

## Analytic integration of real-virtual counterterms in NNLO jet cross sections I

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# Analytic integration of real-virtual counterterms in NNLO jet cross sections I

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**ABSTRACT:** We present analytic evaluations of some integrals needed to give explicitly the integrated real-virtual counterterms, based on a recently proposed subtraction scheme for next-to-next-to-leading order (NNLO) jet cross sections. After an algebraic reduction of the integrals, integration-by-parts identities are used for the reduction to master integrals and for the computation of the master integrals themselves by means of differential equations. The results are written in terms of one- and two-dimensional harmonic polylogarithms, once an extension of the standard basis is made. We expect that the techniques described here will be useful in computing other integrals emerging in calculations in perturbative quantum field theories.

**KEYWORDS:** Jets, QCD.

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\*On leave of absence from INFN, Sezione di Torino.

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## 1. Introduction

LHC physics demands calculating physical observables beyond leading order (LO) accuracy, by including the virtual and real corrections that appear at higher orders. However, the evaluation of phase space integrals beyond LO is not straightforward because it involves infrared singularities that have to be consistently treated before any numerical computation may be performed. At next-to-leading order (NLO), infrared divergences can be handled using a *subtraction scheme* exploiting the fact that the structure of the kinematical singularities of QCD matrix elements is universal and independent of the hard process. This allows us to build process-independent counterterms which regularize the one-loop (or virtual) corrections and real phase space integrals simultaneously [1].

In recent years a lot of effort has been devoted to the extension of the subtraction method to the computation of the radiative corrections at the next-to-next-to-leading order (NNLO) [2–11]. In particular, in ref. [12, 13], a subtraction scheme was defined for computing NNLO corrections to QCD jet cross sections to processes without coloured partons in the initial state and an arbitrary number of massless particles (coloured or colourless) in the final state. This scheme however is of practical utility only after the universal counterterms for the regularization of the real emissions are integrated over the phase space of the unresolved particles. The integrated counterterms can be computed once and for all and their knowledge is necessary to regularize the infrared divergences appearing in virtual corrections. That is indeed the task of this work: we analytically evaluate some of the integrals needed for giving explicitly the counterterms appearing in the scheme [12, 13]. The method is an adaptation of a technique developed in the last two decades to compute multi-loop Feynman diagrams [14–19]. To our knowledge this is the first time that these techniques are applied to integrals of the type

$$F(z) = \int_0^1 \int_0^{\alpha_0} dx dy x^{k_1\epsilon} (1-x)^{k_2\epsilon} y^{k_3\epsilon} (1-y)^{k_4\epsilon} (1-xyz)^{k_5\epsilon} f(x, y, z), \quad (1.1)$$

where

$$f(x, y, z) = \frac{1}{x^{n_1}} \frac{1}{(1-x)^{n_2}} \frac{1}{y^{n_3}} \frac{1}{(1-y)^{n_4}} \frac{1}{(1-xyz)^{n_5}}, \quad (1.2)$$

with  $n_i$  being non-negative integers and  $0 < \alpha_0 \leq 1$ .

An alternative method for computing the  $\epsilon$ -expansion of the integrals is iterated sector decomposition. This approach allows one to express the expansion coefficients of all functions we consider as finite, multidimensional integrals. Integrating these representations numerically, we obtain the expansion coefficients for any fixed value of the arguments. Every integral in this paper was computed numerically as well, with this alternative method for selected values of the parameters. We found that in all cases the analytical and numerical results agreed up to the uncertainty associated with the numerical integration.

The outline of the paper is the following. In section 2 we outline the steps of our method. In section 3 we define the integrals of the subtraction terms that we will consider in the paper. Our analytic results will be presented in terms of one- and two-dimensional harmonic polylogarithms. We summarize those properties of these functions that are important for our computations in sections 4 and 5, respectively. In sections 6 and 7 we calculate analytically the integrals needed for integrating the soft-type counterterms as a series expansion in the dimensional regularization parameter  $\epsilon$ . In section 8 we calculate some of the integrals needed for integrating the collinear counterterms. In sections 9 and 10 we calculate two sets of convoluted integrated counterterms, which can be obtained from a successive integration of the results obtained in section 8. In section 11 we briefly discuss the numerical calculation of the integrated subtraction terms. Finally in section 12 we present the conclusions of this work and we discuss possible developments concerning more complicated classes of integrals. Appendix A contains the spin-averaged splitting functions at tree level and at one-loop, which are needed for the evaluation of the counterterms. There are further appendices containing the (often rather lengthy) expressions of the integrated counterterms.

## 2. The method

Our method of computing the integrals involves the following steps:

**Algebraic reduction of the integrand by means of partial fractioning.** For each class of integrals, we perform a partial fractioning of the integrand in order to obtain a set of independent integrals. For example, for the integrand in eq. (1.2) with  $n_1 = n_2 = n_3 = n_4 = n_5 = 1$  one can perform partial fractioning with respect to the integration variable  $x$  first, so that

$$\frac{1}{x(1-x)(1-xyz)} = \frac{1}{x} + \frac{1}{1-yz} \frac{1}{1-x} - \frac{y^2z^2}{1-yz} \frac{1}{1-xyz}. \quad (2.1)$$

Note the appearance of the new denominator  $1-yz$ , not originally present in the integrand and coming from  $x$  partial fractioning.<sup>1</sup> One then performs partial fractioning with respect to  $y$ , by considering the denominator  $1-xyz$  as a constant: that is because the latter was already involved in the  $x$  partial fractioning and, to avoid an infinite loop, it cannot be subjected to any further transformation. For example:

$$\begin{aligned} \frac{1}{y(1-y)(1-yz)(1-xyz)} &= -\frac{z^2}{1-z} \frac{1}{(1-yz)(1-xyz)} + \frac{1}{y(1-xyz)} + \\ &+ \frac{1}{1-z} \frac{1}{(1-y)(1-xyz)}. \end{aligned} \quad (2.2)$$

After this final partial fractioning over  $y$ , the original integrand  $f$ , depending on five denominators, is transformed into a combination of terms having at most two denominators, out of which at most one depends on  $x$ .<sup>2</sup>

**Reduction to master integrals by means of integration-by-parts identities.** We then write integration-by-parts identities (ibps) for the chosen set of independent amplitudes. If the upper limits in the  $x$  or  $y$  integrals in eq. (1.1) differ from one,  $\alpha_0 < 1$ , surface terms have to be taken into account. That is to be contrasted with the case of loop calculations, in which surface terms always vanish. By solving the ibps with the standard Laporta algorithm, complete reduction to master integrals is accomplished.

**Analytic evaluation of the master integrals.** After having identified for each class of integrals a set of master integrals, we write the corresponding system of differential equations. The  $\epsilon$ -expansion of the master integrals is obtained by solving such systems expanded in powers of  $\epsilon$ . A natural basis consists of one- and two-dimensional harmonic polylogarithms [20, 21]; for representing some master integrals, an extension of the standard basis functions has proved to be necessary.

---

<sup>1</sup>By increasing the number of variables, the number of additional denominators grows very fast.

<sup>2</sup>Performing first the partial fractioning in  $y$  and then in  $x$  results in a different basis of independent amplitudes.

### 3. Integrals needed for the integrated subtraction terms

The subtraction method developed in refs. [12, 13] relies on the universal soft and collinear factorization properties of QCD squared matrix elements. Although the necessary factorization formulae for NNLO computations have been known for almost a decade, the explicit definition of a subtraction scheme has been hampered for several reasons. Firstly, the various factorization formulae overlap in a rather complicated way beyond NLO accuracy and these overlaps have to be disentangled in order to avoid multiple subtractions. At NNLO accuracy this was first achieved in ref. [11]. A general and simple solution to this problem was subsequently given in ref. [22], where a method was described to obtain pure-soft factorization at any order in perturbation theory leading to soft-singular factors without collinear singularities.

Secondly, the factorization formulae are valid only in the strict soft and collinear limits and have to be extended to the whole phase space. A method that works at any order in perturbation theory requires a mapping of the original  $n$  momenta  $\{p\}_n = \{p_1, \dots, p_n\}$  to  $m$  momenta  $\{\tilde{p}\}_m = \{\tilde{p}_1, \dots, \tilde{p}_m\}$  ( $m$  is the number of hard partons and  $n - m$  is the number of unresolved ones) that preserves momentum conservation. Such a mapping leads to an exact factorization of the original  $n$ -particle phase space of total momentum  $Q$ ,

$$d\phi_n(p_1, \dots, p_n; Q) = \prod_{i=1}^n \frac{d^d p_i}{(2\pi)^{d-1}} \delta_+(p_i^2) (2\pi)^d \delta^{(d)} \left( Q - \sum_{i=1}^n p_i \right), \quad (3.1)$$

in the form

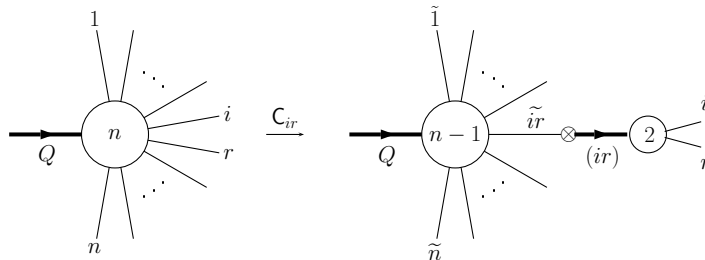
$$d\phi_n(\{p\}_n; Q) = d\phi_m(\{\tilde{p}\}_m; Q) [dp_{n-m;m}(\{p\}_{n-m}; Q)]. \quad (3.2)$$

In the context of computing QCD corrections, this sort of exact phase-space factorization was first introduced in ref. [1], where only three of the original momenta  $\{p\}$  — that of the emitter  $p_i^\mu$ , the spectator  $p_k^\mu$  and the emitted particle  $p_j^\mu$  — were mapped to two momenta,  $\tilde{p}_{ij}^\mu$  and  $\tilde{p}_k^\mu$ , the rest of the phase space was left unchanged. This sort of mapping requires that both  $i$  and  $k$  be hard partons, which is always satisfied in a computation at NLO accuracy because only one parton is unresolved. However, in a computation beyond NLO the spectator momentum may also become unresolved unless this is explicitly avoided by using colour-ordered subamplitudes [7, 8]. In order to take into account the colour degrees of freedom explicitly, as well as to define a phase space mapping valid at any order in perturbation theory, in ref. [23], two types of “democratic” phase-space mappings were introduced. In this paper we are concerned with the integrals of the singly-unresolved counterterms, therefore, in the rest of the paper we deal with the case when  $n - m = 1$ . Symbolically, the mapping

$$\{p\}_n \xrightarrow{C_{ir}} \{\tilde{p}\}_{n-1}^{(ir)} = \{\tilde{p}_1, \dots, \tilde{p}_{ir}, \dots, \tilde{p}_n\}, \quad (3.3)$$

used for collinear subtractions, denotes a mapping where the momenta  $p_i^\mu$  and  $p_r^\mu$  are replaced by a single momentum  $\tilde{p}_{ir}^\mu$  and all other momenta are rescaled, while for soft-type subtractions,

$$\{p\}_n \xrightarrow{S_r} \{\tilde{p}\}_{n-1}^{(r)} = \{\tilde{p}_1, \dots, \tilde{p}_n\} \quad (3.4)$$



**Figure 1:** Graphical representation of the collinear momentum mapping and the implied phase space factorization.

denotes a mapping such that the momentum  $p_r^\mu$ , that may become soft, is missing from the set, and all other momenta are rescaled and transformed by a proper Lorentz transformation. These mappings are defined such that the recoil due to the emission of the unresolved partons is taken by all hard partons. In both cases the factorized phase-space measure can be written in the form of a convolution.

### 3.1 Definition of the collinear integrals

In the case of collinear mapping the factorized phase-space measure can be written as

$$[dp_{1;n-1}^{(ir)}(p_r, \tilde{p}_{ir}; Q)] = d\alpha (1 - \alpha)^{2(n-2)(1-\epsilon)-1} \frac{s_{\tilde{ir}Q}}{2\pi} d\phi_2(p_i, p_r; p_{(ir)}) \Theta(\alpha) \Theta(1 - \alpha), \quad (3.5)$$

where  $s_{\tilde{ir}Q} = 2\tilde{p}_{ir} \cdot Q$  and  $p_{(ir)}^\mu = (1 - \alpha)\tilde{p}_{ir}^\mu + \alpha Q^\mu$ . The collinear momentum mapping and the implied factorization of the phase-space measure are represented graphically in figure 1. The picture on the left shows the  $n$ -particle phase space  $d\phi_n(\{p\}; Q)$ , where in the circle we have indicated the number of momenta. The picture on the right corresponds to eq. (3.2) (with  $n - m = 1$ ) and eq. (3.5): the two circles represent the  $(n - 1)$ -particle phase space  $d\phi_{n-1}(\{\tilde{p}\}^{(ir)}; Q)$  and the two-particle phase space  $d\phi_2(p_i, p_r; p_{(ir)})$  respectively, while the symbol  $\otimes$  stands for the convolution over  $\alpha$ , as precisely defined in eq. (3.5).

Writing the factorized phase space in the form of eq. (3.5) has some advantages:

- It makes the symmetry property of the factorized phase space under the permutation of the factorized momenta manifest. For instance, for any function  $f$ ,

$$\int [dp_{1;n-1}^{(ir)}(p_r, \tilde{p}_{ir}; Q)] f(p_i, p_r) = \int [dp_{1;n-1}^{(ir)}(p_r, \tilde{p}_{ir}; Q)] f(p_r, p_i), \quad (3.6)$$

which can be used to reduce the number of independent integrals.

- It exhibits the  $n$ -dependence of the factorized phase space explicitly. This allows for including  $n$ -dependent factors of  $(1 - \alpha)^{2d_0 - 2(n-2)(1-\epsilon)} \Theta(\alpha_0 - \alpha)$  (with  $d_0|_{\epsilon=0} \geq 2$ ) in the subtraction terms such that the integrated counterterms will be  $n$ -independent (for details see ref. [24]).
- Eq. (3.5) generalizes very straightforwardly for more complicated factorizations. (The formula for the general case when phase-spaces of  $N$  groups of  $r_1, r_2, \dots, r_N$  partons are factorized simultaneously can be given explicitly.)



To write the factorized two-particle phase-space measure we introduce the variable  $v$ ,

$$v = \frac{z_r - z_r^{(-)}}{z_r^{(+)} - z_r^{(-)}}. \tag{3.7}$$

In eq. (3.7)  $z_r$  is the momentum fraction of parton  $r$  in the Altarelli-Parisi splitting function that describes the  $f_{(ir)} \rightarrow f_i + f_r$  collinear splittings ( $f$  denotes the flavour of the partons). This momentum fraction takes values between

$$z_r^{(-)} = \frac{\alpha}{2\alpha + x - \alpha x} \tag{3.8}$$

and  $z_r^{(+)} = 1 - z_r^{(-)}$  ( $x = s_{ir}^{\sim} Q/Q^2$ ). Using the variables  $s_{ir} = 2p_i p_r$ , and  $v$  the two-particle phase-space measure reads

$$\begin{aligned} d\phi_2(p_i, p_r; p_{(ir)}) &= \frac{s_{ir}^{-\epsilon}}{8\pi} \bar{S}_\epsilon ds_{ir} dv \delta(s_{ir} - Q^2 \alpha (\alpha + (1 - \alpha)x)) \\ &\times [v(1 - v)]^{-\epsilon} \Theta(1 - v) \Theta(v), \end{aligned} \tag{3.9}$$

where<sup>3</sup>

$$\bar{S}_\epsilon = \frac{(4\pi)^\epsilon}{\Gamma(1 - \epsilon)}. \tag{3.10}$$

The integrated collinear subtraction terms involve the integrals of the (spin-averaged) Altarelli-Parisi splitting kernels over the unresolved phase space [24]

$$\frac{(4\pi)^2}{\bar{S}_\epsilon} (Q^2)^{(1+\kappa)\epsilon} \int_0^{\alpha_0} d\alpha (1 - \alpha)^{2d_0 - 1} \frac{s_{ir}^{\sim} Q}{2\pi} \int d\phi_2(p_i, p_r; p_{(ir)}) \frac{1}{s_{ir}^{1+\kappa\epsilon}} P_{f_i f_r}^{(\kappa)}(z_i, z_r; \epsilon), \quad \kappa = 0, 1. \tag{3.11}$$

As explained in ref. [24], we include harmless factors of  $(1 - \alpha)^{2d_0 - 2(n-2)(1-\epsilon)} \Theta(\alpha_0 - \alpha)$  (with  $d_0|_{\epsilon=0} \geq 2$ ) in the subtraction terms to make their integrals independent of  $n$  and to restrict the phase space over which the subtractions are non-zero. Thus  $\alpha_0 \in (0, 1]$  is the cut parameter controlling this restriction ( $\alpha_0 = 1$  corresponds to subtracting over the full phase space) and  $d_0$  is an exponent which may be fixed freely (with the constraint that  $d_0|_{\epsilon=0} \geq 2$ ). For a more elaborate discussion, including the explanation of our eventual choice of  $d_0 = 3 - 3\epsilon$ , see appendix A of ref. [24].  $P_{f_i f_r}^{(0)}$  and  $P_{f_i f_r}^{(1)}$  denote the average of the tree-level and one-loop splitting kernels over the spin states of the parent parton (Altarelli-Parisi splitting functions), respectively. These spin-averaged splitting kernels depend, in general, on  $z_i$  and  $z_r$ , with the constraint

$$z_i + z_r = 1, \tag{3.12}$$

---

<sup>3</sup>The  $\overline{\text{MS}}$  renormalization scheme as often employed in the literature uses  $S_\epsilon = (4\pi)^\epsilon e^{-\epsilon\gamma_E}$ . It is not difficult to check that in a computation at the NLO accuracy using  $\bar{S}_\epsilon$  leads to the same expressions as the usual  $\overline{\text{MS}}$  definition. At NNLO the different normalizations lead to slightly different bookkeeping of the IR and UV poles at intermediate steps of the computation, but the physical cross section of infrared-safe observables is the same. Our normalization leads to somewhat simpler bookkeeping at the NNLO level.

$\delta$	Function	$g_I^{(\pm)}(z)$
0	$g_A$	1
$\mp 1$	$g_B^{(\pm)}$	$(1-z)^{\pm\epsilon}$
0	$g_C^{(\pm)}$	$(1-z)^{\pm\epsilon} {}_2F_1(\pm\epsilon, \pm\epsilon, 1 \pm \epsilon, z)$
$\pm 1$	$g_D^{(\pm)}$	${}_2F_1(\pm\epsilon, \pm\epsilon, 1 \pm \epsilon, 1-z)$

**Table 1:** The values of  $\delta$  and  $g_I^{(\pm)}(z_r)$  at which eq. (3.13) needs to be evaluated.

and are listed in appendix A. Inspecting the actual form of the Altarelli-Parisi splitting functions and using the symmetry property of the factorized phase space under the interchange  $i \leftrightarrow r$ , we find that (3.11) can be expressed as a linear combination of the integrals

$$\frac{(4\pi)^2}{\bar{S}_\epsilon} (Q^2)^{(1+\kappa)\epsilon} \int_0^{\alpha_0} d\alpha (1-\alpha)^{2d_0-1} \frac{s_{ir} Q}{2\pi} \int d\phi_2(p_i, p_r; p_{(ir)}) \frac{z_r^{k+\delta\epsilon}}{s_{ir}^{1+\kappa\epsilon}} g_I^{(\pm)}(z_r), \quad (3.13)$$

for  $k = -1, 0, 1, 2$ ,  $\kappa = 0, 1$  and the values of  $\delta$  and functions  $g_I^{(\pm)}$  as given in table 1.

Using eqs. (3.7)–(3.9) and  $z_r$  expressed with  $v$ ,

$$z_r = \frac{\alpha + (1-\alpha)xv}{2\alpha + (1-\alpha)x}, \quad (3.14)$$

we can see that the integrals in eq. (3.13) take the form

$$\begin{aligned} \mathcal{I}(x; \epsilon, \alpha_0, d_0; \kappa, k, \delta, g_I^{(\pm)}) &= x \int_0^{\alpha_0} d\alpha \alpha^{-1-(1+\kappa)\epsilon} (1-\alpha)^{2d_0-1} [\alpha + (1-\alpha)x]^{-1-(1+\kappa)\epsilon} \\ &\times \int_0^1 dv [v(1-v)]^{-\epsilon} \left( \frac{\alpha + (1-\alpha)xv}{2\alpha + (1-\alpha)x} \right)^{k+\delta\epsilon} g_I^{(\pm)} \left( \frac{\alpha + (1-\alpha)xv}{2\alpha + (1-\alpha)x} \right). \end{aligned} \quad (3.15)$$

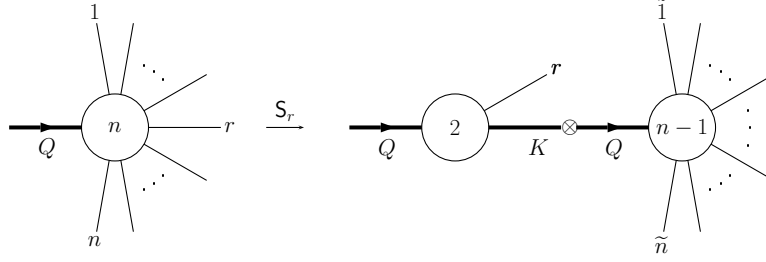
We compute the integrals corresponding to the first two rows of table 1 in section 8.

### 3.2 Definition of the soft-type integrals

In the case of soft mapping the factorized phase-space measure can be written as

$$\begin{aligned} [dp_{1;n-1}^{(r)}(p_r; Q)] &= dy(1-y)^{(n-2)(1-\epsilon)-1} \frac{Q^2}{2\pi} d\phi_2(p_r, K; Q) \\ &\times \Theta(y)\Theta(1-y), \end{aligned} \quad (3.16)$$

where the timelike momentum  $K$  is massive with  $K^2 = (1-y)Q^2$ . We show the soft momentum mapping and the implied phase space factorization in figure 2. The picture on the left shows again the  $n$ -particle phase space  $d\phi_n(\{p\}; Q)$ , while the picture on the right corresponds to eq. (3.2) (with  $n-m=1$ ) and eq. (3.16): the two circles represent the two-particle phase space  $d\phi_2(p_r, K; Q)$  and the  $(n-1)$ -particle phase space  $d\phi_{n-1}(\{\tilde{p}\}^{(r)}; Q)$  respectively. The symbol  $\otimes$  stands for the convolution over  $y$  as defined in eq. (3.16).



**Figure 2:** Graphical representation of the soft momentum mapping and the implied phase space factorization.

The integrated soft and soft-collinear subtraction terms involve the integral of the eikonal factor and its collinear limit over the factorized phase space of eq. (3.16) [24], namely the integrals

$$-\frac{(4\pi)^2}{\bar{S}_\epsilon} (Q^2)^{(1+\kappa)\epsilon} \int_0^{y_0} dy (1-y)^{d'_0-1} \frac{Q^2}{2\pi} \int d\phi_2(p_r, K; Q) \left( \frac{s_{ik}}{s_{ir}s_{kr}} \right)^{1+\kappa\epsilon}, \quad \kappa = 0, 1, \quad (3.17)$$

$$\frac{(4\pi)^2}{\bar{S}_\epsilon} (Q^2)^{(1+\kappa)\epsilon} \int_0^{y_0} dy (1-y)^{d'_0-1} \frac{Q^2}{2\pi} \int d\phi_2(p_r, K; Q) 2 \left( \frac{1}{s_{ir}} \frac{z_i}{z_r} \right)^{1+\kappa\epsilon}, \quad \kappa = 0, 1. \quad (3.18)$$

Here again, we included harmless factors of  $(1-y)^{d'_0-(n-2)(1-\epsilon)}\Theta(y_0-y)$  (with  $d'_0|_{\epsilon=0} \geq 2$ ) in the subtraction terms to make their integrals independent of  $n$  and to restrict the phase space over which the subtractions are non-zero. Thus  $y_0 \in (0, 1]$  is the cut parameter controlling the restriction ( $y_0 = 1$  corresponds to subtracting over the full phase space) and  $d'_0$  is an exponent (in principle independent of  $d_0$ , hence the prime) which may be fixed freely (but with  $d'_0|_{\epsilon=0} \geq 2$ ). See appendix A of ref. [24] for a more detailed discussion, including the explanation of our eventual choice of  $d'_0 = 3 - 3\epsilon$ . The computation of these integrals is fairly straightforward using energy and angle variables.

In order to write the factorized phase-space measure, we choose a frame in which

$$Q^\mu = \sqrt{s}(1, \dots), \quad \tilde{p}_i^\mu = \tilde{E}_i(1, \dots, 1), \quad \tilde{p}_k^\mu = \tilde{E}_k(1, \dots, \sin \chi, \cos \chi), \quad (3.19)$$

and

$$p_r^\mu = E_r(1, \dots, \text{“angles”}, \dots, \sin \vartheta \sin \varphi, \sin \vartheta \cos \varphi, \cos \vartheta). \quad (3.20)$$

In eq. (3.19) the dots stand for vanishing components, while the notation “angles” in eq. (3.20) denotes the dependence of  $p_r$  on the  $d-3$  angular variables that can be trivially integrated. Then in terms of the scaled energy-like variable

$$\varepsilon_r = \frac{2p_r \cdot Q}{Q^2} = \frac{2E_r}{\sqrt{s}} \quad (3.21)$$

and the angular variables  $\vartheta$  and  $\varphi$  the two-particle phase space reads

$$d\phi_2(p_r, K; Q) = \frac{(Q^2)^{-\epsilon}}{16\pi^2} \bar{S}_\epsilon \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} d\varepsilon_r \varepsilon_r^{1-2\epsilon} \delta(y - \varepsilon_r) \times d(\cos \vartheta) d(\cos \varphi) (\sin \vartheta)^{-2\epsilon} (\sin \varphi)^{-1-2\epsilon}, \quad (3.22)$$

where  $y \in (0, 1]$  and the cosines of both angles run from  $-1$  to  $+1$ .

To write the integrands in these variables, we observe that the precise definitions of  $\tilde{p}_i$  and  $\tilde{p}_k$  as given in ref. [12] imply

$$s_{ik} = (1 - \varepsilon_r) s_{\tilde{ik}}, \quad s_{ir} = s_{\tilde{ir}}, \quad s_{kr} = s_{\tilde{kr}}, \quad (3.23)$$

and

$$s_{iQ} = (1 - \varepsilon_r) s_{\tilde{iQ}} + s_{\tilde{ir}}. \quad (3.24)$$

From eqs. (3.19), (3.20), (3.23) and (3.24) we find

$$\frac{s_{ik}}{s_{ir} s_{kr}} = (1 - \varepsilon_r) \frac{s_{\tilde{ik}}}{s_{\tilde{ir}} s_{\tilde{kr}}} = \frac{4Y_{\tilde{ik},Q} (1 - \varepsilon_r)}{Q^2 \varepsilon_r^2} \frac{1}{(1 - \cos \vartheta)(1 - \cos \chi \cos \vartheta - \sin \chi \sin \vartheta \cos \varphi)}, \quad (3.25)$$

and

$$\frac{1}{s_{ir}} \frac{z_i}{z_r} = \frac{1}{s_{\tilde{ir}}} \frac{(1 - \varepsilon_r) s_{\tilde{iQ}} + s_{\tilde{ir}}}{s_{rQ}} = \frac{1}{Q^2} \frac{1}{\varepsilon_r} \left[ 1 + \frac{2(1 - \varepsilon_r)}{\varepsilon_r (1 - \cos \vartheta)} \right]. \quad (3.26)$$

In eq. (3.25) above we have set

$$Y_{\tilde{ik},Q} = \frac{Q^2 s_{\tilde{ik}}}{s_{\tilde{iQ}} s_{\tilde{k}Q}}. \quad (3.27)$$

Using eqs. (3.22), (3.25) and (3.26) we see that the integral of the soft subtraction term in eq. (3.17) may be written as

$$\begin{aligned} \mathcal{J}(Y_{\tilde{ik},Q}; \varepsilon, y_0, d'_0; \kappa) &= - (4Y_{\tilde{ik},Q})^{1+\kappa\epsilon} \frac{\Gamma^2(1 - \epsilon)}{2\pi\Gamma(1 - 2\epsilon)} \Omega^{(1+\kappa\epsilon, 1+\kappa\epsilon)}(\cos \chi) \\ &\times \int_0^{y_0} dy y^{-1-2(1+\kappa)\epsilon} (1 - y)^{d'_0 + \kappa\epsilon}, \end{aligned} \quad (3.28)$$

where  $\Omega^{(i,k)}(\cos \chi)$  denotes the angular integral

$$\begin{aligned} \Omega^{(i,k)}(\cos \chi) &= \int_{-1}^1 d(\cos \vartheta) (\sin \vartheta)^{-2\epsilon} \int_{-1}^1 d(\cos \varphi) (\sin \varphi)^{-1-2\epsilon} \\ &\times (1 - \cos \vartheta)^{-i} (1 - \cos \chi \cos \vartheta - \sin \chi \sin \vartheta \cos \varphi)^{-k}. \end{aligned} \quad (3.29)$$

Furthermore, from eq. (3.19) it is easy to see that

$$\cos \chi = 1 - 2Y_{\tilde{ik},Q} \equiv 1 - \frac{2Q^2 s_{\tilde{ik}}}{s_{\tilde{iQ}} s_{\tilde{k}Q}}. \quad (3.30)$$

We compute the soft integrals  $\mathcal{J}(Y, \varepsilon; y_0, d'_0; \kappa)$  in section 6.

The soft-collinear subtraction term in eq. (3.18) leads to the integral

$$\begin{aligned} \mathcal{K}(\varepsilon, y_0, d'_0; \kappa) &= 2 \int_0^{y_0} dy y^{-(2+\kappa)\epsilon} (1 - y)^{d'_0 - 1} \int_{-1}^1 d(\cos \vartheta) (\sin \vartheta)^{-2\epsilon} \\ &\times \left[ 1 + \frac{2(1 - y)}{y(1 - \cos \vartheta)} \right]^{1+\kappa\epsilon} \frac{\Gamma^2(1 - \epsilon)}{2\pi\Gamma(1 - 2\epsilon)} \int_{-1}^1 d(\cos \varphi) (\sin \varphi)^{-1-2\epsilon}, \end{aligned} \quad (3.31)$$

which we compute in section 7.

### 3.3 Iterated integrals

In an NNLO computation, iterations of the above integrals also appear. In this paper we compute also two of those. The first one is the integration of a soft integral with a collinear one in its argument,

$$\begin{aligned} \mathcal{J}^*\mathcal{I}(Y_{\vec{i}\vec{k},Q}; \epsilon, \alpha_0, d_0, y_0, d'_0; k) &= -4Y_{\vec{i}\vec{k},Q} \frac{\Gamma^2(1-\epsilon)}{2\pi\Gamma(1-2\epsilon)} \Omega^{(1,1)}(\cos \chi) \\ &\times \int_0^{y_0} dy y^{-1-2\epsilon} (1-y)^{d'_0} \mathcal{I}(y; \epsilon, \alpha_0, d_0; 0, k, 0, 1), \end{aligned} \quad (3.32)$$

which we need for  $k = -1, 0, 1, 2$ . Details of the computation are given in section 9. The second case is when the collinear integral appears in the argument of a soft-collinear one,

$$\begin{aligned} \mathcal{K}^*\mathcal{I}(\epsilon, \alpha_0, d_0, y_0, d'_0; k) &= 2 \frac{\Gamma^2(1-\epsilon)}{2\pi\Gamma(1-2\epsilon)} \int_{-1}^1 d(\cos \vartheta) (\sin \vartheta)^{-2\epsilon} \\ &\times \int_{-1}^1 d(\cos \varphi) (\sin \varphi)^{-1-2\epsilon} \int_0^{y_0} dy y^{-1-2\epsilon} (1-y)^{d'_0-1} \\ &\times \frac{2-y(1+\cos \vartheta)}{1-\cos \vartheta} \mathcal{I}(y; \epsilon, \alpha_0, d_0; 0, k, 0, 1), \end{aligned} \quad (3.33)$$

needed again for  $k = -1, 0, 1, 2$ . Details of the computation are given in section 10.

### 4. One-dimensional harmonic polylogarithms

As anticipated in the introduction, it is convenient to represent the integrals depending on a single variable  $x$  in terms of a general class of special functions called harmonic polylogarithms (*HPL*'s) introduced in ref. [20]. The *HPL*'s of weight one, i.e. depending on one index  $w = -1, 0, 1$ , are defined as:

$$H(-1; x) \equiv \log(1+x); \quad H(0; x) \equiv \log(x); \quad H(1; x) \equiv -\log(1-x). \quad (4.1)$$

These functions are then just logarithms of linear functions of  $x$ . The *HPL*'s of higher weight are defined recursively by the relation

$$H(a, \vec{w}; x) \equiv \int_0^x f(a; x') H(\vec{w}; x') dx' \quad \text{for } a \neq 0 \text{ and } \vec{w} \neq \vec{0}_n, \quad (4.2)$$

i.e. in the case in which not all the indices are zero. The left-most index takes the values  $a = -1, 0, 1$  and  $\vec{w}$  is an  $n$ -dimensional vector with components  $w_i = -1, 0, 1$ . We call  $n$  the weight of the *HPL*'s, so the above relation allows one to increase the weight  $w = n \rightarrow n+1$ . The basis functions  $f(a; x)$  are given by

$$f(-1; x) \equiv \frac{1}{1+x}; \quad f(0; x) \equiv \frac{1}{x}; \quad f(1; x) \equiv \frac{1}{1-x}. \quad (4.3)$$

In the case in which all indices are zero, one defines instead,

$$H(\vec{0}_n; x) \equiv \frac{1}{n!} \log^n(x). \quad (4.4)$$

The *HPL*'s introduced above fulfill many interesting relations, one of the most important ones being that of generating a “shuffle algebra”,

$$H(\vec{w}_1; x) H(\vec{w}_2; x) = \sum_{\vec{w}=\vec{w}_1\uplus\vec{w}_2} H(\vec{w}; x), \tag{4.5}$$

where  $\vec{w}_1 \uplus \vec{w}_2$  denotes the merging of the two weight vectors  $\vec{w}_1$  and  $\vec{w}_2$ , i.e. all possible concatenations of  $\vec{w}_1$  and  $\vec{w}_2$  in which relative orderings of  $\vec{w}_1$  and  $\vec{w}_2$  are preserved.

The basis of *HPL*'s can be extended by adding some new basis functions to the set in eq. (4.3); for our computation we have to introduce the function

$$f(2; x) \equiv \frac{1}{x-2}. \tag{4.6}$$

The *HPL*'s can be evaluated numerically in a fast and accurate way; there are various packages available for this purpose [25–27].

### 5. Two-dimensional harmonic polylogarithms

To represent integrals depending on two arguments, an extension of the *HPL*'s to functions of two variables proves to be convenient [21]. Since a harmonic polylogarithm is basically a repeated integration on *one* variable, a second independent variable is introduced as a parameter entering the basis functions:  $f(i; x) \rightarrow f(i, \alpha; x)$ . We may say that in addition to the discrete index  $i$ , we have now a continuous index  $\alpha$ . In ref. [21] the following basis functions were originally introduced:

$$f(c_i(\alpha); x) = \frac{1}{x - c_i(\alpha)}, \tag{5.1}$$

where

$$c_1(\alpha) = 1 - \alpha \quad \text{or} \quad c_2(\alpha) = -\alpha. \tag{5.2}$$

Let us remark that the above extension keeps most of the properties of the one-dimensional *HPL*'s. In this work we have to introduce the following new basis functions, which are slightly more complicated than the ones above,

$$f(c_1(\alpha); x) = \frac{1}{x - c_1(\alpha)} \quad f(c_2(\alpha); x) = \frac{1}{x - c_2(\alpha)}, \tag{5.3}$$

with

$$c_1(\alpha) = \frac{\alpha}{\alpha - 1}, \quad c_2(\alpha) = \frac{2\alpha}{\alpha - 1}. \tag{5.4}$$

The explicit definition of the two-dimensional harmonic polylogarithms (*2dHPL*'s) reads:

$$H(c_i(\alpha), \vec{w}(\alpha); x) \equiv \int_0^x f(c_i(\alpha); x') H(\vec{w}(\alpha); x') dx'. \tag{5.5}$$

In general, the *2dHPL*'s have complicated analyticity properties, with imaginary parts coming from integrating over the zeroes of the basis functions. Our computation does not involve such complications because we can always assume  $0 \leq x, \alpha \leq 1$ . That implies that  $c_k(\alpha) < 0$  for any  $k$ : the denominators are never singular and the *2dHPL*'s are real. The numerical evaluation of our *2dHPL*'s can be achieved by extending the algorithm described and implemented in ref. [28].

**5.1 Special values**

For some special values of the argument, the  $2dHPL$ 's reduce to ordinary one-dimensional  $HPL$ 's. It is easy to see that for  $\alpha = 0$  and  $\alpha = 1$  we have

$$f(c_k(\alpha = 0); x) = f(0; x), \quad \lim_{\alpha \rightarrow 1} f(c_k(\alpha); x) = 0. \tag{5.6}$$

From this it follows that

$$\begin{aligned} H(\dots, c_i(\alpha = 0), \dots; x) &= H(\dots, 0, \dots; x), \\ \lim_{\alpha \rightarrow 1} H(\dots, c_i(\alpha), \dots; x) &= 0. \end{aligned} \tag{5.7}$$

Similarly, for  $x = 1$ , the  $2dHPL$ 's reduce to combinations of one-dimensional  $HPL$ 's in  $\alpha$ . This reduction can be performed using an extension of the algorithm presented in [21]. We first write the  $2dHPL$ 's in  $x = 1$  as the integral of the derivative with respect to  $\alpha$ ,

$$H(\vec{w}(\alpha); 1) = H(\vec{w}(\alpha = 1); 1) + \int_1^\alpha d\alpha' \frac{\partial}{\partial \alpha'} H(\vec{w}(\alpha'); 1). \tag{5.8}$$

In the case where  $\vec{w}$  only contains objects of the type  $c_i$ , we have  $H(\vec{w}(\alpha = 1); x) = 0$ . Thus,

$$H(\vec{w}(\alpha); 1) = \int_1^\alpha d\alpha' \frac{\partial}{\partial \alpha'} H(\vec{w}(\alpha'); 1). \tag{5.9}$$

The derivative is then carried out on the integral representation of  $H(\vec{w}(\alpha'); 1)$ , and integrating back gives the desired reduction of  $H(\vec{w}(\alpha); 1)$  to one-dimensional  $HPL$ 's in  $\alpha$ , e.g.

$$\begin{aligned} H(c_1(\alpha); 1) &= -H(0; \alpha), \\ H(c_2(\alpha); 1) &= H(-1; \alpha) - H(0; \alpha) - \ln 2. \end{aligned} \tag{5.10}$$

**5.2 Interchange of arguments**

The basis of  $2dHPL$ 's introduced above selects  $x$  as the explicit (integration) variable and  $\alpha$  as a parameter, but an alternative representation involving a repeated integration over  $\alpha$  of (different) basis functions depending on  $x$  as an external parameter is also possible. Therefore, we have to deal with the typical problem of analytic computations: multiple representations of the same function. It is well known that a complete analytic control requires the absence of “hidden zeroes” in the formulae. That means that one has to know all the transformation properties (identities) of the functions introduced in order to have a single representative out of each class of identical objects. In ref. [21] an algorithm was presented which allows one to interchange the roles of the two variables. The algorithm is basically the same as the one presented for the special values at  $x = 1$ : let us just replace everywhere  $x = 1$  by  $x$  in eq. (5.9). Then we have to introduce the following set of basis functions for the  $2dHPL$ 's ,

$$f(d_k(x); \alpha) = \frac{1}{\alpha + d_k(x)}, \tag{5.11}$$

where

$$d_k(x) = \frac{x}{x - k}. \tag{5.12}$$

All the properties defined at the beginning of this section can be easily extended to this new class of denominators. One finds for example:

$$\begin{aligned} H(c_1(\alpha); x) &= H(0; x) - H(0; \alpha) + H(d_1(x); \alpha), \\ H(c_2(\alpha); x) &= H(0; x) - H(0; \alpha) - \ln 2 + H(d_2(x); \alpha). \end{aligned} \tag{5.13}$$

### 6. The soft integral $\mathcal{J}$

In this section we present the analytic calculation of the soft integral defined in eq. (3.28) for  $\kappa = 0, 1$  and  $d'_0 = D'_0 + d'_1\epsilon$ , with  $D'_0 \geq 2$  being an integer. The angular integral  $\Omega^{(i,k)}(\cos \chi)$  was evaluated in ref. [29]. The integration over  $y$  leads to a hypergeometric function, and for the complete soft integral (3.28) we obtain the analytic expression

$$\begin{aligned} \mathcal{J}(Y, \epsilon; y_0, d'_0; \kappa) &= -Y^{-(1+\kappa)\epsilon} y_0^{-2(1+\kappa)\epsilon} \frac{1}{(1+\kappa)^2 \epsilon^2} \frac{\Gamma^2(1 - (1+\kappa)\epsilon)}{\Gamma(1 - 2(1+\kappa)\epsilon)} \\ &\times {}_2F_1(-d'_0 - \kappa\epsilon, -2(1+\kappa)\epsilon, 1 - 2(1+\kappa)\epsilon, y_0) \\ &\times {}_2F_1(-(1+\kappa)\epsilon, -(1+\kappa)\epsilon, 1 - \epsilon, 1 - Y), \end{aligned} \tag{6.1}$$

i.e. , we only need to find the  $\epsilon$ -expansion of an integral of the form

$$f(x, \epsilon; n_1, n_2, n_3, r_1, r_2, r_3) = \int_0^1 dt t^{-n_1-r_1\epsilon} (1-t)^{-n_2-r_2\epsilon} (1-xt)^{-n_3-r_3\epsilon}. \tag{6.2}$$

which can be obtained using the HYPEXP *Mathematica* package [30]. Nevertheless, we compute the expansion to show our procedure. The first hypergeometric function on the right hand side of eq. (6.1) is of the specific form  ${}_2F_1(a, b, 1+b; x)$ , whose expansion reduces to the expansion of the incomplete beta function  $B_x$ , which is a simple case to illustrate the steps of our procedure. It involves the integrals

$$\begin{aligned} \beta(x, \epsilon; n_1, n_3, r_1, r_3) &= f(x, \epsilon; n_1, 0, n_3, r_1, 0, r_3) = \int_0^1 dt t^{-n_1-r_1\epsilon} (1-xt)^{-n_3-r_3\epsilon} \\ &= x^{-1+n_1+r_1\epsilon} B_x(1-n_1-r_1\epsilon, 1-n_3-r_3\epsilon). \end{aligned} \tag{6.3}$$

The class of independent integrals can be easily obtained using partial fractioning in  $x$ . However, when writing down the integration-by-parts identities for the independent integrals, we have to take into account a surface term coming from the fact that the denominator in  $(1-xt')$  does not vanish for  $t' = 1$ ,

$$\int_0^1 dt' \frac{\partial}{\partial t'} \left( t'^{-n_1-r_1\epsilon} (1-xt')^{-n_3-r_3\epsilon} \right) = (1-x)^{-n_3-r_3\epsilon}. \tag{6.4}$$

Solving the inhomogeneous linear system we find a single master integral

$$\beta^{(1)}(x, \epsilon) = \beta(x, \epsilon; 0, 0, r_1, r_3), \tag{6.5}$$

which fulfills the differential equation

$$\frac{\partial}{\partial x} \beta^{(1)} = \frac{r_1\epsilon - 1}{x} \beta^{(1)} + \frac{(1-x)^{-r_3\epsilon}}{x}, \tag{6.6}$$



with initial condition

$$\beta^{(1)}(x=0; \epsilon) = \int_0^1 dt' t'^{-r_1\epsilon} = \frac{1}{1-r_1\epsilon} = \sum_{k=0}^{\infty} r_1^k \epsilon^k. \quad (6.7)$$

Solving this differential equation, we obtain the expansion of the incomplete beta function in terms of *HPL*'s and thus the expansion of hypergeometric functions of the form  ${}_2F_1(a, b, 1+b; x)$ .

Turning to the general case, we note that if we want to calculate the integral (6.2) using the integration-by-parts identities, we must require  $r_1 \cdot r_2 \cdot r_3 \neq 0$ , because the integration-by-parts identities can exhibit poles in  $r_i = 0$ . It is also useful to notice that not all of the integrals are independent, but only those where just one of the indices  $n_1, n_2, n_3$  is nonzero and where  $n_2, n_3 \geq 0$ . In fact, all other integrals can be reduced to one of this class using partial fractioning, e.g.

$$f(x, \epsilon; 1, -1, 1, r_1, r_2, r_3) = f(x, \epsilon; 1, 0, 0, r_1, r_2, r_3) - (1-x)f(x, \epsilon; 0, 0, 1, r_1, r_2, r_3). \quad (6.8)$$

If  $r_1 \cdot r_2 \cdot r_3 \neq 0$ , we can write immediately the integration-by-parts identities for the independent integrals for  $f$  obtained by partial fractioning,

$$\int_0^1 dt \frac{\partial}{\partial t} (t^{-n_1-r_1\epsilon} (1-t)^{-n_2-r_2\epsilon} (1-xt)^{-n_3-r_3\epsilon}) = 0. \quad (6.9)$$

Solving the integration-by-parts identities we find that  $f$  has two master integrals,

$$f^{(1)}(x, \epsilon) = f(x, \epsilon; 0, 0, 0, r_1, r_2, r_3), \quad f^{(2)}(x, \epsilon) = f(x, \epsilon; 0, 0, 1, r_1, r_2, r_3). \quad (6.10)$$

The master integrals fulfill the following differential equations

$$\begin{aligned} \frac{\partial}{\partial x} f^{(1)} &= \frac{\epsilon r_3}{x} f^{(2)} - \frac{\epsilon r_3}{x} f^{(1)}, \\ \frac{\partial}{\partial x} f^{(2)} &= f^{(1)} \left( \frac{-\epsilon r_1 - \epsilon r_2 - \epsilon r_3 + 1}{x} + \frac{\epsilon r_1 + \epsilon r_2 + \epsilon r_3 - 1}{x-1} \right) + \\ & f^{(2)} \left( \frac{-\epsilon r_2 - \epsilon r_3}{x-1} + \frac{\epsilon r_1 + \epsilon r_2 + \epsilon r_3 - 1}{x} \right), \end{aligned} \quad (6.11)$$

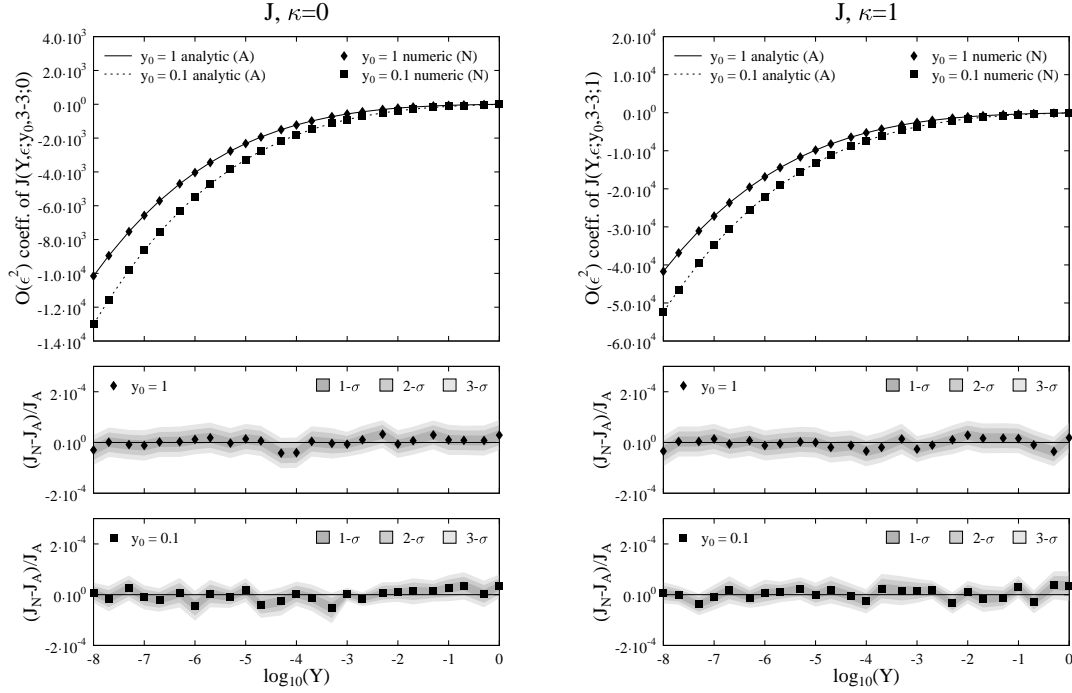
with initial condition

$$f^{(1)}(x=0, \epsilon) = f^{(2)}(x=0, \epsilon) = B(1-r_1\epsilon, 1-r_2\epsilon). \quad (6.12)$$

Solving this set of linear differential equations we can write down the  $\epsilon$ -expansion of the hypergeometric function in terms of *HPL*'s in  $x$ .

The solution for the integral  $\mathcal{J}$  can be easily obtained by using the expansion of the hypergeometric function we just obtained. The results for  $\kappa = 0, 1$  and  $D'_0 = 3$  can be found in appendix B.

As representative examples, in figure 3 we compare the analytic and numeric results for the  $\epsilon^2$  coefficient in the expansion of  $\mathcal{J}(Y, \epsilon; y_0, 3-3\epsilon; \kappa)$  for  $\kappa = 0, 1$  and  $y_0 = 0.1, 1$ . The agreement between the two computations is seen to be excellent for the whole  $Y$ -range. We find a similar agreement for other (lower-order, thus simpler) expansion coefficients and/or other values of the parameters.



**Figure 3:** Representative results for the  $\mathcal{J}$  integral. The plots show the coefficient of the  $O(\epsilon^2)$  term in  $\mathcal{J}(Y, \epsilon; y_0, 3 - 3\epsilon; \kappa)$  for  $\kappa = 0$  (left figure) and  $\kappa = 1$  (right figure) with  $y_0 = 0.1, 1$ .

## 7. The soft-collinear integral $\mathcal{K}$

In this section we calculate analytically the soft-collinear integral defined in eq. (3.31) for  $\kappa = 0, 1$  and  $d'_0 = D'_0 + d'_1\epsilon$ ,  $D'_0$  being an integer. The  $\varphi$  integral is trivial to perform and we find

$$\frac{\Gamma^2(1 - \epsilon)}{2\pi\Gamma(1 - 2\epsilon)} \int_{-1}^1 d(\cos \varphi) (\sin \varphi)^{-1-2\epsilon} = 2^{-1+2\epsilon}. \quad (7.1)$$

Putting  $\cos \vartheta = 2\xi - 1$ , we are left with the integral

$$\mathcal{K}(\epsilon; y_0, d'_0; \kappa) = 2 \int_0^{y_0} dy \int_0^1 d\xi y^{-1-2(1+\kappa)\epsilon} (1-y)^{d'_0-1} \xi^{-\epsilon} (1-\xi)^{-1-(1+\kappa)\epsilon} (1-y\xi)^{1+\kappa\epsilon}. \quad (7.2)$$

### 7.1 Analytic result for $\kappa = 0$

For  $\kappa = 0$ , the integral decouples into a product of two one-dimensional integrals and we get

$$\mathcal{K}(\epsilon; y_0, d'_0; 0) = 2 B_{y_0}(-2\epsilon, d'_0) B(1 - \epsilon, -\epsilon) - 2 B_{y_0}(1 - 2\epsilon, d'_0) B(2 - \epsilon, -\epsilon), \quad (7.3)$$

Using the expansion of the incomplete  $B$ -function, carried out in section 6, we can immediately write down the expansion of  $\mathcal{K}$  for  $\kappa = 0$ . The result for  $D'_0 = 3$  can be found in appendix C.

## 7.2 Analytic result for $\kappa = 1$

The integral (3.31) for  $\kappa = 1$  reads

$$\mathcal{K}(\epsilon; y_0, d'_0; 1) = 2 \int_0^{y_0} dy \int_0^1 d\xi y^{-1-4\epsilon} (1-y)^{d'_0-1} \xi^{-\epsilon} (1-\xi)^{-1-2\epsilon} (1-y\xi)^{1+\epsilon}. \quad (7.4)$$

The analytic solution for this integral cannot be obtained in a straightforward way, due to the presence of the factor  $(1-y\xi)^\epsilon$  that couples the two integrals. Therefore, we rewrite the integral in the form

$$\mathcal{K}(\epsilon; y_0, d'_0; 1) = 2 y_0^{-4\epsilon} K(\epsilon; y_0, d'_1; 1, 1 - D'_0, 0, 1, -1), \quad (7.5)$$

where

$$\begin{aligned} K(\epsilon; y_0, d'_1; n_1, n_2, n_3, n_4, n_5) \\ = \int_0^1 dy \int_0^1 d\xi y^{-n_1-4\epsilon} (1-y_0y)^{-n_2-d'_1\epsilon} \xi^{-n_3-\epsilon} (1-\xi)^{-n_4-2\epsilon} (1-y_0y\xi)^{-n_5+\epsilon}. \end{aligned} \quad (7.6)$$

We now calculate the integral  $K$  using the Laporta algorithm. The independent integrals can be obtained by partial fractioning in  $y$  and  $\xi$ , using the prescription that denominators depending on both integration variables are only partial fractioned in  $\xi$ , e.g.

$$\begin{aligned} \frac{1}{\xi(1-y_0y\xi)} &\rightarrow \frac{1}{\xi} + \frac{y_0y}{1-y_0y\xi}, \\ \frac{1}{y(1-y_0y\xi)} &\rightarrow \frac{1}{y(1-y_0y\xi)}. \end{aligned} \quad (7.7)$$

When writing down the integration-by-parts identities for the independent integrals, we have to take into account a surface term coming from the fact that the denominator in  $(1-y_0y)$  does not vanish in  $y = 1$ ,

$$\begin{aligned} \int_0^1 dy \int_0^1 d\xi \frac{\partial}{\partial \xi} \left( y^{-n_1-4\epsilon} (1-y_0y)^{-n_2-d'_1\epsilon} \xi^{-n_3-\epsilon} (1-\xi)^{-n_4-2\epsilon} (1-y_0y\xi)^{-n_5+\epsilon} \right) \\ = 0 \\ \int_0^1 dy \int_0^1 d\xi \frac{\partial}{\partial y} \left( y^{-n_1-4\epsilon} (1-y_0y)^{-n_2-d'_1\epsilon} \xi^{-n_3-\epsilon} (1-\xi)^{-n_4-2\epsilon} (1-y_0y\xi)^{-n_5+\epsilon} \right) \\ = (1-y_0)^{-n_3-d'_1\epsilon} K_S(\epsilon; y_0, d'_1; n_3, n_4, n_5), \end{aligned} \quad (7.8)$$

with

$$K_S(\epsilon; y_0, d'_1; n_3, n_4, n_5) = \int_0^1 d\xi \xi^{-n_3-\epsilon} (1-\xi)^{-n_4-2\epsilon} (1-y_0\xi)^{-n_5+\epsilon}. \quad (7.9)$$

$K_S$  is just a hypergeometric function,

$$\begin{aligned} K_S(\epsilon; y_0, d'_1; n_3, n_4, n_5) \\ = B(1-n_3-\epsilon, 1-n_4-\epsilon) {}_2F_1(1-n_3-\epsilon, n_5-2\epsilon, 2-n_2-n_4-3\epsilon; y_0), \end{aligned} \quad (7.10)$$

and can thus be calculated using the technique presented in section 6.

Knowing the series expansion for the surface term  $K_S$ , we can solve the integration-by-parts identities for the  $K$  integrals, eq. (7.8). We find the following two master integrals,

$$\begin{aligned} K^{(1)}(\epsilon; y_0, d'_1) &= K(\epsilon; y_0, d'_1; 0, 0, 0, 0, 0), \\ K^{(2)}(\epsilon; y_0, d'_1) &= K(\epsilon; y_0, d'_1; -1, 0, 0, 0, 0), \end{aligned} \tag{7.11}$$

fulfilling the following differential equations,

$$\begin{aligned} \frac{\partial}{\partial y_0} K^{(1)} &= \frac{4\epsilon - 1}{y_0} K^{(1)} + \frac{(1 - y_0)^{d'_1 \epsilon}}{y_0} f^{(1)}, \\ \frac{\partial}{\partial y_0} K^{(2)} &= 2 \frac{2\epsilon - 1}{y_0} K^{(2)} + \frac{(1 - y_0)^{d'_1 \epsilon}}{y_0} f^{(1)}, \end{aligned} \tag{7.12}$$

where  $f^{(1)}$  denotes the master integral of the hypergeometric function calculated in section 6 and where the initial conditions are given by

$$\begin{aligned} K^{(1)}(\epsilon; y_0 = 0, d'_1) &= B(1 - 4\epsilon, 1) B(1 - \epsilon, 1 - 2\epsilon), \\ K^{(2)}(\epsilon; y_0 = 0, d'_1) &= B(2 - 4\epsilon, 1) B(1 - \epsilon, 1 - 2\epsilon). \end{aligned} \tag{7.13}$$

Plugging in the series expansion of  $f^{(1)}$ , and expanding  $(1 - y_0)^{d'_1 \epsilon}$  into a power series in  $\epsilon$ , we can solve for the  $K^{(1)}$  and  $K^{(2)}$  as a power series in  $\epsilon$  whose coefficients are written in terms of *HPL*'s in  $y_0$ .

Knowing the series expansions of  $K^{(1)}$  and  $K^{(2)}$ , we can obtain the integral  $\mathcal{K}(\epsilon; y_0, d'_0; 1)$  for any fixed integer  $D'_0$ . In appendix C we give the explicit result for  $D'_0 = 3$ .

## 8. The collinear integrals $\mathcal{I}$

In this section, we calculate the collinear integrals defined in eq. (3.15) for  $g_I = g_A$  and  $g_I = g_B$  analytically.

### 8.1 The $\mathcal{A}$ -type collinear integrals for $k \geq 0$

The collinear integral for  $g_I = g_A$  requires the evaluation of an integral of the form

$$\begin{aligned} \mathcal{A}(x, \epsilon; \alpha_0, d_0; \kappa, k) &= \frac{1}{x} \mathcal{I}(x, \epsilon; \alpha_0, d_0; \kappa, k, 0, g_A) \\ &= \int_0^{\alpha_0} d\alpha \int_0^1 dv \alpha^{-1-(1+\kappa)\epsilon} (1 - \alpha)^{2d_0-1} [\alpha + (1 - \alpha)x]^{-1-(1+\kappa)\epsilon} \\ &\quad \times v^{-\epsilon} (1 - v)^{-\epsilon} \left( \frac{\alpha + (1 - \alpha)xv}{2\alpha + (1 - \alpha)x} \right)^k, \end{aligned} \tag{8.1}$$

where  $k = -1, 0, 1, 2$ ,  $\kappa = 0, 1$  and  $d_0 = D_0 + d_1 \epsilon$  with  $D_0$  an integer. For  $k \geq 0$  this two-dimensional integral decouples into the product of two one-dimensional integrals, out of which one is straightforward,

$$\begin{aligned} \mathcal{A}(x, \epsilon; \alpha_0, d_0; \kappa, k) &= \sum_{j=0}^k \binom{k}{j} x^j B(1 + j - \epsilon, 1 - \epsilon) \\ &\quad \times \int_0^{\alpha_0} d\alpha \alpha^{k-j-1-(1+\kappa)\epsilon} (1 - \alpha)^{j+2d_0-1} [\alpha + (1 - \alpha)x]^{-1-(1+\kappa)\epsilon} [2\alpha + (1 - \alpha)x]^{-k}. \end{aligned} \tag{8.2}$$

We will therefore treat separately the cases  $k \geq 0$  and  $k < 0$ .

For  $k \geq 0$  the calculation of the  $\mathcal{A}$  integrals reduces to the calculation of a one-dimensional integral of the form

$$\begin{aligned}
 A_+(x, \epsilon; \alpha_0, d_1; \kappa; n_1, n_2, n_3, n_4) \\
 = \int_0^{\alpha_0} d\alpha \alpha^{-n_1-(1+\kappa)\epsilon} (1-\alpha)^{-n_2+2d_1\epsilon} [\alpha + (1-\alpha)x]^{-n_3-(1+\kappa)\epsilon} [2\alpha + (1-\alpha)x]^{-n_4},
 \end{aligned}
 \tag{8.3}$$

$n_i$  being integers. The integration-by-parts identities, including a surface term for the independent integrals, are

$$\begin{aligned}
 \int_0^{\alpha_0} d\alpha \frac{\partial}{\partial \alpha} \left( \alpha^{-n_1-(1+\kappa)\epsilon} (1-\alpha)^{-n_2+2d_1\epsilon} [\alpha + (1-\alpha)x]^{-n_3-(1+\kappa)\epsilon} [2\alpha + (1-\alpha)x]^{-n_4} \right) \\
 = \alpha_0^{-n_1-(1+\kappa)\epsilon} (1-\alpha_0)^{-n_2+2d_1\epsilon} [\alpha_0 + (1-\alpha_0)x]^{-n_3-(1+\kappa)\epsilon} [2\alpha_0 + (1-\alpha_0)x]^{-n_4}.
 \end{aligned}
 \tag{8.4}$$

Using the Laporta algorithm we find three master integrals for  $A_+$ ,

$$\begin{aligned}
 A_+^{(1)}(x, \epsilon; \alpha_0, d_1; \kappa) &= A_+(x, \epsilon; \alpha_0, d_1; \kappa; 0, 0, 0, 0) = \int_0^{\alpha_0} d\mu_\epsilon(\alpha; x), \\
 A_+^{(2)}(x, \epsilon; \alpha_0, d_1; \kappa) &= A_+(x, \epsilon; \alpha_0, d_1; \kappa; -1, 0, 0, 0) = \int_0^{\alpha_0} d\mu_\epsilon(\alpha; x) \alpha, \\
 A_+^{(3)}(x, \epsilon; \alpha_0, d_1; \kappa) &= A_+(x, \epsilon; \alpha_0, d_1; \kappa; 0, 0, 0, 1) = \int_0^{\alpha_0} \frac{d\mu_\epsilon(\alpha; x)}{2\alpha + (1-\alpha)x}.
 \end{aligned}
 \tag{8.5}$$

where

$$\begin{aligned}
 d\mu_\epsilon(\alpha, x) &= d\alpha \alpha^{-(1+\kappa)\epsilon} (1-\alpha)^{2d_1\epsilon} (\alpha + (1-\alpha)x)^{-(1+\kappa)\epsilon}, \\
 &= d\alpha + \epsilon d\alpha (2d_1 \ln(1-\alpha) - (1+\kappa) \ln \alpha - (1+\kappa) \ln(\alpha + x - \alpha x)) + \mathcal{O}(\epsilon^2), \\
 &= d\alpha + \epsilon d\alpha \left( -(1+\kappa)H(0; \alpha) - (1+\kappa)H(0; x) - 2d_1H(1; \alpha) - (\kappa+1)H(d_1(x); \alpha) \right) \\
 &\quad + \mathcal{O}(\epsilon^2).
 \end{aligned}
 \tag{8.6}$$

where we used the  $d$ -representation of the two-dimensional  $HPL$ 's defined in section 5,

$$\begin{aligned}
 H(d_1(x); \alpha) &= \ln \left( 1 + \frac{1-x}{x} \alpha \right), \\
 H(d_1(x), d_1(x); \alpha) &= \frac{1}{2} \ln^2 \left( 1 + \frac{1-x}{x} \alpha \right),
 \end{aligned}
 \tag{8.7}$$

*etc.*

Notice that all three master integrals are finite for  $\epsilon = 0$ . This allows us to expand the integrand into a power series in  $\epsilon$  and integrate order by order in  $\epsilon$ , using the defining property of the  $HPL$ 's, eq. (4.2). We obtain in this way the series expansion of the master integrals as a power series in  $\epsilon$  whose coefficients are written in terms of the  $d$ -representation

of the two-dimensional *HPL*'s . We can then switch back to the  $c$ -representation using the algorithm described in section 5.

Having a representation of the master integrals, we can immediately write down the solutions for  $\mathcal{A}(x, \epsilon; \alpha_0, d_0; \kappa, k)$  for  $k \geq 0$  and fixed  $D_0$  using eq. (8.2). In appendix D we give as an example the series expansions up to order  $\epsilon^2$  for  $D_0 = 3$ .

### 8.2 The $\mathcal{A}$ -type collinear integrals for $k = -1$

For  $k = -1$ , the integral (8.1) does not decouple, so we have to use the Laporta algorithm to calculate the full two-dimensional integral. However, for  $k = -1$ , we can get rid of the denominator in  $(2\alpha + (1 - \alpha)x)$  in the integrand. So we only have to deal with an integral of the form

$$\begin{aligned} A_-(x, \epsilon; \alpha_0, d_1; \kappa; n_1, n_2, n_3, n_4, n_5, n_6) &= \\ &= \int_0^{\alpha_0} d\alpha \int_0^1 dv \alpha^{-n_1-(1+\kappa)\epsilon} (1-\alpha)^{-n_2+2d_1\epsilon} [\alpha + (1-\alpha)x]^{-n_3-(1+\kappa)\epsilon} \\ &\quad \times v^{-n_4-\epsilon} (1-v)^{-n_5-\epsilon} [\alpha + (1-\alpha)xv]^{-n_6}, \end{aligned} \quad (8.8)$$

$n_i$  being integers.

We write down the integration-by-parts identities for  $A_-$  including a surface term for  $\alpha$ ,

$$\begin{aligned} &\int_0^{\alpha_0} d\alpha \int_0^1 dv \frac{\partial}{\partial v} \left( \alpha^{-n_1-(1+\kappa)\epsilon} (1-\alpha)^{-n_2+2d_1\epsilon} [\alpha + (1-\alpha)x]^{-n_3-(1+\kappa)\epsilon} \right. \\ &\quad \left. \times v^{-n_4-\epsilon} (1-v)^{-n_5-\epsilon} [\alpha + (1-\alpha)xv]^{-n_6} \right) = 0, \\ &\int_0^{\alpha_0} d\alpha \int_0^1 dv \frac{\partial}{\partial \alpha} \left( \alpha^{-n_1-(1+\kappa)\epsilon} (1-\alpha)^{-n_2+2d_1\epsilon} [\alpha + (1-\alpha)x]^{-n_3-(1+\kappa)\epsilon} \right. \\ &\quad \left. \times v^{-n_4-\epsilon} (1-v)^{-n_5-\epsilon} [\alpha + (1-\alpha)xv]^{-n_6} \right) \\ &= \alpha_0^{-n_1-(1+\kappa)\epsilon} (1-\alpha_0)^{-n_2+2d_1\epsilon} [\alpha_0 + (1-\alpha_0)x]^{-n_3-(1+\kappa)\epsilon} A_{-,S}(x, \epsilon; \alpha_0, d_1; n_4, n_5, n_6), \end{aligned} \quad (8.9)$$

with

$$\begin{aligned} A_{-,S}(x, \epsilon; \alpha_0, d_1; n_4, n_5, n_6) &= \int_0^1 dv v^{-n_4-\epsilon} (1-v)^{-n_5-\epsilon} [\alpha_0 + (1-\alpha_0)xv]^{-n_6} \\ &= \alpha_0^{-n_6} B(1-n_4-\epsilon, 1-n_5-\epsilon) {}_2F_1 \left( 1-n_4-\epsilon, n_6, 2-n_4-n_5-2\epsilon; \frac{\alpha_0-1}{\alpha_0} x \right). \end{aligned} \quad (8.10)$$

As in the case of  $\mathcal{K}$  we are going to evaluate this surface term using the Laporta algorithm, especially to get rid of the strange argument the hypergeometric function depends on, and to get an expression for  $A_{-,S}$  in terms of two-dimensional *HPL*'s in  $\alpha_0$  and  $x$ .

**Evaluation of the surface term  $A_{-,S}$ .** Because the  $v$  integration is over the whole range  $[0, 1]$ , we do not have to take into account a surface term in the integration-by-parts identities for  $A_{-,S}$ ,

$$\int_0^1 dv \frac{\partial}{\partial v} \left( v^{-n_4-\epsilon} (1-v)^{-n_5-\epsilon} [\alpha_0 + (1-\alpha_0)xv]^{-n_6} \right) = 0. \quad (8.11)$$

Using the Laporta algorithm we see that  $A_{-,S}$  has two master integrals,

$$\begin{aligned} A_{-,S}^{(1)}(x, \epsilon; \alpha_0, d_1) &= A_{-,S}(x, \epsilon; \alpha_0, d_1; 0, 0, 0), \\ A_{-,S}^{(2)}(x, \epsilon; \alpha_0, d_1) &= A_{-,S}(x, \epsilon; \alpha_0, d_1; 0, 0, 1). \end{aligned} \quad (8.12)$$

$A_{-,S}^{(i)}(x, \epsilon; \alpha_0, d_1)$ ,  $i = 1, 2$ , are functions of the two variables  $x$  and  $\alpha_0$  defined on the square  $[0, 1] \times [0, 1]$ , so in principle we should write down a set of partial differential equations for the evolution of both  $\alpha_0$  and  $x$ . However, it is easy to see that in  $x = 0$  we have

$$\begin{aligned} A_{-,S}^{(1)}(x = 0, \epsilon; \alpha_0, d_1) &= B(1 - \epsilon, 1 - \epsilon), \\ A_{-,S}^{(2)}(x = 0, \epsilon; \alpha_0, d_1) &= \frac{1}{\alpha_0} B(1 - \epsilon, 1 - \epsilon), \end{aligned} \quad (8.13)$$

for arbitrary  $\alpha_0$ . So we are in the special situation where we know the solutions on the line  $\{x = 0\} \times [0, 1]$ , and so we only need to consider the evolution for the  $x$  variable. In other words, we consider  $A_{-,S}^{(i)}$  as a function of  $x$  only, keeping  $\alpha_0$  as a parameter.

The differential equations for the evolution in the  $x$  variable read

$$\begin{aligned} \frac{\partial}{\partial x} A_{-,S}^{(1)} &= 0, \\ \frac{\partial}{\partial x} A_{-,S}^{(2)} &= A_{-,S}^{(1)} \left( \frac{1 - 2\epsilon}{\alpha_0 x} + \frac{(\alpha_0 - 1)(2\epsilon - 1)}{\alpha_0(\alpha_0 x - x - \alpha_0)} \right) + A_{-,S}^{(2)} \left( \frac{2\epsilon - 1}{x} - \frac{(\alpha_0 - 1)\epsilon}{\alpha_0 x - x - \alpha_0} \right), \end{aligned} \quad (8.14)$$

and the initial condition for this system is given by eq. (8.13). As the system is already triangular, we can immediately solve for  $A_{-,S}^{(1)}$  and  $A_{-,S}^{(2)}$ . Notice in particular that the denominator in  $(\alpha_0 + x - x\alpha_0)$  will give rise to two-dimensional *HPL*'s of the form  $H(c_1(\alpha_0); x)$ , etc.

**Evaluation of  $A_-$ .** Having an expression for the  $\epsilon$ -expansion of the surface term, we can solve the integration-by-parts identities for  $A_-$ , eq. (8.9). We find four master integrals,

$$\begin{aligned} A_-^{(1)}(x, \epsilon; \alpha_0, d_1; \kappa) &= A_-(x, \epsilon; \alpha_0, d_1; \kappa; 0, 0, 0, 0, 0, 0), \\ A_-^{(2)}(x, \epsilon; \alpha_0, d_1; \kappa) &= A_-(x, \epsilon; \alpha_0, d_1; \kappa; -1, 0, 0, 0, 0, 0), \\ A_-^{(3)}(x, \epsilon; \alpha_0, d_1; \kappa) &= A_-(x, \epsilon; \alpha_0, d_1; \kappa; -1, 0, 0, 0, 0, 1), \\ A_-^{(4)}(x, \epsilon; \alpha_0, d_1; \kappa) &= A_-(x, \epsilon; \alpha_0, d_1; \kappa; -2, 0, 0, 0, 0, 1). \end{aligned} \quad (8.15)$$

It is easy to see that all of the master integrals are finite for  $\epsilon = 0$ .

As in the case of the surface terms, we are only interested in the  $x$  evolution, because the master integrals are known for  $x = 0$  for any value of  $\alpha_0$ ,

$$\begin{aligned} A_-^{(1)}(x = 0, \epsilon; \alpha_0, d_1; \kappa) &= B_{\alpha_0}(1 - 2(1 + \kappa)\epsilon, 1 + 2d_1\epsilon) B(1 - \epsilon, 1 - \epsilon), \\ A_-^{(2)}(x = 0, \epsilon; \alpha_0, d_1; \kappa) &= B_{\alpha_0}(2 - 2(1 + \kappa)\epsilon, 1 + 2d_1\epsilon) B(1 - \epsilon, 1 - \epsilon), \end{aligned} \quad (8.16)$$

and

$$\begin{aligned} A_-^{(3)}(x = 0, \epsilon; \alpha_0, d_1; \kappa) &= A_-^{(1)}(x = 0, \epsilon; \alpha_0, d_1; \kappa), \\ A_-^{(4)}(x = 0, \epsilon; \alpha_0, d_1; \kappa) &= A_-^{(2)}(x = 0, \epsilon; \alpha_0, d_1; \kappa). \end{aligned} \quad (8.17)$$

The master integrals  $A_-^{(1)}$  and  $A_-^{(2)}$  form a subtopology, i.e. the differential equations for these two master integrals close under themselves:

$$\begin{aligned} \frac{\partial}{\partial x} A_-^{(1)} &= \frac{1 - 2(1 + \kappa)\epsilon}{x} A_-^{(1)} - \frac{2(d_1\epsilon - (1 + \kappa)\epsilon + 1)}{x} A_-^{(2)} \\ &\quad - \frac{(1 - \alpha_0)^{1+2d_1\epsilon} (-x\alpha_0 + \alpha_0 + x)^{-(1+\kappa)\epsilon} \alpha_0^{1-(1+\kappa)\epsilon}}{x} A_{-,S}^{(1)}, \\ \frac{\partial}{\partial x} A_-^{(2)} &= \frac{1 - (1 + \kappa)\epsilon}{x - 1} A_-^{(1)} + \frac{-2d_1\epsilon + (1 + \kappa)\epsilon - 2}{x - 1} A_-^{(2)} \\ &\quad - \frac{(1 - \alpha_0)^{1+2d_1\epsilon} (-x\alpha_0 + \alpha_0 + x)^{-(1+\kappa)\epsilon} \alpha_0^{1-(1+\kappa)\epsilon}}{x - 1} A_{-,S}^{(1)}. \end{aligned} \quad (8.18)$$

The two equations can be triangularized by the change of variable

$$\begin{aligned} \tilde{A}_-^{(1)} &= A_-^{(1)} - 2A_-^{(2)}, \\ \tilde{A}_-^{(2)} &= A_-^{(2)}. \end{aligned} \quad (8.19)$$

The equations for the subtopology now take the triangularized form

$$\begin{aligned} \frac{\partial}{\partial x} \tilde{A}_-^{(1)} &= \left( \frac{2\epsilon - 2}{x - 1} + \frac{1 - 2\epsilon}{x} \right) \tilde{A}_-^{(1)} + \left( \frac{4d_1 + 2}{x - 1} - \frac{2d_1 + 2}{x} \right) \epsilon \tilde{A}_-^{(2)} \\ &\quad + (1 - \alpha_0)^{2d_1\epsilon} (-x\alpha_0 + \alpha_0 + x)^{-\epsilon} \alpha_0^{1-\epsilon} \left( \frac{2 - 2\alpha_0}{x - 1} + \frac{\alpha_0 - 1}{x} \right) A_{-,S}^{(1)}, \\ \frac{\partial}{\partial x} \tilde{A}_-^{(2)} &= \frac{1 - \epsilon}{x - 1} \tilde{A}_-^{(1)} - \frac{2d_1 + 1}{x - 1} \epsilon \tilde{A}_-^{(2)} + (1 - \alpha_0)^{2d_1\epsilon} \alpha_0^{1-\epsilon} (-x\alpha_0 + \alpha_0 + x)^{-\epsilon} \frac{\alpha - 1}{x - 1} A_{-,S}^{(1)}. \end{aligned} \quad (8.20)$$

The initial condition for  $\tilde{A}_-^{(2)}$  can be obtained from eq. (8.16). For  $\tilde{A}_-^{(1)}$  however, eq. (8.16) gives only trivial information. Furthermore, the solution of the differential equation has in general a pole in  $x = 1$ , but it is easy to convince oneself that  $\tilde{A}_-^{(1)}$  is finite in  $x = 1$ , which serves as the initial condition.

We can now solve for the remaining two master integrals. The differential equations for  $A_-^{(3)}$  and  $A_-^{(4)}$  read

$$\begin{aligned} \frac{\partial}{\partial x} A_-^{(3)} &= \frac{1 - 2(1 + \kappa)\epsilon}{x} A_-^{(3)} - \frac{2(d_1\epsilon - (1 + \kappa)\epsilon + 1)}{x} A_-^{(4)} \\ &\quad - \frac{(1 - \alpha_0)^{2d_1\epsilon} (-x\alpha_0 + \alpha_0 + x)^{-(1+\kappa)\epsilon} \alpha_0^{2-(1+\kappa)\epsilon}}{x} A_{-,S}^{(2)}, \\ \frac{\partial}{\partial x} A_-^{(4)} &= \left( \frac{1 - 2\epsilon}{x} + \frac{2\epsilon - 1}{x - 1} \right) A_-^{(2)} - \frac{(2 + \kappa)\epsilon - 2}{x - 1} A_-^{(3)} \\ &\quad + \left( \frac{2\epsilon - 1}{x} - \frac{(2d_1 + \kappa)\epsilon}{x - 1} \right) A_-^{(4)} \\ &\quad - \frac{(1 - \alpha_0)^{1+2d_1\epsilon} (-x\alpha_0 + \alpha_0 + x)^{-(1+\kappa)\epsilon} \alpha_0^{2-(1+\kappa)\epsilon}}{x - 1} A_{-,S}^{(2)}. \end{aligned} \quad (8.21)$$

These equations can be brought into a triangularized form via the change of variable

$$\begin{aligned} \tilde{A}_-^{(3)} &= A_-^{(3)} - A_-^{(4)}, \\ \tilde{A}_-^{(4)} &= A_-^{(4)}, \end{aligned} \quad (8.22)$$



and eq. (8.21) now reads

$$\begin{aligned}
\frac{\partial}{\partial x} \tilde{A}_-^{(3)} &= \left( \frac{1-2\epsilon}{x} + \frac{2(\epsilon-1)}{x-1} \right) \tilde{A}_-^{(3)} + \left( \frac{2(d_1+1)\epsilon}{x-1} - \frac{2(d_1+1)\epsilon}{x} \right) \tilde{A}_-^{(4)} \\
&\quad - (1-\alpha_0)^{1+2d_1\epsilon} \alpha_0^{2-\epsilon} (-x\alpha_0 + \alpha_0 + x)^{-\epsilon} \left( \frac{1}{x} - \frac{1}{x-1} \right) A_{-,S}^{(2)} \\
&\quad + \left( \frac{1-2\epsilon}{x-1} + \frac{2\epsilon-1}{x} \right) A_-^{(2)}, \\
\frac{\partial}{\partial x} \tilde{A}_-^{(4)} &= \frac{2-2\epsilon}{x-1} \tilde{A}_-^{(3)} + \left( \frac{2\epsilon-1}{x} - \frac{2(d_1\epsilon+\epsilon)}{x-1} \right) \tilde{A}_-^{(4)} \\
&\quad + \left( \frac{1-2\epsilon}{x} + \frac{2\epsilon-1}{x-1} \right) A_-^{(2)} - \frac{(1-\alpha_0)^{1+2d_1\epsilon} (-x\alpha_0 + \alpha_0 + x)^{-\epsilon} \alpha_0^{2-\epsilon}}{x-1} A_{-,S}^{(2)}.
\end{aligned} \tag{8.23}$$

The initial condition for  $A_-^{(3)}$  and  $A_-^{(4)}$  can again be obtained from eq. (8.16) and requiring  $A_-^{(3)}$  to be finite in  $x = 1$ .

Having the analytic expressions for the master integrals, we can now easily obtain the solutions for  $\mathcal{A}$  for  $k = -1$  for a fixed value of  $D_0$ . The results for  $D_0 = 3$  can be found in appendix D.

In figure 4 we compare the analytic and numeric results for the  $\epsilon^2$  coefficient in the expansion of  $\mathcal{I}(x, \epsilon; \alpha_0, 3 - 3\epsilon; 1, k, 0, g_A)$  for  $k = -1, 2$  and  $\alpha_0 = 0.1, 1$  as representative examples. The dependence on  $\alpha_0$  is not visible on the plots. The agreement between the two computations is excellent for the whole  $x$ -range. We find a similar agreement for other (lower-order, thus simpler) expansion coefficients and/or other values of the parameters.

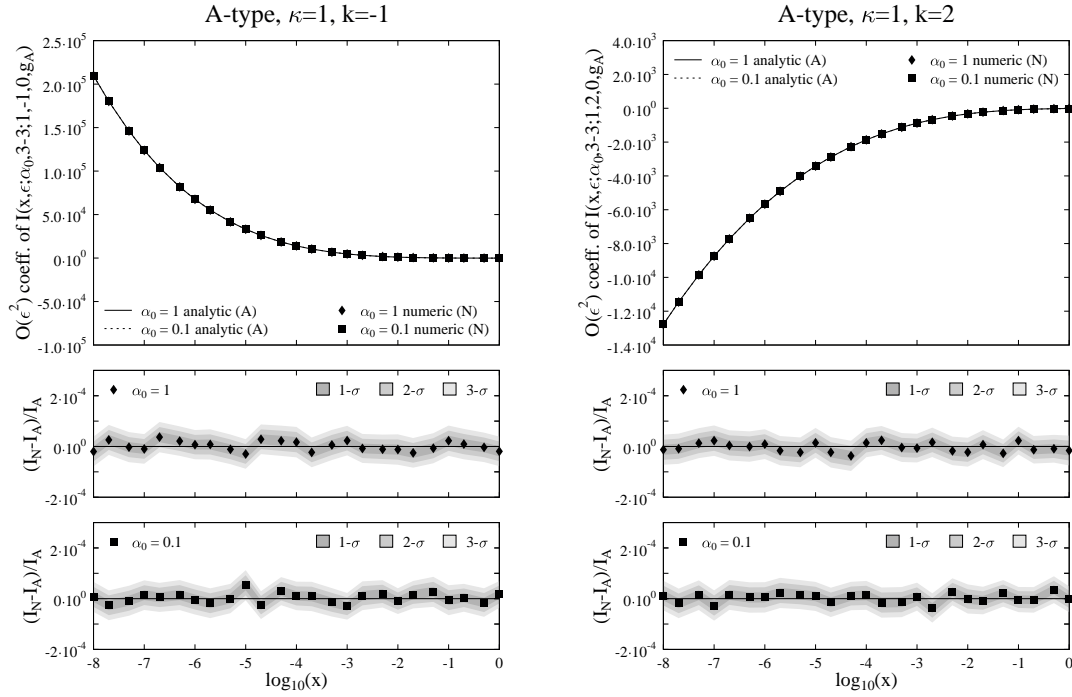
### 8.3 The $\mathcal{B}$ -type collinear integrals

The  $\mathcal{B}$ -type collinear integrals require the evaluation of an integral of the form

$$\begin{aligned}
\mathcal{B}(x, \epsilon; \alpha_0, d_0; \delta, k) &= \frac{1}{x} \mathcal{I}(x, \epsilon; \alpha_0, d_0; 1, k, \delta, g_B) \\
&= \int_0^{\alpha_0} d\alpha \int_0^1 dv \alpha^{-1-2\epsilon} (1-\alpha)^{2d_0-1} [\alpha + (1-\alpha)x]^{-1-2\epsilon} [2\alpha + (1-\alpha)x]^{-k} \\
&\quad \times v^{-\epsilon} (1-v)^{-\epsilon} [\alpha + (1-\alpha)xv]^{k+\delta\epsilon} [\alpha + (1-\alpha)(1-v)x]^{-\delta\epsilon},
\end{aligned} \tag{8.24}$$

where  $k = -1, 0, 1, 2$ ,  $\delta = \pm 1$  and  $d_0 = D_0 + d_1\epsilon$  (as before  $D_0$  is an integer). Unlike the  $\mathcal{A}$ -type integrals, the  $\mathcal{B}$ -type integrals do not decouple for  $k \geq 0$ , due to the appearance of the  $\epsilon$  pieces in the exponents, so we have to consider the denominators altogether, and have to deal with an integral of the form

$$\begin{aligned}
B(x, \epsilon; \alpha_0, d_1; \delta; n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8) &= \\
&= \int_0^{\alpha_0} d\alpha \int_0^1 dv \alpha^{-n_1-2\epsilon} (1-\alpha)^{-n_2+2d_1} [\alpha + (1-\alpha)x]^{-n_3-2\epsilon} [2\alpha + (1-\alpha)x]^{-n_4} \\
&\quad \times v^{-n_5-\epsilon} (1-v)^{-n_6-\epsilon} [\alpha + (1-\alpha)xv]^{-n_7+\delta\epsilon} [\alpha + (1-\alpha)(1-v)x]^{-n_8-\delta\epsilon}.
\end{aligned} \tag{8.25}$$



**Figure 4:** Representative results for the  $\mathcal{A}$ -type integrals. The plots show the coefficient of the  $O(\epsilon^2)$  term in  $\mathcal{I}(x, \epsilon; \alpha_0, 3 - 3\epsilon; 1, k, 0, g_A)$  for  $k = -1$  (left figure) and  $k = 2$  (right figure) with  $\alpha_0 = 0.1, 1$ .

We use again the Laporta algorithm, and write down the integration-by-parts identities for  $B$ ,

$$\begin{aligned}
 & \int_0^{\alpha_0} d\alpha \int_0^1 dv \frac{\partial}{\partial v} \left( \alpha^{-n_1-2\epsilon} (1-\alpha)^{-n_2+2d_1} [\alpha + (1-\alpha)x]^{-n_3-2\epsilon} [2\alpha + (1-\alpha)x]^{-n_4} \right. \\
 & \quad \left. \times v^{-n_5-\epsilon} (1-v)^{-n_6-\epsilon} [\alpha + (1-\alpha)xv]^{-n_7+\delta\epsilon} [\alpha + (1-\alpha)(1-v)x]^{-n_8-\delta\epsilon} \right) \\
 & = 0, \\
 & \int_0^{\alpha_0} d\alpha \int_0^1 dv \frac{\partial}{\partial \alpha} \left( \alpha^{-n_1-2\epsilon} (1-\alpha)^{-n_2+2d_1} [\alpha + (1-\alpha)x]^{-n_3-2\epsilon} [2\alpha + (1-\alpha)x]^{-n_4} \right. \\
 & \quad \left. \times v^{-n_5-\epsilon} (1-v)^{-n_6-\epsilon} [\alpha + (1-\alpha)xv]^{-n_7+\delta\epsilon} [\alpha + (1-\alpha)(1-v)x]^{-n_8-\delta\epsilon} \right) \\
 & = \alpha_0^{-n_1-2\epsilon} (1-\alpha_0)^{-n_2+2d_1} [\alpha_0 + (1-\alpha_0)x]^{-n_3-2\epsilon} [2\alpha_0 + (1-\alpha_0)x]^{-n_4} \\
 & \quad \times B_S(x, \epsilon; \alpha_0, d_1; \delta; n_5, n_6, n_7, n_8),
 \end{aligned}$$

where the surface term is given by

$$\begin{aligned}
 B_S(x, \epsilon; \alpha_0, d_1; \delta; n_5, n_6, n_7, n_8) & = \\
 & = \int_0^1 dv v^{-n_5-\epsilon} (1-v)^{-n_6-\epsilon} [\alpha_0 + (1-\alpha_0)xv]^{-n_7+\delta\epsilon} [\alpha_0 + (1-\alpha_0)(1-v)x]^{-n_8-\delta\epsilon}.
 \end{aligned} \tag{8.26}$$

**Evaluation of the surface term  $B_S$ .** The surface term  $B_S$  is no longer a hypergeometric function as it was the case for the  $\mathcal{K}$  and  $\mathcal{A}$ -type integrals. It can nevertheless be easily calculated using the Laporta algorithm. The integration-by-parts identities for  $B_S$  read

$$\int_0^1 dv \frac{\partial}{\partial v} \left( v^{-n_5-\epsilon} (1-v)^{-n_6-\epsilon} [\alpha_0 + (1-\alpha_0)xv]^{-n_7+\delta\epsilon} [\alpha_0 + (1-\alpha_0)(1-v)x]^{-n_8-\delta\epsilon} \right) = 0. \quad (8.27)$$

We find three master integrals for  $B_S$ ,

$$\begin{aligned} B_S^{(1)}(x, \epsilon; \alpha_0, d_1; \delta) &= B_S(x, \epsilon; \alpha_0, d_1; \delta; 0, 0, 0, 0), \\ B_S^{(2)}(x, \epsilon; \alpha_0, d_1; \delta) &= B_S(x, \epsilon; \alpha_0, d_1; \delta; -1, 0, 0, 0), \\ B_S^{(3)}(x, \epsilon; \alpha_0, d_1; \delta) &= B_S(x, \epsilon; \alpha_0, d_1; \delta; 0, 0, 1, 0), \end{aligned} \quad (8.28)$$

fulfilling the differential equations

$$\begin{aligned} \frac{\partial}{\partial x} B_S^{(1)} &= B_S^{(1)} \left( \frac{2(\epsilon\alpha_0 - \alpha_0 - \epsilon + 1)}{\alpha_0((\alpha_0 - 1)x - 2\alpha_0)} + \frac{-2\alpha_0\epsilon^2 + 2\epsilon^2 + 3\alpha_0\delta\epsilon - 3\delta\epsilon - \alpha_0 + 1}{\alpha_0((\alpha_0 - 1)x - \alpha_0)(\epsilon\delta - 1)} \right. \\ &\quad \left. + \frac{-2\delta\epsilon^2 + 2\epsilon^2 - \delta\epsilon + 2\epsilon - 1}{\alpha_0 x(\epsilon\delta - 1)} \right) \\ &+ B_S^{(2)} \left( \frac{4(\alpha_0 - 1)(\epsilon - 1)}{\alpha_0((\alpha_0 - 1)x - \alpha_0)} - \frac{4(\alpha_0 - 1)(\epsilon - 1)}{\alpha_0((\alpha_0 - 1)x - 2\alpha_0)} \right) \\ &+ B_S^{(3)} \left( \frac{(\alpha_0 - 1)(\delta\epsilon^2 - \epsilon^2 - \epsilon + 1)}{(x\alpha_0 - \alpha_0 - x)(\epsilon\delta - 1)} + \frac{2\delta\epsilon^2 - 2\epsilon^2 + \delta\epsilon - 2\epsilon + 1}{x(\epsilon\delta - 1)} \right. \\ &\quad \left. - \frac{(\alpha_0 - 1)(2\delta\epsilon^2 - 2\epsilon^2 + \delta\epsilon - 2\epsilon + 1)}{((\alpha_0 - 1)x - \alpha_0)(\epsilon\delta - 1)} \right), \\ \frac{\partial}{\partial x} B_S^{(2)} &= -B_S^{(1)} \frac{(-\delta\epsilon + \epsilon - 1)}{x} + B_S^{(2)} \frac{2(\epsilon - 1)}{x} - B_S^{(3)} \frac{\alpha_0\epsilon\delta}{x}, \\ \frac{\partial}{\partial x} B_S^{(3)} &= B_S^{(1)} \left( -\frac{\delta(2\alpha_0\epsilon^2 - 2\epsilon^2 - 2\alpha_0\epsilon - 2\alpha_0\delta\epsilon + 2\delta\epsilon + 2\epsilon + 2\alpha_0\delta - 2\delta)}{\alpha_0((\alpha_0 - 1)x - 2\alpha_0)(1 - \epsilon\delta)} \right. \\ &\quad \left. + \frac{2\delta\epsilon^2 - 2\epsilon^2 + \delta\epsilon - 2\epsilon + 1}{\alpha_0 x(1 - \epsilon\delta)} - \frac{\delta(-2\alpha_0\delta\epsilon^2 + 2\delta\epsilon^2 + 3\alpha_0\epsilon - 3\epsilon - \alpha_0\delta + \delta)}{\alpha_0((\alpha_0 - 1)x - \alpha_0)(1 - \epsilon\delta)} \right) \\ &+ B_S^{(2)} \left( \frac{2(\alpha_0 - 1)(\epsilon - 1)(2\epsilon - 2\delta)\delta}{\alpha_0((\alpha_0 - 1)x - 2\alpha_0)(1 - \epsilon\delta)} - \frac{2(\alpha_0 - 1)(\epsilon - 1)(2\epsilon - 2\delta)\delta}{\alpha_0((\alpha_0 - 1)x - \alpha_0)(1 - \epsilon\delta)} \right) \\ &+ B_S^{(3)} \left( \frac{-2\delta\epsilon^2 + 2\epsilon^2 - \delta\epsilon + 2\epsilon - 1}{x(1 - \epsilon\delta)} - \frac{(\alpha_0 - 1)(-2\delta\epsilon^2 + 2\epsilon^2 - \delta\epsilon + 2\epsilon - 1)}{((\alpha_0 - 1)x - \alpha_0)(1 - \epsilon\delta)} \right. \\ &\quad \left. + \frac{(\alpha_0 - 1)(-\delta\epsilon^2 + \epsilon^2 + \epsilon - 1)}{(x\alpha_0 - \alpha_0 - x)(1 - \epsilon\delta)} \right). \end{aligned} \quad (8.29)$$

The initial conditions for the differential equations are

$$\begin{aligned}
 B_S^{(1)}(x=0, \epsilon; \alpha_0, d_1; \delta) &= B(1-\epsilon, 1-\epsilon), \\
 B_S^{(2)}(x=0, \epsilon; \alpha_0, d_1; \delta) &= B(2-\epsilon, 1-\epsilon), \\
 B_S^{(3)}(x=0, \epsilon; \alpha_0, d_1; \delta) &= \frac{1}{\alpha_0} B(1-\epsilon, 1-\epsilon),
 \end{aligned}
 \tag{8.30}$$

The system can be triangularized by the change of variable

$$\tilde{B}_S^{(1)} = B_S^{(1)} - 2B_S^{(2)}, \quad \tilde{B}_S^{(2)} = B_S^{(2)}, \quad \tilde{B}_S^{(3)} = B_S^{(3)},
 \tag{8.31}$$

and then solved in the usual way.

**Evaluation of the  $B$  integral.** Solving the integration-by-parts identities for the  $B$  integrals, we find nine master integrals

$$\begin{aligned}
 B^{(1)}(x, \epsilon; \alpha_0, d_1; \delta) &= B(x, \epsilon; \alpha_0, d_1; \delta; 0, 0, 0, 0, 0, 0, 0, 0, 0), \\
 B^{(2)}(x, \epsilon; \alpha_0, d_1; \delta) &= B(x, \epsilon; \alpha_0, d_1; \delta; 0, 0, 0, 0, 0, 0, 0, 0, 1), \\
 B^{(3)}(x, \epsilon; \alpha_0, d_1; \delta) &= B(x, \epsilon; \alpha_0, d_1; \delta; 0, 0, 0, 0, 0, 0, 1, 0, 0), \\
 B^{(4)}(x, \epsilon; \alpha_0, d_1; \delta) &= B(x, \epsilon; \alpha_0, d_1; \delta; 0, 0, 0, 1, 0, 0, 0, 0, 0), \\
 B^{(5)}(x, \epsilon; \alpha_0, d_1; \delta) &= B(x, \epsilon; \alpha_0, d_1; \delta; 0, 0, 1, 0, 0, 0, 0, 0, 0), \\
 B^{(6)}(x, \epsilon; \alpha_0, d_1; \delta) &= B(x, \epsilon; \alpha_0, d_1; \delta; -1, 0, 0, 0, 0, 0, 0, 1, 0), \\
 B^{(7)}(x, \epsilon; \alpha_0, d_1; \delta) &= B(x, \epsilon; \alpha_0, d_1; \delta; 0, 0, 0, 1, 0, 0, 1, 0, 0), \\
 B^{(8)}(x, \epsilon; \alpha_0, d_1; \delta) &= B(x, \epsilon; \alpha_0, d_1; \delta; 0, 0, 1, 0, 0, 0, 1, 0, 0), \\
 B^{(9)}(x, \epsilon; \alpha_0, d_1; \delta) &= B(x, \epsilon; \alpha_0, d_1; \delta; 0, 0, 1, 0, 0, 0, 0, 0, 1),
 \end{aligned}
 \tag{8.32}$$

The master integrals  $B^{(i)}$ ,  $i \neq 4, 7$ , form a subtopology, i.e. the differential equations for these master integrals close under themselves. Furthermore the differential equations for  $B^{(1)}, B^{(3)}, B^{(5)}$  and  $B^{(6)}$  have a triangular structure in  $\epsilon$ , i.e. all other master integrals are suppressed by a power of  $\epsilon$ . For  $\delta = +1$ , the corresponding differential equations are

given by

$$\begin{aligned}
 \frac{\partial}{\partial x} B^{(1)} &= \frac{2(\varepsilon - 1)B_S^{(2)}(\alpha_0 - 1)^2}{(2d_1\varepsilon - 4\varepsilon + 1)(x\alpha_0 - \alpha_0 - x)} - \frac{2\varepsilon B^{(1)}}{x - 1} + \left( \frac{4\varepsilon^2}{(2d_1\varepsilon - 4\varepsilon + 1)(x - 1)} - \right. \\
 &\quad \left. \frac{\varepsilon(8\varepsilon - 1)}{(2d_1\varepsilon - 4\varepsilon + 1)x} \right) B^{(2)} + \left( \frac{2\varepsilon^2}{(2d_1\varepsilon - 4\varepsilon + 1)(x - 1)} - \frac{\varepsilon(4\varepsilon - 1)}{(2d_1\varepsilon - 4\varepsilon + 1)x} \right) B^{(3)} + \\
 &\quad \left( \frac{2\varepsilon}{x - 1} + \frac{2(2\varepsilon - 1)\varepsilon}{(2d_1\varepsilon - 4\varepsilon + 1)x} \right) B^{(5)} - \frac{2\varepsilon B^{(6)}}{x} - \frac{2\varepsilon^2 B^{(8)}}{(2d_1\varepsilon - 4\varepsilon + 1)(x - 1)} - \\
 &\quad \frac{4\varepsilon^2 B^{(9)}}{(2d_1\varepsilon - 4\varepsilon + 1)(x - 1)} + \left( - \frac{2\varepsilon(\alpha_0 - 1)^2}{(2d_1\varepsilon - 4\varepsilon + 1)((\alpha_0 - 1)x - \alpha_0)} + \right. \\
 &\quad \left. \frac{(\alpha_0 - 1)^2}{(2d_1\varepsilon - 4\varepsilon + 1)(x\alpha_0 - \alpha_0 - x)} + \frac{2\varepsilon(\alpha_0 - 1)}{(2d_1\varepsilon - 4\varepsilon + 1)x} \right) B_S^{(1)}, \\
 \frac{\partial}{\partial x} B^{(3)} &= -\frac{4\varepsilon B^{(3)}}{x} + \frac{(-2d_1\varepsilon + 4\varepsilon - 1) B^{(6)}}{x} + \frac{(\alpha_0 - 1)\alpha_0 B_S^{(3)}}{x}, \\
 \frac{\partial}{\partial x} B^{(5)} &= \left( \frac{-2d_1\varepsilon + 4\varepsilon - 1}{x} + \frac{2d_1\varepsilon - 4\varepsilon + 1}{x - 1} \right) B^{(1)} + \left( \frac{-2d_1\varepsilon + 4\varepsilon - 1}{x - 1} - \frac{4\varepsilon}{x} \right) B^{(5)} + \\
 &\quad \left( \frac{\alpha_0 - 1}{x} - \frac{(\alpha_0 - 1)^2}{(\alpha_0 - 1)x - \alpha_0} \right) B_S^{(1)}, \\
 \frac{\partial}{\partial x} B^{(6)} &= \left( \frac{1 - 2\varepsilon}{x} + \frac{2\varepsilon - 1}{x - 1} \right) B^{(1)} + \left( \frac{8\varepsilon^2}{(2d_1\varepsilon - 4\varepsilon + 1)(x - 1)} - \frac{\varepsilon}{(2d_1\varepsilon - 4\varepsilon + 1)(x - 2)} - \right. \\
 &\quad \left. \frac{(8\varepsilon - 1)\varepsilon}{(2d_1\varepsilon - 4\varepsilon + 1)x} \right) B^{(2)} + \left( - \frac{2\varepsilon}{(x - 1)^2} + \frac{2d_1\varepsilon - 5\varepsilon + 1}{(2d_1\varepsilon - 4\varepsilon + 1)(x - 2)} - \right. \\
 &\quad \left. \frac{2(2d_1\varepsilon^2 - 6\varepsilon^2 + \varepsilon)}{(2d_1\varepsilon - 4\varepsilon + 1)(x - 1)} + \frac{\varepsilon - 4\varepsilon^2}{(2d_1\varepsilon - 4\varepsilon + 1)x} \right) B^{(3)} + \left( \frac{1 - 2\varepsilon}{x - 1} + \right. \\
 &\quad \left. \frac{4d_1\varepsilon^2 - 12\varepsilon^2 - 2d_1\varepsilon + 8\varepsilon - 1}{(2d_1\varepsilon - 4\varepsilon + 1)(x - 2)} + \frac{2\varepsilon(2\varepsilon - 1)}{(2d_1\varepsilon - 4\varepsilon + 1)x} \right) B^{(5)} + \left( - \frac{2\varepsilon}{x - 1} + \right. \\
 &\quad \left. \frac{-2d_1\varepsilon + 4\varepsilon - 1}{x - 2} - \frac{1}{x} \right) B^{(6)} + \left( \frac{2\varepsilon}{(x - 1)^2} - \frac{4(2d_1\varepsilon^2 - 5\varepsilon^2 + \varepsilon)}{(2d_1\varepsilon - 4\varepsilon + 1)(x - 2)} + \right. \\
 &\quad \left. \frac{4(2d_1\varepsilon^2 - 5\varepsilon^2 + \varepsilon)}{(2d_1\varepsilon - 4\varepsilon + 1)(x - 1)} \right) B^{(8)} + \\
 &\quad \left( \frac{8\varepsilon^2}{(2d_1\varepsilon - 4\varepsilon + 1)(x - 2)} - \frac{8\varepsilon^2}{(2d_1\varepsilon - 4\varepsilon + 1)(x - 1)} \right) B^{(9)} + \\
 &\quad \left( \frac{4(\alpha_0 - 1)^3(\varepsilon - 1)}{(\alpha_0 - 2)(2d_1\varepsilon - 4\varepsilon + 1)((\alpha_0 - 1)x - \alpha_0)} - \right. \\
 &\quad \left. \frac{4(\alpha_0 - 1)^2(\varepsilon - 1)}{(\alpha_0 - 2)(2d_1\varepsilon - 4\varepsilon + 1)(x - 2)} \right) B_S^{(2)} + \\
 &\quad \left( - \frac{2(2\varepsilon - 1)(\alpha_0 - 1)^3}{(\alpha_0 - 2)(2d_1\varepsilon - 4\varepsilon + 1)((\alpha_0 - 1)x - \alpha_0)} + \frac{2\varepsilon(\alpha_0 - 1)}{(2d_1\varepsilon - 4\varepsilon + 1)x} + \right. \\
 &\quad \left. \frac{2(\varepsilon\alpha_0^2 - \alpha_0^2 - \varepsilon\alpha_0 + 2\alpha_0 - 1)}{(\alpha_0 - 2)(2d_1\varepsilon - 4\varepsilon + 1)(x - 2)} \right) B_S^{(1)} + \frac{(\alpha_0 - 1)\alpha_0 B_S^{(3)}}{x - 2},
 \end{aligned}$$

whereas for  $\delta = -1$ , the differential equations are

$$\begin{aligned}
 \frac{\partial}{\partial x} B^{(1)} &= \frac{2(\epsilon - 1)B_S^{(2)}(\alpha_0 - 1)^2}{(2d_1\epsilon - 4\epsilon + 1)(x\alpha_0 - \alpha_0 - x)} - \frac{2\epsilon B^{(1)}}{x-1} + \left( \frac{\epsilon(4\epsilon - 1)}{(2d_1\epsilon - 4\epsilon + 1)x} - \right. \\
 &\quad \left. \frac{2\epsilon^2}{(2d_1\epsilon - 4\epsilon + 1)(x-1)} \right) B^{(2)} + \left( \frac{\epsilon(8\epsilon - 1)}{(2d_1\epsilon - 4\epsilon + 1)x} - \frac{4\epsilon^2}{(2d_1\epsilon - 4\epsilon + 1)(x-1)} \right) \\
 &\quad B^{(3)} + \left( \frac{2\epsilon}{x-1} - \frac{2\epsilon(2\epsilon - 1)}{(2d_1\epsilon - 4\epsilon + 1)x} \right) B^{(5)} + \frac{2\epsilon B^{(6)}}{x} + \frac{4\epsilon^2 B^{(8)}}{(2d_1\epsilon - 4\epsilon + 1)(x-1)} + \\
 &\quad \frac{2\epsilon^2 B^{(9)}}{(2d_1\epsilon - 4\epsilon + 1)(x-1)} + \left( \frac{2\epsilon(\alpha_0 - 1)^2}{(2d_1\epsilon - 4\epsilon + 1)((\alpha_0 - 1)x - \alpha_0)} - \right. \\
 &\quad \left. \frac{(2\epsilon - 1)(\alpha_0 - 1)^2}{(2d_1\epsilon - 4\epsilon + 1)(x\alpha_0 - \alpha_0 - x)} - \frac{2\epsilon(\alpha_0 - 1)}{(2d_1\epsilon - 4\epsilon + 1)x} \right) B_S^{(1)}, \\
 \frac{\partial}{\partial x} B^{(3)} &= -\frac{4\epsilon B^{(3)}}{x} + \frac{(-2d_1\epsilon + 4\epsilon - 1) B^{(6)}}{x} + \frac{(\alpha_0 - 1)\alpha_0 B_S^{(3)}}{x}, \\
 \frac{\partial}{\partial x} B^{(5)} &= \left( \frac{-2d_1\epsilon + 4\epsilon - 1}{x} + \frac{2d_1\epsilon - 4\epsilon + 1}{x-1} \right) B^{(1)} + \left( \frac{-2d_1\epsilon + 4\epsilon - 1}{x-1} - \frac{4\epsilon}{x} \right) B^{(5)} + \\
 &\quad \left( \frac{\alpha_0 - 1}{x} - \frac{(\alpha_0 - 1)^2}{(\alpha_0 - 1)x - \alpha_0} \right) B_S^{(1)}, \\
 \frac{\partial}{\partial x} B^{(6)} &= \left( \frac{1 - 2\epsilon}{x} + \frac{2\epsilon - 1}{x-1} \right) B^{(1)} + \left( -\frac{4\epsilon^2}{(2d_1\epsilon - 4\epsilon + 1)(x-1)} + \right. \\
 &\quad \left. \frac{\epsilon}{(2d_1\epsilon - 4\epsilon + 1)(x-2)} + \frac{(4\epsilon - 1)\epsilon}{(2d_1\epsilon - 4\epsilon + 1)x} \right) B^{(2)} + \left( -\frac{4\epsilon}{(x-1)^2} + \right. \\
 &\quad \left. \frac{2d_1\epsilon - 3\epsilon + 1}{(2d_1\epsilon - 4\epsilon + 1)(x-2)} - \frac{4(2d_1\epsilon^2 - 2\epsilon^2 + \epsilon)}{(2d_1\epsilon - 4\epsilon + 1)(x-1)} + \frac{8\epsilon^2 - \epsilon}{(2d_1\epsilon - 4\epsilon + 1)x} \right) B^{(3)} + \\
 &\quad \left( \frac{1 - 2\epsilon}{x-1} + \frac{4d_1\epsilon^2 - 4\epsilon^2 - 2d_1\epsilon + 4\epsilon - 1}{(2d_1\epsilon - 4\epsilon + 1)(x-2)} - \frac{2\epsilon(2\epsilon - 1)}{(2d_1\epsilon - 4\epsilon + 1)x} \right) B^{(5)} + \left( -\frac{4\epsilon}{x-1} + \right. \\
 &\quad \left. \frac{-2d_1\epsilon + 4\epsilon - 1}{x-2} + \frac{4\epsilon - 1}{x} \right) B^{(6)} + \left( \frac{4\epsilon}{(x-1)^2} - \frac{8(2d_1\epsilon^2 - 3\epsilon^2 + \epsilon)}{(2d_1\epsilon - 4\epsilon + 1)(x-2)} + \right. \\
 &\quad \left. \frac{8(2d_1\epsilon^2 - 3\epsilon^2 + \epsilon)}{(2d_1\epsilon - 4\epsilon + 1)(x-1)} \right) B^{(8)} + \left( \frac{4\epsilon^2}{(2d_1\epsilon - 4\epsilon + 1)(x-1)} - \right. \\
 &\quad \left. \frac{4\epsilon^2}{(2d_1\epsilon - 4\epsilon + 1)(x-2)} \right) B^{(9)} + \left( \frac{4(\alpha_0 - 1)^3(\epsilon - 1)}{(\alpha_0 - 2)(2d_1\epsilon - 4\epsilon + 1)((\alpha_0 - 1)x - \alpha_0)} - \right. \\
 &\quad \left. \frac{4(\alpha_0 - 1)^2(\epsilon - 1)}{(\alpha_0 - 2)(2d_1\epsilon - 4\epsilon + 1)(x-2)} \right) B_S^{(2)} + \\
 &\quad \left( \frac{2(\alpha_0 - 1)^3}{(\alpha_0 - 2)(2d_1\epsilon - 4\epsilon + 1)((\alpha_0 - 1)x - \alpha_0)} - \frac{2\epsilon(\alpha_0 - 1)}{(2d_1\epsilon - 4\epsilon + 1)x} + \right. \\
 &\quad \left. \frac{2(\epsilon\alpha_0^2 - \alpha_0^2 - 3\epsilon\alpha_0 + 2\alpha_0 + 2\epsilon - 1)}{(\alpha_0 - 2)(2d_1\epsilon - 4\epsilon + 1)(x-2)} \right) B_S^{(1)} + \frac{(\alpha_0 - 1)\alpha_0 B_S^{(3)}}{x-2}.
 \end{aligned}$$

The differential equations for  $B^{(2)}$ ,  $B^{(8)}$  and  $B^{(9)}$  read, for  $\delta = +1$ ,

$$\begin{aligned}
 \frac{\partial}{\partial x} B^{(2)} &= \left( \frac{4\epsilon - 1}{x} - \frac{4\epsilon}{x-1} \right) B^{(2)} + \left( \frac{4\epsilon - 1}{x} - \frac{2\epsilon}{x-1} \right) B^{(3)} - \frac{2(2\epsilon - 1)B^{(5)}}{x} + \\
 &\quad \frac{(2d_1\epsilon - 4\epsilon + 1) B^{(6)}}{x} + \frac{2\epsilon B^{(8)}}{x-1} + \frac{4\epsilon B^{(9)}}{x-1} - \frac{(\alpha_0 - 1)\alpha_0 B_S^{(3)}}{x}, \\
 \frac{\partial}{\partial x} B^{(8)} &= \left( \frac{2(d_1 - 2)\epsilon}{x-1} - \frac{2(d_1 - 2)\epsilon}{x} \right) B^{(3)} + \left( \frac{-4\epsilon - 1}{x} - \frac{2(d_1\epsilon - 2\epsilon)}{x-1} \right) B^{(8)} + \\
 &\quad \left( \frac{\alpha_0 - 1}{x} - \frac{(\alpha_0 - 1)^2}{(\alpha_0 - 1)x - \alpha_0} \right) B_S^{(3)}, \\
 \frac{\partial}{\partial x} B^{(9)} &= - \frac{2(\epsilon - 1)B_S^{(2)}(\alpha_0 - 1)^2}{\epsilon(x\alpha_0 - \alpha_0 - x)^2} + \left( \frac{2(d_1 - 2)\epsilon}{x-1} - \frac{2(d_1 - 2)\epsilon}{x} \right) B^{(2)} + \\
 &\quad \left( \frac{-4\epsilon - 1}{x} - \frac{2(d_1\epsilon - 2\epsilon)}{x-1} \right) B^{(9)} + \left( - \frac{2(\alpha_0 - 1)^2}{\alpha_0(x\alpha_0 - \alpha_0 - x)} + \frac{2(\alpha_0 - 1)}{\alpha_0 x} + \right. \\
 &\quad \left. \frac{2\epsilon\alpha_0^2 - \alpha_0^2 - 4\epsilon\alpha_0 + 2\alpha_0 + 2\epsilon - 1}{\epsilon(x\alpha_0 - \alpha_0 - x)^2} \right) B_S^{(1)} + \left( \frac{(\alpha_0 - 1)^2}{(\alpha_0 - 1)x - \alpha_0} + \frac{1 - \alpha_0}{x} \right) B_S^{(3)},
 \end{aligned} \tag{8.33}$$

and for  $\delta = -1$

$$\begin{aligned}
 \frac{\partial}{\partial x} B^{(2)} &= \left( - \frac{2\epsilon}{x-1} - \frac{1}{x} \right) B^{(2)} + \left( \frac{8\epsilon - 1}{x} - \frac{4\epsilon}{x-1} \right) B^{(3)} - \frac{2(2\epsilon - 1) B^{(5)}}{x} + \\
 &\quad \frac{(2d_1\epsilon - 4\epsilon + 1)B^{(6)}}{x} + \frac{4\epsilon B^{(8)}}{x-1} + \frac{2\epsilon B^{(9)}}{x-1} - \frac{(\alpha_0 - 1)\alpha_0 B_S^{(3)}}{x}, \\
 \frac{\partial}{\partial x} B^{(8)} &= \left( \frac{2(d_1 - 2)\epsilon}{x-1} - \frac{2(d_1 - 2)\epsilon}{x} \right) B^{(3)} + \left( \frac{-4\epsilon - 1}{x} - \frac{2(d_1\epsilon - 2\epsilon)}{x-1} \right) B^{(8)} + \\
 &\quad \left( \frac{\alpha_0 - 1}{x} - \frac{(\alpha_0 - 1)^2}{(\alpha_0 - 1)x - \alpha_0} \right) B_S^{(3)}, \\
 \frac{\partial}{\partial x} B^{(9)} &= \frac{2(\epsilon - 1)B_S^{(2)}(\alpha_0 - 1)^2}{\epsilon(x\alpha_0 - \alpha_0 - x)^2} + \left( \frac{2(d_1 - 2)\epsilon}{x-1} - \frac{2(d_1 - 2)\epsilon}{x} \right) B^{(2)} + \\
 &\quad \left( \frac{-4\epsilon - 1}{x} - \frac{2(d_1\epsilon - 2\epsilon)}{x-1} \right) B^{(9)} + \left( - \frac{2(\alpha_0 - 1)^2}{\alpha_0(x\alpha_0 - \alpha_0 - x)} + \frac{(\alpha_0 - 1)^2}{\epsilon(x\alpha_0 - \alpha_0 - x)^2} + \right. \\
 &\quad \left. \frac{2(\alpha_0 - 1)}{\alpha_0 x} \right) B_S^{(1)} + \left( \frac{(\alpha_0 - 1)^2}{(\alpha_0 - 1)x - \alpha_0} + \frac{1 - \alpha_0}{x} \right) B_S^{(3)}.
 \end{aligned} \tag{8.34}$$

Knowing the solutions for the subtopology, we can solve for the remaining two master

integrals  $B^{(4)}$  and  $B^{(7)}$ . They fulfill the following differential equations, for  $\delta = +1$ ,

$$\begin{aligned} \frac{\partial}{\partial x} B^{(4)} &= \left( \frac{-2d_1\epsilon + 4\epsilon - 1}{2x} + \frac{2d_1\epsilon - 4\epsilon + 1}{2(x-2)} \right) B^{(1)} + \left( \frac{-2d_1\epsilon + 4\epsilon - 1}{x-2} - \frac{4\epsilon}{x} \right) B^{(4)} + \\ &\quad \left( \frac{\alpha_0 - 1}{2x} - \frac{(\alpha_0 - 1)^2}{2((\alpha_0 - 1)x - 2\alpha_0)} \right) B_S^{(1)}, \\ \frac{\partial}{\partial x} B^{(7)} &= \left( \frac{(d_1 - 2)\epsilon}{x-2} - \frac{(d_1 - 2)\epsilon}{x} \right) B^{(3)} + \left( \frac{-4\epsilon - 1}{x} - \frac{2(d_1\epsilon - 2\epsilon)}{x-2} \right) B^{(7)} + \\ &\quad \left( \frac{\alpha_0 - 1}{2x} - \frac{(\alpha_0 - 1)^2}{2((\alpha_0 - 1)x - 2\alpha_0)} \right) B_S^{(3)}, \end{aligned} \tag{8.35}$$

whereas for  $\delta = -1$  the differential equations read

$$\begin{aligned} \frac{\partial}{\partial x} B^{(4)} &= \left( \frac{-2d_1\epsilon + 4\epsilon - 1}{2x} + \frac{2d_1\epsilon - 4\epsilon + 1}{2(x-2)} \right) B^{(1)} + \left( \frac{-2d_1\epsilon + 4\epsilon - 1}{x-2} - \frac{4\epsilon}{x} \right) B^{(4)} + \\ &\quad \left( \frac{\alpha_0 - 1}{2x} - \frac{(\alpha_0 - 1)^2}{2((\alpha_0 - 1)x - 2\alpha_0)} \right) B_S^{(1)}, \\ \frac{\partial}{\partial x} B^{(7)} &= \left( \frac{(d_1 - 2)\epsilon}{x-2} - \frac{(d_1 - 2)\epsilon}{x} \right) B^{(3)} + \left( \frac{-4\epsilon - 1}{x} - \frac{2(d_1\epsilon - 2\epsilon)}{x-2} \right) B^{(7)} + \\ &\quad \left( \frac{\alpha_0 - 1}{2x} - \frac{(\alpha_0 - 1)^2}{2((\alpha_0 - 1)x - 2\alpha_0)} \right) B_S^{(3)}. \end{aligned} \tag{8.36}$$

The initial conditions are the following. At  $x = 0$ , we have

$$B^{(1)}(x = 0, \epsilon; \alpha_0, d_1; \delta) = B^{(6)}(x = 0, \epsilon; \alpha_0, d_1; \delta) = B_{\alpha_0}(1 - 4\epsilon, 1 + 2d_1\epsilon) B(1 - \epsilon, 1 - \epsilon). \tag{8.37}$$

At  $x = 1$ , we have

$$\begin{aligned} B^{(5)}(x = 1, \epsilon; \alpha_0, d_1; \delta) &= B^{(1)}(x = 1, \epsilon; \alpha_0, d_1; \delta), \\ B^{(8)}(x = 1, \epsilon; \alpha_0, d_1; \delta) &= B^{(2)}(x = 1, \epsilon; \alpha_0, d_1; \delta), \\ B^{(9)}(x = 1, \epsilon; \alpha_0, d_1; \delta) &= B^{(3)}(x = 1, \epsilon; \alpha_0, d_1; \delta). \end{aligned} \tag{8.38}$$

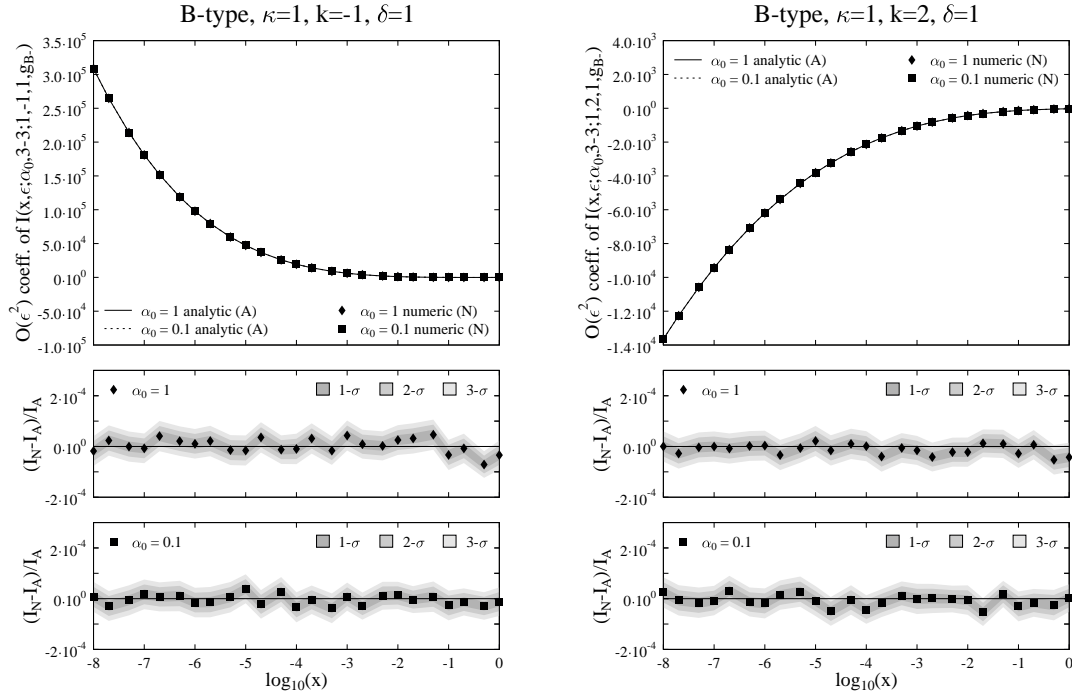
At  $x = 2$ , we have

$$\begin{aligned} B^{(4)}(x = 2, \epsilon; \alpha_0, d_1; \delta) &= \frac{1}{2} B^{(1)}(x = 2, \epsilon; \alpha_0, d_1; \delta), \\ B^{(7)}(x = 2, \epsilon; \alpha_0, d_1; \delta) &= \frac{1}{2} B^{(3)}(x = 2, \epsilon; \alpha_0, d_1; \delta). \end{aligned} \tag{8.39}$$

It is easy to check that  $B^{(1)}$  is finite at  $x = 0$  and  $x = 2$ . The integration constants of  $B^{(2)}$  and  $B^{(3)}$  can then be fixed in an implicit way by requiring the residues of the general solution for  $B^{(1)}$  to vanish at  $x = 0$  and  $x = 2$ .

Having the analytic expression for the master integrals, we can calculate the  $\mathcal{B}$ -type integrals for a fixed integer value of  $D_0$ . We give the explicit results for  $D_0 = 3$  in appendix E.





**Figure 5:** Representative results for the  $\mathcal{B}$ -type integrals. The plots show the coefficient of the  $O(\epsilon^2)$  term in  $\mathcal{I}(x, \epsilon; \alpha_0, 3 - 3\epsilon; 1, k, 1, g_{B-})$  for  $k = -1$  (left figure) and  $k = 2$  (right figure) with  $\alpha_0 = 0.1, 1$ .

In figure 5 we show some representative results of comparing the analytic and numeric computations for the  $\epsilon^2$  coefficient in the expansion of  $\mathcal{I}(x, \epsilon; \alpha_0, 3 - 3\epsilon; 1, k, 1, g_{B-})$  for  $k = -1, 2$  and  $\alpha_0 = 0.1, 1$ . The dependence on  $\alpha_0$  is not visible on the plots. The two sets of results are in excellent agreement for the whole  $x$ -range. For other (lower-order, thus simpler) expansion coefficients and/or other values of the parameters, we find similar agreement.

## 9. The soft $R \times (0)$ -type $\mathcal{J}*\mathcal{I}$ integrals

In this section we calculate the integral defined in eq. (3.32). Substituting the result for the angular integral  $\Omega^{(1,1)}$ , we can rewrite eq. (3.32) as

$$\begin{aligned} \mathcal{J}*\mathcal{I}(Y, \epsilon; y_0, d'_0, \alpha_0, d_0; k) &= -Y B(-\epsilon, -\epsilon) {}_2F_1(1, 1, 1 - \epsilon; 1 - Y) \\ &\times \int_0^{y_0} dy y^{-1-2\epsilon} (1 - y)^{d'_0} \mathcal{I}(y; \epsilon, \alpha_0, d_0; 0, k, 0, g_A). \end{aligned} \quad (9.1)$$

The hypergeometric function can be easily evaluated using the technique described in section 6. The evaluation of the  $y$  integral order by order in  $\epsilon$  is a little bit more cumbersome because the integrand has two kinds of singularities,

1. The pole in  $y = 0$ .

2. The integral  $\mathcal{I}$  is order by order logarithmically divergent for  $y \sim 0$ , as can be easily seen from the  $\epsilon$ -expansion given in appendix D.

The pole in  $y = 0$  can easily be factorized by performing the integration by parts in  $y$ . The logarithmic singularities in  $\mathcal{I}$  however are more subtle. We have to resum all these singularities before expanding the integral. We find that we can write<sup>4</sup>

$$\mathcal{I}(y; \epsilon, \alpha_0, d_0; 0, k, 0, g_A) = y^{-2\epsilon} I(y; \epsilon, \alpha_0, d_0; 0, k, 0, g_A), \tag{9.2}$$

where  $I$  is a function that is order by order finite in  $y = 0$ . Eq. (9.1) can now be written as

$$\begin{aligned} \mathcal{J}\mathcal{I}(Y, \epsilon; y_0, d'_0, \alpha_0, d_0; k) &= -Y B(-\epsilon, -\epsilon) {}_2F_1(1, 1, 1 - \epsilon; 1 - Y) \\ &\times \left\{ -\frac{1}{4\epsilon} y_0^{-4\epsilon} (1 - y_0)^{d'_0} I(y_0; \epsilon, \alpha_0, d_0; 0, k, 0, g_A) \right. \\ &\left. + \frac{1}{4\epsilon} \int_0^{y_0} dy y^{-4\epsilon} \frac{\partial}{\partial y} \left[ (1 - y)^{d'_0} I(y; \epsilon, \alpha_0, d_0; 0, k, 0, g_A) \right] \right\}. \end{aligned} \tag{9.3}$$

As  $I$  does not have logarithmic divergences, the derivative does not produce any poles, and so the integral is uniformly convergent. We can thus just expand the integrand into a power series in  $\epsilon$  and integrate order by order, using the definition of the *HPL*'s,<sup>5</sup> eq. (4.2). The result for  $D_0 = D'_0 = 3$  is given in appendix F.

As representative examples, in figure 6 we compare the analytic and numeric results for the  $\epsilon^0$  coefficient in the expansion of  $\mathcal{J}\mathcal{I}(Y, \epsilon; y_0, 3 - 3\epsilon, \alpha_0, 3 - 3\epsilon; k)$  for  $k = -1, 2$  and  $y_0 = \alpha_0 = 0.1, 1$ . The two computations agree very well over the whole  $Y$ -range. Other (lower-order, thus simpler) expansion coefficients and/or other values of the parameters show similar agreement.

## 10. The soft-collinear $R \times (0)$ -type $\mathcal{K}\mathcal{I}$ integrals

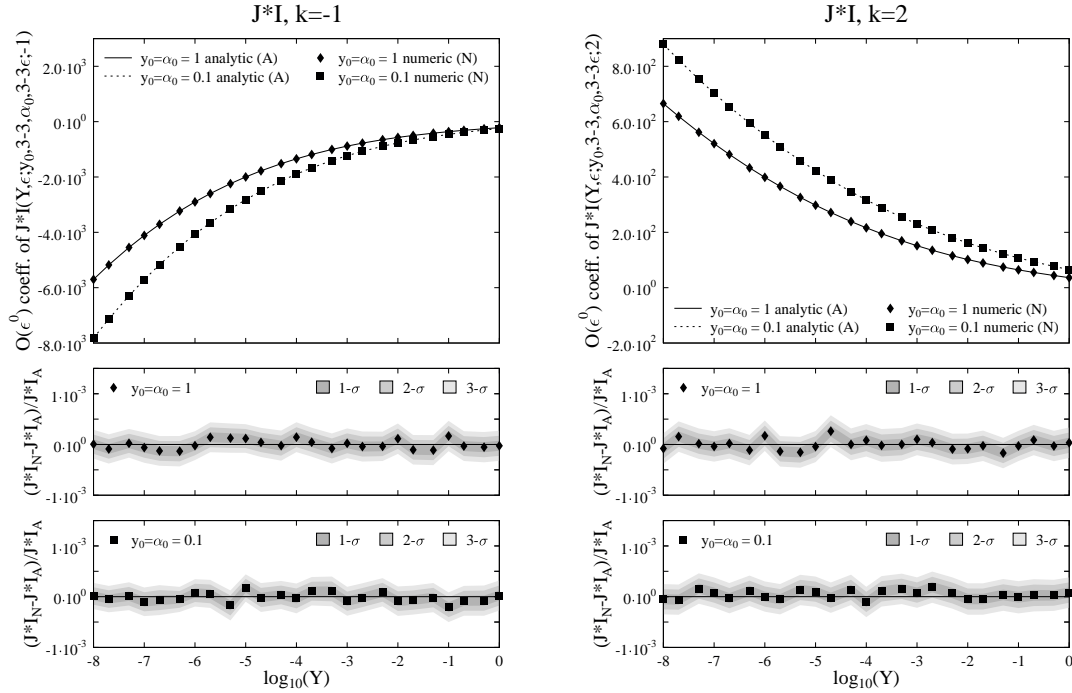
In this section we calculate the integral defined in eq. (3.33). The  $\varphi$  integral is given in eq. (7.1). Putting  $\cos \vartheta = 2\xi - 1$ , the integral can be rewritten as

$$\begin{aligned} \mathcal{K}\mathcal{I}(\epsilon; y_0, d'_0, \alpha_0, d_0; k) &= \\ &2 B(1 - \epsilon, -\epsilon) \int_0^{y_0} dy y^{-1-4\epsilon} (1 - y)^{d'_0} I(y; \epsilon, \alpha_0, d_0; 0, k, 0, g_A) \\ &+ 2 B(1 - \epsilon, 1 - \epsilon) \int_0^{y_0} dy y^{-4\epsilon} (1 - y)^{d'_0-1} I(y; \epsilon, \alpha_0, d_0; 0, k, 0, g_A), \end{aligned} \tag{10.1}$$

where  $I$  was defined in eq. (9.2). The first integral is exactly the same as in section 9. The second integral is uniformly convergent, so we can just expand under the integration sign, and integrate order by order. The result for  $D_0 = D'_0 = 3$  is given in appendix G.

<sup>4</sup>We checked this assumption explicitly on the  $\epsilon$ -expansion of  $\mathcal{I}$  given in appendix D.

<sup>5</sup>Notice that the rational part of  $I$  gives a non-vanishing contribution to the lower integration limit that has to be subtracted.



**Figure 6:** Representative results for the  $\mathcal{J}^*\mathcal{I}$  integrals. The plots show the coefficient of the  $O(\epsilon^0)$  term in  $\mathcal{J}^*\mathcal{L}(Y, \epsilon; y_0, 3 - 3\epsilon, \alpha_0, 3 - 3\epsilon; k)$  for  $k = -1$  (left figure) and  $k = 2$  (right figure) with  $y_0 = \alpha_0 = 0.1, 1$ .

## 11. Numerical evaluation of integrated subtraction terms

Let us briefly discuss the numerical evaluation of the integrals which were analytically computed in the previous sections. First of all, if the singular integrals in the chosen integration variables are non-overlapping and furthermore occur in a single point in the integration region (which can always be mapped to the origin) then we can isolate the poles using standard residuum subtraction. Consider as an example the  $\mathcal{K}(\epsilon; y_0, d'_0; \kappa)$  integral of eq. (7.2):

$$\mathcal{K}(\epsilon; y_0, d'_0; \kappa) = 2 \int_0^{y_0} dy \int_0^1 d\xi y^{-1-2(1+\kappa)\epsilon} (1-y)^{d'_0-1} \xi^{-\epsilon} (1-\xi)^{-1-(1+\kappa)\epsilon} (1-y\xi)^{1+\kappa\epsilon}. \quad (11.1)$$

We see that the singularities come only from  $y \rightarrow 0$  and  $\xi \rightarrow 1$  and there are no overlapping singularities.<sup>6</sup> After remapping the singularity at  $\xi \rightarrow 1$  to the origin by setting  $\xi \rightarrow 1 - \xi$ , we can easily extract the poles using residuum subtraction. The finite integrals that are left over are straightforward to evaluate numerically.

In general, we encounter integrals which contain overlapping divergences. A typical

<sup>6</sup>Overlapping singularities would be signaled by the presence of “composite” denominators, i.e. denominators which vanish only when *both*  $y \rightarrow 0$  and  $\xi \rightarrow 1$ , but not otherwise.

example occurs in the  $\mathcal{A}$ -type collinear integral for  $k = -1$ :

$$\mathcal{A}(x, \epsilon; \alpha_0, d_0; \kappa, -1) = \int_0^{\alpha_0} d\alpha \int_0^1 dv \alpha^{-1-(1+\kappa)\epsilon} (1-\alpha)^{2d_0-1} [\alpha - (1-\alpha)x]^{-1-(1+\kappa)\epsilon} \times v^{-\epsilon} (1-v)^{-\epsilon} \frac{2\alpha + (1-\alpha)x}{\alpha + (1-\alpha)xv}. \tag{11.2}$$

The singularities are at  $\alpha \rightarrow 0$  and  $v \rightarrow 0$ , but the presence of the “composite” denominator  $\alpha + (1-\alpha)xv$  (which vanishes only when both  $\alpha \rightarrow 0$  and  $v \rightarrow 0$ ) does not allow the use of simple residuum subtraction to isolate the poles. Instead one has to disentangle the overlapping singularities first, which we achieve by using sector decomposition [31]. This consists of the following steps

- We first transform the integral so that the range of integration is the unit square. This is easily achieved by setting  $\alpha \rightarrow \alpha_0\alpha$ .
- Then we split the integral into two “sectors” by inserting  $1 = [\Theta(\alpha - v) + \Theta(v - \alpha)]$  in the integrand.
- Next, we transform the variables in each sector such that the integration region is remapped to the unit square. When  $\alpha \geq v$ , we use  $v \rightarrow \alpha v$ , while for  $v \geq \alpha$  we need  $\alpha \rightarrow v\alpha$ .
- Notice that in each sector either  $\alpha$  or  $v$  now factorizes from the composite denominator and the remainder is finite at  $\alpha = 0, v = 0$ .
- We can now apply residuum subtraction in each sector to extract the  $\epsilon$  poles.

As before, the finite integrals left over are straightforward to compute numerically.

We have written a *Mathematica* package for the extraction of poles using these techniques. The program produces FORTRAN codes that may immediately be used in numerical integration programs. To produce the numerical results, we used the Monte Carlo integrator VEGAS [32]. The program SECTORDECOMPOSITION of ref. [33] was used to check our implementation.

## 12. Conclusions

In this work we have analytically evaluated some of the integrals needed for computing the integrated real-virtual counterterms that appear in the subtraction scheme for computing NNLO jet cross sections proposed in refs. [12, 13]. Such integrals have to be computed once and for all and their knowledge is necessary in order to make the subtraction scheme an effective tool. Our method is an adaptation of the current technique used to compute multi-loop Feynman diagrams: after an algebraic reduction to a class of independent amplitudes, integration-by-parts identities are generated and solved with the Laporta algorithm to achieve reduction to master integrals. The latter are computed with the differential equation method and are expressed in terms of one- and two-dimensional harmonic polylogarithms; the  $\epsilon$ -expansion has been performed up to the required order in  $\epsilon$ . The

numerical evaluation of harmonic polylogarithms has been treated in many works, where it has been shown that it can be fast and accurate [25, 28]; there is not any specific problem in our case either. A check of all our analytic results has been made by means of a direct numerical calculation of the integrals, typically with a relative accuracy  $\lesssim \mathcal{O}(10^{-4})$ . Specific properties of the present calculation are:

1. the partial fractioning in many variables of the integrands, which requires in general the introduction of new denominators;
2. the occurrence of surface terms in integration-by-parts identities, consisting of integrals of lower dimensionality than the original ones;
3. a non-trivial basis extension for two-dimensional harmonic polylogarithms, together with corresponding consistency relations in order to have complete analytic control over the results.

Our method can in principle be applied to the analytic evaluation of classes of more complicated real-virtual integrated counter-terms, such as the  $C$  and  $D$ -type integrals of section 8, even though the solution of the ibps in the latter case can be rather lengthy and the explicit expressions of the integrals can become rather cumbersome. For more complicated integrals, it is probably convenient to modify the algorithm used in this work in order to avoid the generation of additional denominators through the multiple partial fractioning. A preliminary study shows for example that our algorithm produces a lot of new denominators in the case of 3-dimensional integrals. For example, in reducing the integral over  $x$  and  $y$  considered in the introduction, one should not subject the “overlapping denominator”  $1/(1 - xy)$  to any partial fractioning; this way one ends up with 3-denominator integrals without any additional denominator.

## Acknowledgments

C.D. is a research fellow of the *Fonds National de la Recherche Scientifique*, Belgium. This work was partly supported by MIUR under contract 2006020509<sub>0</sub>04, by the EC Marie-Curie Research Training Network “Tools and Precision Calculations for Physics Discoveries at Colliders” under contract MRTN-CT-2006-035505, by the Hungarian Scientific Research Fund grand OTKA K-60432 and by the Swiss National Science Foundation (SNF) under contract 200020-117602. We thank the Galileo Galilei Institute for Theoretical Physics and the “Laboratori Nazionali di Frascati” for the hospitality and the INFN for partial support during the completion of this work.

## A. Spin-averaged splitting kernels

In this appendix we recall the explicit expressions for the spin-averaged splitting kernels that enter eq. (3.11).

The azimuthally averaged Altarelli-Parisi splitting kernels read

$$P_{g_i g_r}^{(0)}(z_i, z_r; \epsilon) = 2C_A \left[ \frac{1}{z_i} + \frac{1}{z_r} - 2 + z_i z_r \right], \quad (\text{A.1})$$

$$P_{q_i \bar{q}_r}^{(0)}(z_i, z_r; \epsilon) = T_R \left[ 1 - \frac{2}{1-\epsilon} z_i z_r \right], \quad (\text{A.2})$$

$$P_{q_i g_r}^{(0)}(z_i, z_r; \epsilon) = C_F \left[ \frac{2}{z_r} - 2 + (1-\epsilon) z_r \right], \quad (\text{A.3})$$

while their one-loop generalizations are

$$P_{f_i f_r}^{(1)}(z_i, z_r; \epsilon) = r_{\text{S,ren}}^{f_i f_r}(z_i, z_r; \epsilon) P_{f_i f_r}^{(0)}(z_i, z_r; \epsilon) + \begin{cases} 2C_A r_{\text{NS}}^{gg} \frac{1-2\epsilon z_i z_r}{1-\epsilon}, & \text{if } f_i f_r = gg, \\ 0, & \text{if } f_i f_r = q\bar{q}, \\ C_F r_{\text{NS}}^{qq} (1-\epsilon z_r), & \text{if } f_i f_r = qq. \end{cases} \quad (\text{A.4})$$

The renormalized  $r_{\text{S,ren}}^{f_i f_r}(z_i, z_r; \epsilon)$  functions that appear above are expressed in terms of the corresponding unrenormalized ones as

$$r_{\text{S,ren}}^{f_i f_r}(z_i, z_r; \epsilon) = r_{\text{S}}^{f_i f_r}(z_i, z_r; \epsilon) - \frac{\beta_0}{2\epsilon} \frac{\bar{S}_\epsilon}{(4\pi)^2 c_\Gamma} \left[ \left( \frac{\mu^2}{s_{ir}} \right)^\epsilon \cos(\pi\epsilon) \right]^{-1}, \quad (\text{A.5})$$

where the unrenormalized  $r_{\text{S}}^{f_i f_r}(z_i, z_r; \epsilon)$  factors may be written in the following form

$$r_{\text{S}}^{gg}(z_i, z_r; \epsilon) = \frac{C_A}{\epsilon^2} \left[ -\frac{\pi\epsilon}{\sin(\pi\epsilon)} \left( \frac{z_i}{z_r} \right)^\epsilon + z_i^\epsilon {}_2F_1(\epsilon, \epsilon, 1+\epsilon, z_r) - z_i^{-\epsilon} {}_2F_1(-\epsilon, -\epsilon, 1-\epsilon, z_r) \right], \quad (\text{A.6})$$

$$r_{\text{S}}^{q\bar{q}}(z_i, z_r; \epsilon) = \frac{1}{\epsilon^2} (C_A - 2C_F) + \frac{C_A}{\epsilon^2} \left[ -\frac{\pi\epsilon}{\sin(\pi\epsilon)} \left( \frac{z_i}{z_r} \right)^\epsilon + z_i^\epsilon {}_2F_1(\epsilon, \epsilon, 1+\epsilon, z_r) - \frac{\pi\epsilon}{\sin(\pi\epsilon)} \left( \frac{z_r}{z_i} \right)^\epsilon + z_r^\epsilon {}_2F_1(\epsilon, \epsilon, 1+\epsilon, z_i) \right] + \frac{1}{1-2\epsilon} \left[ \frac{\beta_0 - 3C_F}{\epsilon} + C_A - 2C_F + \frac{C_A + 4T_R(n_f - n_s)}{3(3-2\epsilon)} \right], \quad (\text{A.7})$$

$$r_{\text{S}}^{qq}(z_i, z_r; \epsilon) = -\frac{1}{\epsilon^2} \left[ 2(C_A - C_F) + C_A \frac{\pi\epsilon}{\sin(\pi\epsilon)} \left( \frac{z_i}{z_r} \right)^\epsilon - C_A z_i^\epsilon {}_2F_1(\epsilon, \epsilon, 1+\epsilon, z_r) - (C_A - 2C_F) z_i^{-\epsilon} {}_2F_1(-\epsilon, -\epsilon, 1-\epsilon, z_r) \right]. \quad (\text{A.8})$$

The  $r_{\text{NS}}^{f_i f_r}$  non-singular factors are

$$r_{\text{NS}}^{gg} = \frac{C_A(1-\epsilon) - 2T_R(n_f - n_s)}{(1-2\epsilon)(2-2\epsilon)(3-2\epsilon)}, \quad r_{\text{NS}}^{qq} = \frac{C_A - C_F}{1-2\epsilon}. \quad (\text{A.9})$$

For QCD,  $n_s = 0$ . Finally  $\beta_0$  in eqs. (A.5) and (A.7) is given by

$$\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_R n_f - \frac{2}{3}T_R n_s. \quad (\text{A.10})$$

## B. The $\mathcal{J}$ integrals

The  $\varepsilon$  expansion for this integral reads

$$\mathcal{J}(Y, \varepsilon; y_0, 3 + d'_1 \varepsilon; \kappa) = \frac{1}{\varepsilon^2} i_{-2}^{(\kappa)} + \frac{1}{\varepsilon} i_{-1}^{(\kappa)} + i_0 + \varepsilon i_1^{(\kappa)} + \varepsilon^2 i_2^{(\kappa)} + \mathcal{O}(\varepsilon^3), \quad (\text{B.1})$$

where

$$\begin{aligned} i_{-2}^{(\kappa)} &= -\frac{1}{(\kappa+1)^2}, \\ i_{-1}^{(\kappa)} &= -\frac{2y_0^3}{3(\kappa+1)} + \frac{3y_0^2}{\kappa+1} - \frac{6y_0}{\kappa+1} + \frac{2H(0; y_0)}{\kappa+1} + \frac{H(0; Y)}{\kappa+1}, \\ i_0^{(\kappa)} &= \frac{2d'_1 y_0^3}{9(3\kappa+1)} + \frac{2d'_1 \kappa y_0^3}{9(3\kappa+1)} - \frac{8\kappa y_0^3}{9(3\kappa+1)} - \frac{4y_0^3}{9(3\kappa+1)} - \frac{7d'_1 y_0^2}{6(3\kappa+1)} - \frac{7d'_1 \kappa y_0^2}{6(3\kappa+1)} + \frac{20\kappa y_0^2}{3(3\kappa+1)} + \frac{3y_0^2}{3\kappa+1} + \\ &\frac{11d'_1 y_0}{3(3\kappa+1)} + \frac{11d'_1 \kappa y_0}{3(3\kappa+1)} - \frac{86\kappa y_0}{3(3\kappa+1)} - \frac{12y_0}{3\kappa+1} + \left( \frac{4y_0^3}{3} - 6y_0^2 + 12y_0 \right) H(0; y_0) + \left( \frac{2y_0^3}{3} - 3y_0^2 + 6y_0 - \right. \\ &\left. 2H(0; y_0) \right) H(0; Y) + \left( \frac{2d'_1 y_0^3}{3(3\kappa+1)} + \frac{2d'_1 \kappa y_0^3}{3(3\kappa+1)} + \frac{4\kappa y_0^3}{3(3\kappa+1)} - \frac{3d'_1 y_0^2}{3\kappa+1} - \frac{3d'_1 \kappa y_0^2}{3\kappa+1} - \frac{6\kappa y_0^2}{3\kappa+1} + \frac{6d'_1 y_0}{3\kappa+1} + \right. \\ &\left. \frac{6d'_1 \kappa y_0}{3\kappa+1} + \frac{12\kappa y_0}{3\kappa+1} - \frac{11d'_1}{3(3\kappa+1)} - \frac{11d'_1 \kappa}{3(3\kappa+1)} - \frac{22\kappa}{3(3\kappa+1)} \right) H(1; y_0) - 4H(0, 0; y_0) - H(0, 0; Y) + \left( - \right. \\ &\left. \frac{2\kappa d'_1}{(\kappa+1)^2} - \frac{2d'_1}{(\kappa+1)^2} - \frac{4\kappa}{(\kappa+1)^2} \right) H(0, 1; y_0) - H(1, 0; Y), \\ i_1^{(\kappa)} &= \\ &-\frac{2d_1'^2 y_0^3}{27(3\kappa+1)} + \frac{8d_1' y_0^3}{27(3\kappa+1)} - \frac{2d_1'^2 \kappa y_0^3}{27(3\kappa+1)} + \frac{16 d_1' \kappa y_0^3}{27(3\kappa+1)} - \frac{28\kappa y_0^3}{27(3\kappa+1)} - \frac{8y_0^3}{27(3\kappa+1)} + \frac{17d_1'^2 y_0^2}{36(3\kappa+1)} - \frac{22d_1' y_0^2}{9(3\kappa+1)} + \frac{17d_1'^2 \kappa y_0^2}{36(3\kappa+1)} - \\ &\frac{49d_1' \kappa y_0^2}{9(3\kappa+1)} + \frac{73\kappa y_0^2}{6(3\kappa+1)} + \frac{3y_0^2}{3\kappa+1} - \frac{49d_1'^2 y_0}{18(3\kappa+1)} + \frac{151d_1' y_0}{9(3\kappa+1)} - \frac{49d_1'^2 \kappa y_0}{18(3\kappa+1)} + \frac{355d_1' \kappa y_0}{9(3\kappa+1)} - \frac{319\kappa y_0}{3(3\kappa+1)} - \frac{24y_0}{3\kappa+1} + \left( - \right. \\ &\frac{4d_1' y_0^3}{9(3\kappa+1)} - \frac{4d_1' \kappa y_0^3}{3(3\kappa+1)} + \frac{40\kappa y_0^3}{9(3\kappa+1)} + \frac{8y_0^3}{9(3\kappa+1)} + \frac{7d_1' y_0^2}{3(3\kappa+1)} + \frac{7d_1' \kappa y_0^2}{3\kappa+1} - \frac{98\kappa y_0^2}{3(3\kappa+1)} - \frac{6y_0^2}{3\kappa+1} - \frac{22d_1' y_0}{3(3\kappa+1)} - \frac{22d_1' \kappa y_0}{3\kappa+1} + \\ &\frac{416\kappa y_0}{3(3\kappa+1)} + \frac{24y_0}{3\kappa+1} \left. \right) H(0; y_0) + \left( - \frac{2d_1'^2 y_0^3}{9(3\kappa+1)} + \frac{4d_1' y_0^3}{9(3\kappa+1)} - \frac{2d_1'^2 \kappa y_0^3}{9(3\kappa+1)} + \frac{4d_1' \kappa y_0^3}{9(3\kappa+1)} + \frac{4\kappa y_0^3}{3(3\kappa+1)} + \frac{7d_1'^2 y_0^2}{6(3\kappa+1)} - \right. \\ &\frac{3d_1' y_0^2}{3\kappa+1} + \frac{7d_1'^2 \kappa y_0^2}{6(3\kappa+1)} - \frac{13d_1' \kappa y_0^2}{3(3\kappa+1)} - \frac{29\kappa y_0^2}{3(3\kappa+1)} - \frac{11d_1'^2 y_0}{3(3\kappa+1)} + \frac{12d_1' y_0}{3\kappa+1} - \frac{11d_1'^2 \kappa y_0}{3(3\kappa+1)} + \frac{64d_1' \kappa y_0}{3(3\kappa+1)} + \frac{122\kappa y_0}{3(3\kappa+1)} + \frac{49d_1'^2}{18(3\kappa+1)} - \\ &\frac{85d_1'}{9(3\kappa+1)} + \frac{49d_1'^2 \kappa}{18(3\kappa+1)} - \frac{157d_1' \kappa}{9(3\kappa+1)} - \frac{97\kappa}{3(3\kappa+1)} \left. \right) H(1; y_0) + \frac{1}{6}\pi^2 H(1; Y) + \left( - \frac{56\kappa y_0^3}{3(3\kappa+1)} - \frac{8y_0^3}{3(3\kappa+1)} + \frac{84\kappa y_0^2}{3\kappa+1} + \right. \\ &\frac{12y_0^2}{3\kappa+1} - \frac{168\kappa y_0}{3\kappa+1} - \frac{24y_0}{3\kappa+1} \left. \right) H(0, 0; y_0) + \left( - \frac{14\kappa y_0^3}{3(3\kappa+1)} - \frac{2y_0^3}{3(3\kappa+1)} + \frac{21\kappa y_0^2}{3\kappa+1} + \frac{3y_0^2}{3\kappa+1} - \frac{42\kappa y_0}{3\kappa+1} - \frac{6y_0}{3\kappa+1} + \right. \\ &\left. \left( \frac{14\kappa}{(\kappa+1)^2} + \frac{2}{(\kappa+1)^2} \right) H(0; y_0) \right) H(0, 0; Y) + \left( - \frac{4d_1' y_0^3}{3(3\kappa+1)} - \frac{4d_1' \kappa y_0^3}{3\kappa+1} - \frac{16\kappa y_0^3}{3(3\kappa+1)} + \frac{6d_1' y_0^2}{3\kappa+1} + \frac{18d_1' \kappa y_0^2}{3\kappa+1} + \right. \\ &\frac{24\kappa y_0^2}{3\kappa+1} - \frac{12d_1' y_0}{3\kappa+1} - \frac{36d_1' \kappa y_0}{3\kappa+1} - \frac{48\kappa y_0}{3\kappa+1} \left. \right) H(0, 1; y_0) + H(0; Y) \left( - \frac{2d_1' y_0^3}{9(3\kappa+1)} - \frac{2d_1' \kappa y_0^3}{3(3\kappa+1)} + \frac{20\kappa y_0^3}{9(3\kappa+1)} + \frac{4y_0^3}{9(3\kappa+1)} + \right. \\ &\frac{7d_1' y_0^2}{6(3\kappa+1)} + \frac{7d_1' \kappa y_0^2}{2(3\kappa+1)} - \frac{49\kappa y_0^2}{3(3\kappa+1)} - \frac{3y_0^2}{3\kappa+1} - \frac{11d_1' y_0}{3(3\kappa+1)} - \frac{11d_1' \kappa y_0}{3\kappa+1} + \frac{208\kappa y_0}{3(3\kappa+1)} + \frac{12y_0}{3\kappa+1} + \left( - \frac{28\kappa y_0^3}{3(3\kappa+1)} - \frac{4y_0^3}{3(3\kappa+1)} + \right. \\ &\frac{42\kappa y_0^2}{3\kappa+1} + \frac{6y_0^2}{3\kappa+1} - \frac{84\kappa y_0}{3\kappa+1} - \frac{12y_0}{3\kappa+1} \left. \right) H(0; y_0) + \left( - \frac{2d_1' y_0^3}{3(3\kappa+1)} - \frac{2d_1' \kappa y_0^3}{3\kappa+1} - \frac{8\kappa y_0^3}{3(3\kappa+1)} + \frac{3d_1' y_0^2}{3\kappa+1} + \frac{9d_1' \kappa y_0^2}{3\kappa+1} + \frac{12\kappa y_0^2}{3\kappa+1} - \right. \\ &\frac{6d_1' y_0}{3\kappa+1} - \frac{18d_1' \kappa y_0}{3\kappa+1} - \frac{24\kappa y_0}{3\kappa+1} + \frac{11d_1'}{3(3\kappa+1)} + \frac{11d_1' \kappa}{3\kappa+1} + \frac{44\kappa}{3(3\kappa+1)} \left. \right) H(1; y_0) + \left( \frac{28\kappa}{(\kappa+1)^2} + \frac{4}{(\kappa+1)^2} \right) H(0, 0; y_0) + \\ &\left( \frac{6\kappa d_1'}{(\kappa+1)^2} + \frac{2d_1'}{(\kappa+1)^2} + \frac{8\kappa}{(\kappa+1)^2} \right) H(0, 1; y_0) + \left( - \frac{4d_1' y_0^3}{3(3\kappa+1)} - \frac{4d_1' \kappa y_0^3}{3\kappa+1} - \frac{16\kappa y_0^3}{3(3\kappa+1)} + \frac{6d_1' y_0^2}{3\kappa+1} + \frac{18 d_1' \kappa y_0^2}{3\kappa+1} + \right. \\ &\frac{24\kappa y_0^2}{3\kappa+1} - \frac{12d_1' y_0}{3\kappa+1} - \frac{36d_1' \kappa y_0}{3\kappa+1} - \frac{48\kappa y_0}{3\kappa+1} + \frac{22d_1'}{3(3\kappa+1)} + \frac{22d_1' \kappa}{3\kappa+1} + \frac{88\kappa}{3(3\kappa+1)} \left. \right) H(1, 0; y_0) + \left( - \frac{14\kappa y_0^3}{3(3\kappa+1)} - \right. \\ &\frac{2y_0^3}{3(3\kappa+1)} + \frac{21\kappa y_0^2}{3\kappa+1} + \frac{3y_0^2}{3\kappa+1} - \frac{42\kappa y_0}{3\kappa+1} + \left. \left( \frac{14\kappa}{(\kappa+1)^2} + \frac{2}{(\kappa+1)^2} \right) H(0; y_0) \right) H(1, 0; Y) + \left( - \frac{2d_1'^2 y_0^3}{3(3\kappa+1)} - \right. \\ &\frac{2d_1'^2 \kappa y_0^3}{3(3\kappa+1)} - \frac{8d_1' \kappa y_0^3}{3(3\kappa+1)} - \frac{4\kappa y_0^3}{3(3\kappa+1)} + \frac{3d_1'^2 y_0^2}{3\kappa+1} + \frac{3d_1'^2 \kappa y_0^2}{3\kappa+1} + \frac{12d_1' \kappa y_0^2}{3\kappa+1} + \frac{6\kappa y_0^2}{3\kappa+1} - \frac{6d_1'^2 y_0}{3\kappa+1} - \frac{6d_1'^2 \kappa y_0}{3\kappa+1} - \frac{24d_1' \kappa y_0}{3\kappa+1} - \\ &\frac{12\kappa y_0}{3\kappa+1} + \frac{11d_1'^2}{3(3\kappa+1)} + \frac{11d_1'^2 \kappa}{3(3\kappa+1)} + \frac{44d_1' \kappa}{3(3\kappa+1)} + \frac{22\kappa}{3(3\kappa+1)} \left. \right) H(1, 1; y_0) + \left( \frac{56\kappa}{(\kappa+1)^2} + \frac{8}{(\kappa+1)^2} \right) H(0, 0, 0; y_0) + \end{aligned}$$

$$\left(\frac{7\kappa}{(\kappa+1)^2} + \frac{1}{(\kappa+1)^2}\right)H(0, 0, 0; Y) + \left(\frac{12\kappa d'_1}{(\kappa+1)^2} + \frac{4d'_1}{(\kappa+1)^2} + \frac{16\kappa}{(\kappa+1)^2}\right)H(0, 0, 1; y_0) + \left(\frac{12\kappa d'_1}{(\kappa+1)^2} + \frac{4d'_1}{(\kappa+1)^2} + \frac{16\kappa}{(\kappa+1)^2}\right)H(0, 1, 0; y_0) + \left(\frac{7\kappa}{(\kappa+1)^2} + \frac{1}{(\kappa+1)^2}\right)H(0, 1, 0; Y) + \left(\frac{2\kappa d_1^2}{(\kappa+1)^2} + \frac{2d_1^2}{(\kappa+1)^2} + \frac{8\kappa d'_1}{(\kappa+1)^2} + \frac{4\kappa}{(\kappa+1)^2}\right)H(0, 1, 1; y_0) + H(1, 0, 0; Y) + H(1, 1, 0; Y) + \frac{22\kappa\zeta_3}{3\kappa+1} + \frac{2\zeta_3}{3\kappa+1},$$

$$\begin{aligned} i_2^{(\kappa)} = & \frac{2\kappa y_0^3 d_1^3}{81(3\kappa+1)} + \frac{2y_0^3 d_1^3}{81(3\kappa+1)} - \frac{43\kappa y_0^2 d_1^3}{216(3\kappa+1)} - \frac{43y_0^2 d_1^3}{216(3\kappa+1)} + \frac{251\kappa y_0 d_1^3}{108(3\kappa+1)} + \frac{251y_0 d_1^3}{108(3\kappa+1)} - \frac{8\kappa y_0^3 d_1^2}{27(3\kappa+1)} - \\ & \frac{4y_0^3 d_1^2}{27(3\kappa+1)} + \frac{31\kappa y_0^2 d_1^2}{9(3\kappa+1)} + \frac{167y_0^2 d_1^2}{108(3\kappa+1)} - \frac{833\kappa y_0 d_1^2}{18(3\kappa+1)} - \frac{542y_0 d_1^2}{27(3\kappa+1)} + \frac{28\kappa y_0^3 d_1}{27(3\kappa+1)} + \frac{8y_0^3 d_1}{27(3\kappa+1)} - \frac{1663\kappa y_0^2 d_1}{108(3\kappa+1)} - \\ & \frac{205y_0^2 d_1}{54(3\kappa+1)} + \frac{12815\kappa y_0 d_1}{54(3\kappa+1)} + \frac{1481y_0 d_1}{27(3\kappa+1)} - \frac{81(3\kappa+1)}{81(3\kappa+1)} - \frac{81(3\kappa+1)}{81(3\kappa+1)} + \frac{12(3\kappa+1)}{12(3\kappa+1)} + \frac{3y_0^2}{3\kappa+1} - \frac{2141\kappa y_0}{6(3\kappa+1)} - \frac{48y_0}{3\kappa+1} + \\ & \left(\frac{2\kappa y_0^3 d_1^3}{27(3\kappa+1)} + \frac{2y_0^3 d_1^3}{27(3\kappa+1)} - \frac{17\kappa y_0^2 d_1^3}{36(3\kappa+1)} - \frac{17y_0^2 d_1^3}{36(3\kappa+1)} + \frac{49\kappa y_0 d_1^3}{18(3\kappa+1)} + \frac{49y_0 d_1^3}{18(3\kappa+1)} - \frac{251\kappa d_1^3}{108(3\kappa+1)} - \frac{251d_1^3}{108(3\kappa+1)} - \right. \\ & \left. \frac{4\kappa y_0^3 d_1^2}{9(3\kappa+1)} - \frac{8y_0^3 d_1^2}{27(3\kappa+1)} + \frac{9\kappa y_0^2 d_1^2}{2(3\kappa+1)} + \frac{22y_0^2 d_1^2}{9(3\kappa+1)} - \frac{34\kappa y_0 d_1^2}{3\kappa+1} - \frac{151y_0 d_1^2}{9(3\kappa+1)} + \frac{539\kappa d_1^2}{18(3\kappa+1)} + \frac{395d_1^2}{27(3\kappa+1)} + \frac{4\kappa y_0^3 d_1}{27(3\kappa+1)} + \right. \\ & \left. \frac{8y_0^3 d_1}{27(3\kappa+1)} - \frac{77\kappa y_0^2 d_1}{18(3\kappa+1)} - \frac{3y_0^2 d_1}{3\kappa+1} + \frac{451\kappa y_0 d_1}{9(3\kappa+1)} + \frac{24y_0 d_1}{3\kappa+1} - \frac{54(3\kappa+1)}{54(3\kappa+1)} - \frac{27(3\kappa+1)}{27(3\kappa+1)} + \frac{3(3\kappa+1)}{3(3\kappa+1)} - \frac{6(3\kappa+1)}{6(3\kappa+1)} + \right. \\ & \left. \frac{391\kappa y_0}{3(3\kappa+1)} - \frac{233\kappa}{2(3\kappa+1)}\right)H(1; y_0) + \left(\frac{8d'_1 y_0^3}{9(3\kappa+1)} + \frac{56d'_1 \kappa y_0^3}{9(3\kappa+1)} - \frac{176\kappa y_0^3}{9(3\kappa+1)} - \frac{16y_0^3}{9(3\kappa+1)} - \frac{14d'_1 y_0^2}{3(3\kappa+1)} - \frac{98d'_1 \kappa y_0^2}{3(3\kappa+1)} + \right. \\ & \left. \frac{428\kappa y_0^2}{3(3\kappa+1)} + \frac{12y_0^2}{3\kappa+1} + \frac{44d'_1 y_0}{3(3\kappa+1)} + \frac{308d'_1 \kappa y_0}{3(3\kappa+1)} - \frac{1808\kappa y_0}{3(3\kappa+1)} - \frac{48y_0}{3\kappa+1}\right)H(0, 0; y_0) + \left(\frac{4d_1^2 y_0^3}{9(3\kappa+1)} - \frac{8d_1^2 y_0^3}{9(3\kappa+1)} + \right. \\ & \left. \frac{4d_1^2 \kappa y_0^3}{3(3\kappa+1)} - \frac{8d_1^2 \kappa y_0^3}{3(3\kappa+1)} - \frac{16\kappa y_0^3}{3(3\kappa+1)} - \frac{7d_1^2 y_0^2}{3(3\kappa+1)} + \frac{6d_1^2 y_0^2}{3\kappa+1} - \frac{7d_1^2 \kappa y_0^2}{3\kappa+1} + \frac{70d_1^2 \kappa y_0^2}{3(3\kappa+1)} + \frac{116\kappa y_0^2}{3(3\kappa+1)} + \frac{22d_1^2 y_0}{3(3\kappa+1)} - \right. \\ & \left. \frac{24d_1^2 y_0}{3\kappa+1} + \frac{22d_1^2 \kappa y_0}{3\kappa+1} - \frac{328d_1^2 \kappa y_0}{3(3\kappa+1)} - \frac{488\kappa y_0}{3(3\kappa+1)}\right)H(0, 1; y_0) + H(0, 0; Y) \left(\frac{2d_1^2 y_0^3}{9(3\kappa+1)} + \frac{14d_1^2 \kappa y_0^3}{9(3\kappa+1)} - \frac{44\kappa y_0^3}{9(3\kappa+1)} - \right. \\ & \left. \frac{4y_0^3}{9(3\kappa+1)} - \frac{7d_1^2 y_0^2}{6(3\kappa+1)} - \frac{49d_1^2 \kappa y_0^2}{6(3\kappa+1)} + \frac{107\kappa y_0^2}{3(3\kappa+1)} + \frac{3y_0^2}{3\kappa+1} + \frac{11d_1^2 y_0}{3(3\kappa+1)} + \frac{77d_1^2 \kappa y_0}{3(3\kappa+1)} - \frac{452\kappa y_0}{3(3\kappa+1)} - \frac{12y_0}{3\kappa+1} + \right. \\ & \left. \left(\frac{20\kappa y_0^3}{3\kappa+1} + \frac{4y_0^3}{3(3\kappa+1)} - \frac{90\kappa y_0^2}{3\kappa+1} - \frac{6y_0^2}{3\kappa+1} + \frac{180\kappa y_0}{3\kappa+1} + \frac{12y_0}{3\kappa+1}\right)H(0; y_0) + \left(\frac{2d_1^2 y_0^3}{3(3\kappa+1)} + \frac{14d_1^2 \kappa y_0^3}{3(3\kappa+1)} + \frac{16\kappa y_0^3}{3(3\kappa+1)} - \right. \\ & \left. \frac{3d_1^2 y_0^2}{3\kappa+1} - \frac{21d_1^2 \kappa y_0^2}{3\kappa+1} - \frac{24\kappa y_0^2}{3\kappa+1} + \frac{6d_1^2 y_0}{3\kappa+1} + \frac{42d_1^2 \kappa y_0}{3\kappa+1} + \frac{48\kappa y_0}{3\kappa+1} - \frac{11d_1^2}{3(3\kappa+1)} - \frac{77d_1^2 \kappa}{3(3\kappa+1)} - \frac{88\kappa}{3(3\kappa+1)}\right)H(1; y_0) + \\ & \left(-\frac{60\kappa}{(\kappa+1)^2} - \frac{4}{(\kappa+1)^2}\right)H(0, 0; y_0) + \left(-\frac{14\kappa d_1^2}{(\kappa+1)^2} - \frac{2d_1^2}{(\kappa+1)^2} - \frac{16\kappa}{(\kappa+1)^2}\right)H(0, 1; y_0) + \left(-\frac{7\pi^2 \kappa}{6(3\kappa+1)} - \right. \\ & \left. \frac{\pi^2}{6(3\kappa+1)}\right)H(0, 1; Y) + \left(\frac{4d_1^2 y_0^3}{9(3\kappa+1)} - \frac{8d_1^2 y_0^3}{9(3\kappa+1)} + \frac{4d_1^2 \kappa y_0^3}{3(3\kappa+1)} - \frac{8d_1^2 \kappa y_0^3}{3(3\kappa+1)} - \frac{16\kappa y_0^3}{3(3\kappa+1)} - \frac{7d_1^2 y_0^2}{3(3\kappa+1)} + \frac{6d_1^2 y_0^2}{3\kappa+1} - \right. \\ & \left. \frac{7d_1^2 \kappa y_0^2}{3\kappa+1} + \frac{70d_1^2 \kappa y_0^2}{3(3\kappa+1)} + \frac{116\kappa y_0^2}{3(3\kappa+1)} + \frac{22d_1^2 y_0}{3(3\kappa+1)} - \frac{24d_1^2 y_0}{3\kappa+1} + \frac{22d_1^2 \kappa y_0}{3\kappa+1} - \frac{328d_1^2 \kappa y_0}{3(3\kappa+1)} - \frac{488\kappa y_0}{3(3\kappa+1)} - \frac{49d_1^2}{9(3\kappa+1)} + \right. \\ & \left. \frac{170d_1^2}{9(3\kappa+1)} - \frac{49d_1^2 \kappa}{3(3\kappa+1)} + \frac{266d_1^2 \kappa}{3(3\kappa+1)} + \frac{388\kappa}{3(3\kappa+1)}\right)H(1, 0; y_0) + \left(\frac{2d_1^2 y_0^3}{9(3\kappa+1)} + \frac{14d_1^2 \kappa y_0^3}{9(3\kappa+1)} - \frac{44\kappa y_0^3}{9(3\kappa+1)} - \frac{4y_0^3}{9(3\kappa+1)} - \right. \\ & \left. \frac{7d_1^2 y_0^2}{6(3\kappa+1)} - \frac{49d_1^2 \kappa y_0^2}{6(3\kappa+1)} + \frac{107\kappa y_0^2}{3(3\kappa+1)} + \frac{3y_0^2}{3\kappa+1} + \frac{11d_1^2 y_0}{3(3\kappa+1)} + \frac{77d_1^2 \kappa y_0}{3(3\kappa+1)} - \frac{452\kappa y_0}{3(3\kappa+1)} - \frac{12y_0}{3\kappa+1} + \left(\frac{20\kappa y_0^3}{3\kappa+1} + \frac{4y_0^3}{3(3\kappa+1)} - \right. \right. \\ & \left. \frac{90\kappa y_0^2}{3\kappa+1} - \frac{6y_0^2}{3\kappa+1} + \frac{180\kappa y_0}{3\kappa+1} + \frac{12y_0}{3\kappa+1}\right)H(0; y_0) + \left(\frac{2d_1^2 y_0^3}{3(3\kappa+1)} + \frac{14d_1^2 \kappa y_0^3}{3(3\kappa+1)} + \frac{16\kappa y_0^3}{3(3\kappa+1)} - \frac{3d_1^2 y_0^2}{3\kappa+1} - \frac{21d_1^2 \kappa y_0^2}{3\kappa+1} - \right. \\ & \left. \frac{24\kappa y_0^2}{3\kappa+1} + \frac{6d_1^2 y_0}{3\kappa+1} + \frac{42d_1^2 \kappa y_0}{3\kappa+1} + \frac{48\kappa y_0}{3\kappa+1} - \frac{11d_1^2}{3(3\kappa+1)} - \frac{77d_1^2 \kappa}{3(3\kappa+1)} - \frac{88\kappa}{3(3\kappa+1)}\right)H(1; y_0) + \left(-\frac{60\kappa}{(\kappa+1)^2} - \right. \\ & \left. \frac{4}{(\kappa+1)^2}\right)H(0, 0; y_0) + \left(-\frac{14\kappa d_1^2}{(\kappa+1)^2} - \frac{2d_1^2}{(\kappa+1)^2} - \frac{16\kappa}{(\kappa+1)^2}\right)H(0, 1; y_0) + \frac{2\kappa\pi^2}{3(3\kappa+1)}H(1, 0; Y) + \\ & \left(\frac{2\kappa y_0^3 d_1^3}{9(3\kappa+1)} + \frac{2y_0^3 d_1^3}{9(3\kappa+1)} - \frac{7\kappa y_0^2 d_1^3}{6(3\kappa+1)} - \frac{7y_0^2 d_1^3}{6(3\kappa+1)} + \frac{11\kappa y_0 d_1^3}{3(3\kappa+1)} + \frac{11y_0 d_1^3}{3(3\kappa+1)} - \frac{49\kappa d_1^3}{18(3\kappa+1)} - \frac{49d_1^3}{18(3\kappa+1)} - \frac{4y_0^3 d_1^2}{9(3\kappa+1)} + \right. \\ & \left. \frac{2\kappa y_0^2 d_1^2}{3\kappa+1} + \frac{3y_0^2 d_1^2}{3\kappa+1} - \frac{14\kappa y_0 d_1^2}{3\kappa+1} - \frac{12y_0 d_1^2}{3\kappa+1} + \frac{12\kappa d_1^2}{3\kappa+1} + \frac{85d_1^2}{9(3\kappa+1)} - \frac{20\kappa y_0^3 d_1}{9(3\kappa+1)} + \frac{17\kappa y_0^2 d_1}{3\kappa+1} - \frac{74\kappa y_0 d_1}{3\kappa+1} + \frac{533\kappa d_1}{9(3\kappa+1)} - \right. \\ & \left. \frac{4\kappa y_0^3}{3(3\kappa+1)} + \frac{29\kappa y_0^2}{3(3\kappa+1)} - \frac{122\kappa y_0}{3(3\kappa+1)} + \frac{97\kappa}{3(3\kappa+1)}\right)H(1, 1; y_0) - \frac{1}{6}\pi^2 H(1, 1; Y) + \left(\frac{80\kappa y_0^3}{3\kappa+1} + \frac{16y_0^3}{3(3\kappa+1)} - \right. \\ & \left. \frac{360\kappa y_0^2}{3\kappa+1} - \frac{24y_0^2}{3\kappa+1} + \frac{720\kappa y_0}{3\kappa+1} + \frac{48y_0}{3\kappa+1}\right)H(0, 0, 0; y_0) + \left(\frac{10\kappa y_0^3}{3\kappa+1} + \frac{2y_0^3}{3(3\kappa+1)} - \frac{45\kappa y_0^2}{3\kappa+1} - \frac{3y_0^2}{3\kappa+1} + \frac{90\kappa y_0}{3\kappa+1} + \right. \\ & \left. \frac{6y_0}{3\kappa+1} + \left(-\frac{30\kappa}{(\kappa+1)^2} - \frac{2}{(\kappa+1)^2}\right)H(0; y_0)\right)H(0, 0, 0; Y) + \left(\frac{8d_1^2 y_0^3}{3(3\kappa+1)} + \frac{56d_1^2 \kappa y_0^3}{3(3\kappa+1)} + \frac{64\kappa y_0^3}{3(3\kappa+1)} - \frac{12d_1^2 y_0^2}{3\kappa+1} - \right. \\ & \left. \frac{84d_1^2 \kappa y_0^2}{3\kappa+1} - \frac{96\kappa y_0^2}{3\kappa+1} + \frac{24d_1^2 y_0}{3\kappa+1} + \frac{168d_1^2 \kappa y_0}{3\kappa+1} + \frac{192\kappa y_0}{3\kappa+1}\right)H(0, 0, 1; y_0) + \left(\frac{8d_1^2 y_0^3}{3(3\kappa+1)} + \frac{56d_1^2 \kappa y_0^3}{3(3\kappa+1)} + \frac{64\kappa y_0^3}{3(3\kappa+1)} - \right. \\ & \left. \frac{12d_1^2 y_0^2}{3\kappa+1} - \frac{84d_1^2 \kappa y_0^2}{3\kappa+1} - \frac{96\kappa y_0^2}{3\kappa+1} + \frac{24d_1^2 y_0}{3\kappa+1} + \frac{168d_1^2 \kappa y_0}{3\kappa+1} + \frac{192\kappa y_0}{3\kappa+1}\right)H(0, 1, 0; y_0) + \left(\frac{10\kappa y_0^3}{3\kappa+1} + \frac{2y_0^3}{3(3\kappa+1)} - \right. \end{aligned}$$



$$\begin{aligned}
 & \frac{45\kappa y_0^2}{3\kappa+1} - \frac{3y_0^2}{3\kappa+1} + \frac{90\kappa y_0}{3\kappa+1} + \frac{6y_0}{3\kappa+1} + \left( -\frac{30\kappa}{(\kappa+1)^2} - \frac{2}{(\kappa+1)^2} \right) H(0; y_0) \Big) H(0, 1, 0; Y) + \left( \frac{4d_1^2 y_0^3}{3(3\kappa+1)} + \right. \\
 & \frac{4d_1^2 \kappa y_0^3}{3\kappa+1} + \frac{32d_1^2 \kappa y_0^3}{3(3\kappa+1)} + \frac{16\kappa y_0^3}{3(3\kappa+1)} - \frac{6d_1^2 y_0^2}{3\kappa+1} - \frac{18d_1^2 \kappa y_0^2}{3\kappa+1} - \frac{48d_1^2 \kappa y_0^2}{3\kappa+1} - \frac{24\kappa y_0^2}{3\kappa+1} + \frac{12d_1^2 y_0}{3\kappa+1} + \frac{36d_1^2 \kappa y_0}{3\kappa+1} + \frac{96d_1^2 \kappa y_0}{3\kappa+1} + \\
 & \left. \frac{48\kappa y_0}{3\kappa+1} \right) H(0, 1, 1; y_0) + \left( \frac{8d_1^2 y_0^3}{3(3\kappa+1)} + \frac{56d_1^2 \kappa y_0^3}{3(3\kappa+1)} + \frac{64\kappa y_0^3}{3(3\kappa+1)} - \frac{12d_1^2 y_0^2}{3\kappa+1} - \frac{84d_1^2 \kappa y_0^2}{3\kappa+1} - \frac{96\kappa y_0^2}{3\kappa+1} + \frac{24d_1^2 y_0}{3\kappa+1} + \right. \\
 & \left. \frac{168d_1^2 \kappa y_0}{3\kappa+1} + \frac{192\kappa y_0}{3\kappa+1} - \frac{44d_1^2}{3(3\kappa+1)} - \frac{308d_1^2 \kappa}{3(3\kappa+1)} - \frac{352\kappa}{3(3\kappa+1)} \right) H(1, 0, 0; y_0) + \left( \frac{14\kappa y_0^3}{3(3\kappa+1)} + \frac{2y_0^3}{3(3\kappa+1)} - \frac{21\kappa y_0^2}{3\kappa+1} - \right. \\
 & \frac{3y_0^2}{3\kappa+1} + \frac{42\kappa y_0}{3\kappa+1} + \frac{6y_0}{3\kappa+1} + \left( -\frac{14\kappa}{(\kappa+1)^2} - \frac{2}{(\kappa+1)^2} \right) H(0; y_0) \Big) H(1, 0, 0; Y) + \left( \frac{4d_1^2 y_0^3}{3(3\kappa+1)} + \frac{4d_1^2 \kappa y_0^3}{3\kappa+1} + \right. \\
 & \frac{32d_1^2 \kappa y_0^3}{3(3\kappa+1)} + \frac{16\kappa y_0^3}{3(3\kappa+1)} - \frac{6d_1^2 y_0^2}{3\kappa+1} - \frac{18d_1^2 \kappa y_0^2}{3\kappa+1} - \frac{48d_1^2 \kappa y_0^2}{3\kappa+1} - \frac{24\kappa y_0^2}{3\kappa+1} + \frac{12d_1^2 y_0}{3\kappa+1} + \frac{36d_1^2 \kappa y_0}{3\kappa+1} + \frac{96d_1^2 \kappa y_0}{3\kappa+1} + \frac{48\kappa y_0}{3\kappa+1} - \\
 & \frac{22d_1^2}{3(3\kappa+1)} - \frac{22d_1^2 \kappa}{3\kappa+1} - \frac{176d_1^2 \kappa}{3(3\kappa+1)} - \frac{88\kappa}{3(3\kappa+1)} \Big) H(1, 0, 1; y_0) + \left( \frac{4d_1^2 y_0^3}{3(3\kappa+1)} + \frac{4d_1^2 \kappa y_0^3}{3\kappa+1} + \frac{32d_1^2 \kappa y_0^3}{3(3\kappa+1)} + \frac{16\kappa y_0^3}{3(3\kappa+1)} - \right. \\
 & \frac{6d_1^2 y_0^2}{3\kappa+1} - \frac{18d_1^2 \kappa y_0^2}{3\kappa+1} - \frac{48d_1^2 \kappa y_0^2}{3\kappa+1} - \frac{24\kappa y_0^2}{3\kappa+1} + \frac{12d_1^2 y_0}{3\kappa+1} + \frac{36d_1^2 \kappa y_0}{3\kappa+1} + \frac{96d_1^2 \kappa y_0}{3\kappa+1} + \frac{48\kappa y_0}{3\kappa+1} - \frac{22d_1^2}{3(3\kappa+1)} - \frac{22d_1^2 \kappa}{3\kappa+1} - \\
 & \left. \frac{176d_1^2 \kappa}{3(3\kappa+1)} - \frac{88\kappa}{3(3\kappa+1)} \right) H(1, 1, 0; y_0) + \left( \frac{14\kappa y_0^3}{3(3\kappa+1)} + \frac{2y_0^3}{3(3\kappa+1)} - \frac{21\kappa y_0^2}{3\kappa+1} - \frac{3y_0^2}{3\kappa+1} + \frac{42\kappa y_0}{3\kappa+1} + \frac{6y_0}{3\kappa+1} + \left( -\frac{14\kappa}{(\kappa+1)^2} - \right. \right. \\
 & \left. \left. \frac{2}{(\kappa+1)^2} \right) H(0; y_0) \Big) H(1, 1, 0; Y) + \left( \frac{2\kappa y_0^3 d_1^3}{3(3\kappa+1)} + \frac{2y_0^3 d_1^3}{3(3\kappa+1)} - \frac{3\kappa y_0^2 d_1^3}{3\kappa+1} - \frac{3y_0^2 d_1^3}{3\kappa+1} + \frac{6\kappa y_0 d_1^3}{3\kappa+1} + \frac{6y_0 d_1^3}{3\kappa+1} - \right. \\
 & \frac{11\kappa d_1^3}{3(3\kappa+1)} - \frac{11d_1^3}{3(3\kappa+1)} + \frac{4\kappa y_0^3 d_1^2}{3\kappa+1} - \frac{18\kappa y_0^2 d_1^2}{3\kappa+1} + \frac{36\kappa y_0 d_1^2}{3\kappa+1} - \frac{22\kappa d_1^2}{3\kappa+1} + \frac{4\kappa y_0^2 d_1^2}{3\kappa+1} - \frac{18\kappa y_0 d_1^2}{3\kappa+1} + \frac{36\kappa y_0 d_1^2}{3\kappa+1} - \frac{22\kappa d_1^2}{3\kappa+1} + \\
 & \frac{4\kappa y_0^2}{3(3\kappa+1)} - \frac{6\kappa y_0^2}{3\kappa+1} + \frac{12\kappa y_0}{3\kappa+1} - \frac{22\kappa}{3(3\kappa+1)} \Big) H(1, 1, 1; y_0) + \left( -\frac{240\kappa}{(\kappa+1)^2} - \frac{16}{(\kappa+1)^2} \right) H(0, 0, 0, 0; y_0) + \left( -\frac{15\kappa}{(\kappa+1)^2} - \frac{1}{(\kappa+1)^2} \right) H(0, 0, 0, 0; Y) + \left( -\frac{56\kappa d_1^2}{(\kappa+1)^2} - \frac{8d_1^2}{(\kappa+1)^2} - \frac{64\kappa}{(\kappa+1)^2} \right) H(0, 0, 0, 1; y_0) + \left( -\frac{56\kappa d_1^2}{(\kappa+1)^2} - \frac{8d_1^2}{(\kappa+1)^2} - \frac{64\kappa}{(\kappa+1)^2} \right) H(0, 0, 1, 0; y_0) + \left( -\frac{12\kappa d_1^2}{(\kappa+1)^2} - \frac{4d_1^2}{(\kappa+1)^2} - \frac{32\kappa d_1^2}{(\kappa+1)^2} - \frac{16\kappa}{(\kappa+1)^2} \right) H(0, 0, 1, 0; Y) + \left( -\frac{12\kappa d_1^2}{(\kappa+1)^2} - \frac{4d_1^2}{(\kappa+1)^2} - \frac{32\kappa d_1^2}{(\kappa+1)^2} - \frac{16\kappa}{(\kappa+1)^2} \right) H(0, 1, 0, 0; y_0) + \left( -\frac{7\kappa}{(\kappa+1)^2} - \frac{16\kappa}{(\kappa+1)^2} \right) H(0, 1, 0, 0; Y) + \left( -\frac{12\kappa d_1^2}{(\kappa+1)^2} - \frac{4d_1^2}{(\kappa+1)^2} - \frac{32\kappa d_1^2}{(\kappa+1)^2} - \frac{16\kappa}{(\kappa+1)^2} \right) H(0, 1, 0, 1; y_0) + \left( -\frac{12\kappa d_1^2}{(\kappa+1)^2} - \frac{4d_1^2}{(\kappa+1)^2} - \frac{32\kappa d_1^2}{(\kappa+1)^2} - \frac{16\kappa}{(\kappa+1)^2} \right) H(0, 1, 0, 1; Y) + \left( -\frac{2\kappa d_1^3}{(\kappa+1)^2} - \frac{2d_1^3}{(\kappa+1)^2} - \frac{12\kappa d_1^2}{(\kappa+1)^2} - \frac{12\kappa d_1^2}{(\kappa+1)^2} - \frac{4\kappa}{(\kappa+1)^2} \right) H(0, 1, 1, 1; y_0) + \left( -\frac{11\kappa}{(\kappa+1)^2} - \frac{1}{(\kappa+1)^2} \right) H(1, 0, 0, 0; Y) + \left( -\frac{11\kappa}{(\kappa+1)^2} - \frac{1}{(\kappa+1)^2} \right) H(1, 0, 1, 0; Y) - H(1, 1, 0, 0; Y) - H(1, 1, 1, 0; Y) + H(0; y_0) \left( \frac{4d_1^2 y_0^3}{27(3\kappa+1)} - \frac{16d_1^2 y_0^3}{27(3\kappa+1)} + \frac{4d_1^2 \kappa y_0^3}{9(3\kappa+1)} - \frac{80d_1^2 \kappa y_0^3}{27(3\kappa+1)} + \frac{128\kappa y_0^3}{27(3\kappa+1)} + \frac{16y_0^3}{27(3\kappa+1)} - \frac{17d_1^2 y_0^2}{18(3\kappa+1)} + \frac{44d_1^2 y_0^2}{9(3\kappa+1)} - \frac{17d_1^2 \kappa y_0^2}{6(3\kappa+1)} + \frac{80d_1^2 \kappa y_0^2}{3(3\kappa+1)} - \frac{164\kappa y_0^2}{3(3\kappa+1)} - \frac{6y_0^2}{3\kappa+1} + \frac{49d_1^2 y_0}{9(3\kappa+1)} - \frac{302d_1^2 y_0}{9(3\kappa+1)} + \frac{49d_1^2 \kappa y_0}{3(3\kappa+1)} - \frac{574d_1^2 \kappa y_0}{3(3\kappa+1)} + \frac{1420\kappa y_0}{3(3\kappa+1)} + \frac{48y_0}{3\kappa+1} - \frac{92\kappa \zeta_3}{3\kappa+1} - \frac{4\zeta_3}{3\kappa+1} \right) + H(0; Y) \left( \frac{2d_1^2 y_0^3}{27(3\kappa+1)} - \frac{8d_1^2 y_0^3}{27(3\kappa+1)} + \frac{9d_1^2 \kappa y_0^3}{9(3\kappa+1)} - \frac{2d_1^2 \kappa y_0^3}{27(3\kappa+1)} + \frac{4d_1^2 \kappa y_0^3}{27(3\kappa+1)} + \frac{64\kappa y_0^3}{27(3\kappa+1)} + \frac{8y_0^3}{27(3\kappa+1)} - \frac{17d_1^2 y_0^2}{36(3\kappa+1)} + \frac{22d_1^2 y_0^2}{9(3\kappa+1)} - \frac{17d_1^2 \kappa y_0^2}{12(3\kappa+1)} + \frac{40d_1^2 \kappa y_0^2}{3(3\kappa+1)} - \frac{82\kappa y_0^2}{3(3\kappa+1)} - \frac{3y_0^2}{3\kappa+1} + \frac{49d_1^2 y_0}{18(3\kappa+1)} - \frac{151d_1^2 y_0}{9(3\kappa+1)} + \frac{49d_1^2 \kappa y_0}{6(3\kappa+1)} - \frac{287d_1^2 \kappa y_0}{3(3\kappa+1)} + \frac{710\kappa y_0}{3(3\kappa+1)} + \frac{24y_0}{3\kappa+1} + \left( \frac{4d_1^2 y_0^3}{9(3\kappa+1)} + \frac{28d_1^2 \kappa y_0^3}{9(3\kappa+1)} - \frac{88\kappa y_0^3}{9(3\kappa+1)} - \frac{8y_0^3}{9(3\kappa+1)} - \frac{7d_1^2 y_0^2}{3(3\kappa+1)} - \frac{49d_1^2 \kappa y_0^2}{3(3\kappa+1)} + \frac{214\kappa y_0^2}{3(3\kappa+1)} + \frac{6y_0^2}{3\kappa+1} + \frac{22d_1^2 y_0}{3(3\kappa+1)} + \frac{154d_1^2 \kappa y_0}{3(3\kappa+1)} - \frac{904\kappa y_0}{3(3\kappa+1)} - \frac{24y_0}{3\kappa+1} \right) H(0; y_0) + \left( \frac{2d_1^2 y_0^3}{9(3\kappa+1)} - \frac{4d_1^2 y_0^3}{9(3\kappa+1)} + \frac{2d_1^2 \kappa y_0^3}{3(3\kappa+1)} - \frac{4d_1^2 \kappa y_0^3}{3(3\kappa+1)} - \frac{8\kappa y_0^3}{3(3\kappa+1)} - \frac{7d_1^2 y_0^2}{6(3\kappa+1)} + \frac{3d_1^2 y_0^2}{3\kappa+1} - \frac{7d_1^2 \kappa y_0^2}{2(3\kappa+1)} + \frac{35d_1^2 \kappa y_0^2}{3(3\kappa+1)} + \frac{58\kappa y_0^2}{3(3\kappa+1)} + \frac{11d_1^2 y_0}{3(3\kappa+1)} - \frac{12d_1^2 y_0}{3\kappa+1} + \frac{11d_1^2 \kappa y_0}{3\kappa+1} - \frac{164d_1^2 \kappa y_0}{3(3\kappa+1)} - \frac{244\kappa y_0}{3(3\kappa+1)} - \frac{49d_1^2}{18(3\kappa+1)} + \frac{85d_1^2}{9(3\kappa+1)} - \frac{49d_1^2 \kappa}{6(3\kappa+1)} + \frac{133d_1^2 \kappa}{3(3\kappa+1)} + \frac{194\kappa}{3(3\kappa+1)} \right) H(1; y_0) + \left( \frac{40\kappa y_0^3}{3\kappa+1} + \frac{8y_0^3}{3(3\kappa+1)} - \frac{180\kappa y_0^2}{3\kappa+1} - \frac{12y_0^2}{3\kappa+1} + \frac{360\kappa y_0}{3\kappa+1} + \frac{24y_0}{3\kappa+1} \right) H(0, 0; y_0) + \left( \frac{4d_1^2 y_0^3}{3(3\kappa+1)} + \frac{28d_1^2 \kappa y_0^3}{3(3\kappa+1)} + \frac{32\kappa y_0^3}{3(3\kappa+1)} - \frac{6d_1^2 y_0^2}{3\kappa+1} - \frac{42d_1^2 \kappa y_0^2}{3\kappa+1} - \frac{48\kappa y_0^2}{3\kappa+1} + \frac{12d_1^2 y_0}{3\kappa+1} + \frac{84d_1^2 \kappa y_0}{3\kappa+1} + \frac{96\kappa y_0}{3\kappa+1} \right) H(0, 1; y_0) + \left( \frac{4d_1^2 y_0^3}{3(3\kappa+1)} + \frac{28d_1^2 \kappa y_0^3}{3(3\kappa+1)} + \frac{32\kappa y_0^3}{3(3\kappa+1)} - \frac{6d_1^2 y_0^2}{3\kappa+1} - \frac{42d_1^2 \kappa y_0^2}{3\kappa+1} - \frac{48\kappa y_0^2}{3\kappa+1} + \frac{12d_1^2 y_0}{3\kappa+1} + \frac{84d_1^2 \kappa y_0}{3\kappa+1} + \frac{96\kappa y_0}{3\kappa+1} - \frac{22d_1^2}{3(3\kappa+1)} - \frac{154d_1^2 \kappa}{3(3\kappa+1)} - \frac{176\kappa}{3(3\kappa+1)} \right) H(1, 0; y_0) + \left( \frac{2d_1^2 y_0^3}{3(3\kappa+1)} + \frac{2d_1^2 \kappa y_0^3}{3\kappa+1} + \frac{16d_1^2 \kappa y_0^3}{3(3\kappa+1)} + \frac{8\kappa y_0^3}{3(3\kappa+1)} - \frac{3d_1^2 y_0^2}{3\kappa+1} - \frac{9d_1^2 \kappa y_0^2}{3\kappa+1} - \frac{24d_1^2 \kappa y_0^2}{3\kappa+1} - \frac{12\kappa y_0^2}{3\kappa+1} + \frac{6d_1^2 y_0}{3\kappa+1} + \frac{18d_1^2 \kappa y_0}{3\kappa+1} + \frac{48d_1^2 \kappa y_0}{3\kappa+1} + \frac{24\kappa y_0}{3\kappa+1} - \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \frac{11d_1'^2}{3(3\kappa+1)} - \frac{11d_1'^2 \kappa}{3\kappa+1} - \frac{88d_1' \kappa}{3(3\kappa+1)} - \frac{44\kappa}{3(3\kappa+1)} \right) H(1, 1; y_0) + \left( -\frac{120\kappa}{(\kappa+1)^2} - \frac{8}{(\kappa+1)^2} \right) H(0, 0, 0; y_0) + \\
 & \left( -\frac{28\kappa d_1'}{(\kappa+1)^2} - \frac{4d_1'}{(\kappa+1)^2} - \frac{32\kappa}{(\kappa+1)^2} \right) H(0, 0, 1; y_0) + \left( -\frac{28\kappa d_1'}{(\kappa+1)^2} - \frac{4d_1'}{(\kappa+1)^2} - \frac{32\kappa}{(\kappa+1)^2} \right) H(0, 1, 0; y_0) + \\
 & \left( -\frac{6\kappa d_1'^2}{(\kappa+1)^2} - \frac{2d_1'^2}{(\kappa+1)^2} - \frac{16\kappa d_1'}{(\kappa+1)^2} - \frac{8\kappa}{(\kappa+1)^2} \right) H(0, 1, 1; y_0) - \frac{46\kappa\zeta_3}{3\kappa+1} - \frac{2\zeta_3}{3\kappa+1} + H(1; Y) \left( \frac{7\kappa\pi^2 y_0^3}{9(3\kappa+1)} + \right. \\
 & \left. \frac{\pi^2 y_0^3}{9(3\kappa+1)} - \frac{7\kappa\pi^2 y_0^2}{2(3\kappa+1)} - \frac{\pi^2 y_0^2}{2(3\kappa+1)} + \frac{7\kappa\pi^2 y_0}{3\kappa+1} + \frac{\pi^2 y_0}{3\kappa+1} + \left( -\frac{7\pi^2 \kappa}{3(3\kappa+1)} - \frac{\pi^2}{3(3\kappa+1)} \right) H(0; y_0) - \frac{16\kappa\zeta_3}{3\kappa+1} \right) + \\
 & \frac{92\kappa y_0^3 \zeta_3}{3(3\kappa+1)} + \frac{4y_0^3 \zeta_3}{3(3\kappa+1)} - \frac{138\kappa y_0^2 \zeta_3}{3\kappa+1} - \frac{6y_0^2 \zeta_3}{3\kappa+1} + \frac{276\kappa y_0 \zeta_3}{3\kappa+1} + \frac{12y_0 \zeta_3}{3\kappa+1} + \frac{11\kappa\pi^4}{30(3\kappa+1)} + \frac{\pi^4}{30(3\kappa+1)}.
 \end{aligned}$$

