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Hedge Accounting and Risk Management: An Advanced Prospective Model for Testing Hedge Effectiveness

ANNALISA DI CLEMENTE*

In this work we propose a new prospective model for testing the economic hedge effectiveness. Our model is derived from the initial approach based on the measure of the relative risk reduction (RRR) where the risk is expressed by the standard deviation and a Normal world is assumed. Differently, our model estimates the RRR produced by the hedging strategy in terms of the new risk measures of the value at risk (VaR) and the expected shortfall (ES). Moreover, it fails the traditional hypothesis of a normal distribution for the risk factors generating their return scenarios by Monte Carlo simulation. Because the main hedging issue especially for financial institutions is the portfolio hedging, our model has been implemented to a market risk hedging strategy, the cross hedging, realized by combining a stock index future (short position) with a stock portfolio (long position). We underline that, while our results present a strong significance from an economic viewpoint, they may be utilized only in an experimental way for hedge accounting purposes.

(*J.E.L.*: *G17, G32, C15*).

1. Introduction

The hedge effectiveness has only recently become a main accounting issue. Initially, researchers emphasized the economic rather than the accounting perspective and, in particular, in a context of hedging on futures markets.[1] From an economic viewpoint, hedge effectiveness is generally measured in terms of risk reduction amount achieved through the hedging relationship. This typically involves the comparison between the risk associated with the underlying hedged item and the risk of the package formed by the combination of the underlying item and the hedging instrument. The hedge effectiveness result will depend on the risk

*Financial Markets and Institutions, Department of Social and Economic Sciences, Faculty of Political Sciences, "Sapienza" University of Rome, P.le Aldo Moro n.5, 00185 Rome, Italy. E-mail: annalisa.diclemente@uniroma1.it

[1]Many of the early research papers still provide a useful introduction to the general issues connected with evaluating hedge effectiveness (see Johnson, 1960; Ederington, 1979; Franckle, 1980; Culp and Millar, 1999).

characteristics of the underlying hedged item and of the hedging instrument, as well as on the correlation between them.[2]

The International Accounting Standards Board (IASB) has provided only broad guidelines to hedge effectiveness testing. In the absence of definitive guidance, corporations are expected to devise, apply and defend their own tests. What is required, however, is that the selected approach must be reasonable and consistent with the corporation's risk management strategy. Given that corporations with more stable earnings streams tend to have lower costs of capital, it is essential for them to identify an appropriate methodology for hedge effectiveness testing. But for gaining the maximum accounting benefit it is essential to ensure that the hedge effectiveness methodology is appropriate and reliable.

Traders and portfolio managers judge the hedge effectiveness in terms of volatility reduction. The volatility of the item being hedged in the absence of a hedge is the obvious point of reference against which this reduction should be measured. In order to minimize the operational burden of hedge accounting, high management may wish to consider the methods or tools used for risk management purposes and evaluate whether these ones may be appropriate for hedge effectiveness assessment. Such an approach would be in line with the objective of the new hedge accounting requirements, as described in the Review Draft (RD) issued by IASB in September 2012. The RD aim is to better reflect the effect of the entity's risk management activities in the financial statements. This might include Value-at-Risk (VaR) calculations, volatility reduction methods or similar approaches.[3]

Coherently with these explanations, in this work we propose a new and more adequate prospective model for testing the economic hedge effectiveness. Our method may be defined as advanced because it takes into account, firstly in this context, the empirical characteristics of the risk factors affecting the fair value of the item to be hedged, that is the asymmetry and the leptokurtosis phenomena.

Our model refers to the traditional approach based on the measure of the relative risk reduction (RRR).[4] Differently from the initial approach, in our case the estimate of the RRR by hedging is offered in terms of the new risk measures of the VaR and the expected shortfall (ES). VaR is the risk measure adopted by financial institutions, as well as by a growing number of industrial public companies, in assessing and managing their risk exposures, while the ES has the advantage of being a 'coherent' risk measure, able to capture the 'tail risk'.[5]

[2] Ernest and Young (2011).
[3] KPMG (2012).
[4] Coughlan *et al*. (2004).
[5] Recently, Basel Committee on Banking Supervision (2012) required financial institutions to move from VaR to ES, for determining regulatory capital requirements, given the number of weaknesses identified with using VaR.

Since the central hedging issue in particular for financial institutions is the hedging of the trading portfolio, our advanced prospective model has been implemented to a market risk hedging strategy, named cross hedging, realized by combining a stock index future (short position) with a stock portfolio (long position).

We underline that while our results present a strong significance from an economic viewpoint, they may be utilized only in an experimental way for hedge accounting purposes (waiting for a macro hedge accounting model). However, already the Exposure Draft (2010)[6] proposed that a group of gross positions may be an eligible hedged item if the items in the group are managed together on a group basis for risk management purposes, removing the restrictions under IAS 39[7] regarding the hedges of groups. Therefore, currently, the individual items in the group do not need to move proportionately with the group to allow a hedge of the group. Moreover, the eligibility of groups of net positions for closed portfolios in the ED may also be considered as a promising step in the current debate on macro-hedge accounting, also called open portfolios hedging. In fact, since open portfolios hedging may complicate hedge accounting, IASB decided to address macro hedging separately by a specific forthcoming discussion paper.[8]

In conclusion, the paper has been structured as follows: Section 2 underlines the main advances in hedge accounting proposed by IASB, in particular toward a better alignment with the risk management purposes and techniques; Section 3 describes briefly the hedge accounting background; and Section 4 offers an overview of the traditional methods currently adopted by corporations for testing the effectiveness of their hedging strategies. Section 5 illustrates the statistical characteristics of our advanced prospective model for testing the anticipated future performance of the hedging strategy. In particular, the technical description of the procedure for generating Monte Carlo simulation scenarios of the risk factor log-returns and for estimating the portfolio profit and loss distribution over a one-period (one-day) time horizon is offered in Section 5.1. Section 5.2 describes the methodology adopted for estimating some portfolio's risk metrics (standard deviation, 99 per cent VaR and 99 per cent ES). Section 6 presents the outcomes, in terms of economic hedge effectiveness testing, derived by implementing the advanced prospective method to a market risk hedging strategy realized by combining the stock index future (short position) with the stock portfolio (long position). Section 7 collects some concluding remarks.

[6]IASB ED/2010/13 Hedge Accounting, December 9, 2010, available at www.ifrs.org.
[7]IASB (2003).
[8]Ernest and Young (2011).

2. Hedge Accounting and Risk Management: Toward a Greater Alignment

When companies use derivatives instruments to hedge their economic exposures, they generally desire to apply the hedge accounting. Without this particular treatment, derivatives gains or losses associated with the risk to be hedged would hit earnings in different time periods. The resulting income volatility masks the objectives of the hedging strategy. By the hedge accounting treatment, in IAS 39,[9] issued by the International Accounting Standards Board (IASB) and adopted by corporations in fiscal years beginning after January 1, 2005, this additional volatility may be largely avoided. To qualify for hedge accounting, the derivative's results must be expected to be 'highly effective' in offsetting the changes in fair value or cash flow associated with the risk being hedged. In other words, for qualifying derivative products as hedging instruments, they must be submitted to hedge effectiveness tests measuring their impact in term of risk reduction (of the hedged item).[10]

However, IAS 39 does not provide an objective for hedge accounting, but instead presents various rules and restrictions as to the circumstances under which hedge accounting can be applied.

