in order to guarantee a proper load-carrying capacity and a fast substitution of biomaterial with newly formed bone for health purposes (see, e.g., Lekszycki and dell'Isola 2012; Ancillao and Andreaus 2013; Giorgio et al. 2016, 2015; Andreaus et al. 2013, 2014). In this context, also the modeling of interaction of artificial material with soft tissue, as,

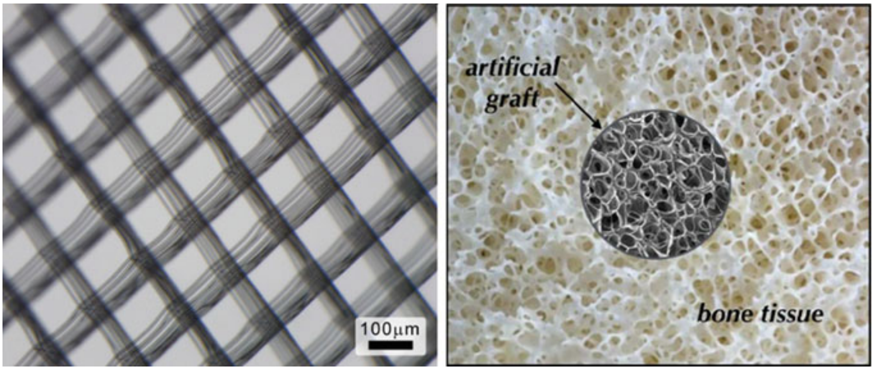

Fig. 1 *Left* an electrospun metamaterial made of polycaprolactone fibers, characterized by extremely slow degradation, and thus suitable for biomedical applications. *Right* the inclusion of a *circular shaped* artificial graft in a bone tissue (scheme). Living bone tissue equipped with an artificial graft can be regarded as a metamaterial whose configuration is described by a displacement field and by an additional kinematical descriptor (e.g., the change of the porosity with respect to the reference configuration)

e.g., cartilage, could be useful (for more details see, e.g., Tomic et al. 2014; Federico and Grillo 2012).

As it is well known that the remodeling process in bone is strictly related to the frequency of the external load. Therefore, the dynamic properties of bones play a key role in the bone functional adaptation. For this reason, a modal analysis should be performed in order to understand how these features changes in the remodeling process and are influenced by external mechanical factors (Fig. 1).

5.1 Risk Assessment and Management

As already mentioned, specific risks are involved in a challenging research activity. In general, the main measure to avoid major problems is closely related with the multidisciplinary approach we propose, since the diversification of the employed approaches makes the team work more robust. For instance, in order to face the possibly enormous difficulties in finding appropriate homogenization methods, the network has to select extremely skilled mathematician which should have, moreover, diversified capabilities and competences within the same theoretical framework constituted by homogenization techniques. In particular, both a *functional* approach (employing gamma- convergence and two-scale convergence methods as their work tools) and a *differential* approach can be carried on, together with the clever employment of formal asymptotic expansion in order to quickly get precious information about the desired continuous limits. To meet the challenge proposed by computational complexity of the considered microstructured materials, different FEMs can be developed and a constant communication between numerical analysts and theoreticians should be in place, which has proven to be so far the best possible insurance against potentially meaningless distortion of the results (Ebinger et al. 2005; Trinh et al. 2012). To avoid the onset of large gaps between theoretical models and experimental results due to a big difference in the accuracy level required, experts in the study of nonlinearity and of instability problems should finally focus their attention to these exact issues, following and developing already existing sophisticated methods (Feyel 2003; Verhoosel et al. 2011).

