# PREVENTING LOW ACHIEVEMENT IN ARITHEMTIC THROUGH THE DIDACTICAL MATERIALS OF THE PERCONTARE PROJECT

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**Abstract**

PerContare is an innovative Italian project, built upon collaboration between cognitive psychologists and mathematics educators, aimed at developing teaching strategies for preventing and addressing early low achievement in arithmetic. The paper describes two emblematic examples of activities proposed within the project to foster the development of number sense that are grounded upon a kinaesthetic and visual-spatial approach. A study within the project was conducted to investigate the effectiveness of the materials used in the experimental classes. Results revealed a higher performance of the experimental group on a number of items of the assessment batteries. Moreover, this group contained half as many subjects with performance below the cut off score on the AC-MT battery compared with the control group. This suggests that the didactical materials developed in PerContare do contribute significantly to diminishing the number of potential false positives in the diagnoses of dyscalculia.

**Key words**: calculation, dyscalculia, fingers, inclusive classroom, part-whole relation

## Introduction

The PerContare project is an Italian inter-regional 3-year project (2011-2014) aimed at developing effective inclusive teaching strategies and materials to help primary school teachers (in Grades 1, 2, and 3) address low achievement, especially of students who are potentially at risk of being diagnosed with developmental dyscalculia (Butterworth, 2005). The teaching strategies and materials developed involve the use of digital and physical artefacts to help students construct mathematical meanings in a solid way, within the Theory of Semiotic Mediation (Bartolini Bussi and Mariotti, 2008).

This paper focuses on two emblematic examples of practices proposed within the PerContare project (also see Baccaglini-Frank and Bartolini Bussi, 2012; Baccaglini-Frank and Scorza, 2013), aimed at fostering interiorization of part-whole relations and awareness of 'structural' aspects of natural numbers (1) through strategies that include particular uses of fingers, and (2) through manipulation of straws in bundles of ten. In the following section I will describe the theoretical grounding of the proposed practices, and then discuss the video on which this paper is based.

## Theoretical Grounding

Studies in mathematics education have highlighted how sensori-motor, perceptive, and kinaesthetic experiences are fundamental for the formation of mathematical concepts – even highly abstract ones (Gallese and Lakoff, 2005; Radford, 2014). Various educators and researchers have designed didactical activities significantly based on bodily experience and on the manipulation of concrete objects. For example, Bartolini Bussi and Mariotti (2008), adopt a semiotic perspective, whereby student's use of specific artefacts in solving mathematical problems contributes to his/her development of mathematical meanings, in a potentially "coherent" way with respect to the mathematical meanings aimed at in the teaching activity. Also research in cognitive psychology – though from a different perspective – has identified specific and preferential channels of access and elaboration of information. For students with learning difficulties these include the non-verbal visual-spatial and the kinesthetic channels (Stella and Grandi, 2011).

Let us think about how these elements can apply to the domain of *number sense*. There is no monolithic interpretation of this notion across the communities of cognitive scientists and of mathematics educators, and not even within the community of mathematics educators alone (e.g. Berch, 2005). However, there seems to be a certain consensus about some features of the notion, which have important implications for mathematics education. The development of number sense is seen as a necessary condition for learning formal arithmetic at the early elementary level (e.g., Griffin, Case and Siegler, 1994; Verschaffel and De Corte, 1996) and it is critical to early algebraic reasoning, particularly in relation to perceiving the "structure" of number (Mulligan and Mitchelmore, 2013).

Moreover, literature from the fields of neuroscience, developmental psychology, and mathematics education indicate that using fingers for counting and representing numbers (Brissiaud, 1992), but also in more basic ways (Butterworth, 2005; Gracia-Bafalluy and Noel, 2008), can have a positive effect on the development of numerical abilities and of number sense. Across fields it is agreed upon that both formal and informal instruction can enhance number sense development prior to entering school. The importance of the role attributed to the use of fingers in the development of number sense by the research literature is highly resonant with the frame of embodied cognition.

