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EXPLORING EXPO GEOMETRIC EXPLOITS

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Abstract. International world fairs are temporary events in which world countries show themselves in a relative narrow space, in buildings constructed on purpose, usually following the main theme of the exposition. These temporary buildings, by their very ephemeral essence, transient, of rapid construction and consumption, express summarily the image of the historical present through a representation of progress and modernity, using the geometry as a means of symbolic abstraction and as materialization of "aesthetic" choices. The different nations participate to the world fairs in order to show their best characteristics, often coinciding with the achievements in the various fields of knowledge, by giving to the architects the task of representing that with amazing pavilions, exploits of imagery and manufacturing competency. From time to time, those buildings became urban icons.
Starting from an historical introduction of some of the most famous and relevant, from a geometric point of view, pavilions, we will explore the last world fair architecture. Expo 2015 was held in Milan, Italy, from May 1st to October 31st, 2015, and its main theme was *Feeding the Planet, Energy for Life*. Although it is noticeable, in this expo just ended, that geometries represent mainly the configuration of the exteriors, some pavilions occur anyway as objects for which the search for the form is addressed starting from strong geometric figures. It may be noted that in some cases the volumes and in others the surfaces determine the external images of the pavilions and in some cases also the character of the interior. We will examine only some of the pavilions - it would occur a book to deal with all – exploring the geometric exploits of this edition of the world fairs.

Keywords: architecture, geometric inspiration, math for building

Mathematics Subject Classification: Primary 00A67.

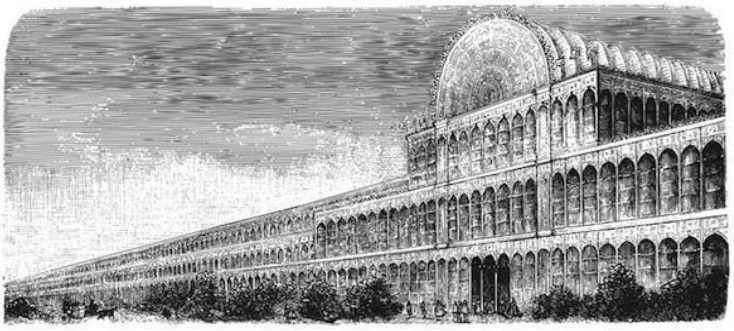
1 Historical overview

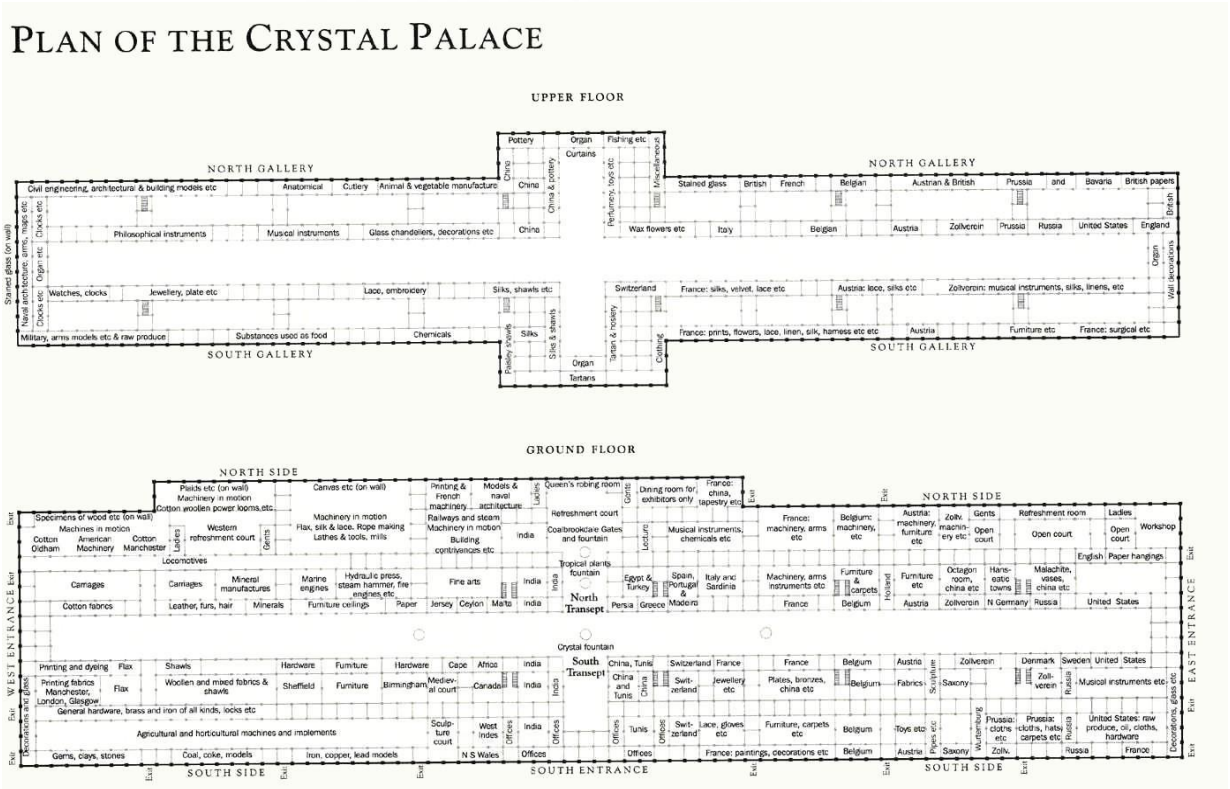
The definition of “Exploit” by Oxford Advanced Learner's Dictionary[1] is: a bold or daring feat, act; an action or a piece of work that needs skill, strength or courage. Middle English: from Old French *esploit* (noun), based on Latin *explicare*, 'unfold'.
The history of the Expo coincides, for architects, with the beginning of modernity. It is the industrial revolution which brought about a real change in the way of conceiving architecture, no

[1] http://www.oxforddictionaries.com

longer addressed only to great works, but oriented to labor and living spaces. This is why Kenneth Frampton [1], one of the most prestigious architecture historians, places the beginning of the modern era to the opening of the *École des Ponts et Chausée*. A School, therefore, not an Academy, which prepares young people for the job of building and not to the art of Architecture.

For the very first Expo, which was held in London in 1851, the Great Exhibition of the Works of Industry of All Nations, Joseph Paxton, architect and botanist of peasant origin, built the Crystal Palace [2], a building of iron and glass so large (four times St. Peter's Basilica in Rome) to hold all the international exhibitors in a large modular space, which extended as far as the eye could see, if you looked from the outside, but even more from the inside. The repetition of the prefabricated square module of side 24 feet (about 7.3 m), was a succession of empty items stacked and put side by side: it modulated the pace of the gaps between the vertical supports and the perimeter of the glass walls, which were also perceived as empty. The dematerialization of the wall, the subsequent opening of space in multiple views and unlimited perspective is a revolutionary concept in the mid-nineteenth century, still not fully explored nowadays.





Joseph Paxton, Crystal Palace: a modular building, potentially growing up to the infinite.

Fig. 1. An unlimited view with more and more of the 77 rows of rectangular modules

Fig. 2. Plan of the two levels with the modular grid and the location of the participating Countries

A different geometrical form of the architectural design, that instead of appealing to the iteration/composition of elements built on the golden ratio to ensure beauty and harmony, is aimed at an essential and optimized geometry, ensuring efficiency and speed of construction. The Crystal Palace, first and only pavilion of the initial universal expo, had a size of 84,000 m² : the main part of the building was made up of 77x17 modules , the entire complex was completed in four months.

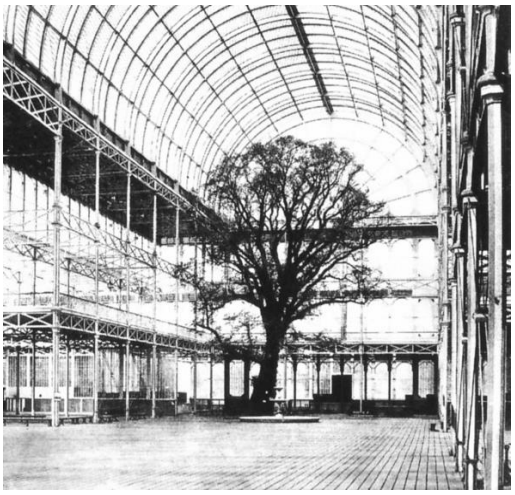
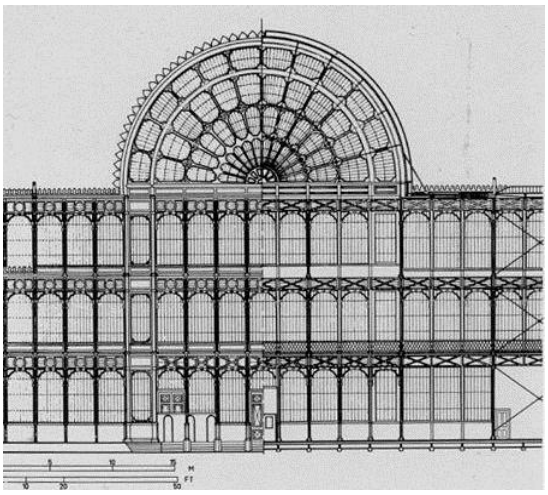



Fig. 3, 4. Joseph Paxton, Crystal Palace: internal view and a detail of the main front and section

Romantic theorists denied any artistic value to the building and John Ruskin dismissed the Crystal Palace as a horrible greenhouse bigger than all those built so far, for which a little of normal algebra was everything that glass could represent of human thought.
Placing in the history of the expos this first act, as part of the history of architecture, makes understand a character that joins the pavilions of the most famous international world fairs. These temporary buildings, by their very ephemeral essence, transient, of rapid construction and consumption, express summarily the image of the historical present through a representation of progress and modernity, using the geometry as a means of symbolic abstraction and as materialization of "aesthetic" choices.
The aesthetics related to the use of geometry is a topic in the field of theories of architecture gradually applied to the compositional process to draw plans and elevations according dimensional relationships, hidden but perceived at a deeper level, probably by virtue of a rooted cultural conditioning; also applied to polygonal modularity, up to the representation of buildings according to new geometries, to the topological transformations of surfaces and volumes, to the dynamic and kinematic parametric representations, and even with reference to René Thom catastrophe theory.
The different nations participate to the world fairs in order to show their best characteristics, often coinciding with the achievements in the various fields of knowledge, by giving to the architects the task of representing that with amazing pavilions, exploits of imagery and manufacturing competency. From time to time, those buildings became urban icons. Now we will briefly examine some of them.