## C. The $\mathcal{K}$ integrals

### C.1 The $\mathcal{K}$ integral for $\kappa = 0$

The  $\varepsilon$  expansion for this integral reads

$$\mathcal{K}(\varepsilon; y_0, 3 + d_1' \varepsilon; 0) = \frac{1}{\varepsilon^2} k_{-2}^{(0)} + \frac{1}{\varepsilon} k_{-1}^{(0)} + k_0^{(0)} + \varepsilon k_1^{(0)} + \varepsilon^2 k_2^{(0)} + \mathcal{O}(\varepsilon^3), \quad (\text{C.1})$$

where

$$k_{-2}^{(0)} = 1,$$

$$k_{-1}^{(0)} = \frac{2y_0^3}{3} - 3y_0^2 + 6y_0 - 2H(0; y_0),$$

$$\begin{aligned}
 k_0^{(0)} = & -\frac{2d_1' x^3}{9} + \frac{10x^3}{9} + \frac{7d_1' x^2}{6} - 5x^2 - \frac{11d_1' x}{3} + 14x + \left( -\frac{4x^3}{3} + 6x^2 - 12x \right) H(0; x) + \\
 & \left( -\frac{2d_1' x^3}{3} + 3d_1' x^2 - 6d_1' x + \frac{11d_1'}{3} \right) H(1; x) + 4H(0, 0; x) + 2d_1' H(0, 1; x) - \frac{\pi^2}{6},
 \end{aligned}$$

$$\begin{aligned}
 k_1^{(0)} = & \frac{2d_1'^2 x^3}{27} - \frac{14d_1' x^3}{27} - \frac{\pi^2 x^3}{9} + \frac{56x^3}{27} - \frac{17d_1'^2 x^2}{36} + \frac{28d_1' x^2}{9} + \frac{\pi^2 x^2}{2} - 9x^2 + \frac{49d_1'^2 x}{18} - \frac{157d_1' x}{9} - \pi^2 x + \\
 & 32x + \left( \frac{4d_1' x^3}{9} - \frac{20x^3}{9} - \frac{7d_1' x^2}{3} + 10x^2 + \frac{22d_1' x}{3} - 28x + \frac{\pi^2}{3} \right) H(0; x) + \left( \frac{2d_1'^2 x^3}{9} - \frac{10d_1' x^3}{9} - \frac{7d_1'^2 x^2}{6} + \right. \\
 & 5d_1' x^2 + \frac{11d_1'^2 x}{3} - 14d_1' x - \frac{49d_1'^2}{18} + \frac{91d_1'}{9} \left. \right) H(1; x) + \left( \frac{8x^3}{3} - 12x^2 + 24x \right) H(0, 0; x) + \left( \frac{4d_1' x^3}{3} - 6d_1' x^2 + \right. \\
 & 12d_1' x \left. \right) H(0, 1; x) + \left( \frac{4d_1' x^3}{3} - 6d_1' x^2 + 12d_1' x - \frac{22d_1'}{3} \right) H(1, 0; x) + \left( \frac{2d_1'^2 x^3}{3} - 3d_1'^2 x^2 + 6d_1'^2 x - \right. \\
 & \left. \frac{11d_1'^2}{3} \right) H(1, 1; x) - 8H(0, 0, 0; x) - 4d_1' H(0, 0, 1; x) - 4d_1' H(0, 1, 0; x) - 2d_1'^2 H(0, 1, 1; x) - 2\zeta_3,
 \end{aligned}$$

$$\begin{aligned}
 k_2^{(0)} = & -\frac{2}{81} x^3 d_1'^3 + \frac{43x^2 d_1'^3}{216} - \frac{251x d_1'^3}{108} + 2H(0, 1, 1, 1; x) d_1'^3 + \frac{2x^3 d_1'^2}{9} - \frac{191x^2 d_1'^2}{108} + \frac{548x d_1'^2}{27} + \\
 & 4H(0, 0, 1, 1; x) d_1'^2 + 4H(0, 1, 0, 1; x) d_1'^2 + 4H(0, 1, 1, 0; x) d_1'^2 + \frac{1}{27} \pi^2 x^3 d_1' - \frac{28x^3 d_1'}{27} - \\
 & \frac{7}{36} \pi^2 x^2 d_1' + \frac{355x^2 d_1'}{54} + \frac{11}{18} \pi^2 x d_1' - \frac{1619x d_1'}{27} + 8H(0, 0, 0, 1; x) d_1' + 8H(0, 0, 1, 0; x) d_1' + \\
 & 8H(0, 1, 0, 0; x) d_1' - \frac{5\pi^2 x^3}{27} + \frac{328x^3}{81} + \frac{5\pi^2 x^2}{6} - 17x^2 - \frac{7\pi^2 x}{3} + 72x + \left( -\frac{2}{27} x^3 d_1'^3 + \frac{17x^2 d_1'^3}{36} - \right. \\
 & \left. \frac{49x d_1'^3}{18} + \frac{251d_1'^3}{108} + \frac{14x^3 d_1'^2}{27} - \frac{28x^2 d_1'^2}{9} + \frac{157x d_1'^2}{9} - \frac{401d_1'^2}{27} + \frac{1}{9} \pi^2 x^3 d_1' - \frac{56x^3 d_1'}{27} - \frac{1}{2} \pi^2 x^2 d_1' + 9x^2 d_1' + \right. \\
 & \left. \pi^2 x d_1' - 32x d_1' - \frac{11\pi^2 d_1'}{18} + \frac{677d_1'}{27} \right) H(1; x) + \left( -\frac{8d_1' x^3}{9} + \frac{40x^3}{9} + \frac{14d_1' x^2}{3} - 20x^2 - \frac{44d_1' x}{3} + 56x - \right. \\
 & \left. \frac{2\pi^2}{3} \right) H(0, 0; x) + \left( -\frac{4}{9} d_1'^2 x^3 + \frac{20d_1' x^3}{9} + \frac{7d_1'^2 x^2}{3} - 10d_1' x^2 - \frac{22d_1'^2 x}{3} + 28d_1' x - \frac{d_1' \pi^2}{3} \right) H(0, 1; x) + \\
 & \left( -\frac{4}{9} d_1'^2 x^3 + \frac{20d_1' x^3}{9} + \frac{7d_1'^2 x^2}{3} - 10d_1' x^2 - \frac{22d_1'^2 x}{3} + 28d_1' x + \frac{49d_1'^2}{9} - \frac{182d_1'}{9} \right) H(1, 0; x) + \left( -\right. \\
 & \left. \frac{2}{9} x^3 d_1'^3 + \frac{7x^2 d_1'^3}{6} - \frac{11x d_1'^3}{3} + \frac{49d_1'^3}{18} + \frac{10x^3 d_1'^2}{9} - 5x^2 d_1'^2 + 14x d_1'^2 - \frac{91d_1'^2}{9} \right) H(1, 1; x) + \left( -\frac{16x^3}{3} + \right.
 \end{aligned}$$

$$\begin{aligned}
 & 24x^2 - 48x)H(0, 0, 0; x) + \left(-\frac{8d_1'x^3}{3} + 12d_1'x^2 - 24d_1'x\right)H(0, 0, 1; x) + \left(-\frac{8d_1'x^3}{3} + 12d_1'x^2 - 24d_1'x\right)H(0, 1, 0; x) \\
 & + \left(-\frac{4}{3}d_1'^2x^3 + 6d_1'^2x^2 - 12d_1'^2x\right)H(0, 1, 1; x) + \left(-\frac{8d_1'x^3}{3} + 12d_1'x^2 - 24d_1'x + \frac{44d_1'}{3}\right)H(1, 0, 0; x) \\
 & + \left(-\frac{4}{3}d_1'^2x^3 + 6d_1'^2x^2 - 12d_1'^2x + \frac{22d_1'^2}{3}\right)H(1, 0, 1; x) + \left(-\frac{4}{3}d_1'^2x^3 + 6d_1'^2x^2 - 12d_1'^2x + \frac{22d_1'^2}{3}\right)H(1, 1, 0; x) \\
 & + \left(-\frac{2}{3}x^3d_1'^3 + 3x^2d_1'^3 - 6xd_1'^3 + \frac{11d_1'^3}{3}\right)H(1, 1, 1; x) + 16H(0, 0, 0, 0; x) + H(0; x) \\
 & \left(-\frac{4}{27}d_1'^2x^3 + \frac{28d_1'x^3}{27} + \frac{2\pi^2x^3}{9} - \frac{112x^3}{27} + \frac{17d_1'^2x^2}{18} - \frac{56d_1'x^2}{9} - \pi^2x^2 + 18x^2 - \frac{49d_1'^2x}{9} + \frac{314d_1'x}{9} + 2\pi^2x - 64x + 4\zeta_3\right) - \frac{4}{3}x^3\zeta_3 + 6x^2\zeta_3 - 12x\zeta_3 - \frac{\pi^4}{40}.
 \end{aligned}$$

## C.2 The $\mathcal{K}$ integral for $\kappa = 1$

The  $\varepsilon$  expansion for this integral reads

$$\mathcal{K}(\varepsilon; y_0, 3 + d_1'\varepsilon; ) = \frac{1}{\varepsilon^2}k_{-2}^{(1)} + \frac{1}{\varepsilon}k_{-1}^{(1)} + k_0^{(1)} + \varepsilon k_1^{(1)} + \varepsilon^2 k_2^{(1)} + \mathcal{O}(\varepsilon^3), \quad (\text{C.2})$$

where

$$\begin{aligned}
 k_{-2}^{(1)} &= \frac{1}{4}, \\
 k_{-1}^{(1)} &= \frac{y_0^3}{3} - \frac{3y_0^2}{2} + 3y_0 - H(0; y_0), \\
 k_0^{(1)} &= -\frac{d_1'x^3}{9} + x^3 + \frac{7d_1'x^2}{12} - \frac{53x^2}{12} - \frac{11d_1'x}{6} + \frac{73x}{6} + \left(-\frac{4x^3}{3} + 6x^2 - 12x\right)H(0; x) + \left(-\frac{d_1'x^3}{3} - \frac{x^3}{3} + \frac{3d_1'x^2}{2} + \frac{3x^2}{2} - 3d_1'x - 3x + \frac{11d_1'}{6} + \frac{11}{6}\right)H(1; x) + 4H(0, 0; x) + (d_1' + 1)H(0, 1; x) - \frac{\pi^2}{12}, \\
 k_1^{(1)} &= \frac{d_1'^2x^3}{27} - \frac{4d_1'x^3}{9} - \frac{\pi^2x^3}{9} + \frac{7x^3}{3} - \frac{17d_1'^2x^2}{72} + \frac{95d_1'x^2}{36} + \frac{\pi^2x^2}{2} - \frac{259x^2}{24} + \frac{49d_1'^2x}{36} - \frac{265d_1'x}{18} - \pi^2x + \frac{515x}{12} + \left(\frac{4d_1'x^3}{9} - 4x^3 - \frac{7d_1'x^2}{3} + \frac{53x^2}{3} + \frac{22d_1'x}{3} - \frac{146x}{3} + \frac{\pi^2}{3}\right)H(0; x) + \left(\frac{d_1'^2x^3}{9} - \frac{8d_1'x^3}{9} - x^3 - \frac{7d_1'x^2}{12} + \frac{23d_1'x^2}{6} + \frac{61x^2}{12} + \frac{11d_1'^2x}{6} - \frac{31d_1'x}{3} - \frac{83x}{6} - \frac{49d_1'^2}{36} + \frac{133d_1'}{18} + \frac{39}{4}\right)H(1; x) + \left(\frac{16x^3}{3} - 24x^2 + 48x\right)H(0, 0; x) + \left(\frac{4d_1'x^3}{3} + \frac{2x^3}{3} - 6d_1'x^2 - 3x^2 + 12d_1'x + 6x + 2\right)H(0, 1; x) + \left(\frac{4d_1'x^3}{3} + \frac{4x^3}{3} - 6d_1'x^2 - 6x^2 + 12d_1'x + 12x - \frac{22d_1'}{3} - \frac{22}{3}\right)H(1, 0; x) + \left(\frac{d_1'^2x^3}{3} + \frac{2d_1'x^3}{3} - \frac{x^3}{3} - \frac{3d_1'^2x^2}{2} - 3d_1'x^2 + \frac{3x^2}{2} + 3d_1'^2x + 6d_1'x - 3x - \frac{11d_1'^2}{6} - \frac{11d_1'}{3} + \frac{11}{6}\right)H(1, 1; x) - 16H(0, 0, 0; x) + (-4d_1' - 2)H(0, 0, 1; x) + (-4d_1' - 4)H(0, 1, 0; x) + \left(-d_1'^2 - 2d_1' + 1\right)H(0, 1, 1; x) - \frac{3\zeta_3}{2}, \\
 k_2^{(1)} &= -\frac{1}{81}x^3d_1'^3 + \frac{43x^2d_1'^3}{432} - \frac{251x d_1'^3}{216} + \frac{5x^3d_1'^2}{27} - \frac{635x^2 d_1'^2}{432} + \frac{3631xd_1'^2}{216} + \frac{1}{27}\pi^2x^3 d_1' - \frac{11x^3d_1'}{9} - \frac{7}{36}\pi^2x^2 d_1' + \frac{1195x^2d_1'}{144} + \frac{11}{18}\pi^2x d_1' - \frac{5831x d_1'}{72} - \frac{\pi^2x^3}{3} + 5x^3 + \frac{53\pi^2x^2}{36} - \frac{1169x^2}{48} - \frac{73\pi^2x}{18} + \frac{1139x}{8} + \left(-\frac{1}{27}x^3d_1'^3 + \frac{17x^2d_1'^3}{72} - \frac{49x d_1'^3}{36} + \frac{251d_1'^3}{216} + \frac{11x^3d_1'^2}{27} - \frac{173x^2d_1'^2}{72} + \frac{481xd_1'^2}{36} - \frac{2455d_1'^2}{216} + \frac{1}{9}\pi^2x^3 d_1' - \frac{17x^3 d_1'}{9} - \frac{1}{2}\pi^2x^2 d_1' + \frac{571x^2d_1'}{72} + \pi^2x d_1' - \frac{1013xd_1'}{36} - \frac{11\pi^2d_1'}{18} + \frac{1591d_1'}{72} + \frac{\pi^2x^3}{9} - \frac{7x^3}{3} - \frac{\pi^2x^2}{2} + \frac{307x^2}{24} + \pi^2x - \frac{191x}{4} - \frac{11\pi^2}{18} + \frac{895}{24}\right)H(1; x) + \left(-\frac{16d_1'x^3}{9} + 16x^3 + \frac{28d_1'x^2}{3} - \frac{212x^2}{3} - \frac{88d_1'x}{3} + \frac{584x}{3} - \frac{4\pi^2}{3}\right)H(0, 0; x) + \left(-\frac{4}{9}d_1'^2x^3 + \frac{34d_1'x^3}{9} + 2x^3 + \frac{7d_1'^2x^2}{3} - \frac{33d_1'x^2}{2} - \frac{19x^2}{2} - \frac{22d_1'^2x}{3} + 45d_1'x + 26x - \frac{d_1'\pi^2}{3} - \frac{\pi^2}{3} + 4\right)H(0, 1; x) + \left(-\frac{4}{9}d_1'^2x^3 + \frac{32d_1'x^3}{9} + 4x^3 + \frac{7d_1'^2x^2}{3} - \frac{46d_1'x^2}{3} - \frac{61x^2}{3} - \frac{22d_1'^2x}{3} + \frac{124d_1'x}{3} + \frac{166x}{3} + \frac{49d_1'^2}{9} - \frac{266d_1'}{9} - 39\right)H(1, 0; x) + \left(-\frac{1}{9}x^3d_1'^3 + \frac{7x^2 d_1'^3}{12} - \frac{11xd_1'^3}{6} + \frac{49d_1'^3}{36} + \frac{7x^3 d_1'^2}{9} - \frac{13x^2d_1'^2}{4} + \frac{17xd_1'^2}{2} - \frac{217d_1'^2}{36} + \frac{19x^3d_1'}{9} - \right)
 \end{aligned}$$

$$\begin{aligned}
& \left. \frac{43x^2d'_1}{4} + \frac{59x d'_1}{2} - \frac{751d'_1}{36} - x^3 + \frac{61x^2}{12} - \frac{83x}{6} + \frac{39}{4} \right) H(1, 1; x) + \left( -\frac{64x^3}{3} + 96x^2 - 192x \right) H(0, 0, 0; x) + \\
& \left( -\frac{16d'_1x^3}{3} - 2x^3 + 24d'_1x^2 + 9x^2 - 48d'_1x - 18x - 2 \right) H(0, 0, 1; x) + \left( -\frac{16d'_1x^3}{3} - \frac{8x^3}{3} + 24d'_1x^2 + \right. \\
& \left. 12x^2 - 48d'_1x - 24x - 8 \right) H(0, 1, 0; x) + \left( -\frac{4}{3}d_1'^2x^3 - \frac{4d_1'x^3}{3} + \frac{2x^3}{3} + 6d_1'^2x^2 + 6d_1'x^2 - 3x^2 - \right. \\
& \left. 12d_1'^2x - 12d_1'x + 6x - 4d_1' + 2 \right) H(0, 1, 1; x) + \left( -\frac{16d_1'x^3}{3} - \frac{16x^3}{3} + 24d_1'x^2 + 24x^2 - 48d_1'x - \right. \\
& \left. 48x + \frac{88d_1'}{3} + \frac{88}{3} \right) H(1, 0, 0; x) + \left( -\frac{4}{3}d_1'^2x^3 - 2d_1'x^3 + 6d_1'^2x^2 + 9d_1'x^2 - 12d_1'^2x - 18d_1'x + \frac{22d_1'^2}{3} + \right. \\
& \left. 11d_1' \right) H(1, 0, 1; x) + \left( -\frac{4}{3}d_1'^2x^3 - \frac{8d_1'x^3}{3} + \frac{4x^3}{3} + 6d_1'^2x^2 + 12d_1'x^2 - 6x^2 - 12d_1'^2x - 24d_1'x + 12x + \right. \\
& \left. \frac{22d_1'^2}{3} + \frac{44d_1'}{3} - \frac{22}{3} \right) H(1, 1, 0; x) + \left( -\frac{1}{3}x^3d_1'^3 + \frac{3x^2d_1'^3}{2} - 3xd_1'^3 + \frac{11d_1'^3}{6} - x^3d_1'^2 + \frac{9x^2d_1'^2}{2} - 9xd_1'^2 + \right. \\
& \left. \frac{11d_1'^2}{2} + x^3d_1' - \frac{9x^2d_1'}{2} + 9xd_1' - \frac{11d_1'}{2} - \frac{x^3}{3} + \frac{3x^2}{2} - 3x + \frac{11}{6} \right) H(1, 1, 1; x) + 64H(0, 0, 0, 0; x) + \\
& (16d_1' + 6) H(0, 0, 0, 1; x) + (16d_1' + 8)H(0, 0, 1, 0; x) + \left( 4d_1'^2 + 4d_1' - 2 \right) H(0, 0, 1, 1; x) + \\
& (16d_1' + 16)H(0, 1, 0, 0; x) + \left( 4d_1'^2 + 6d_1' \right) H(0, 1, 0, 1; x) + \left( 4d_1'^2 + 8d_1' - 4 \right) H(0, 1, 1, 0; x) + \\
& \left( d_1'^3 + 3d_1'^2 - 3d_1' + 1 \right) H(0, 1, 1, 1; x) + H(0; x) \left( -\frac{4}{27}d_1'^2x^3 + \frac{16d_1'x^3}{9} + \frac{4\pi^2x^3}{9} - \frac{28x^3}{3} + \frac{17d_1'^2x^2}{18} - \right. \\
& \left. \frac{95d_1'x^2}{9} - 2\pi^2x^2 + \frac{259x^2}{6} - \frac{49d_1'^2x}{9} + \frac{530d_1'x}{9} + 4\pi^2x - \frac{515x}{3} + 6\zeta_3 \right) - 2x^3\zeta_3 + 9x^2\zeta_3 - 18x\zeta_3 - \frac{11\pi^4}{360}.
\end{aligned}$$

## D. The $\mathcal{A}$ -type collinear integrals

### D.1 The $\mathcal{A}$ integral for $k = 0$ and arbitrary $\kappa$

The  $\varepsilon$  expansion for this integral reads

$$\begin{aligned}
\mathcal{I}(x, \varepsilon; \alpha_0, 3 + d_1\varepsilon; \kappa, 0, 0, g_A) &= x \mathcal{A}(\varepsilon, x; 3 + d_1\varepsilon; \kappa, 2) \\
&= \frac{1}{\varepsilon} a_{-1}^{(\kappa, 0)} + a_0^{(\kappa, 0)} + \varepsilon a_1^{(\kappa, 0)} + \varepsilon^2 a_2^{(\kappa, 0)} + \mathcal{O}(\varepsilon^3), \tag{D.1}
\end{aligned}$$

where

$$\begin{aligned}
a_{-1}^{(\kappa, 0)} &= -\frac{1}{(\kappa+1)}, \\
a_0^{(\kappa, 0)} &= \frac{\alpha_0^4}{4(x-1)} + \frac{\kappa\alpha_0^4}{4(\kappa+1)} + \frac{\alpha_0^4}{4(\kappa+1)} - \frac{\alpha_0^3}{x-1} - \frac{4\kappa\alpha_0^3}{3(\kappa+1)} - \frac{4\alpha_0^3}{3(\kappa+1)} + \frac{\alpha_0^3}{3(x-1)^2} + \frac{3\alpha_0^2}{2(x-1)} + \frac{3\kappa\alpha_0^2}{\kappa+1} + \\
& \frac{3\alpha_0^2}{\kappa+1} - \frac{\alpha_0^2}{(x-1)^2} + \frac{\alpha_0^2}{2(x-1)^3} - \frac{\alpha_0}{x-1} - \frac{4\kappa\alpha_0}{\kappa+1} - \frac{4\alpha_0}{\kappa+1} + \frac{\alpha_0}{(x-1)^2} - \frac{\alpha_0}{(x-1)^3} + \frac{\alpha_0}{(x-1)^4} + \left( 1 + \frac{1}{(x-1)^5} \right) H(0; \alpha_0) + \left( 1 - \frac{1}{(x-1)^5} \right) H(0; x) + \frac{H(c_1(\alpha_0); x)}{(x-1)^5} - \frac{2}{\kappa+1}, \\
a_1^{(\kappa, 0)} &= -\frac{d_1\alpha_0^4}{8(\kappa+1)} - \frac{d_1\kappa\alpha_0^4}{8(\kappa+1)} - \frac{d_1\kappa\alpha_0^4}{8(x-1)(\kappa+1)} + \frac{7\kappa\alpha_0^4}{8(x-1)(\kappa+1)} + \frac{7\kappa\alpha_0^4}{8(\kappa+1)} - \frac{d_1\alpha_0^4}{8(x-1)(\kappa+1)} + \\
& \frac{5\alpha_0^4}{8(x-1)(\kappa+1)} + \frac{5\alpha_0^4}{8(\kappa+1)} + \frac{13d_1\alpha_0^3}{18(\kappa+1)} + \frac{13d_1\kappa\alpha_0^3}{18(\kappa+1)} + \frac{d_1\kappa\alpha_0^3}{2(x-1)(\kappa+1)} - \frac{7\kappa\alpha_0^3}{2(x-1)(\kappa+1)} - \frac{2d_1\kappa\alpha_0^3}{9(x-1)^2(\kappa+1)} + \\
& \frac{19\kappa\alpha_0^3}{12(x-1)^2(\kappa+1)} - \frac{61\kappa\alpha_0^3}{12(\kappa+1)} + \frac{d_1\alpha_0^3}{2(x-1)(\kappa+1)} - \frac{5\alpha_0^3}{2(x-1)(\kappa+1)} - \frac{2d_1\alpha_0^3}{9(x-1)^2(\kappa+1)} + \frac{35\alpha_0^3}{36(x-1)^2(\kappa+1)} - \frac{125\alpha_0^3}{36(\kappa+1)} - \\
& \frac{23d_1\alpha_0^2}{12(\kappa+1)} - \frac{23d_1\kappa\alpha_0^2}{12(\kappa+1)} - \frac{3d_1\kappa\alpha_0^2}{4(x-1)(\kappa+1)} + \frac{41\kappa\alpha_0^2}{8(x-1)(\kappa+1)} + \frac{2d_1\kappa\alpha_0^2}{3(x-1)^2(\kappa+1)} - \frac{39\kappa\alpha_0^2}{8(x-1)^2(\kappa+1)} - \frac{d_1\kappa\alpha_0^2}{2(x-1)^3(\kappa+1)} + \\
& \frac{27\kappa\alpha_0^2}{8(x-1)^3(\kappa+1)} + \frac{107\kappa\alpha_0^2}{8(\kappa+1)} - \frac{3d_1\alpha_0^2}{4(x-1)(\kappa+1)} + \frac{89\alpha_0^2}{24(x-1)(\kappa+1)} + \frac{2d_1\alpha_0^2}{3(x-1)^2(\kappa+1)} - \frac{71\alpha_0^2}{24(x-1)^2(\kappa+1)} - \\
& \frac{d_1\alpha_0^2}{2(x-1)^3(\kappa+1)} + \frac{43\alpha_0^2}{24(x-1)^3(\kappa+1)} + \frac{203\alpha_0^2}{24(\kappa+1)} + \frac{25d_1\alpha_0}{6(\kappa+1)} + \frac{25d_1\kappa\alpha_0}{6(\kappa+1)} + \frac{d_1\kappa\alpha_0}{2(x-1)(\kappa+1)} - \frac{5\kappa\alpha_0}{2(x-1)(\kappa+1)} - \\
& \frac{2d_1\kappa\alpha_0}{3(x-1)^2(\kappa+1)} + \frac{5\kappa\alpha_0}{(x-1)^2(\kappa+1)} + \frac{d_1\kappa\alpha_0}{(x-1)^3(\kappa+1)} - \frac{15\kappa\alpha_0}{2(x-1)^3(\kappa+1)} - \frac{2d_1\kappa\alpha_0}{(x-1)^4(\kappa+1)} + \frac{45\kappa\alpha_0}{4(x-1)^4(\kappa+1)} -
\end{aligned}$$

$$\begin{aligned}
& \frac{105\kappa\alpha_0}{4(\kappa+1)} + \frac{d_1\alpha_0}{2(x-1)(\kappa+1)} - \frac{13\alpha_0}{6(x-1)(\kappa+1)} - \frac{2d_1\alpha_0}{3(x-1)^2(\kappa+1)} + \frac{3\alpha_0}{(x-1)^2(\kappa+1)} + \frac{d_1\alpha_0}{(x-1)^3(\kappa+1)} - \frac{23\alpha_0}{6(x-1)^3(\kappa+1)} - \\
& \frac{2d_1\alpha_0}{(x-1)^4(\kappa+1)} + \frac{61\alpha_0}{12(x-1)^4(\kappa+1)} - \frac{169\alpha_0}{12(\kappa+1)} + \left( -\frac{\kappa\alpha_0^4}{2(x-1)} - \frac{\kappa\alpha_0^4}{2} - \frac{\alpha_0^4}{2(x-1)} - \frac{\alpha_0^4}{2} + \frac{2\kappa\alpha_0^3}{x-1} - \frac{2\kappa\alpha_0^3}{3(x-1)^2} + \frac{8\kappa\alpha_0^3}{3} + \right. \\
& \frac{2\alpha_0^3}{x-1} - \frac{2\alpha_0^3}{3(x-1)^2} + \frac{8\alpha_0^3}{3} - \frac{3\kappa\alpha_0^2}{x-1} + \frac{2\kappa\alpha_0^2}{(x-1)^2} - \frac{\kappa\alpha_0^2}{(x-1)^3} - 6\kappa\alpha_0^2 - \frac{3\alpha_0^2}{x-1} + \frac{2\alpha_0^2}{(x-1)^2} - \frac{\alpha_0^2}{(x-1)^3} - 6\alpha_0^2 + \frac{2\kappa\alpha_0}{x-1} - \\
& \frac{2\kappa\alpha_0}{(x-1)^2} + \frac{2\kappa\alpha_0}{(x-1)^3} - \frac{2\kappa\alpha_0}{(x-1)^4} + 8\kappa\alpha_0 + \frac{2\alpha_0}{x-1} - \frac{2\alpha_0}{(x-1)^2} + \frac{2\alpha_0}{(x-1)^3} - \frac{2\alpha_0}{(x-1)^4} + 8\alpha_0 - \frac{5\kappa}{4(x-1)} + \frac{5\kappa}{6(x-1)^2} - \\
& \frac{5\kappa}{6(x-1)^3} + \frac{5\kappa}{4(x-1)^4} + \frac{25\kappa}{12(x-1)^5} - \frac{25\kappa}{12} - \frac{5}{4(x-1)} + \frac{5}{6(x-1)^2} - \frac{5}{6(x-1)^3} + \frac{5}{4(x-1)^4} + \frac{49}{12(x-1)^5} - \\
& \left. \frac{1}{12} \right) H(0; \alpha_0) + \left( \frac{5\kappa}{4(x-1)} - \frac{5\kappa}{6(x-1)^2} + \frac{5\kappa}{6(x-1)^3} - \frac{5\kappa}{4(x-1)^4} - \frac{25\kappa}{12(x-1)^5} + \frac{25\kappa}{12} + \frac{5}{4(x-1)} - \frac{5}{6(x-1)^2} + \right. \\
& \left. \frac{5}{6(x-1)^3} - \frac{5}{4(x-1)^4} - \frac{49}{12(x-1)^5} + \frac{49}{12} \right) H(0; x) + \left( -\frac{d_1\alpha_0^4}{2} - \frac{d_1\alpha_0^4}{2(x-1)} + \frac{8d_1\alpha_0^3}{3} + \frac{2d_1\alpha_0^3}{x-1} - \frac{2d_1\alpha_0^3}{3(x-1)^2} - \right. \\
& 6d_1\alpha_0^2 - \frac{3d_1\alpha_0^2}{x-1} + \frac{2d_1\alpha_0^2}{(x-1)^2} - \frac{d_1\alpha_0^2}{(x-1)^3} + 8d_1\alpha_0 + \frac{2d_1\alpha_0}{x-1} - \frac{2d_1\alpha_0}{(x-1)^2} + \frac{2d_1\alpha_0}{(x-1)^3} - \frac{2d_1\alpha_0}{(x-1)^4} - \frac{25d_1}{6} - \frac{d_1}{2(x-1)} + \\
& \frac{2d_1}{3(x-1)^2} - \frac{d_1}{(x-1)^3} + \frac{2d_1}{(x-1)^4} \Big) H(1; \alpha_0) + \left( \frac{2d_1}{(x-1)^5} - \frac{\kappa}{(x-1)^5} + \kappa - \frac{1}{(x-1)^5} + 1 \right) H(0; \alpha_0)H(1; x) + \left( -\frac{\kappa\alpha_0^4}{4(x-1)} - \frac{\kappa\alpha_0^4}{4} - \frac{\alpha_0^4}{4(x-1)} - \frac{\alpha_0^4}{4} + \frac{\kappa\alpha_0^3}{x-1} - \frac{\kappa\alpha_0^3}{3(x-1)^2} + \frac{4\kappa\alpha_0^3}{3} + \frac{\alpha_0^3}{x-1} - \frac{\alpha_0^3}{3(x-1)^2} + \frac{4\alpha_0^3}{3} - \frac{3\kappa\alpha_0^2}{2(x-1)} + \frac{\kappa\alpha_0^2}{(x-1)^2} - \right. \\
& \frac{\kappa\alpha_0^2}{2(x-1)^3} - 3\kappa\alpha_0^2 - \frac{3\alpha_0^2}{2(x-1)} + \frac{\alpha_0^2}{(x-1)^2} - \frac{\alpha_0^2}{2(x-1)^3} - 3\alpha_0^2 + \frac{\kappa\alpha_0}{x-1} - \frac{\kappa\alpha_0}{(x-1)^2} + \frac{\kappa\alpha_0}{(x-1)^3} - \frac{\kappa\alpha_0}{(x-1)^4} + 4\kappa\alpha_0 + \\
& \frac{\alpha_0}{x-1} - \frac{\alpha_0}{(x-1)^2} + \frac{\alpha_0}{(x-1)^3} - \frac{\alpha_0}{(x-1)^4} + 4\alpha_0 - \frac{5\kappa}{4(x-1)} + \frac{5\kappa}{6(x-1)^2} - \frac{5\kappa}{6(x-1)^3} + \frac{5\kappa}{4(x-1)^4} + \frac{25\kappa}{12(x-1)^5} - \\
& \frac{25\kappa}{12} + \left( -\frac{2\kappa}{(x-1)^5} - \frac{2}{(x-1)^5} \right) H(0; \alpha_0) - \frac{2d_1H(1; \alpha_0)}{(x-1)^5} - \frac{5}{4(x-1)} + \frac{5}{6(x-1)^2} - \frac{5}{6(x-1)^3} + \frac{5}{4(x-1)^4} + \\
& \frac{49}{12(x-1)^5} - \frac{25}{12} \Big) H(c_1(\alpha_0); x) + \left( -\frac{2\kappa}{(x-1)^5} - 2\kappa - \frac{2}{(x-1)^5} - 2 \right) H(0, 0; \alpha_0) + \left( \frac{2\kappa}{(x-1)^5} - 2\kappa + \frac{2}{(x-1)^5} - \right. \\
& \left. 2 \right) H(0, 0; x) + \left( -\frac{2d_1}{(x-1)^5} - 2d_1 \right) H(0, 1; \alpha_0) + \left( -\frac{\kappa}{(x-1)^5} + \kappa - \frac{1}{(x-1)^5} + 1 \right) H(0, c_1(\alpha_0); x) + \left( -\frac{2d_1}{(x-1)^5} + \frac{\kappa}{(x-1)^5} - \kappa + \frac{1}{(x-1)^5} - 1 \right) H(1, 0; x) + \left( \frac{2d_1}{(x-1)^5} - \frac{\kappa}{(x-1)^5} + \kappa - \frac{1}{(x-1)^5} + 1 \right) H(1, c_1(\alpha_0); x) + \\
& \left( -\frac{\kappa}{(x-1)^5} - \frac{1}{(x-1)^5} \right) H(c_1(\alpha_0), c_1(\alpha_0); x) + \frac{\pi^2\kappa}{2(x-1)^5(\kappa+1)} - \frac{\pi^2\kappa}{2(\kappa+1)} + \frac{\pi^2}{6(x-1)^5(\kappa+1)} - \frac{4}{\kappa+1},
\end{aligned}$$

$$\begin{aligned}
a_2^{(\kappa, 0)} &= \frac{d_1^2\alpha_0^4}{16(\kappa+1)} - \frac{3d_1\alpha_0^4}{8(\kappa+1)} + \frac{d_1^2\kappa\alpha_0^4}{16(\kappa+1)} - \frac{5d_1\kappa\alpha_0^4}{8(\kappa+1)} + \frac{d_1^2\kappa\alpha_0^4}{16(x-1)(\kappa+1)} - \frac{5d_1\kappa\alpha_0^4}{8(x-1)(\kappa+1)} - \frac{\pi^2\kappa\alpha_0^4}{24(x-1)(\kappa+1)} + \\
& \frac{35\kappa\alpha_0^4}{16(x-1)(\kappa+1)} + \frac{35\kappa\alpha_0^4}{16(\kappa+1)} + \frac{d_1^2\alpha_0^4}{16(x-1)(\kappa+1)} - \frac{3d_1\alpha_0^4}{8(x-1)(\kappa+1)} - \frac{\pi^2\alpha_0^4}{24(x-1)(\kappa+1)} + \frac{21\alpha_0^4}{16(x-1)(\kappa+1)} + \frac{21\alpha_0^4}{16(\kappa+1)} - \\
& \frac{\pi^2\alpha_0^4}{24} - \frac{43d_1^2\alpha_0^3}{108(\kappa+1)} + \frac{505d_1\alpha_0^3}{216(\kappa+1)} - \frac{43d_1^2\kappa\alpha_0^3}{108(\kappa+1)} + \frac{33d_1\kappa\alpha_0^3}{8(\kappa+1)} - \frac{d_1^2\kappa\alpha_0^3}{4(x-1)(\kappa+1)} + \frac{5d_1\kappa\alpha_0^3}{2(x-1)(\kappa+1)} + \frac{\pi^2\kappa\alpha_0^3}{6(x-1)(\kappa+1)} - \\
& \frac{35\kappa\alpha_0^3}{4(x-1)(\kappa+1)} + \frac{4d_1^2\kappa\alpha_0^3}{27(x-1)^2(\kappa+1)} - \frac{13d_1\kappa\alpha_0^3}{8(x-1)^2(\kappa+1)} - \frac{\pi^2\kappa\alpha_0^3}{18(x-1)^2(\kappa+1)} + \frac{1055\kappa\alpha_0^3}{216(x-1)^2(\kappa+1)} - \frac{2945\kappa\alpha_0^3}{216(\kappa+1)} - \\
& \frac{d_1^2\alpha_0^3}{4(x-1)(\kappa+1)} + \frac{3d_1\alpha_0^3}{2(x-1)(\kappa+1)} + \frac{\pi^2\alpha_0^3}{6(x-1)(\kappa+1)} - \frac{4(x-1)(\kappa+1)}{27(x-1)^2(\kappa+1)} + \frac{27(x-1)^2(\kappa+1)}{216(x-1)^2(\kappa+1)} - \\
& \frac{\pi^2\alpha_0^3}{18(x-1)^2(\kappa+1)} + \frac{473\alpha_0^3}{216(x-1)^2(\kappa+1)} - \frac{1607\alpha_0^3}{216(\kappa+1)} + \frac{2\pi^2\alpha_0^3}{9} + \frac{95d_1^2\alpha_0^2}{72(\kappa+1)} - \frac{347d_1\alpha_0^2}{48(\kappa+1)} + \frac{95d_1^2\kappa\alpha_0^2}{72(\kappa+1)} - \frac{673d_1\kappa\alpha_0^2}{48(\kappa+1)} + \\
& \frac{3d_1^2\kappa\alpha_0^2}{8(x-1)(\kappa+1)} - \frac{167d_1\kappa\alpha_0^2}{48(x-1)(\kappa+1)} - \frac{\pi^2\kappa\alpha_0^2}{4(x-1)(\kappa+1)} + \frac{1721\kappa\alpha_0^2}{144(x-1)(\kappa+1)} - \frac{4d_1^2\kappa\alpha_0^2}{9(x-1)^2(\kappa+1)} + \frac{247d_1\kappa\alpha_0^2}{48(x-1)^2(\kappa+1)} + \\
& \frac{\pi^2\kappa\alpha_0^2}{6(x-1)^2(\kappa+1)} - \frac{2279\kappa\alpha_0^2}{144(x-1)^2(\kappa+1)} + \frac{d_1^2\kappa\alpha_0^2}{2(x-1)^3(\kappa+1)} - \frac{259d_1\kappa\alpha_0^2}{48(x-1)^3(\kappa+1)} - \frac{\pi^2\kappa\alpha_0^2}{12(x-1)^3(\kappa+1)} + \frac{144(x-1)^3(\kappa+1)}{144(x-1)^3(\kappa+1)} + \\
& \frac{5987\kappa\alpha_0^2}{144(\kappa+1)} + \frac{3d_1^2\alpha_0^2}{8(x-1)(\kappa+1)} - \frac{311d_1\alpha_0^2}{144(x-1)(\kappa+1)} - \frac{\pi^2\alpha_0^2}{4(x-1)(\kappa+1)} + \frac{1103\alpha_0^2}{144(x-1)(\kappa+1)} - \frac{4d_1^2\alpha_0^2}{9(x-1)^2(\kappa+1)} + \\
& \frac{125d_1\alpha_0^2}{48(x-1)^2(\kappa+1)} + \frac{\pi^2\alpha_0^2}{6(x-1)^2(\kappa+1)} - \frac{977\alpha_0^2}{144(x-1)^2(\kappa+1)} + \frac{d_1^2\alpha_0^2}{2(x-1)^3(\kappa+1)} - \frac{355d_1\alpha_0^2}{144(x-1)^3(\kappa+1)} - \frac{\pi^2\alpha_0^2}{12(x-1)^3(\kappa+1)} + \\
& \frac{661\alpha_0^2}{144(x-1)^3(\kappa+1)} + \frac{2741\alpha_0^2}{144(\kappa+1)} - \frac{\pi^2\alpha_0^2}{2} - \frac{205d_1^2\alpha_0}{36(\kappa+1)} + \frac{575d_1\alpha_0}{24(\kappa+1)} - \frac{205d_1^2\kappa\alpha_0}{36(\kappa+1)} + \frac{1325d_1\kappa\alpha_0}{24(\kappa+1)} - \frac{d_1^2\kappa\alpha_0}{4(x-1)(\kappa+1)} - \\
& \frac{2d_1\kappa\alpha_0}{3(x-1)(\kappa+1)} + \frac{\pi^2\kappa\alpha_0}{6(x-1)(\kappa+1)} + \frac{32\kappa\alpha_0}{9(x-1)(\kappa+1)} + \frac{4d_1^2\kappa\alpha_0}{9(x-1)^2(\kappa+1)} - \frac{65d_1\kappa\alpha_0}{12(x-1)^2(\kappa+1)} - \frac{\pi^2\kappa\alpha_0}{6(x-1)^2(\kappa+1)} + \\
& \frac{17\kappa\alpha_0}{(x-1)^2(\kappa+1)} - \frac{d_1^2\kappa\alpha_0}{(x-1)^3(\kappa+1)} + \frac{161d_1\kappa\alpha_0}{12(x-1)^3(\kappa+1)} + \frac{\pi^2\kappa\alpha_0}{6(x-1)^3(\kappa+1)} - \frac{338\kappa\alpha_0}{9(x-1)^3(\kappa+1)} + \frac{4d_1^2\kappa\alpha_0}{(x-1)^4(\kappa+1)} - \\
& \frac{889d_1\kappa\alpha_0}{24(x-1)^4(\kappa+1)} - \frac{\pi^2\kappa\alpha_0}{6(x-1)^4(\kappa+1)} + \frac{668\kappa\alpha_0}{9(x-1)^4(\kappa+1)} - \frac{1127\kappa\alpha_0}{9(\kappa+1)} - \frac{d_1^2\alpha_0}{4(x-1)(\kappa+1)} + \frac{4d_1\alpha_0}{9(x-1)(\kappa+1)} + \frac{\pi^2\alpha_0}{6(x-1)(\kappa+1)} - \\
& \frac{28\alpha_0}{9(x-1)(\kappa+1)} + \frac{4d_1^2\alpha_0}{9(x-1)^2(\kappa+1)} - \frac{97d_1\alpha_0}{36(x-1)^2(\kappa+1)} - \frac{\pi^2\alpha_0}{6(x-1)^2(\kappa+1)} + \frac{7\alpha_0}{(x-1)^2(\kappa+1)} - \frac{d_1^2\alpha_0}{(x-1)^3(\kappa+1)} +
\end{aligned}$$

$$\begin{aligned}
& \frac{209d_1\alpha_0}{36(x-1)^3(\kappa+1)} + \frac{\pi^2\alpha_0}{6(x-1)^3(\kappa+1)} - \frac{98\alpha_0}{9(x-1)^3(\kappa+1)} + \frac{4d_1^2\alpha_0}{(x-1)^4(\kappa+1)} - \frac{1081d_1\alpha_0}{72(x-1)^4(\kappa+1)} - \frac{\pi^2\alpha_0}{6(x-1)^4(\kappa+1)} + \\
& \frac{158\alpha_0}{9(x-1)^4(\kappa+1)} - \frac{347\alpha_0}{9(\kappa+1)} + \frac{2\pi^2\alpha_0}{3} + \left( \frac{d_1\alpha_0^4}{4} + \frac{1}{4}d_1\kappa\alpha_0^4 + \frac{d_1\kappa\alpha_0^4}{4(x-1)} - \frac{7\kappa\alpha_0^4}{4(x-1)} - \frac{7\kappa\alpha_0^4}{4} + \frac{d_1\alpha_0^4}{4(x-1)} - \frac{5\alpha_0^4}{4(x-1)} - \frac{5\alpha_0^4}{4} - \right. \\
& \frac{13d_1\alpha_0^3}{9} - \frac{13}{9}d_1\kappa\alpha_0^3 - \frac{d_1\kappa\alpha_0^3}{x-1} + \frac{7\kappa\alpha_0^3}{x-1} + \frac{4d_1\kappa\alpha_0^3}{9(x-1)^2} - \frac{19\kappa\alpha_0^3}{6(x-1)^2} + \frac{61\kappa\alpha_0^3}{6} - \frac{d_1\alpha_0^3}{x-1} + \frac{5\alpha_0^3}{x-1} + \frac{4d_1\alpha_0^3}{9(x-1)^2} - \frac{35\alpha_0^3}{18(x-1)^2} + \\
& \frac{125\alpha_0^3}{18} + \frac{23}{6}d_1\alpha_0^2 + \frac{23}{6}d_1\kappa\alpha_0^2 + \frac{3d_1\kappa\alpha_0^2}{2(x-1)} - \frac{41\kappa\alpha_0^2}{4(x-1)} - \frac{4d_1\kappa\alpha_0^2}{3(x-1)^2} + \frac{39\kappa\alpha_0^2}{4(x-1)^2} + \frac{d_1\kappa\alpha_0^2}{(x-1)^3} - \frac{27\kappa\alpha_0^2}{4(x-1)^3} - \frac{107\kappa\alpha_0^2}{4} + \\
& \frac{3d_1\alpha_0^2}{2(x-1)} - \frac{89\alpha_0^2}{12(x-1)} - \frac{4d_1\alpha_0^2}{3(x-1)^2} + \frac{71\alpha_0^2}{12(x-1)^2} + \frac{d_1\alpha_0^2}{(x-1)^3} - \frac{43\alpha_0^2}{12(x-1)^3} - \frac{203\alpha_0^2}{12} - \frac{25d_1\alpha_0}{3} - \frac{25d_1\kappa\alpha_0}{3} - \frac{d_1\kappa\alpha_0}{x-1} + \\
& \frac{5\kappa\alpha_0}{x-1} + \frac{4d_1\kappa\alpha_0}{3(x-1)^2} - \frac{10\kappa\alpha_0}{(x-1)^2} - \frac{2d_1\kappa\alpha_0}{(x-1)^3} + \frac{15\kappa\alpha_0}{(x-1)^3} + \frac{4d_1\kappa\alpha_0}{(x-1)^4} - \frac{45\kappa\alpha_0}{2(x-1)^4} + \frac{105\kappa\alpha_0}{2} - \frac{d_1\alpha_0}{x-1} + \frac{13\alpha_0}{3(x-1)} + \frac{4d_1\alpha_0}{3(x-1)^2} - \\
& \frac{6\alpha_0}{(x-1)^2} - \frac{2d_1\alpha_0}{(x-1)^3} + \frac{23\alpha_0}{3(x-1)^3} + \frac{4d_1\alpha_0}{(x-1)^4} - \frac{61\alpha_0}{6(x-1)^4} + \frac{169\alpha_0}{6} + \frac{205d_1}{72} + \frac{205d_1\kappa}{72} + \frac{17d_1\kappa}{8(x-1)} - \frac{45\kappa}{4(x-1)} - \frac{13d_1\kappa}{18(x-1)^2} + \\
& \frac{35\kappa}{6(x-1)^2} + \frac{13d_1\kappa}{18(x-1)^3} - \frac{35\kappa}{6(x-1)^3} - \frac{17d_1\kappa}{8(x-1)^4} + \frac{45\kappa}{4(x-1)^4} - \frac{205d_1\kappa}{72(x-1)^5} + \frac{205\kappa}{12(x-1)^5} - \frac{205\kappa}{12} + \frac{17d_1}{8(x-1)} - \\
& \frac{65}{12(x-1)} - \frac{13d_1}{18(x-1)^2} + \frac{55}{18(x-1)^2} + \frac{13d_1}{18(x-1)^3} - \frac{55}{18(x-1)^3} - \frac{17d_1}{8(x-1)^4} + \frac{65}{12(x-1)^4} - \frac{205d_1}{72(x-1)^5} - \frac{\pi^2}{6(x-1)^5} + \\
& \frac{449}{36(x-1)^5} - \frac{\pi^2}{6} - \frac{161}{36} \Big) H(0; \alpha_0) + \left( -\frac{17\kappa d_1}{8(x-1)} + \frac{13\kappa d_1}{18(x-1)^2} - \frac{13\kappa d_1}{18(x-1)^3} + \frac{17\kappa d_1}{8(x-1)^4} + \frac{205\kappa d_1}{72(x-1)^5} - \frac{205\kappa d_1}{72} - \right. \\
& \frac{17d_1}{8(x-1)} + \frac{13d_1}{18(x-1)^2} - \frac{13d_1}{18(x-1)^3} + \frac{17d_1}{8(x-1)^4} + \frac{205d_1}{72(x-1)^5} - \frac{205d_1}{72} + \frac{45\kappa}{4(x-1)} - \frac{35\kappa}{6(x-1)^2} + \frac{35\kappa}{6(x-1)^3} - \frac{45\kappa}{4(x-1)^4} - \\
& \frac{\pi^2\kappa}{(x-1)^5} - \frac{205\kappa}{12(x-1)^5} + \pi^2\kappa + \frac{205\kappa}{12} + \frac{65}{12(x-1)} - \frac{55}{18(x-1)^2} + \frac{55}{18(x-1)^3} - \frac{65}{12(x-1)^4} - \frac{\pi^2}{6(x-1)^5} - \frac{449}{36(x-1)^5} + \\
& \frac{\pi^2}{6} + \frac{449}{36} \Big) H(0; x) + \left( \frac{d_1^2\alpha_0^4}{4} - \frac{5d_1\alpha_0^4}{4} - \frac{1}{4}d_1\kappa\alpha_0^4 - \frac{d_1\kappa\alpha_0^4}{4(x-1)} + \frac{d_1^2\alpha_0^4}{4(x-1)} - \frac{5d_1\alpha_0^4}{4(x-1)} - \frac{13d_1^2\alpha_0^3}{9} + \frac{125d_1\alpha_0^3}{18} + \right. \\
& \frac{29}{18}d_1\kappa\alpha_0^3 + \frac{d_1\kappa\alpha_0^3}{x-1} - \frac{11d_1\kappa\alpha_0^3}{18(x-1)^2} - \frac{d_1^2\alpha_0^3}{x-1} + \frac{5d_1\alpha_0^3}{x-1} + \frac{4d_1^2\alpha_0^3}{9(x-1)^2} - \frac{35d_1\alpha_0^3}{18(x-1)^2} + \frac{23d_1^2\alpha_0^2}{6} - \frac{203d_1\alpha_0^2}{12} - \frac{59}{12}d_1\kappa\alpha_0^2 - \\
& \frac{17d_1\kappa\alpha_0^2}{12(x-1)} + \frac{23d_1\kappa\alpha_0^2}{12(x-1)^2} - \frac{19d_1\kappa\alpha_0^2}{12(x-1)^3} + \frac{3d_1^2\alpha_0^2}{2(x-1)} - \frac{89d_1\alpha_0^2}{12(x-1)} - \frac{4d_1^2\alpha_0^2}{3(x-1)^2} + \frac{71d_1\alpha_0^2}{12(x-1)^2} + \frac{d_1^2\alpha_0^2}{(x-1)^3} - \frac{43d_1\alpha_0^2}{12(x-1)^3} - \frac{25d_1^2\alpha_0}{3} + \\
& \frac{169d_1\alpha_0}{6} + \frac{73d_1\kappa\alpha_0}{6} + \frac{d_1\kappa\alpha_0}{3(x-1)} - \frac{2d_1\kappa\alpha_0}{(x-1)^2} + \frac{11d_1\kappa\alpha_0}{3(x-1)^3} - \frac{37d_1\kappa\alpha_0}{6(x-1)^4} - \frac{d_1^2\alpha_0}{x-1} + \frac{13d_1\alpha_0}{3(x-1)} + \frac{4d_1^2\alpha_0}{3(x-1)^2} - \frac{6d_1\alpha_0}{(x-1)^2} - \\
& \frac{2d_1^2\alpha_0}{(x-1)^3} + \frac{23d_1\alpha_0}{3(x-1)^3} + \frac{4d_1^2\alpha_0}{(x-1)^4} - \frac{61d_1\alpha_0}{6(x-1)^4} + \frac{205d_1^2}{36} - \frac{305d_1}{18} - \frac{155d_1\kappa}{18} + \frac{d_1\kappa}{3(x-1)} + \frac{25d_1\kappa}{36(x-1)^2} - \frac{25d_1\kappa}{12(x-1)^3} + \\
& \frac{37d_1\kappa}{6(x-1)^4} + \frac{d_1^2}{4(x-1)} - \frac{2d_1}{3(x-1)} - \frac{4d_1^2}{9(x-1)^2} + \frac{73d_1}{36(x-1)^2} + \frac{d_1^2}{(x-1)^3} - \frac{49d_1}{12(x-1)^3} - \frac{4d_1^2}{(x-1)^4} + \frac{61d_1}{6(x-1)^4} \Big) H(1; \alpha_0) + \\
& \left( \frac{3\kappa\alpha_0^4}{x-1} + 3\kappa\alpha_0^4 + \frac{\alpha_0^4}{x-1} + \alpha_0^4 - \frac{12\kappa\alpha_0^3}{x-1} + \frac{4\kappa\alpha_0^3}{(x-1)^2} - 16\kappa\alpha_0^3 - \frac{4\alpha_0^3}{x-1} + \frac{4\alpha_0^3}{3(x-1)^2} - \frac{16\alpha_0^3}{3} + \frac{18\kappa\alpha_0^2}{x-1} - \frac{12\kappa\alpha_0^2}{(x-1)^2} + \right. \\
& \frac{6\kappa\alpha_0^2}{(x-1)^3} + 36\kappa\alpha_0^2 + \frac{6\alpha_0^2}{x-1} - \frac{4\alpha_0^2}{(x-1)^2} + \frac{2\alpha_0^2}{(x-1)^3} + 12\alpha_0^2 - \frac{12\kappa\alpha_0}{x-1} + \frac{12\kappa\alpha_0}{(x-1)^2} - \frac{12\kappa\alpha_0}{(x-1)^3} + \frac{12\kappa\alpha_0}{(x-1)^4} - 48\kappa\alpha_0 - \frac{4\alpha_0}{x-1} + \\
& \frac{4\alpha_0}{(x-1)^2} - \frac{4\alpha_0}{(x-1)^3} + \frac{4\alpha_0}{(x-1)^4} - 16\alpha_0 + \frac{15\kappa}{2(x-1)} - \frac{5\kappa}{(x-1)^2} + \frac{5\kappa}{(x-1)^3} - \frac{15\kappa}{2(x-1)^4} - \frac{33\kappa}{2(x-1)^5} + \frac{17\kappa}{2} + \frac{5}{2(x-1)} - \\
& \frac{5}{3(x-1)^2} + \frac{5}{3(x-1)^3} - \frac{5}{2(x-1)^4} - \frac{49}{6(x-1)^5} + \frac{1}{6} \Big) H(0, 0; \alpha_0) + \left( -\frac{15\kappa}{2(x-1)} + \frac{5\kappa}{(x-1)^2} - \frac{5\kappa}{(x-1)^3} + \frac{15\kappa}{2(x-1)^4} + \right. \\
& \frac{33\kappa}{2(x-1)^5} - \frac{33\kappa}{2} - \frac{5}{2(x-1)} + \frac{5}{3(x-1)^2} - \frac{5}{3(x-1)^3} + \frac{5}{2(x-1)^4} + \frac{49}{6(x-1)^5} - \frac{49}{6} \Big) H(0, 0; x) + \left( d_1\alpha_0^4 + \right. \\
& d_1\kappa\alpha_0^4 + \frac{d_1\kappa\alpha_0^4}{x-1} + \frac{d_1\alpha_0^4}{x-1} - \frac{16d_1\alpha_0^3}{3} - \frac{16}{3}d_1\kappa\alpha_0^3 - \frac{4d_1\kappa\alpha_0^3}{x-1} + \frac{4d_1\kappa\alpha_0^3}{3(x-1)^2} - \frac{4d_1\alpha_0^3}{x-1} + \frac{4d_1\alpha_0^3}{3(x-1)^2} + 12d_1\alpha_0^2 + \\
& 12d_1\kappa\alpha_0^2 + \frac{6d_1\kappa\alpha_0^2}{x-1} - \frac{4d_1\kappa\alpha_0^2}{(x-1)^2} + \frac{2d_1\kappa\alpha_0^2}{(x-1)^3} + \frac{6d_1\alpha_0^2}{x-1} - \frac{4d_1\alpha_0^2}{(x-1)^2} + \frac{2d_1\alpha_0^2}{(x-1)^3} - 16d_1\alpha_0 - 16d_1\kappa\alpha_0 - \frac{4d_1\kappa\alpha_0}{x-1} + \\
& \frac{4d_1\kappa\alpha_0}{(x-1)^2} - \frac{4d_1\kappa\alpha_0}{(x-1)^3} + \frac{4d_1\kappa\alpha_0}{(x-1)^4} - \frac{4d_1\alpha_0}{x-1} + \frac{4d_1\alpha_0}{(x-1)^2} - \frac{4d_1\alpha_0}{(x-1)^3} + \frac{4d_1\alpha_0}{(x-1)^4} + \frac{d_1}{6} + \frac{25d_1\kappa}{6} + \frac{5d_1\kappa}{2(x-1)} - \frac{5d_1\kappa}{3(x-1)^2} + \\
& \frac{5d_1\kappa}{3(x-1)^3} - \frac{5d_1\kappa}{2(x-1)^4} - \frac{25d_1\kappa}{6(x-1)^5} + \frac{5d_1}{2(x-1)} - \frac{5d_1}{3(x-1)^2} + \frac{5d_1}{3(x-1)^3} - \frac{5d_1}{2(x-1)^4} - \frac{49d_1}{6(x-1)^5} \Big) H(0, 1; \alpha_0) + \\
& H(1; x) \left( \frac{\pi^2\kappa d_1}{3(x-1)^5} + \frac{\pi^2 d_1}{3(x-1)^5} - \frac{\pi^2\kappa}{2(x-1)^5} + \frac{\pi^2\kappa}{2} + \left( -\frac{2\kappa d_1}{x-1} + \frac{\kappa d_1}{(x-1)^2} - \frac{2\kappa d_1}{3(x-1)^3} + \frac{\kappa d_1}{2(x-1)^4} + \frac{25\kappa d_1}{6(x-1)^5} - \right. \right. \\
& \frac{2d_1}{x-1} + \frac{d_1}{(x-1)^2} - \frac{2d_1}{3(x-1)^3} + \frac{d_1}{2(x-1)^4} + \frac{49d_1}{6(x-1)^5} + \frac{15\kappa}{4(x-1)} - \frac{5\kappa}{2(x-1)^2} + \frac{5\kappa}{2(x-1)^3} - \frac{15\kappa}{4(x-1)^4} - \frac{33\kappa}{4(x-1)^5} + \\
& \frac{33\kappa}{4} + \frac{5}{4(x-1)} - \frac{5}{6(x-1)^2} + \frac{5}{6(x-1)^3} - \frac{5}{4(x-1)^4} - \frac{49}{12(x-1)^5} + \frac{49}{12} \Big) H(0; \alpha_0) + \left( -\frac{4\kappa d_1}{(x-1)^5} - \frac{4d_1}{(x-1)^5} + \right. \\
& \frac{6\kappa}{(x-1)^5} - 6\kappa + \frac{2}{(x-1)^5} - 2 \Big) H(0, 0; \alpha_0) + \left( -\frac{4d_1^2}{(x-1)^5} + \frac{2\kappa d_1}{(x-1)^5} - 2\kappa d_1 + \frac{2d_1}{(x-1)^5} - 2d_1 \right) H(0, 1; \alpha_0) - \\
& \frac{\pi^2}{6(x-1)^5} + \frac{\pi^2}{6} \Big) + \left( -\frac{2\kappa d_1}{(x-1)^5} + 2\kappa d_1 - \frac{2d_1}{(x-1)^5} + 2d_1 + \frac{6\kappa}{(x-1)^5} - 6\kappa + \frac{2}{(x-1)^5} - 2 \right) H(0; \alpha_0) H(0, 1; x) + \\
& \left( \frac{15\kappa}{4(x-1)} - \frac{5\kappa}{2(x-1)^2} + \frac{5\kappa}{2(x-1)^3} - \frac{15\kappa}{4(x-1)^4} - \frac{33\kappa}{4(x-1)^5} + \frac{33\kappa}{4} + \left( \frac{6\kappa}{(x-1)^5} - 6\kappa + \frac{2}{(x-1)^5} - 2 \right) H(0; \alpha_0) + \right.
\end{aligned}$$

$$\begin{aligned}
 & \left( \frac{2\kappa d_1}{(x-1)^5} - 2\kappa d_1 + \frac{2d_1}{(x-1)^5} - 2d_1 \right) H(1; \alpha_0) + \frac{5}{4(x-1)} - \frac{5}{6(x-1)^2} + \frac{5}{6(x-1)^3} - \frac{5}{4(x-1)^4} - \frac{49}{12(x-1)^5} + \\
 & \frac{49}{12} \left. \right) H(0, c_1(\alpha_0); x) + \left( d_1 \alpha_0^4 + d_1 \kappa \alpha_0^4 + \frac{d_1 \kappa \alpha_0^4}{x-1} + \frac{d_1 \alpha_0^4}{x-1} - \frac{16d_1 \alpha_0^3}{3} - \frac{16}{3} d_1 \kappa \alpha_0^3 - \frac{4d_1 \kappa \alpha_0^3}{x-1} + \frac{4d_1 \kappa \alpha_0^3}{3(x-1)^2} - \right. \\
 & \frac{4d_1 \alpha_0^3}{x-1} + \frac{4d_1 \alpha_0^3}{3(x-1)^2} + 12d_1 \alpha_0^2 + 12d_1 \kappa \alpha_0^2 + \frac{6d_1 \kappa \alpha_0^2}{x-1} - \frac{4d_1 \kappa \alpha_0^2}{(x-1)^2} + \frac{2d_1 \kappa \alpha_0^2}{(x-1)^3} + \frac{6d_1 \alpha_0^2}{x-1} - \frac{4d_1 \alpha_0^2}{(x-1)^2} + \frac{2d_1 \alpha_0^2}{(x-1)^3} - \\
 & 16d_1 \alpha_0 - 16d_1 \kappa \alpha_0 - \frac{4d_1 \kappa \alpha_0}{x-1} + \frac{4d_1 \kappa \alpha_0}{(x-1)^2} - \frac{4d_1 \kappa \alpha_0}{(x-1)^3} + \frac{4d_1 \kappa \alpha_0}{(x-1)^4} - \frac{4d_1 \alpha_0}{x-1} + \frac{4d_1 \alpha_0}{(x-1)^2} - \frac{4}{(x-1)^3} \frac{d_1 \alpha_0}{x-1} + \frac{4d_1 \alpha_0}{(x-1)^4} + \frac{25d_1}{3} + \\
 & \frac{25}{3} \frac{d_1 \kappa}{x-1} + \frac{d_1 \kappa}{x-1} - \frac{4d_1 \kappa}{3(x-1)^2} + \frac{2d_1 \kappa}{(x-1)^3} - \frac{4d_1 \kappa}{(x-1)^4} + \frac{d_1}{x-1} - \frac{4d_1}{3(x-1)^2} + \frac{2d_1}{(x-1)^3} - \frac{4}{(x-1)^4} \frac{d_1}{x-1} \left. \right) H(1, 0; \alpha_0) + \left( \frac{2\kappa d_1}{x-1} - \right. \\
 & \frac{\kappa d_1}{(x-1)^2} + \frac{2\kappa d_1}{3(x-1)^3} - \frac{\kappa d_1}{2(x-1)^4} - \frac{25\kappa d_1}{6(x-1)^5} + \frac{2d_1}{x-1} - \frac{d_1}{(x-1)^2} + \frac{2d_1}{3(x-1)^3} - \frac{d_1}{2(x-1)^4} - \frac{49d_1}{6(x-1)^5} - \frac{15}{4} \frac{\kappa}{(x-1)} + \\
 & \frac{5\kappa}{2(x-1)^2} - \frac{2}{2(x-1)^3} + \frac{5\kappa}{4(x-1)^4} + \frac{15\kappa}{4(x-1)^5} - \frac{33\kappa}{4} - \frac{4}{4(x-1)} + \frac{6}{6(x-1)^2} - \frac{6}{6(x-1)^3} + \frac{4}{4(x-1)^4} + \frac{12}{12(x-1)^5} - \\
 & \frac{49}{12} \left. \right) H(1, 0; x) + \left( d_1^2 \alpha_0^4 + \frac{d_1^2 \alpha_0^4}{x-1} - \frac{16d_1^2 \alpha_0^3}{3} - \frac{4d_1^2 \alpha_0^3}{x-1} + \frac{4d_1^2 \alpha_0^3}{3(x-1)^2} + 12d_1^2 \alpha_0^2 + \frac{6}{x-1} \frac{d_1^2 \alpha_0^2}{x-1} - \frac{4d_1^2 \alpha_0^2}{(x-1)^2} + \frac{2d_1^2 \alpha_0^2}{(x-1)^3} - \right. \\
 & 16d_1^2 \alpha_0 - \frac{4d_1^2 \alpha_0}{x-1} + \frac{4d_1^2 \alpha_0}{(x-1)^2} - \frac{4d_1^2 \alpha_0}{(x-1)^3} + \frac{4d_1^2 \alpha_0}{(x-1)^4} + \frac{25d_1^2}{3} + \frac{d_1^2}{x-1} - \frac{4d_1^2}{3(x-1)^2} + \frac{2d_1^2}{(x-1)^3} - \frac{4d_1^2}{(x-1)^4} \left. \right) H(1, 1; \alpha_0) + \\
 & H(c_1(\alpha_0); x) \left( \frac{d_1 \alpha_0^4}{8} + \frac{1}{8} d_1 \kappa \alpha_0^4 + \frac{d_1 \kappa \alpha_0^4}{8(x-1)} - \frac{7\kappa \alpha_0^4}{8(x-1)} - \frac{7\kappa \alpha_0^4}{8} + \frac{d_1 \alpha_0^4}{8(x-1)} - \frac{5\alpha_0^4}{8(x-1)} - \frac{5\alpha_0^4}{8} - \frac{13d_1 \alpha_0^3}{18} - \right. \\
 & \frac{13}{18} d_1 \kappa \alpha_0^3 - \frac{d_1 \kappa \alpha_0^3}{2(x-1)} + \frac{7\kappa \alpha_0^3}{2(x-1)} + \frac{2d_1 \kappa \alpha_0^3}{9(x-1)^2} - \frac{19\kappa \alpha_0^3}{12(x-1)^2} + \frac{61\kappa \alpha_0^3}{12} - \frac{d_1 \alpha_0^3}{2(x-1)} + \frac{5\alpha_0^3}{2(x-1)} + \frac{2}{9} \frac{d_1 \alpha_0^3}{(x-1)^2} - \frac{35\alpha_0^3}{36(x-1)^2} + \\
 & \frac{125}{36} \frac{\alpha_0^3}{x-1} + \frac{23d_1 \alpha_0^2}{12} + \frac{23}{12} d_1 \kappa \alpha_0^2 + \frac{3d_1 \kappa \alpha_0^2}{4(x-1)} - \frac{41\kappa \alpha_0^2}{8(x-1)} - \frac{2d_1 \kappa \alpha_0^2}{3(x-1)^2} + \frac{39\kappa \alpha_0^2}{8(x-1)^2} + \frac{d_1 \kappa \alpha_0^2}{2(x-1)^3} - \frac{27\kappa \alpha_0^2}{8(x-1)^3} - \frac{107\kappa \alpha_0^2}{8} + \\
 & \frac{3d_1 \alpha_0^2}{4(x-1)} - \frac{89\alpha_0^2}{24(x-1)} - \frac{2d_1 \alpha_0^2}{3(x-1)^2} + \frac{71\alpha_0^2}{24(x-1)^2} + \frac{d_1 \alpha_0^2}{2(x-1)^3} - \frac{43\alpha_0^2}{24(x-1)^3} - \frac{203\alpha_0^2}{24} - \frac{25}{6} \frac{d_1 \alpha_0}{x-1} - \frac{25d_1 \kappa \alpha_0}{6} - \frac{d_1 \kappa \alpha_0}{2(x-1)} + \\
 & \frac{5\kappa \alpha_0}{2(x-1)} + \frac{2d_1 \kappa \alpha_0}{3(x-1)^2} - \frac{5\kappa \alpha_0}{(x-1)^2} - \frac{d_1 \kappa \alpha_0}{(x-1)^3} + \frac{15\kappa \alpha_0}{2(x-1)^3} + \frac{2d_1 \kappa \alpha_0}{(x-1)^4} - \frac{45\kappa \alpha_0}{4(x-1)^4} + \frac{105\kappa \alpha_0}{4} - \frac{d_1 \alpha_0}{2(x-1)} + \frac{13\alpha_0}{6(x-1)} + \\
 & \frac{2d_1 \alpha_0}{3(x-1)^2} - \frac{3\alpha_0}{(x-1)^2} - \frac{d_1 \alpha_0}{(x-1)^3} + \frac{23\alpha_0}{6(x-1)^3} + \frac{2d_1 \alpha_0}{(x-1)^4} - \frac{61\alpha_0}{12(x-1)^4} + \frac{169\alpha_0}{12} + \frac{205}{72} \frac{d_1}{x-1} + \frac{205d_1 \kappa}{72} + \frac{17d_1 \kappa}{8(x-1)} - \frac{45\kappa}{4(x-1)} - \\
 & \frac{13d_1 \kappa}{18(x-1)^2} + \frac{35\kappa}{6(x-1)^2} + \frac{13d_1 \kappa}{18(x-1)^3} - \frac{35\kappa}{6(x-1)^3} - \frac{17d_1 \kappa}{8(x-1)^4} + \frac{45\kappa}{4(x-1)^4} - \frac{205d_1 \kappa}{72(x-1)^5} + \frac{205\kappa}{12(x-1)^5} - \frac{205\kappa}{12} + \\
 & \left( \frac{3\kappa \alpha_0^4}{2(x-1)} + \frac{3\kappa \alpha_0^4}{2} + \frac{\alpha_0^4}{2(x-1)} + \frac{\alpha_0^4}{2} - \frac{6\kappa \alpha_0^3}{x-1} + \frac{2\kappa \alpha_0^3}{(x-1)^2} - 8\kappa \alpha_0^3 - \frac{2}{x-1} \frac{\alpha_0^3}{3(x-1)^2} - \frac{8\alpha_0^3}{3} + \frac{9}{x-1} \frac{\kappa \alpha_0^2}{x-1} - \frac{6\kappa \alpha_0^2}{(x-1)^2} + \right. \\
 & \frac{3\kappa \alpha_0^2}{(x-1)^3} + 18\kappa \alpha_0^2 + \frac{3\alpha_0^2}{x-1} - \frac{2}{(x-1)^2} \frac{\alpha_0^2}{x-1} + \frac{\alpha_0^2}{(x-1)^3} + 6\alpha_0^2 - \frac{6\kappa \alpha_0}{x-1} + \frac{6\kappa \alpha_0}{(x-1)^2} - \frac{6\kappa \alpha_0}{(x-1)^3} + \frac{6\kappa \alpha_0}{(x-1)^4} - 24\kappa \alpha_0 - \frac{2}{x-1} \frac{\alpha_0}{x-1} + \\
 & \frac{2\alpha_0}{(x-1)^2} - \frac{2\alpha_0}{(x-1)^3} + \frac{2}{(x-1)^4} \frac{\alpha_0}{x-1} - 8\alpha_0 + \frac{15\kappa}{2(x-1)} - \frac{5\kappa}{(x-1)^2} + \frac{5\kappa}{(x-1)^3} - \frac{15\kappa}{2(x-1)^4} - \frac{33\kappa}{2(x-1)^5} + \frac{25\kappa}{2} + \frac{5}{2(x-1)} - \\
 & \frac{5}{3(x-1)^2} + \frac{5}{3(x-1)^3} - \frac{5}{2(x-1)^4} - \frac{49}{6(x-1)^5} + \frac{25}{6} \left. \right) H(0; \alpha_0) + \left( \frac{d_1 \alpha_0^4}{2} + \frac{1}{2} d_1 \kappa \alpha_0^4 + \frac{d_1 \kappa \alpha_0^4}{2(x-1)} + \frac{d_1 \alpha_0^4}{2(x-1)} - \right. \\
 & \frac{8d_1 \alpha_0^3}{3} - \frac{8}{3} d_1 \kappa \alpha_0^3 - \frac{2}{3} \frac{d_1 \kappa \alpha_0^3}{x-1} + \frac{2d_1 \kappa \alpha_0^3}{3(x-1)^2} - \frac{2}{x-1} \frac{d_1 \alpha_0^3}{x-1} + \frac{2d_1 \alpha_0^3}{3(x-1)^2} + 6d_1 \alpha_0^2 + 6d_1 \kappa \alpha_0^2 + \frac{3d_1 \kappa \alpha_0^2}{x-1} - \frac{2d_1 \kappa \alpha_0^2}{(x-1)^2} + \\
 & \frac{d_1 \kappa \alpha_0^2}{(x-1)^3} + \frac{3d_1 \alpha_0^2}{x-1} - \frac{2d_1 \alpha_0^2}{(x-1)^2} + \frac{d_1 \alpha_0^2}{(x-1)^3} - 8d_1 \alpha_0 - 8d_1 \kappa \alpha_0 - \frac{2d_1 \kappa \alpha_0}{x-1} + \frac{2d_1 \kappa \alpha_0}{(x-1)^2} - \frac{2d_1 \kappa \alpha_0}{(x-1)^3} + \frac{2d_1 \kappa \alpha_0}{(x-1)^4} - \\
 & \frac{2d_1 \alpha_0}{x-1} + \frac{2d_1 \alpha_0}{(x-1)^2} - \frac{2d_1 \alpha_0}{(x-1)^3} + \frac{2d_1 \alpha_0}{(x-1)^4} + \frac{25d_1}{6} + \frac{25d_1 \kappa}{6} + \frac{5d_1 \kappa}{2(x-1)} - \frac{5d_1 \kappa}{3(x-1)^2} + \frac{5d_1 \kappa}{3(x-1)^3} - \frac{5d_1 \kappa}{2(x-1)^4} - \frac{25d_1 \kappa}{6(x-1)^5} + \\
 & \frac{5d_1}{2(x-1)} - \frac{5d_1}{3(x-1)^2} + \frac{5d_1}{3(x-1)^3} - \frac{5d_1}{2(x-1)^4} - \frac{49d_1}{6(x-1)^5} \left. \right) H(1; \alpha_0) + \left( \frac{12\kappa}{(x-1)^5} + \frac{4}{(x-1)^5} \right) H(0, 0; \alpha_0) + \\
 & \left( \frac{4\kappa d_1}{(x-1)^5} + \frac{4}{(x-1)^5} \right) H(0, 1; \alpha_0) + \left( \frac{4\kappa d_1}{(x-1)^5} + \frac{4d_1}{(x-1)^5} \right) H(1, 0; \alpha_0) + \frac{4d_1^2}{(x-1)^5} H(1, 1; \alpha_0) + \frac{17d_1}{8(x-1)} - \frac{65}{12(x-1)} - \\
 & \frac{13d_1}{18(x-1)^2} + \frac{55}{18(x-1)^2} + \frac{13}{18} \frac{d_1}{(x-1)^3} - \frac{55}{18(x-1)^3} - \frac{17d_1}{8(x-1)^4} + \frac{65}{12(x-1)^4} - \frac{205d_1}{72(x-1)^5} - \frac{\pi^2}{6(x-1)^5} + \frac{449}{36(x-1)^5} - \\
 & \frac{305}{36} \left. \right) + \left( \frac{4d_1^2}{(x-1)^5} - \frac{4}{(x-1)^5} \kappa d_1 - \frac{4d_1}{(x-1)^5} + 2d_1 + \frac{3\kappa}{(x-1)^5} - 3\kappa + \frac{1}{(x-1)^5} - 1 \right) H(0; \alpha_0) H(1, 1; x) + \\
 & \left( -\frac{2\kappa d_1}{x-1} + \frac{\kappa d_1}{(x-1)^2} - \frac{2\kappa d_1}{3(x-1)^3} + \frac{\kappa d_1}{2(x-1)^4} + \frac{25\kappa d_1}{6(x-1)^5} - \frac{2}{x-1} \frac{d_1}{x-1} + \frac{d_1}{(x-1)^2} - \frac{2d_1}{3(x-1)^3} + \frac{d_1}{2(x-1)^4} + \frac{49d_1}{6(x-1)^5} + \right. \\
 & \frac{15\kappa}{4(x-1)} - \frac{5}{2(x-1)^2} \frac{\kappa}{x-1} + \frac{5\kappa}{2(x-1)^3} - \frac{15\kappa}{4(x-1)^4} - \frac{33\kappa}{4(x-1)^5} + \frac{33\kappa}{4} + \left( -\frac{4\kappa d_1}{(x-1)^5} - \frac{4}{(x-1)^5} \frac{d_1}{x-1} + \frac{6\kappa}{(x-1)^5} - 6\kappa + \frac{2}{(x-1)^5} - \right. \\
 & 2 \left. \right) H(0; \alpha_0) + \left( -\frac{4d_1^2}{(x-1)^5} + \frac{2\kappa d_1}{(x-1)^5} - 2\kappa d_1 + \frac{2d_1}{(x-1)^5} - 2d_1 \right) H(1; \alpha_0) + \frac{5}{4(x-1)} - \frac{5}{6(x-1)^2} + \frac{5}{6(x-1)^3} - \\
 & \frac{5}{4(x-1)^4} - \frac{49}{12(x-1)^5} + \frac{49}{12} \left. \right) H(1, c_1(\alpha_0); x) + \left( \frac{3\kappa \alpha_0^4}{4(x-1)} + \frac{3\kappa \alpha_0^4}{4} + \frac{\alpha_0^4}{4(x-1)} + \frac{\alpha_0^4}{4} - \frac{3\kappa \alpha_0^3}{x-1} + \frac{\kappa \alpha_0^3}{(x-1)^2} - \right. \\
 & 4\kappa \alpha_0^3 - \frac{\alpha_0^3}{x-1} + \frac{\alpha_0^3}{3(x-1)^2} - \frac{4}{3} \frac{\alpha_0^3}{x-1} + \frac{9\kappa \alpha_0^2}{2(x-1)} - \frac{3\kappa \alpha_0^2}{(x-1)^2} + \frac{3\kappa \alpha_0^2}{2(x-1)^3} + 9\kappa \alpha_0^2 + \frac{3\alpha_0^2}{2(x-1)} - \frac{\alpha_0^2}{(x-1)^2} + \frac{\alpha_0^2}{2(x-1)^3} + \\
 & 3\alpha_0^2 - \frac{3\kappa \alpha_0}{x-1} + \frac{3\kappa \alpha_0}{(x-1)^2} - \frac{3\kappa \alpha_0}{(x-1)^3} + \frac{3\kappa \alpha_0}{(x-1)^4} - 12\kappa \alpha_0 - \frac{\alpha_0}{x-1} + \frac{\alpha_0}{(x-1)^2} - \frac{\alpha_0}{(x-1)^3} + \frac{\alpha_0}{(x-1)^4} - 4\alpha_0 +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{15\kappa}{4(x-1)} - \frac{5\kappa}{2(x-1)^2} + \frac{5\kappa}{2(x-1)^3} - \frac{15\kappa}{4(x-1)^4} - \frac{33\kappa}{4(x-1)^5} + \frac{25\kappa}{4} + \left( \frac{6\kappa}{(x-1)^5} + \frac{2}{(x-1)^5} \right) H(0; \alpha_0) + \left( \frac{2\kappa d_1}{(x-1)^5} + \frac{2 d_1}{(x-1)^5} \right) H(1; \alpha_0) \\
 & + \frac{5}{4(x-1)} - \frac{5}{6(x-1)^2} + \frac{5}{6(x-1)^3} - \frac{5}{4(x-1)^4} - \frac{49}{12(x-1)^5} + \frac{25}{12} H(c_1(\alpha_0), c_1(\alpha_0); x) + \\
 & \left( \frac{12\kappa}{(x-1)^5} + 12\kappa + \frac{4}{(x-1)^5} + 4 \right) H(0, 0, 0; \alpha_0) + \left( -\frac{12\kappa}{(x-1)^5} + 12\kappa - \frac{4}{(x-1)^5} + 4 \right) H(0, 0, 0; x) + \\
 & \left( \frac{4\kappa d_1}{(x-1)^5} + 4\kappa d_1 + \frac{4d_1}{(x-1)^5} + 4d_1 \right) H(0, 0, 1; \alpha_0) + \left( \frac{6\kappa}{(x-1)^5} - 6\kappa + \frac{2}{(x-1)^5} - 2 \right) H(0, 0, c_1(\alpha_0); x) + \\
 & \left( \frac{4\kappa d_1}{(x-1)^5} + 4\kappa d_1 + \frac{4d_1}{(x-1)^5} + 4d_1 \right) H(0, 1, 0; \alpha_0) + \left( \frac{2\kappa d_1}{(x-1)^5} - 2\kappa d_1 + \frac{2 d_1}{(x-1)^5} - 2d_1 - \frac{6\kappa}{(x-1)^5} + 6\kappa - \frac{2}{(x-1)^5} + 2 \right) H(0, 1, 0; x) \\
 & + \left( \frac{4d_1^2}{(x-1)^5} + 4 d_1^2 \right) H(0, 1, 1; \alpha_0) + \left( -\frac{2\kappa d_1}{(x-1)^5} + 2\kappa d_1 - \frac{2d_1}{(x-1)^5} + 2d_1 + \frac{6\kappa}{(x-1)^5} - 6\kappa + \frac{2}{(x-1)^5} - 2 \right) H(0, 1, c_1(\alpha_0); x) \\
 & + \left( \frac{3\kappa}{(x-1)^5} - 3\kappa + \frac{1}{(x-1)^5} - 1 \right) H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \\
 & \left( \frac{4\kappa d_1}{(x-1)^5} + \frac{4 d_1}{(x-1)^5} - \frac{6\kappa}{(x-1)^5} + 6\kappa - \frac{2}{(x-1)^5} + 2 \right) H(1, 0, 0; x) + \left( -\frac{2\kappa d_1}{(x-1)^5} - \frac{2 d_1}{(x-1)^5} + \frac{3\kappa}{(x-1)^5} - 3\kappa + \frac{1}{(x-1)^5} - 1 \right) H(1, 0, c_1(\alpha_0); x) \\
 & + \left( -\frac{4d_1^2}{(x-1)^5} + \frac{4\kappa d_1}{(x-1)^5} - 2\kappa d_1 + \frac{4d_1}{(x-1)^5} - 2d_1 - \frac{3\kappa}{(x-1)^5} + 3\kappa - \frac{1}{(x-1)^5} + 1 \right) H(1, 1, 0; x) \\
 & + \left( \frac{4d_1^2}{(x-1)^5} - \frac{4\kappa d_1}{(x-1)^5} + 2\kappa d_1 - \frac{4d_1}{(x-1)^5} + 2d_1 + \frac{3\kappa}{(x-1)^5} - 3\kappa + \frac{1}{(x-1)^5} - 1 \right) H(1, 1, c_1(\alpha_0); x) \\
 & + \left( -\frac{2\kappa d_1}{(x-1)^5} - \frac{2d_1}{(x-1)^5} + \frac{3\kappa}{(x-1)^5} - 3\kappa + \frac{1}{(x-1)^5} - 1 \right) H(1, c_1(\alpha_0), c_1(\alpha_0); x) + \\
 & \left( \frac{3\kappa}{(x-1)^5} + \frac{1}{(x-1)^5} \right) H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) - \frac{35\pi^2\kappa}{24(x-1)(\kappa+1)} + \frac{35\pi^2\kappa}{36(x-1)^2(\kappa+1)} - \frac{35\pi^2\kappa}{36(x-1)^3(\kappa+1)} + \\
 & \frac{35\pi^2\kappa}{24(x-1)^4(\kappa+1)} + \frac{247\pi^2\kappa}{72(x-1)^5(\kappa+1)} - \frac{247\pi^2\kappa}{72(\kappa+1)} - \frac{5\pi^2}{24(x-1)(\kappa+1)} + \frac{5\pi^2}{36(x-1)^2(\kappa+1)} - \frac{5\pi^2}{36(x-1)^3(\kappa+1)} + \\
 & \frac{5\pi^2}{24(x-1)^4(\kappa+1)} + \frac{49\pi^2}{72(x-1)^5(\kappa+1)} - \frac{25\pi^2}{72(\kappa+1)} - \frac{8}{\kappa+1} - \frac{7\kappa\zeta_3}{(x-1)^5(\kappa+1)} + \frac{7\kappa\zeta_3}{\kappa+1} - \frac{\zeta_3}{(x-1)^5(\kappa+1)} + \frac{3\zeta_3}{\kappa+1}.
 \end{aligned}$$

## D.2 The $\mathcal{A}$ integral for $k = 1$ and arbitrary $\kappa$

The  $\varepsilon$  expansion for this integral reads

$$\begin{aligned}
 \mathcal{I}(x, \varepsilon; \alpha_0, 3 + d_1\varepsilon; \kappa, 1, 0, g_A) &= x \mathcal{A}(\varepsilon, x; 3 + d_1\varepsilon; \kappa, 1) \\
 &= \frac{1}{\varepsilon} a_{-1}^{(\kappa, 1)} + a_0^{(\kappa, 1)} + \varepsilon a_1^{(\kappa, 1)} + \varepsilon^2 a_2^{(\kappa, 1)} + \mathcal{O}(\varepsilon^3), \tag{D.2}
 \end{aligned}$$

where

$$\begin{aligned}
 a_{-1}^{(\kappa, 1)} &= -\frac{1}{2(\kappa+1)}, \\
 a_0^{(\kappa, 1)} &= \frac{\alpha_0^4}{8(x-1)} + \frac{\kappa\alpha_0^4}{8(\kappa+1)} + \frac{\alpha_0^4}{8(\kappa+1)} - \frac{\alpha_0^3}{2(x-1)} - \frac{2\kappa\alpha_0^3}{3(\kappa+1)} - \frac{2\alpha_0^3}{3(\kappa+1)} + \frac{\alpha_0^3}{6(x-1)^2} + \frac{3\alpha_0^2}{4(x-1)} + \\
 & \frac{3\kappa\alpha_0^2}{2(\kappa+1)} + \frac{3\alpha_0^2}{2(\kappa+1)} - \frac{\alpha_0^2}{2(x-1)^2} + \frac{\alpha_0^2}{4(x-1)^3} - \frac{\alpha_0}{2(x-1)} - \frac{2\kappa\alpha_0}{\kappa+1} - \frac{2\alpha_0}{\kappa+1} + \frac{\alpha_0}{2(x-1)^2} - \frac{\alpha_0}{2(x-1)^3} + \\
 & \frac{\alpha_0}{2(x-1)^4} + \left( \frac{1}{2} + \frac{1}{2(x-1)^5} \right) H(0; \alpha_0) + \left( \frac{1}{2} - \frac{1}{2(x-1)^5} \right) H(0; x) + \frac{H(c_1(\alpha_0); x)}{2(x-1)^5} - \frac{1}{\kappa+1}, \\
 a_1^{(\kappa, 1)} &= -\frac{d_1\alpha_0^4}{16(\kappa+1)} - \frac{d_1\kappa\alpha_0^4}{16(\kappa+1)} - \frac{d_1\kappa\alpha_0^4}{16(x-1)(\kappa+1)} + \frac{7\kappa\alpha_0^4}{16(x-1)(\kappa+1)} + \frac{7\kappa\alpha_0^4}{16(\kappa+1)} - \frac{d_1\alpha_0^4}{16(x-1)(\kappa+1)} + \\
 & \frac{5\alpha_0^4}{16(x-1)(\kappa+1)} + \frac{5\alpha_0^4}{16(\kappa+1)} + \frac{13d_1\alpha_0^3}{36(\kappa+1)} + \frac{13d_1\kappa\alpha_0^3}{36(\kappa+1)} + \frac{d_1\kappa\alpha_0^3}{4(x-1)(\kappa+1)} - \frac{7\kappa\alpha_0^3}{4(x-1)(\kappa+1)} - \frac{d_1\kappa\alpha_0^3}{9(x-1)^2(\kappa+1)} + \\
 & \frac{19\kappa\alpha_0^3}{24(x-1)^2(\kappa+1)} - \frac{24(\kappa+1)}{4(x-1)(\kappa+1)} + \frac{d_1\alpha_0^3}{4(x-1)(\kappa+1)} - \frac{5\alpha_0^3}{4(x-1)(\kappa+1)} - \frac{d_1\alpha_0^3}{9(x-1)^2(\kappa+1)} + \frac{35\alpha_0^3}{72(x-1)^2(\kappa+1)} - \frac{125\alpha_0^3}{72(\kappa+1)} - \\
 & \frac{23d_1\alpha_0^2}{24(\kappa+1)} - \frac{23d_1\kappa\alpha_0^2}{24(\kappa+1)} - \frac{3d_1\kappa\alpha_0^2}{8(x-1)(\kappa+1)} + \frac{41\kappa\alpha_0^2}{16(x-1)(\kappa+1)} + \frac{d_1\kappa\alpha_0^2}{3(x-1)^2(\kappa+1)} - \frac{39\kappa\alpha_0^2}{16(x-1)^2(\kappa+1)} - \frac{d_1\kappa\alpha_0^2}{4(x-1)^3(\kappa+1)} + \\
 & \frac{27\kappa\alpha_0^2}{16(x-1)^3(\kappa+1)} + \frac{107\kappa\alpha_0^2}{16(\kappa+1)} - \frac{3d_1\alpha_0^2}{8(x-1)(\kappa+1)} + \frac{89\alpha_0^2}{48(x-1)(\kappa+1)} + \frac{d_1\alpha_0^2}{3(x-1)^2(\kappa+1)} - \frac{71\alpha_0^2}{48(x-1)^2(\kappa+1)} - \\
 & \frac{d_1\alpha_0^2}{4(x-1)^3(\kappa+1)} + \frac{43\alpha_0^2}{48(x-1)^3(\kappa+1)} + \frac{203\alpha_0^2}{48(\kappa+1)} + \frac{25d_1\alpha_0}{12(\kappa+1)} + \frac{25d_1\kappa\alpha_0}{12(\kappa+1)} + \frac{d_1\kappa\alpha_0}{4(x-1)(\kappa+1)} - \frac{5\kappa\alpha_0}{4(x-1)(\kappa+1)} - \\
 & \frac{d_1\kappa\alpha_0}{3(x-1)^2(\kappa+1)} + \frac{5\kappa\alpha_0}{2(x-1)^2(\kappa+1)} + \frac{d_1\kappa\alpha_0}{2(x-1)^3(\kappa+1)} - \frac{15\kappa\alpha_0}{4(x-1)^3(\kappa+1)} - \frac{d_1\kappa\alpha_0}{(x-1)^4(\kappa+1)} + \frac{45\kappa\alpha_0}{8(x-1)^4(\kappa+1)} -
 \end{aligned}$$



$$\begin{aligned}
& \frac{105\kappa\alpha_0}{8(\kappa+1)} + \frac{d_1\alpha_0}{4(x-1)(\kappa+1)} - \frac{13\alpha_0}{12(x-1)(\kappa+1)} - \frac{d_1\alpha_0}{3(x-1)^2(\kappa+1)} + \frac{3\alpha_0}{2(x-1)^2(\kappa+1)} + \frac{d_1\alpha_0}{2(x-1)^3(\kappa+1)} - \\
& \frac{23\alpha_0}{12(x-1)^3(\kappa+1)} - \frac{d_1\alpha_0}{(x-1)^4(\kappa+1)} + \frac{61\alpha_0}{24(x-1)^4(\kappa+1)} - \frac{169\alpha_0}{24(\kappa+1)} + \left( -\frac{\kappa\alpha_0^4}{4(x-1)} - \frac{\kappa\alpha_0^4}{4} - \frac{\alpha_0^4}{4(x-1)} - \frac{\alpha_0^4}{4} + \frac{\kappa\alpha_0^3}{x-1} - \right. \\
& \frac{\kappa\alpha_0^3}{3(x-1)^2} + \frac{4\kappa\alpha_0^3}{3} + \frac{\alpha_0^3}{x-1} - \frac{\alpha_0^3}{3(x-1)^2} + \frac{4\alpha_0^3}{3} - \frac{3\kappa\alpha_0^2}{2(x-1)} + \frac{\kappa\alpha_0^2}{(x-1)^2} - \frac{\kappa\alpha_0^2}{2(x-1)^3} - 3\kappa\alpha_0^2 - \frac{3\alpha_0^2}{2(x-1)} + \frac{\alpha_0^2}{(x-1)^2} - \\
& \frac{\alpha_0^2}{2(x-1)^3} - 3\alpha_0^2 + \frac{\kappa\alpha_0}{x-1} - \frac{\kappa\alpha_0}{(x-1)^2} + \frac{\kappa\alpha_0}{(x-1)^3} - \frac{\kappa\alpha_0}{(x-1)^4} + 4\kappa\alpha_0 + \frac{\alpha_0}{x-1} - \frac{\alpha_0}{(x-1)^2} + \frac{\alpha_0}{(x-1)^3} - \frac{\alpha_0}{(x-1)^4} + \\
& 4\alpha_0 - \frac{5\kappa}{8(x-1)} + \frac{5\kappa}{12(x-1)^2} - \frac{5\kappa}{12(x-1)^3} + \frac{5\kappa}{8(x-1)^4} + \frac{25\kappa}{24(x-1)^5} - \frac{25\kappa}{24} - \frac{5}{8(x-1)} + \frac{5}{12(x-1)^2} - \frac{5}{12(x-1)^3} + \\
& \frac{5}{8(x-1)^4} + \frac{49}{24(x-1)^5} - \frac{1}{24} \Big) H(0; \alpha_0) + \left( \frac{5\kappa}{8(x-1)} - \frac{5\kappa}{12(x-1)^2} + \frac{5\kappa}{12(x-1)^3} - \frac{5\kappa}{8(x-1)^4} - \frac{25\kappa}{24(x-1)^5} + \frac{25\kappa}{24} + \right. \\
& \frac{5}{8(x-1)} - \frac{5}{12(x-1)^2} + \frac{5}{12(x-1)^3} - \frac{5}{8(x-1)^4} - \frac{49}{24(x-1)^5} + \frac{49}{24} \Big) H(0; x) + \left( -\frac{d_1\alpha_0^4}{4} - \frac{d_1\alpha_0^4}{4(x-1)} + \frac{4d_1\alpha_0^3}{3} + \right. \\
& \frac{d_1\alpha_0^3}{x-1} - \frac{d_1\alpha_0^3}{3(x-1)^2} - 3d_1\alpha_0^2 - \frac{3d_1\alpha_0^2}{2(x-1)} + \frac{d_1\alpha_0^2}{(x-1)^2} - \frac{d_1\alpha_0^2}{2(x-1)^3} + 4d_1\alpha_0 + \frac{d_1\alpha_0}{x-1} - \frac{d_1\alpha_0}{(x-1)^2} + \frac{d_1\alpha_0}{(x-1)^3} - \frac{d_1\alpha_0}{(x-1)^4} - \\
& \frac{25d_1}{12} - \frac{d_1}{4(x-1)} + \frac{d_1}{3(x-1)^2} - \frac{d_1}{2(x-1)^3} + \frac{d_1}{(x-1)^4} \Big) H(1; \alpha_0) + \left( \frac{d_1}{(x-1)^5} - \frac{\kappa}{2(x-1)^5} + \frac{\kappa}{2} - \frac{1}{2(x-1)^5} + \right. \\
& \left. \frac{1}{2} \right) H(0; \alpha_0)H(1; x) + \left( -\frac{\kappa\alpha_0^4}{8(x-1)} - \frac{\kappa\alpha_0^4}{8} - \frac{\alpha_0^4}{8(x-1)} - \frac{\alpha_0^4}{8} + \frac{\kappa\alpha_0^3}{2(x-1)} - \frac{\kappa\alpha_0^3}{6(x-1)^2} + \frac{2\kappa\alpha_0^3}{3} + \frac{\alpha_0^3}{2(x-1)} - \right. \\
& \frac{\alpha_0^3}{6(x-1)^2} + \frac{2\alpha_0^3}{3} - \frac{3\kappa\alpha_0^2}{4(x-1)} + \frac{\kappa\alpha_0^2}{2(x-1)^2} - \frac{\kappa\alpha_0^2}{4(x-1)^3} - \frac{3\kappa\alpha_0^2}{2} - \frac{3\alpha_0^2}{4(x-1)} + \frac{\alpha_0^2}{2(x-1)^2} - \frac{\alpha_0^2}{4(x-1)^3} - \frac{3\alpha_0^2}{2} + \\
& \frac{\kappa\alpha_0}{2(x-1)} - \frac{\kappa\alpha_0}{2(x-1)^2} + \frac{\kappa\alpha_0}{2(x-1)^3} - \frac{\kappa\alpha_0}{2(x-1)^4} + 2\kappa\alpha_0 + \frac{\alpha_0}{2(x-1)} - \frac{\alpha_0}{2(x-1)^2} + \frac{\alpha_0}{2(x-1)^3} - \frac{\alpha_0}{2(x-1)^4} + 2\alpha_0 - \\
& \frac{5\kappa}{8(x-1)} + \frac{5\kappa}{12(x-1)^2} - \frac{5\kappa}{12(x-1)^3} + \frac{5\kappa}{8(x-1)^4} + \frac{25\kappa}{24(x-1)^5} - \frac{25\kappa}{24} + \left( -\frac{\kappa}{(x-1)^5} - \frac{1}{(x-1)^5} \right) H(0; \alpha_0) - \\
& \frac{d_1H(1; \alpha_0)}{(x-1)^5} - \frac{5}{8(x-1)} + \frac{5}{12(x-1)^2} - \frac{5}{12(x-1)^3} + \frac{5}{8(x-1)^4} + \frac{49}{24(x-1)^5} - \frac{25}{24} \Big) H(c_1(\alpha_0); x) + \left( -\frac{\kappa}{(x-1)^5} - \kappa - \frac{1}{(x-1)^5} - 1 \right) \\
& H(0, 0; \alpha_0) + \left( \frac{\kappa}{(x-1)^5} - \kappa + \frac{1}{(x-1)^5} - 1 \right) H(0, 0; x) + \left( -\frac{d_1}{(x-1)^5} - d_1 \right) H(0, 1; \alpha_0) + \left( -\frac{\kappa}{2(x-1)^5} + \frac{\kappa}{2} - \frac{1}{2(x-1)^5} + \frac{1}{2} \right) H(0, c_1(\alpha_0); x) + \left( -\frac{d_1}{(x-1)^5} + \frac{\kappa}{2(x-1)^5} - \right. \\
& \left. \frac{\kappa}{2} + \frac{1}{2(x-1)^5} - \frac{1}{2} \right) H(1, 0; x) + \left( \frac{d_1}{(x-1)^5} - \frac{\kappa}{2(x-1)^5} + \frac{\kappa}{2} - \frac{1}{2(x-1)^5} + \frac{1}{2} \right) H(1, c_1(\alpha_0); x) + \left( -\frac{\kappa}{2(x-1)^5} - \frac{1}{2(x-1)^5} \right) H(c_1(\alpha_0), c_1(\alpha_0); x) + \frac{\pi^2\kappa}{4(x-1)^5(\kappa+1)} - \frac{\pi^2\kappa}{4(\kappa+1)} + \frac{\pi^2}{12(x-1)^5(\kappa+1)} - \frac{2}{\kappa+1},
\end{aligned}$$

$$\begin{aligned}
a_2^{(\kappa,1)} &= \frac{d_1^2\alpha_0^4}{32(\kappa+1)} - \frac{3d_1\alpha_0^4}{16(\kappa+1)} + \frac{d_1^2\kappa\alpha_0^4}{32(\kappa+1)} - \frac{5d_1\kappa\alpha_0^4}{16(\kappa+1)} + \frac{d_1^2\kappa\alpha_0^4}{32(x-1)(\kappa+1)} - \frac{5d_1\kappa\alpha_0^4}{16(x-1)(\kappa+1)} - \frac{\pi^2\kappa\alpha_0^4}{48(x-1)(\kappa+1)} + \\
& \frac{35\kappa\alpha_0^4}{32(x-1)(\kappa+1)} + \frac{35\kappa\alpha_0^4}{32(\kappa+1)} + \frac{d_1^2\alpha_0^4}{32(x-1)(\kappa+1)} - \frac{3d_1\alpha_0^4}{16(x-1)(\kappa+1)} - \frac{\pi^2\alpha_0^4}{48(x-1)(\kappa+1)} + \frac{21\alpha_0^4}{32(x-1)(\kappa+1)} + \frac{21\alpha_0^4}{32(\kappa+1)} - \\
& \frac{\pi^2\alpha_0^4}{48} - \frac{43d_1^2\alpha_0^3}{216(\kappa+1)} + \frac{505d_1\alpha_0^3}{432(\kappa+1)} - \frac{43d_1^2\kappa\alpha_0^3}{216(\kappa+1)} + \frac{33d_1\kappa\alpha_0^3}{16(\kappa+1)} - \frac{d_1^2\kappa\alpha_0^3}{8(x-1)(\kappa+1)} + \frac{5d_1\kappa\alpha_0^3}{4(x-1)(\kappa+1)} + \frac{\pi^2\kappa\alpha_0^3}{12(x-1)(\kappa+1)} - \\
& \frac{35\kappa\alpha_0^3}{8(x-1)(\kappa+1)} + \frac{2d_1^2\kappa\alpha_0^3}{27(x-1)^2(\kappa+1)} - \frac{13d_1\kappa\alpha_0^3}{16(x-1)^2(\kappa+1)} - \frac{\pi^2\kappa\alpha_0^3}{36(x-1)^2(\kappa+1)} + \frac{432(x-1)^2(\kappa+1)}{432(x-1)^2(\kappa+1)} - \frac{432(\kappa+1)}{432(x-1)^2(\kappa+1)} - \\
& \frac{d_1^2\alpha_0^3}{8(x-1)(\kappa+1)} + \frac{3d_1\alpha_0^3}{4(x-1)(\kappa+1)} + \frac{\pi^2\alpha_0^3}{12(x-1)(\kappa+1)} - \frac{21\alpha_0^3}{8(x-1)(\kappa+1)} + \frac{2d_1^2\alpha_0^3}{27(x-1)^2(\kappa+1)} - \frac{181d_1\alpha_0^3}{432(x-1)^2(\kappa+1)} - \\
& \frac{\pi^2\alpha_0^3}{36(x-1)^2(\kappa+1)} + \frac{473\alpha_0^3}{432(x-1)^2(\kappa+1)} - \frac{1607\alpha_0^3}{432(\kappa+1)} + \frac{\pi^2\alpha_0^3}{9} + \frac{95d_1^2\alpha_0^2}{144(\kappa+1)} - \frac{347d_1\alpha_0^2}{96(\kappa+1)} + \frac{95d_1^2\kappa\alpha_0^2}{144(\kappa+1)} - \frac{673d_1\kappa\alpha_0^2}{96(\kappa+1)} + \\
& \frac{3d_1^2\kappa\alpha_0^2}{16(x-1)(\kappa+1)} - \frac{167d_1\kappa\alpha_0^2}{96(x-1)(\kappa+1)} - \frac{\pi^2\kappa\alpha_0^2}{8(x-1)(\kappa+1)} + \frac{1721\kappa\alpha_0^2}{288(x-1)(\kappa+1)} - \frac{2d_1^2\kappa\alpha_0^2}{9(x-1)^2(\kappa+1)} + \frac{247d_1\kappa\alpha_0^2}{96(x-1)^2(\kappa+1)} + \\
& \frac{\pi^2\kappa\alpha_0^2}{2279\kappa\alpha_0^2} - \frac{2279\kappa\alpha_0^2}{2279\kappa\alpha_0^2} + \frac{d_1^2\kappa\alpha_0^2}{259d_1\kappa\alpha_0^2} - \frac{259d_1\kappa\alpha_0^2}{259d_1\kappa\alpha_0^2} - \frac{\pi^2\kappa\alpha_0^2}{\pi^2\kappa\alpha_0^2} + \frac{1987\kappa\alpha_0^2}{1987\kappa\alpha_0^2} + \\
& \frac{12(x-1)^2(\kappa+1)}{288(\kappa+1)} + \frac{288(x-1)^2(\kappa+1)}{288(x-1)(\kappa+1)} + \frac{4(x-1)^3(\kappa+1)}{288(x-1)(\kappa+1)} - \frac{96(x-1)^3(\kappa+1)}{288(x-1)(\kappa+1)} - \frac{24(x-1)^3(\kappa+1)}{288(x-1)(\kappa+1)} + \frac{288(x-1)^3(\kappa+1)}{288(x-1)(\kappa+1)} + \\
& \frac{5987\kappa\alpha_0^2}{288(\kappa+1)} + \frac{3d_1^2\alpha_0^2}{16(x-1)(\kappa+1)} - \frac{311d_1\alpha_0^2}{288(x-1)(\kappa+1)} - \frac{\pi^2\alpha_0^2}{8(x-1)(\kappa+1)} + \frac{1103\alpha_0^2}{288(x-1)(\kappa+1)} - \frac{2d_1^2\alpha_0^2}{9(x-1)^2(\kappa+1)} + \\
& \frac{125d_1\alpha_0^2}{96(x-1)^2(\kappa+1)} + \frac{\pi^2\alpha_0^2}{12(x-1)^2(\kappa+1)} - \frac{288(x-1)^2(\kappa+1)}{288(x-1)^2(\kappa+1)} + \frac{4(x-1)^3(\kappa+1)}{288(x-1)^3(\kappa+1)} - \frac{355d_1\alpha_0^2}{288(x-1)^3(\kappa+1)} - \frac{\pi^2\alpha_0^2}{24(x-1)^3(\kappa+1)} + \\
& \frac{661\alpha_0^2}{288(x-1)^3(\kappa+1)} + \frac{2741\alpha_0^2}{288(\kappa+1)} - \frac{\pi^2\alpha_0^2}{4} - \frac{205d_1^2\alpha_0}{72(\kappa+1)} + \frac{575d_1\alpha_0}{48(\kappa+1)} - \frac{205d_1^2\kappa\alpha_0}{72(\kappa+1)} + \frac{1325d_1\kappa\alpha_0}{48(\kappa+1)} - \frac{d_1^2\kappa\alpha_0}{8(x-1)(\kappa+1)} - \\
& \frac{d_1\kappa\alpha_0}{3(x-1)(\kappa+1)} + \frac{\pi^2\kappa\alpha_0}{12(x-1)(\kappa+1)} + \frac{16\kappa\alpha_0}{9(x-1)(\kappa+1)} + \frac{2d_1^2\kappa\alpha_0}{9(x-1)^2(\kappa+1)} - \frac{65d_1\kappa\alpha_0}{24(x-1)^2(\kappa+1)} - \frac{\pi^2\kappa\alpha_0}{12(x-1)^2(\kappa+1)} + \\
& \frac{17\kappa\alpha_0}{2(x-1)^2(\kappa+1)} - \frac{d_1^2\kappa\alpha_0}{2(x-1)^3(\kappa+1)} + \frac{161d_1\kappa\alpha_0}{24(x-1)^3(\kappa+1)} + \frac{\pi^2\kappa\alpha_0}{12(x-1)^3(\kappa+1)} - \frac{169\kappa\alpha_0}{9(x-1)^3(\kappa+1)} + \frac{2d_1^2\kappa\alpha_0}{(x-1)^4(\kappa+1)} - \\
& \frac{889d_1\kappa\alpha_0}{48(x-1)^4(\kappa+1)} - \frac{\pi^2\kappa\alpha_0}{12(x-1)^4(\kappa+1)} + \frac{334\kappa\alpha_0}{9(x-1)^4(\kappa+1)} - \frac{1127\kappa\alpha_0}{18(\kappa+1)} - \frac{d_1^2\alpha_0}{8(x-1)(\kappa+1)} + \frac{2d_1\alpha_0}{9(x-1)(\kappa+1)} +
\end{aligned}$$

$$\begin{aligned}
& \frac{\pi^2 \alpha_0}{12(x-1)(\kappa+1)} - \frac{14\alpha_0}{9(x-1)(\kappa+1)} + \frac{2d_1^2 \alpha_0}{9(x-1)^2(\kappa+1)} - \frac{97d_1 \alpha_0}{72(x-1)^2(\kappa+1)} - \frac{\pi^2 \alpha_0}{12(x-1)^2(\kappa+1)} + \frac{7\alpha_0}{2(x-1)^2(\kappa+1)} - \\
& \frac{d_1^2 \alpha_0}{2(x-1)^3(\kappa+1)} + \frac{209d_1 \alpha_0}{72(x-1)^3(\kappa+1)} + \frac{\pi^2 \alpha_0}{12(x-1)^3(\kappa+1)} - \frac{49\alpha_0}{9(x-1)^3(\kappa+1)} + \frac{2d_1^2 \alpha_0}{(x-1)^4(\kappa+1)} - \frac{1081d_1 \alpha_0}{144(x-1)^4(\kappa+1)} - \\
& \frac{\pi^2 \alpha_0}{12(x-1)^4(\kappa+1)} + \frac{79\alpha_0}{9(x-1)^4(\kappa+1)} - \frac{347\alpha_0}{18(\kappa+1)} + \frac{\pi^2 \alpha_0}{3} + \left( \frac{d_1 \alpha_0^4}{8} + \frac{1}{8} d_1 \kappa \alpha_0^4 + \frac{d_1 \kappa \alpha_0^4}{8(x-1)} - \frac{7\kappa \alpha_0^4}{8(x-1)} - \frac{7\kappa \alpha_0^4}{8} + \right. \\
& \frac{d_1 \alpha_0^4}{8(x-1)} - \frac{5\alpha_0^4}{8(x-1)} - \frac{5\alpha_0^4}{8} - \frac{13d_1 \alpha_0^3}{18} - \frac{13}{18} d_1 \kappa \alpha_0^3 - \frac{d_1 \kappa \alpha_0^3}{2(x-1)} + \frac{7\kappa \alpha_0^3}{2(x-1)} + \frac{2d_1 \kappa \alpha_0^3}{9(x-1)^2} - \frac{19\kappa \alpha_0^3}{12(x-1)^2} + \frac{61\kappa \alpha_0^3}{12} - \\
& \frac{d_1 \alpha_0^3}{2(x-1)} + \frac{5\alpha_0^3}{2(x-1)} + \frac{2}{9} \frac{d_1 \alpha_0^3}{(x-1)^2} - \frac{35\alpha_0^3}{36(x-1)^2} + \frac{125}{36} \frac{\alpha_0^3}{(x-1)} + \frac{23d_1 \alpha_0^2}{12} + \frac{23}{12} d_1 \kappa \alpha_0^2 + \frac{3d_1 \kappa \alpha_0^2}{4(x-1)} - \frac{41\kappa \alpha_0^2}{8(x-1)} - \frac{2d_1 \kappa \alpha_0^2}{3(x-1)^2} + \\
& \frac{39\kappa \alpha_0^2}{8(x-1)^2} + \frac{d_1 \kappa \alpha_0^2}{2(x-1)^3} - \frac{27\kappa \alpha_0^2}{8(x-1)^3} - \frac{107\kappa \alpha_0^2}{8} + \frac{3d_1 \alpha_0^2}{4(x-1)} - \frac{89\alpha_0^2}{24(x-1)} - \frac{2d_1 \alpha_0^2}{3(x-1)^2} + \frac{71\alpha_0^2}{24(x-1)^2} + \frac{d_1 \alpha_0^2}{2(x-1)^3} - \\
& \frac{43\alpha_0^2}{24(x-1)^3} - \frac{203\alpha_0^2}{24} - \frac{25}{6} \frac{d_1 \alpha_0}{(x-1)} - \frac{25d_1 \kappa \alpha_0}{6} - \frac{d_1 \kappa \alpha_0}{2(x-1)} + \frac{5\kappa \alpha_0}{2(x-1)} + \frac{2d_1 \kappa \alpha_0}{3(x-1)^2} - \frac{5\kappa \alpha_0}{(x-1)^2} - \frac{d_1 \kappa \alpha_0}{(x-1)^3} + \frac{15\kappa \alpha_0}{2(x-1)^3} + \\
& \frac{2d_1 \kappa \alpha_0}{(x-1)^4} - \frac{45\kappa \alpha_0}{4(x-1)^4} + \frac{105\kappa \alpha_0}{4} - \frac{d_1 \alpha_0}{2(x-1)} + \frac{13\alpha_0}{6(x-1)} + \frac{2d_1 \alpha_0}{3(x-1)^2} - \frac{3\alpha_0}{(x-1)^2} - \frac{d_1 \alpha_0}{(x-1)^3} + \frac{23\alpha_0}{6(x-1)^3} + \frac{2d_1 \alpha_0}{(x-1)^4} - \\
& \frac{61\alpha_0}{12(x-1)^4} + \frac{169\alpha_0}{12} + \frac{205}{144} \frac{d_1}{(x-1)} + \frac{205d_1 \kappa}{144} + \frac{17d_1 \kappa}{16(x-1)} - \frac{45\kappa}{8(x-1)} - \frac{13d_1 \kappa}{36(x-1)^2} + \frac{35\kappa}{12(x-1)^2} + \frac{13d_1 \kappa}{36(x-1)^3} - \\
& \frac{35\kappa}{12(x-1)^3} - \frac{17d_1 \kappa}{16(x-1)^4} + \frac{45\kappa}{8(x-1)^4} - \frac{205d_1 \kappa}{144(x-1)^5} + \frac{205\kappa}{24(x-1)^5} - \frac{205\kappa}{24} + \frac{17d_1}{16(x-1)} - \frac{65}{24(x-1)} - \frac{13d_1}{36(x-1)^2} + \\
& \frac{55}{36(x-1)^2} + \frac{13d_1}{36(x-1)^3} - \frac{55}{36(x-1)^3} - \frac{17d_1}{16(x-1)^4} + \frac{65}{24(x-1)^4} - \frac{205}{144(x-1)^5} - \frac{\pi^2}{12(x-1)^5} + \frac{449}{72(x-1)^5} - \frac{\pi^2}{12} \\
& \left. \frac{161}{72} \right) H(0; \alpha_0) + \left( -\frac{17\kappa d_1}{16(x-1)} + \frac{13\kappa d_1}{36(x-1)^2} - \frac{13\kappa d_1}{36(x-1)^3} + \frac{17\kappa d_1}{16(x-1)^4} + \frac{205\kappa d_1}{144(x-1)^5} - \frac{205\kappa d_1}{144} - \frac{17d_1}{16(x-1)} + \right. \\
& \frac{13d_1}{36(x-1)^2} - \frac{13}{36} \frac{d_1}{(x-1)^3} + \frac{17d_1}{16(x-1)^4} + \frac{205d_1}{144(x-1)^5} - \frac{205d_1}{144} + \frac{45\kappa}{8(x-1)} - \frac{35}{12} \frac{\kappa}{(x-1)^2} + \frac{35\kappa}{12(x-1)^3} - \frac{45\kappa}{8(x-1)^4} - \\
& \frac{\pi^2 \kappa}{2(x-1)^5} - \frac{205\kappa}{24(x-1)^5} + \frac{\pi^2 \kappa}{2} + \frac{205\kappa}{24} + \frac{65}{24(x-1)} - \frac{55}{36(x-1)^2} + \frac{55}{36(x-1)^3} - \frac{65}{24(x-1)^4} - \frac{\pi^2}{12(x-1)^5} - \frac{449}{72(x-1)^5} + \\
& \left. \frac{\pi^2}{12} + \frac{449}{72} \right) H(0; x) + \left( \frac{d_1^2 \alpha_0^4}{8} - \frac{5}{8} \frac{d_1 \alpha_0^4}{(x-1)} - \frac{1}{8} d_1 \kappa \alpha_0^4 - \frac{d_1 \kappa \alpha_0^4}{8(x-1)} + \frac{d_1^2 \alpha_0^4}{8(x-1)} - \frac{5d_1 \alpha_0^4}{8(x-1)} - \frac{13d_1^2 \alpha_0^3}{18} + \frac{125d_1 \alpha_0^3}{36} + \right. \\
& \frac{29}{36} d_1 \kappa \alpha_0^3 + \frac{d_1 \kappa \alpha_0^3}{2(x-1)} - \frac{11d_1 \kappa \alpha_0^3}{36(x-1)^2} - \frac{d_1^2 \alpha_0^3}{2(x-1)} + \frac{5d_1 \alpha_0^3}{2(x-1)} + \frac{2d_1^2 \alpha_0^3}{9(x-1)^2} - \frac{35d_1 \alpha_0^3}{36(x-1)^2} + \frac{23d_1^2 \alpha_0^2}{12} - \frac{203d_1 \alpha_0^2}{24} - \\
& \frac{59}{24} d_1 \kappa \alpha_0^2 - \frac{17d_1 \kappa \alpha_0^2}{24(x-1)} + \frac{23d_1 \kappa \alpha_0^2}{24(x-1)^2} - \frac{19d_1 \kappa \alpha_0^2}{24(x-1)^3} + \frac{3}{4} \frac{d_1^2 \alpha_0^2}{(x-1)} - \frac{89d_1 \alpha_0^2}{24(x-1)} - \frac{2d_1^2 \alpha_0^2}{3(x-1)^2} + \frac{71d_1 \alpha_0^2}{24(x-1)^2} + \frac{d_1^2 \alpha_0^2}{2(x-1)^3} - \\
& \frac{43d_1 \alpha_0^2}{24(x-1)^3} - \frac{25d_1^2 \alpha_0}{6} + \frac{169d_1 \alpha_0}{12} + \frac{73d_1 \kappa \alpha_0}{12} + \frac{d_1 \kappa \alpha_0}{6(x-1)} - \frac{d_1 \kappa \alpha_0}{(x-1)^2} + \frac{11d_1 \kappa \alpha_0}{6(x-1)^3} - \frac{37d_1 \kappa \alpha_0}{12(x-1)^4} - \frac{d_1^2 \alpha_0}{2(x-1)} + \frac{13d_1 \alpha_0}{6(x-1)} + \\
& \frac{2d_1^2 \alpha_0}{3(x-1)^2} - \frac{3d_1 \alpha_0}{(x-1)^2} - \frac{d_1^2 \alpha_0}{(x-1)^3} + \frac{23d_1 \alpha_0}{6(x-1)^3} + \frac{2d_1^2 \alpha_0}{(x-1)^4} - \frac{61d_1 \alpha_0}{12(x-1)^4} + \frac{205}{72} \frac{d_1^2}{(x-1)} - \frac{305d_1}{36} - \frac{155d_1 \kappa}{36} + \frac{d_1 \kappa}{6(x-1)} + \\
& \frac{25d_1 \kappa}{72(x-1)^2} - \frac{25d_1 \kappa}{24(x-1)^3} + \frac{37d_1 \kappa}{12(x-1)^4} + \frac{d_1^2}{8(x-1)} - \frac{d_1}{3(x-1)} - \frac{2d_1^2}{9(x-1)^2} + \frac{73d_1}{72(x-1)^2} + \frac{d_1^2}{2(x-1)^3} - \frac{49d_1}{24(x-1)^3} - \\
& \frac{2}{(x-1)^4} + \frac{61d_1}{12(x-1)^4} \Big) H(1; \alpha_0) + \left( \frac{3}{2} \frac{\kappa \alpha_0^4}{(x-1)} + \frac{3\kappa \alpha_0^4}{2} + \frac{\alpha_0^4}{2(x-1)} + \frac{\alpha_0^4}{2} - \frac{6\kappa \alpha_0^3}{x-1} + \frac{2\kappa \alpha_0^3}{(x-1)^2} - 8\kappa \alpha_0^3 - \frac{2\alpha_0^3}{x-1} + \right. \\
& \frac{2\alpha_0^3}{3(x-1)^2} - \frac{8\alpha_0^3}{3} + \frac{9\kappa \alpha_0^2}{x-1} - \frac{6\kappa \alpha_0^2}{(x-1)^2} + \frac{3\kappa \alpha_0^2}{(x-1)^3} + 18\kappa \alpha_0^2 + \frac{3\alpha_0^2}{x-1} - \frac{2\alpha_0^2}{(x-1)^2} + \frac{\alpha_0^2}{(x-1)^3} + 6\alpha_0^2 - \frac{6\kappa \alpha_0}{x-1} + \frac{6\kappa \alpha_0}{(x-1)^2} - \\
& \frac{6\kappa \alpha_0}{(x-1)^3} + \frac{6\kappa \alpha_0}{(x-1)^4} - 24\kappa \alpha_0 - \frac{2}{x-1} \frac{\alpha_0}{(x-1)^2} - \frac{2\alpha_0}{(x-1)^3} + \frac{2}{(x-1)^4} \frac{\alpha_0}{(x-1)} - 8\alpha_0 + \frac{15\kappa}{4(x-1)} - \frac{5\kappa}{2(x-1)^2} + \frac{5\kappa}{2(x-1)^3} - \\
& \frac{15\kappa}{4(x-1)^4} - \frac{33\kappa}{4(x-1)^5} + \frac{17\kappa}{4} + \frac{5}{4(x-1)} - \frac{5}{6(x-1)^2} + \frac{5}{6(x-1)^3} - \frac{5}{4(x-1)^4} - \frac{49}{12(x-1)^5} + \frac{1}{12} \Big) H(0, 0; \alpha_0) + \\
& \left( -\frac{15\kappa}{4(x-1)} + \frac{5\kappa}{2(x-1)^2} - \frac{5\kappa}{2(x-1)^3} + \frac{15\kappa}{4(x-1)^4} + \frac{33\kappa}{4(x-1)^5} - \frac{33\kappa}{4} - \frac{5}{4(x-1)} + \frac{5}{6(x-1)^2} - \frac{5}{6(x-1)^3} + \right. \\
& \frac{5}{4(x-1)^4} + \frac{49}{12(x-1)^5} - \frac{49}{12} \Big) H(0, 0; x) + \left( \frac{d_1 \alpha_0^4}{2} + \frac{1}{2} d_1 \kappa \alpha_0^4 + \frac{d_1 \kappa \alpha_0^4}{2(x-1)} + \frac{d_1 \alpha_0^4}{2(x-1)} - \frac{8d_1 \alpha_0^3}{3} - \frac{8}{3} d_1 \kappa \alpha_0^3 - \right. \\
& \frac{2}{x-1} \frac{d_1 \kappa \alpha_0^3}{(x-1)^2} + \frac{2d_1 \kappa \alpha_0^3}{3(x-1)^2} - \frac{2}{x-1} \frac{d_1 \alpha_0^3}{(x-1)} + \frac{2d_1 \alpha_0^3}{3(x-1)^2} + 6d_1 \alpha_0^2 + 6d_1 \kappa \alpha_0^2 + \frac{3d_1 \kappa \alpha_0^2}{x-1} - \frac{2d_1 \kappa \alpha_0^2}{(x-1)^2} + \frac{d_1 \kappa \alpha_0^2}{(x-1)^3} + \frac{3d_1 \alpha_0^2}{x-1} - \\
& \frac{2d_1 \alpha_0^2}{(x-1)^2} + \frac{d_1 \alpha_0^2}{(x-1)^3} - 8d_1 \alpha_0 - 8d_1 \kappa \alpha_0 - \frac{2d_1 \kappa \alpha_0}{x-1} + \frac{2d_1 \kappa \alpha_0}{(x-1)^2} - \frac{2d_1 \kappa \alpha_0}{(x-1)^3} + \frac{2d_1 \kappa \alpha_0}{(x-1)^4} - \frac{2d_1 \alpha_0}{x-1} + \frac{2d_1 \alpha_0}{(x-1)^2} - \\
& \frac{2d_1 \alpha_0}{(x-1)^3} + \frac{2d_1 \alpha_0}{(x-1)^4} + \frac{d_1}{12} + \frac{25d_1 \kappa}{4(x-1)} + \frac{5d_1 \kappa}{6(x-1)^2} - \frac{5d_1 \kappa}{6(x-1)^3} - \frac{5d_1 \kappa}{4(x-1)^4} - \frac{25d_1 \kappa}{12(x-1)^5} + \frac{5d_1}{4(x-1)} - \frac{5d_1}{6(x-1)^2} + \\
& \frac{5d_1}{6(x-1)^3} - \frac{5d_1}{4(x-1)^4} - \frac{49d_1}{12(x-1)^5} \Big) H(0, 1; \alpha_0) + H(1; x) \left( \frac{\pi^2 \kappa d_1}{6(x-1)^5} + \frac{\pi^2 d_1}{6(x-1)^5} - \frac{\pi^2 \kappa}{4(x-1)^5} + \frac{\pi^2 \kappa}{4} + \left( - \right. \right. \\
& \frac{\kappa d_1}{x-1} + \frac{\kappa d_1}{2(x-1)^2} - \frac{\kappa d_1}{3(x-1)^3} + \frac{\kappa d_1}{4(x-1)^4} + \frac{25\kappa d_1}{12(x-1)^5} - \frac{d_1}{x-1} + \frac{d_1}{2(x-1)^2} - \frac{d_1}{3(x-1)^3} + \frac{d_1}{4(x-1)^4} + \frac{49d_1}{12(x-1)^5} + \\
& \frac{15\kappa}{8(x-1)} - \frac{5\kappa}{4(x-1)^2} + \frac{5\kappa}{4(x-1)^3} - \frac{15\kappa}{8(x-1)^4} - \frac{33\kappa}{8(x-1)^5} + \frac{33\kappa}{8} + \frac{5}{8(x-1)} - \frac{5}{12(x-1)^2} + \frac{5}{12(x-1)^3} - \frac{5}{8(x-1)^4} - \\
& \left. \frac{49}{24(x-1)^5} + \frac{49}{24} \right) H(0; \alpha_0) + \left( -\frac{2}{(x-1)^5} \frac{\kappa d_1}{(x-1)^5} - \frac{2d_1}{(x-1)^5} + \frac{3\kappa}{(x-1)^5} - 3\kappa + \frac{1}{(x-1)^5} - 1 \right) H(0, 0; \alpha_0) + \left( - \right.
\end{aligned}$$

$$\begin{aligned}
 & \frac{2d_1^2}{(x-1)^5} + \frac{\kappa d_1}{(x-1)^5} - \kappa d_1 + \frac{d_1}{(x-1)^5} - d_1 \Big) H(0, 1; \alpha_0) - \frac{\pi^2}{12} \frac{\pi^2}{(x-1)^5} + \frac{\pi^2}{12} \Big) + \left( -\frac{\kappa d_1}{(x-1)^5} + \kappa d_1 - \frac{d_1}{(x-1)^5} + \right. \\
 & d_1 + \frac{3\kappa}{(x-1)^5} - 3\kappa + \frac{1}{(x-1)^5} - 1 \Big) H(0; \alpha_0) H(0, 1; x) + \left( \frac{15\kappa}{8(x-1)} - \frac{5\kappa}{4(x-1)^2} + \frac{5\kappa}{4(x-1)^3} - \frac{15\kappa}{8(x-1)^4} - \right. \\
 & \frac{33\kappa}{8(x-1)^5} + \frac{33\kappa}{8} + \left( \frac{3\kappa}{(x-1)^5} - 3\kappa + \frac{1}{(x-1)^5} - 1 \right) H(0; \alpha_0) + \left( \frac{\kappa d_1}{(x-1)^5} - \kappa d_1 + \frac{d_1}{(x-1)^5} - d_1 \right) H(1; \alpha_0) + \\
 & \frac{5}{8(x-1)} - \frac{5}{12(x-1)^2} + \frac{5}{12(x-1)^3} - \frac{5}{8(x-1)^4} - \frac{49}{24(x-1)^5} + \frac{49}{24} \Big) H(0, c_1(\alpha_0); x) + \left( \frac{d_1 \alpha_0^4}{2} + \frac{1}{2} d_1 \kappa \alpha_0^4 + \right. \\
 & \frac{d_1 \kappa \alpha_0^4}{2(x-1)} + \frac{d_1 \alpha_0^4}{2(x-1)} - \frac{8d_1 \alpha_0^3}{3} - \frac{8}{3} d_1 \kappa \alpha_0^3 - \frac{2}{x-1} \frac{d_1 \kappa \alpha_0^3}{x-1} + \frac{2d_1 \kappa \alpha_0^3}{3(x-1)^2} - \frac{2}{x-1} \frac{d_1 \alpha_0^3}{x-1} + \frac{2d_1 \alpha_0^3}{3(x-1)^2} + 6d_1 \alpha_0^2 + 6d_1 \kappa \alpha_0^2 + \\
 & \frac{3d_1 \kappa \alpha_0^2}{x-1} - \frac{2d_1 \kappa \alpha_0^2}{(x-1)^2} + \frac{d_1 \kappa \alpha_0^2}{(x-1)^3} + \frac{3d_1 \alpha_0^2}{x-1} - \frac{2d_1 \alpha_0^2}{(x-1)^2} + \frac{d_1 \alpha_0^2}{(x-1)^3} - 8 d_1 \alpha_0 - 8d_1 \kappa \alpha_0 - \frac{2d_1 \kappa \alpha_0}{x-1} + \frac{2d_1 \kappa \alpha_0}{(x-1)^2} - \\
 & \frac{2d_1 \kappa \alpha_0}{(x-1)^3} + \frac{2d_1 \kappa \alpha_0}{(x-1)^4} - \frac{2d_1 \alpha_0}{x-1} + \frac{2d_1 \alpha_0}{(x-1)^2} - \frac{2d_1 \alpha_0}{(x-1)^3} + \frac{2d_1 \alpha_0}{(x-1)^4} + \frac{25d_1}{6} + \frac{25d_1 \kappa}{6} + \frac{d_1 \kappa}{2(x-1)} - \frac{2d_1 \kappa}{3(x-1)^2} + \frac{d_1 \kappa}{(x-1)^3} - \\
 & \frac{2d_1 \kappa}{(x-1)^4} + \frac{d_1}{2(x-1)} - \frac{2}{3} \frac{d_1}{(x-1)^2} + \frac{d_1}{(x-1)^3} - \frac{2d_1}{(x-1)^4} \Big) H(1, 0; \alpha_0) + \left( \frac{\kappa d_1}{x-1} - \frac{\kappa d_1}{2(x-1)^2} + \frac{\kappa d_1}{3(x-1)^3} - \frac{\kappa d_1}{4(x-1)^4} - \right. \\
 & \frac{25\kappa d_1}{12(x-1)^5} + \frac{d_1}{x-1} - \frac{d_1}{2(x-1)^2} + \frac{d_1}{3(x-1)^3} - \frac{d_1}{4(x-1)^4} - \frac{49d_1}{12(x-1)^5} - \frac{15\kappa}{8(x-1)} + \frac{5\kappa}{4(x-1)^2} - \frac{5\kappa}{4(x-1)^3} + \frac{15\kappa}{8(x-1)^4} + \\
 & \frac{33\kappa}{8(x-1)^5} - \frac{33\kappa}{8} - \frac{5}{8(x-1)} + \frac{5}{12(x-1)^2} - \frac{5}{12(x-1)^3} + \frac{5}{8(x-1)^4} + \frac{49}{24(x-1)^5} - \frac{49}{24} \Big) H(1, 0; x) + \left( \frac{d_1^2 \alpha_0^4}{2} + \right. \\
 & \frac{d_1^2 \alpha_0^4}{2(x-1)} - \frac{8d_1^2 \alpha_0^3}{3} - \frac{2}{x-1} \frac{d_1^2 \alpha_0^3}{x-1} + \frac{2d_1^2 \alpha_0^3}{3(x-1)^2} + 6d_1^2 \alpha_0^2 + \frac{3d_1^2 \alpha_0^2}{x-1} - \frac{2d_1^2 \alpha_0^2}{(x-1)^2} + \frac{d_1^2 \alpha_0^2}{(x-1)^3} - 8d_1^2 \alpha_0 - \frac{2d_1^2 \alpha_0}{x-1} + \frac{2d_1^2 \alpha_0}{(x-1)^2} - \\
 & \frac{2d_1^2 \alpha_0}{(x-1)^3} + \frac{2d_1^2 \alpha_0}{(x-1)^4} + \frac{25}{6} \frac{d_1^2}{2(x-1)} - \frac{d_1^2}{3(x-1)^2} + \frac{d_1^2}{(x-1)^3} - \frac{2d_1^2}{(x-1)^4} \Big) H(1, 1; \alpha_0) + H(c_1(\alpha_0); x) \left( \frac{d_1 \alpha_0^4}{16} + \right. \\
 & \frac{1}{16} d_1 \kappa \alpha_0^4 + \frac{d_1 \kappa \alpha_0^4}{16(x-1)} - \frac{7\kappa \alpha_0^4}{16(x-1)} - \frac{7\kappa \alpha_0^4}{16} + \frac{d_1 \alpha_0^4}{16(x-1)} - \frac{5\alpha_0^4}{16(x-1)} - \frac{5\alpha_0^4}{16} - \frac{13d_1 \alpha_0^3}{36} - \frac{13}{36} d_1 \kappa \alpha_0^3 - \frac{d_1 \kappa \alpha_0^3}{4(x-1)} + \\
 & \frac{7\kappa \alpha_0^3}{4(x-1)} + \frac{d_1 \kappa \alpha_0^3}{9(x-1)^2} - \frac{19\kappa \alpha_0^3}{24(x-1)^2} + \frac{61\kappa \alpha_0^3}{24} - \frac{d_1 \kappa \alpha_0^3}{4(x-1)} + \frac{5\alpha_0^3}{4(x-1)} + \frac{d_1 \alpha_0^3}{9(x-1)^2} - \frac{35\alpha_0^3}{72(x-1)^2} + \frac{125\alpha_0^3}{72} + \frac{23d_1 \alpha_0^2}{24} + \\
 & \frac{23}{24} d_1 \kappa \alpha_0^2 + \frac{3d_1 \kappa \alpha_0^2}{8(x-1)} - \frac{41\kappa \alpha_0^2}{16(x-1)} - \frac{d_1 \kappa \alpha_0^2}{3(x-1)^2} + \frac{39\kappa \alpha_0^2}{16(x-1)^2} + \frac{d_1 \kappa \alpha_0^2}{4(x-1)^3} - \frac{27\kappa \alpha_0^2}{16(x-1)^3} - \frac{107\kappa \alpha_0^2}{16} + \frac{3d_1 \alpha_0^2}{8(x-1)} - \frac{89\alpha_0^2}{48(x-1)} - \\
 & \frac{d_1 \alpha_0^2}{3(x-1)^2} + \frac{71\alpha_0^2}{48(x-1)^2} + \frac{d_1 \alpha_0^2}{4(x-1)^3} - \frac{43\alpha_0^2}{48(x-1)^3} - \frac{203\alpha_0^2}{48} - \frac{25}{12} d_1 \alpha_0 - \frac{25d_1 \kappa \alpha_0}{12} - \frac{d_1 \kappa \alpha_0}{4(x-1)} + \frac{5\kappa \alpha_0}{4(x-1)} + \frac{d_1 \kappa \alpha_0}{3(x-1)^2} - \\
 & \frac{5\kappa \alpha_0}{2(x-1)^2} - \frac{d_1 \kappa \alpha_0}{2(x-1)^3} + \frac{15\kappa \alpha_0}{4(x-1)^3} + \frac{d_1 \kappa \alpha_0}{(x-1)^4} - \frac{45\kappa \alpha_0}{8(x-1)^4} + \frac{105\kappa \alpha_0}{8} - \frac{d_1 \alpha_0}{4(x-1)} + \frac{13\alpha_0}{12(x-1)} + \frac{d_1 \alpha_0}{3(x-1)^2} - \frac{3\alpha_0}{2(x-1)^2} - \\
 & \frac{d_1 \alpha_0}{2(x-1)^3} + \frac{23\alpha_0}{12(x-1)^3} + \frac{d_1 \alpha_0}{(x-1)^4} - \frac{61\alpha_0}{24(x-1)^4} + \frac{169\alpha_0}{24} + \frac{205d_1}{144} + \frac{205d_1 \kappa}{144} + \frac{17d_1 \kappa}{16(x-1)} - \frac{8}{8(x-1)} - \frac{13d_1 \kappa}{36(x-1)^2} + \\
 & \frac{35\kappa}{12(x-1)^2} + \frac{13d_1 \kappa}{36(x-1)^3} - \frac{35\kappa}{12(x-1)^3} - \frac{17d_1 \kappa}{16(x-1)^4} + \frac{45\kappa}{8(x-1)^4} - \frac{205d_1 \kappa}{144(x-1)^5} + \frac{205\kappa}{24(x-1)^5} - \frac{205\kappa}{24} + \left( \frac{3\kappa \alpha_0^4}{4(x-1)} + \right. \\
 & \frac{3\kappa \alpha_0^4}{4} + \frac{\alpha_0^4}{4(x-1)} + \frac{\alpha_0^4}{4} - \frac{3\kappa \alpha_0^3}{x-1} + \frac{\kappa \alpha_0^3}{(x-1)^2} - 4\kappa \alpha_0^3 - \frac{\alpha_0^3}{x-1} + \frac{\alpha_0^3}{3(x-1)^2} - \frac{4\alpha_0^3}{3} + \frac{9\kappa \alpha_0^2}{2(x-1)} - \frac{3\kappa \alpha_0^2}{(x-1)^2} + \frac{3\kappa \alpha_0^2}{2(x-1)^3} + \\
 & 9\kappa \alpha_0^2 + \frac{3\alpha_0^2}{2(x-1)} - \frac{\alpha_0^2}{(x-1)^2} + \frac{\alpha_0^2}{2(x-1)^3} + 3\alpha_0^2 - \frac{3\kappa \alpha_0}{x-1} + \frac{3\kappa \alpha_0}{(x-1)^2} - \frac{3\kappa \alpha_0}{(x-1)^3} + \frac{3\kappa \alpha_0}{(x-1)^4} - 12\kappa \alpha_0 - \frac{\alpha_0}{x-1} + \frac{\alpha_0}{(x-1)^2} - \\
 & \frac{\alpha_0}{(x-1)^3} + \frac{\alpha_0}{(x-1)^4} - 4\alpha_0 + \frac{15\kappa}{4(x-1)} - \frac{5\kappa}{2(x-1)^2} + \frac{5\kappa}{2(x-1)^3} - \frac{15\kappa}{4(x-1)^4} - \frac{33\kappa}{4(x-1)^5} + \frac{25\kappa}{4} + \frac{5}{4(x-1)} - \\
 & \frac{5}{6(x-1)^2} + \frac{5}{6(x-1)^3} - \frac{5}{4(x-1)^4} - \frac{49}{12(x-1)^5} + \frac{25}{12} \Big) H(0; \alpha_0) + \left( \frac{d_1 \alpha_0^4}{4} + \frac{1}{4} d_1 \kappa \alpha_0^4 + \frac{d_1 \kappa \alpha_0^4}{4(x-1)} + \frac{d_1 \alpha_0^4}{4(x-1)} - \right. \\
 & \frac{4d_1 \alpha_0^3}{3} - \frac{4}{3} d_1 \kappa \alpha_0^3 - \frac{d_1 \kappa \alpha_0^3}{x-1} + \frac{d_1 \kappa \alpha_0^3}{3(x-1)^2} - \frac{d_1 \alpha_0^3}{x-1} + \frac{d_1 \alpha_0^3}{3(x-1)^2} + 3d_1 \alpha_0^2 + 3d_1 \kappa \alpha_0^2 + \frac{3d_1 \kappa \alpha_0^2}{2(x-1)} - \frac{d_1 \kappa \alpha_0^2}{(x-1)^2} + \\
 & \frac{d_1 \kappa \alpha_0^2}{2(x-1)^3} + \frac{3d_1 \alpha_0^2}{2(x-1)} - \frac{d_1 \alpha_0^2}{(x-1)^2} + \frac{d_1 \alpha_0^2}{2(x-1)^3} - 4d_1 \alpha_0 - 4d_1 \kappa \alpha_0 - \frac{d_1 \kappa \alpha_0}{x-1} + \frac{d_1 \kappa \alpha_0}{(x-1)^2} - \frac{d_1 \kappa \alpha_0}{(x-1)^3} + \frac{d_1 \kappa \alpha_0}{(x-1)^4} - \\
 & \frac{d_1 \alpha_0}{x-1} + \frac{d_1 \alpha_0}{(x-1)^2} - \frac{d_1 \alpha_0}{(x-1)^3} + \frac{d_1 \alpha_0}{(x-1)^4} + \frac{25d_1}{12} + \frac{25d_1 \kappa}{12} + \frac{5d_1 \kappa}{4(x-1)} - \frac{5d_1 \kappa}{6(x-1)^2} + \frac{5d_1 \kappa}{6(x-1)^3} - \frac{5d_1 \kappa}{4(x-1)^4} - \frac{25d_1 \kappa}{12(x-1)^5} + \\
 & \frac{5d_1}{4(x-1)} - \frac{5d_1}{6(x-1)^2} + \frac{5d_1}{6(x-1)^3} - \frac{5d_1}{4(x-1)^4} - \frac{49d_1}{12(x-1)^5} \Big) H(1; \alpha_0) + \left( \frac{6\kappa}{(x-1)^5} + \frac{2}{(x-1)^5} \right) H(0, 0; \alpha_0) + \\
 & \left( \frac{2\kappa d_1}{(x-1)^5} + \frac{2d_1}{(x-1)^5} \right) H(0, 1; \alpha_0) + \left( \frac{2\kappa d_1}{(x-1)^5} + \frac{2}{(x-1)^5} \right) H(1, 0; \alpha_0) + \frac{2d_1^2 H(1, 1; \alpha_0)}{(x-1)^5} + \frac{17d_1}{16(x-1)} - \\
 & \frac{65}{24(x-1)} - \frac{13d_1}{36(x-1)^2} + \frac{55}{36(x-1)^2} + \frac{13d_1}{36(x-1)^3} - \frac{55}{36(x-1)^3} - \frac{17d_1}{16(x-1)^4} + \frac{65}{24(x-1)^4} - \frac{205d_1}{144(x-1)^5} - \\
 & \frac{\pi^2}{12(x-1)^5} + \frac{449}{72(x-1)^5} - \frac{305}{72} \Big) + \left( \frac{2d_1^2}{(x-1)^5} - \frac{2}{(x-1)^5} \kappa d_1 + \kappa d_1 - \frac{2d_1}{(x-1)^5} + d_1 + \frac{3\kappa}{2(x-1)^5} - \frac{3\kappa}{2} + \frac{1}{2(x-1)^5} - \right. \\
 & \left. \frac{1}{2} \right) H(0; \alpha_0) H(1, 1; x) + \left( -\frac{\kappa d_1}{x-1} + \frac{\kappa d_1}{2(x-1)^2} - \frac{\kappa d_1}{3(x-1)^3} + \frac{\kappa d_1}{4(x-1)^4} + \frac{25\kappa d_1}{12(x-1)^5} - \frac{d_1}{x-1} + \frac{d_1}{2(x-1)^2} - \right. \\
 & \frac{d_1}{3(x-1)^3} + \frac{d_1}{4(x-1)^4} + \frac{49d_1}{12(x-1)^5} + \frac{15\kappa}{8(x-1)} - \frac{5\kappa}{4(x-1)^2} + \frac{5\kappa}{4(x-1)^3} - \frac{15\kappa}{8(x-1)^4} - \frac{33\kappa}{8(x-1)^5} + \frac{33\kappa}{8} + \left( -\frac{2\kappa d_1}{(x-1)^5} - \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \frac{2d_1}{(x-1)^5} + \frac{3\kappa}{(x-1)^5} - 3\kappa + \frac{1}{(x-1)^5} - 1 \right) H(0; \alpha_0) + \left( -\frac{2d_1^2}{(x-1)^5} + \frac{\kappa d_1}{(x-1)^5} - \kappa d_1 + \frac{d_1}{(x-1)^5} - d_1 \right) H(1; \alpha_0) + \\
 & \frac{5}{8(x-1)} - \frac{5}{12(x-1)^2} + \frac{5}{12(x-1)^3} - \frac{5}{8(x-1)^4} - \frac{49}{24(x-1)^5} + \frac{49}{24} \Big) H(1, c_1(\alpha_0); x) + \left( \frac{3\kappa\alpha_0^4}{8(x-1)} + \frac{3\kappa\alpha_0^4}{8} + \right. \\
 & \frac{\alpha_0^4}{8(x-1)} + \frac{\alpha_0^4}{8} - \frac{3\kappa\alpha_0^3}{2(x-1)} + \frac{\kappa\alpha_0^3}{2(x-1)^2} - 2\kappa\alpha_0^3 - \frac{\alpha_0^3}{2(x-1)} + \frac{\alpha_0^3}{6(x-1)^2} - \frac{2\alpha_0^3}{3} + \frac{9\kappa\alpha_0^2}{4(x-1)} - \frac{3\kappa\alpha_0^2}{2(x-1)^2} + \frac{3\kappa\alpha_0^2}{4(x-1)^3} + \\
 & \frac{9\kappa\alpha_0^2}{2} + \frac{3\alpha_0^2}{4(x-1)} - \frac{\alpha_0^2}{2(x-1)^2} + \frac{\alpha_0^2}{4(x-1)^3} + \frac{3\alpha_0^2}{2} - \frac{3\kappa\alpha_0}{2(x-1)} + \frac{3\kappa\alpha_0}{2(x-1)^2} - \frac{3\kappa\alpha_0}{2(x-1)^3} + \frac{3\kappa\alpha_0}{2(x-1)^4} - 6\kappa\alpha_0 - \\
 & \frac{\alpha_0}{2(x-1)} + \frac{\alpha_0}{2(x-1)^2} - \frac{\alpha_0}{2(x-1)^3} + \frac{\alpha_0}{2(x-1)^4} - 2\alpha_0 + \frac{15\kappa}{8(x-1)} - \frac{5\kappa}{4(x-1)^2} + \frac{5\kappa}{4(x-1)^3} - \frac{15\kappa}{8(x-1)^4} - \frac{33\kappa}{8(x-1)^5} + \\
 & \frac{25\kappa}{8} + \left( \frac{3\kappa}{(x-1)^5} + \frac{1}{(x-1)^5} \right) H(0; \alpha_0) + \left( \frac{\kappa d_1}{(x-1)^5} + \frac{d_1}{(x-1)^5} \right) H(1; \alpha_0) + \frac{5}{8(x-1)} - \frac{5}{12(x-1)^2} + \frac{5}{12(x-1)^3} - \\
 & \frac{5}{8(x-1)^4} - \frac{49}{24(x-1)^5} + \frac{25}{24} \Big) H(c_1(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{6\kappa}{(x-1)^5} + 6\kappa + \frac{2}{(x-1)^5} + 2 \right) H(0, 0, 0; \alpha_0) + \left( -\frac{6\kappa}{(x-1)^5} + 6\kappa - \frac{2}{(x-1)^5} + 2 \right) H(0, 0, 0; x) + \left( \frac{2\kappa d_1}{(x-1)^5} + 2\kappa d_1 + \frac{2d_1}{(x-1)^5} + 2d_1 \right) H(0, 0, 1; \alpha_0) + \left( \frac{3\kappa}{(x-1)^5} - 3\kappa + \frac{1}{(x-1)^5} - 1 \right) H(0, 0, c_1(\alpha_0); x) + \left( \frac{2\kappa d_1}{(x-1)^5} + 2\kappa d_1 + \frac{2d_1}{(x-1)^5} + 2d_1 \right) H(0, 1, 0; \alpha_0) + \left( \frac{\kappa d_1}{(x-1)^5} - \kappa d_1 + \frac{d_1}{(x-1)^5} - d_1 - \frac{3\kappa}{(x-1)^5} + 3\kappa - \frac{1}{(x-1)^5} + 1 \right) H(0, 1, 0; x) + \left( \frac{2d_1^2}{(x-1)^5} + 2d_1^2 \right) H(0, 1, 1; \alpha_0) + \left( -\frac{\kappa d_1}{(x-1)^5} + \kappa d_1 - \frac{d_1}{(x-1)^5} + d_1 + \frac{3\kappa}{(x-1)^5} - 3\kappa + \frac{1}{(x-1)^5} - 1 \right) H(0, 1, c_1(\alpha_0); x) + \left( \frac{3\kappa}{2} + \frac{1}{2(x-1)^5} - \frac{1}{2} \right) H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{2\kappa d_1}{(x-1)^5} + \frac{2d_1}{(x-1)^5} - \frac{3\kappa}{(x-1)^5} + 3\kappa - \frac{1}{(x-1)^5} + 1 \right) H(1, 0, 0; x) + \left( -\frac{\kappa d_1}{(x-1)^5} - \frac{d_1}{(x-1)^5} + \frac{3\kappa}{2(x-1)^5} - \frac{3\kappa}{2} + \frac{1}{2(x-1)^5} - \frac{1}{2} \right) H(1, 0, c_1(\alpha_0); x) + \left( -\frac{2d_1^2}{(x-1)^5} + \frac{2\kappa d_1}{(x-1)^5} - \kappa d_1 + \frac{2d_1}{(x-1)^5} - d_1 - \frac{3\kappa}{2(x-1)^5} + \frac{3\kappa}{2} - \frac{1}{2(x-1)^5} + \frac{1}{2} \right) H(1, 1, 0; x) + \left( \frac{2d_1^2}{(x-1)^5} - \frac{2\kappa d_1}{(x-1)^5} + \kappa d_1 - \frac{2d_1}{(x-1)^5} + d_1 + \frac{3\kappa}{2(x-1)^5} - \frac{3\kappa}{2} + \frac{1}{2(x-1)^5} - \frac{1}{2} \right) H(1, 1, c_1(\alpha_0); x) + \left( -\frac{\kappa d_1}{(x-1)^5} - \frac{d_1}{(x-1)^5} + \frac{3\kappa}{2(x-1)^5} - \frac{3\kappa}{2} + \frac{1}{2(x-1)^5} - \frac{1}{2} \right) H(1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{3\kappa}{2(x-1)^5} + \frac{1}{2(x-1)^5} \right) H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) - \frac{35\pi^2\kappa}{48(x-1)(\kappa+1)} + \frac{35\pi^2\kappa}{72(x-1)^2(\kappa+1)} - \frac{35\pi^2\kappa}{72(x-1)^3(\kappa+1)} + \frac{35\pi^2\kappa}{48(x-1)^4(\kappa+1)} + \frac{247\pi^2\kappa}{144(x-1)^5(\kappa+1)} - \frac{247\pi^2\kappa}{144(\kappa+1)} - \frac{5\pi^2}{48(x-1)(\kappa+1)} + \frac{5\pi^2}{72(x-1)^2(\kappa+1)} - \frac{5\pi^2}{72(x-1)^3(\kappa+1)} + \frac{5\pi^2}{48(x-1)^4(\kappa+1)} + \frac{49\pi^2}{144(x-1)^5(\kappa+1)} - \frac{25\pi^2}{144(\kappa+1)} - \frac{4}{\kappa+1} - \frac{7\kappa\zeta_3}{2(x-1)^5(\kappa+1)} + \frac{7\kappa\zeta_3}{2(\kappa+1)} - \frac{\zeta_3}{2(x-1)^5(\kappa+1)} + \frac{3\zeta_3}{2(\kappa+1)}.
 \end{aligned}$$

### D.3 The $\mathcal{A}$ integral for $k = 2$ and arbitrary $\kappa$

The  $\varepsilon$  expansion for this integral reads

$$\begin{aligned}
 \mathcal{I}(x, \varepsilon; \alpha_0, 3 + d_1\varepsilon; \kappa, 2, 0, g_A) &= x \mathcal{A}(\varepsilon, x; 3 + d_1\varepsilon; \kappa, 2) \\
 &= \frac{1}{\varepsilon} a_{-1}^{(\kappa, 2)} + a_0^{(\kappa, 2)} + \varepsilon a_1^{(\kappa, 2)} + \varepsilon a_2^{(\kappa, 2)} + \mathcal{O}(\varepsilon^3), \tag{D.3}
 \end{aligned}$$

where

$$\begin{aligned}
 a_{-1}^{(\kappa, 2)} &= -\frac{1}{3(\kappa+1)}, \\
 a_0^{(\kappa, 2)} &= -\frac{\alpha_0^6}{3(x\alpha_0 - 2\alpha_0 - x)} + \frac{\alpha_0^5}{3(x-2)} + \frac{5\alpha_0^5}{3(x\alpha_0 - 2\alpha_0 - x)} - \frac{5\alpha_0^4}{4(x-2)} + \frac{\alpha_0^4}{12(x-1)} - \frac{10\alpha_0^4}{3(x\alpha_0 - 2\alpha_0 - x)} + \\
 & \frac{\kappa\alpha_0^4}{12(\kappa+1)} + \frac{\alpha_0^4}{12(\kappa+1)} + \frac{5\alpha_0^4}{6(x-2)^2} + \frac{5\alpha_0^3}{3(x-2)} - \frac{\alpha_0^3}{3(x-1)} + \frac{10\alpha_0^3}{3(x\alpha_0 - 2\alpha_0 - x)} - \frac{4\kappa\alpha_0^3}{9(\kappa+1)} - \frac{4\alpha_0^3}{9(\kappa+1)} - \frac{20\alpha_0^3}{9(x-2)^2} + \\
 & \frac{\alpha_0^3}{9(x-1)^2} + \frac{20\alpha_0^3}{9(x-2)^3} - \frac{5\alpha_0^2}{6(x-2)} + \frac{\alpha_0^2}{2(x-1)} - \frac{5\alpha_0^2}{3(x\alpha_0 - 2\alpha_0 - x)} + \frac{\kappa\alpha_0^2}{\kappa+1} + \frac{\alpha_0^2}{\kappa+1} + \frac{5\alpha_0^2}{3(x-2)^2} - \frac{\alpha_0^2}{3(x-1)^2} - \\
 & \frac{10\alpha_0^2}{3(x-2)^3} + \frac{\alpha_0^2}{6(x-1)^3} + \frac{20\alpha_0^2}{3(x-2)^4} - \frac{\alpha_0}{3(x-1)} + \frac{\alpha_0}{3(x\alpha_0 - 2\alpha_0 - x)} - \frac{4\kappa\alpha_0}{3(\kappa+1)} - \frac{4\alpha_0}{3(\kappa+1)} + \frac{\alpha_0}{3(x-1)^2} - \frac{\alpha_0}{3(x-1)^3} + \\
 & \frac{\alpha_0}{3(x-1)^4} + \frac{80\alpha_0}{3(x-2)^5} + \left( \frac{1}{3(x-1)^5} + \frac{1}{3} + \frac{80}{3(x-2)^5} + \frac{160}{3(x-2)^6} \right) H(0; \alpha_0) + \left( -\frac{1}{3(x-1)^5} + \frac{1}{3} - \frac{80}{3(x-2)^5} - \right. \\
 & \left. \frac{160}{3(x-2)^6} \right) H(0; x) + \frac{H(c_1(\alpha_0); x)}{3(x-1)^5} + \left( \frac{80}{3(x-2)^5} + \frac{160}{3(x-2)^6} \right) H(c_2(\alpha_0); x) - \frac{13}{18(\kappa+1)} + \frac{80 \ln 2}{3(x-2)^5} + \frac{160 \ln 2}{3(x-2)^6},
 \end{aligned}$$

$$\begin{aligned}
 a_1^{(\kappa,2)} = & \frac{1}{\alpha_0(x-2)-x} \left\{ -\frac{1}{24}d_1x\alpha_0^5 - \frac{d_1\alpha_0^5}{12(x-2)} + \frac{d_1\alpha_0^5}{24(x-1)} + \frac{2x\alpha_0^5}{9(\kappa+1)} + \frac{11x\kappa\alpha_0^5}{36(\kappa+1)} + \frac{25\kappa\alpha_0^5}{36(x-2)(\kappa+1)} - \right. \\
 & \frac{11\kappa\alpha_0^5}{36(x-1)(\kappa+1)} + \frac{\kappa\alpha_0^5}{24(\kappa+1)} + \frac{19\alpha_0^5}{36(x-2)(\kappa+1)} - \frac{2\alpha_0^5}{9(x-1)(\kappa+1)} + \frac{\alpha_0^5}{24(\kappa+1)} - \frac{7d_1\alpha_0^4}{108} + \frac{61}{216}d_1x\alpha_0^4 + \frac{11d_1\alpha_0^4}{54(x-2)} - \\
 & \frac{43d_1\alpha_0^4}{216(x-1)} - \frac{157x\alpha_0^4}{108(\kappa+1)} - \frac{56x\kappa\alpha_0^4}{27(\kappa+1)} - \frac{113\kappa\alpha_0^4}{54(x-2)(\kappa+1)} + \frac{149\kappa\alpha_0^4}{108(x-1)(\kappa+1)} + \frac{149\kappa\alpha_0^4}{54(x-2)^2(\kappa+1)} - \frac{59\kappa\alpha_0^4}{108(x-1)^2(\kappa+1)} + \\
 & \frac{41\kappa\alpha_0^4}{216(\kappa+1)} - \frac{91\alpha_0^4}{54(x-2)(\kappa+1)} + \frac{53\alpha_0^4}{54(x-1)(\kappa+1)} + \frac{103\alpha_0^4}{54(x-2)^2(\kappa+1)} - \frac{37\alpha_0^4}{108(x-1)^2(\kappa+1)} + \frac{\alpha_0^4}{216(\kappa+1)} - \frac{23d_1\alpha_0^3}{54(x-2)^2} + \\
 & \frac{2d_1\alpha_0^3}{27(x-1)^2} + \frac{4d_1\alpha_0^3}{9} - \frac{95}{108}d_1x\alpha_0^3 + \frac{d_1\alpha_0^3}{54(x-2)} + \frac{41d_1\alpha_0^3}{108(x-1)} + \frac{911x\alpha_0^3}{216(\kappa+1)} + \frac{1381x\kappa\alpha_0^3}{216(\kappa+1)} + \frac{25\kappa\alpha_0^3}{27(x-2)(\kappa+1)} - \\
 & \frac{239\kappa\alpha_0^3}{108(x-1)(\kappa+1)} - \frac{104\kappa\alpha_0^3}{27(x-2)^2(\kappa+1)} + \frac{467\kappa\alpha_0^3}{216(x-1)^2(\kappa+1)} + \frac{388\kappa\alpha_0^3}{27(x-2)^3(\kappa+1)} - \frac{83\kappa\alpha_0^3}{72(x-1)^3(\kappa+1)} - \frac{83\kappa\alpha_0^3}{36(\kappa+1)} + \\
 & \frac{35\alpha_0^3}{27(x-2)(\kappa+1)} - \frac{108(x-1)(\kappa+1)}{175\alpha_0^3} - \frac{88\alpha_0^3}{277\alpha_0^3} + \frac{277\alpha_0^3}{236\alpha_0^3} - \frac{236\alpha_0^3}{5\alpha_0^3} - \frac{8(x-1)^3(\kappa+1)}{27(x-2)^3(\kappa+1)} - \\
 & \frac{35\alpha_0^3}{36(\kappa+1)} + \frac{8d_1\alpha_0^3}{27(x-2)^2} - \frac{17d_1\alpha_0^3}{54(x-1)^2} - \frac{76d_1\alpha_0^3}{27(x-2)^3} + \frac{d_1\alpha_0^3}{6(x-1)^3} - \frac{35d_1\alpha_0^2}{18} + \frac{73}{36}d_1x\alpha_0^2 - \frac{4d_1\alpha_0^2}{9(x-2)} - \frac{13d_1\alpha_0^2}{36(x-1)} - \\
 & \frac{569x\alpha_0^2}{72(\kappa+1)} - \frac{979x\kappa\alpha_0^2}{72(\kappa+1)} + \frac{16\kappa\alpha_0^2}{3(x-2)(\kappa+1)} - \frac{2\kappa\alpha_0^2}{3(x-1)(\kappa+1)} - \frac{10\kappa\alpha_0^2}{(x-2)^2(\kappa+1)} - \frac{67\kappa\alpha_0^2}{24(x-1)^2(\kappa+1)} + \frac{80\kappa\alpha_0^2}{3(x-2)^3(\kappa+1)} + \\
 & \frac{119\kappa\alpha_0^2}{24(x-1)^3(\kappa+1)} + \frac{1256\kappa\alpha_0^2}{9(x-2)^4(\kappa+1)} - \frac{137\kappa\alpha_0^2}{36(x-1)^4(\kappa+1)} + \frac{32\kappa\alpha_0^2}{3(\kappa+1)} + \frac{16\alpha_0^2}{9(x-2)(\kappa+1)} + \frac{\alpha_0^2}{2(x-1)(\kappa+1)} - \\
 & \frac{10\alpha_0^2}{3(x-2)^2(\kappa+1)} - \frac{119\alpha_0^2}{72(x-1)^2(\kappa+1)} + \frac{9(x-2)^3(\kappa+1)}{80\alpha_0^2} + \frac{19\alpha_0^2}{8(x-1)^3(\kappa+1)} + \frac{632\alpha_0^2}{9(x-2)^4(\kappa+1)} - \frac{7\alpha_0^2}{4(x-1)^4(\kappa+1)} + \\
 & \frac{85\alpha_0^2}{18(\kappa+1)} + \frac{5d_1\alpha_0^2}{3(x-2)^2} + \frac{d_1\alpha_0^2}{2(x-1)^2} - \frac{80d_1\alpha_0^2}{9(x-2)^3} - \frac{5d_1\alpha_0^2}{6(x-1)^3} - \frac{104d_1\alpha_0^2}{3(x-2)^4} + \frac{2d_1\alpha_0^2}{3(x-1)^4} - \frac{d_1\alpha_0}{3(\kappa+1)} - \frac{25d_1x\alpha_0}{18(\kappa+1)} + \\
 & \frac{371x\alpha_0}{108(\kappa+1)} - \frac{d_1\kappa\alpha_0}{3(\kappa+1)} - \frac{25d_1x\kappa\alpha_0}{18(\kappa+1)} - \frac{\pi^2x\kappa\alpha_0}{6(\kappa+1)} + \frac{323x\kappa\alpha_0}{36(\kappa+1)} + \frac{2d_1\kappa\alpha_0}{9(x-2)(\kappa+1)} + \frac{4\kappa\alpha_0}{3(x-2)^2(\kappa+1)} + \frac{d_1\kappa\alpha_0}{18(x-1)(\kappa+1)} - \\
 & \frac{11\kappa\alpha_0}{4(x-1)(\kappa+1)} - \frac{10d_1\kappa\alpha_0}{9(x-2)^2(\kappa+1)} - \frac{d_1\kappa\alpha_0}{9(x-1)^2(\kappa+1)} + \frac{17\kappa\alpha_0}{12(x-1)^2(\kappa+1)} + \frac{80d_1\kappa\alpha_0}{9(x-2)^3(\kappa+1)} - \frac{40\kappa\alpha_0}{3(x-2)^3(\kappa+1)} + \\
 & \frac{d_1\kappa\alpha_0}{3(x-1)^3(\kappa+1)} - \frac{3\kappa\alpha_0}{2(x-1)^3(\kappa+1)} + \frac{208d_1\kappa\alpha_0}{3(x-2)^4(\kappa+1)} + \frac{20\pi^2\kappa\alpha_0}{3(x-2)^4(\kappa+1)} - \frac{2512\kappa\alpha_0}{9(x-2)^4(\kappa+1)} + \frac{2d_1\kappa\alpha_0}{3(x-1)^4(\kappa+1)} + \\
 & \frac{\pi^2\kappa\alpha_0}{6(x-1)^4(\kappa+1)} - \frac{137\kappa\alpha_0}{36(x-1)^4(\kappa+1)} + \frac{256d_1\kappa\alpha_0}{3(x-2)^5(\kappa+1)} + \frac{40\pi^2\kappa\alpha_0}{3(x-2)^5(\kappa+1)} - \frac{3584\kappa\alpha_0}{9(x-2)^5(\kappa+1)} - \frac{\pi^2\kappa\alpha_0}{6(x-1)^5(\kappa+1)} + \\
 & \frac{\pi^2\kappa\alpha_0}{3(\kappa+1)} + \frac{109\kappa\alpha_0}{36(\kappa+1)} + \frac{2d_1\alpha_0}{9(x-2)(\kappa+1)} + \frac{4\alpha_0}{9(x-2)(\kappa+1)} + \frac{d_1\alpha_0}{18(x-1)(\kappa+1)} - \frac{12(x-1)(\kappa+1)}{10d_1\alpha_0} - \frac{10d_1\alpha_0}{9(x-2)^2(\kappa+1)} - \\
 & \frac{d_1\alpha_0}{9(x-1)^2(\kappa+1)} + \frac{17\alpha_0}{36(x-1)^2(\kappa+1)} + \frac{80d_1\alpha_0}{9(x-2)^3(\kappa+1)} - \frac{40\alpha_0}{9(x-2)^3(\kappa+1)} + \frac{d_1\alpha_0}{3(x-1)^3(\kappa+1)} - \frac{\alpha_0}{2(x-1)^3(\kappa+1)} + \\
 & \frac{208d_1\alpha_0}{3(x-2)^4(\kappa+1)} + \frac{20\pi^2\alpha_0}{9(x-2)^4(\kappa+1)} - \frac{1264\alpha_0}{9(x-2)^4(\kappa+1)} + \frac{2d_1\alpha_0}{3(x-1)^4(\kappa+1)} + \frac{\pi^2\alpha_0}{18(x-1)^4(\kappa+1)} - \frac{7\alpha_0}{4(x-1)^4(\kappa+1)} + \\
 & \frac{256d_1\alpha_0}{3(x-2)^5(\kappa+1)} + \frac{40\pi^2\alpha_0}{9(x-2)^5(\kappa+1)} - \frac{2048\alpha_0}{9(x-2)^5(\kappa+1)} - \frac{\pi^2\alpha_0}{18(x-1)^5(\kappa+1)} + \frac{545\alpha_0}{108(\kappa+1)} + \frac{80\kappa\ln^2 2\alpha_0}{(x-2)^4(\kappa+1)} + \\
 & \frac{160\kappa\ln^2 2\alpha_0}{(x-2)^5(\kappa+1)} + \frac{80\ln^2 2\alpha_0}{3(x-2)^4(\kappa+1)} + \frac{160\ln^2 2\alpha_0}{3(x-2)^5(\kappa+1)} + \frac{32d_1\kappa\ln 2\alpha_0}{3(x-2)^4(\kappa+1)} + \frac{352\kappa\ln 2\alpha_0}{9(x-2)^4(\kappa+1)} + \frac{64d_1\kappa\ln 2\alpha_0}{3(x-2)^5(\kappa+1)} + \\
 & \frac{704\kappa\ln 2\alpha_0}{9(x-2)^5(\kappa+1)} + \frac{32d_1\ln 2\alpha_0}{3(x-2)^4(\kappa+1)} + \frac{544\ln 2\alpha_0}{9(x-2)^4(\kappa+1)} + \frac{64d_1\ln 2\alpha_0}{3(x-2)^5(\kappa+1)} + \frac{1088\ln 2\alpha_0}{9(x-2)^5(\kappa+1)} + \left( -\frac{x\alpha_0^5}{6} - \frac{1}{6}x\kappa\alpha_0^5 - \right. \\
 & \frac{\kappa\alpha_0^5}{3(x-2)} + \frac{\kappa\alpha_0^5}{6(x-1)} - \frac{\alpha_0^5}{3(x-2)} + \frac{\alpha_0^5}{6(x-1)} + \frac{19x\alpha_0^4}{18} + \frac{19}{18}x\kappa\alpha_0^4 + \frac{10\kappa\alpha_0^4}{9(x-2)} - \frac{13\kappa\alpha_0^4}{18(x-1)} - \frac{10\kappa\alpha_0^4}{9(x-2)^2} + \frac{2\kappa\alpha_0^4}{9(x-1)^2} - \\
 & \frac{\kappa\alpha_0^4}{9} + \frac{10\alpha_0^4}{9(x-2)} - \frac{13\alpha_0^4}{18(x-1)} - \frac{10\alpha_0^4}{9(x-2)^2} + \frac{2\alpha_0^4}{9(x-1)^2} - \frac{\alpha_0^4}{9} - \frac{26x\alpha_0^3}{9} - \frac{26}{9}x\kappa\alpha_0^3 - \frac{10\kappa\alpha_0^3}{9(x-2)} + \frac{11\kappa\alpha_0^3}{9(x-1)} + \frac{20\kappa\alpha_0^3}{9(x-2)^2} - \\
 & \frac{7\kappa\alpha_0^3}{9(x-1)^2} - \frac{40\kappa\alpha_0^3}{9(x-2)^3} + \frac{\kappa\alpha_0^3}{3(x-1)^3} + \frac{2\kappa\alpha_0^3}{3} - \frac{10\alpha_0^3}{9(x-2)} + \frac{11\alpha_0^3}{9(x-1)} + \frac{20\alpha_0^3}{9(x-2)^2} - \frac{7\alpha_0^3}{9(x-1)^2} - \frac{40\alpha_0^3}{9(x-2)^3} + \frac{\alpha_0^3}{3(x-1)^3} + \\
 & \frac{2\alpha_0^3}{3} + \frac{14x\alpha_0^2}{3} + \frac{14}{3}x\kappa\alpha_0^2 - \frac{\kappa\alpha_0^2}{x-1} + \frac{\kappa\alpha_0^2}{(x-1)^2} - \frac{\kappa\alpha_0^2}{(x-1)^3} - \frac{80\kappa\alpha_0^2}{3(x-2)^4} + \frac{2\kappa\alpha_0^2}{3(x-1)^4} - 2\kappa\alpha_0^2 - \frac{\alpha_0^2}{x-1} + \frac{\alpha_0^2}{(x-1)^2} - \\
 & \frac{\alpha_0^2}{(x-1)^3} - \frac{80\alpha_0^2}{3(x-2)^4} + \frac{2\alpha_0^2}{3(x-1)^4} - 2\alpha_0^2 - \frac{95x\alpha_0}{36} - \frac{121x\kappa\alpha_0}{36} - \frac{40\kappa\alpha_0}{9(x-2)} + \frac{65\kappa\alpha_0}{12(x-1)} + \frac{40\kappa\alpha_0}{9(x-2)^2} - \frac{25\kappa\alpha_0}{18(x-1)^2} + \\
 & \frac{35\kappa\alpha_0}{36(x-1)^3} + \frac{96\kappa\alpha_0}{(x-2)^4} + \frac{31\kappa\alpha_0}{36(x-1)^4} + \frac{256\kappa\alpha_0}{3(x-2)^5} - \frac{25\kappa\alpha_0}{36(x-1)^5} - \frac{4\kappa\alpha_0}{9} - \frac{40\alpha_0}{9(x-2)} + \frac{65\alpha_0}{12(x-1)} + \frac{40\alpha_0}{9(x-2)^2} - \\
 & \frac{25\alpha_0}{18(x-1)^2} + \frac{35\alpha_0}{36(x-1)^3} + \frac{32d_1\alpha_0}{3(x-2)^4} + \frac{1504\alpha_0}{9(x-2)^4} + \frac{19\alpha_0}{12(x-1)^4} + \frac{64d_1\alpha_0}{3(x-2)^5} + \frac{2048\alpha_0}{9(x-2)^5} - \frac{17\alpha_0}{12(x-1)^5} - \frac{17\alpha_0}{9} - \frac{x}{36} + \\
 & \frac{25x\kappa}{36} - \frac{32\kappa}{9(x-2)} + \frac{43\kappa}{12(x-1)} + \frac{40\kappa}{9(x-2)^2} - \frac{9\kappa}{9(x-1)^2} - \frac{9\kappa}{9(x-2)^3} - \frac{36\kappa}{36(x-1)^3} - \frac{3\kappa}{3(x-2)^4} - \frac{128\kappa}{36(x-1)^4} - \frac{43\kappa}{36(x-1)^4} - \frac{64\kappa}{(x-2)^5} - \\
 & \frac{25\kappa}{36(x-1)^5} + \frac{128\kappa}{3(x-2)^6} + \frac{\kappa}{2} - \frac{32}{9(x-2)} + \frac{43}{12(x-1)} + \frac{40}{9(x-2)^2} - \frac{9}{9(x-1)^2} - \frac{9}{9(x-2)^3} - \frac{1}{36(x-1)^3} - \frac{1}{32d_1} - \\
 & \frac{1024}{9(x-2)^4} - \frac{23}{12(x-1)^4} - \frac{128d_1}{3(x-2)^5} - \frac{3136}{9(x-2)^5} - \frac{17}{12(x-1)^5} - \frac{128d_1}{3(x-2)^6} - \frac{2176}{9(x-2)^6} + \frac{1}{2} \Big) H(0; \alpha_0) + \left( - \right. \\
 & \left. \frac{1}{6}d_1x\alpha_0^5 - \frac{d_1\alpha_0^5}{3(x-2)} + \frac{d_1\alpha_0^5}{6(x-1)} - \frac{d_1\alpha_0^4}{9} + \frac{19}{18}d_1x\alpha_0^4 + \frac{10d_1\alpha_0^4}{9(x-2)} - \frac{13d_1\alpha_0^4}{18(x-1)} - \frac{10d_1\alpha_0^4}{9(x-2)^2} + \frac{2d_1\alpha_0^4}{9(x-1)^2} + \frac{2d_1\alpha_0^3}{3} - \right.
 \end{aligned}$$

$$\begin{aligned}
& \frac{26}{9}d_1x\alpha_0^3 - \frac{10d_1}{9}\frac{\alpha_0^3}{(x-2)} + \frac{11d_1\alpha_0^3}{9(x-1)} + \frac{20d_1\alpha_0^3}{9(x-2)^2} - \frac{7d_1\alpha_0^3}{9(x-1)^2} - \frac{40d_1\alpha_0^3}{9(x-2)^3} + \frac{d_1\alpha_0^3}{3(x-1)^3} - 2d_1\alpha_0^2 + \frac{14}{3}d_1x\alpha_0^2 - \frac{d_1\alpha_0^2}{x-1} + \\
& \frac{d_1\alpha_0^2}{(x-1)^2} - \frac{d_1}{(x-1)^3} - \frac{80d_1\alpha_0^2}{3(x-2)^4} + \frac{2d_1\alpha_0^2}{3(x-1)^4} + \frac{10d_1\alpha_0}{9} - \frac{73d_1x\alpha_0}{18} + \frac{5d_1\alpha_0}{9(x-2)} + \frac{7d_1\alpha_0}{18(x-1)} - \frac{20d_1\alpha_0}{9(x-2)^2} - \frac{5d_1\alpha_0}{9(x-1)^2} + \\
& \frac{40d_1\alpha_0}{3(x-2)^3} + \frac{d_1\alpha_0}{(x-1)^3} + \frac{320d_1\alpha_0}{3(x-2)^4} + \frac{320d_1\alpha_0}{3(x-2)^5} + \frac{d_1}{3} + \frac{25d_1x}{18} - \frac{2d_1}{9(x-2)} - \frac{d_1}{18(x-1)} + \frac{10d_1}{9(x-2)^2} + \frac{d_1}{9(x-1)^2} - \\
& \frac{80d_1}{9(x-2)^3} - \frac{d_1}{3(x-1)^3} - \frac{80d_1}{3(x-2)^4} - \frac{2d_1}{3(x-1)^4} - \frac{320d_1}{3(x-2)^5} \Big) H(1; \alpha_0) + \left( \frac{x\alpha_0}{3} + \frac{x\kappa\alpha_0}{3} + \frac{80\kappa\alpha_0}{3(x-2)^4} - \frac{\kappa\alpha_0}{3(x-1)^4} + \right. \\
& \frac{160\kappa\alpha_0}{3(x-2)^5} + \frac{\kappa\alpha_0}{3(x-1)^5} - \frac{2\kappa\alpha_0}{3} + \frac{80\alpha_0}{3(x-2)^4} + \frac{2d_1\alpha_0}{3(x-1)^4} - \frac{\alpha_0}{3(x-1)^4} + \frac{160\alpha_0}{3(x-2)^5} - \frac{2d_1\alpha_0}{3(x-1)^5} + \frac{\alpha_0}{3(x-1)^5} - \frac{2}{3}\frac{\alpha_0}{x} - \frac{x}{3} - \\
& \frac{x\kappa}{3} - \frac{80\kappa}{3(x-2)^4} + \frac{\kappa}{3(x-1)^4} - \frac{320\kappa}{3(x-2)^5} + \frac{\kappa}{3(x-1)^5} - \frac{320\kappa}{3(x-2)^6} - \frac{80}{3(x-2)^4} - \frac{2d_1}{3(x-1)^4} + \frac{1}{3(x-1)^4} - \frac{320}{3(x-2)^5} - \\
& \left. \frac{2d_1}{3(x-1)^5} + \frac{1}{3(x-1)^5} - \frac{320}{3(x-2)^6} \right) H(0; \alpha_0)H(1; x) + \left( -\frac{x\alpha_0^5}{12} - \frac{1}{12}x\kappa\alpha_0^5 - \frac{\kappa\alpha_0^5}{6(x-2)} + \frac{\kappa\alpha_0^5}{12(x-1)} - \frac{\alpha_0^5}{6(x-2)} + \right. \\
& \frac{\alpha_0^5}{12(x-1)} + \frac{19x\alpha_0^4}{36} + \frac{19}{36}x\kappa\alpha_0^4 + \frac{5\kappa\alpha_0^4}{9(x-2)} - \frac{13\kappa\alpha_0^4}{36(x-1)} - \frac{5\kappa\alpha_0^4}{9(x-2)^2} + \frac{\kappa\alpha_0^4}{9(x-1)^2} - \frac{\kappa\alpha_0^4}{18} + \frac{5\alpha_0^4}{9(x-2)} - \frac{13\alpha_0^4}{36(x-1)} - \\
& \frac{5\alpha_0^4}{9(x-2)^2} + \frac{\alpha_0^4}{9(x-1)^2} - \frac{\alpha_0^4}{18} - \frac{13x\alpha_0^3}{9} - \frac{13}{9}x\kappa\alpha_0^3 - \frac{5\kappa\alpha_0^3}{9(x-2)} + \frac{11\kappa\alpha_0^3}{18(x-1)} + \frac{10\kappa\alpha_0^3}{9(x-2)^2} - \frac{7\kappa\alpha_0^3}{18(x-1)^2} - \frac{20\kappa\alpha_0^3}{9(x-2)^3} + \\
& \frac{\kappa\alpha_0^3}{6(x-1)^3} + \frac{\kappa\alpha_0^3}{3} - \frac{5\alpha_0^3}{9(x-2)} + \frac{11\alpha_0^3}{18(x-1)} + \frac{10\alpha_0^3}{9(x-2)^2} - \frac{7\alpha_0^3}{18(x-1)^2} - \frac{20\alpha_0^3}{9(x-2)^3} + \frac{\alpha_0^3}{6(x-1)^3} + \frac{\alpha_0^3}{3} + \frac{7x\alpha_0^2}{3} + \frac{7}{3}x\kappa\alpha_0^2 - \\
& \frac{\kappa\alpha_0^2}{2(x-1)} + \frac{\kappa\alpha_0^2}{2(x-1)^2} - \frac{\kappa\alpha_0^2}{2(x-1)^3} - \frac{40\kappa\alpha_0^2}{3(x-2)^4} + \frac{\kappa\alpha_0^2}{3(x-1)^4} - \kappa\alpha_0^2 - \frac{\alpha_0^2}{2(x-1)} + \frac{\alpha_0^2}{2(x-1)^2} - \frac{\alpha_0^2}{2(x-1)^3} - \frac{40\alpha_0^2}{3(x-2)^4} + \\
& \frac{\alpha_0^2}{3(x-1)^4} - \frac{2}{3}\frac{\alpha_0^2}{(x-1)^4} - \frac{73x\alpha_0}{36} - \frac{73x\kappa\alpha_0}{36} - \frac{40\kappa\alpha_0}{9(x-2)} + \frac{65\kappa\alpha_0}{12(x-1)} + \frac{40\kappa\alpha_0}{9(x-2)^2} - \frac{25\kappa\alpha_0}{18(x-1)^2} + \frac{35\kappa\alpha_0}{36(x-1)^3} + \frac{80\kappa\alpha_0}{(x-2)^4} + \\
& \frac{19\kappa\alpha_0}{36(x-1)^4} + \frac{160\kappa\alpha_0}{3(x-2)^5} - \frac{25\kappa\alpha_0}{36(x-1)^5} + \frac{2\kappa\alpha_0}{9} - \frac{40\alpha_0}{9(x-2)} + \frac{65\alpha_0}{12(x-1)} + \frac{40\alpha_0}{9(x-2)^2} - \frac{25\alpha_0}{18(x-1)^2} + \frac{35\alpha_0}{36(x-1)^3} + \\
& \frac{80\alpha_0}{(x-2)^4} + \frac{5\alpha_0}{4(x-1)^4} + \frac{160\alpha_0}{3(x-2)^5} - \frac{17\alpha_0}{12(x-1)^5} + \frac{2\alpha_0}{9} + \frac{25x}{36} + \frac{25x\kappa}{36} - \frac{32\kappa}{9(x-2)} + \frac{43\kappa}{12(x-1)} + \frac{40\kappa}{9(x-2)^2} - \frac{4\kappa}{9(x-1)^2} - \\
& \frac{80\kappa}{9(x-2)^3} - \frac{\kappa}{36(x-1)^3} - \frac{160\kappa}{3(x-2)^4} - \frac{43\kappa}{36(x-1)^4} - \frac{320\kappa}{3(x-2)^5} - \frac{25\kappa}{36(x-1)^5} + \frac{\kappa}{2} + \left( -\frac{2\kappa\alpha_0}{3(x-1)^4} + \frac{2\kappa\alpha_0}{3(x-1)^5} - \right. \\
& \left. \frac{2\alpha_0}{3(x-1)^4} + \frac{2\alpha_0}{3(x-1)^5} + \frac{2\kappa}{3(x-1)^4} + \frac{2\kappa}{3(x-1)^5} + \frac{2}{3(x-1)^4} + \frac{2}{3(x-1)^5} \right) H(0; \alpha_0) + \left( -\frac{2\alpha_0d_1}{3(x-1)^4} + \frac{2d_1}{3(x-1)^4} + \right. \\
& \left. \frac{2\alpha_0d_1}{3(x-1)^5} + \frac{2d_1}{3(x-1)^5} \right) H(1; \alpha_0) - \frac{32}{9(x-2)} + \frac{43}{12(x-1)} + \frac{40}{9(x-2)^2} - \frac{4}{9(x-1)^2} - \frac{80}{9(x-2)^3} - \frac{1}{36(x-1)^3} - \\
& \frac{160}{3(x-2)^4} - \frac{23}{12(x-1)^4} - \frac{320}{3(x-2)^5} - \frac{17}{12(x-1)^5} + \frac{1}{2} \Big) H(c_1(\alpha_0); x) + \left( -\frac{32\kappa\alpha_0}{3(x-2)^4} - \frac{64\kappa\alpha_0}{3(x-2)^5} + \frac{32d_1\alpha_0}{3(x-2)^4} + \right. \\
& \frac{544\alpha_0}{9(x-2)^4} + \frac{64d_1\alpha_0}{3(x-2)^5} + \frac{1088\alpha_0}{9(x-2)^5} + \frac{32\kappa}{3(x-2)^4} + \frac{128\kappa}{3(x-2)^5} + \frac{128\kappa}{3(x-2)^6} + \left( -\frac{160\kappa\alpha_0}{3(x-2)^4} - \frac{320\kappa\alpha_0}{3(x-2)^5} - \frac{160\alpha_0}{3(x-2)^4} - \right. \\
& \frac{320\alpha_0}{3(x-2)^5} + \frac{160\kappa}{3(x-2)^4} + \frac{640\kappa}{3(x-2)^5} + \frac{640\kappa}{3(x-2)^6} + \frac{160}{3(x-2)^4} + \frac{640}{3(x-2)^5} + \frac{640}{3(x-2)^6} \Big) H(0; \alpha_0) + \left( -\frac{160\alpha_0d_1}{3(x-2)^4} + \right. \\
& \left. \frac{160d_1}{3(x-2)^4} - \frac{320\alpha_0d_1}{3(x-2)^5} + \frac{640d_1}{3(x-2)^5} + \frac{640d_1}{3(x-2)^6} \right) H(1; \alpha_0) - \frac{32d_1}{3(x-2)^4} - \frac{544}{9(x-2)^4} - \frac{128d_1}{3(x-2)^5} - \frac{2176}{9(x-2)^5} - \\
& \frac{128d_1}{3(x-2)^6} - \frac{2176}{9(x-2)^6} \Big) H(c_2(\alpha_0); x) + \left( -\frac{2x\alpha_0}{3} - \frac{2x\kappa\alpha_0}{3} - \frac{160\kappa\alpha_0}{3(x-2)^4} - \frac{2\kappa\alpha_0}{3(x-1)^4} - \frac{320\kappa\alpha_0}{3(x-2)^5} + \frac{2\kappa\alpha_0}{3(x-1)^5} + \right. \\
& \frac{4\kappa\alpha_0}{3} - \frac{160\alpha_0}{3(x-2)^4} - \frac{2\alpha_0}{3(x-1)^4} - \frac{320\alpha_0}{3(x-2)^5} + \frac{2\alpha_0}{3(x-1)^5} + \frac{4\alpha_0}{3} + \frac{2x}{3} + \frac{2x\kappa}{3} + \frac{160\kappa}{3(x-2)^4} + \frac{2\kappa}{3(x-1)^4} + \frac{640\kappa}{3(x-2)^5} + \\
& \frac{2\kappa}{3(x-1)^5} + \frac{640\kappa}{3(x-2)^6} + \frac{160}{3(x-2)^4} + \frac{2}{3(x-1)^4} + \frac{640}{3(x-2)^5} + \frac{2}{3(x-1)^5} + \frac{640}{3(x-2)^6} \Big) H(0, 0; \alpha_0) + \left( -\frac{2x\alpha_0}{3} - \right. \\
& \frac{2x\kappa\alpha_0}{3} + \frac{160\kappa\alpha_0}{3(x-2)^4} + \frac{2\kappa\alpha_0}{3(x-1)^4} + \frac{320\kappa\alpha_0}{3(x-2)^5} - \frac{2\kappa\alpha_0}{3(x-1)^5} + \frac{4\kappa\alpha_0}{3} + \frac{160\alpha_0}{3(x-2)^4} + \frac{2\alpha_0}{3(x-1)^4} + \frac{320\alpha_0}{3(x-2)^5} - \frac{2\alpha_0}{3(x-1)^5} + \\
& \frac{4\alpha_0}{3} + \frac{2x}{3} + \frac{2x\kappa}{3} - \frac{160\kappa}{3(x-2)^4} - \frac{2\kappa}{3(x-1)^4} - \frac{640\kappa}{3(x-2)^5} - \frac{2\kappa}{3(x-1)^5} - \frac{640\kappa}{3(x-2)^6} - \frac{160}{3(x-2)^4} - \frac{2}{3(x-1)^4} - \frac{640}{3(x-2)^5} - \\
& \left. \frac{2}{3(x-1)^5} - \frac{640}{3(x-2)^6} \right) H(0, 0; x) + \left( \frac{4\alpha_0d_1}{3} - \frac{2\alpha_0xd_1}{3} + \frac{2xd_1}{3} - \frac{160\alpha_0d_1}{3(x-2)^4} + \frac{160d_1}{3(x-2)^4} - \frac{2\alpha_0d_1}{3(x-1)^4} + \right. \\
& \left. \frac{2d_1}{3(x-1)^4} - \frac{320\alpha_0d_1}{3(x-2)^5} + \frac{640d_1}{3(x-2)^5} + \frac{2\alpha_0d_1}{3(x-1)^5} + \frac{2d_1}{3(x-1)^5} + \frac{640d_1}{3(x-2)^6} \right) H(0, 1; \alpha_0) + \left( \frac{x\alpha_0}{3} + \frac{x\kappa\alpha_0}{3} + \right. \\
& \frac{80\kappa\alpha_0}{3(x-2)^4} - \frac{\kappa\alpha_0}{3(x-1)^4} + \frac{160\kappa\alpha_0}{3(x-2)^5} + \frac{\kappa\alpha_0}{3(x-1)^5} - \frac{2\kappa\alpha_0}{3} + \frac{80\alpha_0}{3(x-2)^4} - \frac{\alpha_0}{3(x-1)^4} + \frac{160\alpha_0}{3(x-2)^5} + \frac{\alpha_0}{3(x-1)^5} - \frac{2}{3}\frac{\alpha_0}{x} - \frac{x}{3} - \\
& \frac{x\kappa}{3} - \frac{80\kappa}{3(x-2)^4} + \frac{\kappa}{3(x-1)^4} - \frac{320\kappa}{3(x-2)^5} + \frac{\kappa}{3(x-1)^5} - \frac{320\kappa}{3(x-2)^6} - \frac{80}{3(x-2)^4} + \frac{1}{3(x-1)^4} - \frac{320}{3(x-2)^5} + \frac{1}{3(x-1)^5} - \\
& \left. \frac{320}{3(x-2)^6} \right) H(0, c_1(\alpha_0); x) + \left( -\frac{160\kappa\alpha_0}{3(x-2)^4} - \frac{320\kappa\alpha_0}{3(x-2)^5} - \frac{160\alpha_0}{3(x-2)^4} - \frac{320\alpha_0}{3(x-2)^5} + \frac{160\kappa}{3(x-2)^4} + \frac{640\kappa}{3(x-2)^5} + \right. \\
& \frac{640\kappa}{3(x-2)^6} + \frac{160}{3(x-2)^4} + \frac{640}{3(x-2)^5} + \frac{640}{3(x-2)^6} \Big) H(0, c_2(\alpha_0); x) + \left( -\frac{x\alpha_0}{3} - \frac{x\kappa\alpha_0}{3} - \frac{80\kappa\alpha_0}{3(x-2)^4} + \frac{\kappa\alpha_0}{3(x-1)^4} - \right. \\
& \frac{160\kappa\alpha_0}{3(x-2)^5} - \frac{\kappa\alpha_0}{3(x-1)^5} + \frac{2\kappa\alpha_0}{3} - \frac{80\alpha_0}{3(x-2)^4} - \frac{2d_1\alpha_0}{3(x-1)^4} + \frac{\alpha_0}{3(x-1)^4} - \frac{160\alpha_0}{3(x-2)^5} + \frac{2d_1\alpha_0}{3(x-1)^5} - \frac{\alpha_0}{3(x-1)^5} + \frac{2}{3}\frac{\alpha_0}{x} + \frac{x}{3} +
\end{aligned}$$