### *Part-whole relations and numerical structure*

Perceiving pattern and structure is a fundamental way of thinking that should be fostered in young children (e.g. Mulligan and Mitchelmore, 2013). Moreover, lack of the use of this way of thinking seems to characterise children with low mathematical performance. Indeed, Mulligan and her colleagues, over several studies, found that "low achievers" (as defined by their teachers) are more likely to produce poorly organised representations, they tend to use unitary counting exclusively, and appear unable to visualise part-whole relations. This led the researchers to an hypothesis that was confirmed in later studies: "*the more a student's internal representational system has developed structurally, the more coherent, well organised, and stable in its structural aspects will be their external representations and the more mathematically competent the student will be*" (ibid, p. 34).

*Part-whole relations* arise from what Resnick et al. (1991) have described as protoquantitative part-whole schemas that "organise children's knowledge about the ways in which material around them comes apart and goes together" (ibid.,

p. 32). The interiorisation of the part-whole relation between quantities entails understanding of addition and subtraction as dialectically interrelated actions that arise from such relation (Schmittau, 2011), and recognising that numbers are abstract units that can be partitioned and then recombined in different ways to facilitate numerical (also mental) calculation. Hands and fingers can be used to foster development of the part-whole relation, in particular with respect to 5 and 10, in a naturally embodied way.

## Emblematic examples from PerContare

All the didactical materials are collected in an online teachers' guide, accessible for free (at percontare.asphi.it). Each activity is presented as follows: an estimate is given on the time necessary for the activity; then the teacher is guided through the preparation and given a suggestion for the task to propose; the next section briefly describes what the teacher can expect, based on the field-testing of the activity (this section may contain videos and commentaries of actual classroom outcomes); the next section describes the mathematical meanings that the activity intends to promote; then proposals on how to construct these mathematical meanings are given; and finally various student-sheets and possible homework is provided.

The various sections proposed for each activity in the teacher's guide are designed to help the teacher proceed according to the framework of Semiotic Mediation, keeping in mind what the objective-mathematical meanings for each activity are, and giving suggestions about how to help students develop them.

### *The "fingers game"*

The first example comes from a video recorded in a first grade, in November, when the author (A) was proposing the "fingers game". She describes a configuration of fingers saying how many are up or down on each hand, while keeping them behind her back, and asks what number she is representing with the fingers that are up. After about 5 minutes of playing the game, A proposes to ask a 'harder' question.

> A: So now shall we do a harder one?
>
> Class: Yes!
>
> A: So, on one hand… I have three fingers lowered... three fingers lowered… and on the other I have two raised.
>
> Some kids: two.
>
> A: No, how many are raised?... Do it with your hands. [A looks at all students' fingers raised and lowered on each hand.]
>
> A: So, one hand has three lowered, and the other has two raised… How many fingers are raised?
>
> Class: Four, two...four...
>
> A: Let's see how different people did it. [A looks at all students' fingers raised and lowered on each hand.]
>
> A: Do it with your fingers.

Class: [unclear, children say various numbers between one and five. Some say and show 4.]

A: Very good. Three lowered...and two raised.

Child: four.

A: Four. Very good!

In this game the part-whole relation becomes embodied: ten is decomposed into five and five, and five is decomposed in all possible ways on the children's hands. Fig 1 shows representations of how part-whole relations can come into play in determining the total number of fingers raised.

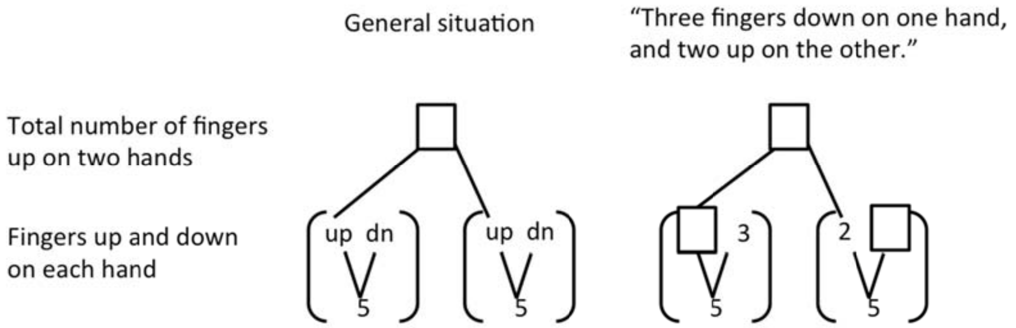


Fig. 1: Representation of the part-whole relation in the "fingers game"