6 Some Examples of Metamaterials with Relevant Potential and Open Questions

6.1 A Quick Look to Parabolic Pantographs

In this section, we will consider two examples of possible metamaterials which are currently under the attention of several researchers for their potentially advantageous characteristics. The first example can be provided by the just mentioned pantographic structures, i.e., a fabric with two families of orthogonal fibers. Herein we will consider a particular geometry for the fibers, in which they are arranged in parabolic curves. The disposition of the fibers, different from the straight ones, is conceived in order to have a greater rigidity for the same weight, but obviously entails greater difficulties and variety of behaviors at both micro- and macroscale. The reason behind the advantageous strength–weight ratio is probably connected to some kind of arch-like response due to the geometry of the fibers, but the exact mathematical formulation of this qualitative behavior is one of the basic theoretical questions to be addressed. In (Fig. 2) we show straight fibers pantographic structures.

Figure 3 shows the current configuration of a rectangular fabric with aspect ratio 6:1 in a standard bias extension test. As usual the plot of the shear strain relative to the initial fiber axes exhibits three zones in which this deformation is almost unchanged

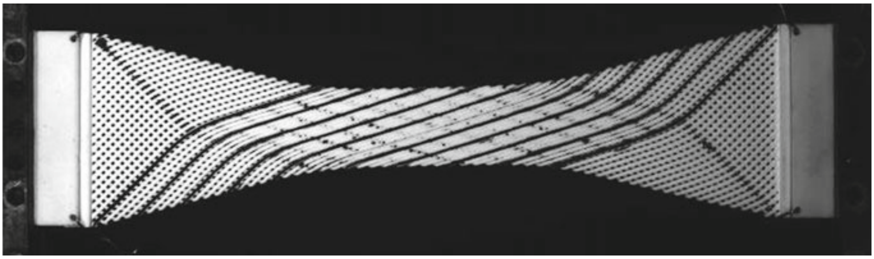

Fig. 2 An architectured material which is lightweight, extremely resistant, and safe in failure. Pantographic sheet under extensional bias test (see dell'Isola et al. 2016b)

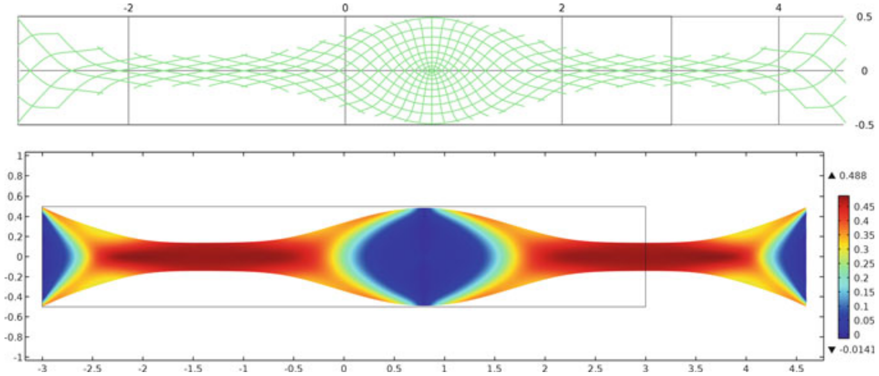

Fig. 3 Bias extension test—aspect ratio of the rectangular lattice 6:1. *Top* Fiber pattern; *bottom* Shear strain relative to the initial fiber axes

with the presence of narrow transition zones due to the presence of a second gradient energy in the model employed to describe bending deformation of fibers (Steigmann and Dell'Isola 2015). Figure 4 shows for a sample of ratio 3:1 a generalized bias test in which there is also an out-of-plane twist deformation imposed on one short edge. The particular arrangement of the fibers, in this case, may result in both out-of-plane and in-plane buckling phenomena if the displacement imposed is beyond a critical value.