$$\begin{aligned}
 & \frac{x\kappa}{3} + \frac{80\kappa}{3(x-2)^4} - \frac{\kappa}{3(x-1)^4} + \frac{320\kappa}{3(x-2)^5} - \frac{\kappa}{3(x-1)^5} + \frac{320\kappa}{3(x-2)^6} + \frac{80}{3(x-2)^4} + \frac{2d_1}{3(x-1)^4} - \frac{1}{3(x-1)^4} + \frac{320}{3(x-2)^5} + \\
 & \frac{2d_1}{3(x-1)^5} - \frac{1}{3(x-1)^5} + \frac{320}{3(x-2)^6} \Big) H(1, 0; x) + \left( \frac{x\alpha_0}{3} + \frac{x\kappa\alpha_0}{3} + \frac{80\kappa\alpha_0}{3(x-2)^4} - \frac{\kappa\alpha_0}{3(x-1)^4} + \frac{160\kappa\alpha_0}{3(x-2)^5} + \frac{\kappa\alpha_0}{3(x-1)^5} - \right. \\
 & \frac{2\kappa\alpha_0}{3} + \frac{80\alpha_0}{3(x-2)^4} + \frac{2d_1\alpha_0}{3(x-1)^4} - \frac{\alpha_0}{3(x-1)^4} + \frac{160\alpha_0}{3(x-2)^5} - \frac{2d_1\alpha_0}{3(x-1)^5} + \frac{\alpha_0}{3(x-1)^5} - \frac{2\alpha_0}{3} - \frac{x}{3} - \frac{x\kappa}{3} - \frac{80\kappa}{3(x-2)^4} + \\
 & \frac{\kappa}{3(x-1)^4} - \frac{320\kappa}{3(x-2)^5} + \frac{\kappa}{3(x-1)^5} - \frac{320\kappa}{3(x-2)^6} - \frac{80}{3(x-2)^4} - \frac{2d_1}{3(x-1)^4} + \frac{1}{3(x-1)^4} - \frac{320}{3(x-2)^5} - \frac{2d_1}{3(x-1)^5} + \\
 & \left. \frac{1}{3(x-1)^5} - \frac{320}{3(x-2)^6} \right) H(1, c_1(\alpha_0); x) + \left( -\frac{160\kappa\alpha_0}{3(x-2)^4} - \frac{320\kappa\alpha_0}{3(x-2)^5} + \frac{160d_1\alpha_0}{3(x-2)^4} - \frac{160\alpha_0}{3(x-2)^4} + \frac{320d_1\alpha_0}{3(x-2)^5} - \right. \\
 & \frac{320\alpha_0}{3(x-2)^5} + \frac{160\kappa}{3(x-2)^4} + \frac{640\kappa}{3(x-2)^5} + \frac{640\kappa}{3(x-2)^6} - \frac{160d_1}{3(x-2)^4} + \frac{160}{3(x-2)^4} - \frac{640d_1}{3(x-2)^5} + \frac{640}{3(x-2)^5} - \frac{640d_1}{3(x-2)^6} + \\
 & \left. \frac{640}{3(x-2)^6} \right) H(2, 0; x) + \left( \frac{160\kappa\alpha_0}{3(x-2)^4} + \frac{320\kappa\alpha_0}{3(x-2)^5} - \frac{160d_1\alpha_0}{3(x-2)^4} + \frac{160\alpha_0}{3(x-2)^4} - \frac{320d_1\alpha_0}{3(x-2)^5} + \frac{320\alpha_0}{3(x-2)^5} - \frac{160\kappa}{3(x-2)^4} - \right. \\
 & \frac{640\kappa}{3(x-2)^5} - \frac{640\kappa}{3(x-2)^6} + \frac{160d_1}{3(x-2)^4} - \frac{160}{3(x-2)^4} + \frac{640d_1}{3(x-2)^5} - \frac{640}{3(x-2)^5} + \frac{640d_1}{3(x-2)^6} - \frac{640}{3(x-2)^6} \Big) H(2, c_2(\alpha_0); x) + \\
 & \left( -\frac{\kappa\alpha_0}{3(x-1)^4} + \frac{\kappa\alpha_0}{3(x-1)^5} - \frac{\alpha_0}{3(x-1)^4} + \frac{\alpha_0}{3(x-1)^5} + \frac{\kappa}{3(x-1)^4} + \frac{\kappa}{3(x-1)^5} + \frac{1}{3(x-1)^4} + \right. \\
 & \left. \frac{1}{3(x-1)^5} \right) H(c_1(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{80\kappa\alpha_0}{3(x-2)^4} - \frac{160\kappa\alpha_0}{3(x-2)^5} - \frac{80\alpha_0}{3(x-2)^4} - \frac{160\alpha_0}{3(x-2)^5} + \frac{80\kappa}{3(x-2)^4} + \frac{320\kappa}{3(x-2)^5} + \right. \\
 & \frac{320\kappa}{3(x-2)^6} + \frac{80}{3(x-2)^4} + \frac{320}{3(x-2)^5} + \frac{320}{3(x-2)^6} \Big) H(c_2(\alpha_0), c_1(\alpha_0); x) + H(0; x) \left( \frac{17x\alpha_0}{12} + \frac{25x\kappa\alpha_0}{36} + \right. \\
 & \frac{40\kappa\alpha_0}{9(x-2)} - \frac{65\kappa\alpha_0}{12(x-1)} - \frac{40\kappa\alpha_0}{9(x-2)^2} + \frac{25\kappa\alpha_0}{18(x-1)^2} - \frac{35\kappa\alpha_0}{36(x-1)^3} - \frac{128\kappa\alpha_0}{3(x-2)^4} - \frac{7\kappa\alpha_0}{36(x-1)^4} + \frac{64\kappa\alpha_0}{3(x-2)^5} + \frac{25\kappa\alpha_0}{36(x-1)^5} - \frac{8\kappa\alpha_0}{9} + \\
 & \frac{40\alpha_0}{9(x-2)} - \frac{65\alpha_0}{12(x-1)} - \frac{40\alpha_0}{9(x-2)^2} + \frac{25\alpha_0}{18(x-1)^2} - \frac{35\alpha_0}{36(x-1)^3} - \frac{32d_1\alpha_0}{3(x-2)^4} - \frac{1024\alpha_0}{9(x-2)^4} - \frac{11\alpha_0}{12(x-1)^4} - \frac{64d_1\alpha_0}{3(x-2)^5} - \\
 & \frac{1088\alpha_0}{9(x-2)^5} + \frac{17\alpha_0}{12(x-1)^5} - \frac{160\kappa\ln 2\alpha_0}{3(x-2)^4} - \frac{320\kappa\ln 2\alpha_0}{3(x-2)^5} - \frac{160\ln 2\alpha_0}{3(x-2)^4} - \frac{320\ln 2\alpha_0}{3(x-2)^5} - \frac{7\alpha_0}{3} - \frac{17x}{12} - \frac{25x\kappa}{36} + \frac{32\kappa}{9(x-2)} - \\
 & \frac{43\kappa}{12(x-1)} - \frac{40\kappa}{9(x-2)^2} + \frac{4\kappa}{9(x-1)^2} + \frac{80\kappa}{9(x-2)^3} + \frac{\kappa}{36(x-1)^3} + \frac{128\kappa}{3(x-2)^4} + \frac{43\kappa}{36(x-1)^4} + \frac{64\kappa}{(x-2)^5} + \frac{25\kappa}{36(x-1)^5} - \\
 & \frac{128\kappa}{3(x-2)^6} - \frac{\kappa}{2} + \frac{32}{9(x-2)} - \frac{43}{12(x-1)} - \frac{40}{9(x-2)^2} + \frac{4}{9(x-1)^2} + \frac{80}{9(x-2)^3} + \frac{1}{36(x-1)^3} + \frac{32d_1}{3(x-2)^4} + \frac{1024}{9(x-2)^4} + \\
 & \frac{23}{12(x-1)^4} + \frac{128d_1}{3(x-2)^5} + \frac{3136}{9(x-2)^5} + \frac{17}{12(x-1)^5} + \frac{128d_1}{3(x-2)^6} + \frac{2176}{9(x-2)^6} + \frac{160\kappa\ln 2}{3(x-2)^4} + \frac{640\kappa\ln 2}{3(x-2)^5} + \frac{640\kappa\ln 2}{3(x-2)^6} + \\
 & \frac{160\ln 2}{3(x-2)^4} + \frac{640\ln 2}{3(x-2)^5} + \frac{640\ln 2}{3(x-2)^6} - \frac{1}{2} \Big) + H(2; x) \left( \frac{160\kappa\ln 2\alpha_0}{3(x-2)^4} + \frac{320\kappa\ln 2\alpha_0}{3(x-2)^5} - \frac{160d_1\ln 2\alpha_0}{3(x-2)^4} + \frac{160\ln 2\alpha_0}{3(x-2)^4} - \right. \\
 & \frac{320d_1\ln 2\alpha_0}{3(x-2)^5} + \frac{320\ln 2\alpha_0}{3(x-2)^5} + \left( \frac{160\kappa\alpha_0}{3(x-2)^4} + \frac{320\kappa\alpha_0}{3(x-2)^5} - \frac{160d_1\alpha_0}{3(x-2)^4} + \frac{160\alpha_0}{3(x-2)^4} - \frac{320d_1\alpha_0}{3(x-2)^5} + \frac{320\alpha_0}{3(x-2)^5} - \frac{160\kappa}{3(x-2)^4} - \right. \\
 & \frac{640\kappa}{3(x-2)^5} - \frac{640\kappa}{3(x-2)^6} + \frac{160d_1}{3(x-2)^4} - \frac{160}{3(x-2)^4} + \frac{640d_1}{3(x-2)^5} - \frac{640}{3(x-2)^5} + \frac{640d_1}{3(x-2)^6} - \frac{640}{3(x-2)^6} \Big) H(0; \alpha_0) - \\
 & \frac{160\kappa\ln 2}{3(x-2)^4} - \frac{640\kappa\ln 2}{3(x-2)^5} - \frac{640\kappa\ln 2}{3(x-2)^6} + \frac{160d_1\ln 2}{3(x-2)^4} - \frac{160\ln 2}{3(x-2)^4} + \frac{640d_1\ln 2}{3(x-2)^5} - \frac{640\ln 2}{3(x-2)^5} + \frac{640d_1\ln 2}{3(x-2)^6} - \\
 & \left. \frac{640\ln 2}{3(x-2)^6} \right) + \frac{40x}{27(\kappa+1)} + \frac{\pi^2 x\kappa}{6(\kappa+1)} - \frac{20\pi^2\kappa}{3(x-2)^4(\kappa+1)} - \frac{\pi^2\kappa}{6(x-1)^4(\kappa+1)} - \frac{80\pi^2\kappa}{3(x-2)^5(\kappa+1)} - \frac{\pi^2\kappa}{6(x-1)^5(\kappa+1)} - \\
 & \frac{80\pi^2\kappa}{3(x-2)^6(\kappa+1)} - \frac{20\pi^2}{9(x-2)^4(\kappa+1)} - \frac{18(x-1)^4(\kappa+1)}{18(x-1)^4(\kappa+1)} - \frac{9(x-2)^5(\kappa+1)}{9(x-2)^5(\kappa+1)} - \frac{18(x-1)^5(\kappa+1)}{18(x-1)^5(\kappa+1)} - \frac{9(x-2)^6(\kappa+1)}{9(x-2)^6(\kappa+1)} - \\
 & \frac{80\kappa\ln^2 2}{(x-2)^4(\kappa+1)} - \frac{320\kappa\ln^2 2}{(x-2)^5(\kappa+1)} - \frac{320\kappa\ln^2 2}{(x-2)^6(\kappa+1)} - \frac{80\ln^2 2}{3(x-2)^4(\kappa+1)} - \frac{320\ln^2 2}{3(x-2)^5(\kappa+1)} - \frac{320\ln^2 2}{3(x-2)^6(\kappa+1)} - \\
 & \frac{32d_1\kappa\ln 2}{3(x-2)^4(\kappa+1)} - \frac{352\kappa\ln 2}{9(x-2)^4(\kappa+1)} - \frac{128d_1\kappa\ln 2}{3(x-2)^5(\kappa+1)} - \frac{1408\kappa\ln 2}{9(x-2)^5(\kappa+1)} - \frac{128d_1\kappa\ln 2}{3(x-2)^6(\kappa+1)} - \frac{1408\kappa\ln 2}{9(x-2)^6(\kappa+1)} - \\
 & \left. \frac{32d_1\ln 2}{3(x-2)^4(\kappa+1)} - \frac{544\ln 2}{9(x-2)^4(\kappa+1)} - \frac{128d_1\ln 2}{3(x-2)^5(\kappa+1)} - \frac{2176\ln 2}{9(x-2)^5(\kappa+1)} - \frac{128d_1\ln 2}{3(x-2)^6(\kappa+1)} - \frac{2176\ln 2}{9(x-2)^6(\kappa+1)} \right\},
 \end{aligned}$$

$$\begin{aligned}
 a_2^{(\kappa, 2)} = & \frac{1}{\alpha_0(x-2)-x} \left\{ \frac{1}{48} d_1^2 x \alpha_0^5 - \frac{1}{72} \pi^2 x \alpha_0^5 + \frac{d_1^2 \alpha_0^5}{24(x-2)} - \frac{\pi^2 \alpha_0^5}{36(x-2)} - \frac{d_1^2 \alpha_0^5}{48(x-1)} + \frac{\pi^2 \alpha_0^5}{72(x-1)} - \frac{d_1 \alpha_0^5}{48(\kappa+1)} - \frac{19d_1 x \alpha_0^5}{144(\kappa+1)} + \right. \\
 & \frac{13x\alpha_0^5}{27(\kappa+1)} - \frac{d_1 \kappa \alpha_0^5}{48(\kappa+1)} - \frac{31d_1 x \kappa \alpha_0^5}{144(\kappa+1)} + \frac{85x\kappa \alpha_0^5}{108(\kappa+1)} - \frac{17d_1 \kappa \alpha_0^5}{36(x-2)(\kappa+1)} + \frac{415 \kappa \alpha_0^5}{216(x-2)(\kappa+1)} + \frac{31d_1 \kappa \alpha_0^5}{144(x-1)(\kappa+1)} - \\
 & \frac{85\kappa \alpha_0^5}{108(x-1)(\kappa+1)} + \frac{25\kappa \alpha_0^5}{144(\kappa+1)} - \frac{11d_1 \alpha_0^5}{36(x-2)(\kappa+1)} + \frac{265\alpha_0^5}{216(x-2)(\kappa+1)} + \frac{19d_1 \alpha_0^5}{144(x-1)(\kappa+1)} - \frac{13\alpha_0^5}{27(x-1)(\kappa+1)} + \\
 & \frac{19\alpha_0^5}{144(\kappa+1)} + \frac{37d_1^2 \alpha_0^4}{648} - \frac{199d_1^2 x \alpha_0^4}{1296} + \frac{19}{216} \pi^2 x \alpha_0^4 - \frac{17d_1^2 \alpha_0^4}{324(x-2)} + \frac{5\pi^2 \alpha_0^4}{54(x-2)} + \frac{145d_1^2 \alpha_0^4}{1296(x-1)} - \frac{13\pi^2 \alpha_0^4}{216(x-1)} - \frac{79d_1 \alpha_0^4}{432(\kappa+1)} + \\
 & \frac{137d_1 x \alpha_0^4}{144(\kappa+1)} - \frac{173x\alpha_0^4}{54(\kappa+1)} - \frac{617d_1 \kappa \alpha_0^4}{1296(\kappa+1)} + \frac{2113d_1 x \kappa \alpha_0^4}{1296(\kappa+1)} - \frac{1835x\kappa \alpha_0^4}{324(\kappa+1)} + \frac{139d_1 \kappa \alpha_0^4}{162(x-2)(\kappa+1)} - \frac{1687\kappa \alpha_0^4}{324(x-2)(\kappa+1)} - \\
 & \frac{1429d_1 \kappa \alpha_0^4}{1296(x-1)(\kappa+1)} + \frac{2359\kappa \alpha_0^4}{648(x-1)(\kappa+1)} - \frac{487d_1 \kappa \alpha_0^4}{162(x-2)^2(\kappa+1)} + \frac{2851\kappa \alpha_0^4}{324(x-2)^2(\kappa+1)} + \frac{359d_1 \kappa \alpha_0^4}{648(x-1)^2(\kappa+1)} -
 \end{aligned}$$

$$\begin{aligned}
& \frac{140\kappa\alpha_0^4}{81(x-1)^2(\kappa+1)} + \frac{733\kappa\alpha_0^4}{1296(\kappa+1)} + \frac{35d_1\alpha_0^4}{54(x-2)(\kappa+1)} - \frac{137\alpha_0^4}{36(x-2)(\kappa+1)} - \frac{283d_1\alpha_0^4}{432(x-1)(\kappa+1)} + \frac{461\alpha_0^4}{216(x-1)(\kappa+1)} - \\
& \frac{54(x-2)^2(\kappa+1)}{95d_1\alpha_0^4} + \frac{108(x-2)^2(\kappa+1)}{503\alpha_0^4} + \frac{24(x-1)^2(\kappa+1)}{7d_1\alpha_0^4} - \frac{43\alpha_0^4}{54(x-1)^2(\kappa+1)} - \frac{59\alpha_0^4}{432(\kappa+1)} + \frac{101d_1^2\alpha_0^4}{324(x-2)^2} - \frac{5\pi^2\alpha_0^4}{54(x-2)^2} - \\
& \frac{4d_1^2\alpha_0^4}{81(x-1)^2} + \frac{\pi^2\alpha_0^4}{54(x-1)^2} - \frac{\pi^2\alpha_0^4}{108} - \frac{25d_1^2\alpha_0^3}{54} + \frac{371}{648}d_1^2x\alpha_0^3 - \frac{13}{54}\pi^2x\alpha_0^3 - \frac{67d_1^2\alpha_0^3}{324(x-2)} - \frac{5\pi^2\alpha_0^3}{54(x-2)} - \frac{155d_1^2\alpha_0^3}{648(x-1)} + \\
& \frac{11\pi^2\alpha_0^3}{108(x-1)} + \frac{437d_1\alpha_0^3}{216(\kappa+1)} - \frac{1441d_1x\alpha_0^3}{432(\kappa+1)} + \frac{521x\alpha_0^3}{54(\kappa+1)} + \frac{985d_1\kappa\alpha_0^3}{216(\kappa+1)} - \frac{8029d_1x\kappa\alpha_0^3}{1296(\kappa+1)} + \frac{6347x\kappa\alpha_0^3}{324(\kappa+1)} + \frac{769d_1\kappa\alpha_0^3}{324(x-2)(\kappa+1)} - \\
& \frac{182\kappa\alpha_0^3}{81(x-2)(\kappa+1)} + \frac{139d_1\kappa\alpha_0^3}{81(x-1)(\kappa+1)} - \frac{3085\kappa\alpha_0^3}{648(x-1)(\kappa+1)} - \frac{110d_1\kappa\alpha_0^3}{81(x-2)^2(\kappa+1)} - \frac{509\kappa\alpha_0^3}{81(x-2)^2(\kappa+1)} - \frac{3677d_1\kappa\alpha_0^3}{1296(x-1)^2(\kappa+1)} + \\
& \frac{10099\kappa\alpha_0^3}{1296(x-1)^2(\kappa+1)} - \frac{2168d_1\kappa\alpha_0^3}{81(x-2)^3(\kappa+1)} + \frac{4684\kappa\alpha_0^3}{81(x-2)^3(\kappa+1)} + \frac{263d_1\kappa\alpha_0^3}{144(x-1)^3(\kappa+1)} - \frac{173\kappa\alpha_0^3}{36(x-1)^3(\kappa+1)} - \frac{4183\kappa\alpha_0^3}{432(\kappa+1)} + \\
& \frac{89d_1\alpha_0^3}{108(x-2)(\kappa+1)} + \frac{58\alpha_0^3}{27(x-2)(\kappa+1)} + \frac{121d_1\alpha_0^3}{108(x-1)(\kappa+1)} - \frac{239\alpha_0^3}{72(x-1)(\kappa+1)} + \frac{2d_1\alpha_0^3}{27(x-2)^2(\kappa+1)} - \frac{185\alpha_0^3}{27(x-2)^2(\kappa+1)} - \\
& \frac{605d_1\alpha_0^3}{605d_1\alpha_0^3} + \frac{135\alpha_0^3}{135\alpha_0^3} + \frac{376d_1\alpha_0^3}{376d_1\alpha_0^3} - \frac{212\alpha_0^3}{212\alpha_0^3} + \frac{367d_1\alpha_0^3}{367d_1\alpha_0^3} - \frac{89\alpha_0^3}{89\alpha_0^3} - \\
& \frac{432(x-1)^2(\kappa+1)}{1021\alpha_0^3} + \frac{432(x-1)^2(\kappa+1)}{29d_1^2\alpha_0^3} + \frac{5\pi^2\alpha_0^3}{27(x-2)^2} + \frac{43d_1^2\alpha_0^3}{162(x-1)^2} - \frac{7\pi^2\alpha_0^3}{108(x-1)^2} + \frac{260d_1^2\alpha_0^3}{81(x-2)^3} - \frac{10\pi^2\alpha_0^3}{27(x-2)^3} - \frac{d_1^2\alpha_0^3}{6(x-1)^3} + \frac{\pi^2\alpha_0^3}{36(x-1)^3} + \\
& \frac{\pi^2\alpha_0^3}{18} + \frac{365d_1^2\alpha_0^2}{108} - \frac{505}{216}d_1^2x\alpha_0^2 + \frac{7}{18}\pi^2x\alpha_0^2 + \frac{14d_1^2\alpha_0^2}{27(x-2)} + \frac{55d_1^2\alpha_0^2}{216(x-1)} - \frac{\pi^2\alpha_0^2}{12(x-1)} - \frac{1445d_1\alpha_0^2}{108(\kappa+1)} + \frac{4637d_1x\alpha_0^2}{432(\kappa+1)} - \\
& \frac{497x\alpha_0^2}{24(\kappa+1)} - \frac{1735d_1\kappa\alpha_0^2}{54(\kappa+1)} + \frac{10115d_1x\kappa\alpha_0^2}{432(\kappa+1)} - \frac{12541x\kappa\alpha_0^2}{216(\kappa+1)} - \frac{458d_1\kappa\alpha_0^2}{27(x-2)(\kappa+1)} + \frac{1280\kappa\alpha_0^2}{27(x-2)(\kappa+1)} + \frac{1619d_1\kappa\alpha_0^2}{216(x-1)(\kappa+1)} - \\
& \frac{575\kappa\alpha_0^2}{24(x-1)(\kappa+1)} + \frac{737d_1\kappa\alpha_0^2}{18(x-2)^2(\kappa+1)} - \frac{821\kappa\alpha_0^2}{9(x-2)^2(\kappa+1)} + \frac{569d_1\kappa\alpha_0^2}{144(x-1)^2(\kappa+1)} - \frac{3731\kappa\alpha_0^2}{432(x-1)^2(\kappa+1)} - \frac{5344d_1\kappa\alpha_0^2}{27(x-2)^3(\kappa+1)} + \\
& \frac{7408\kappa\alpha_0^2}{27(x-2)^3(\kappa+1)} - \frac{2093d_1\kappa\alpha_0^2}{144(x-1)^3(\kappa+1)} + \frac{743\kappa\alpha_0^2}{24(x-1)^3(\kappa+1)} - \frac{9(x-2)^4(\kappa+1)}{9(x-2)^4(\kappa+1)} + \frac{27(x-2)^4(\kappa+1)}{27(x-2)^4(\kappa+1)} + \frac{24(x-1)^4(\kappa+1)}{24(x-1)^4(\kappa+1)} - \\
& \frac{1829\kappa\alpha_0^2}{72(x-1)^4(\kappa+1)} + \frac{28453\kappa\alpha_0^2}{432(\kappa+1)} - \frac{58d_1\alpha_0^2}{9(x-2)(\kappa+1)} + \frac{256\alpha_0^2}{27(x-2)(\kappa+1)} + \frac{427d_1\alpha_0^2}{216(x-1)(\kappa+1)} - \frac{61\alpha_0^2}{24(x-1)(\kappa+1)} + \\
& \frac{299d_1\alpha_0^2}{18(x-2)^2(\kappa+1)} - \frac{163\alpha_0^2}{9(x-2)^2(\kappa+1)} + \frac{881d_1\alpha_0^2}{432(x-1)^2(\kappa+1)} - \frac{1621\alpha_0^2}{432(x-1)^2(\kappa+1)} - \frac{736d_1\alpha_0^2}{9(x-2)^3(\kappa+1)} + \frac{1424\alpha_0^2}{27(x-2)^3(\kappa+1)} - \\
& \frac{871d_1\alpha_0^2}{144(x-1)^3(\kappa+1)} + \frac{569\alpha_0^2}{72(x-1)^3(\kappa+1)} - \frac{2224d_1\alpha_0^2}{9(x-2)^4(\kappa+1)} + \frac{6664\alpha_0^2}{27(x-2)^4(\kappa+1)} + \frac{1105d_1\alpha_0^2}{216(x-1)^4(\kappa+1)} - \frac{1333\alpha_0^2}{216(x-1)^4(\kappa+1)} + \\
& \frac{6791\alpha_0^2}{432(\kappa+1)} - \frac{53d_1^2\alpha_0^2}{18(x-2)^2} - \frac{d_1^2\alpha_0^2}{2(x-1)^2} + \frac{\pi^2\alpha_0^2}{12(x-1)^2} + \frac{784d_1^2\alpha_0^2}{27(x-2)^3} + \frac{3d_1^2\alpha_0^2}{2(x-1)^3} - \frac{\pi^2\alpha_0^2}{12(x-1)^3} + \frac{232d_1^2\alpha_0^2}{3(x-2)^4} - \frac{20\pi^2\alpha_0^2}{9(x-2)^4} - \\
& \frac{4d_1^2\alpha_0^2}{3(x-1)^4} + \frac{\pi^2\alpha_0^2}{18(x-1)^4} - \frac{\pi^2\alpha_0^2}{6} + \frac{d_1^2\alpha_0}{6(\kappa+1)} - \frac{263d_1\alpha_0}{216(\kappa+1)} + \frac{205d_1^2x\alpha_0}{108(\kappa+1)} - \frac{1775d_1x\alpha_0}{216(\kappa+1)} - \frac{73\pi^2x\alpha_0}{216(\kappa+1)} + \frac{6995x\alpha_0}{648(\kappa+1)} + \\
& \frac{d_1^2\kappa\alpha_0}{6(\kappa+1)} - \frac{49d_1\kappa\alpha_0}{24(\kappa+1)} + \frac{205d_1^2x\kappa\alpha_0}{108(\kappa+1)} - \frac{4025d_1x\kappa\alpha_0}{216(\kappa+1)} - \frac{301\pi^2x\kappa\alpha_0}{216(\kappa+1)} + \frac{3121x\kappa\alpha_0}{72(\kappa+1)} - \frac{7d_1^2\kappa\alpha_0}{27(x-2)(\kappa+1)} - \frac{86d_1\kappa\alpha_0}{27(x-2)(\kappa+1)} - \\
& \frac{140\pi^2\kappa\alpha_0}{27(x-2)(\kappa+1)} + \frac{320\kappa\alpha_0}{27(x-2)(\kappa+1)} - \frac{7d_1^2\kappa\alpha_0}{108(x-1)(\kappa+1)} + \frac{1787d_1\kappa\alpha_0}{216(x-1)(\kappa+1)} + \frac{455\pi^2\kappa\alpha_0}{72(x-1)(\kappa+1)} - \frac{865\kappa\alpha_0}{36(x-1)(\kappa+1)} + \\
& \frac{53d_1^2\kappa\alpha_0}{27(x-2)^2(\kappa+1)} - \frac{239d_1\kappa\alpha_0}{27(x-2)^2(\kappa+1)} + \frac{140\pi^2\kappa\alpha_0}{27(x-2)^2(\kappa+1)} + \frac{5d_1^2\kappa\alpha_0}{27(x-1)^2(\kappa+1)} - \frac{889d_1\kappa\alpha_0}{216(x-1)^2(\kappa+1)} - \frac{175\pi^2\kappa\alpha_0}{108(x-1)^2(\kappa+1)} + \\
& \frac{1315\kappa\alpha_0}{108(x-1)^2(\kappa+1)} - \frac{784d_1^2\kappa\alpha_0}{27(x-2)^3(\kappa+1)} + \frac{4168d_1\kappa\alpha_0}{27(x-2)^3(\kappa+1)} - \frac{3704\kappa\alpha_0}{27(x-2)^3(\kappa+1)} - \frac{d_1^2\kappa\alpha_0}{(x-1)^3(\kappa+1)} + \frac{77d_1\kappa\alpha_0}{9(x-1)^3(\kappa+1)} + \\
& \frac{245\pi^2\kappa\alpha_0}{216(x-1)^3(\kappa+1)} - \frac{361\kappa\alpha_0}{24(x-1)^3(\kappa+1)} - \frac{464d_1^2\kappa\alpha_0}{3(x-2)^4(\kappa+1)} + \frac{10016d_1\kappa\alpha_0}{9(x-2)^4(\kappa+1)} + \frac{8d_1\pi^2\kappa\alpha_0}{3(x-2)^4(\kappa+1)} + \frac{704\pi^2\kappa\alpha_0}{9(x-2)^4(\kappa+1)} - \\
& \frac{48368\kappa\alpha_0}{27(x-2)^4(\kappa+1)} - \frac{4d_1^2\kappa\alpha_0}{3(x-1)^4(\kappa+1)} + \frac{299d_1\kappa\alpha_0}{24(x-1)^4(\kappa+1)} + \frac{139\pi^2\kappa\alpha_0}{216(x-1)^4(\kappa+1)} - \frac{1829\kappa\alpha_0}{72(x-1)^4(\kappa+1)} - \frac{512d_1^2\kappa\alpha_0}{3(x-2)^5(\kappa+1)} + \\
& \frac{11264d_1\kappa\alpha_0}{9(x-2)^5(\kappa+1)} + \frac{16d_1\pi^2\kappa\alpha_0}{3(x-2)^5(\kappa+1)} + \frac{32\pi^2\kappa\alpha_0}{(x-2)^5(\kappa+1)} - \frac{59008\kappa\alpha_0}{27(x-2)^5(\kappa+1)} - \frac{253\pi^2\kappa\alpha_0}{216(x-1)^5(\kappa+1)} + \frac{89\pi^2\kappa\alpha_0}{54(\kappa+1)} + \frac{307\kappa\alpha_0}{36(\kappa+1)} - \\
& \frac{7d_1^2\alpha_0}{27(x-2)(\kappa+1)} - \frac{2d_1\alpha_0}{3(x-2)(\kappa+1)} - \frac{20\pi^2\alpha_0}{27(x-2)(\kappa+1)} + \frac{64\alpha_0}{27(x-2)(\kappa+1)} - \frac{7d_1^2\alpha_0}{108(x-1)(\kappa+1)} + \frac{613d_1\alpha_0}{216(x-1)(\kappa+1)} + \\
& \frac{65\pi^2\alpha_0}{72(x-1)(\kappa+1)} - \frac{19\alpha_0}{4(x-1)(\kappa+1)} + \frac{53d_1^2\alpha_0}{27(x-2)^2(\kappa+1)} - \frac{133d_1\alpha_0}{27(x-2)^2(\kappa+1)} + \frac{20\pi^2\alpha_0}{27(x-2)^2(\kappa+1)} + \frac{5d_1^2\alpha_0}{27(x-1)^2(\kappa+1)} - \\
& \frac{331d_1\alpha_0}{216(x-1)^2(\kappa+1)} - \frac{25\pi^2\alpha_0}{108(x-1)^2(\kappa+1)} + \frac{257\alpha_0}{108(x-1)^2(\kappa+1)} - \frac{784d_1^2\alpha_0}{27(x-2)^3(\kappa+1)} + \frac{1816d_1\alpha_0}{27(x-2)^3(\kappa+1)} - \frac{712\alpha_0}{27(x-2)^3(\kappa+1)} - \\
& \frac{d_1^2\alpha_0}{(x-1)^3(\kappa+1)} + \frac{10d_1\alpha_0}{3(x-1)^3(\kappa+1)} + \frac{35\pi^2\alpha_0}{216(x-1)^3(\kappa+1)} - \frac{67\alpha_0}{24(x-1)^3(\kappa+1)} - \frac{464d_1^2\alpha_0}{3(x-2)^4(\kappa+1)} + \frac{4448d_1\alpha_0}{9(x-2)^4(\kappa+1)} + \\
& \frac{8d_1\pi^2\alpha_0}{9(x-2)^4(\kappa+1)} + \frac{496\pi^2\alpha_0}{27(x-2)^4(\kappa+1)} - \frac{13328\alpha_0}{27(x-2)^4(\kappa+1)} - \frac{4d_1^2\alpha_0}{3(x-1)^4(\kappa+1)} + \frac{1105d_1\alpha_0}{216(x-1)^4(\kappa+1)} + \frac{5\pi^2\alpha_0}{24(x-1)^4(\kappa+1)} - \\
& \frac{1333\alpha_0}{216(x-1)^4(\kappa+1)} - \frac{512d_1^2\alpha_0}{3(x-2)^5(\kappa+1)} + \frac{5120d_1\alpha_0}{9(x-2)^5(\kappa+1)} + \frac{16d_1\pi^2\alpha_0}{9(x-2)^5(\kappa+1)} + \frac{512\pi^2\alpha_0}{27(x-2)^5(\kappa+1)} - \frac{19072\alpha_0}{27(x-2)^5(\kappa+1)} - \\
& \frac{17\pi^2\alpha_0}{72(x-1)^5(\kappa+1)} + \frac{\pi^2\alpha_0}{27(\kappa+1)} + \frac{3505\alpha_0}{324(\kappa+1)} + \frac{x\zeta_3\alpha_0}{\kappa+1} + \frac{7x\kappa\zeta_3\alpha_0}{3(\kappa+1)} - \frac{7\kappa\zeta_3\alpha_0}{3(x-1)^4(\kappa+1)} + \frac{7\kappa\zeta_3\alpha_0}{3(x-1)^5(\kappa+1)} - \frac{14\kappa\zeta_3\alpha_0}{3(\kappa+1)} - \\
& \frac{\zeta_3\alpha_0}{3(x-1)^4(\kappa+1)} + \frac{\zeta_3\alpha_0}{3(x-1)^5(\kappa+1)} - \frac{2\zeta_3\alpha_0}{\kappa+1} + \frac{1120\kappa\ln^3 2\alpha_0}{9(x-2)^4(\kappa+1)} + \frac{2240\kappa\ln^3 2\alpha_0}{9(x-2)^5(\kappa+1)} + \frac{160\ln^3 2\alpha_0}{9(x-2)^4(\kappa+1)} + \frac{320\ln^3 2\alpha_0}{9(x-2)^5(\kappa+1)} +
\end{aligned}$$



$$\begin{aligned}
& \frac{32d_1 \kappa \ln^2 2 \alpha_0}{(x-2)^4(\kappa+1)} + \frac{416\kappa \ln^2 2 \alpha_0}{3(x-2)^4(\kappa+1)} + \frac{64d_1 \kappa \ln^2 2 \alpha_0}{(x-2)^5(\kappa+1)} + \frac{832\kappa \ln^2 2 \alpha_0}{3(x-2)^5(\kappa+1)} + \frac{32d_1 \ln^2 2 \alpha_0}{3(x-2)^4(\kappa+1)} + \frac{544 \ln^2 2 \alpha_0}{9(x-2)^4(\kappa+1)} + \\
& \frac{64d_1 \ln^2 2 \alpha_0}{3(x-2)^5(\kappa+1)} + \frac{1088 \ln^2 2 \alpha_0}{9(x-2)^5(\kappa+1)} + \frac{256d_1 \kappa \ln 2 \alpha_0}{9(x-2)^4(\kappa+1)} + \frac{80\pi^2 \kappa \ln 2 \alpha_0}{3(x-2)^4(\kappa+1)} + \frac{1856\kappa \ln 2 \alpha_0}{27(x-2)^4(\kappa+1)} + \frac{512d_1 \kappa \ln 2 \alpha_0}{9(x-2)^5(\kappa+1)} + \\
& \frac{160\pi^2 \kappa \ln 2 \alpha_0}{3(x-2)^5(\kappa+1)} + \frac{3712\kappa \ln 2 \alpha_0}{27(x-2)^5(\kappa+1)} + \frac{256d_1 \ln 2 \alpha_0}{9(x-2)^4(\kappa+1)} + \frac{3392 \ln 2 \alpha_0}{27(x-2)^4(\kappa+1)} + \frac{512d_1 \ln 2 \alpha_0}{9(x-2)^5(\kappa+1)} + \frac{6784 \ln 2 \alpha_0}{27(x-2)^5(\kappa+1)} + \\
& \left( \frac{1}{12} d_1 x \alpha_0^5 - \frac{4x\alpha_0^5}{9} + \frac{1}{12} d_1 x \kappa \alpha_0^5 - \frac{11}{18} x \kappa \alpha_0^5 + \frac{d_1 \kappa \alpha_0^5}{6(x-2)} - \frac{25\kappa\alpha_0^5}{18(x-2)} - \frac{d_1 \kappa \alpha_0^5}{12(x-1)} + \frac{11\kappa\alpha_0^5}{18(x-1)} - \frac{\kappa \alpha_0^5}{12} + \frac{d_1 \alpha_0^5}{6(x-2)} - \right. \\
& \frac{19\alpha_0^5}{18(x-2)} - \frac{d_1 \alpha_0^5}{12(x-1)} + \frac{4\alpha_0^5}{9(x-1)} - \frac{\alpha_0^5}{12} + \frac{7d_1 \alpha_0^4}{54} - \frac{61}{108} d_1 x \alpha_0^4 + \frac{157x\alpha_0^4}{54} + \frac{7}{54} d_1 \kappa \alpha_0^4 - \frac{61}{108} d_1 x \kappa \alpha_0^4 + \\
& \frac{112}{27} x \kappa \alpha_0^4 - \frac{11d_1 \kappa \alpha_0^4}{27(x-2)} + \frac{113\kappa \alpha_0^4}{27(x-2)} + \frac{43d_1 \kappa \alpha_0^4}{108(x-1)} - \frac{149 \kappa \alpha_0^4}{54(x-1)} + \frac{23d_1 \kappa \alpha_0^4}{27(x-2)^2} - \frac{149\kappa \alpha_0^4}{27(x-2)^2} - \frac{4d_1 \kappa \alpha_0^4}{27(x-1)^2} + \frac{59\kappa \alpha_0^4}{54(x-1)^2} - \\
& \frac{41 \kappa \alpha_0^4}{108} - \frac{11d_1 \alpha_0^4}{27(x-2)} + \frac{91\alpha_0^4}{27(x-2)} + \frac{43d_1 \alpha_0^4}{108(x-1)} - \frac{53\alpha_0^4}{27(x-1)} + \frac{23d_1 \alpha_0^4}{27(x-2)^2} - \frac{103\alpha_0^4}{27(x-2)^2} - \frac{4d_1 \alpha_0^4}{27(x-1)^2} + \frac{37\alpha_0^4}{54(x-1)^2} - \\
& \frac{\alpha_0^4}{108} - \frac{8d_1 \alpha_0^3}{9} + \frac{95}{54} d_1 x \alpha_0^3 - \frac{911x\alpha_0^3}{108} - \frac{8}{9} d_1 \kappa \alpha_0^3 + \frac{95}{54} d_1 x \kappa \alpha_0^3 - \frac{1381}{108} x \kappa \alpha_0^3 - \frac{d_1 \kappa \alpha_0^3}{27(x-2)} - \frac{50\kappa \alpha_0^3}{27(x-2)} - \frac{41d_1 \kappa \alpha_0^3}{54(x-1)} + \\
& \frac{239\kappa \alpha_0^3}{54(x-1)} - \frac{16d_1 \kappa \alpha_0^3}{27(x-2)^2} + \frac{208 \kappa \alpha_0^3}{27(x-2)^2} + \frac{17d_1 \kappa \alpha_0^3}{27(x-1)^2} - \frac{467\kappa \alpha_0^3}{108(x-1)^2} + \frac{152d_1 \kappa \alpha_0^3}{27(x-2)^3} - \frac{776\kappa \alpha_0^3}{27(x-2)^3} - \frac{d_1 \kappa \alpha_0^3}{3(x-1)^3} + \frac{83\kappa \alpha_0^3}{36(x-1)^3} + \\
& \frac{83\kappa \alpha_0^3}{18} - \frac{d_1 \alpha_0^3}{27(x-2)} - \frac{70\alpha_0^3}{27(x-2)} - \frac{41d_1 \alpha_0^3}{54(x-1)} + \frac{175\alpha_0^3}{54(x-1)} - \frac{16d_1 \alpha_0^3}{27(x-2)^2} + \frac{176\alpha_0^3}{27(x-2)^2} + \frac{17d_1 \alpha_0^3}{27(x-1)^2} - \frac{277\alpha_0^3}{108(x-1)^2} + \\
& \frac{152d_1 \alpha_0^3}{27(x-2)^3} - \frac{472\alpha_0^3}{27(x-2)^3} - \frac{d_1 \alpha_0^3}{3(x-1)^3} + \frac{5\alpha_0^3}{4(x-1)^3} + \frac{35\alpha_0^3}{18} + \frac{35d_1 \alpha_0^2}{9} - \frac{73}{18} d_1 x \alpha_0^2 + \frac{569x \alpha_0^2}{36} + \frac{35}{9} d_1 \kappa \alpha_0^2 - \\
& \frac{73}{18} d_1 x \kappa \alpha_0^2 + \frac{979}{36} x \kappa \alpha_0^2 + \frac{8d_1 \kappa \alpha_0^2}{9(x-2)} - \frac{32\kappa \alpha_0^2}{3(x-2)} + \frac{13d_1 \kappa \alpha_0^2}{18(x-1)} + \frac{4\kappa \alpha_0^2}{3(x-1)} - \frac{10d_1 \kappa \alpha_0^2}{3(x-2)^2} + \frac{20\kappa \alpha_0^2}{3(x-2)^2} - \frac{d_1 \kappa \alpha_0^2}{(x-1)^2} + \\
& \frac{67\kappa \alpha_0^2}{12(x-1)^2} + \frac{160d_1 \kappa \alpha_0^2}{9(x-2)^3} - \frac{160\kappa \alpha_0^2}{3(x-2)^3} + \frac{5}{3} \frac{d_1 \kappa \alpha_0^2}{(x-1)^3} - \frac{119\kappa \alpha_0^2}{12(x-1)^3} + \frac{208d_1 \kappa \alpha_0^2}{3(x-2)^4} - \frac{2512\kappa \alpha_0^2}{9(x-2)^4} - \frac{4d_1 \kappa \alpha_0^2}{3(x-1)^4} + \frac{137 \kappa \alpha_0^2}{18(x-1)^4} - \\
& \frac{64\kappa \alpha_0^2}{3} + \frac{8d_1 \alpha_0^2}{9(x-2)} - \frac{32\alpha_0^2}{9(x-2)} + \frac{13d_1 \alpha_0^2}{18(x-1)} - \frac{\alpha_0^2}{x-1} - \frac{10d_1 \alpha_0^2}{3(x-2)^2} + \frac{20 \alpha_0^2}{3(x-2)^2} - \frac{d_1 \alpha_0^2}{(x-1)^2} + \frac{119\alpha_0^2}{36(x-1)^2} + \frac{160d_1 \alpha_0^2}{9(x-2)^3} - \\
& \frac{160\alpha_0^2}{9(x-2)^3} + \frac{5d_1 \alpha_0^2}{3(x-1)^3} - \frac{19\alpha_0^2}{4(x-1)^3} + \frac{208}{3} \frac{d_1 \alpha_0^2}{(x-2)^4} - \frac{1264\alpha_0^2}{9(x-2)^4} - \frac{4d_1 \alpha_0^2}{3(x-1)^4} + \frac{7\alpha_0^2}{2(x-1)^4} - \frac{85 \alpha_0^2}{9} - \frac{13d_1 \alpha_0}{27} + \frac{805d_1 x \alpha_0}{216} - \\
& \frac{1}{18} \pi^2 x \alpha_0 - \frac{271x\alpha_0}{24} - \frac{13d_1 \kappa \alpha_0}{27} + \frac{805}{216} d_1 x \kappa \alpha_0 - \frac{5131x \kappa \alpha_0}{216} + \frac{380d_1 \kappa \alpha_0}{27(x-2)} - \frac{1208 \kappa \alpha_0}{27(x-2)} - \frac{379d_1 \kappa \alpha_0}{24(x-1)} + \frac{4009\kappa \alpha_0}{72(x-1)} - \\
& \frac{332d_1 \kappa \alpha_0}{27(x-2)^2} + \frac{1136\kappa \alpha_0}{27(x-2)^2} + \frac{205d_1 \kappa \alpha_0}{108(x-1)^2} - \frac{713\kappa \alpha_0}{54(x-1)^2} - \frac{160d_1 \kappa \alpha_0}{9(x-2)^3} + \frac{80\kappa \alpha_0}{3(x-2)^3} - \frac{395d_1 \kappa \alpha_0}{216(x-1)^3} + \frac{2399 \kappa \alpha_0}{216(x-1)^3} - \\
& \frac{224d_1 \kappa \alpha_0}{(x-2)^4} + \frac{928\kappa \alpha_0}{(x-2)^4} - \frac{331d_1 \kappa \alpha_0}{216(x-1)^4} + \frac{475\kappa \alpha_0}{54(x-1)^4} - \frac{512d_1 \kappa \alpha_0}{3(x-2)^5} + \frac{6656\kappa \alpha_0}{9(x-2)^5} + \frac{205}{216} \frac{d_1 \kappa \alpha_0}{(x-1)^5} - \frac{1255\kappa \alpha_0}{216(x-1)^5} + \frac{203\kappa \alpha_0}{216} + \\
& \frac{380d_1 \alpha_0}{27(x-2)} - \frac{616\alpha_0}{27(x-2)} - \frac{379d_1 \alpha_0}{24(x-1)} + \frac{671\alpha_0}{24(x-1)} - \frac{332d_1 \alpha_0}{27(x-2)^2} + \frac{592\alpha_0}{27(x-2)^2} + \frac{205d_1 \alpha_0}{108(x-1)^2} - \frac{361\alpha_0}{54(x-1)^2} - \frac{160d_1 \alpha_0}{9(x-2)^3} + \\
& \frac{80\alpha_0}{9(x-2)^3} - \frac{395d_1 \alpha_0}{216(x-1)^3} + \frac{1123\alpha_0}{216(x-1)^3} - \frac{1760d_1 \alpha_0}{9(x-2)^4} - \frac{40\pi^2 \alpha_0}{9(x-2)^4} + \frac{17120\alpha_0}{27(x-2)^4} - \frac{331d_1 \alpha_0}{216(x-1)^4} - \frac{\pi^2 \alpha_0}{18(x-1)^4} + \\
& \frac{152\alpha_0}{27(x-1)^4} - \frac{1024d_1 \alpha_0}{9(x-2)^5} - \frac{80\pi^2 \alpha_0}{9(x-2)^5} + \frac{19072\alpha_0}{27(x-2)^5} + \frac{205d_1 \alpha_0}{216(x-1)^5} + \frac{\pi^2 \alpha_0}{18(x-1)^5} - \frac{955\alpha_0}{216(x-1)^5} + \frac{\pi^2 \alpha_0}{9} - \frac{85 \alpha_0}{24} - \frac{3d_1}{4} - \\
& \frac{205d_1 x}{216} + \frac{\pi^2 x}{18} + \frac{35x}{24} - \frac{3d_1 \kappa}{4} - \frac{205d_1 x \kappa}{216} + \frac{1255x \kappa}{216} + \frac{346d_1 \kappa}{27(x-2)} - \frac{952\kappa}{27(x-2)} - \frac{947d_1 \kappa}{72(x-1)} + \frac{2621\kappa}{72(x-1)} - \frac{392d_1 \kappa}{27(x-2)^2} + \\
& \frac{1136\kappa}{27(x-2)^2} + \frac{23d_1 \kappa}{27(x-1)^2} - \frac{92\kappa}{27(x-1)^2} + \frac{784d_1 \kappa}{27(x-2)^3} - \frac{2272\kappa}{27(x-2)^3} + \frac{73d_1 \kappa}{216(x-1)^3} - \frac{247\kappa}{216(x-1)^3} + \frac{256d_1 \kappa}{3(x-2)^4} - \\
& \frac{3328\kappa}{9(x-2)^4} + \frac{367d_1 \kappa}{216(x-1)^4} - \frac{1127\kappa}{108(x-1)^4} + \frac{512d_1 \kappa}{3(x-2)^5} - \frac{2048\kappa}{3(x-2)^5} + \frac{205d_1 \kappa}{216(x-1)^5} - \frac{1255\kappa}{216(x-1)^5} + \frac{1024\kappa}{9(x-2)^6} + \frac{8\kappa}{37} + \\
& \frac{346d_1}{27(x-2)} - \frac{488}{27(x-2)} - \frac{947d_1}{72(x-1)} + \frac{443}{24(x-1)} - \frac{392}{27} \frac{d_1}{(x-2)^2} + \frac{592}{27} \frac{d_1}{(x-2)^2} + \frac{23d_1}{27(x-1)^2} - \frac{52}{27} \frac{d_1}{(x-1)^2} + \frac{784d_1}{27(x-2)^3} - \\
& \frac{1184}{27(x-2)^3} + \frac{73d_1}{216(x-1)^3} - \frac{83}{216(x-1)^3} + \frac{512d_1}{9(x-2)^4} + \frac{40 \pi^2}{9(x-2)^4} - \frac{9536}{27(x-2)^4} + \frac{367d_1}{216(x-1)^4} + \frac{\pi^2}{18(x-1)^4} - \\
& \frac{725}{108(x-1)^4} + \frac{512}{9} \frac{d_1}{(x-2)^5} + \frac{160\pi^2}{9(x-2)^5} - \frac{25856}{27(x-2)^5} + \frac{205d_1}{216(x-1)^5} + \frac{\pi^2}{18(x-1)^5} - \frac{955}{216(x-1)^5} - \frac{1024d_1}{9(x-2)^6} + \frac{160 \pi^2}{9(x-2)^6} - \\
& \frac{13568}{27(x-2)^6} + \frac{55}{24} \Big) H(0; \alpha_0) + \left( -\frac{d_1 \alpha_0^5}{12} + \frac{1}{12} d_1^2 x \alpha_0^5 - \frac{4}{9} d_1 x \alpha_0^5 - \frac{1}{12} d_1 x \kappa \alpha_0^5 - \frac{d_1 \kappa \alpha_0^5}{6(x-2)} + \frac{d_1 \kappa \alpha_0^5}{12(x-1)} + \right. \\
& \frac{d_1^2 \alpha_0^5}{6(x-2)} - \frac{19d_1 \alpha_0^5}{18(x-2)} - \frac{d_1^2 \alpha_0^5}{12(x-1)} + \frac{4d_1 \alpha_0^5}{9(x-1)} + \frac{7d_1^2 \alpha_0^4}{54} - \frac{d_1 \alpha_0^4}{108} - \frac{61}{108} d_1^2 x \alpha_0^4 + \frac{157}{54} d_1 x \alpha_0^4 - \frac{5}{27} d_1 \kappa \alpha_0^4 + \\
& \frac{67}{108} d_1 x \kappa \alpha_0^4 + \frac{11d_1 \kappa \alpha_0^4}{27(x-2)} - \frac{43d_1 \kappa \alpha_0^4}{108(x-1)} - \frac{23d_1 \kappa \alpha_0^4}{27(x-2)^2} + \frac{11d_1 \kappa \alpha_0^4}{54(x-1)^2} - \frac{11d_1^2 \alpha_0^4}{27(x-2)} + \frac{91d_1 \alpha_0^4}{27(x-2)} + \frac{43d_1^2 \alpha_0^4}{108(x-1)} - \frac{53d_1 \alpha_0^4}{27(x-1)} + \\
& \frac{23d_1^2 \alpha_0^4}{27(x-2)^2} - \frac{103d_1 \alpha_0^4}{27(x-2)^2} - \frac{4d_1^2 \alpha_0^4}{27(x-1)^2} + \frac{37d_1 \alpha_0^4}{54(x-1)^2} - \frac{8d_1^2 \alpha_0^3}{9} + \frac{35d_1 \alpha_0^3}{18} + \frac{95}{54} d_1^2 x \alpha_0^3 - \frac{911}{108} d_1 x \alpha_0^3 + \frac{4}{3} d_1 \kappa \alpha_0^3 - \\
& \frac{235}{108} d_1 x \kappa \alpha_0^3 + \frac{10d_1 \kappa \alpha_0^3}{27(x-2)} + \frac{16d_1 \kappa \alpha_0^3}{27(x-1)} + \frac{16d_1 \kappa \alpha_0^3}{27(x-2)^2} - \frac{95d_1 \kappa \alpha_0^3}{108(x-1)^2} - \frac{152d_1 \kappa \alpha_0^3}{27(x-2)^3} + \frac{19d_1 \kappa \alpha_0^3}{36(x-1)^3} - \frac{d_1^2 \alpha_0^3}{27(x-2)} - \\
& \frac{70d_1 \alpha_0^3}{27(x-2)} - \frac{41d_1^2 \alpha_0^3}{54(x-1)} + \frac{175d_1 \alpha_0^3}{54(x-1)} - \frac{16d_1^2 \alpha_0^3}{27(x-2)^2} + \frac{176d_1 \alpha_0^3}{27(x-2)^2} + \frac{17d_1^2 \alpha_0^3}{27(x-1)^2} - \frac{277d_1 \alpha_0^3}{108(x-1)^2} + \frac{152d_1^2 \alpha_0^3}{27(x-2)^3} - \frac{472d_1 \alpha_0^3}{27(x-2)^3} - \\
& \frac{d_1^2 \alpha_0^3}{3(x-1)^3} + \frac{5d_1 \alpha_0^3}{4(x-1)^3} + \frac{35d_1^2 \alpha_0^2}{9} - \frac{85d_1 \alpha_0^2}{9} - \frac{73}{18} d_1^2 x \alpha_0^2 + \frac{569}{36} d_1 x \alpha_0^2 - \frac{107}{18} d_1 \kappa \alpha_0^2 + \frac{205}{36} d_1 x \kappa \alpha_0^2 -
\end{aligned}$$

$$\begin{aligned}
& \frac{32d_1\kappa\alpha_0^2}{9(x-2)} + \frac{7d_1\kappa\alpha_0^2}{6(x-1)} + \frac{20d_1\kappa\alpha_0^2}{3(x-2)^2} + \frac{41d_1\kappa\alpha_0^2}{36(x-1)^2} - \frac{160d_1\kappa\alpha_0^2}{9(x-2)^3} - \frac{31d_1\kappa\alpha_0^2}{12(x-1)^3} - \frac{208d_1\kappa\alpha_0^2}{3(x-2)^4} + \frac{37d_1\kappa\alpha_0^2}{18(x-1)^4} + \frac{8d_1^2\alpha_0^2}{9(x-2)} - \\
& \frac{32d_1\alpha_0^2}{9(x-2)} + \frac{13d_1^2\alpha_0^2}{18(x-1)} - \frac{d_1\alpha_0^2}{x-1} - \frac{10d_1^2\alpha_0^2}{3(x-2)^2} + \frac{20d_1\alpha_0^2}{3(x-2)^2} - \frac{d_1^2\alpha_0^2}{(x-1)^2} + \frac{119d_1\alpha_0^2}{36(x-1)^2} + \frac{160d_1^2\alpha_0^2}{9(x-2)^3} - \frac{160d_1\alpha_0^2}{9(x-2)^3} + \frac{5d_1^2\alpha_0^2}{3(x-1)^3} - \\
& \frac{19d_1\alpha_0^2}{4(x-1)^3} + \frac{208d_1^2\alpha_0^2}{3(x-2)^4} - \frac{1264d_1\alpha_0^2}{9(x-2)^4} - \frac{4d_1^2\alpha_0^2}{3(x-1)^4} + \frac{7d_1\alpha_0^2}{2(x-1)^4} - \frac{80d_1^2\alpha_0}{27} + \frac{703d_1\alpha_0}{108} + \frac{505}{108}d_1^2x\alpha_0 - \frac{1697d_1x\alpha_0}{108} + \\
& \frac{122d_1\kappa\alpha_0}{27} - \frac{187}{27}d_1x\kappa\alpha_0 + \frac{125d_1\kappa\alpha_0}{54(x-2)} - \frac{5d_1\kappa\alpha_0}{108(x-1)} - \frac{196d_1\kappa\alpha_0}{27(x-2)^2} - \frac{133d_1\kappa\alpha_0}{108(x-1)^2} + \frac{392d_1\kappa\alpha_0}{9(x-2)^3} + \frac{43d_1\kappa\alpha_0}{12(x-1)^3} + \\
& \frac{224d_1\kappa\alpha_0}{(x-2)^4} + \frac{512d_1\kappa\alpha_0}{3(x-2)^5} - \frac{47d_1^2\alpha_0}{54(x-2)} + \frac{205d_1\alpha_0}{54(x-2)} - \frac{37d_1^2\alpha_0}{108(x-1)} + \frac{43d_1\alpha_0}{54(x-1)} + \frac{136d_1^2\alpha_0}{27(x-2)^2} - \frac{356d_1\alpha_0}{27(x-2)^2} + \frac{19d_1^2\alpha_0}{27(x-1)^2} - \\
& \frac{263d_1\alpha_0}{108(x-1)^2} - \frac{472d_1^2\alpha_0}{9(x-2)^3} + \frac{712d_1\alpha_0}{9(x-2)^3} - \frac{7d_1^2\alpha_0}{3(x-1)^3} + \frac{23d_1\alpha_0}{4(x-1)^3} - \frac{224d_1^2\alpha_0}{(x-2)^4} + \frac{4576d_1\alpha_0}{9(x-2)^4} - \frac{512d_1^2\alpha_0}{3(x-2)^5} + \frac{4096d_1\alpha_0}{9(x-2)^5} - \\
& \frac{d_1^2}{6} + \frac{13d_1}{12} - \frac{205d_1^2}{108}x + \frac{635d_1x}{108} + \frac{5d_1\kappa}{18} + \frac{155d_1x\kappa}{54} + \frac{17d_1\kappa}{27(x-2)} - \frac{151d_1\kappa}{108(x-1)} + \frac{23d_1\kappa}{27(x-2)^2} + \frac{83d_1\kappa}{108(x-1)^2} - \\
& \frac{544d_1\kappa}{27(x-2)^3} - \frac{55d_1\kappa}{36(x-1)^3} - \frac{464d_1\kappa}{3(x-2)^4} - \frac{37d_1\kappa}{18(x-1)^4} - \frac{512d_1\kappa}{3(x-2)^5} + \frac{7d_1^2}{27(x-2)} + \frac{d_1}{27(x-2)} + \frac{7d_1^2}{108(x-1)} - \frac{41d_1}{27(x-1)} - \\
& \frac{53d_1^2}{27(x-2)^2} + \frac{103d_1}{27(x-2)^2} - \frac{5d_1^2}{27(x-1)^2} + \frac{109d_1}{108(x-1)^2} + \frac{784d_1^2}{27(x-2)^3} - \frac{1184d_1}{27(x-2)^3} + \frac{d_1^2}{(x-1)^3} - \frac{9d_1}{4(x-1)^3} + \frac{464d_1^2}{3(x-2)^4} - \\
& \frac{368d_1}{(x-2)^4} + \frac{4d_1^2}{3(x-1)^4} - \frac{7d_1}{2(x-1)^4} + \frac{512d_1^2}{3(x-2)^5} - \frac{4096d_1}{9(x-2)^5} \Big) H(1; \alpha_0) + \left( \frac{x\alpha_0^5}{3} + x\kappa\alpha_0^5 + \frac{2\kappa\alpha_0^5}{x-2} - \frac{\kappa\alpha_0^5}{x-1} + \frac{2\alpha_0^5}{3(x-2)} - \right. \\
& \left. \frac{\alpha_0^5}{3(x-1)} - \frac{19x\alpha_0^4}{9} - \frac{19}{3}x\kappa\alpha_0^4 - \frac{20\kappa\alpha_0^4}{3(x-2)} + \frac{13\kappa\alpha_0^4}{3(x-1)} + \frac{20\kappa\alpha_0^4}{3(x-2)^2} - \frac{4\kappa\alpha_0^4}{3(x-1)^2} + \frac{2\kappa\alpha_0^4}{3} - \frac{20\alpha_0^4}{9(x-2)} + \frac{13\alpha_0^4}{9(x-1)} + \right. \\
& \left. \frac{20\alpha_0^4}{9(x-2)^2} - \frac{4\alpha_0^4}{9(x-1)^2} + \frac{2\alpha_0^4}{9} + \frac{52x\alpha_0^3}{9} + \frac{52}{3}x\kappa\alpha_0^3 + \frac{20\kappa\alpha_0^3}{3(x-2)} - \frac{22\kappa\alpha_0^3}{3(x-1)} - \frac{40\kappa\alpha_0^3}{3(x-2)^2} + \frac{14\kappa\alpha_0^3}{3(x-1)^2} + \frac{80\kappa\alpha_0^3}{3(x-2)^3} - \right. \\
& \left. \frac{2\kappa\alpha_0^3}{(x-1)^3} - 4\kappa\alpha_0^3 + \frac{20\alpha_0^3}{9(x-2)} - \frac{22\alpha_0^3}{9(x-1)} - \frac{40\alpha_0^3}{9(x-2)^2} + \frac{14\alpha_0^3}{9(x-1)^2} + \frac{80\alpha_0^3}{9(x-2)^3} - \frac{2\alpha_0^3}{3(x-1)^3} - \frac{4\alpha_0^3}{3} - \frac{28x\alpha_0^2}{3} - \right. \\
& \left. 28x\kappa\alpha_0^2 + \frac{6\kappa\alpha_0^2}{x-1} - \frac{6\kappa\alpha_0^2}{(x-1)^2} + \frac{6\kappa\alpha_0^2}{(x-1)^3} + \frac{160\kappa\alpha_0^2}{(x-2)^4} - \frac{4\kappa\alpha_0^2}{(x-1)^4} + 12\kappa\alpha_0^2 + \frac{2\alpha_0^2}{x-1} - \frac{2\alpha_0^2}{(x-1)^2} + \frac{2\alpha_0^2}{(x-1)^3} + \frac{160\alpha_0^2}{3(x-2)^4} - \right. \\
& \left. \frac{4\alpha_0^2}{3(x-1)^4} + 4\alpha_0^2 + \frac{95x\alpha_0}{18} + \frac{337x\kappa\alpha_0}{18} + \frac{80\kappa\alpha_0}{3(x-2)} - \frac{65\kappa\alpha_0}{2(x-1)} - \frac{80\kappa\alpha_0}{3(x-2)^2} + \frac{25\kappa\alpha_0}{3(x-1)^2} - \frac{35\kappa\alpha_0}{6(x-1)^3} - \frac{64d_1\kappa\alpha_0}{3(x-2)^4} - \right. \\
& \left. \frac{6464\kappa\alpha_0}{9(x-2)^4} - \frac{119\kappa\alpha_0}{18(x-1)^4} - \frac{128d_1\kappa\alpha_0}{3(x-2)^5} - \frac{7168\kappa\alpha_0}{9(x-2)^5} + \frac{101\kappa\alpha_0}{18(x-1)^5} + \frac{50\kappa\alpha_0}{9} + \frac{80\alpha_0}{9(x-2)} - \frac{65\alpha_0}{6(x-1)} - \frac{80\alpha_0}{9(x-2)^2} + \right. \\
& \left. \frac{25\alpha_0}{9(x-1)^2} - \frac{35\alpha_0}{18(x-1)^3} - \frac{64d_1\alpha_0}{3(x-2)^4} - \frac{3008\alpha_0}{9(x-2)^4} - \frac{19\alpha_0}{6(x-1)^4} - \frac{128d_1\alpha_0}{3(x-2)^5} - \frac{4096\alpha_0}{9(x-2)^5} + \frac{17\alpha_0}{6(x-1)^5} + \frac{34\alpha_0}{9} + \frac{x}{18} - \right. \\
& \left. \frac{49x\kappa}{18} + \frac{64\kappa}{3(x-2)} - \frac{43\kappa}{2(x-1)} - \frac{80\kappa}{3(x-2)^2} + \frac{8\kappa}{3(x-1)^2} + \frac{160\kappa}{3(x-2)^3} + \frac{\kappa}{6(x-1)^3} + \frac{64d_1\kappa}{3(x-2)^4} + \frac{3584\kappa}{9(x-2)^4} + \frac{155\kappa}{18(x-1)^4} + \right. \\
& \left. \frac{256d_1\kappa}{3(x-2)^5} + \frac{8576\kappa}{9(x-2)^5} + \frac{101\kappa}{18(x-1)^5} + \frac{256d_1\kappa}{3(x-2)^6} + \frac{2816\kappa}{9(x-2)^6} - 3\kappa + \frac{64}{9(x-2)} - \frac{43}{6(x-1)} - \frac{9(x-2)^2}{9(x-1)^2} + \frac{8}{9(x-1)^2} + \right. \\
& \left. \frac{160}{9(x-2)^3} + \frac{1}{18(x-1)^3} + \frac{64d_1}{3(x-2)^4} + \frac{2048}{9(x-2)^4} + \frac{23}{6(x-1)^4} + \frac{256d_1}{3(x-2)^5} + \frac{6272}{9(x-2)^5} + \frac{17}{6(x-1)^5} + \frac{256d_1}{3(x-2)^6} + \right. \\
& \left. \frac{4352}{9(x-2)^6} - 1 \Big) H(0, 0; \alpha_0) + \left( \frac{1}{3}d_1x\alpha_0^5 + \frac{1}{3}d_1x\kappa\alpha_0^5 + \frac{2d_1\kappa\alpha_0^5}{3(x-2)} - \frac{d_1\kappa\alpha_0^5}{3(x-1)} + \frac{2d_1\alpha_0^5}{3(x-2)} - \frac{d_1\alpha_0^5}{3(x-1)} + \frac{2d_1\alpha_0^4}{9} - \right. \\
& \left. \frac{19}{9}d_1x\alpha_0^4 + \frac{2}{9}d_1\kappa\alpha_0^4 - \frac{19}{9}d_1x\kappa\alpha_0^4 - \frac{20d_1\kappa\alpha_0^4}{9(x-2)} + \frac{13d_1\kappa\alpha_0^4}{9(x-1)} + \frac{20d_1\kappa\alpha_0^4}{9(x-2)^2} - \frac{4d_1\kappa\alpha_0^4}{9(x-1)^2} - \frac{20d_1\alpha_0^4}{9(x-2)} + \frac{13d_1\alpha_0^4}{9(x-1)} + \right. \\
& \left. \frac{20d_1\alpha_0^4}{9(x-2)^2} - \frac{4d_1\alpha_0^4}{9(x-1)^2} - \frac{4d_1\alpha_0^3}{3} + \frac{52}{9}d_1x\alpha_0^3 - \frac{4}{3}d_1\kappa\alpha_0^3 + \frac{52}{9}d_1x\kappa\alpha_0^3 + \frac{20d_1\kappa\alpha_0^3}{9(x-2)} - \frac{22d_1\kappa\alpha_0^3}{9(x-1)} - \frac{40d_1\kappa\alpha_0^3}{9(x-2)^2} + \right. \\
& \left. \frac{14d_1\kappa\alpha_0^3}{9(x-1)^2} + \frac{80d_1\kappa\alpha_0^3}{9(x-2)^3} - \frac{2d_1\kappa\alpha_0^3}{3(x-1)^3} + \frac{20d_1\alpha_0^3}{9(x-2)} - \frac{22d_1\alpha_0^3}{9(x-1)} - \frac{40d_1\alpha_0^3}{9(x-2)^2} + \frac{14d_1\alpha_0^3}{9(x-1)^2} + \frac{80d_1\alpha_0^3}{9(x-2)^3} - \frac{2d_1\alpha_0^3}{3(x-1)^3} + 4d_1\alpha_0^2 - \right. \\
& \left. \frac{28}{3}d_1x\alpha_0^2 + 4d_1\kappa\alpha_0^2 - \frac{28}{3}d_1x\kappa\alpha_0^2 + \frac{2d_1\kappa\alpha_0^2}{x-1} - \frac{2d_1\kappa\alpha_0^2}{(x-1)^2} + \frac{2d_1\kappa\alpha_0^2}{(x-1)^3} + \frac{160d_1\kappa\alpha_0^2}{3(x-2)^4} - \frac{4d_1\kappa\alpha_0^2}{3(x-1)^4} + \frac{2d_1\alpha_0^2}{x-1} - \right. \\
& \left. \frac{2d_1\alpha_0^2}{(x-1)^2} + \frac{2d_1\alpha_0^2}{(x-1)^3} + \frac{160d_1\alpha_0^2}{3(x-2)^4} - \frac{4d_1\alpha_0^2}{3(x-1)^4} + \frac{34d_1\alpha_0}{9} + \frac{95d_1x\alpha_0}{18} + \frac{8d_1\kappa\alpha_0}{9} + \frac{121}{18}d_1x\kappa\alpha_0 + \frac{80d_1\kappa\alpha_0}{9(x-2)} - \frac{65d_1\kappa\alpha_0}{6(x-1)} - \right. \\
& \left. \frac{80d_1\kappa\alpha_0}{9(x-2)^2} + \frac{25d_1\kappa\alpha_0}{9(x-1)^2} - \frac{35d_1\kappa\alpha_0}{18(x-1)^3} - \frac{192d_1\kappa\alpha_0}{(x-2)^4} - \frac{31d_1\kappa\alpha_0}{18(x-1)^4} - \frac{512d_1\kappa\alpha_0}{3(x-2)^5} + \frac{25d_1\kappa\alpha_0}{18(x-1)^5} + \frac{80d_1\alpha_0}{9(x-2)} - \frac{65d_1\alpha_0}{6(x-1)} - \right. \\
& \left. \frac{80d_1\alpha_0}{9(x-2)^2} + \frac{25d_1\alpha_0}{9(x-1)^2} - \frac{35d_1\alpha_0}{18(x-1)^3} - \frac{64d_1^2\alpha_0}{3(x-2)^4} - \frac{3008d_1\alpha_0}{9(x-2)^4} - \frac{19d_1\alpha_0}{6(x-1)^4} - \frac{128d_1^2\alpha_0}{3(x-2)^5} - \frac{4096d_1\alpha_0}{9(x-2)^5} + \frac{17d_1\alpha_0}{6(x-1)^5} - \right. \\
& \left. d_1 + \frac{d_1x}{18} - d_1\kappa - \frac{25d_1x\kappa}{18} + \frac{64d_1\kappa}{9(x-2)} - \frac{43d_1\kappa}{6(x-1)} - \frac{80d_1\kappa}{9(x-2)^2} + \frac{8d_1\kappa}{9(x-1)^2} + \frac{160d_1\kappa}{9(x-2)^3} + \frac{d_1\kappa}{18(x-1)^3} + \frac{256d_1\kappa}{3(x-2)^4} + \right. \\
& \left. \frac{43d_1\kappa}{18(x-1)^4} + \frac{128d_1\kappa}{(x-2)^5} + \frac{25d_1\kappa}{18(x-1)^5} - \frac{256d_1\kappa}{3(x-2)^6} + \frac{64d_1}{9(x-2)} - \frac{43d_1}{6(x-1)} - \frac{80d_1}{9(x-2)^2} + \frac{8d_1}{9(x-1)^2} + \frac{160d_1}{9(x-2)^3} + \frac{d_1}{18(x-1)^3} + \frac{256d_1}{3(x-2)^4} + \right. \\
& \left. \frac{64d_1^2}{3(x-2)^4} + \frac{2048d_1}{9(x-2)^4} + \frac{23d_1}{6(x-1)^4} + \frac{256d_1^2}{3(x-2)^5} + \frac{6272d_1}{9(x-2)^5} + \frac{17d_1}{6(x-1)^5} + \frac{256d_1^2}{3(x-2)^6} + \frac{4352d_1}{9(x-2)^6} \Big) H(0, 1; \alpha_0) + \right. \\
& \left. H(1; x) \left( \frac{1}{18}\pi^2x\alpha_0 + \frac{1}{6}\pi^2x\kappa\alpha_0 + \frac{40\pi^2\kappa\alpha_0}{3(x-2)^4} + \frac{d_1\pi^2\kappa\alpha_0}{9(x-1)^4} - \frac{\pi^2\kappa\alpha_0}{6(x-1)^4} + \frac{80\pi^2\kappa\alpha_0}{3(x-2)^5} - \frac{d_1\pi^2\kappa\alpha_0}{9(x-1)^5} + \frac{\pi^2\kappa\alpha_0}{6(x-1)^5} - \right. \right. \\
& \left. \left. \frac{1}{3}\pi^2\kappa\alpha_0 + \frac{40\pi^2\alpha_0}{9(x-2)^4} + \frac{d_1\pi^2\alpha_0}{9(x-1)^4} - \frac{\pi^2\alpha_0}{18(x-1)^4} + \frac{80\pi^2\alpha_0}{9(x-2)^5} - \frac{d_1\pi^2\alpha_0}{9(x-1)^5} + \frac{\pi^2\alpha_0}{18(x-1)^5} - \frac{\pi^2x}{9} - \frac{\pi^2x}{18} - \frac{1}{6}\pi^2x\kappa - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{40\pi^2\kappa}{3(x-2)^4} - \frac{d_1\pi^2\kappa}{9(x-1)^4} + \frac{\pi^2\kappa}{6(x-1)^4} - \frac{160\pi^2\kappa}{3(x-2)^5} - \frac{d_1\pi^2\kappa}{9(x-1)^5} + \frac{\pi^2\kappa}{6(x-1)^5} - \frac{160\pi^2\kappa}{3(x-2)^6} + \left( -\frac{2d_1\alpha_0}{3} + \frac{17x\alpha_0}{12} - \frac{2d_1\kappa\alpha_0}{3} + \right. \\
& \frac{101x\kappa\alpha_0}{36} - \frac{85d_1\kappa\alpha_0}{9(x-2)} + \frac{40\kappa\alpha_0}{3(x-2)} + \frac{94d_1\kappa\alpha_0}{9(x-1)} - \frac{65\kappa\alpha_0}{4(x-1)} + \frac{100d_1\kappa\alpha_0}{9(x-2)^2} - \frac{40\kappa\alpha_0}{3(x-2)^2} - \frac{20d_1\kappa\alpha_0}{9(x-1)^2} + \frac{25\kappa\alpha_0}{6(x-1)^2} - \frac{40d_1\kappa\alpha_0}{3(x-2)^3} + \\
& \frac{17d_1\kappa\alpha_0}{18(x-1)^3} - \frac{35\kappa\alpha_0}{12(x-1)^3} + \frac{64d_1\kappa\alpha_0}{(x-2)^4} - \frac{1088\kappa\alpha_0}{9(x-2)^4} + \frac{19d_1\kappa\alpha_0}{18(x-1)^4} - \frac{47\kappa\alpha_0}{36(x-1)^4} + \frac{64d_1\kappa\alpha_0}{3(x-2)^5} + \frac{704\kappa\alpha_0}{9(x-2)^5} - \frac{25d_1\kappa\alpha_0}{18(x-1)^5} + \\
& \frac{101\kappa\alpha_0}{36(x-1)^5} - \frac{37\kappa\alpha_0}{9} - \frac{85d_1\alpha_0}{9(x-2)} + \frac{40\alpha_0}{9(x-2)} + \frac{94d_1\alpha_0}{9(x-1)} - \frac{65\alpha_0}{12(x-1)} + \frac{100d_1\alpha_0}{9(x-2)^2} - \frac{40\alpha_0}{9(x-2)^2} - \frac{20d_1\alpha_0}{9(x-1)^2} + \frac{25\alpha_0}{18(x-1)^2} - \\
& \frac{40d_1\alpha_0}{3(x-2)^3} + \frac{17d_1\alpha_0}{18(x-1)^3} - \frac{35\alpha_0}{36(x-1)^3} + \frac{64d_1\alpha_0}{(x-2)^4} + \frac{64\alpha_0}{9(x-2)^4} + \frac{5d_1\alpha_0}{2(x-1)^4} - \frac{11\alpha_0}{12(x-1)^4} + \frac{64d_1\alpha_0}{3(x-2)^5} + \frac{1088\alpha_0}{9(x-2)^5} - \\
& \frac{17d_1\alpha_0}{6(x-1)^5} + \frac{17\alpha_0}{12(x-1)^5} - \frac{7\alpha_0}{3} + \frac{2d_1}{3} - \frac{17x}{12} + \frac{2d_1\kappa}{3} - \frac{101x\kappa}{36} - \frac{62d_1\kappa}{9(x-2)} + \frac{32\kappa}{3(x-2)} + \frac{65d_1\kappa}{9(x-1)} - \frac{43\kappa}{4(x-1)} + \\
& \frac{70d_1\kappa}{9(x-2)^2} - \frac{40\kappa}{3(x-1)^2} - \frac{d_1\kappa}{(x-1)^2} + \frac{4\kappa}{3(x-1)^2} - \frac{80d_1\kappa}{9(x-2)^3} + \frac{80\kappa}{3(x-2)^3} + \frac{5d_1\kappa}{18(x-1)^3} + \frac{\kappa}{12(x-1)^3} - \frac{112d_1\kappa}{3(x-2)^4} + \\
& \frac{1088\kappa}{9(x-2)^4} - \frac{31d_1\kappa}{18(x-1)^4} + \frac{155\kappa}{36(x-1)^4} - \frac{448d_1\kappa}{3(x-2)^5} + \frac{1472\kappa}{9(x-2)^5} - \frac{25d_1\kappa}{18(x-1)^5} + \frac{101\kappa}{36(x-1)^5} - \frac{128d_1\kappa}{3(x-2)^6} - \frac{1408\kappa}{9(x-2)^6} - \\
& \frac{3\kappa}{2} - \frac{62d_1}{9(x-2)} + \frac{32}{9(x-2)} + \frac{65d_1}{9(x-1)} - \frac{43}{12(x-1)} + \frac{70d_1}{9(x-2)^2} - \frac{40}{9(x-2)^2} - \frac{d_1}{(x-1)^2} + \frac{4}{9(x-1)^2} - \frac{80d_1}{9(x-2)^3} + \\
& \frac{80}{9(x-2)^3} + \frac{5d_1}{18(x-1)^3} + \frac{1}{36(x-1)^3} - \frac{112d_1}{3(x-2)^4} - \frac{64}{9(x-2)^4} - \frac{19d_1}{6(x-1)^4} + \frac{23}{12(x-1)^4} - \frac{448d_1}{3(x-2)^5} - \frac{1216}{9(x-2)^5} - \\
& \frac{17d_1}{6(x-1)^5} + \frac{17}{12(x-1)^5} - \frac{128d_1}{3(x-2)^6} - \frac{2176}{9(x-2)^6} - \frac{1}{2} \Big) H(0; \alpha_0) + \left( -\frac{2x\alpha_0}{3} - 2x\kappa\alpha_0 - \frac{160\kappa\alpha_0}{(x-2)^4} - \frac{4d_1\kappa\alpha_0}{3(x-1)^4} + \right. \\
& \frac{2\kappa\alpha_0}{(x-1)^4} - \frac{320\kappa\alpha_0}{(x-2)^5} + \frac{4d_1\kappa\alpha_0}{3(x-1)^5} - \frac{2\kappa\alpha_0}{(x-1)^5} + 4\kappa\alpha_0 - \frac{160\alpha_0}{3(x-2)^4} - \frac{4d_1\alpha_0}{3(x-1)^4} + \frac{2\alpha_0}{3(x-1)^4} - \frac{320\alpha_0}{3(x-2)^5} + \frac{4d_1\alpha_0}{3(x-1)^5} - \\
& \frac{2\alpha_0}{3(x-1)^5} + \frac{4\alpha_0}{3} + \frac{2x}{3} + 2x\kappa + \frac{160\kappa}{(x-2)^4} + \frac{4d_1\kappa}{3(x-1)^4} - \frac{2\kappa}{(x-1)^4} + \frac{640\kappa}{(x-2)^5} + \frac{4d_1\kappa}{3(x-1)^5} - \frac{2\kappa}{(x-1)^5} + \frac{640\kappa}{(x-2)^6} + \\
& \left. \frac{160}{3(x-2)^4} + \frac{4d_1}{3(x-1)^4} - \frac{2}{3(x-1)^4} + \frac{640}{3(x-2)^5} + \frac{4d_1}{3(x-1)^5} - \frac{2}{3(x-1)^5} + \frac{640}{3(x-2)^6} \right) H(0, 0; \alpha_0) + \left( -\frac{4\alpha_0 d_1^2}{3(x-1)^4} + \right. \\
& \frac{4d_1^2}{3(x-1)^4} + \frac{4\alpha_0 d_1^2}{3(x-1)^5} + \frac{4d_1^2}{3(x-1)^5} + \frac{4\alpha_0 d_1}{3} - \frac{2\alpha_0 x d_1}{3} + \frac{2x d_1}{3} + \frac{4\alpha_0 \kappa d_1}{3} - \frac{2}{3} \alpha_0 x \kappa d_1 + \frac{2x \kappa d_1}{3} - \frac{160\alpha_0 \kappa d_1}{3(x-2)^4} + \\
& \frac{160\kappa d_1}{3(x-2)^4} + \frac{2\alpha_0 \kappa d_1}{3(x-1)^4} - \frac{2\kappa d_1}{3(x-1)^4} - \frac{320\alpha_0 \kappa d_1}{3(x-2)^5} + \frac{640\kappa d_1}{3(x-2)^5} - \frac{2\alpha_0 \kappa d_1}{3(x-1)^5} - \frac{2\kappa d_1}{3(x-1)^5} + \frac{640\kappa d_1}{3(x-2)^6} - \frac{160\alpha_0 d_1}{3(x-2)^4} + \\
& \left. \frac{160d_1}{3(x-2)^4} + \frac{2\alpha_0 d_1}{3(x-1)^4} - \frac{2d_1}{3(x-1)^4} - \frac{320\alpha_0 d_1}{3(x-2)^5} + \frac{640d_1}{3(x-2)^5} - \frac{2\alpha_0 d_1}{3(x-1)^5} - \frac{2d_1}{3(x-1)^5} + \frac{640d_1}{3(x-2)^6} \right) H(0, 1; \alpha_0) - \\
& \frac{40\pi^2}{9(x-2)^4} - \frac{d_1\pi^2}{9(x-1)^4} + \frac{\pi^2}{18(x-1)^4} - \frac{160\pi^2}{9(x-2)^5} - \frac{d_1\pi^2}{9(x-1)^5} + \frac{\pi^2}{18(x-1)^5} - \frac{160\pi^2}{9(x-2)^6} + \left( -\frac{4d_1\alpha_0}{3} + \frac{2d_1x\alpha_0}{3} - \right. \\
& \frac{2x\alpha_0}{3} - \frac{4d_1\kappa\alpha_0}{3} + \frac{2}{3} d_1 x \kappa \alpha_0 - 2x\kappa\alpha_0 + \frac{160d_1\kappa\alpha_0}{3(x-2)^4} - \frac{160\kappa\alpha_0}{(x-2)^4} - \frac{2d_1\kappa\alpha_0}{3(x-1)^4} + \frac{2\kappa\alpha_0}{(x-1)^4} + \frac{320d_1\kappa\alpha_0}{3(x-2)^5} - \frac{320\kappa\alpha_0}{(x-2)^5} + \\
& \frac{2d_1\kappa\alpha_0}{3(x-1)^5} - \frac{2\kappa\alpha_0}{(x-1)^5} + 4\kappa\alpha_0 + \frac{160d_1\alpha_0}{3(x-2)^4} - \frac{160\alpha_0}{3(x-2)^4} - \frac{2d_1\alpha_0}{3(x-1)^4} + \frac{2\alpha_0}{3(x-1)^4} + \frac{320d_1\alpha_0}{3(x-2)^5} - \frac{320\alpha_0}{(x-2)^5} + \frac{2d_1\alpha_0}{3(x-1)^5} - \\
& \frac{2\alpha_0}{3(x-1)^5} + \frac{4\alpha_0}{3} - \frac{2d_1x}{3} + \frac{2x}{3} - \frac{2d_1x\kappa}{3} + 2x\kappa - \frac{160d_1\kappa}{3(x-2)^4} + \frac{160\kappa}{(x-2)^4} + \frac{2d_1\kappa}{3(x-1)^4} - \frac{2\kappa}{(x-1)^4} - \frac{640d_1\kappa}{3(x-2)^5} + \frac{640\kappa}{(x-2)^5} + \\
& \frac{2d_1\kappa}{3(x-1)^5} - \frac{2\kappa}{(x-1)^5} - \frac{640d_1\kappa}{3(x-2)^6} + \frac{640\kappa}{(x-2)^6} - \frac{160d_1}{3(x-2)^4} + \frac{160}{3(x-2)^4} + \frac{2d_1}{3(x-1)^4} - \frac{2d_1}{3(x-1)^4} - \frac{640d_1}{3(x-2)^5} + \frac{640}{3(x-2)^5} + \\
& \left. \frac{2d_1}{3(x-1)^5} - \frac{2}{3(x-1)^5} - \frac{640d_1}{3(x-2)^6} + \frac{640}{3(x-2)^6} \right) H(0; \alpha_0) H(0, 1; x) + \left( \frac{17x\alpha_0}{12} + \frac{101x\kappa\alpha_0}{36} + \frac{40\kappa\alpha_0}{3(x-2)} - \frac{65\kappa\alpha_0}{4(x-1)} - \right. \\
& \frac{40\kappa\alpha_0}{3(x-2)^2} + \frac{25\kappa\alpha_0}{6(x-1)^2} - \frac{35\kappa\alpha_0}{12(x-1)^3} + \frac{32d_1\kappa\alpha_0}{3(x-2)^4} - \frac{1088\kappa\alpha_0}{9(x-2)^4} - \frac{47\kappa\alpha_0}{36(x-1)^4} + \frac{64d_1\kappa\alpha_0}{3(x-2)^5} + \frac{704\kappa\alpha_0}{9(x-2)^5} + \frac{101\kappa\alpha_0}{36(x-1)^5} - \\
& \frac{37\kappa\alpha_0}{9} + \frac{40\alpha_0}{9(x-2)} - \frac{65\alpha_0}{12(x-1)} - \frac{40\alpha_0}{9(x-2)^2} + \frac{25\alpha_0}{18(x-1)^2} - \frac{35\alpha_0}{36(x-1)^3} + \frac{32d_1\alpha_0}{3(x-2)^4} + \frac{64\alpha_0}{9(x-2)^4} - \frac{11\alpha_0}{12(x-1)^4} + \frac{64d_1\alpha_0}{3(x-2)^5} + \\
& \frac{1088\alpha_0}{9(x-2)^5} + \frac{17\alpha_0}{12(x-1)^5} - \frac{7\alpha_0}{3} - \frac{17x}{12} - \frac{101x\kappa}{36} + \frac{32\kappa}{3(x-2)} - \frac{43\kappa}{4(x-1)} - \frac{40\kappa}{3(x-2)^2} + \frac{4\kappa}{3(x-1)^2} + \frac{80\kappa}{3(x-2)^3} + \frac{\kappa}{12(x-1)^3} - \\
& \frac{32d_1\kappa}{3(x-2)^4} + \frac{1088\kappa}{9(x-2)^4} + \frac{155\kappa}{36(x-1)^4} - \frac{128d_1\kappa}{3(x-2)^5} + \frac{1472\kappa}{9(x-2)^5} + \frac{101\kappa}{36(x-1)^5} - \frac{128d_1\kappa}{3(x-2)^6} - \frac{1408\kappa}{9(x-2)^6} - \frac{3\kappa}{2} + \left( -\frac{2x\alpha_0}{3} - \right. \\
& 2x\kappa\alpha_0 - \frac{160\kappa\alpha_0}{(x-2)^4} + \frac{2\kappa\alpha_0}{(x-1)^4} - \frac{320\kappa\alpha_0}{(x-2)^5} - \frac{2\kappa\alpha_0}{(x-1)^5} + 4\kappa\alpha_0 - \frac{160\alpha_0}{3(x-2)^4} + \frac{2\alpha_0}{3(x-1)^4} - \frac{320\alpha_0}{3(x-2)^5} - \frac{2\alpha_0}{3(x-1)^5} + \frac{4\alpha_0}{3} + \\
& \frac{2x}{3} + 2x\kappa + \frac{160\kappa}{(x-2)^4} - \frac{2\kappa}{(x-1)^4} + \frac{640\kappa}{(x-2)^5} - \frac{2\kappa}{(x-1)^5} + \frac{640\kappa}{(x-2)^6} + \frac{160}{3(x-2)^4} - \frac{2}{3(x-1)^4} + \frac{640}{3(x-2)^5} - \frac{2}{3(x-1)^5} + \\
& \left. \frac{640}{3(x-2)^6} \right) H(0; \alpha_0) + \left( \frac{4\alpha_0 d_1}{3} - \frac{2\alpha_0 x d_1}{3} + \frac{2x d_1}{3} + \frac{4\alpha_0 \kappa d_1}{3} - \frac{2}{3} \alpha_0 x \kappa d_1 + \frac{2x \kappa d_1}{3} - \frac{160\alpha_0 \kappa d_1}{3(x-2)^4} + \frac{160\kappa d_1}{3(x-2)^4} + \right. \\
& \frac{2\alpha_0 \kappa d_1}{3(x-1)^4} - \frac{2\kappa d_1}{3(x-1)^4} - \frac{320\alpha_0 \kappa d_1}{3(x-2)^5} + \frac{640\kappa d_1}{3(x-2)^5} - \frac{2\alpha_0 \kappa d_1}{3(x-1)^5} - \frac{2\kappa d_1}{3(x-1)^5} + \frac{640\kappa d_1}{3(x-2)^6} - \frac{160\alpha_0 d_1}{3(x-2)^4} + \frac{160d_1}{3(x-2)^4} + \frac{2\alpha_0 d_1}{3(x-1)^4} - \\
& \frac{2d_1}{3(x-1)^4} - \frac{320\alpha_0 d_1}{3(x-2)^5} + \frac{640d_1}{3(x-2)^5} - \frac{2\alpha_0 d_1}{3(x-1)^5} - \frac{2d_1}{3(x-1)^5} + \frac{640d_1}{3(x-2)^6} \Big) H(1; \alpha_0) + \frac{32}{9(x-2)} - \frac{43}{12(x-1)} - \\
& \frac{40}{9(x-2)^2} + \frac{4}{9(x-1)^2} + \frac{80}{9(x-2)^3} + \frac{1}{36(x-1)^3} - \frac{32d_1}{3(x-2)^4} - \frac{64}{9(x-2)^4} + \frac{23}{12(x-1)^4} - \frac{128d_1}{3(x-2)^5} - \frac{1216}{9(x-2)^5} + \\
& \frac{17}{12(x-1)^5} - \frac{128d_1}{3(x-2)^6} - \frac{2176}{9(x-2)^6} - \frac{1}{2} \Big) H(0, c_1(\alpha_0); x) + \left( -\frac{64d_1\kappa\alpha_0}{3(x-2)^4} - \frac{704\kappa\alpha_0}{9(x-2)^4} - \frac{128d_1\kappa\alpha_0}{3(x-2)^5} - \frac{1408\kappa\alpha_0}{9(x-2)^5} - \right. \\
& \left. \frac{64d_1\alpha_0}{3(x-2)^4} - \frac{1088\alpha_0}{9(x-2)^4} - \frac{128d_1\alpha_0}{3(x-2)^5} - \frac{2176\alpha_0}{9(x-2)^5} + \frac{64d_1\kappa}{3(x-2)^4} + \frac{704\kappa}{9(x-2)^4} + \frac{256d_1\kappa}{3(x-2)^5} + \frac{2816\kappa}{9(x-2)^5} + \frac{256d_1\kappa}{3(x-2)^6} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2816\kappa}{9(x-2)^6} + \left( \frac{320\kappa\alpha_0}{(x-2)^4} + \frac{640\kappa\alpha_0}{(x-2)^5} + \frac{320\alpha_0}{3(x-2)^4} + \frac{640\alpha_0}{3(x-2)^5} - \frac{320\kappa}{(x-2)^4} - \frac{1280\kappa}{(x-2)^5} - \frac{1280\kappa}{(x-2)^6} - \frac{320}{3(x-2)^4} - \frac{1280}{3(x-2)^5} - \right. \\
& \left. \frac{1280}{3(x-2)^6} \right) H(0; \alpha_0) + \left( \frac{320\alpha_0\kappa d_1}{3(x-2)^4} - \frac{320\kappa d_1}{3(x-2)^4} + \frac{640\alpha_0\kappa d_1}{3(x-2)^5} - \frac{1280\kappa d_1}{3(x-2)^5} - \frac{1280\kappa d_1}{3(x-2)^6} + \frac{320\alpha_0 d_1}{3(x-2)^4} - \frac{320 d_1}{3(x-2)^4} + \right. \\
& \left. \frac{640\alpha_0 d_1}{3(x-2)^5} - \frac{1280 d_1}{3(x-2)^5} - \frac{1280 d_1}{3(x-2)^6} \right) H(1; \alpha_0) + \frac{64 d_1}{3(x-2)^4} + \frac{1088}{9(x-2)^4} + \frac{256 d_1}{3(x-2)^5} + \frac{4352}{9(x-2)^5} + \frac{256 d_1}{3(x-2)^6} + \\
& \left. \frac{4352}{9(x-2)^6} \right) H(0, c_2(\alpha_0); x) + \left( \frac{1}{3} d_1 x \alpha_0^5 + \frac{1}{3} d_1 x \kappa \alpha_0^5 + \frac{2 d_1 \kappa \alpha_0^5}{3(x-2)} - \frac{d_1 \kappa \alpha_0^5}{3(x-1)} + \frac{2 d_1 \alpha_0^5}{3(x-2)} - \frac{d_1 \alpha_0^5}{3(x-1)} + \frac{2 d_1 \alpha_0^4}{9} - \right. \\
& \left. \frac{19}{9} d_1 x \alpha_0^4 + \frac{2}{9} d_1 \kappa \alpha_0^4 - \frac{19}{9} d_1 x \kappa \alpha_0^4 - \frac{20 d_1 \kappa \alpha_0^4}{9(x-2)} + \frac{13 d_1 \kappa \alpha_0^4}{9(x-1)} + \frac{20 d_1 \kappa \alpha_0^4}{9(x-2)^2} - \frac{4 d_1 \kappa \alpha_0^4}{9(x-1)^2} - \frac{20 d_1 \alpha_0^4}{9(x-2)} + \frac{13 d_1 \alpha_0^4}{9(x-1)} + \right. \\
& \left. \frac{20 d_1 \alpha_0^4}{9(x-2)^2} - \frac{4 d_1 \alpha_0^4}{9(x-1)^2} - \frac{4 d_1 \alpha_0^3}{3} + \frac{52}{9} d_1 x \alpha_0^3 - \frac{4}{3} d_1 \kappa \alpha_0^3 + \frac{52}{9} d_1 x \kappa \alpha_0^3 + \frac{20 d_1 \kappa \alpha_0^3}{9(x-2)} - \frac{22 d_1 \kappa \alpha_0^3}{9(x-1)} - \frac{40 d_1 \kappa \alpha_0^3}{9(x-2)^2} + \right. \\
& \left. \frac{14 d_1 \kappa \alpha_0^3}{9(x-1)^2} + \frac{80 d_1 \kappa \alpha_0^3}{9(x-2)^3} - \frac{2 d_1 \kappa \alpha_0^3}{3(x-1)^3} + \frac{20 d_1 \alpha_0^3}{9(x-2)} - \frac{22 d_1 \alpha_0^3}{9(x-1)} - \frac{40 d_1 \alpha_0^3}{9(x-2)^2} + \frac{14 d_1 \alpha_0^3}{9(x-1)^2} + \frac{80 d_1 \alpha_0^3}{9(x-2)^3} - \frac{2 d_1 \alpha_0^3}{3(x-1)^3} + 4 d_1 \alpha_0^2 - \right. \\
& \left. \frac{28}{3} d_1 x \alpha_0^2 + 4 d_1 \kappa \alpha_0^2 - \frac{28}{3} d_1 x \kappa \alpha_0^2 + \frac{2 d_1 \kappa \alpha_0^2}{x-1} - \frac{2 d_1 \kappa \alpha_0^2}{(x-1)^2} + \frac{2 d_1 \kappa \alpha_0^2}{(x-1)^3} + \frac{160 d_1 \kappa \alpha_0^2}{3(x-2)^4} - \frac{4 d_1 \kappa \alpha_0^2}{3(x-1)^4} + \frac{2 d_1 \alpha_0^2}{x-1} - \right. \\
& \left. \frac{2 d_1 \alpha_0^2}{(x-1)^2} + \frac{2 d_1 \alpha_0^2}{(x-1)^3} + \frac{160 d_1 \alpha_0^2}{3(x-2)^4} - \frac{4 d_1 \alpha_0^2}{3(x-1)^4} - \frac{20 d_1 \alpha_0}{9} + \frac{73 d_1 x \alpha_0}{9} - \frac{20 d_1 \kappa \alpha_0}{9} + \frac{73}{9} d_1 x \kappa \alpha_0 - \frac{10 d_1 \kappa \alpha_0}{9(x-2)} - \frac{7 d_1 \kappa \alpha_0}{9(x-1)} + \right. \\
& \left. \frac{40 d_1 \kappa \alpha_0}{9(x-2)^2} + \frac{10 d_1 \kappa \alpha_0}{9(x-1)^2} - \frac{80 d_1 \kappa \alpha_0}{3(x-2)^3} - \frac{2 d_1 \kappa \alpha_0}{3(x-2)^4} - \frac{640 d_1 \kappa \alpha_0}{3(x-2)^5} - \frac{640 d_1 \kappa \alpha_0}{3(x-2)^5} - \frac{10 d_1 \alpha_0}{9(x-2)} - \frac{7 d_1 \alpha_0}{9(x-1)} + \frac{40 d_1 \alpha_0}{9(x-2)^2} + \right. \\
& \left. \frac{10 d_1 \alpha_0}{9(x-1)^2} - \frac{80 d_1 \alpha_0}{3(x-2)^3} - \frac{2 d_1 \alpha_0}{(x-1)^3} - \frac{640 d_1 \alpha_0}{3(x-2)^4} - \frac{640 d_1 \alpha_0}{3(x-2)^5} - \frac{2 d_1}{3} - \frac{25 d_1 x}{9} - \frac{2 d_1 \kappa}{3} - \frac{25 d_1 x \kappa}{9} + \frac{4 d_1 \kappa}{9(x-2)} + \frac{d_1 \kappa}{9(x-1)} - \right. \\
& \left. \frac{20 d_1 \kappa}{9(x-2)^2} - \frac{2 d_1 \kappa}{9(x-1)^2} + \frac{160 d_1 \kappa}{9(x-2)^3} + \frac{2 d_1 \kappa}{3(x-1)^3} + \frac{160 d_1 \kappa}{(x-2)^4} + \frac{4 d_1 \kappa}{3(x-1)^4} + \frac{640 d_1 \kappa}{3(x-2)^5} + \frac{4 d_1}{9(x-2)} + \frac{d_1}{9(x-1)} - \frac{20 d_1}{9(x-2)^2} - \right. \\
& \left. \frac{2 d_1}{9(x-1)^2} + \frac{160 d_1}{9(x-2)^3} + \frac{2 d_1}{3(x-1)^3} + \frac{160 d_1}{(x-2)^4} + \frac{4 d_1}{3(x-1)^4} + \frac{640 d_1}{3(x-2)^5} \right) H(1, 0; \alpha_0) + \left( \frac{2 d_1 \alpha_0}{3} - \frac{17 x \alpha_0}{12} + \frac{2 d_1 \kappa \alpha_0}{3} - \right. \\
& \left. \frac{101 x \kappa \alpha_0}{36} + \frac{85 d_1 \kappa \alpha_0}{9(x-2)} - \frac{40 \kappa \alpha_0}{3(x-2)} - \frac{94 d_1 \kappa \alpha_0}{9(x-1)} + \frac{65 \kappa \alpha_0}{4(x-1)} - \frac{100 d_1 \kappa \alpha_0}{9(x-2)^2} + \frac{40 \kappa \alpha_0}{3(x-2)^2} + \frac{20 d_1 \kappa \alpha_0}{9(x-1)^2} - \frac{25 \kappa \alpha_0}{6(x-1)^2} + \frac{40 d_1 \kappa \alpha_0}{3(x-2)^3} - \right. \\
& \left. \frac{17 d_1 \kappa \alpha_0}{18(x-1)^3} + \frac{35 \kappa \alpha_0}{12(x-1)^3} - \frac{64 d_1 \kappa \alpha_0}{(x-2)^4} + \frac{1088 \kappa \alpha_0}{9(x-2)^4} - \frac{19 d_1 \kappa \alpha_0}{18(x-1)^4} + \frac{47 \kappa \alpha_0}{36(x-1)^4} - \frac{64 d_1 \kappa \alpha_0}{3(x-2)^5} - \frac{704 \kappa \alpha_0}{9(x-2)^5} + \frac{25 d_1 \kappa \alpha_0}{18(x-1)^5} - \right. \\
& \left. \frac{101 \kappa \alpha_0}{36(x-1)^5} + \frac{37 \kappa \alpha_0}{9} + \frac{85 d_1 \alpha_0}{9(x-2)} - \frac{40 \alpha_0}{9(x-2)} - \frac{94 d_1 \alpha_0}{9(x-1)} + \frac{65 \alpha_0}{12(x-1)} - \frac{100 d_1 \alpha_0}{9(x-2)^2} + \frac{40 \alpha_0}{9(x-2)^2} + \frac{20 d_1 \alpha_0}{9(x-1)^2} - \frac{25 \alpha_0}{18(x-1)^2} + \right. \\
& \left. \frac{40 d_1 \alpha_0}{3(x-2)^3} - \frac{17 d_1 \alpha_0}{18(x-1)^3} + \frac{35 \alpha_0}{36(x-1)^3} - \frac{64 d_1 \alpha_0}{(x-2)^4} - \frac{64 \alpha_0}{9(x-2)^4} - \frac{5 d_1 \alpha_0}{2(x-1)^4} + \frac{11 \alpha_0}{12(x-1)^4} - \frac{64 d_1 \alpha_0}{3(x-2)^5} - \frac{1088 \alpha_0}{9(x-2)^5} + \right. \\
& \left. \frac{17 d_1 \alpha_0}{6(x-1)^5} - \frac{17 \alpha_0}{12(x-1)^5} + \frac{7 \alpha_0}{3} - \frac{2 d_1}{3} + \frac{17 x}{12} - \frac{2 d_1 \kappa}{3} + \frac{101 \kappa}{36} + \frac{62 d_1 \kappa}{9(x-2)} - \frac{32 \kappa}{3(x-2)} - \frac{65 d_1 \kappa}{9(x-1)} + \frac{43 \kappa}{4(x-1)} - \right. \\
& \left. \frac{70 d_1 \kappa}{9(x-2)^2} + \frac{40 \kappa}{3(x-2)^2} + \frac{d_1 \kappa}{(x-1)^2} - \frac{4 \kappa}{3(x-1)^2} + \frac{80 d_1 \kappa}{9(x-2)^3} - \frac{80 \kappa}{3(x-2)^3} - \frac{5 d_1 \kappa}{18(x-1)^3} - \frac{\kappa}{12(x-1)^3} + \frac{112 d_1 \kappa}{3(x-2)^4} - \frac{1088 \kappa}{9(x-2)^4} + \right. \\
& \left. \frac{31 d_1 \kappa}{18(x-1)^4} - \frac{155 \kappa}{36(x-1)^4} + \frac{448 d_1 \kappa}{3(x-2)^5} - \frac{1472 \kappa}{9(x-2)^5} + \frac{25 d_1 \kappa}{18(x-1)^5} - \frac{101 \kappa}{36(x-1)^5} + \frac{128 d_1 \kappa}{3(x-2)^6} + \frac{1408 \kappa}{9(x-2)^6} + \frac{3 \kappa}{2} + \frac{62 d_1}{9(x-2)} - \right. \\
& \left. \frac{32}{9(x-2)} - \frac{65 d_1}{9(x-1)} + \frac{43}{12(x-1)} - \frac{70 d_1}{9(x-2)^2} + \frac{40}{9(x-2)^2} + \frac{d_1}{(x-1)^2} - \frac{4}{9(x-1)^2} + \frac{80 d_1}{9(x-2)^3} - \frac{80}{9(x-2)^3} - \frac{5 d_1}{18(x-1)^3} - \right. \\
& \left. \frac{1}{36(x-1)^3} + \frac{112 d_1}{3(x-2)^4} + \frac{64}{9(x-2)^4} + \frac{19 d_1}{6(x-1)^4} - \frac{23}{12(x-1)^4} + \frac{448 d_1}{3(x-2)^5} + \frac{1216}{9(x-2)^5} + \frac{17 d_1}{6(x-1)^5} - \frac{17}{12(x-1)^5} + \right. \\
& \left. \frac{128 d_1}{3(x-2)^6} + \frac{2176}{9(x-2)^6} + \frac{1}{2} \right) H(1, 0; x) + \left( \frac{1}{3} d_1^2 x \alpha_0^5 + \frac{2 d_1^2 \alpha_0^5}{3(x-2)} - \frac{d_1^2 \alpha_0^5}{3(x-1)} + \frac{2 d_1^2 \alpha_0^4}{9} - \frac{19}{9} d_1^2 x \alpha_0^4 - \frac{20 d_1^2 \alpha_0^4}{9(x-2)} + \right. \\
& \left. \frac{13 d_1^2 \alpha_0^4}{9(x-1)} + \frac{20 d_1^2 \alpha_0^4}{9(x-2)^2} - \frac{4 d_1^2 \alpha_0^4}{9(x-1)^2} - \frac{4 d_1^2 \alpha_0^3}{3} + \frac{52}{9} d_1^2 x \alpha_0^3 + \frac{20 d_1^2 \alpha_0^3}{9(x-2)} - \frac{22 d_1^2 \alpha_0^3}{9(x-1)} - \frac{40 d_1^2 \alpha_0^3}{9(x-2)^2} + \frac{14 d_1^2 \alpha_0^3}{9(x-1)^2} + \frac{80 d_1^2 \alpha_0^3}{9(x-2)^3} - \right. \\
& \left. \frac{2 d_1^2 \alpha_0^3}{3(x-1)^3} + 4 d_1^2 \alpha_0^2 - \frac{28}{3} d_1^2 x \alpha_0^2 + \frac{2 d_1^2 \alpha_0^2}{x-1} - \frac{2 d_1^2 \alpha_0^2}{(x-1)^2} + \frac{2 d_1^2 \alpha_0^2}{(x-1)^3} + \frac{160 d_1^2 \alpha_0^2}{3(x-2)^4} - \frac{4 d_1^2 \alpha_0^2}{3(x-1)^4} - \frac{20 d_1^2 \alpha_0}{9} + \frac{73}{9} d_1^2 x \alpha_0 - \right. \\
& \left. \frac{10 d_1^2 \alpha_0}{9(x-2)} - \frac{7 d_1^2 \alpha_0}{9(x-1)} + \frac{40 d_1^2 \alpha_0}{9(x-2)^2} + \frac{10 d_1^2 \alpha_0}{9(x-1)^2} - \frac{80 d_1^2 \alpha_0}{3(x-2)^3} - \frac{2 d_1^2 \alpha_0}{3(x-1)^3} - \frac{640 d_1^2 \alpha_0}{3(x-2)^4} - \frac{640 d_1^2 \alpha_0}{3(x-2)^5} - \frac{2 d_1^2}{3} - \frac{25 d_1^2 x}{9} + \frac{4 d_1^2}{9(x-2)} + \right. \\
& \left. \frac{d_1^2}{9(x-1)} - \frac{20 d_1^2}{9(x-2)^2} - \frac{2 d_1^2}{9(x-1)^2} + \frac{160 d_1^2}{9(x-2)^3} + \frac{2 d_1^2}{3(x-1)^3} + \frac{160 d_1^2}{(x-2)^4} + \frac{4 d_1^2}{3(x-1)^4} + \frac{640 d_1^2}{3(x-2)^5} \right) H(1, 1; \alpha_0) + \\
& H(c_2(\alpha_0); x) \left( -\frac{256 \kappa \alpha_0}{9(x-2)^4} - \frac{512 \kappa \alpha_0}{9(x-2)^5} + \frac{256 d_1 \alpha_0}{9(x-2)^4} - \frac{40 \pi^2 \alpha_0}{9(x-2)^4} + \frac{3392 \alpha_0}{27(x-2)^4} + \frac{512 d_1 \alpha_0}{9(x-2)^5} - \frac{80 \pi^2 \alpha_0}{9(x-2)^5} + \frac{6784 \alpha_0}{27(x-2)^5} + \right. \\
& \left. \frac{256 \kappa}{9(x-2)^4} + \frac{1024 \kappa}{9(x-2)^5} + \frac{1024 \kappa}{9(x-2)^6} + \left( -\frac{64 d_1 \kappa \alpha_0}{3(x-2)^4} - \frac{704 \kappa \alpha_0}{9(x-2)^4} - \frac{128 d_1 \kappa \alpha_0}{3(x-2)^5} - \frac{1408 \kappa \alpha_0}{9(x-2)^5} - \frac{64 d_1 \alpha_0}{3(x-2)^4} - \frac{1088 \alpha_0}{9(x-2)^4} - \right. \right. \\
& \left. \left. \frac{128 d_1 \alpha_0}{3(x-2)^5} - \frac{2176 \alpha_0}{9(x-2)^5} + \frac{64 d_1 \kappa}{3(x-2)^4} + \frac{704 \kappa}{9(x-2)^4} + \frac{256 d_1 \kappa}{3(x-2)^5} + \frac{2816 \kappa}{9(x-2)^5} + \frac{256 d_1 \kappa}{3(x-2)^6} + \frac{2816 \kappa}{9(x-2)^6} + \frac{64 d_1}{3(x-2)^4} + \right. \right. \\
& \left. \left. \frac{1088}{9(x-2)^4} + \frac{256 d_1}{3(x-2)^5} + \frac{4352}{9(x-2)^5} + \frac{256 d_1}{3(x-2)^6} + \frac{4352}{9(x-2)^6} \right) H(0; \alpha_0) + \left( -\frac{64 \alpha_0 d_1^2}{3(x-2)^4} + \frac{64 d_1^2}{3(x-2)^4} - \frac{128 \alpha_0 d_1^2}{3(x-2)^5} + \right. \\
& \left. \frac{256 d_1^2}{3(x-2)^5} + \frac{256 d_1^2}{3(x-2)^6} + \frac{64 \alpha_0 \kappa d_1}{3(x-2)^4} - \frac{64 \kappa d_1}{3(x-2)^4} + \frac{128 \alpha_0 \kappa d_1}{3(x-2)^5} - \frac{256 \kappa d_1}{3(x-2)^5} - \frac{256 \kappa d_1}{3(x-2)^6} - \frac{1088 \alpha_0 d_1}{9(x-2)^4} + \frac{1088 d_1}{9(x-2)^4} - \right. \\
& \left. \frac{2176 \alpha_0 d_1}{9(x-2)^5} + \frac{4352 d_1}{9(x-2)^5} + \frac{4352 d_1}{9(x-2)^6} \right) H(1; \alpha_0) + \left( \frac{320 \kappa \alpha_0}{(x-2)^4} + \frac{640 \kappa \alpha_0}{(x-2)^5} + \frac{320 \alpha_0}{3(x-2)^4} + \frac{640 \alpha_0}{3(x-2)^5} - \frac{320 \kappa}{(x-2)^4} - \frac{1280 \kappa}{(x-2)^5} - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{1280\kappa}{(x-2)^6} - \frac{320}{3(x-2)^4} - \frac{1280}{3(x-2)^5} - \frac{1280}{3(x-2)^6} \right) H(0, 0; \alpha_0) + \left( \frac{320\alpha_0\kappa d_1}{3(x-2)^4} - \frac{320\kappa d_1}{3(x-2)^4} + \frac{640\alpha_0\kappa d_1}{3(x-2)^5} - \frac{1280\kappa d_1}{3(x-2)^5} - \right. \\
& \left. \frac{1280\kappa d_1}{3(x-2)^6} + \frac{320\alpha_0 d_1}{3(x-2)^4} - \frac{320d_1}{3(x-2)^4} + \frac{640\alpha_0 d_1}{3(x-2)^5} - \frac{1280d_1}{3(x-2)^5} - \frac{1280d_1}{3(x-2)^6} \right) H(0, 1; \alpha_0) + \left( \frac{320\alpha_0\kappa d_1}{3(x-2)^4} - \frac{320\kappa d_1}{3(x-2)^4} + \right. \\
& \left. \frac{640\alpha_0\kappa d_1}{3(x-2)^5} - \frac{1280\kappa d_1}{3(x-2)^5} - \frac{1280\kappa d_1}{3(x-2)^6} + \frac{320\alpha_0 d_1}{3(x-2)^4} - \frac{320d_1}{3(x-2)^4} + \frac{640\alpha_0 d_1}{3(x-2)^5} - \frac{1280d_1}{3(x-2)^5} - \frac{1280d_1}{3(x-2)^6} \right) H(1, 0; \alpha_0) + \\
& \left( \frac{320\alpha_0 d_1^2}{3(x-2)^4} - \frac{320d_1^2}{3(x-2)^4} + \frac{640\alpha_0 d_1^2}{3(x-2)^5} - \frac{1280d_1^2}{3(x-2)^5} - \frac{1280d_1^2}{3(x-2)^6} \right) H(1, 1; \alpha_0) - \frac{256 d_1}{9(x-2)^4} + \frac{40\pi^2}{9(x-2)^4} - \frac{3392}{27(x-2)^4} - \\
& \frac{1024d_1}{9(x-2)^5} + \frac{160\pi^2}{9(x-2)^5} - \frac{13568}{27(x-2)^5} - \frac{1024d_1}{9(x-2)^6} + \frac{160\pi^2}{27(x-2)^6} - \frac{13568}{27(x-2)^6} \Big) + H(c_1(\alpha_0); x) \left( \frac{1}{24} d_1 x \alpha_0^5 - \right. \\
& \frac{2x}{9} \alpha_0^5 + \frac{1}{24} d_1 x \kappa \alpha_0^5 - \frac{11}{36} x \kappa \alpha_0^5 + \frac{d_1 \kappa \alpha_0^5}{12(x-2)} - \frac{25\kappa \alpha_0^5}{36(x-2)} - \frac{d_1 \kappa \alpha_0^5}{24(x-1)} + \frac{11\kappa \alpha_0^5}{36(x-1)} - \frac{\kappa \alpha_0^5}{24} + \frac{d_1 \alpha_0^5}{12(x-2)} - \frac{19}{36} \frac{\alpha_0^5}{(x-2)} - \\
& \frac{d_1 \alpha_0^5}{24(x-1)} + \frac{2\alpha_0^5}{9(x-1)} - \frac{\alpha_0^5}{24} + \frac{7d_1 \alpha_0^4}{108} - \frac{61}{216} d_1 x \alpha_0^4 + \frac{157x\alpha_0^4}{108} + \frac{7}{108} d_1 \kappa \alpha_0^4 - \frac{61}{216} d_1 x \kappa \alpha_0^4 + \frac{56}{27} x \kappa \alpha_0^4 - \\
& \frac{11d_1 \kappa \alpha_0^4}{54(x-2)} + \frac{113\kappa \alpha_0^4}{54(x-2)} + \frac{43d_1 \kappa \alpha_0^4}{216(x-1)} - \frac{149\kappa \alpha_0^4}{108(x-1)} + \frac{23d_1 \kappa \alpha_0^4}{54(x-2)^2} - \frac{149\kappa \alpha_0^4}{54(x-2)^2} - \frac{2d_1 \kappa \alpha_0^4}{27(x-1)^2} + \frac{59\kappa \alpha_0^4}{108(x-1)^2} - \frac{41\kappa \alpha_0^4}{216} - \\
& \frac{11d_1 \alpha_0^4}{54(x-2)} + \frac{91\alpha_0^4}{54(x-2)} + \frac{43d_1 \alpha_0^4}{216(x-1)} - \frac{53\alpha_0^4}{54(x-1)} + \frac{23d_1 \alpha_0^4}{54(x-2)^2} - \frac{103\alpha_0^4}{54(x-2)^2} - \frac{2d_1 \alpha_0^4}{27(x-1)^2} + \frac{108(x-1)^2}{216} - \frac{\alpha_0^4}{216} - \\
& \frac{4d_1 \alpha_0^3}{9} + \frac{95}{108} d_1 x \alpha_0^3 - \frac{911x\alpha_0^3}{216} - \frac{4}{9} d_1 \kappa \alpha_0^3 + \frac{95}{108} d_1 x \kappa \alpha_0^3 - \frac{1381}{216} x \kappa \alpha_0^3 - \frac{d_1 \kappa \alpha_0^3}{54(x-2)} - \frac{25\kappa \alpha_0^3}{27(x-2)} - \frac{41d_1 \kappa \alpha_0^3}{108(x-1)} + \\
& \frac{239\kappa \alpha_0^3}{108(x-1)} - \frac{8d_1 \kappa \alpha_0^3}{27(x-2)^2} + \frac{104\kappa \alpha_0^3}{27(x-2)^2} + \frac{17d_1 \kappa \alpha_0^3}{54(x-1)^2} - \frac{467\kappa \alpha_0^3}{216(x-1)^2} + \frac{76d_1 \kappa \alpha_0^3}{27(x-2)^3} - \frac{388\kappa \alpha_0^3}{27(x-2)^3} - \frac{d_1 \kappa \alpha_0^3}{6(x-1)^3} + \frac{83\kappa \alpha_0^3}{72(x-1)^3} + \\
& \frac{83\kappa \alpha_0^3}{36} - \frac{d_1 \alpha_0^3}{54(x-2)} - \frac{35\alpha_0^3}{27(x-2)} - \frac{41d_1 \alpha_0^3}{108(x-1)} + \frac{175\alpha_0^3}{108(x-1)} - \frac{8d_1 \alpha_0^3}{27(x-2)^2} + \frac{88\alpha_0^3}{27(x-2)^2} + \frac{17d_1 \alpha_0^3}{54(x-1)^2} - \frac{277\alpha_0^3}{216(x-1)^2} + \\
& \frac{76d_1 \alpha_0^3}{27(x-2)^3} - \frac{236\alpha_0^3}{27(x-2)^3} - \frac{d_1 \alpha_0^3}{6(x-1)^3} + \frac{5\alpha_0^3}{8(x-1)^3} + \frac{35\alpha_0^3}{36} + \frac{35d_1 \alpha_0^2}{18} - \frac{73}{36} d_1 x \alpha_0^2 + \frac{569x}{72} \alpha_0^2 + \frac{35}{18} d_1 \kappa \alpha_0^2 - \\
& \frac{73}{36} d_1 x \kappa \alpha_0^2 + \frac{979}{72} x \kappa \alpha_0^2 + \frac{4d_1 \kappa \alpha_0^2}{9(x-2)} - \frac{16\kappa \alpha_0^2}{3(x-2)} + \frac{13d_1 \kappa \alpha_0^2}{36(x-1)} + \frac{2\kappa \alpha_0^2}{3(x-1)} - \frac{5d_1 \kappa \alpha_0^2}{3(x-2)^2} + \frac{10\kappa \alpha_0^2}{(x-2)^2} - \frac{d_1 \kappa \alpha_0^2}{2(x-1)^2} + \\
& \frac{67\kappa \alpha_0^2}{24(x-1)^2} + \frac{80d_1 \kappa \alpha_0^2}{9(x-2)^3} - \frac{80\kappa \alpha_0^2}{3(x-2)^3} + \frac{5}{6} \frac{d_1 \kappa \alpha_0^2}{(x-1)^3} - \frac{119\kappa \alpha_0^2}{24(x-1)^3} + \frac{104d_1 \kappa \alpha_0^2}{3(x-2)^4} - \frac{1256\kappa \alpha_0^2}{9(x-2)^4} - \frac{2d_1 \kappa \alpha_0^2}{3(x-1)^4} + \frac{137\kappa \alpha_0^2}{36(x-1)^4} - \\
& \frac{32\kappa \alpha_0^2}{3} + \frac{4d_1 \alpha_0^2}{9(x-2)} - \frac{16\alpha_0^2}{9(x-2)} + \frac{13d_1 \alpha_0^2}{36(x-1)} - \frac{\alpha_0^2}{2(x-1)} - \frac{5d_1 \alpha_0^2}{3(x-2)^2} + \frac{10}{3} \frac{\alpha_0^2}{(x-2)^2} - \frac{d_1 \alpha_0^2}{2(x-1)^2} + \frac{119\alpha_0^2}{72(x-1)^2} + \frac{80d_1 \alpha_0^2}{9(x-2)^3} - \\
& \frac{80\alpha_0^2}{9(x-2)^3} + \frac{5d_1 \alpha_0^2}{6(x-1)^3} - \frac{19\alpha_0^2}{8(x-1)^3} + \frac{104d_1 \alpha_0^2}{3(x-2)^4} - \frac{632\alpha_0^2}{9(x-2)^4} - \frac{2d_1 \alpha_0^2}{3(x-1)^4} + \frac{7\alpha_0^2}{4(x-1)^4} - \frac{85}{18} \frac{\alpha_0^2}{(x-1)^4} - \frac{22d_1 \alpha_0}{27} + \frac{505d_1 x \alpha_0}{216} - \\
& \frac{1697}{216} x \alpha_0 - \frac{22d_1 \kappa \alpha_0}{27} + \frac{505}{216} d_1 x \kappa \alpha_0 - \frac{3193x\kappa \alpha_0}{216} + \frac{386d_1 \kappa \alpha_0}{27(x-2)} - \frac{1172\kappa \alpha_0}{27(x-2)} - \frac{1133d_1 \kappa \alpha_0}{72(x-1)} + \frac{381\kappa \alpha_0}{72(x-1)} - \\
& \frac{362}{27(x-2)^2} d_1 \kappa \alpha_0 + \frac{1136\kappa \alpha_0}{27(x-2)^2} + \frac{193d_1 \kappa \alpha_0}{108(x-1)^2} - \frac{1273\kappa \alpha_0}{108(x-1)^2} - \frac{80d_1 \kappa \alpha_0}{9(x-2)^3} + \frac{40\kappa \alpha_0}{3(x-2)^3} - \frac{323d_1 \kappa \alpha_0}{216(x-1)^3} + \frac{2075\kappa \alpha_0}{216(x-1)^3} - \\
& \frac{464d_1 \kappa \alpha_0}{3(x-2)^4} + \frac{2032\kappa \alpha_0}{3(x-2)^4} - \frac{187d_1 \kappa \alpha_0}{216(x-1)^4} + \frac{539\kappa \alpha_0}{108(x-1)^4} - \frac{256d_1 \kappa \alpha_0}{3(x-2)^5} + \frac{3584\kappa \alpha_0}{9(x-2)^5} + \frac{205d_1 \kappa \alpha_0}{216(x-1)^5} - \frac{1255\kappa \alpha_0}{216(x-1)^5} + \frac{857\kappa \alpha_0}{216} + \\
& \frac{386d_1 \alpha_0}{27(x-2)} - \frac{604\alpha_0}{27(x-2)} - \frac{1133d_1 \alpha_0}{72(x-1)} + \frac{649\alpha_0}{24(x-1)} - \frac{362d_1 \alpha_0}{27(x-2)^2} + \frac{592\alpha_0}{27(x-2)^2} + \frac{193d_1 \alpha_0}{108(x-1)^2} - \frac{671\alpha_0}{108(x-1)^2} - \frac{80d_1 \alpha_0}{9(x-2)^3} + \\
& \frac{40\alpha_0}{9(x-2)^3} - \frac{323d_1 \alpha_0}{216(x-1)^3} + \frac{1015\alpha_0}{216(x-1)^3} - \frac{464d_1 \alpha_0}{3(x-2)^4} + \frac{368\alpha_0}{(x-2)^4} - \frac{187d_1 \alpha_0}{216(x-1)^4} - \frac{\pi^2 \alpha_0}{18(x-1)^4} + \frac{419\alpha_0}{108(x-1)^4} - \frac{256d_1 \alpha_0}{3(x-2)^5} + \\
& \frac{2048\alpha_0}{9(x-2)^5} + \frac{205d_1 \alpha_0}{216(x-1)^5} + \frac{\pi^2 \alpha_0}{18(x-1)^5} - \frac{955\alpha_0}{216(x-1)^5} + \frac{325\alpha_0}{216} - \frac{3}{4} \frac{d_1}{(x-2)} - \frac{205d_1 x}{216} + \frac{635x}{216} - \frac{3d_1 \kappa}{4} - \frac{205d_1 x \kappa}{216} + \\
& \frac{1255x\kappa}{216} + \frac{346d_1 \kappa}{27(x-2)} - \frac{952\kappa}{27(x-2)} - \frac{947d_1 \kappa}{72(x-1)} + \frac{2621\kappa}{72(x-1)} - \frac{392d_1 \kappa}{27(x-2)^2} + \frac{1136\kappa}{27(x-2)^2} + \frac{23d_1 \kappa}{27(x-1)^2} - \frac{92\kappa}{27(x-1)^2} + \\
& \frac{784d_1 \kappa}{27(x-2)^3} - \frac{2272\kappa}{27(x-2)^3} + \frac{73d_1 \kappa}{216(x-1)^3} - \frac{247\kappa}{216(x-1)^3} + \frac{256d_1 \kappa}{3(x-2)^4} - \frac{3584\kappa}{9(x-2)^4} + \frac{367d_1 \kappa}{216(x-1)^4} - \frac{1127\kappa}{108(x-1)^4} + \\
& \frac{512d_1 \kappa}{3(x-2)^5} - \frac{7168\kappa}{9(x-2)^5} + \frac{205d_1 \kappa}{216(x-1)^5} - \frac{1255\kappa}{216(x-1)^5} + \frac{37\kappa}{8} + \left( \frac{x\alpha_0^5}{6} + \frac{1}{2} x \kappa \alpha_0^5 + \frac{\kappa \alpha_0^5}{x-2} - \frac{\kappa \alpha_0^5}{2(x-1)} + \frac{\alpha_0^5}{3(x-2)} - \right. \\
& \frac{\alpha_0^5}{6(x-1)} - \frac{19x}{18} \frac{\alpha_0^4}{(x-2)} - \frac{19}{6} x \kappa \alpha_0^4 - \frac{10\kappa \alpha_0^4}{3(x-2)} + \frac{13\kappa \alpha_0^4}{6(x-1)} + \frac{10\kappa \alpha_0^4}{3(x-2)^2} - \frac{2\kappa \alpha_0^4}{3(x-1)^2} + \frac{\kappa \alpha_0^4}{3} - \frac{10\alpha_0^4}{9(x-2)} + \frac{13\alpha_0^4}{18(x-1)} + \\
& \frac{10}{9} \frac{\alpha_0^4}{(x-2)^2} - \frac{2\alpha_0^4}{9(x-1)^2} + \frac{\alpha_0^4}{9} + \frac{26}{9} \frac{x\alpha_0^3}{(x-2)} + \frac{26}{3} x \kappa \alpha_0^3 + \frac{10\kappa \alpha_0^3}{3(x-2)} - \frac{11\kappa \alpha_0^3}{3(x-1)} - \frac{20\kappa \alpha_0^3}{3(x-2)^2} + \frac{7\kappa \alpha_0^3}{3(x-1)^2} + \frac{40\kappa \alpha_0^3}{3(x-2)^3} - \\
& \frac{\kappa \alpha_0^3}{(x-1)^3} - 2\kappa \alpha_0^3 + \frac{10}{9} \frac{\alpha_0^3}{(x-2)} - \frac{11\alpha_0^3}{9(x-1)} - \frac{20\alpha_0^3}{9(x-2)^2} + \frac{7\alpha_0^3}{9(x-1)^2} + \frac{40\alpha_0^3}{9(x-2)^3} - \frac{\alpha_0^3}{3(x-1)^3} - \frac{2\alpha_0^3}{3} - \frac{14x}{3} \frac{\alpha_0^2}{(x-2)} + 14x\kappa \alpha_0^2 + \\
& \frac{3\kappa \alpha_0^2}{x-1} - \frac{3\kappa \alpha_0^2}{(x-1)^2} + \frac{3\kappa \alpha_0^2}{(x-1)^3} + \frac{80\kappa \alpha_0^2}{(x-2)^4} - \frac{2\kappa \alpha_0^2}{(x-1)^4} + 6\kappa \alpha_0^2 + \frac{\alpha_0^2}{x-1} - \frac{\alpha_0^2}{(x-1)^2} + \frac{\alpha_0^2}{(x-1)^3} + \frac{80\alpha_0^2}{3(x-2)^4} - \frac{2\alpha_0^2}{3(x-1)^4} + 2\alpha_0^2 + \\
& \frac{73x\alpha_0}{18} + \frac{73x\kappa \alpha_0}{6} + \frac{80\kappa \alpha_0}{3(x-2)} - \frac{65\kappa \alpha_0}{2(x-1)} - \frac{80\kappa \alpha_0}{3(x-2)^2} + \frac{25\kappa \alpha_0}{3(x-1)^2} - \frac{35\kappa \alpha_0}{6(x-1)^3} - \frac{480\kappa \alpha_0}{(x-2)^4} - \frac{83\kappa \alpha_0}{18(x-1)^4} - \frac{320\kappa \alpha_0}{(x-2)^5} + \\
& \frac{101\kappa \alpha_0}{18(x-1)^5} - \frac{4\kappa \alpha_0}{3} + \frac{80\alpha_0}{9(x-2)} - \frac{65\alpha_0}{6(x-1)} - \frac{80\alpha_0}{9(x-2)^2} + \frac{25\alpha_0}{9(x-1)^2} - \frac{35\alpha_0}{18(x-1)^3} - \frac{160\alpha_0}{(x-2)^4} - \frac{5\alpha_0}{2(x-1)^4} - \frac{320\alpha_0}{3(x-2)^5} + \\
& \frac{17\alpha_0}{6(x-1)^5} - \frac{4\alpha_0}{9} - \frac{25x}{18} \frac{\kappa}{(x-2)} + \frac{64\kappa}{3(x-2)} - \frac{43\kappa}{2(x-1)} - \frac{80\kappa}{3(x-2)^2} + \frac{8\kappa}{3(x-1)^2} + \frac{160\kappa}{3(x-2)^3} + \frac{\kappa}{6(x-1)^3} + \frac{320\kappa}{(x-2)^4} + \\
& \frac{155\kappa}{18(x-1)^4} + \frac{640\kappa}{(x-2)^5} + \frac{101\kappa}{18(x-1)^5} - 3\kappa + \frac{64}{9(x-2)} - \frac{43}{6(x-1)} - \frac{80}{9(x-2)^2} + \frac{8}{9(x-1)^2} + \frac{160}{9(x-2)^3} + \frac{1}{18(x-1)^3} \Big) +
\end{aligned}$$

$$\begin{aligned}
& \frac{320}{3(x-2)^4} + \frac{23}{6(x-1)^4} + \frac{640}{3(x-2)^5} + \frac{17}{6(x-1)^5} - 1 \Big) H(0; \alpha_0) + \left( \frac{1}{6} d_1 x \alpha_0^5 + \frac{1}{6} d_1 x \kappa \alpha_0^5 + \frac{d_1 \kappa \alpha_0^5}{3(x-2)} - \frac{d_1 \kappa \alpha_0^5}{6(x-1)} + \right. \\
& \frac{d_1 \alpha_0^5}{3(x-2)} - \frac{d_1 \alpha_0^5}{6(x-1)} + \frac{d_1 \alpha_0^4}{9} - \frac{19}{18} d_1 x \alpha_0^4 + \frac{1}{9} d_1 \kappa \alpha_0^4 - \frac{19}{18} d_1 x \kappa \alpha_0^4 - \frac{10 d_1 \kappa \alpha_0^4}{9(x-2)} + \frac{13 d_1 \kappa \alpha_0^4}{18(x-1)} + \frac{10 d_1 \kappa \alpha_0^4}{9(x-2)^2} - \\
& \frac{2 d_1 \kappa \alpha_0^4}{9(x-1)^2} - \frac{10 d_1 \alpha_0^4}{9(x-2)} + \frac{13 d_1 \alpha_0^4}{18(x-1)} + \frac{10 d_1 \alpha_0^4}{9(x-2)^2} - \frac{2 d_1 \alpha_0^4}{9(x-1)^2} - \frac{2 d_1 \alpha_0^3}{3} + \frac{26}{9} d_1 x \alpha_0^3 - \frac{2}{3} d_1 \kappa \alpha_0^3 + \frac{26}{9} d_1 x \kappa \alpha_0^3 + \\
& \frac{10 d_1 \kappa \alpha_0^3}{9(x-2)} - \frac{11 d_1 \kappa \alpha_0^3}{9(x-1)} - \frac{20 d_1 \kappa \alpha_0^3}{9(x-2)^2} + \frac{7 d_1 \kappa \alpha_0^3}{9(x-1)^2} + \frac{40 d_1 \kappa \alpha_0^3}{9(x-2)^3} - \frac{d_1 \kappa \alpha_0^3}{3(x-1)^3} + \frac{10 d_1 \alpha_0^3}{9(x-2)} - \frac{11 d_1 \alpha_0^3}{9(x-1)} - \frac{20 d_1 \alpha_0^3}{9(x-2)^2} + \frac{7 d_1 \alpha_0^3}{9(x-1)^2} + \\
& \frac{40 d_1 \alpha_0^3}{9(x-2)^3} - \frac{d_1 \alpha_0^3}{3(x-1)^3} + 2 d_1 \alpha_0^2 - \frac{14}{3} d_1 x \alpha_0^2 + 2 d_1 \kappa \alpha_0^2 - \frac{14}{3} d_1 x \kappa \alpha_0^2 + \frac{d_1 \kappa \alpha_0^2}{x-1} - \frac{d_1 \kappa \alpha_0^2}{(x-1)^2} + \frac{d_1 \kappa \alpha_0^2}{(x-1)^3} + \frac{80 d_1 \kappa \alpha_0^2}{3(x-2)^4} - \\
& \frac{2 d_1 \kappa \alpha_0^2}{3(x-1)^4} + \frac{d_1 \alpha_0^2}{x-1} - \frac{d_1 \alpha_0^2}{(x-1)^2} + \frac{d_1 \alpha_0^2}{(x-1)^3} + \frac{80 d_1 \alpha_0^2}{3(x-2)^4} - \frac{2 d_1 \alpha_0^2}{3(x-1)^4} - \frac{4 d_1 \alpha_0}{9} + \frac{73 d_1 x \alpha_0}{18} - \frac{4 d_1 \kappa \alpha_0}{9} + \frac{73 d_1 x \kappa \alpha_0}{18} + \\
& \frac{80 d_1 \kappa \alpha_0}{9(x-2)} - \frac{65 d_1 \kappa \alpha_0}{6(x-1)} - \frac{80 d_1 \kappa \alpha_0}{9(x-2)^2} + \frac{25 d_1 \kappa \alpha_0}{9(x-1)^2} - \frac{35 d_1 \kappa \alpha_0}{18(x-1)^3} - \frac{160 d_1 \kappa \alpha_0}{(x-2)^4} - \frac{19 d_1 \kappa \alpha_0}{18(x-1)^4} - \frac{320 d_1 \kappa \alpha_0}{3(x-2)^5} + \frac{25 d_1 \kappa \alpha_0}{18(x-1)^5} + \\
& \frac{80 d_1 \alpha_0}{9(x-2)} - \frac{65 d_1 \alpha_0}{6(x-1)} - \frac{80 d_1 \alpha_0}{9(x-2)^2} + \frac{25 d_1 \alpha_0}{9(x-1)^2} - \frac{35 d_1 \alpha_0}{18(x-1)^3} - \frac{160 d_1 \alpha_0}{(x-2)^4} - \frac{5 d_1 \alpha_0}{2(x-1)^4} - \frac{320 d_1 \alpha_0}{3(x-2)^5} + \frac{17 d_1 \alpha_0}{6(x-1)^5} - d_1 - \\
& \frac{25 d_1 x}{18} - d_1 \kappa - \frac{25 d_1 x \kappa}{18} + \frac{64 d_1 \kappa}{9(x-2)} - \frac{43 d_1 \kappa}{6(x-1)} - \frac{80 d_1 \kappa}{9(x-2)^2} + \frac{8 d_1 \kappa}{9(x-1)^2} + \frac{160 d_1 \kappa}{9(x-2)^3} + \frac{d_1 \kappa}{18(x-1)^3} + \frac{320 d_1 \kappa}{3(x-2)^4} + \\
& \frac{43 d_1 \kappa}{18(x-1)^4} + \frac{640 d_1 \kappa}{3(x-2)^5} + \frac{25 d_1 \kappa}{18(x-1)^5} + \frac{64 d_1}{9(x-2)} - \frac{43 d_1}{6(x-1)} - \frac{80 d_1}{9(x-2)^2} + \frac{8 d_1}{9(x-1)^2} + \frac{160 d_1}{9(x-2)^3} + \frac{d_1}{18(x-1)^3} + \\
& \frac{320 d_1}{3(x-2)^4} + \frac{23 d_1}{6(x-1)^4} + \frac{640 d_1}{3(x-2)^5} + \frac{17 d_1}{6(x-1)^5} \Big) H(1; \alpha_0) + \left( \frac{4 \kappa \alpha_0}{(x-1)^4} - \frac{4 \kappa \alpha_0}{(x-1)^5} + \frac{4 \alpha_0}{3(x-1)^4} - \frac{4 \alpha_0}{3(x-1)^5} - \frac{4 \kappa}{(x-1)^4} - \right. \\
& \left. \frac{4 \kappa}{(x-1)^5} - \frac{4}{3(x-1)^4} - \frac{4}{3(x-1)^5} \right) H(0, 0; \alpha_0) + \left( \frac{4 \alpha_0 \kappa d_1}{3(x-1)^4} - \frac{4 \kappa d_1}{3(x-1)^4} - \frac{4 \alpha_0 \kappa d_1}{3(x-1)^5} - \frac{4 \kappa d_1}{3(x-1)^5} + \frac{4 \alpha_0 d_1}{3(x-1)^4} - \right. \\
& \left. \frac{4 d_1}{3(x-1)^4} - \frac{4 \alpha_0 d_1}{3(x-1)^5} - \frac{4 d_1}{3(x-1)^5} \right) H(0, 1; \alpha_0) + \left( \frac{4 \alpha_0 \kappa d_1^2}{3(x-1)^4} - \frac{4 \kappa d_1^2}{3(x-1)^4} - \frac{4 \alpha_0 d_1^2}{3(x-1)^5} - \frac{4 d_1^2}{3(x-1)^5} \right) H(1, 1; \alpha_0) + \\
& \frac{346 d_1}{27(x-2)} - \frac{488}{27(x-2)} - \frac{947 d_1}{72(x-1)} + \frac{443}{24(x-1)} - \frac{392 d_1}{27(x-2)^2} + \frac{592}{27(x-2)^2} + \frac{23 d_1}{27(x-1)^2} - \frac{52}{27(x-1)^2} + \frac{784 d_1}{27(x-2)^3} - \\
& \frac{1184}{27(x-2)^3} + \frac{73 d_1}{216(x-1)^3} - \frac{83}{216(x-1)^3} + \frac{256 d_1}{3(x-2)^4} - \frac{2048}{9(x-2)^4} + \frac{367 d_1}{216(x-1)^4} + \frac{\pi^2}{18(x-1)^4} - \frac{725}{108(x-1)^4} + \\
& \frac{512 d_1}{3(x-2)^5} - \frac{4096}{9(x-2)^5} + \frac{205 d_1}{216(x-1)^5} + \frac{\pi^2}{18(x-1)^5} - \frac{955}{216(x-1)^5} + \frac{55}{24} \Big) + \left( \frac{4 \alpha_0 d_1^2}{3(x-1)^4} - \frac{4 d_1^2}{3(x-1)^4} - \frac{4 \alpha_0 d_1^2}{3(x-1)^5} - \right. \\
& \left. \frac{4 d_1^2}{3(x-1)^5} - \frac{4 \alpha_0 d_1}{3} + \frac{2 \alpha_0 x d_1}{3} - \frac{2 x d_1}{3} - \frac{4 \alpha_0 \kappa d_1}{3} + \frac{2}{3} \alpha_0 x \kappa d_1 - \frac{2 x \kappa d_1}{3} + \frac{160 \alpha_0 \kappa d_1}{3(x-2)^4} - \frac{160 \kappa d_1}{3(x-2)^4} - \frac{4 \alpha_0 \kappa d_1}{3(x-1)^4} + \right. \\
& \frac{4 \kappa d_1}{3(x-1)^4} + \frac{320 \alpha_0 \kappa d_1}{3(x-2)^5} - \frac{640 \kappa d_1}{3(x-2)^5} + \frac{4 \alpha_0 \kappa d_1}{3(x-1)^5} + \frac{4 \kappa d_1}{3(x-1)^5} - \frac{640 \kappa d_1}{3(x-2)^6} + \frac{160 \alpha_0 d_1}{3(x-2)^4} - \frac{160 d_1}{3(x-2)^4} - \frac{4 \alpha_0 d_1}{3(x-1)^4} + \\
& \frac{4 d_1}{3(x-1)^4} + \frac{320 \alpha_0 d_1}{3(x-2)^5} - \frac{640 d_1}{3(x-2)^5} + \frac{4 \alpha_0 d_1}{3(x-1)^5} + \frac{4 d_1}{3(x-1)^5} - \frac{640 d_1}{3(x-2)^6} + \frac{2 \alpha_0}{3} - \frac{\alpha_0 x}{3} + \frac{x}{3} + 2 \alpha_0 \kappa - \alpha_0 x \kappa + x \kappa - \\
& \frac{80 \alpha_0 \kappa}{(x-2)^4} + \frac{80 \kappa}{(x-2)^4} + \frac{\alpha_0 \kappa}{(x-1)^4} - \frac{\kappa}{(x-1)^4} - \frac{160 \alpha_0 \kappa}{(x-2)^5} + \frac{320 \kappa}{(x-2)^5} - \frac{\alpha_0 \kappa}{(x-1)^5} - \frac{\kappa}{(x-1)^5} + \frac{320 \kappa}{(x-2)^6} - \frac{80 \alpha_0}{3(x-2)^4} + \frac{80}{3(x-2)^4} + \\
& \left. \frac{\alpha_0}{3(x-1)^4} - \frac{1}{3(x-1)^4} - \frac{160 \alpha_0}{3(x-2)^5} + \frac{320}{3(x-2)^5} - \frac{\alpha_0}{3(x-1)^5} - \frac{1}{3(x-1)^5} + \frac{320}{3(x-2)^6} \right) H(0; \alpha_0) H(1, 1; x) + \left( - \right. \\
& \frac{2 d_1 \alpha_0}{3} + \frac{17 x \alpha_0}{12} - \frac{2 d_1 \kappa \alpha_0}{3} + \frac{101 x \kappa \alpha_0}{36} - \frac{85 d_1 \kappa \alpha_0}{9(x-2)} + \frac{40 \kappa \alpha_0}{3(x-2)} + \frac{94 d_1 \kappa \alpha_0}{9(x-1)} - \frac{65 \kappa \alpha_0}{4(x-1)} + \frac{100 d_1 \kappa \alpha_0}{9(x-2)^2} - \frac{40 \kappa \alpha_0}{3(x-2)^2} - \\
& \frac{20 d_1 \kappa \alpha_0}{9(x-1)^2} + \frac{25 \kappa \alpha_0}{6(x-1)^2} - \frac{40 d_1 \kappa \alpha_0}{3(x-2)^3} + \frac{17 d_1 \kappa \alpha_0}{18(x-1)^3} - \frac{35 \kappa \alpha_0}{12(x-1)^3} + \frac{64 d_1 \kappa \alpha_0}{(x-2)^4} - \frac{1088 \kappa \alpha_0}{9(x-2)^4} + \frac{19 d_1 \kappa \alpha_0}{18(x-1)^4} - \frac{47 \kappa \alpha_0}{36(x-1)^4} + \\
& \frac{64 d_1 \kappa \alpha_0}{3(x-2)^5} + \frac{704 \kappa \alpha_0}{9(x-2)^5} - \frac{25 d_1 \kappa \alpha_0}{18(x-1)^5} + \frac{101 \kappa \alpha_0}{36(x-1)^5} - \frac{37 \kappa \alpha_0}{9} - \frac{85 d_1 \alpha_0}{9(x-2)} + \frac{40 \alpha_0}{9(x-2)} + \frac{94 d_1 \alpha_0}{9(x-1)} - \frac{65 \alpha_0}{12(x-1)} + \frac{100 d_1 \alpha_0}{9(x-2)^2} - \\
& \frac{40 \alpha_0}{9(x-2)^2} - \frac{20 d_1 \alpha_0}{9(x-1)^2} + \frac{25 \alpha_0}{18(x-1)^2} - \frac{40 d_1 \alpha_0}{3(x-2)^3} + \frac{17 d_1 \alpha_0}{18(x-1)^3} - \frac{35 \alpha_0}{36(x-1)^3} + \frac{64 d_1 \alpha_0}{(x-2)^4} + \frac{64 \alpha_0}{9(x-2)^4} + \frac{5 d_1 \alpha_0}{2(x-1)^4} - \\
& \frac{11 \alpha_0}{12(x-1)^4} + \frac{64 d_1 \alpha_0}{3(x-2)^5} + \frac{1088 \alpha_0}{9(x-2)^5} - \frac{17 d_1 \alpha_0}{6(x-1)^5} + \frac{17 \alpha_0}{12(x-1)^5} - \frac{7 \alpha_0}{3} + \frac{2 d_1}{3} - \frac{17 x}{12} + \frac{2 d_1 \kappa}{3} - \frac{101 x \kappa}{36} - \frac{62 d_1 \kappa}{9(x-2)} + \\
& \frac{32 \kappa}{3(x-2)} + \frac{65 d_1 \kappa}{9(x-1)} - \frac{43 \kappa}{4(x-1)} + \frac{70 d_1 \kappa}{9(x-2)^2} - \frac{40 \kappa}{3(x-2)^2} - \frac{d_1 \kappa}{(x-1)^2} + \frac{4 \kappa}{3(x-1)^2} - \frac{80 d_1 \kappa}{9(x-2)^3} + \frac{80 \kappa}{3(x-2)^3} + \frac{5 d_1 \kappa}{18(x-1)^3} + \\
& \frac{\kappa}{12(x-1)^3} - \frac{112 d_1 \kappa}{3(x-2)^4} + \frac{1088 \kappa}{9(x-2)^4} - \frac{31 d_1 \kappa}{18(x-1)^4} + \frac{155 \kappa}{36(x-1)^4} - \frac{448 d_1 \kappa}{3(x-2)^5} + \frac{1472 \kappa}{9(x-2)^5} - \frac{25 d_1 \kappa}{18(x-1)^5} + \frac{101 \kappa}{36(x-1)^5} - \\
& \frac{128 d_1 \kappa}{3(x-2)^6} - \frac{1408 \kappa}{9(x-2)^6} - \frac{3 \kappa}{2} + \left( - \frac{2 x \alpha_0}{3} - 2 x \kappa \alpha_0 - \frac{160 \kappa \alpha_0}{(x-2)^4} - \frac{4 d_1 \kappa \alpha_0}{3(x-1)^4} + \frac{2 \kappa \alpha_0}{(x-1)^4} - \frac{320 \kappa \alpha_0}{(x-2)^5} + \frac{4 d_1 \kappa \alpha_0}{3(x-1)^5} - \right. \\
& \frac{2 \kappa \alpha_0}{(x-1)^5} + 4 \kappa \alpha_0 - \frac{160 \alpha_0}{3(x-2)^4} - \frac{4 d_1 \alpha_0}{3(x-1)^4} + \frac{2 \alpha_0}{3(x-1)^4} - \frac{320 \alpha_0}{3(x-2)^5} + \frac{4 d_1 \alpha_0}{3(x-1)^5} - \frac{2 \alpha_0}{3(x-1)^5} + \frac{4 \alpha_0}{3} + \frac{2 x}{3} + 2 x \kappa + \\
& \frac{160 \kappa}{(x-2)^4} + \frac{4 d_1 \kappa}{3(x-1)^4} - \frac{2 \kappa}{(x-1)^4} + \frac{640 \kappa}{(x-2)^5} + \frac{4 d_1 \kappa}{3(x-1)^5} - \frac{2 \kappa}{(x-1)^5} + \frac{640 \kappa}{(x-2)^6} + \frac{160}{3(x-2)^4} + \frac{4 d_1}{3(x-1)^4} - \frac{2}{3(x-1)^4} + \\
& \left. \frac{640}{3(x-2)^5} + \frac{4 d_1}{3(x-1)^5} - \frac{2}{3(x-1)^5} + \frac{640}{3(x-2)^6} \right) H(0; \alpha_0) + \left( - \frac{4 \alpha_0 d_1^2}{3(x-1)^4} + \frac{4 d_1^2}{3(x-1)^4} + \frac{4 \alpha_0 d_1^2}{3(x-1)^5} + \frac{4 d_1^2}{3(x-1)^5} + \right. \\
& \left. \frac{4 \alpha_0 d_1}{3} - \frac{2 \alpha_0 x d_1}{3} + \frac{2 x d_1}{3} + \frac{4 \alpha_0 \kappa d_1}{3} - \frac{2}{3} \alpha_0 x \kappa d_1 + \frac{2 x \kappa d_1}{3} - \frac{160 \alpha_0 \kappa d_1}{3(x-2)^4} + \frac{160 \kappa d_1}{3(x-2)^4} + \frac{2 \alpha_0 \kappa d_1}{3(x-1)^4} - \frac{2 \kappa d_1}{3(x-1)^4} - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{320\alpha_0\kappa d_1}{3(x-2)^5} + \frac{640\kappa d_1}{3(x-2)^5} - \frac{2\alpha_0\kappa d_1}{3(x-1)^5} - \frac{2\kappa d_1}{3(x-1)^5} + \frac{640\kappa d_1}{3(x-2)^6} - \frac{160\alpha_0 d_1}{3(x-2)^4} + \frac{160d_1}{3(x-2)^4} + \frac{2\alpha_0 d_1}{3(x-1)^4} - \frac{2d_1}{3(x-1)^4} - \\
& \frac{320\alpha_0 d_1}{3(x-2)^5} + \frac{640d_1}{3(x-2)^5} - \frac{2\alpha_0 d_1}{3(x-1)^5} - \frac{2d_1}{3(x-1)^5} + \frac{640d_1}{3(x-2)^6} \Big) H(1; \alpha_0) - \frac{62d_1}{9(x-2)} + \frac{32}{9(x-2)} + \frac{65d_1}{9(x-1)} - \frac{43}{12(x-1)} + \\
& \frac{70d_1}{9(x-2)^2} - \frac{40}{9(x-2)^2} - \frac{d_1}{(x-1)^2} + \frac{4}{9(x-1)^2} - \frac{80d_1}{9(x-2)^3} + \frac{80}{9(x-2)^3} + \frac{5d_1}{18(x-1)^3} + \frac{1}{36(x-1)^3} - \frac{112d_1}{3(x-2)^4} - \\
& \frac{64}{9(x-2)^4} - \frac{19d_1}{6(x-1)^4} + \frac{23}{12(x-1)^4} - \frac{448d_1}{3(x-2)^5} - \frac{1216}{9(x-2)^5} - \frac{17d_1}{6(x-1)^5} + \frac{17}{12(x-1)^5} - \frac{128d_1}{3(x-2)^6} - \frac{2176}{9(x-2)^6} - \\
& \frac{1}{2} \Big) H(1, c_1(\alpha_0); x) + \left( -\frac{160d_1\kappa\alpha_0}{3(x-2)^4} + \frac{160\kappa\alpha_0}{(x-2)^4} - \frac{320d_1\kappa\alpha_0}{3(x-2)^5} + \frac{320\kappa\alpha_0}{(x-2)^5} - \frac{160d_1\alpha_0}{3(x-2)^4} + \frac{160\alpha_0}{3(x-2)^4} - \frac{320d_1\alpha_0}{3(x-2)^5} + \right. \\
& \frac{320\alpha_0}{3(x-2)^5} + \frac{160d_1\kappa}{3(x-2)^4} - \frac{160\kappa}{(x-2)^4} + \frac{640d_1\kappa}{3(x-2)^5} - \frac{640\kappa}{(x-2)^5} + \frac{640d_1\kappa}{3(x-2)^6} - \frac{640\kappa}{(x-2)^6} + \frac{160d_1}{3(x-2)^4} - \frac{160}{3(x-2)^4} + \frac{640d_1}{3(x-2)^5} - \\
& \left. \frac{640}{3(x-2)^5} + \frac{640d_1}{3(x-2)^6} - \frac{640}{3(x-2)^6} \right) H(0; \alpha_0) H(2, 1; x) + \left( -\frac{64\alpha_0 d_1^2}{3(x-2)^4} + \frac{64d_1^2}{3(x-2)^4} - \frac{128\alpha_0 d_1^2}{3(x-2)^5} + \frac{256d_1^2}{3(x-2)^5} + \right. \\
& \frac{256d_1^2}{3(x-2)^6} + \frac{128\alpha_0\kappa d_1}{3(x-2)^4} - \frac{128\kappa d_1}{3(x-2)^4} + \frac{256\alpha_0\kappa d_1}{3(x-2)^5} - \frac{512\kappa d_1}{3(x-2)^5} - \frac{512\kappa d_1}{3(x-2)^6} - \frac{896\alpha_0 d_1}{9(x-2)^4} + \frac{896d_1}{9(x-2)^4} - \frac{1792\alpha_0 d_1}{9(x-2)^5} + \\
& \frac{3584d_1}{9(x-2)^5} + \frac{3584d_1}{9(x-2)^6} + \frac{704\alpha_0\kappa}{9(x-2)^4} - \frac{704\kappa}{9(x-2)^4} + \frac{1408\alpha_0\kappa}{9(x-2)^5} - \frac{2816\kappa}{9(x-2)^5} - \frac{2816\kappa}{9(x-2)^6} + \left( \frac{320d_1\kappa\alpha_0}{3(x-2)^4} - \frac{320\kappa\alpha_0}{(x-2)^4} + \right. \\
& \frac{640d_1\kappa\alpha_0}{3(x-2)^5} - \frac{640\kappa\alpha_0}{(x-2)^5} + \frac{320d_1\alpha_0}{3(x-2)^4} - \frac{320\alpha_0}{3(x-2)^4} + \frac{640d_1\alpha_0}{3(x-2)^5} - \frac{640\alpha_0}{3(x-2)^5} - \frac{320d_1\kappa}{3(x-2)^4} + \frac{320\kappa}{(x-2)^4} - \frac{1280d_1\kappa}{3(x-2)^5} + \\
& \left. \frac{1280\kappa}{(x-2)^5} - \frac{1280d_1\kappa}{3(x-2)^6} + \frac{1280\kappa}{(x-2)^6} - \frac{320d_1}{3(x-2)^4} + \frac{320}{3(x-2)^4} - \frac{1280d_1}{3(x-2)^5} + \frac{1280}{3(x-2)^5} - \frac{1280d_1}{3(x-2)^6} + \frac{1280}{3(x-2)^6} \right) H(0; \alpha_0) + \\
& \left( \frac{320\alpha_0 d_1^2}{3(x-2)^4} - \frac{320d_1^2}{3(x-2)^4} + \frac{640\alpha_0 d_1^2}{3(x-2)^5} - \frac{1280d_1^2}{3(x-2)^5} - \frac{1280d_1^2}{3(x-2)^6} - \frac{320\alpha_0\kappa d_1}{3(x-2)^4} + \frac{320\kappa d_1}{3(x-2)^4} - \frac{640\alpha_0\kappa d_1}{3(x-2)^5} + \frac{1280\kappa d_1}{3(x-2)^5} + \right. \\
& \left. \frac{1280\kappa d_1}{3(x-2)^6} - \frac{320\alpha_0 d_1}{3(x-2)^4} + \frac{320d_1}{3(x-2)^4} - \frac{640\alpha_0 d_1}{3(x-2)^5} + \frac{1280d_1}{3(x-2)^5} + \frac{1280d_1}{3(x-2)^6} \right) H(1; \alpha_0) + \frac{1088\alpha_0}{9(x-2)^4} - \frac{1088}{9(x-2)^4} + \\
& \frac{2176\alpha_0}{9(x-2)^5} - \frac{4352}{9(x-2)^5} - \frac{4352}{9(x-2)^6} \Big) H(2, c_2(\alpha_0); x) + \left( \frac{x\alpha_0^5}{12} + \frac{1}{4}x\kappa\alpha_0^5 + \frac{\kappa\alpha_0^5}{2(x-2)} - \frac{\kappa\alpha_0^5}{4(x-1)} + \frac{\alpha_0^5}{6(x-2)} - \right. \\
& \frac{\alpha_0^5}{12(x-1)} - \frac{19x\alpha_0^4}{36} - \frac{19}{12}x\kappa\alpha_0^4 - \frac{5\kappa\alpha_0^4}{3(x-2)} + \frac{13\kappa\alpha_0^4}{12(x-1)} + \frac{5\kappa\alpha_0^4}{3(x-2)^2} - \frac{\kappa\alpha_0^4}{3(x-1)^2} + \frac{\kappa\alpha_0^4}{6} - \frac{5\alpha_0^4}{9(x-2)} + \frac{13\alpha_0^4}{36(x-1)} + \\
& \frac{5\alpha_0^4}{9(x-2)^2} - \frac{\alpha_0^4}{9(x-1)^2} + \frac{\alpha_0^4}{18} + \frac{13x\alpha_0^3}{9} + \frac{13}{3}x\kappa\alpha_0^3 + \frac{5\kappa\alpha_0^3}{3(x-2)} - \frac{11\kappa\alpha_0^3}{6(x-1)} - \frac{10\kappa\alpha_0^3}{3(x-2)^2} + \frac{7\kappa\alpha_0^3}{6(x-1)^2} + \frac{20\kappa\alpha_0^3}{3(x-2)^3} - \\
& \frac{\kappa\alpha_0^3}{2(x-1)^3} - \kappa\alpha_0^3 + \frac{5\alpha_0^3}{9(x-2)} - \frac{11\alpha_0^3}{18(x-1)} - \frac{10\alpha_0^3}{9(x-2)^2} + \frac{7\alpha_0^3}{18(x-1)^2} + \frac{20\alpha_0^3}{9(x-2)^3} - \frac{\alpha_0^3}{6(x-1)^3} - \frac{\alpha_0^3}{3} - \frac{7x\alpha_0^2}{3} - 7x\kappa\alpha_0^2 + \\
& \frac{3\kappa\alpha_0^2}{2(x-1)} - \frac{3\kappa\alpha_0^2}{2(x-1)^2} + \frac{3\kappa\alpha_0^2}{2(x-1)^3} + \frac{40\kappa\alpha_0^2}{(x-2)^4} - \frac{\kappa\alpha_0^2}{(x-1)^4} + 3\kappa\alpha_0^2 + \frac{\alpha_0^2}{2(x-1)} - \frac{\alpha_0^2}{2(x-1)^2} + \frac{\alpha_0^2}{2(x-1)^3} + \frac{40\alpha_0^2}{3(x-2)^4} - \\
& \frac{\alpha_0^2}{3(x-1)^4} + \alpha_0^2 + \frac{73x\alpha_0}{36} + \frac{73x\kappa\alpha_0}{12} + \frac{40\kappa\alpha_0}{3(x-2)} - \frac{65\kappa\alpha_0}{4(x-1)} - \frac{40\kappa\alpha_0}{3(x-2)^2} + \frac{25\kappa\alpha_0}{6(x-1)^2} - \frac{35\kappa\alpha_0}{12(x-1)^3} - \frac{240\kappa\alpha_0}{(x-2)^4} - \\
& \frac{83\kappa\alpha_0}{36(x-1)^4} - \frac{160\kappa\alpha_0}{(x-2)^5} + \frac{101\kappa\alpha_0}{36(x-1)^5} - \frac{2\kappa\alpha_0}{3} + \frac{40\alpha_0}{9(x-2)} - \frac{65\alpha_0}{12(x-1)} - \frac{40\alpha_0}{9(x-2)^2} + \frac{25\alpha_0}{18(x-1)^2} - \frac{35\alpha_0}{36(x-1)^3} - \frac{80\alpha_0}{(x-2)^4} - \\
& \frac{5\alpha_0}{4(x-1)^4} - \frac{160\alpha_0}{3(x-2)^5} + \frac{17\alpha_0}{12(x-1)^5} - \frac{2\alpha_0}{9} - \frac{25x}{36} - \frac{25x\kappa}{12} + \frac{32\kappa}{3(x-2)} - \frac{43\kappa}{4(x-1)} - \frac{40\kappa}{3(x-2)^2} + \frac{4\kappa}{3(x-1)^2} + \frac{80\kappa}{3(x-2)^3} + \\
& \frac{\kappa}{12(x-1)^3} + \frac{160\kappa}{(x-2)^4} + \frac{155\kappa}{36(x-1)^4} + \frac{320\kappa}{(x-2)^5} + \frac{101\kappa}{36(x-1)^5} - \frac{3\kappa}{2} + \left( \frac{2\kappa\alpha_0}{(x-1)^4} - \frac{2\kappa\alpha_0}{(x-1)^5} + \frac{2\alpha_0}{3(x-1)^4} - \frac{2\alpha_0}{3(x-1)^5} - \right. \\
& \left. \frac{2\kappa}{(x-1)^4} - \frac{2\kappa}{(x-1)^5} - \frac{2}{3(x-1)^4} - \frac{2}{3(x-1)^5} \right) H(0; \alpha_0) + \left( \frac{2\alpha_0\kappa d_1}{3(x-1)^4} - \frac{2\kappa d_1}{3(x-1)^4} - \frac{2\alpha_0\kappa d_1}{3(x-1)^5} - \frac{2\kappa d_1}{3(x-1)^5} + \frac{2\alpha_0 d_1}{3(x-1)^4} - \right. \\
& \left. \frac{2d_1}{3(x-1)^4} - \frac{2\alpha_0 d_1}{3(x-1)^5} - \frac{2d_1}{3(x-1)^5} \right) H(1; \alpha_0) + \frac{32}{9(x-2)} - \frac{43}{12(x-1)} - \frac{40}{9(x-2)^2} + \frac{4}{9(x-1)^2} + \frac{80}{9(x-2)^3} + \\
& \frac{1}{36(x-1)^3} + \frac{160}{3(x-2)^4} + \frac{23}{12(x-1)^4} + \frac{320}{3(x-2)^5} + \frac{17}{12(x-1)^5} - \frac{1}{2} \Big) H(c_1(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{32d_1\kappa\alpha_0}{3(x-2)^4} - \right. \\
& \frac{352\kappa\alpha_0}{9(x-2)^4} - \frac{64d_1\kappa\alpha_0}{3(x-2)^5} - \frac{704\kappa\alpha_0}{9(x-2)^5} - \frac{32d_1\alpha_0}{3(x-2)^4} - \frac{544\alpha_0}{9(x-2)^4} - \frac{64d_1\alpha_0}{3(x-2)^5} - \frac{1088\alpha_0}{9(x-2)^5} + \frac{32d_1\kappa}{3(x-2)^4} + \frac{352\kappa}{9(x-2)^4} + \\
& \frac{128d_1\kappa}{3(x-2)^5} + \frac{1408\kappa}{9(x-2)^5} + \frac{128d_1\kappa}{3(x-2)^6} + \frac{1408\kappa}{9(x-2)^6} + \left( \frac{160\kappa\alpha_0}{(x-2)^4} + \frac{320\kappa\alpha_0}{(x-2)^5} + \frac{160\alpha_0}{3(x-2)^4} + \frac{320\alpha_0}{3(x-2)^5} - \frac{160\kappa}{(x-2)^4} - \frac{640\kappa}{(x-2)^5} - \right. \\
& \left. \frac{640\kappa}{(x-2)^6} - \frac{160}{3(x-2)^4} - \frac{640}{3(x-2)^5} - \frac{640}{3(x-2)^6} \right) H(0; \alpha_0) + \left( \frac{160\alpha_0\kappa d_1}{3(x-2)^4} - \frac{160\kappa d_1}{3(x-2)^4} + \frac{320\alpha_0\kappa d_1}{3(x-2)^5} - \frac{640\kappa d_1}{3(x-2)^5} - \right. \\
& \left. \frac{640\kappa d_1}{3(x-2)^6} + \frac{160\alpha_0 d_1}{3(x-2)^4} - \frac{160d_1}{3(x-2)^4} + \frac{320\alpha_0 d_1}{3(x-2)^5} - \frac{640d_1}{3(x-2)^5} - \frac{640d_1}{3(x-2)^6} \right) H(1; \alpha_0) + \frac{32d_1}{3(x-2)^4} + \frac{544}{9(x-2)^4} + \\
& \left. \frac{128d_1}{3(x-2)^5} + \frac{2176}{9(x-2)^5} + \frac{128d_1}{3(x-2)^6} + \frac{2176}{9(x-2)^6} \right) H(c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{4x\alpha_0}{3} + 4x\kappa\alpha_0 + \frac{320\kappa\alpha_0}{(x-2)^4} + \frac{4\kappa\alpha_0}{(x-1)^4} + \right. \\
& \frac{640\kappa\alpha_0}{(x-2)^5} - \frac{4\kappa\alpha_0}{(x-1)^5} - 8\kappa\alpha_0 + \frac{320\alpha_0}{3(x-2)^4} + \frac{4\alpha_0}{3(x-1)^4} + \frac{640\alpha_0}{3(x-2)^5} - \frac{4\alpha_0}{3(x-1)^5} - \frac{8\alpha_0}{3} - \frac{4x}{3} - 4x\kappa - \frac{320\kappa}{(x-2)^4} - \frac{4\kappa}{(x-1)^4} - \\
& \left. \frac{1280\kappa}{(x-2)^5} - \frac{4\kappa}{(x-1)^5} - \frac{1280\kappa}{(x-2)^6} - \frac{320}{3(x-2)^4} - \frac{4}{3(x-1)^4} - \frac{1280}{3(x-2)^5} - \frac{4}{3(x-1)^5} - \frac{1280}{3(x-2)^6} \right) H(0, 0, 0; \alpha_0) +
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{4x\alpha_0}{3} + 4x\kappa\alpha_0 - \frac{320\kappa\alpha_0}{(x-2)^4} - \frac{4\kappa\alpha_0}{(x-1)^4} - \frac{640\kappa\alpha_0}{(x-2)^5} + \frac{4\kappa\alpha_0}{(x-1)^5} - 8\kappa\alpha_0 - \frac{320\alpha_0}{3(x-2)^4} - \frac{4\alpha_0}{3(x-1)^4} - \frac{640\alpha_0}{3(x-2)^5} + \right. \\
& \left. \frac{4\alpha_0}{3(x-1)^5} - \frac{8\alpha_0}{3} - \frac{4x}{3} - 4x\kappa + \frac{320\kappa}{(x-2)^4} + \frac{4\kappa}{(x-1)^4} + \frac{1280\kappa}{(x-2)^5} + \frac{4\kappa}{(x-1)^5} + \frac{1280\kappa}{(x-2)^6} + \frac{320}{3(x-2)^4} + \frac{4}{3(x-1)^4} + \right. \\
& \left. \frac{1280}{3(x-2)^5} + \frac{4}{3(x-1)^5} + \frac{1280}{3(x-2)^6} \right) H(0, 0, 0; x) + \left( -\frac{8\alpha_0 d_1}{3} + \frac{4\alpha_0 x d_1}{3} - \frac{4x d_1}{3} - \frac{8\alpha_0 \kappa d_1}{3} + \frac{4}{3} \alpha_0 x \kappa d_1 - \right. \\
& \left. \frac{4x \kappa d_1}{3} + \frac{320\alpha_0 \kappa d_1}{3(x-2)^4} - \frac{320\kappa d_1}{3(x-2)^4} + \frac{4\alpha_0 \kappa d_1}{3(x-1)^4} - \frac{4\kappa d_1}{3(x-1)^4} + \frac{640\alpha_0 \kappa d_1}{3(x-2)^5} - \frac{1280\kappa d_1}{3(x-2)^5} - \frac{4\alpha_0 \kappa d_1}{3(x-1)^5} - \frac{4\kappa d_1}{3(x-1)^5} - \right. \\
& \left. \frac{1280\kappa d_1}{3(x-2)^6} + \frac{320\alpha_0 d_1}{3(x-2)^4} - \frac{320d_1}{3(x-2)^4} + \frac{4\alpha_0 d_1}{3(x-1)^4} - \frac{4d_1}{3(x-1)^4} + \frac{640\alpha_0 d_1}{3(x-2)^5} - \frac{1280d_1}{3(x-2)^5} - \frac{4\alpha_0 d_1}{3(x-1)^5} - \frac{4d_1}{3(x-1)^5} - \right. \\
& \left. \frac{1280d_1}{3(x-2)^6} \right) H(0, 0, 1; \alpha_0) + \left( -\frac{2x\alpha_0}{3} - 2x\kappa\alpha_0 - \frac{160\kappa\alpha_0}{(x-2)^4} + \frac{2\kappa\alpha_0}{(x-1)^4} - \frac{320\kappa\alpha_0}{(x-2)^5} - \frac{2\kappa\alpha_0}{(x-1)^5} + 4\kappa\alpha_0 - \frac{160\alpha_0}{(x-2)^4} + \right. \\
& \left. \frac{2\alpha_0}{3(x-1)^4} - \frac{320\alpha_0}{3(x-2)^5} - \frac{2\alpha_0}{3(x-1)^5} + \frac{4\alpha_0}{3} + \frac{2x}{3} + 2x\kappa + \frac{160\kappa}{(x-2)^4} - \frac{2\kappa}{(x-1)^4} + \frac{640\kappa}{(x-2)^5} - \frac{2\kappa}{(x-1)^5} + \frac{640\kappa}{(x-2)^6} + \right. \\
& \left. \frac{160}{3(x-2)^4} - \frac{2}{3(x-1)^4} + \frac{640}{3(x-2)^5} - \frac{2}{3(x-1)^5} + \frac{640}{3(x-2)^6} \right) H(0, 0, c_1(\alpha_0); x) + \left( \frac{320\kappa\alpha_0}{(x-2)^4} + \frac{640\kappa\alpha_0}{(x-2)^5} + \frac{320\alpha_0}{3(x-2)^4} + \right. \\
& \left. \frac{640\alpha_0}{3(x-2)^5} - \frac{320\kappa}{(x-2)^4} - \frac{1280\kappa}{(x-2)^5} - \frac{1280\kappa}{(x-2)^6} - \frac{320}{3(x-2)^4} - \frac{1280}{3(x-2)^5} - \frac{1280}{3(x-2)^6} \right) H(0, 0, c_2(\alpha_0); x) + \left( -\frac{8\alpha_0 d_1}{3} + \right. \\
& \left. \frac{4\alpha_0 x d_1}{3} - \frac{4x d_1}{3} - \frac{8\alpha_0 \kappa d_1}{3} + \frac{4}{3} \alpha_0 x \kappa d_1 - \frac{4x \kappa d_1}{3} + \frac{320\alpha_0 \kappa d_1}{3(x-2)^4} - \frac{320\kappa d_1}{3(x-2)^4} + \frac{4\alpha_0 \kappa d_1}{3(x-1)^4} - \frac{4\kappa d_1}{3(x-1)^4} + \frac{640\alpha_0 \kappa d_1}{3(x-2)^5} - \right. \\
& \left. \frac{1280\kappa d_1}{3(x-2)^5} - \frac{4\alpha_0 \kappa d_1}{3(x-1)^5} - \frac{4\kappa d_1}{3(x-1)^5} - \frac{1280\kappa d_1}{3(x-2)^6} + \frac{320\alpha_0 d_1}{3(x-2)^4} - \frac{320d_1}{3(x-2)^4} + \frac{4\alpha_0 d_1}{3(x-1)^4} - \frac{4d_1}{3(x-1)^4} + \frac{640\alpha_0 d_1}{3(x-2)^5} - \right. \\
& \left. \frac{1280d_1}{3(x-2)^5} - \frac{4\alpha_0 d_1}{3(x-1)^5} - \frac{4d_1}{3(x-1)^5} - \frac{1280d_1}{3(x-2)^6} \right) H(0, 1, 0; \alpha_0) + \left( \frac{4d_1\alpha_0}{3} - \frac{2d_1 x \alpha_0}{3} + \frac{2x\alpha_0}{3} + \frac{4d_1\kappa\alpha_0}{3} - \right. \\
& \left. \frac{2}{3} d_1 x \kappa \alpha_0 + 2x\kappa\alpha_0 - \frac{160d_1\kappa\alpha_0}{3(x-2)^4} + \frac{160\kappa\alpha_0}{(x-2)^4} + \frac{2d_1\kappa\alpha_0}{3(x-1)^4} - \frac{2\kappa\alpha_0}{(x-1)^4} - \frac{320d_1\kappa\alpha_0}{3(x-2)^5} + \frac{320\kappa\alpha_0}{(x-2)^5} - \frac{2d_1\kappa\alpha_0}{3(x-1)^5} + \right. \\
& \left. \frac{2\kappa\alpha_0}{(x-1)^5} - 4\kappa\alpha_0 - \frac{160d_1\alpha_0}{3(x-2)^4} + \frac{160\alpha_0}{3(x-2)^4} + \frac{2d_1\alpha_0}{3(x-1)^4} - \frac{2\alpha_0}{3(x-1)^4} - \frac{320d_1\alpha_0}{3(x-2)^5} + \frac{320\alpha_0}{3(x-2)^5} - \frac{2d_1\alpha_0}{3(x-1)^5} + \frac{2\alpha_0}{3(x-1)^5} - \right. \\
& \left. \frac{4\alpha_0}{3} + \frac{2d_1 x}{3} - \frac{2x}{3} + \frac{2d_1 x \kappa}{3} - 2x\kappa + \frac{160d_1\kappa}{3(x-2)^4} - \frac{160\kappa}{(x-2)^4} - \frac{2d_1\kappa}{3(x-1)^4} + \frac{2\kappa}{(x-1)^4} + \frac{640d_1\kappa}{3(x-2)^5} - \frac{640\kappa}{(x-2)^5} - \frac{2d_1\kappa}{3(x-1)^5} + \right. \\
& \left. \frac{2\kappa}{(x-1)^5} + \frac{640d_1\kappa}{3(x-2)^6} - \frac{640\kappa}{(x-2)^6} + \frac{160d_1}{3(x-2)^4} - \frac{160}{3(x-2)^4} - \frac{2d_1}{3(x-1)^4} + \frac{2}{3(x-1)^4} + \frac{640d_1}{3(x-2)^5} - \frac{640}{3(x-2)^5} - \frac{2d_1}{3(x-1)^5} + \right. \\
& \left. \frac{2}{3(x-1)^5} + \frac{640d_1}{3(x-2)^6} - \frac{640}{3(x-2)^6} \right) H(0, 1, 0; x) + \left( -\frac{8\alpha_0 d_1^2}{3} + \frac{4}{3} \alpha_0 x d_1^2 - \frac{4x d_1^2}{3} + \frac{320\alpha_0 d_1^2}{3(x-2)^4} - \frac{320d_1^2}{3(x-2)^4} + \right. \\
& \left. \frac{4\alpha_0 d_1^2}{3(x-1)^4} - \frac{4d_1^2}{3(x-1)^4} + \frac{640\alpha_0 d_1^2}{3(x-2)^5} - \frac{1280d_1^2}{3(x-2)^5} - \frac{4\alpha_0 d_1^2}{3(x-1)^5} - \frac{4d_1^2}{3(x-1)^5} - \frac{1280d_1^2}{3(x-2)^6} \right) H(0, 1, 1; \alpha_0) + \left( -\frac{4d_1\alpha_0}{3} + \right. \\
& \left. \frac{2d_1 x \alpha_0}{3} - \frac{2x\alpha_0}{3} - \frac{4d_1\kappa\alpha_0}{3} + \frac{2}{3} d_1 x \kappa \alpha_0 - 2x\kappa\alpha_0 + \frac{160d_1\kappa\alpha_0}{3(x-2)^4} - \frac{160\kappa\alpha_0}{(x-2)^4} - \frac{2d_1\kappa\alpha_0}{3(x-1)^4} + \frac{2\kappa\alpha_0}{(x-1)^4} + \frac{320d_1\kappa\alpha_0}{3(x-2)^5} - \right. \\
& \left. \frac{320\kappa\alpha_0}{(x-2)^5} + \frac{2d_1\kappa\alpha_0}{3(x-1)^5} - \frac{2\kappa\alpha_0}{(x-1)^5} + 4\kappa\alpha_0 + \frac{160d_1\alpha_0}{3(x-2)^4} - \frac{160\alpha_0}{3(x-2)^4} - \frac{2d_1\alpha_0}{3(x-1)^4} + \frac{2\alpha_0}{3(x-1)^4} + \frac{320d_1\alpha_0}{3(x-2)^5} - \frac{320\alpha_0}{3(x-2)^5} + \right. \\
& \left. \frac{2d_1\alpha_0}{3(x-1)^5} - \frac{2\alpha_0}{3(x-1)^5} + \frac{4\alpha_0}{3} - \frac{2d_1 x}{3} + \frac{2x}{3} - \frac{2d_1 x \kappa}{3} + 2x\kappa - \frac{160d_1\kappa}{3(x-2)^4} + \frac{160\kappa}{(x-2)^4} + \frac{2d_1\kappa}{3(x-1)^4} - \frac{2\kappa}{(x-1)^4} - \frac{640d_1\kappa}{3(x-2)^5} + \right. \\
& \left. \frac{640\kappa}{(x-2)^5} + \frac{2d_1\kappa}{3(x-1)^5} - \frac{2\kappa}{(x-1)^5} - \frac{640d_1\kappa}{3(x-2)^6} + \frac{640\kappa}{(x-2)^6} - \frac{160d_1}{3(x-2)^4} + \frac{160}{3(x-2)^4} + \frac{2d_1}{3(x-1)^4} - \frac{2}{3(x-1)^4} - \frac{640d_1}{3(x-2)^5} + \right. \\
& \left. \frac{640}{3(x-2)^5} + \frac{2d_1}{3(x-1)^5} - \frac{2}{3(x-1)^5} - \frac{640d_1}{3(x-2)^6} + \frac{640}{3(x-2)^6} \right) H(0, 1, c_1(\alpha_0); x) + \left( -\frac{320d_1\kappa\alpha_0}{3(x-2)^4} + \frac{320\kappa\alpha_0}{(x-2)^4} - \right. \\
& \left. \frac{640d_1\kappa\alpha_0}{3(x-2)^5} + \frac{640\kappa\alpha_0}{(x-2)^5} - \frac{320d_1\alpha_0}{3(x-2)^4} + \frac{320\alpha_0}{3(x-2)^4} - \frac{640d_1\alpha_0}{3(x-2)^5} + \frac{640\alpha_0}{3(x-2)^5} + \frac{320d_1\kappa}{3(x-2)^4} - \frac{320\kappa}{(x-2)^4} + \frac{1280d_1\kappa}{3(x-2)^5} - \frac{1280\kappa}{(x-2)^5} + \right. \\
& \left. \frac{1280d_1\kappa}{3(x-2)^6} - \frac{1280\kappa}{(x-2)^6} + \frac{320d_1}{3(x-2)^4} - \frac{320}{3(x-2)^4} + \frac{1280d_1}{3(x-2)^5} - \frac{1280}{3(x-2)^5} + \frac{1280d_1}{3(x-2)^6} - \frac{1280}{3(x-2)^6} \right) H(0, 2, 0; x) + \left( \right. \\
& \left. \frac{320d_1\kappa\alpha_0}{3(x-2)^4} - \frac{320\kappa\alpha_0}{(x-2)^4} + \frac{640d_1\kappa\alpha_0}{3(x-2)^5} - \frac{640\kappa\alpha_0}{(x-2)^5} + \frac{320d_1\alpha_0}{3(x-2)^4} - \frac{320\alpha_0}{3(x-2)^4} + \frac{640d_1\alpha_0}{3(x-2)^5} - \frac{640\alpha_0}{3(x-2)^5} - \frac{320d_1\kappa}{3(x-2)^4} + \right. \\
& \left. \frac{320\kappa}{(x-2)^4} - \frac{1280d_1\kappa}{3(x-2)^5} + \frac{1280\kappa}{(x-2)^5} - \frac{1280d_1\kappa}{3(x-2)^6} + \frac{1280\kappa}{(x-2)^6} - \frac{320d_1}{3(x-2)^4} + \frac{320}{3(x-2)^4} - \frac{1280d_1}{3(x-2)^5} + \frac{1280}{3(x-2)^5} - \frac{1280d_1}{3(x-2)^6} + \right. \\
& \left. \frac{1280}{3(x-2)^6} \right) H(0, 2, c_2(\alpha_0); x) + \left( -\frac{x\alpha_0}{3} - x\kappa\alpha_0 - \frac{80\kappa\alpha_0}{(x-2)^4} + \frac{\kappa\alpha_0}{(x-1)^4} - \frac{160\kappa\alpha_0}{(x-2)^5} - \frac{\kappa\alpha_0}{(x-1)^5} + 2\kappa\alpha_0 - \frac{80\alpha_0}{3(x-2)^4} + \right. \\
& \left. \frac{\alpha_0}{3(x-1)^4} - \frac{160\alpha_0}{3(x-2)^5} - \frac{\alpha_0}{3(x-1)^5} + \frac{2\alpha_0}{3} + \frac{x}{3} + x\kappa + \frac{80\kappa}{(x-2)^4} - \frac{\kappa}{(x-1)^4} + \frac{320\kappa}{(x-2)^5} - \frac{\kappa}{(x-1)^5} + \frac{320\kappa}{(x-2)^6} + \frac{80}{3(x-2)^4} - \right. \\
& \left. \frac{1}{3(x-1)^4} + \frac{320}{3(x-2)^5} - \frac{1}{3(x-1)^5} + \frac{320}{3(x-2)^6} \right) H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{160\kappa\alpha_0}{(x-2)^4} + \frac{320\kappa\alpha_0}{(x-2)^5} + \frac{160\alpha_0}{3(x-2)^4} + \right. \\
& \left. \frac{320\alpha_0}{3(x-2)^5} - \frac{160\kappa}{(x-2)^4} - \frac{640\kappa}{(x-2)^5} - \frac{640\kappa}{(x-2)^6} - \frac{160}{3(x-2)^4} - \frac{640}{3(x-2)^5} - \frac{640}{3(x-2)^6} \right) H(0, c_2(\alpha_0), c_1(\alpha_0); x) + \left( \right. \\
& \left. \frac{2x\alpha_0}{3} + 2x\kappa\alpha_0 + \frac{160\kappa\alpha_0}{(x-2)^4} + \frac{4d_1\kappa\alpha_0}{3(x-1)^4} - \frac{2\kappa\alpha_0}{(x-1)^4} + \frac{320\kappa\alpha_0}{(x-2)^5} - \frac{4d_1\kappa\alpha_0}{3(x-1)^5} + \frac{2\kappa\alpha_0}{(x-1)^5} - 4\kappa\alpha_0 + \frac{160\alpha_0}{3(x-2)^4} + \right. \\
& \left. \frac{4d_1\alpha_0}{3(x-1)^4} - \frac{2\alpha_0}{3(x-1)^4} + \frac{320\alpha_0}{3(x-2)^5} - \frac{4d_1\alpha_0}{3(x-1)^5} + \frac{2\alpha_0}{3(x-1)^5} - \frac{4\alpha_0}{3} - \frac{2x}{3} - 2x\kappa - \frac{160\kappa}{(x-2)^4} - \frac{4d_1\kappa}{3(x-1)^4} + \frac{2\kappa}{(x-1)^4} - \right. \\
& \left. \frac{640\kappa}{(x-2)^5} - \frac{4d_1\kappa}{3(x-1)^5} + \frac{2\kappa}{(x-1)^5} - \frac{640\kappa}{(x-2)^6} - \frac{160}{3(x-2)^4} - \frac{4d_1}{3(x-1)^4} + \frac{2}{3(x-1)^4} - \frac{640}{3(x-2)^5} - \frac{4d_1}{3(x-1)^5} + \frac{2}{3(x-1)^5} - \right.
\end{aligned}$$