The children actively engage in the game, attempting to reproduce instantiations of the described configuration. The game can easily be played with the entire class without leaving any student behind, because the teacher can look at the fingers raised and lowered on each student's hands and use this feedback to choose which students to explicitly engage in mathematical discourse. In classes in which the game was proposed for at least 5 minutes every other day for a month, children no longer needed to move their fingers and look at their hands to respond correctly, suggesting that they had acquired a stronger (dynamic) mental representation of their hands and fingers.

***Mediation of decimal positional notation through bundles of straws***

One of the artefacts introduced in first grade consists of straws that students learn to bundle up in groups of ten, through a process of discovery.

Fig. 2: Representation of the number 36 with straws in bundles of ten

The bundles of straws together with untied straws (Fig. 2) are then used to represent numbers given in different formats. Children discover that to count large numbers of straws it is easier to group the straws in bundles of ten, since this way they can use their ability to count by tens (even though initially there might not be deep meaning associated to the process). Moreover, children are used to making bundles of ten straws from other games proposed. However, children are not explicitly told 'how to' represent numbers with tied up and untied straws. The activity described below introduces this discovery. It is typically proposed around November-December of first grade.

The students are initially given 30 straws each and they are asked to represent the day of the month in which the activity is proposed, using their straws. They are initially invited to come up with ideas and share and discuss them. Once an agreement is reached, phase two proposes to ask students to

a) use the straws to represent a number (up to 30) given orally (verbal code);

b) use the straws to represent a number (up to 30) written in digits (symbolic code) on the blackboard;

c) use the straws to represent a number (up to 30) written in letters (visual-verbal code) on the blackboard;

d) write on their notebooks using digits the numbers represented with straws drawn on the blackboard.

The tasks proposed in this activity involve various transcoding processes (Dehaene, 1992): the verbal code, the symbolic code, and the visual-verbal code are used and put in relation with the structural "straw representation". Such a representation can support students with difficulties because it maintains an analogical format (there is exactly the number of straws that the given number represents) that also recalls symbolic aspects (the tens are grouped) of the numbers involved. Numbers in the "straw representation" maintain a physical connotation, activating the visual and kinaesthetic-tactile channels, and can act as a trampoline for students to pass from one code to the other.

The teacher is also invited to make use of horizontal parentheses under sets of straws to indicate the part-whole relationship s/he is attending to. For example, if the teacher wants to guide the students' attention to the composition of 36 as 'three ten' and 'six' s/he can put a horizontal parenthesis under the three bundles of ten straws on the left and write '3 ten' or '30' and a second one under the six untied straws on the right (see Fig. 2) and write '6'. A final horizontal parenthesis under everything can be used to mark the whole quantity '36'.

Soon after this activity the teacher is invited to use transparent boxes to hold bundles of straws (placed on the left, where the tens digit sits) and free straws (placed on the right, where the unit digits sit). Ten straws can be taken from the container on the far right and bundled up at any time. It is not necessary – like in the case of the abacus – to make a bundle as soon as there are ten straws. Making a bundle and placing it in the tens box makes recognising the number easier, but there is always the same number of straws in total. We have found that for numbers below one hundred the system of straws in boxes works quite well as an alternative for the abacus, which notoriously creates many difficulties for the students. Many of such difficulties seem to arise from the abstraction necessary in seeing a same ball of the abacus as 'one' or 'ten' based on whether it is put on the stick to the far right, or on the next stick to the left. Though the conventionality of the decimal positional notation is present in the representation with boxes of straws (as with the abacus), this artefact maintains a strong connection to the actual numerosity being represented, as it only gives a perceptually different structure to the same number of items being considered.