1.1 Paris 1889. Brussels 1958

Two emblematic examples among all: the Eiffel Tower in Paris [3][4][5] and the Atomium in Brussels [6].

The first was built in 1889 by the Gustave Eiffel for the Expo in Paris celebrating the centenary of the French Revolution. At the beginning the tower had no practical purpose. It was simply representative, and now it is in all respects the symbol of Paris. It had to be removed 20 years after its completion and on the contrary, it was preserved, and because of its height at its top the radio transmission system of the city was installed. The powerful image of the tower is obtained by the iteration of geometrical metallic pattern, a lace whole structure and no decoration, as rich as a Gothic cathedral. Each element is necessary for its static and, seen up close, it is a variation on theme of the auction-node system, with repeated concessions to the semicircular shape at different scales: the minimum coupling pieces and the maximum of the great arches that break off from the ground. The engineering and iron lattice structures in particular combine the geometric properties of spatial configurations with static characteristics, obtaining simple and efficient building systems.

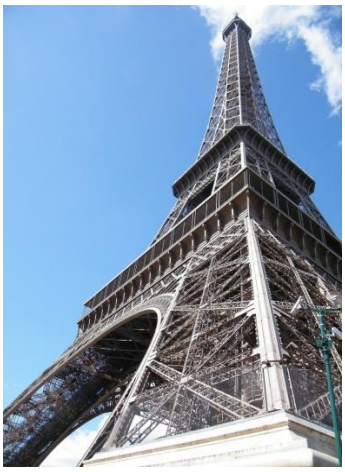



Fig- 5, 6. Paris– Eiffel Tour - © Alessandra Capanna 2012

The second, the Atomium in Brussels, was built as a permanent building in occasion of the 1958 Expo. At the time, the Belgian economy was sustained by the mining and metallurgical industry, so the architects André Waterkeyn, director of commercial Fabrimétal, and André and Jean Polak have designed this particular building that is nothing but the shape of a basic cell of an iron crystal, with its 9 atoms, magnified 165 billion times.

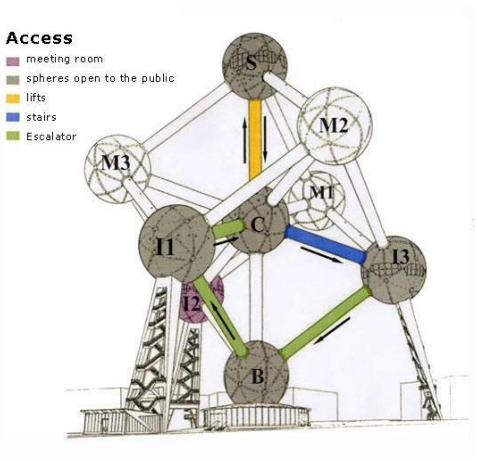



Fig. 7, 8. Bruxelles Atomium 1958. Source: internet - free common media

When the architecture project becomes symbolic, when the function is indifferent, when it it is a matter of event-architectures, the building coincides with the form. The figurative idea prevails. And in this representation of modernity it is precisely the geometric shapes to define the shape of the buildings as pieces of the imaginary of the architect, who has a repertoire of images provided by the cultural context in which he lives.

1.2 Philips Pavilion, Brussels 1958

In the same expo of 1958, the Philips Pavilion of Le Corbusier-Xenakis [7] was realized, that probably is the most mathematical of all, along with that of Buckminster Fuller in 1967, the largest bio-geodesic sphere that was part of the U.S.A. presence to the Expo of Montreal, that we will examine in the next section. The geometry of the Philips Pavilion comes from the challenge that Le Corbusier turned to his young collaborator, to translate with a little of mathematics the concept of "a bottle containing the nectar of the show" [7 pp. 36-59]. The intuition of Xenakis was to use minimal surfaces for an architecture conceived directly in the third dimension, in other words, reconstructing the track of the plant after processing surfaces variously oriented in space and their lines of junction. In particular it was the matter of producing three-dimensional models also very reduced in scale, at the same time to respond to an iconic problem and also to a structural one, that is, to have no pillars or other type of internal supports. Designing a container that was no longer a building, or rather, it was so in the etymological sense of the term, as a built entity, but it was pure geometry, not disguised by the aesthetic harmony of occurrences on which to base the composition of flat surfaces. A bottle, as Le Corbusier called it, defined by portions of hyperbolic paraboloids.

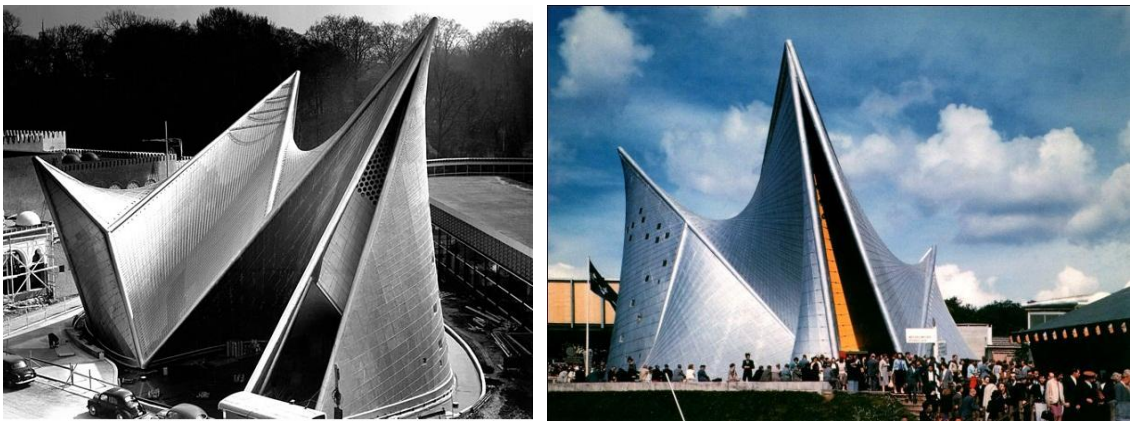


Fig. 9, 10. Philips Pavilion– Bruxelles 1958-59 – postcards Source: internet - free common media

The use of hyperbolic geometry in buildings was at that time on the rise, thanks to the new systems of construction: tensile structures, sometimes thin reinforced concrete, etc. for which the mathematical optimization connected to the study of minimal surfaces corresponded to the double advantage, for the architect, to economize on the amount of material of construction and to obtain self-supporting surfaces. It was only later that an aesthetic value was given to these structures and, by their technical use for sheds, viaducts, and temporary structures, they began to be used for the construction of roofs of buildings that aspired to a greater visibility in the surrounding context.

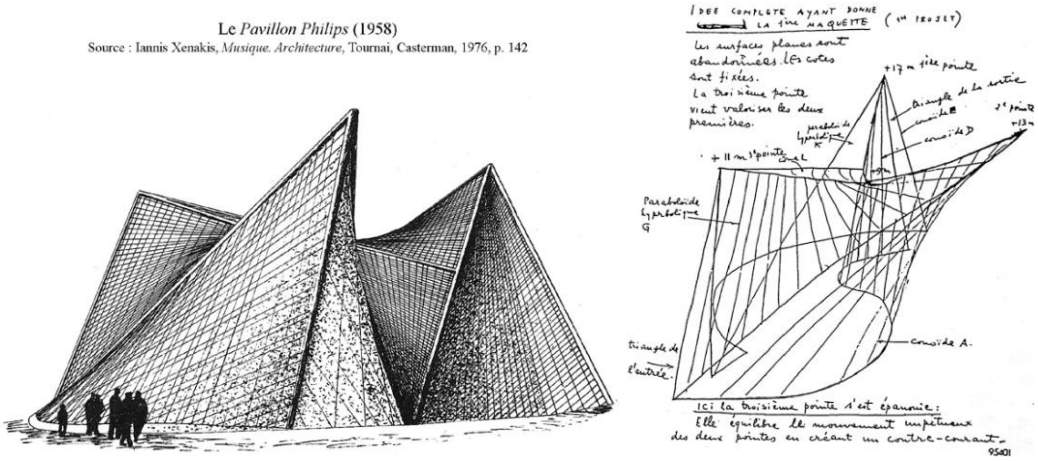


Fig. 11, 12. Iannis Xenakis First project. Sketches. Source: [9]

Iannis Xenakis, engineer, mathematician and musician, who worked for Le Corbusier in the postwar years, studied the shape of this pavilion, intended to be a pure container. A shell for the *Poème Electronique* show. From the statement of Xenakis, published in the Revue Technique Philips [8], the first project of the pavilion was made up of a portion of the conoid indicated by the letter E, a composite surface formed by two conoids that Xenakis in the sketch indicated by the letters A and D, from the hyperbolic paraboloids K and G , by a connection cone L and, finally, a pair of triangles "gaps " that defined the inlet and the outlet .