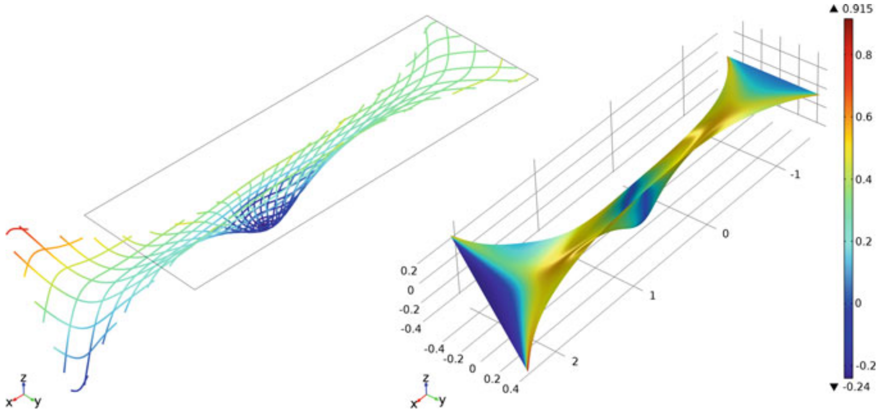

Fig. 4 Test with stretching and twist (45°) (aspect ratio of the rectangular lattice 3:1): Equilibrium shape; the shear strain relative to the initial fiber axes

6.2 Some Preliminary Results on Elastic-Plastic Honeycombs

Another example of metamaterial will be discussed in slightly greater detail, in order to show in a particular case some of the issues discussed before in general terms. Let us, therefore, consider the so-called honeycomb structures.

Honeycomb structures (from now on we will abbreviate the name to honeycombs) are solids with a periodic cellular body which confers them the property of being at the same time very light but still extremely stress resistant. They are produced mainly through two different manufacturing processes: the most used one is the method of expansion, in which the sheets of metal are glued together through binding strips and then expanded, while the less, but still widely, used is the corrugation method (which may be seen as a particularly simple version of roll-to-roll processing), in which the sheets are first deformed in the shape of half-hexagons and then glued together. These structures have been extensively studied and are clearly bio-inspired. The peculiar properties of natural honeycombs were already remarked by Hellenistic scientists (see Russo et al. 2013) and it has been conjectured that one of the first optimization problems was practically solved by bees finding the regular polygon maximizing surface–perimeter ratio. We believe however that there is a lot of room left for technological innovation: inhomogeneities in the microstructure, introduction of active or semi-active components, the addition of composite microscopic substructure are all examples of potentially very fruitful structural modifications which may induce exotic macroscopic behaviors.

Because of their peculiarity, honeycombs are largely used in packaging industries, as well as in computers or electronic components, and among all in aerospace industries or high speed cars. Many authors, such as Gibson and Ashby (1997), Papka and Kyriakides (1998), already attacked the problem of modeling the behavior of such structures with classical balance laws of mechanics, and presented many experimental and numerical results. Here we will show some preliminary results of a wider plan of investigation, approaching the 2D, plane, elastic-plastic problem, trying to focus on the identification of a good representative elementary cell which we will use to perform numerical simulations. From these, we will obtain the stress–strain curves which we will need in future to extrapolate the information necessary to create a mathematical, ‘nonclassical’, model for the homogenized plastic behavior of such structures. As we will see, we will use a parametrization of the stress state which will allow us to catch in the best way the behavior of honeycombs so as to be able, in future investigations, to extend these results to other periodic structures and cellular solids. We would like to remark the importance of cellular solids in nowadays engineering perspective, since their particular structures enable to minimize the costs of production (a very small amount of material is needed to build them), still giving them the properties required from the final industrial scope. Moreover, studying structures, such as honeycombs, foams, wood, cancellous bones, we will be able to modify them in order to optimize their properties and even to create new ones (thanks to the advanced techniques of 3D printing or electrospinning) with exotic behaviors, matching the demand of industries.