## A study on the effectiveness of the didactical materials

Within the greater project, a specific study was carried out with the aim of gaining insight into the effectiveness of the didactical materials developed. A sample of 208 children (10 classes) was selected at the beginning of their first grade and followed until their third grade. No child with IQ score below average was included in the sample. The sample consisted of two groups: an experimental group of 100 children (5 classes) whose teachers used all didactical materials proposed, and a control group of 108 children (5 classes) whose teachers were not aware of the didactical materials. To both groups was administered a set of assessment tests on arithmetical abilities related to numbers and calculation, as in the typical tests used for diagnosing children at risk (Biancardi et al., 2011). The tests were administered three times to the classes of both groups, in the form of a game: in May of the first grade, and in January-February and again in May of the second grade.

The assessment battery for first graders contained the following tasks: (1) writing numbers (numbers under 1000 dictated in random order), (2) subitizing (numerosities from 2 to 7), (3) estimation (two numerosities were compared), (4) enumeration (counting a set of dots and writing the numerosity in symbolic notation), (5) magnitude judgment (choosing the symbol for the greater number), (6) quantity judgment (deciding whether two representations, one analogical and one symbolic, of a number referred to the same number or not), (7) insertions on the number line (placing a number on a number line with tacks and numbers 0 and 20 marked), (8) reverse counting (writing numbers in reverse order on the number line, starting from a given number), (9) additions (written operations, of which three need composition of tens), (10) subtractions (written operations, in which the greater number is within 10). For each task of each test the number of correct answers was collected.

The assessment battery for the second graders in January-February consisted of seven of the same types of tasks (1, 2, 3, 5, 8, 9, 10), that were only made more complex, and of three different tasks (decomposition, ordering increasingly and decreasingly). In May the assessment was the same as in February, only a task on multiplication was added. For each task of each test the number of correct answers was collected.

In order to verify the validity of the results obtained with the newly developed assessment batteries, in November of the third grade, the AC-MT battery

(Cornoldi et al., 2012) was administered to the whole sample of subjects, together with a test for collective evaluation of reading ability DCL (Caldarola et al., 2012), and the dictation of a sequence from a battery for the evaluation of writing and orthographic competence (Tressoldi et al., 2013).

## Results and Conclusion

The results of the assessments at the end of the first grade show a substantially better performance of the experimental group on the following tasks: magnitude judgment, addition and subtraction. Moreover, in the experimental group four subjects of the 100 show low proficiency on at least four tasks of the battery, while in the control group eight subjects of the 108 appeared to be in this condition. The results of the January-February administration in second grade confirmed a significantly higher performance of the experimental group on the addition and subtraction, and also on the tasks on ordering increasingly and decreasingly. The third administration of the assessment battery in May of the second grade again confirmed these results.

As for the results on the validity of the assessment battery developed within the project, data show a significant correlation ($p > 0.05$) between the newly designed tasks and the standardised battery. In particular, there appears to be greater reliability ($\alpha = 0.8$) for the tasks that evaluate numerical knowledge. The comparison between the means of the scores obtained by the two groups on the tasks of the standardised calculation test (AC-MT) show a significant difference ($t$ student $= p > 0.05$) on speed, operations and number knowledge. The experimental group appears to have higher mean scores on every task of the standardised test. Moreover, the percentage of subjects in the experimental group with performances at or below the cut off score on the AC-MT battery was about half of that of these subjects in the control group (7% vs. 13%). These findings in particular suggest that the didactical materials developed in PerContare do contribute significantly to diminishing the number of potential false positives in the diagnoses of dyscalculia.

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### References

Baccaglini-Frank, A., & Scorza, M. (2013). Preventing learning difficulties in early arithmetic: The PerContare Project. In Ramiro-Sànchez, T., & Bermùdez, M.P. (Eds.), *Libro de Actas I Congreso Internacional de Ciencias de la Educatiòn y des Desarrollo* (p. 341). Granada: Universidad de Granada.