To overcome these drawbacks, since November 2008 the International Accounting Standards Board has been working to a general revision of the hedge accounting requirements in order to replace *IAS 39 Financial Instruments: Recognition and Measurement* with a new improved and simplified approach *IFRS 9 Financial Instruments: Recognition and Measurement*.[11] After the Exposure Draft (ED) *Hedge Accounting* published in December 2010, IASB issued a Review Draft (RD) of its hedge accounting model in September 2012. The proposed effective date is annual periods beginning on or after January 1, 2015.

The latest RD's proposals are generally consistent with the ones issued in December 2010, maintaining both the three types of hedging relationships (i.e. fair value hedges, cash flow hedges and hedges of net investments in foreign operations) and the current requirements to measure and recognize hedge ineffectiveness. However, the RD's proposals would mean that more hedging strategies used for risk management would qualify for hedge accounting. The declared objective is to align hedge accounting more closely with risk management.[12] The IASB's new model introduces an objective for hedge accounting which is described as representing 'in the financial statements the effect of an entity's risk management activities that use financial instruments to manage exposures arising from particular risks that could affect profit or loss'.[13]

[9]IASB (2003).
[10]Ernest and Young (2011).
[11]IASB (2009).
[12]KPMG (2012).
[13]Ernest and Young (2011).

IAS 39 already requires that the 'risk management objective' is included within the hedge documentation needed for hedge accounting. However, because there are so many rules concerning what can be a hedging instrument, what may be a hedged item and what sorts of relationships qualify for hedge accounting, the entity's actual risk management strategy may be very different from that which is documented for accounting purposes. Consequently, the documented risk management objective is usually a generic description and interpreted to mean the hedge accounting objective (commonly, the avoidance of profit and loss volatility), rather than the economic strategy that led to hedging for risk management purposes. Although less so than IAS 39, the ED continues to constrain what can constitute a hedging instrument, a hedged item or a qualifying hedge relationship for accounting purposes. As a result, there will continue to be risk management strategies commonly undertaken (especially by financial institutions) that will not be possible to reflect in the entity's hedge accounting. Despite these weaknesses, the new model would allow entities to apply hedge accounting more broadly to manage profit and loss mismatches and improve what might be regarded as being 'artificial' hedge ineffectiveness created from the current IAS 39 model.[14]

By the Review Draft of its hedge accounting model, the IASB attempts to align hedge accounting requirements under International Financial Reporting Standards (IFRS) more closely with risk management, expanding the ability to use hedge accounting furthermore.[15]

The general accounting mechanics of hedge accounting remain largely unchanged with respect to IAS 39. More specifically: (i) the new model retains the cash flow, fair value and net investment hedge accounting mechanics; (ii) entities are still required to measure hedge effectiveness and recognize any ineffectiveness in profit or loss; (iii) hedge documentation is still required; (iv) hedge accounting will remain optional.

However, the new approach presents some fundamental changes to the traditional hedge accounting model. In particular, with regards to the hedge effectiveness testing, under the new model the quantitative retrospective[16] effectiveness test is no longer required.[17] In other words, under ED the hedge effectiveness assessment is purely prospective (i.e. forward looking). Although the ED requires that any retrospective ineffectiveness is measured for accounting purposes in the profit or loss, there is no obligation to pass a retrospective effectiveness test at the end of a reporting period. The hedge effectiveness assessment is required to achieve hedge accounting in subsequent periods. Differently, IAS 39 requires an additional effectiveness

[14]Ernest and Young (2011).

[15]Draft of forthcoming IFRS on general hedge accounting, available at www.ifrs.org.

[16]With respect to historical performance of the hedge.

[17]The new model permits qualitative hedge effectiveness test. This test must meet three criteria to qualify for hedge accounting.

assessment on a retrospective basis by applying the 'bright line' of 80–125 per cent in order to decide whether hedge accounting can be continued or not.[18]

Moreover, the restriction regarding the hedges of groups[19] is removed, while the fair value hedge of a group with offsetting positions is now permitted. With respect to the hedges of groups of gross positions, the new model proposes that a group of gross positions may be an eligible hedged item if: (i) it consists of items that are individually eligible as hedged items, that is the qualification criteria must be satisfied by each individual item within the group; and (ii) the items in the group are managed together on a group basis for risk management purposes. In addition, the individual items in the group no longer need to move proportionately with the group to allow a hedge of the group.

The new hedge accounting proposals draw from the awareness that the restrictions under IAS 39 were not consistent with the economic hedging practices adopted by entities, as for example the macro or portfolio hedging. In fact, the hedge accounting rules were designed, primarily, from a single instrument viewpoint. IAS 39 allows multiple items to be hedged as a group. However, the restrictions are so narrow that the types of groups that are eligible as hedged items under IAS 39 are generally those that would also qualify for hedge accounting in individual hedge relationships.

The RD addresses hedging relationships in which the hedged item is a single item or a closed portfolio (i.e. a portfolio where items cannot be added or removed without creating a new portfolio). However, an entity often uses derivatives and financial instruments to manage risks associated with portfolios from which items are added or removed over time without creating a new portfolio (i.e. an open portfolio). Because open portfolios may complicate hedge accounting, at present the new proposals do not cover open portfolio hedging or macro hedging; even so the IASB has an active project to develop a new macro hedge accounting model.[20]

3. The Accounting Background

The International Accounting Standards Board (IASB) issued Statement 39, or IAS 39, to make an entity's exposure to its derivative positions more transparent. Prior to IAS 39, most derivatives were carried off-balance sheet and reported only in footnotes to the financial statements. The introduction of IAS 39 for International Accounting Standards reporting (so

[18]Ernest and Young (2011).

[19]Under IAS 39 the hedges of groups are permitted only if the change in the fair value attributed to the hedged risk for each individual item in the group is approximately proportional to the overall change in the fair value of the group for the hedged risk.

[20]Ernest and Young (2011).

as FAS 133 for US GAAP reporting) has radically changed the recognition of derivatives. Both these standards require derivatives to be recorded on the balance sheet (as assets or liabilities) at fair value. Derivatives that are not designated as hedges must be adjusted to fair value through income.

Depending on the reason for holding the derivative position and the derivative's effectiveness in hedging, changes in the derivatives' fair value are recorded either in the income statement (in the case of a fair value hedge) or in a component of equity known as other comprehensive income (in the case of a cash flow hedge). Changes in fair value of derivatives that are considered to be 'effective' for hedging aim (as defined by IAS 39) will either offset the change in fair value of the hedged assets, liabilities or entity commitments through earnings or will be recorded in other comprehensive income until the hedged item is recorded in earnings. Any portion of a change in a derivative's fair value that is considered to be ineffective, as defined, may have to be immediately recorded in earnings. Any portion of a change in a derivative's fair value that the entity has elected to exclude from its measurement of effectiveness, such as the change in time value of options contracts, will be recorded in earnings. Consequently, unless they are designed as a part of a hedging relationship which qualifies for hedge accounting treatment, derivative instruments can create additional earnings volatility. Many corporations find this volatility undesirable due to the adverse impact it may have on the views of rating agencies, analysts and investors. By applying this special hedge accounting treatment, managers may avoid this additional volatility largely.[21]

Hedge accounting is elective, but the problem is that companies must qualify for this treatment. To qualify, the manager must measure the effectiveness of the hedge at least each reporting period for the entire life of the hedge relationship. Any ineffective portion or excluded portion of the change in derivative value must be reported directly to earnings.

At this point, it is important to clarify that, for hedge accounting, the effectiveness assessment and the measurement of ineffectiveness have to be distinguished.[22] The effectiveness assessment is performed to determine which hedging relationships qualify for hedge accounting and aims to identify accidental offsetting and prevents hedge accounting in those situations. The measurement of ineffectiveness refers to the calculation of the 'non-offsetting' amounts in accounting for hedge relationships, that is the result in accounting terms. The measurement of ineffectiveness is performed only retrospectively and determines the amount to be recorded in profit or loss. The ED does not propose any change to this requirement currently in IAS 39.[23]

[21]KPMG (2011).
[22]Ernest and Young (2011).
[23]Ernest and Young (2011).