$$\begin{aligned}
& \left. \frac{640}{3(x-2)^6} \right) H(1, 0, 0; x) + \left( -\frac{x\alpha_0}{3} - x\kappa\alpha_0 - \frac{80\kappa\alpha_0}{(x-2)^4} - \frac{2d_1\kappa\alpha_0}{3(x-1)^4} + \frac{\kappa\alpha_0}{(x-1)^4} - \frac{160\kappa\alpha_0}{(x-2)^5} + \frac{2d_1\kappa\alpha_0}{3(x-1)^5} - \right. \\
& \left. \frac{\kappa\alpha_0}{(x-1)^5} + 2\kappa\alpha_0 - \frac{80\alpha_0}{3(x-2)^4} - \frac{2d_1\alpha_0}{3(x-1)^4} + \frac{\alpha_0}{3(x-1)^4} - \frac{160\alpha_0}{3(x-2)^5} + \frac{2d_1\alpha_0}{3(x-1)^5} - \frac{\alpha_0}{3(x-1)^5} + \frac{2\alpha_0}{3} + \frac{x}{3} + x\kappa + \right. \\
& \left. \frac{80\kappa}{(x-2)^4} + \frac{2d_1\kappa}{3(x-1)^4} - \frac{\kappa}{(x-1)^4} + \frac{320\kappa}{(x-2)^5} + \frac{2d_1\kappa}{3(x-1)^5} - \frac{\kappa}{(x-1)^5} + \frac{320\kappa}{(x-2)^6} + \frac{80}{3(x-2)^4} + \frac{2d_1}{3(x-1)^4} - \frac{1}{3(x-1)^4} + \right. \\
& \left. \frac{320}{3(x-2)^5} + \frac{2d_1}{3(x-1)^5} - \frac{1}{3(x-1)^5} + \frac{320}{3(x-2)^6} \right) H(1, 0, c_1(\alpha_0); x) + \left( -\frac{4\alpha_0 d_1^2}{3(x-1)^4} + \frac{4d_1^2}{3(x-1)^4} + \frac{4\alpha_0 d_1^2}{3(x-1)^5} + \right. \\
& \left. \frac{4d_1^2}{3(x-1)^5} + \frac{4\alpha_0 d_1}{3} - \frac{2\alpha_0 x d_1}{3} + \frac{2x d_1}{3} + \frac{4\alpha_0 \kappa d_1}{3} - \frac{2}{3}\alpha_0 x \kappa d_1 + \frac{2x \kappa d_1}{3} - \frac{160\alpha_0 \kappa d_1}{3(x-2)^4} + \frac{160\kappa d_1}{3(x-2)^4} + \frac{4\alpha_0 \kappa d_1}{3(x-1)^4} - \right. \\
& \left. \frac{4\kappa d_1}{3(x-1)^4} - \frac{320\alpha_0 \kappa d_1}{3(x-2)^5} + \frac{640\kappa d_1}{3(x-2)^5} - \frac{4\alpha_0 \kappa d_1}{3(x-1)^5} - \frac{4\kappa d_1}{3(x-1)^5} + \frac{640\kappa d_1}{3(x-2)^6} - \frac{160\alpha_0 d_1}{3(x-2)^4} + \frac{160d_1}{3(x-2)^4} + \frac{4\alpha_0 d_1}{3(x-1)^4} - \right. \\
& \left. \frac{4d_1}{3(x-1)^4} - \frac{320\alpha_0 d_1}{3(x-2)^5} + \frac{640d_1}{3(x-2)^5} - \frac{4\alpha_0 d_1}{3(x-1)^5} - \frac{4d_1}{3(x-1)^5} + \frac{640d_1}{3(x-2)^6} - \frac{2\alpha_0}{3} + \frac{\alpha_0 x}{3} - \frac{x}{3} - 2\alpha_0 \kappa + \alpha_0 x \kappa - \right. \\
& \left. x\kappa + \frac{80\alpha_0 \kappa}{(x-2)^4} - \frac{80\kappa}{(x-1)^4} - \frac{\alpha_0 \kappa}{(x-1)^4} + \frac{\kappa}{(x-1)^4} + \frac{160\alpha_0 \kappa}{(x-2)^5} - \frac{320\kappa}{(x-2)^5} + \frac{\alpha_0 \kappa}{(x-1)^5} + \frac{\kappa}{(x-1)^5} - \frac{320\kappa}{(x-2)^6} + \frac{80\alpha_0}{3(x-2)^4} - \right. \\
& \left. \frac{80}{3(x-2)^4} - \frac{\alpha_0}{3(x-1)^4} + \frac{1}{3(x-1)^4} + \frac{160\alpha_0}{3(x-2)^5} - \frac{320}{3(x-2)^5} + \frac{\alpha_0}{3(x-1)^5} + \frac{1}{3(x-1)^5} - \frac{320}{3(x-2)^6} \right) H(1, 1, 0; x) + \\
& \left( \frac{4\alpha_0 d_1^2}{3(x-1)^4} - \frac{4d_1^2}{3(x-1)^4} - \frac{4\alpha_0 d_1^2}{3(x-1)^5} - \frac{4d_1^2}{3(x-1)^5} - \frac{4\alpha_0 d_1}{3} + \frac{2\alpha_0 x d_1}{3} - \frac{2x d_1}{3} - \frac{4\alpha_0 \kappa d_1}{3} + \frac{2}{3}\alpha_0 x \kappa d_1 - \frac{2x \kappa d_1}{3} + \right. \\
& \left. \frac{160\alpha_0 \kappa d_1}{3(x-2)^4} - \frac{160\kappa d_1}{3(x-2)^4} - \frac{4\alpha_0 \kappa d_1}{3(x-1)^4} + \frac{4\kappa d_1}{3(x-1)^4} + \frac{320\alpha_0 \kappa d_1}{3(x-2)^5} - \frac{640\kappa d_1}{3(x-2)^5} + \frac{4\alpha_0 \kappa d_1}{3(x-1)^5} + \frac{4\kappa d_1}{3(x-1)^5} - \frac{640\kappa d_1}{3(x-2)^6} + \right. \\
& \left. \frac{160\alpha_0 d_1}{3(x-2)^4} - \frac{160d_1}{3(x-2)^4} - \frac{4\alpha_0 d_1}{3(x-1)^4} + \frac{4d_1}{3(x-1)^4} + \frac{320\alpha_0 d_1}{3(x-2)^5} - \frac{640d_1}{3(x-2)^5} + \frac{4\alpha_0 d_1}{3(x-1)^5} + \frac{4d_1}{3(x-1)^5} - \frac{640d_1}{3(x-2)^6} + \frac{2\alpha_0}{3} - \right. \\
& \left. \frac{\alpha_0 x}{3} + \frac{x}{3} + 2\alpha_0 \kappa - \alpha_0 x \kappa + x\kappa - \frac{80\alpha_0 \kappa}{(x-2)^4} + \frac{80\kappa}{(x-2)^4} + \frac{\alpha_0 \kappa}{(x-1)^4} - \frac{\kappa}{(x-1)^4} - \frac{160\alpha_0 \kappa}{(x-2)^5} + \frac{320\kappa}{(x-2)^5} - \frac{\alpha_0 \kappa}{(x-1)^5} - \right. \\
& \left. \frac{\kappa}{(x-1)^5} + \frac{320\kappa}{(x-2)^6} - \frac{80\alpha_0}{3(x-2)^4} + \frac{80}{3(x-2)^4} + \frac{\alpha_0}{3(x-1)^4} - \frac{1}{3(x-1)^4} - \frac{160\alpha_0}{3(x-2)^5} + \frac{320}{3(x-2)^5} - \frac{\alpha_0}{3(x-1)^5} - \frac{1}{3(x-1)^5} + \right. \\
& \left. \frac{320}{3(x-2)^6} \right) H(1, 1, c_1(\alpha_0); x) + \left( -\frac{x\alpha_0}{3} - x\kappa\alpha_0 - \frac{80\kappa\alpha_0}{(x-2)^4} - \frac{2d_1\kappa\alpha_0}{3(x-1)^4} + \frac{\kappa\alpha_0}{(x-1)^4} - \frac{160\kappa\alpha_0}{(x-2)^5} + \frac{2d_1\kappa\alpha_0}{3(x-1)^5} - \right. \\
& \left. \frac{\kappa\alpha_0}{(x-1)^5} + 2\kappa\alpha_0 - \frac{80\alpha_0}{3(x-2)^4} - \frac{2d_1\alpha_0}{3(x-1)^4} + \frac{\alpha_0}{3(x-1)^4} - \frac{160\alpha_0}{3(x-2)^5} + \frac{2d_1\alpha_0}{3(x-1)^5} - \frac{\alpha_0}{3(x-1)^5} + \frac{2\alpha_0}{3} + \frac{x}{3} + x\kappa + \frac{80\kappa}{(x-2)^4} + \right. \\
& \left. \frac{2d_1\kappa}{3(x-1)^4} - \frac{\kappa}{(x-1)^4} + \frac{320\kappa}{(x-2)^5} + \frac{2d_1\kappa}{3(x-1)^5} - \frac{\kappa}{(x-1)^5} + \frac{320\kappa}{(x-2)^6} + \frac{80}{3(x-2)^4} + \frac{2d_1}{3(x-1)^4} - \frac{1}{3(x-1)^4} + \frac{320}{3(x-2)^5} + \right. \\
& \left. \frac{2d_1}{3(x-1)^5} - \frac{1}{3(x-1)^5} + \frac{320}{3(x-2)^6} \right) H(1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{320d_1\kappa\alpha_0}{3(x-2)^4} + \frac{320\kappa\alpha_0}{(x-2)^4} - \frac{640d_1\kappa\alpha_0}{3(x-2)^5} + \frac{640\kappa\alpha_0}{(x-2)^5} - \right. \\
& \left. \frac{320d_1\alpha_0}{3(x-2)^4} + \frac{320\alpha_0}{3(x-2)^4} - \frac{640d_1\alpha_0}{3(x-2)^5} + \frac{640\alpha_0}{3(x-2)^5} + \frac{320d_1\kappa}{(x-2)^4} - \frac{320\kappa}{(x-2)^4} + \frac{1280d_1\kappa}{3(x-2)^5} - \frac{1280\kappa}{(x-2)^5} + \frac{1280d_1\kappa}{3(x-2)^6} - \frac{1280\kappa}{(x-2)^6} + \right. \\
& \left. \frac{320d_1}{3(x-2)^4} - \frac{320}{3(x-2)^4} + \frac{1280d_1}{3(x-2)^5} - \frac{1280}{3(x-2)^5} + \frac{1280d_1}{3(x-2)^6} - \frac{1280}{3(x-2)^6} \right) H(2, 0, 0; x) + \left( -\frac{160d_1\kappa\alpha_0}{3(x-2)^4} + \frac{160\kappa\alpha_0}{(x-2)^4} - \right. \\
& \left. \frac{320d_1\kappa\alpha_0}{3(x-2)^5} + \frac{320\kappa\alpha_0}{(x-2)^5} - \frac{160d_1\alpha_0}{3(x-2)^4} + \frac{160\alpha_0}{3(x-2)^4} - \frac{320d_1\alpha_0}{3(x-2)^5} + \frac{320\alpha_0}{3(x-2)^5} + \frac{160d_1\kappa}{3(x-2)^4} - \frac{160\kappa}{(x-2)^4} + \frac{640d_1\kappa}{3(x-2)^5} - \frac{640\kappa}{(x-2)^5} + \right. \\
& \left. \frac{640d_1\kappa}{3(x-2)^6} - \frac{640\kappa}{(x-2)^6} + \frac{160d_1}{3(x-2)^4} - \frac{160}{3(x-2)^4} + \frac{640d_1}{3(x-2)^5} - \frac{640}{3(x-2)^5} + \frac{640d_1}{3(x-2)^6} - \frac{640}{3(x-2)^6} \right) H(2, 0, c_1(\alpha_0); x) + \\
& \left( \frac{320d_1\kappa\alpha_0}{3(x-2)^4} - \frac{320\kappa\alpha_0}{(x-2)^4} + \frac{640d_1\kappa\alpha_0}{3(x-2)^5} - \frac{640\kappa\alpha_0}{(x-2)^5} + \frac{320d_1\alpha_0}{3(x-2)^4} - \frac{320\alpha_0}{3(x-2)^4} + \frac{640d_1\alpha_0}{3(x-2)^5} - \frac{640\alpha_0}{3(x-2)^5} - \frac{320d_1\kappa}{3(x-2)^4} + \right. \\
& \left. \frac{320\kappa}{(x-2)^4} - \frac{1280d_1\kappa}{3(x-2)^5} + \frac{1280\kappa}{(x-2)^5} - \frac{1280d_1\kappa}{3(x-2)^6} + \frac{1280\kappa}{(x-2)^6} - \frac{320d_1}{3(x-2)^4} + \frac{320}{3(x-2)^4} - \frac{1280d_1}{3(x-2)^5} + \frac{1280}{3(x-2)^5} - \frac{1280d_1}{3(x-2)^6} + \right. \\
& \left. \frac{1280}{3(x-2)^6} \right) H(2, 0, c_2(\alpha_0); x) + \left( \frac{160d_1\kappa\alpha_0}{3(x-2)^4} - \frac{160\kappa\alpha_0}{(x-2)^4} + \frac{320d_1\kappa\alpha_0}{3(x-2)^5} - \frac{320\kappa\alpha_0}{(x-2)^5} + \frac{160d_1\alpha_0}{3(x-2)^4} - \frac{160\alpha_0}{3(x-2)^4} + \right. \\
& \left. \frac{320d_1\alpha_0}{3(x-2)^5} - \frac{320\alpha_0}{3(x-2)^5} - \frac{160d_1\kappa}{3(x-2)^4} + \frac{160\kappa}{(x-2)^4} - \frac{640d_1\kappa}{3(x-2)^5} + \frac{640\kappa}{(x-2)^5} - \frac{640d_1}{3(x-2)^6} + \frac{640\kappa}{(x-2)^6} - \frac{160d_1}{3(x-2)^4} + \frac{160}{3(x-2)^4} - \right. \\
& \left. \frac{640d_1}{3(x-2)^5} + \frac{640}{3(x-2)^5} - \frac{640d_1}{3(x-2)^6} + \frac{640}{3(x-2)^6} \right) H(2, 1, 0; x) + \left( -\frac{160d_1\kappa\alpha_0}{3(x-2)^4} + \frac{160\kappa\alpha_0}{(x-2)^4} - \frac{320d_1\kappa\alpha_0}{3(x-2)^5} + \frac{320\kappa\alpha_0}{(x-2)^5} - \right. \\
& \left. \frac{160d_1\alpha_0}{3(x-2)^4} + \frac{160\alpha_0}{3(x-2)^4} - \frac{320d_1\alpha_0}{3(x-2)^5} + \frac{320\alpha_0}{3(x-2)^5} + \frac{160d_1\kappa}{3(x-2)^4} - \frac{160\kappa}{(x-2)^4} + \frac{640d_1\kappa}{3(x-2)^5} - \frac{640\kappa}{(x-2)^5} + \frac{640d_1\kappa}{3(x-2)^6} - \frac{640\kappa}{(x-2)^6} + \right. \\
& \left. \frac{160d_1}{3(x-2)^4} - \frac{160}{3(x-2)^4} + \frac{640d_1}{3(x-2)^5} - \frac{640}{3(x-2)^5} + \frac{640d_1}{3(x-2)^6} - \frac{640}{3(x-2)^6} \right) H(2, 1, c_1(\alpha_0); x) + \left( -\frac{320\alpha_0 d_1^2}{3(x-2)^4} + \right. \\
& \left. \frac{320d_1^2}{3(x-2)^4} - \frac{640\alpha_0 d_1^2}{3(x-2)^5} + \frac{1280d_1^2}{3(x-2)^5} + \frac{1280d_1^2}{3(x-2)^6} + \frac{640\alpha_0 \kappa d_1}{3(x-2)^4} - \frac{640\kappa d_1}{3(x-2)^4} + \frac{1280\alpha_0 \kappa d_1}{3(x-2)^5} - \frac{2560\kappa d_1}{3(x-2)^5} - \frac{2560\kappa d_1}{3(x-2)^6} + \right. \\
& \left. \frac{640\alpha_0 d_1}{3(x-2)^4} - \frac{640d_1}{3(x-2)^4} + \frac{1280\alpha_0 d_1}{3(x-2)^5} - \frac{2560d_1}{3(x-2)^5} - \frac{2560d_1}{3(x-2)^6} - \frac{320\alpha_0 \kappa}{(x-2)^4} + \frac{320\kappa}{(x-2)^4} - \frac{640\alpha_0 \kappa}{(x-2)^5} + \frac{1280\kappa}{(x-2)^5} + \frac{1280\kappa}{(x-2)^6} - \right. \\
& \left. \frac{320\alpha_0}{3(x-2)^4} + \frac{320}{3(x-2)^4} - \frac{640\alpha_0}{3(x-2)^5} + \frac{1280}{3(x-2)^5} + \frac{1280}{3(x-2)^6} \right) H(2, 2, 0; x) + \left( \frac{320\alpha_0 d_1^2}{3(x-2)^4} - \frac{320d_1^2}{3(x-2)^4} + \frac{640\alpha_0 d_1^2}{3(x-2)^5} - \right. \\
& \left. \frac{1280d_1^2}{3(x-2)^5} - \frac{1280d_1^2}{3(x-2)^6} - \frac{640\alpha_0 \kappa d_1}{3(x-2)^4} + \frac{640\kappa d_1}{3(x-2)^4} - \frac{1280\alpha_0 \kappa d_1}{3(x-2)^5} + \frac{2560\kappa d_1}{3(x-2)^5} + \frac{2560\kappa d_1}{3(x-2)^6} - \frac{640\alpha_0 d_1}{3(x-2)^4} + \frac{640d_1}{3(x-2)^4} - \right. \\
& \left. \frac{1280\alpha_0 d_1}{3(x-2)^5} + \frac{2560d_1}{3(x-2)^5} + \frac{2560d_1}{3(x-2)^6} + \frac{320\alpha_0 \kappa}{(x-2)^4} - \frac{320\kappa}{(x-2)^4} + \frac{640\alpha_0 \kappa}{(x-2)^5} - \frac{1280\kappa}{(x-2)^5} - \frac{1280\kappa}{(x-2)^6} + \frac{320\alpha_0}{3(x-2)^4} - \frac{320}{3(x-2)^4} + \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{640\alpha_0}{3(x-2)^5} - \frac{1280}{3(x-2)^5} - \frac{1280}{3(x-2)^6} \right) H(2, 2, c_2(\alpha_0); x) + \left( \frac{160d_1\kappa\alpha_0}{3(x-2)^4} - \frac{160\kappa\alpha_0}{(x-2)^4} + \frac{320d_1\kappa\alpha_0}{3(x-2)^5} - \frac{320\kappa\alpha_0}{(x-2)^5} + \right. \\
& \left. \frac{160d_1\alpha_0}{3(x-2)^4} - \frac{160\alpha_0}{3(x-2)^4} + \frac{320d_1\alpha_0}{3(x-2)^5} - \frac{320\alpha_0}{3(x-2)^5} - \frac{160d_1\kappa}{3(x-2)^4} + \frac{160\kappa}{(x-2)^4} - \frac{640d_1\kappa}{3(x-2)^5} + \frac{640\kappa}{(x-2)^5} - \frac{640d_1\kappa}{3(x-2)^6} + \frac{640\kappa}{(x-2)^6} - \right. \\
& \left. \frac{160d_1}{3(x-2)^4} + \frac{160}{3(x-2)^4} - \frac{640d_1}{3(x-2)^5} + \frac{640}{3(x-2)^5} - \frac{640d_1}{3(x-2)^6} + \frac{640}{3(x-2)^6} \right) H(2, c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{\kappa\alpha_0}{(x-1)^4} - \right. \\
& \left. \frac{\kappa\alpha_0}{(x-1)^5} + \frac{\alpha_0}{3(x-1)^4} - \frac{\alpha_0}{3(x-1)^5} - \frac{\kappa}{(x-1)^4} - \frac{\kappa}{(x-1)^5} - \frac{1}{3(x-1)^4} - \frac{1}{3(x-1)^5} \right) H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \\
& \left( \frac{80\kappa\alpha_0}{(x-2)^4} + \frac{160\kappa\alpha_0}{3(x-2)^5} + \frac{80\alpha_0}{3(x-2)^4} + \frac{160\alpha_0}{3(x-2)^5} - \frac{80\kappa}{(x-2)^4} - \frac{320\kappa}{(x-2)^5} - \frac{320\kappa}{(x-2)^6} - \frac{80}{3(x-2)^4} - \frac{320}{3(x-2)^5} - \right. \\
& \left. \frac{320}{3(x-2)^6} \right) H(c_2(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + H(0, 0; x) \left( -\frac{17x\alpha_0}{6} - \frac{101x\kappa\alpha_0}{18} - \frac{80\kappa\alpha_0}{3(x-2)} + \frac{65\kappa\alpha_0}{2(x-1)} + \right. \\
& \left. \frac{80\kappa\alpha_0}{3(x-2)^2} - \frac{25\kappa\alpha_0}{3(x-1)^2} + \frac{35\kappa\alpha_0}{6(x-1)^3} + \frac{64d_1\kappa\alpha_0}{3(x-2)^4} + \frac{3584\kappa\alpha_0}{9(x-2)^4} + \frac{47\kappa\alpha_0}{18(x-1)^4} + \frac{128d_1\kappa\alpha_0}{3(x-2)^5} + \frac{1408\kappa\alpha_0}{9(x-2)^5} - \frac{101\kappa\alpha_0}{18(x-1)^5} + \right. \\
& \left. \frac{74\kappa\alpha_0}{9} - \frac{80\alpha_0}{9(x-2)} + \frac{65\alpha_0}{6(x-1)} + \frac{80\alpha_0}{9(x-2)^2} - \frac{25\alpha_0}{9(x-1)^2} + \frac{35\alpha_0}{18(x-1)^3} + \frac{64d_1\alpha_0}{3(x-2)^4} + \frac{2048\alpha_0}{9(x-2)^4} + \frac{11\alpha_0}{6(x-1)^4} + \right. \\
& \left. \frac{128d_1\alpha_0}{3(x-2)^5} + \frac{2176\alpha_0}{9(x-2)^5} - \frac{17\alpha_0}{6(x-1)^5} + \frac{320\kappa\ln 2\alpha_0}{(x-2)^4} + \frac{640\kappa\ln 2\alpha_0}{(x-2)^5} + \frac{320\ln 2\alpha_0}{3(x-2)^4} + \frac{640\ln 2\alpha_0}{3(x-2)^5} + \frac{14\alpha_0}{3} + \frac{17x}{6} + \right. \\
& \left. \frac{101x\kappa}{18} - \frac{64\kappa}{3(x-2)} + \frac{43\kappa}{2(x-1)} + \frac{80\kappa}{3(x-2)^2} - \frac{8\kappa}{3(x-1)^2} - \frac{160\kappa}{3(x-2)^3} - \frac{\kappa}{6(x-1)^3} - \frac{64d_1\kappa}{3(x-2)^4} - \frac{3584\kappa}{9(x-2)^4} - \right. \\
& \left. \frac{155\kappa}{18(x-1)^4} - \frac{256d_1\kappa}{3(x-2)^5} - \frac{8576\kappa}{9(x-2)^5} - \frac{101\kappa}{18(x-1)^5} - \frac{256d_1\kappa}{3(x-2)^6} - \frac{2816\kappa}{9(x-2)^6} + 3\kappa - \frac{64}{9(x-2)} + \frac{43}{6(x-1)} + \right. \\
& \left. \frac{80}{9(x-2)^2} - \frac{8}{9(x-1)^2} - \frac{160}{9(x-2)^3} - \frac{1}{18(x-1)^3} - \frac{64d_1}{3(x-2)^4} - \frac{2048}{9(x-2)^4} - \frac{23}{6(x-1)^4} - \frac{256d_1}{3(x-2)^5} - \frac{6272}{9(x-2)^5} - \right. \\
& \left. \frac{17}{6(x-1)^5} - \frac{256d_1}{3(x-2)^6} - \frac{4352}{9(x-2)^6} - \frac{320\kappa\ln 2}{(x-2)^4} - \frac{1280\kappa\ln 2}{(x-2)^5} - \frac{1280\kappa\ln 2}{(x-2)^6} - \frac{320\ln 2}{3(x-2)^4} - \frac{1280\ln 2}{3(x-2)^5} - \frac{1280\ln 2}{3(x-2)^6} + \right. \\
& \left. 1 \right) + H(2, 2; x) \left( \frac{320\alpha_0\ln 2d_1^2}{3(x-2)^4} - \frac{320\ln 2d_1^2}{3(x-2)^4} + \frac{640\alpha_0\ln 2d_1^2}{3(x-2)^5} - \frac{1280\ln 2d_1^2}{3(x-2)^5} - \frac{1280\ln 2d_1^2}{3(x-2)^6} - \frac{640\alpha_0\kappa\ln 2d_1}{3(x-2)^4} + \right. \\
& \left. \frac{640\kappa\ln 2d_1}{3(x-2)^4} - \frac{1280\alpha_0\kappa\ln 2d_1}{3(x-2)^5} + \frac{2560\kappa\ln 2d_1}{3(x-2)^5} + \frac{2560\kappa\ln 2d_1}{3(x-2)^6} - \frac{640\alpha_0\ln 2d_1}{3(x-2)^4} + \frac{640\ln 2d_1}{3(x-2)^4} - \frac{1280\alpha_0\ln 2d_1}{3(x-2)^5} + \right. \\
& \left. \frac{2560\ln 2d_1}{3(x-2)^5} + \frac{2560\ln 2d_1}{3(x-2)^6} + \left( \frac{320\alpha_0d_1^2}{3(x-2)^4} - \frac{320d_1^2}{3(x-2)^4} + \frac{640\alpha_0d_1^2}{3(x-2)^5} - \frac{1280d_1^2}{3(x-2)^5} - \frac{1280d_1^2}{3(x-2)^6} - \frac{640\alpha_0\kappa d_1}{3(x-2)^4} + \frac{640\kappa d_1}{3(x-2)^4} - \right. \right. \\
& \left. \left. \frac{1280\alpha_0\kappa d_1}{3(x-2)^5} + \frac{2560\kappa d_1}{3(x-2)^5} + \frac{2560\kappa d_1}{3(x-2)^6} - \frac{640\alpha_0 d_1}{3(x-2)^4} + \frac{640d_1}{3(x-2)^4} - \frac{1280\alpha_0 d_1}{3(x-2)^5} + \frac{2560d_1}{3(x-2)^5} + \frac{2560d_1}{3(x-2)^6} + \frac{320\alpha_0\kappa}{(x-2)^4} - \right. \right. \\
& \left. \left. \frac{320\kappa}{(x-2)^4} + \frac{640\alpha_0\kappa}{(x-2)^5} - \frac{1280\kappa}{(x-2)^5} - \frac{1280\kappa}{(x-2)^6} + \frac{320\alpha_0}{3(x-2)^4} - \frac{320}{3(x-2)^4} + \frac{640\alpha_0}{3(x-2)^5} - \frac{1280}{3(x-2)^5} - \frac{1280}{3(x-2)^6} \right) H(0; \alpha_0) + \right. \\
& \left. \frac{320\alpha_0\kappa\ln 2}{(x-2)^4} - \frac{320\kappa\ln 2}{(x-2)^4} + \frac{640\alpha_0\kappa\ln 2}{(x-2)^5} - \frac{1280\kappa\ln 2}{(x-2)^5} - \frac{1280\kappa\ln 2}{(x-2)^6} + \frac{320\alpha_0\ln 2}{3(x-2)^4} - \frac{320\ln 2}{3(x-2)^4} + \frac{640\alpha_0\ln 2}{3(x-2)^5} - \right. \\
& \left. \frac{1280\ln 2}{3(x-2)^5} - \frac{1280\ln 2}{3(x-2)^6} \right) + H(2, 0; x) \left( \frac{64\alpha_0d_1^2}{3(x-2)^4} - \frac{64d_1^2}{3(x-2)^4} + \frac{128\alpha_0d_1^2}{3(x-2)^5} - \frac{256d_1^2}{3(x-2)^5} - \frac{256d_1^2}{3(x-2)^6} - \frac{128\alpha_0\kappa d_1}{3(x-2)^4} + \right. \\
& \left. \frac{128\kappa d_1}{3(x-2)^4} - \frac{256\alpha_0\kappa d_1}{3(x-2)^5} + \frac{512\kappa d_1}{3(x-2)^5} + \frac{512\kappa d_1}{3(x-2)^6} + \frac{896\alpha_0 d_1}{9(x-2)^4} - \frac{896d_1}{9(x-2)^4} + \frac{1792\alpha_0 d_1}{9(x-2)^5} - \frac{3584d_1}{9(x-2)^5} - \frac{3584d_1}{9(x-2)^6} + \right. \\
& \left. \frac{320\alpha_0\kappa\ln 2d_1}{3(x-2)^4} - \frac{320\kappa\ln 2d_1}{3(x-2)^4} + \frac{640\alpha_0\kappa\ln 2d_1}{3(x-2)^5} - \frac{1280\kappa\ln 2d_1}{3(x-2)^5} - \frac{1280\kappa\ln 2d_1}{3(x-2)^6} + \frac{320\alpha_0\ln 2d_1}{3(x-2)^4} - \frac{320\ln 2d_1}{3(x-2)^4} + \right. \\
& \left. \frac{640\alpha_0\ln 2d_1}{3(x-2)^5} - \frac{1280\ln 2d_1}{3(x-2)^5} - \frac{1280\ln 2d_1}{3(x-2)^6} - \frac{704\alpha_0\kappa}{9(x-2)^4} + \frac{704\kappa}{9(x-2)^4} - \frac{1408\alpha_0\kappa}{9(x-2)^5} + \frac{2816\kappa}{9(x-2)^5} + \frac{2816\kappa}{9(x-2)^6} - \right. \\
& \left. \frac{1088\alpha_0}{9(x-2)^4} + \frac{1088}{9(x-2)^4} - \frac{2176\alpha_0}{9(x-2)^5} + \frac{4352}{9(x-2)^5} + \frac{4352}{9(x-2)^6} - \frac{320\alpha_0\kappa\ln 2}{(x-2)^4} + \frac{320\kappa\ln 2}{(x-2)^4} - \frac{640\alpha_0\kappa\ln 2}{(x-2)^5} + \frac{1280\kappa\ln 2}{(x-2)^5} + \right. \\
& \left. \frac{1280\kappa\ln 2}{(x-2)^6} - \frac{320\alpha_0\ln 2}{3(x-2)^4} + \frac{320\ln 2}{3(x-2)^4} - \frac{640\alpha_0\ln 2}{3(x-2)^5} + \frac{1280\ln 2}{3(x-2)^5} + \frac{1280\ln 2}{3(x-2)^6} \right) + H(0, 2; x) \left( \frac{320d_1\kappa\ln 2\alpha_0}{3(x-2)^4} - \right. \\
& \left. \frac{320\kappa\ln 2\alpha_0}{(x-2)^4} + \frac{640d_1\kappa\ln 2\alpha_0}{3(x-2)^5} - \frac{640\kappa\ln 2\alpha_0}{(x-2)^5} + \frac{320d_1\ln 2\alpha_0}{3(x-2)^4} - \frac{320\ln 2\alpha_0}{3(x-2)^4} + \frac{640d_1\ln 2\alpha_0}{3(x-2)^5} - \frac{640\ln 2\alpha_0}{3(x-2)^5} + \right. \\
& \left( \frac{320d_1\kappa\alpha_0}{3(x-2)^4} - \frac{320\kappa\alpha_0}{(x-2)^4} + \frac{640d_1\kappa\alpha_0}{3(x-2)^5} - \frac{640\kappa\alpha_0}{(x-2)^5} + \frac{320d_1\alpha_0}{3(x-2)^4} - \frac{320\alpha_0}{3(x-2)^4} + \frac{640d_1\alpha_0}{3(x-2)^5} - \frac{640\alpha_0}{3(x-2)^5} - \frac{320d_1\kappa}{3(x-2)^4} + \right. \\
& \left. \frac{320\kappa}{(x-2)^4} - \frac{1280d_1\kappa}{3(x-2)^5} + \frac{1280\kappa}{(x-2)^5} - \frac{1280d_1\kappa}{3(x-2)^6} + \frac{1280\kappa}{(x-2)^6} - \frac{320d_1}{3(x-2)^4} + \frac{320}{3(x-2)^4} - \frac{1280d_1}{3(x-2)^5} + \frac{1280}{3(x-2)^5} - \frac{1280d_1}{3(x-2)^6} + \right. \\
& \left. \frac{1280}{3(x-2)^6} \right) H(0; \alpha_0) - \frac{320d_1\kappa\ln 2}{3(x-2)^4} + \frac{320\kappa\ln 2}{(x-2)^4} - \frac{1280d_1\kappa\ln 2}{3(x-2)^5} + \frac{1280\kappa\ln 2}{(x-2)^5} - \frac{1280d_1\kappa\ln 2}{3(x-2)^6} + \frac{1280\kappa\ln 2}{(x-2)^6} - \\
& \left. \frac{320d_1\ln 2}{3(x-2)^4} + \frac{320\ln 2}{3(x-2)^4} - \frac{1280d_1\ln 2}{3(x-2)^5} + \frac{1280\ln 2}{3(x-2)^5} - \frac{1280d_1\ln 2}{3(x-2)^6} + \frac{1280\ln 2}{3(x-2)^6} \right) + H(0; x) \left( \frac{31d_1\alpha_0}{27} - \frac{205d_1x\alpha_0}{216} + \right. \\
& \left. \frac{1}{18}\pi^2x\alpha_0 + \frac{955x\alpha_0}{216} + \frac{31d_1\kappa\alpha_0}{27} - \frac{205}{216}d_1x\kappa\alpha_0 + \frac{1}{3}\pi^2x\kappa\alpha_0 + \frac{1255x\kappa\alpha_0}{216} - \frac{392d_1\kappa\alpha_0}{27(x-2)} + \frac{1136\kappa\alpha_0}{27(x-2)} + \right. \\
& \left. \frac{1129d_1\kappa\alpha_0}{72(x-1)} - \frac{3613\kappa\alpha_0}{72(x-1)} + \frac{392d_1\kappa\alpha_0}{27(x-2)^2} - \frac{1136\kappa\alpha_0}{27(x-2)^2} - \frac{181d_1\kappa\alpha_0}{108(x-1)^2} + \frac{280\kappa\alpha_0}{27(x-1)^2} + \frac{251d_1\kappa\alpha_0}{216(x-1)^3} - \frac{1751\kappa\alpha_0}{216(x-1)^3} + \right. \\
& \left. \frac{256d_1\kappa\alpha_0}{3(x-2)^4} - \frac{40\pi^2\kappa\alpha_0}{3(x-2)^4} - \frac{3328\kappa\alpha_0}{9(x-2)^4} + \frac{43d_1\kappa\alpha_0}{216(x-1)^4} - \frac{\pi^2\kappa\alpha_0}{3(x-1)^4} - \frac{32\kappa\alpha_0}{27(x-1)^4} - \frac{80\pi^2\kappa\alpha_0}{3(x-2)^5} + \frac{512\kappa\alpha_0}{9(x-2)^5} - \frac{205d_1\kappa\alpha_0}{216(x-1)^5} + \right. \\
& \left. \frac{\pi^2\kappa\alpha_0}{3(x-1)^5} + \frac{1255\kappa\alpha_0}{216(x-1)^5} - \frac{2}{3}\pi^2\kappa\alpha_0 - \frac{1511\kappa\alpha_0}{216} - \frac{392d_1\alpha_0}{27(x-2)} + \frac{592\alpha_0}{27(x-2)} + \frac{1129d_1\alpha_0}{72(x-1)} - \frac{209\alpha_0}{8(x-1)} + \frac{392d_1\alpha_0}{27(x-2)^2} - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{592\alpha_0}{27(x-2)^2} - \frac{181d_1\alpha_0}{108(x-1)^2} + \frac{155\alpha_0}{27(x-1)^2} + \frac{251d_1\alpha_0}{216(x-1)^3} - \frac{907\alpha_0}{216(x-1)^3} + \frac{512d_1\alpha_0}{9(x-2)^4} - \frac{9536\alpha_0}{27(x-2)^4} + \frac{43d_1\alpha_0}{216(x-1)^4} - \\
& \frac{\pi^2\alpha_0}{18(x-1)^4} - \frac{115\alpha_0}{54(x-1)^4} - \frac{512d_1\alpha_0}{9(x-2)^5} - \frac{6784\alpha_0}{27(x-2)^5} - \frac{205d_1\alpha_0}{216(x-1)^5} + \frac{\pi^2\alpha_0}{18(x-1)^5} + \frac{955\alpha_0}{216(x-1)^5} - \frac{160\kappa\ln^2 2\alpha_0}{(x-2)^4} - \\
& \frac{320\kappa\ln^2 2\alpha_0}{(x-2)^5} - \frac{160\ln^2 2\alpha_0}{3(x-2)^4} - \frac{320\ln^2 2\alpha_0}{3(x-2)^5} - \frac{64d_1\kappa\ln 2\alpha_0}{3(x-2)^4} - \frac{704\kappa\ln 2\alpha_0}{9(x-2)^4} - \frac{128d_1\kappa\ln 2\alpha_0}{3(x-2)^5} - \frac{1408\kappa\ln 2\alpha_0}{9(x-2)^5} - \\
& \frac{64d_1\ln 2\alpha_0}{3(x-2)^4} - \frac{1088\ln 2\alpha_0}{9(x-2)^4} - \frac{128d_1\ln 2\alpha_0}{3(x-2)^5} - \frac{2176\ln 2\alpha_0}{9(x-2)^5} - \frac{\pi^2\alpha_0}{9} - \frac{1415\alpha_0}{216} + \frac{3d_1}{4} + \frac{205d_1x}{216} - \frac{\pi^2x}{18} - \\
& \frac{955x}{216} + \frac{3d_1\kappa}{4} + \frac{205d_1x\kappa}{216} - \frac{1}{3}\pi^2x\kappa - \frac{1255x\kappa}{216} - \frac{346d_1\kappa}{27(x-2)} + \frac{952\kappa}{27(x-2)} + \frac{947d_1\kappa}{72(x-1)} - \frac{2621\kappa}{72(x-1)} + \frac{392d_1\kappa}{27(x-2)^2} - \\
& \frac{1136\kappa}{27(x-2)^2} - \frac{23d_1\kappa}{27(x-1)^2} + \frac{92\kappa}{27(x-1)^2} - \frac{784d_1\kappa}{27(x-2)^3} + \frac{2272\kappa}{27(x-2)^3} - \frac{73d_1\kappa}{216(x-1)^3} + \frac{247\kappa}{216(x-1)^3} - \frac{256d_1\kappa}{3(x-2)^4} + \\
& \frac{40\pi^2\kappa}{3(x-2)^4} + \frac{3328\kappa}{9(x-2)^4} - \frac{367d_1\kappa}{216(x-1)^4} + \frac{\pi^2\kappa}{3(x-1)^4} + \frac{1127\kappa}{108(x-1)^4} - \frac{512d_1\kappa}{3(x-2)^5} + \frac{160\pi^2\kappa}{3(x-2)^5} + \frac{2048\kappa}{3(x-2)^5} - \frac{205d_1\kappa}{216(x-1)^5} + \\
& \frac{\pi^2\kappa}{3(x-1)^5} + \frac{1255\kappa}{216(x-1)^5} + \frac{160\pi^2\kappa}{3(x-2)^6} - \frac{1024\kappa}{9(x-2)^6} - \frac{37\kappa}{8} - \frac{346d_1}{27(x-2)} + \frac{488}{27(x-2)} + \frac{947d_1}{72(x-1)} - \frac{443}{24(x-1)} + \\
& \frac{392d_1}{27(x-2)^2} - \frac{592}{27(x-2)^2} - \frac{23d_1}{27(x-1)^2} + \frac{52}{27(x-1)^2} - \frac{784d_1}{27(x-2)^3} + \frac{1184}{27(x-2)^3} - \frac{73d_1}{216(x-1)^3} + \frac{83}{216(x-1)^3} - \\
& \frac{512d_1}{9(x-2)^4} + \frac{9536}{27(x-2)^4} - \frac{367d_1}{216(x-1)^4} + \frac{\pi^2}{18(x-1)^4} + \frac{725}{108(x-1)^4} - \frac{512d_1}{9(x-2)^5} + \frac{25856}{27(x-2)^5} - \frac{205d_1}{216(x-1)^5} + \\
& \frac{\pi^2}{18(x-1)^5} + \frac{955}{216(x-1)^5} + \frac{1024d_1}{9(x-2)^6} + \frac{13568}{27(x-2)^6} + \frac{160\kappa\ln^2 2}{(x-2)^5} + \frac{640\kappa\ln^2 2}{9(x-2)^5} + \frac{640\kappa\ln^2 2}{9(x-2)^6} + \frac{160\ln^2 2}{3(x-2)^4} + \\
& \frac{640\ln^2 2}{3(x-2)^5} + \frac{640\ln^2 2}{3(x-2)^6} + \frac{64d_1\kappa\ln 2}{3(x-2)^4} + \frac{704\kappa\ln 2}{9(x-2)^4} + \frac{256d_1\kappa\ln 2}{3(x-2)^5} + \frac{2816\kappa\ln 2}{9(x-2)^5} + \frac{256d_1\kappa\ln 2}{3(x-2)^6} + \frac{2816\kappa\ln 2}{9(x-2)^6} + \\
& \frac{64d_1\ln 2}{3(x-2)^4} + \frac{1088\ln 2}{9(x-2)^4} + \frac{256d_1\ln 2}{3(x-2)^5} + \frac{4352\ln 2}{9(x-2)^5} + \frac{256d_1\ln 2}{3(x-2)^6} + \frac{4352\ln 2}{9(x-2)^6} - \frac{55}{24} + H(2; x) \left( -\frac{64\alpha_0\ln 2d_1^2}{3(x-2)^4} + \right. \\
& \left. \frac{64\ln 2d_1^2}{3(x-2)^4} - \frac{128\alpha_0\ln 2d_1^2}{3(x-2)^5} + \frac{256\ln 2d_1^2}{3(x-2)^5} + \frac{256\ln 2d_1^2}{3(x-2)^6} - \frac{40\alpha_0\pi^2\kappa d_1}{9(x-2)^4} + \frac{40\pi^2\kappa d_1}{9(x-2)^4} - \frac{80\alpha_0\pi^2\kappa d_1}{9(x-2)^5} + \frac{160\pi^2\kappa d_1}{9(x-2)^5} + \right. \\
& \left. \frac{160\pi^2\kappa d_1}{9(x-2)^6} - \frac{40\alpha_0\pi^2 d_1}{9(x-2)^4} + \frac{40\pi^2 d_1}{9(x-2)^4} - \frac{80\alpha_0\pi^2 d_1}{9(x-2)^5} + \frac{160\pi^2 d_1}{9(x-2)^5} + \frac{160\pi^2 d_1}{9(x-2)^6} - \frac{160\alpha_0\kappa\ln^2 2d_1}{3(x-2)^4} + \frac{160\kappa\ln^2 2d_1}{3(x-2)^4} - \right. \\
& \left. \frac{320\alpha_0\kappa\ln^2 2d_1}{3(x-2)^5} + \frac{640\kappa\ln^2 2d_1}{3(x-2)^5} + \frac{640\kappa\ln^2 2d_1}{3(x-2)^6} - \frac{160\alpha_0\ln^2 2d_1}{3(x-2)^4} + \frac{160\ln^2 2d_1}{3(x-2)^4} - \frac{320\alpha_0\ln^2 2d_1}{3(x-2)^5} + \frac{640\ln^2 2d_1}{3(x-2)^5} + \right. \\
& \left. \frac{640\ln^2 2d_1}{3(x-2)^6} + \frac{128\alpha_0\kappa\ln 2d_1}{3(x-2)^4} - \frac{128\kappa\ln 2d_1}{3(x-2)^4} + \frac{256\alpha_0\kappa\ln 2d_1}{3(x-2)^5} - \frac{512\kappa\ln 2d_1}{3(x-2)^5} - \frac{512\kappa\ln 2d_1}{3(x-2)^6} - \frac{896\alpha_0\ln 2d_1}{9(x-2)^4} + \right. \\
& \left. \frac{896\ln 2d_1}{9(x-2)^4} - \frac{1792\alpha_0\ln 2d_1}{9(x-2)^5} + \frac{3584\ln 2d_1}{9(x-2)^5} + \frac{3584\ln 2d_1}{9(x-2)^6} + \frac{40\alpha_0\pi^2\kappa}{3(x-2)^4} - \frac{40\pi^2\kappa}{3(x-2)^4} + \frac{80\alpha_0\pi^2\kappa}{3(x-2)^5} - \frac{160\pi^2\kappa}{3(x-2)^5} - \right. \\
& \left. \frac{160\pi^2\kappa}{3(x-2)^6} + \left( -\frac{64\alpha_0 d_1^2}{3(x-2)^4} + \frac{64d_1^2}{3(x-2)^4} - \frac{128\alpha_0 d_1^2}{3(x-2)^5} + \frac{256d_1^2}{3(x-2)^5} + \frac{256d_1^2}{3(x-2)^6} + \frac{128\alpha_0\kappa d_1}{3(x-2)^4} - \frac{128\kappa d_1}{3(x-2)^4} + \frac{256\alpha_0\kappa d_1}{3(x-2)^5} - \right. \right. \\
& \left. \frac{512\kappa d_1}{3(x-2)^5} - \frac{512\kappa d_1}{3(x-2)^6} - \frac{896\alpha_0 d_1}{9(x-2)^4} + \frac{896d_1}{9(x-2)^4} - \frac{1792\alpha_0 d_1}{9(x-2)^5} + \frac{3584d_1}{9(x-2)^5} + \frac{3584d_1}{9(x-2)^6} + \frac{704\alpha_0\kappa}{9(x-2)^4} - \frac{704\kappa}{9(x-2)^4} + \right. \\
& \left. \frac{1408\alpha_0\kappa}{9(x-2)^5} - \frac{2816\kappa}{9(x-2)^5} - \frac{2816\kappa}{9(x-2)^6} + \frac{1088\alpha_0}{9(x-2)^4} - \frac{1088}{9(x-2)^4} + \frac{2176\alpha_0}{9(x-2)^5} - \frac{4352}{9(x-2)^5} - \frac{4352}{9(x-2)^6} \right) H(0; \alpha_0) + \\
& \left( \frac{320d_1\kappa\alpha_0}{3(x-2)^4} - \frac{320\kappa\alpha_0}{(x-2)^4} + \frac{640d_1\kappa\alpha_0}{3(x-2)^5} - \frac{640\kappa\alpha_0}{(x-2)^5} + \frac{320d_1\alpha_0}{3(x-2)^4} - \frac{320\alpha_0}{3(x-2)^4} + \frac{640d_1\alpha_0}{3(x-2)^5} - \frac{640\alpha_0}{3(x-2)^5} - \frac{320d_1\kappa}{3(x-2)^4} + \right. \\
& \left. \frac{320\kappa}{(x-2)^4} - \frac{1280d_1\kappa}{3(x-2)^5} + \frac{1280\kappa}{(x-2)^5} - \frac{1280d_1\kappa}{3(x-2)^6} + \frac{1280\kappa}{(x-2)^6} - \frac{320d_1}{3(x-2)^4} + \frac{320}{3(x-2)^4} - \frac{1280d_1}{3(x-2)^5} + \frac{1280}{3(x-2)^5} - \frac{1280d_1}{3(x-2)^6} + \right. \\
& \left. \frac{1280}{3(x-2)^6} \right) H(0, 0; \alpha_0) + \left( \frac{320\alpha_0 d_1^2}{3(x-2)^4} - \frac{320d_1^2}{3(x-2)^4} + \frac{640\alpha_0 d_1^2}{3(x-2)^5} - \frac{1280d_1^2}{3(x-2)^5} - \frac{1280d_1^2}{3(x-2)^6} - \frac{320\alpha_0\kappa d_1}{3(x-2)^4} + \frac{320\kappa d_1}{3(x-2)^4} - \right. \\
& \left. \frac{640\alpha_0\kappa d_1}{3(x-2)^5} + \frac{1280\kappa d_1}{3(x-2)^5} + \frac{1280\kappa d_1}{3(x-2)^6} - \frac{320\alpha_0 d_1}{3(x-2)^4} + \frac{320d_1}{3(x-2)^4} - \frac{640\alpha_0 d_1}{3(x-2)^5} + \frac{1280d_1}{3(x-2)^5} + \frac{1280d_1}{3(x-2)^6} \right) H(0, 1; \alpha_0) + \\
& \frac{40\alpha_0\pi^2}{9(x-2)^4} - \frac{40\pi^2}{9(x-2)^4} + \frac{80\alpha_0\pi^2}{9(x-2)^5} - \frac{160\pi^2}{9(x-2)^5} - \frac{160\pi^2}{9(x-2)^6} + \frac{160\alpha_0\kappa\ln^2 2}{(x-2)^4} - \frac{160\kappa\ln^2 2}{(x-2)^4} + \frac{320\alpha_0\kappa\ln^2 2}{(x-2)^5} - \\
& \frac{640\kappa\ln^2 2}{(x-2)^5} - \frac{640\kappa\ln^2 2}{(x-2)^6} + \frac{160\alpha_0\ln^2 2}{3(x-2)^4} - \frac{160\ln^2 2}{3(x-2)^4} + \frac{320\alpha_0\ln^2 2}{3(x-2)^5} - \frac{640\ln^2 2}{3(x-2)^5} - \frac{640\ln^2 2}{3(x-2)^6} + \frac{704\alpha_0\kappa\ln 2}{9(x-2)^4} - \\
& \frac{704\kappa\ln 2}{9(x-2)^4} + \frac{1408\alpha_0\kappa\ln 2}{9(x-2)^5} - \frac{2816\kappa\ln 2}{9(x-2)^5} - \frac{2816\kappa\ln 2}{9(x-2)^6} + \frac{1088\alpha_0\ln 2}{9(x-2)^4} - \frac{1088\ln 2}{9(x-2)^4} + \frac{2176\alpha_0\ln 2}{9(x-2)^5} - \\
& \frac{4352\ln 2}{9(x-2)^5} - \frac{4352\ln 2}{9(x-2)^6} + \frac{25\pi^2x}{216(\kappa+1)} + \frac{242x}{81(\kappa+1)} + \frac{253\pi^2x\kappa}{216(\kappa+1)} - \frac{112\pi^2\kappa}{27(x-2)(\kappa+1)} + \frac{301\pi^2\kappa}{72(x-1)(\kappa+1)} + \\
& \frac{140\pi^2\kappa}{27(x-2)^2(\kappa+1)} - \frac{14\pi^2\kappa}{27(x-1)^2(\kappa+1)} - \frac{280\pi^2\kappa}{27(x-2)^3(\kappa+1)} - \frac{7\pi^2\kappa}{216(x-1)^3(\kappa+1)} - \frac{8d_1\pi^2\kappa}{3(x-2)^4(\kappa+1)} - \frac{664\pi^2\kappa}{9(x-2)^4(\kappa+1)} - \\
& \frac{379\pi^2\kappa}{216(x-1)^4(\kappa+1)} - \frac{32d_1\pi^2\kappa}{3(x-2)^5(\kappa+1)} - \frac{512\pi^2\kappa}{3(x-2)^5(\kappa+1)} - \frac{253\pi^2\kappa}{216(x-1)^5(\kappa+1)} - \frac{32d_1\pi^2\kappa}{3(x-2)^6(\kappa+1)} - \frac{416\pi^2\kappa}{9(x-2)^6(\kappa+1)} + \\
& \frac{7\pi^2\kappa}{12(\kappa+1)} - \frac{16\pi^2}{27(x-2)(\kappa+1)} + \frac{43\pi^2}{72(x-1)(\kappa+1)} + \frac{20\pi^2}{27(x-2)^2(\kappa+1)} - \frac{2\pi^2}{27(x-1)^2(\kappa+1)} - \frac{40\pi^2}{27(x-2)^3(\kappa+1)} - \\
& \frac{\pi^2}{216(x-1)^3(\kappa+1)} - \frac{8d_1\pi^2}{9(x-2)^4(\kappa+1)} - \frac{376\pi^2}{27(x-2)^4(\kappa+1)} - \frac{23\pi^2}{72(x-1)^4(\kappa+1)} - \frac{32d_1\pi^2}{9(x-2)^5(\kappa+1)} - \frac{1024\pi^2}{27(x-2)^5(\kappa+1)} - \\
& \frac{17\pi^2}{72(x-1)^5(\kappa+1)} - \frac{32d_1\pi^2}{9(x-2)^6(\kappa+1)} - \frac{544\pi^2}{27(x-2)^6(\kappa+1)} + \frac{\pi^2}{12(\kappa+1)} - \frac{x\zeta_3}{\kappa+1} - \frac{7x\kappa\zeta_3}{3(\kappa+1)} + \frac{7\kappa\zeta_3}{3(x-1)^4(\kappa+1)} +
\end{aligned}$$

$$\left. \begin{aligned} & \frac{7 \kappa \zeta_3}{3(x-1)^5(\kappa+1)} + \frac{\zeta_3}{3(x-1)^4(\kappa+1)} + \frac{\zeta_3}{3(x-1)^5(\kappa+1)} - \frac{1120\kappa \ln^3 2}{9(x-2)^4(\kappa+1)} - \frac{4480\kappa \ln^3 2}{9(x-2)^5(\kappa+1)} - \frac{4480\kappa \ln^3 2}{9(x-2)^6(\kappa+1)} - \\ & \frac{160 \ln^3 2}{9(x-2)^4(\kappa+1)} - \frac{640 \ln^3 2}{9(x-2)^5(\kappa+1)} - \frac{640 \ln^3 2}{9(x-2)^6(\kappa+1)} - \frac{32d_1 \kappa \ln^2 2}{(x-2)^4(\kappa+1)} - \frac{416 \kappa \ln^2 2}{3(x-2)^4(\kappa+1)} - \frac{128d_1 \kappa \ln^2 2}{(x-2)^5(\kappa+1)} - \\ & \frac{1664\kappa \ln^2 2}{3(x-2)^5(\kappa+1)} - \frac{128d_1 \kappa \ln^2 2}{(x-2)^6(\kappa+1)} - \frac{1664\kappa \ln^2 2}{3(x-2)^6(\kappa+1)} - \frac{32 d_1 \ln^2 2}{3(x-2)^4(\kappa+1)} - \frac{544 \ln^2 2}{9(x-2)^4(\kappa+1)} - \frac{128d_1 \ln^2 2}{3(x-2)^5(\kappa+1)} - \\ & \frac{2176 \ln^2 2}{9(x-2)^5(\kappa+1)} - \frac{128d_1 \ln^2 2}{3(x-2)^6(\kappa+1)} - \frac{2176 \ln^2 2}{9(x-2)^6(\kappa+1)} - \frac{256d_1 \kappa \ln 2}{9(x-2)^4(\kappa+1)} - \frac{80\pi^2 \kappa \ln 2}{3(x-2)^4(\kappa+1)} - \frac{1856\kappa \ln 2}{27(x-2)^4(\kappa+1)} - \\ & \frac{1024 d_1 \kappa \ln 2}{9(x-2)^5(\kappa+1)} - \frac{320\pi^2 \kappa \ln 2}{3(x-2)^5(\kappa+1)} - \frac{7424\kappa \ln 2}{27(x-2)^5(\kappa+1)} - \frac{1024d_1 \kappa \ln 2}{9(x-2)^6(\kappa+1)} - \frac{320\pi^2 \kappa \ln 2}{3(x-2)^6(\kappa+1)} - \frac{7424\kappa \ln 2}{27(x-2)^6(\kappa+1)} - \\ & \frac{256 d_1 \ln 2}{9(x-2)^4(\kappa+1)} - \frac{3392 \ln 2}{27(x-2)^4(\kappa+1)} - \frac{1024d_1 \ln 2}{9(x-2)^5(\kappa+1)} - \frac{13568 \ln 2}{27(x-2)^5(\kappa+1)} - \frac{1024d_1 \ln 2}{9(x-2)^6(\kappa+1)} - \frac{13568 \ln 2}{27(x-2)^6(\kappa+1)} \Big\}. \end{aligned}$$

**D.4 The  $\mathcal{A}$  integral for  $k = -1$  and  $\kappa = 0$**

The  $\varepsilon$  expansion for this integral reads

$$\begin{aligned} \mathcal{I}(x, \varepsilon; \alpha_0, 3 + d_1\varepsilon; 0, 2, 0, g_A) &= x \mathcal{A}(\varepsilon, x; 3 + d_1\varepsilon; 0, 2) \\ &= \frac{1}{\varepsilon^2} a_{-2}^{(0,-1)} + \frac{1}{\varepsilon} a_{-1}^{(0,-1)} + a_0^{(0,-1)} + \varepsilon a_1^{(0,-1)} + \varepsilon^2 a_2^{(0,-1)} \mathcal{O}(\varepsilon^3), \end{aligned} \tag{D.4}$$

where

$$\begin{aligned} a_{-2}^{(0,-1)} &= \frac{1}{2}, \\ a_{-1}^{(0,-1)} &= -H(0; x), \\ a_0^{(0,-1)} &= \frac{\alpha_0^3}{12(x-1)^2} - \frac{\alpha_0^3}{12} + \frac{\alpha_0^2}{24(x-1)} - \frac{5\alpha_0^2}{24(x-1)^2} + \frac{7\alpha_0^2}{24(x-1)^3} + \frac{13}{24} \frac{\alpha_0^2}{\alpha_0} - \frac{\alpha_0}{3(x-1)} + \frac{\alpha_0}{6(x-1)^2} - \\ & \frac{\alpha_0}{3(x-1)^3} + \frac{13\alpha_0}{12(x-1)^4} - \frac{23}{12} \frac{\alpha_0}{\alpha_0} + \left( \frac{25}{12} + \frac{3}{4(x-1)} - \frac{1}{6(x-1)^2} - \frac{1}{6(x-1)^3} + \frac{3}{4(x-1)^4} + \right. \\ & \left. \frac{25}{12(x-1)^5} \right) H(0; \alpha_0) + \left( -\frac{25}{12} - \frac{3}{4(x-1)} + \frac{1}{6(x-1)^2} + \frac{1}{6(x-1)^3} - \frac{3}{4(x-1)^4} - \frac{25}{12(x-1)^5} \right) H(0; x) + \\ & \left( \frac{1}{(x-1)^5} - 1 \right) H(0; \alpha_0) H(1; x) + \left( -\frac{\alpha_0^4}{4(x-1)} + \frac{\alpha_0^4}{4} + \frac{\alpha_0^3}{x-1} - \frac{\alpha_0^3}{3(x-1)^2} - \frac{4\alpha_0^3}{3} - \frac{3\alpha_0^2}{2(x-1)} + \frac{\alpha_0^2}{(x-1)^2} - \right. \\ & \left. \frac{\alpha_0^2}{2(x-1)^3} + 3\alpha_0^2 + \frac{\alpha_0}{x-1} - \frac{\alpha_0}{(x-1)^2} + \frac{\alpha_0}{(x-1)^3} - \frac{\alpha_0}{(x-1)^4} - 4\alpha_0 + \frac{3}{4(x-1)} - \frac{1}{6(x-1)^2} - \frac{1}{6(x-1)^3} + \right. \\ & \left. \frac{3}{4(x-1)^4} + \frac{25}{12(x-1)^5} + \frac{25}{12} \right) H(c_1(\alpha_0); x) + 2H(0, 0; x) + \left( \frac{1}{(x-1)^5} - 1 \right) H(0, c_1(\alpha_0); x) + \left( 1 - \right. \\ & \left. \frac{1}{(x-1)^5} \right) H(1, 0; x) + \left( \frac{1}{(x-1)^5} - 1 \right) H(1, c_1(\alpha_0); x) - \frac{H(c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} - \frac{\pi^2}{6(x-1)^5} - \frac{\pi^2}{12}, \\ a_1^{(0,-1)} &= \frac{7d_1\alpha_0^3}{72} - \frac{7d_1\alpha_0^3}{72(x-1)^2} + \frac{7}{72} \frac{\alpha_0^3}{\alpha_0} - \frac{7\alpha_0^3}{72} - \frac{109d_1}{144} \frac{\alpha_0^2}{\alpha_0} - \frac{13d_1\alpha_0^2}{144(x-1)} - \frac{5\alpha_0^2}{144(x-1)} + \frac{29d_1\alpha_0^2}{144(x-1)^2} - \\ & \frac{47\alpha_0^2}{144(x-1)^2} - \frac{67d_1\alpha_0^2}{144(x-1)^3} + \frac{85\alpha_0^2}{144(x-1)^3} + \frac{127\alpha_0^2}{144} + \frac{305d_1\alpha_0}{72} + \frac{19d_1}{18(x-1)} \frac{\alpha_0}{\alpha_0} - \frac{2\alpha_0}{9(x-1)} - \frac{d_1\alpha_0}{9(x-1)^2} + \frac{13\alpha_0}{36(x-1)^2} + \\ & \frac{d_1\alpha_0}{18(x-1)^3} - \frac{8}{9(x-1)^3} \frac{\alpha_0}{\alpha_0} - \frac{217d_1\alpha_0}{72(x-1)^4} + \frac{149\alpha_0}{36(x-1)^4} - \frac{101\alpha_0}{18} + \left( -\frac{\alpha_0^3}{6(x-1)^2} + \frac{\alpha_0^3}{6} - \frac{\alpha_0^2}{12(x-1)} + \frac{5\alpha_0^2}{12(x-1)^2} - \right. \\ & \left. \frac{7\alpha_0^2}{12(x-1)^3} - \frac{13\alpha_0^2}{12} + \frac{2}{3(x-1)} \frac{\alpha_0}{\alpha_0} - \frac{\alpha_0}{3(x-1)^2} + \frac{2\alpha_0}{3(x-1)^3} - \frac{13}{6(x-1)^4} \frac{\alpha_0}{\alpha_0} + \frac{23\alpha_0}{6} - \frac{205d_1}{72} - \frac{15d_1}{8(x-1)} + \frac{1}{4(x-1)} + \frac{5d_1}{18(x-1)^2} - \right. \\ & \left. \frac{7}{36(x-1)^2} + \frac{5d_1}{18(x-1)^3} - \frac{13}{36(x-1)^3} - \frac{15d_1}{8(x-1)^4} + \frac{3}{(x-1)^4} - \frac{205d_1}{72(x-1)^5} + \frac{65}{9(x-1)^5} + \frac{155}{36} \right) H(0; \alpha_0) + \\ & \left( \frac{15d_1}{8(x-1)} - \frac{5d_1}{18(x-1)^2} - \frac{5d_1}{18(x-1)^3} + \frac{15d_1}{8(x-1)^4} + \frac{205d_1}{72(x-1)^5} + \frac{205d_1}{72} - \frac{1}{4(x-1)} + \frac{7}{36(x-1)^2} + \frac{13}{36(x-1)^3} - \right. \\ & \left. \frac{3}{(x-1)^4} + \frac{\pi^2}{3(x-1)^5} - \frac{65}{9(x-1)^5} + \frac{\pi^2}{6} - \frac{155}{36} \right) H(0; x) + \left( \frac{d_1\alpha_0^3}{6} - \frac{d_1\alpha_0^3}{6(x-1)^2} - \frac{13d_1\alpha_0^2}{12} - \frac{d_1\alpha_0^2}{12(x-1)} + \frac{5d_1\alpha_0^2}{12(x-1)^2} - \right. \\ & \left. \frac{7d_1\alpha_0^2}{12(x-1)^3} + \frac{23d_1\alpha_0}{6} + \frac{2}{3(x-1)} \frac{d_1\alpha_0}{\alpha_0} - \frac{d_1\alpha_0}{3(x-1)^2} + \frac{2d_1\alpha_0}{3(x-1)^3} - \frac{13d_1\alpha_0}{6(x-1)^4} - \frac{35d_1}{12} - \frac{7d_1}{12(x-1)} + \frac{d_1}{12(x-1)^2} - \frac{d_1}{12(x-1)^3} + \right. \\ & \left. \frac{13d_1}{6(x-1)^4} \right) H(1; \alpha_0) + \left( -\frac{d_1\alpha_0^4}{8} + \frac{d_1}{8(x-1)} \frac{\alpha_0^4}{\alpha_0} - \frac{\alpha_0^4}{8(x-1)} + \frac{\alpha_0^4}{8} + \frac{13d_1}{18} \frac{\alpha_0^3}{\alpha_0} - \frac{d_1\alpha_0^3}{2(x-1)} + \frac{5\alpha_0^3}{6(x-1)} + \frac{2}{9(x-1)^2} \frac{d_1\alpha_0^3}{\alpha_0} - \right. \end{aligned}$$

$$\begin{aligned}
& \frac{17\alpha_0^3}{36(x-1)^2} - \frac{29\alpha_0^3}{36} - \frac{23d_1\alpha_0^2}{12} + \frac{3d_1\alpha_0^2}{4(x-1)} - \frac{19\alpha_0^2}{8(x-1)} - \frac{2d_1\alpha_0^2}{3(x-1)^2} + \frac{47\alpha_0^2}{24(x-1)^2} + \frac{d_1\alpha_0^2}{2(x-1)^3} - \frac{11\alpha_0^2}{8(x-1)^3} + \frac{59\alpha_0^2}{24} \\
& \frac{25d_1\alpha_0}{6} - \frac{d_1\alpha_0}{2(x-1)} + \frac{9\alpha_0}{2(x-1)} + \frac{2d_1\alpha_0}{3(x-1)^2} - \frac{23\alpha_0}{6(x-1)^2} - \frac{d_1\alpha_0}{(x-1)^3} + \frac{4\alpha_0}{(x-1)^3} + \frac{2d_1\alpha_0}{(x-1)^4} - \frac{21\alpha_0}{4(x-1)^4} - \frac{73\alpha_0}{12} - \frac{205d_1}{72} + \\
& \left( \frac{\alpha_0^4}{2(x-1)} - \frac{\alpha_0^4}{2} - \frac{2\alpha_0^3}{x-1} + \frac{2\alpha_0^3}{3(x-1)^2} + \frac{8\alpha_0^3}{3} + \frac{3\alpha_0^2}{x-1} - \frac{2\alpha_0^2}{(x-1)^2} + \frac{\alpha_0^2}{(x-1)^3} - 6\alpha_0^2 - \frac{2\alpha_0}{x-1} + \frac{2\alpha_0}{(x-1)^2} - \frac{2\alpha_0}{(x-1)^3} + \frac{2\alpha_0}{(x-1)^4} + \right. \\
& 8\alpha_0 - \frac{3}{2(x-1)} + \frac{1}{3(x-1)^2} + \frac{1}{3(x-1)^3} - \frac{3}{2(x-1)^4} - \frac{25}{6(x-1)^5} - \left. \frac{25}{6} \right) H(0; \alpha_0) + \left( -\frac{d_1\alpha_0^4}{2} + \frac{d_1\alpha_0^4}{2(x-1)} + \frac{8d_1\alpha_0^3}{3} - \right. \\
& \frac{2d_1\alpha_0^3}{x-1} + \frac{2d_1\alpha_0^3}{3(x-1)^2} - 6d_1\alpha_0^2 + \frac{3d_1\alpha_0^2}{x-1} - \frac{2d_1\alpha_0^2}{(x-1)^2} + \frac{d_1\alpha_0^2}{(x-1)^3} + 8d_1\alpha_0 - \frac{2d_1\alpha_0}{x-1} + \frac{2d_1\alpha_0}{(x-1)^2} - \frac{2d_1\alpha_0}{(x-1)^3} + \frac{2d_1\alpha_0}{(x-1)^4} - \\
& \left. \frac{25d_1}{6} - \frac{3d_1}{2(x-1)} + \frac{d_1}{3(x-1)^2} + \frac{d_1}{3(x-1)^3} - \frac{3d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} \right) H(1; \alpha_0) - \frac{15d_1}{8(x-1)} + \frac{1}{4(x-1)} + \frac{5d_1}{18(x-1)^2} - \\
& \frac{7}{36(x-1)^2} + \frac{5d_1}{18(x-1)^3} - \frac{13}{36(x-1)^3} - \frac{15d_1}{8(x-1)^4} + \frac{3}{(x-1)^4} - \frac{205d_1}{72(x-1)^5} + \frac{65}{9(x-1)^5} + \frac{155}{36} \Big) H(c_1(\alpha_0); x) + \\
& \left( -\frac{25}{6} - \frac{3}{2(x-1)} + \frac{1}{3(x-1)^2} + \frac{1}{3(x-1)^3} - \frac{3}{2(x-1)^4} - \frac{25}{6(x-1)^5} \right) H(0, 0; \alpha_0) + \left( \frac{25}{6} + \frac{3}{2(x-1)} - \frac{1}{3(x-1)^2} - \right. \\
& \left. \frac{1}{3(x-1)^3} + \frac{3}{2(x-1)^4} + \frac{25}{6(x-1)^5} \right) H(0, 0; x) + \left( -\frac{3d_1}{2(x-1)} + \frac{d_1}{3(x-1)^2} + \frac{d_1}{3(x-1)^3} - \frac{3d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} - \right. \\
& \left. \frac{25d_1}{6} \right) H(0, 1; \alpha_0) + H(1; x) \left( -\frac{\pi^2 d_1}{3(x-1)^5} + \left( \frac{2d_1}{x-1} - \frac{d_1}{(x-1)^2} + \frac{2d_1}{3(x-1)^3} - \frac{d_1}{2(x-1)^4} + \frac{25d_1}{6(x-1)^5} + \frac{5}{4(x-1)} - \right. \right. \\
& \left. \left. \frac{5}{6(x-1)^2} + \frac{5}{6(x-1)^3} - \frac{5}{4(x-1)^4} + \frac{25}{12(x-1)^5} - \frac{25}{12} \right) H(0; \alpha_0) + \left( 2 - \frac{2}{(x-1)^5} \right) H(0, 0; \alpha_0) + \left( 2d_1 - \right. \\
& \left. \frac{2d_1}{(x-1)^5} \right) H(0, 1; \alpha_0) + \frac{\pi^2}{6(x-1)^5} + \frac{\pi^2}{6} \Big) + \left( \frac{2d_1}{(x-1)^5} - 2d_1 \right) H(0; \alpha_0) H(0, 1; x) + \left( -\frac{\alpha_0^4}{2(x-1)} + \frac{\alpha_0^4}{2} + \right. \\
& \left. \frac{2\alpha_0^3}{x-1} - \frac{2\alpha_0^3}{3(x-1)^2} - \frac{8\alpha_0^3}{3} - \frac{3\alpha_0^2}{x-1} + \frac{2\alpha_0^2}{(x-1)^2} - \frac{\alpha_0^2}{(x-1)^3} + 6\alpha_0^2 + \frac{2\alpha_0}{x-1} - \frac{2\alpha_0}{(x-1)^2} + \frac{2\alpha_0}{(x-1)^3} - \frac{2\alpha_0}{(x-1)^4} - 8\alpha_0 + \left( 2 - \right. \right. \\
& \left. \left. \frac{2}{(x-1)^5} \right) H(0; \alpha_0) + \left( 2d_1 - \frac{2d_1}{(x-1)^5} \right) H(1; \alpha_0) + \frac{3}{4(x-1)} - \frac{1}{6(x-1)^2} - \frac{1}{6(x-1)^3} + \frac{3}{4(x-1)^4} + \frac{25}{12(x-1)^5} + \right. \\
& \left. \frac{25}{12} \right) H(0, c_1(\alpha_0); x) + \left( -\frac{2d_1}{x-1} + \frac{d_1}{(x-1)^2} - \frac{2d_1}{3(x-1)^3} + \frac{d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} - \frac{5}{4(x-1)} + \frac{5}{6(x-1)^2} - \right. \\
& \left. \frac{5}{6(x-1)^3} + \frac{5}{4(x-1)^4} - \frac{25}{12(x-1)^5} + \frac{25}{12} \right) H(1, 0; x) + \left( \frac{4d_1}{(x-1)^5} - 2d_1 - \frac{1}{(x-1)^5} - 1 \right) H(0; \alpha_0) H(1, 1; x) + \\
& \left( \frac{2d_1}{x-1} - \frac{d_1}{(x-1)^2} + \frac{2d_1}{3(x-1)^3} - \frac{d_1}{2(x-1)^4} + \frac{25d_1}{6(x-1)^5} + \left( 2 - \frac{2}{(x-1)^5} \right) H(0; \alpha_0) + \left( 2d_1 - \frac{2d_1}{(x-1)^5} \right) H(1; \alpha_0) + \right. \\
& \left. \frac{5}{4(x-1)} - \frac{5}{6(x-1)^2} + \frac{5}{6(x-1)^3} - \frac{5}{4(x-1)^4} + \frac{25}{12(x-1)^5} - \frac{25}{12} \right) H(1, c_1(\alpha_0); x) + \left( \frac{3\alpha_0^4}{4(x-1)} - \frac{3\alpha_0^4}{4} - \frac{3\alpha_0^3}{x-1} + \right. \\
& \left. \frac{\alpha_0^3}{(x-1)^2} + 4\alpha_0^3 + \frac{9\alpha_0^2}{2(x-1)} - \frac{3\alpha_0^2}{(x-1)^2} + \frac{3\alpha_0^2}{2(x-1)^3} - 9\alpha_0^2 - \frac{3\alpha_0}{x-1} + \frac{3\alpha_0}{(x-1)^2} - \frac{3\alpha_0}{(x-1)^3} + \frac{3\alpha_0}{(x-1)^4} + 12\alpha_0 + \frac{2H(0; \alpha_0)}{(x-1)^5} + \right. \\
& \left. \frac{2d_1 H(1; \alpha_0)}{(x-1)^5} - \frac{9}{4(x-1)} + \frac{1}{2(x-1)^2} + \frac{1}{2(x-1)^3} - \frac{9}{4(x-1)^4} - \frac{25}{4(x-1)^5} - \frac{25}{4} \right) H(c_1(\alpha_0), c_1(\alpha_0); x) - \\
& 4H(0, 0, 0; x) + \left( 2d_1 - \frac{2d_1}{(x-1)^5} \right) H(0, 1, 0; x) + \left( \frac{2d_1}{(x-1)^5} - 2d_1 \right) H(0, 1, c_1(\alpha_0); x) + \left( 3 - \right. \\
& \left. \frac{1}{(x-1)^5} \right) H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{2}{(x-1)^5} - 2 \right) H(1, 0, 0; x) + \left( \frac{2d_1}{(x-1)^5} - \frac{1}{(x-1)^5} - \right. \\
& \left. 1 \right) H(1, 0, c_1(\alpha_0); x) + \left( -\frac{4d_1}{(x-1)^5} + 2d_1 + \frac{1}{(x-1)^5} + 1 \right) H(1, 1, 0; x) + \left( \frac{4d_1}{(x-1)^5} - 2d_1 - \frac{1}{(x-1)^5} - \right. \\
& \left. 1 \right) H(1, 1, c_1(\alpha_0); x) + \left( -\frac{2d_1}{(x-1)^5} - \frac{1}{(x-1)^5} + 3 \right) H(1, c_1(\alpha_0), c_1(\alpha_0); x) - \frac{2H(c_1(\alpha_0), 0, c_1(\alpha_0); x)}{(x-1)^5} + \\
& \frac{3H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} - \frac{\pi^2}{8(x-1)} + \frac{\pi^2}{36(x-1)^2} + \frac{\pi^2}{36(x-1)^3} - \frac{\pi^2}{8(x-1)^4} - \frac{25\pi^2}{72(x-1)^5} - \frac{3\zeta_3}{(x-1)^5} - 5\zeta_3 - \frac{25\pi^2}{72},
\end{aligned}$$

$$\begin{aligned}
a_2^{(0,-1)} = & -\frac{37}{432} d_1^2 \alpha_0^3 + \frac{37d_1\alpha_0^3}{216} + \frac{37d_1^2\alpha_0^3}{432(x-1)^2} - \frac{37d_1\alpha_0^3}{216(x-1)^2} - \frac{\pi^2\alpha_0^3}{72(x-1)^2} + \frac{37\alpha_0^3}{432(x-1)^2} + \frac{\pi^2\alpha_0^3}{72} - \frac{37\alpha_0^3}{432} + \\
& \frac{715d_1^2\alpha_0^2}{864} - \frac{52d_1\alpha_0^2}{27} + \frac{115d_1^2\alpha_0^2}{864(x-1)} + \frac{d_1\alpha_0^2}{216(x-1)} - \frac{\pi^2\alpha_0^2}{144(x-1)} - \frac{119\alpha_0^2}{864(x-1)} - \frac{107d_1^2\alpha_0^2}{864(x-1)^2} + \frac{14d_1\alpha_0^2}{27(x-1)^2} + \frac{5\pi^2\alpha_0^2}{144(x-1)^2} - \\
& \frac{341\alpha_0^2}{864(x-1)^2} + \frac{493d_1^2\alpha_0^2}{864(x-1)^3} - \frac{305d_1\alpha_0^2}{216(x-1)^3} - \frac{7\pi^2\alpha_0^2}{144(x-1)^3} + \frac{727\alpha_0^2}{864(x-1)^3} - \frac{13\pi^2\alpha_0^2}{144} + \frac{949\alpha_0^2}{864} - \frac{3515d_1^2\alpha_0}{432} + \\
& \frac{8965d_1\alpha_0}{432} - \frac{265d_1^2\alpha_0}{108(x-1)} + \frac{263d_1\alpha_0}{108(x-1)} + \frac{\pi^2\alpha_0}{18(x-1)} + \frac{65\alpha_0}{108(x-1)} - \frac{d_1^2\alpha_0}{108(x-1)^2} - \frac{113d_1\alpha_0}{216(x-1)^2} - \frac{\pi^2\alpha_0}{36(x-1)^2} + \\
& \frac{115\alpha_0}{216(x-1)^2} + \frac{113d_1^2\alpha_0}{108(x-1)^3} + \frac{41d_1\alpha_0}{108(x-1)^3} + \frac{\pi^2\alpha_0}{18(x-1)^3} - \frac{217\alpha_0}{108(x-1)^3} + \frac{2911d_1^2\alpha_0}{432(x-1)^4} - \frac{7523d_1\alpha_0}{432(x-1)^4} - \frac{13\pi^2\alpha_0}{72(x-1)^4} + \\
& \frac{2369\alpha_0}{216(x-1)^4} + \frac{23\pi^2\alpha_0}{72} - \frac{697\alpha_0}{54} + \left( -\frac{7d_1\alpha_0^3}{36} + \frac{7d_1\alpha_0^3}{36(x-1)^2} - \frac{7\alpha_0^3}{36(x-1)^2} + \frac{7\alpha_0^3}{36} + \frac{109d_1\alpha_0^2}{72} + \frac{13d_1\alpha_0^2}{72(x-1)} + \frac{5\alpha_0^2}{72(x-1)} - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{29d_1\alpha_0^2}{72(x-1)^2} + \frac{47\alpha_0^2}{72(x-1)^2} + \frac{67d_1\alpha_0^2}{72(x-1)^3} - \frac{85\alpha_0^2}{72(x-1)^3} - \frac{127\alpha_0^2}{72} - \frac{305d_1\alpha_0}{36} - \frac{19d_1\alpha_0}{9(x-1)} + \frac{4\alpha_0}{9(x-1)} + \frac{2d_1\alpha_0}{9(x-1)^2} - \\
& \frac{13\alpha_0}{18(x-1)^2} - \frac{d_1\alpha_0}{9(x-1)^3} + \frac{16\alpha_0}{9(x-1)^3} + \frac{217d_1\alpha_0}{36(x-1)^4} - \frac{149\alpha_0}{18(x-1)^4} + \frac{101\alpha_0}{9} + \frac{2035d_1^2}{432} - \frac{5615d_1}{432} + \frac{63d_1^2}{16(x-1)} - \frac{209d_1}{48(x-1)} - \\
& \frac{\pi^2}{8(x-1)} - \frac{2}{x-1} - \frac{19d_1^2}{54(x-1)^2} + \frac{347d_1}{216(x-1)^2} + \frac{\pi^2}{36(x-1)^2} - \frac{19}{216(x-1)^2} - \frac{19d_1^2}{54(x-1)^3} + \frac{173d_1}{216(x-1)^3} + \frac{\pi^2}{36(x-1)^3} - \\
& \frac{205}{216(x-1)^3} + \frac{63d_1^2}{16(x-1)^4} - \frac{197d_1}{16(x-1)^4} - \frac{\pi^2}{8(x-1)^4} + \frac{163}{24(x-1)^4} + \frac{2035d_1^2}{432(x-1)^5} - \frac{8705d_1}{432(x-1)^5} - \frac{25\pi^2}{72(x-1)^5} + \\
& \frac{3965}{216(x-1)^5} - \frac{25\pi^2}{72} + \frac{235}{27} \Big) H(0; \alpha_0) + \left( -\frac{7}{36}d_1^2\alpha_0^3 + \frac{7d_1\alpha_0^3}{36} + \frac{7d_1^2\alpha_0^3}{36(x-1)^2} - \frac{7d_1\alpha_0^3}{36(x-1)^2} + \frac{109d_1^2\alpha_0^2}{72} - \right. \\
& \frac{127d_1\alpha_0^2}{72} + \frac{13d_1^2\alpha_0^2}{72(x-1)} + \frac{5d_1\alpha_0^2}{72(x-1)} - \frac{29d_1^2\alpha_0^2}{72(x-1)^2} + \frac{47d_1\alpha_0^2}{72(x-1)^2} + \frac{67d_1^2\alpha_0^2}{72(x-1)^3} - \frac{85d_1\alpha_0^2}{72(x-1)^3} - \frac{305d_1^2\alpha_0}{36} + \frac{101d_1\alpha_0}{9} - \\
& \frac{19d_1^2\alpha_0}{9(x-1)} + \frac{4d_1\alpha_0}{9(x-1)} + \frac{2d_1^2\alpha_0}{9(x-1)^2} - \frac{13d_1\alpha_0}{18(x-1)^2} - \frac{d_1^2\alpha_0}{9(x-1)^3} + \frac{16d_1\alpha_0}{9(x-1)^3} + \frac{217d_1^2\alpha_0}{36(x-1)^4} - \frac{149d_1\alpha_0}{18(x-1)^4} + \frac{515d_1^2}{72} - \frac{695d_1}{72} + \\
& \frac{139d_1^2}{72(x-1)} - \frac{37d_1}{72(x-1)} - \frac{d_1^2}{72(x-1)^2} + \frac{19d_1}{72(x-1)^2} - \frac{59d_1^2}{72(x-1)^3} - \frac{43d_1}{72(x-1)^3} - \frac{217d_1^2}{36(x-1)^4} + \frac{149d_1}{18(x-1)^4} \Big) H(1; \alpha_0) + \\
& \left( \frac{\alpha_0^3}{3(x-1)^2} - \frac{\alpha_0^3}{3} + \frac{\alpha_0^2}{6(x-1)} - \frac{5\alpha_0^2}{6(x-1)^2} + \frac{7\alpha_0^2}{6(x-1)^3} + \frac{13\alpha_0^2}{6} - \frac{4\alpha_0}{3(x-1)} + \frac{2\alpha_0}{3(x-1)^2} - \frac{4\alpha_0}{3(x-1)^3} + \frac{13\alpha_0}{3(x-1)^4} - \frac{23\alpha_0}{3} + \right. \\
& \frac{205d_1}{36} + \frac{15d_1}{4(x-1)} - \frac{1}{2(x-1)} - \frac{5d_1}{9(x-1)^2} + \frac{7}{18(x-1)^2} - \frac{5d_1}{9(x-1)^3} + \frac{13}{18(x-1)^3} + \frac{15d_1}{4(x-1)^4} - \frac{6}{(x-1)^4} + \frac{205d_1}{36(x-1)^5} - \\
& \frac{130}{9(x-1)^5} - \frac{155}{18} \Big) H(0, 0; \alpha_0) + \left( -\frac{15d_1}{4(x-1)} + \frac{5d_1}{9(x-1)^2} + \frac{5d_1}{9(x-1)^3} - \frac{15d_1}{4(x-1)^4} - \frac{205d_1}{36(x-1)^5} - \frac{205d_1}{36} + \frac{1}{2(x-1)} - \right. \\
& \frac{7}{18(x-1)^2} - \frac{13}{18(x-1)^3} + \frac{6}{(x-1)^4} - \frac{2\pi^2}{3(x-1)^5} + \frac{130}{9(x-1)^5} - \frac{\pi^2}{3} + \frac{155}{18} \Big) H(0, 0; x) + \left( -\frac{d_1\alpha_0^3}{3} + \frac{d_1\alpha_0^3}{3(x-1)^2} + \right. \\
& \frac{13d_1\alpha_0^2}{6} + \frac{d_1\alpha_0^2}{6(x-1)} - \frac{5d_1\alpha_0^2}{6(x-1)^2} + \frac{7d_1\alpha_0^2}{6(x-1)^3} - \frac{23d_1\alpha_0}{3} - \frac{4d_1\alpha_0}{3(x-1)} + \frac{2d_1\alpha_0}{3(x-1)^2} - \frac{4d_1\alpha_0}{3(x-1)^3} + \frac{13d_1\alpha_0}{3(x-1)^4} + \frac{205d_1^2}{36} - \\
& \frac{155d_1^2}{18} + \frac{15d_1^2}{4(x-1)} - \frac{d_1}{2(x-1)} - \frac{5d_1^2}{9(x-1)^2} + \frac{7d_1}{18(x-1)^2} - \frac{5d_1^2}{9(x-1)^3} + \frac{13d_1}{18(x-1)^3} + \frac{15d_1^2}{4(x-1)^4} - \frac{6d_1}{(x-1)^4} + \frac{205d_1^2}{36(x-1)^5} - \\
& \frac{130d_1}{9(x-1)^5} \Big) H(0, 1; \alpha_0) + \left( \frac{\pi^2 d_1}{3(x-1)^5} + \frac{\pi^2 d_1}{3} + \left( \frac{5d_1}{2(x-1)} - \frac{5d_1}{3(x-1)^2} + \frac{5d_1}{3(x-1)^3} - \frac{5d_1}{2(x-1)^4} + \frac{25d_1}{6(x-1)^5} - \frac{25d_1}{6} + \right. \right. \\
& \frac{5}{2(x-1)} - \frac{5}{3(x-1)^2} + \frac{5}{3(x-1)^3} - \frac{5}{2(x-1)^4} + \frac{25}{6(x-1)^5} - \frac{25}{6} \Big) H(0; \alpha_0) + \left( 4d_1 - \frac{4d_1}{(x-1)^5} \right) H(0, 0; \alpha_0) + \\
& \left( 4d_1^2 - \frac{4d_1^2}{(x-1)^5} \right) H(0, 1; \alpha_0) - \frac{2\pi^2}{3(x-1)^5} H(0, 1; x) + \left( -\frac{d_1\alpha_0^3}{3} + \frac{d_1\alpha_0^3}{3(x-1)^2} + \frac{13d_1\alpha_0^2}{6} + \frac{d_1\alpha_0^2}{6(x-1)} - \frac{5d_1\alpha_0^2}{6(x-1)^2} + \right. \\
& \frac{7d_1\alpha_0^2}{6(x-1)^3} - \frac{23d_1\alpha_0}{3} - \frac{4d_1\alpha_0}{3(x-1)} + \frac{2d_1\alpha_0}{3(x-1)^2} - \frac{4d_1\alpha_0}{3(x-1)^3} + \frac{13d_1\alpha_0}{3(x-1)^4} + \frac{35d_1}{6} + \frac{7d_1}{6(x-1)} - \frac{d_1}{6(x-1)^2} + \frac{d_1}{6(x-1)^3} - \\
& \frac{13d_1}{3(x-1)^4} \Big) H(1, 0; \alpha_0) + \left( \frac{4d_1^2}{x-1} - \frac{d_1^2}{(x-1)^2} + \frac{4d_1^2}{9(x-1)^3} - \frac{d_1^2}{4(x-1)^4} + \frac{205d_1^2}{36(x-1)^5} - \frac{97d_1}{24(x-1)} + \frac{157d_1}{36(x-1)^2} - \right. \\
& \frac{137d_1}{36(x-1)^3} + \frac{19d_1}{8(x-1)^4} + \frac{2\pi^2 d_1}{3(x-1)^5} - \frac{835d_1}{72(x-1)^5} - \frac{205d_1}{72} - \frac{97}{12(x-1)} + \frac{179}{36(x-1)^2} - \frac{179}{36(x-1)^3} + \frac{97}{12(x-1)^4} - \\
& \frac{\pi^2}{6(x-1)^5} - \frac{155}{36(x-1)^5} - \frac{\pi^2}{2} + \frac{155}{36} \Big) H(1, 0; x) + \left( -\frac{1}{3}d_1^2\alpha_0^3 + \frac{d_1^2\alpha_0^3}{3(x-1)^2} + \frac{13d_1^2\alpha_0^2}{6} + \frac{d_1^2\alpha_0^2}{6(x-1)} - \frac{5d_1^2\alpha_0^2}{6(x-1)^2} + \right. \\
& \frac{7d_1^2\alpha_0^2}{6(x-1)^3} - \frac{23d_1^2\alpha_0}{3} - \frac{4d_1^2\alpha_0}{3(x-1)} + \frac{2d_1^2\alpha_0}{3(x-1)^2} - \frac{4d_1^2\alpha_0}{3(x-1)^3} + \frac{13d_1^2\alpha_0}{3(x-1)^4} + \frac{35d_1^2}{6} + \frac{7d_1^2}{6(x-1)} - \frac{d_1^2}{6(x-1)^2} + \frac{d_1^2}{6(x-1)^3} - \\
& \frac{13d_1^2}{3(x-1)^4} \Big) H(1, 1; \alpha_0) + H(0, c_1(\alpha_0); x) \left( -\frac{d_1\alpha_0^4}{4} + \frac{d_1\alpha_0^4}{4(x-1)} - \frac{\alpha_0^4}{4(x-1)} + \frac{\alpha_0^4}{4} + \frac{13d_1\alpha_0^3}{9} - \frac{d_1\alpha_0^3}{x-1} + \frac{5\alpha_0^3}{3(x-1)} + \right. \\
& \frac{4d_1\alpha_0^3}{9(x-1)^2} - \frac{7\alpha_0^3}{9(x-1)^2} - \frac{16\alpha_0^3}{9} - \frac{23d_1\alpha_0^2}{6} + \frac{3d_1\alpha_0^2}{2(x-1)} - \frac{14\alpha_0^2}{3(x-1)} - \frac{4d_1\alpha_0^2}{3(x-1)^2} + \frac{7\alpha_0^2}{2(x-1)^2} + \frac{d_1\alpha_0^2}{(x-1)^3} - \frac{13\alpha_0^2}{6(x-1)^3} + 6\alpha_0^2 + \\
& \frac{25d_1\alpha_0}{3} - \frac{d_1\alpha_0}{x-1} + \frac{25\alpha_0}{3(x-1)} + \frac{4d_1\alpha_0}{3(x-1)^2} - \frac{22\alpha_0}{3(x-1)^2} - \frac{2d_1\alpha_0}{(x-1)^3} + \frac{22\alpha_0}{3(x-1)^3} + \frac{4d_1\alpha_0}{(x-1)^4} - \frac{25\alpha_0}{3(x-1)^4} - 16\alpha_0 - \frac{205d_1}{72} + \\
& \left( \frac{\alpha_0^4}{x-1} - \alpha_0^4 - \frac{4\alpha_0^3}{x-1} + \frac{4\alpha_0^3}{3(x-1)^2} + \frac{16\alpha_0^3}{3} + \frac{6\alpha_0^2}{x-1} - \frac{4\alpha_0^2}{(x-1)^2} + \frac{2\alpha_0^2}{(x-1)^3} - 12\alpha_0^2 - \frac{4\alpha_0}{x-1} + \frac{4\alpha_0}{(x-1)^2} - \frac{4\alpha_0}{(x-1)^3} + \frac{4\alpha_0}{(x-1)^4} + \right. \\
& 16\alpha_0 - \frac{3}{2(x-1)} + \frac{1}{3(x-1)^2} + \frac{1}{3(x-1)^3} - \frac{3}{2(x-1)^4} - \frac{25}{6(x-1)^5} - \frac{25}{6} \Big) H(0; \alpha_0) + \left( -d_1\alpha_0^4 + \frac{d_1\alpha_0^4}{x-1} + \frac{16d_1\alpha_0^3}{3} - \right. \\
& \frac{4d_1\alpha_0^3}{x-1} + \frac{4d_1\alpha_0^3}{3(x-1)^2} - 12d_1\alpha_0^2 + \frac{6d_1\alpha_0^2}{x-1} - \frac{4d_1\alpha_0^2}{(x-1)^2} + \frac{2d_1\alpha_0^2}{(x-1)^3} + 16d_1\alpha_0 - \frac{4d_1\alpha_0}{x-1} + \frac{4d_1\alpha_0}{(x-1)^2} - \frac{4d_1\alpha_0}{(x-1)^3} + \frac{4d_1\alpha_0}{(x-1)^4} - \\
& \frac{25d_1}{6} - \frac{3d_1}{2(x-1)} + \frac{d_1}{3(x-1)^2} + \frac{d_1}{3(x-1)^3} - \frac{3d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} \Big) H(1; \alpha_0) + \left( \frac{4}{(x-1)^5} - 4 \right) H(0, 0; \alpha_0) + \\
& \left( \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(0, 1; \alpha_0) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(1, 0; \alpha_0) + \left( \frac{4d_1^2}{(x-1)^5} - 4d_1^2 \right) H(1, 1; \alpha_0) - \frac{15d_1}{8(x-1)} + \\
& \frac{3}{x-1} + \frac{5d_1}{18(x-1)^2} - \frac{13}{36(x-1)^2} + \frac{5d_1}{18(x-1)^3} - \frac{7}{36(x-1)^3} - \frac{15d_1}{8(x-1)^4} + \frac{1}{4(x-1)^4} - \frac{205d_1}{72(x-1)^5} - \frac{\pi^2}{6(x-1)^5} + \\
& \frac{155}{36(x-1)^5} + \frac{\pi^2}{6} + \frac{65}{9} \Big) + H(c_1(\alpha_0); x) \left( \frac{d_1^2\alpha_0^4}{16} - \frac{d_1\alpha_0^4}{8} - \frac{d_1^2\alpha_0^4}{16(x-1)} + \frac{d_1\alpha_0^4}{8(x-1)} + \frac{\pi^2\alpha_0^4}{24(x-1)} - \frac{\alpha_0^4}{16(x-1)} - \frac{\pi^2\alpha_0^4}{24} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\alpha_0^4}{16} - \frac{43 d_1^2 \alpha_0^3}{108} + \frac{193 d_1 \alpha_0^3}{216} + \frac{d_1^2 \alpha_0^3}{4(x-1)} - \frac{8 d_1 \alpha_0^3}{9(x-1)} - \frac{\pi^2 \alpha_0^3}{6(x-1)} + \frac{23 \alpha_0^3}{36(x-1)} - \frac{4 d_1^2 \alpha_0^3}{27(x-1)^2} + \frac{127 d_1 \alpha_0^3}{216(x-1)^2} + \frac{\pi^2 \alpha_0^3}{18(x-1)^2} - \\
& \frac{95 \alpha_0^3}{216(x-1)^2} + \frac{2 \pi^2 \alpha_0^3}{9} - \frac{107 \alpha_0^3}{216} + \frac{95 d_1^2 \alpha_0^2}{72} - \frac{163 d_1 \alpha_0^2}{48} - \frac{3 d_1^2 \alpha_0^2}{8(x-1)} + \frac{47 d_1 \alpha_0^2}{16(x-1)} + \frac{\pi^2 \alpha_0^2}{4(x-1)} - \frac{47 \alpha_0^2}{16(x-1)} + \frac{4 d_1^2 \alpha_0^2}{9(x-1)^2} - \\
& \frac{125 d_1 \alpha_0^2}{48(x-1)^2} - \frac{\pi^2 \alpha_0^2}{6(x-1)^2} + \frac{401 \alpha_0^2}{144(x-1)^2} - \frac{d_1^2 \alpha_0^2}{2(x-1)^3} + \frac{115 d_1 \alpha_0^2}{48(x-1)^3} + \frac{\pi^2 \alpha_0^2}{12(x-1)^3} - \frac{35 \alpha_0^2}{16(x-1)^3} - \frac{\pi^2 \alpha_0^2}{2} + \frac{305 \alpha_0^2}{144} - \\
& \frac{205 d_1^2 \alpha_0}{36} + \frac{125 d_1 \alpha_0}{8} + \frac{d_1^2 \alpha_0}{4(x-1)} - \frac{53 d_1 \alpha_0}{6(x-1)} - \frac{\pi^2 \alpha_0}{6(x-1)} + \frac{47 \alpha_0}{4(x-1)} - \frac{4 d_1^2 \alpha_0}{9(x-1)^2} + \frac{7 d_1 \alpha_0}{(x-1)^2} + \frac{\pi^2 \alpha_0}{6(x-1)^2} - \frac{389 \alpha_0}{36(x-1)^2} + \\
& \frac{d_1^2 \alpha_0}{(x-1)^3} - \frac{49 d_1 \alpha_0}{6(x-1)^3} - \frac{\pi^2 \alpha_0}{6(x-1)^3} + \frac{34 \alpha_0}{3(x-1)^3} - \frac{4 d_1^2 \alpha_0}{(x-1)^4} + \frac{409 d_1 \alpha_0}{24(x-1)^4} + \frac{\pi^2 \alpha_0}{6(x-1)^4} - \frac{47 \alpha_0}{3(x-1)^4} + \frac{2 \pi^2 \alpha_0}{3} - \frac{187 \alpha_0}{18} + \\
& \frac{2035 d_1^2}{432} - \frac{5615 d_1}{432} + \left( \frac{d_1 \alpha_0^4}{4} - \frac{d_1 \alpha_0^4}{4(x-1)} + \frac{\alpha_0^4}{4(x-1)} - \frac{\alpha_0^4}{4} - \frac{13 d_1 \alpha_0^3}{9} + \frac{d_1 \alpha_0^3}{x-1} - \frac{5 \alpha_0^3}{3(x-1)} - \frac{4 d_1 \alpha_0^3}{9(x-1)^2} + \frac{17 \alpha_0^3}{18(x-1)^2} + \right. \\
& \left. \frac{29 \alpha_0^3}{18} + \frac{23 d_1 \alpha_0^2}{6} - \frac{3 d_1 \alpha_0^2}{2(x-1)} + \frac{19 \alpha_0^2}{4(x-1)} + \frac{4 d_1 \alpha_0^2}{3(x-1)^2} - \frac{47 \alpha_0^2}{12(x-1)^2} - \frac{d_1 \alpha_0^2}{(x-1)^3} + \frac{11 \alpha_0^2}{4(x-1)^3} - \frac{59 \alpha_0^2}{12} - \frac{25 d_1 \alpha_0}{3} + \frac{d_1 \alpha_0}{x-1} - \right. \\
& \left. \frac{9 \alpha_0}{x-1} - \frac{4 d_1 \alpha_0}{3(x-1)^2} + \frac{23 \alpha_0}{3(x-1)^2} + \frac{2 d_1 \alpha_0}{(x-1)^3} - \frac{8 \alpha_0}{(x-1)^3} - \frac{4 d_1 \alpha_0}{(x-1)^4} + \frac{21 \alpha_0}{2(x-1)^4} + \frac{73 \alpha_0}{6} + \frac{205 d_1}{36} + \frac{15 d_1}{4(x-1)} - \frac{1}{2(x-1)} - \right. \\
& \left. \frac{5 d_1}{9(x-1)^2} + \frac{7}{18(x-1)^2} - \frac{5 d_1}{9(x-1)^3} + \frac{13}{18(x-1)^3} + \frac{15 d_1}{4(x-1)^4} - \frac{6}{(x-1)^4} + \frac{205 d_1}{36(x-1)^5} - \frac{130}{9(x-1)^5} - \frac{155}{18} \right) H(0; \alpha_0) + \\
& \left( \frac{d_1^2 \alpha_0^4}{4} - \frac{d_1 \alpha_0^4}{4} - \frac{d_1^2 \alpha_0^4}{4(x-1)} + \frac{d_1 \alpha_0^4}{4(x-1)} - \frac{13 d_1^2 \alpha_0^3}{9} + \frac{29 d_1 \alpha_0^3}{18} + \frac{d_1^2 \alpha_0^3}{x-1} - \frac{5 d_1 \alpha_0^3}{3(x-1)} - \frac{4 d_1^2 \alpha_0^3}{9(x-1)^2} + \frac{17 d_1 \alpha_0^3}{18(x-1)^2} + \frac{23 d_1^2 \alpha_0^2}{6} - \right. \\
& \left. \frac{59 d_1 \alpha_0^2}{12} - \frac{3 d_1^2 \alpha_0^2}{2(x-1)} + \frac{19 d_1 \alpha_0^2}{4(x-1)} + \frac{4 d_1^2 \alpha_0^2}{3(x-1)^2} - \frac{47 d_1 \alpha_0^2}{12(x-1)^2} - \frac{d_1^2 \alpha_0^2}{(x-1)^3} + \frac{11 d_1 \alpha_0^2}{4(x-1)^3} - \frac{25 d_1^2 \alpha_0}{3} + \frac{73 d_1 \alpha_0}{6} + \frac{d_1^2 \alpha_0}{x-1} - \right. \\
& \left. \frac{9 d_1 \alpha_0}{x-1} - \frac{4 d_1^2 \alpha_0}{3(x-1)^2} + \frac{23 d_1 \alpha_0}{3(x-1)^2} + \frac{2 d_1^2 \alpha_0}{(x-1)^3} - \frac{8 d_1 \alpha_0}{(x-1)^3} - \frac{4 d_1^2 \alpha_0}{(x-1)^4} + \frac{21 d_1 \alpha_0}{2(x-1)^4} + \frac{205 d_1^2}{36} - \frac{155 d_1}{18} + \frac{15 d_1^2}{4(x-1)} - \frac{d_1}{2(x-1)} - \right. \\
& \left. \frac{5 d_1^2}{9(x-1)^2} + \frac{7 d_1}{18(x-1)^2} - \frac{5 d_1^2}{9(x-1)^3} + \frac{13 d_1}{18(x-1)^3} + \frac{15 d_1^2}{4(x-1)^4} - \frac{6 d_1}{(x-1)^4} + \frac{205 d_1^2}{36(x-1)^5} - \frac{130 d_1}{9(x-1)^5} \right) H(1; \alpha_0) + \left( - \right. \\
& \left. \frac{\alpha_0^4}{x-1} + \alpha_0^4 + \frac{4 \alpha_0^3}{x-1} - \frac{4 \alpha_0^3}{3(x-1)^2} - \frac{16 \alpha_0^3}{3} - \frac{6 \alpha_0^2}{x-1} + \frac{4 \alpha_0^2}{(x-1)^2} - \frac{2 \alpha_0^2}{(x-1)^3} + 12 \alpha_0^2 + \frac{4 \alpha_0}{x-1} - \frac{4 \alpha_0}{(x-1)^2} + \frac{4 \alpha_0}{(x-1)^3} - \frac{4 \alpha_0}{(x-1)^4} - \right. \\
& \left. 16 \alpha_0 + \frac{3}{x-1} - \frac{2}{3(x-1)^2} - \frac{2}{3(x-1)^3} + \frac{3}{(x-1)^4} + \frac{25}{3(x-1)^5} + \frac{25}{3} \right) H(0, 0; \alpha_0) + \left( d_1 \alpha_0^4 - \frac{d_1 \alpha_0^4}{x-1} - \frac{16 d_1 \alpha_0^3}{3} + \right. \\
& \left. \frac{4 d_1 \alpha_0^3}{x-1} - \frac{4 d_1 \alpha_0^3}{3(x-1)^2} + 12 d_1 \alpha_0^2 - \frac{6 d_1 \alpha_0^2}{x-1} + \frac{4 d_1 \alpha_0^2}{(x-1)^2} - \frac{2 d_1 \alpha_0^2}{(x-1)^3} - 16 d_1 \alpha_0 + \frac{4 d_1 \alpha_0}{x-1} - \frac{4 d_1 \alpha_0}{(x-1)^2} + \frac{4 d_1 \alpha_0}{(x-1)^3} - \frac{4 d_1 \alpha_0}{(x-1)^4} + \right. \\
& \left. \frac{25 d_1}{3} + \frac{3 d_1}{x-1} - \frac{2 d_1}{3(x-1)^2} - \frac{2 d_1}{3(x-1)^3} + \frac{3 d_1}{(x-1)^4} + \frac{25 d_1}{3(x-1)^5} \right) H(0, 1; \alpha_0) + \left( d_1 \alpha_0^4 - \frac{d_1 \alpha_0^4}{x-1} - \frac{16 d_1 \alpha_0^3}{3} + \frac{4 d_1 \alpha_0^3}{x-1} - \right. \\
& \left. \frac{4 d_1 \alpha_0^3}{3(x-1)^2} + 12 d_1 \alpha_0^2 - \frac{6 d_1 \alpha_0^2}{x-1} + \frac{4 d_1 \alpha_0^2}{(x-1)^2} - \frac{2 d_1 \alpha_0^2}{(x-1)^3} - 16 d_1 \alpha_0 + \frac{4 d_1 \alpha_0}{x-1} - \frac{4 d_1 \alpha_0}{(x-1)^2} + \frac{4 d_1 \alpha_0}{(x-1)^3} - \frac{4 d_1 \alpha_0}{(x-1)^4} + \frac{25 d_1}{3} + \right. \\
& \left. \frac{3 d_1}{x-1} - \frac{2 d_1}{3(x-1)^2} - \frac{2 d_1}{3(x-1)^3} + \frac{3 d_1}{(x-1)^4} + \frac{25 d_1}{3(x-1)^5} \right) H(1, 0; \alpha_0) + \left( d_1^2 \alpha_0^4 - \frac{d_1^2 \alpha_0^4}{x-1} - \frac{16 d_1^2 \alpha_0^3}{3} + \frac{4 d_1^2 \alpha_0^3}{x-1} - \right. \\
& \left. \frac{4 d_1^2 \alpha_0^3}{3(x-1)^2} + 12 d_1^2 \alpha_0^2 - \frac{6 d_1^2 \alpha_0^2}{x-1} + \frac{4 d_1^2 \alpha_0^2}{(x-1)^2} - \frac{2 d_1^2 \alpha_0^2}{(x-1)^3} - 16 d_1^2 \alpha_0 + \frac{4 d_1^2 \alpha_0}{x-1} - \frac{4 d_1^2 \alpha_0}{(x-1)^2} + \frac{4 d_1^2 \alpha_0}{(x-1)^3} - \frac{4 d_1^2 \alpha_0}{(x-1)^4} + \frac{25 d_1^2}{3} + \right. \\
& \left. \frac{3 d_1^2}{x-1} - \frac{2 d_1^2}{3(x-1)^2} - \frac{2 d_1^2}{3(x-1)^3} + \frac{3 d_1^2}{(x-1)^4} + \frac{25 d_1^2}{3(x-1)^5} \right) H(1, 1; \alpha_0) + \frac{63 d_1^2}{16(x-1)} - \frac{209 d_1}{48(x-1)} - \frac{\pi^2}{8(x-1)} - \frac{2}{x-1} - \\
& \frac{19 d_1^2}{54(x-1)^2} + \frac{347 d_1}{216(x-1)^2} + \frac{\pi^2}{36(x-1)^2} - \frac{19}{216(x-1)^2} - \frac{19 d_1^2}{54(x-1)^3} + \frac{173 d_1}{216(x-1)^3} + \frac{\pi^2}{36(x-1)^3} - \frac{205}{216(x-1)^3} + \\
& \frac{63 d_1^2}{16(x-1)^4} - \frac{197 d_1}{16(x-1)^4} - \frac{\pi^2}{8(x-1)^4} + \frac{163}{24(x-1)^4} + \frac{2035 d_1^2}{432(x-1)^5} - \frac{8705 d_1}{432(x-1)^5} - \frac{25 \pi^2}{72(x-1)^5} + \frac{3965}{216(x-1)^5} - \frac{25 \pi^2}{72} + \\
& \frac{235}{27} \Big) + \left( - \frac{2 \pi^2 d_1^2}{3(x-1)^5} + \frac{2 \pi^2 d_1}{3(x-1)^5} + \frac{\pi^2 d_1}{3} + \left( \frac{4 d_1^2}{x-1} - \frac{2 d_1^2}{(x-1)^2} + \frac{4 d_1^2}{3(x-1)^3} - \frac{d_1^2}{(x-1)^4} + \frac{25 d_1^2}{3(x-1)^5} + \frac{9 d_1}{2(x-1)} - \right. \right. \\
& \left. \frac{8 d_1}{3(x-1)^2} + \frac{7 d_1}{3(x-1)^3} - \frac{3 d_1}{(x-1)^4} + \frac{25 d_1}{3(x-1)^5} - \frac{25 d_1}{6} - \frac{1}{4(x-1)} - \frac{1}{2(x-1)^2} + \frac{7}{6(x-1)^3} - \frac{11}{4(x-1)^4} - \frac{25}{12(x-1)^5} - \right. \\
& \left. \frac{25}{4} \right) H(0; \alpha_0) + \left( - \frac{8 d_1}{(x-1)^5} + 4 d_1 + \frac{2}{(x-1)^5} + 2 \right) H(0, 0; \alpha_0) + \left( - \frac{8 d_1^2}{(x-1)^5} + 4 d_1^2 + \frac{2 d_1}{(x-1)^5} + \right. \\
& \left. 2 d_1 \right) H(0, 1; \alpha_0) - \frac{\pi^2}{6(x-1)^5} - \frac{\pi^2}{6} \Big) H(1, 1; x) + \left( - \frac{4 d_1^2}{x-1} + \frac{d_1^2}{(x-1)^2} - \frac{4 d_1^2}{9(x-1)^3} + \frac{d_1^2}{4(x-1)^4} - \frac{205 d_1^2}{36(x-1)^5} + \right. \\
& \left. \frac{97 d_1}{24(x-1)} - \frac{157 d_1}{36(x-1)^2} + \frac{137 d_1}{36(x-1)^3} - \frac{19 d_1}{8(x-1)^4} + \frac{835 d_1}{72(x-1)^5} + \frac{205 d_1}{72} + \left( - \frac{4 d_1}{x-1} + \frac{2 d_1}{(x-1)^2} - \frac{4 d_1}{3(x-1)^3} + \frac{d_1}{(x-1)^4} - \right. \right. \\
& \left. \frac{25 d_1}{3(x-1)^5} - \frac{5}{2(x-1)} + \frac{5}{3(x-1)^2} - \frac{5}{3(x-1)^3} + \frac{5}{2(x-1)^4} - \frac{25}{6(x-1)^5} + \frac{25}{6} \right) H(0; \alpha_0) + \left( - \frac{4 d_1^2}{x-1} + \frac{2 d_1^2}{(x-1)^2} - \right. \\
& \left. \frac{4 d_1^2}{3(x-1)^3} + \frac{d_1^2}{(x-1)^4} - \frac{25 d_1^2}{3(x-1)^5} - \frac{5 d_1}{2(x-1)} + \frac{5 d_1}{3(x-1)^2} - \frac{5 d_1}{3(x-1)^3} + \frac{5 d_1}{2(x-1)^4} - \frac{25 d_1}{6(x-1)^5} + \frac{25 d_1}{6} \right) H(1; \alpha_0) + \\
& \left( \frac{4}{(x-1)^5} - 4 \right) H(0, 0; \alpha_0) + \left( \frac{4 d_1}{(x-1)^5} - 4 d_1 \right) H(0, 1; \alpha_0) + \left( \frac{4 d_1}{(x-1)^5} - 4 d_1 \right) H(1, 0; \alpha_0) + \left( \frac{4 d_1^2}{(x-1)^5} - \right. \\
& \left. 4 d_1^2 \right) H(1, 1; \alpha_0) + \frac{97}{12(x-1)} - \frac{179}{36(x-1)^2} + \frac{179}{36(x-1)^3} - \frac{97}{12(x-1)^4} - \frac{\pi^2}{6(x-1)^5} + \frac{155}{36(x-1)^5} + \frac{\pi^2}{6} -
\end{aligned}$$

$$\begin{aligned}
 & \frac{155}{36} \Big) H(1, c_1(\alpha_0); x) + \left( \frac{3d_1\alpha_0^4}{8} - \frac{3d_1\alpha_0^4}{8(x-1)} + \frac{3\alpha_0^4}{8(x-1)} - \frac{3\alpha_0^4}{8} - \frac{13d_1\alpha_0^3}{6} + \frac{3d_1\alpha_0^3}{2(x-1)} - \frac{5\alpha_0^3}{2(x-1)} - \frac{2d_1\alpha_0^3}{3(x-1)^2} + \right. \\
 & \frac{5\alpha_0^3}{4(x-1)^2} + \frac{31\alpha_0^3}{12} + \frac{23d_1\alpha_0^2}{4} - \frac{9d_1\alpha_0^2}{4(x-1)} + \frac{169\alpha_0^2}{24(x-1)} + \frac{2d_1\alpha_0^2}{(x-1)^2} - \frac{131\alpha_0^2}{24(x-1)^2} - \frac{3d_1\alpha_0^2}{2(x-1)^3} + \frac{85\alpha_0^2}{24(x-1)^3} - \frac{203\alpha_0^2}{24} - \\
 & \frac{25d_1\alpha_0}{2} + \frac{3d_1\alpha_0}{2(x-1)} - \frac{77\alpha_0}{6(x-1)} - \frac{2d_1\alpha_0}{(x-1)^2} + \frac{67\alpha_0}{6(x-1)^2} + \frac{3d_1\alpha_0}{(x-1)^3} - \frac{34\alpha_0}{3(x-1)^3} - \frac{6d_1\alpha_0}{(x-1)^4} + \frac{163\alpha_0}{12(x-1)^4} + \frac{265\alpha_0}{12} + \\
 & \frac{205d_1}{24} + \left( -\frac{3\alpha_0^4}{2(x-1)} + \frac{3\alpha_0^4}{2} + \frac{6\alpha_0^3}{x-1} - \frac{2\alpha_0^3}{(x-1)^2} - 8\alpha_0^3 - \frac{9\alpha_0^2}{x-1} + \frac{6\alpha_0^2}{(x-1)^2} - \frac{3\alpha_0^2}{(x-1)^3} + 18\alpha_0^2 + \frac{6\alpha_0}{x-1} - \frac{6\alpha_0}{(x-1)^2} + \right. \\
 & \frac{6\alpha_0}{(x-1)^3} - \frac{6\alpha_0}{(x-1)^4} - 24\alpha_0 + \frac{9}{2(x-1)} - \frac{1}{(x-1)^2} - \frac{1}{(x-1)^3} + \frac{9}{2(x-1)^4} + \frac{25}{2(x-1)^5} + \frac{25}{2} \Big) H(0; \alpha_0) + \left( \frac{3d_1\alpha_0^4}{2} - \right. \\
 & \frac{3d_1\alpha_0^4}{2(x-1)} - 8d_1\alpha_0^3 + \frac{6d_1\alpha_0^3}{x-1} - \frac{2d_1\alpha_0^3}{(x-1)^2} + 18d_1\alpha_0^2 - \frac{9d_1\alpha_0^2}{x-1} + \frac{6d_1\alpha_0^2}{(x-1)^2} - \frac{3d_1\alpha_0^2}{(x-1)^3} - 24d_1\alpha_0 + \frac{6d_1\alpha_0}{x-1} - \frac{6d_1\alpha_0}{(x-1)^2} + \\
 & \frac{6d_1\alpha_0}{(x-1)^3} - \frac{6d_1\alpha_0}{(x-1)^4} + \frac{25d_1}{2} + \frac{9d_1}{2(x-1)} - \frac{d_1}{(x-1)^2} - \frac{d_1}{(x-1)^3} + \frac{9d_1}{2(x-1)^4} + \frac{25d_1}{2(x-1)^5} \Big) H(1; \alpha_0) - \frac{4H(0,0;\alpha_0)}{(x-1)^5} - \\
 & \frac{4d_1H(0,1;\alpha_0)}{(x-1)^5} - \frac{4d_1H(1,0;\alpha_0)}{(x-1)^5} - \frac{4d_1^2H(1,1;\alpha_0)}{(x-1)^5} + \frac{45d_1}{8(x-1)} - \frac{7}{2(x-1)} - \frac{5d_1}{6(x-1)^2} + \frac{3}{4(x-1)^2} - \frac{5d_1}{6(x-1)^3} + \\
 & \frac{11}{12(x-1)^3} + \frac{45d_1}{8(x-1)^4} - \frac{25}{4(x-1)^4} + \frac{205d_1}{24(x-1)^5} + \frac{\pi^2}{6(x-1)^5} - \frac{75}{4(x-1)^5} - \frac{95}{6} \Big) H(c_1(\alpha_0), c_1(\alpha_0); x) + \\
 & \left( \frac{25}{3} + \frac{3}{x-1} - \frac{2}{3(x-1)^2} - \frac{2}{3(x-1)^3} + \frac{3}{(x-1)^4} + \frac{25}{3(x-1)^5} \right) H(0, 0, 0; \alpha_0) + \left( -\frac{25}{3} - \frac{3}{x-1} + \frac{2}{3(x-1)^2} + \right. \\
 & \frac{2}{3(x-1)^3} - \frac{3}{(x-1)^4} - \frac{25}{3(x-1)^5} \Big) H(0, 0, 0; x) + \left( \frac{3d_1}{x-1} - \frac{2d_1}{3(x-1)^2} - \frac{2d_1}{3(x-1)^3} + \frac{3d_1}{(x-1)^4} + \frac{25d_1}{3(x-1)^5} + \right. \\
 & \frac{25d_1}{3} \Big) H(0, 0, 1; \alpha_0) + \left( \frac{4}{(x-1)^5} - 4 \right) H(0; \alpha_0) H(0, 0, 1; x) + \left( -\frac{\alpha_0^4}{x-1} + \alpha_0^4 + \frac{4\alpha_0^3}{x-1} - \frac{4\alpha_0^3}{3(x-1)^2} - \right. \\
 & \frac{16\alpha_0^3}{3} - \frac{6\alpha_0^2}{x-1} + \frac{4\alpha_0^2}{(x-1)^2} - \frac{2\alpha_0^2}{(x-1)^3} + 12\alpha_0^2 + \frac{4\alpha_0}{x-1} - \frac{4\alpha_0}{(x-1)^2} + \frac{4\alpha_0}{(x-1)^3} - \frac{4\alpha_0}{(x-1)^4} - 16\alpha_0 + \frac{3}{2(x-1)} - \frac{1}{3(x-1)^2} - \\
 & \frac{1}{3(x-1)^3} + \frac{3}{2(x-1)^4} + \frac{25}{6(x-1)^5} + \frac{25}{6} \Big) H(0, 0, c_1(\alpha_0); x) + \left( \frac{3d_1}{x-1} - \frac{2d_1}{3(x-1)^2} - \frac{2d_1}{3(x-1)^3} + \frac{3d_1}{(x-1)^4} + \right. \\
 & \frac{25d_1}{3(x-1)^5} + \frac{25d_1}{3} \Big) H(0, 1, 0; \alpha_0) + \left( -\frac{5d_1}{2(x-1)} + \frac{5d_1}{3(x-1)^2} - \frac{5d_1}{3(x-1)^3} + \frac{5d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} + \frac{25d_1}{6} - \frac{5}{2(x-1)} + \right. \\
 & \frac{5}{3(x-1)^2} - \frac{5}{3(x-1)^3} + \frac{5}{2(x-1)^4} - \frac{25}{6(x-1)^5} + \frac{25}{6} \Big) H(0, 1, 0; x) + \left( \frac{3d_1^2}{x-1} - \frac{2d_1^2}{3(x-1)^2} - \frac{2d_1^2}{3(x-1)^3} + \frac{3d_1^2}{(x-1)^4} + \right. \\
 & \frac{25d_1^2}{3(x-1)^5} + \frac{25d_1^2}{3} \Big) H(0, 1, 1; \alpha_0) + \left( \frac{4d_1^2}{(x-1)^5} - 4d_1^2 - \frac{2d_1}{(x-1)^5} - 2d_1 + \frac{4}{(x-1)^5} \right) H(0; \alpha_0) H(0, 1, 1; x) + \\
 & \left( \frac{5d_1}{2(x-1)} - \frac{5d_1}{3(x-1)^2} + \frac{5d_1}{3(x-1)^3} - \frac{5d_1}{2(x-1)^4} + \frac{25d_1}{6(x-1)^5} - \frac{25d_1}{6} + \left( 4d_1 - \frac{4d_1}{(x-1)^5} \right) H(0; \alpha_0) + \left( 4d_1^2 - \right. \right. \\
 & \left. \left. \frac{4d_1^2}{(x-1)^5} \right) H(1; \alpha_0) + \frac{5}{2(x-1)} - \frac{5}{3(x-1)^2} + \frac{5}{3(x-1)^3} - \frac{5}{2(x-1)^4} + \frac{25}{6(x-1)^5} - \frac{25}{6} \right) H(0, 1, c_1(\alpha_0); x) + \\
 & \left( \frac{3\alpha_0^4}{2(x-1)} - \frac{3\alpha_0^4}{2} - \frac{6\alpha_0^3}{x-1} + \frac{2\alpha_0^3}{(x-1)^2} + 8\alpha_0^3 + \frac{9\alpha_0^2}{x-1} - \frac{6\alpha_0^2}{(x-1)^2} + \frac{3\alpha_0^2}{(x-1)^3} - 18\alpha_0^2 - \frac{6\alpha_0}{x-1} + \frac{6\alpha_0}{(x-1)^2} - \frac{6\alpha_0}{(x-1)^3} + \right. \\
 & \frac{6\alpha_0}{(x-1)^4} + 24\alpha_0 + \left( \frac{2}{(x-1)^5} - 6 \right) H(0; \alpha_0) + \left( \frac{2d_1}{(x-1)^5} - 6d_1 \right) H(1; \alpha_0) - \frac{3}{4(x-1)} + \frac{1}{6(x-1)^2} + \frac{1}{6(x-1)^3} - \\
 & \frac{3}{4(x-1)^4} - \frac{25}{12(x-1)^5} - \frac{25}{12} \Big) H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{4d_1}{x-1} - \frac{2d_1}{(x-1)^2} + \frac{4d_1}{3(x-1)^3} - \frac{d_1}{(x-1)^4} + \frac{25d_1}{3(x-1)^5} + \right. \\
 & \frac{5}{2(x-1)} - \frac{5}{3(x-1)^2} + \frac{5}{3(x-1)^3} - \frac{5}{2(x-1)^4} + \frac{25}{6(x-1)^5} - \frac{25}{6} \Big) H(1, 0, 0; x) + \left( \frac{4d_1^2}{(x-1)^5} - \frac{2d_1}{(x-1)^5} - 2d_1 + \right. \\
 & \left. \frac{2}{(x-1)^5} - 2 \right) H(0; \alpha_0) H(1, 0, 1; x) + \left( \frac{2d_1}{x-1} - \frac{d_1}{(x-1)^2} + \frac{2d_1}{3(x-1)^3} - \frac{d_1}{2(x-1)^4} + \frac{25d_1}{6(x-1)^5} + \left( -\frac{4d_1}{(x-1)^5} + \right. \right. \\
 & \left. \left. \frac{2}{(x-1)^5} + 2 \right) H(0; \alpha_0) + \left( -\frac{4d_1^2}{(x-1)^5} + \frac{2d_1}{(x-1)^5} + 2d_1 \right) H(1; \alpha_0) - \frac{1}{4(x-1)} - \frac{1}{2(x-1)^2} + \frac{7}{6(x-1)^3} - \right. \\
 & \frac{11}{4(x-1)^4} - \frac{25}{12(x-1)^5} - \frac{25}{4} \Big) H(1, 0, c_1(\alpha_0); x) + \left( -\frac{4d_1^2}{x-1} + \frac{2d_1^2}{(x-1)^2} - \frac{4d_1^2}{3(x-1)^3} + \frac{d_1^2}{(x-1)^4} - \frac{25d_1^2}{3(x-1)^5} - \right. \\
 & \frac{9d_1}{2(x-1)} + \frac{8d_1}{3(x-1)^2} - \frac{7d_1}{3(x-1)^3} + \frac{3d_1}{(x-1)^4} - \frac{25d_1}{3(x-1)^5} + \frac{25d_1}{6} + \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2} - \frac{7}{6(x-1)^3} + \frac{11}{4(x-1)^4} + \\
 & \frac{25}{12(x-1)^5} + \frac{25}{4} \Big) H(1, 1, 0; x) + \left( \frac{12d_1^2}{(x-1)^5} - 4d_1^2 - \frac{6d_1}{(x-1)^5} - 4d_1 + \frac{1}{(x-1)^5} + 1 \right) H(0; \alpha_0) H(1, 1, 1; x) + \\
 & \left( \frac{4d_1^2}{x-1} - \frac{2d_1^2}{(x-1)^2} + \frac{4d_1^2}{3(x-1)^3} - \frac{d_1^2}{(x-1)^4} + \frac{25d_1^2}{3(x-1)^5} + \frac{9d_1}{2(x-1)} - \frac{8d_1}{3(x-1)^2} + \frac{7d_1}{3(x-1)^3} - \frac{3d_1}{(x-1)^4} + \frac{25d_1}{3(x-1)^5} - \right. \\
 & \left. \frac{25d_1}{6} + \left( -\frac{8d_1}{(x-1)^5} + 4d_1 + \frac{2}{(x-1)^5} + 2 \right) H(0; \alpha_0) + \left( -\frac{8d_1^2}{(x-1)^5} + 4d_1^2 + \frac{2d_1}{(x-1)^5} + 2d_1 \right) H(1; \alpha_0) - \right. \\
 & \left. \frac{1}{4(x-1)} - \frac{1}{2(x-1)^2} + \frac{7}{6(x-1)^3} - \frac{11}{4(x-1)^4} - \frac{25}{12(x-1)^5} - \frac{25}{4} \right) H(1, 1, c_1(\alpha_0); x) + \left( -\frac{6d_1}{x-1} + \frac{3d_1}{(x-1)^2} - \right.
 \end{aligned}$$



$$\begin{aligned}
& \frac{2d_1}{(x-1)^3} + \frac{3d_1}{2(x-1)^4} - \frac{25d_1}{2(x-1)^5} + \left( \frac{4d_1}{(x-1)^5} + \frac{2}{(x-1)^5} - 6 \right) H(0; \alpha_0) + \left( \frac{4d_1^2}{(x-1)^5} + \frac{2d_1}{(x-1)^5} - 6d_1 \right) H(1; \alpha_0) - \\
& \frac{9}{4(x-1)} + \frac{13}{6(x-1)^2} - \frac{17}{6(x-1)^3} + \frac{21}{4(x-1)^4} - \frac{25}{12(x-1)^5} + \frac{125}{12} \Big) H(1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{\alpha_0^4}{x-1} - \alpha_0^4 - \right. \\
& \frac{4\alpha_0^3}{x-1} + \frac{4\alpha_0^3}{3(x-1)^2} + \frac{16\alpha_0^3}{3} + \frac{6\alpha_0^2}{x-1} - \frac{4\alpha_0^2}{(x-1)^2} + \frac{2\alpha_0^2}{(x-1)^3} - 12\alpha_0^2 - \frac{4\alpha_0}{x-1} + \frac{4\alpha_0}{(x-1)^2} - \frac{4\alpha_0}{(x-1)^3} + \frac{4\alpha_0}{(x-1)^4} + 16\alpha_0 + \\
& \frac{4H(0; \alpha_0)}{(x-1)^5} + \frac{4d_1 H(1; \alpha_0)}{(x-1)^5} - \frac{3}{x-1} + \frac{2}{3(x-1)^2} + \frac{2}{3(x-1)^3} - \frac{3}{(x-1)^4} - \frac{25}{3(x-1)^5} - \frac{25}{3} \Big) H(c_1(\alpha_0), 0, c_1(\alpha_0); x) + \\
& \left( -\frac{7\alpha_0^4}{4(x-1)} + \frac{7\alpha_0^4}{4} + \frac{7\alpha_0^3}{x-1} - \frac{7\alpha_0^3}{3(x-1)^2} - \frac{28\alpha_0^3}{3} - \frac{21\alpha_0^2}{2(x-1)} + \frac{7\alpha_0^2}{(x-1)^2} - \frac{7\alpha_0^2}{2(x-1)^3} + 21\alpha_0^2 + \frac{7\alpha_0}{x-1} - \frac{7\alpha_0}{(x-1)^2} + \right. \\
& \frac{7\alpha_0}{(x-1)^3} - \frac{7\alpha_0}{(x-1)^4} - 28\alpha_0 - \frac{6H(0; \alpha_0)}{(x-1)^5} - \frac{6d_1 H(1; \alpha_0)}{(x-1)^5} + \frac{21}{4(x-1)} - \frac{7}{6(x-1)^2} - \frac{7}{6(x-1)^3} + \frac{21}{4(x-1)^4} + \frac{175}{12(x-1)^5} + \\
& \left. \frac{175}{12} \right) H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + 8 H(0, 0, 0, 0; x) + \left( \frac{4}{(x-1)^5} - 4 \right) H(0, 0, 0, c_1(\alpha_0); x) + \\
& \left( 4 - \frac{4}{(x-1)^5} \right) H(0, 0, 1, 0; x) + \left( \frac{4}{(x-1)^5} - 4 \right) H(0, 0, 1, c_1(\alpha_0); x) - \frac{4 H(0, 0, c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \\
& \left( \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(0, 1, 0, 0; x) + \left( -\frac{2d_1}{(x-1)^5} - 2d_1 + \frac{4}{(x-1)^5} \right) H(0, 1, 0, c_1(\alpha_0); x) + \left( -\frac{4d_1^2}{(x-1)^5} + 4d_1^2 + \frac{2d_1}{(x-1)^5} + 2d_1 - \frac{4}{(x-1)^5} \right) H(0, 1, 1, 0; x) + \left( \frac{4d_1^2}{(x-1)^5} - 4d_1^2 - \frac{2d_1}{(x-1)^5} - 2d_1 + \frac{4}{(x-1)^5} \right) H(0, 1, 1, c_1(\alpha_0); x) + \left( -\frac{2d_1}{(x-1)^5} + 6d_1 - \frac{4}{(x-1)^5} \right) H(0, 1, c_1(\alpha_0), c_1(\alpha_0); x) + 4 H(0, c_1(\alpha_0), 0, c_1(\alpha_0); x) + \left( \frac{1}{(x-1)^5} - 7 \right) H(0, c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \left( 4 - \frac{4}{(x-1)^5} \right) H(1, 0, 0, 0; x) + \left( \frac{2}{(x-1)^5} - 2 \right) H(1, 0, 0, c_1(\alpha_0); x) + \left( -\frac{4d_1^2}{(x-1)^5} + \frac{2d_1}{(x-1)^5} + 2d_1 - \frac{2}{(x-1)^5} + 2 \right) H(1, 0, 1, 0; x) + \left( \frac{4d_1^2}{(x-1)^5} - \frac{2d_1}{(x-1)^5} - 2d_1 + \frac{2}{(x-1)^5} - 2 \right) H(1, 0, 1, c_1(\alpha_0); x) + \left( -\frac{2d_1}{(x-1)^5} + \frac{1}{(x-1)^5} + 1 \right) H(1, 0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{8d_1}{(x-1)^5} - 4d_1 - \frac{2}{(x-1)^5} - 2 \right) H(1, 1, 0, 0; x) + \left( \frac{4d_1^2}{(x-1)^5} - \frac{4d_1}{(x-1)^5} - 2d_1 + \frac{1}{(x-1)^5} + 1 \right) H(1, 1, 0, c_1(\alpha_0); x) + \left( -\frac{12d_1^2}{(x-1)^5} + 4d_1^2 + \frac{6d_1}{(x-1)^5} + 4d_1 - \frac{1}{(x-1)^5} - 1 \right) H(1, 1, 1, 0; x) + \left( \frac{12d_1^2}{(x-1)^5} - 4d_1^2 - \frac{6d_1}{(x-1)^5} - 4d_1 + \frac{1}{(x-1)^5} + 1 \right) H(1, 1, 1, c_1(\alpha_0); x) + \left( -\frac{4d_1^2}{(x-1)^5} - \frac{4d_1}{(x-1)^5} + 6d_1 + \frac{1}{(x-1)^5} + 1 \right) H(1, 1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( 4 - \frac{4d_1}{(x-1)^5} \right) H(1, c_1(\alpha_0), 0, c_1(\alpha_0); x) + \left( \frac{6d_1}{(x-1)^5} + \frac{1}{(x-1)^5} - 7 \right) H(1, c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) - \frac{4 H(c_1(\alpha_0), 0, 0, c_1(\alpha_0); x)}{(x-1)^5} + \frac{6 H(c_1(\alpha_0), 0, c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \frac{4 H(c_1(\alpha_0), c_1(\alpha_0), 0, c_1(\alpha_0); x)}{(x-1)^5} - \frac{7 H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + H(0; x) \left( -\frac{63d_1^2}{16(x-1)} + \frac{19d_1^2}{54(x-1)^2} + \frac{19d_1^2}{54(x-1)^3} - \frac{63d_1^2}{16(x-1)^4} - \frac{2035d_1^2}{432(x-1)^5} - \frac{2035d_1^2}{432} + \frac{209d_1}{48(x-1)} - \frac{347d_1}{216(x-1)^2} - \frac{173d_1}{216(x-1)^3} + \frac{197d_1}{16(x-1)^4} + \frac{8705d_1}{432(x-1)^5} + \frac{5615d_1}{432} + \frac{3\pi^2}{8(x-1)} + \frac{2}{x-1} - \frac{\pi^2}{12(x-1)^2} + \frac{19}{216(x-1)^2} - \frac{\pi^2}{12(x-1)^3} + \frac{205}{216(x-1)^3} + \frac{3\pi^2}{8(x-1)^4} - \frac{163}{24(x-1)^4} + \frac{25\pi^2}{24(x-1)^5} - \frac{3965}{216(x-1)^5} + \frac{6\zeta_3}{(x-1)^5} + 10\zeta_3 + \frac{25\pi^2}{24} - \frac{235}{27} \right) + H(1; x) \left( -\frac{\pi^2 d_1}{3(x-1)} + \frac{\pi^2 d_1}{6(x-1)^2} - \frac{\pi^2 d_1}{9(x-1)^3} + \frac{\pi^2 d_1}{12(x-1)^4} - \frac{25\pi^2 d_1}{36(x-1)^5} - \frac{6\zeta_3 d_1}{(x-1)^5} + \left( -\frac{4d_1^2}{x-1} + \frac{d_1^2}{(x-1)^2} - \frac{4d_1^2}{9(x-1)^3} + \frac{d_1^2}{4(x-1)^4} - \frac{205d_1^2}{36(x-1)^5} + \frac{97d_1}{24(x-1)} - \frac{157d_1}{36(x-1)^2} + \frac{137d_1}{36(x-1)^3} - \frac{19d_1}{8(x-1)^4} + \frac{835d_1}{72(x-1)^5} + \frac{205d_1}{72} + \frac{97}{12(x-1)} - \frac{179}{36(x-1)^2} + \frac{179}{36(x-1)^3} - \frac{97}{12(x-1)^4} - \frac{\pi^2}{6(x-1)^5} + \frac{155}{36(x-1)^5} + \frac{\pi^2}{6} - \frac{155}{36} \right) H(0; \alpha_0) + \left( -\frac{4d_1}{x-1} + \frac{2d_1}{(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \frac{d_1}{(x-1)^4} - \frac{25d_1}{3(x-1)^5} - \frac{5}{2(x-1)} + \frac{5}{3(x-1)^2} - \frac{5}{3(x-1)^3} + \frac{5}{2(x-1)^4} - \frac{25}{6(x-1)^5} + \frac{25}{6} \right) H(0, 0; \alpha_0) + \left( -\frac{4d_1^2}{x-1} + \frac{2d_1^2}{(x-1)^2} - \frac{4d_1^2}{3(x-1)^3} + \frac{d_1^2}{(x-1)^4} - \frac{25d_1^2}{3(x-1)^5} - \frac{5d_1}{2(x-1)} + \frac{5d_1}{3(x-1)^2} - \frac{5d_1}{3(x-1)^3} + \frac{5d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} + \frac{25d_1}{6} \right) H(0, 1; \alpha_0) + \left( \frac{4}{(x-1)^5} - 4 \right) H(0, 0, 0; \alpha_0) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(0, 0, 1; \alpha_0) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(0, 1, 0; \alpha_0) + \left( \frac{4d_1^2}{(x-1)^5} - 4d_1^2 \right) H(0, 1, 1; \alpha_0) + \frac{\pi^2}{24(x-1)} + \frac{\pi^2}{12(x-1)^2} - \frac{7\pi^2}{36(x-1)^3} + \frac{11\pi^2}{24(x-1)^4} + \frac{25\pi^2}{72(x-1)^5} - \frac{\zeta_3}{(x-1)^5} + 7\zeta_3 + \frac{25\pi^2}{24} \Big) + \frac{5d_1\pi^2}{16(x-1)} - \frac{\pi^2}{2(x-1)} -
\end{aligned}$$

$$\frac{5d_1\pi^2}{108(x-1)^2} + \frac{13\pi^2}{216(x-1)^2} - \frac{5d_1\pi^2}{108(x-1)^3} + \frac{7\pi^2}{216(x-1)^3} + \frac{5d_1\pi^2}{16(x-1)^4} - \frac{\pi^2}{24(x-1)^4} - \frac{23\pi^4}{180(x-1)^5} + \frac{205d_1\pi^2}{432(x-1)^5} - \frac{155\pi^2}{216(x-1)^5} - \frac{21\zeta_3}{4(x-1)} + \frac{7\zeta_3}{6(x-1)^2} + \frac{7\zeta_3}{6(x-1)^3} - \frac{21\zeta_3}{4(x-1)^4} - \frac{175\zeta_3}{12(x-1)^5} - \frac{175\zeta_3}{12} - \frac{17\pi^4}{144} + \frac{205d_1\pi^2}{432} - \frac{65\pi^2}{54}.$$

**D.5 The  $\mathcal{A}$  integral for  $k = -1$  and  $\kappa = 1$**

The  $\varepsilon$  expansion for this integral reads

$$\begin{aligned} \mathcal{I}(x, \varepsilon; \alpha_0, 3 + d_1\varepsilon; 1, 2, 0, g_A) &= x \mathcal{A}(\varepsilon, x; 3 + d_1\varepsilon; 1, 2) \\ &= \frac{1}{\varepsilon^2} a_{-2}^{(1,-1)} + \frac{1}{\varepsilon} a_{-1}^{(1,-1)} + a_0^{(1,-1)} + \varepsilon a_1^{(1,-1)} + \varepsilon^2 a_2^{(1,-1)} + \mathcal{O}(\varepsilon^3), \end{aligned} \tag{D.5}$$

where

$$\begin{aligned} a_{-2}^{(1,-1)} &= \frac{1}{6}, \\ a_{-1}^{(1,-1)} &= -\frac{2}{3} H(0; x), \\ a_0^{(1,-1)} &= \frac{\alpha_0^3}{12(x-1)^2} - \frac{\alpha_0^3}{12} + \frac{\alpha_0^2}{24(x-1)} - \frac{5\alpha_0^2}{24(x-1)^2} + \frac{7\alpha_0^2}{24(x-1)^3} + \frac{13\alpha_0^2}{24} - \frac{\alpha_0}{3(x-1)} + \frac{\alpha_0}{6(x-1)^2} - \frac{\alpha_0}{3(x-1)^3} + \frac{13\alpha_0}{12(x-1)^4} - \frac{23\alpha_0}{12} + \left( \frac{25}{12} + \frac{3}{4(x-1)} - \frac{1}{6(x-1)^2} - \frac{1}{6(x-1)^3} + \frac{3}{4(x-1)^4} + \frac{25}{12(x-1)^5} \right) H(0; \alpha_0) + \left( -\frac{25}{12} - \frac{3}{4(x-1)} + \frac{1}{6(x-1)^2} + \frac{1}{6(x-1)^3} - \frac{3}{4(x-1)^4} - \frac{25}{12(x-1)^5} \right) H(0; x) + \left( \frac{1}{(x-1)^5} - 1 \right) H(0; \alpha_0) H(1; x) + \left( -\frac{\alpha_0^4}{4(x-1)} + \frac{\alpha_0^4}{4} + \frac{\alpha_0^3}{x-1} - \frac{\alpha_0^3}{3(x-1)^2} - \frac{4\alpha_0^3}{3} - \frac{3\alpha_0^2}{2(x-1)} + \frac{\alpha_0^2}{(x-1)^2} - \frac{\alpha_0^2}{2(x-1)^3} + 3\alpha_0^2 + \frac{\alpha_0}{x-1} - \frac{\alpha_0}{(x-1)^2} + \frac{\alpha_0}{(x-1)^3} - \frac{\alpha_0}{(x-1)^4} - 4\alpha_0 + \frac{3}{4(x-1)} - \frac{1}{6(x-1)^2} - \frac{1}{6(x-1)^3} + \frac{3}{4(x-1)^4} + \frac{25}{12(x-1)^5} + \frac{25}{12} \right) H(c_1(\alpha_0); x) + \frac{8}{3} H(0, 0; x) + \left( \frac{1}{(x-1)^5} - 1 \right) H(0, c_1(\alpha_0); x) + \left( 1 - \frac{1}{(x-1)^5} \right) H(1, 0; x) + \left( \frac{1}{(x-1)^5} - 1 \right) H(1, c_1(\alpha_0); x) - \frac{H(c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} - \frac{\pi^2}{6(x-1)^5} + \frac{\pi^2}{36}, \\ a_1^{(1,-1)} &= \frac{7d_1\alpha_0^3}{72} - \frac{7d_1\alpha_0^3}{72(x-1)^2} + \frac{7\alpha_0^3}{36(x-1)^2} - \frac{7\alpha_0^3}{36} - \frac{109d_1\alpha_0^2}{144} - \frac{13d_1\alpha_0^2}{144(x-1)} - \frac{\alpha_0^2}{36(x-1)} + \frac{29d_1\alpha_0^2}{144(x-1)^2} - \frac{11\alpha_0^2}{18(x-1)^2} - \frac{67d_1\alpha_0^2}{144(x-1)^3} + \frac{41\alpha_0^2}{36(x-1)^3} + \frac{31\alpha_0^2}{18} + \frac{305d_1\alpha_0}{72} + \frac{19d_1\alpha_0}{18(x-1)} - \frac{25\alpha_0}{36(x-1)} - \frac{d_1\alpha_0}{9(x-1)^2} + \frac{23\alpha_0}{36(x-1)^2} + \frac{d_1\alpha_0}{18(x-1)^3} - \frac{55\alpha_0}{36(x-1)^3} - \frac{217d_1\alpha_0}{72(x-1)^4} + \frac{569\alpha_0}{72(x-1)^4} - \frac{775\alpha_0}{72} + \left( -\frac{\alpha_0^3}{3(x-1)^2} + \frac{\alpha_0^3}{3} - \frac{\alpha_0^2}{6(x-1)} + \frac{5\alpha_0^2}{6(x-1)^2} - \frac{7\alpha_0^2}{6(x-1)^3} - \frac{13\alpha_0^2}{6} + \frac{4\alpha_0}{3(x-1)} - \frac{2\alpha_0}{3(x-1)^2} + \frac{4\alpha_0}{3(x-1)^3} - \frac{13\alpha_0}{3(x-1)^4} + \frac{23\alpha_0}{3} - \frac{205d_1}{72} - \frac{15d_1}{8(x-1)} + \frac{5}{8(x-1)} + \frac{5d_1}{18(x-1)^2} - \frac{7}{18(x-1)^2} + \frac{5d_1}{18(x-1)^3} - \frac{13}{18(x-1)^3} - \frac{15d_1}{8(x-1)^4} + \frac{49}{8(x-1)^4} - \frac{205d_1}{72(x-1)^5} + \frac{935}{72(x-1)^5} + \frac{515}{72} \right) H(0; \alpha_0) + \left( \frac{15d_1}{8(x-1)} - \frac{5d_1}{18(x-1)^2} - \frac{5d_1}{18(x-1)^3} + \frac{15d_1}{8(x-1)^4} + \frac{205d_1}{72(x-1)^5} + \frac{205d_1}{72} - \frac{5}{8(x-1)} + \frac{7}{18(x-1)^2} + \frac{13}{18(x-1)^3} - \frac{49}{8(x-1)^4} + \frac{2\pi^2}{3(x-1)^5} - \frac{935}{72(x-1)^5} - \frac{\pi^2}{9} - \frac{515}{72} \right) H(0; x) + \left( \frac{d_1\alpha_0^3}{6} - \frac{d_1\alpha_0^3}{6(x-1)^2} - \frac{13d_1\alpha_0^2}{12} - \frac{d_1\alpha_0^2}{12(x-1)} + \frac{5d_1\alpha_0^2}{12(x-1)^2} - \frac{7d_1\alpha_0^2}{12(x-1)^3} + \frac{23d_1\alpha_0}{6} + \frac{2d_1\alpha_0}{3(x-1)} - \frac{d_1\alpha_0}{3(x-1)^2} + \frac{2d_1\alpha_0}{3(x-1)^3} - \frac{13d_1\alpha_0}{6(x-1)^4} - \frac{35d_1}{12} - \frac{7d_1}{12(x-1)} + \frac{d_1}{12(x-1)^2} - \frac{d_1}{12(x-1)^3} + \frac{13d_1}{6(x-1)^4} \right) H(1; \alpha_0) + \left( -\frac{d_1\alpha_0^4}{8} + \frac{d_1\alpha_0^4}{8(x-1)} - \frac{\alpha_0^4}{4(x-1)} + \frac{\alpha_0^4}{4} + \frac{13d_1\alpha_0^3}{18} - \frac{d_1\alpha_0^3}{2(x-1)} + \frac{3\alpha_0^3}{2(x-1)} + \frac{2d_1\alpha_0^3}{9(x-1)^2} - \frac{31\alpha_0^3}{36(x-1)^2} - \frac{55\alpha_0^3}{36} - \frac{23d_1\alpha_0^2}{12} + \frac{3d_1\alpha_0^2}{4(x-1)} - \frac{95\alpha_0^2}{24(x-1)} - \frac{2d_1\alpha_0^2}{3(x-1)^2} + \frac{27\alpha_0^2}{8(x-1)^2} + \frac{d_1\alpha_0^2}{2(x-1)^3} - \frac{59\alpha_0^2}{24(x-1)^3} + \frac{35\alpha_0^2}{8} + \frac{25d_1\alpha_0}{6} - \frac{d_1\alpha_0}{2(x-1)} + \frac{43\alpha_0}{6(x-1)} + \frac{2d_1\alpha_0}{3(x-1)^2} - \frac{37\alpha_0}{6(x-1)^2} - \frac{d_1\alpha_0}{(x-1)^3} + \frac{20\alpha_0}{3(x-1)^3} + \frac{2d_1\alpha_0}{(x-1)^4} - \frac{113\alpha_0}{12(x-1)^4} - \frac{41\alpha_0}{4} - \frac{205d_1}{72} + \left( \frac{\alpha_0^4}{x-1} - \alpha_0^4 - \frac{4\alpha_0^3}{x-1} + \frac{4\alpha_0^3}{3(x-1)^2} + \frac{16\alpha_0^3}{3} + \frac{6\alpha_0^2}{x-1} - \frac{4\alpha_0^2}{(x-1)^2} + \frac{2\alpha_0^2}{(x-1)^3} - 12\alpha_0^2 - \frac{4\alpha_0}{x-1} + \frac{4\alpha_0}{(x-1)^2} - \frac{4\alpha_0}{(x-1)^3} + \frac{4\alpha_0}{(x-1)^4} + 16\alpha_0 - \frac{3}{x-1} + \frac{2}{3(x-1)^2} + \frac{2}{3(x-1)^3} - \frac{3}{(x-1)^4} - \frac{25}{3(x-1)^5} - \frac{25}{3} \right) H(0; \alpha_0) + \left( -\frac{d_1\alpha_0^4}{2} + \right. \end{aligned}$$

$$\begin{aligned}
& \frac{d_1 \alpha_0^4}{2(x-1)} + \frac{8 d_1 \alpha_0^3}{3} - \frac{2d_1 \alpha_0^3}{x-1} + \frac{2d_1 \alpha_0^3}{3(x-1)^2} - 6 d_1 \alpha_0^2 + \frac{3d_1 \alpha_0^2}{x-1} - \frac{2d_1 \alpha_0^2}{(x-1)^2} + \frac{d_1 \alpha_0^2}{(x-1)^3} + 8d_1 \alpha_0 - \frac{2d_1 \alpha_0}{x-1} + \frac{2d_1 \alpha_0}{(x-1)^2} - \\
& \frac{2d_1 \alpha_0}{(x-1)^3} + \frac{2 d_1 \alpha_0}{(x-1)^4} - \frac{25d_1}{6} - \frac{3d_1}{2(x-1)} + \frac{d_1}{3(x-1)^2} + \frac{d_1}{3(x-1)^3} - \frac{3d_1}{2(x-1)^4} - \frac{25 d_1}{6(x-1)^5} \Big) H(1; \alpha_0) - \frac{15d_1}{8(x-1)} + \\
& \frac{5}{8(x-1)} + \frac{5d_1}{18(x-1)^2} - \frac{7}{18(x-1)^2} + \frac{5d_1}{18(x-1)^3} - \frac{13}{18(x-1)^3} - \frac{15d_1}{8(x-1)^4} + \frac{49}{8(x-1)^4} - \frac{205d_1}{72(x-1)^5} + \frac{935}{72(x-1)^5} + \\
& \frac{515}{72} \Big) H(c_1(\alpha_0); x) + \left( -\frac{25}{3} - \frac{3}{x-1} + \frac{2}{3(x-1)^2} + \frac{2}{3(x-1)^3} - \frac{3}{(x-1)^4} - \frac{25}{3(x-1)^5} \right) H(0, 0; \alpha_0) + \left( \frac{25}{3} + \right. \\
& \left. \frac{3}{x-1} - \frac{2}{3(x-1)^2} - \frac{2}{3(x-1)^3} + \frac{3}{(x-1)^4} + \frac{25}{3(x-1)^5} \right) H(0, 0; x) + \left( -\frac{3d_1}{2(x-1)} + \frac{d_1}{3(x-1)^2} + \frac{d_1}{3(x-1)^3} - \frac{3d_1}{2(x-1)^4} - \right. \\
& \left. \frac{25 d_1}{6(x-1)^5} - \frac{25d_1}{6} \right) H(0, 1; \alpha_0) + H(1; x) \left( -\frac{\pi^2 d_1}{3(x-1)^5} + \left( \frac{2 d_1}{x-1} - \frac{d_1}{(x-1)^2} + \frac{2d_1}{3(x-1)^3} - \frac{d_1}{2(x-1)^4} + \frac{25d_1}{6(x-1)^5} + \right. \right. \\
& \left. \left. \frac{5}{4(x-1)} - \frac{5}{6(x-1)^2} + \frac{5}{6(x-1)^3} - \frac{5}{4(x-1)^4} + \frac{25}{12(x-1)^5} - \frac{25}{12} \right) H(0; \alpha_0) + \left( 4 - \frac{4}{(x-1)^5} \right) H(0, 0; \alpha_0) + \\
& \left( 2d_1 - \frac{2d_1}{(x-1)^5} \right) H(0, 1; \alpha_0) + \frac{\pi^2}{2(x-1)^5} + \frac{\pi^2}{6} \Big) + \left( \frac{2d_1}{(x-1)^5} - 2 d_1 - \frac{2}{(x-1)^5} + 2 \right) H(0; \alpha_0) H(0, 1; x) + \\
& \left( -\frac{\alpha_0^4}{2(x-1)} + \frac{\alpha_0^4}{2} + \frac{2\alpha_0^3}{x-1} - \frac{2\alpha_0^3}{3(x-1)^2} - \frac{8\alpha_0^3}{3} - \frac{3\alpha_0^2}{x-1} + \frac{2 \alpha_0^2}{(x-1)^2} - \frac{\alpha_0^2}{(x-1)^3} + 6\alpha_0^2 + \frac{2 \alpha_0}{x-1} - \frac{2\alpha_0}{(x-1)^2} + \frac{2\alpha_0}{(x-1)^3} - \right. \\
& \left. \frac{2 \alpha_0}{(x-1)^4} - 8\alpha_0 + \left( 4 - \frac{4}{(x-1)^5} \right) H(0; \alpha_0) + \left( 2 d_1 - \frac{2d_1}{(x-1)^5} \right) H(1; \alpha_0) + \frac{3}{4(x-1)} - \frac{1}{6(x-1)^2} - \frac{1}{6(x-1)^3} + \right. \\
& \left. \frac{3}{4(x-1)^4} + \frac{25}{12(x-1)^5} + \frac{25}{12} \right) H(0, c_1(\alpha_0); x) + \left( -\frac{2 d_1}{x-1} + \frac{d_1}{(x-1)^2} - \frac{2d_1}{3(x-1)^3} + \frac{d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} - \right. \\
& \left. \frac{5}{4(x-1)} + \frac{5}{6(x-1)^2} - \frac{5}{6(x-1)^3} + \frac{5}{4(x-1)^4} - \frac{25}{12(x-1)^5} + \frac{25}{12} \right) H(1, 0; x) + \left( \frac{4d_1}{(x-1)^5} - 2 d_1 - \frac{3}{(x-1)^5} - \right. \\
& \left. 1 \right) H(0; \alpha_0) H(1, 1; x) + \left( \frac{2 d_1}{x-1} - \frac{d_1}{(x-1)^2} + \frac{2d_1}{3(x-1)^3} - \frac{d_1}{2(x-1)^4} + \frac{25d_1}{6(x-1)^5} + \left( 4 - \frac{4}{(x-1)^5} \right) H(0; \alpha_0) + \right. \\
& \left( 2d_1 - \frac{2d_1}{(x-1)^5} \right) H(1; \alpha_0) + \frac{5}{4(x-1)} - \frac{5}{6(x-1)^2} + \frac{5}{6(x-1)^3} - \frac{5}{4(x-1)^4} + \frac{25}{12(x-1)^5} - \frac{25}{12} \Big) H(1, c_1(\alpha_0); x) + \\
& \left( \frac{5\alpha_0^4}{4(x-1)} - \frac{5 \alpha_0^4}{4} - \frac{5\alpha_0^3}{x-1} + \frac{5\alpha_0^3}{3(x-1)^2} + \frac{20 \alpha_0^3}{3} + \frac{15\alpha_0^2}{2(x-1)} - \frac{5\alpha_0^2}{(x-1)^2} + \frac{5 \alpha_0^2}{2(x-1)^3} - 15\alpha_0^2 - \frac{5\alpha_0}{x-1} + \frac{5 \alpha_0}{(x-1)^2} - \right. \\
& \left. \frac{5\alpha_0}{(x-1)^3} + \frac{5\alpha_0}{(x-1)^4} + 20 \alpha_0 + \frac{4H(0; \alpha_0)}{(x-1)^5} + \frac{2d_1 H(1; \alpha_0)}{(x-1)^5} - \frac{15}{4(x-1)} + \frac{5}{6(x-1)^2} + \frac{5}{6(x-1)^3} - \frac{15}{4(x-1)^4} - \right. \\
& \left. \frac{125}{12(x-1)^5} - \frac{125}{12} \right) H(c_1(\alpha_0), c_1(\alpha_0); x) - \frac{32}{3} H(0, 0, 0; x) + \left( 2 - \frac{2}{(x-1)^5} \right) H(0, 0, c_1(\alpha_0); x) + \left( - \right. \\
& \left. \frac{2d_1}{(x-1)^5} + 2 d_1 + \frac{2}{(x-1)^5} - 2 \right) H(0, 1, 0; x) + \left( \frac{2d_1}{(x-1)^5} - 2 d_1 - \frac{2}{(x-1)^5} + 2 \right) H(0, 1, c_1(\alpha_0); x) + \\
& \left( 5 - \frac{1}{(x-1)^5} \right) H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{4}{(x-1)^5} - 4 \right) H(1, 0, 0; x) + \left( \frac{2d_1}{(x-1)^5} - \frac{3}{(x-1)^5} - \right. \\
& \left. 1 \right) H(1, 0, c_1(\alpha_0); x) + \left( -\frac{4d_1}{(x-1)^5} + 2 d_1 + \frac{3}{(x-1)^5} + 1 \right) H(1, 1, 0; x) + \left( \frac{4d_1}{(x-1)^5} - 2 d_1 - \frac{3}{(x-1)^5} - \right. \\
& \left. 1 \right) H(1, 1, c_1(\alpha_0); x) + \left( -\frac{2 d_1}{(x-1)^5} - \frac{1}{(x-1)^5} + 5 \right) H(1, c_1(\alpha_0), c_1(\alpha_0); x) - \frac{2 H(c_1(\alpha_0), 0, c_1(\alpha_0); x)}{(x-1)^5} + \\
& \frac{5 H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} - \frac{\pi^2}{8(x-1)} + \frac{\pi^2}{36(x-1)^2} + \frac{\pi^2}{36(x-1)^3} - \frac{\pi^2}{8(x-1)^4} - \frac{25\pi^2}{72(x-1)^5} - \frac{3 \zeta_3}{(x-1)^5} - 6\zeta_3 - \frac{25\pi^2}{72},
\end{aligned}$$

$$\begin{aligned}
a_2^{(1,-1)} = & -\frac{37}{432} d_1^2 \alpha_0^3 + \frac{37d_1 \alpha_0^3}{108} + \frac{37d_1^2 \alpha_0^3}{432(x-1)^2} - \frac{37d_1 \alpha_0^3}{108(x-1)^2} - \frac{\pi^2 \alpha_0^3}{72(x-1)^2} + \frac{37\alpha_0^3}{108(x-1)^2} + \frac{\pi^2 \alpha_0^3}{72} - \\
& \frac{37\alpha_0^3}{108} + \frac{715d_1^2 \alpha_0^2}{864} - \frac{1625d_1 \alpha_0^2}{432} + \frac{115d_1^2 \alpha_0^2}{864(x-1)} - \frac{35d_1 \alpha_0^2}{432(x-1)} - \frac{\pi^2 \alpha_0^2}{144(x-1)} - \frac{10\alpha_0^2}{27(x-1)} - \frac{107d_1^2 \alpha_0^2}{864(x-1)^2} + \frac{409d_1 \alpha_0^2}{432(x-1)^2} + \\
& \frac{5\pi^2 \alpha_0^2}{144(x-1)^2} - \frac{151\alpha_0^2}{108(x-1)^2} + \frac{493d_1^2 \alpha_0^2}{864(x-1)^3} - \frac{1181d_1 \alpha_0^2}{432(x-1)^3} - \frac{7\pi^2 \alpha_0^2}{144(x-1)^3} + \frac{86\alpha_0^2}{27(x-1)^3} - \frac{13\pi^2 \alpha_0^2}{144} + \frac{455\alpha_0^2}{108} - \\
& \frac{3515d_1^2 \alpha_0}{432} + \frac{17285d_1 \alpha_0}{432} - \frac{265d_1^2 \alpha_0}{108(x-1)} + \frac{1205d_1 \alpha_0}{216(x-1)} + \frac{\pi^2 \alpha_0}{18(x-1)} + \frac{13\alpha_0}{54(x-1)} - \frac{d_1^2 \alpha_0}{108(x-1)^2} - \frac{187d_1 \alpha_0}{216(x-1)^2} - \\
& \frac{\pi^2 \alpha_0}{36(x-1)^2} + \frac{191\alpha_0}{108(x-1)^2} + \frac{113d_1^2 \alpha_0}{108(x-1)^3} + \frac{11d_1 \alpha_0}{216(x-1)^3} + \frac{\pi^2 \alpha_0}{18(x-1)^3} - \frac{317\alpha_0}{54(x-1)^3} + \frac{2911d_1^2 \alpha_0}{432(x-1)^4} - \frac{14479d_1 \alpha_0}{432(x-1)^4} - \\
& \frac{13\pi^2 \alpha_0}{72(x-1)^4} + \frac{2207\alpha_0}{54(x-1)^4} + \frac{23\pi^2 \alpha_0}{72} - \frac{5213\alpha_0}{108} + \left( -\frac{7d_1 \alpha_0^3}{18} + \frac{7 d_1 \alpha_0^3}{18(x-1)^2} - \frac{7\alpha_0^3}{9(x-1)^2} + \frac{7 \alpha_0^3}{9} + \frac{109d_1 \alpha_0^2}{36} + \frac{13d_1 \alpha_0^2}{36(x-1)} + \right. \\
& \left. \frac{\alpha_0^2}{9(x-1)} - \frac{29d_1 \alpha_0^2}{36(x-1)^2} + \frac{22 \alpha_0^2}{9(x-1)^2} + \frac{67d_1 \alpha_0^2}{36(x-1)^3} - \frac{41\alpha_0^2}{9(x-1)^3} - \frac{62\alpha_0^2}{9} - \frac{305d_1 \alpha_0}{18} - \frac{38d_1 \alpha_0}{9(x-1)} + \frac{25\alpha_0}{9(x-1)} + \frac{4d_1 \alpha_0}{9(x-1)^2} - \right. \\
& \left. \frac{23\alpha_0}{9(x-1)^2} - \frac{2d_1 \alpha_0}{9(x-1)^3} + \frac{55\alpha_0}{9(x-1)^3} + \frac{217d_1 \alpha_0}{18(x-1)^4} - \frac{569\alpha_0}{18(x-1)^4} + \frac{775\alpha_0}{18} + \frac{2035 d_1^2}{432} - \frac{9685d_1}{432} + \frac{63d_1^2}{16(x-1)} - \frac{407 d_1}{48(x-1)} - \right. \\
& \left. \frac{\pi^2}{8(x-1)} - \frac{37}{8(x-1)} - \frac{19 d_1^2}{54(x-1)^2} + \frac{80d_1}{27(x-1)^2} + \frac{\pi^2}{36(x-1)^2} - \frac{101}{108(x-1)^2} - \frac{19d_1^2}{54(x-1)^3} + \frac{73 d_1}{54(x-1)^3} + \frac{\pi^2}{36(x-1)^3} - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{365}{108(x-1)^3} + \frac{63d_1^2}{16(x-1)^4} - \frac{1171d_1}{48(x-1)^4} - \frac{\pi^2}{8(x-1)^4} + \frac{239}{8(x-1)^4} + \frac{2035d_1^2}{432(x-1)^5} - \frac{15865d_1}{432(x-1)^5} - \frac{25\pi^2}{72(x-1)^5} + \\
& \frac{13505}{216(x-1)^5} - \frac{25\pi^2}{72} + \frac{5525}{216} \Big) H(0; \alpha_0) + \left( -\frac{7}{36}d_1^2\alpha_0^3 + \frac{7d_1\alpha_0^3}{18} + \frac{7d_1^2\alpha_0^3}{36(x-1)^2} - \frac{7d_1\alpha_0^3}{18(x-1)^2} + \frac{109d_1^2\alpha_0^2}{72} - \frac{31d_1\alpha_0^2}{9} + \right. \\
& \frac{13d_1^2\alpha_0^2}{72(x-1)} + \frac{d_1\alpha_0^2}{18(x-1)} - \frac{29d_1^2\alpha_0^2}{72(x-1)^2} + \frac{11d_1\alpha_0^2}{9(x-1)^2} + \frac{67d_1^2\alpha_0^2}{72(x-1)^3} - \frac{41d_1\alpha_0^2}{18(x-1)^3} - \frac{305d_1^2\alpha_0}{36} + \frac{775d_1\alpha_0}{36} - \frac{19d_1^2\alpha_0}{9(x-1)} + \\
& \frac{25d_1\alpha_0}{18(x-1)} + \frac{2d_1^2\alpha_0}{9(x-1)^2} - \frac{23d_1\alpha_0}{18(x-1)^2} - \frac{d_1^2\alpha_0}{9(x-1)^3} + \frac{55d_1\alpha_0}{18(x-1)^3} + \frac{217d_1^2\alpha_0}{36(x-1)^4} - \frac{569d_1\alpha_0}{36(x-1)^4} + \frac{515d_1^2}{72} - \frac{665d_1}{36} + \\
& \left. \frac{139d_1^2}{72(x-1)} - \frac{13d_1}{9(x-1)} - \frac{d_1^2}{72(x-1)^2} + \frac{4d_1}{9(x-1)^2} - \frac{59d_1^2}{72(x-1)^3} - \frac{7d_1}{9(x-1)^3} - \frac{217d_1^2}{36(x-1)^4} + \frac{569d_1}{36(x-1)^4} \right) H(1; \alpha_0) + \\
& \left( \frac{4\alpha_0^3}{3(x-1)^2} - \frac{4\alpha_0^3}{3} + \frac{2\alpha_0^2}{3(x-1)} - \frac{10\alpha_0^2}{3(x-1)^2} + \frac{14\alpha_0^2}{3(x-1)^3} + \frac{26\alpha_0^2}{3} - \frac{16\alpha_0}{3(x-1)} + \frac{8\alpha_0}{3(x-1)^2} - \frac{16\alpha_0}{3(x-1)^3} + \frac{52\alpha_0}{3(x-1)^4} - \right. \\
& \frac{92\alpha_0}{3} + \frac{205d_1}{18} + \frac{15d_1}{2(x-1)} - \frac{5}{2(x-1)} - \frac{10d_1}{9(x-1)^2} + \frac{14}{9(x-1)^2} - \frac{10d_1}{9(x-1)^3} + \frac{26}{9(x-1)^3} + \frac{15d_1}{2(x-1)^4} - \frac{49}{2(x-1)^4} + \\
& \left. \frac{205d_1}{18(x-1)^5} - \frac{935}{18(x-1)^5} - \frac{515}{18} \right) H(0, 0; \alpha_0) + \left( -\frac{15d_1}{2(x-1)} + \frac{10d_1}{9(x-1)^2} + \frac{10d_1}{9(x-1)^3} - \frac{15d_1}{2(x-1)^4} - \frac{205d_1}{18(x-1)^5} - \right. \\
& \frac{205d_1}{18} + \frac{5}{2(x-1)} - \frac{14}{9(x-1)^2} - \frac{26}{9(x-1)^3} + \frac{49}{2(x-1)^4} - \frac{8\pi^2}{3(x-1)^5} + \frac{935}{18(x-1)^5} + \frac{4\pi^2}{9} + \frac{515}{18} \Big) H(0, 0; x) + \left( -\right. \\
& \frac{2d_1\alpha_0^3}{3} + \frac{2d_1\alpha_0^3}{3(x-1)^2} + \frac{13d_1\alpha_0^2}{3} + \frac{d_1\alpha_0^2}{3(x-1)} - \frac{5d_1\alpha_0^2}{3(x-1)^2} + \frac{7d_1\alpha_0^2}{3(x-1)^3} - \frac{46d_1\alpha_0}{3} - \frac{8d_1\alpha_0}{3(x-1)} + \frac{4d_1\alpha_0}{3(x-1)^2} - \frac{8d_1\alpha_0}{3(x-1)^3} + \\
& \frac{26d_1\alpha_0}{3(x-1)^4} + \frac{205d_1^2}{36} - \frac{515d_1}{36} + \frac{15d_1^2}{4(x-1)} - \frac{5d_1}{4(x-1)} - \frac{5d_1^2}{9(x-1)^2} + \frac{7d_1}{9(x-1)^2} - \frac{5d_1^2}{9(x-1)^3} + \frac{13d_1}{9(x-1)^3} + \frac{15d_1^2}{4(x-1)^4} - \\
& \left. \frac{49d_1}{4(x-1)^4} + \frac{205d_1^2}{36(x-1)^5} - \frac{935d_1}{36(x-1)^5} \right) H(0, 1; \alpha_0) + \left( \frac{\pi^2 d_1}{(x-1)^5} + \frac{\pi^2 d_1}{3} + \left( \frac{5d_1}{2(x-1)} - \frac{5d_1}{3(x-1)^2} + \frac{5d_1}{3(x-1)^3} - \right. \right. \\
& \left. \left. \frac{5d_1}{2(x-1)^4} + \frac{25d_1}{6(x-1)^5} - \frac{25d_1}{6} + \frac{5}{2(x-1)} - \frac{5}{3(x-1)^2} + \frac{5}{3(x-1)^3} - \frac{5}{2(x-1)^4} + \frac{25}{6(x-1)^5} - \frac{25}{6} \right) H(0; \alpha_0) + \left( -\right. \\
& \left. \frac{8d_1}{(x-1)^5} + 8d_1 + \frac{8}{(x-1)^5} - 8 \right) H(0, 0; \alpha_0) + \left( -\frac{4d_1^2}{(x-1)^5} + 4d_1^2 + \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(0, 1; \alpha_0) - \frac{7\pi^2}{3(x-1)^5} - \\
& \frac{\pi^2}{3} \Big) H(0, 1; x) + \left( -\frac{2d_1\alpha_0^3}{3} + \frac{2d_1\alpha_0^3}{3(x-1)^2} + \frac{13d_1\alpha_0^2}{3} + \frac{d_1\alpha_0^2}{3(x-1)} - \frac{5d_1\alpha_0^2}{3(x-1)^2} + \frac{7d_1\alpha_0^2}{3(x-1)^3} - \frac{46d_1\alpha_0}{3} - \frac{8d_1\alpha_0}{3(x-1)} + \right. \\
& \left. \frac{4d_1\alpha_0}{3(x-1)^2} - \frac{8d_1\alpha_0}{3(x-1)^3} + \frac{26d_1\alpha_0}{3(x-1)^4} + \frac{35d_1}{3} + \frac{7d_1}{3(x-1)} - \frac{d_1}{3(x-1)^2} + \frac{d_1}{3(x-1)^3} - \frac{26d_1}{3(x-1)^4} \right) H(1, 0; \alpha_0) + \\
& \left( \frac{4d_1^2}{x-1} - \frac{d_1^2}{(x-1)^2} + \frac{4d_1^2}{9(x-1)^3} - \frac{d_1^2}{4(x-1)^4} + \frac{205d_1^2}{36(x-1)^5} - \frac{193d_1}{24(x-1)} + \frac{265d_1}{36(x-1)^2} - \frac{25d_1}{4(x-1)^3} + \frac{107d_1}{24(x-1)^4} + \right. \\
& \left. \frac{4\pi^2 d_1}{3(x-1)^5} - \frac{185d_1}{8(x-1)^5} - \frac{205d_1}{72} - \frac{133}{8(x-1)} + \frac{289}{36(x-1)^2} - \frac{289}{36(x-1)^3} + \frac{133}{8(x-1)^4} - \frac{11\pi^2}{6(x-1)^5} - \frac{305}{72(x-1)^5} - \frac{5\pi^2}{6} + \right. \\
& \left. \frac{305}{72} \right) H(1, 0; x) + \left( -\frac{1}{3}d_1^2\alpha_0^3 + \frac{d_1^2\alpha_0^3}{3(x-1)^2} + \frac{13d_1^2\alpha_0^2}{6} + \frac{d_1^2\alpha_0^2}{6(x-1)} - \frac{5d_1^2\alpha_0^2}{6(x-1)^2} + \frac{7d_1^2\alpha_0^2}{6(x-1)^3} - \frac{23d_1^2\alpha_0}{3} - \frac{4d_1^2\alpha_0}{3(x-1)} + \right. \\
& \left. \frac{2d_1^2\alpha_0}{3(x-1)^2} - \frac{4d_1^2\alpha_0}{3(x-1)^3} + \frac{13d_1^2\alpha_0}{3(x-1)^4} + \frac{35d_1^2}{6} + \frac{7d_1^2}{6(x-1)} - \frac{d_1^2}{6(x-1)^2} + \frac{d_1^2}{6(x-1)^3} - \frac{13d_1^2}{3(x-1)^4} \right) H(1, 1; \alpha_0) + \\
& H(0, c_1(\alpha_0); x) \left( -\frac{d_1\alpha_0^4}{4} + \frac{d_1\alpha_0^4}{4(x-1)} - \frac{\alpha_0^4}{2(x-1)} + \frac{\alpha_0^4}{2} + \frac{13d_1\alpha_0^3}{9} - \frac{d_1\alpha_0^3}{x-1} + \frac{3\alpha_0^3}{x-1} + \frac{4d_1\alpha_0^3}{9(x-1)^2} - \frac{25\alpha_0^3}{18(x-1)^2} - \right. \\
& \frac{61\alpha_0^3}{18} - \frac{23d_1\alpha_0^2}{6} + \frac{3d_1\alpha_0^2}{2(x-1)} - \frac{31\alpha_0^2}{4(x-1)} - \frac{4d_1\alpha_0^2}{3(x-1)^2} + \frac{71\alpha_0^2}{12(x-1)^2} + \frac{d_1\alpha_0^2}{(x-1)^3} - \frac{15\alpha_0^2}{4(x-1)^3} + \frac{131\alpha_0^2}{12} + \frac{25d_1\alpha_0}{3} - \frac{d_1\alpha_0}{x-1} + \\
& \frac{13\alpha_0}{x-1} + \frac{4d_1\alpha_0}{3(x-1)^2} - \frac{35\alpha_0}{3(x-1)^2} - \frac{2d_1\alpha_0}{(x-1)^3} + \frac{12\alpha_0}{(x-1)^3} + \frac{4d_1\alpha_0}{(x-1)^4} - \frac{29\alpha_0}{2(x-1)^4} - \frac{169\alpha_0}{6} - \frac{205d_1}{72} + \left( \frac{2\alpha_0^4}{x-1} - 2\alpha_0^4 - \right. \\
& \frac{8\alpha_0^3}{x-1} + \frac{8\alpha_0^3}{3(x-1)^2} + \frac{32\alpha_0^3}{3} + \frac{12\alpha_0^2}{x-1} - \frac{8\alpha_0^2}{(x-1)^2} + \frac{4\alpha_0^2}{(x-1)^3} - 24\alpha_0^2 - \frac{8\alpha_0}{x-1} + \frac{8\alpha_0}{(x-1)^2} - \frac{8\alpha_0}{(x-1)^3} + \frac{8\alpha_0}{(x-1)^4} + 32\alpha_0 - \\
& \left. \frac{3}{x-1} + \frac{2}{3(x-1)^2} + \frac{2}{3(x-1)^3} - \frac{3}{(x-1)^4} - \frac{25}{3(x-1)^5} - \frac{25}{3} \right) H(0; \alpha_0) + \left( -d_1\alpha_0^4 + \frac{d_1\alpha_0^4}{x-1} + \frac{16d_1\alpha_0^3}{3} - \frac{4d_1\alpha_0^3}{x-1} + \right. \\
& \frac{4d_1\alpha_0^3}{3(x-1)^2} - 12d_1\alpha_0^2 + \frac{6d_1\alpha_0^2}{x-1} - \frac{4d_1\alpha_0^2}{(x-1)^2} + \frac{2d_1\alpha_0^2}{(x-1)^3} + 16d_1\alpha_0 - \frac{4d_1\alpha_0}{x-1} + \frac{4d_1\alpha_0}{(x-1)^2} - \frac{4d_1\alpha_0}{(x-1)^3} + \frac{4d_1\alpha_0}{(x-1)^4} - \frac{25d_1}{6} - \\
& \left. \frac{3d_1}{2(x-1)} + \frac{d_1}{3(x-1)^2} + \frac{d_1}{3(x-1)^3} - \frac{3d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} \right) H(1; \alpha_0) + \left( \frac{16}{(x-1)^5} - 16 \right) H(0, 0; \alpha_0) + \left( \frac{8d_1}{(x-1)^5} - \right. \\
& \left. 8d_1 \right) H(0, 1; \alpha_0) + \left( \frac{8d_1}{(x-1)^5} - 8d_1 \right) H(1, 0; \alpha_0) + \left( \frac{4d_1^2}{(x-1)^5} - 4d_1^2 \right) H(1, 1; \alpha_0) - \frac{15d_1}{8(x-1)} + \frac{71}{8(x-1)} + \\
& \frac{5d_1}{18(x-1)^2} - \frac{8}{9(x-1)^2} + \frac{5d_1}{18(x-1)^3} - \frac{2}{9(x-1)^3} - \frac{15d_1}{8(x-1)^4} - \frac{17}{8(x-1)^4} - \frac{205d_1}{72(x-1)^5} - \frac{\pi^2}{6(x-1)^5} + \frac{305}{72(x-1)^5} + \frac{\pi^2}{6} + \\
& \frac{1145}{72} \Big) + H(c_1(\alpha_0); x) \left( \frac{d_1^2\alpha_0^4}{16} - \frac{d_1\alpha_0^4}{4} - \frac{d_1^2\alpha_0^4}{16(x-1)} + \frac{d_1\alpha_0^4}{4(x-1)} + \frac{\pi^2\alpha_0^4}{24(x-1)} - \frac{\alpha_0^4}{4(x-1)} - \frac{\pi^2\alpha_0^4}{24} + \frac{\alpha_0^4}{4} - \frac{43d_1^2\alpha_0^3}{108} + \right. \\
& \frac{365d_1\alpha_0^3}{216} + \frac{d_1^2\alpha_0^3}{4(x-1)} - \frac{19d_1\alpha_0^3}{12(x-1)} - \frac{\pi^2\alpha_0^3}{6(x-1)} + \frac{13\alpha_0^3}{6(x-1)} - \frac{4d_1^2\alpha_0^3}{27(x-1)^2} + \frac{233d_1\alpha_0^3}{216(x-1)^2} + \frac{\pi^2\alpha_0^3}{18(x-1)^2} - \frac{169\alpha_0^3}{108(x-1)^2} + \\
& \left. \frac{2\pi^2\alpha_0^3}{9} - \frac{193\alpha_0^3}{108} + \frac{95d_1^2\alpha_0^2}{72} - \frac{869d_1\alpha_0^2}{144} - \frac{3d_1^2\alpha_0^2}{8(x-1)} + \frac{695d_1\alpha_0^2}{144(x-1)} + \frac{\pi^2\alpha_0^2}{4(x-1)} - \frac{319\alpha_0^2}{36(x-1)} + \frac{4d_1^2\alpha_0^2}{9(x-1)^2} - \frac{641d_1\alpha_0^2}{144(x-1)^2} - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\pi^2 \alpha_0^2}{6(x-1)^2} + \frac{26\alpha_0^2}{3(x-1)^2} - \frac{d_1^2 \alpha_0^2}{2(x-1)^3} + \frac{623d_1 \alpha_0^2}{144(x-1)^3} + \frac{\pi^2 \alpha_0^2}{12(x-1)^3} - \frac{67\alpha_0^2}{9(x-1)^3} - \frac{\pi^2 \alpha_0^2}{2} + \frac{27\alpha_0^2}{4} - \frac{205d_1^2 \alpha_0}{36} + \frac{1945d_1 \alpha_0}{72} + \\
& \frac{d_1^2 \alpha_0}{4(x-1)} - \frac{505 d_1 \alpha_0}{36(x-1)} - \frac{\pi^2 \alpha_0}{6(x-1)} + \frac{299\alpha_0}{9(x-1)} - \frac{4d_1^2 \alpha_0}{9(x-1)^2} + \frac{11d_1 \alpha_0}{(x-1)^2} + \frac{\pi^2 \alpha_0}{6(x-1)^2} - \frac{541\alpha_0}{18(x-1)^2} + \frac{d_1^2 \alpha_0}{(x-1)^3} - \frac{239d_1 \alpha_0}{18(x-1)^3} - \\
& \frac{\pi^2 \alpha_0}{6(x-1)^3} + \frac{296\alpha_0}{9(x-1)^3} - \frac{4d_1^2 \alpha_0}{(x-1)^4} + \frac{2237d_1 \alpha_0}{72(x-1)^4} + \frac{\pi^2 \alpha_0}{6(x-1)^4} - \frac{3835\alpha_0}{72(x-1)^4} + \frac{2\pi^2 \alpha_0}{3} - \frac{739\alpha_0}{24} + \frac{2035d_1^2}{432} - \frac{9685 d_1}{432} + \\
& \left( \frac{d_1 \alpha_0^4}{2} - \frac{d_1 \alpha_0^4}{2(x-1)} + \frac{\alpha_0^4}{x-1} - \alpha_0^4 - \frac{26d_1 \alpha_0^3}{9} + \frac{2d_1 \alpha_0^3}{x-1} - \frac{6\alpha_0^3}{x-1} - \frac{8d_1 \alpha_0^3}{9(x-1)^2} + \frac{31\alpha_0^3}{9(x-1)^2} + \frac{55\alpha_0^3}{9} + \frac{23d_1 \alpha_0^2}{3} - \frac{3d_1 \alpha_0^2}{x-1} + \right. \\
& \left. \frac{95\alpha_0^2}{6(x-1)} + \frac{8 d_1 \alpha_0^2}{3(x-1)^2} - \frac{27\alpha_0^2}{2(x-1)^2} - \frac{2d_1 \alpha_0^2}{(x-1)^3} + \frac{59\alpha_0^2}{6(x-1)^3} - \frac{35 \alpha_0^2}{2} - \frac{50d_1 \alpha_0}{3} + \frac{2d_1 \alpha_0}{x-1} - \frac{86\alpha_0}{3(x-1)} - \frac{8d_1 \alpha_0}{3(x-1)^2} + \frac{74\alpha_0}{3(x-1)^2} + \right. \\
& \left. \frac{4 d_1 \alpha_0}{(x-1)^3} - \frac{80\alpha_0}{3(x-1)^3} - \frac{8d_1 \alpha_0}{(x-1)^4} + \frac{113\alpha_0}{3(x-1)^4} + 41\alpha_0 + \frac{205d_1}{18} + \frac{15d_1}{2(x-1)} - \frac{5}{2(x-1)} - \frac{10d_1}{9(x-1)^2} + \frac{14}{9(x-1)^2} - \right. \\
& \left. \frac{10d_1}{9(x-1)^3} + \frac{26}{9(x-1)^3} + \frac{15 d_1}{18(x-1)^4} - \frac{49}{2(x-1)^4} + \frac{205d_1}{18(x-1)^5} - \frac{935}{18(x-1)^5} - \frac{515}{18} \right) H(0; \alpha_0) + \left( \frac{d_1^2 \alpha_0^4}{4} - \frac{d_1 \alpha_0^4}{2} - \right. \\
& \left. \frac{d_1^2 \alpha_0^4}{4(x-1)} + \frac{d_1 \alpha_0^4}{2(x-1)} - \frac{13d_1^2 \alpha_0^3}{9} + \frac{55d_1 \alpha_0^3}{18} + \frac{d_1^2 \alpha_0^3}{x-1} - \frac{3d_1 \alpha_0^3}{x-1} - \frac{4d_1^2 \alpha_0^3}{9(x-1)^2} + \frac{31d_1 \alpha_0^3}{18(x-1)^2} + \frac{23d_1^2 \alpha_0^2}{6} - \frac{35d_1 \alpha_0^2}{4} - \frac{3 d_1^2 \alpha_0^2}{2(x-1)} + \right. \\
& \left. \frac{95d_1 \alpha_0^2}{12(x-1)} + \frac{4d_1^2 \alpha_0^2}{3(x-1)^2} - \frac{27d_1 \alpha_0^2}{4(x-1)^2} - \frac{d_1^2 \alpha_0^2}{(x-1)^3} + \frac{59d_1 \alpha_0^2}{12(x-1)^3} - \frac{25d_1^2 \alpha_0}{3} + \frac{41d_1 \alpha_0}{2} + \frac{d_1^2 \alpha_0}{x-1} - \frac{43d_1 \alpha_0}{3(x-1)} - \frac{4d_1^2 \alpha_0}{3(x-1)^2} + \right. \\
& \left. \frac{37d_1 \alpha_0}{3(x-1)^2} + \frac{2d_1^2 \alpha_0}{(x-1)^3} - \frac{40d_1 \alpha_0}{3(x-1)^3} - \frac{4d_1^2 \alpha_0}{(x-1)^4} + \frac{113d_1 \alpha_0}{6(x-1)^4} + \frac{205d_1^2}{36} - \frac{515d_1}{36} + \frac{15d_1^2}{4(x-1)} - \frac{5d_1}{4(x-1)} - \frac{5d_1^2}{9(x-1)^2} + \right. \\
& \left. \frac{7d_1}{9(x-1)^2} - \frac{5d_1^2}{9(x-1)^3} + \frac{13d_1}{9(x-1)^3} + \frac{15 d_1^2}{4(x-1)^4} - \frac{49d_1}{4(x-1)^4} + \frac{205d_1^2}{36(x-1)^5} - \frac{935d_1}{36(x-1)^5} \right) H(1; \alpha_0) + \left( -\frac{4 \alpha_0^4}{x-1} + \right. \\
& \left. 4\alpha_0^4 + \frac{16\alpha_0^3}{x-1} - \frac{16\alpha_0^3}{3(x-1)^2} - \frac{64\alpha_0^3}{3} - \frac{24\alpha_0^2}{x-1} + \frac{16 \alpha_0^2}{(x-1)^2} - \frac{8\alpha_0^2}{(x-1)^3} + 48\alpha_0^2 + \frac{16 \alpha_0}{x-1} - \frac{16\alpha_0}{(x-1)^2} + \frac{16\alpha_0}{(x-1)^3} - \frac{16 \alpha_0}{(x-1)^4} - \right. \\
& \left. 64\alpha_0 + \frac{12}{x-1} - \frac{8}{3(x-1)^2} - \frac{8}{3(x-1)^3} + \frac{12}{(x-1)^4} + \frac{100}{3(x-1)^5} + \frac{100}{3} \right) H(0, 0; \alpha_0) + \left( 2d_1 \alpha_0^4 - \frac{2d_1 \alpha_0^4}{x-1} - \frac{32d_1 \alpha_0^3}{3} + \right. \\
& \left. \frac{8d_1 \alpha_0^3}{x-1} - \frac{8d_1 \alpha_0^3}{3(x-1)^2} + 24 d_1 \alpha_0^2 - \frac{12d_1 \alpha_0^2}{x-1} + \frac{8d_1 \alpha_0^2}{(x-1)^2} - \frac{4d_1 \alpha_0^2}{(x-1)^3} - 32d_1 \alpha_0 + \frac{8d_1 \alpha_0}{x-1} - \frac{8d_1 \alpha_0}{(x-1)^2} + \frac{8d_1 \alpha_0}{(x-1)^3} - \frac{8 d_1 \alpha_0}{(x-1)^4} + \right. \\
& \left. \frac{50d_1}{3} + \frac{6d_1}{x-1} - \frac{4d_1}{3(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \frac{6d_1}{(x-1)^4} + \frac{50 d_1}{3(x-1)^5} \right) H(0, 1; \alpha_0) + \left( 2d_1 \alpha_0^4 - \frac{2d_1 \alpha_0^4}{x-1} - \frac{32d_1 \alpha_0^3}{3} + \right. \\
& \left. \frac{8d_1 \alpha_0^3}{x-1} - \frac{8 d_1 \alpha_0^3}{3(x-1)^2} + 24d_1 \alpha_0^2 - \frac{12d_1 \alpha_0^2}{x-1} + \frac{8 d_1 \alpha_0^2}{(x-1)^2} - \frac{4d_1 \alpha_0^2}{(x-1)^3} - 32d_1 \alpha_0 + \frac{8 d_1 \alpha_0}{x-1} - \frac{8d_1 \alpha_0}{(x-1)^2} + \frac{8d_1 \alpha_0}{(x-1)^3} - \frac{8d_1 \alpha_0}{(x-1)^4} + \right. \\
& \left. \frac{50d_1}{3} + \frac{6 d_1}{x-1} - \frac{4d_1}{3(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \frac{6 d_1}{(x-1)^4} + \frac{50d_1}{3(x-1)^5} \right) H(1, 0; \alpha_0) + \left( d_1^2 \alpha_0^4 - \frac{d_1^2 \alpha_0^4}{x-1} - \frac{16d_1^2 \alpha_0^3}{3} + \frac{4d_1^2 \alpha_0^3}{x-1} - \right. \\
& \left. \frac{4d_1^2 \alpha_0^3}{3(x-1)^2} + 12d_1^2 \alpha_0^2 - \frac{6 d_1^2 \alpha_0^2}{x-1} + \frac{4d_1^2 \alpha_0^2}{(x-1)^2} - \frac{2d_1^2 \alpha_0^2}{(x-1)^3} - 16d_1^2 \alpha_0 + \frac{4d_1^2 \alpha_0}{x-1} - \frac{4d_1^2 \alpha_0}{(x-1)^2} + \frac{4d_1^2 \alpha_0}{(x-1)^3} - \frac{4d_1^2 \alpha_0}{(x-1)^4} + \frac{25d_1^2}{3} + \right. \\
& \left. \frac{3d_1^2}{x-1} - \frac{2d_1^2}{3(x-1)^2} - \frac{2d_1^2}{3(x-1)^3} + \frac{3d_1^2}{(x-1)^4} + \frac{25 d_1^2}{3(x-1)^5} \right) H(1, 1; \alpha_0) + \frac{63d_1^2}{16(x-1)} - \frac{407 d_1}{48(x-1)} - \frac{\pi^2}{8(x-1)} - \frac{37}{8(x-1)} - \\
& \frac{19 d_1^2}{54(x-1)^2} + \frac{80d_1}{27(x-1)^2} + \frac{\pi^2}{36(x-1)^2} - \frac{101}{108(x-1)^2} - \frac{19d_1^2}{54(x-1)^3} + \frac{73 d_1}{54(x-1)^3} + \frac{\pi^2}{36(x-1)^3} - \frac{365}{108(x-1)^3} + \frac{63d_1^2}{16(x-1)^4} - \\
& \frac{1171d_1}{48(x-1)^4} - \frac{\pi^2}{8(x-1)^4} + \frac{239}{8(x-1)^4} + \frac{2035 d_1^2}{432(x-1)^5} - \frac{15865d_1}{432(x-1)^5} - \frac{25\pi^2}{72(x-1)^5} + \frac{13505}{216(x-1)^5} - \frac{25\pi^2}{72} + \frac{5525}{216} \left) + \left( - \right. \\
& \left. \frac{2\pi^2 d_1^2}{3(x-1)^5} + \frac{2\pi^2 d_1}{(x-1)^5} + \frac{\pi^2 d_1}{3} + \left( \frac{4 d_1^2}{x-1} - \frac{2d_1^2}{(x-1)^2} + \frac{4d_1^2}{3(x-1)^3} - \frac{d_1^2}{(x-1)^4} + \frac{25d_1^2}{3(x-1)^5} + \frac{9 d_1}{2(x-1)} - \frac{8d_1}{3(x-1)^2} + \frac{7d_1}{3(x-1)^3} - \right. \right. \\
& \left. \left. \frac{3 d_1}{(x-1)^4} + \frac{25d_1}{3(x-1)^5} - \frac{25d_1}{6} - \frac{13}{4(x-1)} + \frac{1}{6(x-1)^2} + \frac{11}{6(x-1)^3} - \frac{23}{4(x-1)^4} - \frac{125}{12(x-1)^5} - \frac{175}{12} \right) H(0; \alpha_0) + \left( - \right. \\
& \left. \frac{16 d_1}{(x-1)^5} + 8d_1 + \frac{12}{(x-1)^5} + 4 \right) H(0, 0; \alpha_0) + \left( -\frac{8d_1^2}{(x-1)^5} + 4d_1^2 + \frac{6 d_1}{(x-1)^5} + 2d_1 \right) H(0, 1; \alpha_0) - \frac{7\pi^2}{6(x-1)^5} - \\
& \frac{5\pi^2}{6} \left) H(1, 1; x) + \left( -\frac{4 d_1^2}{x-1} + \frac{d_1^2}{(x-1)^2} - \frac{4d_1^2}{9(x-1)^3} + \frac{d_1^2}{4(x-1)^4} - \frac{205d_1^2}{36(x-1)^5} + \frac{193d_1}{24(x-1)} - \frac{265d_1}{36(x-1)^2} + \frac{25 d_1}{4(x-1)^3} - \right. \\
& \left. \frac{107d_1}{24(x-1)^4} + \frac{185d_1}{8(x-1)^5} + \frac{205d_1}{72} + \left( -\frac{8d_1}{x-1} + \frac{4 d_1}{(x-1)^2} - \frac{8d_1}{3(x-1)^3} + \frac{2d_1}{(x-1)^4} - \frac{50 d_1}{3(x-1)^5} - \frac{5}{x-1} + \frac{10}{3(x-1)^2} - \right. \right. \\
& \left. \left. \frac{10}{3(x-1)^3} + \frac{5}{(x-1)^4} - \frac{25}{3(x-1)^5} + \frac{25}{3} \right) H(0; \alpha_0) + \left( -\frac{4d_1^2}{x-1} + \frac{2d_1^2}{(x-1)^2} - \frac{4 d_1^2}{3(x-1)^3} + \frac{d_1^2}{(x-1)^4} - \frac{25d_1^2}{3(x-1)^5} - \right. \right. \\
& \left. \left. \frac{5d_1}{2(x-1)} + \frac{5d_1}{3(x-1)^2} - \frac{5d_1}{3(x-1)^3} + \frac{5d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} + \frac{25 d_1}{6} \right) H(1; \alpha_0) + \left( \frac{16}{(x-1)^5} - 16 \right) H(0, 0; \alpha_0) + \\
& \left( \frac{8d_1}{(x-1)^5} - 8d_1 \right) H(0, 1; \alpha_0) + \left( \frac{8d_1}{(x-1)^5} - 8d_1 \right) H(1, 0; \alpha_0) + \left( \frac{4d_1^2}{(x-1)^5} - 4d_1^2 \right) H(1, 1; \alpha_0) + \\
& \frac{133}{8(x-1)} - \frac{289}{36(x-1)^2} + \frac{289}{36(x-1)^3} - \frac{133}{8(x-1)^4} - \frac{\pi^2}{6(x-1)^5} + \frac{305}{72(x-1)^5} + \frac{\pi^2}{6} - \frac{305}{72} \left) H(1, c_1(\alpha_0); x) + \right. \\
& \left. \left( \frac{5d_1 \alpha_0^4}{8} - \frac{5d_1 \alpha_0^4}{8(x-1)} + \frac{5\alpha_0^4}{4(x-1)} - \frac{5\alpha_0^4}{4} - \frac{65d_1 \alpha_0^3}{18} + \frac{5d_1 \alpha_0^3}{2(x-1)} - \frac{15\alpha_0^3}{2(x-1)} - \frac{10d_1 \alpha_0^3}{9(x-1)^2} + \frac{137\alpha_0^3}{36(x-1)^2} + \frac{293\alpha_0^3}{36} + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{115d_1\alpha_0^2}{12} - \frac{15d_1\alpha_0^2}{4(x-1)} + \frac{469\alpha_0^2}{24(x-1)} + \frac{10d_1\alpha_0^2}{3(x-1)^2} - \frac{125\alpha_0^2}{8(x-1)^2} - \frac{5d_1\alpha_0^2}{2(x-1)^3} + \frac{253\alpha_0^2}{24(x-1)^3} - \frac{201\alpha_0^2}{8} - \frac{125d_1\alpha_0}{6} + \frac{5d_1\alpha_0}{2(x-1)} - \\
& \frac{203\alpha_0}{6(x-1)} - \frac{10d_1\alpha_0}{3(x-1)^2} + \frac{179\alpha_0}{6(x-1)^2} + \frac{5d_1\alpha_0}{(x-1)^3} - \frac{94\alpha_0}{3(x-1)^3} - \frac{10d_1\alpha_0}{(x-1)^4} + \frac{487\alpha_0}{12(x-1)^4} + \frac{251\alpha_0}{4} + \frac{1025d_1}{72} + \left( -\frac{5\alpha_0^4}{x-1} + \right. \\
& 5\alpha_0^4 + \frac{20\alpha_0^3}{x-1} - \frac{20\alpha_0^3}{3(x-1)^2} - \frac{80\alpha_0^3}{3} - \frac{30\alpha_0^2}{x-1} + \frac{20\alpha_0^2}{(x-1)^2} - \frac{10\alpha_0^2}{(x-1)^3} + 60\alpha_0^2 + \frac{20\alpha_0}{x-1} - \frac{20\alpha_0}{(x-1)^2} + \frac{20\alpha_0}{(x-1)^3} - \frac{20\alpha_0}{(x-1)^4} - \\
& 80\alpha_0 + \frac{15}{x-1} - \frac{10}{3(x-1)^2} - \frac{10}{3(x-1)^3} + \frac{15}{(x-1)^4} + \frac{125}{3(x-1)^5} + \frac{125}{3} \Big) H(0; \alpha_0) + \left( \frac{5d_1\alpha_0^4}{2} - \frac{5d_1\alpha_0^4}{2(x-1)} - \frac{40d_1\alpha_0^3}{3} + \right. \\
& \frac{10d_1\alpha_0^3}{x-1} - \frac{10d_1\alpha_0^3}{3(x-1)^2} + 30d_1\alpha_0^2 - \frac{15d_1\alpha_0^2}{x-1} + \frac{10d_1\alpha_0^2}{(x-1)^2} - \frac{5d_1\alpha_0^2}{(x-1)^3} - 40d_1\alpha_0 + \frac{10d_1\alpha_0}{x-1} - \frac{10d_1\alpha_0}{(x-1)^2} + \frac{10d_1\alpha_0}{(x-1)^3} - \\
& \frac{10d_1\alpha_0}{(x-1)^4} + \frac{125d_1}{6} + \frac{15d_1}{2(x-1)} - \frac{5d_1}{3(x-1)^2} - \frac{5d_1}{3(x-1)^3} + \frac{15d_1}{2(x-1)^4} + \frac{125d_1}{6(x-1)^5} \Big) H(1; \alpha_0) - \frac{16H(0,0;\alpha_0)}{(x-1)^5} - \\
& \frac{8d_1H(0,1;\alpha_0)}{(x-1)^5} - \frac{8d_1H(1,0;\alpha_0)}{(x-1)^5} - \frac{4d_1^2H(1,1;\alpha_0)}{(x-1)^5} + \frac{75d_1}{8(x-1)} - \frac{91}{8(x-1)} - \frac{25d_1}{18(x-1)^2} + \frac{22}{9(x-1)^2} - \frac{25d_1}{18(x-1)^3} + \\
& \frac{28}{9(x-1)^3} + \frac{75d_1}{8(x-1)^4} - \frac{179}{8(x-1)^4} + \frac{1025d_1}{72(x-1)^5} + \frac{\pi^2}{6(x-1)^5} - \frac{4045}{72(x-1)^5} - \frac{3205}{72} \Big) H(c_1(\alpha_0), c_1(\alpha_0); x) + \\
& \left( \frac{100}{3} + \frac{12}{x-1} - \frac{8}{3(x-1)^2} - \frac{8}{3(x-1)^3} + \frac{12}{(x-1)^4} + \frac{100}{3(x-1)^5} \right) H(0, 0, 0; \alpha_0) + \left( -\frac{100}{3} - \frac{12}{x-1} + \frac{8}{3(x-1)^2} + \right. \\
& \frac{8}{3(x-1)^3} - \frac{12}{(x-1)^4} - \frac{100}{3(x-1)^5} \Big) H(0, 0, 0; x) + \left( \frac{6d_1}{x-1} - \frac{4d_1}{3(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \frac{6d_1}{(x-1)^4} + \frac{50d_1}{3(x-1)^5} + \right. \\
& \frac{50d_1}{3} \Big) H(0, 0, 1; \alpha_0) + \left( -\frac{4d_1}{(x-1)^5} + 4d_1 + \frac{12}{(x-1)^5} - 12 \right) H(0; \alpha_0) H(0, 0, 1; x) + \left( -\frac{\alpha_0^4}{x-1} + \alpha_0^4 + \frac{4\alpha_0^3}{x-1} - \right. \\
& \frac{4\alpha_0^3}{3(x-1)^2} - \frac{16\alpha_0^3}{3} - \frac{6\alpha_0^2}{x-1} + \frac{4\alpha_0^2}{(x-1)^2} - \frac{2\alpha_0^2}{(x-1)^3} + 12\alpha_0^2 + \frac{4\alpha_0}{x-1} - \frac{4\alpha_0}{(x-1)^2} + \frac{4\alpha_0}{(x-1)^3} - \frac{4\alpha_0}{(x-1)^4} - 16\alpha_0 + \left( \frac{8}{(x-1)^5} - \right. \\
& 8 \Big) H(0; \alpha_0) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(1; \alpha_0) + \frac{3}{2(x-1)} - \frac{1}{3(x-1)^2} - \frac{1}{3(x-1)^3} + \frac{3}{2(x-1)^4} + \frac{25}{6(x-1)^5} + \\
& \frac{25}{6} \Big) H(0, 0, c_1(\alpha_0); x) + \left( \frac{6d_1}{x-1} - \frac{4d_1}{3(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \frac{6d_1}{(x-1)^4} + \frac{50d_1}{3(x-1)^5} + \frac{50d_1}{3} \right) H(0, 1, 0; \alpha_0) + \left( -\frac{5d_1}{2(x-1)} + \right. \\
& \frac{5d_1}{3(x-1)^2} - \frac{5d_1}{3(x-1)^3} + \frac{5d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} + \frac{25d_1}{6} - \frac{5}{2(x-1)} + \frac{5}{3(x-1)^2} - \frac{5}{3(x-1)^3} + \frac{5}{2(x-1)^4} - \\
& \frac{25}{6(x-1)^5} + \frac{25}{6} \Big) H(0, 1, 0; x) + \left( \frac{3d_1^2}{x-1} - \frac{2d_1^2}{3(x-1)^2} - \frac{2d_1^2}{3(x-1)^3} + \frac{3d_1^2}{(x-1)^4} + \frac{25d_1^2}{3(x-1)^5} + \frac{25d_1^2}{3} \right) H(0, 1, 1; \alpha_0) + \\
& \left( \frac{4d_1^2}{(x-1)^5} - 4d_1^2 - \frac{10d_1}{(x-1)^5} + 2d_1 + \frac{14}{(x-1)^5} + 2 \right) H(0; \alpha_0) H(0, 1, 1; x) + \left( \frac{5d_1}{2(x-1)} - \frac{5d_1}{3(x-1)^2} + \frac{5d_1}{3(x-1)^3} - \right. \\
& \frac{5d_1}{2(x-1)^4} + \frac{25d_1}{6(x-1)^5} - \frac{25d_1}{6} + \left( -\frac{8d_1}{(x-1)^5} + 8d_1 + \frac{8}{(x-1)^5} - 8 \right) H(0; \alpha_0) + \left( -\frac{4d_1^2}{(x-1)^5} + 4d_1^2 + \frac{4d_1}{(x-1)^5} - \right. \\
& 4d_1 \Big) H(1; \alpha_0) + \frac{5}{2(x-1)} - \frac{5}{3(x-1)^2} + \frac{5}{3(x-1)^3} - \frac{5}{2(x-1)^4} + \frac{25}{6(x-1)^5} - \frac{25}{6} \Big) H(0, 1, c_1(\alpha_0); x) + \\
& \left( \frac{5\alpha_0^4}{2(x-1)} - \frac{5\alpha_0^4}{2} - \frac{10\alpha_0^3}{x-1} + \frac{10\alpha_0^3}{3(x-1)^2} + \frac{40\alpha_0^3}{3} + \frac{15\alpha_0^2}{x-1} - \frac{10\alpha_0^2}{(x-1)^2} + \frac{5\alpha_0^2}{(x-1)^3} - 30\alpha_0^2 - \frac{10\alpha_0}{x-1} + \frac{10\alpha_0}{(x-1)^2} - \frac{10\alpha_0}{(x-1)^3} + \right. \\
& \frac{10\alpha_0}{(x-1)^4} + 40\alpha_0 + \left( \frac{4}{(x-1)^5} - 20 \right) H(0; \alpha_0) + \left( \frac{2d_1}{(x-1)^5} - 10d_1 \right) H(1; \alpha_0) + \frac{3}{4(x-1)} - \frac{1}{6(x-1)^2} - \frac{1}{6(x-1)^3} + \\
& \frac{3}{4(x-1)^4} + \frac{25}{12(x-1)^5} + \frac{25}{12} \Big) H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{8d_1}{x-1} - \frac{4d_1}{(x-1)^2} + \frac{8d_1}{3(x-1)^3} - \frac{2d_1}{(x-1)^4} + \frac{50d_1}{3(x-1)^5} + \right. \\
& \frac{5}{x-1} - \frac{10}{3(x-1)^2} + \frac{10}{3(x-1)^3} - \frac{5}{(x-1)^4} + \frac{25}{3(x-1)^5} - \frac{25}{3} \Big) H(1, 0, 0; x) + \left( \frac{4d_1^2}{(x-1)^5} - \frac{10d_1}{(x-1)^5} - 2d_1 + \frac{10}{(x-1)^5} - \right. \\
& 2 \Big) H(0; \alpha_0) H(1, 0, 1; x) + \left( \frac{2d_1}{x-1} - \frac{d_1}{(x-1)^2} + \frac{2d_1}{3(x-1)^3} - \frac{d_1}{2(x-1)^4} + \frac{25d_1}{6(x-1)^5} + \left( -\frac{8d_1}{(x-1)^5} + \frac{12}{(x-1)^5} + \right. \right. \\
& 4 \Big) H(0; \alpha_0) + \left( -\frac{4d_1^2}{(x-1)^5} + \frac{6d_1}{(x-1)^5} + 2d_1 \right) H(1; \alpha_0) - \frac{13}{4(x-1)} + \frac{1}{6(x-1)^2} + \frac{11}{6(x-1)^3} - \frac{23}{4(x-1)^4} - \\
& \frac{125}{12(x-1)^5} - \frac{175}{12} \Big) H(1, 0, c_1(\alpha_0); x) + \left( -\frac{4d_1^2}{x-1} + \frac{2d_1^2}{(x-1)^2} - \frac{4d_1^2}{3(x-1)^3} + \frac{d_1^2}{(x-1)^4} - \frac{25d_1^2}{3(x-1)^5} - \frac{9d_1}{2(x-1)} + \right. \\
& \frac{8d_1}{3(x-1)^2} - \frac{7d_1}{3(x-1)^3} + \frac{3d_1}{(x-1)^4} - \frac{25d_1}{3(x-1)^5} + \frac{25d_1}{6} + \frac{13}{4(x-1)} - \frac{1}{6(x-1)^2} - \frac{11}{6(x-1)^3} + \frac{23}{4(x-1)^4} + \frac{125}{12(x-1)^5} + \\
& \frac{175}{12} \Big) H(1, 1, 0; x) + \left( \frac{12d_1^2}{(x-1)^5} - 4d_1^2 - \frac{18d_1}{(x-1)^5} - 4d_1 + \frac{7}{(x-1)^5} + 5 \right) H(0; \alpha_0) H(1, 1, 1; x) + \left( \frac{4d_1^2}{x-1} - \right. \\
& \frac{2d_1^2}{(x-1)^2} + \frac{4d_1^2}{3(x-1)^3} - \frac{d_1^2}{(x-1)^4} + \frac{25d_1^2}{3(x-1)^5} + \frac{9d_1}{2(x-1)} - \frac{8d_1}{3(x-1)^2} + \frac{7d_1}{3(x-1)^3} - \frac{3d_1}{(x-1)^4} + \frac{25d_1}{3(x-1)^5} - \frac{25d_1}{6} + \\
& \left( -\frac{16d_1}{(x-1)^5} + 8d_1 + \frac{12}{(x-1)^5} + 4 \right) H(0; \alpha_0) + \left( -\frac{8d_1^2}{(x-1)^5} + 4d_1^2 + \frac{6d_1}{(x-1)^5} + 2d_1 \right) H(1; \alpha_0) - \frac{13}{4(x-1)} + \\
& \frac{1}{6(x-1)^2} + \frac{11}{6(x-1)^3} - \frac{23}{4(x-1)^4} - \frac{125}{12(x-1)^5} - \frac{175}{12} \Big) H(1, 1, c_1(\alpha_0); x) + \left( -\frac{10d_1}{x-1} + \frac{5d_1}{(x-1)^2} - \frac{10d_1}{3(x-1)^3} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{5 d_1}{2(x-1)^4} - \frac{125d_1}{6(x-1)^5} + \left( \frac{8d_1}{(x-1)^5} + \frac{4}{(x-1)^5} - 20 \right) H(0; \alpha_0) + \left( \frac{4 d_1^2}{(x-1)^5} + \frac{2d_1}{(x-1)^5} - 10d_1 \right) H(1; \alpha_0) - \\
& \frac{7}{4(x-1)} + \frac{19}{6(x-1)^2} - \frac{31}{6(x-1)^3} + \frac{43}{4(x-1)^4} + \frac{25}{12(x-1)^5} + \frac{275}{12} \Big) H(1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{3\alpha_0^4}{2(x-1)} - \right. \\
& \frac{3 \alpha_0^4}{2} - \frac{6\alpha_0^3}{x-1} + \frac{2\alpha_0^3}{(x-1)^2} + 8\alpha_0^3 + \frac{9 \alpha_0^2}{x-1} - \frac{6\alpha_0^2}{(x-1)^2} + \frac{3\alpha_0^2}{(x-1)^3} - 18\alpha_0^2 - \frac{6\alpha_0}{x-1} + \frac{6\alpha_0}{(x-1)^2} - \frac{6\alpha_0}{(x-1)^3} + \\
& \frac{6 \alpha_0}{(x-1)^4} + 24\alpha_0 + \frac{8H(0; \alpha_0)}{(x-1)^5} + \frac{4d_1 H(1; \alpha_0)}{(x-1)^5} - \frac{9}{2(x-1)} + \frac{1}{(x-1)^2} + \frac{1}{(x-1)^3} - \frac{9}{2(x-1)^4} - \frac{25}{2(x-1)^5} - \\
& \left. \frac{25}{2} \right) H(c_1(\alpha_0), 0, c_1(\alpha_0); x) + \left( -\frac{19\alpha_0^4}{4(x-1)} + \frac{19 \alpha_0^4}{4} + \frac{19\alpha_0^3}{x-1} - \frac{19\alpha_0^3}{3(x-1)^2} - \frac{76 \alpha_0^3}{3} - \frac{57\alpha_0^2}{2(x-1)} + \frac{19\alpha_0^2}{(x-1)^2} - \right. \\
& \frac{19 \alpha_0^2}{2(x-1)^3} + 57\alpha_0^2 + \frac{19\alpha_0}{x-1} - \frac{19\alpha_0}{(x-1)^2} + \frac{19\alpha_0}{(x-1)^3} - \frac{19\alpha_0}{(x-1)^4} - 76\alpha_0 - \frac{20 H(0; \alpha_0)}{(x-1)^5} - \frac{10d_1 H(1; \alpha_0)}{(x-1)^5} + \frac{57}{4(x-1)} - \\
& \frac{19}{6(x-1)^2} - \frac{19}{6(x-1)^3} + \frac{57}{4(x-1)^4} + \frac{475}{12(x-1)^5} + \frac{475}{12} \Big) H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \frac{128}{3} H(0, 0, 0, 0; x) + \\
& \left( \frac{12}{(x-1)^5} - 12 \right) H(0, 0, 0, c_1(\alpha_0); x) + \left( \frac{4d_1}{(x-1)^5} - 4 d_1 - \frac{12}{(x-1)^5} + 12 \right) H(0, 0, 1, 0; x) + \left( -\frac{4 d_1}{(x-1)^5} + \right. \\
& 4d_1 + \frac{12}{(x-1)^5} - 12 \Big) H(0, 0, 1, c_1(\alpha_0); x) + \left( -10 - \frac{6}{(x-1)^5} \right) H(0, 0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{8d_1}{(x-1)^5} - \right. \\
& 8 d_1 - \frac{8}{(x-1)^5} + 8 \Big) H(0, 1, 0, 0; x) + \left( -\frac{6 d_1}{(x-1)^5} - 2d_1 + \frac{14}{(x-1)^5} + 2 \right) H(0, 1, 0, c_1(\alpha_0); x) + \left( -\frac{4d_1^2}{(x-1)^5} + \right. \\
& 4d_1^2 + \frac{10 d_1}{(x-1)^5} - 2d_1 - \frac{14}{(x-1)^5} - 2 \Big) H(0, 1, 1, 0; x) + \left( \frac{4d_1^2}{(x-1)^5} - 4d_1^2 - \frac{10 d_1}{(x-1)^5} + 2d_1 + \frac{14}{(x-1)^5} + \right. \\
& 2 \Big) H(0, 1, 1, c_1(\alpha_0); x) + \left( -\frac{2d_1}{(x-1)^5} + 10 d_1 - \frac{6}{(x-1)^5} - 10 \right) H(0, 1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( 6 + \frac{2}{(x-1)^5} \right) \\
& H(0, c_1(\alpha_0), 0, c_1(\alpha_0); x) + \left( -19 - \frac{1}{(x-1)^5} \right) H(0, c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \left( 16 - \frac{16}{(x-1)^5} \right) \\
& H(1, 0, 0, 0; x) + \left( -\frac{4d_1}{(x-1)^5} + \frac{10}{(x-1)^5} - 2 \right) H(1, 0, 0, c_1(\alpha_0); x) + \left( -\frac{4d_1^2}{(x-1)^5} + \frac{10 d_1}{(x-1)^5} + \right. \\
& 2d_1 - \frac{10}{(x-1)^5} + 2 \Big) H(1, 0, 1, 0; x) + \left( \frac{4d_1^2}{(x-1)^5} - \frac{10d_1}{(x-1)^5} - 2 d_1 + \frac{10}{(x-1)^5} - 2 \right) H(1, 0, 1, c_1(\alpha_0); x) + \\
& \left( -\frac{2 d_1}{(x-1)^5} + \frac{5}{(x-1)^5} - 1 \right) H(1, 0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{16d_1}{(x-1)^5} - 8 d_1 - \frac{12}{(x-1)^5} - 4 \right) H(1, 1, 0, 0; x) + \\
& \left( \frac{4 d_1^2}{(x-1)^5} - \frac{12d_1}{(x-1)^5} - 2d_1 + \frac{7}{(x-1)^5} + 5 \right) H(1, 1, 0, c_1(\alpha_0); x) + \left( -\frac{12d_1^2}{(x-1)^5} + 4d_1^2 + \frac{18 d_1}{(x-1)^5} + 4d_1 - \right. \\
& \left. \frac{7}{(x-1)^5} - 5 \right) H(1, 1, 1, 0; x) + \left( \frac{12d_1^2}{(x-1)^5} - 4d_1^2 - \frac{18 d_1}{(x-1)^5} - 4d_1 + \frac{7}{(x-1)^5} + 5 \right) H(1, 1, 1, c_1(\alpha_0); x) + \\
& \left( -\frac{4d_1^2}{(x-1)^5} - \frac{4 d_1}{(x-1)^5} + 10d_1 + \frac{5}{(x-1)^5} - 1 \right) H(1, 1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{4 d_1}{(x-1)^5} + \frac{2}{(x-1)^5} + \right. \\
& 6 \Big) H(1, c_1(\alpha_0), 0, c_1(\alpha_0); x) + \left( \frac{10 d_1}{(x-1)^5} - \frac{1}{(x-1)^5} - 19 \right) H(1, c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) - \\
& \frac{4 H(c_1(\alpha_0), 0, 0, c_1(\alpha_0); x)}{(x-1)^5} + \frac{10 H(c_1(\alpha_0), 0, c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \frac{6 H(c_1(\alpha_0), c_1(\alpha_0), 0, c_1(\alpha_0); x)}{(x-1)^5} - \\
& \frac{19 H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + H(0; x) \left( -\frac{63d_1^2}{16(x-1)} + \frac{19d_1^2}{54(x-1)^2} + \frac{19 d_1^2}{54(x-1)^3} - \frac{63d_1^2}{16(x-1)^4} - \right. \\
& \frac{2035d_1^2}{432(x-1)^5} - \frac{2035d_1^2}{432} + \frac{407d_1}{48(x-1)} - \frac{80 d_1}{27(x-1)^2} - \frac{73d_1}{54(x-1)^3} + \frac{1171d_1}{48(x-1)^4} + \frac{15865d_1}{432(x-1)^5} + \frac{9685d_1}{432} + \frac{5 \pi^2}{8(x-1)} + \\
& \frac{37}{8(x-1)} - \frac{5\pi^2}{36(x-1)^2} + \frac{101}{108(x-1)^2} - \frac{5\pi^2}{36(x-1)^3} + \frac{365}{108(x-1)^3} + \frac{5\pi^2}{8(x-1)^4} - \frac{239}{8(x-1)^4} + \frac{125\pi^2}{72(x-1)^5} - \frac{13505}{216(x-1)^5} + \\
& \frac{12\zeta_3}{(x-1)^5} + 24 \zeta_3 + \frac{125\pi^2}{72} - \frac{5525}{216} \Big) + H(1; x) \left( -\frac{\pi^2 d_1}{3(x-1)} + \frac{\pi^2 d_1}{6(x-1)^2} - \frac{\pi^2 d_1}{9(x-1)^3} + \frac{\pi^2 d_1}{12(x-1)^4} - \frac{25\pi^2 d_1}{36(x-1)^5} - \right. \\
& \frac{6\zeta_3 d_1}{(x-1)^5} + \left( -\frac{4d_1^2}{x-1} + \frac{d_1^2}{(x-1)^2} - \frac{4 d_1^2}{9(x-1)^3} + \frac{d_1^2}{4(x-1)^4} - \frac{205d_1^2}{36(x-1)^5} + \frac{193d_1}{24(x-1)} - \frac{265d_1}{36(x-1)^2} + \frac{25 d_1}{4(x-1)^3} - \right. \\
& \frac{107d_1}{24(x-1)^4} + \frac{185d_1}{8(x-1)^5} + \frac{205d_1}{72} + \frac{133}{8(x-1)} - \frac{289}{36(x-1)^2} + \frac{289}{36(x-1)^3} - \frac{133}{8(x-1)^4} - \frac{\pi^2}{6(x-1)^5} + \frac{305}{72(x-1)^5} + \frac{\pi^2}{6} - \\
& \left. \frac{305}{72} \right) H(0; \alpha_0) + \left( -\frac{8d_1}{x-1} + \frac{4d_1}{(x-1)^2} - \frac{8d_1}{3(x-1)^3} + \frac{2d_1}{(x-1)^4} - \frac{50d_1}{3(x-1)^5} - \frac{5}{x-1} + \frac{10}{3(x-1)^2} - \frac{10}{3(x-1)^3} + \right. \\
& \frac{5}{(x-1)^4} - \frac{25}{3(x-1)^5} + \frac{25}{3} \Big) H(0, 0; \alpha_0) + \left( -\frac{4d_1^2}{x-1} + \frac{2d_1^2}{(x-1)^2} - \frac{4 d_1^2}{3(x-1)^3} + \frac{d_1^2}{(x-1)^4} - \frac{25d_1^2}{3(x-1)^5} - \frac{5d_1}{2(x-1)} + \right. \\
& \frac{5d_1}{3(x-1)^2} - \frac{5d_1}{3(x-1)^3} + \frac{5d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} + \frac{25 d_1}{6} \Big) H(0, 1; \alpha_0) + \left( \frac{16}{(x-1)^5} - 16 \right) H(0, 0, 0; \alpha_0) + \\
& \left( \frac{8d_1}{(x-1)^5} - 8d_1 \right) H(0, 0, 1; \alpha_0) + \left( \frac{8 d_1}{(x-1)^5} - 8d_1 \right) H(0, 1, 0; \alpha_0) + \left( \frac{4d_1^2}{(x-1)^5} - 4 d_1^2 \right) H(0, 1, 1; \alpha_0) +
\end{aligned}$$

$$\frac{13\pi^2}{24(x-1)} - \frac{\pi^2}{36(x-1)^2} - \frac{11\pi^2}{36(x-1)^3} + \frac{23\pi^2}{24(x-1)^4} + \frac{125\pi^2}{72(x-1)^5} - \frac{\zeta_3}{(x-1)^5} + 13\zeta_3 + \frac{175\pi^2}{72} \Big) + \frac{5d_1\pi^2}{16(x-1)} - \frac{71\pi^2}{48(x-1)} - \frac{5d_1\pi^2}{108(x-1)^2} + \frac{4\pi^2}{27(x-1)^2} - \frac{5d_1\pi^2}{108(x-1)^3} + \frac{\pi^2}{27(x-1)^3} + \frac{5d_1\pi^2}{16(x-1)^4} + \frac{17\pi^2}{48(x-1)^4} - \frac{61\pi^4}{180(x-1)^5} + \frac{205d_1\pi^2}{432(x-1)^5} - \frac{305\pi^2}{432(x-1)^5} - \frac{39\zeta_3}{4(x-1)} + \frac{13\zeta_3}{6(x-1)^2} + \frac{13\zeta_3}{6(x-1)^3} - \frac{39\zeta_3}{4(x-1)^4} - \frac{325\zeta_3}{12(x-1)^5} - \frac{325\zeta_3}{12} - \frac{49\pi^4}{432} + \frac{205d_1\pi^2}{432} - \frac{1145\pi^2}{432}.$$