Baccaglini-Frank, A., & Bartolini Bussi, M.G (2012). The PerContare Project: Proposed teaching strategies and some developed materials. In Dellai, F., Mammarella, I.C., & Re, A.M. (Eds.), *International Academy for Research on Learning Disabilities 36th Annual Conference* (pp. 194-196). Trento: Erickson.

Bartolini Bussi, M.G., & Mariotti, M.A. (2008). Semiotic mediation in the mathematics classroom artifacts and signs after a Vygotskian perspective. In English, L. (Ed.), *Handbook of international research in mathematics education* (2nd ed) (pp. 746-783). New York: Routledge.

Berch, D. B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities,* 38(4), 333-339.

Biancardi A., Mariani E., & Pieretti M. (2011). *La discalculia evolutiva. Dai modelli neuropsicologici alla riabilitazione*. Milano: Franco Angeli.

Brissiaud, R. (1992). A toll for number construction: Finger symbol sets. In Bidaud, J., Meljac, C., & Fischer, J.-P. (Eds.), *Pathways to number: Children's developing numerical abilities*. New Jersey: Lawrence Erlbaum Associates.

Butterworth, B. (2005). Developmental dyscalculia. In Campbell, J.I.D (Ed.), *Handbook of mathematical cognition* (pp. 455-467). New York: Psychology Press.

Caldarola, N., Perini, N., & Cornoldi, C. (2012). DLC: una prova di decisione lessicale per la valutazione collettiva delle abilità di lettura. *Dislessia,* 9(1), 89-104.

Cornoldi, C., Lucangeli, D., & Bellina, M. (2012). *Test AC-MT 6-11 – Test di Valutazione delle abilità di calcolo e soluzione di problemi*. Trento: Erickson.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition,* 44, 1-42.

Gallese, V., & Lakoff, G. (2005). The brain's concepts: The role of the sensory-motor system in conceptual knowledge. *Cognitive Neuropsychology,* 22(3-4), 455-479.

Gracia-Bafalluy, M.G., & Noël, M.P. (2008). Does finger training increase young children's numerical performance? *Cortex,* 44, 368-375.

Griffin, S. A., Case, R., & Siegler, R. S. (1994). Rightstart: Providing the central conceptual prerequisites for first formal learning of arithmetic to students at risk for school failure. In McGilly, K. (Ed.), *Classroom lessons: Integrating cognitive theory and classroom practice* (pp. 24–49). Cambridge, MA: MIT Press.

Mulligan, J. T., & Mitchelmore, M.C. (2013). Early awareness of mathematical pattern and structure. In English, L., & Mulligan, J. (Eds.), *Reconceptualizing Early Mathematics Learning* (pp. 29-46). Dordrecht: Springer Science-Business Media.

Radford, L. (2014). Towards an embodied, cultural, and material conception of mathematics cognition. *ZDM Mathematics Education,* 46(3), 349-361.

Resnick, L.B., Bill, V.L., Lesgold, S. B., & Leer, N.M. (1991). Thinking in arithmetic class. In Means, B., Chelemer, C., & Knapp, M.S. (Eds.), *Teaching advanced skills to at-risk students* (pp. 27-53). SRI international.

Schmittau, J. (2011). The Role of theoretical analysis in developing algebraic thinking: A Vygotskian Perspective. In Cai, J., & Knuth, E. (Eds.), *Early algebraization A global dialogue from multiple perspectives* (pp. 71-86). Berlin: Springer.

Stella G., & Grandi, L. (2011). *Conoscere la dislessia e i DSA*. Milano: Giunti Editore.

Tressoldi, P.E., Cornoldi, C., & Re, A.M. (2013). *BVSCO-2. Batteria per la valutazione della scrittura e della competenza ortografica – 2*. Milano: Giunti Editore.

Verschaffel, L., & De Corte, E. (1996). Number and arithmetic. In Bishop, A. J., Clements, K., Keitel, C., Kilpatrick, J., & Laborde, C. (Eds.), *International handbook of mathematics education* (pp. 99–137). Dordrecht: Kluwer.