The objective of the hedge effectiveness assessment is to 'ensure that the hedging relationship will produce an unbiased result and minimize expected hedge ineffectiveness'. An unbiased outcome does not mean that the hedging relationship is expected to be perfectly effective at all times; however, the entity should have an expectation that changes in the value of the hedging instrument will not be systematically higher or lower than changes in value of the hedged item. Under ER the objective to minimize hedge ineffectiveness does not introduce a requirement to use the best possible hedging instrument. In other words, the entity may prefer to use the less effective instrument because it is cheaper or easer to transact.

4. Traditional Methods for Testing Hedge Effectiveness

Traditionally, the hedge effectiveness is typically determined using the 'dollar-offset' rule and the regression method (or *R*-squared method). The simplest method is the 'dollar-offset' or the so-called '80/125 rule'. This method, which has some historical significance for the accounting profession (Kawaller and Koch, 2000; and DIG, 2000 Issue E7), compares the changes in the value of the derivative[24] to the changes in the value of the hedged item.[25] The dollar-offset method can be applied either period-by-period or cumulatively (DIG, 2000 Issue E8). For a perfect hedge, the change in the value of the derivative exactly offsets the change in the value of the hedged item, and the negative of their ratio is 1.00. Of course, perfection is not necessary to qualify for hedge accounting. In practice, a hedge is deemed effective if the ratio of the change in value of the derivative to that of the hedged item is between 80 and 125 per cent (Swad, 1995). In other words, the 80/125 standard requires that the derivative's change in value offsets at least 80 per cent and not more than 125 per cent of the value of the hedged item.[26]

The drawback of this method is that during a period of market stability virtually any hedge is likely to fail. This is due to the high sensitivity of this ratio test to small changes in the value of the hedged item or the derivative. Under very reasonable assumptions about the distribution of changes in prices, Canabarro (1999) has shown that the 80/125 standard rejects as ineffective 36 per cent of all hedges when the coefficient of determination (correlation squared) R^2 is 0.98. In conclusion, this test does not seem to be sufficiently consistent (Kawaller and Koch, 2000; Kawaller, 2001; Althoff and Finnerty, 2001; and Finnerty and Grand, 2003).

[24]Derivative refers to any derivative or combination of derivatives used to hedge changes in fair value or cash flow.

[25]Hedged item refers to an asset or a liability or a prospective cash inflow or outflow.

[26]The logic underlying 80/125 is that the standard is independent of the arbitrary choice of numerator and denominator because 80 per cent = 4/5 and 125 per cent = 5/4.

Another traditional approach is the regression method, based on the correlation of the change in value of the hedged item and that of the derivative. Regression analysis is a statistical technique that provides quantitative information about the relationship between two or more variables. In this context, the need to show that a derivative will be highly effective translates to showing that the price (or interest rate or currency exchange rate) associated with the hedged item bears a close relationship to the price associated with the hedging derivative. Simple regression provides a summary statistic, the correlation coefficient, which quantifies the closeness of the linear relationship. The correlation coefficient may range from –1 to +1, where 1.0 is indicative of a case where the two variables (the changes in fair value respectively of the hedged item and of hedging derivative) are perfectly correlated. A related concept to the correlation coefficient is the coefficient of determination, *R*-squared, found simply by squaring the correlation coefficient, so the possible range of the *R*-squared is from 0 to 1. It can be thought of as measuring the proportion of the variance of the dependent variable that can be explained by independent variable. In practice, for an effective hedge (Kawaller and Steinberg, 2002), the *R*-squared must be at least 80 per cent (that is, at least equal to 0.80).

Traders and portfolio managers judge the effectiveness of a hedge in terms of volatility reduction. The volatility of the item being hedged in the absence of a hedge is the obvious point of reference against which this reduction should be measured. In contrast, the IASB guidelines focus on *pair-wise* (date-by-date) comparison of changes in value, rather than on overall volatility with and without the hedge. The method based on the volatility reduction measure (Kalotay and Abreo, 2001)[27] allows practitioners to capture the significance of hedging retaining the basic intent of IASB. Specifically, the volatility reduction measure (VRM) method compares the variability of the fair value or cash flow of the hedged combined position (or hedge package)[28] to the variability of the fair value or cash flow of the item being hedged alone. More formally, the VRM is defined as: $\text{VRM} = 1 - [\text{stdev (of hedge package)}/\text{stdev(of item being hedged)}]$. The critical value for determining how large a reduction in variability is sufficient to demonstrate hedge effectiveness must be specified in order for this measure to be useful. Because of the similarity of this test to the regression method test, a standard of 80 per cent may be correct consistently with the suggestion of Lipe (1996), Kalotay and Abreo (2001) and Finnerty and Grand (2003).

The VRM approach is similar to the idea of variance reduction introduced by Ederington in 1979 for assessing hedging performance. As

[27]The Volatility Reduction Measure (VRM) for hedge effectiveness testing was invented by Andrew Kalotay Associates, Inc. It has been implemented at a leading telecommunication company and has been accepted by a Big-5 accounting firm in USA.

[28]Hedge package means hedge item plus derivative.

implied by its name, Ederington's method measures volatility reduction from a ratio of variances. Differently, the VRM method prefers using standard deviations because they tend to be more meaningful to management and for being in accord with the 80/125 rule. In fact, a VRM result of 80 per cent is equivalent to a variance reduction of 96 per cent, which can be misleading if one is focused on an 80 per cent threshold. Or, in other words, a variance reduction of 80 per cent is equivalent to a VRM of 55 per cent. Clearly, in the spirit of the 80/125 rule, this last case fails for hedge accounting treatment. Moreover, the VRM method has the added advantage of showing a common analytical framework with VaR, a widely used risk measure representing monetary exposure (Kalotay and Abreo, 2001). The traditional VRM models use standard deviations-based formulas that are applied to historical or simulated changes in value of the corporation's positions. In comparison with the other traditional methods (dollar-offset and regression) the VRM approach appears superior in its simplicity as well as for its rigor and defensibility (Kalotay and Abreo, 2001). In fact, the standard deviation is the traditional measure of volatility and, when expressed in monetary terms, it reflects the actual business risk of a corporation and is more familiar to higher management.

Hence, assuming that the volatility of the changes in fair value is the appropriate measure of risk, by this risk metric (σ) the hedge relationship will be effective (but not necessarily highly effective) if it reduces the risk of the underlying.

In mathematical terms: if $\sigma_{\text{hedge package}} < \sigma_{\text{underlying}}$, then the hedge is effective. The degree of economic hedge effectiveness is given by the RRR value of the hedge. Analytically: $\text{RRR} = (\sigma_{\text{underlying}} - \sigma_{\text{package}})/\sigma_{\text{underlying}}$. If $\text{RRR} > 0$, then the hedge is effective because it reduces risk relative to the unhedged risk of the underlying (Coughlan *et al.*, 2004). An effective hedge in economic terms corresponds to an RRR between 0 (not effective) and 1 (perfectly effective). Assuming a Normal world for the financial returns, Coughlan *et al.* (2004) analysed the relationship between the correlation[29] and the RRR measure for the optimal hedge, finding that a correlation of –0.8 leads to an RRR of 40 per cent (while a correlation of –0.9 leads to 56 per cent RRR). Their results suggest to define a value of 40 per cent as the minimal threshold for RRR method (and for a hedge 'highly effective').