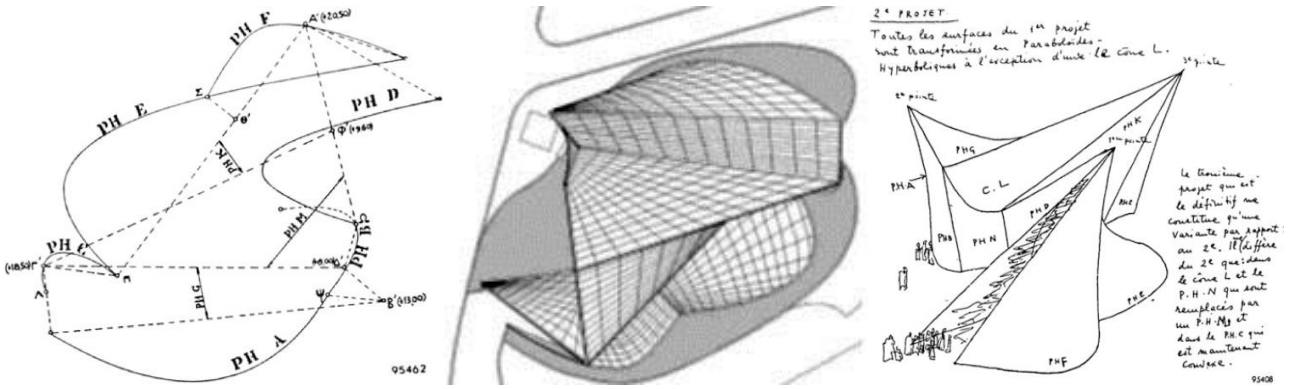


Fig. 13, 14. Philips Pavilion – geometric design of the plan. Source: [7][9]

Fig. 1.5. Sketch by Xenakis with notes about the construction of the volume. Source:[9]

At the end of the planning process, in order to obtain a pure shell, without internal supports, the outer surfaces of which were self-supporting, the pavilion was realized by keeping unchanged the hyperbolic paraboloids G and K. These constitute the main surfaces on which images were projected while the cone L was significantly expanded and the conoids A, E and D were transformed into 5 paraboloids: A, E and B, N and D and also paraboloids C and F were added [10].

1.3 Montreal 1967

Fig. 16. Buckminster Fuller. Biosphere for the U.S.A. pavilion. Source: internet - free common media.

Dealing with the most famous pavilions of the expos of the past, it is worth mentioning Montreal 1967.
In this expo Xenakis realized his first polytope within the French pavilion, while Buckminster Fuller [12] built the Biosphere geodesic (the US Pavilion) [17]. Fifteen years earlier, in 1952, he had already made his first geodesic dome, with a diameter of 18 meters in Massachusetts, the Dome Restaurant [11]. This presented several problems and then was removed, but later research led Fuller to refine the system deepening its geometric properties and the potential of the structural composition of triangles that he called perfect and "natural" figures that, in combination with other triangles, can provide the maximum efficiency with a minimum structural effort.

1.4 Osaka 1970

For the German Pavilion at the Osaka World's Fair Karlheinz Stockhausen with Fritz Bornemann designed a spherical space well-lighted by a starry vault created by Otto Pien. It is a spherical concert hall in which people are seating in the center, surrounded by loudspeakers gathered together in seven rings at different "latitudes" around the interior walls of the sphere (a synthetic description at [14], see also [13] and [15]).

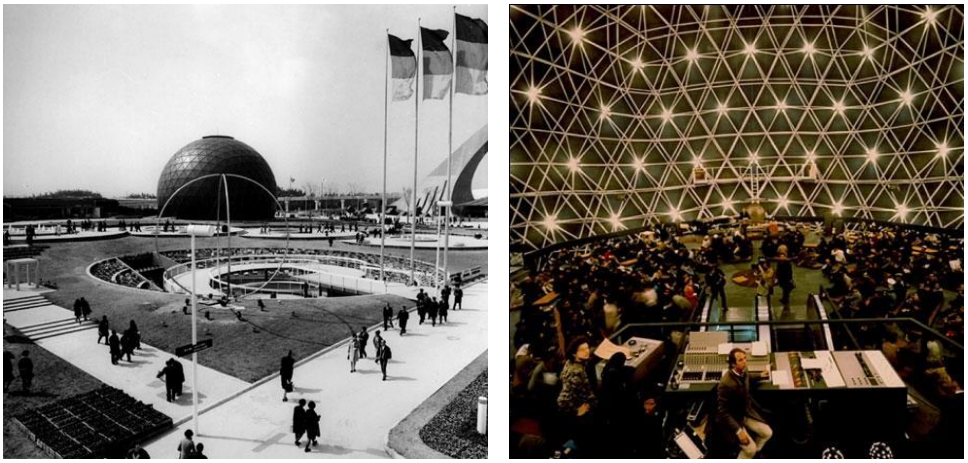


Fig. 17,18. Karlheinz Stockhausen, «Spherical Concert Hall» Source: internet - free common media.

1.5 Lisboa 1998

Alvaro Siza, together with the colleague Eduardo Souto de Moura, built the portuguese pavilion for this Expo. A description of the mathematical properties of the shape adopted for this building was presented in a previous Aplimat conference to which we refer [16].

Fig. 19. Alvaro Siza, Portuguese pavilion

“The use of catenaries in Architecture has been further incremented by the use of concrete, which allows an unprecedented levity. As an example, we mention the work of Alvaro Siza who designed the Portugal Pavilion for Expo 1998, where levity is realized by hanging chains filled by a thin layer of (white) concrete. An area of 65x58 mt is topped by a geometrical shape molded in one piece 20 cm thick; the orthogonal independent sections form a family of (parallel) catenaries and straight lines, respectively, so that the resulting Gaussian curvature vanishes. Therefore it is a ruled and developable surface. In particular it is not "minimal", (otherwise its MEAN curvature would have to be zero)” [16, pp. 104-105].

2 EXPO MILANO 2015

Expo 2015 was a Universal Exposition hosted by Milan, Italy. Although it is noticeable, in this expo just ended, that geometries represent mainly the configuration of the exteriors, some pavilions occur anyway as objects for which the search for the form is addressed starting from strong geometric figures. It may be noted that in some cases the volumes and in others the surfaces determine the external images of the pavilions and in some cases also the character of the interior. France pavilion, for example, in particular.
If we analyze the method of composition typical of the architects - the figurative idea passes even before the implementation in terms of design, the abstraction and the search for the meaning of the subject of study, and then they analyze the actual requirements of the functional program – we can understand the continuous tension that the project action brings between the idea-form and necessary requirements.
It must be said, however, that the ephemeral architecture is made of shells, thin and light skins intended to be built and removed quickly; packaging, clothing, for what is internally exposed.

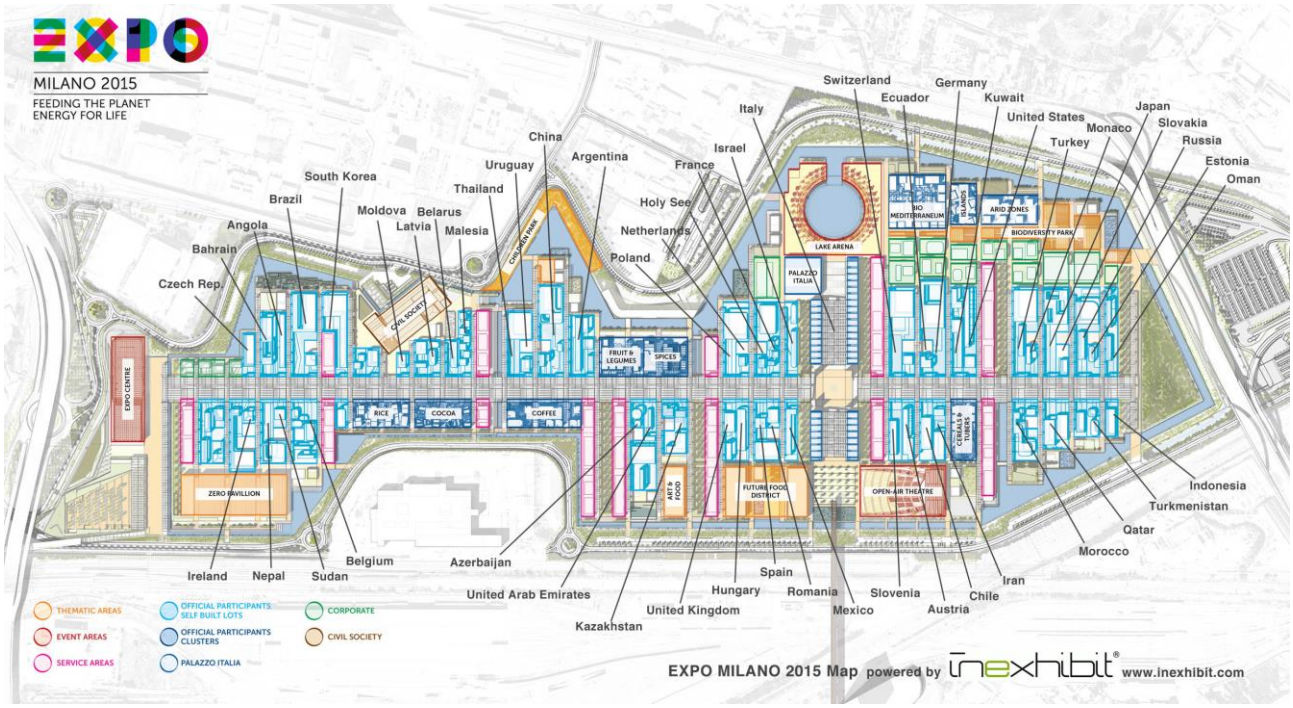


Fig. 20. Map of the Expo Milan 2015

Expo 2015 was held under the theme *Feeding the Planet, Energy for Life*. The opening took place on May 1, 2015 and the Expo closed on October 31, 2015. The site is located about 15 kilometres (9.3 mi) northwest of Milan in the municipalities of Rho and Pero, and covers an area of 1.1 km^2 (0.42 sq mi). The chosen area is oblong in shape with an overall length of nearly 3 km (1.9 mi), immediately suggesting the idea of a main axis conceived as a boulevard along which the pavilions would be located. The design of pools and waterways in and around the Expo area has been an element of primary importance. The main idea was to trace out within the area two distinct lines of demarcation, two avenues, a main avenue and a secondary avenue representing the ancient Roman layout comprising a cardo and a decumanus. The exhibition areas are completely identical for each country. The entire area was covered by large tent-like structures to convey the idea of a global marketplace.[2]

2.1 Geometric structures within Expo2015

We will now briefly take into consideration some of the iconic buildings of this Expo, starting from its symbols, to proceed to the examination of some of the pavilions. Of course it is impossible to deal with everything in such a brief space, it would occur a book to describe them all in detail. We will see how geometry/math is important in the exemplified cases, and from different points of view (inspiration, design, building techniques, digital management of the construction etc.).