## E. The $\mathcal{B}$ -type collinear integrals

### E.1 The $\mathcal{B}$ integral for $k = 0$ and $\delta = -1$

The  $\varepsilon$  expansion for this integral reads

$$\begin{aligned} \mathcal{I}(x, \varepsilon; \alpha_0, 3 + d_1\varepsilon; 1, 0, -1, g_B) &= x \mathcal{B}(\varepsilon, x; 3 + d_1\varepsilon; -1, 0) \\ &= \frac{1}{\varepsilon} b_{-1}^{(-1,0)} + b_0^{(-1,0)} + \varepsilon b_1^{(-1,0)} + \varepsilon^2 b_2^{(-1,0)} + \mathcal{O}(\varepsilon^3), \end{aligned} \quad (\text{E.1})$$

where

$$\begin{aligned} b_{-1}^{(-1,0)} &= -\frac{1}{2}, \\ b_0^{(-1,0)} &= \frac{\alpha_0^4}{4(x-1)} + \frac{\alpha_0^4}{4} - \frac{\alpha_0^3}{x-1} + \frac{\alpha_0^3}{3(x-1)^2} - \frac{4\alpha_0^3}{3} + \frac{3\alpha_0^2}{2(x-1)} - \frac{\alpha_0^2}{(x-1)^2} + \frac{\alpha_0^2}{2(x-1)^3} + 3\alpha_0^2 - \frac{\alpha_0}{x-1} + \frac{\alpha_0}{(x-1)^2} - \frac{\alpha_0}{(x-1)^3} + \frac{\alpha_0}{(x-1)^4} - 4\alpha_0 + \left(1 + \frac{1}{(x-1)^5}\right) H(0; \alpha_0) + \left(1 - \frac{1}{(x-1)^5}\right) H(0; x) + \frac{H(c_1(\alpha_0); x)}{(x-1)^5} - 1, \\ b_1^{(-1,0)} &= -\frac{d_1\alpha_0^4}{8} - \frac{d_1\alpha_0^4}{8(x-1)} + \frac{3\alpha_0^4}{4} + \frac{3\alpha_0^4}{4} + \frac{13d_1\alpha_0^3}{18} + \frac{d_1\alpha_0^3}{2(x-1)} - \frac{3\alpha_0^3}{x-1} - \frac{2d_1\alpha_0^3}{9(x-1)^2} + \frac{23\alpha_0^3}{18(x-1)^2} - \frac{77\alpha_0^3}{18} - \frac{23d_1\alpha_0^2}{12} - \frac{3d_1\alpha_0^2}{4(x-1)} + \frac{53\alpha_0^2}{12(x-1)} + \frac{2d_1\alpha_0^2}{3(x-1)^2} - \frac{47\alpha_0^2}{12(x-1)^2} - \frac{d_1\alpha_0^2}{2(x-1)^3} + \frac{31\alpha_0^2}{12(x-1)^3} + \frac{131\alpha_0^2}{12} + \frac{25d_1\alpha_0}{6} + \frac{d_1\alpha_0}{2(x-1)} - \frac{7\alpha_0}{3(x-1)} - \frac{2d_1\alpha_0}{3(x-1)^2} + \frac{4\alpha_0}{(x-1)^2} + \frac{d_1\alpha_0}{(x-1)^3} - \frac{17\alpha_0}{3(x-1)^3} - \frac{2d_1\alpha_0}{(x-1)^4} + \frac{49\alpha_0}{6(x-1)^4} - \frac{121\alpha_0}{6} + \left(-\frac{\alpha_0^4}{x-1} - \alpha_0^4 + \frac{4\alpha_0^3}{x-1} - \frac{4\alpha_0^3}{3(x-1)^2} + \frac{16\alpha_0^3}{3} - \frac{6\alpha_0^2}{x-1} + \frac{4\alpha_0^2}{(x-1)^2} - \frac{2\alpha_0^2}{(x-1)^3} - 12\alpha_0^2 + \frac{4\alpha_0}{x-1} - \frac{4\alpha_0}{(x-1)^2} + \frac{4\alpha_0}{(x-1)^3} - \frac{4\alpha_0}{(x-1)^4} + 16\alpha_0 - \frac{5}{2(x-1)} + \frac{5}{3(x-1)^2} - \frac{5}{3(x-1)^3} + \frac{5}{2(x-1)^4} + \frac{37}{6(x-1)^5} - \frac{13}{6}\right) H(0; \alpha_0) + \left(\frac{37}{6} + \frac{5}{2(x-1)} - \frac{5}{3(x-1)^2} + \frac{5}{3(x-1)^3} - \frac{5}{2(x-1)^4} - \frac{37}{6(x-1)^5}\right) H(0; x) + \left(-\frac{d_1\alpha_0^4}{2} - \frac{d_1\alpha_0^4}{2(x-1)} + \frac{8d_1\alpha_0^3}{3} + \frac{2d_1\alpha_0^3}{x-1} - \frac{2d_1\alpha_0^3}{3(x-1)^2} - 6d_1\alpha_0^2 - \frac{3d_1\alpha_0^2}{x-1} + \frac{2d_1\alpha_0^2}{(x-1)^2} - \frac{d_1\alpha_0^2}{(x-1)^3} + 8d_1\alpha_0 + \frac{2d_1\alpha_0}{x-1} - \frac{2d_1\alpha_0}{(x-1)^2} + \frac{2d_1\alpha_0}{(x-1)^3} - \frac{2d_1\alpha_0}{(x-1)^4} - \frac{25d_1}{6} - \frac{d_1}{2(x-1)} + \frac{2d_1}{3(x-1)^2} - \frac{d_1}{(x-1)^3} + \frac{2d_1}{(x-1)^4}\right) H(1; \alpha_0) + \left(\frac{2d_1}{(x-1)^5} - \frac{2}{(x-1)^5} + 2\right) H(0; \alpha_0) H(1; x) + \left(-\frac{\alpha_0^4}{2(x-1)} - \frac{\alpha_0^4}{2} + \frac{2\alpha_0^3}{x-1} - \frac{2\alpha_0^3}{3(x-1)^2} + \frac{8\alpha_0^3}{3} - \frac{3\alpha_0^2}{x-1} + \frac{2\alpha_0^2}{(x-1)^2} - \frac{\alpha_0^2}{(x-1)^3} - 6\alpha_0^2 + \frac{2\alpha_0}{x-1} - \frac{2\alpha_0}{(x-1)^2} + \frac{2\alpha_0}{(x-1)^3} - \frac{2\alpha_0}{(x-1)^4} + 8\alpha_0 - \frac{4H(0; \alpha_0)}{(x-1)^5} - \frac{2d_1H(1; \alpha_0)}{(x-1)^5} - \frac{5}{2(x-1)} + \frac{5}{3(x-1)^2} - \frac{5}{3(x-1)^3} + \frac{5}{2(x-1)^4} + \frac{37}{6(x-1)^5} - \frac{25}{6}\right) H(c_1(\alpha_0); x) + \left(-4 - \frac{4}{(x-1)^5}\right) H(0, 0; \alpha_0) + \left(\frac{4}{(x-1)^5} - 4\right) H(0, 0; x) + \left(-\frac{2d_1}{(x-1)^5} - 2d_1\right) H(0, 1; \alpha_0) + \left(2 - \frac{2}{(x-1)^5}\right) H(0, c_1(\alpha_0); x) + \left(-\frac{2d_1}{(x-1)^5} + \frac{2}{(x-1)^5} - 2\right) H(1, 0; x) + \left(\frac{2d_1}{(x-1)^5} - \frac{2}{(x-1)^5} + 2\right) H(1, c_1(\alpha_0); x) - \frac{2H(c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \frac{\pi^2}{3(x-1)^5} - \frac{\pi^2}{3} - 2, \\ b_2^{(-1,0)} &= \frac{d_1^2\alpha_0^4}{16} - \frac{d_1\alpha_0^4}{2} + \frac{d_1^2\alpha_0^4}{16(x-1)} - \frac{d_1\alpha_0^4}{2(x-1)} - \frac{\pi^2\alpha_0^4}{24(x-1)} + \frac{7\alpha_0^4}{4(x-1)} - \frac{\pi^2\alpha_0^4}{24} + \frac{7\alpha_0^4}{4} - \frac{43d_1^2\alpha_0^3}{108} + \frac{349d_1\alpha_0^3}{108} - \frac{d_1^2\alpha_0^3}{4(x-1)} + \frac{2d_1\alpha_0^3}{x-1} + \frac{\pi^2\alpha_0^3}{6(x-1)} - \frac{7\alpha_0^3}{x-1} + \frac{4d_1^2\alpha_0^3}{27(x-1)^2} - \frac{133d_1\alpha_0^3}{108(x-1)^2} - \frac{\pi^2\alpha_0^3}{18(x-1)^2} + \frac{191\alpha_0^3}{54(x-1)^2} + \frac{2\pi^2\alpha_0^3}{9} - \frac{569\alpha_0^3}{54} + \frac{95d_1^2\alpha_0^2}{72} - \frac{85d_1\alpha_0^2}{8} + \frac{3d_1^2\alpha_0^2}{8(x-1)} - \frac{203d_1\alpha_0^2}{72(x-1)} - \frac{\pi^2\alpha_0^2}{4(x-1)} + \frac{353\alpha_0^2}{36(x-1)} - \frac{4d_1^2\alpha_0^2}{9(x-1)^2} + \frac{31d_1\alpha_0^2}{8(x-1)^2} + \frac{\pi^2\alpha_0^2}{6(x-1)^2} - \frac{407\alpha_0^2}{36(x-1)^2} + \frac{d_1^2\alpha_0^2}{2(x-1)^3} - \frac{283d_1\alpha_0^2}{72(x-1)^3} - \frac{\pi^2\alpha_0^2}{12(x-1)^3} + \frac{331\alpha_0^2}{36(x-1)^3} - \frac{\pi^2\alpha_0^2}{2} + \frac{1091\alpha_0^2}{36} - \frac{205d_1^2\alpha_0}{36} + \frac{475d_1\alpha_0}{12} + \end{aligned}$$



$$\begin{aligned}
& \frac{2\alpha_0}{3(x-2)} - \frac{d_1^2\alpha_0}{4(x-1)} - \frac{d_1\alpha_0}{9(x-1)} + \frac{\pi^2\alpha_0}{6(x-1)} - \frac{13\alpha_0}{36(x-1)} + \frac{4d_1^2\alpha_0}{9(x-1)^2} - \frac{73d_1\alpha_0}{18(x-1)^2} - \frac{\pi^2\alpha_0}{6(x-1)^2} + \frac{35\alpha_0}{3(x-1)^2} - \frac{d_1^2\alpha_0}{(x-1)^3} + \\
& \frac{173d_1\alpha_0}{18(x-1)^3} + \frac{\pi^2\alpha_0}{6(x-1)^3} - \frac{875\alpha_0}{36(x-1)^3} + \frac{4d_1^2\alpha_0}{(x-1)^4} - \frac{937d_1\alpha_0}{36(x-1)^4} - \frac{\pi^2\alpha_0}{6(x-1)^4} + \frac{413\alpha_0}{9(x-1)^4} + \frac{2\pi^2\alpha_0}{9} - \frac{737\alpha_0}{9} + \left( \frac{d_1}{2}\alpha_0^4 + \right. \\
& \frac{d_1\alpha_0^4}{2(x-1)} - \frac{3\alpha_0^4}{x-1} - 3\alpha_0^4 - \frac{26d_1\alpha_0^3}{9} - \frac{2d_1\alpha_0^3}{x-1} + \frac{12\alpha_0^3}{x-1} + \frac{8d_1\alpha_0^3}{9(x-1)^2} - \frac{46\alpha_0^3}{9(x-1)^2} + \frac{154\alpha_0^3}{9} + \frac{23d_1\alpha_0^2}{3} + \frac{3d_1\alpha_0^2}{x-1} - \\
& \frac{53\alpha_0^2}{3(x-1)} - \frac{8d_1\alpha_0^2}{3(x-1)^2} + \frac{47\alpha_0^2}{3(x-1)^2} + \frac{2d_1\alpha_0^2}{(x-1)^3} - \frac{31\alpha_0^2}{3(x-1)^3} - \frac{131\alpha_0^2}{3} - \frac{50d_1\alpha_0}{3} - \frac{2d_1\alpha_0}{x-1} + \frac{28\alpha_0}{3(x-1)} + \frac{8d_1\alpha_0}{3(x-1)^2} - \\
& \frac{16\alpha_0}{(x-1)^2} - \frac{4d_1\alpha_0}{(x-1)^3} + \frac{68\alpha_0}{3(x-1)^3} + \frac{8d_1\alpha_0}{(x-1)^4} - \frac{98\alpha_0}{3(x-1)^4} + \frac{242\alpha_0}{3} + \frac{205d_1}{36} - \frac{4}{x-2} + \frac{17d_1}{4(x-1)} - \frac{53}{4(x-1)} + \frac{8}{3(x-2)^2} - \\
& \frac{13d_1}{9(x-1)^2} + \frac{317}{36(x-1)^2} + \frac{13d_1}{9(x-1)^3} - \frac{371}{36(x-1)^3} - \frac{17d_1}{4(x-1)^4} + \frac{193}{12(x-1)^4} - \frac{205d_1}{36(x-1)^5} - \frac{\pi^2}{6(x-1)^5} + \frac{266}{9(x-1)^5} - \\
& \frac{\pi^2}{6} - \frac{194}{9} \Big) H(0; \alpha_0) + \left( -\frac{17d_1}{4(x-1)} + \frac{13d_1}{9(x-1)^2} - \frac{13d_1}{9(x-1)^3} + \frac{17d_1}{4(x-1)^4} + \frac{205d_1}{36(x-1)^5} - \frac{205d_1}{36} - \frac{4}{x-2} + \frac{53}{4(x-1)} - \right. \\
& \frac{8}{3(x-2)^2} - \frac{317}{36(x-1)^2} + \frac{371}{36(x-1)^3} - \frac{193}{12(x-1)^4} - \frac{7\pi^2}{6(x-1)^5} - \frac{266}{9(x-1)^5} + \frac{3\pi^2}{2} + \frac{266}{9} \Big) H(0; x) + \left( \frac{d_1^2\alpha_0^4}{4} - \right. \\
& \frac{3d_1\alpha_0^4}{2} + \frac{d_1^2\alpha_0^4}{4(x-1)} - \frac{3d_1\alpha_0^4}{2(x-1)} - \frac{13d_1^2\alpha_0^3}{9} + \frac{77d_1\alpha_0^3}{9} - \frac{d_1^2\alpha_0^3}{x-1} + \frac{6d_1\alpha_0^3}{x-1} + \frac{4d_1^2\alpha_0^3}{9(x-1)^2} - \frac{23d_1\alpha_0^3}{9(x-1)^2} + \frac{23d_1^2\alpha_0^2}{6} - \frac{131d_1\alpha_0^2}{6} + \\
& \frac{3d_1^2\alpha_0^2}{2(x-1)} - \frac{53d_1\alpha_0^2}{6(x-1)} - \frac{4d_1^2\alpha_0^2}{3(x-1)^2} + \frac{47d_1\alpha_0^2}{6(x-1)^2} + \frac{d_1^2\alpha_0^2}{(x-1)^3} - \frac{31d_1\alpha_0^2}{6(x-1)^3} - \frac{25d_1^2\alpha_0}{3} + \frac{121d_1\alpha_0}{3} - \frac{d_1^2\alpha_0}{x-1} + \frac{14d_1\alpha_0}{3(x-1)} + \\
& \frac{4d_1^2\alpha_0}{3(x-1)^2} - \frac{8d_1\alpha_0}{(x-1)^2} - \frac{2d_1^2\alpha_0}{(x-1)^3} + \frac{34d_1\alpha_0}{3(x-1)^3} + \frac{4d_1^2\alpha_0}{(x-1)^4} - \frac{49d_1\alpha_0}{3(x-1)^4} + \frac{205d_1^2}{36} - \frac{230d_1}{9} + \frac{d_1^2}{4(x-1)} - \frac{d_1}{3(x-1)} - \frac{4d_1^2}{9(x-1)^2} + \\
& \frac{49d_1}{18(x-1)^2} + \frac{d_1^2}{(x-1)^3} - \frac{37d_1}{6(x-1)^3} - \frac{4d_1^2}{(x-1)^4} + \frac{49d_1}{3(x-1)^4} \Big) H(1; \alpha_0) + \left( \frac{\pi^2}{2(x-1)^5} - \frac{\pi^2}{2} \right) H(2; x) + \left( \frac{4\alpha_0^4}{x-1} + \right. \\
& 4\alpha_0^4 - \frac{16\alpha_0^3}{x-1} + \frac{16\alpha_0^3}{3(x-1)^2} - \frac{64\alpha_0^3}{3} + \frac{24\alpha_0^3}{x-1} - \frac{16\alpha_0^2}{(x-1)^2} + \frac{8\alpha_0^2}{(x-1)^3} + 48\alpha_0^2 - \frac{16\alpha_0}{x-1} + \frac{16\alpha_0}{(x-1)^2} - \frac{16\alpha_0}{(x-1)^3} + \frac{16\alpha_0}{(x-1)^4} - \\
& 64\alpha_0 + \frac{10}{x-1} - \frac{20}{3(x-1)^2} + \frac{20}{3(x-1)^3} - \frac{10}{(x-1)^4} - \frac{74}{3(x-1)^5} + \frac{26}{3} \Big) H(0, 0; \alpha_0) + \left( -\frac{74}{3} - \frac{10}{x-1} + \frac{20}{3(x-1)^2} - \right. \\
& \frac{20}{3(x-1)^3} + \frac{10}{(x-1)^4} + \frac{74}{3(x-1)^5} \Big) H(0, 0; x) + \left( 2d_1\alpha_0^4 + \frac{2d_1\alpha_0^4}{x-1} - \frac{32d_1\alpha_0^3}{3} - \frac{8d_1\alpha_0^3}{x-1} + \frac{8d_1\alpha_0^3}{3(x-1)^2} + 24d_1\alpha_0^2 + \right. \\
& \frac{12d_1\alpha_0^2}{x-1} - \frac{8d_1\alpha_0^2}{(x-1)^2} + \frac{4d_1\alpha_0^2}{(x-1)^3} - 32d_1\alpha_0 - \frac{8d_1\alpha_0}{x-1} + \frac{8d_1\alpha_0}{(x-1)^2} - \frac{8d_1\alpha_0}{(x-1)^3} + \frac{8d_1\alpha_0}{(x-1)^4} + \frac{13d_1}{3} + \frac{5d_1}{x-1} - \frac{10d_1}{3(x-1)^2} + \\
& \frac{10d_1}{3(x-1)^3} - \frac{5d_1}{(x-1)^4} - \frac{37d_1}{3(x-1)^5} \Big) H(0, 1; \alpha_0) + H(1; x) \left( \frac{2\pi^2 d_1}{3(x-1)^5} + \left( -\frac{4d_1}{x-1} + \frac{2d_1}{(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \right. \right. \\
& \frac{d_1}{(x-1)^4} + \frac{37d_1}{3(x-1)^5} + \frac{4}{x-2} + \frac{5}{2(x-1)} - \frac{8}{3(x-2)^2} - \frac{3}{(x-1)^2} + \frac{8}{3(x-2)^3} + \frac{5}{3(x-1)^3} - \frac{13}{2(x-1)^4} - \frac{37}{3(x-1)^5} + \\
& \left. \frac{37}{3} \right) H(0; \alpha_0) + \left( -\frac{8d_1}{(x-1)^5} + \frac{8}{(x-1)^5} - 8 \right) H(0, 0; \alpha_0) + \left( -\frac{4d_1^2}{(x-1)^5} + \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(0, 1; \alpha_0) - \\
& \frac{2\pi^2}{3(x-1)^5} + \frac{\pi^2}{3} + \left( -\frac{4d_1}{(x-1)^5} + 4d_1 + \frac{8}{(x-1)^5} - 8 \right) H(0; \alpha_0) H(0, 1; x) + \left( \left( \frac{8}{(x-1)^5} - 8 \right) H(0; \alpha_0) + \right. \\
& \left. \left( \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(1; \alpha_0) + \frac{4}{x-2} + \frac{5}{2(x-1)} - \frac{8}{3(x-2)^2} - \frac{3}{(x-1)^2} + \frac{8}{3(x-2)^3} + \frac{5}{3(x-1)^3} - \frac{13}{2(x-1)^4} - \right. \\
& \left. \frac{37}{3(x-1)^5} + \frac{37}{3} \right) H(0, c_1(\alpha_0); x) + \left( 2d_1\alpha_0^4 + \frac{2d_1\alpha_0^4}{x-1} - \frac{32d_1\alpha_0^3}{3} - \frac{8d_1\alpha_0^3}{x-1} + \frac{8d_1\alpha_0^3}{3(x-1)^2} + 24d_1\alpha_0^2 + \frac{12d_1\alpha_0^2}{x-1} - \right. \\
& \frac{8d_1\alpha_0^2}{(x-1)^2} + \frac{4d_1\alpha_0^2}{(x-1)^3} - 32d_1\alpha_0 - \frac{8d_1\alpha_0}{x-1} + \frac{8d_1\alpha_0}{(x-1)^2} - \frac{8d_1\alpha_0}{(x-1)^3} + \frac{8d_1\alpha_0}{(x-1)^4} + \frac{50d_1}{3} + \frac{2d_1}{x-1} - \frac{8d_1}{3(x-1)^2} + \frac{4d_1}{(x-1)^3} - \\
& \left. \frac{8d_1}{(x-1)^4} \right) H(1, 0; \alpha_0) + \left( \frac{4d_1}{x-1} - \frac{2d_1}{(x-1)^2} + \frac{4d_1}{3(x-1)^3} - \frac{d_1}{(x-1)^4} - \frac{37d_1}{3(x-1)^5} - \frac{4}{x-2} - \frac{5}{2(x-1)} + \frac{8}{3(x-2)^2} + \right. \\
& \frac{3}{(x-1)^2} - \frac{8}{3(x-2)^3} - \frac{5}{3(x-1)^3} + \frac{13}{2(x-1)^4} + \frac{37}{3(x-1)^5} - \frac{37}{3} \Big) H(1, 0; x) + \left( d_1^2\alpha_0^4 + \frac{d_1^2\alpha_0^4}{x-1} - \frac{16d_1^2\alpha_0^3}{3} - \frac{4d_1^2\alpha_0^3}{x-1} + \right. \\
& \frac{4d_1^2\alpha_0^3}{3(x-1)^2} + 12d_1^2\alpha_0^2 + \frac{6d_1^2\alpha_0^2}{x-1} - \frac{4d_1^2\alpha_0^2}{(x-1)^2} + \frac{2d_1^2\alpha_0^2}{(x-1)^3} - 16d_1^2\alpha_0 - \frac{4d_1^2\alpha_0}{x-1} + \frac{4d_1^2\alpha_0}{(x-1)^2} - \frac{4d_1^2\alpha_0}{(x-1)^3} + \frac{4d_1^2\alpha_0}{(x-1)^4} + \frac{25d_1^2}{3} + \\
& \frac{d_1^2}{x-1} - \frac{4d_1^2}{3(x-1)^2} + \frac{2d_1^2}{(x-1)^3} - \frac{4d_1^2}{(x-1)^4} \Big) H(1, 1; \alpha_0) + H(c_1(\alpha_0); x) \left( \frac{d_1\alpha_0^4}{4} + \frac{d_1\alpha_0^4}{4(x-1)} - \frac{3\alpha_0^4}{2(x-1)} - \frac{3\alpha_0^4}{2} - \right. \\
& \frac{13d_1\alpha_0^3}{9} - \frac{d_1\alpha_0^3}{x-1} + \frac{6\alpha_0^3}{x-1} + \frac{4d_1\alpha_0^3}{9(x-1)^2} - \frac{23\alpha_0^3}{9(x-1)^2} + \frac{77\alpha_0^3}{9} + \frac{23d_1\alpha_0^2}{6} - \frac{\alpha_0^2}{3(x-2)} + \frac{3d_1\alpha_0^2}{2(x-1)} - \frac{103\alpha_0^2}{12(x-1)} - \frac{4d_1\alpha_0^2}{3(x-1)^2} + \\
& \frac{95\alpha_0^2}{12(x-1)^2} + \frac{d_1\alpha_0^2}{(x-1)^3} - \frac{31\alpha_0^2}{6(x-1)^3} - \frac{131\alpha_0^2}{6} - \frac{25d_1\alpha_0}{3} + \frac{2\alpha_0}{x-2} - \frac{d_1\alpha_0}{x-1} + \frac{10\alpha_0}{3(x-1)} - \frac{4\alpha_0}{3(x-2)^2} + \frac{4d_1\alpha_0}{3(x-1)^2} - \\
& \frac{15\alpha_0}{2(x-1)^2} - \frac{2d_1\alpha_0}{(x-1)^3} + \frac{71\alpha_0}{6(x-1)^3} + \frac{4d_1\alpha_0}{(x-1)^4} - \frac{49\alpha_0}{3(x-1)^4} + \frac{121\alpha_0}{3} + \frac{205d_1}{36} + \left( \frac{2\alpha_0^4}{x-1} + 2\alpha_0^4 - \frac{8\alpha_0^3}{x-1} + \frac{8\alpha_0^3}{3(x-1)^2} - \right. \\
& \left. \frac{32\alpha_0^3}{3} + \frac{12\alpha_0^3}{x-1} - \frac{8\alpha_0^2}{(x-1)^2} + \frac{4\alpha_0^2}{(x-1)^3} + 24\alpha_0^2 - \frac{8\alpha_0}{x-1} + \frac{8\alpha_0}{(x-1)^2} - \frac{8\alpha_0}{(x-1)^3} + \frac{8\alpha_0}{(x-1)^4} - 32\alpha_0 + \frac{10}{x-1} - \frac{20}{3(x-1)^2} + \right. \\
& \left. \frac{20}{3(x-1)^3} - \frac{10}{(x-1)^4} - \frac{74}{3(x-1)^5} + \frac{50}{3} \right) H(0; \alpha_0) + \left( d_1\alpha_0^4 + \frac{d_1\alpha_0^4}{x-1} - \frac{16d_1\alpha_0^3}{3} - \frac{4d_1\alpha_0^3}{x-1} + \frac{4d_1\alpha_0^3}{3(x-1)^2} + 12d_1\alpha_0^2 + \right.
\end{aligned}$$

$$\begin{aligned}
 & \frac{6d_1\alpha_0^2}{x-1} - \frac{4d_1\alpha_0^2}{(x-1)^2} + \frac{2d_1\alpha_0^2}{(x-1)^3} - 16d_1\alpha_0 - \frac{4d_1\alpha_0}{x-1} + \frac{4d_1\alpha_0}{(x-1)^2} - \frac{4d_1\alpha_0}{(x-1)^3} + \frac{4d_1\alpha_0}{(x-1)^4} + \frac{25d_1}{3} + \frac{5d_1}{x-1} - \frac{10d_1}{3(x-1)^2} + \\
 & \frac{10d_1}{3(x-1)^3} - \frac{5d_1}{(x-1)^4} - \frac{37d_1}{3(x-1)^5} \Big) H(1; \alpha_0) + \frac{16H(0,0;\alpha_0)}{(x-1)^5} + \frac{8d_1H(0,1;\alpha_0)}{(x-1)^5} + \frac{8d_1H(1,0;\alpha_0)}{(x-1)^5} + \frac{4d_1^2H(1,1;\alpha_0)}{(x-1)^5} - \\
 & \frac{4}{x-2} + \frac{17d_1}{4(x-1)} - \frac{53}{4(x-1)} + \frac{8}{3(x-2)^2} - \frac{13d_1}{9(x-1)^2} + \frac{317}{36(x-1)^2} + \frac{13d_1}{9(x-1)^3} - \frac{371}{36(x-1)^3} - \frac{17d_1}{4(x-1)^4} + \frac{193}{12(x-1)^4} - \\
 & \frac{205d_1}{36(x-1)^5} - \frac{\pi^2}{6(x-1)^5} + \frac{266}{9(x-1)^5} - \frac{230}{9} \Big) + \left( \frac{4d_1^2}{(x-1)^5} - \frac{8d_1}{(x-1)^5} + 4d_1 + \frac{4}{(x-1)^5} - 2 \right) H(0; \alpha_0)H(1, 1; x) + \\
 & \left( -\frac{4d_1}{x-1} + \frac{2d_1}{(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \frac{d_1}{(x-1)^4} + \frac{37d_1}{3(x-1)^5} + \left( -\frac{8d_1}{(x-1)^5} + \frac{8}{(x-1)^5} - 8 \right) H(0; \alpha_0) + \left( -\frac{4d_1^2}{(x-1)^5} + \right. \right. \\
 & \left. \left. \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(1; \alpha_0) + \frac{4}{x-2} + \frac{5}{2(x-1)} - \frac{8}{3(x-2)^2} - \frac{3}{(x-1)^2} + \frac{8}{3(x-2)^3} + \frac{5}{3(x-1)^3} - \frac{13}{2(x-1)^4} - \right. \\
 & \left. \frac{37}{3(x-1)^5} + \frac{37}{3} \right) H(1, c_1(\alpha_0); x) + \left( 2 - \frac{2}{(x-1)^5} \right) H(0; \alpha_0)H(2, 1; x) + \left( \frac{\alpha_0^4}{x-1} + \frac{3\alpha_0^4}{2} - \frac{4\alpha_0^3}{x-1} + \frac{4\alpha_0^3}{3(x-1)^2} - \right. \\
 & \left. 8\alpha_0^3 + \frac{6\alpha_0^2}{x-1} - \frac{4\alpha_0^2}{(x-1)^2} + \frac{2\alpha_0^2}{(x-1)^3} + 18\alpha_0^2 - \frac{4\alpha_0}{x-1} + \frac{4\alpha_0}{(x-1)^2} - \frac{4\alpha_0}{(x-1)^3} + \frac{4\alpha_0}{(x-1)^4} - 24\alpha_0 + \frac{8H(0;\alpha_0)}{(x-1)^5} + \right. \\
 & \left. \frac{4d_1H(1;\alpha_0)}{(x-1)^5} + \frac{7}{x-1} - \frac{13}{3(x-1)^2} + \frac{4}{(x-1)^3} - \frac{11}{2(x-1)^4} - \frac{37}{3(x-1)^5} + \frac{25}{2} \right) H(c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{\alpha_0^4}{2(x-1)} - \right. \\
 & \left. \frac{\alpha_0^4}{2} - \frac{2\alpha_0^3}{x-1} + \frac{2\alpha_0^3}{3(x-1)^2} + \frac{8\alpha_0^3}{3} + \frac{3\alpha_0^2}{x-1} - \frac{2\alpha_0^2}{(x-1)^2} + \frac{\alpha_0^2}{(x-1)^3} - 6\alpha_0^2 - \frac{2\alpha_0}{x-1} + \frac{2\alpha_0}{(x-1)^2} - \frac{2\alpha_0}{(x-1)^3} + \frac{2\alpha_0}{(x-1)^4} + 8\alpha_0 - \right. \\
 & \left. \frac{4}{x-2} + \frac{1}{2(x-1)} + \frac{8}{3(x-2)^2} + \frac{2}{3(x-1)^2} - \frac{8}{3(x-2)^3} + \frac{1}{(x-1)^3} + \frac{2}{(x-1)^4} - \frac{25}{6} \right) H(c_2(\alpha_0), c_1(\alpha_0); x) + \left( 16 + \right. \\
 & \left. \frac{16}{(x-1)^5} \right) H(0, 0, 0; \alpha_0) + \left( 16 - \frac{16}{(x-1)^5} \right) H(0, 0, 0; x) + \left( \frac{8d_1}{(x-1)^5} + 8d_1 \right) H(0, 0, 1; \alpha_0) + \left( \frac{8}{(x-1)^5} - \right. \\
 & \left. 8 \right) H(0, 0, c_1(\alpha_0); x) + \left( \frac{8d_1}{(x-1)^5} + 8d_1 \right) H(0, 1, 0; \alpha_0) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 - \frac{8}{(x-1)^5} + 8 \right) H(0, 1, 0; x) + \\
 & \left( \frac{4d_1^2}{(x-1)^5} + 4d_1^2 \right) H(0, 1, 1; \alpha_0) + \left( -\frac{4d_1}{(x-1)^5} + 4d_1 + \frac{8}{(x-1)^5} - 8 \right) H(0, 1, c_1(\alpha_0); x) + \left( \frac{4}{(x-1)^5} - \right. \\
 & \left. 6 \right) H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( 2 - \frac{2}{(x-1)^5} \right) H(0, c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{8d_1}{(x-1)^5} - \frac{8}{(x-1)^5} + \right. \\
 & \left. 8 \right) H(1, 0, 0; x) + \left( -\frac{4d_1}{(x-1)^5} + \frac{4}{(x-1)^5} - 2 \right) H(1, 0, c_1(\alpha_0); x) + \left( -\frac{4d_1^2}{(x-1)^5} + \frac{8d_1}{(x-1)^5} - 4d_1 - \right. \\
 & \left. \frac{4}{(x-1)^5} + 2 \right) H(1, 1, 0; x) + \left( \frac{4d_1^2}{(x-1)^5} - \frac{8d_1}{(x-1)^5} + 4d_1 + \frac{4}{(x-1)^5} - 2 \right) H(1, 1, c_1(\alpha_0); x) + \left( -\frac{4d_1}{(x-1)^5} + \frac{4}{(x-1)^5} - 6 \right) \\
 & H(1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( 2 - \frac{2}{(x-1)^5} \right) H(2, 0, c_1(\alpha_0); x) + \left( \frac{2}{(x-1)^5} - 2 \right) H(2, c_2(\alpha_0), c_1(\alpha_0); x) + \\
 & \frac{4}{(x-1)^5} H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \frac{2}{(x-1)^5} H(c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x) - \frac{\pi^2}{x-2} - \frac{3\pi^2}{8(x-1)} + \frac{2\pi^2}{3(x-2)^2} + \frac{5\pi^2}{9(x-1)^2} - \\
 & \frac{2\pi^2}{3(x-2)^3} - \frac{7\pi^2}{36(x-1)^3} + \frac{5\pi^2}{4(x-1)^4} + \frac{37\pi^2}{18(x-1)^5} - \frac{21\zeta_3}{4(x-1)^5} + \frac{17\zeta_3}{4} + \frac{\pi^2 \ln 2}{2(x-1)^5} - \frac{1}{2}\pi^2 \ln 2 - \frac{173\pi^2}{72} - 4.
 \end{aligned}$$

## E.2 The $\mathcal{B}$ integral for $k = 1$ and $\delta = -1$

The  $\varepsilon$  expansion for this integral reads

$$\begin{aligned}
 \mathcal{I}(x, \varepsilon; \alpha_0, 3 + d_1\varepsilon; 1, 1, -1, g_B) &= x \mathcal{B}(\varepsilon, x; 3 + d_1\varepsilon; -1, 1) \\
 &= \frac{1}{\varepsilon} b_{-1}^{(-1,1)} + b_0^{(-1,1)} + \varepsilon b_1^{(-1,1)} + \varepsilon^2 b_2^{(-1,1)} + \mathcal{O}(\varepsilon^3), \tag{E.2}
 \end{aligned}$$

where

$$\begin{aligned}
 b_{-1}^{(-1,1)} &= -\frac{1}{4}, \\
 b_0^{(-1,1)} &= \\
 & \frac{\alpha_0^4}{8(x-1)} + \frac{\alpha_0^4}{8} - \frac{\alpha_0^3}{2(x-1)} + \frac{\alpha_0^3}{6(x-1)^2} - \frac{2\alpha_0^3}{3} + \frac{3\alpha_0^2}{4(x-1)} - \frac{\alpha_0^2}{2(x-1)^2} + \frac{\alpha_0^2}{4(x-1)^3} + \frac{3\alpha_0^2}{2} - \frac{\alpha_0}{2(x-1)} + \frac{\alpha_0}{2(x-1)^2} - \\
 & \frac{\alpha_0}{2(x-1)^3} + \frac{\alpha_0}{2(x-1)^4} - 2\alpha_0 + \left( \frac{1}{2} + \frac{1}{2(x-1)^5} \right) H(0; \alpha_0) + \left( \frac{1}{2} - \frac{1}{2(x-1)^5} \right) H(0; x) + \frac{H(c_1(\alpha_0); x)}{2(x-1)^5} - \frac{1}{4},
 \end{aligned}$$

$$\begin{aligned}
 b_1^{(-1,1)} = & -\frac{d_1\alpha_0^4}{16} - \frac{d_1\alpha_0^4}{16(x-1)} + \frac{\alpha_0^4}{4(x-1)} + \frac{\alpha_0^4}{4} + \frac{13d_1\alpha_0^3}{36} + \frac{\alpha_0^3}{6(x-2)} + \frac{d_1\alpha_0^3}{4(x-1)} - \frac{13\alpha_0^3}{12(x-1)} - \frac{d_1\alpha_0^3}{9(x-1)^2} + \\
 & \frac{17\alpha_0^3}{36(x-1)^2} - \frac{53\alpha_0^3}{36} - \frac{23d_1\alpha_0^2}{24} - \frac{5\alpha_0^2}{6(x-2)} - \frac{3d_1\alpha_0^2}{8(x-1)} + \frac{23\alpha_0^2}{12(x-1)} + \frac{2\alpha_0^2}{3(x-2)^2} + \frac{d_1\alpha_0^2}{3(x-1)^2} - \frac{19\alpha_0^2}{12(x-1)^2} - \frac{d_1\alpha_0^2}{4(x-1)^3} + \\
 & \frac{25\alpha_0^2}{24(x-1)^3} + \frac{95\alpha_0^2}{24} + \frac{25d_1\alpha_0}{12} + \frac{7\alpha_0}{3(x-2)} + \frac{d_1\alpha_0}{4(x-1)} - \frac{9\alpha_0}{4(x-1)} - \frac{10\alpha_0}{3(x-2)^2} - \frac{d_1\alpha_0}{3(x-1)^2} + \frac{13\alpha_0}{6(x-1)^2} + \frac{4\alpha_0}{(x-2)^3} + \\
 & \frac{d_1\alpha_0}{2(x-1)^3} - \frac{31\alpha_0}{12(x-1)^3} - \frac{d_1\alpha_0}{(x-1)^4} + \frac{43\alpha_0}{12(x-1)^4} - \frac{97\alpha_0}{12} + \left( -\frac{\alpha_0^4}{2(x-1)} - \frac{\alpha_0^4}{2} + \frac{2\alpha_0^3}{x-1} - \frac{2\alpha_0^3}{3(x-1)^2} + \frac{8\alpha_0^3}{3} - \frac{3\alpha_0^2}{x-1} + \right. \\
 & \left. \frac{2\alpha_0^2}{(x-1)^2} - \frac{\alpha_0^2}{(x-1)^3} - 6\alpha_0^2 + \frac{2\alpha_0}{x-1} - \frac{2\alpha_0}{(x-1)^2} + \frac{2\alpha_0}{(x-1)^3} - \frac{2\alpha_0}{(x-1)^4} + 8\alpha_0 - \frac{4}{x-2} + \frac{5}{2(x-1)} + \frac{16}{3(x-2)^2} - \right. \\
 & \left. \frac{1}{12(x-1)^2} - \frac{8}{(x-2)^3} - \frac{5}{12(x-1)^3} + \frac{16}{(x-2)^4} + \frac{1}{(x-1)^4} + \frac{31}{12(x-1)^5} - \frac{19}{12} \right) H(0; \alpha_0) + \left( -\frac{5}{2(x-1)} + \right. \\
 & \left. \frac{1}{12(x-1)^2} + \frac{5}{12(x-1)^3} - \frac{1}{(x-1)^4} - \frac{31}{12(x-1)^5} + \frac{31}{12} + \frac{4}{x-2} - \frac{16}{3(x-2)^2} + \frac{8}{(x-2)^3} - \frac{16}{(x-2)^4} \right) H(0; x) + \left( -\frac{d_1\alpha_0^4}{4} - \frac{d_1\alpha_0^4}{4(x-1)} + \frac{4d_1\alpha_0^3}{3} + \frac{d_1\alpha_0^3}{3(x-1)^2} - \frac{d_1\alpha_0^3}{3(x-1)^2} - 3d_1\alpha_0^2 - \frac{3d_1\alpha_0^2}{2(x-1)} + \frac{d_1\alpha_0^2}{(x-1)^2} - \frac{d_1\alpha_0^2}{2(x-1)^3} + 4d_1\alpha_0 + \frac{d_1\alpha_0}{x-1} - \right. \\
 & \left. \frac{d_1\alpha_0}{(x-1)^2} + \frac{d_1\alpha_0}{(x-1)^3} - \frac{d_1\alpha_0}{(x-1)^4} - \frac{25d_1}{12} - \frac{d_1}{4(x-1)} + \frac{d_1}{3(x-1)^2} - \frac{d_1}{2(x-1)^3} + \frac{d_1}{(x-1)^4} \right) H(1; \alpha_0) + \left( \frac{d_1}{(x-1)^5} + \right. \\
 & \left. \frac{16}{(x-2)^5} - \frac{1}{(x-1)^5} + 1 \right) H(0; \alpha_0)H(1; x) + \left( -\frac{\alpha_0^4}{4(x-2)} - \frac{\alpha_0^4}{4(x-1)} - \frac{\alpha_0^4}{4} + \frac{\alpha_0^3}{x-2} + \frac{\alpha_0^3}{x-1} - \frac{2\alpha_0^3}{3(x-2)^2} - \right. \\
 & \left. \frac{\alpha_0^3}{3(x-1)^2} + \frac{4\alpha_0^3}{3} - \frac{3\alpha_0^2}{2(x-2)} - \frac{3\alpha_0^2}{2(x-1)} + \frac{2\alpha_0^2}{(x-2)^2} + \frac{\alpha_0^2}{(x-1)^2} - \frac{2\alpha_0^2}{(x-2)^3} - \frac{\alpha_0^2}{2(x-1)^3} - 3\alpha_0^2 + \frac{\alpha_0}{x-2} + \frac{\alpha_0}{x-1} - \frac{2\alpha_0}{(x-2)^2} - \right. \\
 & \left. \frac{\alpha_0}{(x-1)^2} + \frac{4\alpha_0}{(x-2)^3} + \frac{\alpha_0}{(x-1)^3} - \frac{8\alpha_0}{(x-2)^4} - \frac{\alpha_0}{(x-1)^4} + 4\alpha_0 - \frac{2H(0; \alpha_0)}{(x-1)^5} - \frac{d_1H(1; \alpha_0)}{(x-1)^5} - \frac{4}{x-2} + \frac{5}{2(x-1)} + \right. \\
 & \left. \frac{16}{3(x-2)^2} - \frac{1}{12(x-1)^2} - \frac{8}{(x-2)^3} - \frac{5}{12(x-1)^3} + \frac{16}{(x-2)^4} + \frac{1}{(x-1)^4} + \frac{31}{12(x-1)^5} - \frac{25}{12} \right) H(c_1(\alpha_0); x) + \left( -\right. \\
 & \left. 2 - \frac{2}{(x-1)^5} \right) H(0, 0; \alpha_0) + \left( \frac{2}{(x-1)^5} - 2 \right) H(0, 0; x) + \left( -\frac{d_1}{(x-1)^5} - d_1 \right) H(0, 1; \alpha_0) + \left( -\frac{1}{(x-1)^5} + \right. \\
 & \left. 1 + \frac{16}{(x-2)^5} \right) H(0, c_1(\alpha_0); x) + \left( -\frac{d_1}{(x-1)^5} - \frac{16}{(x-2)^5} + \frac{1}{(x-1)^5} - 1 \right) H(1, 0; x) + \left( \frac{d_1}{(x-1)^5} + \frac{16}{(x-2)^5} - \right. \\
 & \left. \frac{1}{(x-1)^5} + 1 \right) H(1, c_1(\alpha_0); x) - \frac{H(c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} - \frac{16H(c_2(\alpha_0), c_1(\alpha_0); x)}{(x-2)^5} - \frac{4\pi^2}{(x-2)^5} + \frac{\pi^2}{6(x-1)^5} - \frac{\pi^2}{6} - \frac{1}{4},
 \end{aligned}$$

$$\begin{aligned}
 b_2^{(-1,1)} = & \frac{d_1^2\alpha_0^4}{32} - \frac{3d_1\alpha_0^4}{16} + \frac{d_1^2\alpha_0^4}{32(x-1)} - \frac{3d_1\alpha_0^4}{16(x-1)} - \frac{\pi^2\alpha_0^4}{48(x-1)} + \frac{3\alpha_0^4}{8(x-1)} - \frac{\pi^2\alpha_0^4}{48} + \frac{3\alpha_0^4}{8} - \frac{43d_1^2\alpha_0^3}{216} + \frac{271d_1\alpha_0^3}{216} - \frac{7d_1\alpha_0^3}{36(x-2)} + \\
 & \frac{5\alpha_0^3}{9(x-2)} - \frac{d_1^2\alpha_0^3}{8(x-1)} + \frac{61d_1\alpha_0^3}{72(x-1)} + \frac{\pi^2\alpha_0^3}{12(x-1)} - \frac{16\alpha_0^3}{9(x-1)} + \frac{2d_1^2\alpha_0^3}{27(x-1)^2} - \frac{109d_1\alpha_0^3}{216(x-1)^2} - \frac{\pi^2\alpha_0^3}{36(x-1)^2} + \frac{26\alpha_0^3}{27(x-1)^2} + \frac{\pi^2\alpha_0^3}{9} - \\
 & \frac{133\alpha_0^3}{54} + \frac{95d_1^2\alpha_0^2}{144} - \frac{209d_1\alpha_0^2}{48} + \frac{41d_1\alpha_0^2}{36(x-2)} - \frac{28\alpha_0^2}{9(x-2)} + \frac{3d_1^2\alpha_0^2}{16(x-1)} - \frac{61d_1\alpha_0^2}{36(x-1)} - \frac{\pi^2\alpha_0^2}{8(x-1)} + \frac{253\alpha_0^2}{72(x-1)} - \frac{10d_1\alpha_0^2}{9(x-2)^2} + \\
 & \frac{26\alpha_0^2}{9(x-2)^2} - \frac{2d_1^2\alpha_0^2}{9(x-1)^2} + \frac{43d_1\alpha_0^2}{24(x-1)^2} + \frac{\pi^2\alpha_0^2}{12(x-1)^2} - \frac{281\alpha_0^2}{72(x-1)^2} + \frac{d_1^2\alpha_0^2}{4(x-1)^3} - \frac{247d_1\alpha_0^2}{144(x-1)^3} - \frac{\pi^2\alpha_0^2}{24(x-1)^3} + \frac{55\alpha_0^2}{18(x-1)^3} - \\
 & \frac{\pi^2\alpha_0^2}{4} + \frac{295\alpha_0^2}{36} - \frac{205d_1^2\alpha_0}{72} + \frac{425d_1\alpha_0}{24} - \frac{97d_1\alpha_0}{18(x-2)} + \frac{221\alpha_0}{18(x-2)} - \frac{d_1^2\alpha_0}{8(x-1)} + \frac{91d_1\alpha_0}{24(x-1)} + \frac{\pi^2\alpha_0}{12(x-1)} - \frac{17\alpha_0}{3(x-1)} + \\
 & \frac{74d_1\alpha_0}{9(x-2)^2} - \frac{178\alpha_0}{9(x-2)^2} + \frac{2d_1^2\alpha_0}{9(x-1)^2} - \frac{29d_1\alpha_0}{9(x-1)^2} - \frac{\pi^2\alpha_0}{12(x-1)^2} + \frac{145\alpha_0}{18(x-1)^2} - \frac{12d_1\alpha_0}{(x-2)^3} + \frac{28\alpha_0}{(x-2)^3} - \frac{d_1^2\alpha_0}{2(x-1)^3} + \\
 & \frac{355d_1\alpha_0}{72(x-1)^3} + \frac{\pi^2\alpha_0}{12(x-1)^3} - \frac{409\alpha_0}{36(x-1)^3} + \frac{2d_1^2\alpha_0}{(x-1)^4} - \frac{865d_1\alpha_0}{72(x-1)^4} - \frac{\pi^2\alpha_0}{12(x-1)^4} + \frac{661\alpha_0}{36(x-1)^4} + \frac{\pi^2\alpha_0}{3} - \frac{1039\alpha_0}{36} + \left( \frac{d_1\alpha_0^4}{4} + \right. \\
 & \left. \frac{d_1\alpha_0^4}{4(x-1)} - \frac{\alpha_0^4}{x-1} - \alpha_0^4 - \frac{13d_1\alpha_0^3}{9} - \frac{2\alpha_0^3}{3(x-2)} - \frac{d_1\alpha_0^3}{x-1} + \frac{13\alpha_0^3}{3(x-1)} + \frac{4d_1\alpha_0^3}{9(x-1)^2} - \frac{17\alpha_0^3}{9(x-1)^2} + \frac{53\alpha_0^3}{9} + \frac{23d_1\alpha_0^2}{6} + \frac{10\alpha_0^2}{3(x-2)} + \right. \\
 & \left. \frac{3d_1\alpha_0^2}{2(x-1)} - \frac{23\alpha_0^2}{3(x-1)} - \frac{8\alpha_0^2}{3(x-2)^2} - \frac{4d_1\alpha_0^2}{3(x-1)^2} + \frac{19\alpha_0^2}{3(x-1)^2} + \frac{d_1\alpha_0^2}{(x-1)^3} - \frac{25\alpha_0^2}{6(x-1)^3} - \frac{95\alpha_0^2}{6} - \frac{25d_1\alpha_0}{3} - \frac{28\alpha_0}{3(x-2)} - \frac{d_1\alpha_0}{x-1} - \right. \\
 & \left. \frac{9\alpha_0}{x-1} + \frac{40\alpha_0}{3(x-2)^2} + \frac{4d_1\alpha_0}{3(x-1)^2} - \frac{26\alpha_0}{3(x-1)^2} - \frac{16\alpha_0}{(x-2)^3} - \frac{2d_1\alpha_0}{(x-1)^3} + \frac{31\alpha_0}{3(x-1)^3} + \frac{4d_1\alpha_0}{(x-1)^4} - \frac{43\alpha_0}{3(x-1)^4} + \frac{97\alpha_0}{3} + \frac{205d_1}{72} + \right. \\
 & \left. \frac{34d_1}{3(x-2)} - \frac{8}{x-2} - \frac{109d_1}{12(x-1)} - \frac{1}{3(x-1)} - \frac{116d_1}{9(x-2)^2} + \frac{112}{9(x-2)^2} + \frac{37d_1}{72(x-1)^2} + \frac{2}{9(x-1)^2} + \frac{16d_1}{(x-2)^3} - \frac{24}{(x-2)^3} + \right. \\
 & \left. \frac{29d_1}{72(x-1)^3} - \frac{31}{18(x-1)^3} - \frac{32d_1}{(x-2)^4} + \frac{80}{(x-2)^4} - \frac{2d_1}{(x-1)^4} + \frac{4}{(x-1)^4} - \frac{205d_1}{72(x-1)^5} - \frac{\pi^2}{12(x-1)^5} + \frac{403}{36(x-1)^5} - \frac{\pi^2}{12} - \right. \\
 & \left. \frac{367}{36} \right) H(0; \alpha_0) + \left( -\frac{34d_1}{3(x-2)} + \frac{109d_1}{12(x-1)} + \frac{116d_1}{9(x-2)^2} - \frac{37d_1}{72(x-1)^2} - \frac{16d_1}{(x-2)^3} - \frac{29d_1}{72(x-1)^3} + \frac{32d_1}{(x-2)^4} + \frac{2d_1}{(x-1)^4} + \right. \\
 & \left. \frac{205d_1}{72(x-1)^5} - \frac{205d_1}{72} + \frac{8}{x-2} + \frac{1}{3(x-1)} - \frac{112}{9(x-2)^2} - \frac{2}{9(x-1)^2} + \frac{24}{(x-2)^3} + \frac{31}{18(x-1)^3} - \frac{80}{(x-2)^4} - \frac{4}{(x-1)^4} + \right. \\
 & \left. \frac{16\pi^2}{(x-2)^5} - \frac{7\pi^2}{12(x-1)^5} - \frac{403}{36(x-1)^5} + \frac{3\pi^2}{4} + \frac{403}{36} \right) H(0; x) + \left( \frac{d_1^2\alpha_0^4}{8} - \frac{d_1\alpha_0^4}{2} + \frac{d_1^2\alpha_0^4}{8(x-1)} - \frac{d_1\alpha_0^4}{2(x-1)} - \frac{13d_1^2\alpha_0^3}{18} + \right.
 \end{aligned}$$

$$\begin{aligned}
& \frac{53d_1 \alpha_0^3}{18} - \frac{d_1 \alpha_0^3}{3(x-2)} - \frac{d_1^2 \alpha_0^3}{2(x-1)} + \frac{13d_1 \alpha_0^3}{6(x-1)} + \frac{2d_1^2 \alpha_0^3}{9(x-1)^2} - \frac{17d_1 \alpha_0^3}{18(x-1)^2} + \frac{23d_1^2 \alpha_0^2}{12} - \frac{95d_1 \alpha_0^2}{12} + \frac{5d_1 \alpha_0^2}{3(x-2)} + \frac{3d_1^2 \alpha_0^2}{4(x-1)} - \\
& \frac{23d_1 \alpha_0^2}{6(x-1)} - \frac{4d_1 \alpha_0^2}{3(x-2)^2} - \frac{2d_1^2 \alpha_0^2}{3(x-1)^2} + \frac{19d_1 \alpha_0^2}{6(x-1)^2} + \frac{d_1^2 \alpha_0^2}{2(x-1)^3} - \frac{25d_1 \alpha_0^2}{12(x-1)^3} - \frac{25d_1^2 \alpha_0}{6} + \frac{97d_1 \alpha_0}{6} - \frac{14d_1 \alpha_0}{3(x-2)} - \frac{d_1^2 \alpha_0}{2(x-1)} + \\
& \frac{9d_1 \alpha_0}{2(x-1)} + \frac{20d_1 \alpha_0}{3(x-2)^2} + \frac{2d_1^2 \alpha_0}{3(x-1)^2} - \frac{13d_1 \alpha_0}{3(x-1)^2} - \frac{8d_1 \alpha_0}{(x-2)^3} - \frac{d_1^2 \alpha_0}{(x-1)^3} + \frac{31d_1 \alpha_0}{6(x-1)^3} + \frac{2d_1^2 \alpha_0}{(x-1)^4} - \frac{43d_1 \alpha_0}{6(x-1)^4} + \frac{205d_1^2}{72} - \\
& \frac{385d_1}{36} + \frac{10d_1}{3(x-2)} + \frac{d_1^2}{8(x-1)} - \frac{7d_1}{3(x-1)} - \frac{16d_1}{3(x-2)^2} - \frac{2d_1^2}{9(x-1)^2} + \frac{19d_1}{9(x-1)^2} + \frac{8d_1}{(x-2)^3} + \frac{d_1^2}{2(x-1)^3} - \frac{37d_1}{12(x-1)^3} - \\
& \frac{2d_1^2}{(x-1)^4} + \frac{43d_1}{6(x-1)^4} \Big) H(1; \alpha_0) + \left( \frac{8\pi^2 d_1}{(x-2)^5} - \frac{16\pi^2}{(x-2)^5} + \frac{\pi^2}{4(x-1)^5} - \frac{\pi^2}{4} \right) H(2; x) + \left( \frac{2\alpha_0^4}{x-1} + 2\alpha_0^4 - \frac{8\alpha_0^3}{x-1} + \right. \\
& \left. \frac{8\alpha_0^3}{3(x-1)^2} - \frac{32\alpha_0^3}{3} + \frac{12\alpha_0^2}{x-1} - \frac{8\alpha_0^2}{(x-1)^2} + \frac{4\alpha_0^2}{(x-1)^3} + 24\alpha_0^2 - \frac{8\alpha_0}{x-1} + \frac{8\alpha_0}{(x-1)^2} - \frac{8\alpha_0}{(x-1)^3} + \frac{8\alpha_0}{(x-1)^4} - 32\alpha_0 + \frac{16}{x-2} - \right. \\
& \left. \frac{10}{x-1} - \frac{64}{3(x-2)^2} + \frac{1}{3(x-1)^2} + \frac{32}{(x-2)^3} + \frac{5}{3(x-1)^3} - \frac{64}{(x-2)^4} - \frac{4}{(x-1)^4} - \frac{31}{3(x-1)^5} + \frac{19}{3} \right) H(0, 0; \alpha_0) + \\
& \left( \frac{10}{x-1} - \frac{1}{3(x-1)^2} - \frac{5}{3(x-1)^3} + \frac{4}{(x-1)^4} + \frac{31}{3(x-1)^5} - \frac{31}{3} - \frac{16}{x-2} + \frac{64}{3(x-2)^2} - \frac{32}{(x-2)^3} + \frac{64}{(x-2)^4} \right) H(0, 0; x) + \\
& \left( d_1 \alpha_0^4 + \frac{d_1 \alpha_0^4}{x-1} - \frac{16d_1 \alpha_0^3}{3} - \frac{4d_1 \alpha_0^3}{x-1} + \frac{4d_1 \alpha_0^3}{3(x-1)^2} + 12d_1 \alpha_0^2 + \frac{6d_1 \alpha_0^2}{x-1} - \frac{4d_1 \alpha_0^2}{(x-1)^2} + \frac{2d_1 \alpha_0^2}{(x-1)^3} - 16d_1 \alpha_0 - \frac{4d_1 \alpha_0}{x-1} + \right. \\
& \left. \frac{4d_1 \alpha_0}{(x-1)^2} - \frac{4d_1 \alpha_0}{(x-1)^3} + \frac{4d_1 \alpha_0}{(x-1)^4} + \frac{19d_1}{6} + \frac{8d_1}{x-2} - \frac{5d_1}{x-1} - \frac{32d_1}{3(x-2)^2} + \frac{d_1}{6(x-1)^2} + \frac{16d_1}{(x-2)^3} + \frac{5d_1}{6(x-1)^3} - \frac{32d_1}{(x-2)^4} - \right. \\
& \left. \frac{2d_1}{(x-1)^4} - \frac{31d_1}{6(x-1)^5} \right) H(0, 1; \alpha_0) + H(1; x) \left( \frac{\pi^2 d_1}{3(x-1)^5} + \left( -\frac{15d_1}{2(x-2)} + \frac{11d_1}{2(x-1)} + \frac{28d_1}{3(x-2)^2} - \frac{5d_1}{6(x-1)^2} - \right. \right. \\
& \left. \left. \frac{12d_1}{(x-2)^3} + \frac{d_1}{6(x-1)^3} + \frac{16d_1}{(x-2)^4} + \frac{31d_1}{6(x-1)^5} - \frac{3}{2(x-2)} + \frac{5}{x-1} + \frac{8}{3(x-2)^2} - \frac{29}{12(x-1)^2} - \frac{8}{3(x-2)^3} + \frac{5}{4(x-1)^3} - \right. \right. \\
& \left. \left. \frac{7}{2(x-1)^4} + \frac{16}{(x-2)^5} - \frac{31}{6(x-1)^5} + \frac{31}{6} \right) H(0; \alpha_0) + \left( -\frac{4d_1}{(x-1)^5} - \frac{64}{(x-2)^5} + \frac{4}{(x-1)^5} - 4 \right) H(0, 0; \alpha_0) + \\
& \left( -\frac{2d_1^2}{(x-1)^5} - \frac{32d_1}{(x-2)^5} + \frac{2d_1}{(x-1)^5} - 2d_1 \right) H(0, 1; \alpha_0) - \frac{8\pi^2}{3(x-2)^5} - \frac{\pi^2}{3(x-1)^5} + \frac{\pi^2}{6} \Big) + \left( \frac{32d_1}{(x-2)^5} - \frac{2d_1}{(x-1)^5} + \right. \\
& \left. 2d_1 - \frac{32}{(x-2)^5} + \frac{4}{(x-1)^5} - 4 \right) H(0; \alpha_0) H(0, 1; x) + \left( -\frac{\alpha_0^4}{2(x-2)} + \frac{2\alpha_0^3}{x-2} - \frac{4\alpha_0^3}{3(x-2)^2} - \frac{3\alpha_0^2}{x-2} + \frac{4\alpha_0^2}{(x-2)^2} - \right. \\
& \left. \frac{4\alpha_0^2}{(x-2)^3} + \frac{2\alpha_0}{x-2} - \frac{4\alpha_0}{(x-2)^2} + \frac{8\alpha_0}{(x-2)^3} - \frac{16\alpha_0}{(x-2)^4} + \left( \frac{4}{(x-1)^5} - 4 - \frac{64}{(x-2)^5} \right) H(0; \alpha_0) + \left( -\frac{32d_1}{(x-2)^5} + \frac{2d_1}{(x-1)^5} - \right. \right. \\
& \left. \left. 2d_1 \right) H(1; \alpha_0) - \frac{2}{x-2} + \frac{5}{x-1} + \frac{4}{(x-2)^2} - \frac{29}{12(x-1)^2} - \frac{20}{3(x-2)^3} + \frac{5}{4(x-1)^3} + \frac{16}{(x-2)^4} - \frac{7}{2(x-1)^4} + \frac{16}{(x-2)^5} - \right. \\
& \left. \frac{31}{6(x-1)^5} + \frac{31}{6} \right) H(0, c_1(\alpha_0); x) + \left( d_1 \alpha_0^4 + \frac{d_1 \alpha_0^4}{x-1} - \frac{16d_1 \alpha_0^3}{3} - \frac{4d_1 \alpha_0^3}{x-1} + \frac{4d_1 \alpha_0^3}{3(x-1)^2} + 12d_1 \alpha_0^2 + \frac{6d_1 \alpha_0^2}{x-1} - \right. \\
& \left. \frac{4d_1 \alpha_0^2}{(x-1)^2} + \frac{2d_1 \alpha_0^2}{(x-1)^3} - 16d_1 \alpha_0 - \frac{4d_1 \alpha_0}{x-1} + \frac{4d_1 \alpha_0}{(x-1)^2} - \frac{4d_1 \alpha_0}{(x-1)^3} + \frac{4d_1 \alpha_0}{(x-1)^4} + \frac{25d_1}{3} + \frac{d_1}{x-1} - \frac{4d_1}{3(x-1)^2} + \frac{2d_1}{(x-1)^3} - \right. \\
& \left. \frac{4d_1}{(x-1)^4} \right) H(1, 0; \alpha_0) + \left( \frac{15d_1}{2(x-2)} - \frac{11d_1}{2(x-1)} - \frac{28d_1}{3(x-2)^2} + \frac{5d_1}{6(x-1)^2} + \frac{12d_1}{(x-2)^3} - \frac{d_1}{6(x-1)^3} - \frac{16d_1}{(x-2)^4} - \right. \\
& \left. \frac{31d_1}{6(x-1)^5} + \frac{3}{2(x-2)} - \frac{5}{x-1} - \frac{8}{3(x-2)^2} + \frac{29}{12(x-1)^2} + \frac{8}{3(x-2)^3} - \frac{5}{4(x-1)^3} + \frac{7}{2(x-1)^4} - \frac{16}{(x-2)^5} + \frac{31}{6(x-1)^5} - \right. \\
& \left. \frac{31}{6} \right) H(1, 0; x) + \left( \frac{d_1^2 \alpha_0^4}{2} + \frac{d_1^2 \alpha_0^4}{2(x-1)} - \frac{8d_1^2 \alpha_0^3}{3} - \frac{2d_1^2 \alpha_0^3}{x-1} + \frac{2d_1^2 \alpha_0^3}{3(x-1)^2} + 6d_1^2 \alpha_0^2 + \frac{3d_1^2 \alpha_0^2}{x-1} - \frac{2d_1^2 \alpha_0^2}{(x-1)^2} + \frac{d_1^2 \alpha_0^2}{(x-1)^3} - \right. \\
& \left. 8d_1^2 \alpha_0 - \frac{2d_1^2 \alpha_0}{x-1} + \frac{2d_1^2 \alpha_0}{(x-1)^2} - \frac{2d_1^2 \alpha_0}{(x-1)^3} + \frac{2d_1^2 \alpha_0}{(x-1)^4} + \frac{25d_1^2}{6} + \frac{d_1^2}{2(x-1)} - \frac{2d_1^2}{3(x-1)^2} + \frac{d_1^2}{(x-1)^3} - \frac{2d_1^2}{(x-1)^4} \right) H(1, 1; \alpha_0) + \\
& H(c_1(\alpha_0); x) \left( \frac{d_1 \alpha_0^4}{8} + \frac{d_1 \alpha_0^4}{8(x-2)} - \frac{\alpha_0^4}{2(x-2)} + \frac{d_1 \alpha_0^4}{8(x-1)} - \frac{\alpha_0^4}{2(x-1)} - \frac{\alpha_0^4}{2} - \frac{13d_1 \alpha_0^3}{18} - \frac{d_1 \alpha_0^3}{2(x-2)} + \frac{5\alpha_0^3}{3(x-2)} - \frac{d_1 \alpha_0^3}{2(x-1)} + \right. \\
& \left. \frac{29\alpha_0^3}{12(x-1)} + \frac{4d_1 \alpha_0^3}{9(x-2)^2} - \frac{14\alpha_0^3}{9(x-2)^2} + \frac{2d_1 \alpha_0^3}{9(x-1)^2} - \frac{17\alpha_0^3}{18(x-1)^2} + \frac{53\alpha_0^3}{18} + \frac{23d_1 \alpha_0^2}{12} + \frac{3d_1 \alpha_0^2}{4(x-2)} - \frac{5\alpha_0^2}{4(x-2)} + \frac{3d_1 \alpha_0^2}{4(x-1)} - \right. \\
& \left. \frac{61\alpha_0^2}{12(x-1)} - \frac{4d_1 \alpha_0^2}{3(x-2)^2} + \frac{10\alpha_0^2}{3(x-2)^2} - \frac{2d_1 \alpha_0^2}{3(x-1)^2} + \frac{43\alpha_0^2}{12(x-1)^2} + \frac{2d_1 \alpha_0^2}{(x-2)^3} - \frac{6\alpha_0^2}{(x-2)^3} + \frac{d_1 \alpha_0^2}{2(x-1)^3} - \frac{25\alpha_0^2}{12(x-1)^3} - \frac{95\alpha_0^2}{12} - \right. \\
& \left. \frac{25d_1 \alpha_0}{6} - \frac{d_1 \alpha_0}{2(x-2)} - \frac{25\alpha_0}{6(x-2)} - \frac{d_1 \alpha_0}{2(x-1)} + \frac{103\alpha_0}{12(x-1)} + \frac{4d_1 \alpha_0}{3(x-2)^2} + \frac{10\alpha_0}{3(x-2)^2} + \frac{2d_1 \alpha_0}{3(x-1)^2} - \frac{73\alpha_0}{12(x-1)^2} - \frac{4d_1 \alpha_0}{(x-2)^3} + \right. \\
& \left. \frac{4\alpha_0}{(x-2)^3} - \frac{d_1 \alpha_0}{(x-1)^3} + \frac{37\alpha_0}{6(x-1)^3} + \frac{16d_1 \alpha_0}{(x-2)^4} - \frac{40\alpha_0}{(x-2)^4} + \frac{2d_1 \alpha_0}{(x-1)^4} - \frac{43\alpha_0}{6(x-1)^4} + \frac{97\alpha_0}{6} + \frac{205d_1}{72} + \left( \frac{\alpha_0^4}{x-2} + \frac{\alpha_0^4}{x-1} + \alpha_0^4 - \right. \right. \\
& \left. \left. \frac{4\alpha_0^3}{x-2} - \frac{4\alpha_0^3}{x-1} + \frac{8\alpha_0^3}{3(x-2)^2} + \frac{4\alpha_0^3}{3(x-1)^2} - \frac{16\alpha_0^3}{3} + \frac{6\alpha_0^2}{x-2} + \frac{6\alpha_0^2}{x-1} - \frac{8\alpha_0^2}{(x-2)^2} - \frac{4\alpha_0^2}{(x-1)^2} + \frac{8\alpha_0^2}{(x-2)^3} + \frac{2\alpha_0^2}{(x-1)^3} + 12\alpha_0^2 - \right. \right. \\
& \left. \left. \frac{4\alpha_0}{x-2} - \frac{4\alpha_0}{x-1} + \frac{8\alpha_0}{(x-2)^2} + \frac{4\alpha_0}{(x-1)^2} - \frac{16\alpha_0}{(x-2)^3} - \frac{4\alpha_0}{(x-1)^3} + \frac{32\alpha_0}{(x-2)^4} + \frac{4\alpha_0}{(x-1)^4} - 16\alpha_0 + \frac{16}{x-2} - \frac{10}{x-1} - \frac{64}{3(x-2)^2} + \right. \right. \\
& \left. \left. \frac{1}{3(x-1)^2} + \frac{32}{(x-2)^3} + \frac{5}{3(x-1)^3} - \frac{64}{(x-2)^4} - \frac{4}{(x-1)^4} - \frac{31}{3(x-1)^5} + \frac{25}{3} \right) H(0; \alpha_0) + \left( \frac{d_1 \alpha_0^4}{2} + \frac{d_1 \alpha_0^4}{2(x-2)} + \right. \\
& \left. \frac{d_1 \alpha_0^4}{2(x-1)} - \frac{8d_1 \alpha_0^3}{3} - \frac{2d_1 \alpha_0^3}{x-2} - \frac{2d_1 \alpha_0^3}{x-1} + \frac{4d_1 \alpha_0^3}{3(x-2)^2} + \frac{2d_1 \alpha_0^3}{3(x-1)^2} + 6d_1 \alpha_0^2 + \frac{3d_1 \alpha_0^2}{x-2} + \frac{3d_1 \alpha_0^2}{x-1} - \frac{4d_1 \alpha_0^2}{(x-2)^2} - \frac{2d_1 \alpha_0^2}{(x-1)^2} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{4d_1 \alpha_0^2}{(x-2)^3} + \frac{d_1 \alpha_0^2}{(x-1)^3} - 8d_1 \alpha_0 - \frac{2d_1 \alpha_0}{x-2} - \frac{2d_1 \alpha_0}{x-1} + \frac{4d_1 \alpha_0}{(x-2)^2} + \frac{2d_1 \alpha_0}{(x-1)^2} - \frac{8d_1 \alpha_0}{(x-2)^3} - \frac{2d_1 \alpha_0}{(x-1)^3} + \frac{16d_1 \alpha_0}{(x-2)^4} + \frac{2d_1 \alpha_0}{(x-1)^4} + \\
& \frac{25d_1}{6} + \frac{8d_1}{x-2} - \frac{5d_1}{x-1} - \frac{32d_1}{3(x-2)^2} + \frac{d_1}{6(x-1)^2} + \frac{16d_1}{(x-2)^3} + \frac{5d_1}{6(x-1)^3} - \frac{32d_1}{(x-2)^4} - \frac{2d_1}{(x-1)^4} - \frac{31d_1}{6(x-1)^5} \Big) H(1; \alpha_0) + \\
& \frac{8}{(x-1)^5} H(0,0;\alpha_0) + \frac{4d_1 H(0,1;\alpha_0)}{(x-1)^5} + \frac{4d_1 H(1,0;\alpha_0)}{(x-1)^5} + \frac{2d_1^2 H(1,1;\alpha_0)}{(x-1)^5} + \frac{34d_1}{3(x-2)} - \frac{8}{x-2} - \frac{109d_1}{12(x-1)} - \frac{1}{3(x-1)} - \\
& \frac{116d_1}{9(x-2)^2} + \frac{112}{9(x-2)^2} + \frac{37d_1}{72(x-1)^2} + \frac{2}{9(x-1)^2} + \frac{16d_1}{(x-2)^3} - \frac{24}{(x-2)^3} + \frac{29d_1}{72(x-1)^3} - \frac{31}{18(x-1)^3} - \frac{32d_1}{(x-2)^4} + \frac{80}{(x-2)^4} - \\
& \frac{2d_1}{(x-1)^4} + \frac{4}{(x-1)^4} - \frac{205d_1}{72(x-1)^5} - \frac{\pi^2}{12(x-1)^5} + \frac{403}{36(x-1)^5} - \frac{385}{36} \Big) + \left( \frac{2d_1^2}{(x-1)^5} + \frac{32d_1}{(x-2)^5} - \frac{4d_1}{(x-1)^5} + 2d_1 + \right. \\
& \left. \frac{16}{(x-2)^5} + \frac{2}{(x-1)^5} - 1 \right) H(0; \alpha_0) H(1, 1; x) + \left( -\frac{15d_1}{2(x-2)} + \frac{11d_1}{2(x-1)} + \frac{28d_1}{3(x-2)^2} - \frac{5d_1}{6(x-1)^2} - \frac{12d_1}{(x-2)^3} + \right. \\
& \left. \frac{d_1}{6(x-1)^3} + \frac{16d_1}{(x-2)^4} + \frac{31d_1}{6(x-1)^5} + \left( -\frac{4d_1}{(x-1)^5} - \frac{64}{(x-2)^5} + \frac{4}{(x-1)^5} - 4 \right) H(0; \alpha_0) + \left( -\frac{2d_1^2}{(x-1)^5} - \frac{32d_1}{(x-2)^5} + \right. \right. \\
& \left. \left. \frac{2d_1}{(x-1)^5} - 2d_1 \right) H(1; \alpha_0) - \frac{3}{2(x-2)} + \frac{5}{x-1} + \frac{8}{3(x-2)^2} - \frac{29}{12(x-1)^2} - \frac{8}{3(x-2)^3} + \frac{5}{4(x-1)^3} - \frac{7}{2(x-1)^4} + \right. \\
& \left. \frac{16}{(x-2)^5} - \frac{31}{6(x-1)^5} + \frac{31}{6} \right) H(1, c_1(\alpha_0); x) + \left( -\frac{32d_1}{(x-2)^5} + \frac{64}{(x-2)^5} - \frac{1}{(x-1)^5} + 1 \right) H(0; \alpha_0) H(2, 1; x) + \\
& \left( \frac{5\alpha_0^4}{4(x-2)} + \frac{\alpha_0^4}{2(x-1)} + \frac{3\alpha_0^4}{4} - \frac{5\alpha_0^3}{x-2} - \frac{2\alpha_0^3}{x-1} + \frac{10\alpha_0^3}{3(x-2)^2} + \frac{2\alpha_0^3}{3(x-1)^2} - 4\alpha_0^3 + \frac{15\alpha_0^2}{2(x-2)} + \frac{3\alpha_0^2}{x-1} - \frac{10\alpha_0^2}{(x-2)^2} - \frac{2\alpha_0^2}{(x-1)^2} + \right. \\
& \left. \frac{10\alpha_0^2}{(x-2)^3} + \frac{\alpha_0^2}{(x-1)^3} + 9\alpha_0^2 - \frac{5\alpha_0}{x-2} - \frac{2\alpha_0}{x-1} + \frac{10\alpha_0}{(x-2)^2} + \frac{2\alpha_0}{(x-1)^2} - \frac{20\alpha_0}{(x-2)^3} - \frac{2\alpha_0}{(x-1)^3} + \frac{40\alpha_0}{(x-2)^4} + \frac{2\alpha_0}{(x-1)^4} - 12\alpha_0 + \right. \\
& \left. \frac{4H(0;\alpha_0)}{(x-1)^5} + \frac{2d_1 H(1;\alpha_0)}{(x-1)^5} + \frac{20}{x-2} - \frac{61}{4(x-1)} - \frac{80}{3(x-2)^2} + \frac{29}{12(x-1)^2} + \frac{40}{(x-2)^3} - \frac{1}{12(x-1)^3} - \frac{80}{(x-2)^4} - \frac{3}{2(x-1)^4} - \right. \\
& \left. \frac{31}{6(x-1)^5} + \frac{25}{4} \right) H(c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{\alpha_0^4}{4(x-1)} - \frac{\alpha_0^4}{4} - \frac{\alpha_0^3}{x-1} + \frac{\alpha_0^3}{3(x-1)^2} + \frac{4\alpha_0^3}{3} + \frac{3\alpha_0^2}{2(x-1)} - \frac{\alpha_0^2}{(x-1)^2} + \right. \\
& \left. \frac{\alpha_0^2}{2(x-1)^3} - 3\alpha_0^2 - \frac{\alpha_0}{x-1} + \frac{\alpha_0}{(x-1)^2} - \frac{\alpha_0}{(x-1)^3} + \frac{\alpha_0}{(x-1)^4} + 4\alpha_0 + \frac{64H(0;\alpha_0)}{(x-2)^5} + \frac{32d_1 H(1;\alpha_0)}{(x-2)^5} - \frac{2}{x-2} + \frac{1}{4(x-1)} + \right. \\
& \left. \frac{4}{3(x-2)^2} + \frac{1}{3(x-1)^2} - \frac{4}{3(x-2)^3} + \frac{1}{2(x-1)^3} + \frac{1}{(x-1)^4} - \frac{16}{(x-2)^5} - \frac{25}{12} \right) H(c_2(\alpha_0), c_1(\alpha_0); x) + \left( 8 + \right. \\
& \left. \frac{8}{(x-1)^5} \right) H(0, 0, 0; \alpha_0) + \left( 8 - \frac{8}{(x-1)^5} \right) H(0, 0, 0; x) + \left( \frac{4d_1}{(x-1)^5} + 4d_1 \right) H(0, 0, 1; \alpha_0) + \left( \frac{4}{(x-1)^5} - 4 - \right. \\
& \left. \frac{32}{(x-2)^5} \right) H(0, 0, c_1(\alpha_0); x) + \left( \frac{4d_1}{(x-1)^5} + 4d_1 \right) H(0, 1, 0; \alpha_0) + \left( -\frac{32d_1}{(x-2)^5} + \frac{2d_1}{(x-1)^5} - 2d_1 + \frac{32}{(x-2)^5} - \right. \\
& \left. \frac{4}{(x-1)^5} + 4 \right) H(0, 1, 0; x) + \left( \frac{2d_1^2}{(x-1)^5} + 2d_1^2 \right) H(0, 1, 1; \alpha_0) + \left( \frac{32d_1}{(x-2)^5} - \frac{2d_1}{(x-1)^5} + 2d_1 - \frac{32}{(x-2)^5} + \right. \\
& \left. \frac{4}{(x-1)^5} - 4 \right) H(0, 1, c_1(\alpha_0); x) + \left( \frac{2}{(x-1)^5} - 3 - \frac{80}{(x-2)^5} \right) H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{1}{(x-1)^5} + \right. \\
& \left. 1 + \frac{64}{(x-2)^5} \right) H(0, c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{4d_1}{(x-1)^5} + \frac{64}{(x-2)^5} - \frac{4}{(x-1)^5} + 4 \right) H(1, 0, 0; x) + \left( -\frac{2d_1}{(x-1)^5} + \right. \\
& \left. \frac{16}{(x-2)^5} + \frac{2}{(x-1)^5} - 1 \right) H(1, 0, c_1(\alpha_0); x) + \left( -\frac{2d_1^2}{(x-1)^5} - \frac{32d_1}{(x-2)^5} + \frac{4d_1}{(x-1)^5} - 2d_1 - \frac{16}{(x-2)^5} - \frac{2}{(x-1)^5} + \right. \\
& \left. 1 \right) H(1, 1, 0; x) + \left( \frac{2d_1^2}{(x-1)^5} + \frac{32d_1}{(x-2)^5} - \frac{4d_1}{(x-1)^5} + 2d_1 + \frac{16}{(x-2)^5} + \frac{2}{(x-1)^5} - 1 \right) H(1, 1, c_1(\alpha_0); x) + \\
& \left( -\frac{2d_1}{(x-1)^5} - \frac{80}{(x-2)^5} + \frac{2}{(x-1)^5} - 3 \right) H(1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{32d_1}{(x-2)^5} + \frac{64}{(x-2)^5} - \frac{1}{(x-1)^5} + \right. \\
& \left. 1 \right) H(2, 0, c_1(\alpha_0); x) + \left( \frac{32d_1}{(x-2)^5} - \frac{64}{(x-2)^5} + \frac{1}{(x-1)^5} - 1 \right) H(2, 1, 0; x) + \left( -\frac{32d_1}{(x-2)^5} + \frac{64}{(x-2)^5} - \right. \\
& \left. \frac{1}{(x-1)^5} + 1 \right) H(2, 1, c_1(\alpha_0); x) + \left( \frac{32d_1}{(x-2)^5} - \frac{64}{(x-2)^5} + \frac{1}{(x-1)^5} - 1 \right) H(2, c_2(\alpha_0), c_1(\alpha_0); x) + \\
& \frac{2}{(x-1)^5} H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \frac{H(c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} - \frac{32}{(x-2)^5} H(c_2(\alpha_0), 0, c_1(\alpha_0); x) + \\
& \frac{80}{3} \frac{H(c_2(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x)}{(x-2)^5} + \frac{\pi^2}{6(x-2)} - \frac{13\pi^2}{16(x-1)} - \frac{5\pi^2}{9(x-2)^2} + \frac{31\pi^2}{72(x-1)^2} + \frac{\pi^2}{(x-2)^3} - \frac{\pi^2}{6(x-1)^3} - \frac{8\pi^2}{3(x-2)^4} + \\
& \frac{2\pi^2}{3(x-1)^4} - \frac{4\pi^2}{(x-2)^5} + \frac{31\pi^2}{36(x-1)^5} + \frac{28\zeta_3}{(x-2)^5} - \frac{21\zeta_3}{8(x-1)^5} + \frac{17\zeta_3}{8} - \frac{24\pi^2 \ln 2}{(x-2)^5} + \frac{\pi^2 \ln 2}{4(x-1)^5} - \frac{1}{4}\pi^2 \ln 2 - \frac{149\pi^2}{144} - \frac{1}{4}
\end{aligned}$$

**E.3 The  $\mathcal{B}$  integral for  $k = 2$  and  $\delta = -1$  and  $d_1 = -3$** 

The  $\varepsilon$  expansion for this integral reads

$$\begin{aligned} \mathcal{I}(x, \varepsilon; \alpha_0, 3 + d_1\varepsilon; 1, 2, -1, g_B) &= x \mathcal{B}(\varepsilon, x; 3 + d_1\varepsilon; -1, 2) \\ &= \frac{1}{\varepsilon} b_{-1}^{(-1,2)} + b_0^{(-1,2)} + \varepsilon b_1^{(-1,2)} + \varepsilon^2 b_2^{(-1,2)} + \mathcal{O}(\varepsilon^3), \end{aligned} \quad (\text{E.3})$$

where

$$\begin{aligned} b_{-1}^{(-1,2)} &= -\frac{1}{6}, \\ b_0^{(-1,2)} &= -\frac{\alpha_0^{10}}{3(\alpha_0+1)^4(x\alpha_0-2\alpha_0-x)} + \frac{\alpha_0^9}{3(\alpha_0+1)^4(x\alpha_0-2\alpha_0-x)} + \frac{\alpha_0^8}{12(\alpha_0+1)^4(x-1)} + \\ &\quad \frac{4\alpha_0^8}{3(\alpha_0+1)^4(x\alpha_0-2\alpha_0-x)} - \frac{4\alpha_0^7}{3(\alpha_0+1)^4(x\alpha_0-2\alpha_0-x)} - \frac{\alpha_0^6}{3(\alpha_0+1)^4(x-1)} - \frac{2\alpha_0^6}{(\alpha_0+1)^4(x\alpha_0-2\alpha_0-x)} + \\ &\quad \frac{\alpha_0^6}{9(\alpha_0+1)^3(x-1)^2} + \frac{\alpha_0^5}{3(x-2)} + \frac{\alpha_0^5}{(\alpha_0+1)^4(x\alpha_0-2\alpha_0-x)} - \frac{5\alpha_0^4}{4(x-2)} + \frac{5\alpha_0^4}{12(\alpha_0+1)^4(x-1)} + \frac{4\alpha_0^4}{3(\alpha_0+1)^4(x\alpha_0-2\alpha_0-x)} + \\ &\quad \frac{5\alpha_0^4}{6(x-2)^2} - \frac{\alpha_0^4}{3(\alpha_0+1)^3(x-1)^2} + \frac{\alpha_0^4}{6(\alpha_0+1)^2(x-1)^3} + \frac{\alpha_0^4}{12} + \frac{5\alpha_0^3}{3(x-2)} - \frac{\alpha_0^3}{3(\alpha_0+1)^4(x-1)} - \frac{4\alpha_0^3}{3(\alpha_0+1)^4(x\alpha_0-2\alpha_0-x)} - \\ &\quad \frac{20\alpha_0^3}{9(x-2)^2} + \frac{\alpha_0^3}{9(\alpha_0+1)^3(x-1)^2} + \frac{20\alpha_0^3}{9(x-2)^3} - \frac{4\alpha_0^3}{9} - \frac{5\alpha_0^2}{6(x-2)} - \frac{5\alpha_0^2}{6(\alpha_0+1)^4(x-1)} - \frac{\alpha_0^2}{3(\alpha_0+1)^4(x\alpha_0-2\alpha_0-x)} + \\ &\quad \frac{5\alpha_0^2}{3(x-2)^2} + \frac{2\alpha_0^2}{3(\alpha_0+1)^3(x-1)^2} - \frac{10\alpha_0^2}{3(x-2)^3} - \frac{\alpha_0^2}{2(\alpha_0+1)^2(x-1)^3} + \frac{\alpha_0^2}{3(x-2)^4} + \frac{\alpha_0^2}{3(\alpha_0+1)(x-1)^4} + \alpha_0^2 - \\ &\quad \frac{\alpha_0}{3(\alpha_0+1)^4(x-1)} + \frac{\alpha_0}{3(\alpha_0+1)^4(x\alpha_0-2\alpha_0-x)} + \frac{\alpha_0}{3(\alpha_0+1)^3(x-1)^2} - \frac{\alpha_0}{3(\alpha_0+1)^2(x-1)^3} + \frac{\alpha_0}{3(\alpha_0+1)(x-1)^4} + \\ &\quad \frac{80\alpha_0}{3(x-2)^5} - \frac{4\alpha_0}{3} + \left( \frac{1}{3(x-1)^5} + \frac{1}{3} + \frac{80}{3(x-2)^5} + \frac{160}{3(x-2)^6} \right) H(0; \alpha_0) + \left( -\frac{1}{3(x-1)^5} + \frac{1}{3} - \frac{80}{3(x-2)^5} - \right. \\ &\quad \left. \frac{160}{3(x-2)^6} \right) H(0; x) + \frac{H(c_1(\alpha_0); x)}{3(x-1)^5} + \left( \frac{80}{3(x-2)^5} + \frac{160}{3(x-2)^6} \right) H(c_2(\alpha_0); x) + \frac{80 \ln 2}{3(x-2)^5} + \frac{160 \ln 2}{3(x-2)^6} - \frac{1}{9}, \\ b_1^{(-1,2)} &= \frac{1}{\alpha_0 x - x - 2\alpha_0} \left\{ \frac{19x\alpha_0^5}{72} + \frac{31\alpha_0^5}{36(x-2)} - \frac{19\alpha_0^5}{72(x-1)} + \frac{\alpha_0^5}{6} - \frac{131x\alpha_0^4}{72} - \frac{5\alpha_0^4}{2(x-2)} + \frac{95\alpha_0^4}{72(x-1)} + \right. \\ &\quad \frac{65\alpha_0^4}{18(x-2)^2} - \frac{\alpha_0^4}{2(x-1)^2} - \frac{\alpha_0^4}{3} + \frac{52x\alpha_0^3}{9} + \frac{25\alpha_0^3}{18(x-2)} - \frac{95\alpha_0^3}{36(x-1)} - \frac{40\alpha_0^3}{9(x-2)^2} + \frac{53\alpha_0^3}{24(x-1)^2} + \frac{20\alpha_0^3}{(x-2)^3} - \frac{41\alpha_0^3}{36(x-1)^3} - \\ &\quad \frac{121\alpha_0^3}{72} - \frac{40x\alpha_0^2}{3} + \frac{23\alpha_0^2}{9(x-2)} + \frac{13\alpha_0^2}{6(x-1)} - \frac{9\alpha_0^2}{(x-2)^2} - \frac{271\alpha_0^2}{72(x-1)^2} + \frac{400\alpha_0^2}{9(x-2)^3} + \frac{17\alpha_0^2}{3(x-1)^3} + \frac{1880\alpha_0^2}{9(x-2)^4} - \frac{77\alpha_0^2}{18(x-1)^4} + \\ &\quad \frac{857\alpha_0^2}{72} - \frac{1}{9}\pi^2 x \alpha_0 + \frac{244x\alpha_0}{27} - \frac{10\alpha_0}{9(x-2)} - \frac{13\alpha_0}{12(x-1)} + \frac{6\alpha_0}{(x-2)^2} + \frac{31\alpha_0}{36(x-1)^2} - \frac{392\alpha_0}{9(x-2)^3} - \frac{7\alpha_0}{4(x-1)^3} + \frac{4\pi^2\alpha_0}{9(x-2)^4} - \\ &\quad \frac{3760\alpha_0}{9(x-2)^4} + \frac{\pi^2\alpha_0}{9(x-1)^4} - \frac{77\alpha_0}{18(x-1)^4} + \frac{80\pi^2\alpha_0}{9(x-2)^5} - \frac{5120\alpha_0}{9(x-2)^5} - \frac{\pi^2\alpha_0}{9(x-1)^5} + \frac{160 \ln^2 2 \alpha_0}{3(x-2)^4} + \frac{320 \ln^2 2 \alpha_0}{3(x-2)^5} + \frac{160 \ln 2 \alpha_0}{9(x-2)^4} + \\ &\quad \frac{320 \ln 2 \alpha_0}{9(x-2)^5} + \frac{2\pi^2\alpha_0}{9} + \frac{265\alpha_0}{108} + \frac{\pi^2 x}{27} + \frac{2x}{27} + \left( -\frac{x\alpha_0^5}{3} - \frac{2\alpha_0^5}{3(x-2)} + \frac{\alpha_0^5}{3(x-1)} + \frac{19x\alpha_0^4}{9} + \frac{20\alpha_0^4}{9(x-2)} - \frac{13\alpha_0^4}{9(x-1)} - \right. \\ &\quad \frac{20\alpha_0^4}{9(x-2)^2} + \frac{4\alpha_0^4}{9(x-1)^2} - \frac{2\alpha_0^4}{9} - \frac{52x\alpha_0^3}{9} - \frac{20\alpha_0^3}{9(x-2)} + \frac{22\alpha_0^3}{9(x-1)} + \frac{40\alpha_0^3}{9(x-2)^2} - \frac{14\alpha_0^3}{9(x-1)^2} - \frac{80\alpha_0^3}{9(x-2)^3} + \frac{2\alpha_0^3}{3(x-1)^3} + \frac{4\alpha_0^3}{3} + \\ &\quad \frac{28x\alpha_0^2}{3} - \frac{2\alpha_0^2}{x-1} + \frac{2\alpha_0^2}{(x-1)^2} - \frac{2\alpha_0^2}{(x-1)^3} - \frac{160\alpha_0^2}{3(x-2)^4} + \frac{4\alpha_0^2}{3(x-1)^4} - 4\alpha_0^2 - \frac{13x\alpha_0}{2} - \frac{32\alpha_0}{9(x-2)} + \frac{37\alpha_0}{6(x-1)} + \frac{8\alpha_0}{9(x-2)^2} - \\ &\quad \frac{13\alpha_0}{9(x-1)^2} + \frac{16\alpha_0}{(x-2)^3} + \frac{23\alpha_0}{18(x-1)^3} + \frac{2080\alpha_0}{9(x-2)^4} + \frac{79\alpha_0}{36(x-1)^4} + \frac{2240\alpha_0}{9(x-2)^5} - \frac{29\alpha_0}{18(x-1)^5} - \frac{19\alpha_0}{12} + \frac{7x}{6} - \frac{40}{9(x-2)} + \\ &\quad \frac{13}{3(x-1)} + \frac{56}{9(x-2)^2} - \frac{18}{(x-1)^2} - \frac{160}{9(x-2)^3} - \frac{2}{9(x-1)^3} - \frac{1408}{9(x-2)^4} - \frac{85}{36(x-1)^4} - \frac{2560}{9(x-2)^5} - \frac{29}{18(x-1)^5} - \\ &\quad \frac{640}{9(x-2)^6} + \frac{5}{4} \Big) H(0; \alpha_0) + \left( \frac{x\alpha_0^5}{2} + \frac{\alpha_0^5}{x-2} - \frac{\alpha_0^5}{2(x-1)} - \frac{19x\alpha_0^4}{6} - \frac{10\alpha_0^4}{3(x-2)} + \frac{13\alpha_0^4}{6(x-1)} + \frac{10\alpha_0^4}{3(x-2)^2} - \frac{2\alpha_0^4}{3(x-1)^2} + \right. \\ &\quad \frac{\alpha_0^4}{3} + \frac{26x\alpha_0^3}{3} + \frac{10\alpha_0^3}{3(x-2)} - \frac{11\alpha_0^3}{3(x-1)} - \frac{20\alpha_0^3}{3(x-2)^2} + \frac{7\alpha_0^3}{3(x-1)^2} + \frac{40\alpha_0^3}{3(x-2)^3} - \frac{\alpha_0^3}{(x-1)^3} - 2\alpha_0^3 - 14x\alpha_0^2 + \frac{3\alpha_0^2}{x-1} - \\ &\quad \frac{3\alpha_0^2}{(x-1)^2} + \frac{3\alpha_0^2}{(x-1)^3} + \frac{80\alpha_0^2}{(x-2)^4} - \frac{2\alpha_0^2}{(x-1)^4} + 6\alpha_0^2 + \frac{73x\alpha_0}{6} - \frac{5\alpha_0}{3(x-2)} - \frac{7\alpha_0}{6(x-1)} + \frac{20\alpha_0}{3(x-2)^2} + \frac{5\alpha_0}{3(x-1)^2} - \frac{40\alpha_0}{(x-2)^3} - \\ &\quad \frac{3\alpha_0}{(x-1)^3} - \frac{320\alpha_0}{(x-2)^4} - \frac{320\alpha_0}{(x-2)^5} - \frac{10\alpha_0}{3} - \frac{25x}{6} + \frac{2}{3(x-2)} + \frac{1}{6(x-1)} - \frac{1}{3(x-2)^2} - \frac{1}{3(x-1)^2} + \frac{80}{3(x-2)^3} + \frac{1}{(x-1)^3} + \\ &\quad \frac{240}{(x-2)^4} + \frac{2}{(x-1)^4} + \frac{320}{(x-2)^5} - 1 \Big) H(1; \alpha_0) + \left( \frac{2x\alpha_0}{3} + \frac{208\alpha_0}{3(x-2)^4} - \frac{8\alpha_0}{3(x-1)^4} + \frac{320\alpha_0}{3(x-2)^5} + \frac{8\alpha_0}{3(x-1)^5} - \frac{4\alpha_0}{3} - \right. \\ &\quad \left. \frac{2x}{3} - \frac{208}{3(x-2)^4} + \frac{8}{3(x-1)^4} - \frac{736}{3(x-2)^5} + \frac{8}{3(x-1)^5} - \frac{640}{3(x-2)^6} \right) H(0; \alpha_0) H(1; x) + \left( -\frac{x\alpha_0^5}{6} - \frac{\alpha_0^5}{3(x-2)} + \right. \end{aligned}$$

$$\begin{aligned}
& \frac{\alpha_0^5}{6(x-1)} - \frac{\alpha_0^5}{4} + \frac{19x\alpha_0^4}{18} + \frac{17\alpha_0^4}{18(x-2)} - \frac{13\alpha_0^4}{18(x-1)} - \frac{10\alpha_0^4}{9(x-2)^2} + \frac{2\alpha_0^4}{9(x-1)^2} + \frac{41\alpha_0^4}{36} - \frac{26x\alpha_0^3}{9} - \frac{4\alpha_0^3}{9(x-2)} + \frac{11\alpha_0^3}{9(x-1)} + \\
& \frac{14\alpha_0^3}{9(x-2)^2} - \frac{7\alpha_0^3}{9(x-1)^2} - \frac{40\alpha_0^3}{9(x-2)^3} + \frac{\alpha_0^3}{3(x-1)^3} - \frac{11\alpha_0^3}{6} + \frac{14x\alpha_0^2}{3} - \frac{\alpha_0^2}{x-2} - \frac{\alpha_0^2}{x-1} + \frac{2\alpha_0^2}{(x-2)^2} + \frac{\alpha_0^2}{(x-1)^2} - \frac{4\alpha_0^2}{(x-2)^3} - \\
& \frac{\alpha_0^2}{(x-1)^3} - \frac{80\alpha_0^2}{3(x-2)^4} + \frac{2\alpha_0^2}{3(x-1)^4} + \frac{\alpha_0^2}{2} - \frac{73x\alpha_0}{18} - \frac{32\alpha_0}{9(x-2)} + \frac{37\alpha_0}{6(x-1)} + \frac{8\alpha_0}{9(x-2)^2} - \frac{13\alpha_0}{9(x-1)^2} + \frac{16\alpha_0}{(x-2)^3} + \\
& \frac{23\alpha_0}{18(x-1)^3} + \frac{176\alpha_0}{(x-2)^4} + \frac{55\alpha_0}{36(x-1)^4} + \frac{320\alpha_0}{3(x-2)^5} - \frac{29\alpha_0}{18(x-1)^5} - \frac{29\alpha_0}{36} + \frac{25x}{18} + \left( -\frac{4\alpha_0}{3(x-1)^4} + \frac{4\alpha_0}{3(x-1)^5} + \frac{4}{3(x-1)^4} + \right. \\
& \left. \frac{4}{3(x-1)^5} \right) H(0; \alpha_0) + \left( \frac{2\alpha_0}{(x-1)^4} - \frac{2\alpha_0}{(x-1)^5} - \frac{2}{(x-1)^4} - \frac{2}{(x-1)^5} \right) H(1; \alpha_0) - \frac{40}{9(x-2)} + \frac{13}{3(x-1)} + \frac{56}{9(x-2)^2} - \\
& \frac{7}{18(x-1)^2} - \frac{160}{9(x-2)^3} - \frac{2}{9(x-1)^3} - \frac{416}{3(x-2)^4} - \frac{85}{36(x-1)^4} - \frac{640}{3(x-2)^5} - \frac{29}{18(x-1)^5} + \frac{5}{4} \Big) H(c_1(\alpha_0); x) + \\
& \left( \frac{160\alpha_0}{9(x-2)^4} + \frac{320\alpha_0}{9(x-2)^5} + \left( -\frac{320\alpha_0}{3(x-2)^4} - \frac{640\alpha_0}{3(x-2)^5} + \frac{320}{3(x-2)^4} + \frac{1280}{3(x-2)^5} + \frac{1280}{3(x-2)^6} \right) H(0; \alpha_0) + \left( \frac{160\alpha_0}{(x-2)^4} + \right. \right. \\
& \left. \left. \frac{320\alpha_0}{(x-2)^5} - \frac{160}{(x-2)^4} - \frac{640}{(x-2)^5} - \frac{640}{(x-2)^6} \right) H(1; \alpha_0) - \frac{160}{9(x-2)^4} - \frac{640}{9(x-2)^5} - \frac{640}{9(x-2)^6} \right) H(c_2(\alpha_0); x) + \\
& \left( -\frac{4x\alpha_0}{3} - \frac{320\alpha_0}{3(x-2)^4} - \frac{4\alpha_0}{3(x-1)^4} - \frac{640\alpha_0}{3(x-2)^5} + \frac{4\alpha_0}{3(x-1)^5} + \frac{8\alpha_0}{3} + \frac{4x}{3} + \frac{320}{3(x-2)^4} + \frac{4}{3(x-1)^4} + \frac{1280}{3(x-2)^5} + \right. \\
& \left. \frac{4}{3(x-1)^5} + \frac{1280}{3(x-2)^6} \right) H(0, 0; \alpha_0) + \left( -\frac{4x\alpha_0}{3} + \frac{320\alpha_0}{3(x-2)^4} + \frac{4\alpha_0}{3(x-1)^4} + \frac{640\alpha_0}{3(x-2)^5} - \frac{4\alpha_0}{3(x-1)^5} + \frac{8\alpha_0}{3} + \right. \\
& \left. \frac{4x}{3} - \frac{320}{3(x-2)^4} - \frac{4}{3(x-1)^4} - \frac{1280}{3(x-2)^5} - \frac{4}{3(x-1)^5} - \frac{1280}{3(x-2)^6} \right) H(0, 0; x) + \left( 2x\alpha_0 + \frac{160\alpha_0}{(x-2)^4} + \frac{2\alpha_0}{(x-1)^4} + \right. \\
& \left. \frac{320\alpha_0}{(x-2)^5} - \frac{2\alpha_0}{(x-1)^5} - 4\alpha_0 - 2x - \frac{160}{(x-2)^4} - \frac{2}{(x-1)^4} - \frac{640}{(x-2)^5} - \frac{2}{(x-1)^5} - \frac{640}{(x-2)^6} \right) H(0, 1; \alpha_0) + \left( \frac{2x\alpha_0}{3} + \frac{208\alpha_0}{3(x-2)^4} - \frac{2\alpha_0}{3(x-1)^4} + \frac{320\alpha_0}{3(x-2)^5} + \frac{2\alpha_0}{3(x-1)^5} - \frac{4\alpha_0}{3} - \frac{2x}{3} - \right. \\
& \left. \frac{208\alpha_0}{3(x-2)^4} - \frac{2\alpha_0}{3(x-1)^4} + \frac{320\alpha_0}{3(x-2)^5} + \frac{2\alpha_0}{3(x-1)^5} - \frac{4\alpha_0}{3} - \frac{2x}{3} - \frac{208}{3(x-2)^4} + \frac{2}{3(x-1)^4} - \frac{736}{3(x-2)^5} + \frac{2}{3(x-1)^5} - \right. \\
& \left. \frac{640}{3(x-2)^6} \right) H(0, c_1(\alpha_0); x) + \left( -\frac{320\alpha_0}{3(x-2)^4} - \frac{640\alpha_0}{3(x-2)^5} + \frac{320}{3(x-2)^4} + \frac{1280}{3(x-2)^5} + \frac{1280}{3(x-2)^6} \right) H(0, c_2(\alpha_0); x) + \\
& \left( -\frac{2x\alpha_0}{3} - \frac{208\alpha_0}{3(x-2)^4} + \frac{8\alpha_0}{3(x-1)^4} - \frac{320\alpha_0}{3(x-2)^5} - \frac{8\alpha_0}{3(x-1)^5} + \frac{4\alpha_0}{3} + \frac{2x}{3} + \frac{208}{3(x-2)^4} - \frac{8}{3(x-1)^4} + \frac{736}{3(x-2)^5} - \right. \\
& \left. \frac{8}{3(x-1)^5} + \frac{640}{3(x-2)^6} \right) H(1, 0; x) + \left( \frac{2x\alpha_0}{3} + \frac{208\alpha_0}{3(x-2)^4} - \frac{8\alpha_0}{3(x-1)^4} + \frac{320\alpha_0}{3(x-2)^5} + \frac{8\alpha_0}{3(x-1)^5} - \frac{4\alpha_0}{3} - \frac{2x}{3} - \right. \\
& \left. \frac{208}{3(x-2)^4} + \frac{8}{3(x-1)^4} - \frac{736}{3(x-2)^5} + \frac{8}{3(x-1)^5} - \frac{640}{3(x-2)^6} \right) H(1, c_1(\alpha_0); x) + \left( -\frac{800\alpha_0}{3(x-2)^4} - \frac{1600\alpha_0}{3(x-2)^5} + \right. \\
& \left. \frac{800}{3(x-2)^4} + \frac{3200}{3(x-2)^5} + \frac{3200}{3(x-2)^6} \right) H(2, 0; x) + \left( \frac{800\alpha_0}{3(x-2)^4} + \frac{1600\alpha_0}{3(x-2)^5} - \frac{800}{3(x-2)^4} - \frac{3200}{3(x-2)^5} - \right. \\
& \left. \frac{3200}{3(x-2)^6} \right) H(2, c_2(\alpha_0); x) + \left( -\frac{2\alpha_0}{3(x-1)^4} + \frac{2\alpha_0}{3(x-1)^5} + \frac{2}{3(x-1)^4} + \frac{2}{3(x-1)^5} \right) H(c_1(\alpha_0), c_1(\alpha_0); x) + \\
& \left( -\frac{208\alpha_0}{3(x-2)^4} - \frac{320\alpha_0}{3(x-2)^5} + \frac{208}{3(x-2)^4} + \frac{736}{3(x-2)^5} + \frac{640}{3(x-2)^6} \right) H(c_2(\alpha_0), c_1(\alpha_0); x) + H(0; x) \left( \frac{29x\alpha_0}{18} + \right. \\
& \frac{32\alpha_0}{9(x-2)} - \frac{37\alpha_0}{6(x-1)} - \frac{8\alpha_0}{9(x-2)^2} + \frac{13\alpha_0}{9(x-1)^2} - \frac{16\alpha_0}{(x-2)^3} - \frac{23\alpha_0}{18(x-1)^3} - \frac{1120\alpha_0}{9(x-2)^4} - \frac{31\alpha_0}{36(x-1)^4} - \frac{320\alpha_0}{9(x-2)^5} + \\
& \frac{29\alpha_0}{18(x-1)^5} - \frac{320 \ln 2 \alpha_0}{3(x-2)^4} - \frac{640 \ln 2 \alpha_0}{3(x-2)^5} - \frac{71\alpha_0}{36} - \frac{29x}{18} + \frac{40}{9(x-2)} - \frac{13}{3(x-1)} - \frac{56}{9(x-2)^2} + \frac{7}{18(x-1)^2} + \\
& \frac{160}{9(x-2)^3} + \frac{2}{9(x-1)^3} + \frac{1408}{9(x-2)^4} + \frac{85}{36(x-1)^4} + \frac{2560}{9(x-2)^5} + \frac{29}{18(x-1)^5} + \frac{640}{9(x-2)^6} + \frac{320 \ln 2}{3(x-2)^4} + \frac{1280 \ln 2}{3(x-2)^5} + \\
& \left. \frac{1280 \ln 2}{3(x-2)^6} - \frac{5}{4} \right) + H(2; x) \left( \frac{800 \ln 2 \alpha_0}{3(x-2)^4} + \frac{1600 \ln 2 \alpha_0}{3(x-2)^5} + \left( \frac{800\alpha_0}{3(x-2)^4} + \frac{1600\alpha_0}{3(x-2)^5} - \frac{800}{3(x-2)^4} - \frac{3200}{3(x-2)^5} - \right. \right. \\
& \left. \left. \frac{3200}{3(x-2)^6} \right) H(0; \alpha_0) - \frac{800 \ln 2}{3(x-2)^4} - \frac{3200 \ln 2}{3(x-2)^5} - \frac{3200 \ln 2}{3(x-2)^6} \right) - \frac{4\pi^2}{9(x-2)^4} - \frac{\pi^2}{9(x-1)^4} - \frac{88\pi^2}{9(x-2)^5} - \\
& \left. \frac{\pi^2}{9(x-1)^5} - \frac{160\pi^2}{9(x-2)^6} - \frac{160 \ln^2 2}{3(x-2)^4} - \frac{640 \ln^2 2}{3(x-2)^5} - \frac{640 \ln^2 2}{3(x-2)^6} - \frac{160 \ln 2}{9(x-2)^4} - \frac{640 \ln 2}{9(x-2)^5} - \frac{640 \ln 2}{9(x-2)^6} \right\},
\end{aligned}$$

$$\begin{aligned}
b_2^{(-1,2)} = & \frac{1}{\alpha_0 x - x - 2\alpha_0} \left\{ -\frac{1}{72} \pi^2 x \alpha_0^5 + \frac{301x\alpha_0^5}{432} - \frac{\pi^2 \alpha_0^5}{36(x-2)} + \frac{673\alpha_0^5}{216(x-2)} + \frac{\pi^2 \alpha_0^5}{72(x-1)} - \frac{301\alpha_0^5}{432(x-1)} + \right. \\
& \frac{31\alpha_0^5}{36} + \frac{19}{216} \pi^2 x \alpha_0^4 - \frac{6967x\alpha_0^4}{1296} + \frac{5\pi^2 \alpha_0^4}{54(x-2)} - \frac{2345\alpha_0^4}{324(x-2)} - \frac{13\pi^2 \alpha_0^4}{216(x-1)} + \frac{5251\alpha_0^4}{1296(x-1)} - \frac{5\pi^2 \alpha_0^4}{54(x-2)^2} + \frac{5405\alpha_0^4}{324(x-2)^2} + \\
& \frac{\pi^2 \alpha_0^4}{54(x-1)^2} - \frac{613\alpha_0^4}{324(x-1)^2} - \frac{\pi^2 \alpha_0^4}{108} - \frac{299\alpha_0^4}{162} - \frac{13}{54} \pi^2 x \alpha_0^3 + \frac{3413x\alpha_0^3}{162} - \frac{5\pi^2 \alpha_0^3}{54(x-2)} - \frac{895\alpha_0^3}{324(x-2)} + \frac{11\pi^2 \alpha_0^3}{108(x-1)} - \\
& \frac{5807\alpha_0^3}{648(x-1)} + \frac{27\pi^2 \alpha_0^3}{27(x-2)^2} - \frac{81\pi^2 \alpha_0^3}{81(x-2)^2} - \frac{7\pi^2 \alpha_0^3}{108(x-1)^2} + \frac{14261\alpha_0^3}{1296(x-1)^2} - \frac{10\pi^2 \alpha_0^3}{27(x-2)^3} + \frac{10580\alpha_0^3}{81(x-2)^3} + \frac{\pi^2 \alpha_0^3}{36(x-1)^3} - \\
& \frac{1411\alpha_0^3}{216(x-1)^3} + \frac{\pi^2 \alpha_0^3}{18} - \frac{4837\alpha_0^3}{432} + \frac{7}{18} \pi^2 x \alpha_0^2 - \frac{4571x\alpha_0^2}{54} + \frac{545\alpha_0^2}{18(x-2)} - \frac{\pi^2 \alpha_0^2}{12(x-1)} - \frac{121\alpha_0^2}{36(x-1)} - \frac{2075\alpha_0^2}{18(x-2)^2} + \\
& \left. \frac{\pi^2 \alpha_0^2}{12(x-1)^2} - \frac{3577\alpha_0^2}{144(x-1)^2} + \frac{7600\alpha_0^2}{9(x-2)^3} - \frac{\pi^2 \alpha_0^2}{12(x-1)^3} + \frac{524\alpha_0^2}{9(x-1)^3} - \frac{20\pi^2 \alpha_0^2}{9(x-2)^4} + \frac{66760\alpha_0^2}{27(x-2)^4} + \frac{\pi^2 \alpha_0^2}{18(x-1)^4} - \frac{5047\alpha_0^2}{108(x-1)^4} - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\pi^2 \alpha_0^2}{6} + \frac{17483\alpha_0^2}{144} - \frac{7\pi^2 x \alpha_0}{8} + \frac{5525 x \alpha_0}{81} - \frac{26\pi^2 \alpha_0}{9(x-2)} - \frac{17\alpha_0}{9(x-2)} + \frac{745\pi^2 \alpha_0}{216(x-1)} - \frac{467\alpha_0}{24(x-1)} + \frac{10\pi^2 \alpha_0}{3(x-2)^2} + \\
& \frac{565\alpha_0}{9(x-2)^2} - \frac{8\pi^2 \alpha_0}{9(x-1)^2} + \frac{91\alpha_0}{8(x-1)^2} - \frac{8\pi^2 \alpha_0}{3(x-2)^3} - \frac{7240\alpha_0}{9(x-2)^3} + \frac{79\pi^2 \alpha_0}{108(x-1)^3} - \frac{689\alpha_0}{24(x-1)^3} + \frac{920\pi^2 \alpha_0}{27(x-2)^4} - \frac{133520\alpha_0}{27(x-2)^4} + \\
& \frac{5\pi^2 \alpha_0}{54(x-1)^4} - \frac{5047\alpha_0}{108(x-1)^4} + \frac{400\pi^2 \alpha_0}{27(x-2)^5} - \frac{154240\alpha_0}{27(x-2)^5} - \frac{29\pi^2 \alpha_0}{54(x-1)^5} + \frac{17x\zeta_3 \alpha_0}{12} + \frac{224\zeta_3 \alpha_0}{3(x-2)^4} - \frac{7\zeta_3 \alpha_0}{4(x-1)^4} + \\
& \frac{280\zeta_3 \alpha_0}{3(x-2)^5} + \frac{7\zeta_3 \alpha_0}{4(x-1)^5} - \frac{17\zeta_3 \alpha_0}{6} + \frac{640 \ln^3 2 \alpha_0}{9(x-2)^4} + \frac{1280 \ln^3 2 \alpha_0}{9(x-2)^5} + \frac{320 \ln^2 2 \alpha_0}{9(x-2)^4} + \frac{640 \ln^2 2 \alpha_0}{9(x-2)^5} - \frac{1}{6} \pi^2 x \ln 2 \alpha_0 - \\
& \frac{32\pi^2 \ln 2 \alpha_0}{3(x-2)^4} + \frac{320 \ln 2 \alpha_0}{27(x-2)^4} + \frac{\pi^2 \ln 2 \alpha_0}{6(x-1)^4} + \frac{80\pi^2 \ln 2 \alpha_0}{3(x-2)^5} + \frac{640 \ln 2 \alpha_0}{27(x-2)^5} - \frac{\pi^2 \ln 2 \alpha_0}{6(x-1)^5} + \frac{1}{3} \pi^2 \ln 2 \alpha_0 + \frac{53\pi^2 \alpha_0}{72} + \\
& \frac{1669 \alpha_0}{648} + \frac{47\pi^2 x}{72} + \frac{4x}{81} + \left( -\frac{19x \alpha_0^5}{18} - \frac{31\alpha_0^5}{9(x-2)} + \frac{19\alpha_0^5}{18(x-1)} - \frac{2 \alpha_0^5}{3} + \frac{131x\alpha_0^4}{18} + \frac{10\alpha_0^4}{x-2} - \frac{95 \alpha_0^4}{18(x-1)} - \frac{130\alpha_0^4}{9(x-2)^2} + \right. \\
& \frac{2\alpha_0^4}{(x-1)^2} + \frac{4\alpha_0^4}{3} - \frac{208x\alpha_0^3}{9} - \frac{50\alpha_0^3}{9(x-2)} + \frac{95\alpha_0^3}{9(x-1)} + \frac{160\alpha_0^3}{9(x-2)^2} - \frac{53 \alpha_0^3}{6(x-1)^2} - \frac{80\alpha_0^3}{(x-2)^3} + \frac{41\alpha_0^3}{9(x-1)^3} + \frac{121\alpha_0^3}{18} + \frac{160x\alpha_0^2}{3} - \\
& \frac{92\alpha_0^2}{9(x-2)} - \frac{26\alpha_0^2}{3(x-1)} + \frac{36\alpha_0^2}{(x-2)^2} + \frac{271 \alpha_0^2}{18(x-1)^2} - \frac{1600\alpha_0^2}{9(x-2)^3} - \frac{68\alpha_0^2}{3(x-1)^3} - \frac{7520\alpha_0^2}{9(x-2)^4} + \frac{154\alpha_0^2}{9(x-1)^4} - \frac{857\alpha_0^2}{18} - \frac{1}{18} \pi^2 x \alpha_0 - \\
& \frac{5255x \alpha_0}{108} - \frac{812\alpha_0}{9(x-2)} + \frac{239\alpha_0}{2(x-1)} + \frac{448 \alpha_0}{9(x-2)^2} - \frac{18\alpha_0}{(x-1)^2} + \frac{3248\alpha_0}{9(x-2)^3} + \frac{164\alpha_0}{9(x-1)^3} - \frac{40\pi^2 \alpha_0}{9(x-2)^4} + \frac{76160\alpha_0}{27(x-2)^4} - \frac{\pi^2 \alpha_0}{18(x-1)^4} + \\
& \frac{4877\alpha_0}{216(x-1)^4} - \frac{80\pi^2 \alpha_0}{9(x-2)^5} + \frac{62080\alpha_0}{27(x-2)^5} + \frac{\pi^2 \alpha_0}{18(x-1)^5} - \frac{1351\alpha_0}{108(x-1)^5} + \frac{\pi^2 \alpha_0}{9} + \frac{809 \alpha_0}{216} + \frac{\pi^2 x}{18} + \frac{1319 x}{108} - \frac{92}{x-2} + \\
& \frac{1133}{12(x-1)} + \frac{1040}{9(x-2)^2} - \frac{223}{36(x-1)^2} - \frac{3008}{9(x-2)^3} - \frac{103}{36(x-1)^3} + \frac{40\pi^2}{9(x-2)^4} - \frac{41120}{27(x-2)^4} + \frac{\pi^2}{18(x-1)^4} - \frac{4223}{216(x-1)^4} + \\
& \frac{160 \pi^2}{9(x-2)^5} - \frac{62720}{27(x-2)^5} + \frac{\pi^2}{18(x-1)^5} - \frac{1351}{108(x-1)^5} + \frac{160\pi^2}{9(x-2)^6} - \frac{1280}{27(x-2)^6} + \frac{275}{24} \Big) H(0; \alpha_0) + \left( \frac{19x\alpha_0^5}{12} + \right. \\
& \frac{31\alpha_0^5}{6(x-2)} - \frac{19 \alpha_0^5}{12(x-1)} + \alpha_0^5 - \frac{131x\alpha_0^4}{12} - \frac{15 \alpha_0^4}{x-2} + \frac{95\alpha_0^4}{12(x-1)} + \frac{65\alpha_0^4}{3(x-2)^2} - \frac{3\alpha_0^4}{(x-1)^2} - 2\alpha_0^4 + \frac{104x \alpha_0^3}{3} + \frac{25\alpha_0^3}{3(x-2)} - \\
& \frac{95\alpha_0^3}{6(x-1)} - \frac{80 \alpha_0^3}{3(x-2)^2} + \frac{53\alpha_0^3}{4(x-1)^2} + \frac{120 \alpha_0^3}{(x-2)^3} - \frac{41\alpha_0^3}{6(x-1)^3} - \frac{121\alpha_0^3}{12} - 80x \alpha_0^2 + \frac{46\alpha_0^2}{3(x-2)} + \frac{13\alpha_0^2}{x-1} - \frac{54 \alpha_0^2}{(x-2)^2} - \frac{271\alpha_0^2}{12(x-1)^2} + \\
& \frac{800\alpha_0^2}{3(x-2)^3} + \frac{34\alpha_0^2}{(x-1)^3} + \frac{3760\alpha_0^2}{3(x-2)^4} - \frac{77\alpha_0^2}{3(x-1)^4} + \frac{857\alpha_0^2}{12} + \frac{367x \alpha_0}{4} - \frac{109\alpha_0}{6(x-2)} - \frac{103\alpha_0}{12(x-1)} + \frac{296 \alpha_0}{3(x-2)^2} + \frac{205\alpha_0}{12(x-1)^2} - \\
& \frac{888 \alpha_0}{(x-2)^3} - \frac{89\alpha_0}{2(x-1)^3} - \frac{12640\alpha_0}{3(x-2)^4} - \frac{10240\alpha_0}{3(x-2)^5} - \frac{233\alpha_0}{4} - 445x + \frac{13}{3(x-2)} + \frac{61}{12(x-1)} - \frac{119}{3(x-2)^2} - \frac{19}{4(x-1)^2} + \\
& \frac{1504}{3(x-2)^3} + \frac{52}{3(x-1)^3} + \frac{2960}{(x-2)^4} + \frac{77}{3(x-1)^4} + \frac{10240}{3(x-2)^5} - \frac{25}{12} \Big) H(1; \alpha_0) + \left( \frac{4x\alpha_0^5}{3} + \frac{8 \alpha_0^5}{3(x-2)} - \frac{4\alpha_0^5}{3(x-1)} - \right. \\
& \frac{76x\alpha_0^4}{9} - \frac{80 \alpha_0^4}{9(x-2)} + \frac{52\alpha_0^4}{9(x-1)} + \frac{80\alpha_0^4}{9(x-2)^2} - \frac{16\alpha_0^4}{9(x-1)^2} + \frac{8\alpha_0^4}{9} + \frac{208x \alpha_0^3}{9} + \frac{80\alpha_0^3}{9(x-2)} - \frac{88\alpha_0^3}{9(x-1)} - \frac{160 \alpha_0^3}{9(x-2)^2} + \frac{56\alpha_0^3}{9(x-1)^2} + \\
& \frac{320\alpha_0^3}{9(x-2)^3} - \frac{8\alpha_0^3}{3(x-1)^3} - \frac{16\alpha_0^3}{3} - \frac{112x \alpha_0^2}{3} + \frac{8\alpha_0^2}{x-1} - \frac{8\alpha_0^2}{(x-1)^2} + \frac{8 \alpha_0^2}{(x-1)^3} + \frac{640\alpha_0^2}{3(x-2)^4} - \frac{16\alpha_0^2}{3(x-1)^4} + 16\alpha_0^2 + 26x\alpha_0 + \\
& \frac{128\alpha_0}{9(x-2)} - \frac{74\alpha_0}{3(x-1)} - \frac{32\alpha_0}{9(x-2)^2} + \frac{52\alpha_0}{9(x-1)^2} - \frac{64 \alpha_0}{(x-2)^3} - \frac{46\alpha_0}{9(x-1)^3} - \frac{8320\alpha_0}{9(x-2)^4} - \frac{79\alpha_0}{9(x-1)^4} - \frac{8960\alpha_0}{9(x-2)^5} + \frac{58 \alpha_0}{9(x-1)^5} + \\
& \frac{19\alpha_0}{3} - \frac{14x}{3} + \frac{160}{9(x-2)} - \frac{52}{3(x-1)} - \frac{224}{9(x-2)^2} + \frac{14}{9(x-1)^2} + \frac{640}{9(x-2)^3} + \frac{8}{9(x-1)^3} + \frac{5632}{9(x-2)^4} + \frac{85}{9(x-1)^4} + \frac{10240}{9(x-2)^5} + \\
& \frac{58}{9(x-1)^5} + \frac{2560}{9(x-2)^6} - 5 \Big) H(0, 0; \alpha_0) + \left( -2x \alpha_0^5 - \frac{4\alpha_0^5}{x-2} + \frac{2\alpha_0^5}{x-1} + \frac{38x \alpha_0^4}{3} + \frac{40\alpha_0^4}{3(x-2)} - \frac{26\alpha_0^4}{3(x-1)} - \frac{40 \alpha_0^4}{3(x-2)^2} + \right. \\
& \frac{8\alpha_0^4}{3(x-1)^2} - \frac{4 \alpha_0^4}{3} - \frac{104x\alpha_0^3}{3} - \frac{40\alpha_0^3}{3(x-2)} + \frac{44 \alpha_0^3}{3(x-1)} + \frac{80\alpha_0^3}{3(x-2)^2} - \frac{28\alpha_0^3}{3(x-1)^2} - \frac{160\alpha_0^3}{3(x-2)^3} + \frac{4\alpha_0^3}{(x-1)^3} + 8 \alpha_0^3 + 56x\alpha_0^2 - \\
& \frac{12\alpha_0^2}{x-1} + \frac{12 \alpha_0^2}{(x-1)^2} - \frac{12\alpha_0^2}{(x-1)^3} - \frac{320 \alpha_0^2}{(x-2)^4} + \frac{8\alpha_0^2}{(x-1)^4} - 24\alpha_0^2 - 39x\alpha_0 - \frac{64 \alpha_0}{3(x-2)} + \frac{37\alpha_0}{x-1} + \frac{16\alpha_0}{3(x-2)^2} - \frac{26 \alpha_0}{3(x-1)^2} + \\
& \frac{96\alpha_0}{(x-2)^3} + \frac{23\alpha_0}{3(x-1)^3} + \frac{4160\alpha_0}{3(x-2)^4} + \frac{79\alpha_0}{6(x-1)^4} + \frac{4480\alpha_0}{3(x-2)^5} - \frac{29\alpha_0}{3(x-1)^5} - \frac{19 \alpha_0}{2} + 7x - \frac{80}{3(x-2)} + \frac{26}{x-1} + \frac{112}{3(x-2)^2} - \\
& \frac{7}{3(x-1)^2} - \frac{320}{3(x-2)^3} - \frac{4}{3(x-1)^3} - \frac{2816}{3(x-2)^4} - \frac{85}{6(x-1)^4} - \frac{5120}{3(x-2)^5} - \frac{29}{3(x-1)^5} - \frac{1280}{3(x-2)^6} + \frac{15}{2} \Big) H(0, 1; \alpha_0) + \\
& H(1; x) \left( \frac{1}{9} \pi^2 x \alpha_0 + \frac{32\pi^2 \alpha_0}{3(x-2)^4} - \frac{8\pi^2 \alpha_0}{9(x-1)^4} + \frac{80\pi^2 \alpha_0}{3(x-2)^5} + \frac{8\pi^2 \alpha_0}{9(x-1)^5} - \frac{2\pi^2 \alpha_0}{9} - \frac{\pi^2 x}{9} + \left( \frac{29x \alpha_0}{9} + \frac{406\alpha_0}{9(x-2)} - \right. \right. \\
& \frac{166\alpha_0}{3(x-1)} - \frac{424 \alpha_0}{9(x-2)^2} + \frac{193\alpha_0}{18(x-1)^2} + \frac{32 \alpha_0}{(x-2)^3} - \frac{53\alpha_0}{9(x-1)^3} - \frac{4000\alpha_0}{9(x-2)^4} - \frac{175\alpha_0}{18(x-1)^4} + \frac{640\alpha_0}{9(x-2)^5} + \frac{116\alpha_0}{9(x-1)^5} + \frac{2\alpha_0}{9} - \\
& \frac{29 x}{9} + \frac{326}{9(x-2)} - \frac{118}{3(x-1)} - \frac{388}{9(x-2)^2} + \frac{95}{18(x-1)^2} + \frac{560}{9(x-2)^3} + \frac{4}{9(x-1)^3} + \frac{3424}{9(x-2)^4} + \frac{289}{18(x-1)^4} + \frac{7360}{9(x-2)^5} + \\
& \frac{116}{9(x-1)^5} - \frac{1280}{9(x-2)^6} - \frac{20}{3} \Big) H(0; \alpha_0) + \left( -\frac{8x\alpha_0}{3} - \frac{832 \alpha_0}{3(x-2)^4} + \frac{32\alpha_0}{3(x-1)^4} - \frac{1280\alpha_0}{3(x-2)^5} - \frac{32\alpha_0}{3(x-1)^5} + \frac{16\alpha_0}{3} + \right. \\
& \frac{8 x}{3} + \frac{832}{3(x-2)^4} - \frac{32}{3(x-1)^4} + \frac{2944}{3(x-2)^5} - \frac{32}{3(x-1)^5} + \frac{2560}{3(x-2)^6} \Big) H(0, 0; \alpha_0) + \left( 4x\alpha_0 + \frac{416\alpha_0}{(x-2)^4} - \frac{16\alpha_0}{(x-1)^4} + \right. \\
& \frac{640 \alpha_0}{(x-2)^5} + \frac{16\alpha_0}{(x-1)^5} - 8\alpha_0 - 4x - \frac{416}{(x-2)^4} + \frac{16}{(x-1)^4} - \frac{1472}{(x-2)^5} + \frac{16}{(x-1)^5} - \frac{1280}{(x-2)^6} \Big) H(0, 1; \alpha_0) - \frac{32\pi^2}{3(x-2)^4} + \\
& \frac{8\pi^2}{9(x-1)^4} - \frac{48\pi^2}{(x-2)^5} + \frac{8 \pi^2}{9(x-1)^5} - \frac{160\pi^2}{3(x-2)^6} \Big) + \left( -\frac{20x \alpha_0}{3} - \frac{1984\alpha_0}{3(x-2)^4} + \frac{20\alpha_0}{3(x-1)^4} - \frac{3200\alpha_0}{3(x-2)^5} - \frac{20\alpha_0}{3(x-1)^5} + \right.
\end{aligned}$$



$$\begin{aligned}
& \left. \frac{40}{3} \alpha_0 + \frac{20x}{3} + \frac{1984}{3(x-2)^4} - \frac{20}{3(x-1)^4} + \frac{7168}{3(x-2)^5} - \frac{20}{3(x-1)^5} + \frac{6400}{3(x-2)^6} \right) H(0; \alpha_0) H(0, 1; x) + \left( -\frac{\alpha_0^5}{2} - \right. \\
& \left. \frac{\alpha_0^4}{3(x-2)} + \frac{5\alpha_0^4}{2} + \frac{4\alpha_0^3}{3(x-2)} - \frac{4\alpha_0^3}{3(x-2)^2} - 5\alpha_0^3 - \frac{2\alpha_0^2}{x-2} + \frac{4\alpha_0^2}{(x-2)^2} - \frac{8\alpha_0^2}{(x-2)^3} + 5\alpha_0^2 + \frac{29x\alpha_0}{9} + \frac{160\alpha_0}{9(x-2)} - \frac{62\alpha_0}{3(x-1)} - \right. \\
& \left. \frac{184\alpha_0}{9(x-2)^2} + \frac{97\alpha_0}{18(x-1)^2} + \frac{16\alpha_0}{(x-2)^3} - \frac{38\alpha_0}{9(x-1)^3} - \frac{832\alpha_0}{9(x-2)^4} - \frac{5\alpha_0}{9(x-1)^4} + \frac{640\alpha_0}{9(x-2)^5} + \frac{29\alpha_0}{9(x-1)^5} - \frac{113\alpha_0}{18} - \frac{29x}{9} + \right. \\
& \left( -\frac{8x\alpha_0}{3} - \frac{832\alpha_0}{3(x-2)^4} + \frac{8\alpha_0}{3(x-1)^4} - \frac{1280\alpha_0}{3(x-2)^5} - \frac{8\alpha_0}{3(x-1)^5} + \frac{16\alpha_0}{3} + \frac{8x}{3} + \frac{832}{3(x-2)^4} - \frac{8}{3(x-1)^4} + \frac{2944}{3(x-2)^5} - \right. \\
& \left. \frac{8}{3(x-1)^5} + \frac{2560}{3(x-2)^6} \right) H(0; \alpha_0) + \left( 4x\alpha_0 + \frac{416\alpha_0}{(x-2)^4} - \frac{4\alpha_0}{(x-1)^4} + \frac{640\alpha_0}{(x-2)^5} + \frac{4\alpha_0}{(x-1)^5} - 8\alpha_0 - 4x - \frac{416}{(x-2)^4} + \right. \\
& \left. \frac{4}{(x-1)^4} - \frac{1472}{(x-2)^5} + \frac{4}{(x-1)^5} - \frac{1280}{(x-2)^6} \right) H(1; \alpha_0) + \frac{104}{9(x-2)} - \frac{13}{x-1} - \frac{136}{9(x-2)^2} + \frac{41}{18(x-1)^2} + \frac{224}{9(x-2)^3} + \\
& \frac{10}{9(x-1)^3} + \frac{832}{9(x-2)^4} + \frac{53}{9(x-1)^4} + \frac{1600}{9(x-2)^5} + \frac{29}{9(x-1)^5} - \frac{1280}{9(x-2)^6} - \frac{13}{6} \Big) H(0, c_1(\alpha_0); x) + \left( -\frac{640\alpha_0}{9(x-2)^4} - \right. \\
& \left. \frac{1280\alpha_0}{9(x-2)^5} + \left( \frac{1280\alpha_0}{3(x-2)^4} + \frac{2560\alpha_0}{3(x-2)^5} - \frac{1280}{3(x-2)^4} - \frac{5120}{3(x-2)^5} - \frac{5120}{3(x-2)^6} \right) H(0; \alpha_0) + \left( -\frac{640\alpha_0}{(x-2)^4} - \frac{1280\alpha_0}{(x-2)^5} + \right. \\
& \left. \frac{640}{(x-2)^4} + \frac{2560}{(x-2)^5} + \frac{2560}{(x-2)^6} \right) H(1; \alpha_0) + \frac{640}{9(x-2)^4} + \frac{2560}{9(x-2)^5} + \frac{2560}{9(x-2)^6} \Big) H(0, c_2(\alpha_0); x) + \left( -2x\alpha_0^5 - \right. \\
& \left. \frac{4\alpha_0^5}{x-2} + \frac{2\alpha_0^5}{x-1} + \frac{38x\alpha_0^4}{3} + \frac{40\alpha_0^4}{3(x-2)} - \frac{26\alpha_0^4}{3(x-1)} - \frac{40\alpha_0^4}{3(x-2)^2} + \frac{8\alpha_0^4}{3(x-1)^2} - \frac{4\alpha_0^4}{3} - \frac{104x\alpha_0^3}{3} - \frac{40\alpha_0^3}{3(x-2)} + \frac{44\alpha_0^3}{3(x-1)} + \right. \\
& \left. \frac{80\alpha_0^3}{3(x-2)^2} - \frac{28\alpha_0^3}{3(x-1)^2} - \frac{160\alpha_0^3}{3(x-2)^3} + \frac{4\alpha_0^3}{(x-1)^3} + 8\alpha_0^3 + 56x\alpha_0^2 - \frac{12\alpha_0^2}{x-1} + \frac{12\alpha_0^2}{(x-1)^2} - \frac{12\alpha_0^2}{(x-1)^3} - \frac{320\alpha_0^2}{(x-2)^4} + \frac{8\alpha_0^2}{(x-1)^4} - \right. \\
& \left. 24\alpha_0^2 - \frac{146x\alpha_0}{3} + \frac{20\alpha_0}{3(x-2)} + \frac{14\alpha_0}{3(x-1)} - \frac{80\alpha_0}{3(x-2)^2} - \frac{20\alpha_0}{3(x-1)^2} + \frac{160\alpha_0}{(x-2)^3} + \frac{12\alpha_0}{(x-1)^3} + \frac{1280\alpha_0}{(x-2)^4} + \frac{1280\alpha_0}{(x-2)^5} + \right. \\
& \left. \frac{40\alpha_0}{3} + \frac{50x}{3} - \frac{8}{3(x-2)} - \frac{2}{3(x-1)} + \frac{40}{3(x-2)^2} + \frac{4}{3(x-1)^2} - \frac{320}{3(x-2)^3} - \frac{4}{(x-1)^3} - \frac{960}{(x-2)^4} - \frac{8}{(x-1)^4} - \frac{1280}{(x-2)^5} + \right. \\
& \left. 4 \right) H(1, 0; \alpha_0) + \left( -\frac{29x\alpha_0}{9} - \frac{406\alpha_0}{9(x-2)} + \frac{166\alpha_0}{3(x-1)} + \frac{424\alpha_0}{9(x-2)^2} - \frac{193\alpha_0}{18(x-1)^2} - \frac{32\alpha_0}{(x-2)^3} + \frac{53\alpha_0}{9(x-1)^3} + \frac{4000\alpha_0}{9(x-2)^4} + \right. \\
& \left. \frac{175\alpha_0}{18(x-1)^4} - \frac{640\alpha_0}{9(x-2)^5} - \frac{116\alpha_0}{9(x-1)^5} - \frac{2\alpha_0}{9} + \frac{29x}{9} - \frac{326}{9(x-2)} + \frac{118}{3(x-1)} + \frac{388}{9(x-2)^2} - \frac{95}{18(x-1)^2} - \frac{560}{9(x-2)^3} - \right. \\
& \left. \frac{4}{9(x-1)^3} - \frac{3424}{9(x-2)^4} - \frac{289}{18(x-1)^4} - \frac{7360}{9(x-2)^5} - \frac{116}{9(x-1)^5} + \frac{1280}{9(x-2)^6} + \frac{20}{3} \right) H(1, 0; x) + \left( 3x\alpha_0^5 + \frac{6\alpha_0^5}{x-2} - \right. \\
& \left. \frac{3\alpha_0^5}{x-1} - 19x\alpha_0^4 - \frac{20\alpha_0^4}{x-2} + \frac{13\alpha_0^4}{x-1} + \frac{20\alpha_0^4}{(x-2)^2} - \frac{4\alpha_0^4}{(x-1)^2} + 2\alpha_0^4 + 52x\alpha_0^3 + \frac{20\alpha_0^3}{x-2} - \frac{22\alpha_0^3}{x-1} - \frac{40\alpha_0^3}{(x-2)^2} + \frac{14\alpha_0^3}{(x-1)^2} + \right. \\
& \left. \frac{80\alpha_0^3}{(x-2)^3} - \frac{6\alpha_0^3}{(x-1)^3} - 12\alpha_0^3 - 84x\alpha_0^2 + \frac{18\alpha_0^2}{x-1} - \frac{18\alpha_0^2}{(x-1)^2} + \frac{18\alpha_0^2}{(x-1)^3} + \frac{480\alpha_0^2}{(x-2)^4} - \frac{12\alpha_0^2}{(x-1)^4} + 36\alpha_0^2 + 73x\alpha_0 - \right. \\
& \left. \frac{10\alpha_0}{x-2} - \frac{7\alpha_0}{x-1} + \frac{40\alpha_0}{(x-2)^2} + \frac{10\alpha_0}{(x-1)^2} - \frac{240\alpha_0}{(x-2)^3} - \frac{18\alpha_0}{(x-1)^3} - \frac{1920\alpha_0}{(x-2)^4} - \frac{1920\alpha_0}{(x-2)^5} - 20\alpha_0 - 25x + \frac{4}{x-2} + \frac{1}{x-1} - \right. \\
& \left. \frac{20}{(x-2)^2} - \frac{2}{(x-1)^2} + \frac{160}{(x-2)^3} + \frac{6}{(x-1)^3} + \frac{1440}{(x-2)^4} + \frac{12}{(x-1)^4} + \frac{1920}{(x-2)^5} - 6 \right) H(1, 1; \alpha_0) + H(c_2(\alpha_0); x) \left( - \right. \\
& \left. \frac{40\pi^2\alpha_0}{9(x-2)^4} + \frac{320\alpha_0}{27(x-2)^4} - \frac{80\pi^2\alpha_0}{9(x-2)^5} + \frac{640\alpha_0}{27(x-2)^5} + \left( -\frac{640\alpha_0}{9(x-2)^4} - \frac{1280\alpha_0}{9(x-2)^5} + \frac{640}{9(x-2)^4} + \frac{2560}{9(x-2)^5} + \right. \right. \\
& \left. \left. \frac{2560}{9(x-2)^6} \right) H(0; \alpha_0) + \left( \frac{320\alpha_0}{3(x-2)^4} + \frac{640\alpha_0}{3(x-2)^5} - \frac{320}{3(x-2)^4} - \frac{1280}{3(x-2)^5} - \frac{1280}{3(x-2)^6} \right) H(1; \alpha_0) + \left( \frac{1280\alpha_0}{3(x-2)^4} + \right. \\
& \left. \frac{2560\alpha_0}{3(x-2)^5} - \frac{1280}{3(x-2)^4} - \frac{5120}{3(x-2)^5} - \frac{5120}{3(x-2)^6} \right) H(0, 0; \alpha_0) + \left( -\frac{640\alpha_0}{(x-2)^4} - \frac{1280\alpha_0}{(x-2)^5} + \frac{640}{(x-2)^4} + \frac{2560}{(x-2)^5} + \right. \\
& \left. \frac{2560}{(x-2)^6} \right) H(0, 1; \alpha_0) + \left( -\frac{640\alpha_0}{(x-2)^4} - \frac{1280\alpha_0}{(x-2)^5} + \frac{640}{(x-2)^4} + \frac{2560}{(x-2)^5} + \frac{2560}{(x-2)^6} \right) H(1, 0; \alpha_0) + \left( \frac{960\alpha_0}{(x-2)^4} + \right. \\
& \left. \frac{1920\alpha_0}{(x-2)^5} - \frac{960}{(x-2)^4} - \frac{3840}{(x-2)^5} - \frac{3840}{(x-2)^6} \right) H(1, 1; \alpha_0) + \frac{40\pi^2}{9(x-2)^4} - \frac{320}{27(x-2)^4} + \frac{160\pi^2}{9(x-2)^5} - \frac{1280}{27(x-2)^5} + \\
& \left. \frac{160\pi^2}{9(x-2)^6} - \frac{1280}{27(x-2)^6} \right) + H(c_1(\alpha_0); x) \left( -\frac{19x\alpha_0^5}{36} - \frac{31\alpha_0^5}{18(x-2)} + \frac{19\alpha_0^5}{36(x-1)} - \frac{9\alpha_0^5}{8} + \frac{131x\alpha_0^4}{36} + \frac{137\alpha_0^4}{36(x-2)} - \right. \\
& \left. \frac{103\alpha_0^4}{36(x-1)} - \frac{65\alpha_0^4}{9(x-2)^2} + \frac{\alpha_0^4}{(x-1)^2} + \frac{349\alpha_0^4}{72} - \frac{104x\alpha_0^3}{9} + \frac{2\alpha_0^3}{x-2} + \frac{235\alpha_0^3}{36(x-1)} + \frac{20\alpha_0^3}{9(x-2)^2} - \frac{43\alpha_0^3}{9(x-1)^2} - \frac{40\alpha_0^3}{(x-2)^3} + \right. \\
& \left. \frac{41\alpha_0^3}{18(x-1)^3} - \frac{209\alpha_0^3}{36} + \frac{80x\alpha_0^2}{3} - \frac{103\alpha_0^2}{9(x-2)} - \frac{307\alpha_0^2}{36(x-1)} + \frac{38\alpha_0^2}{(x-2)^2} + \frac{86\alpha_0^2}{9(x-1)^2} - \frac{1420\alpha_0^2}{9(x-2)^3} - \frac{73\alpha_0^2}{6(x-1)^3} - \frac{3760\alpha_0^2}{9(x-2)^4} + \right. \\
& \left. \frac{77\alpha_0^2}{9(x-1)^4} - \frac{455\alpha_0^2}{36} - \frac{367x\alpha_0}{12} - \frac{802\alpha_0}{9(x-2)} + \frac{231\alpha_0}{2(x-1)} + \frac{52\alpha_0}{(x-2)^2} - \frac{140\alpha_0}{9(x-1)^2} + \frac{2864\alpha_0}{9(x-2)^3} + \frac{125\alpha_0}{9(x-1)^3} + \frac{2160\alpha_0}{(x-2)^4} - \right. \\
& \left. \frac{\pi^2\alpha_0}{18(x-1)^4} + \frac{3029\alpha_0}{216(x-1)^4} + \frac{10240\alpha_0}{9(x-2)^5} + \frac{\pi^2\alpha_0}{18(x-1)^5} - \frac{1351\alpha_0}{108(x-1)^5} + \frac{235\alpha_0}{72} + \frac{445x}{36} + \left( \frac{2x\alpha_0^5}{3} + \frac{4\alpha_0^5}{3(x-2)} - \right. \right. \\
& \left. \left. \frac{2\alpha_0^5}{3(x-1)} + \alpha_0^5 - \frac{38x\alpha_0^4}{9} - \frac{34\alpha_0^4}{9(x-2)} + \frac{26\alpha_0^4}{9(x-1)} + \frac{40\alpha_0^4}{9(x-2)^2} - \frac{8\alpha_0^4}{9(x-1)^2} - \frac{41\alpha_0^4}{9} + \frac{104x\alpha_0^3}{9} + \frac{16\alpha_0^3}{9(x-2)} - \frac{44\alpha_0^3}{9(x-1)} - \right. \right. \\
& \left. \left. \frac{56\alpha_0^3}{9(x-2)^2} + \frac{28\alpha_0^3}{9(x-1)^2} + \frac{160\alpha_0^3}{9(x-2)^3} - \frac{4\alpha_0^3}{3(x-1)^3} + \frac{22\alpha_0^3}{3} - \frac{56x\alpha_0^2}{3} + \frac{4\alpha_0^2}{x-2} + \frac{4\alpha_0^2}{x-1} - \frac{8\alpha_0^2}{(x-2)^2} - \frac{4\alpha_0^2}{(x-1)^2} + \frac{16\alpha_0^2}{(x-2)^3} + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{4\alpha_0^2}{(x-1)^3} + \frac{320\alpha_0^2}{3(x-2)^4} - \frac{8\alpha_0^2}{3(x-1)^4} - 2\alpha_0^2 + \frac{146x\alpha_0}{9} + \frac{128\alpha_0}{9(x-2)} - \frac{74\alpha_0}{3(x-1)} - \frac{32\alpha_0}{9(x-2)^2} + \frac{52\alpha_0}{9(x-1)^2} - \frac{64\alpha_0}{(x-2)^3} - \\
& \frac{46\alpha_0}{9(x-1)^3} - \frac{704\alpha_0}{(x-2)^4} - \frac{55\alpha_0}{9(x-1)^4} - \frac{1280\alpha_0}{3(x-2)^5} + \frac{58\alpha_0}{9(x-1)^5} + \frac{29\alpha_0}{9} - \frac{50x}{9} + \frac{160}{9(x-2)} - \frac{52}{3(x-1)} - \frac{224}{9(x-2)^2} + \\
& \frac{14}{9(x-1)^2} + \frac{640}{9(x-2)^3} + \frac{8}{9(x-1)^3} + \frac{1664}{3(x-2)^4} + \frac{85}{9(x-1)^4} + \frac{2560}{3(x-2)^5} + \frac{58}{9(x-1)^5} - 5 \Big) H(0; \alpha_0) + \left( -x\alpha_0^5 - \right. \\
& \frac{2\alpha_0^5}{x-2} + \frac{\alpha_0^5}{x-1} - \frac{3\alpha_0^5}{2} + \frac{19x\alpha_0^4}{3} + \frac{17\alpha_0^4}{3(x-2)} - \frac{13\alpha_0^4}{3(x-1)} - \frac{20\alpha_0^4}{3(x-2)^2} + \frac{4\alpha_0^4}{3(x-1)^2} + \frac{41\alpha_0^4}{6} - \frac{52x\alpha_0^3}{3} - \frac{8\alpha_0^3}{3(x-2)} + \\
& \frac{22\alpha_0^3}{3(x-1)} + \frac{28\alpha_0^3}{3(x-2)^2} - \frac{14\alpha_0^3}{3(x-1)^2} - \frac{80\alpha_0^3}{3(x-2)^3} + \frac{2\alpha_0^3}{(x-1)^3} - 11\alpha_0^3 + 28x\alpha_0^2 - \frac{6\alpha_0^2}{x-2} - \frac{6\alpha_0^2}{x-1} + \frac{12\alpha_0^2}{(x-2)^2} + \frac{6\alpha_0^2}{(x-1)^2} - \\
& \frac{24\alpha_0^2}{(x-2)^3} - \frac{6\alpha_0^2}{(x-1)^3} - \frac{160\alpha_0^2}{(x-2)^4} + \frac{4\alpha_0^2}{(x-1)^4} + 3\alpha_0^2 - \frac{73x\alpha_0}{3} - \frac{64\alpha_0}{3(x-2)} + \frac{37\alpha_0}{x-1} + \frac{16\alpha_0}{3(x-2)^2} - \frac{26\alpha_0}{3(x-1)^2} + \frac{96\alpha_0}{(x-2)^3} + \\
& \frac{23\alpha_0}{3(x-1)^3} + \frac{1056\alpha_0}{(x-2)^4} + \frac{55\alpha_0}{6(x-1)^4} + \frac{64\alpha_0}{(x-2)^5} - \frac{29\alpha_0}{3(x-1)^5} - \frac{29\alpha_0}{6} + \frac{25x}{3} - \frac{80}{3(x-2)} + \frac{26}{x-1} + \frac{112}{3(x-2)^2} - \frac{7}{3(x-1)^2} - \\
& \frac{320}{3(x-2)^3} - \frac{4}{3(x-1)^3} - \frac{832}{(x-2)^4} - \frac{85}{6(x-1)^4} - \frac{1280}{(x-2)^5} - \frac{29}{3(x-1)^5} + \frac{15}{2} \Big) H(1; \alpha_0) + \left( \frac{16\alpha_0}{3(x-1)^4} - \frac{16\alpha_0}{3(x-1)^5} - \right. \\
& \frac{16}{3(x-1)^4} + \frac{16}{3(x-1)^5} \Big) H(0, 0; \alpha_0) + \left( -\frac{8\alpha_0}{(x-1)^4} + \frac{8\alpha_0}{(x-1)^5} + \frac{8}{(x-1)^4} + \frac{8}{(x-1)^5} \right) H(0, 1; \alpha_0) + \left( -\frac{8\alpha_0}{(x-1)^4} + \right. \\
& \left. \frac{8\alpha_0}{(x-1)^5} + \frac{8}{(x-1)^4} + \frac{8}{(x-1)^5} \right) H(1, 0; \alpha_0) + \left( \frac{12\alpha_0}{(x-1)^4} - \frac{12\alpha_0}{(x-1)^5} - \frac{12}{(x-1)^4} - \frac{12}{(x-1)^5} \right) H(1, 1; \alpha_0) - \frac{92}{x-2} + \\
& \frac{1133}{12(x-1)} + \frac{1040}{9(x-2)^2} - \frac{223}{36(x-1)^2} - \frac{3008}{9(x-2)^3} - \frac{103}{36(x-1)^3} - \frac{13600}{9(x-2)^4} + \frac{\pi^2}{18(x-1)^4} - \frac{4223}{216(x-1)^4} - \frac{20480}{9(x-2)^5} + \\
& \frac{\pi^2}{18(x-1)^5} - \frac{1351}{108(x-1)^5} + \frac{275}{24} \Big) + \left( -\frac{14x\alpha_0}{3} - \frac{480\alpha_0}{(x-2)^4} + \frac{64\alpha_0}{3(x-1)^4} - \frac{800\alpha_0}{(x-2)^5} - \frac{64\alpha_0}{3(x-1)^5} + \frac{28\alpha_0}{3} + \frac{14x}{3} + \right. \\
& \frac{480}{(x-2)^4} - \frac{64}{3(x-1)^4} + \frac{1760}{(x-2)^5} - \frac{64}{3(x-1)^5} + \frac{1600}{(x-2)^6} \Big) H(0; \alpha_0) H(1, 1; x) + \left( \frac{29x\alpha_0}{9} + \frac{406\alpha_0}{9(x-2)} - \frac{166\alpha_0}{3(x-1)} - \right. \\
& \frac{424\alpha_0}{9(x-2)^2} + \frac{193\alpha_0}{18(x-1)^2} + \frac{32\alpha_0}{(x-2)^3} - \frac{53\alpha_0}{9(x-1)^3} - \frac{4000\alpha_0}{9(x-2)^4} - \frac{175\alpha_0}{18(x-1)^4} + \frac{640\alpha_0}{9(x-2)^5} + \frac{116\alpha_0}{9(x-1)^5} + \frac{2\alpha_0}{9} - \frac{29x}{9} + \\
& \left( -\frac{8x\alpha_0}{3} - \frac{832\alpha_0}{3(x-2)^4} + \frac{32\alpha_0}{3(x-1)^4} - \frac{1280\alpha_0}{3(x-2)^5} - \frac{32\alpha_0}{3(x-1)^5} + \frac{16\alpha_0}{3} + \frac{8x}{3} + \frac{832}{3(x-2)^4} - \frac{32}{3(x-1)^4} + \frac{2944}{3(x-2)^5} - \right. \\
& \left. \frac{32}{3(x-1)^5} + \frac{2560}{3(x-2)^6} \right) H(0; \alpha_0) + \left( 4x\alpha_0 + \frac{416\alpha_0}{(x-2)^4} - \frac{16\alpha_0}{(x-1)^4} + \frac{640\alpha_0}{(x-2)^5} + \frac{16\alpha_0}{(x-1)^5} - 8\alpha_0 - 4x - \frac{416}{(x-2)^4} + \right. \\
& \left. \frac{16}{(x-1)^4} - \frac{1472}{(x-2)^5} + \frac{16}{(x-1)^5} - \frac{1280}{(x-2)^6} \right) H(1; \alpha_0) + \frac{326}{9(x-2)} - \frac{118}{3(x-1)} - \frac{388}{9(x-2)^2} + \frac{95}{18(x-1)^2} + \frac{560}{9(x-2)^3} + \\
& \frac{4}{9(x-1)^3} + \frac{3424}{9(x-2)^4} + \frac{289}{18(x-1)^4} + \frac{7360}{9(x-2)^5} + \frac{116}{9(x-1)^5} - \frac{1280}{9(x-2)^6} - \frac{20}{3} \Big) H(1, c_1(\alpha_0); x) + \left( \frac{2x\alpha_0}{3} + \right. \\
& \frac{2080\alpha_0}{3(x-2)^4} - \frac{2\alpha_0}{3(x-1)^4} + \frac{3200\alpha_0}{3(x-2)^5} + \frac{2\alpha_0}{3(x-1)^5} - \frac{4\alpha_0}{3} - \frac{2x}{3} - \frac{2080}{3(x-2)^4} + \frac{2}{3(x-1)^4} - \frac{7360}{3(x-2)^5} + \frac{2}{3(x-1)^5} - \\
& \frac{6400}{3(x-2)^6} \Big) H(0; \alpha_0) H(2, 1; x) + \left( \frac{1600\alpha_0}{9(x-2)^4} + \frac{3200\alpha_0}{9(x-2)^5} + \left( -\frac{3200\alpha_0}{3(x-2)^4} - \frac{6400\alpha_0}{3(x-2)^5} + \frac{3200}{3(x-2)^4} + \frac{12800}{3(x-2)^5} + \right. \right. \\
& \left. \frac{12800}{3(x-2)^6} \right) H(0; \alpha_0) + \left( \frac{1600\alpha_0}{(x-2)^4} + \frac{3200\alpha_0}{(x-2)^5} - \frac{1600}{(x-2)^4} - \frac{6400}{(x-2)^5} - \frac{6400}{(x-2)^6} \right) H(1; \alpha_0) - \frac{1600}{9(x-2)^4} - \\
& \frac{6400}{9(x-2)^5} - \frac{6400}{9(x-2)^6} \Big) H(2, c_2(\alpha_0); x) + \left( \frac{x\alpha_0^5}{2} + \frac{5\alpha_0^5}{6(x-2)} - \frac{\alpha_0^5}{3(x-1)} + \alpha_0^5 - \frac{19x\alpha_0^4}{6} - \frac{35\alpha_0^4}{18(x-2)} + \frac{13\alpha_0^4}{9(x-1)} + \right. \\
& \frac{25\alpha_0^4}{9(x-2)^2} - \frac{4\alpha_0^4}{9(x-1)^2} - \frac{14\alpha_0^4}{3} + \frac{26x\alpha_0^3}{3} - \frac{5\alpha_0^3}{9(x-2)} - \frac{22\alpha_0^3}{9(x-1)} - \frac{20\alpha_0^3}{9(x-2)^2} + \frac{14\alpha_0^3}{9(x-1)^2} + \frac{100\alpha_0^3}{9(x-2)^3} - \frac{2\alpha_0^3}{3(x-1)^3} + \\
& 8\alpha_0^3 - 14x\alpha_0^2 + \frac{5\alpha_0^2}{x-2} + \frac{2\alpha_0^2}{x-1} - \frac{10\alpha_0^2}{(x-2)^2} - \frac{2\alpha_0^2}{(x-1)^2} + \frac{20\alpha_0^2}{(x-2)^3} + \frac{2\alpha_0^2}{(x-1)^3} + \frac{200\alpha_0^2}{3(x-2)^4} - \frac{4\alpha_0^2}{3(x-1)^4} - 4\alpha_0^2 + \frac{73x\alpha_0}{6} - \\
& \frac{40\alpha_0}{9(x-2)} - \frac{73\alpha_0}{18(x-1)} + \frac{160\alpha_0}{9(x-2)^2} + \frac{5\alpha_0}{18(x-1)^2} - \frac{80\alpha_0}{(x-2)^3} - \frac{11\alpha_0}{9(x-1)^3} - \frac{480\alpha_0}{(x-2)^4} - \frac{32\alpha_0}{9(x-1)^4} - \frac{800\alpha_0}{3(x-2)^5} + \frac{29\alpha_0}{9(x-1)^5} + \\
& \frac{11\alpha_0}{3} - \frac{25x}{6} + \left( \frac{8\alpha_0}{3(x-1)^4} - \frac{8\alpha_0}{3(x-1)^5} - \frac{8}{3(x-1)^4} - \frac{8}{3(x-1)^5} \right) H(0; \alpha_0) + \left( -\frac{4\alpha_0}{(x-1)^4} + \frac{4\alpha_0}{(x-1)^5} + \frac{4}{(x-1)^4} + \right. \\
& \left. \frac{4}{(x-1)^5} \right) H(1; \alpha_0) + \frac{40}{9(x-2)} - \frac{71}{18(x-1)} - \frac{80}{9(x-2)^2} - \frac{1}{6(x-1)^2} + \frac{400}{9(x-2)^3} + \frac{7}{9(x-1)^3} + \frac{1280}{3(x-2)^4} + \frac{38}{9(x-1)^4} + \\
& \frac{1600}{3(x-2)^5} + \frac{29}{9(x-1)^5} - 4 \Big) H(c_1(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{x\alpha_0^5}{6} - \frac{\alpha_0^5}{6(x-1)} + \frac{\alpha_0^5}{2} + \frac{19x\alpha_0^4}{18} + \frac{13\alpha_0^4}{18(x-1)} - \frac{2\alpha_0^4}{9(x-1)^2} - \right. \\
& \frac{47\alpha_0^4}{18} - \frac{26x\alpha_0^3}{9} - \frac{11\alpha_0^3}{9(x-1)} + \frac{7\alpha_0^3}{9(x-1)^2} - \frac{\alpha_0^3}{3(x-1)^3} + \frac{17\alpha_0^3}{3} + \frac{14x\alpha_0^2}{3} + \frac{\alpha_0^2}{x-1} - \frac{\alpha_0^2}{(x-1)^2} + \frac{\alpha_0^2}{(x-1)^3} - \frac{2\alpha_0^2}{3(x-1)^4} - 7\alpha_0^2 - \\
& \frac{73x\alpha_0}{18} + \frac{8\alpha_0}{9(x-2)} + \frac{\alpha_0}{18(x-1)} - \frac{8\alpha_0}{9(x-2)^2} + \frac{\alpha_0}{9(x-1)^2} + \frac{\alpha_0}{3(x-1)^3} - \frac{320\alpha_0}{9(x-2)^4} - \frac{4\alpha_0}{3(x-1)^4} - \frac{640\alpha_0}{9(x-2)^5} + \frac{41\alpha_0}{18} + \\
& \frac{25x}{18} + \left( \frac{832\alpha_0}{3(x-2)^4} + \frac{1280\alpha_0}{3(x-2)^5} - \frac{832}{3(x-2)^4} - \frac{2944}{3(x-2)^5} - \frac{2560}{3(x-2)^6} \right) H(0; \alpha_0) + \left( -\frac{416\alpha_0}{(x-2)^4} - \frac{640\alpha_0}{(x-2)^5} + \frac{416}{(x-2)^4} + \right. \\
& \left. \frac{1472}{(x-2)^5} + \frac{1280}{(x-2)^6} \right) H(1; \alpha_0) + \frac{16}{9(x-2)} - \frac{7}{18(x-1)} - \frac{8}{9(x-2)^2} - \frac{5}{9(x-1)^2} + \frac{16}{9(x-2)^3} - \frac{1}{(x-1)^3} + \frac{320}{9(x-2)^4} -
\end{aligned}$$

$$\begin{aligned}
& \frac{2}{3(x-1)^4} + \frac{1280}{9(x-2)^5} + \frac{1280}{9(x-2)^6} + \frac{7}{6} \Big) H(c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{16x\alpha_0}{3} + \frac{1280\alpha_0}{3(x-2)^4} + \frac{16\alpha_0}{3(x-1)^4} + \frac{2560\alpha_0}{3(x-2)^5} - \right. \\
& \left. \frac{16\alpha_0}{3(x-1)^5} - \frac{32\alpha_0}{3} - \frac{16x}{3} - \frac{1280}{3(x-2)^4} - \frac{16}{3(x-1)^4} - \frac{5120}{3(x-2)^5} - \frac{16}{3(x-1)^5} - \frac{5120}{3(x-2)^6} \right) H(0, 0, 0; \alpha_0) + \left( \frac{16x\alpha_0}{3} - \right. \\
& \left. \frac{1280\alpha_0}{3(x-2)^4} - \frac{16\alpha_0}{3(x-1)^4} - \frac{2560\alpha_0}{3(x-2)^5} + \frac{16\alpha_0}{3(x-1)^5} - \frac{32\alpha_0}{3} - \frac{16x}{3} + \frac{1280}{3(x-2)^4} + \frac{16}{3(x-1)^4} + \frac{5120}{3(x-2)^5} + \frac{16}{3(x-1)^5} + \right. \\
& \left. \frac{5120}{3(x-2)^6} \right) H(0, 0, 0; x) + \left( -8x\alpha_0 - \frac{640\alpha_0}{(x-2)^4} - \frac{8\alpha_0}{(x-1)^4} - \frac{1280\alpha_0}{(x-2)^5} + \frac{8\alpha_0}{(x-1)^5} + 16\alpha_0 + 8x + \frac{640}{(x-2)^4} + \right. \\
& \left. \frac{8}{(x-1)^4} + \frac{2560}{(x-2)^5} + \frac{8}{(x-1)^5} + \frac{2560}{(x-2)^6} \right) H(0, 0, 1; \alpha_0) + \left( -\frac{8x\alpha_0}{3} - \frac{736\alpha_0}{3(x-2)^4} + \frac{8\alpha_0}{3(x-1)^4} - \frac{1280\alpha_0}{3(x-2)^5} - \right. \\
& \left. \frac{8\alpha_0}{3(x-1)^5} + \frac{16\alpha_0}{3} + \frac{8x}{3} + \frac{736}{3(x-2)^4} - \frac{8}{3(x-1)^4} + \frac{2752}{3(x-2)^5} - \frac{8}{3(x-1)^5} + \frac{2560}{3(x-2)^6} \right) H(0, 0, c_1(\alpha_0); x) + \\
& \left( \frac{1280\alpha_0}{3(x-2)^4} + \frac{2560\alpha_0}{3(x-2)^5} - \frac{1280}{3(x-2)^4} - \frac{5120}{3(x-2)^5} - \frac{5120}{3(x-2)^6} \right) H(0, 0, c_2(\alpha_0); x) + \left( -8x\alpha_0 - \frac{640\alpha_0}{(x-2)^4} - \right. \\
& \left. \frac{8\alpha_0}{(x-1)^4} - \frac{1280\alpha_0}{(x-2)^5} + \frac{8\alpha_0}{(x-1)^5} + 16\alpha_0 + 8x + \frac{640}{(x-2)^4} + \frac{8}{(x-1)^4} + \frac{2560}{(x-2)^5} + \frac{8}{(x-1)^5} + \right. \\
& \left. \frac{2560}{(x-2)^6} \right) H(0, 1, 0; \alpha_0) + \left( \frac{20x\alpha_0}{3} + \frac{1984\alpha_0}{3(x-2)^4} - \frac{20\alpha_0}{3(x-1)^4} + \frac{3200\alpha_0}{3(x-2)^5} + \frac{20\alpha_0}{3(x-1)^5} - \frac{40\alpha_0}{3} - \frac{20x}{3} - \frac{1984}{3(x-2)^4} + \right. \\
& \left. \frac{20}{3(x-1)^4} - \frac{7168}{3(x-2)^5} + \frac{20}{3(x-1)^5} - \frac{6400}{3(x-2)^6} \right) H(0, 1, 0; x) + \left( 12x\alpha_0 + \frac{960\alpha_0}{(x-2)^4} + \frac{12\alpha_0}{(x-1)^4} + \frac{1920\alpha_0}{(x-2)^5} - \right. \\
& \left. \frac{12\alpha_0}{(x-1)^5} - 24\alpha_0 - 12x - \frac{960}{(x-2)^4} - \frac{12}{(x-1)^4} - \frac{3840}{(x-2)^5} - \frac{12}{(x-1)^5} - \frac{3840}{(x-2)^6} \right) H(0, 1, 1; \alpha_0) + \\
& \left( -\frac{20x\alpha_0}{3} - \frac{1984\alpha_0}{3(x-2)^4} + \frac{20\alpha_0}{3(x-1)^4} - \frac{3200\alpha_0}{3(x-2)^5} - \frac{20\alpha_0}{3(x-1)^5} + \frac{40\alpha_0}{3} + \frac{20x}{3} + \frac{1984}{3(x-2)^4} - \frac{20}{3(x-1)^4} + \right. \\
& \left. \frac{7168}{3(x-2)^5} - \frac{20}{3(x-1)^5} + \frac{6400}{3(x-2)^6} \right) H(0, 1, c_1(\alpha_0); x) + \left( \frac{3200\alpha_0}{3(x-2)^4} + \frac{6400\alpha_0}{3(x-2)^5} - \frac{3200}{3(x-2)^4} - \frac{12800}{3(x-2)^5} - \right. \\
& \left. \frac{12800}{3(x-2)^6} \right) H(0, 2, 0; x) + \left( -\frac{3200\alpha_0}{3(x-2)^4} - \frac{6400\alpha_0}{3(x-2)^5} + \frac{3200}{3(x-2)^4} + \frac{12800}{3(x-2)^5} + \frac{12800}{3(x-2)^6} \right) H(0, 2, c_2(\alpha_0); x) + \\
& \left( -2x\alpha_0 - \frac{640\alpha_0}{3(x-2)^4} + \frac{4\alpha_0}{3(x-1)^4} - \frac{800\alpha_0}{3(x-2)^5} - \frac{4\alpha_0}{3(x-1)^5} + 4\alpha_0 + 2x + \frac{640}{3(x-2)^4} - \frac{4}{3(x-1)^4} + \frac{2080}{3(x-2)^5} - \right. \\
& \left. \frac{4}{3(x-1)^5} + \frac{1600}{3(x-2)^6} \right) H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{2x\alpha_0}{3} + \frac{832\alpha_0}{3(x-2)^4} - \frac{2\alpha_0}{3(x-1)^4} + \frac{1280\alpha_0}{3(x-2)^5} + \frac{2\alpha_0}{3(x-1)^5} - \right. \\
& \left. \frac{4\alpha_0}{3} - \frac{2x}{3} - \frac{832}{3(x-2)^4} + \frac{2}{3(x-1)^4} - \frac{2944}{3(x-2)^5} + \frac{2}{3(x-1)^5} - \frac{2560}{3(x-2)^6} \right) H(0, c_2(\alpha_0), c_1(\alpha_0); x) + \\
& \left( \frac{8x\alpha_0}{3} + \frac{832\alpha_0}{3(x-2)^4} - \frac{32\alpha_0}{3(x-1)^4} + \frac{1280\alpha_0}{3(x-2)^5} + \frac{32\alpha_0}{3(x-1)^5} - \frac{16\alpha_0}{3} - \frac{8x}{3} - \frac{832}{3(x-2)^4} + \frac{32}{3(x-1)^4} - \frac{2944}{3(x-2)^5} + \right. \\
& \left. \frac{32}{3(x-1)^5} - \frac{2560}{3(x-2)^6} \right) H(1, 0, 0; x) + \left( -\frac{2x\alpha_0}{3} - \frac{64\alpha_0}{(x-2)^4} + \frac{16\alpha_0}{3(x-1)^4} - \frac{160\alpha_0}{(x-2)^5} - \frac{16\alpha_0}{3(x-1)^5} + \frac{4\alpha_0}{3} + \frac{2x}{3} + \right. \\
& \left. \frac{64}{(x-2)^4} - \frac{16}{3(x-1)^4} + \frac{288}{(x-2)^5} - \frac{16}{3(x-1)^5} + \frac{320}{(x-2)^6} \right) H(1, 0, c_1(\alpha_0); x) + \left( \frac{14x\alpha_0}{3} + \frac{480\alpha_0}{(x-2)^4} - \frac{64\alpha_0}{3(x-1)^4} + \right. \\
& \left. \frac{800\alpha_0}{(x-2)^5} + \frac{64\alpha_0}{3(x-1)^5} - \frac{28\alpha_0}{3} - \frac{14x}{3} - \frac{480}{(x-2)^4} + \frac{64}{3(x-1)^4} - \frac{1760}{(x-2)^5} + \frac{64}{3(x-1)^5} - \frac{1600}{(x-2)^6} \right) H(1, 1, 0; x) + \\
& \left( -\frac{14x\alpha_0}{3} - \frac{480\alpha_0}{(x-2)^4} + \frac{64\alpha_0}{3(x-1)^4} - \frac{800\alpha_0}{(x-2)^5} - \frac{64\alpha_0}{3(x-1)^5} + \frac{28\alpha_0}{3} + \frac{14x}{3} + \frac{480}{(x-2)^4} - \frac{64}{3(x-1)^4} + \frac{1760}{(x-2)^5} - \right. \\
& \left. \frac{64}{3(x-1)^5} + \frac{1600}{(x-2)^6} \right) H(1, 1, c_1(\alpha_0); x) + \left( -2x\alpha_0 - \frac{640\alpha_0}{3(x-2)^4} + \frac{16\alpha_0}{3(x-1)^4} - \frac{800\alpha_0}{3(x-2)^5} - \frac{16\alpha_0}{3(x-1)^5} + 4\alpha_0 + \right. \\
& \left. 2x + \frac{640}{3(x-2)^4} - \frac{16}{3(x-1)^4} + \frac{2080}{3(x-2)^5} - \frac{16}{3(x-1)^5} + \frac{1600}{3(x-2)^6} \right) H(1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{3200\alpha_0}{3(x-2)^4} + \right. \\
& \left. \frac{6400\alpha_0}{3(x-2)^5} - \frac{3200}{3(x-2)^4} - \frac{12800}{3(x-2)^5} - \frac{12800}{3(x-2)^6} \right) H(2, 0, 0; x) + \left( \frac{2x\alpha_0}{3} + \frac{2080\alpha_0}{3(x-2)^4} - \frac{2\alpha_0}{3(x-1)^4} + \frac{3200\alpha_0}{3(x-2)^5} + \right. \\
& \left. \frac{2\alpha_0}{3(x-1)^5} - \frac{4\alpha_0}{3} - \frac{2x}{3} - \frac{2080}{3(x-2)^4} + \frac{2}{3(x-1)^4} - \frac{7360}{3(x-2)^5} + \frac{2}{3(x-1)^5} - \frac{6400}{3(x-2)^6} \right) H(2, 0, c_1(\alpha_0); x) + \\
& \left( -\frac{3200\alpha_0}{3(x-2)^4} - \frac{6400\alpha_0}{3(x-2)^5} + \frac{3200}{3(x-2)^4} + \frac{12800}{3(x-2)^5} + \frac{12800}{3(x-2)^6} \right) H(2, 0, c_2(\alpha_0); x) + \left( -\frac{2x\alpha_0}{3} - \right. \\
& \left. \frac{2080\alpha_0}{3(x-2)^4} + \frac{2\alpha_0}{3(x-1)^4} - \frac{3200\alpha_0}{3(x-2)^5} - \frac{2\alpha_0}{3(x-1)^5} + \frac{4\alpha_0}{3} + \frac{2x}{3} + \frac{2080}{3(x-2)^4} - \frac{2}{3(x-1)^4} + \frac{7360}{3(x-2)^5} - \frac{2}{3(x-1)^5} + \right. \\
& \left. \frac{6400}{3(x-2)^6} \right) H(2, 1, 0; x) + \left( \frac{2x\alpha_0}{3} + \frac{2080\alpha_0}{3(x-2)^4} - \frac{2\alpha_0}{3(x-1)^4} + \frac{3200\alpha_0}{3(x-2)^5} + \frac{2\alpha_0}{3(x-1)^5} - \frac{4\alpha_0}{3} - \frac{2x}{3} - \frac{2080}{3(x-2)^4} + \right. \\
& \left. \frac{2}{3(x-1)^4} - \frac{7360}{3(x-2)^5} + \frac{2}{3(x-1)^5} - \frac{6400}{3(x-2)^6} \right) H(2, 1, c_1(\alpha_0); x) + \left( -\frac{8000\alpha_0}{3(x-2)^4} - \frac{16000\alpha_0}{3(x-2)^5} + \right. \\
& \left. \frac{8000}{3(x-2)^4} + \frac{32000}{3(x-2)^5} + \frac{32000}{3(x-2)^6} \right) H(2, 2, 0; x) + \left( \frac{8000\alpha_0}{3(x-2)^4} + \frac{16000\alpha_0}{3(x-2)^5} - \frac{8000}{3(x-2)^4} - \frac{32000}{3(x-2)^5} - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{32000}{3(x-2)^6} \right) H(2, 2, c_2(\alpha_0); x) + \left( -\frac{2x\alpha_0}{3} - \frac{2080\alpha_0}{3(x-2)^4} + \frac{2\alpha_0}{3(x-1)^4} - \frac{3200\alpha_0}{3(x-2)^5} - \frac{2\alpha_0}{3(x-1)^5} + \frac{4\alpha_0}{3} + \frac{2x}{3} + \right. \\
& \left. \frac{2080}{3(x-2)^4} - \frac{2}{3(x-1)^4} + \frac{7360}{3(x-2)^5} - \frac{2}{3(x-1)^5} + \frac{6400}{3(x-2)^6} \right) H(2, c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{4\alpha_0}{3(x-1)^4} - \right. \\
& \left. \frac{4\alpha_0}{3(x-1)^5} - \frac{4}{3(x-1)^4} - \frac{4}{3(x-1)^5} \right) H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{2\alpha_0}{3(x-1)^4} - \frac{2\alpha_0}{3(x-1)^5} - \frac{2}{3(x-1)^4} - \right. \\
& \left. \frac{2}{3(x-1)^5} \right) H(c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{32\alpha_0}{(x-2)^4} + \frac{32}{(x-2)^4} + \frac{64}{(x-2)^5} \right) H(c_2(\alpha_0), 0, c_1(\alpha_0); x) + \\
& \left( \frac{640\alpha_0}{3(x-2)^4} + \frac{800\alpha_0}{3(x-2)^5} - \frac{640}{3(x-2)^4} - \frac{2080}{3(x-2)^5} - \frac{1600}{3(x-2)^6} \right) H(c_2(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + H(2, 0; x) \left( -\frac{1600\alpha_0}{9(x-2)^4} - \frac{3200\alpha_0}{9(x-2)^5} - \frac{3200 \ln 2 \alpha_0}{3(x-2)^4} - \frac{6400 \ln 2 \alpha_0}{3(x-2)^5} + \frac{1600}{9(x-2)^4} + \frac{6400}{9(x-2)^5} + \frac{6400}{9(x-2)^6} + \frac{3200 \ln 2}{3(x-2)^4} + \frac{12800 \ln 2}{3(x-2)^5} + \frac{12800 \ln 2}{3(x-2)^6} \right) + H(0, 2; x) \left( -\frac{3200 \ln 2 \alpha_0}{3(x-2)^4} - \frac{6400 \ln 2 \alpha_0}{3(x-2)^5} + \left( -\frac{3200\alpha_0}{3(x-2)^4} - \frac{6400\alpha_0}{3(x-2)^5} + \frac{3200}{3(x-2)^4} + \frac{12800}{3(x-2)^5} + \frac{12800}{3(x-2)^6} \right) H(0; \alpha_0) + \frac{3200 \ln 2}{3(x-2)^4} + \frac{12800 \ln 2}{3(x-2)^5} + \frac{12800 \ln 2}{3(x-2)^6} \right) + H(0, 0; x) \left( -\frac{58x\alpha_0}{9} - \frac{128\alpha_0}{9(x-2)} + \frac{74\alpha_0}{3(x-1)} + \frac{32\alpha_0}{9(x-2)^2} - \frac{52\alpha_0}{9(x-1)^2} + \frac{64\alpha_0}{(x-2)^3} + \frac{46\alpha_0}{9(x-1)^3} + \frac{4480\alpha_0}{9(x-2)^4} + \frac{31\alpha_0}{9(x-1)^4} + \frac{1280\alpha_0}{9(x-2)^5} - \frac{58\alpha_0}{9(x-1)^5} + \frac{1280 \ln 2 \alpha_0}{3(x-2)^4} + \frac{2560 \ln 2 \alpha_0}{3(x-2)^5} + \frac{71\alpha_0}{9} + \frac{58x}{9} - \frac{160}{9(x-2)} + \frac{52}{3(x-1)} + \frac{224}{9(x-2)^2} - \frac{14}{9(x-1)^2} - \frac{640}{9(x-2)^3} - \frac{8}{9(x-1)^3} - \frac{5632}{9(x-2)^4} - \frac{85}{9(x-1)^4} - \frac{10240}{9(x-2)^5} - \frac{58}{9(x-1)^5} - \frac{2560}{9(x-2)^6} - \frac{1280 \ln 2}{3(x-2)^4} - \frac{5120 \ln 2}{3(x-2)^5} - \frac{5120 \ln 2}{3(x-2)^6} + 5 \right) + H(2, 2; x) \left( \frac{8000 \ln 2 \alpha_0}{3(x-2)^4} + \frac{16000 \ln 2 \alpha_0}{3(x-2)^5} + \left( \frac{8000\alpha_0}{3(x-2)^4} + \frac{16000\alpha_0}{3(x-2)^5} - \frac{8000}{3(x-2)^4} - \frac{32000}{3(x-2)^5} - \frac{32000}{3(x-2)^6} \right) H(0; \alpha_0) - \frac{8000 \ln 2}{3(x-2)^4} - \frac{32000 \ln 2}{3(x-2)^5} - \frac{32000 \ln 2}{3(x-2)^6} \right) + H(0; x) \left( \frac{1}{2}\pi^2 x \alpha_0 + \frac{1351x\alpha_0}{108} + \frac{284\alpha_0}{3(x-2)} - \frac{691\alpha_0}{6(x-1)} - \frac{664\alpha_0}{9(x-2)^2} + \frac{131\alpha_0}{9(x-1)^2} - \frac{560\alpha_0}{3(x-2)^3} - \frac{101\alpha_0}{9(x-1)^3} + \frac{8\pi^2\alpha_0}{3(x-2)^4} - \frac{31040\alpha_0}{27(x-2)^4} - \frac{7\pi^2\alpha_0}{18(x-1)^4} - \frac{1181\alpha_0}{216(x-1)^4} - \frac{80\pi^2\alpha_0}{3(x-2)^5} - \frac{640\alpha_0}{27(x-2)^5} + \frac{7\pi^2\alpha_0}{18(x-1)^5} + \frac{1351\alpha_0}{108(x-1)^5} - \frac{640 \ln^2 2 \alpha_0}{3(x-2)^4} - \frac{1280 \ln^2 2 \alpha_0}{3(x-2)^5} - \frac{640 \ln 2 \alpha_0}{9(x-2)^4} - \frac{1280 \ln 2 \alpha_0}{9(x-2)^5} - \pi^2 \alpha_0 - \frac{2929\alpha_0}{216} - \frac{\pi^2 x}{2} - \frac{1351 x}{108} + \frac{92}{x-2} - \frac{1133}{12(x-1)} - \frac{1040}{9(x-2)^2} + \frac{223}{36(x-1)^2} + \frac{3008}{9(x-2)^3} + \frac{103}{36(x-1)^3} - \frac{8\pi^2}{3(x-2)^4} + \frac{41120}{27(x-2)^4} + \frac{7\pi^2}{18(x-1)^4} + \frac{4223}{216(x-1)^4} + \frac{64\pi^2}{3(x-2)^5} + \frac{62720}{27(x-2)^5} + \frac{7\pi^2}{18(x-1)^5} + \frac{1351}{108(x-1)^5} + \frac{160\pi^2}{3(x-2)^6} + \frac{1280}{27(x-2)^6} + \frac{640 \ln^2 2}{3(x-2)^4} + \frac{2560 \ln^2 2}{3(x-2)^5} + \frac{2560 \ln^2 2}{3(x-2)^6} + \frac{640 \ln 2}{9(x-2)^4} + \frac{2560 \ln 2}{9(x-2)^5} + \frac{2560 \ln 2}{9(x-2)^6} - \frac{275}{24} \right) + H(2; x) \left( -\frac{1}{6}\pi^2 x \alpha_0 + \frac{40\pi^2\alpha_0}{9(x-2)^4} + \frac{\pi^2\alpha_0}{6(x-1)^4} + \frac{800\pi^2\alpha_0}{9(x-2)^5} - \frac{\pi^2\alpha_0}{6(x-1)^5} + \frac{1600 \ln^2 2 \alpha_0}{3(x-2)^4} + \frac{3200 \ln^2 2 \alpha_0}{3(x-2)^5} + \frac{1600 \ln 2 \alpha_0}{9(x-2)^4} + \frac{3200 \ln 2 \alpha_0}{9(x-2)^5} + \frac{\pi^2\alpha_0}{3} + \frac{\pi^2 x}{6} + \left( \frac{1600\alpha_0}{9(x-2)^4} + \frac{3200\alpha_0}{9(x-2)^5} - \frac{1600}{9(x-2)^4} - \frac{6400}{9(x-2)^5} - \frac{6400}{9(x-2)^6} \right) H(0; \alpha_0) + \left( -\frac{3200\alpha_0}{3(x-2)^4} - \frac{6400\alpha_0}{3(x-2)^5} + \frac{3200}{3(x-2)^4} + \frac{12800}{3(x-2)^5} + \frac{12800}{3(x-2)^6} \right) H(0, 0; \alpha_0) + \left( \frac{1600\alpha_0}{(x-2)^4} + \frac{3200\alpha_0}{(x-2)^5} - \frac{1600}{(x-2)^4} - \frac{6400}{(x-2)^5} - \frac{6400}{(x-2)^6} \right) H(0, 1; \alpha_0) - \frac{40\pi^2}{9(x-2)^4} - \frac{\pi^2}{6(x-1)^4} - \frac{880\pi^2}{9(x-2)^5} - \frac{\pi^2}{6(x-1)^5} - \frac{1600\pi^2}{9(x-2)^6} - \frac{1600 \ln^2 2}{3(x-2)^4} - \frac{6400 \ln^2 2}{3(x-2)^5} - \frac{6400 \ln^2 2}{3(x-2)^6} - \frac{1600 \ln 2}{9(x-2)^4} - \frac{6400 \ln 2}{9(x-2)^5} - \frac{6400 \ln 2}{9(x-2)^6} - \frac{16\pi^2}{9(x-2)} + \frac{461\pi^2}{216(x-1)} + \frac{22\pi^2}{9(x-2)^2} - \frac{23\pi^2}{54(x-1)^2} - \frac{4\pi^2}{(x-2)^3} - \frac{29\pi^2}{108(x-1)^3} - \frac{656\pi^2}{27(x-2)^4} - \frac{28\pi^2}{27(x-1)^4} - \frac{1760\pi^2}{27(x-2)^5} - \frac{29\pi^2}{54(x-1)^5} - \frac{320\pi^2}{27(x-2)^6} - \frac{17}{12}x\zeta_3 - \frac{224\zeta_3}{3(x-2)^4} + \frac{7\zeta_3}{4(x-1)^4} - \frac{728\zeta_3}{3(x-2)^5} + \frac{7\zeta_3}{4(x-1)^5} - \frac{560\zeta_3}{3(x-2)^6} - \frac{640 \ln^3 2}{9(x-2)^4} - \frac{2560 \ln^3 2}{9(x-2)^5} - \frac{2560 \ln^3 2}{9(x-2)^6} - \frac{320 \ln^2 2}{9(x-2)^4} - \frac{1280 \ln^2 2}{9(x-2)^5} - \frac{1280 \ln^2 2}{9(x-2)^6} + \frac{1}{6}\pi^2 x \ln 2 + \frac{32\pi^2 \ln 2}{3(x-2)^4} - \frac{320 \ln 2}{27(x-2)^4} - \frac{\pi^2 \ln 2}{6(x-1)^4} - \frac{16\pi^2 \ln 2}{3(x-2)^5} - \frac{1280 \ln 2}{27(x-2)^5} - \frac{\pi^2 \ln 2}{6(x-1)^5} - \frac{160\pi^2 \ln 2}{3(x-2)^6} - \frac{1280 \ln 2}{27(x-2)^6} + \frac{11\pi^2}{24} \}.
\end{aligned}$$