Concluding this brief overview of the most common hedge effectiveness methods used nowadays by corporations, we underline that from an economic viewpoint the most intuitive technique for testing the hedge effectiveness is the RRR method. As earlier mentioned, the RRR method has traditionally been implemented choosing the volatility (variance and/or standard deviation) as appropriate measure of risk. Differently, in this paper, we drop the traditional hypothesis of a Normal world for the financial returns

[29]Between the item to be hedged and the hedging instrument.

of the market risk factors and utilize the VAR and the ES as metrics of risk (in addition to the standard deviation).

5. An Advanced Prospective Model for Testing Hedge Effectiveness

To qualify a derivative position for hedge accounting, the entity must specify the hedged item, identify the type of risk to be hedged and choose the relative hedging strategy and the appropriate derivative instrument. Moreover, for assessing hedge effectiveness, the entity must select an approach reasonable and consistent with its own risk management strategy.

Under the ED, hedge effectiveness will have to be assessed prospectively at inception and prospectively every reporting period on an ongoing basis. An effectiveness assessment must be performed, as a minimum, at each reporting date or upon a significant change in circumstances, whichever comes first. Nevertheless, the IASB does not prevent an entity from assessing effectiveness more frequently in accordance with its risk management practices.[30]

For a robust prospective (i.e. forward-looking) evaluation of hedge effectiveness it is generally necessary to perform a risk simulation. This implies generating a large number of realistic scenarios of future market risk factor returns based on historical data. In implementing the risk simulation, historical returns of the risk factors are used to build up scenarios for future risk factor returns. This can be done in a number of different ways.

Assuming as the hedged item a stock portfolio (long position) and identifying the risk to be hedged as the market risk (that is, the portfolio's market value losses resulting from adverse stock price movements), the original contribution of this work is measuring the degree of stock portfolio's market risk reduction produced by a cross hedging strategy based on a financial derivative, the stock index future, utilizing different risk measures: the traditional standard deviation, the VaR and the ES and assuming a non-Normal world.

Given the well-known statistical characteristics of risk factor returns affecting the fair value changes of stock portfolio, such as the asymmetry, leptokurtosis and heteroskedasticity phenomena, the 'coherent'[31] risk measure of ES would be preferred.[32] In fact, in a non-Normal world, ES is a risk metric superior to volatility and VaR, since it is able to capture 'tail risk' of the profit and loss distribution. It is evident that the adoption of different risk measures by corporations may lead to different evaluations of risk reduction degree produced from the same hedging strategy.

[30] KPMG (2012).
[31] Artzner *et al*. (1997, 1999).
[32] See Basel Committee on Banking Supervision (2012), Consultative Document.

For estimating the stock portfolio's profit and loss distribution, taking into account the real statistical characteristics of market risk factors returns, a Monte Carlo simulation model may be utilized.[33] In particular, the volatility of the market risk factors log-returns has been estimated by using a stochastic model of the GARCH[34] family that permits us to consider for the heteroskedasticity phenomenon. Our advanced simulation model estimates the multivariate probability distribution of the standardized returns by building the margins from the empirical distributions and the dependence structure from a Gaussian copula function[35] whose parameter is a dynamic correlation matrix. In this way the model takes into account the leptokurtosis of the market risk factor returns. Generally, a model for estimating the portfolio's profit and loss distribution by Monte Carlo simulation can be summarized in the following two steps: (i) generating Monte Carlo scenarios for risk factors returns over the designated time horizon; and (ii) revaluating portfolio value at the end of the reference time horizon in different scenarios by using adequate pricing formulae.[36] A detailed technical description of step (i) is reported in Section 5.1. A brief explanation of step (ii), along with the analytical description of the risk measures extracted from the portfolio's profit and loss distribution, is offered in Section 5.2.

5.1. Generating Monte Carlo Scenarios for Market Risk Factor Returns

We suppose to be in time t. The concerned time horizon is $[t, t+1]$. Let $\mathbf{x}_{t+1} = \left(x_{1,t+1}, \dots, x_{n,t+1}\right)$ be the random vector for the n risk factor log-returns influencing portfolio value in $[t,\ t+1]$. Precisely:

$$x_{i,t+1} = \ln\left(\frac{P_{i,t+1}}{P_{i,t}}\right) = \ln P_{i,t+1} - \ln P_{i,t},\ i = 1, \dots, n, \tag{1}$$

where $P_{i,t}$ is the price of risk factor i in time t.

We also assume to afford a set S with T historical data for each of the n risk factor log-returns: $S = \{\mathbf{x}_{t-i+1}\}_{i=1,\dots,T}$.

The model adopted for simulating scenarios for risk factor log-returns in the time range $[t,\ t+1]$ is the following:

$$x_{i,t+1} = \sigma_{i,t+1} \cdot \varepsilon_{i,t+1},\ i = 1, \dots, n, \tag{2}$$

[33] We remember as in the recent financial literature other different models have been proposed for the same purposes, for example Barone-Adesi *et al.* (1999, 2002), Gibson (2001), Bingaham *et al.* (2003), Ivanov *et al.* (2003), Di Clemente and Romano (2005) and Di Clemente (2006).

[34] Generalized Auto-Regressive Conditional Heteroskedasticity (see Bollerslev, 1986).

[35] The selection of a particular type of copula function (Gaussian copula or Student's *t*-copula) does not influence the VaR estimates at the 99% probability level. Differently, it is not true for very high quantiles (see Di Clemente and Romano, 2005).

[36] It is useful remarking how the implementation of the pricing formulae in the case of particularly complex structured financial instruments could be a matter of challenging solution.

where the random vector $\boldsymbol{\varepsilon}_{t+1} = \left(\varepsilon_{1,t+1}, \ldots, \varepsilon_{n,t+1}\right)$ has got an n-dimensional distribution with margins equal to the empirical distributions of the standardized log-returns, $\hat{F}_i$, $i = 1, \ldots, n$, and the dependence structure is given by a Gaussian copula function[37] with parameter the correlation matrix $\mathbf{R_{t+1}}$.

The volatility term in model (2) is assessed by a stochastic process belonging to the family GARCH(1,1). Analytically:

$$\sigma^2_{i,t+1} = c_i + a_i x^2_{i,t} + b_i \sigma^2_{i,t},\ i = 1, \ldots, n, \tag{3}$$

where the parameters, $a_i > 0$, $b_i \geq 0$ and $c_i \geq 0$, are estimated by maximum likelihood method using the set S of historical data.[38]

The empirical cumulative distribution function (c.d.f.) is obtained by using historical data[39] of the standardized log-returns, $z_{i,t-j+1} = \frac{x_{i,t-j+1}}{\sigma_{i,t-j+1}}$, $i = 1, \ldots, n$; $j = 1, \ldots, T$.

Analytically:

$$\hat{F}_i(x) = \frac{1}{T} \sum_{j=1}^{T} \mathbf{I}\left\{z_{i,t-j+1} \leq x\right\}\ i = 1, \ldots, n, \tag{4}$$

where $\mathbf{I}\left\{z_{i,t-j+1} \leq x\right\}$ is the indicator function assuming value equal to 1 if $z_{i,t-j+1} \leq x$, 0 otherwise.