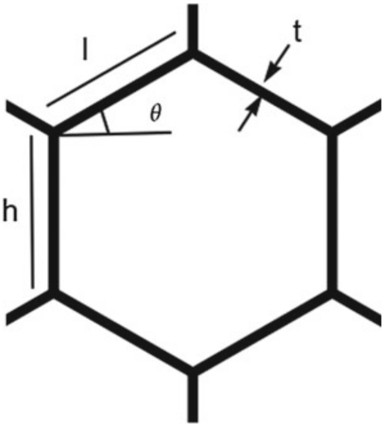

Fig. 5 Hexagonal unitary cell

A preliminary analysis can be performed on the mechanical properties of the core material. Here we will consider Aluminum-5052 (Young's modulus = 68.97 GPa, Poisson's ratio = 0.3, yield strength = 292 MPa). For such a material we consider an elastic-perfectly plastic model. We consider a structure made of regular hexagons with $h = l = 1$, $t = 0.216$ and $\theta = \pi/6$ (see Fig. 5).

We perform a first simulation on the whole structure, that we fix at the bottom, and to which we impose a strain in the y direction and periodic boundary conditions (so as to have an average strain in x direction = 0). From the numerical result,

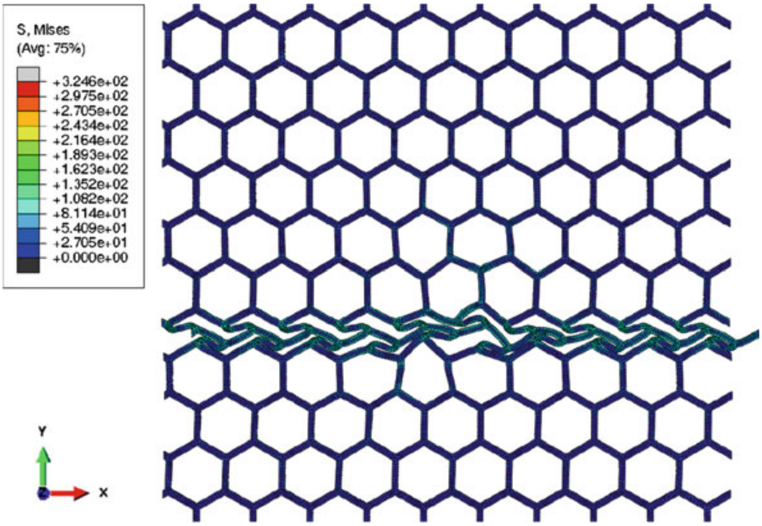

Fig. 6 Localization effect of plastic deformations

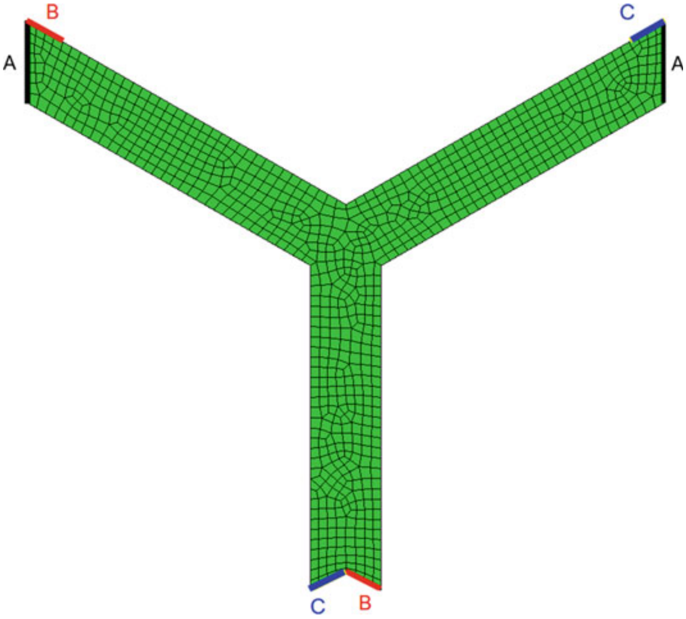

Fig. 7 Elementary cell with highlighted boundary conditions

shown in Fig. 6, we can already see the nontrivial behavior of the honeycombs, in the localization effect of the deformation. Indeed, after the initial elastic and local isotropic (Gibson and Ashby 1997) regime, the plastic deformation (which we initialised through a small concentrated pressure), is localized in rows adjacent one to each other.