[2] http://www.expo2015.org/; https://en.wikipedia.org/wiki/Expo_2015

2.1.1 Tree of Life

Let's start from the symbol of this Expo, located in the Lake Arena: the Tree of Life installation, designed by Marco Balich. The inspiration for this giant in wood and steel is the pavement in Piazza del Campidoglio (Capitoline Hill) in Rome, by Michelangelo (1538 to ~ 1650), symbol of the Italian Renaissance, but not only, present in different epochs, from the Romans to the XX century, as ornament for pavements, etc.

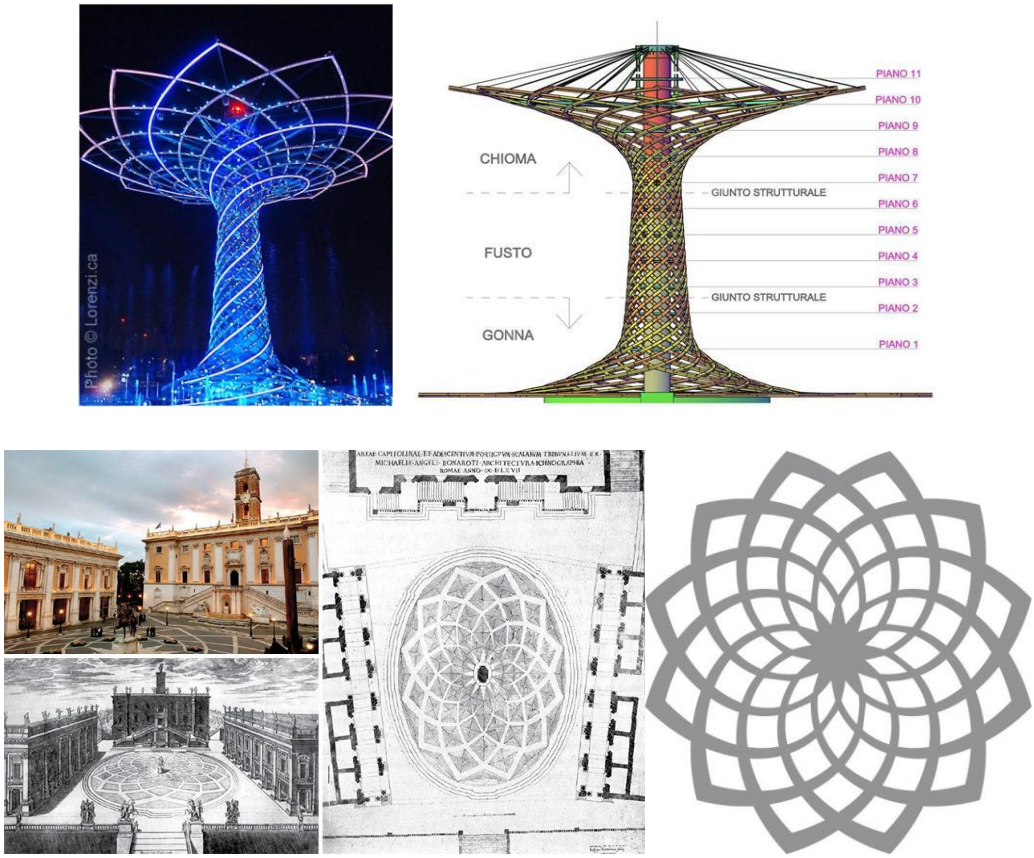


Fig. 21. The tree of Life; lateral section of the plan; inspiration from Capitoline Hill in Rome; design of the branches

This place was very important for the Romans, it was the arrival of the triumphal path of the Imperial Forums, and it was to be considered *Umbilicus Caput Mundi*, the most sacred place of the city. At its center there was an oval stone denominated "*Omphalos*" (navel), of which the ornament is a symbol, reminding the one contained into the ancient Temple of Apollo in Delphi, Greece.[3] This has been united to another important symbol, the "tree of life", generator of Nature, the great force from which everything has originated. Symbols and forms that persist through the centuries, as we suggested in another paper [16]. The novelty is the realization in 3D of these two symbols in a unique artifact, as strong symbol both of the event and of its theme.
In order to design it, sophisticated finite elements software packages (that implement the finite element method for solving partial differential equations or aid in the pre- and post-processing of finite element models) have been used, as well as, for graphic modelling and production technical

[3] W.H. Rocher, *Neue Omphalosstudien*, Lipsia, 1915

drafts, dedicated 3D software with output for numerical control machines. Just a few numbers: its height is 36 m, the wooden part of the tree weighs in total about 90 tons, the central column or trunk has a diameter of 2,5 m, the top of 40 m. There are 24 whorls, 12 clockwise and 12 counterclockwise, of about 124 m length each. Light, music (composed by Roberto Cacciapaglia), water and 3d effects complete this emotional "Show of life"[4].

2.1.2 Italia Pavilion

Another symbol of this Expo is the Italian Pavilion, the only one to remain permanently on site.
Four compositional elements represent in a figurative sense big trees, that elevate to form an intricate net of branches, culminating with a glass canopy, that lets natural light filter.

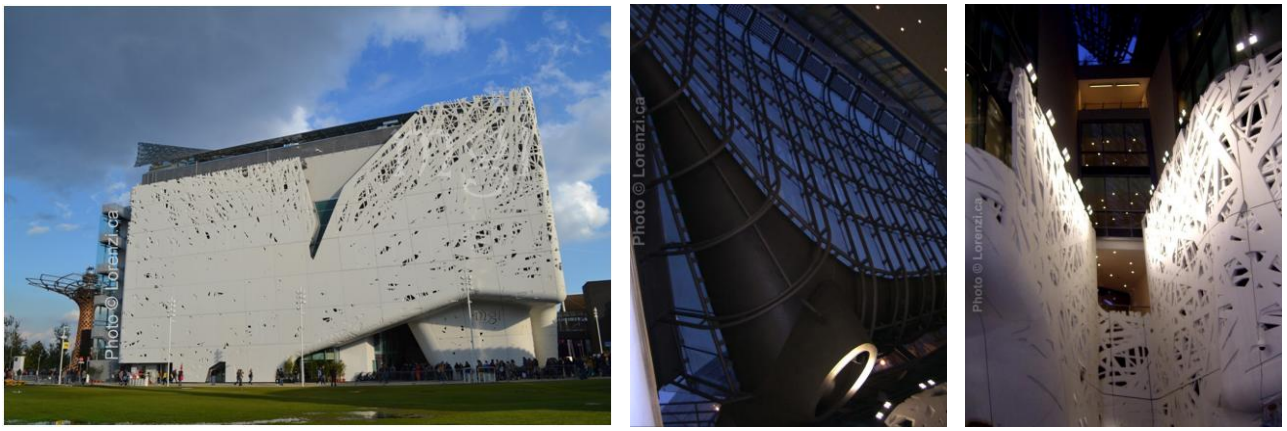
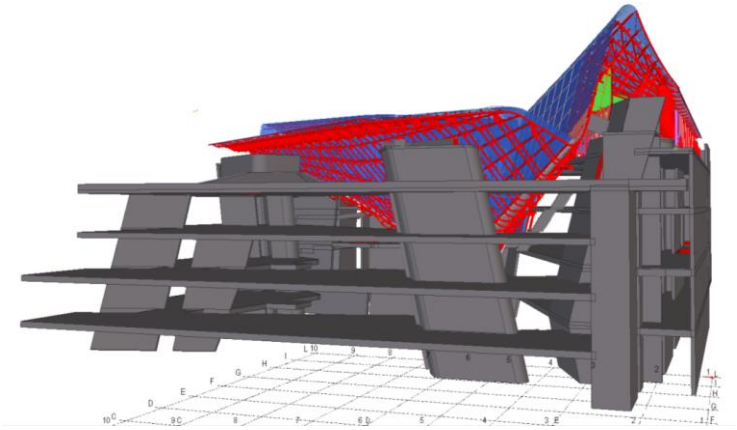


Fig. 22. Pavilion Italia: view and particulars from the interior. Photo © Marcella Giulia Lorenzi

Its steel and glass roofing is a unique suspended mold curved in a complex form, split by a net that adapts to the curvatures of such a created architecture.[5]
"Each creation is the result of a trade-off between the freedom of an idea and the restraints of matter. However, there are materials that create their own shapes. One of them is the biodynamic cement by Italcementi used to build the Italian Pavilion at Expo Milano 2015. A structure that evokes a forest made up of very complex elements that only i.active BIODYNAMIC with its excellent plasticity could achieve.
What Pier Luigi Nervi called *the most beautiful material that humanity has ever invented* has demonstrated that matter has its own aesthetics when the designer and the producer accept the ongoing challenge of Research and Innovation".[6]
"Palazzo Italia is a unique work of architecture. Every component was designed as craftwork and then made industrially, overturning all the rules" Enrico Borgarello, Director of Research and Innovation at Italcementi[7]

[4] http://www.farecultura.net/wordpress/senza-categoria/801/nutrire-il-pianeta-con-expo-2015-seconda-parte/
[5] http://www.promozioneacciaio.it/cms/it6120-expo-2015-la-vela-in-acciaio-di-palazzo-italia.asp
[6] http://www.padiglioneitaliaexpo2015.com/media/news/pdf/leaflet-biodynamic-cement.pdf
[7] http://www.padiglioneitaliaexpo2015.com/media/news/pdf/brochure-palazzo-italia-the-matter-of-beauty.pdf



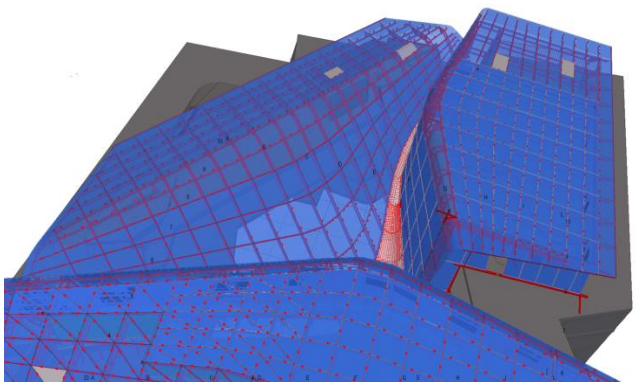


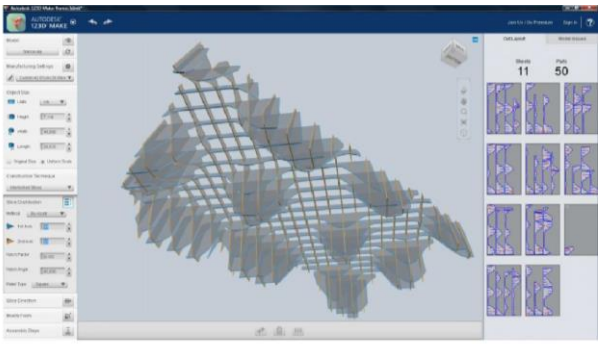
Fig. 23, 24, 25. Rendering and picture of the glass roofing. Source: http://www.harpaceas.it/project/palazzoitaliaexpo/

The design concept developed by Nemesi Studio & Partners has been processed by Harpaceas engineering, the first Italian BIM (Building Information Programing) Specialist. The use of BIM tools was crucial: it would be almost impossible to translate the complex geometries of the whole structure in 2D drawings. In particular the BIM methodology has supported the coordination of the structural model with the design of the facades, with the "outer skin" and with the "sail" of coverage, assembled by different companies.