A Monte Carlo scenario for the n risk factor log-returns over the time horizon $[t, t+1]$ is obtained by the following algorithm:

(i) generating a scenario with n random numbers uniformly distributed in [0,1], $(u_1, \ldots, u_n)$, from the Gaussian copula;
(ii) generating a scenario for the standardized log-returns over the time span $[t, t+1]$ by inverting the values u_i by using the empirical c.d.f. (3): $z_i = \hat{F}_i^{-1}(u_i),\ i = 1, \ldots, n;$ [40]
(iii) a scenario for risk factor log-returns over time horizon $[t, t+1]$ is generated by multiplying the values z_i by the volatility assessed by Equation (3): $x_i = z_i \cdot \sigma_{i,t+1}$.

Steps (i)–(iii) are repeated a great number of times, s.

Now we illustrate how random numbers from the Gaussian copula function can be generated. Remember that a copula function describes the

[37] The selection of a particular type of copula function (Gaussian copula or Student's t-copula) does not influence the VaR estimates at the 99 per cent probability level. Differently, it is not true for very high quantiles (see Di Clemente and Romano, 2005).

[38] By using a statistical software like MatLab™.

[39] We have filtered the risk factors returns by a GARCH model (see Barone-Adesi *et al.*, 1999, 2002).

[40] Since the empirical c.d.f. is not absolutely continuous, by $\hat{F}_i^{-1}$ we consider the generalized inverse function: $\hat{F}_i^{-1}(u) = \min\left\{z : \hat{F}_i(z) \geq u\right\}$.

dependence structure among n random variables. It is a function linking the marginal distributions of the n random variables in order to create the multivariate distribution. The concept of copula goes back to Sklar (1959). Copula is a function of several variables and describes, in a powerful way, how the joint distribution is linked to its univariate margins. A n-dimensional copula function is a multivariate c.d.f., C, with margins uniformly distributed on [0,1] and with the following properties:

(i) $C: [0,1]^n \rightarrow [0,1]$;
(ii) C is grounded and n-increasing;
(iii) C has margins C_i $(i = 1, \ldots, n)$ satisfying: $C_i(u) = C(1, \ldots, 1, u, 1, \ldots, 1) = u$ for all $u \in [0,1]$.

If $u_1, \ldots, u_n$ are values of n univariate distribution functions, so each $u_i \in [0,1]$, then a copula is a function $C(u_1, \ldots, u_n) \rightarrow [0,1]$. Copulas are used to combine marginal distributions into a multivariate distribution. They are unique: for any given multivariate distribution (with continuous marginal distributions) there is a unique copula that represents it. They are also invariant under strictly increasing transformations of the marginal distributions. Moreover, copulas have long been recognized as a powerful tool for modelling dependence between random variables (Nelsen, 1999). The basic idea behind copulas is to separate the dependence and marginal behaviour of the univariates.

By copula functions we may construct and simulate multivariate c.d.f. The most important theorem about copula functions is Sklar's theorem, used in many practical financial applications.

Sklar's Theorem: *Let F be an n-dimensional c.d.f. with continuous margins $F_1, \ldots, F_n$. Then it has the following unique copula representation:*

$$F(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n)). \tag{5}$$

From Sklar's theorem we see that, for continuous multivariate distribution functions, the univariate margins and the multivariate dependence structure can be separated. The copula reveals the link between the joint distribution and its margins. Copulas are thus multivariate uniform distributions, which describe the dependence structure of random variables. The main advantage of copulas consists in representing the joint distribution by separating the impact of the margins from the association structure, explained by the copula functional form.

An important corollary of Sklar's theorem is the following:

Corollary: *Let F be an n-dimensional c.d.f. with continuous margins $F_1, \ldots, F_n$ and copula C (satisfying (5)). Then, for any $\mathbf{u} = (u_1, \ldots, u_n)$ in $[0,1]^n$:*

$$C(u_1, \ldots, u_n) = F\left(F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n)\right), \tag{6}$$

where F_i^{-1} is the generalized inverse of F_i.

The *Gaussian copula* is the copula of the multivariate Normal distribution. In fact, the random vector $\mathbf{X} = (X_1, \ldots, X_n)$ is multivariate normal if and only if:

(i) the univariate margins $F_1, \ldots, F_n$ are Gaussians;
(ii) the dependence structure among the margins is described by a unique copula function C (the Gaussian copula) such that:[41]

$$C_{\mathbf{R}}^{Ga}(u_1, \ldots, u_n) = \Phi_{\mathbf{R}}\left(\phi^{-1}(u_1), \ldots, \phi^{-1}(u_n)\right), \tag{7}$$

where Φ_R is the standard multivariate normal c.d.f. with linear correlation matrix $\mathbf{R}$ and ϕ^{-1} is the inverse of the standard univariate Gaussian c.d.f.

We can dynamically estimate the parameter $\mathbf{R}$ of the Gaussian copula by using the set of T historical data about the standardized log-returns, $z_{i,t-j+1}$, $i = 1, \ldots, n; j = 1, \ldots, T$, obtained how it has been described previously. With the term dynamic estimate we intend that the estimated value of parameter $\mathbf{R}$ depends on the exact point of time in which the estimate is performed. The algorithm used for estimating the parameter over the time horizon $[t, t+1]$ **($\mathbf{R_{t+1}}$)** is the following:

1. Transforming historical data into variates uniformly distributed on [0,1] by using the empirical distributions (4): $u_{i,t-j+1} = \hat{F}_i(z_{i,t-j+1})$, $i = 1, \ldots, n$, $j = 1, \ldots, T$.
2. Historical data are further transformed by using the inverse of the standard Normal c.d.f.: $\tilde{z}_{i,t-j+1} = \Phi^{-1}(u_{i,t-j+1})$, $i = 1, \ldots, n$, $j = 1, \ldots, T$.
3. The matrix $\mathbf{R_{t+1}}$ is estimated by the EWMA[42] method.

The variances over the time horizon $[t, t+1]$ are estimated by the EWMA methodology (see RiskMetrics Group, 1994) in the following way:

$$\sigma_{i,t+1}^2 = \frac{\sum_{j=1}^{T^*} \lambda^{j-1} \tilde{z}_{i,t-j+1}^2}{\sum_{j=1}^{T^*} \lambda^{j-1}}, \ \ldots i = 1, \ldots, n, \tag{8}$$

where $0 < \lambda \leq 1$ is the decay factor, that is a value indicating the weight of historical data: the lower is λ, the higher is the weight of the most recent data respectively to the weight of the less recent data.

[41] As one can easily deduce from Equation (6).
[42] Exponentially weighted moving averages.

The covariances over the time horizon $[t, t+1]$ are estimated in the following way:

$$\sigma_{ij,t+1} = \frac{\sum_{k=1}^{T^*} \lambda^{k-1} \tilde{z}_{i,t-k+1} \tilde{z}_{j,t-k+1}}{\sum_{k=1}^{T^*} \lambda^{k-1}}, \; i,j = 1,\ldots,n, \; i \neq j. \tag{9}$$

In RiskMetrics Group (1994) it is recommended to use the value $\lambda = 0.94$ and $T^* = 74$ for historical data.

The generic element $r_{ij,t+1}, i,j = 1,\ldots,n$ in the matrix $\mathbf{R_{t+1}}$ is obtained in the following way from Equations (8) and (9):

$$r_{ij,t+1} = \frac{\sigma_{ij,t+1}}{\sigma_{i,t+1}\sigma_{j,t+1}} \tag{10}$$

assuming $\sigma_{ij,t+1} = \sigma_{i,t+1}^2$ when $i = j$.