This simple result already suggests that a generalized continuum approach is probably suitable in order to model the plastic behavior of the structure.

As an elementary cell, we select the one shown in Fig. 7. Its behavior may be extended to the macrostructure, if periodicity is assumed.

The periodic boundary conditions that we imposed are also shown: the same displacement for the parts highlighted with the same color and named by the same letter is set.

Biaxial loads, which prescribe the average plane stress state, is enforced, so as to let the elementary cell to deform freely. Twenty-node quadratic isoparametric elements, with reduced integration, are used in the program ABAQUS.

Next, we see how we parametrize the stress state, in order to keep in mind the geometry of the structure and to better understand and interpret the obtained results. We parametrize the plane stress state

$$\mathbb{T} = \begin{bmatrix} T_{11} & T_{12} \\ T_{12} & T_{22} \end{bmatrix} \mathbf{e_i} \otimes \mathbf{e_j} \,,$$

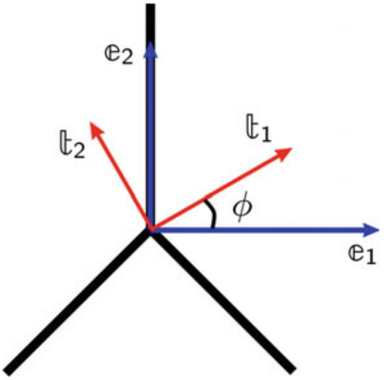

Fig. 8 In black the structure; in *blue* the standard base vectors; in *red* the ones which we use for the parametrization

trough:

- *orientation angle* ϕ
- *magnitude of the stress* $m = \sqrt{\lambda_1^2 + \lambda_2^2}$
- *biaxiality measure* χ s.t. $\lambda_1 = \cos(\chi)$, $\lambda_2 = \sin(\chi)$

Figure 8 may be useful to visualize the parametrization.

For the sake of simplicity we will consider only three kinds of tests:

- compression–compression test, $\lambda_1 < 0$, $\lambda_2 < 0$
- tension–tension test, $\lambda_1 > 0$, $\lambda_2 > 0$
- tension–compression test, $\lambda_1 > 0$, $\lambda_2 < 0$

Moreover, we will show the results for only 21 angles ϕ and 6 angles χ.

The power of using the elementary cell stands behind the fact that the time of computation is drastically reduced and that it allows us to really visualize which are the deformation experienced locally, at the microscale level, by the whole structure as we can see from Fig. 9.

Apart from the visualization argument, what is important are the data that we can extrapolate from each of such simulations. For example, particularly relevant are the stress–strain curves for this basic cases. We can see them in Fig. 10. Only small deformations are considered and on the axes the norms of the linear strain tensor and Cauchy stress tensor are reported. We can clearly distinguish the 6 groups of simulations for each type of test, for the values that χ assumes (notice that $\chi = 0$ is a simple uniaxial test while $\chi = \pi/4$ is a 'isotropic' biaxial test), and the 21 angles per each value of χ, that ϕ assumes. It is already possible to extract some conclusions:

- increasing χ (which means increasing the 'biaxiality' of the tests), increases the stiffness of the response;
- tension–compression tests are, in general, the weakest, tension–tension ones the strongest and compression–compression are in between the former two;
- the isotropy of the elastic regime is visible by the fact that the response for different values of the variables tend to coincide;