2.1.3 France Pavilion

“A hexagonal shape that tectonic uplift have more or less upset”, we read in the report of the authors of this project. In fact, the low-tech market built to present the theme of the French food is a world volume that turns inside out. A structural laminated grid filled with food that become textures, tessellations of curved surfaces, which are perceived as images reproduced by distorting mirrors. Externally it shows the transparency of the sequence of the structural rhythm, as if the ensuing sections of the building constituted a constantly interrupted volume, made of empty as the inside is made of obsessive filling of the blanks with items almost deprived of their peculiar nature: bottles, cheese, loaves of bread, etc. to become pure elements, entities, quantities. …

Fig. 26, 27. France Pavilion: exterior and particular from the interior. Photo © Marcella Giulia Lorenzi



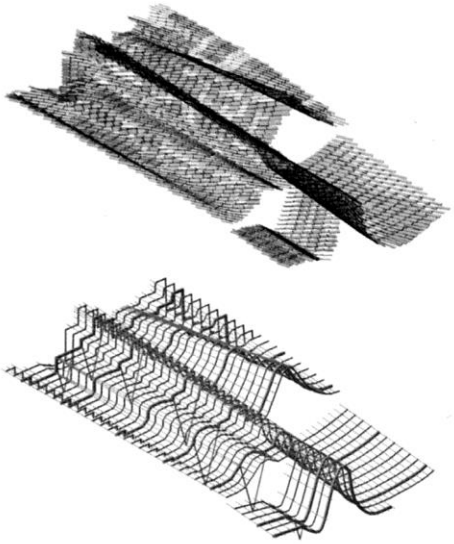
midiEXPO @ Liceo Brera Design survey- Milano 2015
Design Workshop and concepts: Prof.: Federico Brunetti,
kind support of: Arch. Marco Valentino; pavillon **FRANCE** 5F : student: GINEVRA
123Dmake model reconstruction

Fig. 28. France Pavilion. Geometry of the structure. Photo courtesy © Prof. Federico Brunetti

2.1.4 China pavilion

A large undulating roof supported by a structure of wood and steel, covered with a network of perforated metal panels. In this project, designed and developed by a multidisciplinary team leaded by Yichen Lueach at Tsinghua University & Studio Link-Arc, the elements that constitute the coverage are different one from the other to adapt to the continuous change of curvature of the wooden beams that form the double coverage. The rectangular elements, slightly curved to follow the shape, have a function of energy control, and were cut using a numerical control machine. These roof panels were designed via a uniquely digital process, which began by programming their geometry directly using Processing, followed by an intense optimization and rationalization effort to ensure they would closely follow the roof form in construction.

Fig. 29, 30. Pavilion China, Photo © Marcella Giulia Lorenzi. Interiors © Alessandra Capanna



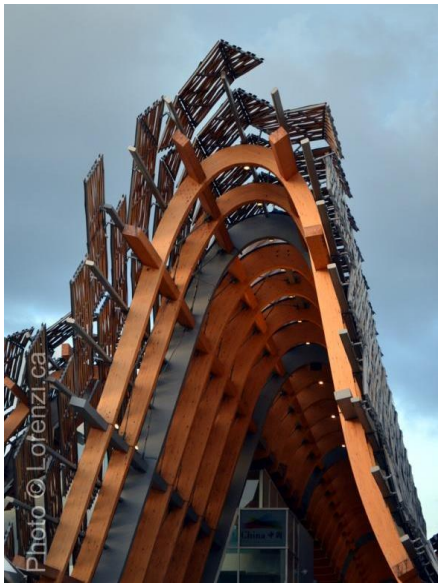


Fig. 31, 32. Geometrical outline of the roofing structure[8]. Source:[1] Industria delle costruzioni n.444 lug-ago 2015 pag 46. Detail of the roofing © Marcella Giulia Lorenzi

2.1.5 United Arab Emirates Pavilion

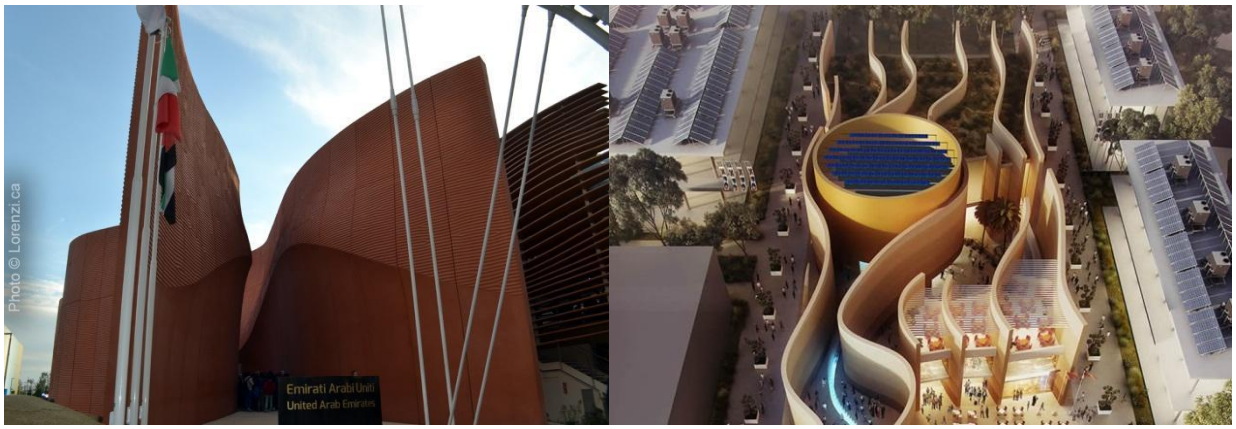


Fig. 33. United Arab Emirates Pavilion. Photo © Marcella Giulia Lorenzi
Fig. 34. United Arab Emirates Pavilion. Photo © Foster & Partners.[9]

United Arab Emirates united pavilion designed by Foster & Partners, is characterized by an impressive 12 meters-high structure that evokes both the half-shaded streets of the historic settlements of the United Arab Emirates than the sand dunes of its deserts.
The configuration of this hall is apparently made possible by the use of software for modeling of parametric surfaces and the particularities of these spaces is the perfect identity between external and internal (which in reality are also external) surfaces, to be among the other open-air rooms leading into the central core of the circular exhibition area. It is a walk-through architecture, as it is good practice for the spaces for temporary exhibitions, for which the identity of figurative internal and external offers a possible interpretation in terms of architectural geometries of continuous.

[8] http://link-arc.com/project/china-pavilion/
[9] Source: http://www.archinfo.it/il-padiglione-degli-emirati-arabi-uniti-a-expo-2015

The undulating walls recall the pedestrian ways and enclosure of the desert cities. Such surfaces were very difficult to draw before the advent of parametric modeling software and now are widely used in architecture.