In order to generate random variates from the Gaussian copula (7), we can use the following procedure. If the matrix $\mathbf{R}$ is positive definite, then there are some $n \times n$ matrices $\mathbf{A}$ such as $\mathbf{R} = \mathbf{A}\mathbf{A}^{\mathrm{T}}$. It is also assumed that the random variables $Z_1, \ldots, Z_n$ are independent standard normal. Then, the random vector $\boldsymbol{\mu} + \mathbf{AZ}$ (where $\mathbf{Z} = (Z_1, \ldots, Z_n)^{\mathrm{T}}$ and the vector $\boldsymbol{\mu} \in R^n$) is multi-normally distributed with mean vector $\boldsymbol{\mu}$ and covariance matrix $\mathbf{R}$.

The matrix $\mathbf{A}$ can be easily determined with the Cholesky decomposition of $\mathbf{R}$. This decomposition is the unique lower-triangular matrix $\mathbf{L}$ such as $\mathbf{L}\mathbf{L}^{\mathrm{T}} = \mathbf{R}$. Hence, one can generate random variates from the n-dimensional Gaussian copula running the following algorithm:

(i) find the Cholesky decomposition $\mathbf{A}$ of the matrix $\mathbf{R}$;
(ii) simulate n independent standard normal random variates $\mathbf{z} = (z_1, \ldots, z_n)^{\mathrm{T}}$ from N(0, 1);
(iii) set $\mathbf{x} = \mathbf{Az}$;
(iv) determine the components $u_i = \phi(x_i), \; i = 1, \ldots, n$;

the vector $(u_1, \ldots, u_n)^{\mathrm{T}}$ is a random variate from the n-dimensional Gaussian copula, $C_{\mathbf{R}}^{\mathrm{Ga}}$.

5.2. *Estimating Portfolio Risk Measures*

After having generated s scenarios for the n market risk factors log-returns the affecting fair value of the portfolio over the time horizon $[t, t+1]$, a scenarios of portfolio losses over the same time horizon may be obtained.

Analytically, the portfolio loss in scenario j, L_j, is given by the following equation:

$$L_j = f\left(x_1^j, \ldots, x_n^j\right), j = 1, \ldots, s, \tag{11}$$

where $x_1^j, \ldots, x_n^j$ is the jth scenario for risk factor log-returns and f is the 'mapping' function.

By means of the mapping function, the portfolio is revalued in time $t+1$ by applying the correct pricing function. The pricing function allows us to express the price of m assets in portfolio as a function of n risk factors log-returns. Portfolio loss in scenario j will be equal to the difference between portfolio value in time t (basis scenario) and the jth scenario of portfolio value in time $t+1$. In this way, possible profits are considered as negative values of losses.

The c.d.f. of the portfolio profits and losses,[43] G, is obtained in the following way:

$$G(x) = \frac{1}{s}\sum_{j=1}^{s} \mathbf{I}\left\{L_j \le x\right\}, \tag{12}$$

where $\mathbf{I}\left\{L_j \le x\right\} = 1$ if $L_j \le x$, 0 otherwise.

Over the time horizon $[t, t+1]$ portfolio risk measures can be extracted from the portfolio's profit and loss distribution. By risk measure, the whole profit and loss distribution may be synthesized in one number alone. From G, we extract the following portfolio risk measures: expected loss (EL), volatility (vol, that is standard deviation), VaR and ES. For each of these metrics, we show the analytical equations below:

- Expected loss:

$$\mathrm{EL} = \frac{1}{s}\sum_{j=1}^{s} L_j. \tag{13}$$

- Volatility (standard deviation of portfolio losses):

$$\mathrm{vol} = \sqrt{\frac{1}{s}\sum_{j=1}^{s} \left(L_j - \mathrm{EL}\right)^2}. \tag{14}$$

[43]Profits are considered as negative losses.

- VaR at probability level p. Where s scenarios L_j (representing portfolio losses) are ordered in non-decreasing order: $L_{(1)} \leq L_{(2)} \leq \cdots \leq L_{(s)}$.

$$\text{VaR}_p = \min\left\{ L_{(j)}, j = 1, \ldots, s : \frac{j}{s} \geq p \right\} \tag{15}$$

ES[44] at the probability level p:

$$\text{ES}_p = \text{VaR}_p + \frac{1}{(1-p)s} \sum_{j=1}^{s} \left(L_j - \text{VaR}_p\right)^+, \tag{16}$$

where $\left(L_j - \text{VaR}_p\right)^+ = L_j - \text{VaR}_p$ if $L_j - \text{VaR}_p > 0$, or $= 0$ otherwise.

6. Implementing the Advanced Model to a Cross Hedging on the Futures Market

In this section we implement the advanced prospective model to a cross hedging on the futures market in order to test the effectiveness degree of this typical risk management strategy. For these purposes, we assume as item to be hedged a hypothetical stock portfolio composed with Italian equities, and as hedging instrument a stock index future traded on the Italian Derivative Market.

The hedged position (or package) is a combination of a long position on a stock portfolio with N short positions on a Ftse-Mib future, that is the Italian stock index future traded on the Italian Derivatives Market (IDEM) nowadays. The hypothetical stock portfolio is composed with 10,000 units of each of 15 Italian equities (listed in Table 1) and its market value on October 15, 2012 is of 1,404,200 euros. Table 1 collects some information about each of the 15 equities composing this diversified stock portfolio. The first column lists equity labels, the second column shows the closing prices (in euros) for each equity on October 15, 2012, and the third column presents the correlation values[45] of each equity with the Italian stock index (Ftse-Mib). For estimating the parameters of the GARCH model and the log-return marginal distributions for every market risk factor (that is, the 15 equities and the stock index) a 6-year data set of daily log-returns has been utilized (precisely since October 15, 2012 to October 16, 2006).[46] For assessing the

[44] ES_p may be also calculated as the mean value of the loss scenarios exceeding VaR_p.

[45] More precisely, the correlation between the standardized log-returns of each equity and the standardized log-returns of Ftse-Mib index is calculated following the procedure described in Section 5.1.

[46] The length of the data set permit us to obtain a greater number of extreme values.

Table 1: Characteristics of 15 Italian Equities Composing the Stock Portfolio: Denomination, Closing Price (in Euros), Correlation Values with Ftse-Mib Index Calculated by EWMA Method

Equities	Price (October 15, 2012)	Correlation (i,m)
1 - FIAT	4.25	69.19%
2 - Impregilo	3.33	66.51%
3 - Luxottica	28.81	46.25%
4 - Telecom Italia	0.76	70.59%
5 - ENEL	2.86	86.62%
6 - Mediaset	1.53	67.06%
7 - Generali	11.89	89.96%
8 - ENI	17.5	87.41%
9 - Intesa-San Paolo	1.27	92.31%
10 - Danieli	20.64	73.17%
11 - MedioBanca	4.32	80.82%
12 - Banca Pop. Milano	0.43	68.86%
13 - Banca MPS	0.23	57.05%
14 - BNP Paribas	39.01	86.72%
15 - Mediolanum	3.59	85.56%

correlations among the market risk factor log-returns by means of the EWMA[47] method the last 74 log-returns of the data set have been used. Given that the type of risk to be hedged is the market risk, the reference time horizon adopted is of 1 day.