**E.4 The  $\mathcal{B}$  integral for  $k = -1$  and  $\delta = -1$**

The  $\varepsilon$  expansion for this integral reads

$$\begin{aligned}
\mathcal{I}(x, \varepsilon; \alpha_0, 3 + d_1\varepsilon; 1, -1, -1, g_B) &= x \mathcal{B}(\varepsilon, x; 3 + d_1\varepsilon; -1, -1) \\
&= \frac{1}{\varepsilon^2} b_{-2}^{(-1, -1)} + \frac{1}{\varepsilon} b_{-1}^{(-1, -1)} + b_0^{(-1, -1)} + \varepsilon b_1^{(-1, -1)} + \varepsilon^2 b_2^{(-1, -1)} + \mathcal{O}(\varepsilon^3),
\end{aligned} \tag{E.4}$$

where

$$b_{-2}^{(-1,-1)} = \frac{1}{8},$$

$$b_{-1}^{(-1,-1)} = -\frac{1}{2}H(0; x),$$

$$\begin{aligned} b_0^{(-1,-1)} &= \frac{\alpha_0^3}{12(x-1)^2} - \frac{\alpha_0^3}{12} + \frac{\alpha_0^2}{24(x-1)} - \frac{5\alpha_0^2}{24(x-1)^2} + \frac{7\alpha_0^2}{24(x-1)^3} + \frac{13\alpha_0^2}{24} - \frac{\alpha_0}{3(x-1)} + \frac{\alpha_0}{6(x-1)^2} - \\ &\frac{\alpha_0}{3(x-1)^3} + \frac{13\alpha_0}{12(x-1)^4} - \frac{23\alpha_0}{12} + \left( \frac{25}{12} + \frac{3}{4(x-1)} - \frac{1}{6(x-1)^2} - \frac{1}{6(x-1)^3} + \frac{3}{4(x-1)^4} + \right. \\ &\left. \frac{25}{12(x-1)^5} \right) H(0; \alpha_0) + \left( -\frac{25}{12} - \frac{3}{4(x-1)} + \frac{1}{6(x-1)^2} + \frac{1}{6(x-1)^3} - \frac{3}{4(x-1)^4} - \frac{25}{12(x-1)^5} \right) H(0; x) + \\ &\left( \frac{1}{(x-1)^5} - 1 \right) H(0; \alpha_0) H(1; x) + \left( -\frac{\alpha_0^4}{4(x-1)} + \frac{\alpha_0^4}{4} + \frac{\alpha_0^3}{x-1} - \frac{\alpha_0^3}{3(x-1)^2} - \frac{4\alpha_0^3}{3} - \frac{3\alpha_0^2}{2(x-1)} + \frac{\alpha_0^2}{(x-1)^2} - \right. \\ &\left. \frac{\alpha_0^2}{2(x-1)^3} + 3\alpha_0^2 + \frac{\alpha_0}{x-1} - \frac{\alpha_0}{(x-1)^2} + \frac{\alpha_0}{(x-1)^3} - \frac{\alpha_0}{(x-1)^4} - 4\alpha_0 + \frac{3}{4(x-1)} - \frac{1}{6(x-1)^2} - \frac{1}{6(x-1)^3} + \right. \\ &\left. \frac{3}{4(x-1)^4} + \frac{25}{12(x-1)^5} + \frac{25}{12} \right) H(c_1(\alpha_0); x) + 2H(0, 0; x) + \left( \frac{1}{(x-1)^5} - 1 \right) H(0, c_1(\alpha_0); x) + \left( 1 - \right. \\ &\left. \frac{1}{(x-1)^5} \right) H(1, 0; x) + \left( \frac{1}{(x-1)^5} - 1 \right) H(1, c_1(\alpha_0); x) - \frac{H(c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} - \frac{\pi^2}{6(x-1)^5} + \frac{\pi^2}{8}, \end{aligned}$$

$$\begin{aligned} b_1^{(-1,-1)} &= \\ &\frac{7d_1\alpha_0^3}{72} - \frac{7d_1\alpha_0^3}{72(x-1)^2} + \frac{7\alpha_0^3}{36(x-1)^2} - \frac{7\alpha_0^3}{36} - \frac{109d_1\alpha_0^2}{144} - \frac{13d_1\alpha_0^2}{144(x-1)} + \frac{7\alpha_0^2}{72(x-1)} + \frac{29d_1\alpha_0^2}{144(x-1)^2} - \frac{35\alpha_0^2}{72(x-1)^2} - \\ &\frac{67d_1\alpha_0^2}{144(x-1)^3} + \frac{85\alpha_0^2}{72(x-1)^3} + \frac{127\alpha_0^2}{72} + \frac{305d_1\alpha_0}{72} - \frac{2\alpha_0}{3(x-2)} + \frac{19d_1\alpha_0}{18(x-1)} - \frac{10\alpha_0}{9(x-1)} - \frac{d_1\alpha_0}{9(x-1)^2} + \frac{13\alpha_0}{18(x-1)^2} + \frac{d_1\alpha_0}{18(x-1)^3} - \\ &\frac{7\alpha_0}{9(x-1)^3} - \frac{217d_1\alpha_0}{72(x-1)^4} + \frac{149\alpha_0}{18(x-1)^4} - \frac{101\alpha_0}{9} + \left( -\frac{\alpha_0^3}{3(x-1)^2} + \frac{\alpha_0^3}{3} - \frac{\alpha_0^2}{6(x-1)} + \frac{5\alpha_0^2}{6(x-1)^2} - \frac{7\alpha_0^2}{6(x-1)^3} - \frac{13\alpha_0^2}{6} + \right. \\ &\left. \frac{4\alpha_0}{3(x-1)} - \frac{2\alpha_0}{3(x-1)^2} + \frac{4\alpha_0}{3(x-1)^3} - \frac{13\alpha_0}{3(x-1)^4} + \frac{23\alpha_0}{3} - \frac{205d_1}{72} + \frac{4}{x-2} - \frac{15d_1}{8(x-1)} - \frac{1}{2(x-1)} - \frac{8}{3(x-2)^2} + \frac{5d_1}{18(x-1)^2} - \right. \\ &\left. \frac{13}{18(x-1)^2} + \frac{5d_1}{18(x-1)^3} + \frac{5}{18(x-1)^3} - \frac{15d_1}{8(x-1)^4} + \frac{9}{(x-1)^4} - \frac{205d_1}{72(x-1)^5} + \frac{130}{9(x-1)^5} + \frac{155}{18} \right) H(0; \alpha_0) + \\ &\left( \frac{15d_1}{8(x-1)} - \frac{5d_1}{18(x-1)^2} - \frac{5d_1}{18(x-1)^3} + \frac{15d_1}{8(x-1)^4} + \frac{205d_1}{72(x-1)^5} + \frac{205d_1}{72} - \frac{4}{x-2} + \frac{1}{2(x-1)} + \frac{8}{3(x-2)^2} + \frac{13}{18(x-1)^2} - \right. \\ &\left. \frac{5}{18(x-1)^3} - \frac{9}{(x-1)^4} + \frac{2\pi^2}{3(x-1)^5} - \frac{130}{9(x-1)^5} - \frac{\pi^2}{2} - \frac{155}{18} \right) H(0; x) + \left( \frac{d_1\alpha_0^3}{6} - \frac{d_1\alpha_0^3}{6(x-1)^2} - \frac{13d_1\alpha_0^2}{12} - \right. \\ &\left. \frac{d_1\alpha_0^2}{12(x-1)} + \frac{5d_1\alpha_0^2}{12(x-1)^2} - \frac{7d_1\alpha_0^2}{12(x-1)^3} + \frac{23d_1\alpha_0}{6} + \frac{2d_1\alpha_0}{3(x-1)} - \frac{d_1\alpha_0}{3(x-1)^2} + \frac{2d_1\alpha_0}{3(x-1)^3} - \frac{13d_1\alpha_0}{6(x-1)^4} - \frac{35d_1}{12} - \frac{7d_1}{12(x-1)} + \right. \\ &\left. \frac{d_1}{12(x-1)^2} - \frac{d_1}{12(x-1)^3} + \frac{13d_1}{6(x-1)^4} \right) H(1; \alpha_0) + \left( \frac{\pi^2}{2} - \frac{\pi^2}{2(x-1)^5} \right) H(2; x) + \left( -\frac{d_1\alpha_0^4}{8} + \frac{d_1\alpha_0^4}{8(x-1)} - \frac{\alpha_0^4}{4(x-1)} + \right. \\ &\left. \frac{\alpha_0^4}{4} + \frac{13d_1\alpha_0^3}{18} - \frac{d_1\alpha_0^3}{2(x-1)} + \frac{4\alpha_0^3}{3(x-1)} + \frac{2d_1\alpha_0^3}{9(x-1)^2} - \frac{17\alpha_0^3}{18(x-1)^2} - \frac{29\alpha_0^3}{18} - \frac{23d_1\alpha_0^2}{12} + \frac{\alpha_0^2}{3(x-2)} + \frac{3d_1\alpha_0^2}{4(x-1)} - \frac{41\alpha_0^2}{12(x-1)} - \right. \\ &\left. \frac{2d_1\alpha_0^2}{3(x-1)^2} + \frac{13\alpha_0^2}{4(x-1)^2} + \frac{d_1\alpha_0^2}{2(x-1)^3} - \frac{11\alpha_0^2}{4(x-1)^3} + \frac{59\alpha_0^2}{12} + \frac{25d_1\alpha_0}{6} - \frac{2\alpha_0}{x-2} - \frac{d_1\alpha_0}{2(x-1)} + \frac{20\alpha_0}{3(x-1)} + \frac{4\alpha_0}{3(x-2)^2} + \right. \\ &\left. \frac{2d_1\alpha_0}{3(x-1)^2} - \frac{17\alpha_0}{3(x-1)^2} - \frac{d_1\alpha_0}{(x-1)^3} + \frac{6\alpha_0}{(x-1)^3} + \frac{2d_1\alpha_0}{(x-1)^4} - \frac{21\alpha_0}{2(x-1)^4} - \frac{73\alpha_0}{6} - \frac{205d_1}{72} + \left( \frac{\alpha_0^4}{x-1} - \alpha_0^4 - \frac{4\alpha_0^3}{x-1} + \frac{4\alpha_0^3}{3(x-1)^2} + \right. \right. \\ &\left. \left. \frac{16\alpha_0^3}{3} + \frac{6\alpha_0^2}{x-1} - \frac{4\alpha_0^2}{(x-1)^2} + \frac{2\alpha_0^2}{(x-1)^3} - 12\alpha_0^2 - \frac{4\alpha_0}{x-1} + \frac{4\alpha_0}{(x-1)^2} - \frac{4\alpha_0}{(x-1)^3} + \frac{4\alpha_0}{(x-1)^4} + 16\alpha_0 - \frac{3}{x-1} + \frac{2}{3(x-1)^2} + \right. \right. \\ &\left. \left. \frac{2}{3(x-1)^3} - \frac{3}{(x-1)^4} - \frac{25}{3(x-1)^5} - \frac{25}{3} \right) H(0; \alpha_0) + \left( -\frac{d_1\alpha_0^4}{2} + \frac{d_1\alpha_0^4}{2(x-1)} + \frac{8d_1\alpha_0^3}{3} - \frac{2d_1\alpha_0^3}{x-1} + \frac{2d_1\alpha_0^3}{3(x-1)^2} - 6d_1\alpha_0^2 + \right. \\ &\left. \frac{3d_1\alpha_0^2}{x-1} - \frac{2d_1\alpha_0^2}{(x-1)^2} + \frac{d_1\alpha_0^2}{(x-1)^3} + 8d_1\alpha_0 - \frac{2d_1\alpha_0}{x-1} + \frac{2d_1\alpha_0}{(x-1)^2} - \frac{2d_1\alpha_0}{(x-1)^3} + \frac{2d_1\alpha_0}{(x-1)^4} - \frac{25d_1}{6} - \frac{3d_1}{2(x-1)} + \frac{d_1}{3(x-1)^2} + \right. \\ &\left. \frac{d_1}{3(x-1)^3} - \frac{3d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} \right) H(1; \alpha_0) + \frac{4}{x-2} - \frac{15d_1}{8(x-1)} - \frac{1}{2(x-1)} - \frac{8}{3(x-2)^2} + \frac{5d_1}{18(x-1)^2} - \frac{13}{18(x-1)^2} + \\ &\frac{5d_1}{18(x-1)^3} + \frac{5}{18(x-1)^3} - \frac{15d_1}{8(x-1)^4} + \frac{9}{(x-1)^4} - \frac{205d_1}{72(x-1)^5} + \frac{130}{9(x-1)^5} + \frac{155}{18} \right) H(c_1(\alpha_0); x) + \left( -\frac{25}{3} - \frac{3}{x-1} + \right. \\ &\left. \frac{2}{3(x-1)^2} + \frac{2}{3(x-1)^3} - \frac{3}{(x-1)^4} - \frac{25}{3(x-1)^5} \right) H(0, 0; \alpha_0) + \left( \frac{25}{3} + \frac{3}{x-1} - \frac{2}{3(x-1)^2} - \frac{2}{3(x-1)^3} + \frac{3}{(x-1)^4} + \right. \\ &\left. \frac{25}{3(x-1)^5} \right) H(0, 0; x) + \left( -\frac{3d_1}{2(x-1)} + \frac{d_1}{3(x-1)^2} + \frac{d_1}{3(x-1)^3} - \frac{3d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} - \frac{25d_1}{6} \right) H(0, 1; \alpha_0) + \\ &H(1; x) \left( -\frac{\pi^2 d_1}{3(x-1)^5} + \left( \frac{2d_1}{x-1} - \frac{d_1}{(x-1)^2} + \frac{2d_1}{3(x-1)^3} - \frac{d_1}{2(x-1)^4} + \frac{25d_1}{6(x-1)^5} - \frac{4}{x-2} + \frac{5}{2(x-1)} + \frac{8}{3(x-2)^2} - \right. \right. \\ &\left. \left. \frac{1}{3(x-1)^2} - \frac{8}{3(x-2)^3} + \frac{5}{3(x-1)^3} + \frac{3}{2(x-1)^4} + \frac{25}{6(x-1)^5} - \frac{25}{6} \right) H(0; \alpha_0) + \left( 4 - \frac{4}{(x-1)^5} \right) H(0, 0; \alpha_0) + \end{aligned}$$

$$\begin{aligned}
 & \left(2d_1 - \frac{2d_1}{(x-1)^5}\right)H(0, 1; \alpha_0) + \frac{\pi^2}{3(x-1)^5} + \frac{\pi^2}{3} \Big) + \left(\frac{2d_1}{(x-1)^5} - 2d_1 - \frac{2}{(x-1)^5} + 2\right)H(0; \alpha_0)H(0, 1; x) + \\
 & \left(-\frac{\alpha_0^4}{2(x-1)} + \frac{\alpha_0^4}{2} + \frac{2\alpha_0^3}{x-1} - \frac{2\alpha_0^3}{3(x-1)^2} - \frac{8\alpha_0^3}{3} - \frac{3\alpha_0^2}{x-1} + \frac{2\alpha_0^2}{(x-1)^2} - \frac{\alpha_0^2}{(x-1)^3} + 6\alpha_0^2 + \frac{2\alpha_0}{x-1} - \frac{2\alpha_0}{(x-1)^2} + \frac{2\alpha_0}{(x-1)^3} - \right. \\
 & \left. \frac{2\alpha_0}{(x-1)^4} - 8\alpha_0 + \left(4 - \frac{4}{(x-1)^5}\right)H(0; \alpha_0) + \left(2d_1 - \frac{2d_1}{(x-1)^5}\right)H(1; \alpha_0) - \frac{4}{x-2} + \frac{2}{x-1} + \frac{8}{3(x-2)^2} + \right. \\
 & \left. \frac{1}{3(x-1)^2} - \frac{8}{3(x-2)^3} + \frac{2}{3(x-1)^3} + \frac{7}{2(x-1)^4} + \frac{25}{6(x-1)^5}\right)H(0, c_1(\alpha_0); x) + \left(-\frac{2d_1}{x-1} + \frac{d_1}{(x-1)^2} - \frac{2d_1}{3(x-1)^3} + \right. \\
 & \left. \frac{d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} + \frac{4}{x-2} - \frac{5}{2(x-1)} - \frac{8}{3(x-2)^2} + \frac{1}{3(x-1)^2} + \frac{8}{3(x-2)^3} - \frac{5}{3(x-1)^3} - \frac{3}{2(x-1)^4} - \frac{25}{6(x-1)^5} + \right. \\
 & \left. \frac{25}{6}\right)H(1, 0; x) + \left(\frac{4d_1}{(x-1)^5} - 2d_1 - \frac{2}{(x-1)^5} - 2\right)H(0; \alpha_0)H(1, 1; x) + \left(\frac{2d_1}{x-1} - \frac{d_1}{(x-1)^2} + \frac{2d_1}{3(x-1)^3} - \right. \\
 & \left. \frac{d_1}{2(x-1)^4} + \frac{25d_1}{6(x-1)^5} + \left(4 - \frac{4}{(x-1)^5}\right)H(0; \alpha_0) + \left(2d_1 - \frac{2d_1}{(x-1)^5}\right)H(1; \alpha_0) - \frac{4}{x-2} + \frac{5}{2(x-1)} + \right. \\
 & \left. \frac{8}{3(x-2)^2} - \frac{1}{3(x-1)^2} - \frac{8}{3(x-2)^3} + \frac{5}{3(x-1)^3} + \frac{3}{2(x-1)^4} + \frac{25}{6(x-1)^5} - \frac{25}{6}\right)H(1, c_1(\alpha_0); x) + \left(\frac{2}{(x-1)^5} - \right. \\
 & \left. 2\right)H(0; \alpha_0)H(2, 1; x) + \left(\frac{3\alpha_0^4}{2(x-1)} - \frac{3\alpha_0^4}{2} - \frac{6\alpha_0^3}{x-1} + \frac{2\alpha_0^3}{(x-1)^2} + 8\alpha_0^3 + \frac{9\alpha_0^2}{x-1} - \frac{6\alpha_0^2}{(x-1)^2} + \frac{3\alpha_0^2}{(x-1)^3} - 18\alpha_0^2 - \frac{6\alpha_0}{x-1} + \right. \\
 & \left. \frac{6\alpha_0}{(x-1)^2} - \frac{6\alpha_0}{(x-1)^3} + \frac{6\alpha_0}{(x-1)^4} + 24\alpha_0 + \frac{4H(0; \alpha_0)}{(x-1)^5} + \frac{2d_1H(1; \alpha_0)}{(x-1)^5} - \frac{9}{2(x-1)} + \frac{1}{(x-1)^2} + \frac{1}{(x-1)^3} - \frac{9}{2(x-1)^4} - \right. \\
 & \left. \frac{25}{2(x-1)^5} - \frac{25}{2}\right)H(c_1(\alpha_0), c_1(\alpha_0); x) + \left(-\frac{\alpha_0^4}{2(x-1)} + \frac{\alpha_0^4}{2} + \frac{2\alpha_0^3}{x-1} - \frac{2\alpha_0^3}{3(x-1)^2} - \frac{8\alpha_0^3}{3} - \frac{3\alpha_0^2}{x-1} + \frac{2\alpha_0^2}{(x-1)^2} - \right. \\
 & \left. \frac{\alpha_0^2}{(x-1)^3} + 6\alpha_0^2 + \frac{2\alpha_0}{x-1} - \frac{2\alpha_0}{(x-1)^2} + \frac{2\alpha_0}{(x-1)^3} - \frac{2\alpha_0}{(x-1)^4} - 8\alpha_0 + \frac{4}{x-2} - \frac{1}{2(x-1)} - \frac{8}{3(x-2)^2} - \frac{2}{3(x-1)^2} + \frac{8}{3(x-2)^3} - \right. \\
 & \left. \frac{1}{(x-1)^3} - \frac{2}{(x-1)^4} + \frac{25}{6}\right)H(c_2(\alpha_0), c_1(\alpha_0); x) - 8H(0, 0, 0; x) + \left(2 - \frac{2}{(x-1)^5}\right)H(0, 0, c_1(\alpha_0); x) + \\
 & \left(-\frac{2d_1}{(x-1)^5} + 2d_1 + \frac{2}{(x-1)^5} - 2\right)H(0, 1, 0; x) + \left(\frac{2d_1}{(x-1)^5} - 2d_1 - \frac{2}{(x-1)^5} + 2\right)H(0, 1, c_1(\alpha_0); x) + \\
 & \left(6 - \frac{2}{(x-1)^5}\right)H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \left(\frac{2}{(x-1)^5} - 2\right)H(0, c_2(\alpha_0), c_1(\alpha_0); x) + \left(\frac{4}{(x-1)^5} - \right. \\
 & \left. 4\right)H(1, 0, 0; x) + \left(\frac{2d_1}{(x-1)^5} - \frac{2}{(x-1)^5} - 2\right)H(1, 0, c_1(\alpha_0); x) + \left(-\frac{4d_1}{(x-1)^5} + 2d_1 + \frac{2}{(x-1)^5} + \right. \\
 & \left. 2\right)H(1, 1, 0; x) + \left(\frac{4d_1}{(x-1)^5} - 2d_1 - \frac{2}{(x-1)^5} - 2\right)H(1, 1, c_1(\alpha_0); x) + \left(-\frac{2d_1}{(x-1)^5} - \frac{2}{(x-1)^5} + \right. \\
 & \left. 6\right)H(1, c_1(\alpha_0), c_1(\alpha_0); x) + \left(\frac{2}{(x-1)^5} - 2\right)H(2, 0, c_1(\alpha_0); x) + \left(2 - \frac{2}{(x-1)^5}\right)H(2, 1, 0; x) + \\
 & \left(\frac{2}{(x-1)^5} - 2\right)H(2, 1, c_1(\alpha_0); x) + \left(2 - \frac{2}{(x-1)^5}\right)H(2, c_2(\alpha_0), c_1(\alpha_0); x) - \frac{2H(c_1(\alpha_0), 0, c_1(\alpha_0); x)}{(x-1)^5} + \\
 & \frac{6H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} - \frac{2H(c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \frac{\pi^2}{x-2} - \frac{3\pi^2}{8(x-1)} - \frac{2\pi^2}{3(x-2)^2} - \frac{\pi^2}{9(x-1)^2} + \\
 & \frac{2\pi^2}{3(x-2)^3} - \frac{7\pi^2}{36(x-1)^3} - \frac{3\pi^2}{4(x-1)^4} - \frac{25\pi^2}{36(x-1)^5} - \frac{3\zeta_3}{4(x-1)^5} - 4\zeta_3 - \frac{\pi^2 \ln 2}{2(x-1)^5} + \frac{1}{2}\pi^2 \ln 2 + \frac{25\pi^2}{72},
 \end{aligned}$$

$$\begin{aligned}
 b_2^{(-1, -1)} = & -\frac{37}{432}d_1^2\alpha_0^3 + \frac{37d_1\alpha_0^3}{108} + \frac{37d_1^2\alpha_0^3}{432(x-1)^2} - \frac{37d_1\alpha_0^3}{108(x-1)^2} - \frac{\pi^2\alpha_0^3}{72(x-1)^2} + \frac{37\alpha_0^3}{108(x-1)^2} + \frac{\pi^2\alpha_0^3}{72} - \frac{37\alpha_0^3}{108} + \\
 & \frac{715d_1^2\alpha_0^2}{864} - \frac{104d_1\alpha_0^2}{27} + \frac{115d_1^2\alpha_0^2}{864(x-1)} - \frac{19d_1\alpha_0^2}{54(x-1)} - \frac{\pi^2\alpha_0^2}{144(x-1)} + \frac{37\alpha_0^2}{216(x-1)} - \frac{107d_1^2\alpha_0^2}{864(x-1)^2} + \frac{73d_1\alpha_0^2}{108(x-1)^2} + \frac{5\pi^2\alpha_0^2}{144(x-1)^2} - \\
 & \frac{185\alpha_0^2}{216(x-1)^2} + \frac{493d_1^2\alpha_0^2}{864(x-1)^3} - \frac{305d_1\alpha_0^2}{108(x-1)^3} - \frac{7\pi^2\alpha_0^2}{144(x-1)^3} + \frac{727\alpha_0^2}{216(x-1)^3} - \frac{13\pi^2\alpha_0^2}{144} + \frac{949\alpha_0^2}{216} - \frac{3515d_1^2\alpha_0}{432} + \frac{8965d_1\alpha_0}{216} + \\
 & \frac{25d_1\alpha_0}{9(x-2)} - \frac{50\alpha_0}{9(x-2)} - \frac{265d_1^2\alpha_0}{108(x-1)} + \frac{341d_1\alpha_0}{54(x-1)} + \frac{\pi^2\alpha_0}{18(x-1)} - \frac{107\alpha_0}{54(x-1)} - \frac{d_1^2\alpha_0}{108(x-1)^2} - \frac{185d_1\alpha_0}{108(x-1)^2} - \frac{\pi^2\alpha_0}{36(x-1)^2} + \\
 & \frac{187\alpha_0}{54(x-1)^2} + \frac{113d_1^2\alpha_0}{108(x-1)^3} - \frac{74d_1\alpha_0}{27(x-1)^3} + \frac{\pi^2\alpha_0}{18(x-1)^3} + \frac{25\alpha_0}{54(x-1)^3} + \frac{2911d_1^2\alpha_0}{432(x-1)^4} - \frac{7523d_1\alpha_0}{216(x-1)^4} - \frac{13\pi^2\alpha_0}{72(x-1)^4} + \\
 & \frac{2369\alpha_0}{54(x-1)^4} + \frac{23\pi^2\alpha_0}{72} - \frac{1394\alpha_0}{27} + \left(-\frac{7d_1\alpha_0^3}{18} + \frac{7d_1\alpha_0^3}{18(x-1)^2} - \frac{7\alpha_0^3}{9(x-1)^2} + \frac{7\alpha_0^3}{9} + \frac{109d_1\alpha_0^2}{36} + \frac{13d_1\alpha_0^2}{36(x-1)} - \frac{7\alpha_0^2}{18(x-1)} - \right. \\
 & \left. \frac{29d_1\alpha_0^2}{36(x-1)^2} + \frac{35\alpha_0^2}{18(x-1)^2} + \frac{67d_1\alpha_0^2}{36(x-1)^3} - \frac{85\alpha_0^2}{18(x-1)^3} - \frac{127\alpha_0^2}{18} - \frac{305d_1\alpha_0}{18} + \frac{8\alpha_0}{3(x-2)} - \frac{38d_1\alpha_0}{9(x-1)} + \frac{40\alpha_0}{9(x-1)} + \frac{4d_1\alpha_0}{9(x-1)^2} - \right. \\
 & \left. \frac{26\alpha_0}{9(x-1)^2} - \frac{2d_1\alpha_0}{9(x-1)^3} + \frac{28\alpha_0}{9(x-1)^3} + \frac{217d_1\alpha_0}{18(x-1)^4} - \frac{298\alpha_0}{9(x-1)^4} + \frac{404\alpha_0}{9} + \frac{2035d_1^2}{432} - \frac{5615d_1}{216} - \frac{38d_1}{3(x-2)} + \frac{68}{3(x-2)} + \right. \\
 & \left. \frac{63d_1^2}{16(x-1)} - \frac{85d_1}{24(x-1)} - \frac{\pi^2}{8(x-1)} - \frac{37}{4(x-1)} + \frac{76d_1}{9(x-2)^2} - \frac{152}{9(x-2)^2} - \frac{19d_1^2}{54(x-1)^2} + \frac{317d_1}{108(x-1)^2} + \frac{\pi^2}{36(x-1)^2} + \right. \\
 & \left. \frac{49}{108(x-1)^2} - \frac{19d_1^2}{54(x-1)^3} - \frac{313d_1}{108(x-1)^3} + \frac{\pi^2}{36(x-1)^3} + \frac{949}{108(x-1)^3} + \frac{63d_1^2}{16(x-1)^4} - \frac{257d_1}{8(x-1)^4} - \frac{\pi^2}{8(x-1)^4} + \frac{213}{4(x-1)^4} + \right.
 \end{aligned}$$

$$\begin{aligned}
& \left. \frac{2035d_1^2}{432(x-1)^5} - \frac{8705d_1}{216(x-1)^5} - \frac{25\pi^2}{72(x-1)^5} + \frac{3965}{54(x-1)^5} - \frac{25\pi^2}{72} + \frac{940}{27} \right) H(0; \alpha_0) + \left( -\frac{7}{36}d_1^2\alpha_0^3 + \frac{7d_1\alpha_0^3}{18} + \right. \\
& \left. \frac{7d_1^2\alpha_0^3}{36(x-1)^2} - \frac{7d_1\alpha_0^3}{18(x-1)^2} + \frac{109d_1^2\alpha_0^2}{72} - \frac{127d_1\alpha_0^2}{36} + \frac{13d_1^2\alpha_0^2}{72(x-1)} - \frac{7d_1\alpha_0^2}{36(x-1)} - \frac{29d_1^2\alpha_0^2}{72(x-1)^2} + \frac{35d_1\alpha_0^2}{36(x-1)^2} + \frac{67d_1^2\alpha_0^2}{72(x-1)^3} - \right. \\
& \left. \frac{85d_1\alpha_0^2}{36(x-1)^3} - \frac{305d_1^2\alpha_0}{36} + \frac{202d_1\alpha_0}{9} + \frac{4d_1\alpha_0}{3(x-2)} - \frac{19d_1^2\alpha_0}{9(x-1)} + \frac{20d_1\alpha_0}{9(x-1)} + \frac{2d_1^2\alpha_0}{9(x-1)^2} - \frac{13d_1\alpha_0}{9(x-1)^2} - \frac{d_1^2\alpha_0}{9(x-1)^3} + \frac{14d_1\alpha_0}{9(x-1)^3} + \right. \\
& \left. \frac{217d_1^2\alpha_0}{36(x-1)^4} - \frac{149d_1\alpha_0}{9(x-1)^4} + \frac{515d_1^2}{72} - \frac{695d_1}{36} - \frac{4d_1}{3(x-2)} + \frac{139d_1^2}{72(x-1)} - \frac{73d_1}{36(x-1)} - \frac{d_1^2}{72(x-1)^2} + \frac{31d_1}{36(x-1)^2} - \frac{59d_1^2}{72(x-1)^3} + \right. \\
& \left. \frac{29d_1}{36(x-1)^3} - \frac{217d_1^2}{36(x-1)^4} + \frac{149d_1}{9(x-1)^4} \right) H(1; \alpha_0) + \left( \frac{4\alpha_0^3}{3(x-1)^2} - \frac{4\alpha_0^3}{3} + \frac{2\alpha_0^2}{3(x-1)} - \frac{10\alpha_0^2}{3(x-1)^2} + \frac{14\alpha_0^2}{3(x-1)^3} + \frac{26\alpha_0^2}{3} - \right. \\
& \left. \frac{16\alpha_0}{3(x-1)} + \frac{8\alpha_0}{3(x-1)^2} - \frac{16\alpha_0}{3(x-1)^3} + \frac{52\alpha_0}{3(x-1)^4} - \frac{92\alpha_0}{3} + \frac{205d_1}{18} - \frac{16}{x-2} + \frac{15d_1}{2(x-1)} + \frac{2}{x-1} + \frac{32}{3(x-2)^2} - \frac{10d_1}{9(x-1)^2} + \right. \\
& \left. \frac{26}{9(x-1)^2} - \frac{10d_1}{9(x-1)^3} - \frac{10}{9(x-1)^3} + \frac{15d_1}{2(x-1)^4} - \frac{36}{(x-1)^4} + \frac{205d_1}{18(x-1)^5} - \frac{520}{9(x-1)^5} - \frac{310}{9} \right) H(0, 0; \alpha_0) + \left( -\frac{15d_1}{2(x-1)} + \frac{10d_1}{9(x-1)^2} + \frac{10d_1}{9(x-1)^3} - \frac{15d_1}{2(x-1)^4} - \frac{205d_1}{18(x-1)^5} - \frac{205d_1}{18} + \frac{16}{x-2} - \frac{2}{x-1} - \frac{32}{3(x-2)^2} - \frac{26}{9(x-1)^2} + \right. \\
& \left. \frac{10}{9(x-1)^3} + \frac{36}{(x-1)^4} - \frac{8\pi^2}{3(x-1)^5} + \frac{520}{9(x-1)^5} + 2\pi^2 + \frac{310}{9} \right) H(0, 0; x) + \left( -\frac{2d_1\alpha_0^3}{3} + \frac{2d_1\alpha_0^3}{3(x-1)^2} + \frac{13d_1\alpha_0^2}{3} + \frac{d_1\alpha_0^2}{3(x-1)} - \frac{5d_1\alpha_0^2}{3(x-1)^2} + \frac{7d_1\alpha_0^2}{3(x-1)^3} - \frac{46d_1\alpha_0}{3} - \frac{8d_1\alpha_0}{3(x-1)} + \frac{4d_1\alpha_0}{3(x-1)^2} - \frac{8d_1\alpha_0}{3(x-1)^3} + \frac{26d_1\alpha_0}{3(x-1)^4} + \frac{205d_1^2}{36} - \frac{155d_1}{9} - \frac{8d_1}{x-2} + \right. \\
& \left. \frac{15d_1^2}{4(x-1)} + \frac{d_1}{x-1} + \frac{16d_1}{3(x-2)^2} - \frac{5d_1^2}{9(x-1)^2} + \frac{13d_1}{9(x-1)^2} - \frac{5d_1^2}{9(x-1)^3} - \frac{5d_1}{9(x-1)^3} + \frac{15d_1^2}{4(x-1)^4} - \frac{18d_1}{(x-1)^4} + \frac{205d_1^2}{36(x-1)^5} - \right. \\
& \left. \frac{260d_1}{9(x-1)^5} \right) H(0, 1; \alpha_0) + \left( \frac{2\pi^2 d_1}{3(x-1)^5} + \frac{2\pi^2 d_1}{3} + \left( -\frac{8d_1}{x-2} + \frac{5d_1}{x-1} + \frac{16d_1}{3(x-2)^2} - \frac{2d_1}{3(x-1)^2} - \frac{16d_1}{3(x-2)^3} + \frac{10d_1}{3(x-1)^3} + \right. \right. \\
& \left. \left. \frac{3d_1}{(x-1)^4} + \frac{25d_1}{3(x-1)^5} - \frac{25d_1}{3} + \frac{8}{x-2} - \frac{16}{3(x-2)^2} - \frac{8}{3(x-1)^2} + \frac{16}{3(x-2)^3} - \frac{8}{(x-1)^4} \right) H(0; \alpha_0) + \left( -\frac{8d_1}{(x-1)^5} + 8d_1 + \frac{8}{(x-1)^5} - 8 \right) H(0, 0; \alpha_0) + \left( -\frac{4d_1^2}{(x-1)^5} + 4d_1^2 + \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(0, 1; \alpha_0) - \frac{2\pi^2}{(x-1)^5} - \frac{2\pi^2}{3} \right) H(0, 1; x) + \\
& \left( -\frac{\pi^2 d_1}{(x-1)^5} + \pi^2 d_1 + \frac{\pi^2}{(x-1)^5} - \pi^2 \right) H(0, 2; x) + \left( -\frac{2d_1\alpha_0^3}{3} + \frac{2d_1\alpha_0^3}{3(x-1)^2} + \frac{13d_1\alpha_0^2}{3} + \frac{d_1\alpha_0^2}{3(x-1)} - \frac{5d_1\alpha_0^2}{3(x-1)^2} + \frac{7d_1\alpha_0^2}{3(x-1)^3} - \frac{46d_1\alpha_0}{3} - \frac{8d_1\alpha_0}{3(x-1)} + \frac{4d_1\alpha_0}{3(x-1)^2} - \frac{8d_1\alpha_0}{3(x-1)^3} + \frac{26d_1\alpha_0}{3(x-1)^4} + \frac{35d_1}{3} + \frac{7d_1}{3(x-1)} - \frac{d_1}{3(x-1)^2} + \frac{d_1}{3(x-1)^3} - \right. \\
& \left. \frac{26d_1}{3(x-1)^4} \right) H(1, 0; \alpha_0) + \left( \frac{4d_1^2}{x-1} - \frac{d_1^2}{(x-1)^2} + \frac{4d_1^2}{9(x-1)^3} - \frac{d_1^2}{4(x-1)^4} + \frac{205d_1^2}{36(x-1)^5} - \frac{46d_1}{3(x-2)} - \frac{3d_1}{4(x-1)} + \frac{52d_1}{9(x-2)^2} + \right. \\
& \left. \frac{185d_1}{18(x-1)^2} - \frac{28d_1}{9(x-2)^3} - \frac{29d_1}{18(x-1)^3} + \frac{27d_1}{4(x-1)^4} + \frac{4\pi^2 d_1}{3(x-1)^5} - \frac{835d_1}{36(x-1)^5} - \frac{205d_1}{36} + \frac{80}{3(x-2)} - \frac{70}{3(x-1)} - \frac{56}{9(x-2)^2} - \right. \\
& \left. \frac{13}{18(x-1)^2} + \frac{56}{9(x-2)^3} - \frac{355}{18(x-1)^3} - \frac{10}{3(x-1)^4} - \frac{7\pi^2}{6(x-1)^5} - \frac{155}{9(x-1)^5} - \frac{3\pi^2}{2} + \frac{155}{9} \right) H(1, 0; x) + \left( -\frac{1}{3}d_1^2\alpha_0^3 + \frac{d_1^2\alpha_0^3}{3(x-1)^2} + \frac{13d_1^2\alpha_0^2}{6} + \frac{d_1^2\alpha_0^2}{6(x-1)} - \frac{5d_1^2\alpha_0^2}{6(x-1)^2} + \frac{7d_1^2\alpha_0^2}{6(x-1)^3} - \frac{23d_1^2\alpha_0}{3} - \frac{4d_1^2\alpha_0}{3(x-1)} + \frac{2d_1^2\alpha_0}{3(x-1)^2} - \frac{4d_1^2\alpha_0}{3(x-1)^3} + \right. \\
& \left. \frac{13d_1^2\alpha_0}{3(x-1)^4} + \frac{35d_1^2}{6} + \frac{7d_1^2}{6(x-1)} - \frac{d_1^2}{6(x-1)^2} + \frac{d_1^2}{6(x-1)^3} - \frac{13d_1^2}{3(x-1)^4} \right) H(1, 1; \alpha_0) + H(0, c_1(\alpha_0); x) \left( -\frac{d_1\alpha_0^4}{4} + \frac{d_1\alpha_0^4}{4(x-1)} - \frac{\alpha_0^4}{2(x-1)} + \frac{\alpha_0^4}{2} + \frac{13d_1\alpha_0^3}{9} - \frac{d_1\alpha_0^3}{x-1} + \frac{8\alpha_0^3}{3(x-1)} + \frac{4d_1\alpha_0^3}{9(x-1)^2} - \frac{14\alpha_0^3}{9(x-1)^2} - \frac{32\alpha_0^3}{9} - \frac{23d_1\alpha_0^2}{6} + \frac{2\alpha_0^2}{3(x-2)} + \right. \\
& \left. \frac{3d_1\alpha_0^2}{2(x-1)} - \frac{20\alpha_0^2}{3(x-1)} - \frac{4d_1\alpha_0^2}{3(x-1)^2} + \frac{17\alpha_0^2}{3(x-1)^2} + \frac{d_1\alpha_0^2}{(x-1)^3} - \frac{13\alpha_0^2}{3(x-1)^3} + 12\alpha_0^2 + \frac{25d_1\alpha_0}{3} - \frac{4\alpha_0}{x-2} - \frac{d_1\alpha_0}{x-1} + \frac{12\alpha_0}{x-1} + \frac{8\alpha_0}{3(x-2)^2} + \frac{4d_1\alpha_0}{3(x-1)^2} - \frac{32\alpha_0}{3(x-1)^2} - \frac{2d_1\alpha_0}{(x-1)^3} + \frac{32\alpha_0}{3(x-1)^3} + \frac{4d_1\alpha_0}{(x-1)^4} - \frac{50\alpha_0}{3(x-1)^4} - 32\alpha_0 + \left( \frac{2\alpha_0^4}{x-1} - 2\alpha_0^4 - \frac{8\alpha_0^3}{x-1} + \frac{8\alpha_0^3}{3(x-1)^2} + \frac{32\alpha_0^3}{3} + \frac{12\alpha_0^2}{x-1} - \frac{8\alpha_0^2}{(x-1)^2} + \frac{12\alpha_0^2}{(x-1)^2} - \frac{8\alpha_0^2}{(x-1)^2} + \frac{4\alpha_0^2}{(x-1)^3} - 24\alpha_0^2 - \frac{8\alpha_0}{x-1} + \frac{8\alpha_0}{(x-1)^2} - \frac{8\alpha_0}{(x-1)^3} + \frac{8\alpha_0}{(x-1)^4} + 32\alpha_0 + \frac{16}{x-2} - \right. \\
& \left. \frac{8}{x-1} - \frac{32}{3(x-2)^2} - \frac{4}{3(x-1)^2} + \frac{32}{3(x-2)^3} - \frac{8}{3(x-1)^3} - \frac{14}{(x-1)^4} - \frac{50}{3(x-1)^5} \right) H(0; \alpha_0) + \left( -d_1\alpha_0^4 + \frac{d_1\alpha_0^4}{x-1} + \frac{16d_1\alpha_0^3}{3} - \frac{4d_1\alpha_0^3}{x-1} + \frac{4d_1\alpha_0^3}{3(x-1)^2} - 12d_1\alpha_0^2 + \frac{6d_1\alpha_0^2}{x-1} - \frac{4d_1\alpha_0^2}{(x-1)^2} + \frac{2d_1\alpha_0^2}{(x-1)^3} + 16d_1\alpha_0 - \frac{4d_1\alpha_0}{x-1} + \frac{4d_1\alpha_0}{(x-1)^2} - \frac{4d_1\alpha_0}{(x-1)^3} + \right. \\
& \left. \frac{4d_1\alpha_0}{(x-1)^4} + \frac{8d_1}{x-2} - \frac{4d_1}{x-1} - \frac{16d_1}{3(x-2)^2} - \frac{2d_1}{3(x-1)^2} + \frac{16d_1}{3(x-2)^3} - \frac{4d_1}{3(x-1)^3} - \frac{7d_1}{(x-1)^4} - \frac{25d_1}{3(x-1)^5} \right) H(1; \alpha_0) + \\
& \left( \frac{16}{(x-1)^5} - 16 \right) H(0, 0; \alpha_0) + \left( \frac{8d_1}{(x-1)^5} - 8d_1 \right) H(0, 1; \alpha_0) + \left( \frac{8d_1}{(x-1)^5} - 8d_1 \right) H(1, 0; \alpha_0) + \left( \frac{4d_1^2}{(x-1)^5} - \right. \\
& \left. 4d_1^2 \right) H(1, 1; \alpha_0) + \frac{32d_1}{3(x-2)} - \frac{70}{3(x-2)} - \frac{20d_1}{3(x-1)} + \frac{95}{6(x-1)} - \frac{28d_1}{9(x-2)^2} + \frac{32}{9(x-2)^2} - \frac{23d_1}{9(x-1)^2} + \frac{131}{18(x-1)^2} + \\
& \frac{28d_1}{9(x-2)^3} - \frac{56}{9(x-2)^3} - \frac{40d_1}{9(x-1)^3} + \frac{241}{18(x-1)^3} - \frac{31d_1}{4(x-1)^4} + \frac{20}{(x-1)^4} - \frac{205d_1}{36(x-1)^5} - \frac{\pi^2}{6(x-1)^5} + \frac{155}{9(x-1)^5} + \frac{\pi^2}{6} + \\
& \left. \frac{35}{6} \right) + H(c_1(\alpha_0); x) \left( \frac{d_1^2\alpha_0^4}{16} - \frac{d_1\alpha_0^4}{4} - \frac{d_1^2\alpha_0^4}{16(x-1)} + \frac{d_1\alpha_0^4}{4(x-1)} + \frac{\pi^2\alpha_0^4}{24(x-1)} - \frac{\alpha_0^4}{4(x-1)} - \frac{\pi^2\alpha_0^4}{24} + \frac{\alpha_0^4}{4} - \frac{43d_1^2\alpha_0^3}{108} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{193d_1\alpha_0^3}{108} + \frac{d_1^2\alpha_0^3}{4(x-1)} - \frac{25d_1\alpha_0^3}{18(x-1)} - \frac{\pi^2\alpha_0^3}{6(x-1)} + \frac{16\alpha_0^3}{9(x-1)} - \frac{4d_1^2\alpha_0^3}{27(x-1)^2} + \frac{127d_1\alpha_0^3}{108(x-1)^2} + \frac{\pi^2\alpha_0^3}{18(x-1)^2} - \frac{95\alpha_0^3}{54(x-1)^2} + \\
& \frac{2\pi^2\alpha_0^3}{9} - \frac{107\alpha_0^3}{54} + \frac{95d_1^2\alpha_0^2}{72} - \frac{163d_1\alpha_0^2}{24} - \frac{13d_1\alpha_0^2}{18(x-2)} + \frac{13\alpha_0^2}{9(x-2)} - \frac{3d_1^2\alpha_0^2}{8(x-1)} + \frac{311d_1\alpha_0^2}{72(x-1)} + \frac{\pi^2\alpha_0^2}{4(x-1)} - \frac{281\alpha_0^2}{36(x-1)} + \\
& \frac{4d_1^2\alpha_0^2}{9(x-1)^2} - \frac{295d_1\alpha_0^2}{72(x-1)^2} - \frac{\pi^2\alpha_0^2}{6(x-1)^2} + \frac{89\alpha_0^2}{12(x-1)^2} - \frac{d_1^2\alpha_0^2}{2(x-1)^3} + \frac{115d_1\alpha_0^2}{24(x-1)^3} + \frac{\pi^2\alpha_0^2}{12(x-1)^3} - \frac{35\alpha_0^2}{4(x-1)^3} - \frac{\pi^2\alpha_0^2}{2} + \\
& \frac{305\alpha_0^2}{36} - \frac{205d_1^2\alpha_0}{36} + \frac{125d_1\alpha_0}{4} + \frac{17d_1\alpha_0}{3(x-2)} - \frac{10\alpha_0}{x-2} + \frac{d_1^2\alpha_0}{4(x-1)} - \frac{251d_1\alpha_0}{18(x-1)} - \frac{\pi^2\alpha_0}{6(x-1)} + \frac{1115\alpha_0}{36(x-1)} - \frac{38d_1\alpha_0}{9(x-2)^2} + \\
& \frac{76\alpha_0}{9(x-2)^2} - \frac{4d_1^2\alpha_0}{9(x-1)^2} + \frac{32d_1\alpha_0}{3(x-1)^2} + \frac{\pi^2\alpha_0}{6(x-1)^2} - \frac{511\alpha_0}{18(x-1)^2} + \frac{d_1^2\alpha_0}{(x-1)^3} - \frac{31d_1\alpha_0}{3(x-1)^3} - \frac{\pi^2\alpha_0}{6(x-1)^3} + \frac{95\alpha_0}{4(x-1)^3} - \\
& \frac{4d_1^2\alpha_0}{(x-1)^4} + \frac{409d_1\alpha_0}{12(x-1)^4} + \frac{\pi^2\alpha_0}{6(x-1)^4} - \frac{188\alpha_0}{3(x-1)^4} + \frac{2\pi^2\alpha_0}{3} - \frac{374\alpha_0}{9} + \frac{2035d_1^2}{432} - \frac{5615d_1}{216} + \left( \frac{d_1\alpha_0^4}{2} - \frac{d_1\alpha_0^4}{2(x-1)} + \frac{\alpha_0^4}{x-1} - \right. \\
& \left. \alpha_0^4 - \frac{26d_1\alpha_0^3}{9} + \frac{2d_1\alpha_0^3}{x-1} - \frac{16\alpha_0^3}{3(x-1)} - \frac{8d_1\alpha_0^3}{9(x-1)^2} + \frac{34\alpha_0^3}{9(x-1)^2} + \frac{58\alpha_0^3}{9} + \frac{23d_1\alpha_0^2}{3} - \frac{4\alpha_0^2}{3(x-2)} - \frac{3d_1\alpha_0^2}{x-1} + \frac{41\alpha_0^2}{3(x-1)} + \right. \\
& \left. \frac{8d_1\alpha_0^2}{3(x-1)^2} - \frac{13\alpha_0^2}{(x-1)^2} - \frac{2d_1\alpha_0^2}{(x-1)^3} + \frac{11\alpha_0^2}{(x-1)^3} - \frac{59\alpha_0^2}{3} - \frac{50d_1\alpha_0}{3} + \frac{8\alpha_0}{x-2} + \frac{2d_1\alpha_0}{x-1} - \frac{80\alpha_0}{3(x-1)} - \frac{16\alpha_0}{3(x-2)^2} - \frac{8d_1\alpha_0}{3(x-1)^2} + \right. \\
& \left. \frac{68\alpha_0}{3(x-1)^2} + \frac{4d_1\alpha_0}{(x-1)^3} - \frac{24\alpha_0}{(x-1)^3} - \frac{8d_1\alpha_0}{(x-1)^4} + \frac{42\alpha_0}{(x-1)^4} + \frac{146\alpha_0}{3} + \frac{205d_1}{18} - \frac{16}{x-2} + \frac{15d_1}{2(x-1)} + \frac{2}{x-1} + \frac{32}{3(x-2)^2} - \right. \\
& \left. \frac{10d_1}{9(x-1)^2} + \frac{26}{9(x-1)^2} - \frac{10d_1}{9(x-1)^3} - \frac{10}{9(x-1)^3} + \frac{15d_1}{2(x-1)^4} - \frac{36}{(x-1)^4} + \frac{205d_1}{18(x-1)^5} - \frac{520}{9(x-1)^5} - \frac{310}{9} \right) H(0; \alpha_0) + \\
& \left( \frac{d_1^4\alpha_0^4}{4} - \frac{d_1\alpha_0^4}{2} - \frac{d_1^2\alpha_0^4}{4(x-1)} + \frac{d_1\alpha_0^4}{2(x-1)} - \frac{13d_1^2\alpha_0^3}{9} + \frac{29d_1\alpha_0^3}{9} + \frac{d_1^2\alpha_0^3}{x-1} - \frac{8d_1\alpha_0^3}{3(x-1)} - \frac{4d_1^2\alpha_0^3}{9(x-1)^2} + \frac{17d_1\alpha_0^3}{9(x-1)^2} + \frac{23d_1^2\alpha_0^2}{6} - \right. \\
& \left. \frac{59d_1\alpha_0^2}{6} - \frac{2d_1\alpha_0^2}{3(x-2)} - \frac{3d_1^2\alpha_0^2}{2(x-1)} + \frac{41d_1\alpha_0^2}{6(x-1)} + \frac{4d_1^2\alpha_0^2}{3(x-1)^2} - \frac{13d_1\alpha_0^2}{2(x-1)^2} - \frac{d_1^2\alpha_0^2}{(x-1)^3} + \frac{11d_1\alpha_0^2}{2(x-1)^3} - \frac{25d_1^2\alpha_0}{3} + \frac{73d_1\alpha_0}{3} + \right. \\
& \left. \frac{4d_1\alpha_0}{x-2} + \frac{d_1^2\alpha_0}{x-1} - \frac{40d_1\alpha_0}{3(x-2)^2} - \frac{8d_1\alpha_0}{3(x-2)^2} - \frac{4d_1^2\alpha_0}{3(x-1)^2} + \frac{34d_1\alpha_0}{3(x-1)^2} + \frac{2d_1^2\alpha_0}{(x-1)^3} - \frac{12d_1\alpha_0}{(x-1)^3} - \frac{4d_1^2\alpha_0}{(x-1)^4} + \frac{21d_1\alpha_0}{(x-1)^4} + \frac{205d_1^2}{36} - \right. \\
& \left. \frac{155d_1}{9} - \frac{8d_1}{x-2} + \frac{15d_1^2}{4(x-1)} + \frac{d_1}{x-1} + \frac{16d_1}{3(x-2)^2} - \frac{5d_1^2}{9(x-1)^2} + \frac{13d_1}{9(x-1)^2} - \frac{5d_1^2}{9(x-1)^3} - \frac{5d_1}{9(x-1)^3} + \frac{15d_1^2}{4(x-1)^4} - \frac{18d_1}{(x-1)^4} + \right. \\
& \left. \frac{205d_1^2}{36(x-1)^5} - \frac{260d_1}{9(x-1)^5} \right) H(1; \alpha_0) + \left( -\frac{4\alpha_0^4}{x-1} + 4\alpha_0^4 + \frac{16\alpha_0^3}{x-1} - \frac{16\alpha_0^3}{3(x-1)^2} - \frac{64\alpha_0^3}{3} - \frac{24\alpha_0^2}{x-1} + \frac{16\alpha_0^2}{(x-1)^2} - \frac{8\alpha_0^2}{(x-1)^3} + \right. \\
& \left. 48\alpha_0^2 + \frac{16\alpha_0}{x-1} - \frac{16\alpha_0}{(x-1)^2} + \frac{16\alpha_0}{(x-1)^3} - \frac{16\alpha_0}{(x-1)^4} - 64\alpha_0 + \frac{12}{x-1} - \frac{8}{3(x-1)^2} - \frac{8}{3(x-1)^3} + \frac{12}{(x-1)^4} + \frac{100}{3(x-1)^5} + \right. \\
& \left. \frac{100}{3} \right) H(0, 0; \alpha_0) + \left( 2d_1\alpha_0^4 - \frac{2d_1\alpha_0^4}{x-1} - \frac{32d_1\alpha_0^3}{3} + \frac{8d_1\alpha_0^3}{x-1} - \frac{8d_1\alpha_0^3}{3(x-1)^2} + 24d_1\alpha_0^2 - \frac{12d_1\alpha_0^2}{x-1} + \frac{8d_1\alpha_0^2}{(x-1)^2} - \right. \\
& \left. \frac{4d_1\alpha_0^2}{(x-1)^3} - 32d_1\alpha_0 + \frac{8d_1\alpha_0}{x-1} - \frac{8d_1\alpha_0}{(x-1)^2} + \frac{8d_1\alpha_0}{(x-1)^3} - \frac{8d_1\alpha_0}{(x-1)^4} + \frac{50d_1}{3} + \frac{6d_1}{x-1} - \frac{4d_1}{3(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \frac{6d_1}{(x-1)^4} + \right. \\
& \left. \frac{50d_1}{3(x-1)^5} \right) H(0, 1; \alpha_0) + \left( 2d_1\alpha_0^4 - \frac{2d_1\alpha_0^4}{x-1} - \frac{32d_1\alpha_0^3}{3} + \frac{8d_1\alpha_0^3}{x-1} - \frac{8d_1\alpha_0^3}{3(x-1)^2} + 24d_1\alpha_0^2 - \frac{12d_1\alpha_0^2}{x-1} + \frac{8d_1\alpha_0^2}{(x-1)^2} - \right. \\
& \left. \frac{4d_1\alpha_0^2}{(x-1)^3} - 32d_1\alpha_0 + \frac{8d_1\alpha_0}{x-1} - \frac{8d_1\alpha_0}{(x-1)^2} + \frac{8d_1\alpha_0}{(x-1)^3} - \frac{8d_1\alpha_0}{(x-1)^4} + \frac{50d_1}{3} + \frac{6d_1}{x-1} - \frac{4d_1}{3(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \frac{6d_1}{(x-1)^4} + \right. \\
& \left. \frac{50d_1}{3(x-1)^5} \right) H(1, 0; \alpha_0) + \left( d_1^2\alpha_0^4 - \frac{d_1^2\alpha_0^4}{x-1} - \frac{16d_1^2\alpha_0^3}{3} + \frac{4d_1^2\alpha_0^3}{x-1} - \frac{4d_1^2\alpha_0^3}{3(x-1)^2} + 12d_1^2\alpha_0^2 - \frac{6d_1^2\alpha_0^2}{x-1} + \frac{4d_1^2\alpha_0^2}{(x-1)^2} - \right. \\
& \left. \frac{2d_1^2\alpha_0^2}{(x-1)^3} - 16d_1^2\alpha_0 + \frac{4d_1^2\alpha_0}{x-1} - \frac{4d_1^2\alpha_0}{(x-1)^2} + \frac{4d_1^2\alpha_0}{(x-1)^3} - \frac{4d_1^2\alpha_0}{(x-1)^4} + \frac{25d_1^2}{3} + \frac{3d_1^2}{x-1} - \frac{2d_1^2}{3(x-1)^2} - \frac{2d_1^2}{3(x-1)^3} + \frac{3d_1^2}{(x-1)^4} + \right. \\
& \left. \frac{25d_1^2}{3(x-1)^5} \right) H(1, 1; \alpha_0) - \frac{38d_1}{3(x-2)} + \frac{68}{3(x-2)} + \frac{63d_1^2}{16(x-1)} - \frac{85d_1}{24(x-1)} - \frac{\pi^2}{8(x-1)} - \frac{37}{4(x-1)} + \frac{76d_1}{9(x-2)^2} - \frac{152}{9(x-2)^2} - \\
& \frac{19d_1^2}{54(x-1)^2} + \frac{317d_1}{108(x-1)^2} + \frac{\pi^2}{36(x-1)^2} + \frac{49}{108(x-1)^2} - \frac{19d_1^2}{54(x-1)^3} - \frac{313d_1}{108(x-1)^3} + \frac{\pi^2}{36(x-1)^3} + \frac{949}{108(x-1)^3} + \\
& \frac{63d_1^2}{16(x-1)^4} - \frac{257d_1}{8(x-1)^4} - \frac{\pi^2}{8(x-1)^4} + \frac{213}{4(x-1)^4} + \frac{2035d_1^2}{432(x-1)^5} - \frac{8705d_1}{216(x-1)^5} - \frac{25\pi^2}{72(x-1)^5} + \frac{3965}{54(x-1)^5} - \\
& \frac{25\pi^2}{72} + \frac{940}{27} \Big) + \left( -\frac{2\pi^2d_1^2}{3(x-1)^5} + \frac{4\pi^2d_1}{3(x-1)^5} + 2\pi^2\frac{d_1}{3} + \left( \frac{4d_1^2}{x-1} - \frac{2d_1^2}{(x-1)^2} + \frac{4d_1^2}{3(x-1)^3} - \frac{d_1^2}{(x-1)^4} + \frac{25d_1^2}{3(x-1)^5} - \right. \right. \\
& \left. \frac{8d_1}{x-2} + \frac{9d_1}{x-1} + \frac{16d_1}{3(x-2)^2} - \frac{8d_1}{3(x-1)^2} - \frac{16d_1}{3(x-2)^3} + \frac{14d_1}{3(x-1)^3} + \frac{2d_1}{(x-1)^4} + \frac{50d_1}{3(x-1)^5} - \frac{25d_1}{3} - \frac{4}{x-2} - \frac{3}{2(x-1)} + \right. \\
& \left. \frac{8}{3(x-2)^2} - \frac{8}{3(x-2)^3} + \frac{11}{3(x-1)^3} - \frac{5}{(x-1)^4} - \frac{25}{3(x-1)^5} - \frac{125}{6} \right) H(0; \alpha_0) + \left( -\frac{16d_1}{(x-1)^5} + 8d_1 + \frac{8}{(x-1)^5} + \right. \\
& \left. 8 \right) H(0, 0; \alpha_0) + \left( -\frac{8d_1^2}{(x-1)^5} + 4d_1^2 + \frac{4d_1}{(x-1)^5} + 4d_1 \right) H(0, 1; \alpha_0) - \frac{2\pi^2}{3(x-1)^5} - \frac{2\pi^2}{3} \Big) H(1, 1; x) + \left( -\right. \\
& \left. \frac{\pi^2d_1}{(x-1)^5} + \frac{\pi^2}{2(x-1)^5} + \frac{3\pi^2}{2} \right) H(1, 2; x) + \left( -\frac{4d_1^2}{x-1} + \frac{d_1^2}{(x-1)^2} - \frac{4d_1^2}{9(x-1)^3} + \frac{d_1^2}{4(x-1)^4} - \frac{205d_1^2}{36(x-1)^5} + \frac{46d_1}{3(x-2)} + \right. \\
& \left. \frac{3d_1}{4(x-1)} - \frac{52d_1}{9(x-2)^2} - \frac{185d_1}{18(x-1)^2} + \frac{28d_1}{9(x-2)^3} + \frac{29d_1}{18(x-1)^3} - \frac{27d_1}{4(x-1)^4} + \frac{835d_1}{36(x-1)^5} + \frac{205d_1}{36} + \left( -\frac{8d_1}{x-1} + \frac{4d_1}{(x-1)^2} - \right. \right. \\
& \left. \left. \frac{8d_1}{3(x-1)^3} + \frac{2d_1}{(x-1)^4} - \frac{50d_1}{3(x-1)^5} + \frac{16}{x-2} - \frac{10}{x-1} - \frac{32}{3(x-2)^2} + \frac{4}{3(x-1)^2} + \frac{32}{3(x-2)^3} - \frac{20}{3(x-1)^3} - \frac{6}{(x-1)^4} - \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \frac{50}{3(x-1)^5} + \frac{50}{3} \Big) H(0; \alpha_0) + \left( -\frac{4d_1^2}{x-1} + \frac{2d_1^2}{(x-1)^2} - \frac{4d_1^2}{3(x-1)^3} + \frac{d_1^2}{(x-1)^4} - \frac{25d_1^2}{3(x-1)^5} + \frac{8d_1}{x-2} - \frac{5d_1}{x-1} - \frac{16d_1}{3(x-2)^2} + \right. \\
& \frac{2d_1}{3(x-1)^2} + \frac{16d_1}{3(x-2)^3} - \frac{10d_1}{3(x-1)^3} - \frac{3d_1}{(x-1)^4} - \frac{25d_1}{3(x-1)^5} + \frac{25d_1}{3} \Big) H(1; \alpha_0) + \left( \frac{16}{(x-1)^5} - 16 \right) H(0, 0; \alpha_0) + \\
& \left( \frac{8d_1}{(x-1)^5} - 8d_1 \right) H(0, 1; \alpha_0) + \left( \frac{4d_1^2}{(x-1)^5} - 4d_1^2 \right) H(1, 1; \alpha_0) - \\
& \frac{80}{3(x-2)} + \frac{70}{3(x-1)} + \frac{56}{9(x-2)^2} + \frac{13}{18(x-1)^2} - \frac{56}{9(x-2)^3} + \frac{355}{18(x-1)^3} + \frac{10}{3(x-1)^4} - \frac{\pi^2}{6(x-1)^5} + \frac{155}{9(x-1)^5} + \frac{\pi^2}{6} - \\
& \frac{155}{9} \Big) H(1, c_1(\alpha_0); x) + \left( \frac{2\pi^2}{(x-1)^5} - 2\pi^2 \right) H(2, 0; x) + \left( \left( \frac{8d_1}{x-2} - \frac{16d_1}{3(x-2)^2} - \frac{8d_1}{3(x-1)^2} + \frac{16d_1}{3(x-2)^3} - \right. \right. \\
& \left. \frac{8d_1}{(x-1)^4} - \frac{16}{x-2} + \frac{15}{2(x-1)} + \frac{32}{3(x-2)^2} + \frac{1}{3(x-1)^2} - \frac{32}{3(x-2)^3} + \frac{5}{(x-1)^3} + \frac{17}{2(x-1)^4} + \frac{25}{2(x-1)^5} - \frac{25}{2} \right) H(0; \alpha_0) + \\
& \left( 8 - \frac{8}{(x-1)^5} \right) H(0, 0; \alpha_0) + \left( 4d_1 - \frac{4d_1}{(x-1)^5} \right) H(0, 1; \alpha_0) - \frac{\pi^2}{3(x-1)^5} + \frac{\pi^2}{3} \Big) H(2, 1; x) + \left( \frac{\pi^2 d_1}{(x-1)^5} - \right. \\
& \left. \pi^2 d_1 - \frac{2\pi^2}{(x-1)^5} + 2\pi^2 \right) H(2, 2; x) + \left( \frac{3d_1\alpha_0^4}{4} - \frac{3d_1\alpha_0^4}{4(x-1)} + \frac{3\alpha_0^4}{2(x-1)} - \frac{3\alpha_0^4}{2} - \frac{13d_1\alpha_0^3}{3} + \frac{3d_1\alpha_0^3}{x-1} - \frac{49\alpha_0^3}{6(x-1)} - \right. \\
& \frac{4d_1\alpha_0^3}{3(x-1)^2} + \frac{5\alpha_0^3}{(x-1)^2} + \frac{61\alpha_0^3}{6} + \frac{23d_1\alpha_0^2}{2} - \frac{5\alpha_0^2}{3(x-2)} - \frac{9d_1\alpha_0^2}{2(x-1)} + \frac{83\alpha_0^2}{4(x-1)} + \frac{4d_1\alpha_0^2}{(x-1)^2} - \frac{109\alpha_0^2}{6(x-1)^2} - \frac{3d_1\alpha_0^2}{(x-1)^3} + \frac{85\alpha_0^2}{6(x-1)^3} - \\
& \frac{131\alpha_0^2}{4} - 25d_1\alpha_0 + \frac{10\alpha_0}{x-2} + \frac{3d_1\alpha_0}{x-1} - \frac{229\alpha_0}{6(x-1)} - \frac{20\alpha_0}{3(x-2)^2} - \frac{4d_1\alpha_0}{(x-1)^2} + \frac{100\alpha_0}{3(x-1)^2} + \frac{6d_1\alpha_0}{(x-1)^3} - \frac{103\alpha_0}{3(x-1)^3} - \frac{12d_1\alpha_0}{(x-1)^4} + \\
& \frac{163\alpha_0}{3(x-1)^4} + \frac{169\alpha_0}{2} + \frac{205d_1}{12} + \left( -\frac{6\alpha_0^4}{x-1} + 6\alpha_0^4 + \frac{24\alpha_0^3}{x-1} - \frac{8\alpha_0^3}{(x-1)^2} - 32\alpha_0^3 - \frac{36\alpha_0^2}{x-1} + \frac{24\alpha_0^2}{(x-1)^2} - \frac{12\alpha_0^2}{(x-1)^3} + 72\alpha_0^2 + \right. \\
& \frac{24\alpha_0}{x-1} - \frac{24\alpha_0}{(x-1)^2} + \frac{24\alpha_0}{(x-1)^3} - \frac{24\alpha_0}{(x-1)^4} - 96\alpha_0 + \frac{18}{x-1} - \frac{4}{(x-1)^2} - \frac{18}{(x-1)^3} + \frac{18}{(x-1)^4} + \frac{50}{(x-1)^5} + 50 \Big) H(0; \alpha_0) + \\
& \left( 3d_1\alpha_0^4 - \frac{3d_1\alpha_0^4}{x-1} - 16d_1\alpha_0^3 + \frac{12d_1\alpha_0^3}{x-1} - \frac{4d_1\alpha_0^3}{(x-1)^2} + 36d_1\alpha_0^2 - \frac{18d_1\alpha_0^2}{x-1} + \frac{12d_1\alpha_0^2}{(x-1)^2} - \frac{6d_1\alpha_0^2}{(x-1)^3} - 48d_1\alpha_0 + \right. \\
& \frac{12d_1\alpha_0}{x-1} - \frac{12d_1\alpha_0}{(x-1)^2} + \frac{12d_1\alpha_0}{(x-1)^3} - \frac{12d_1\alpha_0}{(x-1)^4} + 25d_1 + \frac{9d_1}{x-1} - \frac{2d_1}{(x-1)^2} - \frac{2d_1}{(x-1)^3} + \frac{9d_1}{(x-1)^4} + \frac{25d_1}{(x-1)^5} \Big) H(1; \alpha_0) - \\
& \frac{16}{(x-1)^5} H(0, 0; \alpha_0) - \frac{8d_1 H(0, 1; \alpha_0)}{(x-1)^5} - \frac{8d_1 H(1, 0; \alpha_0)}{(x-1)^5} - \frac{4d_1^2 H(1, 1; \alpha_0)}{(x-1)^5} - \frac{20}{x-2} + \frac{45d_1}{4(x-1)} - \frac{31}{4(x-1)} + \frac{40}{3(x-2)^2} - \\
& \frac{5d_1}{3(x-1)^2} + \frac{55}{12(x-1)^2} - \frac{5d_1}{3(x-1)^3} - \frac{5}{4(x-1)^3} + \frac{45d_1}{4(x-1)^4} - \frac{83}{2(x-1)^4} + \frac{205d_1}{12(x-1)^5} + \frac{\pi^2}{6(x-1)^5} - \frac{75}{(x-1)^5} - \\
& \frac{725}{12} \Big) H(c_1(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{d_1\alpha_0^4}{4} + \frac{d_1\alpha_0^4}{4(x-1)} - \frac{\alpha_0^4}{2(x-1)} + \frac{\alpha_0^4}{2} + \frac{13d_1\alpha_0^3}{9} - \frac{d_1\alpha_0^3}{x-1} + \frac{3\alpha_0^3}{x-1} + \frac{4d_1\alpha_0^3}{9(x-1)^2} - \right. \\
& \frac{25\alpha_0^3}{18(x-1)^2} - \frac{61\alpha_0^3}{18} - \frac{23d_1\alpha_0^2}{6} + \frac{3d_1\alpha_0^2}{2(x-1)} - \frac{31\alpha_0^2}{4(x-1)} - \frac{4d_1\alpha_0^2}{3(x-1)^2} + \frac{71\alpha_0^2}{12(x-1)^2} + \frac{d_1\alpha_0^2}{(x-1)^3} - \frac{15\alpha_0^2}{4(x-1)^3} + \frac{131\alpha_0^2}{12} + \\
& \frac{25d_1\alpha_0}{3} - \frac{d_1\alpha_0}{x-1} + \frac{13\alpha_0}{x-1} + \frac{4d_1\alpha_0}{3(x-1)^2} - \frac{35\alpha_0}{3(x-1)^2} - \frac{2d_1\alpha_0}{(x-1)^3} + \frac{12\alpha_0}{(x-1)^3} + \frac{4d_1\alpha_0}{(x-1)^4} - \frac{29\alpha_0}{2(x-1)^4} - \frac{169\alpha_0}{6} - \frac{205d_1}{36} + \\
& \left( \frac{2\alpha_0^4}{x-1} - 2\alpha_0^4 - \frac{8\alpha_0^3}{x-1} + \frac{8\alpha_0^3}{3(x-1)^2} + \frac{32\alpha_0^3}{3} + \frac{12\alpha_0^2}{x-1} - \frac{8\alpha_0^2}{(x-1)^2} + \frac{4\alpha_0^2}{(x-1)^3} - 24\alpha_0^2 - \frac{8\alpha_0}{x-1} + \frac{8\alpha_0}{(x-1)^2} - \frac{8\alpha_0}{(x-1)^3} + \right. \\
& \frac{8\alpha_0}{(x-1)^4} + 32\alpha_0 - \frac{16}{x-2} + \frac{2}{x-1} + \frac{32}{3(x-2)^2} + \frac{8}{3(x-1)^2} - \frac{32}{3(x-2)^3} + \frac{4}{(x-1)^3} + \frac{8}{(x-1)^4} - \frac{50}{3} \Big) H(0; \alpha_0) + \\
& \left( -d_1\alpha_0^4 + \frac{d_1\alpha_0^4}{x-1} + \frac{16d_1\alpha_0^3}{3} - \frac{4d_1\alpha_0^3}{x-1} + \frac{4d_1\alpha_0^3}{3(x-1)^2} - 12d_1\alpha_0^2 + \frac{6d_1\alpha_0^2}{x-1} - \frac{4d_1\alpha_0^2}{(x-1)^2} + \frac{2d_1\alpha_0^2}{(x-1)^3} + 16d_1\alpha_0 - \right. \\
& \frac{4d_1\alpha_0}{x-1} + \frac{4d_1\alpha_0}{(x-1)^2} - \frac{4d_1\alpha_0}{(x-1)^3} + \frac{4d_1\alpha_0}{(x-1)^4} - \frac{25d_1}{3} - \frac{8d_1}{x-2} + \frac{d_1}{x-1} + \frac{16d_1}{3(x-2)^2} + \frac{4d_1}{3(x-1)^2} - \frac{16d_1}{3(x-2)^3} + \frac{2d_1}{(x-1)^3} + \\
& \frac{4d_1}{(x-1)^4} \Big) H(1; \alpha_0) - \frac{32d_1}{3(x-2)} + \frac{82}{3(x-2)} + \frac{35d_1}{12(x-1)} - \frac{73}{12(x-1)} + \frac{28d_1}{9(x-2)^2} - \frac{56}{9(x-2)^2} + \frac{28d_1}{9(x-1)^2} - \frac{323}{36(x-1)^2} - \\
& \frac{28d_1}{9(x-2)^3} + \frac{56}{9(x-2)^3} + \frac{5d_1}{(x-1)^3} - \frac{53}{4(x-1)^3} + \frac{4d_1}{(x-1)^4} - \frac{29}{2(x-1)^4} + \frac{725}{36} \Big) H(c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{100}{3} + \frac{12}{x-1} - \right. \\
& \frac{8}{3(x-1)^2} - \frac{8}{3(x-1)^3} + \frac{12}{(x-1)^4} + \frac{100}{3(x-1)^5} \Big) H(0, 0, 0; \alpha_0) + \left( -\frac{100}{3} - \frac{12}{x-1} + \frac{8}{3(x-1)^2} + \frac{8}{3(x-1)^3} - \frac{12}{(x-1)^4} - \right. \\
& \left. \frac{100}{3(x-1)^5} \right) H(0, 0, 0; x) + \left( \frac{6d_1}{x-1} - \frac{4d_1}{3(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \frac{6d_1}{(x-1)^4} + \frac{50d_1}{3(x-1)^5} + \frac{50d_1}{3} \right) H(0, 0, 1; \alpha_0) + \\
& \left( -\frac{4d_1}{(x-1)^5} + 4d_1 + \frac{12}{(x-1)^5} - 12 \right) H(0; \alpha_0) H(0, 0, 1; x) + \left( -\frac{\alpha_0^4}{x-1} + \alpha_0^4 + \frac{4\alpha_0^3}{x-1} - \frac{4\alpha_0^3}{3(x-1)^2} - \frac{16\alpha_0^3}{3} - \right. \\
& \frac{6\alpha_0^2}{x-1} + \frac{4\alpha_0^2}{(x-1)^2} - \frac{2\alpha_0^2}{(x-1)^3} + 12\alpha_0^2 + \frac{4\alpha_0}{x-1} - \frac{4\alpha_0}{(x-1)^2} + \frac{4\alpha_0}{(x-1)^3} - \frac{4\alpha_0}{(x-1)^4} - 16\alpha_0 + \left( \frac{8}{(x-1)^5} - 8 \right) H(0; \alpha_0) + \\
& \left( \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(1; \alpha_0) + \frac{8}{x-2} - \frac{1}{x-1} - \frac{16}{3(x-2)^2} - \frac{4}{3(x-1)^2} + \frac{16}{3(x-2)^3} - \frac{2}{(x-1)^3} - \frac{4}{(x-1)^4} + \\
& \frac{25}{3} \Big) H(0, 0, c_1(\alpha_0); x) + \left( \frac{6d_1}{x-1} - \frac{4d_1}{3(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \frac{6d_1}{(x-1)^4} + \frac{50d_1}{3(x-1)^5} + \frac{50d_1}{3} \right) H(0, 1, 0; \alpha_0) +
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{8 d_1}{x-2} - \frac{5d_1}{x-1} - \frac{16d_1}{3(x-2)^2} + \frac{2d_1}{3(x-1)^2} + \frac{16d_1}{3(x-2)^3} - \frac{10d_1}{3(x-1)^3} - \frac{3 d_1}{(x-1)^4} - \frac{25d_1}{3(x-1)^5} + \frac{25 d_1}{3} - \frac{8}{x-2} + \frac{16}{3(x-2)^2} + \right. \\
& \left. \frac{8}{3(x-1)^2} - \frac{16}{3(x-2)^3} + \frac{8}{(x-1)^4} \right) H(0, 1, 0; x) + \left( \frac{3d_1^2}{x-1} - \frac{2d_1^2}{3(x-1)^2} - \frac{2d_1^2}{3(x-1)^3} + \frac{3d_1^2}{(x-1)^4} + \frac{25d_1^2}{3(x-1)^5} + \right. \\
& \left. \frac{25 d_1^2}{3} \right) H(0, 1, 1; \alpha_0) + \left( \frac{4d_1^2}{(x-1)^5} - 4 d_1^2 - \frac{8d_1}{(x-1)^5} + \frac{12}{(x-1)^5} + 4 \right) H(0; \alpha_0) H(0, 1, 1; x) + \left( -\frac{8d_1}{x-2} + \right. \\
& \left. \frac{5d_1}{x-1} + \frac{16d_1}{3(x-2)^2} - \frac{2d_1}{3(x-1)^2} - \frac{16d_1}{3(x-2)^3} + \frac{10 d_1}{3(x-1)^3} + \frac{3d_1}{(x-1)^4} + \frac{25d_1}{3(x-1)^5} - \frac{25d_1}{3} + \left( -\frac{8d_1}{(x-1)^5} + 8 d_1 + \frac{8}{(x-1)^5} - \right. \right. \\
& \left. \left. 8 \right) H(0; \alpha_0) + \left( -\frac{4d_1^2}{(x-1)^5} + 4 d_1^2 + \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(1; \alpha_0) + \frac{8}{x-2} - \frac{16}{3(x-2)^2} - \frac{8}{3(x-1)^2} + \frac{16}{3(x-2)^3} - \right. \\
& \left. \frac{8}{(x-1)^4} \right) H(0, 1, c_1(\alpha_0); x) + \left( \frac{4d_1}{(x-1)^5} - 4 d_1 - \frac{4}{(x-1)^5} + 4 \right) H(0; \alpha_0) H(0, 2, 1; x) + \left( \frac{3 \alpha_0^4}{x-1} - 3\alpha_0^4 - \right. \\
& \left. \frac{12\alpha_0^3}{x-1} + \frac{4\alpha_0^3}{(x-1)^2} + 16 \alpha_0^3 + \frac{18\alpha_0^2}{x-1} - \frac{12\alpha_0^2}{(x-1)^2} + \frac{6 \alpha_0^2}{(x-1)^3} - 36\alpha_0^2 - \frac{12\alpha_0}{x-1} + \frac{12 \alpha_0}{(x-1)^2} - \frac{12\alpha_0}{(x-1)^3} + \frac{12\alpha_0}{(x-1)^4} + 48 \alpha_0 + \right. \\
& \left( \frac{8}{(x-1)^5} - 24 \right) H(0; \alpha_0) + \left( \frac{4 d_1}{(x-1)^5} - 12d_1 \right) H(1; \alpha_0) + \frac{20}{x-2} - \frac{11}{2(x-1)} - \frac{40}{3(x-2)^2} - \frac{8}{3(x-1)^2} + \frac{40}{3(x-2)^3} - \\
& \frac{13}{3(x-1)^3} - \frac{13}{(x-1)^4} - \frac{25}{3(x-1)^5} + \frac{25}{2} \right) H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{\alpha_0^4}{x-1} + \alpha_0^4 + \frac{4 \alpha_0^3}{x-1} - \frac{4\alpha_0^3}{3(x-1)^2} - \frac{16\alpha_0^3}{3} - \right. \\
& \left. \frac{6 \alpha_0^2}{x-1} + \frac{4\alpha_0^2}{(x-1)^2} - \frac{2\alpha_0^2}{(x-1)^3} + 12\alpha_0^2 + \frac{4\alpha_0}{x-1} - \frac{4\alpha_0}{(x-1)^2} + \frac{4\alpha_0}{(x-1)^3} - \frac{4 \alpha_0}{(x-1)^4} - 16\alpha_0 + \left( 8 - \frac{8}{(x-1)^5} \right) H(0; \alpha_0) + \right. \\
& \left( 4 d_1 - \frac{4d_1}{(x-1)^5} \right) H(1; \alpha_0) - \frac{16}{x-2} + \frac{13}{2(x-1)} + \frac{32}{3(x-2)^2} + \frac{5}{3(x-1)^2} - \frac{32}{3(x-2)^3} + \frac{3}{(x-1)^3} + \frac{25}{2(x-1)^4} + \\
& \left. \frac{25}{2(x-1)^5} - \frac{25}{6} \right) H(0, c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{8 d_1}{x-1} - \frac{4d_1}{(x-1)^2} + \frac{8d_1}{3(x-1)^3} - \frac{2 d_1}{(x-1)^4} + \frac{50d_1}{3(x-1)^5} - \frac{16}{x-2} + \right. \\
& \left. \frac{10}{x-1} + \frac{32}{3(x-2)^2} - \frac{4}{3(x-1)^2} - \frac{32}{3(x-2)^3} + \frac{20}{3(x-1)^3} + \frac{6}{(x-1)^4} + \frac{50}{3(x-1)^5} - \frac{50}{3} \right) H(1, 0, 0; x) + \left( \frac{4d_1^2}{(x-1)^5} - \right. \\
& \left. \frac{8 d_1}{(x-1)^5} - 4d_1 + \frac{8}{(x-1)^5} \right) H(0; \alpha_0) H(1, 0, 1; x) + \left( \frac{4d_1}{x-1} - \frac{2d_1}{(x-1)^2} + \frac{4 d_1}{3(x-1)^3} - \frac{d_1}{(x-1)^4} + \frac{25d_1}{3(x-1)^5} + \left( -\frac{8d_1}{(x-1)^5} + \frac{8}{(x-1)^5} + 8 \right) H(0; \alpha_0) + \left( -\frac{4d_1^2}{(x-1)^5} + \frac{4d_1}{(x-1)^5} + 4 d_1 \right) H(1; \alpha_0) - \frac{4}{x-2} - \frac{3}{2(x-1)} + \frac{8}{3(x-2)^2} - \frac{8}{3(x-2)^3} + \frac{11}{3(x-1)^3} - \frac{5}{(x-1)^4} - \frac{25}{3(x-1)^5} - \frac{125}{6} \right) H(1, 0, c_1(\alpha_0); x) + \left( -\frac{4d_1^2}{x-1} + \frac{2 d_1^2}{(x-1)^2} - \frac{4d_1^2}{3(x-1)^3} + \frac{d_1^2}{(x-1)^4} - \frac{25d_1^2}{3(x-1)^5} + \frac{8 d_1}{x-2} - \frac{9d_1}{x-1} - \frac{16d_1}{3(x-2)^2} + \frac{8d_1}{3(x-1)^2} + \frac{16d_1}{3(x-2)^3} - \frac{14d_1}{3(x-1)^3} - \frac{2 d_1}{(x-1)^4} - \frac{50d_1}{3(x-1)^5} + \frac{25 d_1}{3} + \frac{4}{x-2} + \frac{3}{2(x-1)} - \frac{8}{3(x-2)^2} + \frac{8}{3(x-2)^3} - \frac{11}{3(x-1)^3} + \frac{5}{(x-1)^4} + \frac{25}{3(x-1)^5} + \frac{125}{6} \right) H(1, 1, 0; x) + \left( \frac{12d_1^2}{(x-1)^5} - 4d_1^2 - \frac{12 d_1}{(x-1)^5} - 8d_1 + \frac{4}{(x-1)^5} + 4 \right) H(0; \alpha_0) H(1, 1, 1; x) + \left( \frac{4d_1^2}{x-1} - \frac{2d_1^2}{(x-1)^2} + \frac{4 d_1^2}{3(x-1)^3} - \frac{d_1^2}{(x-1)^4} + \frac{25d_1^2}{3(x-1)^5} - \frac{8d_1}{x-2} + \frac{9d_1}{x-1} + \frac{16d_1}{3(x-2)^2} - \frac{8d_1}{3(x-1)^2} - \frac{16d_1}{3(x-2)^3} + \frac{14 d_1}{3(x-1)^3} + \frac{2d_1}{(x-1)^4} + \frac{50d_1}{3(x-1)^5} - \frac{25d_1}{3} + \left( -\frac{16d_1}{(x-1)^5} + 8 d_1 + \frac{8}{(x-1)^5} + 8 \right) H(0; \alpha_0) + \left( -\frac{8d_1^2}{(x-1)^5} + 4 d_1^2 + \frac{4d_1}{(x-1)^5} + 4d_1 \right) H(1; \alpha_0) - \frac{4}{x-2} - \frac{3}{2(x-1)} + \frac{8}{3(x-2)^2} - \frac{8}{3(x-2)^3} + \frac{11}{3(x-1)^3} - \frac{5}{(x-1)^4} - \frac{25}{3(x-1)^5} - \frac{125}{6} \right) H(1, 1, c_1(\alpha_0); x) + \left( \frac{4d_1}{(x-1)^5} - \frac{2}{(x-1)^5} - 6 \right) H(0; \alpha_0) H(1, 2, 1; x) + \left( -\frac{12d_1}{x-1} + \frac{6 d_1}{(x-1)^2} - \frac{4d_1}{(x-1)^3} + \frac{3d_1}{(x-1)^4} - \frac{25 d_1}{(x-1)^5} + \left( \frac{8d_1}{(x-1)^5} + \frac{8}{(x-1)^5} - 24 \right) H(0; \alpha_0) + \left( \frac{4d_1^2}{(x-1)^5} + \frac{4d_1}{(x-1)^5} - 12 d_1 \right) H(1; \alpha_0) + \frac{20}{x-2} - \frac{17}{2(x-1)} - \frac{40}{3(x-2)^2} + \frac{4}{3(x-1)^2} + \frac{40}{3(x-2)^3} - \frac{31}{3(x-1)^3} - \frac{1}{(x-1)^4} - \frac{25}{3(x-1)^5} + \frac{75}{2} \right) H(1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{4d_1}{(x-1)^5} - 4 d_1 - \frac{4}{(x-1)^5} + 4 \right) H(0; \alpha_0) H(2, 0, 1; x) + \left( \frac{8 d_1}{x-2} - \frac{16d_1}{3(x-2)^2} - \frac{8d_1}{3(x-1)^2} + \frac{16 d_1}{3(x-2)^3} - \frac{8d_1}{(x-1)^4} + \left( 8 - \frac{8}{(x-1)^5} \right) H(0; \alpha_0) + \left( 4d_1 - \frac{4d_1}{(x-1)^5} \right) H(1; \alpha_0) - \frac{16}{x-2} + \frac{15}{2(x-1)} + \frac{32}{3(x-2)^2} + \frac{1}{3(x-1)^2} - \frac{32}{3(x-2)^3} + \frac{5}{(x-1)^3} + \frac{17}{2(x-1)^4} + \frac{25}{2(x-1)^5} - \frac{25}{2} \right) H(2, 0, c_1(\alpha_0); x) + \left( -\frac{8d_1}{x-2} + \frac{16 d_1}{3(x-2)^2} + \frac{8d_1}{3(x-1)^2} - \frac{16d_1}{3(x-2)^3} + \frac{8d_1}{(x-1)^4} + \frac{16}{x-2} - \frac{15}{2(x-1)} - \frac{32}{3(x-2)^2} - \frac{1}{3(x-1)^2} + \frac{32}{3(x-2)^3} - \frac{5}{(x-1)^3} - \frac{17}{2(x-1)^4} - \frac{25}{2(x-1)^5} + \frac{25}{2} \right) H(2, 1, 0; x) + \left( \frac{4d_1}{(x-1)^5} - 4 d_1 + \frac{2}{(x-1)^5} - 2 \right) H(0; \alpha_0) H(2, 1, 1; x) + \left( \frac{8 d_1}{x-2} - \frac{16d_1}{3(x-2)^2} - \frac{8d_1}{3(x-1)^2} + \frac{16 d_1}{3(x-2)^3} - \frac{8d_1}{(x-1)^4} + \left( 8 - \frac{8}{(x-1)^5} \right) H(0; \alpha_0) + \left( 4d_1 - \frac{4d_1}{(x-1)^5} \right) H(1; \alpha_0) - \frac{16}{x-2} + \frac{15}{2(x-1)} + \frac{32}{3(x-2)^2} + \frac{1}{3(x-1)^2} - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{32}{3(x-2)^3} + \frac{5}{(x-1)^3} + \frac{17}{2(x-1)^4} + \frac{25}{2(x-1)^5} - \frac{25}{2} \Big) H(2, 1, c_1(\alpha_0); x) + \left( -\frac{4d_1}{(x-1)^5} + 4d_1 + \frac{8}{(x-1)^5} - \right. \\
& \left. 8 \right) H(0; \alpha_0) H(2, 2, 1; x) + \left( -\frac{8d_1}{x-2} + \frac{16d_1}{3(x-2)^2} + \frac{8d_1}{3(x-1)^2} - \frac{16d_1}{3(x-2)^3} + \frac{8d_1}{(x-1)^4} + \left( \frac{8}{(x-1)^5} - 8 \right) H(0; \alpha_0) + \right. \\
& \left( \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(1; \alpha_0) + \frac{16}{x-2} - \frac{15}{2(x-1)} - \frac{32}{3(x-2)^2} - \frac{1}{3(x-1)^2} + \frac{32}{3(x-2)^3} - \frac{5}{(x-1)^3} - \frac{17}{2(x-1)^4} - \\
& \frac{25}{2(x-1)^5} + \frac{25}{2} \Big) H(2, c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{2\alpha_0^4}{x-1} - 2\alpha_0^4 - \frac{8\alpha_0^3}{x-1} + \frac{8\alpha_0^3}{3(x-1)^2} + \frac{32}{3} \frac{\alpha_0^3}{x-1} + \frac{12\alpha_0^2}{x-1} - \frac{8\alpha_0^2}{(x-1)^2} + \right. \\
& \frac{4}{(x-1)^3} - 24\alpha_0^2 - \frac{8\alpha_0}{x-1} + \frac{8\alpha_0}{(x-1)^2} - \frac{8\alpha_0}{(x-1)^3} + \frac{8\alpha_0}{(x-1)^4} + 32\alpha_0 + \frac{8H(0; \alpha_0)}{(x-1)^5} + \frac{4d_1 H(1; \alpha_0)}{(x-1)^5} - \frac{6}{x-1} + \frac{4}{3(x-1)^2} + \\
& \frac{4}{3(x-1)^3} - \frac{6}{(x-1)^4} - \frac{50}{3(x-1)^5} - \frac{50}{3} \Big) H(c_1(\alpha_0), 0, c_1(\alpha_0); x) + \left( -\frac{7\alpha_0^4}{x-1} + 7\alpha_0^4 + \frac{28}{x-1} \frac{\alpha_0^3}{x-1} - \frac{28\alpha_0^3}{3(x-1)^2} - \right. \\
& \frac{112\alpha_0^3}{3} - \frac{42}{x-1} \frac{\alpha_0^2}{x-1} + \frac{28\alpha_0^2}{(x-1)^2} - \frac{14\alpha_0^2}{(x-1)^3} + 84\alpha_0^2 + \frac{28\alpha_0}{x-1} - \frac{28\alpha_0}{(x-1)^2} + \frac{28}{(x-1)^3} \frac{\alpha_0}{x-1} - \frac{28\alpha_0}{(x-1)^4} - 112\alpha_0 - \frac{24}{(x-1)^5} \frac{H(0; \alpha_0)}{x-1} - \\
& \frac{12d_1}{(x-1)^5} \frac{H(1; \alpha_0)}{x-1} + \frac{21}{x-1} - \frac{14}{3(x-1)^2} - \frac{14}{3(x-1)^3} + \frac{21}{(x-1)^4} + \frac{175}{3(x-1)^5} + \frac{175}{3} \Big) H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \\
& \left( \frac{3\alpha_0^4}{2(x-1)} - \frac{3}{2} \frac{\alpha_0^4}{x-1} - \frac{6\alpha_0^3}{x-1} + \frac{2\alpha_0^3}{(x-1)^2} + 8\alpha_0^3 + \frac{9}{x-1} \frac{\alpha_0^2}{x-1} - \frac{6\alpha_0^2}{(x-1)^2} + \frac{3\alpha_0^2}{(x-1)^3} - 18\alpha_0^2 - \frac{6\alpha_0}{x-1} + \frac{6\alpha_0}{(x-1)^2} - \frac{6\alpha_0}{(x-1)^3} + \right. \\
& \frac{6}{(x-1)^4} \frac{\alpha_0}{x-1} + 24\alpha_0 + \frac{8H(0; \alpha_0)}{(x-1)^5} + \frac{4d_1 H(1; \alpha_0)}{(x-1)^5} - \frac{9}{2(x-1)} + \frac{1}{(x-1)^2} + \frac{1}{(x-1)^3} - \frac{9}{2(x-1)^4} - \frac{25}{2(x-1)^5} - \\
& \frac{25}{2} \Big) H(c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{\alpha_0^4}{x-1} + \alpha_0^4 + \frac{4}{x-1} \frac{\alpha_0^3}{x-1} - \frac{4\alpha_0^3}{3(x-1)^2} - \frac{16\alpha_0^3}{3} - \frac{6}{x-1} \frac{\alpha_0^2}{x-1} + \frac{4\alpha_0^2}{(x-1)^2} - \frac{2\alpha_0^2}{(x-1)^3} + \right. \\
& 12\alpha_0^2 + \frac{4\alpha_0}{x-1} - \frac{4\alpha_0}{(x-1)^2} + \frac{4\alpha_0}{(x-1)^3} - \frac{4}{(x-1)^4} - 16\alpha_0 + \frac{8}{x-2} - \frac{1}{x-1} - \frac{16}{3(x-2)^2} - \frac{4}{3(x-1)^2} + \frac{16}{3(x-2)^3} - \\
& \frac{2}{(x-1)^3} - \frac{4}{(x-1)^4} + \frac{25}{3} \Big) H(c_2(\alpha_0), 0, c_1(\alpha_0); x) + \left( \frac{5\alpha_0^4}{2(x-1)} - \frac{5}{2} \frac{\alpha_0^4}{x-1} - \frac{10\alpha_0^3}{x-1} + \frac{10\alpha_0^3}{3(x-1)^2} + \frac{40}{3} \frac{\alpha_0^3}{x-1} + \frac{15\alpha_0^2}{x-1} - \right. \\
& \frac{10\alpha_0^2}{(x-1)^2} + \frac{5}{(x-1)^3} \frac{\alpha_0^2}{x-1} - 30\alpha_0^2 - \frac{10\alpha_0}{x-1} + \frac{10}{(x-1)^2} \frac{\alpha_0}{x-1} - \frac{10\alpha_0}{(x-1)^3} + \frac{10\alpha_0}{(x-1)^4} + 40\alpha_0 - \frac{20}{x-2} + \frac{5}{2(x-1)} + \frac{40}{3(x-2)^2} + \\
& \frac{10}{3(x-1)^2} - \frac{40}{3(x-2)^3} + \frac{5}{(x-1)^3} + \frac{10}{(x-1)^4} - \frac{125}{6} \Big) H(c_2(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + 32 H(0, 0, 0, 0; x) + \\
& \left( \frac{12}{(x-1)^5} - 12 \right) H(0, 0, 0, c_1(\alpha_0); x) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 - \frac{12}{(x-1)^5} + 12 \right) H(0, 0, 1, 0; x) + \left( -\frac{4d_1}{(x-1)^5} + \right. \\
& \left. 4d_1 + \frac{12}{(x-1)^5} - 12 \right) H(0, 0, 1, c_1(\alpha_0); x) + \left( -12 - \frac{4}{(x-1)^5} \right) H(0, 0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( 4 - \right. \\
& \left. \frac{4}{(x-1)^5} \right) H(0, 0, c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{8d_1}{(x-1)^5} - 8d_1 - \frac{8}{(x-1)^5} + 8 \right) H(0, 1, 0, 0; x) + \left( -\frac{4d_1}{(x-1)^5} - \right. \\
& \left. 4d_1 + \frac{12}{(x-1)^5} + 4 \right) H(0, 1, 0, c_1(\alpha_0); x) + \left( -\frac{4d_1^2}{(x-1)^5} + 4d_1^2 + \frac{8d_1}{(x-1)^5} - \frac{12}{(x-1)^5} - 4 \right) H(0, 1, 1, 0; x) + \\
& \left( \frac{4d_1^2}{(x-1)^5} - 4d_1^2 - \frac{8d_1}{(x-1)^5} + \frac{12}{(x-1)^5} + 4 \right) H(0, 1, 1, c_1(\alpha_0); x) + \left( -\frac{4d_1}{(x-1)^5} + 12d_1 - \frac{4}{(x-1)^5} - \right. \\
& \left. 12 \right) H(0, 1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 - \frac{4}{(x-1)^5} + 4 \right) H(0, 2, 0, c_1(\alpha_0); x) + \left( -\frac{4d_1}{(x-1)^5} + \right. \\
& \left. 4d_1 + \frac{4}{(x-1)^5} - 4 \right) H(0, 2, 1, 0; x) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 - \frac{4}{(x-1)^5} + 4 \right) H(0, 2, 1, c_1(\alpha_0); x) + \left( -\frac{4d_1}{(x-1)^5} + \right. \\
& \left. \frac{4d_1}{(x-1)^5} + 4d_1 + \frac{4}{(x-1)^5} - 4 \right) H(0, 2, c_2(\alpha_0), c_1(\alpha_0); x) + 8 H(0, c_1(\alpha_0), 0, c_1(\alpha_0); x) + \left( \frac{4}{(x-1)^5} - \right. \\
& \left. 28 \right) H(0, c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \left( 6 + \frac{2}{(x-1)^5} \right) H(0, c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{4}{(x-1)^5} - \right. \\
& \left. 4 \right) H(0, c_2(\alpha_0), 0, c_1(\alpha_0); x) + \left( 10 - \frac{10}{(x-1)^5} \right) H(0, c_2(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \left( 16 - \right. \\
& \left. \frac{16}{(x-1)^5} \right) H(1, 0, 0, 0; x) + \left( \frac{8}{(x-1)^5} - \frac{4d_1}{(x-1)^5} \right) H(1, 0, 0, c_1(\alpha_0); x) + \left( -\frac{4d_1^2}{(x-1)^5} + \frac{8d_1}{(x-1)^5} + 4d_1 - \right. \\
& \left. \frac{8}{(x-1)^5} \right) H(1, 0, 1, 0; x) + \left( \frac{4d_1^2}{(x-1)^5} - \frac{8d_1}{(x-1)^5} - 4d_1 + \frac{8}{(x-1)^5} \right) H(1, 0, 1, c_1(\alpha_0); x) + \left( -\frac{4d_1}{(x-1)^5} + \right. \\
& \left. \frac{4}{(x-1)^5} + 4 \right) H(1, 0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{4d_1}{(x-1)^5} - \frac{2}{(x-1)^5} - 6 \right) H(1, 0, c_2(\alpha_0), c_1(\alpha_0); x) + \\
& \left( \frac{16d_1}{(x-1)^5} - 8d_1 - \frac{8}{(x-1)^5} - 8 \right) H(1, 1, 0, 0; x) + \left( \frac{4d_1^2}{(x-1)^5} - \frac{8d_1}{(x-1)^5} - 4d_1 + \frac{4}{(x-1)^5} + \right. \\
& \left. 4 \right) H(1, 1, 0, c_1(\alpha_0); x) + \left( -\frac{12d_1^2}{(x-1)^5} + 4d_1^2 + \frac{12d_1}{(x-1)^5} + 8d_1 - \frac{4}{(x-1)^5} - 4 \right) H(1, 1, 1, 0; x) + \\
& \left( \frac{12d_1^2}{(x-1)^5} - 4d_1^2 - \frac{12d_1}{(x-1)^5} - 8d_1 + \frac{4}{(x-1)^5} + 4 \right) H(1, 1, 1, c_1(\alpha_0); x) + \left( -\frac{4d_1^2}{(x-1)^5} - \frac{8d_1}{(x-1)^5} + 12d_1 + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{4}{(x-1)^5} + 4\right) H(1, 1, c_1(\alpha_0), c_1(\alpha_0); x) + \left(\frac{4 d_1}{(x-1)^5} - \frac{2}{(x-1)^5} - 6\right) H(1, 2, 0, c_1(\alpha_0); x) + \left(-\frac{4 d_1}{(x-1)^5} + \frac{2}{(x-1)^5} + 6\right) H(1, 2, 1, 0; x) + \left(\frac{4 d_1}{(x-1)^5} - \frac{2}{(x-1)^5} - 6\right) H(1, 2, 1, c_1(\alpha_0); x) + \left(-\frac{4 d_1}{(x-1)^5} + \frac{2}{(x-1)^5} + 6\right) H(1, 2, c_2(\alpha_0), c_1(\alpha_0); x) + \left(8 - \frac{4d_1}{(x-1)^5}\right) H(1, c_1(\alpha_0), 0, c_1(\alpha_0); x) + \left(\frac{12 d_1}{(x-1)^5} + \frac{4}{(x-1)^5} - 28\right) H(1, c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \left(-\frac{4 d_1}{(x-1)^5} + \frac{2}{(x-1)^5} + 6\right) H(1, c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x) + \left(4 - \frac{4}{(x-1)^5}\right) H(2, 0, 0, c_1(\alpha_0); x) + \left(-\frac{4d_1}{(x-1)^5} + 4 d_1 + \frac{4}{(x-1)^5} - 4\right) H(2, 0, 1, 0; x) + \left(\frac{4 d_1}{(x-1)^5} - 4d_1 - \frac{4}{(x-1)^5} + 4\right) H(2, 0, 1, c_1(\alpha_0); x) + \left(10 - \frac{10}{(x-1)^5}\right) H(2, 0, c_1(\alpha_0), c_1(\alpha_0); x) + \left(\frac{8}{(x-1)^5} - 8\right) H(2, 0, c_2(\alpha_0), c_1(\alpha_0); x) + \left(\frac{8}{(x-1)^5} - 8\right) H(2, 1, 0, 0; x) + \left(\frac{2}{(x-1)^5} - 2\right) H(2, 1, 0, c_1(\alpha_0); x) + \left(-\frac{4d_1}{(x-1)^5} + 4d_1 - \frac{2}{(x-1)^5} + 2\right) H(2, 1, 1, 0; x) + \left(\frac{4d_1}{(x-1)^5} - 4 d_1 + \frac{2}{(x-1)^5} - 2\right) H(2, 1, 1, c_1(\alpha_0); x) + \left(10 - \frac{10}{(x-1)^5}\right) H(2, 1, c_1(\alpha_0), c_1(\alpha_0); x) + \left(-\frac{4d_1}{(x-1)^5} + 4 d_1 + \frac{8}{(x-1)^5} - 8\right) H(2, 2, 0, c_1(\alpha_0); x) + \left(\frac{4 d_1}{(x-1)^5} - 4d_1 - \frac{8}{(x-1)^5} + 8\right) H(2, 2, 1, 0; x) + \left(-\frac{4d_1}{(x-1)^5} + 4 d_1 + \frac{8}{(x-1)^5} - 8\right) H(2, 2, 1, c_1(\alpha_0); x) + \left(\frac{4 d_1}{(x-1)^5} - 4d_1 - \frac{8}{(x-1)^5} + 8\right) H(2, 2, c_2(\alpha_0), c_1(\alpha_0); x) + \left(4 - \frac{4}{(x-1)^5}\right) H(2, c_2(\alpha_0), 0, c_1(\alpha_0); x) + \left(\frac{10}{(x-1)^5} - 10\right) H(2, c_2(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) - \frac{4 H(c_1(\alpha_0), 0, 0, c_1(\alpha_0); x)}{(x-1)^5} + \frac{12 H(c_1(\alpha_0), 0, c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} - \frac{4 H(c_1(\alpha_0), 0, c_2(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \frac{8 H(c_1(\alpha_0), c_1(\alpha_0), 0, c_1(\alpha_0); x)}{(x-1)^5} - \frac{28 H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \frac{6 H(c_1(\alpha_0), c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} - \frac{4 H(c_1(\alpha_0), c_2(\alpha_0), 0, c_1(\alpha_0); x)}{(x-1)^5} + \frac{10 H(c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + H(0; x) \left(-\frac{63d_1^2}{16(x-1)} + \frac{19d_1^2}{54(x-1)^2} + \frac{19 d_1^2}{54(x-1)^3} - \frac{63d_1^2}{16(x-1)^4} - \frac{2035d_1^2}{432(x-1)^5} - \frac{2035d_1^2}{432} + \frac{38d_1}{3(x-2)} + \frac{85 d_1}{24(x-1)} - \frac{76d_1}{9(x-2)^2} - \frac{317d_1}{108(x-1)^2} + \frac{313d_1}{108(x-1)^3} + \frac{257d_1}{8(x-1)^4} + \frac{8705d_1}{216(x-1)^5} + \frac{5615d_1}{216} - \frac{4 \pi^2}{x-2} - \frac{68}{3(x-2)} + \frac{13\pi^2}{8(x-1)} + \frac{37}{4(x-1)} + \frac{8\pi^2}{3(x-2)^2} + \frac{152}{9(x-2)^2} + \frac{5\pi^2}{12(x-1)^2} - \frac{49}{108(x-1)^2} - \frac{8\pi^2}{3(x-2)^3} + \frac{3\pi^2}{4(x-1)^3} - \frac{949}{108(x-1)^3} + \frac{25 \pi^2}{8(x-1)^4} - \frac{213}{4(x-1)^4} + \frac{25\pi^2}{8(x-1)^5} - \frac{3965}{54(x-1)^5} + \frac{3\zeta_3}{(x-1)^5} + 16\zeta_3 + \frac{2 \pi^2 \ln 2}{(x-1)^5} - 2\pi^2 \ln 2 - \frac{25\pi^2}{24} - \frac{940}{27}\right) + H(2; x) \left(-\frac{2\pi^2 d_1}{x-2} + \frac{4\pi^2 d_1}{3(x-2)^2} + \frac{2\pi^2 d_1}{3(x-1)^2} - \frac{4\pi^2 d_1}{3(x-2)^3} + \frac{2\pi^2 d_1}{(x-1)^4} + \frac{4\pi^2}{x-2} - \frac{15\pi^2}{8(x-1)} - \frac{8\pi^2}{3(x-2)^2} - \frac{8\pi^2}{12(x-1)^2} + \frac{8\pi^2}{3(x-2)^3} - \frac{5 \pi^2}{4(x-1)^3} - \frac{17\pi^2}{8(x-1)^4} - \frac{25\pi^2}{8(x-1)^5} + \frac{7\zeta_3}{2(x-1)^5} - \frac{7\zeta_3}{2} - \frac{3\pi^2 \ln 2}{(x-1)^5} + 3\pi^2 \ln 2 + \frac{25\pi^2}{8}\right) + H(1; x) \left(-\frac{2\pi^2 d_1}{3(x-1)} + \frac{\pi^2 d_1}{3(x-1)^2} - \frac{2\pi^2 d_1}{9(x-1)^3} + \frac{\pi^2 d_1}{6(x-1)^4} - \frac{25\pi^2 d_1}{18(x-1)^5} - \frac{3\zeta_3 d_1}{2(x-1)^5} - \frac{\pi^2 \ln 2 d_1}{(x-1)^5} + \left(-\frac{4 d_1^2}{x-1} + \frac{d_1^2}{(x-1)^2} - \frac{4d_1^2}{9(x-1)^3} + \frac{d_1^2}{4(x-1)^4} - \frac{205d_1^2}{36(x-1)^5} + \frac{46d_1}{3(x-2)} + \frac{3d_1}{4(x-1)} - \frac{52d_1}{9(x-2)^2} - \frac{185d_1}{18(x-1)^2} + \frac{28d_1}{9(x-2)^3} + \frac{29 d_1}{18(x-1)^3} - \frac{27d_1}{4(x-1)^4} + \frac{835d_1}{36(x-1)^5} + \frac{205d_1}{36} - \frac{80}{3(x-2)} + \frac{70}{3(x-1)} + \frac{56}{9(x-2)^2} + \frac{13}{18(x-1)^2} - \frac{56}{9(x-2)^3} + \frac{355}{18(x-1)^3} + \frac{10}{3(x-1)^4} - \frac{\pi^2}{6(x-1)^5} + \frac{155}{9(x-1)^5} + \frac{\pi^2}{6} - \frac{155}{9}\right) H(0; \alpha_0) + \left(-\frac{8 d_1}{x-1} + \frac{4d_1}{(x-1)^2} - \frac{8d_1}{3(x-1)^3} + \frac{2 d_1}{(x-1)^4} - \frac{3(50d_1)}{3(x-1)^5} + \frac{16}{x-2} - \frac{10}{x-1} - \frac{32}{3(x-2)^2} + \frac{4}{3(x-1)^2} + \frac{32}{3(x-2)^3} - \frac{3(x-1)^3}{(x-1)^4} - \frac{50}{3(x-1)^5} + \frac{50}{3}\right) H(0, 0; \alpha_0) + \left(-\frac{4d_1^2}{x-1} + \frac{2 d_1^2}{(x-1)^2} - \frac{4d_1^2}{3(x-1)^3} + \frac{d_1^2}{(x-1)^4} - \frac{25d_1^2}{3(x-1)^5} + \frac{8 d_1}{x-2} - \frac{5d_1}{x-1} - \frac{16d_1}{3(x-2)^2} + \frac{2d_1}{3(x-1)^2} + \frac{16d_1}{3(x-2)^3} - \frac{10d_1}{3(x-1)^3} - \frac{3 d_1}{(x-1)^4} - \frac{25d_1}{3(x-1)^5} + \frac{25d_1}{3}\right) H(0, 1; \alpha_0) + \left(\frac{16}{(x-1)^5} - 16\right) H(0, 0, 0; \alpha_0) + \left(\frac{8 d_1}{(x-1)^5} - 8d_1\right) H(0, 0, 1; \alpha_0) + \left(\frac{8d_1}{(x-1)^5} - 8 d_1\right) H(0, 1, 0; \alpha_0) + \left(\frac{4d_1^2}{(x-1)^5} - 4d_1^2\right) H(0, 1, 1; \alpha_0) + \frac{2\pi^2}{3(x-2)} + \frac{\pi^2}{4(x-1)} - \frac{4 \pi^2}{9(x-2)^2} + \frac{4\pi^2}{9(x-2)^3} - \frac{11\pi^2}{18(x-1)^3} + \frac{5\pi^2}{6(x-1)^4} + \frac{25\pi^2}{18(x-1)^5} - \frac{21\zeta_3}{4(x-1)^5} + \frac{33\zeta_3}{4} + \frac{\pi^2 \ln 2}{2(x-1)^5} + \frac{3}{2} \pi^2 \ln 2 + \frac{125\pi^2}{36} - \frac{8d_1 \pi^2}{3(x-2)} + \frac{37\pi^2}{6(x-2)} + \frac{65d_1 \pi^2}{48(x-1)} - \frac{151\pi^2}{48(x-1)} + \frac{7d_1 \pi^2}{9(x-2)^2} - \frac{10\pi^2}{9(x-2)^2} + \frac{37d_1 \pi^2}{54(x-1)^2} - \frac{847\pi^2}{432(x-1)^2} - \frac{7d_1 \pi^2}{9(x-2)^3} + \frac{14\pi^2}{9(x-2)^3} + \frac{125d_1 \pi^2}{108(x-1)^3} - \frac{1441\pi^2}{432(x-1)^3} + \frac{13d_1 \pi^2}{8(x-1)^4} - \frac{109\pi^2}{24(x-1)^4} - \frac{241\pi^4}{720(x-1)^5} + \frac{205d_1 \pi^2}{216(x-1)^5} - \frac{155\pi^2}{54(x-1)^5} - \frac{7\zeta_3}{x-2} - \frac{85 \zeta_3}{16(x-1)} + \frac{14\zeta_3}{3(x-2)^2} + \frac{61\zeta_3}{24(x-1)^2} - \frac{14\zeta_3}{3(x-2)^3} + \frac{25\zeta_3}{8(x-1)^3} - \frac{43\zeta_3}{16(x-1)^4} -
\end{aligned}$$

$$\frac{275\zeta_3}{16(x-1)^5} - \frac{1175\zeta_3}{48} - \frac{4\text{Li}_4\frac{1}{2}}{(x-1)^5} + 4\text{Li}_4\frac{1}{2} - \frac{\ln^4 2}{6(x-1)^5} + \frac{\ln^4 2}{6} - \frac{4\pi^2 \ln^2 2}{3(x-1)^5} + \frac{4}{3}\pi^2 \ln^2 2 + \frac{6\pi^2 \ln 2}{x-2} - \frac{15\pi^2 \ln 2}{8(x-1)} - \frac{4\pi^2 \ln 2}{(x-2)^2} - \frac{3\pi^2 \ln 2}{4(x-1)^2} + \frac{4\pi^2 \ln 2}{(x-2)^3} - \frac{5\pi^2 \ln 2}{4(x-1)^3} - \frac{33\pi^2 \ln 2}{8(x-1)^4} - \frac{25\pi^2 \ln 2}{8(x-1)^5} + \frac{25}{8}\pi^2 \ln 2 + \frac{\pi^4}{30} - \frac{205}{432} \frac{d_1 \pi^2}{x-1} + \frac{305\pi^2}{432}.$$

### E.5 The $\mathcal{B}$ integral for $k = 0$ and $\delta = 1$

The  $\varepsilon$  expansion for this integral reads

$$\begin{aligned} \mathcal{I}(x, \varepsilon; \alpha_0, 3 + d_1 \varepsilon; 1, 0, 1, g_B) &= x \mathcal{B}(\varepsilon, x; 3 + d_1 \varepsilon; 1, 0) \\ &= \frac{1}{\varepsilon} b_{-1}^{(1,0)} + b_0^{(1,0)} + \varepsilon b_1^{(1,0)} + \varepsilon^2 b_2^{(1,0)} + \mathcal{O}(\varepsilon^3), \end{aligned} \quad (\text{E.5})$$

where

$$\begin{aligned} b_{-1}^{(1,0)} &= -\frac{1}{2}, \\ b_0^{(1,0)} &= \frac{\alpha_0^4}{4(x-1)} + \frac{\alpha_0^4}{4} - \frac{\alpha_0^3}{x-1} + \frac{\alpha_0^3}{3(x-1)^2} - \frac{4\alpha_0^3}{3} + \frac{3\alpha_0^2}{2(x-1)} - \frac{\alpha_0^2}{(x-1)^2} + \frac{\alpha_0^2}{2(x-1)^3} + 3\alpha_0^2 - \frac{\alpha_0}{x-1} + \frac{\alpha_0}{(x-1)^2} - \frac{\alpha_0}{(x-1)^3} + \frac{\alpha_0}{(x-1)^4} - 4\alpha_0 + \left(1 + \frac{1}{(x-1)^5}\right) H(0; \alpha_0) + \left(1 - \frac{1}{(x-1)^5}\right) H(0; x) + \frac{H(c_1(\alpha_0); x)}{(x-1)^5} - 1, \\ b_1^{(1,0)} &= \\ &= -\frac{d_1 \alpha_0^4}{8} - \frac{d_1 \alpha_0^4}{8(x-1)} + \frac{3\alpha_0^4}{4(x-1)} + \frac{3\alpha_0^4}{4} + \frac{13d_1 \alpha_0^3}{18} + \frac{d_1 \alpha_0^3}{2(x-1)} - \frac{3\alpha_0^3}{x-1} - \frac{2d_1 \alpha_0^3}{9(x-1)^2} + \frac{23}{18} \frac{\alpha_0^3}{(x-1)^2} - \frac{77\alpha_0^3}{18} - \frac{23d_1}{12} \frac{\alpha_0^2}{(x-1)} - \frac{3d_1 \alpha_0^2}{4(x-1)} + \frac{53\alpha_0^2}{12(x-1)} + \frac{2d_1 \alpha_0^2}{3(x-1)^2} - \frac{47\alpha_0^2}{12(x-1)^2} - \frac{d_1 \alpha_0^2}{2(x-1)^3} + \frac{31\alpha_0^2}{12(x-1)^3} + \frac{131\alpha_0^2}{12} + \frac{25d_1 \alpha_0}{6} + \frac{d_1 \alpha_0}{2(x-1)} - \frac{7\alpha_0}{3(x-1)} - \frac{2d_1 \alpha_0}{3(x-1)^2} + \frac{4}{3} \frac{\alpha_0}{(x-1)^2} + \frac{d_1 \alpha_0}{3(x-1)^3} - \frac{17\alpha_0}{3(x-1)^3} - \frac{2d_1 \alpha_0}{(x-1)^4} + \frac{49\alpha_0}{6(x-1)^4} - \frac{121}{6} \alpha_0 + \left(-\frac{\alpha_0^4}{x-1} - \alpha_0^4 + \frac{4\alpha_0^3}{x-1} - \frac{4}{3} \frac{\alpha_0^3}{(x-1)^2} + \frac{16\alpha_0^3}{3} - \frac{6\alpha_0^2}{x-1} + \frac{4}{(x-1)^2} - \frac{2\alpha_0^2}{(x-1)^3} - 12\alpha_0^2 + \frac{4}{x-1} - \frac{4\alpha_0}{(x-1)^2} + \frac{4\alpha_0}{(x-1)^3} - \frac{4}{(x-1)^4} + 16\alpha_0 - \frac{5}{2(x-1)} + \frac{5}{3(x-1)^2} - \frac{5}{3(x-1)^3} + \frac{5}{2(x-1)^4} + \frac{37}{6(x-1)^5} - \frac{13}{6}\right) H(0; \alpha_0) + \left(\frac{37}{6} + \frac{5}{2(x-1)} - \frac{5}{3(x-1)^2} + \frac{5}{3(x-1)^3} - \frac{5}{2(x-1)^4} - \frac{37}{6(x-1)^5}\right) H(0; x) + \left(-\frac{d_1 \alpha_0^4}{2} - \frac{d_1 \alpha_0^4}{2(x-1)} + \frac{8d_1 \alpha_0^3}{3} + \frac{2d_1 \alpha_0^3}{x-1} - \frac{2d_1 \alpha_0^3}{3(x-1)^2} - 6d_1 \alpha_0^2 - \frac{3d_1 \alpha_0^2}{x-1} + \frac{2d_1 \alpha_0^2}{(x-1)^2} - \frac{d_1 \alpha_0^2}{(x-1)^3} + 8d_1 \alpha_0 + \frac{2d_1 \alpha_0}{x-1} - \frac{2d_1 \alpha_0}{(x-1)^2} + \frac{2d_1 \alpha_0}{(x-1)^3} - \frac{2}{(x-1)^4} - \frac{25d_1}{6} - \frac{d_1}{2(x-1)} + \frac{2d_1}{3(x-1)^2} - \frac{d_1}{(x-1)^3} + \frac{2d_1}{(x-1)^4}\right) H(1; \alpha_0) + \left(\frac{2d_1}{(x-1)^5} - \frac{2}{(x-1)^5} + 2\right) H(0; \alpha_0) H(1; x) + \left(-\frac{\alpha_0^4}{2(x-1)} - \frac{\alpha_0^4}{2} + \frac{2}{x-1} - \frac{2\alpha_0^3}{3(x-1)^2} + \frac{8\alpha_0^3}{3} - \frac{3}{x-1} + \frac{2\alpha_0^2}{(x-1)^2} - \frac{\alpha_0^2}{(x-1)^3} - 6\alpha_0^2 + \frac{2\alpha_0}{x-1} - \frac{2\alpha_0}{(x-1)^2} + \frac{2}{(x-1)^3} - \frac{2\alpha_0}{(x-1)^4} + 8\alpha_0 - \frac{4H(0; \alpha_0)}{(x-1)^5} - \frac{2d_1 H(1; \alpha_0)}{(x-1)^5} - \frac{5}{2(x-1)} + \frac{5}{3(x-1)^2} - \frac{5}{3(x-1)^3} + \frac{5}{2(x-1)^4} + \frac{37}{6(x-1)^5} - \frac{25}{6}\right) H(c_1(\alpha_0); x) + \left(-4 - \frac{4}{(x-1)^5}\right) H(0, 0; \alpha_0) + \left(\frac{4}{(x-1)^5} - 4\right) H(0, 0; x) + \left(-\frac{2}{(x-1)^5} - 2d_1\right) H(0, 1; \alpha_0) + \left(2 - \frac{2}{(x-1)^5}\right) H(0, c_1(\alpha_0); x) + \left(-\frac{2d_1}{(x-1)^5} + \frac{2}{(x-1)^5} - 2\right) H(1, 0; x) + \left(\frac{2d_1}{(x-1)^5} - \frac{2}{(x-1)^5} + 2\right) H(1, c_1(\alpha_0); x) - \frac{2H(c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \frac{\pi^2}{3(x-1)^5} - \frac{\pi^2}{3} - 2, \\ b_2^{(1,0)} &= \frac{d_1^2 \alpha_0^4}{16} - \frac{d_1 \alpha_0^4}{2} + \frac{d_1^2 \alpha_0^4}{16(x-1)} - \frac{d_1 \alpha_0^4}{2(x-1)} - \frac{\pi^2 \alpha_0^4}{24(x-1)} + \frac{7\alpha_0^4}{4(x-1)} - \frac{\pi^2 \alpha_0^4}{24} + \frac{7}{4} \frac{\alpha_0^4}{(x-1)} - \frac{43d_1^2 \alpha_0^3}{108} + \frac{349d_1}{108} \frac{\alpha_0^3}{(x-1)} - \frac{d_1^2 \alpha_0^3}{4(x-1)} + \frac{2d_1}{x-1} \frac{\alpha_0^3}{(x-1)} + \frac{\pi^2 \alpha_0^3}{6(x-1)} - \frac{7\alpha_0^3}{x-1} + \frac{4}{27} \frac{d_1^2 \alpha_0^3}{(x-1)^2} - \frac{133d_1 \alpha_0^3}{108(x-1)^2} - \frac{\pi^2 \alpha_0^3}{18(x-1)^2} + \frac{191\alpha_0^3}{54(x-1)^2} + \frac{2\pi^2}{9} \frac{\alpha_0^3}{(x-1)^2} - \frac{569\alpha_0^3}{54} + \frac{95d_1^2 \alpha_0^2}{72} - \frac{85d_1}{8} \frac{\alpha_0^2}{(x-1)} + \frac{3d_1^2 \alpha_0^2}{8(x-1)} - \frac{203d_1 \alpha_0^2}{72(x-1)} - \frac{\pi^2 \alpha_0^2}{4(x-1)} + \frac{353\alpha_0^2}{36(x-1)} - \frac{4d_1^2 \alpha_0^2}{9(x-1)^2} + \frac{31d_1 \alpha_0^2}{8(x-1)^2} + \frac{\pi^2 \alpha_0^2}{6(x-1)^2} - \frac{407\alpha_0^2}{36(x-1)^2} + \frac{d_1^2 \alpha_0^2}{2(x-1)^3} - \frac{283d_1 \alpha_0^2}{72(x-1)^3} - \frac{\pi^2 \alpha_0^2}{12(x-1)^3} + \frac{331\alpha_0^2}{36(x-1)^3} - \frac{\pi^2}{2} \frac{\alpha_0^2}{(x-1)^3} + \frac{1091\alpha_0^2}{36} - \frac{205d_1^2 \alpha_0}{36} + \frac{475}{12} \frac{d_1 \alpha_0}{(x-1)} + \frac{2\alpha_0}{3(x-2)} - \frac{d_1^2 \alpha_0}{4(x-1)} - \frac{d_1 \alpha_0}{9(x-1)} + \frac{\pi^2 \alpha_0}{6(x-1)} - \frac{13}{36} \frac{\alpha_0}{(x-1)} + \frac{4d_1^2 \alpha_0}{9(x-1)^2} - \frac{73d_1 \alpha_0}{18(x-1)^2} - \frac{\pi^2 \alpha_0}{6(x-1)^2} + \frac{35\alpha_0}{3(x-1)^2} - \frac{d_1^2 \alpha_0}{(x-1)^3} + \frac{173d_1 \alpha_0}{18(x-1)^3} + \frac{\pi^2 \alpha_0}{6(x-1)^3} - \frac{875\alpha_0}{36(x-1)^3} + \frac{4d_1^2 \alpha_0}{(x-1)^4} - \frac{937d_1 \alpha_0}{36(x-1)^4} - \frac{\pi^2 \alpha_0}{6(x-1)^4} + \frac{413\alpha_0}{9(x-1)^4} + \frac{2\pi^2 \alpha_0}{3} - \frac{737\alpha_0}{9} + \left(\frac{d_1}{2} \frac{\alpha_0^4}{(x-1)} + \frac{d_1 \alpha_0^4}{2(x-1)} - \frac{3\alpha_0^4}{x-1} - 3\alpha_0^4\right) \end{aligned}$$

$$\begin{aligned}
& \frac{26d_1\alpha_0^3}{9} - \frac{2d_1\alpha_0^3}{x-1} + \frac{12\alpha_0^3}{x-1} + \frac{8d_1\alpha_0^3}{9(x-1)^2} - \frac{46\alpha_0^3}{9(x-1)^2} + \frac{154\alpha_0^3}{9} + \frac{23d_1\alpha_0^2}{3} + \frac{3d_1\alpha_0^2}{x-1} - \frac{53\alpha_0^2}{3(x-1)} - \frac{8d_1\alpha_0^2}{3(x-1)^2} + \frac{47\alpha_0^2}{3(x-1)^2} + \\
& \frac{2d_1\alpha_0^2}{(x-1)^3} - \frac{31\alpha_0^2}{3(x-1)^3} - \frac{131\alpha_0^2}{3} - \frac{50d_1\alpha_0}{3} - \frac{2d_1\alpha_0}{x-1} + \frac{28\alpha_0}{3(x-1)} + \frac{8d_1\alpha_0}{3(x-1)^2} - \frac{16\alpha_0}{(x-1)^2} - \frac{4d_1\alpha_0}{(x-1)^3} + \frac{68\alpha_0}{3(x-1)^3} + \frac{8d_1\alpha_0}{(x-1)^4} - \\
& \frac{98\alpha_0}{3(x-1)^4} + \frac{242\alpha_0}{3} + \frac{205d_1}{36} - \frac{4}{x-2} + \frac{17d_1}{4(x-1)} - \frac{53}{4(x-1)} + \frac{8}{3(x-2)^2} - \frac{13d_1}{9(x-1)^2} + \frac{317}{36(x-1)^2} + \frac{13d_1}{9(x-1)^3} - \\
& \frac{371}{36(x-1)^3} - \frac{17d_1}{4(x-1)^4} + \frac{193}{12(x-1)^4} - \frac{205d_1}{36(x-1)^5} - \frac{\pi^2}{6(x-1)^5} + \frac{266}{9(x-1)^5} - \frac{\pi^2}{6} - \frac{194}{9} \Big) H(0; \alpha_0) + \left( -\frac{17d_1}{4(x-1)} + \right. \\
& \frac{13d_1}{9(x-1)^2} - \frac{13d_1}{9(x-1)^3} + \frac{17d_1}{4(x-1)^4} + \frac{205d_1}{36(x-1)^5} - \frac{205d_1}{36} + \frac{4}{x-2} + \frac{53}{4(x-1)} - \frac{8}{3(x-2)^2} - \frac{317}{36(x-1)^2} + \frac{371}{36(x-1)^3} - \\
& \left. \frac{193}{12(x-1)^4} - \frac{7\pi^2}{6(x-1)^5} - \frac{266}{9(x-1)^5} + \frac{3\pi^2}{2} + \frac{266}{9} \right) H(0; x) + \left( \frac{d_1^2\alpha_0^4}{4} - \frac{3d_1\alpha_0^4}{2} + \frac{d_1^2\alpha_0^4}{4(x-1)} - \frac{3d_1\alpha_0^4}{4(x-1)} - \frac{13d_1^2\alpha_0^3}{9} + \right. \\
& \frac{77d_1\alpha_0^3}{9} - \frac{d_1^2\alpha_0^3}{x-1} + \frac{6d_1\alpha_0^3}{x-1} + \frac{4d_1^2\alpha_0^3}{9(x-1)^2} - \frac{23d_1\alpha_0^3}{9(x-1)^2} + \frac{23d_1^2\alpha_0^2}{6} - \frac{131d_1\alpha_0^2}{6} + \frac{3d_1^2\alpha_0^2}{2(x-1)} - \frac{53d_1\alpha_0^2}{6(x-1)} - \frac{4d_1^2\alpha_0^2}{3(x-1)^2} + \\
& \frac{47d_1\alpha_0^2}{6(x-1)^2} + \frac{d_1^2\alpha_0^2}{(x-1)^3} - \frac{31d_1\alpha_0^2}{6(x-1)^3} - \frac{25d_1^2\alpha_0}{3} + \frac{121d_1\alpha_0}{3} - \frac{d_1^2\alpha_0}{x-1} + \frac{14d_1\alpha_0}{3(x-1)} + \frac{4d_1^2\alpha_0}{3(x-1)^2} - \frac{8d_1\alpha_0}{(x-1)^2} - \frac{2d_1^2\alpha_0}{(x-1)^3} + \\
& \frac{34d_1\alpha_0}{3(x-1)^3} + \frac{4d_1^2\alpha_0}{(x-1)^4} - \frac{49d_1\alpha_0}{3(x-1)^4} + \frac{205d_1^2}{36} - \frac{230d_1}{9} + \frac{d_1^2}{4(x-1)} - \frac{d_1}{3(x-1)} - \frac{4d_1^2}{9(x-1)^2} + \frac{49d_1}{18(x-1)^2} + \frac{d_1^2}{(x-1)^3} - \\
& \left. \frac{37d_1}{6(x-1)^3} - \frac{4d_1^2}{(x-1)^4} + \frac{49d_1}{3(x-1)^4} \right) H(1; \alpha_0) + \left( \frac{\pi^2}{2(x-1)^5} - \frac{\pi^2}{2} \right) H(2; x) + \left( \frac{4\alpha_0^4}{x-1} + 4\alpha_0^4 - \frac{16\alpha_0^3}{x-1} + \frac{16\alpha_0^3}{3(x-1)^2} - \right. \\
& \frac{64\alpha_0^3}{3} + \frac{24\alpha_0^2}{x-1} - \frac{16\alpha_0^2}{(x-1)^2} + \frac{8\alpha_0^2}{(x-1)^3} + 48\alpha_0^2 - \frac{16\alpha_0}{x-1} + \frac{16\alpha_0}{(x-1)^2} - \frac{16\alpha_0}{(x-1)^3} + \frac{16\alpha_0}{(x-1)^4} - 64\alpha_0 + \frac{10}{x-1} - \frac{20}{3(x-1)^2} + \\
& \left. \frac{20}{3(x-1)^3} - \frac{10}{(x-1)^4} - \frac{74}{3(x-1)^5} + \frac{26}{3} \right) H(0, 0; \alpha_0) + \left( -\frac{74}{3} - \frac{10}{x-1} + \frac{20}{3(x-1)^2} - \frac{20}{3(x-1)^3} + \frac{10}{(x-1)^4} + \right. \\
& \left. \frac{74}{3(x-1)^5} \right) H(0, 0; x) + \left( 2d_1\alpha_0^4 + \frac{2d_1\alpha_0^4}{x-1} - \frac{32d_1\alpha_0^3}{3} - \frac{8d_1\alpha_0^3}{x-1} + \frac{8d_1\alpha_0^3}{3(x-1)^2} + 24d_1\alpha_0^2 + \frac{12d_1\alpha_0^2}{x-1} - \frac{8d_1\alpha_0^2}{(x-1)^2} + \right. \\
& \frac{4d_1\alpha_0^2}{(x-1)^3} - 32d_1\alpha_0 - \frac{8d_1\alpha_0}{x-1} + \frac{8d_1\alpha_0}{(x-1)^2} - \frac{8d_1\alpha_0}{(x-1)^3} + \frac{8d_1\alpha_0}{(x-1)^4} + \frac{13d_1}{3} + \frac{5d_1}{x-1} - \frac{10d_1}{3(x-1)^2} + \frac{10d_1}{3(x-1)^3} - \frac{5d_1}{(x-1)^4} - \\
& \left. \frac{37d_1}{3(x-1)^5} \right) H(0, 1; \alpha_0) + H(1; x) \left( \frac{2\pi^2 d_1}{3(x-1)^5} + \left( -\frac{4d_1}{x-1} + \frac{2d_1}{(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \frac{d_1}{(x-1)^4} + \frac{37d_1}{3(x-1)^5} + \frac{4}{x-2} + \right. \right. \\
& \left. \frac{5}{2(x-1)} - \frac{8}{3(x-2)^2} - \frac{3}{(x-1)^2} + \frac{8}{3(x-2)^3} + \frac{5}{3(x-1)^3} - \frac{13}{2(x-1)^4} - \frac{37}{3(x-1)^5} + \frac{37}{3} \right) H(0; \alpha_0) + \left( -\frac{8d_1}{(x-1)^5} + \right. \\
& \left. \frac{8}{(x-1)^5} - 8 \right) H(0, 0; \alpha_0) + \left( -\frac{4d_1^2}{(x-1)^5} + \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(0, 1; \alpha_0) - \frac{2\pi^2}{3(x-1)^5} + \frac{\pi^2}{3} + \left( -\frac{4d_1}{(x-1)^5} + \right. \\
& \left. 4d_1 + \frac{8}{(x-1)^5} - 8 \right) H(0; \alpha_0) H(0, 1; x) + \left( \left( \frac{8}{(x-1)^5} - 8 \right) H(0; \alpha_0) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(1; \alpha_0) + \right. \\
& \left. \frac{4}{x-2} + \frac{5}{2(x-1)} - \frac{8}{3(x-2)^2} - \frac{3}{(x-1)^2} + \frac{8}{3(x-2)^3} + \frac{5}{3(x-1)^3} - \frac{13}{2(x-1)^4} - \frac{37}{3(x-1)^5} + \frac{37}{3} \right) H(0, c_1(\alpha_0); x) + \\
& \left( 2d_1\alpha_0^4 + \frac{2d_1\alpha_0^4}{x-1} - \frac{32d_1\alpha_0^3}{3} - \frac{8d_1\alpha_0^3}{x-1} + \frac{8d_1\alpha_0^3}{3(x-1)^2} + 24d_1\alpha_0^2 + \frac{12d_1\alpha_0^2}{x-1} - \frac{8d_1\alpha_0^2}{(x-1)^2} + \frac{4d_1\alpha_0^2}{(x-1)^3} - 32d_1\alpha_0 - \right. \\
& \left. \frac{8d_1\alpha_0}{x-1} + \frac{8d_1\alpha_0}{(x-1)^2} - \frac{8d_1\alpha_0}{(x-1)^3} + \frac{8d_1\alpha_0}{(x-1)^4} + \frac{50d_1}{3} + \frac{2d_1}{x-1} - \frac{8d_1}{3(x-1)^2} + \frac{4d_1}{(x-1)^3} - \frac{8d_1}{(x-1)^4} \right) H(1, 0; \alpha_0) + \left( \frac{4d_1}{x-1} - \right. \\
& \left. \frac{2d_1}{(x-1)^2} + \frac{4d_1}{3(x-1)^3} - \frac{d_1}{(x-1)^4} - \frac{37d_1}{3(x-1)^5} - \frac{4}{x-2} - \frac{5}{2(x-1)} + \frac{8}{3(x-2)^2} + \frac{3}{(x-1)^2} - \frac{8}{3(x-2)^3} - \frac{5}{3(x-1)^3} + \right. \\
& \left. \frac{13}{2(x-1)^4} + \frac{37}{3(x-1)^5} - \frac{37}{3} \right) H(1, 0; x) + \left( d_1^2\alpha_0^4 + \frac{d_1^2\alpha_0^4}{x-1} - \frac{16d_1^2\alpha_0^3}{3} - \frac{4d_1^2\alpha_0^3}{x-1} + \frac{4d_1^2\alpha_0^3}{3(x-1)^2} + 12d_1^2\alpha_0^2 + \right. \\
& \frac{6d_1^2\alpha_0^2}{x-1} - \frac{4d_1^2\alpha_0^2}{(x-1)^2} + \frac{2d_1^2\alpha_0^2}{(x-1)^3} - 16d_1^2\alpha_0 - \frac{4d_1^2\alpha_0}{x-1} + \frac{4d_1^2\alpha_0}{(x-1)^2} - \frac{4d_1^2\alpha_0}{(x-1)^3} + \frac{4d_1^2\alpha_0}{(x-1)^4} + \frac{25d_1^2}{3} + \frac{d_1^2}{x-1} - \frac{4d_1^2}{3(x-1)^2} + \\
& \left. \frac{2d_1^2}{(x-1)^3} - \frac{4d_1^2}{(x-1)^4} \right) H(1, 1; \alpha_0) + H(c_1(\alpha_0); x) \left( \frac{d_1\alpha_0^4}{4} + \frac{d_1\alpha_0^4}{4(x-1)} - \frac{3\alpha_0^4}{2(x-1)} - \frac{3\alpha_0^4}{2} - \frac{13d_1\alpha_0^3}{9} - \frac{d_1\alpha_0^3}{x-1} + \right. \\
& \frac{6\alpha_0^3}{x-1} + \frac{4d_1\alpha_0^3}{9(x-1)^2} - \frac{23\alpha_0^3}{9(x-1)^2} + \frac{77\alpha_0^3}{9} + \frac{23d_1\alpha_0^2}{6} - \frac{\alpha_0^2}{3(x-2)} + \frac{3d_1\alpha_0^2}{2(x-1)} - \frac{103\alpha_0^2}{12(x-1)} - \frac{4d_1\alpha_0^2}{3(x-1)^2} + \frac{95\alpha_0^2}{12(x-1)^2} + \\
& \frac{d_1\alpha_0^2}{(x-1)^3} - \frac{31\alpha_0^2}{6(x-1)^3} - \frac{131\alpha_0^2}{6} - \frac{25d_1\alpha_0}{3} + \frac{2\alpha_0}{x-2} - \frac{d_1\alpha_0}{x-1} + \frac{10\alpha_0}{3(x-1)} - \frac{4\alpha_0}{3(x-2)^2} + \frac{4d_1\alpha_0}{3(x-1)^2} - \frac{15\alpha_0}{2(x-1)^2} - \frac{2d_1\alpha_0}{(x-1)^3} + \\
& \left. \frac{71\alpha_0}{6(x-1)^3} + \frac{4d_1\alpha_0}{(x-1)^4} - \frac{49\alpha_0}{3(x-1)^4} + \frac{121\alpha_0}{3} + \frac{205d_1}{36} + \left( \frac{2\alpha_0^4}{x-1} + 2\alpha_0^4 - \frac{8\alpha_0^3}{x-1} + \frac{8\alpha_0^3}{3(x-1)^2} - \frac{32\alpha_0^3}{3} + \frac{12\alpha_0^2}{x-1} - \frac{8\alpha_0^2}{(x-1)^2} + \right. \right. \\
& \left. \frac{4\alpha_0^2}{(x-1)^3} + 24\alpha_0^2 - \frac{8\alpha_0}{x-1} + \frac{8\alpha_0}{(x-1)^2} - \frac{8\alpha_0}{(x-1)^3} + \frac{8\alpha_0}{(x-1)^4} - 32\alpha_0 + \frac{10}{x-1} - \frac{20}{3(x-1)^2} + \frac{20}{3(x-1)^3} - \frac{10}{(x-1)^4} - \right. \\
& \left. \frac{74}{3(x-1)^5} + \frac{50}{3} \right) H(0; \alpha_0) + \left( d_1\alpha_0^4 + \frac{d_1\alpha_0^4}{x-1} - \frac{16d_1\alpha_0^3}{3} - \frac{4d_1\alpha_0^3}{x-1} + \frac{4d_1\alpha_0^3}{3(x-1)^2} + 12d_1\alpha_0^2 + \frac{6d_1\alpha_0^2}{x-1} - \frac{4d_1\alpha_0^2}{(x-1)^2} + \right. \\
& \left. \frac{2d_1\alpha_0^2}{(x-1)^3} - 16d_1\alpha_0 - \frac{4d_1\alpha_0}{x-1} + \frac{4d_1\alpha_0}{(x-1)^2} - \frac{4d_1\alpha_0}{(x-1)^3} + \frac{4d_1\alpha_0}{(x-1)^4} + \frac{25d_1}{3} + \frac{5d_1}{x-1} - \frac{10d_1}{3(x-1)^2} + \frac{10d_1}{3(x-1)^3} - \frac{5d_1}{(x-1)^4} - \right. \\
& \left. \frac{37d_1}{3(x-1)^5} \right) H(1; \alpha_0) + \frac{16H(0,0;\alpha_0)}{(x-1)^5} + \frac{8d_1H(0,1;\alpha_0)}{(x-1)^5} + \frac{8d_1H(1,0;\alpha_0)}{(x-1)^5} + \frac{4d_1^2H(1,1;\alpha_0)}{(x-1)^5} - \frac{4}{x-2} + \frac{17d_1}{4(x-1)} -
\end{aligned}$$

$$\begin{aligned}
& \frac{53}{4(x-1)} + \frac{8}{3(x-2)^2} - \frac{13d_1}{9(x-1)^2} + \frac{317}{36(x-1)^2} + \frac{13d_1}{9(x-1)^3} - \frac{371}{36(x-1)^3} - \frac{17d_1}{4(x-1)^4} + \frac{193}{12(x-1)^4} - \frac{205d_1}{36(x-1)^5} - \\
& \frac{\pi^2}{6(x-1)^5} + \frac{266}{9(x-1)^5} - \frac{230}{9} + \left( \frac{4d_1^2}{(x-1)^5} - \frac{8d_1}{(x-1)^5} + 4d_1 + \frac{4}{(x-1)^5} - 2 \right) H(0; \alpha_0) H(1, 1; x) + \left( -\frac{4d_1}{x-1} + \right. \\
& \left. \frac{2d_1}{(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \frac{d_1}{(x-1)^4} + \frac{37d_1}{3(x-1)^5} + \left( -\frac{8d_1}{(x-1)^5} + \frac{8}{(x-1)^5} - 8 \right) H(0; \alpha_0) + \left( -\frac{4d_1^2}{(x-1)^5} + \frac{4d_1}{(x-1)^5} - \right. \right. \\
& \left. \left. 4d_1 \right) H(1; \alpha_0) + \frac{4}{x-2} + \frac{5}{2(x-1)} - \frac{8}{3(x-2)^2} - \frac{3}{(x-1)^2} + \frac{8}{3(x-2)^3} + \frac{5}{3(x-1)^3} - \frac{13}{2(x-1)^4} - \frac{37}{3(x-1)^5} + \right. \\
& \left. \frac{37}{3} \right) H(1, c_1(\alpha_0); x) + \left( 2 - \frac{2}{(x-1)^5} \right) H(0; \alpha_0) H(2, 1; x) + \left( \frac{\alpha_0^4}{x-1} + \frac{3\alpha_0^4}{2} - \frac{4\alpha_0^3}{x-1} + \frac{4\alpha_0^3}{3(x-1)^2} - 8\alpha_0^3 + \right. \\
& \left. \frac{6\alpha_0^2}{x-1} - \frac{4\alpha_0^2}{(x-1)^2} + \frac{2\alpha_0^2}{(x-1)^3} + 18\alpha_0^2 - \frac{4\alpha_0}{x-1} + \frac{4\alpha_0}{(x-1)^2} - \frac{4\alpha_0}{(x-1)^3} + \frac{4\alpha_0}{(x-1)^4} - 24\alpha_0 + \frac{8H(0; \alpha_0)}{(x-1)^5} + \frac{4d_1 H(1; \alpha_0)}{(x-1)^5} + \right. \\
& \left. \frac{7}{x-1} - \frac{13}{3(x-1)^2} + \frac{4}{(x-1)^3} - \frac{11}{2(x-1)^4} - \frac{37}{3(x-1)^5} + \frac{25}{2} \right) H(c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{\alpha_0^4}{2(x-1)} - \frac{\alpha_0^4}{2} - \frac{2\alpha_0^3}{x-1} + \right. \\
& \left. \frac{2\alpha_0^3}{3(x-1)^2} + \frac{8\alpha_0^3}{3} + \frac{3\alpha_0^2}{x-1} - \frac{2\alpha_0^2}{(x-1)^2} + \frac{\alpha_0^2}{(x-1)^3} - 6\alpha_0^2 - \frac{2\alpha_0}{x-1} + \frac{2\alpha_0}{(x-1)^2} - \frac{2\alpha_0}{(x-1)^3} + \frac{2\alpha_0}{(x-1)^4} + 8\alpha_0 - \frac{4}{x-2} + \right. \\
& \left. \frac{1}{2(x-1)} + \frac{8}{3(x-2)^2} + \frac{2}{3(x-1)^2} - \frac{8}{3(x-2)^3} + \frac{1}{(x-1)^3} + \frac{2}{(x-1)^4} - \frac{25}{6} \right) H(c_2(\alpha_0), c_1(\alpha_0); x) + \left( 16 + \right. \\
& \left. \frac{16}{(x-1)^5} \right) H(0, 0, 0; \alpha_0) + \left( 16 - \frac{16}{(x-1)^5} \right) H(0, 0, 0; x) + \left( \frac{8d_1}{(x-1)^5} + 8d_1 \right) H(0, 0, 1; \alpha_0) + \left( \frac{8}{(x-1)^5} - \right. \\
& \left. 8 \right) H(0, 0, c_1(\alpha_0); x) + \left( \frac{8d_1}{(x-1)^5} + 8d_1 \right) H(0, 1, 0; \alpha_0) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 - \frac{8}{(x-1)^5} + 8 \right) H(0, 1, 0; x) + \\
& \left( \frac{4d_1^2}{(x-1)^5} + 4d_1^2 \right) H(0, 1, 1; \alpha_0) + \left( -\frac{4d_1}{(x-1)^5} + 4d_1 + \frac{8}{(x-1)^5} - 8 \right) H(0, 1, c_1(\alpha_0); x) + \left( \frac{4}{(x-1)^5} - \right. \\
& \left. 6 \right) H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( 2 - \frac{2}{(x-1)^5} \right) H(0, c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{8d_1}{(x-1)^5} - \frac{8}{(x-1)^5} + \right. \\
& \left. 8 \right) H(1, 0, 0; x) + \left( -\frac{4d_1}{(x-1)^5} + \frac{4}{(x-1)^5} - 2 \right) H(1, 0, c_1(\alpha_0); x) + \left( -\frac{4d_1^2}{(x-1)^5} + \frac{8d_1}{(x-1)^5} - 4d_1 - \right. \\
& \left. \frac{4}{(x-1)^5} + 2 \right) H(1, 1, 0; x) + \left( \frac{4d_1^2}{(x-1)^5} - \frac{8d_1}{(x-1)^5} + 4d_1 + \frac{4}{(x-1)^5} - 2 \right) H(1, 1, c_1(\alpha_0); x) + \left( -\frac{4d_1}{(x-1)^5} + \frac{4}{(x-1)^5} - 6 \right) \\
& H(1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( 2 - \frac{2}{(x-1)^5} \right) H(2, 0, c_1(\alpha_0); x) + \left( \frac{2}{(x-1)^5} - 2 \right) H(2, 1, 0; x) + \left( 2 - \frac{2}{(x-1)^5} \right) H(2, 1, c_1(\alpha_0); x) + \\
& \left( \frac{2}{(x-1)^5} - 2 \right) H(2, c_2(\alpha_0), c_1(\alpha_0); x) + \frac{4 H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \frac{2 H(c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} - \frac{\pi^2}{x-2} - \frac{3\pi^2}{8(x-1)} + \frac{2\pi^2}{3(x-2)^2} + \frac{5\pi^2}{9(x-1)^2} - \\
& \frac{2\pi^2}{3(x-2)^3} - \frac{7\pi^2}{36(x-1)^3} + \frac{5\pi^2}{4(x-1)^4} + \frac{37\pi^2}{18(x-1)^5} - \frac{21\zeta_3}{4(x-1)^5} + \frac{17\zeta_3}{4} + \frac{\pi^2 \ln 2}{2(x-1)^5} - \frac{1}{2}\pi^2 \ln 2 - \frac{173\pi^2}{72} - 4.
\end{aligned}$$

**E.6 The  $\mathcal{B}$  integral for  $k = 1$  and  $\delta = 1$**

The  $\varepsilon$  expansion for this integral reads

$$\begin{aligned}
\mathcal{I}(x, \varepsilon; \alpha_0, 3 + d_1\varepsilon; 1, 1, 1, g_B) &= x \mathcal{B}(\varepsilon, x; 3 + d_1\varepsilon; 1, 1) \\
&= \frac{1}{\varepsilon} b_{-1}^{(1,1)} + b_0^{(1,1)} + \varepsilon b_1^{(1,1)} + \varepsilon^2 b_2^{(1,1)} + \mathcal{O}(\varepsilon^3), \tag{E.6}
\end{aligned}$$

where

$$\begin{aligned}
b_{-1}^{(1,1)} &= -\frac{1}{4}, \\
b_0^{(1,1)} &= \frac{\alpha_0^4}{8(x-1)} + \frac{\alpha_0^4}{8} - \frac{\alpha_0^3}{2(x-1)} + \frac{\alpha_0^3}{6(x-1)^2} - \frac{2\alpha_0^3}{3} + \frac{3\alpha_0^2}{4(x-1)} - \frac{\alpha_0^2}{2(x-1)^2} + \frac{\alpha_0^2}{4(x-1)^3} + \frac{3\alpha_0^2}{2} - \frac{\alpha_0}{2(x-1)} + \frac{\alpha_0}{2(x-1)^2} - \\
& \frac{\alpha_0}{2(x-1)^3} + \frac{\alpha_0}{2(x-1)^4} - 2\alpha_0 + \left( \frac{1}{2} + \frac{1}{2(x-1)^5} \right) H(0; \alpha_0) + \left( \frac{1}{2} - \frac{1}{2(x-1)^5} \right) H(0; x) + \frac{H(c_1(\alpha_0); x)}{2(x-1)^5} - \frac{3}{4}, \\
b_1^{(1,1)} &= -\frac{d_1\alpha_0^4}{16} - \frac{d_1\alpha_0^4}{16(x-1)} + \frac{\alpha_0^4}{2(x-1)} + \frac{\alpha_0^4}{2} + \frac{13d_1\alpha_0^3}{36} - \frac{\alpha_0^3}{6(x-2)} + \frac{d_1\alpha_0^3}{4(x-1)} - \frac{23\alpha_0^3}{12(x-1)} - \frac{d_1\alpha_0^3}{9(x-1)^2} + \\
& \frac{29\alpha_0^3}{36(x-1)^2} - \frac{101\alpha_0^3}{36} - \frac{23d_1\alpha_0^2}{24} + \frac{5\alpha_0^2}{6(x-2)} - \frac{3d_1\alpha_0^2}{8(x-1)} + \frac{5\alpha_0^2}{2(x-1)} - \frac{2\alpha_0^2}{3(x-2)^2} + \frac{d_1\alpha_0^2}{3(x-1)^2} - \frac{7\alpha_0^2}{3(x-1)^2} - \frac{d_1\alpha_0^2}{4(x-1)^3} +
\end{aligned}$$

$$\begin{aligned}
 & \frac{37\alpha_0^2}{24(x-1)^3} + \frac{167\alpha_0^2}{24} + \frac{25d_1\alpha_0}{12} - \frac{7\alpha_0}{3(x-2)} + \frac{d_1\alpha_0}{4(x-1)} - \frac{\alpha_0}{12(x-1)} + \frac{10\alpha_0}{3(x-2)^2} - \frac{d_1\alpha_0}{3(x-1)^2} + \frac{11\alpha_0}{6(x-1)^2} - \frac{4\alpha_0}{(x-2)^3} + \\
 & \frac{d_1\alpha_0}{2(x-1)^3} - \frac{37\alpha_0}{12(x-1)^3} - \frac{d_1\alpha_0}{(x-1)^4} + \frac{55\alpha_0}{12(x-1)^4} - \frac{145\alpha_0}{12} + \left( -\frac{\alpha_0^4}{2(x-1)} - \frac{\alpha_0^4}{2} + \frac{2\alpha_0^3}{x-1} - \frac{2\alpha_0^3}{3(x-1)^2} + \frac{8\alpha_0^3}{3} - \right. \\
 & \left. \frac{3\alpha_0^2}{x-1} + \frac{2\alpha_0^2}{(x-1)^2} - \frac{\alpha_0^2}{(x-1)^3} - 6\alpha_0^2 + \frac{2\alpha_0}{x-1} - \frac{2\alpha_0}{(x-1)^2} + \frac{2\alpha_0}{(x-1)^3} - \frac{2\alpha_0}{(x-1)^4} + 8\alpha_0 + \frac{4}{x-2} - \frac{5}{x-1} - \frac{16}{3(x-2)^2} + \right. \\
 & \left. \frac{7}{4(x-1)^2} + \frac{8}{(x-2)^3} - \frac{5}{4(x-1)^3} - \frac{16}{(x-2)^4} + \frac{3}{2(x-1)^4} + \frac{43}{12(x-1)^5} - \frac{7}{12} \right) H(0; \alpha_0) + \left( \frac{5}{x-1} - \frac{7}{4(x-1)^2} + \right. \\
 & \left. \frac{5}{4(x-1)^3} - \frac{3}{2(x-1)^4} - \frac{43}{12(x-1)^5} + \frac{43}{12} - \frac{4}{x-2} + \frac{16}{3(x-2)^2} - \frac{8}{(x-2)^3} + \frac{16}{(x-2)^4} \right) H(0; x) + \left( -\frac{d_1\alpha_0^4}{4} - \right. \\
 & \left. \frac{d_1\alpha_0^4}{4(x-1)} + \frac{4d_1\alpha_0^3}{3} + \frac{d_1\alpha_0^3}{x-1} - \frac{d_1\alpha_0^3}{3(x-1)^2} - 3d_1\alpha_0^2 - \frac{3d_1\alpha_0^2}{2(x-1)} + \frac{d_1\alpha_0^2}{(x-1)^2} - \frac{d_1\alpha_0^2}{2(x-1)^3} + 4d_1\alpha_0 + \frac{d_1\alpha_0}{x-1} - \frac{d_1\alpha_0}{(x-1)^2} + \right. \\
 & \left. \frac{d_1\alpha_0}{(x-1)^3} - \frac{d_1\alpha_0}{(x-1)^4} - \frac{25d_1}{12} - \frac{d_1}{4(x-1)} + \frac{d_1}{3(x-1)^2} - \frac{d_1}{2(x-1)^3} + \frac{d_1}{(x-1)^4} \right) H(1; \alpha_0) + \left( \frac{d_1}{(x-1)^5} - \frac{16}{(x-2)^5} - \right. \\
 & \left. \frac{1}{(x-1)^5} + 1 \right) H(0; \alpha_0) H(1; x) + \left( \frac{\alpha_0^4}{4(x-2)} - \frac{\alpha_0^4}{4(x-1)} - \frac{\alpha_0^4}{4} - \frac{\alpha_0^3}{x-2} + \frac{\alpha_0^3}{x-1} + \frac{2\alpha_0^3}{3(x-2)^2} - \frac{\alpha_0^3}{3(x-1)^2} + \frac{4\alpha_0^3}{3} + \right. \\
 & \left. \frac{3\alpha_0^2}{2(x-2)} - \frac{3\alpha_0^2}{2(x-1)} - \frac{2\alpha_0^2}{(x-2)^2} + \frac{\alpha_0^2}{(x-1)^2} + \frac{2\alpha_0^2}{(x-2)^3} - \frac{\alpha_0^2}{2(x-1)^3} - 3\alpha_0^2 - \frac{\alpha_0}{x-2} + \frac{\alpha_0}{x-1} + \frac{2\alpha_0}{(x-2)^2} - \frac{\alpha_0}{(x-1)^2} - \right. \\
 & \left. \frac{4\alpha_0}{(x-2)^3} + \frac{\alpha_0}{(x-1)^3} + \frac{8\alpha_0}{(x-2)^4} - \frac{\alpha_0}{(x-1)^4} + 4\alpha_0 - \frac{2H(0; \alpha_0)}{(x-1)^5} - \frac{d_1H(1; \alpha_0)}{(x-1)^5} + \frac{4}{x-2} - \frac{5}{x-1} - \frac{16}{3(x-2)^2} + \right. \\
 & \left. \frac{7}{4(x-1)^2} + \frac{8}{(x-2)^3} - \frac{5}{4(x-1)^3} - \frac{16}{(x-2)^4} + \frac{3}{2(x-1)^4} + \frac{43}{12(x-1)^5} - \frac{25}{12} \right) H(c_1(\alpha_0); x) + \left( -2 - \right. \\
 & \left. \frac{2}{(x-1)^5} \right) H(0, 0; \alpha_0) + \left( \frac{2}{(x-1)^5} - 2 \right) H(0, 0; x) + \left( -\frac{d_1}{(x-1)^5} - d_1 \right) H(0, 1; \alpha_0) + \left( -\frac{1}{(x-1)^5} + \right. \\
 & \left. 1 - \frac{16}{(x-2)^5} \right) H(0, c_1(\alpha_0); x) + \left( -\frac{d_1}{(x-1)^5} + \frac{16}{(x-2)^5} + \frac{1}{(x-1)^5} - 1 \right) H(1, 0; x) + \left( \frac{d_1}{(x-1)^5} - \frac{16}{(x-2)^5} - \right. \\
 & \left. \frac{1}{(x-1)^5} + 1 \right) H(1, c_1(\alpha_0); x) - \frac{H(c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \frac{16H(c_2(\alpha_0), c_1(\alpha_0); x)}{(x-2)^5} + \frac{4\pi^2}{(x-2)^5} + \frac{\pi^2}{6(x-1)^5} - \frac{\pi^2}{6} - \frac{7}{4},
 \end{aligned}$$

$$\begin{aligned}
 b_2^{(1,1)} = & \frac{d_1^2\alpha_0^4}{32} - \frac{5d_1\alpha_0^4}{16} + \frac{d_1^2\alpha_0^4}{32(x-1)} - \frac{5d_1\alpha_0^4}{16(x-1)} - \frac{\pi^2\alpha_0^4}{48(x-1)} + \frac{11\alpha_0^4}{8(x-1)} - \frac{\pi^2\alpha_0^4}{48} + \frac{11\alpha_0^4}{8} - \frac{43d_1^2\alpha_0^3}{216} + \\
 & \frac{427d_1\alpha_0^3}{216} + \frac{7d_1\alpha_0^3}{36(x-2)} - \frac{5\alpha_0^3}{9(x-2)} - \frac{d_1^2\alpha_0^3}{8(x-1)} + \frac{83d_1\alpha_0^3}{72(x-1)} + \frac{\pi^2\alpha_0^3}{12(x-1)} - \frac{47\alpha_0^3}{9(x-1)} + \frac{2d_1^2\alpha_0^3}{27(x-1)^2} - \frac{157d_1\alpha_0^3}{216(x-1)^2} - \\
 & \frac{\pi^2\alpha_0^3}{36(x-1)^2} + \frac{139\alpha_0^3}{54(x-1)^2} + \frac{\pi^2\alpha_0^3}{9} - \frac{218\alpha_0^3}{27} + \frac{95d_1^2\alpha_0^2}{144} - \frac{301d_1\alpha_0^2}{48} - \frac{41d_1\alpha_0^2}{36(x-2)} + \frac{28\alpha_0^2}{9(x-2)} + \frac{3d_1^2\alpha_0^2}{16(x-1)} - \frac{9d_1\alpha_0^2}{8(x-1)} - \\
 & \frac{\pi^2\alpha_0^2}{8(x-1)} + \frac{151\alpha_0^2}{24(x-1)} + \frac{10d_1\alpha_0^2}{9(x-2)^2} - \frac{26\alpha_0^2}{9(x-2)^2} - \frac{2d_1^2\alpha_0^2}{9(x-1)^2} + \frac{25d_1\alpha_0^2}{12(x-1)^2} + \frac{\pi^2\alpha_0^2}{12(x-1)^2} - \frac{533\alpha_0^2}{72(x-1)^2} + \frac{d_1^2\alpha_0^2}{4(x-1)^3} - \\
 & \frac{319d_1\alpha_0^2}{144(x-1)^3} - \frac{\pi^2\alpha_0^2}{24(x-1)^3} + \frac{221\alpha_0^2}{36(x-1)^3} - \frac{\pi^2\alpha_0^2}{4} + \frac{199\alpha_0^2}{9} - \frac{205d_1^2\alpha_0}{72} + \frac{175d_1\alpha_0}{8} + \frac{97d_1\alpha_0}{18(x-2)} - \frac{209\alpha_0}{18(x-2)} - \frac{d_1^2\alpha_0}{8(x-1)} - \\
 & \frac{281d_1\alpha_0}{72(x-1)} + \frac{\pi^2\alpha_0}{12(x-1)} + \frac{191\alpha_0}{36(x-1)} - \frac{74d_1\alpha_0}{9(x-2)^2} + \frac{178\alpha_0}{9(x-2)^2} + \frac{2d_1^2\alpha_0}{9(x-1)^2} - \frac{5d_1\alpha_0}{6(x-1)^2} - \frac{\pi^2\alpha_0}{12(x-1)^2} + \frac{65\alpha_0}{18(x-1)^2} + \\
 & \frac{12d_1\alpha_0}{(x-2)^3} - \frac{28\alpha_0}{(x-2)^3} - \frac{d_1^2\alpha_0}{2(x-1)^3} + \frac{337d_1\alpha_0}{72(x-1)^3} + \frac{\pi^2\alpha_0}{12(x-1)^3} - \frac{233\alpha_0}{18(x-1)^3} + \frac{2d_1^2\alpha_0}{(x-1)^4} - \frac{1009d_1\alpha_0}{72(x-1)^4} - \frac{\pi^2\alpha_0}{12(x-1)^4} + \\
 & \frac{991\alpha_0}{36(x-1)^4} + \frac{\pi^2\alpha_0}{3} - \frac{1909\alpha_0}{36} + \left( \frac{d_1\alpha_0^4}{4} + \frac{d_1\alpha_0^4}{4(x-1)} - \frac{2\alpha_0^4}{x-1} - 2\alpha_0^4 - \frac{13d_1\alpha_0^3}{9} + \frac{2\alpha_0^3}{3(x-2)} - \frac{d_1\alpha_0^3}{x-1} + \frac{23\alpha_0^3}{3(x-1)} + \right. \\
 & \left. \frac{4d_1\alpha_0^3}{9(x-1)^2} - \frac{29\alpha_0^3}{9(x-1)^2} + \frac{101\alpha_0^3}{9} + \frac{23d_1\alpha_0^2}{6} - \frac{10\alpha_0^2}{3(x-2)} + \frac{3d_1\alpha_0^2}{2(x-1)} - \frac{10\alpha_0^2}{x-1} + \frac{8\alpha_0^2}{3(x-2)^2} - \frac{4d_1\alpha_0^2}{3(x-1)^2} + \frac{28\alpha_0^2}{3(x-1)^2} + \right. \\
 & \left. \frac{d_1\alpha_0^2}{(x-1)^3} - \frac{37\alpha_0^2}{6(x-1)^3} - \frac{167\alpha_0^2}{6} - \frac{25d_1\alpha_0}{3} + \frac{28\alpha_0}{3(x-2)} - \frac{d_1\alpha_0}{x-1} + \frac{\alpha_0}{3(x-1)} - \frac{40\alpha_0}{3(x-2)^2} + \frac{4d_1\alpha_0}{3(x-1)^2} - \frac{22\alpha_0}{3(x-1)^2} + \right. \\
 & \left. \frac{16\alpha_0}{(x-2)^3} - \frac{2d_1\alpha_0}{(x-1)^3} + \frac{37\alpha_0}{3(x-1)^3} + \frac{4d_1\alpha_0}{(x-1)^4} - \frac{55\alpha_0}{3(x-1)^4} + \frac{145\alpha_0}{3} + \frac{205d_1}{72} - \frac{34d_1}{3(x-2)} + \frac{4}{x-2} + \frac{40d_1}{3(x-1)} - \frac{155}{12(x-1)} + \right. \\
 & \left. \frac{116d_1}{9(x-2)^2} - \frac{88}{9(x-2)^2} - \frac{47d_1}{24(x-1)^2} + \frac{103}{12(x-1)^2} - \frac{16d_1}{(x-2)^3} + \frac{24}{(x-2)^3} + \frac{25d_1}{24(x-1)^3} - \frac{103}{12(x-1)^3} + \frac{32d_1}{(x-2)^4} - \frac{80}{(x-2)^4} - \right. \\
 & \left. \frac{9d_1}{4(x-1)^4} + \frac{145}{12(x-1)^4} - \frac{205d_1}{72(x-1)^5} - \frac{\pi^2}{12(x-1)^5} + \frac{661}{36(x-1)^5} - \frac{\pi^2}{12} - \frac{409}{36} \right) H(0; \alpha_0) + \left( \frac{34d_1}{3(x-2)} - \frac{40d_1}{3(x-1)} - \right. \\
 & \left. \frac{116d_1}{9(x-2)^2} + \frac{47d_1}{24(x-1)^2} + \frac{16d_1}{(x-2)^3} - \frac{25d_1}{24(x-1)^3} - \frac{32d_1}{(x-2)^4} + \frac{9d_1}{4(x-1)^4} + \frac{205d_1}{72(x-1)^5} - \frac{205d_1}{72} - \frac{4}{x-2} + \frac{155}{12(x-1)} + \right. \\
 & \left. \frac{88}{9(x-2)^2} - \frac{103}{12(x-1)^2} - \frac{24}{(x-2)^3} + \frac{103}{12(x-1)^3} + \frac{80}{(x-2)^4} - \frac{145}{12(x-1)^4} - \frac{16\pi^2}{(x-2)^5} - \frac{7\pi^2}{12(x-1)^5} - \frac{661}{36(x-1)^5} + \frac{3\pi^2}{4} + \right. \\
 & \left. \frac{661}{36} \right) H(0; x) + \left( \frac{d_1^2\alpha_0^4}{8} - d_1\alpha_0^4 + \frac{d_1^2\alpha_0^4}{8(x-1)} - \frac{d_1\alpha_0^4}{x-1} - \frac{13d_1^2\alpha_0^3}{18} + \frac{101d_1\alpha_0^3}{18} + \frac{d_1\alpha_0^3}{3(x-2)} - \frac{d_1^2\alpha_0^3}{2(x-1)} + \frac{23d_1\alpha_0^3}{6(x-1)} + \right. \\
 & \left. \frac{2d_1^2\alpha_0^3}{9(x-1)^2} - \frac{29d_1\alpha_0^3}{18(x-1)^2} + \frac{23d_1^2\alpha_0^2}{12} - \frac{167d_1\alpha_0^2}{12} - \frac{5d_1\alpha_0^2}{3(x-2)} + \frac{3d_1^2\alpha_0^2}{4(x-1)} - \frac{5d_1\alpha_0^2}{x-1} + \frac{4d_1\alpha_0^2}{3(x-2)^2} - \frac{2d_1^2\alpha_0^2}{3(x-1)^2} + \frac{14d_1\alpha_0^2}{3(x-1)^2} + \right. \\
 & \left. \frac{d_1^2\alpha_0^2}{2(x-1)^3} - \frac{37d_1\alpha_0^2}{12(x-1)^3} - \frac{25d_1^2\alpha_0}{6} + \frac{145d_1\alpha_0}{6} + \frac{14d_1\alpha_0}{3(x-2)} - \frac{d_1^2\alpha_0}{2(x-1)} + \frac{d_1\alpha_0}{6(x-1)} - \frac{20d_1\alpha_0}{3(x-2)^2} + \frac{2d_1^2\alpha_0}{3(x-1)^2} - \frac{11d_1\alpha_0}{3(x-1)^2} + \right.
 \end{aligned}$$



$$\begin{aligned}
& \frac{8d_1 \alpha_0}{(x-2)^3} - \frac{d_1^2 \alpha_0}{(x-1)^3} + \frac{37d_1 \alpha_0}{6(x-1)^3} + \frac{2d_1^2 \alpha_0}{(x-1)^4} - \frac{55d_1 \alpha_0}{6(x-1)^4} + \frac{205 d_1^2}{72} - \frac{535d_1}{36} - \frac{10d_1}{3(x-2)} + \frac{d_1^2}{8(x-1)} + \frac{2d_1}{x-1} + \frac{16d_1}{3(x-2)^2} - \\
& \frac{2d_1^2}{9(x-1)^2} + \frac{11d_1}{18(x-1)^2} - \frac{8 d_1}{(x-2)^3} + \frac{d_1^2}{2(x-1)^3} - \frac{37d_1}{12(x-1)^3} - \frac{2d_1^2}{(x-1)^4} + \frac{55d_1}{6(x-1)^4} \Big) H(1; \alpha_0) + \left( -\frac{8\pi^2 d_1}{(x-2)^5} + \right. \\
& \left. \frac{16\pi^2}{(x-2)^5} + \frac{\pi^2}{4(x-1)^5} - \frac{\pi^2}{4} \right) H(2; x) + \left( \frac{2\alpha_0^4}{x-1} + 2\alpha_0^4 - \frac{8\alpha_0^3}{x-1} + \frac{8 \alpha_0^3}{3(x-1)^2} - \frac{32\alpha_0^3}{3} + \frac{12\alpha_0^2}{x-1} - \frac{8 \alpha_0^2}{(x-1)^2} + \frac{4\alpha_0^2}{(x-1)^3} + \right. \\
& \left. 24\alpha_0^2 - \frac{8 \alpha_0}{x-1} + \frac{8\alpha_0}{(x-1)^2} - \frac{8\alpha_0}{(x-1)^3} + \frac{8 \alpha_0}{(x-1)^4} - 32\alpha_0 - \frac{16}{x-2} + \frac{20}{x-1} + \frac{64}{3(x-2)^2} - \frac{7}{(x-1)^2} - \frac{32}{(x-2)^3} + \frac{5}{(x-1)^3} + \right. \\
& \left. \frac{64}{(x-2)^4} - \frac{6}{(x-1)^4} - \frac{43}{3(x-1)^5} + \frac{7}{3} \right) H(0, 0; \alpha_0) + \left( -\frac{20}{x-1} + \frac{7}{(x-1)^2} - \frac{5}{(x-1)^3} + \frac{6}{(x-1)^4} + \frac{43}{3(x-1)^5} - \right. \\
& \left. \frac{43}{3} + \frac{16}{x-2} - \frac{64}{3(x-2)^2} + \frac{32}{(x-2)^3} - \frac{64}{(x-2)^4} \right) H(0, 0; x) + \left( d_1 \alpha_0^4 + \frac{d_1 \alpha_0^4}{x-1} - \frac{16d_1 \alpha_0^3}{3} - \frac{4d_1 \alpha_0^3}{x-1} + \frac{4d_1 \alpha_0^3}{3(x-1)^2} + \right. \\
& \left. 12d_1 \alpha_0^2 + \frac{6d_1 \alpha_0^2}{x-1} - \frac{4d_1 \alpha_0^2}{(x-1)^2} + \frac{2d_1 \alpha_0^2}{(x-1)^3} - 16d_1 \alpha_0 - \frac{4d_1 \alpha_0}{x-1} + \frac{4d_1 \alpha_0}{(x-1)^2} - \frac{4d_1 \alpha_0}{(x-1)^3} + \frac{4d_1 \alpha_0}{(x-1)^4} + \frac{7d_1}{6} - \frac{8d_1}{x-2} + \frac{10 d_1}{x-1} + \right. \\
& \left. \frac{32d_1}{3(x-2)^2} - \frac{7d_1}{2(x-1)^2} - \frac{16 d_1}{3(x-1)^3} + \frac{5d_1}{2(x-1)^3} + \frac{32d_1}{(x-2)^4} - \frac{3 d_1}{(x-1)^4} - \frac{43d_1}{6(x-1)^5} \right) H(0, 1; \alpha_0) + H(1; x) \left( \frac{\pi^2 d_1}{3(x-1)^5} + \right. \\
& \left( \frac{15d_1}{2(x-2)} - \frac{19d_1}{2(x-1)} - \frac{28d_1}{3(x-2)^2} + \frac{17 d_1}{6(x-1)^2} + \frac{12d_1}{(x-2)^3} - \frac{3d_1}{2(x-1)^3} - \frac{16d_1}{(x-2)^4} + \frac{d_1}{(x-1)^4} + \frac{43d_1}{6(x-1)^5} + \frac{11}{2(x-2)} - \right. \\
& \left. \frac{5}{2(x-1)} - \frac{16}{3(x-2)^2} - \frac{7}{12(x-1)^2} + \frac{16}{3(x-2)^3} + \frac{5}{12(x-1)^3} - \frac{3}{(x-1)^4} - \frac{16}{(x-2)^5} - \frac{43}{6(x-1)^5} + \frac{43}{6} \right) H(0; \alpha_0) + \\
& \left( -\frac{4 d_1}{(x-1)^5} + \frac{64}{(x-1)^5} + \frac{4}{(x-1)^5} - 4 \right) H(0, 0; \alpha_0) + \left( -\frac{2d_1^2}{(x-1)^5} + \frac{32d_1}{(x-2)^5} + \frac{2 d_1}{(x-1)^5} - 2d_1 \right) H(0, 1; \alpha_0) + \\
& \left( \frac{8\pi^2}{3(x-2)^5} - \frac{\pi^2}{3(x-1)^5} + \frac{\pi^2}{6} \right) + \left( -\frac{32 d_1}{(x-2)^5} - \frac{2d_1}{(x-1)^5} + 2 d_1 + \frac{32}{(x-2)^5} + \frac{4}{(x-1)^5} - 4 \right) H(0; \alpha_0) H(0, 1; x) + \\
& \left( \frac{\alpha_0^4}{2(x-2)} - \frac{2\alpha_0^3}{x-2} + \frac{4 \alpha_0^3}{3(x-2)^2} + \frac{3\alpha_0^2}{x-2} - \frac{4\alpha_0^2}{(x-2)^2} + \frac{4 \alpha_0^2}{(x-2)^3} - \frac{2\alpha_0}{x-2} + \frac{4\alpha_0}{(x-2)^2} - \frac{8 \alpha_0}{(x-2)^3} + \frac{16 \alpha_0}{(x-2)^4} + \left( \frac{4}{(x-1)^5} - \right. \right. \\
& \left. \left. 4 + \frac{64}{(x-2)^5} \right) H(0; \alpha_0) + \left( \frac{32d_1}{(x-2)^5} + \frac{2d_1}{(x-1)^5} - 2d_1 \right) H(1; \alpha_0) + \frac{6}{x-2} - \frac{5}{2(x-1)} - \frac{20}{3(x-2)^2} - \frac{7}{12(x-1)^2} + \right. \\
& \left. \frac{28}{3(x-2)^3} + \frac{5}{12(x-1)^3} - \frac{16}{(x-2)^4} - \frac{3}{(x-1)^4} - \frac{16}{(x-2)^5} - \frac{43}{6(x-1)^5} + \frac{43}{6} \right) H(0, c_1(\alpha_0); x) + \left( d_1 \alpha_0^4 + \frac{d_1 \alpha_0^4}{x-1} - \right. \\
& \left. \frac{16d_1 \alpha_0^3}{3} - \frac{4d_1 \alpha_0^3}{x-1} + \frac{4d_1 \alpha_0^3}{3(x-1)^2} + 12d_1 \alpha_0^2 + \frac{6d_1 \alpha_0^2}{x-1} - \frac{4d_1 \alpha_0^2}{(x-1)^2} + \frac{2d_1 \alpha_0^2}{(x-1)^3} - 16d_1 \alpha_0 - \frac{4d_1 \alpha_0}{x-1} + \frac{4d_1 \alpha_0}{(x-1)^2} - \right. \\
& \left. \frac{4d_1 \alpha_0}{(x-1)^3} + \frac{4d_1 \alpha_0}{(x-1)^4} + \frac{25d_1}{3} + \frac{d_1}{x-1} - \frac{4d_1}{3(x-1)^2} + \frac{2d_1}{(x-1)^3} - \frac{4d_1}{(x-1)^4} \right) H(1, 0; \alpha_0) + \left( -\frac{15d_1}{2(x-2)} + \frac{19d_1}{2(x-1)} + \right. \\
& \left. \frac{28d_1}{3(x-2)^2} - \frac{17d_1}{6(x-1)^2} - \frac{12d_1}{(x-2)^3} + \frac{3 d_1}{2(x-1)^3} + \frac{16d_1}{(x-2)^4} - \frac{d_1}{(x-1)^4} - \frac{43 d_1}{6(x-1)^5} - \frac{11}{2(x-2)} + \frac{5}{2(x-1)} + \frac{16}{3(x-2)^2} + \right. \\
& \left. \frac{7}{12(x-1)^2} - \frac{16}{3(x-2)^3} - \frac{5}{12(x-1)^3} + \frac{3}{(x-1)^4} + \frac{16}{(x-2)^5} + \frac{43}{6(x-1)^5} - \frac{43}{6} \right) H(1, 0; x) + \left( \frac{d_1^2 \alpha_0^4}{2} + \frac{d_1^2 \alpha_0^4}{2(x-1)} - \right. \\
& \left. \frac{8d_1^2 \alpha_0^3}{3} - \frac{2 d_1^2 \alpha_0^3}{x-1} + \frac{2d_1^2 \alpha_0^3}{3(x-1)^2} + 6d_1^2 \alpha_0^2 + \frac{3d_1^2 \alpha_0^2}{x-1} - \frac{2d_1^2 \alpha_0^2}{(x-1)^2} + \frac{d_1^2 \alpha_0^2}{(x-1)^3} - 8d_1^2 \alpha_0 - \frac{2d_1^2 \alpha_0}{x-1} + \frac{2d_1^2 \alpha_0}{(x-1)^2} - \frac{2d_1^2 \alpha_0}{(x-1)^3} + \right. \\
& \left. \frac{2d_1^2 \alpha_0}{(x-1)^4} + \frac{25 d_1^2}{6} + \frac{d_1^2}{2(x-1)} - \frac{2d_1^2}{3(x-1)^2} + \frac{d_1^2}{(x-1)^3} - \frac{2d_1^2}{(x-1)^4} \right) H(1, 1; \alpha_0) + H(c_1(\alpha_0); x) \left( \frac{d_1 \alpha_0^4}{8} - \right. \\
& \left. \frac{d_1 \alpha_0^4}{8(x-2)} + \frac{\alpha_0^4}{2(x-2)} + \frac{d_1 \alpha_0^4}{8(x-1)} - \frac{\alpha_0^4}{x-1} - \alpha_0^4 - \frac{13d_1 \alpha_0^3}{18} + \frac{d_1 \alpha_0^3}{2(x-2)} - \frac{5\alpha_0^3}{3(x-2)} - \frac{d_1 \alpha_0^3}{2(x-1)} + \frac{43\alpha_0^3}{12(x-1)} - \frac{4d_1 \alpha_0^3}{9(x-2)^2} + \right. \\
& \left. \frac{14\alpha_0^3}{9(x-2)^2} + \frac{2d_1 \alpha_0^3}{9(x-1)^2} - \frac{29\alpha_0^3}{18(x-1)^2} + \frac{101\alpha_0^3}{18} + \frac{23 d_1 \alpha_0^2}{12} - \frac{3d_1 \alpha_0^2}{4(x-2)} + \frac{11\alpha_0^2}{12(x-2)} + \frac{3d_1 \alpha_0^2}{4(x-1)} - \frac{7\alpha_0^2}{2(x-1)} + \frac{4d_1 \alpha_0^2}{3(x-2)^2} - \right. \\
& \left. \frac{10\alpha_0^2}{3(x-2)^2} - \frac{2d_1 \alpha_0^2}{3(x-1)^2} + \frac{13\alpha_0^2}{3(x-1)^2} - \frac{2d_1 \alpha_0^2}{(x-2)^3} + \frac{6\alpha_0^2}{(x-2)^3} + \frac{d_1 \alpha_0^2}{2(x-1)^3} - \frac{37\alpha_0^2}{12(x-1)^3} - \frac{167\alpha_0^2}{12} - \frac{25 d_1 \alpha_0}{6} + \frac{d_1 \alpha_0}{2(x-2)} + \right. \\
& \left. \frac{37\alpha_0}{6(x-2)} - \frac{d_1 \alpha_0}{2(x-1)} - \frac{21\alpha_0}{4(x-1)} - \frac{4d_1 \alpha_0}{3(x-2)^2} - \frac{14\alpha_0}{3(x-2)^2} + \frac{2d_1 \alpha_0}{3(x-1)^2} - \frac{17\alpha_0}{12(x-1)^2} + \frac{4d_1 \alpha_0}{(x-2)^3} - \frac{4 \alpha_0}{(x-2)^3} - \frac{d_1 \alpha_0}{(x-1)^3} + \right. \\
& \left. \frac{17\alpha_0}{3(x-1)^3} - \frac{16d_1 \alpha_0}{(x-2)^4} + \frac{40\alpha_0}{(x-2)^4} + \frac{2 d_1 \alpha_0}{(x-1)^4} - \frac{55\alpha_0}{6(x-1)^4} + \frac{145 \alpha_0}{6} + \frac{205d_1}{72} + \left( -\frac{\alpha_0^4}{x-2} + \frac{\alpha_0^4}{x-1} + \alpha_0^4 + \frac{4\alpha_0^3}{x-2} - \frac{4\alpha_0^3}{x-1} - \right. \right. \\
& \left. \left. \frac{8\alpha_0^3}{3(x-2)^2} + \frac{4\alpha_0^3}{3(x-1)^2} - \frac{16\alpha_0^3}{3} - \frac{6 \alpha_0^2}{x-2} + \frac{6\alpha_0^2}{x-1} + \frac{8\alpha_0^2}{(x-2)^2} - \frac{4 \alpha_0^2}{(x-1)^2} - \frac{8\alpha_0^2}{(x-2)^3} + \frac{2\alpha_0^2}{(x-1)^3} + 12 \alpha_0^2 + \frac{4\alpha_0}{x-2} - \frac{4\alpha_0}{x-1} - \right. \right. \\
& \left. \left. \frac{8 \alpha_0}{(x-2)^2} + \frac{4\alpha_0}{(x-1)^2} + \frac{16\alpha_0}{(x-2)^3} - \frac{4 \alpha_0}{(x-1)^3} - \frac{32\alpha_0}{(x-2)^4} + \frac{4\alpha_0}{(x-1)^4} - 16 \alpha_0 - \frac{16}{x-2} + \frac{20}{x-1} + \frac{64}{3(x-2)^2} - \frac{7}{(x-1)^2} - \right. \right. \\
& \left. \left. \frac{32}{(x-2)^3} + \frac{5}{(x-1)^3} + \frac{64}{(x-2)^4} - \frac{6}{(x-1)^4} - \frac{43}{3(x-1)^5} + \frac{25}{3} \right) H(0; \alpha_0) + \left( \frac{d_1 \alpha_0^4}{2} - \frac{d_1 \alpha_0^4}{2(x-2)} + \frac{d_1 \alpha_0^4}{2(x-1)} - \frac{8d_1 \alpha_0^3}{3} + \right. \\
& \left. \frac{2d_1 \alpha_0^3}{x-2} - \frac{2 d_1 \alpha_0^3}{x-1} - \frac{4d_1 \alpha_0^3}{3(x-1)^2} + \frac{2d_1 \alpha_0^3}{3(x-1)^2} + 6d_1 \alpha_0^2 - \frac{3d_1 \alpha_0^2}{x-2} + \frac{3d_1 \alpha_0^2}{x-1} + \frac{4d_1 \alpha_0^2}{(x-2)^2} - \frac{2d_1 \alpha_0^2}{(x-1)^2} - \frac{4d_1 \alpha_0^2}{(x-2)^3} + \frac{d_1 \alpha_0^2}{(x-1)^3} - \right. \\
& \left. 8d_1 \alpha_0 + \frac{2d_1 \alpha_0}{x-2} - \frac{4d_1 \alpha_0}{x-1} - \frac{4d_1 \alpha_0}{(x-2)^2} + \frac{2d_1 \alpha_0}{(x-1)^2} + \frac{8d_1 \alpha_0}{(x-2)^3} - \frac{2d_1 \alpha_0}{(x-1)^3} - \frac{16d_1 \alpha_0}{(x-2)^4} + \frac{2d_1 \alpha_0}{(x-1)^4} + \frac{25d_1}{6} - \frac{8d_1}{x-2} + \right. \\
& \left. \frac{10 d_1}{x-1} + \frac{32d_1}{3(x-2)^2} - \frac{7d_1}{2(x-1)^2} - \frac{16 d_1}{(x-2)^3} + \frac{5d_1}{2(x-1)^3} + \frac{32d_1}{(x-2)^4} - \frac{3 d_1}{(x-1)^4} - \frac{43d_1}{6(x-1)^5} \right) H(1; \alpha_0) + \frac{8 H(0,0;\alpha_0)}{(x-1)^5} +
\end{aligned}$$

$$\begin{aligned}
 & \frac{4d_1 H(0,1;\alpha_0)}{(x-1)^5} + \frac{4d_1 H(1,0;\alpha_0)}{(x-1)^5} + \frac{2d_1^2 H(1,1;\alpha_0)}{(x-1)^5} - \frac{34 d_1}{3(x-2)} + \frac{4}{x-2} + \frac{40d_1}{3(x-1)} - \frac{155}{12(x-1)} + \frac{116d_1}{9(x-2)^2} - \frac{88}{9(x-2)^2} - \\
 & \frac{47 d_1}{24(x-1)^2} + \frac{103}{12(x-1)^2} - \frac{16 d_1}{(x-2)^3} + \frac{24}{(x-2)^3} + \frac{25d_1}{24(x-1)^3} - \frac{103}{12(x-1)^3} + \frac{32 d_1}{(x-2)^4} - \frac{80}{(x-2)^4} - \frac{9d_1}{4(x-1)^4} + \\
 & \frac{145}{12(x-1)^4} - \frac{205d_1}{72(x-1)^5} - \frac{\pi^2}{12(x-1)^5} + \frac{661}{36(x-1)^5} - \frac{535}{36} \left) + \left( \frac{2d_1^2}{(x-1)^5} - \frac{32 d_1}{(x-2)^5} - \frac{4d_1}{(x-1)^5} + 2 d_1 - \frac{16}{(x-2)^5} + \right. \right. \\
 & \left. \left. \frac{2}{(x-1)^5} - 1 \right) H(0; \alpha_0) H(1, 1; x) + \left( \frac{15d_1}{2(x-2)} - \frac{19d_1}{2(x-1)} - \frac{28 d_1}{3(x-2)^2} + \frac{17d_1}{6(x-1)^2} + \frac{12 d_1}{(x-2)^3} - \frac{3d_1}{2(x-1)^3} - \right. \right. \\
 & \left. \left. \frac{16 d_1}{(x-2)^4} + \frac{d_1}{(x-1)^4} + \frac{43d_1}{6(x-1)^5} + \left( -\frac{4 d_1}{(x-1)^5} + \frac{64}{(x-2)^5} + \frac{4}{(x-1)^5} - 4 \right) H(0; \alpha_0) + \left( -\frac{2d_1^2}{(x-1)^5} + \frac{32d_1}{(x-2)^5} + \right. \right. \right. \\
 & \left. \left. \frac{2 d_1}{(x-1)^5} - 2d_1 \right) H(1; \alpha_0) + \frac{11}{2(x-2)} - \frac{5}{2(x-1)} - \frac{16}{3(x-2)^2} - \frac{7}{12(x-1)^2} + \frac{16}{3(x-2)^3} + \frac{5}{12(x-1)^3} - \frac{3}{(x-1)^4} - \right. \\
 & \left. \frac{16}{(x-2)^5} - \frac{43}{6(x-1)^5} + \frac{43}{6} \right) H(1, c_1(\alpha_0); x) + \left( \frac{32 d_1}{(x-2)^5} - \frac{64}{(x-2)^5} - \frac{1}{(x-1)^5} + 1 \right) H(0; \alpha_0) H(2, 1; x) + \left( -\frac{5\alpha_0^4}{4(x-2)^3} + \frac{\alpha_0^4}{2(x-1)^3} + \frac{3\alpha_0^4}{4} + \frac{5\alpha_0^3}{x-2} - \frac{2\alpha_0^3}{x-1} - \frac{10\alpha_0^3}{3(x-2)^2} + \frac{2\alpha_0^3}{3(x-1)^2} - 4\alpha_0^3 - \frac{15\alpha_0^2}{2(x-2)} + \frac{3\alpha_0^2}{x-1} + \frac{10\alpha_0^2}{(x-2)^2} - \frac{2\alpha_0^2}{(x-1)^2} - \right. \\
 & \left. \frac{10\alpha_0^2}{(x-2)^3} + \frac{\alpha_0^2}{(x-1)^3} + 9\alpha_0^2 + \frac{5\alpha_0}{x-2} - \frac{2\alpha_0}{x-1} - \frac{10\alpha_0}{(x-2)^2} + \frac{2\alpha_0}{(x-1)^2} + \frac{20\alpha_0}{(x-2)^3} - \frac{2\alpha_0}{(x-1)^3} - \frac{40\alpha_0}{(x-2)^4} + \frac{2\alpha_0}{(x-1)^4} - 12\alpha_0 + \right. \\
 & \left. \frac{4H(0;\alpha_0)}{(x-1)^5} + \frac{2d_1 H(1;\alpha_0)}{(x-1)^5} - \frac{20}{x-2} + \frac{89}{4(x-1)} + \frac{80}{3(x-2)^2} - \frac{27}{4(x-1)^2} - \frac{40}{(x-2)^3} + \frac{49}{12(x-1)^3} + \frac{80}{(x-2)^4} - \frac{4}{(x-1)^4} - \right. \\
 & \left. \frac{43}{6(x-1)^5} + \frac{25}{4} \right) H(c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{\alpha_0^4}{4(x-1)} - \frac{\alpha_0^4}{4} - \frac{\alpha_0^3}{x-1} + \frac{\alpha_0^3}{3(x-1)^2} + \frac{4\alpha_0^3}{3} + \frac{3\alpha_0^2}{2(x-1)} - \frac{\alpha_0^2}{(x-1)^2} + \right. \\
 & \left. \frac{\alpha_0^2}{2(x-1)^3} - 3\alpha_0^2 - \frac{\alpha_0}{x-1} + \frac{\alpha_0}{(x-1)^2} - \frac{\alpha_0}{(x-1)^3} + \frac{\alpha_0}{(x-1)^4} + 4\alpha_0 - \frac{64H(0;\alpha_0)}{(x-2)^5} - \frac{32d_1 H(1;\alpha_0)}{(x-2)^5} - \frac{2}{x-2} + \frac{1}{4(x-1)} + \right. \\
 & \left. \frac{4}{3(x-2)^2} + \frac{1}{3(x-1)^2} - \frac{4}{3(x-2)^3} + \frac{1}{2(x-1)^3} + \frac{1}{(x-1)^4} + \frac{16}{(x-2)^5} - \frac{25}{12} \right) H(c_2(\alpha_0), c_1(\alpha_0); x) + \left( 8 + \frac{8}{(x-1)^5} \right) H(0, 0, 0; \alpha_0) + \left( 8 - \frac{8}{(x-1)^5} \right) H(0, 0, 0; x) + \left( \frac{4 d_1}{(x-1)^5} + 4d_1 \right) H(0, 0, 1; \alpha_0) + \left( \frac{4}{(x-1)^5} - 4 + \frac{32}{(x-2)^5} \right) H(0, 0, c_1(\alpha_0); x) + \left( \frac{4d_1}{(x-1)^5} + 4d_1 \right) H(0, 1, 0; \alpha_0) + \left( \frac{32d_1}{(x-2)^5} + \frac{2d_1}{(x-1)^5} - 2 d_1 - \frac{32}{(x-2)^5} - \frac{4}{(x-1)^5} + 4 \right) H(0, 1, 0; x) + \left( \frac{2d_1^2}{(x-1)^5} + 2d_1^2 \right) H(0, 1, 1; \alpha_0) + \left( -\frac{32d_1}{(x-2)^5} - \frac{2d_1}{(x-1)^5} + 2 d_1 + \frac{32}{(x-2)^5} + \frac{4}{(x-1)^5} - 4 \right) H(0, 1, c_1(\alpha_0); x) + \left( \frac{2}{(x-1)^5} - 3 + \frac{80}{(x-2)^5} \right) H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{1}{(x-1)^5} + 1 - \frac{64}{(x-2)^5} \right) H(0, c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{4 d_1}{(x-1)^5} - \frac{64}{(x-2)^5} - \frac{4}{(x-1)^5} + 4 \right) H(1, 0, 0; x) + \left( -\frac{2 d_1}{(x-1)^5} - \frac{16}{(x-2)^5} + \frac{2}{(x-1)^5} - 1 \right) H(1, 0, c_1(\alpha_0); x) + \left( -\frac{2d_1^2}{(x-1)^5} + \frac{32 d_1}{(x-2)^5} + \frac{4d_1}{(x-1)^5} - 2 d_1 + \frac{16}{(x-2)^5} - \frac{2}{(x-1)^5} + 1 \right) H(1, 1, 0; x) + \left( \frac{2d_1^2}{(x-1)^5} - \frac{32 d_1}{(x-2)^5} - \frac{4d_1}{(x-1)^5} + 2 d_1 - \frac{16}{(x-2)^5} + \frac{2}{(x-1)^5} - 1 \right) H(1, 1, c_1(\alpha_0); x) + \left( -\frac{2 d_1}{(x-1)^5} + \frac{80}{(x-2)^5} + \frac{2}{(x-1)^5} - 3 \right) H(1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{32 d_1}{(x-2)^5} - \frac{64}{(x-2)^5} - \frac{1}{(x-1)^5} + 1 \right) H(2, 0, c_1(\alpha_0); x) + \left( -\frac{32 d_1}{(x-2)^5} + \frac{64}{(x-2)^5} + \frac{1}{(x-1)^5} - 1 \right) H(2, 1, 0; x) + \left( \frac{32 d_1}{(x-2)^5} - \frac{64}{(x-2)^5} - \frac{1}{(x-1)^5} + 1 \right) H(2, 1, c_1(\alpha_0); x) + \left( -\frac{32 d_1}{(x-2)^5} + \frac{64}{(x-2)^5} + \frac{1}{(x-1)^5} - 1 \right) H(2, c_2(\alpha_0), c_1(\alpha_0); x) + \frac{2 H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \frac{H(c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \frac{32 H(c_2(\alpha_0), 0, c_1(\alpha_0); x)}{(x-2)^5} - \frac{80 H(c_2(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x)}{(x-2)^5} - \frac{7\pi^2}{6(x-2)} + \frac{7\pi^2}{16(x-1)} + \frac{11\pi^2}{9(x-2)^2} + \frac{\pi^2}{8(x-1)^2} - \frac{5\pi^2}{3(x-2)^3} - \frac{\pi^2}{36(x-1)^3} + \frac{8\pi^2}{3(x-2)^4} + \frac{7\pi^2}{12(x-1)^4} + \frac{4\pi^2}{(x-2)^5} + \frac{43\pi^2}{36(x-1)^5} - \frac{28\zeta_3}{(x-2)^5} - \frac{21\zeta_3}{8(x-1)^5} + \frac{17\zeta_3}{8} + \frac{24\pi^2 \ln 2}{(x-2)^5} + \frac{\pi^2 \ln 2}{4(x-1)^5} - \frac{1}{4}\pi^2 \ln 2 - \frac{197\pi^2}{144} - \frac{15}{4}
 \end{aligned}$$