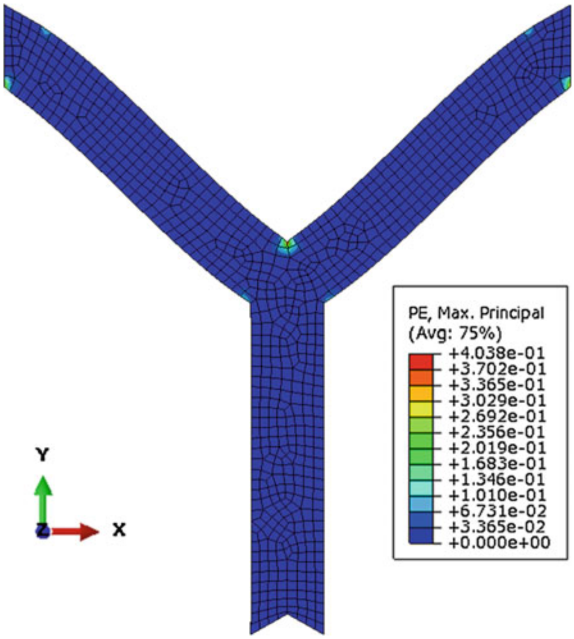

Fig. 9 Final configuration of a compression–compression test

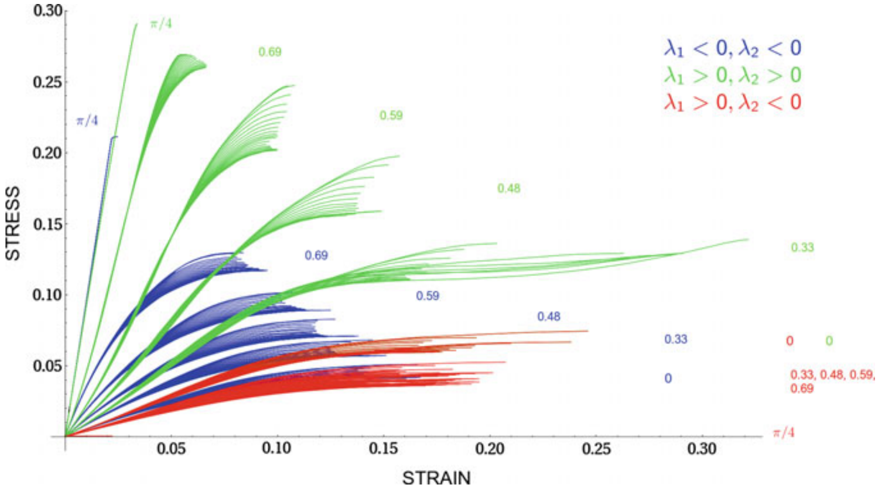

Fig. 10 Stress–strain response of all tests: in *blue* compression–compression; in *green* tension–tension; in *red* tension–compression. Tests reported for different values of χ

- the anisotropy emerges only within the nonlinear elastic part and it is more evident in plasticity

Finally, we would like to point out the unusual nonconvex behavior of such curves, which may be of interest for further investigation with nonlinear theory.

To sum up, we found a candidate elementary cell which should be validated by means of future investigations so as to make sure that it is capable to describe well the behavior of the macrostructure. Obviously these are preliminary results, which will be used in future to investigate the yield limit of such structures (i.e., the limit strength which they can sustain before undergoing plastic deformations) and the following plastic behavior of honeycombs. This last purpose is still challenging if one wants to face it with a nonclassical approach, introducing second gradient theories, which may provide a better description, capable of capturing the effect of localization and therefore predict the collapsing modes of such structures.

7 Conclusions. the Leader's Role: An Eye Kept on Past, Present and Future

Due attention should finally be paid to global research management issues. The researchers should completely agree to work together, toward the achievement of the aforementioned objectives, and a harmonious and fruitful cooperation is simply necessary if good results are expected. As already mentioned, the leading figures in the networks should coordinate and drive the research work, providing the overall strategy. The leaders of such a challenging scientific project have to be fully responsible for the actual realization, ensuring that every collaborating researcher always have a precise idea of the results obtained by the others and of the scientific work that they are expected to do. The network as a whole should always behave as a problem-oriented unity where different competences will coalesce to supply a timely, innovative, and potentially ground-breaking scientific understanding.

We expect that the proposed approach will emerge gradually as the only one really viable in order to make the so needed advancements in metamaterial science. Those researchers and groups that will understand the intrinsically multidisciplinary nature of the required skills, and will be open to enlarge their horizons to new problems ad techniques, will result as the winners in this fascinating competition.

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