For calculating the value of the stock index future on October 15, 2012 (that is, on t, the evaluation time), the following pricing formula has been implemented:[48]

$$F(t) = S(t) \cdot \mathrm{e}^{r(t)\cdot(T-t)}, \tag{17}$$

where $S(t)$ is the value of the Ftse-Mib stock index in time t, $r(t)$ is the risk-free interest rate in time t and T is the future maturity date. The value of the Ftse-Mib on October 15, 2012 (time t) is of 15590.72 index points. We assume a flat yearly risk-free interest rate equal to 3 per cent and a maturity for the future contract equal to 2 months ($T - t = 60/360$). Given these assumptions, we generate 10,000 Monte Carlo (MC) simulation scenarios for the log-returns of the 15 equities and of the Ftse-Mib index over the daily time horizon. The unhedged diversified portfolio is composed of w_i units for each equity i ($i = 1, \ldots, 15$). A portfolio loss scenario is calculated as the difference between the portfolio values in t and in $t + 1$ respectively in the following way:

$$L_j = \sum_{i=1}^{15} w_i\left(P_i(t) - P_i(t) \cdot \mathrm{e}^{x_i^j(t+1)}\right), j = 1, \ldots, 10{,}000, \tag{18}$$

[47]Exponentially weighted moving averages.
[48]Assuming absence of market frictions and of arbitrage opportunities.

where $P_i(t)$ is the closing price of equity i in time t, and $x_i^j(t+1)$ is the value of the log-return of equity i over the daily time horizon $[t, t+1]$ in scenario j.

The hedged diversified portfolio is composed of w_i units for each equity i ($i = 1, \ldots, 15$) and N short positions on the Ftse-Mib future. The optimal number of futures contracts for hedging is N^* calculated by historical data following this equation:[49] $N^* = \rho \cdot (\sigma_i/\sigma_d)$, where ρ is the correlation coefficient between the item to be hedged and the hedging derivative, and σ_i and σ_d are the standard deviations of the values of the item and the derivative, respectively.[50] The loss scenarios for the hedged diversified portfolio are calculated in the following way:

$$L_j = \sum_{i=1}^{15} w_i\left(P_i(t) - P_i(t)e^{x_i^j(t+1)}\right) - N^* \cdot 5 \cdot \left(F(t) - F_j(t+1)\right), j = 1, \ldots, 10{,}000, \quad (19)$$

where $F_j(t+1) = S(t)\,e^{x_j(t+1)}\,e^{r(t+1)(T-t-1)}$ is the future value in $t+1$ in scenario j, and $x_j(t+1)$ is the log-return of the Ftse-Mib index over time horizon $[t, t+1]$ in scenario j. In Equation (19), we multiply the index point quotation of the future by its monetary multiplier, equal to 5 euros per index point, for obtaining the monetary value of the future.

Some risk measures, such as volatility, $\text{VaR}_{99\text{ per cent}}$ and $\text{ES}_{99\text{ per cent}}$ (both calculated for a confidence level of 99 per cent), on a daily time horizon, have been estimated for both unhedged and hedged stock portfolio (as described in Section 5.2). Successively, three different RRR measures, estimated by using volatility (standard deviation), VaR and ES respectively, have been generated: and RRR_1 is the relative risk reduction measure expressed in terms of portfolio volatility; RRR_2 is the relative risk reduction measure expressed in terms of portfolio $\text{VAR}_{99\text{ per cent}}$; and RRR_3 is the relative risk reduction measure expressed in terms of portfolio $\text{ES}_{99\text{ per cent}}$. The analytical expressions for RRR_1, RRR_2 and RRR_3 are offered by the following equations respectively:

$$\text{RRR}_1 = \frac{\text{vol(unhedged ptf)} - \text{vol(hedged ptf)}}{\text{vol(unhedged ptf)}}, \quad (20)$$

$$\text{RRR}_2 = \frac{\text{VaR}_{99\,\text{percent}}(\text{unhedged ptf}) - \text{VaR}_{99\,\text{percent}}(\text{hedged ptf})}{\text{VaR}_{99\,\text{percent}}(\text{unhedged ptf})}, \quad (21)$$

$$\text{RRR}_3 = \frac{\text{ES}_{99\,\text{percent}}(\text{unhedged ptf}) - \text{ES}_{99\,\text{percent}}(\text{hedged ptf})}{\text{ES}_{99\,\text{percent}}(\text{unhedged ptf})}. \quad (22)$$

[49]Rounding the value to the closer integer.
[50]The optimal N^* provides the maximal risk reduction.

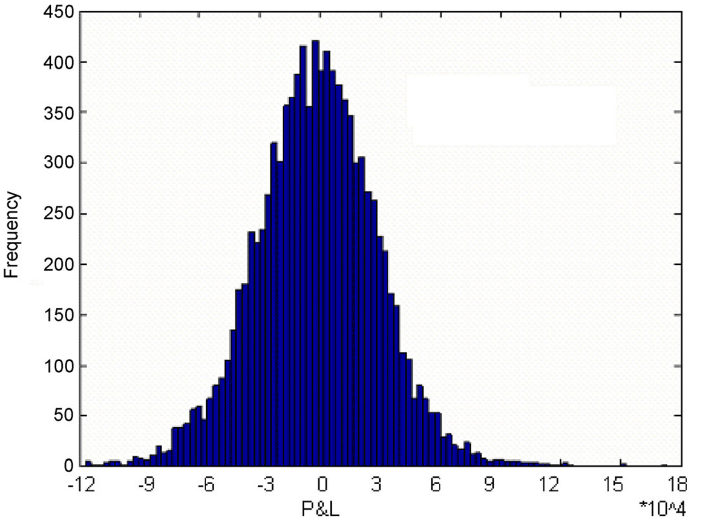

Figure 1: Profit and Loss Distribution of Unhedged Diversified Stock Portfolio

The profit and loss distributions[51] of the unhedged diversified stock portfolio and of the hedged diversified stock portfolio are pictured respectively in Figures 1 and 2.

It is evident, observing Figures 1 and 2, that the as profit and loss distribution (P&L) of the hedged diversified portfolio (Figure 2) is remarkably less asymmetric, volatile and fat tailed than the one of for the unhedged diversified portfolio (Figure 1). This first result confirms the considerable and "highly" effective risk reduction in terms of all the risk metrics utilized in this example. In addition, we consider 15 different stock portfolios concentrated on a single equity: portfolio 1 is composed with 100,000 units of equity 1 (FIAT), portfolio 2 with 100,000 units of equity 2 (Impregilo), …, portfolio 15 with 100,000 units of equity 15 (Mediolanum). Only portfolio 16 is diversified including all the considered 15 equities. For all the 16 unhedged stock portfolios, Table 2 collects their monetary values and the estimates of their respective three risk metrics (volatility, $VaR_{99\text{ per cent}}$, $ES_{99\text{ per cent}}$).

Also for these 15 concentrated portfolios we attempt to realize a cross hedging by adding to each of these unhedged portfolios *N* short positions on the Ftse-Mib index future. The total results in terms of hedged portfolio risk measures and RRRs for all the 16 portfolios are reported in Table 3.

From the results in Table 3, the diversified portfolio 16 records the highest RRR with respect to all the three risk measures. This outcome is

[51]In Figures 1 and 2, portfolio losses are presented as positive values, while portfolio profits are presented as negative values of P&L distribution.