### E.7 The $\mathcal{B}$ integral for $k = 2$ and $\delta = 1$ and $d_1 = -3$

The  $\varepsilon$  expansion for this integral reads

$$\begin{aligned}
 \mathcal{I}(x, \varepsilon; \alpha_0, 3 + d_1\varepsilon; 1, 2, 1, g_B) &= x \mathcal{B}(\varepsilon, x; 3 + d_1\varepsilon; 1, 2) \\
 &= \frac{1}{\varepsilon} b_{-1}^{(1,2)} + b_0^{(1,2)} + \varepsilon b_1^{(1,2)} + \varepsilon^2 b_2^{(1,2)} + \mathcal{O}(\varepsilon^3), \tag{E.7}
 \end{aligned}$$

where

$$b_{-1}^{(1,2)} = -\frac{1}{6},$$

$$b_0^{(1,2)} = -\frac{\alpha_0^6}{3(x\alpha_0-2\alpha_0-x)} + \frac{\alpha_0^5}{3(x-2)} + \frac{5\alpha_0^5}{3(x\alpha_0-2\alpha_0-x)} - \frac{5\alpha_0^4}{4(x-2)} + \frac{\alpha_0^4}{12(x-1)} - \frac{10\alpha_0^4}{3(x\alpha_0-2\alpha_0-x)} + \frac{5\alpha_0^4}{6(x-2)^2} + \frac{\alpha_0^4}{12} + \frac{5\alpha_0^3}{3(x-2)} - \frac{\alpha_0^3}{3(x-1)} + \frac{10\alpha_0^3}{3(x\alpha_0-2\alpha_0-x)} - \frac{20\alpha_0^3}{9(x-2)^2} + \frac{\alpha_0^3}{9(x-1)^2} + \frac{20\alpha_0^3}{9(x-2)^3} - \frac{4\alpha_0^3}{9} - \frac{5\alpha_0^2}{6(x-2)} + \frac{\alpha_0^2}{2(x-1)} - \frac{5\alpha_0^2}{3(x\alpha_0-2\alpha_0-x)} + \frac{5\alpha_0^2}{3(x-2)^2} - \frac{\alpha_0^2}{3(x-1)^2} - \frac{10\alpha_0^2}{3(x-2)^3} + \frac{\alpha_0^2}{6(x-1)^3} + \frac{20\alpha_0^2}{3(x-2)^4} + \alpha_0^2 - \frac{\alpha_0}{3(x-1)} + \frac{\alpha_0}{3(x\alpha_0-2\alpha_0-x)} + \frac{\alpha_0}{3(x-1)^2} - \frac{\alpha_0}{3(x-1)^3} + \frac{\alpha_0}{3(x-1)^4} + \frac{80\alpha_0}{3(x-2)^5} - \frac{4\alpha_0}{3} + \left( \frac{1}{3(x-1)^5} + \frac{1}{3} + \frac{80}{3(x-2)^5} + \frac{160}{3(x-2)^6} \right) H(0; \alpha_0) + \left( -\frac{1}{3(x-1)^5} + \frac{1}{3} - \frac{80}{3(x-2)^5} - \frac{160}{3(x-2)^6} \right) H(0; x) + \frac{H(c_1(\alpha_0); x)}{3(x-1)^5} + \left( \frac{80}{3(x-2)^5} + \frac{160}{3(x-2)^6} \right) H(c_2(\alpha_0); x) + \frac{80 \ln 2}{3(x-2)^5} + \frac{160 \ln 2}{3(x-2)^6} - \frac{11}{18},$$

$$b_1^{(1,2)} = \frac{1}{\alpha_0 x - x - 2\alpha_0} \left\{ \frac{37x\alpha_0^5}{72} + \frac{31\alpha_0^5}{36(x-2)} - \frac{37\alpha_0^5}{72(x-1)} - \frac{\alpha_0^5}{12} - \frac{245x\alpha_0^4}{72} - \frac{5\alpha_0^4}{2(x-2)} + \frac{161\alpha_0^4}{72(x-1)} + \frac{65\alpha_0^4}{18(x-2)^2} - \frac{5\alpha_0^4}{6(x-1)^2} + \frac{11\alpha_0^4}{12} + \frac{91x\alpha_0^3}{9} + \frac{13\alpha_0^3}{18(x-2)} - \frac{125\alpha_0^3}{36(x-1)} - \frac{40\alpha_0^3}{9(x-2)^2} + \frac{25\alpha_0^3}{8(x-1)^2} + \frac{20\alpha_0^3}{(x-2)^3} - \frac{59\alpha_0^3}{36(x-1)^3} - \frac{307\alpha_0^3}{72} - \frac{61x\alpha_0^2}{3} + \frac{65\alpha_0^2}{9(x-2)} - \frac{\alpha_0^2}{6(x-1)} - \frac{43\alpha_0^2}{3(x-2)^2} - \frac{265\alpha_0^2}{72(x-1)^2} + \frac{400\alpha_0^2}{9(x-2)^3} + \frac{20\alpha_0^2}{3(x-1)^3} + \frac{1880\alpha_0^2}{9(x-2)^4} - \frac{95\alpha_0^2}{18(x-1)^4} + \frac{1091\alpha_0^2}{72} - \frac{1}{9}\pi^2 x \alpha_0 + \frac{623x\alpha_0}{54} + \frac{14\alpha_0}{9(x-2)} - \frac{35\alpha_0}{12(x-1)} + \frac{2\alpha_0}{3(x-2)^2} + \frac{61\alpha_0}{36(x-1)^2} - \frac{248\alpha_0}{9(x-2)^3} - \frac{9\alpha_0}{4(x-1)^3} + \frac{76\pi^2\alpha_0}{9(x-2)^4} - \frac{3760\alpha_0}{9(x-2)^4} + \frac{\pi^2\alpha_0}{9(x-1)^4} - \frac{95\alpha_0}{18(x-1)^4} + \frac{80\pi^2\alpha_0}{9(x-2)^5} - \frac{5120\alpha_0}{9(x-2)^5} - \frac{\pi^2\alpha_0}{9(x-1)^5} + \frac{160 \ln^2 2 \alpha_0}{3(x-2)^4} + \frac{320 \ln^2 2 \alpha_0}{3(x-2)^5} + \frac{160 \ln 2 \alpha_0}{9(x-2)^4} + \frac{320 \ln 2 \alpha_0}{9(x-2)^5} + \frac{2\pi^2 \alpha_0}{9} + \frac{859\alpha_0}{108} + \frac{\pi^2 x}{9} + \frac{85x}{54} + \left( -\frac{x\alpha_0^5}{3} - \frac{2\alpha_0^5}{3(x-2)} + \frac{\alpha_0^5}{3(x-1)} + \frac{19x\alpha_0^4}{9} + \frac{20\alpha_0^4}{9(x-2)} - \frac{13\alpha_0^4}{9(x-1)} - \frac{20\alpha_0^4}{9(x-2)^2} + \frac{4\alpha_0^4}{9(x-1)^2} - \frac{2\alpha_0^4}{9} - \frac{52x\alpha_0^3}{9} - \frac{20\alpha_0^3}{9(x-2)} + \frac{22\alpha_0^3}{9(x-1)} + \frac{40\alpha_0^3}{9(x-2)^2} - \frac{14\alpha_0^3}{9(x-1)^2} - \frac{80\alpha_0^3}{9(x-2)^3} + \frac{2\alpha_0^3}{3(x-1)^3} + \frac{4\alpha_0^3}{3} + \frac{28x\alpha_0^2}{3} - \frac{2\alpha_0^2}{x-1} + \frac{2\alpha_0^2}{(x-1)^2} - \frac{2\alpha_0^2}{(x-1)^3} - \frac{160\alpha_0^2}{3(x-2)^4} + \frac{4\alpha_0^2}{3(x-1)^4} - 4\alpha_0^2 - \frac{11x\alpha_0}{2} - \frac{128\alpha_0}{9(x-2)} + \frac{31\alpha_0}{2(x-1)} + \frac{152\alpha_0}{9(x-2)^2} - \frac{37\alpha_0}{9(x-1)^2} - \frac{16\alpha_0}{(x-2)^3} + \frac{47\alpha_0}{18(x-1)^3} + \frac{2080\alpha_0}{9(x-2)^4} + \frac{97\alpha_0}{36(x-1)^4} + \frac{2240\alpha_0}{9(x-2)^5} - \frac{18(x-1)^5}{12} + \frac{x}{6} - \frac{88}{9(x-2)} + \frac{10}{x-1} + \frac{104}{9(x-2)^2} - \frac{25}{18(x-1)^2} - \frac{160}{9(x-2)^3} + \frac{1}{9(x-1)^3} - \frac{832}{9(x-2)^4} - \frac{139}{36(x-1)^4} - \frac{2560}{9(x-2)^5} - \frac{47}{18(x-1)^5} - \frac{640}{9(x-2)^6} + \frac{3}{4} \right) H(0; \alpha_0) + \left( \frac{x\alpha_0^5}{2} + \frac{\alpha_0^5}{x-2} - \frac{\alpha_0^5}{2(x-1)} - \frac{19x\alpha_0^4}{6} - \frac{10\alpha_0^4}{3(x-2)} + \frac{13\alpha_0^4}{6(x-1)} + \frac{10\alpha_0^4}{3(x-2)^2} - \frac{2\alpha_0^4}{3(x-1)^2} + \frac{\alpha_0^4}{3} + \frac{26x\alpha_0^3}{3} + \frac{10\alpha_0^3}{3(x-2)} - \frac{11\alpha_0^3}{3(x-1)} - \frac{20\alpha_0^3}{3(x-2)^2} + \frac{7\alpha_0^3}{3(x-1)^2} + \frac{40\alpha_0^3}{3(x-2)^3} - \frac{\alpha_0^3}{(x-1)^3} - 2\alpha_0^3 - 14x\alpha_0^2 + \frac{3\alpha_0^2}{x-1} - \frac{3\alpha_0^2}{(x-1)^2} + \frac{3\alpha_0^2}{(x-1)^3} + \frac{80\alpha_0^2}{(x-2)^4} - \frac{2\alpha_0^2}{(x-1)^4} + 6\alpha_0^2 + \frac{73x\alpha_0}{6} - \frac{5\alpha_0}{3(x-2)} - \frac{7\alpha_0}{6(x-1)} + \frac{20\alpha_0}{3(x-2)^2} + \frac{5\alpha_0}{3(x-1)^2} - \frac{40\alpha_0}{(x-2)^3} - \frac{3\alpha_0}{(x-1)^3} - \frac{320\alpha_0}{(x-2)^4} - \frac{320\alpha_0}{(x-2)^5} - \frac{10\alpha_0}{3} - \frac{25x}{6} + \frac{2}{3(x-2)} + \frac{1}{6(x-1)} - \frac{10}{3(x-2)^2} - \frac{1}{3(x-1)^2} + \frac{80}{3(x-2)^3} + \frac{1}{(x-1)^3} + \frac{240}{(x-2)^4} + \frac{2}{(x-1)^4} + \frac{320}{(x-2)^5} - 1 \right) H(1; \alpha_0) + \left( \frac{2x\alpha_0}{3} + \frac{112\alpha_0}{3(x-2)^4} - \frac{8\alpha_0}{3(x-1)^4} + \frac{320\alpha_0}{3(x-2)^5} + \frac{8\alpha_0}{3(x-1)^5} - \frac{4\alpha_0}{3} - \frac{2x}{3} - \frac{112}{3(x-2)^4} + \frac{8}{3(x-1)^4} - \frac{544}{3(x-2)^5} + \frac{8}{3(x-1)^5} - \frac{640}{3(x-2)^6} \right) H(0; \alpha_0) H(1; x) + \left( -\frac{x\alpha_0^5}{6} - \frac{\alpha_0^5}{3(x-2)} + \frac{\alpha_0^5}{6(x-1)} + \frac{\alpha_0^5}{4} + \frac{19x\alpha_0^4}{18} + \frac{23\alpha_0^4}{18(x-2)} - \frac{13\alpha_0^4}{18(x-1)} - \frac{10\alpha_0^4}{9(x-2)^2} + \frac{2\alpha_0^4}{9(x-1)^2} - \frac{49\alpha_0^4}{36} - \frac{26x\alpha_0^3}{9} - \frac{16\alpha_0^3}{9(x-2)} + \frac{11\alpha_0^3}{9(x-1)} + \frac{26\alpha_0^3}{9(x-2)^2} - \frac{7\alpha_0^3}{9(x-1)^2} - \frac{40\alpha_0^3}{9(x-2)^3} + \frac{\alpha_0^3}{3(x-1)^3} + \frac{19\alpha_0^3}{6} + \frac{14x\alpha_0^2}{3} + \frac{\alpha_0^2}{x-2} - \frac{\alpha_0^2}{x-1} - \frac{2\alpha_0^2}{(x-2)^2} + \frac{\alpha_0^2}{(x-1)^2} + \frac{4\alpha_0^2}{(x-2)^3} - \frac{\alpha_0^2}{(x-1)^3} - \frac{80\alpha_0^2}{3(x-2)^4} + \frac{2\alpha_0^2}{3(x-1)^4} - \frac{9\alpha_0^2}{2} - \frac{73x\alpha_0}{18} - \frac{128\alpha_0}{9(x-2)} + \frac{31\alpha_0}{2(x-1)} + \frac{152\alpha_0}{9(x-2)^2} - \frac{37\alpha_0}{9(x-1)^2} - \frac{16\alpha_0}{(x-2)^3} + \frac{47\alpha_0}{18(x-1)^3} + \frac{144\alpha_0}{(x-2)^4} + \frac{73\alpha_0}{36(x-1)^4} + \frac{320\alpha_0}{3(x-2)^5} - \frac{47\alpha_0}{18(x-1)^5} + \frac{61\alpha_0}{36} + \frac{25x}{18} + \left( -\frac{4\alpha_0}{3(x-1)^4} + \frac{4\alpha_0}{3(x-1)^5} + \frac{4}{3(x-1)^4} + \frac{4}{3(x-1)^5} \right) H(0; \alpha_0) + \left( \frac{2\alpha_0}{(x-1)^4} - \frac{2\alpha_0}{(x-1)^5} - \frac{2}{(x-1)^4} - \frac{2}{(x-1)^5} \right) H(1; \alpha_0) - \frac{88}{9(x-2)} + \frac{10}{x-1} + \frac{104}{9(x-2)^2} - \frac{25}{18(x-1)^2} - \frac{160}{9(x-2)^3} + \frac{1}{9(x-1)^3} - \frac{224}{3(x-2)^4} - \frac{139}{36(x-1)^4} - \frac{640}{3(x-2)^5} - \frac{47}{18(x-1)^5} + \frac{3}{4} \right) H(c_1(\alpha_0); x) + \left( \frac{160\alpha_0}{9(x-2)^4} + \frac{320\alpha_0}{9(x-2)^5} + \left( -\frac{320\alpha_0}{3(x-2)^4} - \frac{640\alpha_0}{3(x-2)^5} + \frac{320}{3(x-2)^4} + \frac{1280}{3(x-2)^5} + \frac{1280}{3(x-2)^6} \right) H(0; \alpha_0) + \left( \frac{160\alpha_0}{(x-2)^4} + \frac{320\alpha_0}{(x-2)^5} - \frac{160}{(x-2)^4} - \frac{640}{(x-2)^5} - \frac{640}{(x-2)^6} \right) H(1; \alpha_0) - \frac{160}{9(x-2)^4} - \frac{640}{9(x-2)^5} - \frac{640}{9(x-2)^6} \right) H(c_2(\alpha_0); x) + \left( -\frac{4x\alpha_0}{3} - \frac{320\alpha_0}{3(x-2)^4} - \frac{4\alpha_0}{3(x-1)^4} - \frac{640\alpha_0}{3(x-2)^5} + \frac{4\alpha_0}{3(x-1)^5} + \frac{8\alpha_0}{3} + \frac{4x}{3} + \frac{320}{3(x-2)^4} + \frac{4}{3(x-1)^4} + \frac{1280}{3(x-2)^5} + \frac{4}{3(x-1)^5} + \frac{1280}{3(x-2)^6} \right) H(0, 0; \alpha_0) +$$

$$\left( -\frac{4x\alpha_0}{3} + \frac{320\alpha_0}{3(x-2)^4} + \frac{4\alpha_0}{3(x-1)^4} + \frac{640\alpha_0}{3(x-2)^5} - \frac{4\alpha_0}{3(x-1)^5} + \frac{8\alpha_0}{3} + \frac{4x}{3} - \frac{320}{3(x-2)^4} - \frac{4}{3(x-1)^4} - \frac{1280}{3(x-2)^5} - \frac{4}{3(x-1)^5} - \frac{1280}{3(x-2)^6} \right) H(0, 0; x) + \left( 2x\alpha_0 + \frac{160\alpha_0}{(x-2)^4} + \frac{2\alpha_0}{(x-1)^4} + \frac{320\alpha_0}{(x-2)^5} - \frac{2\alpha_0}{(x-1)^5} - 4\alpha_0 - 2x - \frac{160}{(x-2)^4} - \frac{2}{(x-1)^4} - \frac{640}{(x-2)^5} - \frac{2}{(x-1)^5} - \frac{640}{(x-2)^6} \right) H(0, 1; \alpha_0) + \left( \frac{2x\alpha_0}{3} + \frac{112\alpha_0}{3(x-2)^4} - \frac{2\alpha_0}{3(x-1)^4} + \frac{320\alpha_0}{3(x-2)^5} + \frac{2\alpha_0}{3(x-1)^5} - \frac{4\alpha_0}{3} - \frac{2x}{3} - \frac{112}{3(x-2)^4} + \frac{2}{3(x-1)^4} - \frac{544}{3(x-2)^5} + \frac{2}{3(x-1)^5} - \frac{640}{3(x-2)^6} \right) H(0, c_1(\alpha_0); x) + \left( -\frac{320\alpha_0}{3(x-2)^4} - \frac{640\alpha_0}{3(x-2)^5} + \frac{320}{3(x-2)^4} + \frac{1280}{3(x-2)^5} + \frac{1280}{3(x-2)^6} \right) H(0, c_2(\alpha_0); x) + \left( -\frac{2x\alpha_0}{3} - \frac{112\alpha_0}{3(x-2)^4} + \frac{8\alpha_0}{3(x-1)^4} - \frac{320\alpha_0}{3(x-2)^5} - \frac{8\alpha_0}{3(x-1)^5} + \frac{4\alpha_0}{3} + \frac{2x}{3} + \frac{112}{3(x-2)^4} - \frac{8}{3(x-1)^4} + \frac{544}{3(x-2)^5} - \frac{8}{3(x-1)^5} + \frac{640}{3(x-2)^6} \right) H(1, 0; x) + \left( \frac{2x\alpha_0}{3} + \frac{112\alpha_0}{3(x-2)^4} - \frac{8\alpha_0}{3(x-1)^4} + \frac{320\alpha_0}{3(x-2)^5} + \frac{8\alpha_0}{3(x-1)^5} - \frac{4\alpha_0}{3} - \frac{2x}{3} - \frac{112}{3(x-2)^4} + \frac{8}{3(x-1)^4} - \frac{544}{3(x-2)^5} + \frac{8}{3(x-1)^5} - \frac{640}{3(x-2)^6} \right) H(1, c_1(\alpha_0); x) + \left( -\frac{800\alpha_0}{3(x-2)^4} - \frac{1600\alpha_0}{3(x-2)^5} + \frac{800}{3(x-2)^4} + \frac{3200}{3(x-2)^5} + \frac{3200}{3(x-2)^6} \right) H(2, 0; x) + \left( \frac{800\alpha_0}{3(x-2)^4} + \frac{1600\alpha_0}{3(x-2)^5} - \frac{800}{3(x-2)^4} - \frac{3200}{3(x-2)^5} - \frac{3200}{3(x-2)^6} \right) H(2, c_2(\alpha_0); x) + \left( -\frac{2\alpha_0}{3(x-1)^4} + \frac{2\alpha_0}{3(x-1)^5} + \frac{2}{3(x-1)^4} + \frac{2}{3(x-1)^5} \right) H(c_1(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{112\alpha_0}{3(x-2)^4} - \frac{320\alpha_0}{3(x-2)^5} + \frac{112}{3(x-2)^4} + \frac{544}{3(x-2)^5} + \frac{640}{3(x-2)^6} \right) H(c_2(\alpha_0), c_1(\alpha_0); x) + H(0; x) \left( \frac{47x\alpha_0}{18} + \frac{128\alpha_0}{9(x-2)} - \frac{31\alpha_0}{2(x-1)} - \frac{152\alpha_0}{9(x-2)^2} + \frac{37\alpha_0}{9(x-1)^2} + \frac{16\alpha_0}{(x-2)^3} - \frac{47\alpha_0}{18(x-1)^3} - \frac{1120\alpha_0}{9(x-2)^4} - \frac{49\alpha_0}{36(x-1)^4} - \frac{320\alpha_0}{9(x-2)^5} + \frac{47\alpha_0}{18(x-1)^5} - \frac{320\ln 2\alpha_0}{3(x-2)^4} - \frac{640\ln 2\alpha_0}{3(x-2)^5} - \frac{161\alpha_0}{36} - \frac{47x}{18} + \frac{88}{9(x-2)} - \frac{10}{x-1} - \frac{104}{9(x-2)^2} + \frac{25}{18(x-1)^2} + \frac{160}{9(x-2)^3} - \frac{1}{9(x-1)^3} + \frac{832}{9(x-2)^4} + \frac{139}{36(x-1)^4} + \frac{2560}{9(x-2)^5} + \frac{47}{18(x-1)^5} + \frac{640}{9(x-2)^6} + \frac{320\ln 2}{3(x-2)^4} + \frac{1280\ln 2}{3(x-2)^5} + \frac{1280\ln 2}{3(x-2)^6} - \frac{3}{4} \right) + H(2; x) \left( \frac{800\ln 2\alpha_0}{3(x-2)^4} + \frac{1600\ln 2\alpha_0}{3(x-2)^5} + \left( \frac{800\alpha_0}{3(x-2)^4} + \frac{1600\alpha_0}{3(x-2)^5} - \frac{800}{3(x-2)^4} - \frac{3200}{3(x-2)^5} - \frac{3200}{3(x-2)^6} \right) H(0; \alpha_0) - \frac{800\ln 2}{3(x-2)^4} - \frac{3200\ln 2}{3(x-2)^5} - \frac{3200\ln 2}{3(x-2)^6} \right) - \frac{76\pi^2}{9(x-2)^4} - \frac{\pi^2}{9(x-1)^4} - \frac{232\pi^2}{9(x-2)^5} - \frac{\pi^2}{9(x-1)^5} - \frac{160\pi^2}{9(x-2)^6} - \frac{160\ln^2 2}{3(x-2)^4} - \frac{640\ln^2 2}{3(x-2)^5} - \frac{640\ln^2 2}{3(x-2)^6} - \frac{160\ln 2}{9(x-2)^4} - \frac{640\ln 2}{9(x-2)^5} - \frac{640\ln 2}{9(x-2)^6} \},$$

$$b_2^{(1,2)} = \frac{1}{\alpha_0 x - 2\alpha_0} \left\{ -\frac{1}{72}\pi^2 x \alpha_0^5 + \frac{895\alpha_0^5}{432} - \frac{\pi^2 \alpha_0^5}{36(x-2)} + \frac{673\alpha_0^5}{216(x-2)} + \frac{\pi^2 \alpha_0^5}{72(x-1)} - \frac{895\alpha_0^5}{432(x-1)} - \frac{37\alpha_0^5}{72} + \frac{19}{216}\pi^2 x \alpha_0^4 - \frac{18829x\alpha_0^4}{1296} + \frac{5\pi^2 \alpha_0^4}{54(x-2)} - \frac{2345\alpha_0^4}{324(x-2)} - \frac{13\pi^2 \alpha_0^4}{216(x-1)} + \frac{12073\alpha_0^4}{1296(x-1)} - \frac{5\pi^2 \alpha_0^4}{54(x-2)^2} + \frac{5405\alpha_0^4}{324(x-2)^2} + \frac{\pi^2 \alpha_0^4}{54(x-1)^2} - \frac{1351\alpha_0^4}{324(x-1)^2} - \frac{\pi^2 \alpha_0^4}{108} + \frac{3691\alpha_0^4}{648} - \frac{13\pi^2 x \alpha_0^3}{54} + \frac{7859x\alpha_0^3}{162} - \frac{5\pi^2 \alpha_0^3}{54(x-2)} - \frac{3451\alpha_0^3}{324(x-2)} + \frac{11\pi^2 \alpha_0^3}{108(x-1)} - \frac{7985\alpha_0^3}{648(x-1)} + \frac{5\pi^2 \alpha_0^3}{27(x-2)^2} - \frac{115\alpha_0^3}{81(x-2)^2} - \frac{7\pi^2 \alpha_0^3}{108(x-1)^2} + \frac{22919\alpha_0^3}{1296(x-1)^2} - \frac{10\pi^2 \alpha_0^3}{27(x-2)^3} + \frac{10580\alpha_0^3}{81(x-2)^3} + \frac{\pi^2 \alpha_0^3}{36(x-1)^3} - \frac{2401\alpha_0^3}{216(x-1)^3} + \frac{\pi^2 \alpha_0^3}{18} - \frac{12859\alpha_0^3}{432} + \frac{7\pi^2 x \alpha_0^2}{18} - \frac{7613x\alpha_0^2}{54} + \frac{211\alpha_0^2}{2(x-2)} - \frac{\pi^2 \alpha_0^2}{12(x-1)} - \frac{823\alpha_0^2}{18(x-1)} - \frac{3931\alpha_0^2}{18(x-2)^2} + \frac{\pi^2 \alpha_0^2}{12(x-1)^2} - \frac{685\alpha_0^2}{48(x-1)^2} + \frac{7600\alpha_0^2}{9(x-2)^3} - \frac{\pi^2 \alpha_0^2}{12(x-1)^3} + \frac{2507\alpha_0^2}{36(x-1)^3} - \frac{20\pi^2 \alpha_0^2}{9(x-2)^4} + \frac{66760\alpha_0^2}{27(x-2)^4} + \frac{\pi^2 \alpha_0^2}{18(x-1)^4} - \frac{6685\alpha_0^2}{108(x-1)^4} - \frac{\pi^2 \alpha_0^2}{6} + \frac{22817\alpha_0^2}{144} - \frac{29\pi^2 x \alpha_0}{24} + \frac{16423x\alpha_0}{162} - \frac{10\pi^2 \alpha_0}{9(x-2)} + \frac{65\alpha_0}{3(x-2)} + \frac{409\pi^2 \alpha_0}{216(x-1)} - \frac{3017\alpha_0}{72(x-1)} + \frac{2\pi^2 \alpha_0}{3(x-2)^2} + \frac{13\alpha_0}{(x-2)^2} - \frac{4\pi^2 \alpha_0}{9(x-1)^2} + \frac{1715\alpha_0}{72(x-1)^2} + \frac{8\pi^2 \alpha_0}{3(x-2)^3} - \frac{4936\alpha_0}{9(x-2)^3} + \frac{55\pi^2 \alpha_0}{108(x-1)^3} - \frac{1033\alpha_0}{24(x-1)^3} + \frac{1136\pi^2 \alpha_0}{27(x-2)^4} - \frac{133520\alpha_0}{27(x-2)^4} + \frac{55\pi^2 \alpha_0}{108(x-1)^4} - \frac{6685\alpha_0}{108(x-1)^4} + \frac{400\pi^2 \alpha_0}{27(x-2)^5} - \frac{154240\alpha_0}{27(x-2)^5} - \frac{47\pi^2 \alpha_0}{54(x-1)^5} + \frac{17}{12}x\zeta_3\alpha_0 + \frac{56\zeta_3\alpha_0}{3(x-2)^4} - \frac{7\zeta_3\alpha_0}{4(x-1)^4} + \frac{280\zeta_3\alpha_0}{3(x-2)^5} + \frac{7\zeta_3\alpha_0}{4(x-1)^5} - \frac{17}{6}\zeta_3\alpha_0 + \frac{640\ln^3 2\alpha_0}{9(x-2)^4} + \frac{1280\ln^3 2\alpha_0}{9(x-2)^5} + \frac{320\ln^2 2\alpha_0}{9(x-2)^4} + \frac{640\ln^2 2\alpha_0}{9(x-2)^5} - \frac{1}{6}\pi^2 x \ln 2\alpha_0 + \frac{112\pi^2 \ln 2\alpha_0}{3(x-2)^4} + \frac{320\ln 2\alpha_0}{27(x-2)^4} + \frac{\pi^2 \ln 2\alpha_0}{6(x-1)^4} + \frac{80\pi^2 \ln 2\alpha_0}{3(x-2)^5} + \frac{640\ln 2\alpha_0}{27(x-2)^5} - \frac{\pi^2 \ln 2\alpha_0}{6(x-1)^5} + \frac{1}{3}\pi^2 \ln 2\alpha_0 + \frac{95\pi^2 \alpha_0}{72} + \frac{20569\alpha_0}{648} + \frac{71\pi^2 x}{72} + \frac{575x}{162} + \left( -\frac{37x\alpha_0^5}{18} - \frac{31\alpha_0^5}{9(x-2)} + \frac{37\alpha_0^5}{18(x-1)} + \frac{\alpha_0^5}{3} + \frac{245x\alpha_0^4}{18} + \frac{10\alpha_0^4}{x-2} - \frac{161\alpha_0^4}{18(x-1)} - \frac{130\alpha_0^4}{9(x-2)^2} + \frac{10\alpha_0^4}{3(x-1)^2} - \frac{11\alpha_0^4}{3} - \frac{364x\alpha_0^3}{9} - \frac{26\alpha_0^3}{9(x-2)} + \frac{125\alpha_0^3}{9(x-1)} + \frac{160\alpha_0^3}{9(x-2)^2} - \frac{25\alpha_0^3}{2(x-1)^2} - \frac{80\alpha_0^3}{(x-2)^3} + \frac{59\alpha_0^3}{9(x-1)^3} + \frac{307\alpha_0^3}{18} + \frac{244x\alpha_0^2}{3} - \frac{260\alpha_0^2}{9(x-2)} + \frac{2\alpha_0^2}{3(x-1)} + \frac{172\alpha_0^2}{3(x-2)^2} + \frac{265\alpha_0^2}{18(x-1)^2} - \frac{1600\alpha_0^2}{9(x-2)^3} - \frac{80\alpha_0^2}{3(x-1)^3} - \frac{7520\alpha_0^2}{9(x-2)^4} + \frac{190\alpha_0^2}{9(x-1)^4} - \frac{1091\alpha_0^2}{18} - \frac{1}{18}\pi^2 x \alpha_0 - \frac{7109x\alpha_0}{108} - \frac{1804\alpha_0}{9(x-2)} + \frac{2002\alpha_0}{9(x-1)} + \frac{1936\alpha_0}{9(x-2)^2} - \frac{413\alpha_0}{9(x-1)^2} - \frac{496\alpha_0}{9(x-2)^3} + \frac{227\alpha_0}{6(x-1)^3} - \frac{40\pi^2 \alpha_0}{9(x-2)^4} + \frac{76160\alpha_0}{27(x-2)^4} - \frac{\pi^2 \alpha_0}{18(x-1)^4} + \frac{5381\alpha_0}{216(x-1)^4} - \frac{80\pi^2 \alpha_0}{9(x-2)^5} + \frac{62080\alpha_0}{27(x-2)^5} + \frac{\pi^2 \alpha_0}{18(x-1)^5} - \frac{2125\alpha_0}{108(x-1)^5} + \frac{\pi^2 \alpha_0}{9} - \frac{811\alpha_0}{216} + \right.$$

$$\begin{aligned}
& \frac{\pi^2 x}{18} + \frac{1445x}{108} - \frac{1372}{9(x-2)} + \frac{5705}{36(x-1)} + \frac{512}{3(x-2)^2} - \frac{475}{36(x-1)^2} - \frac{2432}{9(x-2)^3} - \frac{35}{12(x-1)^3} + \frac{40\pi^2}{9(x-2)^4} - \frac{22112}{27(x-2)^4} + \\
& \frac{\pi^2}{18(x-1)^4} - \frac{7679}{216(x-1)^4} + \frac{160\pi^2}{9(x-2)^5} - \frac{62720}{27(x-2)^5} + \frac{\pi^2}{18(x-1)^5} - \frac{2125}{108(x-1)^5} + \frac{160\pi^2}{9(x-2)^6} - \frac{1280}{27(x-2)^6} + \\
& \frac{271}{24} \Big) H(0; \alpha_0) + \left( \frac{37x\alpha_0^5}{12} + \frac{31\alpha_0^5}{6(x-2)} - \frac{37\alpha_0^5}{12(x-1)} - \frac{\alpha_0^5}{2} - \frac{245x\alpha_0^4}{12} - \frac{15\alpha_0^4}{x-2} + \frac{161\alpha_0^4}{12(x-1)} + \frac{65\alpha_0^4}{3(x-2)^2} - \frac{5\alpha_0^4}{(x-1)^2} + \right. \\
& \frac{11\alpha_0^4}{2} + \frac{182x\alpha_0^3}{3} + \frac{13\alpha_0^3}{3(x-2)} - \frac{125\alpha_0^3}{6(x-1)} - \frac{80\alpha_0^3}{3(x-2)^2} + \frac{75\alpha_0^3}{4(x-1)^2} + \frac{120\alpha_0^3}{(x-2)^3} - \frac{59\alpha_0^3}{6(x-1)^3} - \frac{307\alpha_0^3}{12} - 122x\alpha_0^2 + \frac{130\alpha_0^2}{3(x-2)} - \\
& \frac{\alpha_0^2}{x-1} - \frac{86\alpha_0^2}{(x-2)^2} - \frac{265\alpha_0^2}{12(x-1)^2} + \frac{800\alpha_0^2}{3(x-2)^3} + \frac{40\alpha_0^2}{(x-1)^3} + \frac{3760\alpha_0^2}{3(x-2)^4} - \frac{95\alpha_0^2}{3(x-1)^4} + \frac{1091\alpha_0^2}{12} + \frac{513x\alpha_0}{4} - \frac{205\alpha_0}{6(x-2)} - \\
& \frac{25\alpha_0}{12(x-1)} + \frac{344\alpha_0}{3(x-2)^2} + \frac{211\alpha_0}{12(x-1)^2} - \frac{792\alpha_0}{(x-2)^3} - \frac{107\alpha_0}{2(x-1)^3} - \frac{12640\alpha_0}{3(x-2)^4} - \frac{10240\alpha_0}{3(x-2)^5} - \frac{245\alpha_0}{4} - \frac{595}{12}x - \frac{11}{3(x-2)} + \\
& \frac{163}{12(x-1)} - \frac{71}{3(x-2)^2} - \frac{37}{4(x-1)^2} + \frac{1216}{3(x-2)^3} + \frac{70}{3(x-1)^3} + \frac{2960}{(x-2)^4} + \frac{95}{3(x-1)^4} + \frac{10240}{3(x-2)^5} - \frac{109}{12} \Big) H(1; \alpha_0) + \\
& \left( \frac{4x\alpha_0^5}{3} + \frac{8\alpha_0^5}{3(x-2)} - \frac{4\alpha_0^5}{3(x-1)} - \frac{76x\alpha_0^4}{9} - \frac{80\alpha_0^4}{9(x-2)} + \frac{52\alpha_0^4}{9(x-1)} + \frac{80\alpha_0^4}{9(x-2)^2} - \frac{16\alpha_0^4}{9(x-1)^2} + \frac{8\alpha_0^4}{9} + \frac{208x\alpha_0^3}{9} + \frac{80\alpha_0^3}{9(x-2)} - \right. \\
& \frac{88\alpha_0^3}{9(x-1)} - \frac{160\alpha_0^3}{9(x-2)^2} + \frac{56\alpha_0^3}{9(x-1)^2} + \frac{320\alpha_0^3}{9(x-2)^3} - \frac{8\alpha_0^3}{3(x-1)^3} - \frac{16\alpha_0^3}{3} - \frac{112x\alpha_0^2}{3} + \frac{8\alpha_0^2}{x-1} - \frac{8\alpha_0^2}{(x-1)^2} + \frac{8\alpha_0^2}{(x-1)^3} + \\
& \frac{640\alpha_0^2}{3(x-2)^4} - \frac{16\alpha_0^2}{3(x-1)^4} + 16\alpha_0^2 + 22x\alpha_0 + \frac{512\alpha_0}{9(x-2)} - \frac{62\alpha_0}{x-1} - \frac{608\alpha_0}{9(x-2)^2} + \frac{148\alpha_0}{9(x-1)^2} + \frac{64\alpha_0}{(x-2)^3} - \frac{94\alpha_0}{9(x-1)^3} - \\
& \frac{8320\alpha_0}{9(x-2)^4} - \frac{97\alpha_0}{9(x-1)^4} - \frac{8960\alpha_0}{9(x-2)^5} + \frac{94\alpha_0}{9(x-1)^5} + \frac{37\alpha_0}{3} - \frac{2x}{3} + \frac{352}{9(x-2)} - \frac{40}{x-1} - \frac{416}{9(x-2)^2} + \frac{50}{9(x-1)^2} + \frac{640}{9(x-2)^3} - \\
& \frac{4}{9(x-1)^3} + \frac{3328}{9(x-2)^4} + \frac{139}{9(x-1)^4} + \frac{10240}{9(x-2)^5} + \frac{94}{9(x-1)^5} + \frac{2560}{9(x-2)^6} - 3 \Big) H(0, 0; \alpha_0) + \left( -2x\alpha_0^5 - \frac{4\alpha_0^5}{x-2} + \right. \\
& \frac{2\alpha_0^5}{x-1} + \frac{38x\alpha_0^4}{3} + \frac{40\alpha_0^4}{3(x-2)} - \frac{26\alpha_0^4}{3(x-1)} - \frac{40\alpha_0^4}{3(x-2)^2} + \frac{8\alpha_0^4}{3(x-1)^2} - \frac{4\alpha_0^4}{3} - \frac{104x\alpha_0^3}{3} - \frac{40\alpha_0^3}{3(x-2)} + \frac{44\alpha_0^3}{3(x-1)} + \frac{80\alpha_0^3}{3(x-2)^2} - \\
& \frac{28\alpha_0^3}{3(x-1)^2} - \frac{160\alpha_0^3}{3(x-2)^3} + \frac{4\alpha_0^3}{(x-1)^3} + 8\alpha_0^3 + 56x\alpha_0^2 - \frac{12\alpha_0^2}{x-1} + \frac{12\alpha_0^2}{(x-1)^2} - \frac{12\alpha_0^2}{(x-1)^3} - \frac{320\alpha_0^2}{(x-2)^4} + \frac{8\alpha_0^2}{(x-1)^4} - 24\alpha_0^2 - \\
& 33x\alpha_0 - \frac{256\alpha_0}{3(x-2)} + \frac{93\alpha_0}{x-1} + \frac{304\alpha_0}{3(x-2)^2} - \frac{74\alpha_0}{3(x-1)^2} - \frac{96\alpha_0}{(x-2)^3} + \frac{47\alpha_0}{3(x-1)^3} + \frac{4160\alpha_0}{3(x-2)^4} + \frac{97\alpha_0}{6(x-1)^4} + \frac{4480\alpha_0}{3(x-2)^5} - \\
& \frac{47\alpha_0}{3(x-1)^5} - \frac{37\alpha_0}{2} + x - \frac{176}{3(x-2)} + \frac{60}{x-1} + \frac{208}{3(x-2)^2} - \frac{25}{3(x-1)^2} - \frac{320}{3(x-2)^3} + \frac{2}{3(x-1)^3} - \frac{1664}{3(x-2)^4} - \frac{139}{6(x-1)^4} - \\
& \frac{5120}{3(x-2)^5} - \frac{47}{3(x-1)^5} - \frac{1280}{3(x-2)^6} + \frac{9}{2} \Big) H(0, 1; \alpha_0) + H(1; x) \left( \frac{1}{9}\pi^2 x\alpha_0 + \frac{16\pi^2\alpha_0}{(x-2)^4} - \frac{8\pi^2\alpha_0}{9(x-1)^4} + \frac{80\pi^2\alpha_0}{3(x-2)^5} + \right. \\
& \frac{8\pi^2\alpha_0}{9(x-1)^5} - \frac{2\pi^2\alpha_0}{9} - \frac{\pi^2 x}{9} + \left( \frac{47x\alpha_0}{9} + \frac{838\alpha_0}{9(x-2)} - \frac{102\alpha_0}{x-1} - \frac{1000\alpha_0}{9(x-2)^2} + \frac{433\alpha_0}{18(x-1)^2} + \frac{128\alpha_0}{(x-2)^3} - \frac{113\alpha_0}{9(x-1)^3} - \right. \\
& \frac{4288\alpha_0}{9(x-2)^4} - \frac{137\alpha_0}{9(x-1)^4} + \frac{640\alpha_0}{9(x-2)^5} + \frac{188\alpha_0}{9(x-1)^5} - \frac{77\alpha_0}{18} - \frac{47x}{9} + \frac{578}{9(x-2)} - \frac{203}{3(x-1)} - \frac{676}{9(x-2)^2} + \frac{185}{18(x-1)^2} + \\
& \frac{848}{9(x-2)^3} - \frac{11}{9(x-1)^3} + \frac{1984}{9(x-2)^4} + \frac{239}{9(x-1)^4} + \frac{7936}{9(x-2)^5} + \frac{188}{9(x-1)^5} - \frac{1280}{9(x-2)^6} - \frac{37}{6} \Big) H(0; \alpha_0) + \left( -\frac{8x\alpha_0}{3} - \right. \\
& \frac{448\alpha_0}{3(x-2)^4} + \frac{32\alpha_0}{3(x-1)^4} - \frac{1280\alpha_0}{3(x-2)^5} - \frac{32\alpha_0}{3(x-1)^5} + \frac{16\alpha_0}{3} + \frac{8x}{3} + \frac{448}{3(x-2)^4} - \frac{32}{3(x-1)^4} + \frac{2176}{3(x-2)^5} - \frac{32}{3(x-1)^5} + \\
& \left. \frac{2560}{3(x-2)^6} \right) H(0, 0; \alpha_0) + \left( 4x\alpha_0 + \frac{224\alpha_0}{(x-2)^4} - \frac{16\alpha_0}{(x-1)^4} + \frac{640\alpha_0}{(x-2)^5} + \frac{16\alpha_0}{(x-1)^5} - 8\alpha_0 - 4x - \frac{224}{(x-2)^4} + \frac{16}{(x-1)^4} - \right. \\
& \frac{1088}{(x-2)^5} + \frac{16}{(x-1)^5} - \frac{1280}{(x-2)^6} \Big) H(0, 1; \alpha_0) - \frac{16\pi^2}{(x-2)^4} + \frac{8\pi^2}{9(x-1)^4} - \frac{176\pi^2}{3(x-2)^5} + \frac{8\pi^2}{9(x-1)^5} - \frac{160\pi^2}{3(x-2)^6} \Big) + \left( - \right. \\
& \frac{20x\alpha_0}{3} - \frac{1216\alpha_0}{3(x-2)^4} + \frac{20\alpha_0}{3(x-1)^4} - \frac{3200\alpha_0}{3(x-2)^5} - \frac{20\alpha_0}{3(x-1)^5} + \frac{40\alpha_0}{3} + \frac{20x}{3} + \frac{1216}{3(x-2)^4} - \frac{20}{3(x-1)^4} + \frac{5632}{3(x-2)^5} - \\
& \frac{20}{3(x-1)^5} + \frac{6400}{3(x-2)^6} \Big) H(0; \alpha_0) H(0, 1; x) + \left( \frac{\alpha_0^5}{2} + \frac{\alpha_0^4}{3(x-2)} - \frac{5\alpha_0^4}{2} - \frac{4\alpha_0^3}{3(x-2)} + \frac{4\alpha_0^3}{3(x-2)^2} + 5\alpha_0^3 + \frac{2\alpha_0^2}{x-2} - \right. \\
& \frac{4\alpha_0^2}{(x-2)^2} + \frac{8\alpha_0^2}{(x-2)^3} - 5\alpha_0^2 + \frac{47x\alpha_0}{9} + \frac{64\alpha_0}{9(x-2)} - \frac{34\alpha_0}{3(x-1)} - \frac{40\alpha_0}{9(x-2)^2} + \frac{49\alpha_0}{18(x-1)^2} - \frac{16\alpha_0}{(x-2)^3} - \frac{26\alpha_0}{9(x-1)^3} - \frac{1696\alpha_0}{9(x-2)^4} - \\
& \frac{55\alpha_0}{18(x-1)^4} + \frac{640\alpha_0}{9(x-2)^5} + \frac{47\alpha_0}{9(x-1)^5} - \frac{52\alpha_0}{9} - \frac{47x}{9} + \left( -\frac{8x\alpha_0}{3} - \frac{448\alpha_0}{3(x-2)^4} + \frac{8\alpha_0}{3(x-1)^4} - \frac{1280\alpha_0}{3(x-2)^5} - \frac{8\alpha_0}{3(x-1)^5} + \right. \\
& \frac{16\alpha_0}{3} + \frac{8x}{3} + \frac{448}{3(x-2)^4} - \frac{8}{3(x-1)^4} + \frac{2176}{3(x-2)^5} - \frac{8}{3(x-1)^5} + \frac{2560}{3(x-2)^6} \Big) H(0; \alpha_0) + \left( 4x\alpha_0 + \frac{224\alpha_0}{(x-2)^4} - \right. \\
& \frac{4\alpha_0}{(x-1)^4} + \frac{640\alpha_0}{(x-2)^5} + \frac{4\alpha_0}{(x-1)^5} - 8\alpha_0 - 4x - \frac{224}{(x-2)^4} + \frac{4}{(x-1)^4} - \frac{1088}{(x-2)^5} + \frac{4}{(x-1)^5} - \frac{1280}{(x-2)^6} \Big) H(1; \alpha_0) + \\
& \frac{56}{9(x-2)} - \frac{22}{3(x-1)} - \frac{88}{9(x-2)^2} + \frac{23}{18(x-1)^2} + \frac{224}{9(x-2)^3} + \frac{13}{9(x-1)^3} + \frac{1696}{9(x-2)^4} + \frac{133}{18(x-1)^4} + \frac{2176}{9(x-2)^5} + \\
& \frac{47}{9(x-1)^5} - \frac{1280}{9(x-2)^6} - \frac{8}{3} \Big) H(0, c_1(\alpha_0); x) + \left( -\frac{640\alpha_0}{9(x-2)^4} - \frac{1280\alpha_0}{9(x-2)^5} + \left( \frac{1280\alpha_0}{3(x-2)^4} + \frac{2560\alpha_0}{3(x-2)^5} - \frac{1280}{3(x-2)^4} - \right. \right. \\
& \left. \frac{5120}{3(x-2)^5} - \frac{5120}{3(x-2)^6} \right) H(0; \alpha_0) + \left( -\frac{640\alpha_0}{(x-2)^4} - \frac{1280\alpha_0}{(x-2)^5} + \frac{640}{(x-2)^4} + \frac{2560}{(x-2)^5} + \frac{2560}{(x-2)^6} \right) H(1; \alpha_0) + \\
& \frac{640}{9(x-2)^4} + \frac{2560}{9(x-2)^5} + \frac{2560}{9(x-2)^6} \Big) H(0, c_2(\alpha_0); x) + \left( -2x\alpha_0^5 - \frac{4\alpha_0^5}{x-2} + \frac{2\alpha_0^5}{x-1} + \frac{38x\alpha_0^4}{3} + \frac{40\alpha_0^4}{3(x-2)} - \frac{26\alpha_0^4}{3(x-1)} - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{40\alpha_0^4}{3(x-2)^2} + \frac{8\alpha_0^4}{3(x-1)^2} - \frac{4\alpha_0^4}{3} - \frac{104x\alpha_0^3}{3} - \frac{40\alpha_0^3}{3(x-2)} + \frac{44\alpha_0^3}{3(x-1)} + \frac{80\alpha_0^3}{3(x-2)^2} - \frac{28\alpha_0^3}{3(x-1)^2} - \frac{160\alpha_0^3}{3(x-2)^3} + \frac{4\alpha_0^3}{(x-1)^3} + \\
& 8\alpha_0^3 + 56x\alpha_0^2 - \frac{12\alpha_0^2}{x-1} + \frac{12\alpha_0^2}{(x-1)^2} - \frac{12\alpha_0^2}{(x-1)^3} - \frac{320\alpha_0^2}{(x-2)^4} + \frac{8\alpha_0^2}{(x-1)^4} - 24\alpha_0^2 - \frac{146x\alpha_0}{3} + \frac{20\alpha_0}{3(x-2)} + \frac{14\alpha_0}{3(x-1)} - \\
& \frac{80\alpha_0}{3(x-2)^2} - \frac{20\alpha_0}{3(x-1)^2} + \frac{160\alpha_0}{(x-2)^3} + \frac{12\alpha_0}{(x-1)^3} + \frac{1280\alpha_0}{(x-2)^4} + \frac{1280\alpha_0}{(x-2)^5} + \frac{40\alpha_0}{3} + \frac{50x}{3} - \frac{8}{3(x-2)} - \frac{2}{3(x-1)} + \frac{40}{3(x-2)^2} + \\
& \frac{4}{3(x-1)^2} - \frac{320}{3(x-2)^3} - \frac{4}{(x-1)^3} - \frac{960}{(x-2)^4} - \frac{8}{(x-1)^4} - \frac{1280}{(x-2)^5} + 4) H(1, 0; \alpha_0) + \left( -\frac{47x\alpha_0}{9} - \frac{838\alpha_0}{9(x-2)} + \right. \\
& \frac{102\alpha_0}{x-1} + \frac{1000\alpha_0}{9(x-2)^2} - \frac{433\alpha_0}{18(x-1)^2} - \frac{128\alpha_0}{(x-2)^3} + \frac{113\alpha_0}{9(x-1)^3} + \frac{4288\alpha_0}{9(x-2)^4} + \frac{137\alpha_0}{9(x-1)^4} - \frac{640\alpha_0}{9(x-2)^5} - \frac{188\alpha_0}{9(x-1)^5} + \frac{77\alpha_0}{18} + \\
& \frac{47x}{9} - \frac{578}{9(x-2)} + \frac{203}{3(x-1)} + \frac{676}{9(x-2)^2} - \frac{185}{18(x-1)^2} - \frac{848}{9(x-2)^3} + \frac{11}{9(x-1)^3} - \frac{1984}{9(x-2)^4} - \frac{239}{9(x-1)^4} - \frac{7936}{9(x-2)^5} - \\
& \frac{188}{9(x-1)^5} + \frac{1280}{9(x-2)^6} + \frac{37}{6} \left. \right) H(1, 0; x) + \left( 3x\alpha_0^5 + \frac{6\alpha_0^5}{x-2} - \frac{3\alpha_0^5}{x-1} - 19x\alpha_0^4 - \frac{20\alpha_0^4}{x-2} + \frac{13\alpha_0^4}{x-1} + \frac{20\alpha_0^4}{(x-2)^2} - \right. \\
& \frac{4\alpha_0^4}{(x-1)^2} + 2\alpha_0^4 + 52x\alpha_0^3 + \frac{20\alpha_0^3}{x-2} - \frac{22\alpha_0^3}{x-1} - \frac{40\alpha_0^3}{(x-2)^2} + \frac{14\alpha_0^3}{(x-1)^2} + \frac{80\alpha_0^3}{(x-2)^3} - \frac{6\alpha_0^3}{(x-1)^3} - 12\alpha_0^3 - 84x\alpha_0^2 + \frac{18\alpha_0^2}{x-1} - \\
& \frac{18\alpha_0^2}{(x-1)^2} + \frac{18\alpha_0^2}{(x-1)^3} + \frac{480\alpha_0^2}{(x-2)^4} - \frac{12\alpha_0^2}{(x-1)^4} + 36\alpha_0^2 + 73x\alpha_0 - \frac{10\alpha_0}{x-2} - \frac{7\alpha_0}{x-1} + \frac{40\alpha_0}{(x-2)^2} + \frac{10\alpha_0}{(x-1)^2} - \frac{240\alpha_0}{(x-2)^3} - \\
& \frac{18\alpha_0}{(x-1)^3} - \frac{1920\alpha_0}{(x-2)^4} - \frac{1920\alpha_0}{(x-2)^5} - 20\alpha_0 - 25x + \frac{4}{x-2} + \frac{1}{x-1} - \frac{20}{(x-2)^2} - \frac{2}{(x-1)^2} + \frac{160}{(x-2)^3} + \frac{6}{(x-1)^3} + \frac{1440}{(x-2)^4} + \\
& \frac{12}{(x-1)^4} + \frac{1920}{(x-2)^5} - 6) H(1, 1; \alpha_0) + H(c_2(\alpha_0); x) \left( -\frac{40\pi^2\alpha_0}{9(x-2)^4} + \frac{320\alpha_0}{27(x-2)^4} - \frac{80\pi^2\alpha_0}{9(x-2)^5} + \frac{640\alpha_0}{27(x-2)^5} + \right. \\
& \left( -\frac{640\alpha_0}{9(x-2)^4} - \frac{1280\alpha_0}{9(x-2)^5} + \frac{640}{9(x-2)^4} + \frac{2560}{9(x-2)^5} + \frac{2560}{9(x-2)^6} \right) H(0; \alpha_0) + \left( \frac{320\alpha_0}{3(x-2)^4} + \frac{640\alpha_0}{3(x-2)^5} - \frac{320}{3(x-2)^4} - \right. \\
& \frac{1280}{3(x-2)^5} - \frac{1280}{3(x-2)^6} \left. \right) H(1; \alpha_0) + \left( \frac{1280\alpha_0}{3(x-2)^4} + \frac{2560\alpha_0}{3(x-2)^5} - \frac{1280}{3(x-2)^4} - \frac{5120}{3(x-2)^5} - \frac{5120}{3(x-2)^6} \right) H(0, 0; \alpha_0) + \\
& \left( -\frac{640\alpha_0}{(x-2)^4} - \frac{1280\alpha_0}{(x-2)^5} + \frac{640}{(x-2)^4} + \frac{2560}{(x-2)^5} + \frac{2560}{(x-2)^6} \right) H(0, 1; \alpha_0) + \left( -\frac{640\alpha_0}{(x-2)^4} - \frac{1280\alpha_0}{(x-2)^5} + \frac{640}{(x-2)^4} + \right. \\
& \frac{2560}{(x-2)^5} + \frac{2560}{(x-2)^6} \left. \right) H(1, 0; \alpha_0) + \left( \frac{960\alpha_0}{(x-2)^4} + \frac{1920\alpha_0}{(x-2)^5} - \frac{960}{(x-2)^4} - \frac{3840}{(x-2)^5} - \frac{3840}{(x-2)^6} \right) H(1, 1; \alpha_0) + \\
& \frac{40\pi^2}{9(x-2)^4} - \frac{320}{27(x-2)^4} + \frac{160\pi^2}{9(x-2)^5} - \frac{1280}{27(x-2)^5} + \frac{160\pi^2}{9(x-2)^6} - \frac{1280}{27(x-2)^6} \left. \right) + H(c_1(\alpha_0); x) \left( -\frac{37x\alpha_0^5}{36} - \frac{31\alpha_0^5}{18(x-2)} + \right. \\
& \frac{37\alpha_0^5}{36(x-1)} + \frac{9\alpha_0^5}{8} + \frac{245x\alpha_0^4}{36} + \frac{73\alpha_0^4}{12(x-2)} - \frac{151\alpha_0^4}{36(x-1)} - \frac{65\alpha_0^4}{9(x-2)^2} + \frac{5\alpha_0^4}{3(x-1)^2} - \frac{497\alpha_0^4}{72} - \frac{182x\alpha_0^3}{9} - \frac{52\alpha_0^3}{9(x-2)} + \\
& \frac{187\alpha_0^3}{36(x-1)} + \frac{44\alpha_0^3}{3(x-2)^2} - \frac{211\alpha_0^3}{36(x-1)^2} - \frac{40\alpha_0^3}{(x-2)^3} + \frac{59\alpha_0^3}{18(x-1)^3} + \frac{353\alpha_0^3}{18} + \frac{122x\alpha_0^2}{3} - \frac{91\alpha_0^2}{9(x-2)} + \frac{275\alpha_0^2}{36(x-1)} + \frac{34\alpha_0^2}{3(x-2)^2} + \\
& \frac{167\alpha_0^2}{36(x-1)^2} - \frac{268\alpha_0^2}{9(x-2)^3} - \frac{38\alpha_0^2}{3(x-1)^3} - \frac{3760\alpha_0^2}{9(x-2)^4} + \frac{95\alpha_0^2}{9(x-1)^4} - \frac{781\alpha_0^2}{18} - \frac{171x\alpha_0}{4} - \frac{610\alpha_0}{3(x-2)} + \frac{3965\alpha_0}{18(x-1)} + \frac{228\alpha_0}{(x-2)^2} - \\
& \frac{797\alpha_0}{18(x-1)^2} - \frac{1312\alpha_0}{9(x-2)^3} + \frac{34\alpha_0}{(x-1)^3} + \frac{1808\alpha_0}{(x-2)^4} - \frac{\pi^2\alpha_0}{18(x-1)^4} + \frac{3101\alpha_0}{216(x-1)^4} + \frac{10240\alpha_0}{9(x-2)^5} + \frac{\pi^2\alpha_0}{18(x-1)^5} - \frac{2125\alpha_0}{108(x-1)^5} + \\
& \frac{1315\alpha_0}{72} + \frac{595x}{36} + \left( \frac{2x\alpha_0^5}{3} + \frac{4\alpha_0^5}{3(x-2)} - \frac{2\alpha_0^5}{3(x-1)} - \alpha_0^5 - \frac{38x\alpha_0^4}{9} - \frac{46\alpha_0^4}{9(x-2)} + \frac{26\alpha_0^4}{9(x-1)} + \frac{40\alpha_0^4}{9(x-2)^2} - \frac{8\alpha_0^4}{9(x-1)^2} + \frac{49\alpha_0^4}{9} + \right. \\
& \frac{104x\alpha_0^3}{9} + \frac{64\alpha_0^3}{9(x-2)} - \frac{44\alpha_0^3}{9(x-1)} - \frac{104\alpha_0^3}{9(x-2)^2} + \frac{28\alpha_0^3}{9(x-1)^2} + \frac{160\alpha_0^3}{9(x-2)^3} - \frac{4\alpha_0^3}{3(x-1)^3} - \frac{38\alpha_0^3}{3} - \frac{56x\alpha_0^2}{3} - \frac{4\alpha_0^2}{x-2} + \frac{4\alpha_0^2}{x-1} + \\
& \frac{8\alpha_0^2}{(x-2)^2} - \frac{4\alpha_0^2}{(x-1)^2} - \frac{16\alpha_0^2}{(x-2)^3} + \frac{4\alpha_0^2}{(x-1)^3} + \frac{320\alpha_0^2}{3(x-2)^4} - \frac{8\alpha_0^2}{3(x-1)^4} + 18\alpha_0^2 + \frac{146x\alpha_0}{9} + \frac{512\alpha_0}{9(x-2)} - \frac{62\alpha_0}{x-1} - \frac{608\alpha_0}{9(x-2)^2} + \\
& \frac{148\alpha_0}{9(x-1)^2} + \frac{64\alpha_0}{(x-2)^3} - \frac{94\alpha_0}{9(x-1)^3} - \frac{576\alpha_0}{(x-2)^4} - \frac{73\alpha_0}{9(x-1)^4} - \frac{1280\alpha_0}{3(x-2)^5} + \frac{94\alpha_0}{9(x-1)^5} - \frac{61\alpha_0}{9} - \frac{50x}{9} + \frac{352}{9(x-2)} - \frac{40}{x-1} - \\
& \frac{416}{9(x-2)^2} + \frac{50}{9(x-1)^2} + \frac{640}{9(x-2)^3} - \frac{4}{9(x-1)^3} + \frac{896}{3(x-2)^4} + \frac{139}{9(x-1)^4} + \frac{2560}{3(x-2)^5} + \frac{94}{9(x-1)^5} - 3) H(0; \alpha_0) + \left( - \right. \\
& x\alpha_0^5 - \frac{2\alpha_0^5}{x-2} + \frac{\alpha_0^5}{x-1} + \frac{3\alpha_0^5}{2} + \frac{19x\alpha_0^4}{3} + \frac{23\alpha_0^4}{3(x-2)} - \frac{13\alpha_0^4}{3(x-1)} - \frac{20\alpha_0^4}{3(x-2)^2} + \frac{4\alpha_0^4}{3(x-1)^2} - \frac{49\alpha_0^4}{6} - \frac{52x\alpha_0^3}{3} - \frac{32\alpha_0^3}{3(x-2)} + \\
& \frac{22\alpha_0^3}{3(x-1)} + \frac{52\alpha_0^3}{3(x-2)^2} - \frac{14\alpha_0^3}{3(x-1)^2} - \frac{80\alpha_0^3}{3(x-2)^3} + \frac{2\alpha_0^3}{(x-1)^3} + 19\alpha_0^3 + 28x\alpha_0^2 + \frac{6\alpha_0^2}{x-2} - \frac{6\alpha_0^2}{x-1} - \frac{12\alpha_0^2}{(x-2)^2} + \frac{6\alpha_0^2}{(x-1)^2} + \\
& \frac{24\alpha_0^2}{(x-2)^3} - \frac{6\alpha_0^2}{(x-1)^3} - \frac{160\alpha_0^2}{(x-2)^4} + \frac{4\alpha_0^2}{(x-1)^4} - 27\alpha_0^2 - \frac{73x\alpha_0}{3} - \frac{256\alpha_0}{3(x-2)} + \frac{93\alpha_0}{x-1} + \frac{304\alpha_0}{3(x-2)^2} - \frac{74\alpha_0}{3(x-1)^2} - \frac{96\alpha_0}{(x-2)^3} + \\
& \frac{47\alpha_0}{3(x-1)^3} + \frac{864\alpha_0}{(x-2)^4} + \frac{73\alpha_0}{6(x-1)^4} + \frac{640\alpha_0}{(x-2)^5} - \frac{47\alpha_0}{3(x-1)^5} + \frac{61\alpha_0}{6} + \frac{25x}{3} - \frac{176}{3(x-2)} + \frac{60}{x-1} + \frac{208}{3(x-2)^2} - \frac{25}{3(x-1)^2} - \\
& \frac{320}{3(x-2)^3} + \frac{2}{3(x-1)^3} - \frac{448}{(x-2)^4} - \frac{139}{6(x-1)^4} - \frac{1280}{(x-2)^5} - \frac{47}{3(x-1)^5} + \frac{9}{2} \left. \right) H(1; \alpha_0) + \left( \frac{16\alpha_0}{3(x-1)^4} - \frac{16\alpha_0}{3(x-1)^5} - \right. \\
& \frac{16}{3(x-1)^4} - \frac{16}{3(x-1)^5} \left. \right) H(0, 0; \alpha_0) + \left( -\frac{8\alpha_0}{(x-1)^4} + \frac{8\alpha_0}{(x-1)^5} + \frac{8}{(x-1)^4} + \frac{8}{(x-1)^5} \right) H(0, 1; \alpha_0) + \left( -\frac{8\alpha_0}{(x-1)^4} + \right. \\
& \frac{8\alpha_0}{(x-1)^5} + \frac{8}{(x-1)^4} + \frac{8}{(x-1)^5} \left. \right) H(1, 0; \alpha_0) + \left( \frac{12\alpha_0}{(x-1)^4} - \frac{12\alpha_0}{(x-1)^5} - \frac{12}{(x-1)^4} - \frac{12}{(x-1)^5} \right) H(1, 1; \alpha_0) - \\
& \frac{1372}{9(x-2)} + \frac{5705}{36(x-1)} + \frac{512}{3(x-2)^2} - \frac{475}{36(x-1)^2} - \frac{2432}{9(x-2)^3} - \frac{35}{12(x-1)^3} - \frac{7264}{9(x-2)^4} + \frac{\pi^2}{18(x-1)^4} - \frac{7679}{216(x-1)^4} -
\end{aligned}$$

$$\begin{aligned}
& \frac{20480}{9(x-2)^5} + \frac{\pi^2}{18(x-1)^5} - \frac{2125}{108(x-1)^5} + \frac{271}{24} \Big) + \left( -\frac{14x\alpha_0}{3} - \frac{320\alpha_0}{(x-2)^4} + \frac{64\alpha_0}{3(x-1)^4} - \frac{800\alpha_0}{(x-2)^5} - \frac{64\alpha_0}{3(x-1)^5} + \frac{28\alpha_0}{3} + \right. \\
& \frac{14x}{3} + \frac{320}{(x-2)^4} - \frac{64}{3(x-1)^4} + \frac{1440}{(x-2)^5} - \frac{64}{3(x-1)^5} + \frac{1600}{(x-2)^6} \Big) H(0; \alpha_0) H(1, 1; x) + \left( \frac{47x\alpha_0}{9} + \frac{838\alpha_0}{9(x-2)} - \right. \\
& \frac{102\alpha_0}{x-1} - \frac{1000\alpha_0}{9(x-2)^2} + \frac{433\alpha_0}{18(x-1)^2} + \frac{128\alpha_0}{(x-2)^3} - \frac{113\alpha_0}{9(x-1)^3} - \frac{4288\alpha_0}{9(x-2)^4} - \frac{137\alpha_0}{9(x-1)^4} + \frac{640\alpha_0}{9(x-2)^5} + \frac{188\alpha_0}{9(x-1)^5} - \frac{77\alpha_0}{18} - \\
& \frac{47x}{9} + \left( -\frac{8x\alpha_0}{3} - \frac{448\alpha_0}{3(x-2)^4} + \frac{32\alpha_0}{3(x-1)^4} - \frac{1280\alpha_0}{3(x-2)^5} - \frac{32\alpha_0}{3(x-1)^5} + \frac{16\alpha_0}{3} + \frac{8x}{3} + \frac{448}{3(x-2)^4} - \frac{32}{3(x-1)^4} + \frac{2176}{3(x-2)^5} - \right. \\
& \left. \frac{32}{3(x-1)^5} + \frac{2560}{3(x-2)^6} \right) H(0; \alpha_0) + \left( 4x\alpha_0 + \frac{224\alpha_0}{(x-2)^4} - \frac{16\alpha_0}{(x-1)^4} + \frac{640\alpha_0}{(x-2)^5} + \frac{16\alpha_0}{(x-1)^5} - 8\alpha_0 - 4x - \frac{224}{(x-2)^4} + \right. \\
& \left. \frac{16}{(x-1)^4} - \frac{1088}{(x-2)^5} + \frac{16}{(x-1)^5} - \frac{1280}{(x-2)^6} \right) H(1; \alpha_0) + \frac{578}{9(x-2)} - \frac{203}{3(x-1)} - \frac{676}{9(x-2)^2} + \frac{185}{18(x-1)^2} + \frac{848}{9(x-2)^3} - \\
& \frac{11}{9(x-1)^3} + \frac{1984}{9(x-2)^4} + \frac{239}{9(x-1)^4} + \frac{7936}{9(x-2)^5} + \frac{188}{9(x-1)^5} - \frac{1280}{9(x-2)^6} - \frac{37}{6} \Big) H(1, c_1(\alpha_0); x) + \left( \frac{2x\alpha_0}{3} + \right. \\
& \frac{1120\alpha_0}{3(x-2)^4} - \frac{2\alpha_0}{3(x-1)^4} + \frac{3200\alpha_0}{3(x-2)^5} + \frac{2\alpha_0}{3(x-1)^5} - \frac{4\alpha_0}{3} - \frac{2x}{3} - \frac{1120}{3(x-2)^4} + \frac{2}{3(x-1)^4} - \frac{5440}{3(x-2)^5} + \frac{2}{3(x-1)^5} - \\
& \frac{6400}{3(x-2)^6} \Big) H(0; \alpha_0) H(2, 1; x) + \left( \frac{1600\alpha_0}{9(x-2)^4} + \frac{3200\alpha_0}{9(x-2)^5} + \left( -\frac{3200\alpha_0}{3(x-2)^4} - \frac{6400\alpha_0}{3(x-2)^5} + \frac{3200}{3(x-2)^4} + \frac{12800}{3(x-2)^5} + \right. \right. \\
& \left. \left. \frac{12800}{3(x-2)^6} \right) H(0; \alpha_0) + \left( \frac{1600\alpha_0}{(x-2)^4} + \frac{3200\alpha_0}{(x-2)^5} - \frac{1600}{(x-2)^4} - \frac{6400}{(x-2)^5} - \frac{6400}{(x-2)^6} \right) H(1; \alpha_0) - \frac{1600}{9(x-2)^4} - \frac{6400}{9(x-2)^5} - \right. \\
& \frac{6400}{9(x-2)^6} \Big) H(2, c_2(\alpha_0); x) + \left( \frac{x\alpha_0^5}{2} + \frac{5\alpha_0^5}{6(x-2)} - \frac{\alpha_0^5}{3(x-1)} - \frac{3\alpha_0^5}{2} - \frac{19x\alpha_0^4}{6} - \frac{65\alpha_0^4}{18(x-2)} + \frac{13\alpha_0^4}{9(x-1)} + \frac{25\alpha_0^4}{9(x-2)^2} - \right. \\
& \frac{4\alpha_0^4}{9(x-1)^2} + \frac{47\alpha_0^4}{6} + \frac{26x\alpha_0^3}{3} + \frac{55\alpha_0^3}{9(x-2)} - \frac{22\alpha_0^3}{9(x-1)} - \frac{80\alpha_0^3}{9(x-2)^2} + \frac{14\alpha_0^3}{9(x-1)^2} + \frac{100\alpha_0^3}{9(x-2)^3} - \frac{2\alpha_0^3}{3(x-1)^3} - 17\alpha_0^3 - \\
& 14x\alpha_0^2 - \frac{5\alpha_0^2}{x-2} + \frac{2\alpha_0^2}{x-1} + \frac{10\alpha_0^2}{(x-2)^2} - \frac{2\alpha_0^2}{(x-1)^2} - \frac{20\alpha_0^2}{(x-2)^3} + \frac{2\alpha_0^2}{(x-1)^3} + \frac{200\alpha_0^2}{3(x-2)^4} - \frac{4\alpha_0^2}{3(x-1)^4} + 21\alpha_0^2 + \frac{73x\alpha_0}{6} + \\
& \frac{440\alpha_0}{9(x-2)} - \frac{913\alpha_0}{18(x-1)} - \frac{560\alpha_0}{9(x-2)^2} + \frac{245\alpha_0}{18(x-1)^2} + \frac{80\alpha_0}{(x-2)^3} - \frac{71\alpha_0}{9(x-1)^3} - \frac{320\alpha_0}{(x-2)^4} - \frac{55\alpha_0}{18(x-1)^4} - \frac{800\alpha_0}{3(x-2)^5} + \frac{47\alpha_0}{9(x-1)^5} - \\
& \frac{53\alpha_0}{6} - \frac{25x}{6} + \left( \frac{8\alpha_0}{3(x-1)^4} - \frac{8\alpha_0}{3(x-1)^5} - \frac{8}{3(x-1)^4} - \frac{8}{3(x-1)^5} \right) H(0; \alpha_0) + \left( -\frac{4\alpha_0}{(x-1)^4} + \frac{4\alpha_0}{(x-1)^5} + \frac{4}{(x-1)^4} + \right. \\
& \left. \frac{4}{(x-1)^5} \right) H(1; \alpha_0) + \frac{280}{9(x-2)} - \frac{581}{18(x-1)} - \frac{320}{9(x-2)^2} + \frac{29}{6(x-1)^2} + \frac{400}{9(x-2)^3} - \frac{8}{9(x-1)^3} + \frac{320}{3(x-2)^4} + \frac{157}{18(x-1)^4} + \\
& \frac{1600}{3(x-2)^5} + \frac{47}{9(x-1)^5} - \frac{3}{2} \Big) H(c_1(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{x\alpha_0^5}{6} - \frac{\alpha_0^5}{6(x-1)} + \frac{\alpha_0^5}{2} + \frac{19x\alpha_0^4}{18} + \frac{13\alpha_0^4}{18(x-1)} - \frac{2\alpha_0^4}{9(x-1)^2} - \right. \\
& \frac{47\alpha_0^4}{18} - \frac{26x\alpha_0^3}{9} - \frac{11\alpha_0^3}{9(x-1)} + \frac{7\alpha_0^3}{9(x-1)^2} - \frac{\alpha_0^3}{3(x-1)^3} + \frac{17\alpha_0^3}{3} + \frac{14x\alpha_0^2}{3} + \frac{\alpha_0^2}{x-1} - \frac{\alpha_0^2}{(x-1)^2} + \frac{\alpha_0^2}{(x-1)^3} - \frac{2\alpha_0^2}{3(x-1)^4} - 7\alpha_0^2 - \\
& \frac{73x\alpha_0}{18} + \frac{8\alpha_0}{9(x-2)} + \frac{\alpha_0}{18(x-1)} - \frac{8\alpha_0}{9(x-2)^2} + \frac{\alpha_0}{9(x-1)^2} + \frac{\alpha_0}{3(x-1)^3} - \frac{32\alpha_0}{9(x-2)^4} - \frac{4\alpha_0}{3(x-1)^4} - \frac{640\alpha_0}{9(x-2)^5} + \frac{41\alpha_0}{18} + \\
& \frac{25x}{18} + \left( \frac{448\alpha_0}{3(x-2)^4} + \frac{1280\alpha_0}{3(x-2)^5} - \frac{448}{3(x-2)^4} - \frac{2176}{3(x-2)^5} - \frac{2560}{3(x-2)^6} \right) H(0; \alpha_0) + \left( -\frac{224\alpha_0}{(x-2)^4} - \frac{640\alpha_0}{(x-2)^5} + \frac{224}{(x-2)^4} + \right. \\
& \left. \frac{1088}{(x-2)^5} + \frac{1280}{(x-2)^6} \right) H(1; \alpha_0) + \frac{16}{9(x-2)} - \frac{7}{18(x-1)} - \frac{8}{9(x-2)^2} - \frac{5}{9(x-1)^2} + \frac{16}{9(x-2)^3} - \frac{1}{(x-1)^3} + \frac{32}{9(x-2)^4} - \\
& \frac{2}{3(x-1)^4} + \frac{704}{9(x-2)^5} + \frac{1280}{9(x-2)^6} + \frac{7}{6} \Big) H(c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{16x\alpha_0}{3} + \frac{1280\alpha_0}{3(x-2)^4} + \frac{16\alpha_0}{3(x-1)^4} + \frac{2560\alpha_0}{3(x-2)^5} - \right. \\
& \frac{16\alpha_0}{3(x-1)^5} - \frac{32\alpha_0}{3} - \frac{16x}{3} - \frac{1280}{3(x-2)^4} - \frac{16}{3(x-1)^4} - \frac{5120}{3(x-2)^5} - \frac{16}{3(x-1)^5} - \frac{5120}{3(x-2)^6} \Big) H(0, 0, 0; \alpha_0) + \left( \frac{16x\alpha_0}{3} - \right. \\
& \frac{1280\alpha_0}{3(x-2)^4} - \frac{16\alpha_0}{3(x-1)^4} - \frac{2560\alpha_0}{3(x-2)^5} + \frac{16\alpha_0}{3(x-1)^5} - \frac{32\alpha_0}{3} - \frac{16x}{3} + \frac{1280}{3(x-2)^4} + \frac{16}{3(x-1)^4} + \frac{5120}{3(x-2)^5} + \frac{16}{3(x-1)^5} + \\
& \frac{5120}{3(x-2)^6} \Big) H(0, 0, 0; x) + \left( -8x\alpha_0 - \frac{640\alpha_0}{(x-2)^4} - \frac{8\alpha_0}{(x-1)^4} - \frac{1280\alpha_0}{(x-2)^5} + \frac{8\alpha_0}{(x-1)^5} + 16\alpha_0 + 8x + \frac{640}{(x-2)^4} + \right. \\
& \frac{8}{(x-1)^4} + \frac{2560}{(x-2)^5} + \frac{8}{(x-1)^5} + \frac{2560}{(x-2)^6} \Big) H(0, 0, 1; \alpha_0) + \left( -\frac{8x\alpha_0}{3} - \frac{544\alpha_0}{3(x-2)^4} + \frac{8\alpha_0}{3(x-1)^4} - \frac{1280\alpha_0}{3(x-2)^5} - \right. \\
& \frac{8\alpha_0}{3(x-1)^5} + \frac{16\alpha_0}{3} + \frac{8x}{3} + \frac{544}{3(x-2)^4} - \frac{8}{3(x-1)^4} + \frac{2368}{3(x-2)^5} - \frac{8}{3(x-1)^5} + \frac{2560}{3(x-2)^6} \Big) H(0, 0, c_1(\alpha_0); x) + \\
& \left( \frac{1280\alpha_0}{3(x-2)^4} + \frac{2560\alpha_0}{3(x-2)^5} - \frac{1280}{3(x-2)^4} - \frac{5120}{3(x-2)^5} - \frac{5120}{3(x-2)^6} \right) H(0, 0, c_2(\alpha_0); x) + \left( -8x\alpha_0 - \frac{640\alpha_0}{(x-2)^4} - \right. \\
& \frac{8\alpha_0}{(x-1)^4} - \frac{1280\alpha_0}{(x-2)^5} + \frac{8\alpha_0}{(x-1)^5} + 16\alpha_0 + 8x + \frac{640}{(x-2)^4} + \frac{8}{(x-1)^4} + \frac{2560}{(x-2)^5} + \frac{8}{(x-1)^5} + \\
& \left. \frac{2560}{(x-2)^6} \right) H(0, 1, 0; \alpha_0) + \left( \frac{20x\alpha_0}{3} + \frac{1216\alpha_0}{3(x-2)^4} - \frac{20\alpha_0}{3(x-1)^4} + \frac{3200\alpha_0}{3(x-2)^5} + \frac{20\alpha_0}{3(x-1)^5} - \frac{40\alpha_0}{3} - \frac{20x}{3} - \frac{1216}{3(x-2)^4} + \right. \\
& \frac{20}{3(x-1)^4} - \frac{5632}{3(x-2)^5} + \frac{20}{3(x-1)^5} - \frac{6400}{3(x-2)^6} \Big) H(0, 1, 0; x) + \left( 12x\alpha_0 + \frac{960\alpha_0}{(x-2)^4} + \frac{12\alpha_0}{(x-1)^4} + \frac{1920\alpha_0}{(x-2)^5} - \right. \\
& \left. \frac{12\alpha_0}{(x-1)^5} - 24\alpha_0 - 12x - \frac{960}{(x-2)^4} - \frac{12}{(x-1)^4} - \frac{3840}{(x-2)^5} - \frac{12}{(x-1)^5} - \frac{3840}{(x-2)^6} \right) H(0, 1, 1; \alpha_0) +
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{20x\alpha_0}{3} - \frac{1216\alpha_0}{3(x-2)^4} + \frac{20\alpha_0}{3(x-1)^4} - \frac{3200\alpha_0}{3(x-2)^5} - \frac{20\alpha_0}{3(x-1)^5} + \frac{40\alpha_0}{3} + \frac{20x}{3} + \frac{1216}{3(x-2)^4} - \frac{20}{3(x-1)^4} + \right. \\
& \left. \frac{5632}{3(x-2)^5} - \frac{20}{3(x-1)^5} + \frac{6400}{3(x-2)^6} \right) H(0, 1, c_1(\alpha_0); x) + \left( \frac{3200\alpha_0}{3(x-2)^4} + \frac{6400\alpha_0}{3(x-2)^5} - \frac{3200}{3(x-2)^4} - \frac{12800}{3(x-2)^5} - \right. \\
& \left. \frac{12800}{3(x-2)^6} \right) H(0, 2, 0; x) + \left( -\frac{3200\alpha_0}{3(x-2)^4} - \frac{6400\alpha_0}{3(x-2)^5} + \frac{3200}{3(x-2)^4} + \frac{12800}{3(x-2)^5} + \frac{12800}{3(x-2)^6} \right) H(0, 2, c_2(\alpha_0); x) + \\
& \left( -2x\alpha_0 - \frac{160\alpha_0}{3(x-2)^4} + \frac{4\alpha_0}{3(x-1)^4} - \frac{800\alpha_0}{3(x-2)^5} - \frac{4\alpha_0}{3(x-1)^5} + 4\alpha_0 + 2x + \frac{160}{3(x-2)^4} - \frac{4}{3(x-1)^4} + \frac{1120}{3(x-2)^5} - \right. \\
& \left. \frac{4}{3(x-1)^5} + \frac{1600}{3(x-2)^6} \right) H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{2x\alpha_0}{3} + \frac{448\alpha_0}{3(x-2)^4} - \frac{2\alpha_0}{3(x-1)^4} + \frac{1280\alpha_0}{3(x-2)^5} + \frac{2\alpha_0}{3(x-1)^5} - \right. \\
& \left. \frac{4\alpha_0}{3} - \frac{2x}{3} - \frac{448}{3(x-2)^4} + \frac{2}{3(x-1)^4} - \frac{2176}{3(x-2)^5} + \frac{2}{3(x-1)^5} - \frac{2560}{3(x-2)^6} \right) H(0, c_2(\alpha_0), c_1(\alpha_0); x) + \\
& \left( \frac{8x\alpha_0}{3} + \frac{448\alpha_0}{3(x-2)^4} - \frac{32\alpha_0}{3(x-1)^4} + \frac{1280\alpha_0}{3(x-2)^5} + \frac{32\alpha_0}{3(x-1)^5} - \frac{16\alpha_0}{3} - \frac{8x}{3} - \frac{448}{3(x-2)^4} + \frac{32}{3(x-1)^4} - \frac{2176}{3(x-2)^5} + \right. \\
& \left. \frac{32}{3(x-1)^5} - \frac{2560}{3(x-2)^6} \right) H(1, 0, 0; x) + \left( -\frac{2x\alpha_0}{3} - \frac{96\alpha_0}{(x-2)^4} + \frac{16\alpha_0}{3(x-1)^4} - \frac{160\alpha_0}{(x-2)^5} - \frac{16\alpha_0}{3(x-1)^5} + \frac{4\alpha_0}{3} + \frac{2x}{3} + \right. \\
& \left. \frac{96}{(x-2)^4} - \frac{16}{3(x-1)^4} + \frac{352}{(x-2)^5} - \frac{16}{3(x-1)^5} + \frac{320}{(x-2)^6} \right) H(1, 0, c_1(\alpha_0); x) + \left( \frac{14x\alpha_0}{3} + \frac{320\alpha_0}{(x-2)^4} - \frac{64\alpha_0}{3(x-1)^4} + \right. \\
& \left. \frac{800\alpha_0}{(x-2)^5} + \frac{64\alpha_0}{3(x-1)^5} - \frac{28\alpha_0}{3} - \frac{14x}{3} - \frac{320}{(x-2)^4} + \frac{64}{3(x-1)^4} - \frac{1440}{(x-2)^5} + \frac{64}{3(x-1)^5} - \frac{1600}{(x-2)^6} \right) H(1, 1, 0; x) + \\
& \left( -\frac{14x\alpha_0}{3} - \frac{320\alpha_0}{(x-2)^4} + \frac{64\alpha_0}{3(x-1)^4} - \frac{800\alpha_0}{(x-2)^5} - \frac{64\alpha_0}{3(x-1)^5} + \frac{28\alpha_0}{3} + \frac{14x}{3} + \frac{320}{(x-2)^4} - \frac{64}{3(x-1)^4} + \frac{1440}{(x-2)^5} - \right. \\
& \left. \frac{64}{3(x-1)^5} + \frac{1600}{(x-2)^6} \right) H(1, 1, c_1(\alpha_0); x) + \left( -2x\alpha_0 - \frac{160\alpha_0}{3(x-2)^4} + \frac{16\alpha_0}{3(x-1)^4} - \frac{800\alpha_0}{3(x-2)^5} - \frac{16\alpha_0}{3(x-1)^5} + 4\alpha_0 + \right. \\
& \left. 2x + \frac{160}{3(x-2)^4} - \frac{16}{3(x-1)^4} + \frac{1120}{3(x-2)^5} - \frac{16}{3(x-1)^5} + \frac{1600}{3(x-2)^6} \right) H(1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{3200\alpha_0}{3(x-2)^4} + \right. \\
& \left. \frac{6400\alpha_0}{3(x-2)^5} - \frac{3200}{3(x-2)^4} - \frac{12800}{3(x-2)^5} - \frac{12800}{3(x-2)^6} \right) H(2, 0, 0; x) + \left( \frac{2x\alpha_0}{3} + \frac{1120\alpha_0}{3(x-2)^4} - \frac{2\alpha_0}{3(x-1)^4} + \frac{3200\alpha_0}{3(x-2)^5} + \right. \\
& \left. \frac{2\alpha_0}{3(x-1)^5} - \frac{4\alpha_0}{3} - \frac{2x}{3} - \frac{1120}{3(x-2)^4} + \frac{2}{3(x-1)^4} - \frac{5440}{3(x-2)^5} + \frac{2}{3(x-1)^5} - \frac{6400}{3(x-2)^6} \right) H(2, 0, c_1(\alpha_0); x) + \\
& \left( -\frac{3200\alpha_0}{3(x-2)^4} - \frac{6400\alpha_0}{3(x-2)^5} + \frac{3200}{3(x-2)^4} + \frac{12800}{3(x-2)^5} + \frac{12800}{3(x-2)^6} \right) H(2, 0, c_2(\alpha_0); x) + \left( -\frac{2x\alpha_0}{3} - \right. \\
& \left. \frac{1120\alpha_0}{3(x-2)^4} + \frac{2\alpha_0}{3(x-1)^4} - \frac{3200\alpha_0}{3(x-2)^5} - \frac{2\alpha_0}{3(x-1)^5} + \frac{4\alpha_0}{3} + \frac{2x}{3} + \frac{1120}{3(x-2)^4} - \frac{2}{3(x-1)^4} + \frac{5440}{3(x-2)^5} - \frac{2}{3(x-1)^5} + \right. \\
& \left. \frac{6400}{3(x-2)^6} \right) H(2, 1, 0; x) + \left( \frac{2x\alpha_0}{3} + \frac{1120\alpha_0}{3(x-2)^4} - \frac{2\alpha_0}{3(x-1)^4} + \frac{3200\alpha_0}{3(x-2)^5} + \frac{2\alpha_0}{3(x-1)^5} - \frac{4\alpha_0}{3} - \frac{2x}{3} - \frac{1120}{3(x-2)^4} + \right. \\
& \left. \frac{2}{3(x-1)^4} - \frac{5440}{3(x-2)^5} + \frac{2}{3(x-1)^5} - \frac{6400}{3(x-2)^6} \right) H(2, 1, c_1(\alpha_0); x) + \left( -\frac{8000\alpha_0}{3(x-2)^4} - \frac{16000\alpha_0}{3(x-2)^5} + \right. \\
& \left. \frac{8000}{3(x-2)^4} + \frac{32000}{3(x-2)^5} + \frac{32000}{3(x-2)^6} \right) H(2, 2, 0; x) + \left( \frac{8000\alpha_0}{3(x-2)^4} + \frac{16000\alpha_0}{3(x-2)^5} - \frac{8000}{3(x-2)^4} - \frac{32000}{3(x-2)^5} - \right. \\
& \left. \frac{32000}{3(x-2)^6} \right) H(2, 2, c_2(\alpha_0); x) + \left( -\frac{2x\alpha_0}{3} - \frac{1120\alpha_0}{3(x-2)^4} + \frac{2\alpha_0}{3(x-1)^4} - \frac{3200\alpha_0}{3(x-2)^5} - \frac{2\alpha_0}{3(x-1)^5} + \frac{4\alpha_0}{3} + \frac{2x}{3} + \right. \\
& \left. \frac{1120}{3(x-2)^4} - \frac{2}{3(x-1)^4} + \frac{5440}{3(x-2)^5} - \frac{2}{3(x-1)^5} + \frac{6400}{3(x-2)^6} \right) H(2, c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{4\alpha_0}{3(x-1)^4} - \right. \\
& \left. \frac{4\alpha_0}{3(x-1)^5} - \frac{4}{3(x-1)^4} - \frac{4}{3(x-1)^5} \right) H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{2\alpha_0}{3(x-1)^4} - \frac{2\alpha_0}{3(x-1)^5} - \frac{2}{3(x-1)^4} - \right. \\
& \left. \frac{2}{3(x-1)^5} \right) H(c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{32\alpha_0}{(x-2)^4} - \frac{32}{(x-2)^4} - \frac{64}{(x-2)^5} \right) H(c_2(\alpha_0), 0, c_1(\alpha_0); x) + \\
& \left( \frac{160\alpha_0}{3(x-2)^4} + \frac{800\alpha_0}{3(x-2)^5} - \frac{160}{3(x-2)^4} - \frac{1120}{3(x-2)^5} - \frac{1600}{3(x-2)^6} \right) H(c_2(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + H(2, 0; x) \left( -\right. \\
& \left. \frac{1600\alpha_0}{9(x-2)^4} - \frac{3200\alpha_0}{9(x-2)^5} - \frac{3200 \ln 2 \alpha_0}{3(x-2)^4} - \frac{6400 \ln 2 \alpha_0}{3(x-2)^5} + \frac{1600}{9(x-2)^4} + \frac{6400}{9(x-2)^5} + \frac{6400}{9(x-2)^6} + \frac{3200 \ln 2}{3(x-2)^4} + \frac{12800 \ln 2}{3(x-2)^5} + \right. \\
& \left. \frac{12800 \ln 2}{3(x-2)^6} \right) + H(0, 2; x) \left( -\frac{3200 \ln 2 \alpha_0}{3(x-2)^4} - \frac{6400 \ln 2 \alpha_0}{3(x-2)^5} + \left( -\frac{3200\alpha_0}{3(x-2)^4} - \frac{6400\alpha_0}{3(x-2)^5} + \frac{3200}{3(x-2)^4} + \frac{12800}{3(x-2)^5} + \right. \right. \\
& \left. \left. \frac{12800}{3(x-2)^6} \right) H(0; \alpha_0) + \frac{3200 \ln 2}{3(x-2)^4} + \frac{12800 \ln 2}{3(x-2)^5} + \frac{12800 \ln 2}{3(x-2)^6} \right) + H(0, 0; x) \left( -\frac{94x\alpha_0}{9} - \frac{512\alpha_0}{9(x-2)} + \frac{62\alpha_0}{x-1} + \right. \\
& \left. \frac{608\alpha_0}{9(x-2)^2} - \frac{148\alpha_0}{9(x-1)^2} - \frac{64\alpha_0}{(x-2)^3} + \frac{94\alpha_0}{9(x-1)^3} + \frac{4480\alpha_0}{9(x-2)^4} + \frac{49\alpha_0}{9(x-1)^4} + \frac{1280\alpha_0}{9(x-2)^5} - \frac{94\alpha_0}{9(x-1)^5} + \frac{1280 \ln 2 \alpha_0}{3(x-2)^4} + \right. \\
& \left. \frac{2560 \ln 2 \alpha_0}{3(x-2)^5} + \frac{161}{9} + \frac{94x}{9} - \frac{352}{9(x-2)} + \frac{40}{x-1} + \frac{416}{9(x-2)^2} - \frac{50}{9(x-1)^2} - \frac{640}{9(x-2)^3} + \frac{4}{9(x-1)^3} - \frac{3328}{9(x-2)^4} - \right. \\
& \left. \frac{139}{9(x-1)^4} - \frac{10240}{9(x-2)^5} - \frac{94}{9(x-1)^5} - \frac{2560}{9(x-2)^6} - \frac{1280 \ln 2}{3(x-2)^4} - \frac{5120 \ln 2}{3(x-2)^5} - \frac{5120 \ln 2}{3(x-2)^6} + 3 \right) + H(2, 2; x) \left( \frac{8000 \ln 2 \alpha_0}{3(x-2)^4} + \right.
\end{aligned}$$



$$\begin{aligned}
 & \frac{16000 \ln 2 \alpha_0}{3(x-2)^5} + \left( \frac{8000 \alpha_0}{3(x-2)^4} + \frac{16000 \alpha_0}{3(x-2)^5} - \frac{8000}{3(x-2)^4} - \frac{32000}{3(x-2)^5} - \frac{32000}{3(x-2)^6} \right) H(0; \alpha_0) - \frac{8000 \ln 2}{3(x-2)^4} - \\
 & \frac{32000 \ln 2}{3(x-2)^5} - \frac{32000 \ln 2}{3(x-2)^6} \Big) + H(0; x) \left( \frac{1}{2} \pi^2 x \alpha_0 + \frac{2125 x \alpha_0}{108} + \frac{1748 \alpha_0}{9(x-2)} - \frac{1897 \alpha_0}{9(x-1)} - \frac{1960 \alpha_0}{9(x-2)^2} + \frac{352 \alpha_0}{9(x-1)^2} + \right. \\
 & \frac{496 \alpha_0}{3(x-2)^3} - \frac{173 \alpha_0}{6(x-1)^3} - \frac{88 \pi^2 \alpha_0}{3(x-2)^4} - \frac{31040 \alpha_0}{27(x-2)^4} - \frac{7 \pi^2 \alpha_0}{18(x-1)^4} - \frac{821 \alpha_0}{216(x-1)^4} - \frac{80 \pi^2 \alpha_0}{3(x-2)^5} - \frac{640 \alpha_0}{27(x-2)^5} + \frac{7 \pi^2 \alpha_0}{18(x-1)^5} + \\
 & \frac{2125 \alpha_0}{108(x-1)^5} - \frac{640 \ln^2 2 \alpha_0}{3(x-2)^4} - \frac{1280 \ln^2 2 \alpha_0}{3(x-2)^5} - \frac{640 \ln 2 \alpha_0}{9(x-2)^4} - \frac{1280 \ln 2 \alpha_0}{9(x-2)^5} - \pi^2 \alpha_0 - \frac{6061 \alpha_0}{216} - \frac{\pi^2 x}{2} - \frac{2125 x}{108} + \\
 & \frac{1372}{9(x-2)} - \frac{5705}{36(x-1)} - \frac{512}{3(x-2)^2} + \frac{475}{36(x-1)^2} + \frac{2432}{9(x-2)^3} + \frac{35}{12(x-1)^3} + \frac{88 \pi^2}{3(x-2)^4} + \frac{22112}{27(x-2)^4} + \frac{7 \pi^2}{18(x-1)^4} + \\
 & \frac{7679}{216(x-1)^4} + \frac{256 \pi^2}{3(x-2)^5} + \frac{62720}{27(x-2)^5} + \frac{7 \pi^2}{18(x-1)^5} + \frac{2125}{108(x-1)^5} + \frac{160 \pi^2}{3(x-2)^6} + \frac{1280}{27(x-2)^6} + \frac{640 \ln^2 2}{3(x-2)^4} + \\
 & \frac{2560 \ln^2 2}{3(x-2)^5} + \frac{2560 \ln^2 2}{3(x-2)^6} + \frac{640 \ln 2}{9(x-2)^4} + \frac{2560 \ln 2}{9(x-2)^5} + \frac{2560 \ln 2}{9(x-2)^6} - \frac{271}{24} \Big) + H(2; x) \left( -\frac{1}{6} \pi^2 x \alpha_0 + \frac{760 \pi^2 \alpha_0}{9(x-2)^4} + \right. \\
 & \frac{\pi^2 \alpha_0}{6(x-1)^4} + \frac{800 \pi^2 \alpha_0}{9(x-2)^5} - \frac{\pi^2 \alpha_0}{6(x-1)^5} + \frac{1600 \ln^2 2 \alpha_0}{3(x-2)^4} + \frac{3200 \ln^2 2 \alpha_0}{3(x-2)^5} + \frac{1600 \ln 2 \alpha_0}{9(x-2)^4} + \frac{3200 \ln 2 \alpha_0}{9(x-2)^5} + \frac{\pi^2 \alpha_0}{3} + \frac{\pi^2 x}{6} + \\
 & \left. \left( \frac{1600 \alpha_0}{9(x-2)^4} + \frac{3200 \alpha_0}{9(x-2)^5} - \frac{1600}{9(x-2)^4} - \frac{6400}{9(x-2)^5} - \frac{6400}{9(x-2)^6} \right) H(0; \alpha_0) + \left( -\frac{3200 \alpha_0}{3(x-2)^4} - \frac{6400 \alpha_0}{3(x-2)^5} + \frac{3200}{3(x-2)^4} + \right. \right. \\
 & \frac{12800}{3(x-2)^5} + \left. \frac{12800}{3(x-2)^6} \right) H(0, 0; \alpha_0) + \left( \frac{1600 \alpha_0}{(x-2)^4} + \frac{3200 \alpha_0}{(x-2)^5} - \frac{1600}{(x-2)^4} - \frac{6400}{(x-2)^5} - \frac{6400}{(x-2)^6} \right) H(0, 1; \alpha_0) - \\
 & \frac{760 \pi^2}{9(x-2)^4} - \frac{\pi^2}{6(x-1)^4} - \frac{2320 \pi^2}{9(x-2)^5} - \frac{\pi^2}{6(x-1)^5} - \frac{1600 \pi^2}{9(x-2)^6} - \frac{1600 \ln^2 2}{3(x-2)^4} - \frac{6400 \ln^2 2}{3(x-2)^5} - \frac{6400 \ln^2 2}{3(x-2)^6} - \frac{1600 \ln 2}{9(x-2)^4} - \\
 & \frac{6400 \ln 2}{9(x-2)^5} - \frac{6400 \ln 2}{9(x-2)^6} \Big) - \frac{8 \pi^2}{9(x-2)} + \frac{257 \pi^2}{216(x-1)} + \frac{14 \pi^2}{9(x-2)^2} - \frac{7 \pi^2}{27(x-1)^2} - \frac{4 \pi^2}{(x-2)^3} - \frac{35 \pi^2}{108(x-1)^3} - \frac{1160 \pi^2}{27(x-2)^4} - \\
 & \frac{139 \pi^2}{108(x-1)^4} - \frac{2192 \pi^2}{27(x-2)^5} - \frac{47 \pi^2}{54(x-1)^5} - \frac{320 \pi^2}{27(x-2)^6} - \frac{17}{12} x \zeta_3 - \frac{56 \zeta_3}{3(x-2)^4} + \frac{7 \zeta_3}{4(x-1)^4} - \frac{392 \zeta_3}{3(x-2)^5} + \frac{7 \zeta_3}{4(x-1)^5} - \\
 & \frac{560 \zeta_3}{3(x-2)^6} - \frac{640 \ln^3 2}{9(x-2)^4} - \frac{2560 \ln^3 2}{9(x-2)^5} - \frac{2560 \ln^3 2}{9(x-2)^6} - \frac{320 \ln^2 2}{9(x-2)^4} - \frac{1280 \ln^2 2}{9(x-2)^5} - \frac{1280 \ln^2 2}{9(x-2)^6} + \frac{1}{6} \pi^2 x \ln 2 - \\
 & \frac{112 \pi^2 \ln 2}{3(x-2)^4} - \frac{320 \ln 2}{27(x-2)^4} - \frac{\pi^2 \ln 2}{6(x-1)^4} - \frac{304 \pi^2 \ln 2}{3(x-2)^5} - \frac{1280 \ln 2}{27(x-2)^5} - \frac{\pi^2 \ln 2}{6(x-1)^5} - \frac{160 \pi^2 \ln 2}{3(x-2)^6} - \frac{1280 \ln 2}{27(x-2)^6} + \frac{13 \pi^2}{24} \Big\}.
 \end{aligned}$$

### E.8 The $\mathcal{B}$ integral for $k = -1$ and $\delta = 1$

The  $\varepsilon$  expansion for this integral reads

$$\begin{aligned}
 \mathcal{I}(x, \varepsilon; \alpha_0, 3 + d_1 \varepsilon; 1, -1, 1, g_B) &= x \mathcal{B}(\varepsilon, x; 3 + d_1 \varepsilon; 1, -1) \\
 &= \frac{1}{\varepsilon^2} b_{-2}^{(1,-1)} + \frac{1}{\varepsilon} b_{-1}^{(1,-1)} + b_0^{(1,-1)} + \varepsilon b_1^{(1,-1)} + \varepsilon^2 b_2^{(1,-1)} + \mathcal{O}(\varepsilon^3), \tag{E.8}
 \end{aligned}$$

where

$$\begin{aligned}
 b_{-2}^{(1,-1)} &= \frac{1}{8}, \\
 b_{-1}^{(1,-1)} &= -H(0; x), \\
 b_0^{(1,-1)} &= \frac{\alpha_0^3}{12(x-1)^2} - \frac{\alpha_0^3}{12} + \frac{\alpha_0^2}{24(x-1)} - \frac{5\alpha_0^2}{24(x-1)^2} + \frac{7\alpha_0^2}{24(x-1)^3} + \frac{13}{24} \frac{\alpha_0^2}{(x-1)} - \frac{\alpha_0}{3(x-1)} + \frac{\alpha_0}{6(x-1)^2} - \\
 & \frac{\alpha_0}{3(x-1)^3} + \frac{13\alpha_0}{12(x-1)^4} - \frac{23}{12} \frac{\alpha_0}{(x-1)} + \left( \frac{25}{12} + \frac{3}{4(x-1)} - \frac{1}{6(x-1)^2} - \frac{1}{6(x-1)^3} + \frac{3}{4(x-1)^4} + \right. \\
 & \left. \frac{25}{12(x-1)^5} \right) H(0; \alpha_0) + \left( -\frac{25}{12} - \frac{3}{4(x-1)} + \frac{1}{6(x-1)^2} + \frac{1}{6(x-1)^3} - \frac{3}{4(x-1)^4} - \frac{25}{12(x-1)^5} \right) H(0; x) + \\
 & \left( \frac{1}{(x-1)^5} - 1 \right) H(0; \alpha_0) H(1; x) + \left( -\frac{\alpha_0^4}{4(x-1)} + \frac{\alpha_0^4}{4} + \frac{\alpha_0^3}{x-1} - \frac{\alpha_0^3}{3(x-1)^2} - \frac{4\alpha_0^3}{3} - \frac{3\alpha_0^2}{2(x-1)} + \frac{\alpha_0^2}{(x-1)^2} - \right. \\
 & \frac{\alpha_0^2}{2(x-1)^3} + 3\alpha_0^2 + \frac{\alpha_0}{x-1} - \frac{\alpha_0}{(x-1)^2} + \frac{\alpha_0}{(x-1)^3} - \frac{\alpha_0}{(x-1)^4} - 4\alpha_0 + \frac{3}{4(x-1)} - \frac{1}{6(x-1)^2} - \frac{1}{6(x-1)^3} + \\
 & \left. \frac{3}{4(x-1)^4} + \frac{25}{12(x-1)^5} + \frac{25}{12} \right) H(c_1(\alpha_0); x) + 4H(0, 0; x) + \left( \frac{1}{(x-1)^5} - 1 \right) H(0, c_1(\alpha_0); x) + \left( 1 - \right. \\
 & \left. \frac{1}{(x-1)^5} \right) H(1, 0; x) + \left( \frac{1}{(x-1)^5} - 1 \right) H(1, c_1(\alpha_0); x) - \frac{H(c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} - \frac{\pi^2}{6(x-1)^5} - \frac{\pi^2}{12},
 \end{aligned}$$

$$\begin{aligned}
 b_1^{(1,-1)} = & \frac{7d_1\alpha_0^3}{72} - \frac{7d_1\alpha_0^3}{72(x-1)^2} + \frac{7\alpha_0^3}{36(x-1)^2} - \frac{7\alpha_0^3}{36} - \frac{109d_1\alpha_0^2}{144} - \frac{13d_1\alpha_0^2}{144(x-1)} - \frac{11\alpha_0^2}{72(x-1)} + \frac{29d_1\alpha_0^2}{144(x-1)^2} - \frac{53\alpha_0^2}{72(x-1)^2} - \\
 & \frac{67d_1\alpha_0^2}{144(x-1)^3} + \frac{79\alpha_0^2}{72(x-1)^3} + \frac{121\alpha_0^2}{72} + \frac{305d_1\alpha_0}{72} + \frac{2\alpha_0}{3(x-2)} + \frac{19d_1\alpha_0}{18(x-1)} - \frac{5\alpha_0}{18(x-1)} - \frac{d_1\alpha_0}{9(x-1)^2} + \frac{5\alpha_0}{9(x-1)^2} + \frac{d_1\alpha_0}{18(x-1)^3} - \\
 & \frac{41\alpha_0}{18(x-1)^3} - \frac{217d_1\alpha_0}{72(x-1)^4} + \frac{271\alpha_0}{36(x-1)^4} - \frac{371\alpha_0}{36} + \left( -\frac{\alpha_0^3}{3(x-1)^2} + \frac{\alpha_0^3}{3} - \frac{\alpha_0^2}{6(x-1)} + \frac{5\alpha_0^2}{6(x-1)^2} - \frac{7\alpha_0^2}{6(x-1)^3} - \frac{13\alpha_0^2}{6} + \right. \\
 & \left. \frac{4\alpha_0}{3(x-1)} - \frac{2\alpha_0}{3(x-1)^2} + \frac{4\alpha_0}{3(x-1)^3} - \frac{13\alpha_0}{3(x-1)^4} + \frac{23\alpha_0}{3} - \frac{205d_1}{72} - \frac{4}{x-2} - \frac{15d_1}{8(x-1)} + \frac{7}{4(x-1)} + \frac{8}{3(x-2)^2} + \frac{5d_1}{18(x-1)^2} - \right. \\
 & \left. \frac{1}{18(x-1)^2} + \frac{5d_1}{18(x-1)^3} - \frac{31}{18(x-1)^3} - \frac{15d_1}{8(x-1)^4} + \frac{13}{4(x-1)^4} - \frac{205d_1}{72(x-1)^5} + \frac{415}{36(x-1)^5} + \frac{205}{36} \right) H(0; \alpha_0) + \\
 & \left( \frac{15d_1}{8(x-1)} - \frac{5d_1}{18(x-1)^2} - \frac{5d_1}{18(x-1)^3} + \frac{15d_1}{8(x-1)^4} + \frac{205d_1}{72(x-1)^5} + \frac{205d_1}{72} + \frac{4}{x-2} - \frac{7}{4(x-1)} - \frac{8}{3(x-2)^2} + \frac{1}{18(x-1)^2} + \right. \\
 & \left. \frac{31}{18(x-1)^3} - \frac{13}{4(x-1)^4} + \frac{2\pi^2}{3(x-1)^5} - \frac{415}{36(x-1)^5} + \frac{\pi^2}{3} - \frac{205}{36} \right) H(0; x) + \left( \frac{d_1\alpha_0^3}{6} - \frac{d_1\alpha_0^3}{6(x-1)^2} - \frac{13d_1\alpha_0^2}{12} - \frac{d_1\alpha_0^2}{12(x-1)} + \right. \\
 & \left. \frac{5d_1\alpha_0^2}{12(x-1)^2} - \frac{7d_1\alpha_0^2}{12(x-1)^3} + \frac{23d_1\alpha_0}{6} + \frac{2d_1\alpha_0}{3(x-1)} - \frac{d_1\alpha_0}{3(x-1)^2} + \frac{2d_1\alpha_0}{3(x-1)^3} - \frac{13d_1\alpha_0}{6(x-1)^4} - \frac{35d_1}{12} - \frac{7d_1}{12(x-1)} + \frac{d_1}{12(x-1)^2} - \right. \\
 & \left. \frac{d_1}{12(x-1)^3} + \frac{13d_1}{6(x-1)^4} \right) H(1; \alpha_0) + \left( \frac{\pi^2}{2(x-1)^5} - \frac{\pi^2}{2} \right) H(2; x) + \left( -\frac{d_1\alpha_0^4}{8} + \frac{d_1\alpha_0^4}{8(x-1)} - \frac{\alpha_0^4}{4(x-1)} + \frac{\alpha_0^4}{4} + \right. \\
 & \left. \frac{13d_1\alpha_0^3}{18} - \frac{d_1\alpha_0^3}{2(x-1)} + \frac{5\alpha_0^3}{3(x-1)} + \frac{2d_1\alpha_0^3}{9(x-1)^2} - \frac{7\alpha_0^3}{9(x-1)^2} - \frac{13\alpha_0^3}{9} - \frac{23d_1\alpha_0^2}{12} - \frac{\alpha_0^2}{3(x-2)} + \frac{3d_1\alpha_0^2}{4(x-1)} - \frac{9\alpha_0^2}{2(x-1)} - \frac{2d_1\alpha_0^2}{3(x-1)^2} + \right. \\
 & \left. \frac{7\alpha_0^2}{2(x-1)^2} + \frac{d_1\alpha_0^2}{2(x-1)^3} - \frac{13\alpha_0^2}{6(x-1)^3} + \frac{23\alpha_0^2}{6} + \frac{25d_1\alpha_0}{6} + \frac{2\alpha_0}{x-2} - \frac{d_1\alpha_0}{2(x-1)} + \frac{23\alpha_0}{3(x-1)} - \frac{4\alpha_0}{3(x-2)^2} + \frac{2d_1\alpha_0}{3(x-1)^2} - \frac{20\alpha_0}{3(x-1)^2} - \right. \\
 & \left. \frac{d_1\alpha_0}{(x-1)^3} + \frac{22\alpha_0}{3(x-1)^3} + \frac{2d_1\alpha_0}{(x-1)^4} - \frac{25\alpha_0}{3(x-1)^4} - \frac{25\alpha_0}{3} - \frac{205d_1}{72} + \left( \frac{\alpha_0^4}{x-1} - \alpha_0^4 - \frac{4\alpha_0^3}{x-1} + \frac{4\alpha_0^3}{3(x-1)^2} + \frac{16\alpha_0^3}{3} + \frac{6\alpha_0^2}{x-1} - \right. \right. \\
 & \left. \left. \frac{4\alpha_0^2}{(x-1)^2} + \frac{2\alpha_0^2}{(x-1)^3} - 12\alpha_0^2 - \frac{4\alpha_0}{x-1} + \frac{4\alpha_0}{(x-1)^2} - \frac{4\alpha_0}{(x-1)^3} + \frac{4\alpha_0}{(x-1)^4} + 16\alpha_0 - \frac{3}{x-1} + \frac{2}{3(x-1)^2} + \frac{2}{3(x-1)^3} - \frac{3}{(x-1)^4} - \right. \right. \\
 & \left. \left. \frac{25}{3(x-1)^5} - \frac{25}{3} \right) H(0; \alpha_0) + \left( -\frac{d_1\alpha_0^4}{2} + \frac{d_1\alpha_0^4}{2(x-1)} + \frac{8d_1\alpha_0^3}{3} - \frac{2d_1\alpha_0^3}{x-1} + \frac{2d_1\alpha_0^3}{3(x-1)^2} - 6d_1\alpha_0^2 + \frac{3d_1\alpha_0^2}{x-1} - \frac{2d_1\alpha_0^2}{(x-1)^2} + \right. \right. \\
 & \left. \left. \frac{d_1\alpha_0^2}{(x-1)^3} + 8d_1\alpha_0 - \frac{2d_1\alpha_0}{x-1} + \frac{2d_1\alpha_0}{(x-1)^2} - \frac{2d_1\alpha_0}{(x-1)^3} + \frac{2d_1\alpha_0}{(x-1)^4} - \frac{25d_1}{6} - \frac{3d_1}{2(x-1)} + \frac{d_1}{3(x-1)^2} + \frac{d_1}{3(x-1)^3} - \frac{3d_1}{2(x-1)^4} - \right. \right. \\
 & \left. \left. \frac{25d_1}{6(x-1)^5} \right) H(1; \alpha_0) - \frac{4}{x-2} - \frac{15d_1}{8(x-1)} + \frac{7}{4(x-1)} + \frac{8}{3(x-2)^2} + \frac{5d_1}{18(x-1)^2} - \frac{1}{18(x-1)^2} + \frac{5d_1}{18(x-1)^3} - \frac{31}{18(x-1)^3} - \right. \\
 & \left. \frac{15d_1}{8(x-1)^4} + \frac{13}{4(x-1)^4} - \frac{205d_1}{72(x-1)^5} + \frac{415}{36(x-1)^5} + \frac{205}{36} \right) H(c_1(\alpha_0); x) + \left( -\frac{25}{3} - \frac{3}{x-1} + \frac{2}{3(x-1)^2} + \frac{2}{3(x-1)^3} - \right. \\
 & \left. \frac{3}{(x-1)^4} - \frac{25}{3(x-1)^5} \right) H(0, 0; \alpha_0) + \left( \frac{25}{3} + \frac{3}{x-1} - \frac{2}{3(x-1)^2} - \frac{2}{3(x-1)^3} + \frac{3}{(x-1)^4} + \frac{25}{3(x-1)^5} \right) H(0, 0; x) + \left( -\right. \\
 & \left. \frac{3d_1}{2(x-1)} + \frac{d_1}{3(x-1)^2} + \frac{d_1}{3(x-1)^3} - \frac{3d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} - \frac{25d_1}{6} \right) H(0, 1; \alpha_0) + H(1; x) \left( -\frac{\pi^2 d_1}{3(x-1)^5} + \left( \frac{2d_1}{x-1} - \right. \right. \\
 & \left. \left. \frac{d_1}{(x-1)^2} + \frac{2d_1}{3(x-1)^3} - \frac{d_1}{2(x-1)^4} + \frac{25d_1}{6(x-1)^5} + \frac{4}{x-2} - \frac{8}{3(x-2)^2} - \frac{4}{3(x-1)^2} + \frac{8}{3(x-2)^3} - \frac{4}{(x-1)^4} \right) H(0; \alpha_0) + \right. \\
 & \left( 4 - \frac{4}{(x-1)^5} \right) H(0, 0; \alpha_0) + \left( 2d_1 - \frac{2d_1}{(x-1)^5} \right) H(0, 1; \alpha_0) + \frac{2\pi^2}{3(x-1)^5} + \left( \frac{2d_1}{(x-1)^5} - 2d_1 - \frac{2}{(x-1)^5} + \right. \\
 & \left. 2 \right) H(0; \alpha_0) H(0, 1; x) + \left( -\frac{\alpha_0^4}{2(x-1)} + \frac{\alpha_0^4}{2} + \frac{2\alpha_0^3}{x-1} - \frac{2\alpha_0^3}{3(x-1)^2} - \frac{8\alpha_0^3}{3} - \frac{3\alpha_0^2}{x-1} + \frac{2\alpha_0^2}{(x-1)^2} - \frac{\alpha_0^2}{(x-1)^3} + 6\alpha_0^2 + \right. \\
 & \left. \frac{2\alpha_0}{x-1} - \frac{2\alpha_0}{(x-1)^2} + \frac{2\alpha_0}{(x-1)^3} - \frac{2\alpha_0}{(x-1)^4} - 8\alpha_0 + \left( 4 - \frac{4}{(x-1)^5} \right) H(0; \alpha_0) + \left( 2d_1 - \frac{2d_1}{(x-1)^5} \right) H(1; \alpha_0) + \right. \\
 & \left. \frac{4}{x-2} - \frac{1}{2(x-1)} - \frac{8}{3(x-2)^2} - \frac{2}{3(x-1)^2} + \frac{8}{3(x-2)^3} - \frac{1}{(x-1)^3} - \frac{2}{(x-1)^4} + \frac{25}{6} \right) H(0, c_1(\alpha_0); x) + \left( -\frac{2d_1}{x-1} + \right. \\
 & \left. \frac{d_1}{(x-1)^2} - \frac{2d_1}{3(x-1)^3} + \frac{d_1}{2(x-1)^4} - \frac{25d_1}{6(x-1)^5} - \frac{4}{x-2} + \frac{8}{3(x-2)^2} + \frac{4}{3(x-1)^2} - \frac{8}{3(x-2)^3} + \frac{4}{(x-1)^4} \right) H(1, 0; x) + \\
 & \left( \frac{4d_1}{(x-1)^5} - 2d_1 - \frac{4}{(x-1)^5} \right) H(0; \alpha_0) H(1, 1; x) + \left( \frac{2d_1}{x-1} - \frac{d_1}{(x-1)^2} + \frac{2d_1}{3(x-1)^3} - \frac{d_1}{2(x-1)^4} + \frac{25d_1}{6(x-1)^5} + \right. \\
 & \left( 4 - \frac{4}{(x-1)^5} \right) H(0; \alpha_0) + \left( 2d_1 - \frac{2d_1}{(x-1)^5} \right) H(1; \alpha_0) + \frac{4}{x-2} - \frac{8}{3(x-2)^2} - \frac{4}{3(x-1)^2} + \frac{8}{3(x-2)^3} - \\
 & \left. \frac{4}{(x-1)^4} \right) H(1, c_1(\alpha_0); x) + \left( 2 - \frac{2}{(x-1)^5} \right) H(0; \alpha_0) H(2, 1; x) + \left( \frac{\alpha_0^4}{x-1} - \alpha_0^4 - \frac{4\alpha_0^3}{x-1} + \frac{4\alpha_0^3}{3(x-1)^2} + \frac{16\alpha_0^3}{3} + \right. \\
 & \left. \frac{6\alpha_0^2}{x-1} - \frac{4\alpha_0^2}{(x-1)^2} + \frac{2\alpha_0^2}{(x-1)^3} - 12\alpha_0^2 - \frac{4\alpha_0}{x-1} + \frac{4\alpha_0}{(x-1)^2} - \frac{4\alpha_0}{(x-1)^3} + \frac{4\alpha_0}{(x-1)^4} + 16\alpha_0 + \frac{4H(0; \alpha_0)}{(x-1)^5} + \frac{2d_1 H(1; \alpha_0)}{(x-1)^5} - \right. \\
 & \left. \frac{3}{x-1} + \frac{2}{3(x-1)^2} + \frac{2}{3(x-1)^3} - \frac{3}{(x-1)^4} - \frac{25}{3(x-1)^5} - \frac{25}{3} \right) H(c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{\alpha_0^4}{2(x-1)} - \frac{\alpha_0^4}{2} - \frac{2\alpha_0^3}{x-1} + \right.
 \end{aligned}$$

$$\begin{aligned}
& \frac{2\alpha_0^3}{3(x-1)^2} + \frac{8\alpha_0^3}{3} + \frac{3\alpha_0^2}{x-1} - \frac{2\alpha_0^2}{(x-1)^2} + \frac{\alpha_0^2}{(x-1)^3} - 6\alpha_0^2 - \frac{2\alpha_0}{x-1} + \frac{2\alpha_0}{(x-1)^2} - \frac{2\alpha_0}{(x-1)^3} + \frac{2\alpha_0}{(x-1)^4} + 8\alpha_0 - \frac{4}{x-2} + \frac{1}{2(x-1)} + \\
& \frac{8}{3(x-2)^2} + \frac{2}{3(x-1)^2} - \frac{8}{3(x-2)^3} + \frac{1}{(x-1)^3} + \frac{2}{(x-1)^4} - \frac{25}{6} \Big) H(c_2(\alpha_0), c_1(\alpha_0); x) - 16H(0, 0, 0; x) + \\
& \left( 2 - \frac{2}{(x-1)^5} \right) H(0, 0, c_1(\alpha_0); x) + \left( -\frac{2d_1}{(x-1)^5} + 2d_1 + \frac{2}{(x-1)^5} - 2 \right) H(0, 1, 0; x) + \left( \frac{2d_1}{(x-1)^5} - 2d_1 - \right. \\
& \left. \frac{2}{(x-1)^5} + 2 \right) H(0, 1, c_1(\alpha_0); x) + 4H(0, c_1(\alpha_0), c_1(\alpha_0); x) + \left( 2 - \frac{2}{(x-1)^5} \right) H(0, c_2(\alpha_0), c_1(\alpha_0); x) + \\
& \left( \frac{4}{(x-1)^5} - 4 \right) H(1, 0, 0; x) + \left( \frac{2d_1}{(x-1)^5} - \frac{4}{(x-1)^5} \right) H(1, 0, c_1(\alpha_0); x) + \left( -\frac{4d_1}{(x-1)^5} + 2d_1 + \right. \\
& \left. \frac{4}{(x-1)^5} \right) H(1, 1, 0; x) + \left( \frac{4d_1}{(x-1)^5} - 2d_1 - \frac{4}{(x-1)^5} \right) H(1, 1, c_1(\alpha_0); x) + \left( 4 - \right. \\
& \left. \frac{2d_1}{(x-1)^5} \right) H(1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( 2 - \frac{2}{(x-1)^5} \right) H(2, 0, c_1(\alpha_0); x) + \left( \frac{2}{(x-1)^5} - \right. \\
& \left. 2 \right) H(2, 1, 0; x) + \left( 2 - \frac{2}{(x-1)^5} \right) H(2, 1, c_1(\alpha_0); x) + \left( \frac{2}{(x-1)^5} - 2 \right) H(2, c_2(\alpha_0), c_1(\alpha_0); x) - \\
& \frac{2H(c_1(\alpha_0), 0, c_1(\alpha_0); x)}{(x-1)^5} + \frac{4H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \frac{2H(c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} - \frac{\pi^2}{x-2} + \frac{\pi^2}{8(x-1)} + \\
& \frac{2\pi^2}{3(x-2)^2} + \frac{\pi^2}{6(x-1)^2} - \frac{2\pi^2}{3(x-2)^3} + \frac{\pi^2}{4(x-1)^3} + \frac{\pi^2}{2(x-1)^4} - \frac{21\zeta_3}{4(x-1)^5} - \frac{33\zeta_3}{4} + \frac{\pi^2 \ln 2}{2(x-1)^5} - \frac{1}{2}\pi^2 \ln 2 - \frac{25\pi^2}{24}, \\
& b_2^{(1,-1)} = -\frac{37}{432}d_1^2\alpha_0^3 + \frac{37d_1\alpha_0^3}{108} + \frac{37d_1^2\alpha_0^3}{432(x-1)^2} - \frac{37d_1\alpha_0^3}{108(x-1)^2} - \frac{\pi^2\alpha_0^3}{72(x-1)^2} + \frac{37\alpha_0^3}{108(x-1)^2} + \frac{\pi^2\alpha_0^3}{72} - \frac{37\alpha_0^3}{108} + \\
& \frac{715d_1^2\alpha_0^2}{864} - \frac{793d_1\alpha_0^2}{216} + \frac{115d_1^2\alpha_0^2}{864(x-1)} + \frac{41d_1\alpha_0^2}{216(x-1)} - \frac{\pi^2\alpha_0^2}{144(x-1)} - \frac{197\alpha_0^2}{216(x-1)} - \frac{107d_1^2\alpha_0^2}{864(x-1)^2} + \frac{263d_1\alpha_0^2}{216(x-1)^2} + \frac{5\pi^2\alpha_0^2}{144(x-1)^2} - \\
& \frac{419\alpha_0^2}{216(x-1)^2} + \frac{493d_1^2\alpha_0^2}{864(x-1)^3} - \frac{571d_1\alpha_0^2}{216(x-1)^3} - \frac{7\pi^2\alpha_0^2}{144(x-1)^3} + \frac{649\alpha_0^2}{216(x-1)^3} - \frac{13\pi^2\alpha_0^2}{144} + \frac{871\alpha_0^2}{216} - \frac{3515d_1^2\alpha_0}{432} + \frac{1040d_1\alpha_0}{27} - \\
& \frac{25d_1\alpha_0}{9(x-2)} + \frac{50\alpha_0}{9(x-2)} - \frac{265d_1^2\alpha_0}{108(x-1)} + \frac{523d_1\alpha_0}{108(x-1)} + \frac{\pi^2\alpha_0}{18(x-1)} + \frac{62\alpha_0}{27(x-1)} - \frac{d_1^2\alpha_0}{108(x-1)^2} - \frac{d_1\alpha_0}{54(x-1)^2} - \frac{\pi^2\alpha_0}{36(x-1)^2} + \\
& \frac{2\alpha_0}{27(x-1)^2} + \frac{113d_1^2\alpha_0}{108(x-1)^3} + \frac{307d_1\alpha_0}{108(x-1)^3} + \frac{\pi^2\alpha_0}{18(x-1)^3} - \frac{325\alpha_0}{27(x-1)^3} + \frac{2911d_1^2\alpha_0}{432(x-1)^4} - \frac{1739d_1\alpha_0}{54(x-1)^4} - \frac{13\pi^2\alpha_0}{72(x-1)^4} + \\
& \frac{4099\alpha_0}{108(x-1)^4} + \frac{23\pi^2\alpha_0}{72} - \frac{4859\alpha_0}{108} + \left( -\frac{7d_1\alpha_0^3}{18} + \frac{7d_1\alpha_0^3}{18(x-1)^2} - \frac{7\alpha_0^3}{9(x-1)^2} + \frac{7\alpha_0^3}{9} + \frac{109d_1\alpha_0^2}{36} + \frac{13d_1\alpha_0^2}{36(x-1)} + \frac{11\alpha_0^2}{18(x-1)} - \right. \\
& \left. \frac{29d_1\alpha_0^2}{36(x-1)^2} + \frac{53\alpha_0^2}{18(x-1)^2} + \frac{67d_1\alpha_0^2}{36(x-1)^3} - \frac{79\alpha_0^2}{18(x-1)^3} - \frac{121\alpha_0^2}{18} - \frac{305d_1\alpha_0}{18} - \frac{8\alpha_0}{3(x-2)} - \frac{38d_1\alpha_0}{9(x-1)} + \frac{10\alpha_0}{9(x-1)} + \frac{4d_1\alpha_0}{9(x-1)^2} - \right. \\
& \left. \frac{20\alpha_0}{9(x-1)^2} - \frac{2d_1\alpha_0}{9(x-1)^3} + \frac{82\alpha_0}{9(x-1)^3} + \frac{217d_1\alpha_0}{18(x-1)^4} - \frac{271\alpha_0}{9(x-1)^4} + \frac{371\alpha_0}{9} + \frac{2035d_1^2}{432} - \frac{2035d_1}{108} + \frac{38d_1}{3(x-2)} - \frac{68}{3(x-2)} + \right. \\
& \left. \frac{63d_1^2}{16(x-1)} - \frac{161d_1}{12(x-1)} - \frac{\pi^2}{8(x-1)} + \frac{3}{2(x-1)} - \frac{76d_1}{9(x-2)^2} + \frac{152}{9(x-2)^2} - \frac{19d_1^2}{54(x-1)^2} + \frac{323d_1}{108(x-1)^2} + \frac{\pi^2}{36(x-1)^2} - \right. \\
& \left. \frac{215}{108(x-1)^2} - \frac{19d_1^2}{54(x-1)^3} + \frac{605d_1}{108(x-1)^3} + \frac{\pi^2}{36(x-1)^3} - \frac{1643}{108(x-1)^3} + \frac{63d_1^2}{16(x-1)^4} - \frac{50d_1}{3(x-1)^4} - \frac{\pi^2}{8(x-1)^4} + \frac{8}{(x-1)^4} + \right. \\
& \left. \frac{2035d_1^2}{432(x-1)^5} - \frac{895d_1}{27(x-1)^5} - \frac{25\pi^2}{72(x-1)^5} + \frac{5665}{108(x-1)^5} - \frac{25\pi^2}{72} + \frac{1855}{108} \right) H(0; \alpha_0) + \left( -\frac{7}{36}d_1^2\alpha_0^3 + \frac{7d_1\alpha_0^3}{18} + \right. \\
& \left. \frac{7d_1^2\alpha_0^2}{36(x-1)^2} - \frac{7d_1\alpha_0^2}{18(x-1)^2} + \frac{109d_1^2\alpha_0^2}{72} - \frac{121d_1\alpha_0^2}{36} + \frac{13d_1^2\alpha_0^2}{72(x-1)} + \frac{11d_1\alpha_0^2}{36(x-1)} - \frac{29d_1^2\alpha_0^2}{72(x-1)^2} + \frac{53d_1\alpha_0^2}{36(x-1)^2} + \frac{67d_1^2\alpha_0^2}{72(x-1)^3} - \right. \\
& \left. \frac{79d_1\alpha_0^2}{36(x-1)^3} - \frac{305d_1^2\alpha_0}{36} + \frac{371d_1\alpha_0}{18} - \frac{4d_1\alpha_0}{3(x-2)} - \frac{19d_1^2\alpha_0}{9(x-1)} + \frac{5d_1\alpha_0}{9(x-1)} + \frac{2d_1^2\alpha_0}{9(x-1)^2} - \frac{10d_1\alpha_0}{9(x-1)^2} - \frac{d_1^2\alpha_0}{9(x-1)^3} + \frac{41d_1\alpha_0}{9(x-1)^3} + \right. \\
& \left. \frac{217d_1^2\alpha_0}{36(x-1)^4} - \frac{271d_1\alpha_0}{18(x-1)^4} + \frac{515d_1^2}{72} - \frac{635d_1}{36} + \frac{4d_1}{3(x-2)} + \frac{139d_1^2}{72(x-1)} - \frac{31d_1}{36(x-1)} - \frac{d_1^2}{72(x-1)^2} + \frac{d_1}{36(x-1)^2} - \frac{59d_1^2}{72(x-1)^3} - \right. \\
& \left. \frac{85d_1}{36(x-1)^3} - \frac{217d_1^2}{36(x-1)^4} + \frac{271d_1}{18(x-1)^4} \right) H(1; \alpha_0) + \left( \frac{4\alpha_0^3}{3(x-1)^2} - \frac{4\alpha_0^3}{3} + \frac{2\alpha_0^2}{3(x-1)} - \frac{10\alpha_0^2}{3(x-1)^2} + \frac{14\alpha_0^2}{3(x-1)^3} + \frac{26\alpha_0^2}{3} - \right. \\
& \left. \frac{16\alpha_0}{3(x-1)} + \frac{8\alpha_0}{3(x-1)^2} - \frac{16\alpha_0}{3(x-1)^3} + \frac{52\alpha_0}{3(x-1)^4} - \frac{92\alpha_0}{3} + \frac{205d_1}{18} + \frac{16}{x-2} + \frac{15d_1}{2(x-1)} - \frac{7}{x-1} - \frac{32}{3(x-2)^2} - \frac{10d_1}{9(x-1)^2} + \right. \\
& \left. \frac{2}{9(x-1)^2} - \frac{10d_1}{9(x-1)^3} + \frac{62}{9(x-1)^3} + \frac{15d_1}{2(x-1)^4} - \frac{13}{(x-1)^4} + \frac{205d_1}{18(x-1)^5} - \frac{415}{9(x-1)^5} - \frac{205}{9} \right) H(0, 0; \alpha_0) + \left( - \right. \\
& \left. \frac{15d_1}{2(x-1)} + \frac{10d_1}{9(x-1)^2} + \frac{10d_1}{9(x-1)^3} - \frac{15d_1}{2(x-1)^4} - \frac{205d_1}{18(x-1)^5} - \frac{205d_1}{18} - \frac{16}{x-2} + \frac{7}{x-1} + \frac{32}{3(x-2)^2} - \frac{2}{9(x-1)^2} - \right. \\
& \left. \frac{62}{9(x-1)^3} + \frac{13}{(x-1)^4} - \frac{8\pi^2}{3(x-1)^5} + \frac{415}{9(x-1)^5} - \frac{4\pi^2}{3} + \frac{205}{9} \right) H(0, 0; x) + \left( -\frac{2d_1\alpha_0^3}{3} + \frac{2d_1\alpha_0^3}{3(x-1)^2} + \frac{13d_1\alpha_0^2}{3} + \right. \\
& \left. \frac{d_1\alpha_0^2}{3(x-1)} - \frac{5d_1\alpha_0^2}{3(x-1)^2} + \frac{7d_1\alpha_0^2}{3(x-1)^3} - \frac{46d_1\alpha_0}{3} - \frac{8d_1\alpha_0}{3(x-1)} + \frac{4d_1\alpha_0}{3(x-1)^2} - \frac{8d_1\alpha_0}{3(x-1)^3} + \frac{26d_1\alpha_0}{3(x-1)^4} + \frac{205d_1^2}{36} - \frac{205d_1}{18} + \frac{8d_1}{x-2} + \right. \\
& \left. \frac{15d_1^2}{4(x-1)} - \frac{7d_1}{2(x-1)} - \frac{16d_1}{3(x-2)^2} - \frac{5d_1^2}{9(x-1)^2} + \frac{d_1}{9(x-1)^2} - \frac{5d_1^2}{9(x-1)^3} + \frac{31d_1}{9(x-1)^3} + \frac{15d_1^2}{4(x-1)^4} - \frac{13d_1}{2(x-1)^4} + \frac{205d_1^2}{36(x-1)^5} - \right. \\
& \left. \frac{415d_1}{18(x-1)^5} \right) H(0, 1; \alpha_0) + \left( \frac{4\pi^2 d_1}{3(x-1)^5} + \left( \frac{8d_1}{x-2} - \frac{16d_1}{3(x-2)^2} - \frac{8d_1}{3(x-1)^2} + \frac{16d_1}{3(x-2)^3} - \frac{8d_1}{(x-1)^4} - \frac{8}{x-2} + \frac{5}{x-1} + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{16}{3(x-2)^2} - \frac{2}{3(x-1)^2} - \frac{16}{3(x-2)^3} + \frac{10}{3(x-1)^3} + \frac{3}{(x-1)^4} + \frac{25}{3(x-1)^5} - \frac{25}{3} \Big) H(0; \alpha_0) + \left( -\frac{8d_1}{(x-1)^5} + 8d_1 + \right. \\
& \left. \frac{8}{(x-1)^5} - 8 \right) H(0, 0; \alpha_0) + \left( -\frac{4d_1^2}{(x-1)^5} + 4d_1^2 + \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(0, 1; \alpha_0) - \frac{8\pi^2}{3(x-1)^5} \Big) H(0, 1; x) + \\
& \left( \frac{\pi^2 d_1}{(x-1)^5} - \pi^2 d_1 - \frac{\pi^2}{(x-1)^5} + \pi^2 \right) H(0, 2; x) + \left( -\frac{2d_1 \alpha_0^3}{3} + \frac{2d_1 \alpha_0^3}{3(x-1)^2} + \frac{13d_1 \alpha_0^2}{3} + \frac{d_1 \alpha_0^2}{3(x-1)} - \frac{5d_1 \alpha_0^2}{3(x-1)^2} + \right. \\
& \frac{7d_1 \alpha_0^2}{3(x-1)^3} - \frac{46d_1 \alpha_0}{3} - \frac{8d_1 \alpha_0}{3(x-1)} + \frac{4d_1 \alpha_0}{3(x-1)^2} - \frac{8d_1 \alpha_0}{3(x-1)^3} + \frac{26d_1 \alpha_0}{3(x-1)^4} + \frac{35d_1}{3} + \frac{7d_1}{3(x-1)} - \frac{d_1}{3(x-1)^2} + \frac{d_1}{3(x-1)^3} - \\
& \left. \frac{26d_1}{3(x-1)^4} \right) H(1, 0; \alpha_0) + \left( \frac{4d_1^2}{x-1} - \frac{d_1^2}{(x-1)^2} + \frac{4d_1^2}{9(x-1)^3} - \frac{d_1^2}{4(x-1)^4} + \frac{205d_1^2}{36(x-1)^5} + \frac{46d_1}{3(x-2)} - \frac{46d_1}{3(x-1)} - \frac{52d_1}{9(x-2)^2} + \right. \\
& \frac{40d_1}{9(x-1)^2} + \frac{28d_1}{9(x-2)^3} - \frac{98d_1}{9(x-1)^3} + \frac{13d_1}{6(x-1)^4} + \frac{4\pi^2 d_1}{3(x-1)^5} - \frac{415d_1}{18(x-1)^5} - \frac{80}{3(x-2)} - \frac{43}{6(x-1)} + \frac{56}{9(x-2)^2} + \frac{299}{18(x-1)^2} - \\
& \frac{56}{9(x-2)^3} + \frac{23}{6(x-1)^3} + \frac{203}{6(x-1)^4} - \frac{5\pi^2}{2(x-1)^5} + \frac{35}{6(x-1)^5} - \frac{\pi^2}{6} - \frac{35}{6} \Big) H(1, 0; x) + \left( -\frac{1}{3}d_1^2 \alpha_0^3 + \frac{d_1^2 \alpha_0^3}{3(x-1)^2} + \right. \\
& \frac{13d_1^2 \alpha_0^2}{6} + \frac{d_1^2 \alpha_0^2}{6(x-1)} - \frac{5d_1^2 \alpha_0^2}{6(x-1)^2} + \frac{7d_1^2 \alpha_0^2}{6(x-1)^3} - \frac{23d_1^2 \alpha_0}{3} - \frac{4d_1^2 \alpha_0}{3(x-1)} + \frac{2d_1^2 \alpha_0}{3(x-1)^2} - \frac{4d_1^2 \alpha_0}{3(x-1)^3} + \frac{13d_1^2 \alpha_0}{3(x-1)^4} + \frac{35d_1^2}{6} + \\
& \left. \frac{7d_1^2}{6(x-1)} - \frac{d_1^2}{6(x-1)^2} + \frac{d_1^2}{6(x-1)^3} - \frac{13d_1^2}{3(x-1)^4} \right) H(1, 1; \alpha_0) + H(0, c_1(\alpha_0); x) \left( -\frac{d_1 \alpha_0^4}{4} + \frac{d_1 \alpha_0^4}{4(x-1)} - \frac{\alpha_0^4}{2(x-1)} + \right. \\
& \frac{\alpha_0^4}{2} + \frac{13d_1 \alpha_0^3}{9} - \frac{d_1 \alpha_0^3}{x-1} + \frac{10\alpha_0^3}{3(x-1)} + \frac{4d_1 \alpha_0^3}{9(x-1)^2} - \frac{11\alpha_0^3}{9(x-1)^2} - \frac{29\alpha_0^3}{9} - \frac{23d_1 \alpha_0^2}{6} - \frac{2\alpha_0^2}{3(x-2)} + \frac{3d_1 \alpha_0^2}{2(x-1)} - \frac{53\alpha_0^2}{6(x-1)} - \\
& \frac{4d_1 \alpha_0^2}{3(x-1)^2} + \frac{37\alpha_0^2}{6(x-1)^2} + \frac{d_1 \alpha_0^2}{(x-1)^3} - \frac{19\alpha_0^2}{6(x-1)^3} + \frac{59\alpha_0^2}{6} + \frac{25d_1 \alpha_0}{3} + \frac{4\alpha_0}{x-2} - \frac{d_1 \alpha_0}{x-1} + \frac{14\alpha_0}{x-1} - \frac{8\alpha_0}{3(x-2)^2} + \frac{4d_1 \alpha_0}{3(x-1)^2} - \\
& \frac{38\alpha_0}{3(x-1)^2} - \frac{2d_1 \alpha_0}{(x-1)^3} + \frac{40\alpha_0}{3(x-1)^3} + \frac{4d_1 \alpha_0}{(x-1)^4} - \frac{37\alpha_0}{3(x-1)^4} - \frac{73\alpha_0}{3} - \frac{205d_1}{36} + \left( \frac{2\alpha_0^4}{x-1} - 2\alpha_0^4 - \frac{8\alpha_0^3}{x-1} + \frac{8\alpha_0^3}{3(x-1)^2} + \frac{32\alpha_0^3}{3} + \right. \\
& \frac{12\alpha_0^2}{x-1} - \frac{8\alpha_0^2}{(x-1)^2} + \frac{4\alpha_0^2}{(x-1)^3} - 24\alpha_0^2 - \frac{8\alpha_0}{x-1} + \frac{8\alpha_0}{(x-1)^2} - \frac{8\alpha_0}{(x-1)^3} + \frac{8\alpha_0}{(x-1)^4} + 32\alpha_0 - \frac{16}{x-2} + \frac{2}{x-1} + \frac{32}{3(x-2)^2} + \\
& \left. \frac{8}{3(x-1)^2} - \frac{32}{3(x-1)^3} + \frac{4}{(x-1)^3} + \frac{8}{(x-1)^4} - \frac{50}{3} \right) H(0; \alpha_0) + \left( -d_1 \alpha_0^4 + \frac{d_1 \alpha_0^4}{x-1} + \frac{16d_1 \alpha_0^3}{3} - \frac{4d_1 \alpha_0^3}{x-1} + \frac{4d_1 \alpha_0^3}{3(x-1)^2} - \right. \\
& 12d_1 \alpha_0^2 + \frac{6d_1 \alpha_0^2}{x-1} - \frac{4d_1 \alpha_0^2}{(x-1)^2} + \frac{2d_1 \alpha_0^2}{(x-1)^3} + 16d_1 \alpha_0 - \frac{4d_1 \alpha_0}{x-1} + \frac{4d_1 \alpha_0}{(x-1)^2} - \frac{4d_1 \alpha_0}{(x-1)^3} + \frac{4d_1 \alpha_0}{(x-1)^4} - \frac{25d_1}{3} - \frac{8d_1}{x-2} + \frac{d_1}{x-1} + \\
& \frac{16d_1}{3(x-2)^2} + \frac{4d_1}{3(x-1)^2} - \frac{16d_1}{3(x-2)^3} + \frac{2d_1}{(x-1)^3} + \frac{4d_1}{(x-1)^4} \Big) H(1; \alpha_0) + \left( \frac{16}{(x-1)^5} - 16 \right) H(0, 0; \alpha_0) + \left( \frac{8d_1}{(x-1)^5} - \right. \\
& 8d_1 \Big) H(0, 1; \alpha_0) + \left( \frac{8d_1}{(x-1)^5} - 8d_1 \right) H(1, 0; \alpha_0) + \left( \frac{4d_1^2}{(x-1)^5} - 4d_1^2 \right) H(1, 1; \alpha_0) - \frac{32d_1}{3(x-2)} + \frac{70}{3(x-2)} + \\
& \frac{35d_1}{12(x-1)} - \frac{5}{6(x-1)} + \frac{28d_1}{9(x-2)^2} - \frac{32}{9(x-2)^2} + \frac{28d_1}{9(x-1)^2} - \frac{80}{9(x-1)^2} - \frac{28d_1}{9(x-2)^3} + \frac{56}{9(x-2)^3} + \frac{5d_1}{(x-1)^3} - \frac{14}{(x-1)^3} + \\
& \frac{4d_1}{(x-1)^4} - \frac{43}{2(x-1)^4} - \frac{\pi^2}{6(x-1)^5} - \frac{35}{6(x-1)^5} + \frac{\pi^2}{6} + \frac{415}{18} \Big) + H(c_1(\alpha_0); x) \left( \frac{d_1^2 \alpha_0^4}{16} - \frac{d_1 \alpha_0^4}{4} - \frac{d_1^2 \alpha_0^4}{16(x-1)} + \right. \\
& \frac{d_1 \alpha_0^4}{4(x-1)} + \frac{\pi^2 \alpha_0^4}{24(x-1)} - \frac{\alpha_0^4}{4(x-1)} - \frac{\pi^2 \alpha_0^4}{24} + \frac{\alpha_0^4}{4} - \frac{43d_1^2 \alpha_0^3}{108} + \frac{43d_1 \alpha_0^3}{27} + \frac{d_1^2 \alpha_0^3}{4(x-1)} - \frac{16d_1 \alpha_0^3}{9(x-1)} - \frac{\pi^2 \alpha_0^3}{6(x-1)} + \frac{23\alpha_0^3}{9(x-1)} - \\
& \frac{4d_1^2 \alpha_0^3}{27(x-1)^2} + \frac{53d_1 \alpha_0^3}{54(x-1)^2} + \frac{\pi^2 \alpha_0^3}{18(x-1)^2} - \frac{37\alpha_0^3}{27(x-1)^2} + \frac{2\pi^2 \alpha_0^3}{9} - \frac{43\alpha_0^3}{27} + \frac{95d_1^2 \alpha_0^2}{72} - \frac{95d_1 \alpha_0^2}{18} + \frac{13d_1 \alpha_0^2}{18(x-2)} - \frac{13\alpha_0^2}{9(x-2)} - \\
& \frac{3d_1^2 \alpha_0^2}{8(x-1)} + \frac{16d_1 \alpha_0^2}{3(x-1)} + \frac{\pi^2 \alpha_0^2}{4(x-1)} - \frac{59\alpha_0^2}{6(x-1)} + \frac{4d_1^2 \alpha_0^2}{9(x-1)^2} - \frac{173d_1 \alpha_0^2}{36(x-1)^2} - \frac{\pi^2 \alpha_0^2}{6(x-1)^2} + \frac{59\alpha_0^2}{6(x-1)^2} - \frac{d_1^2 \alpha_0^2}{2(x-1)^3} + \frac{139d_1 \alpha_0^2}{36(x-1)^3} + \\
& \frac{\pi^2 \alpha_0^2}{12(x-1)^3} - \frac{56\alpha_0^2}{9(x-1)^3} - \frac{\pi^2 \alpha_0^2}{2} + \frac{46\alpha_0^2}{9} - \frac{205d_1^2 \alpha_0}{36} + \frac{205d_1 \alpha_0}{9} - \frac{17d_1 \alpha_0}{3(x-2)} + \frac{10\alpha_0}{x-2} + \frac{d_1^2 \alpha_0}{4(x-1)} - \frac{127d_1 \alpha_0}{9(x-1)} - \\
& \frac{\pi^2 \alpha_0}{6(x-1)} + \frac{1247\alpha_0}{36(x-1)} + \frac{38d_1 \alpha_0}{9(x-2)^2} - \frac{76\alpha_0}{9(x-2)^2} - \frac{4d_1^2 \alpha_0}{9(x-1)^2} + \frac{34d_1 \alpha_0}{3(x-1)^2} + \frac{\pi^2 \alpha_0}{6(x-1)^2} - \frac{565\alpha_0}{18(x-1)^2} + \frac{d_1^2 \alpha_0}{(x-1)^3} - \frac{146d_1 \alpha_0}{9(x-1)^3} - \\
& \frac{\pi^2 \alpha_0}{6(x-1)^3} + \frac{1495\alpha_0}{36(x-1)^3} - \frac{4d_1^2 \alpha_0}{(x-1)^4} + \frac{505d_1 \alpha_0}{18(x-1)^4} + \frac{\pi^2 \alpha_0}{6(x-1)^4} - \frac{803\alpha_0}{18(x-1)^4} + \frac{2\pi^2 \alpha_0}{3} - \frac{377\alpha_0}{18} + \frac{2035d_1}{432} - \frac{2035d_1}{108} + \\
& \left( \frac{d_1 \alpha_0^4}{2} - \frac{d_1 \alpha_0^4}{2(x-1)} + \frac{\alpha_0^4}{x-1} - \alpha_0^4 - \frac{26d_1 \alpha_0^3}{9} + \frac{2d_1 \alpha_0^3}{x-1} - \frac{20\alpha_0^3}{3(x-1)} - \frac{8d_1 \alpha_0^3}{9(x-1)^2} + \frac{28\alpha_0^3}{9(x-1)^2} + \frac{52\alpha_0^3}{9} + \frac{23d_1 \alpha_0^2}{3} + \frac{4\alpha_0^2}{3(x-2)} - \right. \\
& \frac{3d_1 \alpha_0^2}{x-1} + \frac{18\alpha_0^2}{x-1} + \frac{8d_1 \alpha_0^2}{3(x-1)^2} - \frac{14\alpha_0^2}{(x-1)^2} - \frac{2d_1 \alpha_0^2}{(x-1)^3} + \frac{26\alpha_0^2}{3(x-1)^3} - \frac{46\alpha_0^2}{3} - \frac{50d_1 \alpha_0}{3} - \frac{8\alpha_0}{x-2} + \frac{2d_1 \alpha_0}{x-1} - \frac{92\alpha_0}{3(x-1)} + \\
& \frac{16\alpha_0}{3(x-2)^2} - \frac{8d_1 \alpha_0}{3(x-1)^2} + \frac{80\alpha_0}{3(x-1)^2} + \frac{4d_1 \alpha_0}{(x-1)^3} - \frac{88\alpha_0}{3(x-1)^3} - \frac{8d_1 \alpha_0}{(x-1)^4} + \frac{100\alpha_0}{3(x-1)^4} + \frac{100\alpha_0}{3} + \frac{205d_1}{18} + \frac{16}{x-2} + \frac{15d_1}{2(x-1)} - \\
& \frac{7}{x-1} - \frac{32}{3(x-2)^2} - \frac{10d_1}{9(x-1)^2} + \frac{2}{9(x-1)^2} - \frac{10d_1}{9(x-1)^3} + \frac{62}{9(x-1)^3} + \frac{15d_1}{2(x-1)^4} - \frac{13}{(x-1)^4} + \frac{205d_1}{18(x-1)^5} - \frac{415}{9(x-1)^5} - \\
& \left. \frac{205}{9} \right) H(0; \alpha_0) + \left( \frac{d_1^2 \alpha_0^4}{4} - \frac{d_1 \alpha_0^4}{2} - \frac{d_1^2 \alpha_0^4}{4(x-1)} + \frac{d_1 \alpha_0^4}{2(x-1)} - \frac{13d_1^2 \alpha_0^3}{9} + \frac{26d_1 \alpha_0^3}{9} + \frac{d_1^2 \alpha_0^3}{x-1} - \frac{10d_1 \alpha_0^3}{3(x-1)} - \frac{4d_1^2 \alpha_0^3}{9(x-1)^2} + \right. \\
& \frac{14d_1 \alpha_0^3}{9(x-1)^2} + \frac{23d_1^2 \alpha_0^2}{6} - \frac{23d_1 \alpha_0^2}{3} + \frac{2d_1 \alpha_0^2}{3(x-2)} - \frac{3d_1^2 \alpha_0^2}{2(x-1)} + \frac{9d_1 \alpha_0^2}{x-1} + \frac{4d_1^2 \alpha_0^2}{3(x-1)^2} - \frac{7d_1 \alpha_0^2}{(x-1)^2} - \frac{d_1^2 \alpha_0^2}{(x-1)^3} + \frac{13d_1 \alpha_0^2}{3(x-1)^3} - \\
& \left. \frac{25d_1^2 \alpha_0}{3} + \frac{50d_1 \alpha_0}{3} - \frac{4d_1 \alpha_0}{x-2} + \frac{d_1^2 \alpha_0}{x-1} - \frac{46d_1 \alpha_0}{3(x-1)} + \frac{8d_1 \alpha_0}{3(x-2)^2} - \frac{4d_1^2 \alpha_0}{3(x-1)^2} + \frac{40d_1 \alpha_0}{3(x-1)^2} + \frac{2d_1^2 \alpha_0}{(x-1)^3} - \frac{44d_1 \alpha_0}{3(x-1)^3} - \frac{4d_1^2 \alpha_0}{(x-1)^4} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{50d_1\alpha_0}{3(x-1)^4} + \frac{205d_1^2}{36} - \frac{205d_1}{18} + \frac{8d_1}{x-2} + \frac{15d_1^2}{4(x-1)} - \frac{7d_1}{2(x-1)} - \frac{16d_1}{3(x-2)^2} - \frac{5d_1^2}{9(x-1)^2} + \frac{d_1}{9(x-1)^2} - \frac{5d_1^2}{9(x-1)^3} + \frac{31d_1}{9(x-1)^3} + \\
& \frac{15d_1^2}{4(x-1)^4} - \frac{13d_1}{2(x-1)^4} + \frac{205d_1^2}{36(x-1)^5} - \frac{415d_1}{18(x-1)^5} \Big) H(1; \alpha_0) + \left( -\frac{4\alpha_0^4}{x-1} + 4\alpha_0^4 + \frac{16\alpha_0^3}{x-1} - \frac{16\alpha_0^3}{3(x-1)^2} - \frac{64\alpha_0^3}{3} - \right. \\
& \frac{24\alpha_0^2}{x-1} + \frac{16\alpha_0^2}{(x-1)^2} - \frac{8\alpha_0^2}{(x-1)^3} + 48\alpha_0^2 + \frac{16\alpha_0}{x-1} - \frac{16\alpha_0}{(x-1)^2} + \frac{16\alpha_0}{(x-1)^3} - \frac{16\alpha_0}{(x-1)^4} - 64\alpha_0 + \frac{12}{x-1} - \frac{8}{3(x-1)^2} - \frac{8}{3(x-1)^3} + \\
& \left. \frac{12}{(x-1)^4} + \frac{100}{3(x-1)^5} + \frac{100}{3} \right) H(0, 0; \alpha_0) + \left( 2d_1\alpha_0^4 - \frac{2d_1\alpha_0^4}{x-1} - \frac{32d_1\alpha_0^3}{3} + \frac{8d_1\alpha_0^3}{x-1} - \frac{8d_1\alpha_0^3}{3(x-1)^2} + 24d_1\alpha_0^2 - \right. \\
& \frac{12d_1\alpha_0^2}{x-1} + \frac{8d_1\alpha_0^2}{(x-1)^2} - \frac{4d_1\alpha_0^2}{(x-1)^3} - 32d_1\alpha_0 + \frac{8d_1\alpha_0}{x-1} - \frac{8d_1\alpha_0}{(x-1)^2} + \frac{8d_1\alpha_0}{(x-1)^3} - \frac{8d_1\alpha_0}{(x-1)^4} + \frac{50d_1}{3} + \frac{6d_1}{x-1} - \frac{4d_1}{3(x-1)^2} - \\
& \frac{4d_1}{3(x-1)^3} + \frac{6d_1}{(x-1)^4} + \frac{50d_1}{3(x-1)^5} \Big) H(0, 1; \alpha_0) + \left( 2d_1\alpha_0^4 - \frac{2d_1\alpha_0^4}{x-1} - \frac{32d_1\alpha_0^3}{3} + \frac{8d_1\alpha_0^3}{x-1} - \frac{8d_1\alpha_0^3}{3(x-1)^2} + 24d_1\alpha_0^2 - \right. \\
& \frac{12d_1\alpha_0^2}{x-1} + \frac{8d_1\alpha_0^2}{(x-1)^2} - \frac{4d_1\alpha_0^2}{(x-1)^3} - 32d_1\alpha_0 + \frac{8d_1\alpha_0}{x-1} - \frac{8d_1\alpha_0}{(x-1)^2} + \frac{8d_1\alpha_0}{(x-1)^3} - \frac{8d_1\alpha_0}{(x-1)^4} + \frac{50d_1}{3} + \frac{6d_1}{x-1} - \frac{4d_1}{3(x-1)^2} - \\
& \frac{4d_1}{3(x-1)^3} + \frac{6d_1}{(x-1)^4} + \frac{50d_1}{3(x-1)^5} \Big) H(1, 0; \alpha_0) + \left( d_1^2\alpha_0^4 - \frac{d_1^2\alpha_0^4}{x-1} - \frac{16d_1^2\alpha_0^3}{3} + \frac{4d_1^2\alpha_0^3}{x-1} - \frac{4d_1^2\alpha_0^3}{3(x-1)^2} + 12d_1^2\alpha_0^2 - \right. \\
& \frac{6d_1^2\alpha_0^2}{x-1} + \frac{4d_1^2\alpha_0^2}{(x-1)^2} - \frac{2d_1^2\alpha_0^2}{(x-1)^3} - 16d_1^2\alpha_0 + \frac{4d_1^2\alpha_0}{x-1} - \frac{4d_1^2\alpha_0}{(x-1)^2} + \frac{4d_1^2\alpha_0}{(x-1)^3} - \frac{4d_1^2\alpha_0}{(x-1)^4} + \frac{25d_1^2}{3} + \frac{3d_1^2}{x-1} - \frac{2d_1^2}{3(x-1)^2} - \\
& \left. \frac{2d_1^2}{3(x-1)^3} + \frac{3d_1^2}{(x-1)^4} + \frac{25d_1^2}{3(x-1)^5} \right) H(1, 1; \alpha_0) + \frac{38d_1}{3(x-2)} - \frac{68}{3(x-2)} + \frac{63d_1^2}{16(x-1)} - \frac{161d_1}{12(x-1)} - \frac{\pi^2}{8(x-1)} + \frac{3}{2(x-1)} - \\
& \frac{76d_1}{9(x-2)^2} + \frac{152}{9(x-2)^2} - \frac{19d_1^2}{54(x-1)^2} + \frac{323d_1}{108(x-1)^2} + \frac{\pi^2}{36(x-1)^2} - \frac{215}{108(x-1)^2} - \frac{19d_1^2}{54(x-1)^3} + \frac{605d_1}{108(x-1)^3} + \frac{\pi^2}{36(x-1)^3} - \\
& \frac{1643}{108(x-1)^3} + \frac{63d_1^2}{16(x-1)^4} - \frac{50d_1}{3(x-1)^4} - \frac{\pi^2}{8(x-1)^4} + \frac{8}{(x-1)^4} + \frac{2035d_1^2}{432(x-1)^5} - \frac{895d_1}{27(x-1)^5} - \frac{25\pi^2}{72(x-1)^5} + \frac{5665}{108(x-1)^5} - \\
& \frac{25\pi^2}{72} + \frac{1855}{108} \Big) + \left( -\frac{2\pi^2 d_1^2}{3(x-1)^5} + \frac{8\pi^2 d_1}{3(x-1)^5} + \left( \frac{4d_1^2}{x-1} - \frac{2d_1^2}{(x-1)^2} + \frac{4d_1^2}{3(x-1)^3} - \frac{d_1^2}{(x-1)^4} + \frac{25d_1^2}{3(x-1)^5} + \frac{8d_1}{x-2} - \frac{16d_1}{3(x-2)^2} - \right. \right. \\
& \frac{8d_1}{3(x-1)^2} + \frac{16d_1}{3(x-2)^3} - \frac{8d_1}{(x-1)^4} + \frac{4}{x-2} - \frac{7}{2(x-1)} - \frac{8}{3(x-2)^2} + \frac{8}{3(x-2)^3} - \frac{1}{3(x-1)^3} - \frac{5}{(x-1)^4} - \frac{25}{3(x-1)^5} - \\
& \left. \frac{25}{6} \right) H(0; \alpha_0) + \left( -\frac{16d_1}{(x-1)^5} + 8d_1 + \frac{16}{(x-1)^5} \right) H(0, 0; \alpha_0) + \left( -\frac{8d_1^2}{(x-1)^5} + 4d_1^2 + \frac{8d_1}{(x-1)^5} \right) H(0, 1; \alpha_0) - \\
& \frac{2\pi^2}{(x-1)^5} - \frac{2\pi^2}{3} \Big) H(1, 1; x) + \left( \frac{\pi^2 d_1}{(x-1)^5} - \frac{\pi^2}{2(x-1)^5} - \frac{3\pi^2}{2} \right) H(1, 2; x) + \left( -\frac{4d_1^2}{x-1} + \frac{d_1^2}{(x-1)^2} - \frac{4d_1^2}{9(x-1)^3} + \right. \\
& \frac{d_1^2}{4(x-1)^4} - \frac{205d_1^2}{36(x-1)^5} - \frac{46d_1}{3(x-2)} + \frac{46d_1}{3(x-1)} + \frac{52d_1}{9(x-2)^2} - \frac{40d_1}{9(x-1)^2} - \frac{28d_1}{9(x-2)^3} + \frac{98d_1}{9(x-1)^3} - \frac{13d_1}{6(x-1)^4} + \\
& \frac{415d_1}{18(x-1)^5} + \left( -\frac{8d_1}{x-1} + \frac{4d_1}{(x-1)^2} - \frac{8d_1}{3(x-1)^3} + \frac{2d_1}{(x-1)^4} - \frac{50d_1}{3(x-1)^5} - \frac{16}{x-2} + \frac{32}{3(x-2)^2} + \frac{16}{3(x-1)^2} - \frac{32}{3(x-2)^3} + \right. \\
& \left. \frac{16}{(x-1)^4} \right) H(0; \alpha_0) + \left( -\frac{4d_1^2}{x-1} + \frac{2d_1^2}{(x-1)^2} - \frac{4d_1^2}{3(x-1)^3} + \frac{d_1^2}{(x-1)^4} - \frac{25d_1^2}{3(x-1)^5} - \frac{8d_1}{x-2} + \frac{16d_1}{3(x-2)^2} + \frac{8d_1}{3(x-1)^2} - \right. \\
& \left. \frac{16d_1}{3(x-2)^3} + \frac{8d_1}{(x-1)^4} \right) H(1; \alpha_0) + \left( \frac{16}{(x-1)^5} - 16 \right) H(0, 0; \alpha_0) + \left( \frac{8d_1}{(x-1)^5} - 8d_1 \right) H(0, 1; \alpha_0) + \left( \frac{8d_1}{(x-1)^5} - \right. \\
& \left. 8d_1 \right) H(1, 0; \alpha_0) + \left( \frac{4d_1^2}{(x-1)^5} - 4d_1^2 \right) H(1, 1; \alpha_0) + \frac{80}{3(x-2)} + \frac{43}{6(x-1)} - \frac{56}{9(x-2)^2} - \frac{299}{18(x-1)^2} + \frac{56}{9(x-2)^3} - \\
& \frac{23}{6(x-1)^3} - \frac{203}{6(x-1)^4} - \frac{\pi^2}{6(x-1)^5} - \frac{35}{6(x-1)^5} + \frac{\pi^2}{6} + \frac{35}{6} \Big) H(1, c_1(\alpha_0); x) + \left( 2\pi^2 - \frac{2\pi^2}{(x-1)^5} \right) H(2, 0; x) + \\
& \left( \left( -\frac{8d_1}{x-2} + \frac{16d_1}{3(x-2)^2} + \frac{8d_1}{3(x-1)^2} - \frac{16d_1}{3(x-2)^3} + \frac{8d_1}{(x-1)^4} + \frac{16}{x-2} - \frac{15}{2(x-1)} - \frac{32}{3(x-2)^2} - \frac{1}{3(x-1)^2} + \right. \right. \\
& \left. \frac{32}{3(x-2)^3} - \frac{5}{(x-1)^3} - \frac{17}{2(x-1)^4} - \frac{25}{2(x-1)^5} + \frac{25}{2} \right) H(0; \alpha_0) + \left( \frac{8}{(x-1)^5} - 8 \right) H(0, 0; \alpha_0) + \left( \frac{4d_1}{(x-1)^5} - \right. \\
& \left. 4d_1 \right) H(0, 1; \alpha_0) + \frac{\pi^2}{3(x-1)^5} - \frac{\pi^2}{3} \Big) H(2, 1; x) + \left( -\frac{\pi^2 d_1}{(x-1)^5} + \pi^2 d_1 + \frac{2\pi^2}{(x-1)^5} - 2\pi^2 \right) H(2, 2; x) + \\
& \left( \frac{d_1\alpha_0^4}{2} - \frac{d_1\alpha_0^4}{2(x-1)} + \frac{\alpha_0^4}{x-1} - \alpha_0^4 - \frac{26d_1\alpha_0^3}{9} + \frac{2d_1\alpha_0^3}{x-1} - \frac{41\alpha_0^3}{6(x-1)} - \frac{8d_1\alpha_0^3}{9(x-1)^2} + \frac{25\alpha_0^3}{9(x-1)^2} + \frac{107\alpha_0^3}{18} + \frac{23d_1\alpha_0^2}{3} + \right. \\
& \frac{5\alpha_0^2}{3(x-2)} - \frac{3d_1\alpha_0^2}{x-1} + \frac{221\alpha_0^2}{12(x-1)} + \frac{8d_1\alpha_0^2}{3(x-1)^2} - \frac{27\alpha_0^2}{2(x-1)^2} - \frac{2d_1\alpha_0^2}{(x-1)^3} + \frac{15\alpha_0^2}{2(x-1)^3} - \frac{197\alpha_0^2}{12} - \frac{50d_1\alpha_0}{3} - \frac{10\alpha_0}{x-2} + \frac{2d_1\alpha_0}{x-1} - \\
& \frac{181\alpha_0}{6(x-1)} + \frac{20\alpha_0}{3(x-2)^2} - \frac{8d_1\alpha_0}{3(x-1)^2} + \frac{80\alpha_0}{3(x-1)^2} + \frac{4d_1\alpha_0}{(x-1)^3} - \frac{29\alpha_0}{(x-1)^3} - \frac{8d_1\alpha_0}{(x-1)^4} + \frac{29\alpha_0}{(x-1)^4} + \frac{223\alpha_0}{6} + \frac{205d_1}{18} + \left( - \right. \\
& \frac{4\alpha_0^4}{x-1} + 4\alpha_0^4 + \frac{16\alpha_0^3}{x-1} - \frac{16\alpha_0^3}{3(x-1)^2} - \frac{64\alpha_0^3}{3} - \frac{24\alpha_0^2}{x-1} + \frac{16\alpha_0^2}{(x-1)^2} - \frac{8\alpha_0^2}{(x-1)^3} + 48\alpha_0^2 + \frac{16\alpha_0}{x-1} - \frac{16\alpha_0}{(x-1)^2} + \frac{16\alpha_0}{(x-1)^3} - \\
& \left. \frac{16\alpha_0}{(x-1)^4} - 64\alpha_0 + \frac{12}{x-1} - \frac{8}{3(x-1)^2} - \frac{8}{3(x-1)^3} + \frac{12}{(x-1)^4} + \frac{100}{3(x-1)^5} + \frac{100}{3} \right) H(0; \alpha_0) + \left( 2d_1\alpha_0^4 - \frac{2d_1\alpha_0^4}{x-1} - \right. \\
& \left. \frac{32d_1\alpha_0^3}{3} + \frac{8d_1\alpha_0^3}{x-1} - \frac{8d_1\alpha_0^3}{3(x-1)^2} + 24d_1\alpha_0^2 - \frac{12d_1\alpha_0^2}{x-1} + \frac{8d_1\alpha_0^2}{(x-1)^2} - \frac{4d_1\alpha_0^2}{(x-1)^3} - 32d_1\alpha_0 + \frac{8d_1\alpha_0}{x-1} - \frac{8d_1\alpha_0}{(x-1)^2} + \frac{8d_1\alpha_0}{(x-1)^3} - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{8d_1\alpha_0}{(x-1)^4} + \frac{50d_1}{3} + \frac{6d_1}{x-1} - \frac{4d_1}{3(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \frac{6d_1}{(x-1)^4} + \frac{50d_1}{3(x-1)^5} \right) H(1; \alpha_0) - \frac{16}{(x-1)^5} H(0,0;\alpha_0) - \frac{8d_1 H(0,1;\alpha_0)}{(x-1)^5} - \\
& \frac{8d_1}{(x-1)^5} H(1,0;\alpha_0) - \frac{4d_1^2}{(x-1)^5} H(1,1;\alpha_0) + \frac{20}{x-2} + \frac{15d_1}{2(x-1)} - \frac{49}{4(x-1)} - \frac{40}{3(x-2)^2} - \frac{10d_1}{9(x-1)^2} + \frac{5}{36(x-1)^2} - \frac{10d_1}{9(x-1)^3} + \\
& \frac{275}{36(x-1)^3} + \frac{15d_1}{2(x-1)^4} - \frac{6}{(x-1)^4} + \frac{205d_1}{18(x-1)^5} + \frac{\pi^2}{6(x-1)^5} - \frac{725}{18(x-1)^5} - \frac{925}{36} \Big) H(c_1(\alpha_0), c_1(\alpha_0); x) + \\
& \left( \frac{d_1\alpha_0^4}{4} - \frac{d_1\alpha_0^4}{4(x-1)} + \frac{\alpha_0^4}{2(x-1)} - \frac{\alpha_0^4}{2} - \frac{13d_1\alpha_0^3}{9} + \frac{d_1\alpha_0^3}{x-1} - \frac{3\alpha_0^3}{x-1} - \frac{4d_1\alpha_0^3}{9(x-1)^2} + \frac{25\alpha_0^3}{18(x-1)^2} + \frac{61\alpha_0^3}{18} + \frac{23d_1\alpha_0^2}{6} - \right. \\
& \frac{3d_1\alpha_0^2}{2(x-1)} + \frac{31\alpha_0^2}{4(x-1)} + \frac{4d_1\alpha_0^2}{3(x-1)^2} - \frac{71\alpha_0^2}{12(x-1)^2} - \frac{d_1\alpha_0^2}{(x-1)^3} + \frac{15\alpha_0^2}{4(x-1)^3} - \frac{131\alpha_0^2}{12} - \frac{25d_1\alpha_0}{3} + \frac{d_1\alpha_0}{x-1} - \frac{13\alpha_0}{x-1} - \\
& \frac{4d_1\alpha_0}{3(x-1)^2} + \frac{35\alpha_0}{3(x-1)^2} + \frac{2d_1\alpha_0}{(x-1)^3} - \frac{12\alpha_0}{(x-1)^3} - \frac{4d_1\alpha_0}{(x-1)^4} + \frac{29\alpha_0}{2(x-1)^4} + \frac{169\alpha_0}{6} + \frac{205d_1}{36} + \left( -\frac{2\alpha_0^4}{x-1} + 2\alpha_0^4 + \frac{8\alpha_0^3}{x-1} - \right. \\
& \frac{8\alpha_0^3}{3(x-1)^2} - \frac{32\alpha_0^3}{3} - \frac{12\alpha_0^2}{x-1} + \frac{8\alpha_0^2}{(x-1)^2} - \frac{4\alpha_0^2}{(x-1)^3} + 24\alpha_0^2 + \frac{8\alpha_0}{x-1} - \frac{8\alpha_0}{(x-1)^2} + \frac{8\alpha_0}{(x-1)^3} - \frac{8\alpha_0}{(x-1)^4} - 32\alpha_0 + \frac{16}{x-2} - \\
& \frac{2}{x-1} - \frac{32}{3(x-2)^2} - \frac{8}{3(x-1)^2} + \frac{32}{3(x-2)^3} - \frac{4}{(x-1)^3} - \frac{8}{(x-1)^4} + \frac{50}{3} \Big) H(0; \alpha_0) + \left( d_1\alpha_0^4 - \frac{d_1\alpha_0^4}{x-1} - \frac{16d_1\alpha_0^3}{3} + \right. \\
& \frac{4d_1\alpha_0^3}{x-1} - \frac{4d_1\alpha_0^3}{3(x-1)^2} + 12d_1\alpha_0^2 - \frac{6d_1\alpha_0^2}{x-1} + \frac{4d_1\alpha_0^2}{(x-1)^2} - \frac{2d_1\alpha_0^2}{(x-1)^3} - 16d_1\alpha_0 + \frac{4d_1\alpha_0}{x-1} - \frac{4d_1\alpha_0}{(x-1)^2} + \frac{4d_1\alpha_0}{(x-1)^3} - \frac{4d_1\alpha_0}{(x-1)^4} + \\
& \frac{25d_1}{3} + \frac{8d_1}{x-2} - \frac{d_1}{x-1} - \frac{16d_1}{3(x-2)^2} - \frac{4d_1}{3(x-1)^2} + \frac{16d_1}{3(x-2)^3} - \frac{2d_1}{(x-1)^3} - \frac{4d_1}{(x-1)^4} \Big) H(1; \alpha_0) + \frac{32d_1}{3(x-2)} - \frac{82}{3(x-2)} - \\
& \frac{35d_1}{12(x-1)} + \frac{73}{12(x-1)} - \frac{28d_1}{9(x-2)^2} + \frac{56}{9(x-2)^2} - \frac{28d_1}{9(x-1)^2} + \frac{323}{36(x-1)^2} + \frac{28d_1}{9(x-2)^3} - \frac{56}{9(x-2)^3} - \frac{5d_1}{(x-1)^3} + \frac{53}{4(x-1)^3} - \\
& \frac{4d_1}{(x-1)^4} + \frac{29}{2(x-1)^4} - \frac{725}{36} \Big) H(c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{100}{3} + \frac{12}{x-1} - \frac{8}{3(x-1)^2} - \frac{8}{3(x-1)^3} + \frac{12}{(x-1)^4} + \right. \\
& \frac{100}{3(x-1)^5} \Big) H(0,0,0;\alpha_0) + \left( -\frac{100}{3} - \frac{12}{x-1} + \frac{8}{3(x-1)^2} + \frac{8}{3(x-1)^3} - \frac{12}{(x-1)^4} - \frac{100}{3(x-1)^5} \right) H(0,0,0;x) + \\
& \left( \frac{6d_1}{x-1} - \frac{4d_1}{3(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \frac{6d_1}{(x-1)^4} + \frac{50d_1}{3(x-1)^5} + \frac{50d_1}{3} \right) H(0,0,1;\alpha_0) + \left( -\frac{4d_1}{(x-1)^5} + 4d_1 + \frac{12}{(x-1)^5} - \right. \\
& 12 \Big) H(0;\alpha_0) H(0,0,1;x) + \left( -\frac{\alpha_0^4}{x-1} + \alpha_0^4 + \frac{4\alpha_0^3}{x-1} - \frac{4\alpha_0^3}{3(x-1)^2} - \frac{16\alpha_0^3}{3} - \frac{6\alpha_0^2}{x-1} + \frac{4\alpha_0^2}{(x-1)^2} - \frac{2\alpha_0^2}{(x-1)^3} + 12\alpha_0^2 + \right. \\
& \frac{4\alpha_0}{x-1} - \frac{4\alpha_0}{(x-1)^2} + \frac{4\alpha_0}{(x-1)^3} - \frac{4\alpha_0}{(x-1)^4} - 16\alpha_0 + \left( \frac{8}{(x-1)^5} - 8 \right) H(0;\alpha_0) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(1;\alpha_0) - \frac{8}{x-2} + \\
& \frac{4}{x-1} + \frac{16}{3(x-2)^2} + \frac{2}{3(x-1)^2} - \frac{16}{3(x-2)^3} + \frac{4}{3(x-1)^3} + \frac{7}{(x-1)^4} + \frac{25}{3(x-1)^5} \Big) H(0,0,c_1(\alpha_0); x) + \left( \frac{6d_1}{x-1} - \right. \\
& \frac{4d_1}{3(x-1)^2} - \frac{4d_1}{3(x-1)^3} + \frac{6d_1}{(x-1)^4} + \frac{50d_1}{3(x-1)^5} + \frac{50d_1}{3} \Big) H(0,1,0;\alpha_0) + \left( -\frac{8d_1}{x-2} + \frac{16d_1}{3(x-2)^2} + \frac{8d_1}{3(x-1)^2} - \frac{16d_1}{3(x-2)^3} + \right. \\
& \frac{8d_1}{(x-1)^4} + \frac{8}{x-2} - \frac{5}{x-1} - \frac{16}{3(x-2)^2} + \frac{2}{3(x-1)^2} + \frac{16}{3(x-2)^3} - \frac{10}{3(x-1)^3} - \frac{3}{(x-1)^4} - \frac{25}{3(x-1)^5} + \frac{25}{3} \Big) H(0,1,0;x) + \\
& \left( \frac{3d_1^2}{x-1} - \frac{2d_1^2}{3(x-1)^2} - \frac{2d_1^2}{3(x-1)^3} + \frac{3d_1^2}{(x-1)^4} + \frac{25d_1^2}{3(x-1)^5} + \frac{25d_1^2}{3} \right) H(0,1,1;\alpha_0) + \left( \frac{4d_1^2}{(x-1)^5} - 4d_1^2 - \frac{12d_1}{(x-1)^5} + \right. \\
& 4d_1 + \frac{16}{(x-1)^5} \Big) H(0;\alpha_0) H(0,1,1;x) + \left( \frac{8d_1}{x-2} - \frac{16d_1}{3(x-2)^2} - \frac{8d_1}{3(x-1)^2} + \frac{16d_1}{3(x-2)^3} - \frac{8d_1}{(x-1)^4} + \left( -\frac{8d_1}{(x-1)^5} + \right. \right. \\
& 8d_1 + \frac{8}{(x-1)^5} - 8 \Big) H(0;\alpha_0) + \left( -\frac{4d_1^2}{(x-1)^5} + 4d_1^2 + \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(1;\alpha_0) - \frac{8}{x-2} + \frac{5}{x-1} + \frac{16}{3(x-2)^2} - \\
& \frac{2}{3(x-1)^2} - \frac{16}{3(x-2)^3} + \frac{10}{3(x-1)^3} + \frac{3}{(x-1)^4} + \frac{25}{3(x-1)^5} - \frac{25}{3} \Big) H(0,1,c_1(\alpha_0); x) + \left( -\frac{4d_1}{(x-1)^5} + 4d_1 + \right. \\
& \frac{4}{(x-1)^5} - 4 \Big) H(0;\alpha_0) H(0,2,1;x) + \left( \frac{2\alpha_0^4}{x-1} - 2\alpha_0^4 - \frac{8\alpha_0^3}{x-1} + \frac{8\alpha_0^3}{3(x-1)^2} + \frac{32\alpha_0^3}{3} + \frac{12\alpha_0^2}{x-1} - \frac{8\alpha_0^2}{(x-1)^2} + \frac{4\alpha_0^2}{(x-1)^3} - \right. \\
& 24\alpha_0^2 - \frac{8\alpha_0}{x-1} + \frac{8\alpha_0}{(x-1)^2} - \frac{8\alpha_0}{(x-1)^3} + \frac{8\alpha_0}{(x-1)^4} + 32\alpha_0 - 16H(0;\alpha_0) - 8d_1 H(1;\alpha_0) - \frac{20}{x-2} + \frac{11}{2(x-1)} + \\
& \frac{40}{3(x-2)^2} + \frac{8}{3(x-1)^2} - \frac{40}{3(x-2)^3} + \frac{13}{3(x-1)^3} + \frac{13}{(x-1)^4} + \frac{25}{3(x-1)^5} - \frac{25}{2} \Big) H(0,c_1(\alpha_0), c_1(\alpha_0); x) + \left( \frac{\alpha_0^4}{x-1} - \right. \\
& \alpha_0^4 - \frac{4\alpha_0^3}{x-1} + \frac{4\alpha_0^3}{3(x-1)^2} + \frac{16\alpha_0^3}{3} + \frac{6\alpha_0^2}{x-1} - \frac{4\alpha_0^2}{(x-1)^2} + \frac{2\alpha_0^2}{(x-1)^3} - 12\alpha_0^2 - \frac{4\alpha_0}{x-1} + \frac{4\alpha_0}{(x-1)^2} - \frac{4\alpha_0}{(x-1)^3} + \frac{4\alpha_0}{(x-1)^4} + \\
& 16\alpha_0 + \left( \frac{8}{(x-1)^5} - 8 \right) H(0;\alpha_0) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(1;\alpha_0) + \frac{16}{x-2} - \frac{13}{2(x-1)} - \frac{32}{3(x-2)^2} - \frac{5}{3(x-1)^2} + \\
& \frac{32}{3(x-2)^3} - \frac{3}{(x-1)^3} - \frac{25}{2(x-1)^4} - \frac{25}{2(x-1)^5} + \frac{25}{6} \Big) H(0,c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{8d_1}{x-1} - \frac{4d_1}{(x-1)^2} + \frac{8d_1}{3(x-1)^3} - \right. \\
& \frac{2d_1}{(x-1)^4} + \frac{50d_1}{3(x-1)^5} + \frac{16}{x-2} - \frac{32}{3(x-2)^2} - \frac{16}{3(x-1)^2} + \frac{32}{3(x-2)^3} - \frac{16}{(x-1)^4} \Big) H(1,0,0;x) + \left( \frac{4d_1^2}{(x-1)^5} - \frac{12d_1}{(x-1)^5} + \right. \\
& \frac{12}{(x-1)^5} - 4 \Big) H(0;\alpha_0) H(1,0,1;x) + \left( \left( \frac{16}{(x-1)^5} - \frac{8d_1}{(x-1)^5} \right) H(0;\alpha_0) + \left( \frac{8d_1}{(x-1)^5} - \frac{4d_1^2}{(x-1)^5} \right) H(1;\alpha_0) + \right.
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{4}{x-2} - \frac{7}{2(x-1)} - \frac{8}{3(x-2)^2} + \frac{8}{3(x-2)^3} - \frac{1}{3(x-1)^3} - \frac{5}{(x-1)^4} - \frac{25}{3(x-1)^5} - \frac{25}{6} \right) H(1, 0, c_1(\alpha_0); x) + \left( -\frac{4}{x-1} + \frac{2d_1^2}{(x-1)^2} - \frac{4d_1^2}{3(x-1)^3} + \frac{d_1^2}{(x-1)^4} - \frac{25d_1^2}{3(x-1)^5} - \frac{8d_1}{x-2} + \frac{16d_1}{3(x-2)^2} + \frac{8d_1}{3(x-1)^2} - \frac{16d_1}{3(x-2)^3} + \frac{8d_1}{(x-1)^4} - \frac{4}{x-2} + \frac{7}{2(x-1)} + \frac{8}{3(x-2)^2} - \frac{8}{3(x-2)^3} + \frac{1}{3(x-1)^3} + \frac{5}{(x-1)^4} + \frac{25}{3(x-1)^5} + \frac{25}{6} \right) H(1, 1, 0; x) + \left( \frac{12d_1^2}{(x-1)^5} - 4d_1^2 - \frac{24d_1}{(x-1)^5} + \frac{12}{(x-1)^5} + 4 \right) H(0; \alpha_0) H(1, 1, 1; x) + \left( \frac{4d_1^2}{x-1} - \frac{2d_1^2}{(x-1)^2} + \frac{4d_1^2}{3(x-1)^3} - \frac{d_1^2}{(x-1)^4} + \frac{25d_1^2}{3(x-1)^5} + \frac{8d_1}{x-2} - \frac{16d_1}{3(x-2)^2} - \frac{8d_1}{3(x-1)^2} + \frac{16d_1}{3(x-2)^3} - \frac{8d_1}{(x-1)^4} + \left( -\frac{16d_1}{(x-1)^5} + 8d_1 + \frac{16}{(x-1)^5} \right) H(0; \alpha_0) + \left( -\frac{8d_1^2}{(x-1)^5} + 4d_1^2 + \frac{8d_1}{(x-1)^5} \right) H(1; \alpha_0) + \frac{4}{x-2} - \frac{7}{2(x-1)} - \frac{8}{3(x-2)^2} + \frac{8}{3(x-2)^3} - \frac{1}{3(x-1)^3} - \frac{5}{(x-1)^4} - \frac{25}{3(x-1)^5} - \frac{25}{6} \right) H(1, 1, c_1(\alpha_0); x) + \left( -\frac{4d_1}{(x-1)^5} + \frac{2}{(x-1)^5} + 6 \right) H(0; \alpha_0) H(1, 2, 1; x) + \left( -\frac{8d_1}{x-1} + \frac{4d_1}{(x-1)^2} - \frac{8d_1}{3(x-1)^3} + \frac{2d_1}{(x-1)^4} - \frac{50d_1}{3(x-1)^5} + \left( \frac{8d_1}{(x-1)^5} - 16 \right) H(0; \alpha_0) + \left( \frac{4d_1^2}{(x-1)^5} - 8d_1 \right) H(1; \alpha_0) - \frac{20}{x-2} + \frac{7}{2(x-1)} + \frac{40}{3(x-2)^2} + \frac{16}{3(x-2)^2} - \frac{40}{3(x-2)^3} + \frac{1}{3(x-1)^3} + \frac{21}{(x-1)^4} + \frac{25}{3(x-1)^5} + \frac{25}{6} \right) H(1, c_1(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{4d_1}{(x-1)^5} + 4d_1 + \frac{4}{(x-1)^5} - 4 \right) H(0; \alpha_0) H(2, 0, 1; x) + \left( -\frac{8d_1}{x-2} + \frac{16d_1}{3(x-2)^2} + \frac{8d_1}{3(x-1)^2} - \frac{16d_1}{3(x-2)^3} + \frac{8d_1}{(x-1)^4} + \left( \frac{8}{(x-1)^5} - 8 \right) H(0; \alpha_0) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(1; \alpha_0) + \frac{16}{x-2} - \frac{15}{2(x-1)} - \frac{32}{3(x-2)^2} - \frac{1}{3(x-1)^2} + \frac{32}{3(x-2)^3} - \frac{5}{(x-1)^3} - \frac{17}{2(x-1)^4} - \frac{25}{2(x-1)^5} + \frac{25}{2} \right) H(2, 0, c_1(\alpha_0); x) + \left( \frac{8d_1}{x-2} - \frac{16d_1}{3(x-2)^2} - \frac{8d_1}{3(x-1)^2} + \frac{16d_1}{3(x-2)^3} - \frac{8d_1}{(x-1)^4} - \frac{16}{x-2} + \frac{15}{2(x-1)} + \frac{32}{3(x-2)^2} + \frac{1}{3(x-1)^2} - \frac{32}{3(x-2)^3} + \frac{5}{(x-1)^3} + \frac{17}{2(x-1)^4} + \frac{25}{2(x-1)^5} - \frac{25}{2} \right) H(2, 1, 0; x) + \left( -\frac{4d_1}{(x-1)^5} + 4d_1 - \frac{2}{(x-1)^5} + 2 \right) H(0; \alpha_0) H(2, 1, 1; x) + \left( -\frac{8d_1}{x-2} + \frac{16d_1}{3(x-2)^2} + \frac{8d_1}{3(x-1)^2} - \frac{16d_1}{3(x-2)^3} + \frac{8d_1}{(x-1)^4} + \left( \frac{8}{(x-1)^5} - 8 \right) H(0; \alpha_0) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 \right) H(1; \alpha_0) + \frac{16}{x-2} - \frac{15}{2(x-1)} - \frac{32}{3(x-2)^2} - \frac{1}{3(x-1)^2} + \frac{32}{3(x-2)^3} - \frac{5}{(x-1)^3} - \frac{17}{2(x-1)^4} - \frac{25}{2(x-1)^5} + \frac{25}{2} \right) H(2, 1, c_1(\alpha_0); x) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 - \frac{8}{(x-1)^5} + 8 \right) H(0; \alpha_0) H(2, 2, 1; x) + \left( \frac{8d_1}{x-2} - \frac{16d_1}{3(x-2)^2} - \frac{8d_1}{3(x-1)^2} + \frac{16d_1}{3(x-2)^3} - \frac{8d_1}{(x-1)^4} + \left( 8 - \frac{8}{(x-1)^5} \right) H(0; \alpha_0) + \left( 4d_1 - \frac{4d_1}{(x-1)^5} \right) H(1; \alpha_0) - \frac{16}{x-2} + \frac{15}{2(x-1)} + \frac{32}{3(x-2)^2} + \frac{1}{3(x-1)^2} - \frac{32}{3(x-2)^3} + \frac{5}{(x-1)^3} + \frac{17}{2(x-1)^4} + \frac{25}{2(x-1)^5} - \frac{25}{2} \right) H(2, c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{\alpha_0^4}{x-1} - \alpha_0^4 - \frac{4\alpha_0^3}{x-1} + \frac{4\alpha_0^3}{3(x-1)^2} + \frac{16\alpha_0^3}{3} + \frac{6\alpha_0^2}{x-1} - \frac{4\alpha_0^2}{(x-1)^2} + \frac{2\alpha_0^2}{(x-1)^3} - 12\alpha_0^2 - \frac{4\alpha_0}{x-1} + \frac{4\alpha_0}{(x-1)^2} - \frac{4\alpha_0}{(x-1)^3} + \frac{4\alpha_0}{(x-1)^4} + 16\alpha_0 + \frac{8H(0; \alpha_0)}{(x-1)^5} + \frac{4d_1 H(1; \alpha_0)}{(x-1)^5} - \frac{3}{x-1} + \frac{2}{3(x-1)^2} + \frac{2}{3(x-1)^3} - \frac{3}{(x-1)^4} - \frac{25}{3(x-1)^5} - \frac{25}{3} \right) H(c_1(\alpha_0), 0, c_1(\alpha_0); x) + \left( -\frac{3\alpha_0^4}{x-1} + 3\alpha_0^4 + \frac{12\alpha_0^3}{x-1} - \frac{4\alpha_0^3}{(x-1)^2} - 16\alpha_0^3 - \frac{18\alpha_0^2}{x-1} + \frac{12\alpha_0^2}{(x-1)^2} - \frac{6\alpha_0^2}{(x-1)^3} + 36\alpha_0^2 + \frac{12\alpha_0}{x-1} - \frac{12\alpha_0}{(x-1)^2} + \frac{12\alpha_0}{(x-1)^3} - \frac{12\alpha_0}{(x-1)^4} - 48\alpha_0 - \frac{16H(0; \alpha_0)}{(x-1)^5} - \frac{8d_1 H(1; \alpha_0)}{(x-1)^5} + \frac{9}{x-1} - \frac{2}{(x-1)^2} - \frac{2}{(x-1)^3} + \frac{9}{(x-1)^4} + \frac{25}{(x-1)^5} + 25 \right) H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \left( -\frac{3\alpha_0^4}{2(x-1)} + \frac{3\alpha_0^4}{2} + \frac{6\alpha_0^3}{x-1} - \frac{2\alpha_0^3}{(x-1)^2} - 8\alpha_0^3 - \frac{9\alpha_0^2}{x-1} + \frac{6\alpha_0^2}{(x-1)^2} - \frac{3\alpha_0^2}{(x-1)^3} + 18\alpha_0^2 + \frac{6\alpha_0}{x-1} - \frac{6\alpha_0}{(x-1)^2} + \frac{6\alpha_0}{(x-1)^3} - \frac{6\alpha_0}{(x-1)^4} - 24\alpha_0 - \frac{8H(0; \alpha_0)}{(x-1)^5} - \frac{4d_1 H(1; \alpha_0)}{(x-1)^5} + \frac{9}{2(x-1)} - \frac{1}{(x-1)^2} - \frac{1}{(x-1)^3} + \frac{9}{2(x-1)^4} + \frac{25}{2(x-1)^5} + \frac{25}{2} \right) H(c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x) + \left( \frac{\alpha_0^4}{x-1} - \alpha_0^4 - \frac{4\alpha_0^3}{x-1} + \frac{4\alpha_0^3}{3(x-1)^2} + \frac{16\alpha_0^3}{3} + \frac{6\alpha_0^2}{x-1} - \frac{4\alpha_0^2}{(x-1)^2} + \frac{2\alpha_0^2}{(x-1)^3} - 12\alpha_0^2 - \frac{4\alpha_0}{x-1} + \frac{4\alpha_0}{(x-1)^2} - \frac{4\alpha_0}{(x-1)^3} + \frac{4\alpha_0}{(x-1)^4} + 16\alpha_0 - \frac{8}{x-2} + \frac{1}{x-1} + \frac{16}{3(x-2)^2} + \frac{4}{3(x-1)^2} - \frac{16}{3(x-2)^3} + \frac{2}{(x-1)^3} + \frac{4}{(x-1)^4} - \frac{25}{3} \right) H(c_2(\alpha_0), 0, c_1(\alpha_0); x) + \left( -\frac{5\alpha_0^4}{2(x-1)} + \frac{5\alpha_0^4}{2} + \frac{10\alpha_0^3}{x-1} - \frac{10\alpha_0^3}{3(x-1)^2} - \frac{40\alpha_0^3}{3} - \frac{15\alpha_0^2}{x-1} + \frac{10\alpha_0^2}{(x-1)^2} - \frac{5\alpha_0^2}{(x-1)^3} + 30\alpha_0^2 + \frac{10\alpha_0}{x-1} - \frac{10\alpha_0}{(x-1)^2} + \frac{10\alpha_0}{(x-1)^3} - \frac{10\alpha_0}{(x-1)^4} - 40\alpha_0 + \frac{20}{x-2} - \frac{5}{2(x-1)} - \frac{40}{3(x-2)^2} - \frac{10}{3(x-1)^2} + \frac{40}{3(x-2)^3} - \frac{5}{(x-1)^3} - \frac{10}{(x-1)^4} + \frac{125}{6} \right) H(c_2(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + 64 H(0, 0, 0, 0; x) + \left( \frac{12}{(x-1)^5} - 12 \right) H(0, 0, 0, c_1(\alpha_0); x) + \left( \frac{4d_1}{(x-1)^5} - 4d_1 - \frac{12}{(x-1)^5} + 12 \right) H(0, 0, 1, 0; x) + \left( -\frac{4d_1}{(x-1)^5} + \right)
\end{aligned}$$

$$\begin{aligned}
& 4d_1 + \frac{12}{(x-1)^5} - 12) H(0, 0, 1, c_1(\alpha_0); x) + \left(-8 - \frac{8}{(x-1)^5}\right) H(0, 0, c_1(\alpha_0), c_1(\alpha_0); x) + \left(\frac{4}{(x-1)^5} - \right. \\
& 4) H(0, 0, c_2(\alpha_0), c_1(\alpha_0); x) + \left(\frac{8d_1}{(x-1)^5} - 8d_1 - \frac{8}{(x-1)^5} + 8\right) H(0, 1, 0, 0; x) + \left(\frac{16}{(x-1)^5} - \right. \\
& \left.\frac{8d_1}{(x-1)^5}\right) H(0, 1, 0, c_1(\alpha_0); x) + \left(-\frac{4d_1^2}{(x-1)^5} + 4d_1^2 + \frac{12d_1}{(x-1)^5} - 4d_1 - \frac{16}{(x-1)^5}\right) H(0, 1, 1, 0; x) + \\
& \left(\frac{4d_1^2}{(x-1)^5} - 4d_1^2 - \frac{12d_1}{(x-1)^5} + 4d_1 + \frac{16}{(x-1)^5}\right) H(0, 1, 1, c_1(\alpha_0); x) + \left(8d_1 - \frac{8}{(x-1)^5} - \right. \\
& 8) H(0, 1, c_1(\alpha_0), c_1(\alpha_0); x) + \left(-\frac{4d_1}{(x-1)^5} + 4d_1 + \frac{4}{(x-1)^5} - 4\right) H(0, 2, 0, c_1(\alpha_0); x) + \left(\frac{4d_1}{(x-1)^5} - \right. \\
& 4d_1 - \frac{4}{(x-1)^5} + 4) H(0, 2, 1, 0; x) + \left(-\frac{4d_1}{(x-1)^5} + 4d_1 + \frac{4}{(x-1)^5} - 4\right) H(0, 2, 1, c_1(\alpha_0); x) + \left(\frac{4d_1}{(x-1)^5} - \right. \\
& 4d_1 - \frac{4}{(x-1)^5} + 4) H(0, 2, c_2(\alpha_0), c_1(\alpha_0); x) + \left(4 + \frac{4}{(x-1)^5}\right) H(0, c_1(\alpha_0), 0, c_1(\alpha_0); x) + \left(- \right. \\
& 12 - \frac{4}{(x-1)^5}) H(0, c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \left(-6 - \frac{2}{(x-1)^5}\right) H(0, c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x) + \\
& \left(4 - \frac{4}{(x-1)^5}\right) H(0, c_2(\alpha_0), 0, c_1(\alpha_0); x) + \left(\frac{10}{(x-1)^5} - 10\right) H(0, c_2(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \\
& \left(16 - \frac{16}{(x-1)^5}\right) H(1, 0, 0, 0; x) + \left(-\frac{4d_1}{(x-1)^5} + \frac{12}{(x-1)^5} - 4\right) H(1, 0, 0, c_1(\alpha_0); x) + \left(-\frac{4d_1^2}{(x-1)^5} + \right. \\
& \left.\frac{12d_1}{(x-1)^5} - \frac{12}{(x-1)^5} + 4\right) H(1, 0, 1, 0; x) + \left(\frac{4d_1^2}{(x-1)^5} - \frac{12d_1}{(x-1)^5} + \frac{12}{(x-1)^5} - 4\right) H(1, 0, 1, c_1(\alpha_0); x) + \\
& \left(\frac{4}{(x-1)^5} - 4\right) H(1, 0, c_1(\alpha_0), c_1(\alpha_0); x) + \left(-\frac{4d_1}{(x-1)^5} + \frac{2}{(x-1)^5} + 6\right) H(1, 0, c_2(\alpha_0), c_1(\alpha_0); x) + \\
& \left(\frac{16d_1}{(x-1)^5} - 8d_1 - \frac{16}{(x-1)^5}\right) H(1, 1, 0, 0; x) + \left(\frac{4d_1^2}{(x-1)^5} - \frac{16d_1}{(x-1)^5} + \frac{12}{(x-1)^5} + 4\right) H(1, 1, 0, c_1(\alpha_0); x) + \\
& \left(-\frac{12d_1^2}{(x-1)^5} + 4d_1^2 + \frac{24d_1}{(x-1)^5} - \frac{12}{(x-1)^5} - 4\right) H(1, 1, 1, 0; x) + \left(\frac{12d_1^2}{(x-1)^5} - 4d_1^2 - \frac{24d_1}{(x-1)^5} + \frac{12}{(x-1)^5} + \right. \\
& 4) H(1, 1, 1, c_1(\alpha_0); x) + \left(-\frac{4d_1^2}{(x-1)^5} + 8d_1 + \frac{4}{(x-1)^5} - 4\right) H(1, 1, c_1(\alpha_0), c_1(\alpha_0); x) + \left(-\frac{4d_1}{(x-1)^5} + \right. \\
& \left.\frac{2}{(x-1)^5} + 6\right) H(1, 2, 0, c_1(\alpha_0); x) + \left(\frac{4d_1}{(x-1)^5} - \frac{2}{(x-1)^5} - 6\right) H(1, 2, 1, 0; x) + \left(-\frac{4d_1}{(x-1)^5} + \frac{2}{(x-1)^5} + \right. \\
& 6) H(1, 2, 1, c_1(\alpha_0); x) + \left(\frac{4d_1}{(x-1)^5} - \frac{2}{(x-1)^5} - 6\right) H(1, 2, c_2(\alpha_0), c_1(\alpha_0); x) + \left(-\frac{4d_1}{(x-1)^5} + \right. \\
& \left.\frac{4}{(x-1)^5} + 4\right) H(1, c_1(\alpha_0), 0, c_1(\alpha_0); x) + \left(\frac{8d_1}{(x-1)^5} - \frac{4}{(x-1)^5} - 12\right) H(1, c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) + \\
& \left(\frac{4d_1}{(x-1)^5} - \frac{2}{(x-1)^5} - 6\right) H(1, c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x) + \left(\frac{4}{(x-1)^5} - 4\right) H(2, 0, 0, c_1(\alpha_0); x) + \\
& \left(\frac{4d_1}{(x-1)^5} - 4d_1 - \frac{4}{(x-1)^5} + 4\right) H(2, 0, 1, 0; x) + \left(-\frac{4d_1}{(x-1)^5} + 4d_1 + \frac{4}{(x-1)^5} - 4\right) H(2, 0, 1, c_1(\alpha_0); x) + \\
& \left(\frac{10}{(x-1)^5} - 10\right) H(2, 0, c_1(\alpha_0), c_1(\alpha_0); x) + \left(8 - \frac{8}{(x-1)^5}\right) H(2, 0, c_2(\alpha_0), c_1(\alpha_0); x) + \left(8 - \right. \\
& \left.\frac{8}{(x-1)^5}\right) H(2, 1, 0, 0; x) + \left(2 - \frac{2}{(x-1)^5}\right) H(2, 1, 0, c_1(\alpha_0); x) + \left(\frac{4d_1}{(x-1)^5} - 4d_1 + \frac{2}{(x-1)^5} - \right. \\
& 2) H(2, 1, 1, 0; x) + \left(-\frac{4d_1}{(x-1)^5} + 4d_1 - \frac{2}{(x-1)^5} + 2\right) H(2, 1, 1, c_1(\alpha_0); x) + \left(\frac{10}{(x-1)^5} - \right. \\
& 10) H(2, 1, c_1(\alpha_0), c_1(\alpha_0); x) + \left(\frac{4d_1}{(x-1)^5} - 4d_1 - \frac{8}{(x-1)^5} + 8\right) H(2, 2, 0, c_1(\alpha_0); x) + \left(-\frac{4d_1}{(x-1)^5} + \right. \\
& 4d_1 + \frac{8}{(x-1)^5} - 8) H(2, 2, 1, 0; x) + \left(\frac{4d_1}{(x-1)^5} - 4d_1 - \frac{8}{(x-1)^5} + 8\right) H(2, 2, 1, c_1(\alpha_0); x) + \left(-\frac{4d_1}{(x-1)^5} + \right. \\
& 4d_1 + \frac{8}{(x-1)^5} - 8) H(2, 2, c_2(\alpha_0), c_1(\alpha_0); x) + \left(\frac{4}{(x-1)^5} - 4\right) H(2, c_2(\alpha_0), 0, c_1(\alpha_0); x) + \left(10 - \right. \\
& \left.\frac{10}{(x-1)^5}\right) H(2, c_2(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x) - \frac{4H(c_1(\alpha_0), 0, 0, c_1(\alpha_0); x)}{(x-1)^5} + \frac{8H(c_1(\alpha_0), 0, c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \\
& \frac{4H(c_1(\alpha_0), 0, c_2(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \frac{4H(c_1(\alpha_0), c_1(\alpha_0), 0, c_1(\alpha_0); x)}{(x-1)^5} - \frac{12H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} - \\
& \frac{6H(c_1(\alpha_0), c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \frac{4H(c_1(\alpha_0), c_2(\alpha_0), 0, c_1(\alpha_0); x)}{(x-1)^5} - \frac{10H(c_1(\alpha_0), c_2(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); x)}{(x-1)^5} + \\
& H(2; x) \left(\frac{2\pi^2 d_1}{x-2} - \frac{4\pi^2 d_1}{3(x-2)^2} - \frac{2\pi^2 d_1}{3(x-1)^2} + \frac{4\pi^2 d_1}{3(x-2)^3} - \frac{2\pi^2 d_1}{(x-1)^4} - \frac{4\pi^2}{x-2} + \frac{15\pi^2}{8(x-1)} + \frac{8\pi^2}{3(x-2)^2} + \frac{\pi^2}{12(x-1)^2} - \right. \\
& \left. \frac{8\pi^2}{3(x-2)^3} + \frac{5\pi^2}{4(x-1)^3} + \frac{17\pi^2}{8(x-1)^4} + \frac{25\pi^2}{8(x-1)^5} - \frac{7\zeta_3}{2(x-1)^5} + \frac{7\zeta_3}{2} + \frac{3\pi^2 \ln 2}{(x-1)^5} - 3\pi^2 \ln 2 - \frac{25\pi^2}{8}\right) + H(0; x) \left(- \right.
\end{aligned}$$



$$\begin{aligned}
& \frac{63d_1^2}{16(x-1)} + \frac{19d_1^2}{54(x-1)^2} + \frac{19d_1^2}{54(x-1)^3} - \frac{63d_1^2}{16(x-1)^4} - \frac{2035d_1^2}{432(x-1)^5} - \frac{2035d_1^2}{432} - \frac{38d_1}{3(x-2)} + \frac{161d_1}{12(x-1)} + \frac{76d_1}{9(x-2)^2} - \\
& \frac{323d_1}{108(x-1)^2} - \frac{605d_1}{108(x-1)^3} + \frac{50d_1}{3(x-1)^4} + \frac{895d_1}{27(x-1)^5} + \frac{2035d_1}{108} + \frac{4\pi^2}{x-2} + \frac{68}{3(x-2)} - \frac{3\pi^2}{8(x-1)} - \frac{3}{2(x-1)} - \frac{8\pi^2}{3(x-2)^2} - \\
& \frac{152}{9(x-2)^2} - \frac{25\pi^2}{36(x-1)^2} + \frac{215}{108(x-1)^2} + \frac{8\pi^2}{3(x-2)^3} - \frac{37\pi^2}{36(x-1)^3} + \frac{1643}{108(x-1)^3} - \frac{15\pi^2}{8(x-1)^4} - \frac{8}{(x-1)^4} + \frac{25\pi^2}{72(x-1)^5} - \\
& \frac{5665}{108(x-1)^5} + \frac{21\zeta_3}{(x-1)^5} + 33\zeta_3 - \frac{2\pi^2 \ln 2}{(x-1)^5} + 2\pi^2 \ln 2 + \frac{325\pi^2}{72} - \frac{1855}{108} \Big) + H(1; x) \Big( - \frac{21\zeta_3 d_1}{2(x-1)^5} + \frac{\pi^2 \ln 2 d_1}{(x-1)^5} + \\
& \Big( - \frac{4d_1^2}{x-1} + \frac{d_1^2}{(x-1)^2} - \frac{4d_1^2}{9(x-1)^3} + \frac{d_1^2}{4(x-1)^4} - \frac{205d_1^2}{36(x-1)^5} - \frac{46d_1}{3(x-2)} + \frac{46d_1}{3(x-1)} + \frac{52d_1}{9(x-2)^2} - \frac{40d_1}{9(x-1)^2} - \\
& \frac{28d_1}{9(x-2)^3} + \frac{98d_1}{9(x-1)^3} - \frac{13d_1}{6(x-1)^4} + \frac{415d_1}{18(x-1)^5} + \frac{80}{3(x-2)} + \frac{43}{6(x-1)} - \frac{56}{9(x-2)^2} - \frac{299}{18(x-1)^2} + \frac{56}{9(x-2)^3} - \\
& \frac{23}{6(x-1)^3} - \frac{203}{6(x-1)^4} - \frac{\pi^2}{6(x-1)^5} - \frac{35}{6(x-1)^5} + \frac{\pi^2}{6} + \frac{35}{6} \Big) H(0; \alpha_0) + \Big( - \frac{8d_1}{x-1} + \frac{4d_1}{(x-1)^2} - \frac{8d_1}{3(x-1)^3} + \\
& \frac{2d_1}{(x-1)^4} - \frac{50d_1}{3(x-1)^5} - \frac{16}{x-2} + \frac{32}{3(x-2)^2} + \frac{16}{3(x-1)^2} - \frac{32}{3(x-2)^3} + \frac{16}{(x-1)^4} \Big) H(0, 0; \alpha_0) + \Big( - \frac{4d_1^2}{x-1} + \frac{2d_1^2}{(x-1)^2} - \\
& \frac{4d_1^2}{3(x-1)^3} + \frac{d_1^2}{(x-1)^4} - \frac{25d_1^2}{3(x-1)^5} - \frac{8d_1}{x-2} + \frac{16d_1}{3(x-2)^2} + \frac{8d_1}{3(x-1)^2} - \frac{16d_1}{3(x-2)^3} + \frac{8d_1}{(x-1)^4} \Big) H(0, 1; \alpha_0) + \Big( \frac{16}{(x-1)^5} - \\
& 16 \Big) H(0, 0, 0; \alpha_0) + \Big( \frac{8d_1}{(x-1)^5} - 8d_1 \Big) H(0, 0, 1; \alpha_0) + \Big( \frac{8d_1}{(x-1)^5} - 8d_1 \Big) H(0, 1, 0; \alpha_0) + \Big( \frac{4d_1^2}{(x-1)^5} - \\
& 4d_1^2 \Big) H(0, 1, 1; \alpha_0) - \frac{2\pi^2}{3(x-2)} + \frac{7\pi^2}{12(x-1)} + \frac{4\pi^2}{9(x-2)^2} - \frac{4\pi^2}{9(x-2)^3} + \frac{\pi^2}{18(x-1)^3} + \frac{5\pi^2}{6(x-1)^4} + \frac{25\pi^2}{18(x-1)^5} + \\
& \frac{21\zeta_3}{4(x-1)^5} + \frac{63\zeta_3}{4} - \frac{\pi^2 \ln 2}{2(x-1)^5} - \frac{3}{2}\pi^2 \ln 2 + \frac{25\pi^2}{36} \Big) + \frac{8d_1\pi^2}{3(x-2)} - \frac{37\pi^2}{6(x-2)} - \frac{35d_1\pi^2}{48(x-1)} + \frac{31\pi^2}{48(x-1)} - \frac{7d_1\pi^2}{9(x-2)^2} + \\
& \frac{10\pi^2}{9(x-2)^2} - \frac{7d_1\pi^2}{9(x-1)^2} + \frac{107\pi^2}{48(x-1)^2} + \frac{7d_1\pi^2}{9(x-2)^3} - \frac{14\pi^2}{9(x-2)^3} - \frac{5d_1\pi^2}{4(x-1)^3} + \frac{55\pi^2}{16(x-1)^3} - \frac{d_1\pi^2}{(x-1)^4} + \frac{115\pi^2}{24(x-1)^4} - \\
& \frac{263\pi^4}{720(x-1)^5} + \frac{35\pi^2}{36(x-1)^5} + \frac{7\zeta_3}{x-2} - \frac{203\zeta_3}{16(x-1)} - \frac{14\zeta_3}{3(x-2)^2} + \frac{35\zeta_3}{24(x-1)^2} + \frac{14\zeta_3}{3(x-2)^3} + \frac{7\zeta_3}{8(x-1)^3} - \frac{245\zeta_3}{16(x-1)^4} - \\
& \frac{525\zeta_3}{16(x-1)^5} - \frac{1225\zeta_3}{48} + \frac{4\text{Li}_4\left(\frac{1}{2}\right)}{(x-1)^5} - 4\text{Li}_4\frac{1}{2} + \frac{\ln^4 2}{6(x-1)^5} - \frac{\ln^4 2}{6} + \frac{4\pi^2 \ln^2 2}{3(x-1)^5} - \frac{4}{3}\pi^2 \ln^2 2 - \frac{6\pi^2 \ln 2}{x-2} + \frac{15\pi^2 \ln 2}{8(x-1)} + \\
& \frac{4\pi^2 \ln 2}{(x-2)^2} + \frac{3\pi^2 \ln 2}{4(x-1)^2} - \frac{4\pi^2 \ln 2}{(x-2)^3} + \frac{5\pi^2 \ln 2}{4(x-1)^3} + \frac{33\pi^2 \ln 2}{8(x-1)^4} + \frac{25\pi^2 \ln 2}{8(x-1)^5} - \frac{25}{8}\pi^2 \ln 2 - \frac{21\pi^4}{80} + \frac{205d_1\pi^2}{144} - \frac{265\pi^2}{48}.
\end{aligned}$$