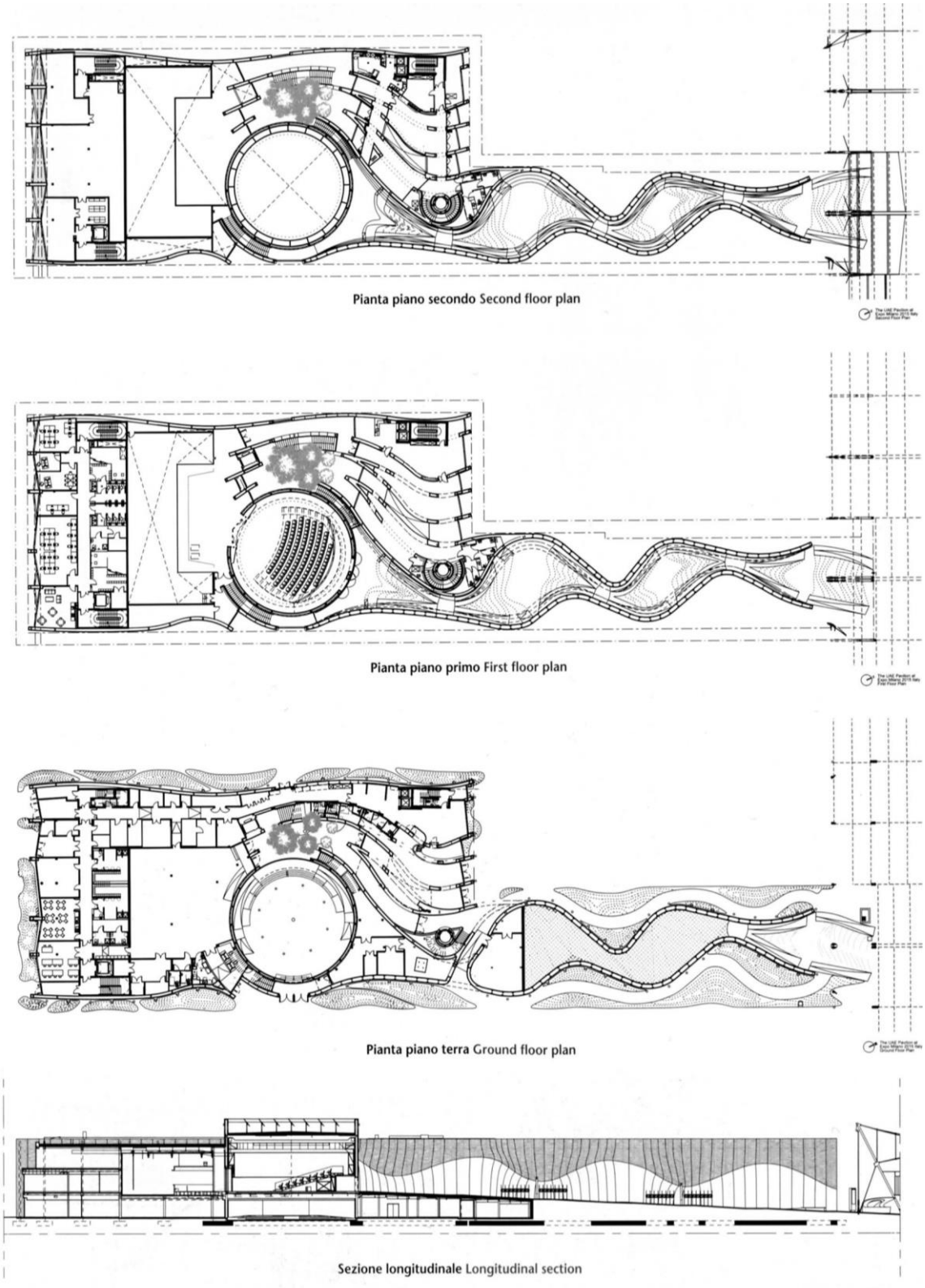


Fig. 35. Plan and section of the pavilion. Source: Industria delle costruzioni n.444 lug-ago 2015 pag. 55

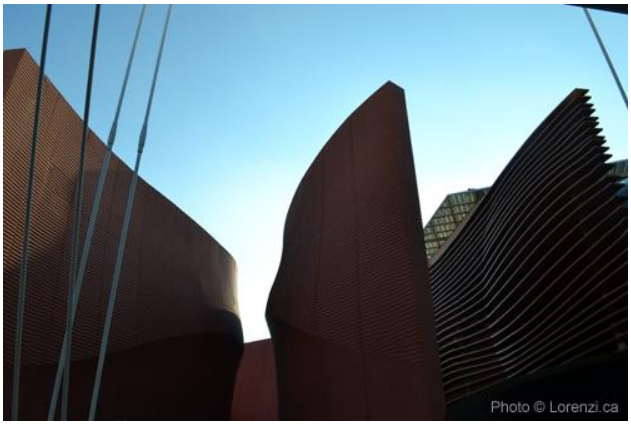


Fig. 36, 37. United Arab Emirates Pavilion next to Azerbaijan pavilion. Photo © Marcella Giulia Lorenzi

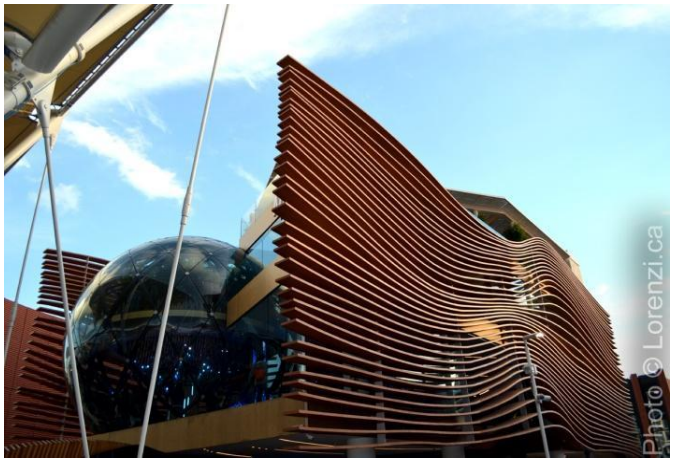


Fig. 38, 39. Azerbaijan pavilion, Photo © Marcella Giulia Lorenzi. Particular of the Interiors. Photo © Alessandra Capanna

Next to Emirates is the Pavilion of Azerbaijan. Notice the continuity effect given by the same curves and colors, even if by using totally different materials. In addition, other geometric elements are the curved front desk (Emirates) and the sphere as main building (Azerbaijan).

Patrick Schumacher (associate with Zaha Hadid) in his text "Parametricism, Candidate Epochal Style for the 21st Century"[10] states that: "It has now become both possible and necessary to enhance architecture's capacity to organize and articulate the increasing complexity of (the most advanced centres of) "Post-fordist" network society through the re-foundation of architectural semiology under the auspices of parametricism".

That means that parametricism offers a flexible set of components to manipulate, up to an infinite amount of variation, shapes for dynamic, and interactive surfaces that become "shells" splines, nurbs, and subdivs—perfect for temporary architectures.

So we can say that parametric surfaces represent new interpretative models of complexity, born "inside" the computer as Greg Lynn well summarized when he describes Digital Architectures which «use cybernetics to generate forms that are no more than calculus»[11], and that organic is natural and rational as well, because today all natural forms have calculus based construction and representation.

[10]http://www.patrikschumacher.com/Texts/The%20Historical%20Pertinence%20of%20Parametricism%20and%20the%20Prospect%20of%20a%20Free%20Market%20Urban%20Order.html

[11] Greg Lynn, How calculus is changing architecture, 2009 http://www.ted.com/talks/greg_lynn_on_organic_design

Moreover optimization of the relationship between form and structure are distinctive themes of an architectural language whose generative code is expressed by scripts and formulations that translate into alphanumeric code notions of technical physics, structural and energy performance, etc. All the potentiality of this method, deep rooted in the mathematical researches, are evident and very important in architecture studies in form-finding.

Fig. 40, 41. Sculptures and Vanke pavilion by Libeskind. Photo © Marcella Giulia Lorenzi

Also Libeskind makes extensive use of parametric softwares for his designs in this expo: four sculptures and Vanke Pavilion.
His Wings sculptures[12] are at the four corners of the crossing of Decumano and Cardo, the two principal roads of the Expo. Situated in the heart of the Expo in the Piazza Italia, four 10 m-high shimmering tree-like sculptures will anchor the four corners of the central square. Conceived as gates, each structure's dynamic form spiral out of the ground and spread into two branches spanning 10 meters over the square. Crafted out of brushed aluminum and fitted with innovative LED technology, the Wings animated the public space with a constant flow of pulsating patterns and imagery related to the theme of the Expo: health, energy, sustainability and technology.
Also the corporate pavilion for the leading real-estate company Vanke China explores key issues related to the theme of the Expo Milano 2015 drawn from Chinese culture related to food: the *shi-tang*, a traditional Chinese dining hall; the landscape, the fundamental element to life; and the dragon, which is metaphorically related to farming and sustenance. The twisting form of this pavilion set a number of complex challenges to structural practicality and facade geometry which were solved with some creative thought, in-house coding and complex parametric modelling. Even though no part of the building geometry was the same the structure itself needed to be simple and rational to limit cost and on-site complexity.[13]
The design features a sinuous geometrical pattern that flows between inside and outside. A grand staircase, clad in warm grey concrete, carves through the red serpentine form and guides visitors to the upper level.
The pavilion is clad in more than 4,000 red metalized tiles that Libeskind designed with the Italian company Casalgrande Padana. The geometric ceramic panels not only create an expressive pattern that is evocative of a dragon-like skin, but also possess highly sustainable self-cleaning and air purification properties. The three-dimensional surface is coated with a metallic coloration that changes as light and viewpoints shift. At times it will appear as deep crimson, then a dazzling gold, and even, at certain angles, a brilliant white. The tiles are installed with a state-of-the-art cladding support system that gives a rhythmic pattern and mathematical form to an otherwise supple,

[12] http://libeskind.com/related/sculpture/
[13] http://formatengineers.com/Vanke-Pavilion-Milan-Expo-2015: here you can find also examples of the codes.

torquing shape. Two spiraling stairs, echoing the form, ascend the pavilion to the south, and to the north from the Lake Arena entrance, serving both as circulation and seating.[14]

2.2 Exploring Expo2015 geometries

It would occur an entire book to describe in detail all the pavilions. So we will continue our tour simply showing other pavilions, grouped following their most evident geometric properties.

2.2.1 Triangles, hexagons

The UK pavilion by Wolfgang Buttress was awarded BIE Gold Award for Architecture and Landscape the Expo's highest design accolade and the International Prize for Best Pavilion Architecture.[15] Its structure contains a sphere surrounded by metallic structure made of hexagonal wires and represents a honeycomb.[16]


Fig. 42, 43, 44. UK Pavilion; particulars. Photo © Marcella Giulia Lorenzi

Fig.45, 46. UK Pavilion, particulars. Photo © Elena Paolucci

[14] http://libeskind.com/work/vanke-pavilion/
[15] http://www.ukpavilion2015.com/uk-pavilion/
[16] http://www.bdp.com/en/latest/news/2015/uk-pavilion-is-best-at-milan-expo/

Fig. 47, 48, 49. Turkey Pavilion , particulars. Photo © Marcella Giulia Lorenzi

Fig. 50, 51. Malaysia pavilion. Photo © Marcella Giulia Lorenzi

Fig. 52. Dome by Biagio di Carlo[17]

[17] http://www.biagiodicarlo.com/iweb/la_cupola_del_terzo_paradiso.html

The project “Coltivare la Città – La Risaia sul Tetto” – “Cultivating the city – The rise field on the roof” by novacivitas, is a urban agriculture field on the roof of Superstudio Più in Milan, Italy, one of the Events of Expo City. It has been an effective example of how nature and construction can coexist in perfect equilibrium, thanks to an architecture capable of sustaining the quality of the environment. The dome is designed by Biagio di Carlo.