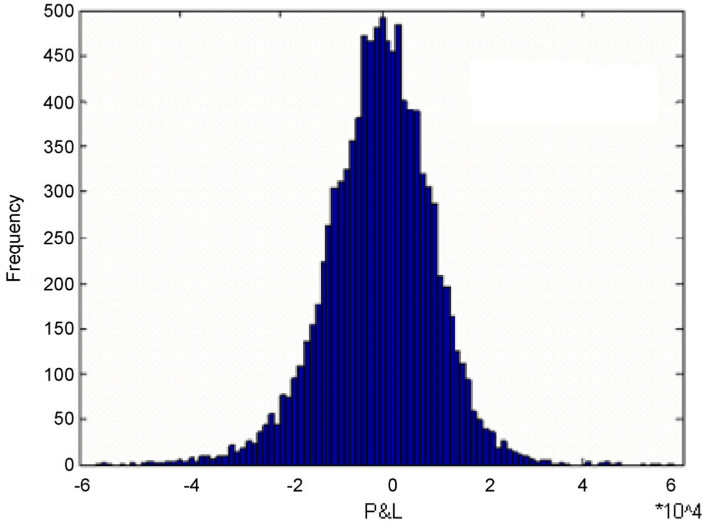

Figure 2: Profit and Loss Distribution of the Hedged Diversified Stock Portfolio

Table 2: Values in *t* and Risk Measures for the 16 Unhedged Portfolios (in Euros)

Portfolio	Value	Volatility	99% VaR	99% ES
1	425,000	11,920	35,356	48,551
2	333,000	7,989	21,942	33,050
3	2,881,000	45,778	122,616	156,008
4	76,000	2,115	4,463	8,739
5	286,000	5,486	16,468	23,693
6	153,000	5,202	13,695	16,819
7	1,189,000	28,467	110,716	136,290
8	1,750,000	30,233	88,448	115,528
9	127,000	3,792	10,565	14,775
10	2,064,000	35,382	106,455	130,125
11	432,000	10,482	28,602	33,845
12	43,000	1,401	3,782	5,135
13	23,000	969	2,638	3,793
14	3,901,000	84,655	226,396	298,786
15	359,000	10,101	25,901	28,968
16	1,404,200	22,900	66,179	78,002

consistent. In fact, the Ftse-Mib index is built similar to a basket of equities and therefore it will be more highly correlated to the return of a diversified portfolio (portfolio 16) than to the return of a concentrated portfolio (in our case composed by a single equity alone). With regard to the other 15 concentrated portfolios, the following considerations can be carried out: (a) the RRR is generally higher for portfolios composed with equity highly

Table 3: Risk Measures and Relative Risk Reductions for the 16 Hedged Portfolios

Portfolio	*N*	Volatility	Vol RRR	99% VaR	VaR RRR	99% ES	ES RRR
1	6	8,560	28.18%	25,16	28.83%	34,26	29.43%
2	4	5,982	25.12%	15,88	27.60%	23,65	28.42%
3	14	40,97	10.48%	105,916	13.62%	132,185	15.27%
4	1	1,514	28.43%	3,120	30.09%	6,141	29.73%
5	3	2,801	48.94%	8,305	49.57%	11,83	50.04%
6	2	3,929	24.47%	10,00	26.98%	12,32	26.70%
7	18	12,04	57.69%	46,10	58.36%	55,48	59.29%
8	18	14,98	50.42%	43,03	51.34%	55,85	51.65%
9	2	1,608	57.59%	4,316	59.15%	6,068	58.93%
10	18	24,26	31.42%	72,06	32.31%	87,37	32.85%
11	6	6,449	38.47%	17,06	40.35%	19,40	42.68%
12	1	1,103	21.28%	2,883	23.77%	3,808	25.83%
13	0	969	0.00	2,638	0.00	3,793	0.00
14	51	41,80	50.62%	106,814	52.82%	143,596	51.94%
15	6	5,406	46.48%	13,512	47.83%	15,428	46.74%
16	15	7,340	67.95%	21,098	68.12%	24,368	68.76%

correlated to the Ftse-Mib index; and (b) the RRR is generally worse for portfolios with a lower market value given that the approximation error realized by rounding N^* to the closer integer (N) is higher.

Moreover, we find, for each of the 16 portfolios, that the values of RRR measures calculated with respect to ES and VaR are better than the respective RRR measure value calculated with refer to volatility. Because ES is a risk measure that is particularly sensitive to the tail risk, these results seem to show that cross hedging by stock index future is able to manage tail risk. In particular, observing the hedged portfolio 11, we find that the RRR measure value calculated for volatility (Vol RRR) is equal to 38.47 per cent, while the RRR measure value calculated for ES (ES RRR) is equal to 42.68 per cent (RRR measure value calculated for VaR is equal to 40.35 per cent). By adopting a threshold value for the RRR measure equal to 40 per cent (as in Coughlan *et al.*, 2004) the hedging strategy for portfolio 11 might be recognized as effective only when ES and VaR are used as metrics of risk, not when volatility is utilized. For portfolios 5, 7, 8, 9, 14, 15, and 16, the cross hedging strategy might be judged as effective regardless of the risk metrics adopted. Portfolios 1, 2, 3, 4, 6, 10, and 12 fail the RRR measure testing in correspondence of all the three risk metrics. Portfolio 13 gives the worst results, given that N^* is 0.2. In this case, the best choice is the absence of a hedge.

7. Concluding Remarks

In this work for testing the prospective economic effectiveness of the hedging relationship, we drop the traditional hypothesis of a Normal world for financial data and apply an advanced Monte Carlo simulation model for estimating the profit and loss distribution of the stock portfolio without and with hedging. In particular, we focus on the degree of the economic effectiveness of a cross hedging strategy performed by a stock index future traded on IDEM, that is the Ftse-Mib index future. We utilize this hedging strategy for managing the market risk of different hypothetical stock portfolios composed with Italian equities.

The RRR performed by the cross hedging strategy has been estimated utilizing as risk metrics not only the traditional standard deviation but also the VaR and the ES, both these metrics calculated at a confidence level of 99 per cent. All hedge examples (here described) underline that the RRR measure calculated in terms of ES is constantly larger than the RRR measure expressed in terms of standard deviation. In one case (for portfolio 11) the hedge could be recognized as effective in RRR method only if ES and VaR were used as risk metrics; differently (that is, utilizing the st.dev. metric) the hedge effectiveness test fails.

In conclusion, given the failure of the Normal-world assumption for financial data, in order to realize an effective economic hedging we underline the need of: (i) implementing advanced simulative models for estimating financial profit and loss distributions taking into account the real statistical phenomena of financial returns, such as asymmetry and leptokurtosis; and (ii) adopting a risk metric more reliable and coherent, as the ES, able to take into account the dangerous tail risk of the profit and loss distribution.

In this way, we may promote a greater alignment of the best practices used in risk management with those adopted in accounting, in particular, for testing the hedge effectiveness.

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Non Technical Summary

Traders and portfolio managers judge the hedge effectiveness in terms of volatility reduction. In order to minimize the operational burden of hedge accounting, high management may wish to consider the methods or tools used for risk management purposes and evaluate whether these ones may be appropriate for hedge effectiveness assessment. Such an approach would be in line with the objective of the new hedge accounting requirements, as described in the Review Draft (RD) issued by IASB in September 2012. The RD aim is to better reflect the effect of the entity's risk management activities in the financial statements. This might include Value at Risk calculations, volatility reduction methods or similar approaches.

In order to encourage a greater alignment between the hedge accounting and the best financial risk management practices, we propose a new and more adequate prospective model for testing the economic hedge effectiveness. Our method may be defined as advanced because it takes into account, firstly in this operational context, the empirical characteristics of the risk factors affecting the fair value of the item to be hedged, that is the asymmetry and the leptokurtosis phenomena, dropping the unrealistic hypothesis of a Normal world (for the daily financial data).

Although our model refers to the traditional approach based on the measure of the relative risk reduction (RRR), differently from the original method, in our case the estimate of the RRR obtained by hedging is offered in terms of the new financial risk measures, such as Value at Risk and Expected Shortfall.

Because the central hedging issue, mainly for financial institutions, is the hedging of the trading portfolio, our advanced prospective model has been implemented to a market risk hedging strategy, that is the cross hedging, realized by combining a stock index future (short position) with a stock portfolio (long position).

We underline that our results present a strong significance from an economic viewpoint, but they may be utilized only in an experimental way for hedge accounting purposes (waiting for a macro hedge accounting model).