Fig. 53, 54, 55. Domes as service structures. Photo © Marcella Giulia Lorenzi

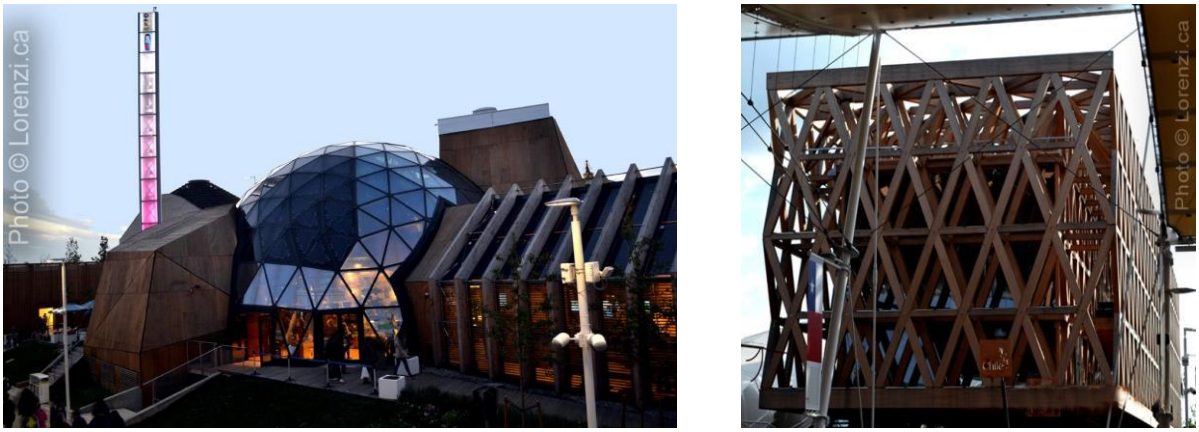


Fig. 56, 57, 58. Belgium Pavilion; fig. 59: Chile Pavilion. Photo © Marcella Giulia Lorenzi

2.1.2 Forms of organic inspiration

Fig. 60, 61. Mexico Pavilion. Photo © Marcella Giulia Lorenzi; Germany Pavilion. Photo © Alessandra Capanna

Fig. 62, 63. Vietnam, Indonesia pavilions. Photo © Marcella Giulia Lorenzi

2.1.3 Other geometries at Expo 2015

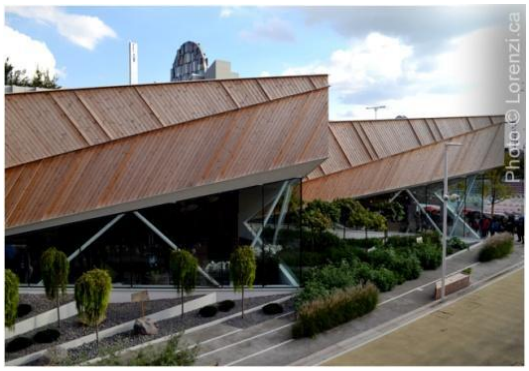


Fig. 64, 65. Slovenia and Slovakia Pavilions. Photo © Marcella Giulia Lorenzi

Fig. 66, 67. Qatar pavilion Photo © Marcella Giulia Lorenzi

Fig.68, 69. Kazakistan Pavilion. Photo © Marcella Giulia Lorenzi

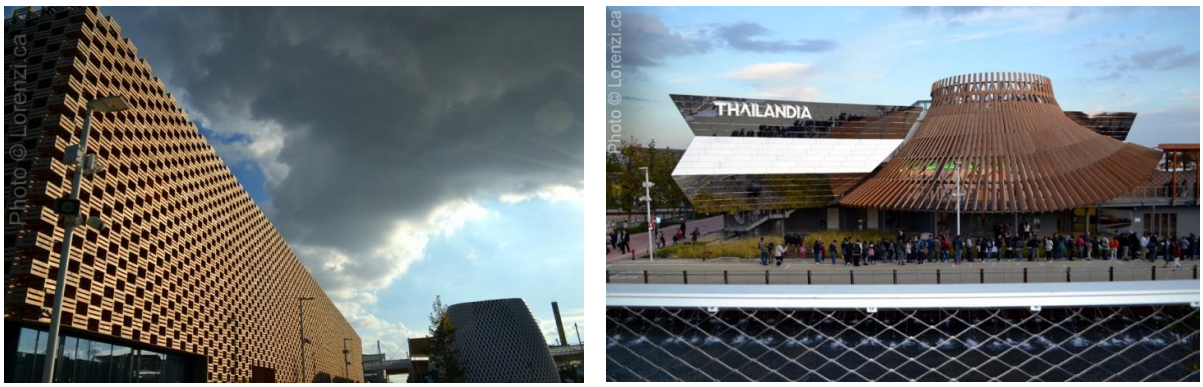


Fig. 70, 71. Poland and Intesa bank pavilions; Thailandia pavilion Photo © Marcella Giulia Lorenzi

Fig. 72, 73. Uruguay, Irpinia pavilions. Photo © Marcella Giulia Lorenzi

Fig. 74, 75. Alitalia/Ethiad pavilion; Cluster seas[18]. Photo © Marcella Giulia Lorenzi

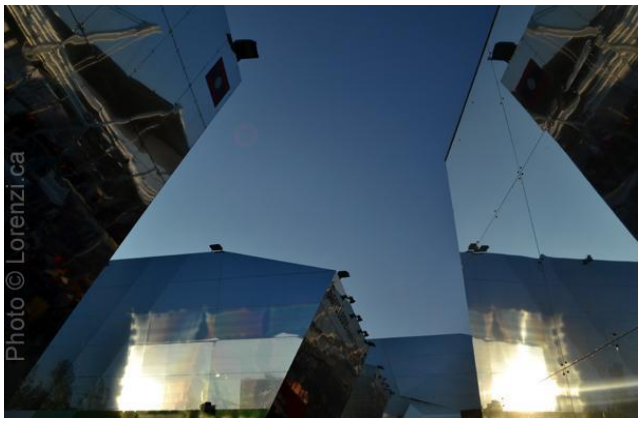


Fig. 76, 77. **Mirrors**. Russia pavilion; Rice Cluster. Photo © Marcella Giulia Lorenzi

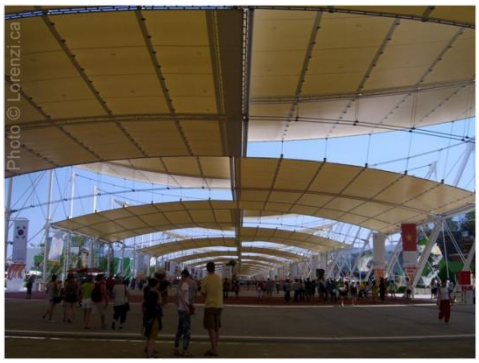



Fig. 78, 79, 80. **Tensile structures**. Photo © Marcella Giulia Lorenzi

[18] http://www.laps-a.com/Milan-EXPO;
http://www.polimi.it/fileadmin/user_upload/Ateneo/progetti/cluster_expo/Islands.pdf

Fig. 81-92. **Various.** Photo © Marcella Giulia Lorenzi

3 Educational projects

The enthusiasm for this important event and the not-to-miss occasion to see the cutting edge buildings and technologies attracted the interest of scholars, researchers and teachers, that made the Expo a topic on strong interest, especially from a didactic point of view.

3.1 PHOTO COMPETITION FOR STUDENTS

"Un punto di vista sull'Expo" (A point of view at the Expo) has been a photographic competition open to all the students of the faculty of Architecture of Rome "La Sapienza" with the aim to describe the Expo Milano 2015. Here a selection among the about 300 photos participating to the competition.
The focus is with evidence on the geometries characterizing the favorite pavilions for students in architecture. Even when the subject is a detail of the inside exhibitions, frame for pictures are geometrical regularities and ruled surfaces, 3d shapes embodying national pavilions.

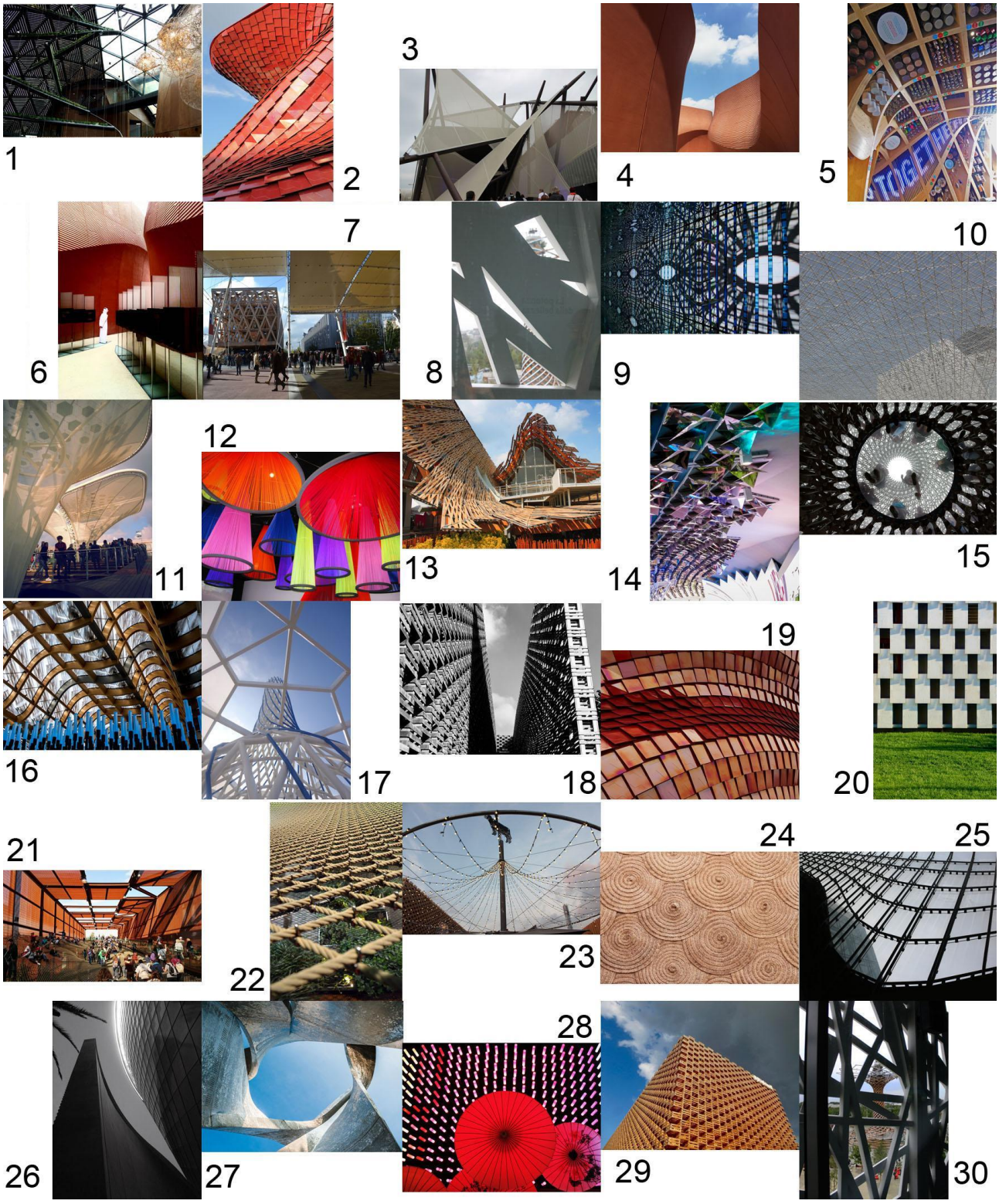


Fig. 93. A poster with selected pictures by students in Architecture[19]

[19] 1 Alessia Gallo IMPREVEDIBILI ARMONIE Belgio; 2 Silvia Cristofari PELLE padoglione Vanke; 3 Arianna Monaci I VELI DELL'ARCHITETTURA Kwait; 4 Ippolita Barnato the UAE pavilion BUT INSIDE A- PARALLEL SPACE Emirati; 5 Sara Linardi ALL TOGETHER NOW! Francia; 6 Fanny Ciufo ASPETTANDO DUBAI Emirati; 7 Andrea Di Mattia CULTURE A CONFRONTO padiglione Cile-Austria; 8 Noemi Domizi L'ALBERO DELLA VITA; 9 Diana Carolina LemaGuaman SPECCHIO RIFLESSO Inside padiglione Italia; 10 Valerio Ventura INTRECCIO TRA

3.2 MIDIEXPO

Another didactic experience concerning the geometries of the Pavilions at the EXPO2015 was held just in Milan. Prof.: Federico Brunetti, with proff.: Laura Stringini, Gianni Veneziano and kind support of Marco Valentino; Klima Hotel has organized a design workshop for young students of the Liceo Artistico Statale di Brera – Milano. They produced models (scale 1:100) of the most important pavilions as sampling and analysis of the most significant emerging projects and related geometries.

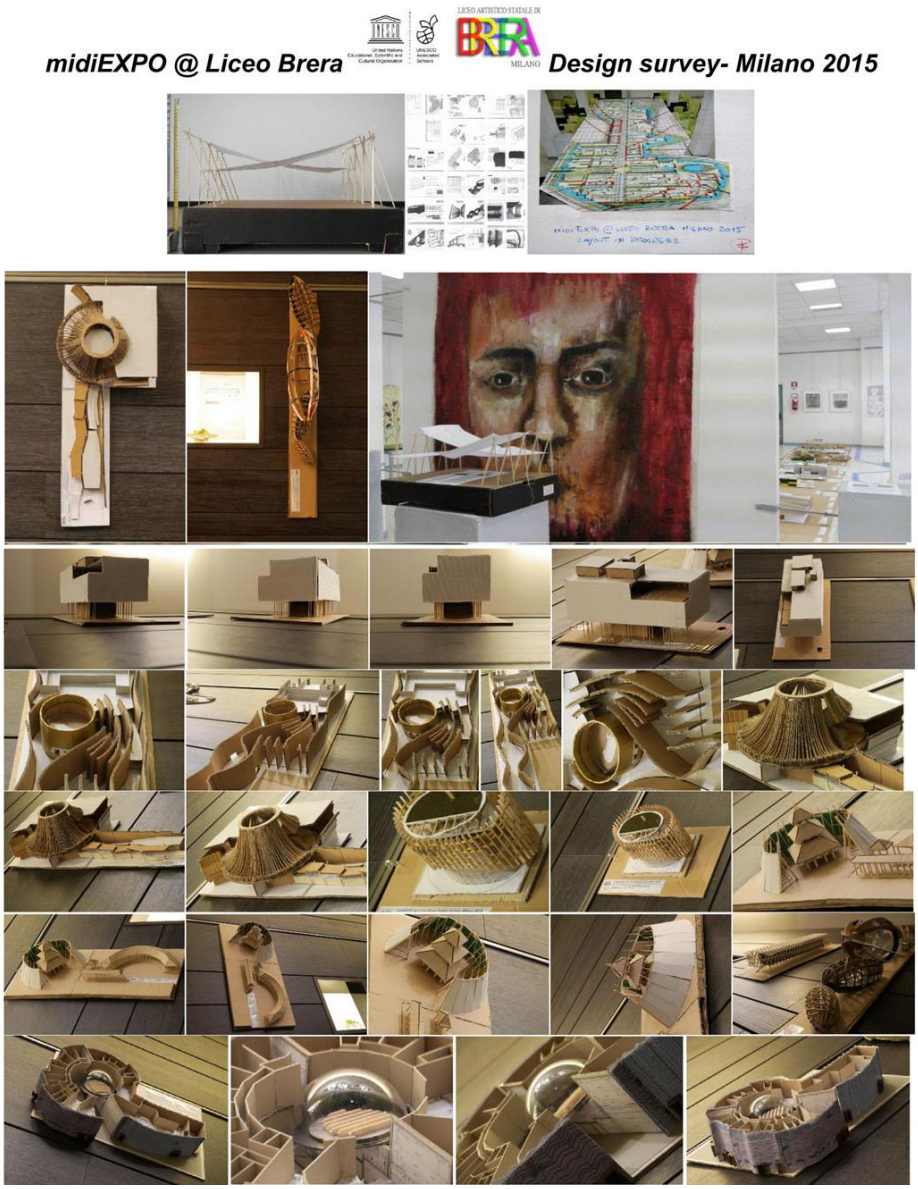


Fig. 94. A poster with selected models by students.

CULTURE padiglione Italia; 11 Silvia Cimino TERRAZZA DEL PADIGLIONE DELLA GERMANIA; 12 Camilla Gironi VIBRANCE Cluster del Caffè Repubblica Dominicana; 13 Giuseppe Longo - CINA; 14 Diana Ferro FRAGMENTS Iran; 15 Raissa D'uffizi IL CENTRO DELL'ALVEARE UK; 16 Irene Antonelli IL PASSATO RURALE NEL FUTURO TECNOLOGICO Cina; 17 Patrícia Mendes Araujo LO STRAORDINARIO CONTRADDITTORIO Turchia; 18 Iris Gjoni SCATOLE DI LEGNO Polonia; 19 Fabio Liberati ARCHITETTURA IN ONDA padiglione Vanke; 20 Viviana Cirulli GEOMETRIE-SOSTENIBILI Padiglione Intesa San Paolo; 21 Chiara Bartoli CONNESSIONE Brasile; 22 Leticia De Marchi IL TOCCO BRASILIANO Brasile; 23 Roberta Colonna EXPO FESTA TRA PAESI Olanda; 24 Roberta Colonna IL CERCHIO DELLA-VITA Spagna; 25 Valentina Diana LA CHIAVE DI SOL padiglione Italia; 26 Valentina Diana AFFINITA' ELETTIVE Emirati; 27 Iris Gjoni LIBESKIND AD EXPO piazza Italia; 28 Damiana Catalano TRADIZIONE E MODERNITA' CINESE Cina; 29 Martina Cellanetti KOSTKA Polonia; 30 Giuseppe Longo PADIGLIONE ITALIA

4 Conclusions

Examining the world fairs from their beginning we have seen that the temporary nature of these events offer limits but also freedom to experiment with buildings, very often using a geometric inspiration as materialization of "aesthetic" choices.
We have seen infact how geometry/math is important in the exemplified cases, and from different points of view (inspiration, design, building techniques, digital management of the construction etc.).
The projects follow, as in the case of Philips pavilion, new geometries, new technologies (hyperbolic geometry in buildings was at that time on the rise, thanks to the new systems of construction) or, in the case of Italia Pavilion, the possibility offered by math tools (bim) as well as new materials.
We have examined only some of the pavilions, further studies will be published in a book, exploring all the geometric exploits of this edition of the world fairs.

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This article and the entire session within the Aplimat conference is dedicated to Prof. Mauro Francaviglia, first co-organizer of the Math&Art session.
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PAVILION DESIGNERS

AZERBAIJAN - Studio Arassociati, sul concept di Daniele Zambelli, fondatore di Simmetrico;
BELGIUM - Patrick Genard & ass. and Marc Belderbos; CHILE - Cristian Undurraga; CHINA – Tsingua University + Studio Link-Arc; EMIRATES – Foster & partners; FRANCE – XTU architects: GERMANY – Schmiduber studio; ITALY - Nemesi & partners (Michele Molé - Susanna Tradati); MALAYSIA - Hijjas Architects + Planners; MEXICO – Francisco López Guerra Almada; JAPAN - Atsushi Kitagawara; KAZAKHSTAN - gtp2 Architekten; KUWAIT - Studio Italo Rota, POLAND - Piotr Musiałowski dello studio 2PM in collab. con i progettisti dello studio WXCA; RUSSIA - Studio Speech di Sergei Tchoban
SLOVENIA - SoNo Arhitekti; THAILANDIA - OBA The Office of Bangkok Architects; TURKEY – dDf / dream Design factory, UK – Wolfgang Buttress / BDP, URUGUAY - Javier Diaz, Y Arquitectura; VIETNAM - Vo Trong Nghia Architects

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