## F. The $\mathcal{J}*\mathcal{I}$ -type integrals

### F.1 The $\mathcal{J}*\mathcal{I}$ integral for $k = 0$

The  $\varepsilon$  expansion for this integral reads

$$\mathcal{J}*\mathcal{I}(Y, \varepsilon; y_0, d'_0, \alpha_0, d_0; 0) = \frac{1}{\varepsilon^3}(j * i)_{-3}^{(0)} + \frac{1}{\varepsilon^2}(j * i)_{-2}^{(0)} + \frac{1}{\varepsilon}(j * i)_{-1}^{(0)} + (j * i)_0^{(0)} + \mathcal{O}(\varepsilon), \quad (\text{F.1})$$

where

$$(j * i)_{-3}^{(0)} = \frac{1}{2},$$

$$(j * i)_{-2}^{(0)} = \frac{2y_0^3}{3} - 3y_0^2 + 6y_0 - \frac{1}{2}H(0; Y) - 2H(0; y_0) + 1,$$

$$\begin{aligned}
(j * i)_{-1}^{(0)} = & -\frac{1}{6}y_0^3\alpha_0^4 + \frac{y_0^2\alpha_0^4}{2} - \frac{y_0\alpha_0^4}{2} + \frac{8y_0^3\alpha_0^3}{9} - 3y_0^2\alpha_0^3 + \frac{10y_0\alpha_0^3}{3} - 2y_0^3\alpha_0^2 + \frac{15y_0^2\alpha_0^2}{2} - \\
& 10y_0\alpha_0^2 + \frac{8y_0^3\alpha_0}{3} - 11y_0^2\alpha_0 + 18y_0\alpha_0 - \frac{2d'_1y_0^3}{9} + 2y_0^3 + \frac{7d'_1y_0^2}{6} - \frac{21y_0^2}{2} - \frac{11d'_1y_0}{3} + 30y_0 + \left( - \frac{2y_0^3}{3} + \right. \\
& \left. 3y_0^2 - 6y_0 + \frac{2}{y_0-1} + 2 \right) H(0; \alpha_0) - \frac{2}{3}y_0^3H(0; Y) + 3y_0^2H(0; Y) - 6y_0H(0; Y) - H(0; Y) + \left( - \right. \\
& \left. 2y_0^3 + 9y_0^2 - 18y_0 + 2H(0; Y) - \frac{2}{y_0-1} - 6 \right) H(0; y_0) + \left( - \frac{2d'_1y_0^3}{3} + 3d'_1y_0^2 - 6d'_1y_0 + \frac{11d'_1}{3} - \right. \\
& \left. 2H(0; \alpha_0) \right) H(1; y_0) + \left( - 2\alpha_0 + \frac{2}{y_0-1} + 2 \right) H(c_1(\alpha_0); y_0) + \frac{1}{2}H(0, 0; Y) + 8H(0, 0; y_0) + \\
& 2d'_1H(0, 1; y_0) - 2H(0, c_1(\alpha_0); y_0) + \frac{1}{2}H(1, 0; Y) + 2H(1, 0; y_0) - 2H(1, c_1(\alpha_0); y_0) + \frac{\pi^2}{12} + 2,
\end{aligned}$$

$$\begin{aligned}
 (j * i)_0^{(0)} = & \frac{1}{12}d_1y_0^3\alpha_0^4 + \frac{1}{18}d_1'y_0^3\alpha_0^4 - \frac{7}{12}\frac{y_0^3\alpha_0^4}{y_0} - \frac{1}{4}d_1y_0^2\alpha_0^4 - \frac{1}{6}d_1'y_0^2\alpha_0^4 + \frac{23y_0^2\alpha_0^4}{12} + \frac{1}{4}d_1y_0\alpha_0^4 + \frac{1}{6}d_1'y_0\alpha_0^4 - \frac{29y_0\alpha_0^4}{12} + \\
 & \frac{1}{6}y_0^3H(0; Y)\alpha_0^4 - \frac{1}{2}y_0^2H(0; Y)\alpha_0^4 + \frac{1}{2}y_0H(0; Y)\alpha_0^4 - \frac{13}{27}d_1y_0^3\alpha_0^3 - \frac{8}{27}d_1'y_0^3\alpha_0^3 + \frac{173y_0^3\alpha_0^3}{54} + \frac{5}{3}d_1y_0^2\alpha_0^3 + \\
 & \frac{19}{18}d_1'y_0^2\alpha_0^3 - \frac{217y_0^2\alpha_0^3}{18} - \frac{17}{9}d_1y_0\alpha_0^3 - \frac{11}{9}d_1'y_0\alpha_0^3 + \frac{305y_0\alpha_0^3}{18} - \frac{8}{9}y_0^3H(0; Y)\alpha_0^3 + 3y_0^2H(0; Y)\alpha_0^3 - \\
 & \frac{10}{3}y_0H(0; Y)\alpha_0^3 + \frac{23}{18}d_1y_0^3\alpha_0^2 + \frac{2}{3}d_1'y_0^3\alpha_0^2 - \frac{275y_0^3\alpha_0^2}{36} - 5d_1y_0^2\alpha_0^2 - \frac{11}{4}d_1'y_0^2\alpha_0^2 + \frac{581y_0^2\alpha_0^2}{18} + \frac{43}{6}d_1y_0\alpha_0^2 + \\
 & \frac{9}{2}d_1'y_0\alpha_0^2 - \frac{1003y_0\alpha_0^2}{18} + 2y_0^3H(0; Y)\alpha_0^2 - \frac{15}{2}y_0^2H(0; Y)\alpha_0^2 + 10y_0H(0; Y)\alpha_0^2 - \frac{25}{9}d_1y_0^3\alpha_0 - \\
 & \frac{8}{9}d_1'y_0^3\alpha_0 + \frac{217y_0^3\alpha_0}{18} + 12d_1y_0^2\alpha_0 + \frac{25}{6}d_1'y_0^2\alpha_0 - \frac{503y_0^2\alpha_0}{9} - \frac{65d_1y_0\alpha_0}{3} - \frac{29d_1'y_0\alpha_0}{3} + \frac{237y_0\alpha_0}{2} - \\
 & \frac{8}{3}y_0^3H(0; Y)\alpha_0 + 11y_0^2H(0; Y)\alpha_0 - 18y_0H(0; Y)\alpha_0 + \frac{2d_1^2y_0^3}{27} - \frac{8d_1'y_0^3}{9} + \frac{14y_0^3}{3} - \frac{17d_1^2y_0^2}{36} + 6d_1'y_0^2 - \\
 & \frac{111y_0^2}{4} + \frac{49d_1^2y_0}{18} - \frac{65d_1'y_0}{2} + 114y_0 + \frac{2}{9}d_1'y_0^3H(0; Y) - 2y_0^3H(0; Y) - \frac{7}{6}d_1'y_0^2H(0; Y) + \frac{21}{2}y_0^2H(0; Y) + \\
 & \frac{11}{3}d_1'y_0H(0; Y) - 30y_0H(0; Y) - \frac{1}{12}\pi^2H(0; Y) - 2H(0; Y) + H(0; \alpha_0)\left(\frac{y_0^3\alpha_0^4}{3} - y_0^2\alpha_0^4 + y_0\alpha_0^4 - \right. \\
 & \left. \frac{16y_0^3\alpha_0^3}{9} + 6y_0^2\alpha_0^3 - \frac{20y_0\alpha_0^3}{3} + 4y_0^3\alpha_0^2 - 15y_0^2\alpha_0^2 + 20y_0\alpha_0^2 - \frac{16y_0^3\alpha_0}{3} + 22y_0^2\alpha_0 - 36y_0\alpha_0 + \frac{2d_1'y_0^3}{9} - \frac{11y_0^3}{18} - \right. \\
 & \left. \frac{7d_1'y_0^2}{6} + \frac{31y_0^2}{6} - 4d_1 + 2d_1' + \frac{11d_1'y_0}{3} - \frac{131y_0}{6} + \frac{2}{3}y_0^3H(0; Y) - 3y_0^2H(0; Y) + 6y_0H(0; Y) - \frac{2H(0; Y)}{y_0-1} - \right. \\
 & \left. 2H(0; Y) - \frac{4d_1}{y_0-1} + \frac{2d_1'}{y_0-1} + \frac{61}{6(y_0-1)} + \frac{61}{6}\right) + \left(\frac{1}{3}d_1y_0^3\alpha_0^4 - d_1y_0^2\alpha_0^4 + d_1y_0\alpha_0^4 - \frac{16}{9}d_1y_0^3\alpha_0^3 + 6d_1y_0^2\alpha_0^3 - \right. \\
 & \left. \frac{20}{3}d_1y_0\alpha_0^3 + 4d_1y_0^3\alpha_0^2 - 15d_1y_0^2\alpha_0^2 + 20d_1y_0\alpha_0^2 - \frac{16}{3}d_1y_0^3\alpha_0 + 22d_1y_0^2\alpha_0 - 36d_1y_0\alpha_0 + \frac{25d_1y_0^3}{9} - \right. \\
 & \left. 12d_1y_0^2 + \frac{65d_1y_0}{3}\right)H(1; \alpha_0) - \frac{1}{12}\pi^2H(1; Y) + \left(\frac{y_0^3\alpha_0^4}{6} - \frac{y_0^2\alpha_0^4}{2} + \frac{y_0\alpha_0^4}{2} - \frac{\alpha_0^4}{6} - \frac{8y_0^3\alpha_0^3}{9} + 3y_0^2\alpha_0^3 - \frac{10y_0\alpha_0^3}{3} + \right. \\
 & \left. \frac{11\alpha_0^3}{9} + 2y_0^3\alpha_0^2 - \frac{15y_0^2\alpha_0^2}{2} + 10y_0\alpha_0^2 - \frac{9\alpha_0^2}{2} - \frac{8y_0^3\alpha_0}{3} + 11y_0^2\alpha_0 + 4d_1\alpha_0 - 2d_1'\alpha_0 - 18y_0\alpha_0 + 2H(0; Y)\alpha_0 - \right. \\
 & \left. 2\alpha_0 + \frac{25y_0^3}{18} - \frac{16y_0^2}{3} - 4d_1 + 2d_1' + \frac{49y_0}{6} + \left(4\alpha_0 - \frac{4}{y_0-1} - 4\right)H(0; \alpha_0) - \frac{2H(0; Y)}{y_0-1} - 2H(0; Y) + \right. \\
 & \left. \left(4\alpha_0d_1 - \frac{4d_1}{y_0-1} - 4d_1\right)H(1; \alpha_0) - \frac{4d_1}{y_0-1} + \frac{2d_1'}{y_0-1} + \frac{61}{6(y_0-1)} + \frac{61}{6}\right)H(c_1(\alpha_0); y_0) + \left(\frac{4y_0^3}{3} - 6y_0^2 + \right. \\
 & \left. 12y_0 - \frac{4}{y_0-1} - 4\right)H(0, 0; \alpha_0) + \frac{2}{3}y_0^3H(0, 0; Y) - 3y_0^2H(0, 0; Y) + 6y_0H(0, 0; Y) + H(0, 0; Y) + \\
 & \left(\frac{20y_0^3}{3} - 30y_0^2 + 60y_0 - 8H(0; Y) + \frac{12}{y_0-1} + 28\right)H(0, 0; y_0) + \left(\frac{4d_1y_0^3}{3} - 6d_1y_0^2 + 12d_1y_0 - 4d_1 - \right. \\
 & \left. \frac{4d_1}{y_0-1}\right)H(0, 1; \alpha_0) + H(1; y_0)\left(\frac{1}{6}d_1'y_0^3\alpha_0^4 - \frac{1}{2}d_1'y_0^2\alpha_0^4 - \frac{d_1'\alpha_0^4}{6} + \frac{1}{2}d_1'y_0\alpha_0^4 - \frac{8}{9}d_1'y_0^3\alpha_0^3 + 3d_1'y_0^2\alpha_0^3 + \right. \\
 & \left. \frac{11d_1'\alpha_0^3}{9} - \frac{10}{3}d_1'y_0\alpha_0^3 + 2d_1'y_0^3\alpha_0^2 - \frac{15}{2}d_1'y_0^2\alpha_0^2 - \frac{9d_1'\alpha_0^2}{2} + 10d_1'y_0\alpha_0^2 - \frac{8}{3}d_1'y_0^3\alpha_0 + 11d_1'y_0^2\alpha_0 + \frac{23d_1'\alpha_0}{3} - \right. \\
 & \left. 18d_1'y_0\alpha_0 + \frac{2d_1^2y_0^3}{9} - 2d_1'y_0^3 - \frac{49d_1^2}{18} - \frac{7d_1^2y_0^2}{6} + \frac{21d_1'y_0^2}{2} + \frac{43d_1'}{2} + \frac{11d_1^2y_0}{3} - 30d_1'y_0 + \frac{2}{3}d_1'y_0^3H(0; Y) - \right. \\
 & \left. 3d_1'y_0^2H(0; Y) - \frac{11}{3}d_1'H(0; Y) + 6d_1'y_0H(0; Y) + H(0; \alpha_0)\left(\frac{2d_1'y_0^3}{3} - \frac{2y_0^3}{3} - 3d_1'y_0^2 + 3y_0^2 + \right. \right. \\
 & \left. \left. 6d_1'y_0 - 6y_0 - \frac{11d_1'}{3} + 2H(0; Y) + \frac{4d_1}{y_0-1} - \frac{2d_1'}{y_0-1} - \frac{2}{y_0-1} - 10\right) + 4H(0, 0; \alpha_0) + 4d_1H(0, 1; \alpha_0) - \right. \\
 & \left. \frac{\pi^2}{3}\right) + \left(2d_1'y_0^3 - 9d_1'y_0^2 + 18d_1'y_0 + 6d_1' + (4 - 4d_1)H(0; \alpha_0) - 2d_1'H(0; Y) + \frac{2d_1'}{y_0-1}\right)H(0, 1; y_0) + \\
 & \left(-\frac{2y_0^3}{3} + 3y_0^2 - 6y_0 + 4H(0; \alpha_0) + 2H(0; Y) + 4d_1H(1; \alpha_0) - \frac{6}{y_0-1} - 10\right)H(0, c_1(\alpha_0); y_0) + \\
 & H(0; y_0)\left(\frac{y_0^3\alpha_0^4}{3} - y_0^2\alpha_0^4 + y_0\alpha_0^4 - \frac{16y_0^3\alpha_0^3}{9} + 6y_0^2\alpha_0^3 - \frac{20y_0\alpha_0^3}{3} + 4y_0^3\alpha_0^2 - 15y_0^2\alpha_0^2 + 20y_0\alpha_0^2 - \frac{16y_0^3\alpha_0}{3} + \right. \\
 & \left. 22y_0^2\alpha_0 - 36y_0\alpha_0 + \frac{2d_1'y_0^3}{3} - \frac{133y_0^3}{18} - \frac{7d_1'y_0^2}{2} + \frac{221y_0^2}{6} + 4d_1 - 2d_1' + 11d_1'y_0 - \frac{589y_0}{6} + \left(\frac{4y_0^3}{3} - \right. \right. \\
 & \left. \left. 6y_0^2 + 12y_0 - \frac{4}{y_0-1} - 4\right)H(0; \alpha_0) + 2y_0^3H(0; Y) - 9y_0^2H(0; Y) + 18y_0H(0; Y) + \frac{2H(0; Y)}{y_0-1} + \right. \\
 & \left. 6H(0; Y) - 2H(0, 0; Y) - 2H(1, 0; Y) + \frac{4d_1}{y_0-1} - \frac{2d_1'}{y_0-1} - \frac{61}{6(y_0-1)} - \frac{\pi^2}{3} - \frac{109}{6}\right) + \frac{2}{3}y_0^3H(1, 0; Y) - \\
 & 3y_0^2H(1, 0; Y) + 6y_0H(1, 0; Y) + H(1, 0; Y) + \left(2d_1'y_0^3 + \frac{2y_0^3}{3} - 9d_1'y_0^2 - 3y_0^2 + 18d_1'y_0 + 6y_0 - \right. \\
 & \left. 11d_1' + 4H(0; \alpha_0) - 2H(0; Y) - \frac{4d_1}{y_0-1} + \frac{2d_1'}{y_0-1} + \frac{2}{y_0-1} + 10\right)H(1, 0; y_0) + \left(\frac{2d_1^2y_0^3}{3} - 3d_1^2y_0^2 + 6d_1^2y_0 - \right.
 \end{aligned}$$

$$\begin{aligned} & \frac{11d_1'^2}{3} + (-4d_1 + 2d_1' + 2)H(0; \alpha_0) \Big) H(1, 1; y_0) + \left( -\frac{2y_0^3}{3} + 3y_0^2 - 6y_0 + 4H(0; \alpha_0) + 2H(0; Y) + \right. \\ & 4d_1 H(1; \alpha_0) + \frac{4d_1}{y_0-1} - \frac{2d_1'}{y_0-1} - \frac{2}{y_0-1} - 10 \Big) H(1, c_1(\alpha_0); y_0) + \left( 4\alpha_0 - \frac{4}{y_0-1} - 4 \right) H(c_1(\alpha_0), 0; y_0) + \\ & \left( 2\alpha_0 d_1' - \frac{2d_1'}{y_0-1} - 2d_1' \right) H(c_1(\alpha_0), 1; y_0) + \left( 2\alpha_0 - \frac{2}{y_0-1} - 2 \right) H(c_1(\alpha_0), c_1(\alpha_0); y_0) - \\ & \frac{1}{2} H(0, 0, 0; Y) - 32H(0, 0, 0; y_0) - 8d_1' H(0, 0, 1; y_0) + 8H(0, 0, c_1(\alpha_0); y_0) - \frac{1}{2} H(0, 1, 0; Y) + \\ & (4d_1 - 8d_1' - 4)H(0, 1, 0; y_0) - 2d_1'^2 H(0, 1, 1; y_0) + (-4d_1 + 2d_1' + 4)H(0, 1, c_1(\alpha_0); y_0) + \\ & 4 H(0, c_1(\alpha_0), 0; y_0) + 2d_1' H(0, c_1(\alpha_0), 1; y_0) + 2 H(0, c_1(\alpha_0), c_1(\alpha_0); y_0) - \frac{1}{2} H(1, 0, 0; Y) - \\ & 12H(1, 0, 0; y_0) - 2d_1' H(1, 0, 1; y_0) + 6H(1, 0, c_1(\alpha_0); y_0) - \frac{1}{2} H(1, 1, 0; Y) + (4d_1 - 2d_1' - \\ & 2)H(1, 1, 0; y_0) + (-4d_1 + 2d_1' + 2)H(1, 1, c_1(\alpha_0); y_0) + 4 H(1, c_1(\alpha_0), 0; y_0) + \\ & 2d_1' H(1, c_1(\alpha_0), 1; y_0) + 2 H(1, c_1(\alpha_0), c_1(\alpha_0); y_0) + \frac{\pi^2}{3(y_0-1)} - 3\zeta_3 + \frac{\pi^2}{2} + 4. \end{aligned}$$

**F.2 The  $\mathcal{J}\mathcal{I}$  integral for  $k = 1$**

The  $\varepsilon$  expansion for this integral reads

$$\mathcal{J}\mathcal{I}(Y, \varepsilon; y_0, d_0', \alpha_0, d_0; 1) = \frac{1}{\varepsilon^3} (j * i)_{-3}^{(1)} + \frac{1}{\varepsilon^2} (j * i)_{-2}^{(1)} + \frac{1}{\varepsilon} (j * i)_{-1}^{(1)} + (j * i)_0^{(1)} + \mathcal{O}(\varepsilon), \quad (\text{F.2})$$

where

$$(j * i)_{-3}^{(1)} = \frac{1}{4},$$

$$(j * i)_{-2}^{(1)} = \frac{y_0^3}{3} - \frac{3y_0^2}{2} + 3y_0 - \frac{1}{4} H(0; Y) - H(0; y_0) + \frac{1}{2},$$

$$\begin{aligned} (j * i)_{-1}^{(1)} = & -\frac{1}{12} y_0^3 \alpha_0^4 + \frac{y_0^2 \alpha_0^4}{4} - \frac{y_0 \alpha_0^4}{4} + \frac{4y_0^3 \alpha_0^3}{9} - \frac{3y_0^2 \alpha_0^3}{2} + \frac{5y_0 \alpha_0^3}{3} - y_0^3 \alpha_0^2 + \frac{15y_0^2 \alpha_0^2}{4} - 5y_0 \alpha_0^2 + \\ & \frac{4y_0^3 \alpha_0}{3} - \frac{11y_0^2 \alpha_0}{2} + 9y_0 \alpha_0 - \frac{d_1' y_0^3}{9} + y_0^3 + \frac{7d_1' y_0^2}{12} - \frac{21y_0^2}{4} - \frac{11d_1' y_0}{6} + 15y_0 + \left( -\frac{y_0^3}{3} + \frac{3y_0^2}{2} - 3y_0 + \right. \\ & \left. \frac{1}{y_0-1} + 1 \right) H(0; \alpha_0) - \frac{1}{3} y_0^3 H(0; Y) + \frac{3}{2} y_0^2 H(0; Y) - 3y_0 H(0; Y) - \frac{1}{2} H(0; Y) + \left( -y_0^3 + \right. \\ & \left. \frac{9y_0^2}{2} - 9y_0 + H(0; Y) - \frac{1}{y_0-1} - 3 \right) H(0; y_0) + \left( -\frac{d_1' y_0^3}{3} + \frac{3d_1' y_0^2}{2} - 3d_1' y_0 + \frac{11d_1'}{6} - \right. \\ & \left. H(0; \alpha_0) \right) H(1; y_0) + \left( -\alpha_0 + \frac{1}{y_0-1} + 1 \right) H(c_1(\alpha_0); y_0) + \frac{1}{4} H(0, 0; Y) + 4H(0, 0; y_0) + \\ & d_1' H(0, 1; y_0) - H(0, c_1(\alpha_0); y_0) + \frac{1}{4} H(1, 0; Y) + H(1, 0; y_0) - H(1, c_1(\alpha_0); y_0) + \frac{\pi^2}{24} + 1, \end{aligned}$$

$$\begin{aligned} (j * i)_0^{(1)} = & \frac{1}{24} d_1 y_0^3 \alpha_0^4 + \frac{1}{36} d_1' y_0^3 \alpha_0^4 - \frac{7y_0^3 \alpha_0^4}{24} - \frac{1}{8} d_1 y_0^2 \alpha_0^4 - \frac{1}{12} d_1' y_0^2 \alpha_0^4 + \frac{23y_0^2 \alpha_0^4}{24} + \frac{1}{8} d_1 y_0 \alpha_0^4 + \\ & \frac{1}{12} d_1' y_0 \alpha_0^4 - \frac{29y_0 \alpha_0^4}{24} + \frac{1}{12} y_0^3 H(0; Y) \alpha_0^4 - \frac{1}{4} y_0^2 H(0; Y) \alpha_0^4 + \frac{1}{4} y_0 H(0; Y) \alpha_0^4 - \frac{13}{54} d_1 y_0^3 \alpha_0^3 - \\ & \frac{4}{27} d_1' y_0^3 \alpha_0^3 + \frac{173y_0^3 \alpha_0^3}{108} + \frac{5}{6} d_1 y_0^2 \alpha_0^3 + \frac{19}{36} d_1' y_0^2 \alpha_0^3 - \frac{217y_0^2 \alpha_0^3}{36} - \frac{17}{18} d_1 y_0 \alpha_0^3 - \frac{11}{18} d_1' y_0 \alpha_0^3 + \frac{305y_0 \alpha_0^3}{36} - \\ & \frac{4}{9} y_0^3 H(0; Y) \alpha_0^3 + \frac{3}{2} y_0^2 H(0; Y) \alpha_0^3 - \frac{5}{3} y_0 H(0; Y) \alpha_0^3 + \frac{23}{36} d_1 y_0^3 \alpha_0^2 + \frac{1}{3} d_1' y_0^3 \alpha_0^2 - \frac{275y_0^3 \alpha_0^2}{72} - \frac{5}{2} d_1 y_0^2 \alpha_0^2 - \\ & \frac{11}{8} d_1' y_0^2 \alpha_0^2 + \frac{581y_0^2 \alpha_0^2}{36} + \frac{43}{12} d_1 y_0 \alpha_0^2 + \frac{9}{4} d_1' y_0 \alpha_0^2 - \frac{1003y_0 \alpha_0^2}{36} + y_0^3 H(0; Y) \alpha_0^2 - \frac{15}{4} y_0^2 H(0; Y) \alpha_0^2 + \\ & 5y_0 H(0; Y) \alpha_0^2 - \frac{25}{18} d_1 y_0^3 \alpha_0 - \frac{4}{9} d_1' y_0^3 \alpha_0 + \frac{217y_0^3 \alpha_0}{36} + 6d_1 y_0^2 \alpha_0 + \frac{25}{12} d_1' y_0^2 \alpha_0 - \frac{503y_0^2 \alpha_0}{18} - \frac{65d_1 y_0 \alpha_0}{6} - \\ & \frac{29d_1' y_0 \alpha_0}{6} + \frac{237y_0 \alpha_0}{4} - \frac{4}{3} y_0^3 H(0; Y) \alpha_0 + \frac{11}{2} y_0^2 H(0; Y) \alpha_0 - 9y_0 H(0; Y) \alpha_0 + \frac{d_1'^2 y_0^3}{27} - \frac{4d_1' y_0^3}{9} + \\ & \frac{7y_0^3}{3} - \frac{17d_1'^2 y_0^2}{72} + 3d_1' y_0^2 - \frac{111y_0^2}{8} + \frac{49d_1'^2 y_0}{36} - \frac{65d_1' y_0}{4} + 57y_0 + \frac{1}{9} d_1' y_0^3 H(0; Y) - y_0^3 H(0; Y) - \\ & \frac{7}{12} d_1' y_0^2 H(0; Y) + \frac{21}{4} y_0^2 H(0; Y) + \frac{11}{6} d_1' y_0 H(0; Y) - 15y_0 H(0; Y) - \frac{1}{24} \pi^2 H(0; Y) - H(0; Y) + \\ & H(0; \alpha_0) \left( \frac{y_0^3 \alpha_0^4}{6} - \frac{y_0^2 \alpha_0^4}{2} + \frac{y_0 \alpha_0^4}{2} - \frac{8y_0^3 \alpha_0^3}{9} + 3y_0^2 \alpha_0^3 - \frac{10y_0 \alpha_0^3}{3} + 2y_0^3 \alpha_0^2 - \frac{15y_0^2 \alpha_0^2}{2} + 10y_0 \alpha_0^2 - \frac{8y_0^3 \alpha_0}{3} + \right. \\ & \left. 11y_0^2 \alpha_0 - 18y_0 \alpha_0 + \frac{d_1' y_0^3}{9} - \frac{11y_0^3}{36} - \frac{7d_1' y_0^2}{12} + \frac{31y_0^2}{12} - 2d_1 + d_1' + \frac{11d_1' y_0}{6} - \frac{131y_0}{12} + \frac{1}{3} y_0^3 H(0; Y) - \right. \end{aligned}$$

$$\begin{aligned}
 & \frac{3}{2}y_0^2 H(0; Y) + 3y_0 H(0; Y) - \frac{H(0; Y)}{y_0-1} - H(0; Y) - \frac{2}{y_0-1} \frac{d_1}{y_0-1} + \frac{d_1'}{y_0-1} + \frac{61}{12} \frac{1}{(y_0-1)} + \frac{61}{12} \Big) + \left( \frac{1}{6} d_1 y_0^3 \alpha_0^4 - \right. \\
 & \frac{1}{2} d_1 y_0^2 \alpha_0^4 + \frac{1}{2} d_1 y_0 \alpha_0^4 - \frac{8}{9} d_1 y_0^3 \alpha_0^3 + 3 d_1 y_0^2 \alpha_0^3 - \frac{10}{3} d_1 y_0 \alpha_0^3 + 2 d_1 y_0^3 \alpha_0^2 - \frac{15}{2} d_1 y_0^2 \alpha_0^2 + 10 d_1 y_0 \alpha_0^2 - \\
 & \left. \frac{8}{3} d_1 y_0^3 \alpha_0 + 11 d_1 y_0^2 \alpha_0 - 18 d_1 y_0 \alpha_0 + \frac{25 d_1 y_0^3}{18} - 6 d_1 y_0^2 + \frac{65 d_1 y_0}{6} \right) H(1; \alpha_0) - \frac{1}{24} \pi^2 H(1; Y) + \left( \frac{y_0^3}{12} \frac{\alpha_0^4}{12} - \right. \\
 & \frac{y_0^2 \alpha_0^4}{4} + \frac{y_0 \alpha_0^4}{4} - \frac{\alpha_0^4}{12} - \frac{4 y_0^3 \alpha_0^3}{9} + \frac{3 y_0^2 \alpha_0^3}{2} - \frac{5 y_0 \alpha_0^3}{3} + \frac{11 \alpha_0^3}{18} + y_0^3 \alpha_0^2 - \frac{15 y_0^2 \alpha_0^2}{4} + 5 y_0 \alpha_0^2 - \frac{9 \alpha_0^2}{4} - \frac{4 y_0^3 \alpha_0}{3} + \\
 & \left. \frac{11 y_0^2 \alpha_0}{2} + 2 d_1 \alpha_0 - d_1' \alpha_0 - 9 y_0 \alpha_0 + H(0; Y) \alpha_0 - \alpha_0 + \frac{25 y_0^3}{36} - \frac{8 y_0^2}{3} - 2 d_1 + d_1' + \frac{49 y_0}{12} \right) + \left( 2 \alpha_0 - \frac{2}{y_0-1} - \right. \\
 & \left. 2 \right) H(0; \alpha_0) - \frac{H(0; Y)}{y_0-1} - H(0; Y) + \left( 2 \alpha_0 d_1 - \frac{2}{y_0-1} d_1 - 2 d_1 \right) H(1; \alpha_0) - \frac{2}{y_0-1} \frac{d_1}{y_0-1} + \frac{d_1'}{y_0-1} + \frac{61}{12} \frac{1}{(y_0-1)} + \\
 & \frac{61}{12} \Big) H(c_1(\alpha_0); y_0) + \left( \frac{2 y_0^3}{3} - 3 y_0^2 + 6 y_0 - \frac{2}{y_0-1} - 2 \right) H(0, 0; \alpha_0) + \frac{1}{3} y_0^3 H(0, 0; Y) - \frac{3}{2} y_0^2 H(0, 0; Y) + \\
 & 3 y_0 H(0, 0; Y) + \frac{1}{2} H(0, 0; Y) + \left( \frac{10 y_0^3}{3} - 15 y_0^2 + 30 y_0 - 4 H(0; Y) + \frac{6}{y_0-1} + 14 \right) H(0, 0; y_0) + \\
 & \left( \frac{2 d_1 y_0^3}{3} - 3 d_1 y_0^2 + 6 d_1 y_0 - 2 d_1 - \frac{2 d_1}{y_0-1} \right) H(0, 1; \alpha_0) + H(1; y_0) \left( \frac{1}{12} d_1' y_0^3 \alpha_0^4 - \frac{1}{4} d_1' y_0^2 \alpha_0^4 - \frac{d_1' \alpha_0^4}{12} + \right. \\
 & \frac{1}{4} d_1' y_0 \alpha_0^4 - \frac{4}{9} d_1' y_0^3 \alpha_0^3 + \frac{3}{2} d_1' y_0^2 \alpha_0^3 + \frac{11 d_1' \alpha_0^3}{18} - \frac{5}{3} d_1' y_0 \alpha_0^3 + d_1' y_0^3 \alpha_0^2 - \frac{15}{4} d_1' y_0^2 \alpha_0^2 - \frac{9 d_1' \alpha_0^2}{4} + 5 d_1' y_0 \alpha_0^2 - \\
 & \left. \frac{4}{3} d_1' y_0^3 \alpha_0 + \frac{11}{2} d_1' y_0^2 \alpha_0 + \frac{23}{6} d_1' \alpha_0 - 9 d_1' y_0 \alpha_0 + \frac{d_1' y_0^3}{9} - d_1' y_0^3 - \frac{49 d_1'^2}{36} - \frac{7 d_1'^2 y_0^2}{12} + \frac{21 d_1' y_0^2}{4} + \frac{43 d_1'}{4} + \frac{11 d_1' y_0}{6} - \right. \\
 & 15 d_1' y_0 + \frac{1}{3} d_1' y_0^3 H(0; Y) - \frac{3}{2} d_1' y_0^2 H(0; Y) - \frac{11}{6} d_1' H(0; Y) + 3 d_1' y_0 H(0; Y) + H(0; \alpha_0) \left( \frac{d_1' y_0^3}{3} - \right. \\
 & \left. \frac{y_0^3}{3} - \frac{3 d_1' y_0^2}{2} + \frac{3 y_0^2}{2} + 3 d_1' y_0 - 3 y_0 - \frac{11}{6} \frac{d_1'}{y_0-1} + H(0; Y) + \frac{2}{y_0-1} \frac{d_1}{y_0-1} - \frac{d_1'}{y_0-1} - \frac{1}{y_0-1} - 5 \right) + 2 H(0, 0; \alpha_0) + \\
 & 2 d_1 H(0, 1; \alpha_0) - \frac{\pi^2}{6} \Big) + \left( d_1' y_0^3 - \frac{9 d_1' y_0^2}{2} + 9 d_1' y_0 + 3 d_1' + (2 - 2 d_1) H(0; \alpha_0) - d_1' H(0; Y) + \right. \\
 & \left. \frac{d_1'}{y_0-1} \right) H(0, 1; y_0) + \left( - \frac{y_0^3}{3} + \frac{3 y_0^2}{2} - 3 y_0 + 2 H(0; \alpha_0) + H(0; Y) + 2 d_1 H(1; \alpha_0) - \frac{3}{y_0-1} - \right. \\
 & \left. 5 \right) H(0, c_1(\alpha_0); y_0) + H(0; y_0) \left( \frac{y_0^3 \alpha_0^4}{6} - \frac{y_0^2 \alpha_0^4}{2} + \frac{y_0 \alpha_0^4}{2} - \frac{8 y_0^3 \alpha_0^3}{9} + 3 y_0^2 \alpha_0^3 - \frac{10 y_0 \alpha_0^3}{3} + 2 y_0^3 \alpha_0^2 - \frac{15 y_0^2 \alpha_0^2}{2} + \right. \\
 & 10 y_0 \alpha_0^2 - \frac{8 y_0^3 \alpha_0}{3} + 11 y_0^2 \alpha_0 - 18 y_0 \alpha_0 + \frac{d_1' y_0^3}{3} - \frac{133 y_0^3}{36} - \frac{7 d_1' y_0^2}{4} + \frac{221 y_0^2}{12} + 2 d_1 - d_1' + \frac{11 d_1' y_0}{2} - \frac{589 y_0}{12} + \\
 & \left. \left( \frac{2 y_0^3}{3} - 3 y_0^2 + 6 y_0 - \frac{2}{y_0-1} - 2 \right) H(0; \alpha_0) + y_0^3 H(0; Y) - \frac{9}{2} y_0^2 H(0; Y) + 9 y_0 H(0; Y) + \frac{H(0; Y)}{y_0-1} + \right. \\
 & 3 H(0; Y) - H(0, 0; Y) - H(1, 0; Y) + \frac{2}{y_0-1} \frac{d_1}{y_0-1} - \frac{d_1'}{y_0-1} - \frac{61}{12} \frac{1}{(y_0-1)} - \frac{\pi^2}{6} - \frac{109}{12} \Big) + \frac{1}{3} y_0^3 H(1, 0; Y) - \\
 & \frac{3}{2} y_0^2 H(1, 0; Y) + 3 y_0 H(1, 0; Y) + \frac{1}{2} H(1, 0; Y) + \left( d_1' y_0^3 + \frac{y_0^3}{3} - \frac{9 d_1' y_0^2}{2} - \frac{3 y_0^2}{2} + 9 d_1' y_0 + 3 y_0 - \right. \\
 & \left. \frac{11 d_1'}{2} + 2 H(0; \alpha_0) - H(0; Y) - \frac{2}{y_0-1} \frac{d_1}{y_0-1} + \frac{d_1'}{y_0-1} + \frac{1}{y_0-1} + 5 \right) H(1, 0; y_0) + \left( \frac{d_1'^2 y_0^3}{3} - \frac{3 d_1'^2 y_0^2}{2} + 3 d_1'^2 y_0 - \right. \\
 & \left. \frac{11 d_1'^2}{6} + (-2 d_1 + d_1' + 1) H(0; \alpha_0) \right) H(1, 1; y_0) + \left( - \frac{y_0^3}{3} + \frac{3 y_0^2}{2} - 3 y_0 + 2 H(0; \alpha_0) + H(0; Y) + \right. \\
 & 2 d_1 H(1; \alpha_0) + \frac{2}{y_0-1} \frac{d_1}{y_0-1} - \frac{d_1'}{y_0-1} - \frac{1}{y_0-1} - 5 \Big) H(1, c_1(\alpha_0); y_0) + \left( 2 \alpha_0 - \frac{2}{y_0-1} - 2 \right) H(c_1(\alpha_0), 0; y_0) + \\
 & \left( \alpha_0 d_1' - \frac{d_1'}{y_0-1} - d_1' \right) H(c_1(\alpha_0), 1; y_0) + \left( \alpha_0 - \frac{1}{y_0-1} - 1 \right) H(c_1(\alpha_0), c_1(\alpha_0); y_0) - \\
 & \frac{1}{4} H(0, 0, 0; Y) - 16 H(0, 0, 0; y_0) - 4 d_1' H(0, 0, 1; y_0) + 4 H(0, 0, c_1(\alpha_0); y_0) - \frac{1}{4} H(0, 1, 0; Y) + \\
 & (2 d_1 - 4 d_1' - 2) H(0, 1, 0; y_0) - d_1'^2 H(0, 1, 1; y_0) + (-2 d_1 + d_1' + 2) H(0, 1, c_1(\alpha_0); y_0) + \\
 & 2 H(0, c_1(\alpha_0), 0; y_0) + d_1' H(0, c_1(\alpha_0), 1; y_0) + H(0, c_1(\alpha_0), c_1(\alpha_0); y_0) - \frac{1}{4} H(1, 0, 0; Y) - \\
 & 6 H(1, 0, 0; y_0) - d_1' H(1, 0, 1; y_0) + 3 H(1, 0, c_1(\alpha_0); y_0) - \frac{1}{4} H(1, 1, 0; Y) + (2 d_1 - d_1' - \\
 & 1) H(1, 1, 0; y_0) + (-2 d_1 + d_1' + 1) H(1, 1, c_1(\alpha_0); y_0) + 2 H(1, c_1(\alpha_0), 0; y_0) + \\
 & d_1' H(1, c_1(\alpha_0), 1; y_0) + H(1, c_1(\alpha_0), c_1(\alpha_0); y_0) + \frac{\pi^2}{6} \frac{1}{(y_0-1)} - \frac{3 \zeta_3}{2} + \frac{\pi^2}{4} + 2.
 \end{aligned}$$

### F.3 The $\mathcal{J}^* \mathcal{I}$ integral for $k = 2$

The  $\varepsilon$  expansion for this integral reads

$$\mathcal{J}^* \mathcal{I}(Y, \varepsilon; y_0, d_1', \alpha_0, d_0; 2) = \frac{1}{\varepsilon^3} (j * i)_{-3}^{(2)} + \frac{1}{\varepsilon^2} (j * i)_{-2}^{(2)} + \frac{1}{\varepsilon} (j * i)_{-1}^{(2)} + (j * i)_0^{(2)} + \mathcal{O}(\varepsilon), \quad (\text{F.3})$$

where

$$(j * i)_{-3}^{(2)} = \frac{1}{6},$$

$$(j * i)_{-2}^{(2)} = \frac{2y_0^3}{9} - y_0^2 + 2y_0 - \frac{1}{6}H(0; Y) - \frac{2}{3}H(0; y_0) + \frac{4}{9},$$

$$\begin{aligned} (j * i)_{-1}^{(2)} = & -\frac{1}{18}y_0^3\alpha_0^4 + \frac{y_0^2\alpha_0^4}{12} - \frac{y_0\alpha_0^4}{3} + \frac{\alpha_0^4}{6(y_0-2)} + \frac{\alpha_0^4}{12} + \frac{8y_0^3\alpha_0^3}{27} - \frac{2y_0^2\alpha_0^3}{3} + \frac{8y_0\alpha_0^3}{9} + \frac{\alpha_0^3}{y_0-2} + \\ & \frac{4\alpha_0^3}{9(y_0-2)^2} + \frac{7\alpha_0^3}{18} - \frac{2y_0^3\alpha_0^2}{3} + 2y_0^2\alpha_0^2 - \frac{5y_0\alpha_0^2}{3} + \frac{8\alpha_0^2}{3(y_0-2)} + \frac{11\alpha_0^2}{3(y_0-2)^2} + \frac{4\alpha_0^2}{3(y_0-2)^3} + \frac{7\alpha_0^2}{12} + \frac{8y_0^3\alpha_0}{9} - \\ & \frac{10y_0^2\alpha_0}{3} + 4y_0\alpha_0 + \frac{13\alpha_0}{3(y_0-2)} + \frac{18\alpha_0}{(y_0-2)^2} + \frac{52\alpha_0}{3(y_0-2)^3} + \frac{16\alpha_0}{3(y_0-2)^4} - \frac{\alpha_0}{2} - \frac{2d_1'y_0^3}{27} + \frac{19y_0^3}{27} + \frac{7d_1'y_0^2}{18} - \frac{11y_0^2}{3} - \\ & \frac{11d_1'y_0}{9} + \frac{31y_0}{3} + \left( -\frac{2y_0^3}{9} + y_0^2 - 2y_0 + \frac{2}{3(y_0-1)} + \frac{80}{3(y_0-2)^2} + \frac{160}{3(y_0-2)^3} + \frac{40}{(y_0-2)^4} + \frac{32}{3(y_0-2)^5} - \right. \\ & \left. \frac{3}{2} \right) H(0; \alpha_0) - \frac{2}{9}y_0^3H(0; Y) + y_0^2H(0; Y) - 2y_0H(0; Y) - \frac{4}{9}H(0; Y) + \left( -\frac{2y_0^3}{3} + 3y_0^2 - 6y_0 + \right. \\ & \left. \frac{2}{3}H(0; Y) - \frac{2}{3(y_0-1)} - \frac{80}{3(y_0-2)^2} - \frac{160}{3(y_0-2)^3} - \frac{40}{(y_0-2)^4} - \frac{32}{3(y_0-2)^5} - \frac{5}{18} \right) H(0; y_0) + \left( -\frac{2d_1'y_0^3}{9} + \right. \\ & \left. d_1'y_0^2 - 2d_1'y_0 + \frac{11d_1'}{9} - \frac{2}{3}H(0; \alpha_0) \right) H(1; y_0) + \left( -\frac{2\alpha_0}{3} + \frac{2}{3(y_0-1)} + \frac{2}{3} \right) H(c_1(\alpha_0); y_0) + \left( -\frac{\alpha_0^4}{2} - \frac{2\alpha_0^3}{3} + \alpha_0^2 + 2\alpha_0 + \frac{80}{3(y_0-2)^2} + \frac{160}{3(y_0-2)^3} + \frac{40}{(y_0-2)^4} + \frac{32}{3(y_0-2)^5} - \frac{11}{6} \right) H(c_2(\alpha_0); y_0) + \\ & \frac{1}{6}H(0, 0; Y) + \frac{8}{3}H(0, 0; y_0) + \frac{2}{3}d_1'H(0, 1; y_0) - \frac{2}{3}H(0, c_1(\alpha_0); y_0) + \frac{1}{6}H(1, 0; Y) + \\ & \frac{2}{3}H(1, 0; y_0) - \frac{2}{3}H(1, c_1(\alpha_0); y_0) + \frac{80 \ln 2}{3(y_0-2)^2} + \frac{160 \ln 2}{3(y_0-2)^3} + \frac{40 \ln 2}{(y_0-2)^4} + \frac{32 \ln 2}{3(y_0-2)^5} - \frac{13 \ln 2}{6} + \frac{\pi^2}{36} + \frac{26}{27}, \end{aligned}$$

$$\begin{aligned} (j * i)_0^{(2)} = & \frac{1}{36}d_1y_0^3\alpha_0^4 + \frac{1}{54}d_1'y_0^3\alpha_0^4 - \frac{11y_0^3\alpha_0^4}{54} - \frac{1}{24}d_1y_0^2\alpha_0^4 - \frac{1}{72}d_1'y_0^2\alpha_0^4 + \frac{5y_0^2\alpha_0^4}{18} - \frac{d_1\alpha_0^4}{24} + \\ & \frac{1}{6}d_1y_0\alpha_0^4 + \frac{1}{36}d_1'y_0\alpha_0^4 - \frac{35y_0\alpha_0^4}{18} + \frac{1}{18}y_0^3H(0; Y)\alpha_0^4 - \frac{1}{12}y_0^2H(0; Y)\alpha_0^4 + \frac{1}{3}y_0H(0; Y)\alpha_0^4 - \frac{H(0; Y)\alpha_0^4}{6(y_0-2)} - \\ & \frac{1}{12}H(0; Y)\alpha_0^4 - \frac{d_1\alpha_0^4}{12(y_0-2)} + \frac{19\alpha_0^4}{36(y_0-2)} + \frac{19\alpha_0^4}{72} - \frac{13d_1y_0^3\alpha_0^3}{81} - \frac{8d_1'y_0^3\alpha_0^3}{81} + \frac{181y_0^3\alpha_0^3}{162} + \frac{7d_1y_0^2\alpha_0^3}{18} + \\ & \frac{5}{27}d_1'y_0^2\alpha_0^3 - \frac{281y_0^2\alpha_0^3}{108} - \frac{43d_1\alpha_0^3}{108} + \frac{d_1'\alpha_0^3}{18} - \frac{10}{27}d_1y_0\alpha_0^3 - \frac{14}{27}d_1'y_0\alpha_0^3 + \frac{493y_0\alpha_0^3}{108} - \frac{8}{27}y_0^3H(0; Y)\alpha_0^3 + \\ & \frac{2}{3}y_0^2H(0; Y)\alpha_0^3 - \frac{8}{9}y_0H(0; Y)\alpha_0^3 - \frac{H(0; Y)\alpha_0^3}{y_0-2} - \frac{4H(0; Y)\alpha_0^3}{9(y_0-2)^2} - \frac{7}{18}H(0; Y)\alpha_0^3 - \frac{17d_1\alpha_0^3}{18(y_0-2)} + \frac{d_1'\alpha_0^3}{9(y_0-2)} + \\ & \frac{61\alpha_0^3}{18(y_0-2)} - \frac{8d_1\alpha_0^3}{27(y_0-2)^2} + \frac{40\alpha_0^3}{27(y_0-2)^2} + \frac{143\alpha_0^3}{108} + \frac{23}{54}d_1y_0^3\alpha_0^2 + \frac{2}{9}d_1'y_0^3\alpha_0^2 - \frac{287y_0^3\alpha_0^2}{108} - \frac{17}{12}d_1y_0^2\alpha_0^2 - \\ & \frac{2}{3}d_1'y_0^2\alpha_0^2 + \frac{1883y_0^2\alpha_0^2}{216} - \frac{113d_1\alpha_0^2}{72} + \frac{13d_1'\alpha_0^2}{36} + \frac{10}{9}d_1y_0\alpha_0^2 + \frac{1}{3}d_1'y_0\alpha_0^2 - \frac{224y_0\alpha_0^2}{27} + \frac{2}{3}y_0^3H(0; Y)\alpha_0^2 - \\ & 2y_0^2H(0; Y)\alpha_0^2 + \frac{5}{3}y_0H(0; Y)\alpha_0^2 - \frac{8H(0; Y)\alpha_0^2}{3(y_0-2)} - \frac{11H(0; Y)\alpha_0^2}{3(y_0-2)^2} - \frac{4H(0; Y)\alpha_0^2}{3(y_0-2)^3} - \frac{7}{12}H(0; Y)\alpha_0^2 - \frac{46d_1\alpha_0^2}{9(y_0-2)} + \\ & \frac{8d_1'\alpha_0^2}{9(y_0-2)} + \frac{179\alpha_0^2}{18(y_0-2)} - \frac{83d_1\alpha_0^2}{18(y_0-2)^2} + \frac{d_1'\alpha_0^2}{3(y_0-2)^2} + \frac{247\alpha_0^2}{18(y_0-2)^2} - \frac{4d_1\alpha_0^2}{3(y_0-2)^3} + \frac{44\alpha_0^2}{9(y_0-2)^3} + \frac{155\alpha_0^2}{72} - \frac{25}{27}d_1y_0^3\alpha_0 - \\ & \frac{8}{27}d_1'y_0^3\alpha_0 + \frac{25y_0^3\alpha_0}{6} + \frac{23}{6}d_1y_0^2\alpha_0 + \frac{11}{9}d_1'y_0^2\alpha_0 - \frac{1883y_0^2\alpha_0}{108} - \frac{83d_1\alpha_0}{36} + \frac{19d_1'\alpha_0}{18} - \frac{52d_1y_0\alpha_0}{9} - \frac{14d_1'y_0\alpha_0}{9} + \\ & \frac{949y_0\alpha_0}{36} - \frac{8}{9}y_0^3H(0; Y)\alpha_0 + \frac{10}{3}y_0^2H(0; Y)\alpha_0 - 4y_0H(0; Y)\alpha_0 - \frac{13H(0; Y)\alpha_0}{3(y_0-2)} - \frac{18H(0; Y)\alpha_0}{(y_0-2)^2} - \\ & \frac{52H(0; Y)\alpha_0}{3(y_0-2)^3} - \frac{16H(0; Y)\alpha_0}{3(y_0-2)^4} + \frac{1}{2}H(0; Y)\alpha_0 - \frac{383d_1\alpha_0}{18(y_0-2)} + \frac{35d_1'\alpha_0}{9(y_0-2)} + \frac{113\alpha_0}{6(y_0-2)} - \frac{151d_1\alpha_0}{3(y_0-2)^2} + \frac{38d_1'\alpha_0}{9(y_0-2)^2} + \\ & \frac{263\alpha_0}{3(y_0-2)^2} - \frac{118d_1\alpha_0}{3(y_0-2)^3} + \frac{4d_1'\alpha_0}{3(y_0-2)^3} + \frac{746\alpha_0}{9(y_0-2)^3} - \frac{32d_1\alpha_0}{3(y_0-2)^4} + \frac{224\alpha_0}{9(y_0-2)^4} - \frac{133\alpha_0}{36} + \frac{2d_1^2y_0^3}{81} - \frac{25d_1'y_0^3}{81} + \\ & \frac{137y_0^3}{81} - \frac{17d_1^2y_0^2}{108} + \frac{223d_1'y_0^2}{108} - \frac{179y_0^2}{18} + \frac{49d_1^2y_0}{54} - \frac{298d_1'y_0}{27} + \frac{359y_0}{9} + \frac{2}{27}d_1'y_0^3H(0; Y) - \frac{19}{27}y_0^3H(0; Y) - \\ & \frac{7}{18}d_1'y_0^2H(0; Y) + \frac{11}{3}y_0^2H(0; Y) + \frac{11}{9}d_1'y_0H(0; Y) - \frac{31}{3}y_0H(0; Y) - \frac{1}{36}\pi^2H(0; Y) - \frac{26}{27}H(0; Y) + \\ & H(0; \alpha_0) \left( \frac{y_0^3\alpha_0^4}{9} - \frac{y_0^2\alpha_0^4}{6} + \frac{2y_0\alpha_0^4}{3} - \frac{\alpha_0^4}{3(y_0-2)} - \frac{\alpha_0^4}{6} - \frac{16y_0^3\alpha_0^3}{27} + \frac{4y_0^2\alpha_0^3}{3} - \frac{16y_0\alpha_0^3}{9} - \frac{2\alpha_0^3}{y_0-2} - \frac{8\alpha_0^3}{9(y_0-2)^2} - \right. \\ & \left. \frac{7\alpha_0^3}{9} + \frac{4y_0^2\alpha_0^2}{3} - 4y_0\alpha_0^2 + \frac{10y_0\alpha_0^2}{3} - \frac{16\alpha_0^2}{3(y_0-2)} - \frac{22\alpha_0^2}{3(y_0-2)^2} - \frac{8\alpha_0^2}{3(y_0-2)^3} - \frac{7\alpha_0^2}{6} - \frac{16y_0^3\alpha_0}{9} + \frac{20y_0^2\alpha_0}{3} - 8y_0\alpha_0 - \right. \\ & \left. \frac{26\alpha_0}{3(y_0-2)} - \frac{36\alpha_0}{(y_0-2)^2} - \frac{104\alpha_0}{3(y_0-2)^3} - \frac{32\alpha_0}{3(y_0-2)^4} + \alpha_0 + \frac{2d_1'y_0^3}{27} - \frac{13y_0^3}{54} - \frac{7d_1'y_0^2}{18} + \frac{71y_0^2}{36} + \frac{41d_1}{36} + \frac{d_1'}{18} + \frac{11d_1'y_0}{9} - \right. \\ & \left. \frac{25y_0}{3} + \frac{2}{9}y_0^3H(0; Y) - y_0^2H(0; Y) + 2y_0H(0; Y) - \frac{2H(0; Y)}{3(y_0-1)} - \frac{80H(0; Y)}{3(y_0-2)^2} - \frac{160H(0; Y)}{3(y_0-2)^3} - \frac{40H(0; Y)}{(y_0-2)^4} - \right. \end{aligned}$$

$$\begin{aligned}
& \frac{32H(0;Y)}{3(y_0-2)^5} + \frac{3}{2} H(0; Y) + \frac{8d'_1}{3(y_0-2)} - \frac{20}{3(y_0-2)} - \frac{4d_1}{3(y_0-1)} + \frac{2d'_1}{3(y_0-1)} + \frac{7}{2(y_0-1)} - \frac{16d_1}{(y_0-2)^2} + \frac{12d'_1}{(y_0-2)^2} + \\
& \left( \frac{88}{(y_0-2)^2} - \frac{128d_1}{9(y_0-2)^3} + \frac{88d'_1}{9(y_0-2)^3} + \frac{168}{(y_0-2)^3} - \frac{4d_1}{(y_0-2)^4} + \frac{8d'_1}{3(y_0-2)^4} + \frac{116}{(y_0-2)^4} + \frac{256}{9(y_0-2)^5} - \frac{259}{36} \right) + \\
& \left( \frac{1}{9}d_1y_0^3\alpha_0^4 - \frac{1}{6}d_1y_0^2\alpha_0^4 - \frac{d_1\alpha_0^4}{6} + \frac{2}{3}d_1y_0\alpha_0^4 - \frac{d_1\alpha_0^4}{3(y_0-2)} - \frac{16}{27}d_1y_0^3\alpha_0^3 + \frac{4}{3}d_1y_0^2\alpha_0^3 - \frac{7d_1\alpha_0^3}{9} - \frac{16}{9}d_1y_0\alpha_0^3 - \right. \\
& \left. \frac{2d_1\alpha_0^3}{y_0-2} - \frac{8d_1\alpha_0^3}{9(y_0-2)^2} + \frac{4}{3}d_1y_0^3\alpha_0^2 - 4d_1y_0^2\alpha_0^2 - \frac{7d_1\alpha_0^2}{6} + \frac{10}{3}d_1y_0\alpha_0^2 - \frac{16d_1\alpha_0^2}{3(y_0-2)} - \frac{22d_1\alpha_0^2}{3(y_0-2)^2} - \frac{8d_1\alpha_0^2}{3(y_0-2)^3} - \right. \\
& \left. \frac{16}{9}d_1y_0^3\alpha_0 + \frac{20}{3}d_1y_0^2\alpha_0 + d_1\alpha_0 - 8d_1y_0\alpha_0 - \frac{26d_1\alpha_0}{3(y_0-2)} - \frac{36d_1\alpha_0}{(y_0-2)^2} - \frac{104d_1\alpha_0}{3(y_0-2)^3} - \frac{32d_1\alpha_0}{3(y_0-2)^4} + \frac{25d_1y_0^3}{27} - \right. \\
& \left. \frac{23d_1y_0^2}{6} + \frac{10d_1}{9} + \frac{52d_1y_0}{9} + \frac{49d_1}{3(y_0-2)} + \frac{398d_1}{9(y_0-2)^2} + \frac{112d_1}{3(y_0-2)^3} + \frac{32d_1}{3(y_0-2)^4} \right) H(1; \alpha_0) - \frac{1}{36}\pi^2 H(1; Y) + \\
& \left( \frac{y_0^3\alpha_0^4}{18} - \frac{y_0^2\alpha_0^4}{12} + \frac{y_0\alpha_0^4}{3} - \frac{\alpha_0^4}{6(y_0-2)} - \frac{17\alpha_0^4}{36} - \frac{8y_0^3\alpha_0^3}{27} + \frac{2y_0^2\alpha_0^3}{3} - \frac{8y_0\alpha_0^3}{9} - \frac{\alpha_0^3}{y_0-2} - \frac{4\alpha_0^3}{9(y_0-2)^2} - \frac{29\alpha_0^3}{54} + \right. \\
& \left. \frac{2y_0^3\alpha_0^2}{3} - 2y_0^2\alpha_0^2 + \frac{5y_0\alpha_0^2}{3} - \frac{8\alpha_0^2}{3(y_0-2)} - \frac{11\alpha_0^2}{3(y_0-2)^2} - \frac{4\alpha_0^2}{3(y_0-2)^3} - \frac{13\alpha_0^2}{12} - \frac{8y_0^3\alpha_0}{9} + \frac{10y_0^2\alpha_0}{3} + \frac{4d_1\alpha_0}{3} - \frac{2d'_1\alpha_0}{3} - \right. \\
& \left. 4y_0\alpha_0 + \frac{2}{3}H(0; Y)\alpha_0 - \frac{13\alpha_0}{3(y_0-2)} - \frac{18\alpha_0}{(y_0-2)^2} - \frac{52\alpha_0}{3(y_0-2)^3} - \frac{16\alpha_0}{3(y_0-2)^4} + \frac{7\alpha_0}{18} + \frac{25y_0^3}{54} - \frac{61y_0^2}{36} - \frac{4d_1}{3} + \frac{2d'_1}{3} + \right. \\
& \left. 2y_0 + \left( \frac{4\alpha_0}{3} - \frac{4}{3(y_0-1)} - \frac{4}{3} \right) H(0; \alpha_0) - \frac{2H(0;Y)}{3(y_0-1)} - \frac{2}{3}H(0; Y) + \left( \frac{4\alpha_0d_1}{3} - \frac{4d_1}{3(y_0-1)} - \frac{4d_1}{3} \right) H(1; \alpha_0) + \right. \\
& \left. \frac{32}{3(y_0-2)} - \frac{4d_1}{3(y_0-1)} + \frac{2d'_1}{3(y_0-1)} + \frac{7}{2(y_0-1)} + \frac{332}{9(y_0-2)^2} + \frac{104}{3(y_0-2)^3} + \frac{32}{3(y_0-2)^4} + \frac{59}{18} \right) H(c_1(\alpha_0); y_0) + \\
& \left( \frac{d_1\alpha_0^4}{4} - \frac{d'_1\alpha_0^4}{6} + \frac{1}{2}H(0; Y)\alpha_0^4 - \frac{5\alpha_0^4}{4} + \frac{7d_1\alpha_0^3}{9} - \frac{5d'_1\alpha_0^3}{9} + \frac{2}{3}H(0; Y)\alpha_0^3 - \alpha_0^3 + \frac{d_1\alpha_0^2}{6} - \frac{d'_1\alpha_0^2}{3} - \right. \\
& \left. H(0; Y)\alpha_0^2 + \frac{29\alpha_0^2}{6} - \frac{11d_1\alpha_0}{3} + \frac{5d'_1\alpha_0}{3} - 2H(0; Y)\alpha_0 + 7\alpha_0 + \frac{89d_1}{36} - \frac{11d'_1}{18} + \left( \alpha_0^4 + \frac{4\alpha_0^3}{3} - 2\alpha_0^2 - 4\alpha_0 - \right. \right. \\
& \left. \left. \frac{160}{3(y_0-2)^2} - \frac{320}{3(y_0-2)^3} - \frac{80}{(y_0-2)^4} - \frac{64}{3(y_0-2)^5} + \frac{11}{3} \right) H(0; \alpha_0) - \frac{80H(0;Y)}{3(y_0-2)^2} - \frac{160H(0;Y)}{3(y_0-2)^3} - \frac{40H(0;Y)}{(y_0-2)^4} - \right. \\
& \left. \frac{32H(0;Y)}{3(y_0-2)^5} + \frac{11}{6}H(0; Y) + \left( d_1\alpha_0^4 + \frac{4d_1\alpha_0^3}{3} - 2d_1\alpha_0^2 - 4d_1\alpha_0 + \frac{11d_1}{3} - \frac{160d_1}{3(y_0-2)^2} - \frac{320d_1}{3(y_0-2)^3} - \frac{80d_1}{(y_0-2)^4} - \right. \right. \\
& \left. \left. \frac{64d_1}{3(y_0-2)^5} \right) H(1; \alpha_0) + \frac{8d'_1}{3(y_0-2)} - \frac{52}{3(y_0-2)} - \frac{16d_1}{(y_0-2)^2} + \frac{12d'_1}{(y_0-2)^2} + \frac{460}{9(y_0-2)^2} - \frac{128d_1}{9(y_0-2)^3} + \frac{88d'_1}{9(y_0-2)^3} + \right. \\
& \left. \frac{400}{3(y_0-2)^3} - \frac{4d_1}{(y_0-2)^4} + \frac{8d'_1}{3(y_0-2)^4} + \frac{316}{3(y_0-2)^4} + \frac{256}{9(y_0-2)^5} - \frac{115}{12} \right) H(c_2(\alpha_0); y_0) + \left( \frac{4y_0^3}{9} - 2y_0^2 + \right. \\
& \left. 4y_0 - \frac{4}{3(y_0-1)} - \frac{160}{3(y_0-2)^2} - \frac{320}{3(y_0-2)^3} - \frac{80}{(y_0-2)^4} - \frac{64}{3(y_0-2)^5} + 3 \right) H(0, 0; \alpha_0) + \frac{2}{9}y_0^3H(0, 0; Y) - \\
& y_0^2H(0, 0; Y) + 2y_0H(0, 0; Y) + \frac{4}{9}H(0, 0; Y) + \left( \frac{20y_0^3}{9} - 10y_0^2 + 20y_0 - \frac{8}{3}H(0; Y) + \frac{4}{y_0-1} + \frac{160}{(y_0-2)^2} + \right. \\
& \left. \frac{320}{(y_0-2)^3} + \frac{240}{(y_0-2)^4} + \frac{64}{(y_0-2)^5} - \frac{17}{9} \right) H(0, 0; y_0) + \left( \frac{4d_1y_0^3}{9} - 2d_1y_0^2 + 4d_1y_0 + 3d_1 - \frac{4d_1}{3(y_0-1)} - \frac{160d_1}{3(y_0-2)^2} - \right. \\
& \left. \frac{320d_1}{3(y_0-2)^3} - \frac{80d_1}{(y_0-2)^4} - \frac{64d_1}{3(y_0-2)^5} \right) H(0, 1; \alpha_0) + \left( \frac{2d'_1y_0^3}{3} - 3d'_1y_0^2 + 6d'_1y_0 + \frac{5d'_1}{18} + \left( \frac{4}{3} - \frac{4d_1}{3} \right) H(0; \alpha_0) - \right. \\
& \left. \frac{2}{3}d'_1H(0; Y) + \frac{2d'_1}{3(y_0-1)} + \frac{80d'_1}{3(y_0-2)^2} + \frac{160d'_1}{3(y_0-2)^3} + \frac{40d'_1}{(y_0-2)^4} + \frac{32d'_1}{3(y_0-2)^5} \right) H(0, 1; y_0) + \left( -\frac{2y_0^3}{9} + y_0^2 - \right. \\
& \left. 2y_0 + \frac{4}{3}H(0; \alpha_0) + \frac{2}{3}H(0; Y) + \frac{4}{3}d_1H(1; \alpha_0) - \frac{2}{y_0-1} + \frac{80}{3(y_0-2)^2} + \frac{160}{3(y_0-2)^3} + \frac{40}{(y_0-2)^4} + \frac{32}{3(y_0-2)^5} - \right. \\
& \left. \frac{107}{18} \right) H(0, c_1(\alpha_0); y_0) + \left( \frac{26}{3} - \frac{320}{3(y_0-2)^2} - \frac{640}{3(y_0-2)^3} - \frac{160}{(y_0-2)^4} - \frac{128}{3(y_0-2)^5} \right) H(0, c_2(\alpha_0); y_0) + \\
& \frac{2}{9}y_0^3H(1, 0; Y) - y_0^2H(1, 0; Y) + 2y_0H(1, 0; Y) + \frac{4}{9}H(1, 0; Y) + \left( \frac{2d'_1y_0^3}{3} + \frac{2y_0^3}{9} - 3d'_1y_0^2 - y_0^2 + \right. \\
& \left. 6d'_1y_0 + 2y_0 - \frac{19d'_1}{3} + \frac{4}{3}H(0; \alpha_0) - \frac{2}{3}H(0; Y) - \frac{4d_1}{3(y_0-1)} + \frac{2d'_1}{3(y_0-1)} + \frac{2}{3(y_0-1)} + \frac{80d'_1}{3(y_0-2)^2} - \frac{80}{3(y_0-2)^2} + \right. \\
& \left. \frac{160d'_1}{3(y_0-2)^3} - \frac{160}{3(y_0-2)^3} + \frac{40d'_1}{(y_0-2)^4} - \frac{40}{(y_0-2)^4} + \frac{32d'_1}{3(y_0-2)^5} - \frac{32}{3(y_0-2)^5} + \frac{107}{18} \right) H(1, 0; y_0) + \left( \frac{2d_1^2y_0^3}{9} - d_1^2y_0^2 + \right. \\
& \left. 2d_1^2y_0 - \frac{11d_1^2}{9} + \left( -\frac{4d_1}{3} + \frac{2d'_1}{3} + \frac{2}{3} \right) H(0; \alpha_0) \right) H(1, 1; y_0) + \left( -\frac{2y_0^3}{9} + y_0^2 - 2y_0 + \frac{4}{3}H(0; \alpha_0) + \right. \\
& \left. \frac{2}{3}H(0; Y) + \frac{4}{3}d_1H(1; \alpha_0) + \frac{4d_1}{3(y_0-1)} - \frac{2d'_1}{3(y_0-1)} - \frac{2}{3(y_0-1)} + \frac{80}{3(y_0-2)^2} + \frac{160}{3(y_0-2)^3} + \frac{40}{(y_0-2)^4} + \frac{32}{3(y_0-2)^5} - \right. \\
& \left. \frac{107}{18} \right) H(1, c_1(\alpha_0); y_0) + \left( -\frac{80d'_1}{3(y_0-2)^2} - \frac{160d'_1}{3(y_0-2)^3} - \frac{40d'_1}{(y_0-2)^4} - \frac{32d'_1}{3(y_0-2)^5} + \frac{8d'_1}{3} \right) H(1, c_2(\alpha_0); y_0) + \\
& \left( \frac{160d_1}{3(y_0-2)^2} + \frac{320d_1}{3(y_0-2)^3} + \frac{80d_1}{(y_0-2)^4} + \frac{64d_1}{3(y_0-2)^5} - \frac{8d'_1}{3} - \frac{160}{3(y_0-2)^2} - \frac{320}{3(y_0-2)^3} - \frac{80}{(y_0-2)^4} - \frac{64}{3(y_0-2)^5} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{26}{3} \Big) H(2, 0; y_0) + \left( -\frac{160d_1}{3(y_0-2)^2} - \frac{320d_1}{3(y_0-2)^3} - \frac{80d_1}{(y_0-2)^4} - \frac{64d_1}{3(y_0-2)^5} + \frac{8d_1'}{3} + \frac{160}{3(y_0-2)^2} + \frac{320}{3(y_0-2)^3} + \right. \\
& \left. \frac{80}{(y_0-2)^4} + \frac{64}{3(y_0-2)^5} - \frac{26}{3} \right) H(2, c_2(\alpha_0); y_0) + \left( \frac{4}{3} \frac{\alpha_0}{3} - \frac{4}{3(y_0-1)} - \frac{4}{3} \right) H(c_1(\alpha_0), 0; y_0) + \left( \frac{2\alpha_0 d_1'}{3} - \right. \\
& \left. \frac{2d_1'}{3(y_0-1)} - \frac{2d_1'}{3} \right) H(c_1(\alpha_0), 1; y_0) + \left( \frac{2}{3} \frac{\alpha_0}{3} - \frac{2}{3(y_0-1)} - \frac{2}{3} \right) H(c_1(\alpha_0), c_1(\alpha_0); y_0) + \left( \alpha_0^4 + \frac{4\alpha_0^3}{3} - \right. \\
& \left. 2\alpha_0^2 - 4\alpha_0 - \frac{160}{3(y_0-2)^2} - \frac{320}{3(y_0-2)^3} - \frac{80}{(y_0-2)^4} - \frac{64}{3(y_0-2)^5} + \frac{11}{3} \right) H(c_2(\alpha_0), 0; y_0) + \left( \frac{d_1' \alpha_0^4}{2} + \right. \\
& \left. \frac{2d_1' \alpha_0^3}{3} - d_1' \alpha_0^2 - 2d_1' \alpha_0 + \frac{11d_1'}{6} - \frac{80d_1'}{3(y_0-2)^2} - \frac{160d_1'}{3(y_0-2)^3} - \frac{40d_1'}{(y_0-2)^4} - \frac{32d_1'}{3(y_0-2)^5} \right) H(c_2(\alpha_0), 1; y_0) + \\
& \left( \frac{\alpha_0^4}{2} + \frac{2\alpha_0^3}{3} - \alpha_0^2 - 2\alpha_0 - \frac{80}{3(y_0-2)^2} - \frac{160}{3(y_0-2)^3} - \frac{40}{(y_0-2)^4} - \frac{32}{3(y_0-2)^5} + \frac{11}{6} \right) H(c_2(\alpha_0), c_1(\alpha_0); y_0) - \\
& \frac{1}{6} H(0, 0, 0; Y) - \frac{32}{3} H(0, 0, 0; y_0) - \frac{8}{3} d_1' H(0, 0, 1; y_0) + \frac{8}{3} H(0, 0, c_1(\alpha_0); y_0) - \frac{1}{6} H(0, 1, 0; Y) + \\
& \left( \frac{4d_1}{3} - \frac{8d_1'}{3} - \frac{4}{3} \right) H(0, 1, 0; y_0) - \frac{2}{3} d_1'^2 H(0, 1, 1; y_0) + \left( -\frac{4}{3} \frac{d_1}{3} + \frac{2d_1'}{3} + \frac{4}{3} \right) H(0, 1, c_1(\alpha_0); y_0) + \\
& \frac{4}{3} H(0, c_1(\alpha_0), 0; y_0) + \frac{2}{3} d_1' H(0, c_1(\alpha_0), 1; y_0) + \frac{2}{3} H(0, c_1(\alpha_0), c_1(\alpha_0); y_0) - \frac{1}{6} H(1, 0, 0; Y) - \\
& 4 H(1, 0, 0; y_0) - \frac{2}{3} d_1' H(1, 0, 1; y_0) + 2 H(1, 0, c_1(\alpha_0); y_0) - \frac{1}{6} H(1, 1, 0; Y) + \left( \frac{4}{3} \frac{d_1}{3} - \frac{2d_1'}{3} - \right. \\
& \left. \frac{2}{3} \right) H(1, 1, 0; y_0) + \left( -\frac{4}{3} \frac{d_1}{3} + \frac{2d_1'}{3} + \frac{2}{3} \right) H(1, 1, c_1(\alpha_0); y_0) + \frac{4}{3} H(1, c_1(\alpha_0), 0; y_0) + \\
& \frac{2}{3} d_1' H(1, c_1(\alpha_0), 1; y_0) + \frac{2}{3} H(1, c_1(\alpha_0), c_1(\alpha_0); y_0) + H(0; y_0) \left( \frac{y_0^3 \alpha_0^4}{9} - \frac{y_0^2 \alpha_0^4}{6} + \frac{2y_0 \alpha_0^4}{3} - \frac{\alpha_0^4}{3(y_0-2)} - \right. \\
& \left. \frac{\alpha_0^4}{6} - \frac{16y_0^3 \alpha_0^3}{27} + \frac{4y_0^2 \alpha_0^3}{3} - \frac{16y_0 \alpha_0^3}{9} - \frac{2\alpha_0^3}{y_0-2} - \frac{8\alpha_0^3}{9(y_0-2)^2} - \frac{7\alpha_0^3}{9} + \frac{4y_0^3 \alpha_0^2}{3} - 4y_0^2 \alpha_0^2 + \frac{10y_0 \alpha_0^2}{3} - \frac{16\alpha_0^2}{3(y_0-2)} - \frac{22\alpha_0^2}{3(y_0-2)^2} - \right. \\
& \left. \frac{8\alpha_0^2}{3(y_0-2)^3} - \frac{7\alpha_0^2}{6} - \frac{16y_0^3 \alpha_0}{9} + \frac{20y_0^2 \alpha_0}{3} - 8y_0 \alpha_0 - \frac{26\alpha_0}{3(y_0-2)} - \frac{36\alpha_0}{(y_0-2)^2} - \frac{104\alpha_0}{3(y_0-2)^3} - \frac{32\alpha_0}{3(y_0-2)^4} + \alpha_0 + \frac{2d_1' y_0^3}{9} - \right. \\
& \left. \frac{139y_0^3}{54} - \frac{7d_1' y_0^3}{6} + \frac{457y_0^2}{36} - \frac{41d_1}{36} - \frac{d_1'}{18} + \frac{11d_1' y_0}{3} - 33y_0 + \left( \frac{4}{9} \frac{y_0^3}{3} - 2y_0^2 + 4y_0 - \frac{4}{3(y_0-1)} - \frac{160}{3(y_0-2)^2} - \right. \right. \\
& \left. \left. \frac{320}{3(y_0-2)^3} - \frac{80}{(y_0-2)^4} - \frac{64}{3(y_0-2)^5} + 3 \right) H(0; \alpha_0) + \frac{2}{3} y_0^3 H(0; Y) - 3y_0^2 H(0; Y) + 6y_0 H(0; Y) + \frac{2H(0; Y)}{3(y_0-1)} + \right. \\
& \left. \frac{80H(0; Y)}{3(y_0-2)^2} + \frac{160H(0; Y)}{3(y_0-2)^3} + \frac{40H(0; Y)}{(y_0-2)^4} + \frac{32H(0; Y)}{3(y_0-2)^5} + \frac{5}{18} H(0; Y) - \frac{2}{3} H(0, 0; Y) - \frac{2}{3} H(1, 0; Y) - \frac{8d_1'}{3(y_0-2)} + \right. \\
& \left. \frac{20}{3(y_0-2)} + \frac{4d_1}{3(y_0-1)} - \frac{2d_1'}{3(y_0-1)} - \frac{7}{2(y_0-1)} + \frac{16d_1}{(y_0-2)^2} - \frac{12d_1'}{(y_0-2)^2} - \frac{88}{(y_0-2)^2} + \frac{128d_1}{9(y_0-2)^3} - \frac{88d_1'}{9(y_0-2)^3} - \frac{168}{(y_0-2)^3} + \right. \\
& \left. \frac{4d_1}{(y_0-2)^4} - \frac{8d_1'}{3(y_0-2)^4} - \frac{116}{(y_0-2)^4} - \frac{256}{9(y_0-2)^5} - \frac{320 \ln 2}{3(y_0-2)^2} - \frac{640 \ln 2}{3(y_0-2)^3} - \frac{160 \ln 2}{(y_0-2)^4} - \frac{128 \ln 2}{3(y_0-2)^5} + \frac{26 \ln 2}{3} - \right. \\
& \left. \frac{\pi^2}{9} + \frac{361}{108} \right) + H(2; y_0) \left( -\frac{160 \ln 2 d_1}{3(y_0-2)^2} - \frac{320 \ln 2 d_1}{3(y_0-2)^3} - \frac{80 \ln 2 d_1}{(y_0-2)^4} - \frac{64 \ln 2 d_1}{3(y_0-2)^5} + \left( -\frac{160d_1}{3(y_0-2)^2} - \frac{320d_1}{3(y_0-2)^3} - \right. \right. \\
& \left. \left. \frac{80d_1}{(y_0-2)^4} - \frac{64d_1}{3(y_0-2)^5} + \frac{8d_1'}{3} + \frac{160}{3(y_0-2)^2} + \frac{320}{3(y_0-2)^3} + \frac{80}{(y_0-2)^4} + \frac{64}{3(y_0-2)^5} - \frac{26}{3} \right) H(0; \alpha_0) + \frac{8}{3} d_1' \ln 2 + \right. \\
& \left. \frac{160 \ln 2}{3(y_0-2)^2} + \frac{320 \ln 2}{3(y_0-2)^3} + \frac{80 \ln 2}{(y_0-2)^4} + \frac{64 \ln 2}{3(y_0-2)^5} - \frac{26 \ln 2}{3} \right) + H(1; y_0) \left( \frac{1}{18} d_1' y_0^3 \alpha_0^4 - \frac{1}{12} d_1' y_0^2 \alpha_0^4 - \frac{17d_1' \alpha_0^4}{36} + \right. \\
& \left. \frac{1}{3} d_1' y_0 \alpha_0^4 - \frac{d_1' \alpha_0^4}{6(y_0-2)} - \frac{8}{27} d_1' y_0^3 \alpha_0^3 + \frac{2}{3} d_1' y_0^2 \alpha_0^3 - \frac{d_1' \alpha_0^3}{27} - \frac{8}{9} d_1' y_0 \alpha_0^3 - \frac{d_1' \alpha_0^3}{y_0-2} - \frac{4d_1' \alpha_0^3}{9(y_0-2)^2} + \frac{2}{3} d_1' y_0^3 \alpha_0^2 - \right. \\
& \left. 2d_1' y_0^2 \alpha_0^2 - \frac{2d_1' \alpha_0^2}{3} + \frac{5}{3} d_1' y_0 \alpha_0^2 - \frac{8d_1' \alpha_0^2}{3(y_0-2)} - \frac{11d_1' \alpha_0^2}{3(y_0-2)^2} - \frac{4d_1' \alpha_0^2}{3(y_0-2)^3} - \frac{8}{9} d_1' y_0^3 \alpha_0 + \frac{10}{3} d_1' y_0^2 \alpha_0 + \frac{23d_1' \alpha_0}{9} - \right. \\
& \left. 4d_1' y_0 \alpha_0 - \frac{13d_1' \alpha_0}{3(y_0-2)} - \frac{18d_1' \alpha_0}{(y_0-2)^2} - \frac{52d_1' \alpha_0}{3(y_0-2)^3} - \frac{16d_1' \alpha_0}{3(y_0-2)^4} + \frac{2d_1'^2 y_0^3}{27} - \frac{19d_1' y_0^3}{27} - \frac{49d_1'^2}{54} - \frac{7d_1' y_0^2}{18} + \frac{11d_1' y_0^2}{3} + \right. \\
& \left. \frac{199d_1'}{27} + \frac{11d_1'^2 y_0}{9} - \frac{31d_1' y_0}{3} + H(0; \alpha_0) \left( \frac{2d_1' y_0^3}{9} - \frac{2y_0^3}{9} - d_1' y_0^2 + y_0^2 + 2d_1' y_0 - 2y_0 + \frac{13d_1'}{9} + \frac{2}{3} H(0; Y) + \right. \right. \\
& \left. \left. \frac{4d_1}{3(y_0-1)} - \frac{2d_1'}{3(y_0-1)} - \frac{2}{3(y_0-1)} - \frac{80d_1'}{3(y_0-2)^2} + \frac{80}{3(y_0-2)^2} - \frac{160d_1'}{3(y_0-2)^3} + \frac{160}{3(y_0-2)^3} - \frac{40d_1'}{(y_0-2)^4} + \frac{40}{(y_0-2)^4} - \right. \right. \\
& \left. \left. \frac{32d_1'}{3(y_0-2)^5} + \frac{32}{3(y_0-2)^5} - \frac{107}{18} \right) + \frac{2}{9} d_1' y_0^3 H(0; Y) - d_1' y_0^2 H(0; Y) - \frac{11}{9} d_1' H(0; Y) + 2d_1' y_0 H(0; Y) + \right. \\
& \left. \frac{4}{3} H(0, 0; \alpha_0) + \frac{4}{3} d_1' H(0, 1; \alpha_0) + \frac{8}{3} d_1' \ln 2 - \frac{80d_1' \ln 2}{3(y_0-2)^2} - \frac{160d_1' \ln 2}{3(y_0-2)^3} - \frac{40d_1' \ln 2}{(y_0-2)^4} - \frac{32d_1' \ln 2}{3(y_0-2)^5} - \frac{\pi^2}{9} \right) + \\
& \frac{\pi^2}{9(y_0-1)} + \frac{20\pi^2}{9(y_0-2)^2} + \frac{40\pi^2}{9(y_0-2)^3} + \frac{10\pi^2}{3(y_0-2)^4} + \frac{8\pi^2}{9(y_0-2)^5} - \zeta_3 + \frac{80 \ln^2 2}{3(y_0-2)^2} + \frac{160 \ln^2 2}{3(y_0-2)^3} + \frac{40 \ln^2 2}{(y_0-2)^4} + \\
& \frac{32 \ln^2 2}{3(y_0-2)^5} - \frac{13 \ln^2 2}{6} + \frac{89}{36} d_1 \ln 2 - \frac{11}{18} d_1' \ln 2 - \frac{80 H(0; Y) \ln 2}{3(y_0-2)^2} - \frac{160 H(0; Y) \ln 2}{3(y_0-2)^3} - \frac{40 H(0; Y) \ln 2}{(y_0-2)^4} - \\
& \frac{32 H(0; Y) \ln 2}{3(y_0-2)^5} + \frac{13}{6} H(0; Y) \ln 2 + \frac{8d_1' \ln 2}{3(y_0-2)} - \frac{52 \ln 2}{3(y_0-2)} - \frac{16d_1 \ln 2}{(y_0-2)^2} + \frac{12d_1' \ln 2}{(y_0-2)^2} + \frac{460 \ln 2}{9(y_0-2)^2} -
\end{aligned}$$

$$\frac{128d_1 \ln 2}{9(y_0-2)^3} + \frac{88d_1' \ln 2}{9(y_0-2)^3} + \frac{400 \ln 2}{3(y_0-2)^3} - \frac{4d_1 \ln 2}{(y_0-2)^4} + \frac{8d_1' \ln 2}{3(y_0-2)^4} + \frac{316 \ln 2}{3(y_0-2)^4} + \frac{256 \ln 2}{9(y_0-2)^5} - \frac{377 \ln 2}{36} + \frac{\pi^2}{216} + \frac{160}{81}.$$

#### F.4 The $\mathcal{J}\mathcal{L}$ integral for $k = -1$

The  $\varepsilon$  expansion for this integral reads

$$\mathcal{JL}(Y, \varepsilon; y_0, d_0', \alpha_0, d_0; -1) = \frac{1}{\varepsilon^4} (j*i)_{-4}^{(-1)} + \frac{1}{\varepsilon^3} (j*i)_{-3}^{(-1)} + \frac{1}{\varepsilon^2} (j*i)_{-2}^{(-1)} + \frac{1}{\varepsilon} (j*i)_{-1}^{(-1)} + (j*i)_0^{(-1)} + \mathcal{O}(\varepsilon), \quad (\text{F.4})$$

where

$$(j * i)_{-4}^{(-1)} = -\frac{1}{4},$$

$$(j * i)_{-3}^{(-1)} = -\frac{y_0^3}{3} + \frac{3y_0^2}{2} - 3y_0 + \frac{1}{4}H(0; Y) + H(0; y_0),$$

$$(j * i)_{-2}^{(-1)} = \frac{d_1' y_0^3}{9} + \frac{1}{3}H(0; Y)y_0^3 - \frac{4}{9}y_0^3 - \frac{7d_1' y_0^2}{12} - \frac{3}{2}H(0; Y)y_0^2 + 3y_0^2 + \frac{11d_1' y_0}{6} + 3H(0; Y)y_0 - 12y_0 + \left( \frac{4}{3}y_0^3 - 6y_0^2 + 12y_0 - H(0; Y) \right) H(0; y_0) + \left( \frac{d_1' y_0^3}{3} - \frac{3d_1' y_0^2}{2} + 3d_1' y_0 - \frac{11d_1'}{6} \right) H(1; y_0) - \frac{1}{4}H(0, 0; Y) - 4H(0, 0; y_0) - d_1' H(0, 1; y_0) - \frac{1}{4}H(1, 0; Y) - \frac{\pi^2}{24},$$

$$(j * i)_{-1}^{(-1)} = \frac{y_0^3 \alpha_0^4}{18} - \frac{5y_0^2 \alpha_0^4}{12} + \frac{5y_0 \alpha_0^4}{3} - \frac{13y_0^3 \alpha_0^3}{54} + \frac{17y_0^2 \alpha_0^3}{9} - \frac{80 y_0 \alpha_0^3}{9} + \frac{11y_0^3 \alpha_0^2}{36} - \frac{109y_0^2 \alpha_0^2}{36} + \frac{158y_0 \alpha_0^2}{9} + \frac{7y_0^3 \alpha_0}{18} + \frac{7 y_0^2 \alpha_0}{18} - \frac{43y_0 \alpha_0}{3} - \frac{d_1'^2 y_0^3}{27} + \frac{8 d_1' y_0^3}{27} + \frac{\pi^2 y_0^3}{18} - \frac{14y_0^3}{27} + \frac{17 d_1'^2 y_0^2}{72} - \frac{22d_1' y_0^2}{9} - \frac{\pi^2 y_0^2}{4} + \frac{21y_0^2}{4} - \frac{49d_1' y_0}{36} + \frac{151d_1' y_0}{9} + \frac{\pi^2 y_0}{2} - 42y_0 + \left( -\frac{7y_0^3}{6} + \frac{13 y_0^2}{3} - \frac{11y_0}{2} + \frac{13}{6(y_0-1)} + \frac{13}{6} \right) H(0; \alpha_0) - \frac{1}{9}d_1' y_0^3 H(0; Y) + \frac{4}{9}y_0^3 H(0; Y) + \frac{7}{12}d_1' y_0^2 H(0; Y) - 3y_0^2 H(0; Y) - \frac{11}{6} d_1' y_0 H(0; Y) + 12y_0 H(0; Y) + \frac{1}{24}\pi^2 H(0; Y) + \frac{1}{24} \pi^2 H(1; Y) + \left( -\frac{1}{9}d_1'^2 y_0^3 + \frac{4d_1' y_0^3}{9} - \frac{1}{3}d_1' H(0; Y)y_0^3 + \frac{7d_1'^2 y_0^2}{12} - 3 d_1' y_0^2 + \frac{3}{2}d_1' H(0; Y)y_0^2 - \frac{11d_1'^2 y_0}{6} + 12 d_1' y_0 - 3d_1' H(0; Y)y_0 + \frac{49d_1'^2}{36} - \frac{85 d_1'}{9} + \left( \frac{2y_0^3}{3} - 3y_0^2 + 6 y_0 + \frac{2}{y_0-1} - \frac{19}{3} \right) H(0; \alpha_0) + \frac{11}{6}d_1' H(0; Y) + \frac{\pi^2}{3} \right) H(1; y_0) + \left( -\frac{1}{6}y_0^3 \alpha_0^4 + y_0^2 \alpha_0^4 - \frac{5y_0 \alpha_0^4}{2} + \frac{5\alpha_0^4}{3} + \frac{8}{9}y_0^3 \alpha_0^3 - 5y_0^2 \alpha_0^3 + \frac{38y_0 \alpha_0^3}{3} - \frac{68}{9}\alpha_0^3 - 2y_0^3 \alpha_0^2 + \frac{21y_0^2 \alpha_0^2}{2} - 26y_0 \alpha_0^2 + \frac{38\alpha_0^2}{3} + \frac{8y_0^3 \alpha_0}{3} - 13y_0^2 \alpha_0 + 30y_0 \alpha_0 - \frac{41\alpha_0}{3} - \frac{25y_0^3}{18} + \frac{35y_0^2}{6} - \frac{23 y_0}{2} + \frac{13}{6(y_0-1)} + \frac{13}{6} \right) H(c_1(\alpha_0); y_0) - \frac{1}{3}y_0^3 H(0, 0; Y) + \frac{3}{2}y_0^2 H(0, 0; Y) - 3y_0 H(0, 0; Y) + \left( -\frac{16y_0^3}{3} + 24y_0^2 - 48y_0 + 4 H(0; Y) \right) H(0, 0; y_0) + \left( -\frac{4d_1' y_0^3}{3} + 6d_1' y_0^2 - 12 d_1' y_0 - 4H(0; \alpha_0) + d_1' H(0; Y) \right) H(0, 1; y_0) + \left( \alpha_0^4 - \frac{16}{3}\alpha_0^3 + 12\alpha_0^2 - 16\alpha_0 + \frac{2y_0^3}{3} - 3y_0^2 + 6 y_0 + \frac{2}{y_0-1} + 2 \right) H(0, c_1(\alpha_0); y_0) - \frac{1}{3}y_0^3 H(1, 0; Y) + \frac{3}{2}y_0^2 H(1, 0; Y) - 3y_0 H(1, 0; Y) + H(0; y_0) \left( -\frac{4d_1' y_0^3}{9} - \frac{4}{3}H(0; Y)y_0^3 + \frac{53}{18}y_0^3 + \frac{7d_1' y_0^2}{3} + 6H(0; Y)y_0^2 - \frac{49}{3}y_0^2 - \frac{22d_1' y_0}{3} - 12H(0; Y)y_0 + \frac{107}{2}y_0 + H(0, 0; Y) + H(1, 0; Y) - \frac{13}{6(y_0-1)} + \frac{\pi^2}{6} - \frac{13}{6} \right) + \left( -\frac{4d_1' y_0^3}{3} - \frac{2}{3}y_0^3 + 6d_1' y_0^2 + 3y_0^2 - 12d_1' y_0 - 6y_0 + \frac{22}{3}d_1' - \frac{2}{y_0-1} + \frac{19}{3} \right) H(1, 0; y_0) + \left( -\frac{1}{3}d_1'^2 y_0^3 + \frac{3d_1' y_0^2}{2} - 3 d_1' y_0 + \frac{11d_1'^2}{6} - 2H(0; \alpha_0) \right) H(1, 1; y_0) + \left( \frac{2y_0^3}{3} - 3y_0^2 + 6 y_0 + \frac{2}{y_0-1} - \frac{19}{3} \right) H(1, c_1(\alpha_0); y_0) + \left( 2 \alpha_0 - \frac{2}{y_0-1} - 2 \right) H(c_1(\alpha_0), c_1(\alpha_0); y_0) + \frac{1}{4} H(0, 0, 0; Y) + 16H(0, 0, 0; y_0) + 4d_1' H(0, 0, 1; y_0) - 4H(0, 0, c_1(\alpha_0); y_0) + \frac{1}{4}H(0, 1, 0; Y) + (4d_1' + 4)H(0, 1, 0; y_0) + d_1'^2 H(0, 1, 1; y_0) - 4 H(0, 1, c_1(\alpha_0); y_0) + 2H(0, c_1(\alpha_0), c_1(\alpha_0); y_0) + \frac{1}{4} H(1, 0, 0; Y) - 2H(1, 0, c_1(\alpha_0); y_0) + \frac{1}{4}H(1, 1, 0; Y) + 2 H(1, 1, 0; y_0) - 2H(1, 1, c_1(\alpha_0); y_0) + 2 H(1, c_1(\alpha_0), c_1(\alpha_0); y_0) - \frac{\pi^2}{3(y_0-1)} + \frac{3}{2}\zeta_3 - \frac{\pi^2}{3},$$



$$\begin{aligned}
(j * i)_0^{(-1)} = & -\frac{1}{36}d_1y_0^3\alpha_0^4 - \frac{1}{27}d_1'y_0^3\alpha_0^4 + \frac{13}{108}y_0^3\alpha_0^4 + \frac{5}{24}d_1y_0^2\alpha_0^4 + \frac{13}{36}d_1'y_0^2\alpha_0^4 - \frac{7y_0^2\alpha_0^4}{6} - \\
& \frac{5}{6}d_1y_0\alpha_0^4 - \frac{47}{18}d_1'y_0\alpha_0^4 + \frac{97y_0\alpha_0^4}{12} - \frac{1}{18}y_0^3H(0;Y)\alpha_0^4 + \frac{5}{12}y_0^2H(0;Y)\alpha_0^4 - \frac{5}{3}y_0H(0;Y)\alpha_0^4 + \\
& \frac{31}{324}d_1y_0^3\alpha_0^3 + \frac{29}{162}d_1'y_0^3\alpha_0^3 - \frac{185y_0^3\alpha_0^3}{324} - \frac{91}{108}d_1y_0^2\alpha_0^3 - \frac{187}{108}d_1'y_0^2\alpha_0^3 + \frac{631y_0^2\alpha_0^3}{108} + \frac{245}{54}d_1y_0\alpha_0^3 + \\
& \frac{1549}{108}d_1'y_0\alpha_0^3 - \frac{5095y_0\alpha_0^3}{108} + \frac{13}{54}y_0^3H(0;Y)\alpha_0^3 - \frac{17}{9}y_0^2H(0;Y)\alpha_0^3 + \frac{80}{9}y_0H(0;Y)\alpha_0^3 + \frac{17}{216}d_1y_0^3\alpha_0^2 - \\
& \frac{35}{108}d_1'y_0^3\alpha_0^2 + \frac{179y_0^3\alpha_0^2}{216} + \frac{17}{27}d_1y_0^2\alpha_0^2 + \frac{707}{216}d_1'y_0^2\alpha_0^2 - \frac{589y_0^2\alpha_0^2}{54} - \frac{895}{108}d_1y_0\alpha_0^2 - \frac{809}{27}d_1'y_0\alpha_0^2 + \\
& \frac{2836y_0\alpha_0^2}{27} - \frac{11}{36}y_0^3H(0;Y)\alpha_0^2 + \frac{109}{36}y_0^2H(0;Y)\alpha_0^2 - \frac{158}{9}y_0H(0;Y)\alpha_0^2 - \frac{205}{108}d_1y_0^3\alpha_0 + \frac{1}{6}d_1'y_0^3\alpha_0 + \\
& \frac{95y_0^3\alpha_0}{54} - \frac{659}{108}d_1y_0^2\alpha_0 - \frac{74}{27}d_1'y_0^2\alpha_0 + \frac{187y_0^2\alpha_0}{108} + \frac{d_1y_0\alpha_0}{36} + \frac{278d_1'y_0\alpha_0}{9} - \frac{219y_0\alpha_0}{2} - \frac{7}{18}y_0^3H(0;Y)\alpha_0 - \\
& \frac{7}{18}y_0^2H(0;Y)\alpha_0 + \frac{43}{3}y_0H(0;Y)\alpha_0 + \frac{d_1^3y_0^3}{81} - \frac{4d_1^2y_0^3}{27} + \frac{14d_1'y_0^3}{27} - \frac{1}{54}d_1'\pi^2y_0^3 + \frac{29\pi^2y_0^3}{108} - \frac{46y_0^3}{81} - \frac{43d_1^3y_0^2}{432} + \\
& \frac{167}{108}d_1^2y_0^2 - \frac{1435d_1'y_0^2}{216} + \frac{7}{72}d_1'\pi^2y_0^2 - \frac{11\pi^2y_0^2}{9} + \frac{69y_0^2}{8} + \frac{251}{216}d_1^3y_0 - \frac{542d_1^2y_0}{27} + \frac{10367d_1'y_0}{108} - \frac{11}{36}d_1'\pi^2y_0 + \\
& \frac{35\pi^2y_0}{12} - 138y_0 + \frac{1}{27}d_1^2y_0^3H(0;Y) - \frac{8}{27}d_1'y_0^3H(0;Y) - \frac{1}{18}\pi^2y_0^3H(0;Y) + \frac{14}{27}y_0^3H(0;Y) - \\
& \frac{17}{72}d_1^2y_0^2H(0;Y) + \frac{22}{9}d_1'y_0^2H(0;Y) + \frac{1}{4}\pi^2y_0^2H(0;Y) - \frac{21}{4}y_0^2H(0;Y) + \frac{49}{36}d_1^2y_0H(0;Y) - \\
& \frac{151}{9}d_1'y_0H(0;Y) - \frac{1}{2}\pi^2y_0H(0;Y) + 42y_0H(0;Y) + \frac{\pi^2H(0;Y)}{3(y_0-1)} + \frac{1}{3}\pi^2H(0;Y) + H(0;\alpha_0) \left( - \right. \\
& \frac{1}{9}y_0^3\alpha_0^4 + \frac{5y_0^2\alpha_0^4}{6} - \frac{10y_0\alpha_0^4}{3} + \frac{13y_0^3\alpha_0^3}{27} - \frac{34y_0^2\alpha_0^3}{9} + \frac{160y_0\alpha_0^3}{9} - \frac{11}{18}y_0^3\alpha_0^2 + \frac{109y_0^2\alpha_0^2}{18} - \frac{316y_0\alpha_0^2}{9} - \frac{7y_0^3\alpha_0}{9} - \\
& \frac{7y_0^2\alpha_0}{9} + \frac{86y_0\alpha_0}{3} + \frac{205d_1y_0^3}{108} + \frac{17d_1'y_0^3}{54} - \frac{35}{9}y_0^3 - \frac{22d_1y_0^2}{3} - \frac{10d_1'y_0^2}{9} + \frac{649}{36}y_0^2 - \frac{217d_1}{36} + \frac{d_1'}{6} + \frac{469d_1'y_0}{36} - \\
& \frac{7d_1'y_0}{18} - \frac{299y_0}{9} + \frac{7}{6}y_0^3H(0;Y) - \frac{13}{3}y_0^2H(0;Y) + \frac{11}{2}y_0H(0;Y) - \frac{13}{6}H(0;Y) - \frac{13}{6}H(0;Y) - \frac{217d_1}{36(y_0-1)} + \\
& \left. \frac{d_1'}{6(y_0-1)} + \frac{149}{18(y_0-1)} + \frac{149}{18} \right) + \left( -\frac{1}{9}d_1y_0^3\alpha_0^4 + \frac{5}{6}d_1y_0^2\alpha_0^4 - \frac{10}{3}d_1y_0\alpha_0^4 + \frac{13}{27}d_1y_0^3\alpha_0^3 - \frac{34}{9}d_1y_0^2\alpha_0^3 + \right. \\
& \frac{160}{9}d_1y_0\alpha_0^3 - \frac{11}{18}d_1y_0^3\alpha_0^2 + \frac{109}{18}d_1y_0^2\alpha_0^2 - \frac{316}{9}d_1y_0\alpha_0^2 - \frac{7}{9}d_1y_0^3\alpha_0 - \frac{7}{9}d_1y_0^2\alpha_0 + \frac{86d_1y_0\alpha_0}{3} + \frac{55d_1'y_0^3}{54} - \\
& \left. \frac{7d_1y_0^2}{3} - 8d_1y_0 \right) H(1;\alpha_0) + \frac{1}{18}\pi^2y_0^3H(1;Y) - \frac{1}{4}\pi^2y_0^2H(1;Y) + \frac{1}{2}\pi^2y_0H(1;Y) + \left( \frac{1}{12}d_1y_0^3\alpha_0^4 + \right. \\
& \frac{1}{18}d_1'y_0^3\alpha_0^4 - \frac{y_0^2\alpha_0^4}{4} - \frac{1}{2}d_1y_0^2\alpha_0^4 - \frac{5}{12}d_1'y_0^2\alpha_0^4 + \frac{7y_0^2\alpha_0^4}{4} - \frac{5}{6}d_1\alpha_0^4 - \frac{47d_1'\alpha_0^4}{36} + \frac{5}{4}d_1y_0\alpha_0^4 + \frac{5}{3}d_1'y_0\alpha_0^4 - \\
& \frac{25y_0\alpha_0^4}{4} + \frac{1}{6}y_0^3H(0;Y)\alpha_0^4 - y_0^2H(0;Y)\alpha_0^4 + \frac{5}{2}y_0H(0;Y)\alpha_0^4 - \frac{5}{3}H(0;Y)\alpha_0^4 + \frac{19\alpha_0^4}{4} - \frac{13}{27}d_1y_0^3\alpha_0^3 - \\
& \frac{8}{27}d_1'y_0^3\alpha_0^3 + \frac{77y_0^3\alpha_0^3}{54} + \frac{8}{3}d_1y_0^2\alpha_0^3 + \frac{37}{18}d_1'y_0^2\alpha_0^3 - \frac{29y_0^2\alpha_0^3}{3} + \frac{221d_1\alpha_0^3}{54} + \frac{313}{54}d_1'\alpha_0^3 - \frac{61}{9}d_1y_0\alpha_0^3 - \\
& \frac{77}{9}d_1'y_0\alpha_0^3 + \frac{331y_0\alpha_0^3}{9} - \frac{8}{9}y_0^3H(0;Y)\alpha_0^3 + 5y_0^2H(0;Y)\alpha_0^3 - \frac{38}{3}y_0H(0;Y)\alpha_0^3 + \frac{68}{9}H(0;Y)\alpha_0^3 - \\
& \frac{676\alpha_0^3}{27} + \frac{23}{18}d_1y_0^3\alpha_0^2 + \frac{2}{3}d_1'y_0^3\alpha_0^2 - \frac{131y_0^3\alpha_0^2}{36} - \frac{13}{2}d_1y_0^2\alpha_0^2 - \frac{17}{4}d_1'y_0^2\alpha_0^2 + \frac{283y_0^2\alpha_0^2}{12} - \frac{287d_1\alpha_0^2}{36} - \frac{28d_1'\alpha_0^2}{3} + \\
& \frac{95}{6}d_1y_0\alpha_0^2 + \frac{35}{2}d_1'y_0\alpha_0^2 - \frac{533y_0\alpha_0^2}{6} + 2y_0^3H(0;Y)\alpha_0^2 - \frac{21}{2}y_0^2H(0;Y)\alpha_0^2 + 26y_0H(0;Y)\alpha_0^2 - \\
& \frac{38}{3}H(0;Y)\alpha_0^2 + \frac{149}{3}\alpha_0^2 - \frac{25}{9}d_1y_0^3\alpha_0 - \frac{8}{9}d_1'y_0^3\alpha_0 + \frac{121y_0^3\alpha_0}{18} + 13d_1y_0^2\alpha_0 + \frac{31}{6}d_1'y_0^2\alpha_0 - \frac{241y_0^2\alpha_0}{6} + \\
& \frac{343d_1\alpha_0}{18} + \frac{47d_1'\alpha_0}{6} - \frac{85d_1y_0\alpha_0}{3} - \frac{59d_1'y_0\alpha_0}{3} + \frac{404}{3}y_0\alpha_0 - \frac{8}{3}y_0^3H(0;Y)\alpha_0 + 13y_0^2H(0;Y)\alpha_0 - \\
& 30y_0H(0;Y)\alpha_0 + \frac{41}{3}H(0;Y)\alpha_0 - \frac{143\alpha_0}{2} + \frac{205d_1'y_0^3}{108} + \frac{25d_1'y_0^3}{54} - \frac{115y_0^3}{27} - \frac{22}{3}d_1y_0^2 - \frac{7d_1'y_0^2}{3} + \frac{196y_0^2}{9} - \\
& \frac{217}{36}d_1 + \frac{d_1'}{6} + \frac{469d_1y_0}{36} + 8d_1'y_0 - \frac{569}{9}y_0 + \left( \frac{y_0^3\alpha_0^4}{3} - 2y_0^2\alpha_0^4 + 5y_0\alpha_0^4 - \frac{10}{3}\alpha_0^4 - \frac{16y_0^3\alpha_0^3}{9} + 10y_0^2\alpha_0^3 - \right. \\
& \frac{76y_0\alpha_0^3}{3} + \frac{136\alpha_0^3}{9} + 4y_0^3\alpha_0^2 - 21y_0^2\alpha_0^2 + 52y_0\alpha_0^2 - \frac{76\alpha_0^2}{3} - \frac{16y_0^3\alpha_0}{3} + 26y_0^2\alpha_0 - 60y_0\alpha_0 + \frac{82\alpha_0}{3} + \\
& \left. \frac{25y_0^3}{9} - \frac{35y_0^2}{3} + 23y_0 - \frac{13}{3(y_0-1)} - \frac{13}{3} \right) H(0;\alpha_0) + \frac{25}{18}y_0^3H(0;Y) - \frac{35}{6}y_0^2H(0;Y) + \frac{23}{2}y_0H(0;Y) - \\
& \frac{13H(0;Y)}{6(y_0-1)} - \frac{13}{6}H(0;Y) + \left( \frac{1}{3}d_1y_0^3\alpha_0^4 - 2d_1y_0^2\alpha_0^4 - \frac{10}{3}d_1\alpha_0^4 + 5d_1y_0\alpha_0^4 - \frac{16}{9}d_1y_0^3\alpha_0^3 + 10d_1y_0^2\alpha_0^3 + \right. \\
& \frac{136d_1\alpha_0^3}{9} - \frac{76}{3}d_1y_0\alpha_0^3 + 4d_1y_0^3\alpha_0^2 - 21d_1y_0^2\alpha_0^2 - \frac{76d_1\alpha_0^2}{3} + 52d_1y_0\alpha_0^2 - \frac{16}{3}d_1y_0^3\alpha_0 + 26d_1y_0^2\alpha_0 + \\
& \left. \frac{82d_1\alpha_0}{3} - 60d_1y_0\alpha_0 + \frac{25d_1y_0^3}{9} - \frac{35d_1y_0^2}{3} - \frac{13d_1}{3} + 23d_1y_0 - \frac{13d_1}{3(y_0-1)} \right) H(1;\alpha_0) - \frac{217d_1}{36(y_0-1)} + \\
& \left. \frac{d_1'}{6(y_0-1)} + \frac{149}{18(y_0-1)} + \frac{149}{18} \right) H(c_1(\alpha_0);y_0) + \left( \frac{7y_0^3}{3} - \frac{26y_0^2}{3} + 11y_0 - \frac{13}{3(y_0-1)} - \frac{13}{3} \right) H(0,0;\alpha_0) + \\
& \frac{1}{9}d_1'y_0^3H(0,0;Y) - \frac{4}{9}y_0^3H(0,0;Y) - \frac{7}{12}d_1'y_0^2H(0,0;Y) + 3y_0^2H(0,0;Y) + \frac{11}{6}d_1'y_0H(0,0;Y) -
\end{aligned}$$

$$\begin{aligned}
& 12y_0 H(0, 0; Y) - \frac{1}{24}\pi^2 H(0, 0; Y) + \left( \frac{7d_1 y_0^3}{3} - \frac{26d_1 y_0^2}{3} + 11d_1 y_0 - \frac{13d_1}{3} - \frac{13 d_1}{3(y_0-1)} \right) H(0, 1; \alpha_0) - \\
& \frac{1}{24}\pi^2 H(0, 1; Y) + \left( -\frac{d_1 \alpha_0^4}{2} - H(0; Y) \alpha_0^4 + \frac{\alpha_0^4}{2} + \frac{26d_1 \alpha_0^3}{9} + \frac{16}{3} H(0; Y) \alpha_0^3 - \frac{38\alpha_0^3}{9} - \frac{23d_1 \alpha_0^2}{3} - \right. \\
& 12H(0; Y) \alpha_0^2 + \frac{49\alpha_0^2}{3} + \frac{50d_1 \alpha_0}{3} + 16H(0; Y) \alpha_0 - \frac{142 \alpha_0}{3} - \frac{2d_1^2 y_0^3}{9} + \frac{37y_0^3}{18} + \frac{7d_1^2 y_0^2}{6} - \frac{31y_0^2}{3} - 4d_1 + \\
& 2d_1' - \frac{11d_1' y_0}{3} + \frac{59y_0}{2} + \left( -2\alpha_0^4 + \frac{32\alpha_0^3}{3} - 24\alpha_0^2 + 32 \alpha_0 - \frac{4y_0^3}{3} + 6y_0^2 - 12y_0 - \frac{4}{y_0-1} - 4 \right) H(0; \alpha_0) - \\
& \frac{2}{3}y_0^3 H(0; Y) + 3y_0^2 H(0; Y) - 6y_0 H(0; Y) - \frac{2H(0; Y)}{y_0-1} - 2H(0; Y) + \left( -2d_1 \alpha_0^4 + \frac{32d_1 \alpha_0^3}{3} - \right. \\
& 24d_1 \alpha_0^2 + 32d_1 \alpha_0 - \frac{4d_1 y_0^3}{3} + 6d_1 y_0^2 - 4d_1 - 12d_1 y_0 - \frac{4d_1}{y_0-1} \left. \right) H(1; \alpha_0) - \frac{4d_1}{y_0-1} + \frac{2 d_1'}{y_0-1} + \frac{11}{6(y_0-1)} + \\
& \frac{11}{6} \left. \right) H(0, c_1(\alpha_0); y_0) + H(0, 0; y_0) \left( \frac{16d_1' y_0^3}{9} + \frac{16}{3} H(0; Y) y_0^3 - \frac{127y_0^3}{9} - \frac{28d_1' y_0^2}{3} - 24H(0; Y) y_0^2 + \right. \\
& 74y_0^2 + \frac{88d_1' y_0}{3} + 48H(0; Y) y_0 - 225y_0 - 4H(0, 0; Y) - 4H(1, 0; Y) + \frac{13}{y_0-1} - \frac{2\pi^2}{3} + 13 \left. \right) + \\
& \frac{1}{9}d_1' y_0^3 H(1, 0; Y) - \frac{4}{9}y_0^3 H(1, 0; Y) - \frac{7}{12}d_1' y_0^2 H(1, 0; Y) + 3y_0^2 H(1, 0; Y) + \frac{11}{6}d_1' y_0 H(1, 0; Y) - \\
& 12y_0 H(1, 0; Y) - \frac{1}{24} \pi^2 H(1, 0; Y) + H(0, 1; y_0) \left( \frac{4d_1^2 y_0^3}{9} - \frac{53 d_1' y_0^3}{18} + \frac{4}{3}d_1' H(0; Y) y_0^3 - \frac{7d_1^2 y_0^2}{3} + \right. \\
& \frac{49d_1^2 y_0^2}{3} - 6d_1' H(0; Y) y_0^2 + \frac{22d_1^2 y_0}{3} - \frac{107d_1' y_0}{2} + 12d_1' H(0; Y) y_0 + \frac{13 d_1'}{6} + H(0; \alpha_0) \left( \frac{4d_1 y_0^3}{3} - \right. \\
& \frac{4y_0^3}{3} - 6d_1 y_0^2 + 6y_0^2 + 12d_1 y_0 - 12y_0 - \frac{38d_1}{3} + 4H(0; Y) + \frac{4 d_1}{y_0-1} - \frac{4}{y_0-1} - \frac{62}{3} \left. \right) + 8H(0, 0; \alpha_0) - \\
& d_1' H(0, 0; Y) + 8d_1' H(0, 1; \alpha_0) - d_1' H(1, 0; Y) + \frac{13d_1'}{6(y_0-1)} + \frac{2d_1 \pi^2}{3} - \frac{d_1 \pi^2}{6} \left. \right) + \left( \frac{4d_1^2 y_0^3}{9} - \frac{49d_1' y_0^3}{18} + \right. \\
& \frac{4}{3}d_1' H(0; Y) y_0^3 + \frac{2}{3}H(0; Y) y_0^3 - \frac{37y_0^3}{18} - \frac{7d_1^2 y_0^2}{3} + \frac{4d_1 y_0^2}{3} + \frac{91d_1' y_0^2}{6} - 6d_1' H(0; Y) y_0^2 - 3H(0; Y) y_0^2 + \\
& \frac{35y_0^2}{3} + \frac{22d_1^2 y_0}{3} - \frac{16d_1 y_0}{3} - \frac{299d_1' y_0}{6} + 12d_1' H(0; Y) y_0 + 6H(0; Y) y_0 - \frac{209y_0}{6} - \frac{49d_1^2}{9} + \frac{37d_1}{18} + \frac{673 d_1'}{18} + \\
& \left( -\frac{4y_0^3}{3} + 6y_0^2 - 12 y_0 - \frac{4}{y_0-1} + \frac{38}{3} \right) H(0; \alpha_0) - \frac{22}{3}d_1' H(0; Y) + \frac{2H(0; Y)}{y_0-1} - \frac{19}{3}H(0; Y) - \frac{d_1}{3(y_0-1)} + \\
& \frac{d_1'}{6(y_0-1)} - \frac{37}{6(y_0-1)} - \frac{4\pi^2}{3} + \frac{127}{3} \left. \right) H(1, 0; y_0) - \frac{1}{24}\pi^2 H(1, 1; Y) + \left( \frac{y_0^3 d_1^3}{9} - \frac{7y_0^2 d_1^3}{12} + \frac{11y_0 d_1^3}{6} - \frac{49d_1^3}{36} - \right. \\
& \frac{4}{9}y_0^3 d_1^2 + 3y_0^2 d_1^2 - 12y_0 d_1^2 + \frac{1}{3}y_0^3 H(0; Y) d_1^2 - \frac{3}{2}y_0^2 H(0; Y) d_1^2 + 3y_0 H(0; Y) d_1^2 - \frac{11}{6}H(0; Y) d_1^2 + \\
& \frac{85d_1^2}{9} - \frac{\pi^2 d_1}{3} + H(0; \alpha_0) \left( \frac{4d_1 y_0^3}{3} - \frac{4d_1' y_0^3}{3} + \frac{2y_0^3}{3} - 6d_1 y_0^2 + 6d_1' y_0^2 - 3y_0^2 + 12d_1 y_0 - 12d_1' y_0 + 6y_0 - \right. \\
& \frac{38d_1}{3} + 10d_1' + 2H(0; Y) + \frac{8d_1}{y_0-1} - \frac{4d_1'}{y_0-1} - \frac{2}{y_0-1} - \frac{43}{3} \left. \right) + 4 H(0, 0; \alpha_0) + 4d_1 H(0, 1; \alpha_0) + \frac{2d_1 \pi^2}{3} - \\
& \frac{\pi^2}{3} \left. \right) H(1, 1; y_0) + \left( \frac{1}{6}d_1' y_0^3 \alpha_0^4 - d_1' y_0^2 \alpha_0^4 - \frac{5d_1' \alpha_0^4}{3} + \frac{5}{2}d_1' y_0 \alpha_0^4 - \frac{8}{9} d_1' y_0^3 \alpha_0^3 + 5d_1' y_0^2 \alpha_0^3 + \frac{77d_1' \alpha_0^3}{9} - \right. \\
& \frac{38}{3}d_1' y_0 \alpha_0^3 + 2d_1' y_0^3 \alpha_0^2 - \frac{21}{2} d_1' y_0^2 \alpha_0^2 - \frac{35d_1' \alpha_0^2}{2} + 26d_1' y_0 \alpha_0^2 - \frac{8}{3}d_1' y_0^3 \alpha_0 + 13d_1' y_0^2 \alpha_0 + \frac{65d_1' \alpha_0}{3} - \\
& 30d_1' y_0 \alpha_0 + \frac{7d_1' y_0^3}{6} + \frac{37 y_0^3}{18} - \frac{4d_1 y_0^2}{3} - \frac{14d_1' y_0^2}{3} - \frac{35 y_0^2}{3} - \frac{37d_1}{18} - \frac{13d_1'}{3} + \frac{16d_1 y_0}{3} + \frac{47d_1' y_0}{6} + \frac{209y_0}{6} + \left( - \right. \\
& \frac{4}{3}y_0^3 + 6y_0^2 - 12y_0 - \frac{4}{y_0-1} + \frac{38}{3} \left. \right) H(0; \alpha_0) - \frac{2}{3}y_0^3 H(0; Y) + 3y_0^2 H(0; Y) - 6y_0 H(0; Y) - \frac{2 H(0; Y)}{y_0-1} + \\
& \frac{19}{3}H(0; Y) + \left( -\frac{4d_1 y_0^3}{3} + 6d_1 y_0^2 - 12d_1 y_0 + \frac{38d_1}{3} - \frac{4d_1}{y_0-1} \right) H(1; \alpha_0) + \frac{d_1}{3(y_0-1)} - \frac{d_1'}{6(y_0-1)} + \frac{37}{6(y_0-1)} - \\
& \frac{127}{3} \left. \right) H(1, c_1(\alpha_0); y_0) + \left( \frac{y_0^3 \alpha_0^4}{3} - 2y_0^2 \alpha_0^4 + 5y_0 \alpha_0^4 - \frac{10\alpha_0^4}{3} - \frac{16y_0^3 \alpha_0^3}{9} + 10y_0^2 \alpha_0^3 - \frac{76y_0 \alpha_0^3}{3} + \frac{136\alpha_0^3}{9} + \right. \\
& 4 y_0^3 \alpha_0^2 - 21y_0^2 \alpha_0^2 + 52y_0 \alpha_0^2 - \frac{76\alpha_0^2}{3} - \frac{16 y_0^3 \alpha_0}{3} + 26y_0^2 \alpha_0 - 60y_0 \alpha_0 + \frac{82\alpha_0}{3} + \frac{25 y_0^3}{9} - \frac{35y_0^2}{3} + 23y_0 - \\
& \frac{13}{3(y_0-1)} - \frac{13}{3} \left. \right) H(c_1(\alpha_0), 0; y_0) + \left( \frac{1}{6}d_1' y_0^3 \alpha_0^4 - d_1' y_0^2 \alpha_0^4 - \frac{5d_1' \alpha_0^4}{3} + \frac{5}{2}d_1' y_0 \alpha_0^4 - \frac{8}{9} d_1' y_0^3 \alpha_0^3 + 5d_1' y_0^2 \alpha_0^3 + \right. \\
& \frac{68d_1' \alpha_0^3}{9} - \frac{38}{3}d_1' y_0 \alpha_0^3 + 2d_1' y_0^3 \alpha_0^2 - \frac{21}{2} d_1' y_0^2 \alpha_0^2 - \frac{38d_1' \alpha_0^2}{3} + 26d_1' y_0 \alpha_0^2 - \frac{8}{3}d_1' y_0^3 \alpha_0 + 13d_1' y_0^2 \alpha_0 + \\
& \frac{41d_1' \alpha_0}{3} - 30d_1' y_0 \alpha_0 + \frac{25d_1' y_0^3}{18} - \frac{35d_1' y_0^2}{6} - \frac{13d_1'}{6} + \frac{23d_1' y_0}{2} - \frac{13d_1'}{6(y_0-1)} \left. \right) H(c_1(\alpha_0), 1; y_0) + \left( \frac{y_0^3 \alpha_0^4}{2} - \right. \\
& 3y_0^2 \alpha_0^4 + \frac{15y_0 \alpha_0^4}{2} - 5\alpha_0^4 - \frac{8y_0^3 \alpha_0^3}{3} + 15y_0^2 \alpha_0^3 - 38y_0 \alpha_0^3 + \frac{68\alpha_0^3}{3} + 6y_0^3 \alpha_0^2 - \frac{63y_0^2 \alpha_0^2}{2} + 78y_0 \alpha_0^2 - 38\alpha_0^2 - \\
& 8y_0^3 \alpha_0 + 39y_0^2 \alpha_0 - 4d_1 \alpha_0 + 2 d_1' \alpha_0 - 90y_0 \alpha_0 - 2H(0; Y) \alpha_0 + 45\alpha_0 + \frac{25y_0^3}{6} - \frac{35 y_0^2}{2} + 4d_1 - 2d_1' +
\end{aligned}$$

$$\begin{aligned}
& \frac{69y_0}{2} + \left( -4\alpha_0 + \frac{4}{y_0-1} + 4 \right) H(0; \alpha_0) + \frac{2H(0;Y)}{y_0-1} + 2H(0; Y) + \left( -4\alpha_0 d_1 + \frac{4d_1}{y_0-1} + 4d_1 \right) H(1; \alpha_0) + \\
& \frac{4}{y_0-1} \frac{d_1}{y_0-1} - \frac{2d'_1}{y_0-1} - \frac{21}{2(y_0-1)} - \frac{21}{2} \Big) H(c_1(\alpha_0), c_1(\alpha_0); y_0) + \frac{1}{3}y_0^3 H(0, 0, 0; Y) - \frac{3}{2}y_0^2 H(0, 0, 0; Y) + \\
& 3y_0 H(0, 0, 0; Y) + \left( \frac{64y_0^3}{3} - 96y_0^2 + 192y_0 - 16H(0; Y) \right) H(0, 0, 0; y_0) + \left( \frac{16d'_1 y_0^3}{3} - 24d'_1 y_0^2 + \right. \\
& \left. 48d'_1 y_0 + (8 - 8d_1)H(0; \alpha_0) - 4d'_1 H(0; Y) \right) H(0, 0, 1; y_0) + \left( -\frac{8}{3} \frac{y_0^3}{3} + 12y_0^2 - 24y_0 + 8H(0; \alpha_0) + \right. \\
& \left. 4H(0; Y) + 8d_1 H(1; \alpha_0) - \frac{8}{y_0-1} - 8 \right) H(0, 0, c_1(\alpha_0); y_0) + \frac{1}{3}y_0^3 H(0, 1, 0; Y) - \frac{3}{2}y_0^2 H(0, 1, 0; Y) + \\
& 3y_0 H(0, 1, 0; Y) + \left( -\frac{4d_1 y_0^3}{3} + \frac{16d'_1 y_0^3}{3} + \frac{4y_0^3}{3} + 6d_1 y_0^2 - 24d'_1 y_0^2 - 6y_0^2 - 12d_1 y_0 + 48d'_1 y_0 + 12y_0 + \right. \\
& \left. \frac{38d_1}{3} + 8H(0; \alpha_0) - 4d'_1 H(0; Y) - 4 H(0; Y) - \frac{4d_1}{y_0-1} + \frac{4}{y_0-1} + \frac{62}{3} \right) H(0, 1, 0; y_0) + \left( \frac{4d_1^2 y_0^3}{3} - \right. \\
& \left. 6d_1^2 y_0^2 + 12d_1^2 y_0 + (8 d'_1 - 12d_1)H(0; \alpha_0) - d_1^2 H(0; Y) \right) H(0, 1, 1; y_0) + \left( -d'_1 \alpha_0^4 + \frac{16d'_1 \alpha_0^3}{3} - \right. \\
& \left. 12d'_1 \alpha_0^2 + 16d'_1 \alpha_0 + \frac{4d_1}{3} \frac{y_0^3}{3} - \frac{2d'_1 y_0^3}{3} - \frac{4y_0^3}{3} - 6d_1 y_0^2 + 3d'_1 y_0^2 + 6y_0^2 - \frac{38d_1}{3} - 2d'_1 + 12d_1 y_0 - 6d'_1 y_0 - \right. \\
& \left. 12y_0 + 8 H(0; \alpha_0) + 4H(0; Y) + 8d_1 H(1; \alpha_0) + \frac{4d_1}{y_0-1} - \frac{2}{y_0-1} \frac{d'_1}{y_0-1} - \frac{4}{y_0-1} - \frac{62}{3} \right) H(0, 1, c_1(\alpha_0); y_0) + \\
& \left( -2\alpha_0^4 + \frac{32\alpha_0^3}{3} - 24\alpha_0^2 + 32\alpha_0 - \frac{4y_0^3}{3} + 6y_0^2 - 12y_0 - \frac{4}{y_0-1} - 4 \right) H(0, c_1(\alpha_0), 0; y_0) + \left( -d'_1 \alpha_0^4 + \right. \\
& \left. \frac{16d'_1 \alpha_0^3}{3} - 12d'_1 \alpha_0^2 + 16d'_1 \alpha_0 - \frac{2d'_1 y_0^3}{3} + 3d'_1 y_0^2 - 2d'_1 - 6d'_1 y_0 - \frac{2d'_1}{y_0-1} \right) H(0, c_1(\alpha_0), 1; y_0) + \left( - \right. \\
& \left. 3\alpha_0^4 + 16 \alpha_0^3 - 36\alpha_0^2 + 48\alpha_0 - 2y_0^3 + 9y_0^2 - 18y_0 - 4H(0; \alpha_0) - 2H(0; Y) - 4 d_1 H(1; \alpha_0) + \frac{2}{y_0-1} + \right. \\
& \left. 2 \right) H(0, c_1(\alpha_0), c_1(\alpha_0); y_0) + \frac{1}{3}y_0^3 H(1, 0, 0; Y) - \frac{3}{2} y_0^2 H(1, 0, 0; Y) + 3y_0 H(1, 0, 0; Y) + \left( \frac{16d'_1 y_0^3}{3} + \right. \\
& \left. 4 y_0^3 - 24d'_1 y_0^2 - 18y_0^2 + 48d'_1 y_0 + 36y_0 - \frac{88}{3} \frac{d'_1}{y_0-1} + \frac{12}{y_0-1} - 38 \right) H(1, 0, 0; y_0) + \left( \frac{4d_1^2 y_0^3}{3} + \frac{2d_1 y_0^3}{3} - \right. \\
& \left. 6d_1^2 y_0^2 - 3d_1 y_0^2 + 12 d_1^2 y_0 + 6d_1 y_0 - \frac{22d_1^2}{3} - \frac{19d_1}{3} + (4 - 4d_1) H(0; \alpha_0) + \frac{2d'_1}{y_0-1} \right) H(1, 0, 1; y_0) + \left( - \right. \\
& \left. \frac{2d'_1}{3} \frac{y_0^3}{3} - \frac{2y_0^3}{3} + 3d'_1 y_0^2 + 3y_0^2 - 6d'_1 y_0 - 6 y_0 + \frac{11d'_1}{3} + 4H(0; \alpha_0) + 2H(0; Y) + 4d_1 H(1; \alpha_0) + \frac{4}{y_0-1} \frac{d_1}{y_0-1} - \right. \\
& \left. \frac{2d'_1}{y_0-1} - \frac{6}{y_0-1} - \frac{5}{3} \right) H(1, 0, c_1(\alpha_0); y_0) + \frac{1}{3}y_0^3 H(1, 1, 0; Y) - \frac{3}{2}y_0^2 H(1, 1, 0; Y) + 3y_0 H(1, 1, 0; Y) + \\
& \left( \frac{4d_1^2 y_0^3}{3} - \frac{4d_1 y_0^3}{3} + \frac{4d_1 y_0^3}{3} - \frac{2y_0^3}{3} - 6d_1^2 y_0^2 + 6 d_1 y_0^2 - 6d'_1 y_0^2 + 3y_0^2 + 12d_1^2 y_0 - 12d_1 y_0 + 12d'_1 y_0 - 6 y_0 - \right. \\
& \left. \frac{22d_1^2}{3} + \frac{38d_1}{3} - 10d'_1 + 4H(0; \alpha_0) - 2H(0; Y) - \frac{8d_1}{y_0-1} + \frac{4d'_1}{y_0-1} + \frac{2}{y_0-1} + \frac{43}{3} \right) H(1, 1, 0; y_0) + \left( \frac{y_0^3 d_1^3}{3} - \right. \\
& \left. \frac{3y_0^2 d_1^3}{2} + 3y_0 d_1^3 - \frac{11d_1^3}{6} + (-8d_1 + 4d'_1 + 2) H(0; \alpha_0) \right) H(1, 1, 1; y_0) + \left( \frac{4d_1 y_0^3}{3} - \frac{4d'_1 y_0^3}{3} + \frac{2y_0^3}{3} - 6d_1 y_0^2 + \right. \\
& \left. 6d'_1 y_0^2 - 3y_0^2 + 12d_1 y_0 - 12d'_1 y_0 + 6y_0 - \frac{38d_1}{3} + 10d'_1 + 4H(0; \alpha_0) + 2H(0; Y) + 4 d_1 H(1; \alpha_0) + \frac{8d_1}{y_0-1} - \right. \\
& \left. \frac{4}{y_0-1} \frac{d'_1}{y_0-1} - \frac{2}{y_0-1} - \frac{43}{3} \right) H(1, 1, c_1(\alpha_0); y_0) + \left( -\frac{4y_0^3}{3} + 6y_0^2 - 12 y_0 - \frac{4}{y_0-1} + \frac{38}{3} \right) H(1, c_1(\alpha_0), 0; y_0) + \\
& \left( -\frac{2d'_1 y_0^3}{3} + 3d'_1 y_0^2 - 6d'_1 y_0 + \frac{19d'_1}{3} - \frac{2d'_1}{y_0-1} \right) H(1, c_1(\alpha_0), 1; y_0) + \left( -2y_0^3 + 9y_0^2 - 18y_0 - 4H(0; \alpha_0) - \right. \\
& \left. 2 H(0; Y) - 4d_1 H(1; \alpha_0) - \frac{4d_1}{y_0-1} + \frac{2}{y_0-1} \frac{d'_1}{y_0-1} - \frac{2}{y_0-1} + 27 \right) H(1, c_1(\alpha_0), c_1(\alpha_0); y_0) + \left( -2\alpha_0 d'_1 + \right. \\
& \left. \frac{2d'_1}{y_0-1} + 2 d'_1 \right) H(c_1(\alpha_0), 1, c_1(\alpha_0); y_0) + \left( -4 \alpha_0 + \frac{4}{y_0-1} + 4 \right) H(c_1(\alpha_0), c_1(\alpha_0), 0; y_0) + \left( -2\alpha_0 d'_1 + \right. \\
& \left. \frac{2d'_1}{y_0-1} + 2d'_1 \right) H(c_1(\alpha_0), c_1(\alpha_0), 1; y_0) + \left( -6\alpha_0 + \frac{6}{y_0-1} + 6 \right) H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); y_0) - \\
& \frac{1}{4}H(0, 0, 0, 0; Y) - 64 H(0, 0, 0, 0; y_0) - 16d'_1 H(0, 0, 0, 1; y_0) + 16 H(0, 0, 0, c_1(\alpha_0); y_0) - \\
& \frac{1}{4}H(0, 0, 1, 0; Y) + (8d_1 - 16d'_1 - 8) H(0, 0, 1, 0; y_0) - 4d_1^2 H(0, 0, 1, 1; y_0) + (-8d_1 + 4d'_1 + \\
& 8)H(0, 0, 1, c_1(\alpha_0); y_0) + 8H(0, 0, c_1(\alpha_0), 0; y_0) + 4d_1^2 H(0, 0, c_1(\alpha_0), 1; y_0) + \\
& 4 H(0, 0, c_1(\alpha_0), c_1(\alpha_0); y_0) - \frac{1}{4}H(0, 1, 0, 0; Y) + (-16d'_1 - 24) H(0, 1, 0, 0; y_0) + \left( -4d_1^2 - \right. \\
& \left. 4d'_1 \right) H(0, 1, 0, 1; y_0) + (-4d_1 + 4 d'_1 + 8)H(0, 1, 0, c_1(\alpha_0); y_0) - \frac{1}{4}H(0, 1, 1, 0; Y) + \left( -4 d_1^2 - \right.
\end{aligned}$$

$$\begin{aligned}
 & 8d'_1 + 12d_1)H(0, 1, 1, 0; y_0) - d_1^3 H(0, 1, 1, 1; y_0) + (8d'_1 - 12d_1)H(0, 1, 1, c_1(\alpha_0); y_0) + \\
 & 8H(0, 1, c_1(\alpha_0), 0; y_0) + 4d'_1 H(0, 1, c_1(\alpha_0), 1; y_0) + (4d_1 - 2d'_1 + 8)H(0, 1, c_1(\alpha_0), c_1(\alpha_0); y_0) - \\
 & 2d'_1 H(0, c_1(\alpha_0), 1, c_1(\alpha_0); y_0) - 4H(0, c_1(\alpha_0), c_1(\alpha_0), 0; y_0) - 2d'_1 H(0, c_1(\alpha_0), c_1(\alpha_0), 1; y_0) - \\
 & 6H(0, c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); y_0) - \frac{1}{4}H(1, 0, 0, 0; Y) + 8H(1, 0, 0, c_1(\alpha_0); y_0) - \\
 & \frac{1}{4}H(1, 0, 1, 0; Y) + (4d_1 - 4)H(1, 0, 1, 0; y_0) + (-4d_1 + 2d'_1 + 4)H(1, 0, 1, c_1(\alpha_0); y_0) + \\
 & 4H(1, 0, c_1(\alpha_0), 0; y_0) + 2d'_1 H(1, 0, c_1(\alpha_0), 1; y_0) - 2H(1, 0, c_1(\alpha_0), c_1(\alpha_0); y_0) - \\
 & \frac{1}{4}H(1, 1, 0, 0; Y) - 12H(1, 1, 0, 0; y_0) - 2d'_1 H(1, 1, 0, 1; y_0) + (-4d_1 + 2d'_1 + \\
 & 6)H(1, 1, 0, c_1(\alpha_0); y_0) - \frac{1}{4}H(1, 1, 1, 0; Y) + (8d_1 - 4d'_1 - 2)H(1, 1, 1, 0; y_0) + (-8d_1 + 4d'_1 + \\
 & 2)H(1, 1, 1, c_1(\alpha_0); y_0) + 4H(1, 1, c_1(\alpha_0), 0; y_0) + 2d'_1 H(1, 1, c_1(\alpha_0), 1; y_0) + (4d_1 - 2d'_1 + \\
 & 2)H(1, 1, c_1(\alpha_0), c_1(\alpha_0); y_0) - 2d'_1 H(1, c_1(\alpha_0), 1, c_1(\alpha_0); y_0) - 4H(1, c_1(\alpha_0), c_1(\alpha_0), 0; y_0) - \\
 & 2d'_1 H(1, c_1(\alpha_0), c_1(\alpha_0), 1; y_0) - 6H(1, c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); y_0) + H(0; y_0) \left( -\frac{1}{9}y_0^3 \alpha_0^4 + \right. \\
 & \frac{5y_0^2 \alpha_0^4}{6} - \frac{10y_0 \alpha_0^4}{3} + \frac{13y_0^3 \alpha_0^3}{27} - \frac{34y_0^2 \alpha_0^3}{9} + \frac{160y_0 \alpha_0^3}{9} - \frac{11y_0^3 \alpha_0^2}{18} + \frac{109y_0^2 \alpha_0^2}{18} - \frac{316y_0 \alpha_0^2}{9} - \frac{7y_0^3 \alpha_0}{9} - \frac{7y_0^2 \alpha_0}{9} + \\
 & \frac{86y_0 \alpha_0}{3} + \frac{4d_1^2 y_0^3}{27} - \frac{205d_1 y_0^3}{108} - \frac{3d_1' y_0^3}{2} - \frac{2\pi^2 y_0^3}{9} + \frac{161y_0^3}{27} - \frac{17d_1^2 y_0^2}{18} + \frac{22d_1 y_0^2}{3} + \frac{98d_1' y_0^2}{9} + \pi^2 y_0^2 - \\
 & \frac{1405y_0^2}{36} + \frac{217d_1}{36} - \frac{d_1'}{6} + \frac{49d_1^2 y_0}{9} - \frac{469d_1 y_0}{36} - \frac{1201d_1' y_0}{18} - 2\pi^2 y_0 + \frac{1811y_0}{9} + \left( \frac{7y_0^3}{3} - \frac{26y_0^2}{3} + 11y_0 - \right. \\
 & \left. \frac{13}{3(y_0-1)} - \frac{13}{3} \right) H(0; \alpha_0) + \frac{4}{9}d_1' y_0^3 H(0; Y) - \frac{53}{18}y_0^3 H(0; Y) - \frac{7}{3}d_1' y_0^2 H(0; Y) + \frac{49}{3}y_0^2 H(0; Y) + \\
 & \frac{22}{3}d_1' y_0 H(0; Y) - \frac{107}{2}y_0 H(0; Y) + \frac{13H(0; Y)}{6(y_0-1)} - \frac{1}{6}\pi^2 H(0; Y) + \frac{13}{6}H(0; Y) - \frac{1}{6}\pi^2 H(1; Y) + \\
 & \frac{4}{3}y_0^3 H(0, 0; Y) - 6y_0^2 H(0, 0; Y) + 12y_0 H(0, 0; Y) + \frac{4}{3}y_0^3 H(1, 0; Y) - 6y_0^2 H(1, 0; Y) + \\
 & 12y_0 H(1, 0; Y) - H(0, 0, 0; Y) - H(0, 1, 0; Y) - H(1, 0, 0; Y) - H(1, 1, 0; Y) + \frac{217d_1}{36(y_0-1)} - \\
 & \frac{d_1'}{6(y_0-1)} + \frac{4\pi^2}{3(y_0-1)} - \frac{149}{18(y_0-1)} - 6\zeta_3 + \frac{4\pi^2}{3} - \frac{149}{18} \Big) + H(1; y_0) \left( -\frac{1}{18}d_1' y_0^3 \alpha_0^4 + \frac{5}{12}d_1' y_0^2 \alpha_0^4 + \frac{47d_1' \alpha_0^4}{36} - \right. \\
 & \frac{5}{3}d_1' y_0 \alpha_0^4 + \frac{13}{54}d_1' y_0^3 \alpha_0^3 - \frac{17}{9}d_1' y_0^2 \alpha_0^3 - \frac{391d_1' \alpha_0^3}{54} + \frac{80}{9}d_1' y_0 \alpha_0^3 - \frac{11}{36}d_1' y_0^3 \alpha_0^2 + \frac{109}{36}d_1' y_0^2 \alpha_0^2 + \frac{89d_1' \alpha_0^2}{6} - \\
 & \frac{158}{9}d_1' y_0 \alpha_0^2 - \frac{7}{18}d_1' y_0^3 \alpha_0 - \frac{7}{18}d_1' y_0^2 \alpha_0 - \frac{247d_1' \alpha_0}{18} + \frac{43d_1' y_0 \alpha_0}{27} - \frac{251d_1'^3}{216} + \frac{d_1'^3 y_0^3}{27} - \frac{8d_1'^2 y_0^3}{27} + \frac{14d_1' y_0^3}{27} - \\
 & \frac{1}{18}d_1' \pi^2 y_0^3 - \frac{\pi^2 y_0^3}{9} + \frac{395d_1'^2}{27} - \frac{17d_1'^3 y_0^2}{72} + \frac{22d_1'^2 y_0^2}{9} - \frac{21d_1' y_0^2}{4} + \frac{1}{4}d_1' \pi^2 y_0^2 + \frac{\pi^2 y_0^2}{2} - \frac{4025d_1'}{108} + \frac{49d_1'^3 y_0}{36} - \\
 & \frac{151d_1'^2 y_0}{9} + 42d_1' y_0 - \frac{1}{2}d_1' \pi^2 y_0 - \pi^2 y_0 + \frac{1}{9}d_1'^2 y_0^3 H(0; Y) - \frac{4}{9}d_1' y_0^3 H(0; Y) - \frac{49}{36}d_1'^2 H(0; Y) - \\
 & \frac{7}{12}d_1'^2 y_0^2 H(0; Y) + 3d_1' y_0^2 H(0; Y) + \frac{85}{9}d_1' H(0; Y) + \frac{11}{6}d_1'^2 y_0 H(0; Y) - 12d_1' y_0 H(0; Y) - \\
 & \frac{1}{3}\pi^2 H(0; Y) + H(0; \alpha_0) \left( \frac{17d_1' y_0^3}{18} - \frac{2}{3}H(0; Y)y_0^3 + \frac{37y_0^3}{18} - \frac{4d_1 y_0^2}{3} - \frac{19d_1' y_0^2}{6} + 3H(0; Y)y_0^2 - \right. \\
 & \left. \frac{35y_0^2}{3} + \frac{16d_1 y_0}{3} + \frac{11d_1' y_0}{6} - 6H(0; Y)y_0 + \frac{209y_0}{6} - \frac{37d_1}{18} + \frac{7d_1'}{18} - \frac{2H(0; Y)}{y_0-1} + \frac{19}{3}H(0; Y) + \frac{d_1}{3(y_0-1)} - \right. \\
 & \left. \frac{d_1'}{6(y_0-1)} + \frac{37}{6(y_0-1)} - \frac{127}{3} \right) + \left( -\frac{4y_0^3}{3} + 6y_0^2 - 12y_0 - \frac{4}{y_0-1} + \frac{38}{3} \right) H(0, 0; \alpha_0) + \frac{1}{3}d_1' y_0^3 H(0, 0; Y) - \\
 & \frac{3}{2}d_1' y_0^2 H(0, 0; Y) - \frac{11}{6}d_1' H(0, 0; Y) + 3d_1' y_0 H(0, 0; Y) + \left( -\frac{4d_1 y_0^3}{3} + 6d_1 y_0^2 - 12d_1 y_0 + \right. \\
 & \left. \frac{38d_1}{3} - \frac{4d_1}{y_0-1} \right) H(0, 1; \alpha_0) + \frac{1}{3}d_1' y_0^3 H(1, 0; Y) - \frac{3}{2}d_1' y_0^2 H(1, 0; Y) - \frac{11}{6}d_1' H(1, 0; Y) + \\
 & 3d_1' y_0 H(1, 0; Y) - \frac{2d_1 \pi^2}{3(y_0-1)} + \frac{d_1' \pi^2}{3(y_0-1)} + \frac{\pi^2}{3(y_0-1)} + 6\zeta_3 + \frac{11d_1' \pi^2}{36} + \frac{43\pi^2}{18} \Big) + \frac{2d_1 \pi^2}{3(y_0-1)} - \frac{d_1' \pi^2}{3(y_0-1)} - \\
 & \frac{37\pi^2}{36(y_0-1)} + 4y_0^3 \zeta_3 - 18y_0^2 \zeta_3 + 36y_0 \zeta_3 - \frac{3}{2}H(0; Y)\zeta_3 - \frac{6\zeta_3}{y_0-1} - 6\zeta_3 + \frac{\pi^4}{288} + \frac{2d_1 \pi^2}{3} - \frac{d_1' \pi^2}{3} - \frac{37\pi^2}{36}.
 \end{aligned}$$

## G. The $\mathcal{K}\ast\mathcal{I}$ -type integrals

### G.1 The $\mathcal{K}\ast\mathcal{I}$ integral for $k = 0$

The  $\varepsilon$  expansion for this integral reads

$$\mathcal{K}\ast\mathcal{I}(\varepsilon; y_0, d'_0, \alpha_0, d_0; 0) = \frac{1}{\varepsilon^3}(k \ast i)_{-3}^{(0)} + \frac{1}{\varepsilon^2}(k \ast i)_{-2}^{(0)} + \frac{1}{\varepsilon}(k \ast i)_{-1}^{(0)} + (k \ast i)_0^{(0)} + \mathcal{O}(\varepsilon), \quad (\text{G.1})$$

where

$$(k * i)_{-3}^{(0)} = -\frac{1}{2},$$

$$(k * i)_{-2}^{(0)} = -\frac{2y_0^3}{3} + 3y_0^2 - 6y_0 + 2H(0; y_0) - 1,$$

$$(k * i)_{-1}^{(0)} = \frac{y_0^3 \alpha_0^4}{6} - \frac{y_0^2 \alpha_0^4}{2} + \frac{y_0 \alpha_0^4}{2} - \frac{8y_0^3 \alpha_0^3}{9} + 3y_0^2 \alpha_0^3 - \frac{10y_0 \alpha_0^3}{3} + 2y_0^3 \alpha_0^2 - \frac{15y_0^2 \alpha_0^2}{2} + 10y_0 \alpha_0^2 - \frac{8y_0^3 \alpha_0}{3} + 11y_0^2 \alpha_0 - 18y_0 \alpha_0 + \frac{2d_1' y_0^3}{9} - \frac{8y_0^3}{3} - \frac{7d_1' y_0^2}{6} + \frac{25y_0^2}{2} + \frac{11d_1' y_0}{3} - 32y_0 + \left( \frac{2y_0^3}{3} - 3y_0^2 + 6y_0 - \frac{2}{y_0-1} - 2 \right) H(0; \alpha_0) + \left( 2y_0^3 - 9y_0^2 + 18y_0 + \frac{2}{y_0-1} + 6 \right) H(0; y_0) + \left( \frac{2d_1' y_0^3}{3} - 3d_1' y_0^2 + 6d_1' y_0 - \frac{11d_1'}{3} + 2H(0; \alpha_0) \right) H(1; y_0) + \left( 2\alpha_0 - \frac{2}{y_0-1} - 2 \right) H(c_1(\alpha_0); y_0) - 8H(0, 0; y_0) - 2d_1' H(0, 1; y_0) + 2H(0, c_1(\alpha_0); y_0) - 2H(1, 0; y_0) + 2H(1, c_1(\alpha_0); y_0) - 2,$$

$$(k * i)_0^{(0)} = -\frac{1}{12} d_1 y_0^3 \alpha_0^4 - \frac{1}{18} d_1' y_0^3 \alpha_0^4 + \frac{3}{4} y_0^3 \alpha_0^4 + \frac{1}{4} d_1 y_0^2 \alpha_0^4 + \frac{1}{6} d_1' y_0^2 \alpha_0^4 - \frac{13y_0^2 \alpha_0^4}{6} - \frac{1}{4} d_1 y_0 \alpha_0^4 - \frac{1}{6} d_1' y_0 \alpha_0^4 + \frac{29y_0 \alpha_0^4}{12} + \frac{13}{27} d_1 y_0^3 \alpha_0^3 + \frac{8}{27} d_1' y_0^3 \alpha_0^3 - \frac{221y_0^3 \alpha_0^3}{54} - \frac{5}{3} d_1 y_0^2 \alpha_0^3 - \frac{19}{18} d_1' y_0^2 \alpha_0^3 + \frac{247y_0^2 \alpha_0^3}{18} + \frac{17}{9} d_1 y_0 \alpha_0^3 + \frac{11}{9} d_1' y_0 \alpha_0^3 - \frac{305y_0 \alpha_0^3}{18} - \frac{23}{18} d_1 y_0^3 \alpha_0^2 - \frac{2}{3} d_1' y_0^3 \alpha_0^2 + \frac{347y_0^3 \alpha_0^2}{36} + 5d_1 y_0^2 \alpha_0^2 + \frac{11}{4} d_1' y_0^2 \alpha_0^2 - \frac{331y_0^2 \alpha_0^2}{9} - \frac{43}{6} d_1 y_0 \alpha_0^2 - \frac{9}{2} d_1' y_0 \alpha_0^2 + \frac{1021y_0 \alpha_0^2}{18} + \frac{25}{9} d_1 y_0^3 \alpha_0 + \frac{8}{9} d_1' y_0^3 \alpha_0 - \frac{265y_0^3 \alpha_0}{18} - 12d_1 y_0^2 \alpha_0 - \frac{25}{6} d_1' y_0^2 \alpha_0 + \frac{566y_0^2 \alpha_0}{9} + \frac{65d_1 y_0 \alpha_0}{3} + \frac{29d_1' y_0 \alpha_0}{3} - \frac{245y_0 \alpha_0}{2} - \frac{\alpha_0}{y_0-1} - \alpha_0 - \frac{2d_1' y_0^3}{27} + \frac{10d_1' y_0^3}{9} + \frac{\pi^2 y_0^3}{9} - 8y_0^3 + \frac{17d_1' y_0^2}{36} - \frac{20d_1' y_0^2}{3} - \frac{\pi^2 y_0^2}{2} + \frac{155y_0^2}{4} - \frac{49d_1' y_0^2}{18} + \frac{199d_1' y_0}{6} + \pi^2 y_0 - 128y_0 + \left( -\frac{1}{3} y_0^3 \alpha_0^4 + y_0^2 \alpha_0^4 - y_0 \alpha_0^4 + \frac{16y_0^3 \alpha_0^3}{9} - 6y_0^2 \alpha_0^3 + \frac{20y_0 \alpha_0^3}{3} - 4y_0^3 \alpha_0^2 + 15y_0^2 \alpha_0^2 - 20y_0 \alpha_0^2 + \frac{16y_0^3 \alpha_0}{3} - 22y_0^2 \alpha_0 + 36y_0 \alpha_0 - \frac{2d_1' y_0^3}{9} + \frac{23y_0^3}{18} + \frac{7d_1' y_0^2}{6} - \frac{43y_0^2}{6} + 4d_1 - 2d_1' - \frac{11d_1' y_0}{3} + \frac{143y_0}{6} + \frac{4d_1}{y_0-1} - \frac{2d_1'}{y_0-1} - \frac{61}{6(y_0-1)} - \frac{1}{(y_0-1)^2} - \frac{55}{6} \right) H(0; \alpha_0) + \left( -\frac{1}{3} y_0^3 \alpha_0^4 + y_0^2 \alpha_0^4 - y_0 \alpha_0^4 + \frac{16y_0^3 \alpha_0^3}{9} - 6y_0^2 \alpha_0^3 + \frac{20y_0 \alpha_0^3}{3} - 4y_0^3 \alpha_0^2 + 15y_0^2 \alpha_0^2 - 20y_0 \alpha_0^2 + \frac{16y_0^3 \alpha_0}{3} - 22y_0^2 \alpha_0 + 36y_0 \alpha_0 - \frac{2d_1' y_0^3}{3} + \frac{169y_0^3}{18} + \frac{7d_1' y_0^2}{2} - \frac{25y_0^2}{6} - 4d_1 + 2d_1' - 11d_1' y_0 + \frac{625y_0}{6} + \left( -\frac{4y_0^3}{3} + 6y_0^2 - 12y_0 + \frac{4}{y_0-1} + 4 \right) H(0; \alpha_0) - \frac{4d_1}{y_0-1} + \frac{2d_1'}{y_0-1} + \frac{61}{6(y_0-1)} + \frac{1}{(y_0-1)^2} + \frac{103}{6} \right) H(0; y_0) + \left( -\frac{1}{3} d_1 y_0^3 \alpha_0^4 + d_1 y_0^2 \alpha_0^4 - d_1 y_0 \alpha_0^4 + \frac{16}{9} d_1 y_0^3 \alpha_0^3 - 6d_1 y_0^2 \alpha_0^3 + \frac{20}{3} d_1 y_0 \alpha_0^3 - 4d_1 y_0^3 \alpha_0^2 + 15d_1 y_0^2 \alpha_0^2 - 20d_1 y_0 \alpha_0^2 + \frac{16}{3} d_1 y_0^3 \alpha_0 - 22d_1 y_0^2 \alpha_0 + 36d_1 y_0 \alpha_0 - \frac{25d_1 y_0^3}{9} + 12d_1 y_0^2 - \frac{65d_1 y_0}{3} \right) H(1; \alpha_0) + \left( -\frac{1}{6} y_0^3 \alpha_0^4 + \frac{y_0^2 \alpha_0^4}{2} - \frac{y_0 \alpha_0^4}{2} + \frac{\alpha_0^4}{6} + \frac{8y_0^3 \alpha_0^3}{9} - 3y_0^2 \alpha_0^3 + \frac{10y_0 \alpha_0^3}{3} - \frac{11\alpha_0^3}{9} - 2y_0^3 \alpha_0^2 + \frac{15y_0^2 \alpha_0^2}{2} - 10y_0 \alpha_0^2 + \frac{11\alpha_0^2}{2} + \frac{8y_0^3 \alpha_0}{3} - 11y_0^2 \alpha_0 - 4d_1 \alpha_0 + 2d_1' \alpha_0 + 18y_0 \alpha_0 - \frac{25y_0^3}{18} + \frac{16y_0^2}{3} + 4d_1 - 2d_1' - \frac{49y_0}{6} + \left( -4\alpha_0 + \frac{4}{y_0-1} + 4 \right) H(0; \alpha_0) + \left( -4\alpha_0 d_1 + \frac{4d_1}{y_0-1} + 4d_1 \right) H(1; \alpha_0) + \frac{4d_1}{y_0-1} - \frac{2d_1'}{y_0-1} - \frac{61}{6(y_0-1)} - \frac{1}{(y_0-1)^2} - \frac{55}{6} \right) H(c_1(\alpha_0); y_0) + \left( -\frac{4y_0^3}{3} + 6y_0^2 - 12y_0 + \frac{4}{y_0-1} + 4 \right) H(0, 0; \alpha_0) + \left( -\frac{20y_0^3}{3} + 30y_0^2 - 60y_0 - \frac{12}{y_0-1} - 28 \right) H(0, 0; y_0) + \left( -\frac{4d_1 y_0^3}{3} + 6d_1 y_0^2 - 12d_1 y_0 + 4d_1 + \frac{4d_1}{y_0-1} \right) H(0, 1; \alpha_0) + H(1; y_0) \left( -\frac{1}{6} d_1 y_0^3 \alpha_0^4 + \frac{1}{2} d_1' y_0^2 \alpha_0^4 + \frac{d_1' \alpha_0^4}{6} - \frac{1}{2} d_1 y_0 \alpha_0^4 + \frac{8}{9} d_1' y_0^3 \alpha_0^3 - 3d_1' y_0^2 \alpha_0^3 - \frac{11d_1' \alpha_0^3}{9} + \frac{10}{3} d_1' y_0 \alpha_0^3 - 2d_1' y_0^3 \alpha_0^2 + \frac{15}{2} d_1' y_0^2 \alpha_0^2 + \frac{9d_1' \alpha_0^2}{2} - 10d_1' y_0 \alpha_0^2 + \frac{8}{3} d_1' y_0^3 \alpha_0 - 11d_1' y_0^2 \alpha_0 - \frac{23d_1' \alpha_0}{3} + 18d_1' y_0 \alpha_0 - \frac{2d_1' y_0^3}{9} + \frac{8d_1' y_0^3}{3} + \frac{49d_1'}{18} + \frac{7d_1' y_0^2}{6} - \frac{25d_1' y_0^2}{2} - \frac{133d_1'}{6} - \frac{11d_1' y_0}{3} + 32d_1' y_0 + \left( -\frac{2d_1' y_0^3}{3} + \frac{2y_0^3}{3} + 3d_1' y_0^2 - 3y_0^2 - 6d_1' y_0 + 6y_0 + \frac{11d_1'}{3} - \frac{4d_1}{y_0-1} + \frac{2d_1'}{y_0-1} + \frac{2}{y_0-1} + 10 \right) H(0; \alpha_0) - 4H(0, 0; \alpha_0) - 4d_1 H(0, 1; \alpha_0) + \frac{\pi^2}{3} \right) + \left( -2d_1' y_0^3 + 9d_1' y_0^2 - 18d_1' y_0 - 6d_1' + (4d_1 - 4)H(0; \alpha_0) - \frac{2d_1'}{y_0-1} \right) H(0, 1; y_0) + \left( \frac{2y_0^3}{3} - 3y_0^2 + 6y_0 - 4H(0; \alpha_0) - 4d_1 H(1; \alpha_0) + \frac{6}{y_0-1} + 10 \right) H(0, c_1(\alpha_0); y_0) + \left( -2d_1' y_0^3 - \frac{2y_0^3}{3} + 9d_1' y_0^2 + 3y_0^2 - 18d_1' y_0 - \right.$$

$$\begin{aligned}
& 6y_0 + 11 d'_1 - 4H(0; \alpha_0) + \frac{4d_1}{y_0-1} - \frac{2 d'_1}{y_0-1} - \frac{2}{y_0-1} - 10 \Big) H(1, 0; y_0) + \left( -\frac{2}{3} d_1'^2 y_0^3 + 3d_1'^2 y_0^2 - 6d_1'^2 y_0 + \right. \\
& \left. \frac{11d_1'^2}{3} + (4d_1 - 2 d'_1 - 2)H(0; \alpha_0) \right) H(1, 1; y_0) + \left( \frac{2y_0^3}{3} - 3y_0^2 + 6 y_0 - 4H(0; \alpha_0) - 4d_1 H(1; \alpha_0) - \right. \\
& \left. \frac{4d_1}{y_0-1} + \frac{2 d'_1}{y_0-1} + \frac{2}{y_0-1} + 10 \right) H(1, c_1(\alpha_0); y_0) + \left( -4 \alpha_0 + \frac{4}{y_0-1} + 4 \right) H(c_1(\alpha_0), 0; y_0) + \left( -2\alpha_0 d'_1 + \right. \\
& \left. \frac{2 d'_1}{y_0-1} + 2d'_1 \right) H(c_1(\alpha_0), 1; y_0) + \left( -2 \alpha_0 + \frac{2}{y_0-1} + 2 \right) H(c_1(\alpha_0), c_1(\alpha_0); y_0) + 32H(0, 0, 0; y_0) + \\
& 8 d'_1 H(0, 0, 1; y_0) - 8H(0, 0, c_1(\alpha_0); y_0) + (-4d_1 + 8d'_1 + 4) H(0, 1, 0; y_0) + 2d_1'^2 H(0, 1, 1; y_0) + \\
& (4d_1 - 2d'_1 - 4) H(0, 1, c_1(\alpha_0); y_0) - 4H(0, c_1(\alpha_0), 0; y_0) - 2d'_1 H(0, c_1(\alpha_0), 1; y_0) - \\
& 2H(0, c_1(\alpha_0), c_1(\alpha_0); y_0) + 12H(1, 0, 0; y_0) + 2 d'_1 H(1, 0, 1; y_0) - 6H(1, 0, c_1(\alpha_0); y_0) + \\
& (-4d_1 + 2d'_1 + 2) H(1, 1, 0; y_0) + (4d_1 - 2d'_1 - 2)H(1, 1, c_1(\alpha_0); y_0) - 4 H(1, c_1(\alpha_0), 0; y_0) - \\
& 2d'_1 H(1, c_1(\alpha_0), 1; y_0) - 2 H(1, c_1(\alpha_0), c_1(\alpha_0); y_0) - \frac{\pi^2}{3(y_0-1)} + 3 \zeta_3 - \frac{\pi^2}{3} - 4.
\end{aligned}$$

### G.2 The $\mathcal{K}\mathcal{I}$ integral for $k = 1$

The  $\epsilon$  expansion for this integral reads

$$\mathcal{K}\mathcal{I}(\epsilon; y_0, d'_0, \alpha_0, d_0; 1) = \frac{1}{\epsilon^3} (k * i)_{-3}^{(1)} + \frac{1}{\epsilon^2} (k * i)_{-2}^{(1)} + \frac{1}{\epsilon} (k * i)_{-1}^{(1)} + (k * i)_0^{(1)} + \mathcal{O}(\epsilon), \quad (\text{G.2})$$

where

$$(k * i)_{-3}^{(1)} = -\frac{1}{4},$$

$$(k * i)_{-2}^{(1)} = -\frac{y_0^3}{3} + \frac{3y_0^2}{2} - 3y_0 + H(0; y_0) - \frac{1}{2},$$

$$\begin{aligned}
(k * i)_{-1}^{(1)} &= \frac{y_0^3 \alpha_0^4}{12} - \frac{y_0^2 \alpha_0^4}{4} + \frac{y_0 \alpha_0^4}{4} - \frac{4y_0^3 \alpha_0^3}{9} + \frac{3y_0^2 \alpha_0^3}{2} - \frac{5y_0 \alpha_0^3}{3} + y_0^3 \alpha_0^2 - \frac{15y_0^2 \alpha_0^2}{4} + 5y_0 \alpha_0^2 - \\
& \frac{4 y_0^3 \alpha_0}{3} + \frac{11y_0^2 \alpha_0}{2} - 9y_0 \alpha_0 + \frac{d'_1 y_0^3}{9} - \frac{4y_0^3}{3} - \frac{7d'_1 y_0^2}{12} + \frac{25 y_0^2}{4} + \frac{11d'_1 y_0}{6} - 16y_0 + \left( \frac{y_0^3}{3} - \frac{3 y_0^2}{2} + 3y_0 - \right. \\
& \left. \frac{1}{y_0-1} - 1 \right) H(0; \alpha_0) + \left( y_0^3 - \frac{9 y_0^2}{2} + 9y_0 + \frac{1}{y_0-1} + 3 \right) H(0; y_0) + \left( \frac{d'_1 y_0^3}{3} - \frac{3d'_1 y_0^2}{2} + 3d'_1 y_0 - \right. \\
& \left. \frac{11 d'_1}{6} + H(0; \alpha_0) \right) H(1; y_0) + \left( \alpha_0 - \frac{1}{y_0-1} - 1 \right) H(c_1(\alpha_0); y_0) - 4H(0, 0; y_0) - d'_1 H(0, 1; y_0) + \\
& H(0, c_1(\alpha_0); y_0) - H(1, 0; y_0) + H(1, c_1(\alpha_0); y_0) - 1,
\end{aligned}$$

$$\begin{aligned}
(k * i)_0^{(1)} &= \\
& -\frac{1}{24} d_1 y_0^3 \alpha_0^4 - \frac{1}{36} d_1' y_0^3 \alpha_0^4 + \frac{3 y_0^3 \alpha_0^4}{8} + \frac{1}{8} d_1 y_0^2 \alpha_0^4 + \frac{1}{12} d_1' y_0^2 \alpha_0^4 - \frac{13y_0^2 \alpha_0^4}{12} - \frac{1}{8} d_1 y_0 \alpha_0^4 - \frac{1}{12} d_1' y_0 \alpha_0^4 + \\
& \frac{29y_0 \alpha_0^4}{24} + \frac{13}{54} d_1 y_0^3 \alpha_0^3 + \frac{4}{27} d_1' y_0^3 \alpha_0^3 - \frac{221y_0^3 \alpha_0^3}{108} - \frac{5}{6} d_1 y_0^2 \alpha_0^3 - \frac{19}{36} d_1' y_0^2 \alpha_0^3 + \frac{247y_0^2 \alpha_0^3}{36} + \frac{17}{18} d_1 y_0 \alpha_0^3 + \\
& \frac{11}{18} d_1' y_0 \alpha_0^3 - \frac{305y_0 \alpha_0^3}{36} - \frac{23}{36} d_1 y_0^3 \alpha_0^2 - \frac{1}{3} d_1' y_0^3 \alpha_0^2 + \frac{347y_0^3 \alpha_0^2}{72} + \frac{5}{2} d_1 y_0^2 \alpha_0^2 + \frac{11}{8} d_1' y_0^2 \alpha_0^2 - \frac{331y_0^2 \alpha_0^2}{18} - \\
& \frac{43}{12} d_1 y_0 \alpha_0^2 - \frac{9}{4} d_1' y_0 \alpha_0^2 + \frac{1021y_0 \alpha_0^2}{36} + \frac{25}{18} d_1 y_0^3 \alpha_0 + \frac{4}{9} d_1' y_0^3 \alpha_0 - \frac{265y_0^3 \alpha_0}{36} - 6d_1 y_0^2 \alpha_0 - \frac{25}{12} d_1' y_0^2 \alpha_0 + \\
& \frac{283y_0^2 \alpha_0}{9} + \frac{65d_1 y_0 \alpha_0}{6} + \frac{29d_1' y_0 \alpha_0}{6} - \frac{245y_0 \alpha_0}{4} - \frac{\alpha_0}{2(y_0-1)} - \frac{\alpha_0}{2} - \frac{d_1'^2 y_0^3}{27} + \frac{5d_1' y_0^3}{9} + \frac{\pi^2 y_0^3}{18} - 4 y_0^3 + \frac{17d_1'^2 y_0^2}{72} - \\
& \frac{10d_1' y_0^2}{3} - \frac{\pi^2 y_0^2}{4} + \frac{155y_0^2}{8} - \frac{49d_1'^2 y_0}{36} + \frac{199d_1' y_0}{12} + \frac{\pi^2 y_0}{2} - 64y_0 + \left( -\frac{1}{6} y_0^3 \alpha_0^4 + \frac{y_0^2 \alpha_0^4}{2} - \frac{y_0 \alpha_0^4}{2} + \frac{8y_0^3 \alpha_0^3}{9} - \right. \\
& 3y_0^2 \alpha_0^3 + \frac{10y_0 \alpha_0^3}{3} - 2y_0^3 \alpha_0^2 + \frac{15 y_0^2 \alpha_0^2}{2} - 10y_0 \alpha_0^2 + \frac{8y_0^3 \alpha_0}{3} - 11y_0^2 \alpha_0 + 18 y_0 \alpha_0 - \frac{d_1' y_0^3}{9} + \frac{23y_0^3}{36} + \frac{7d_1' y_0^2}{12} - \\
& \frac{43y_0^2}{12} + 2d_1 - d_1' - \frac{11d_1' y_0}{6} + \frac{143y_0}{12} + \frac{2 d_1}{y_0-1} - \frac{d_1'}{y_0-1} - \frac{61}{12(y_0-1)} - \frac{1}{2(y_0-1)^2} - \frac{55}{12} \Big) H(0; \alpha_0) + \left( -\frac{1}{6} y_0^3 \alpha_0^4 + \right. \\
& \left. \frac{y_0^2 \alpha_0^4}{2} - \frac{y_0 \alpha_0^4}{2} + \frac{8y_0^3 \alpha_0^3}{9} - 3 y_0^2 \alpha_0^3 + \frac{10y_0 \alpha_0^3}{3} - 2y_0^3 \alpha_0^2 + \frac{15y_0^2 \alpha_0^2}{2} - 10y_0 \alpha_0^2 + \frac{8y_0^3 \alpha_0}{3} - 11y_0^2 \alpha_0 + 18y_0 \alpha_0 - \right. \\
& \left. \frac{d_1' y_0^3}{3} + \frac{169y_0^3}{36} + \frac{7d_1' y_0^2}{4} - \frac{257y_0^2}{12} - 2d_1 + d_1' - \frac{11d_1' y_0}{2} + \frac{625y_0}{12} + \left( -\frac{2y_0^3}{3} + 3y_0^2 - 6 y_0 + \frac{2}{y_0-1} + 2 \right) H(0; \alpha_0) - \right. \\
& \left. \frac{2 d_1}{y_0-1} + \frac{d_1'}{y_0-1} + \frac{61}{12(y_0-1)} + \frac{1}{2(y_0-1)^2} + \frac{103}{12} \right) H(0; y_0) + \left( -\frac{1}{6} d_1 y_0^3 \alpha_0^4 + \frac{1}{2} d_1 y_0^2 \alpha_0^4 - \frac{1}{2} d_1 y_0 \alpha_0^4 + \right.
\end{aligned}$$

$$\begin{aligned}
 & \frac{8}{9}d_1y_0^3\alpha_0^3 - 3d_1y_0^2\alpha_0^3 + \frac{10}{3}d_1y_0\alpha_0^3 - 2d_1y_0^3\alpha_0^2 + \frac{15}{2}d_1y_0^2\alpha_0^2 - 10d_1y_0\alpha_0^2 + \frac{8}{3}d_1y_0^3\alpha_0 - 11d_1y_0^2\alpha_0 + \\
 & 18d_1y_0\alpha_0 - \frac{25d_1y_0^3}{18} + 6d_1y_0^2 - \frac{65d_1y_0}{6} \Big) H(1; \alpha_0) + \left( -\frac{1}{12}y_0^3\alpha_0^4 + \frac{y_0^2\alpha_0^4}{4} - \frac{y_0\alpha_0^4}{4} + \frac{\alpha_0^4}{12} + \frac{4y_0^3\alpha_0^3}{9} - \right. \\
 & \frac{3y_0^2\alpha_0^3}{2} + \frac{5y_0\alpha_0^3}{3} - \frac{11\alpha_0^3}{18} - y_0^3\alpha_0^2 + \frac{15y_0^2\alpha_0^2}{4} - 5y_0\alpha_0^2 + \frac{11\alpha_0^2}{4} + \frac{4y_0^3\alpha_0}{3} - \frac{11y_0^2\alpha_0}{2} - 2d_1\alpha_0 + d_1' \alpha_0 + 9y_0\alpha_0 - \\
 & \frac{25y_0^3}{36} + \frac{8y_0^2}{3} + 2d_1 - d_1' - \frac{49y_0}{12} + \left( -2\alpha_0 + \frac{2}{y_0-1} + 2 \right) H(0; \alpha_0) + \left( -2\alpha_0d_1 + \frac{2d_1}{y_0-1} + 2d_1 \right) H(1; \alpha_0) + \\
 & \frac{2d_1}{y_0-1} - \frac{d_1'}{y_0-1} - \frac{61}{12(y_0-1)} - \frac{1}{2(y_0-1)^2} - \frac{55}{12} \Big) H(c_1(\alpha_0); y_0) + \left( -\frac{2y_0^3}{3} + 3y_0^2 - 6y_0 + \frac{2}{y_0-1} + \right. \\
 & \left. 2 \right) H(0, 0; \alpha_0) + \left( -\frac{10y_0^3}{3} + 15y_0^2 - 30y_0 - \frac{6}{y_0-1} - 14 \right) H(0, 0; y_0) + \left( -\frac{2d_1y_0^3}{3} + 3d_1y_0^2 - 6d_1y_0 + \right. \\
 & \left. 2d_1 + \frac{2d_1}{y_0-1} \right) H(0, 1; \alpha_0) + H(1; y_0) \left( -\frac{1}{12}d_1'y_0^3\alpha_0^4 + \frac{1}{4}d_1'y_0^2\alpha_0^4 + \frac{d_1'\alpha_0^4}{12} - \frac{1}{4}d_1'y_0\alpha_0^4 + \frac{4}{9}d_1'y_0^3\alpha_0^3 - \right. \\
 & \frac{3}{2}d_1'y_0^2\alpha_0^3 - \frac{11d_1'\alpha_0^3}{18} + \frac{5}{3}d_1'y_0\alpha_0^3 - d_1'y_0^3\alpha_0^2 + \frac{15}{4}d_1'y_0^2\alpha_0^2 + \frac{9d_1'\alpha_0^2}{4} - 5d_1'y_0\alpha_0^2 + \frac{4}{3}d_1'y_0^3\alpha_0 - \frac{11}{2}d_1'y_0^2\alpha_0 - \\
 & \frac{23}{6}d_1'\alpha_0 + 9d_1'y_0\alpha_0 - \frac{d_1'^2y_0^3}{9} + \frac{4d_1'y_0^3}{3} + \frac{49d_1'^2}{36} + \frac{7d_1'^2y_0^2}{12} - \frac{25}{4}d_1'y_0^2 - \frac{133d_1'}{12} - \frac{11d_1'y_0}{6} + 16d_1'y_0 + \left( -\frac{d_1'y_0^3}{3} + \right. \\
 & \left. \frac{y_0^3}{3} + \frac{3d_1'y_0^2}{2} - \frac{3y_0^2}{2} - 3d_1'y_0 + 3y_0 + \frac{11}{6}d_1' - \frac{2d_1}{y_0-1} + \frac{d_1'}{y_0-1} + \frac{1}{y_0-1} + 5 \right) H(0; \alpha_0) - 2H(0, 0; \alpha_0) - \\
 & 2d_1H(0, 1; \alpha_0) + \frac{\pi^2}{6} \Big) + \left( -d_1'y_0^3 + \frac{9d_1'y_0^2}{2} - 9d_1'y_0 - 3d_1' + (2d_1 - 2)H(0; \alpha_0) - \frac{d_1'}{y_0-1} \right) H(0, 1; y_0) + \\
 & \left( \frac{y_0^3}{3} - \frac{3y_0^2}{2} + 3y_0 - 2H(0; \alpha_0) - 2d_1H(1; \alpha_0) + \frac{3}{y_0-1} + 5 \right) H(0, c_1(\alpha_0); y_0) + \left( -d_1'y_0^3 - \frac{y_0^3}{3} + \right. \\
 & \left. \frac{9d_1'y_0^2}{2} + \frac{3y_0^2}{2} - 9d_1'y_0 - 3y_0 + \frac{11d_1'}{2} - 2H(0; \alpha_0) + \frac{2d_1}{y_0-1} - \frac{d_1'}{y_0-1} - \frac{1}{y_0-1} - 5 \right) H(1, 0; y_0) + \left( -\frac{1}{3}d_1'^2y_0^3 + \right. \\
 & \left. \frac{3d_1'^2y_0^2}{2} - 3d_1'^2y_0 + \frac{11d_1'^2}{6} + (2d_1 - d_1' - 1)H(0; \alpha_0) \right) H(1, 1; y_0) + \left( \frac{y_0^3}{3} - \frac{3y_0^2}{2} + 3y_0 - 2H(0; \alpha_0) - \right. \\
 & \left. 2d_1H(1; \alpha_0) - \frac{2d_1}{y_0-1} + \frac{d_1'}{y_0-1} + \frac{1}{y_0-1} + 5 \right) H(1, c_1(\alpha_0); y_0) + \left( -2\alpha_0 + \frac{2}{y_0-1} + 2 \right) H(c_1(\alpha_0), 0; y_0) + \\
 & \left( -\alpha_0d_1' + \frac{d_1'}{y_0-1} + d_1' \right) H(c_1(\alpha_0), 1; y_0) + \left( -\alpha_0 + \frac{1}{y_0-1} + 1 \right) H(c_1(\alpha_0), c_1(\alpha_0); y_0) + \\
 & 16H(0, 0, 0; y_0) + 4d_1'H(0, 0, 1; y_0) - 4H(0, 0, c_1(\alpha_0); y_0) + (-2d_1 + 4d_1' + 2)H(0, 1, 0; y_0) + \\
 & d_1'^2H(0, 1, 1; y_0) + (2d_1 - d_1' - 2)H(0, 1, c_1(\alpha_0); y_0) - 2H(0, c_1(\alpha_0), 0; y_0) - \\
 & d_1'H(0, c_1(\alpha_0), 1; y_0) - H(0, c_1(\alpha_0), c_1(\alpha_0); y_0) + 6H(1, 0, 0; y_0) + d_1'H(1, 0, 1; y_0) - \\
 & 3H(1, 0, c_1(\alpha_0); y_0) + (-2d_1 + d_1' + 1)H(1, 1, 0; y_0) + (2d_1 - d_1' - 1)H(1, 1, c_1(\alpha_0); y_0) - \\
 & 2H(1, c_1(\alpha_0), 0; y_0) - d_1'H(1, c_1(\alpha_0), 1; y_0) - H(1, c_1(\alpha_0), c_1(\alpha_0); y_0) - \frac{\pi^2}{6(y_0-1)} + \frac{3\zeta_3}{2} - \frac{\pi^2}{6} - 2.
 \end{aligned}$$

### G.3 The $\mathcal{K}\mathcal{I}$ integral for $k = 2$

The  $\epsilon$  expansion for this integral reads

$$\mathcal{K}\mathcal{I}(\epsilon; y_0, d_0', \alpha_0, d_0; 1) = \frac{1}{\epsilon^3}(k * i)_{-3}^{(2)} + \frac{1}{\epsilon^2}(k * i)_{-2}^{(2)} + \frac{1}{\epsilon}(k * i)_{-1}^{(2)} + (k * i)_0^{(2)} + \mathcal{O}(\epsilon), \quad (\text{G.3})$$

where

$$(k * i)_{-3}^{(2)} = -\frac{1}{6},$$

$$(k * i)_{-2}^{(2)} = -\frac{2y_0^3}{9} + y_0^2 - 2y_0 + \frac{2}{3}H(0; y_0) - \frac{4}{9},$$

$$\begin{aligned}
 (k * i)_{-1}^{(2)} = & \frac{y_0^3\alpha_0^4}{18} - \frac{y_0^2\alpha_0^4}{12} + \frac{y_0\alpha_0^4}{3} - \frac{\alpha_0^4}{6(y_0-2)} - \frac{\alpha_0^4}{12} - \frac{8y_0^3\alpha_0^3}{27} + \frac{2y_0^2\alpha_0^3}{3} - \frac{8y_0\alpha_0^3}{9} - \frac{\alpha_0^3}{y_0-2} - \\
 & \frac{4\alpha_0^3}{9(y_0-2)^2} - \frac{7\alpha_0^3}{18} + \frac{2y_0^3\alpha_0^2}{3} - 2y_0^2\alpha_0^2 + \frac{5y_0\alpha_0^2}{3} - \frac{8\alpha_0^2}{3(y_0-2)} - \frac{11\alpha_0^2}{3(y_0-2)^2} - \frac{4\alpha_0^2}{3(y_0-2)^3} - \frac{7\alpha_0^2}{12} - \frac{8}{9}y_0^3\alpha_0 + \\
 & \frac{10y_0^2\alpha_0}{3} - 4y_0\alpha_0 - \frac{13\alpha_0}{3(y_0-2)} - \frac{18\alpha_0}{(y_0-2)^2} - \frac{52\alpha_0}{3(y_0-2)^3} - \frac{16\alpha_0}{3(y_0-2)^4} + \frac{\alpha_0}{2} + \frac{2d_1'y_0^3}{27} - \frac{25y_0^3}{27} - \frac{7d_1'y_0^2}{18} + \\
 & \frac{13y_0^2}{3} + \frac{11d_1'y_0}{9} - 11y_0 + \left( \frac{2y_0^3}{9} - y_0^2 + 2y_0 - \frac{2}{3(y_0-1)} - \frac{80}{3(y_0-2)^2} - \frac{160}{3(y_0-2)^3} - \frac{40}{(y_0-2)^4} - \right.
 \end{aligned}$$

$$\begin{aligned} & \left. \frac{32}{3(y_0-2)^5} + \frac{3}{2} \right) H(0; \alpha_0) + \left( \frac{2y_0^3}{3} - 3y_0^2 + 6y_0 + \frac{2}{3(y_0-1)} + \frac{80}{3(y_0-2)^2} + \frac{160}{3(y_0-2)^3} + \frac{40}{(y_0-2)^4} + \right. \\ & \left. \frac{32}{3(y_0-2)^5} + \frac{5}{18} \right) H(0; y_0) + \left( \frac{2d'_1 y_0^3}{9} - d'_1 y_0^2 + 2d'_1 y_0 - \frac{11d'_1}{9} + \frac{2}{3} H(0; \alpha_0) \right) H(1; y_0) + \left( \frac{2\alpha_0}{3} - \right. \\ & \left. \frac{2}{3(y_0-1)} - \frac{2}{3} \right) H(c_1(\alpha_0); y_0) + \left( \frac{\alpha_0^4}{2} + \frac{2\alpha_0^3}{3} - \alpha_0^2 - 2\alpha_0 - \frac{80}{3(y_0-2)^2} - \frac{160}{3(y_0-2)^3} - \frac{40}{(y_0-2)^4} - \right. \\ & \left. \frac{32}{3(y_0-2)^5} + \frac{11}{6} \right) H(c_2(\alpha_0); y_0) - \frac{8}{3} H(0, 0; y_0) - \frac{2}{3} d'_1 H(0, 1; y_0) + \frac{2}{3} H(0, c_1(\alpha_0); y_0) - \\ & \frac{2}{3} H(1, 0; y_0) + \frac{2}{3} H(1, c_1(\alpha_0); y_0) - \frac{80 \ln 2}{3(y_0-2)^2} - \frac{160 \ln 2}{3(y_0-2)^3} - \frac{40 \ln 2}{(y_0-2)^4} - \frac{32 \ln 2}{3(y_0-2)^5} + \frac{13 \ln 2}{6} - \frac{26}{27}, \end{aligned}$$

$$\begin{aligned} (k * i)_0^{(2)} = & -\frac{1}{36} d_1 y_0^3 \alpha_0^4 - \frac{1}{54} d'_1 y_0^3 \alpha_0^4 + \frac{7}{27} y_0^3 \alpha_0^4 + \frac{1}{24} d_1 y_0^2 \alpha_0^4 + \frac{1}{72} d'_1 y_0^2 \alpha_0^4 - \frac{5y_0^2 \alpha_0^4}{18} + \frac{d_1 \alpha_0^4}{24} - \\ & \frac{1}{6} d_1 y_0 \alpha_0^4 - \frac{11}{36} d'_1 y_0 \alpha_0^4 + \frac{41y_0 \alpha_0^4}{18} + \frac{d_1 \alpha_0^4}{12(y_0-2)} - \frac{31\alpha_0^4}{36(y_0-2)} - \frac{31\alpha_0^4}{72} + \frac{13}{81} d_1 y_0^3 \alpha_0^3 + \frac{8}{81} d'_1 y_0^3 \alpha_0^3 - \frac{229y_0^3 \alpha_0^3}{162} - \\ & \frac{7}{18} d_1 y_0^2 \alpha_0^3 - \frac{5}{27} d'_1 y_0^2 \alpha_0^3 + \frac{305y_0^2 \alpha_0^3}{108} + \frac{43d_1 \alpha_0^3}{108} - \frac{d'_1 \alpha_0^3}{18} + \frac{10}{27} d_1 y_0 \alpha_0^3 + \frac{14}{27} d'_1 y_0 \alpha_0^3 - \frac{541y_0 \alpha_0^3}{108} + \frac{17d_1 \alpha_0^3}{18(y_0-2)} - \\ & \frac{d'_1 \alpha_0^3}{9(y_0-2)} - \frac{29\alpha_0^3}{6(y_0-2)} + \frac{8d_1 \alpha_0^3}{27(y_0-2)^2} - \frac{64\alpha_0^3}{27(y_0-2)^2} - \frac{197\alpha_0^3}{108} - \frac{23}{54} d_1 y_0^3 \alpha_0^2 - \frac{2}{9} d'_1 y_0^3 \alpha_0^2 + \frac{359y_0^3 \alpha_0^2}{108} + \frac{17}{12} d_1 y_0^2 \alpha_0^2 + \\ & \frac{2}{3} d'_1 y_0^2 \alpha_0^2 - \frac{2099y_0^2 \alpha_0^2}{216} + \frac{113d_1 \alpha_0^2}{72} - \frac{13}{36} d'_1 \alpha_0^2 - \frac{10}{9} d_1 y_0 \alpha_0^2 - \frac{1}{3} d'_1 y_0 \alpha_0^2 + \frac{215y_0 \alpha_0^2}{27} + \frac{46d_1 \alpha_0^2}{9(y_0-2)} - \frac{8}{9} d'_1 \alpha_0^2 - \\ & \frac{25\alpha_0^2}{2(y_0-2)} + \frac{83d_1 \alpha_0^2}{18(y_0-2)^2} - \frac{d'_1 \alpha_0^2}{3(y_0-2)^2} - \frac{349 \alpha_0^2}{18(y_0-2)^2} + \frac{4d_1 \alpha_0^2}{3(y_0-2)^3} - \frac{68 \alpha_0^2}{9(y_0-2)^3} - \frac{169\alpha_0^2}{72} + \frac{25}{27} d_1 y_0^3 \alpha_0 + \frac{8}{27} d'_1 y_0^3 \alpha_0 - \\ & \frac{91y_0^3 \alpha_0}{18} - \frac{23}{6} d_1 y_0^2 \alpha_0 - \frac{11}{9} d'_1 y_0^2 \alpha_0 + \frac{2099y_0^2 \alpha_0}{108} + \frac{83d_1 \alpha_0}{36} - \frac{19d'_1 \alpha_0}{18} + \frac{52d_1 y_0 \alpha_0}{9} + \frac{14d'_1 y_0 \alpha_0}{9} - \frac{949y_0 \alpha_0}{36} + \\ & \frac{383d_1 \alpha_0}{18(y_0-2)} - \frac{35d'_1 \alpha_0}{9(y_0-2)} - \frac{385\alpha_0}{18(y_0-2)} - \frac{\alpha_0}{3(y_0-1)} + \frac{151}{3(y_0-2)^2} - \frac{38d'_1 \alpha_0}{9(y_0-2)^2} - \frac{983 \alpha_0}{9(y_0-2)^2} + \frac{118d_1 \alpha_0}{3(y_0-2)^3} - \frac{4d'_1 \alpha_0}{3(y_0-2)^3} - \\ & \frac{998\alpha_0}{9(y_0-2)^3} + \frac{32d_1 \alpha_0}{3(y_0-2)^4} - \frac{320\alpha_0}{9(y_0-2)^4} + \frac{167\alpha_0}{36} - \frac{2}{81} d_1^2 y_0^3 + \frac{31d'_1 y_0^3}{81} + \frac{\pi^2 y_0^3}{27} - \frac{230y_0^3}{81} + \frac{17d_1^2 y_0^3}{108} - \frac{247}{108} d'_1 y_0^3 - \\ & \frac{\pi^2 y_0^2}{6} + \frac{247y_0^2}{18} - \frac{49}{54} d_1^2 y_0 + \frac{304d'_1 y_0}{27} + \frac{\pi^2 y_0}{3} - \frac{134y_0}{3} + \left( -\frac{1}{9} y_0^3 \alpha_0^4 + \frac{y_0^2 \alpha_0^4}{6} - \frac{2y_0 \alpha_0^4}{3} + \frac{\alpha_0^4}{3(y_0-2)} + \frac{\alpha_0^4}{6} + \right. \\ & \left. \frac{16y_0^3 \alpha_0^3}{27} - \frac{4y_0^2 \alpha_0^3}{3} + \frac{16y_0 \alpha_0^3}{9} + \frac{2\alpha_0^3}{y_0-2} + \frac{8 \alpha_0^3}{9(y_0-2)^2} + \frac{7\alpha_0^3}{9} - \frac{4y_0^3 \alpha_0^2}{3} + 4y_0^2 \alpha_0^2 - \frac{10y_0 \alpha_0^2}{3} + \frac{16\alpha_0^2}{3(y_0-2)} + \frac{22 \alpha_0^2}{3(y_0-2)^2} + \right. \\ & \left. \frac{8\alpha_0^2}{3(y_0-2)^3} + \frac{7 \alpha_0^2}{6} + \frac{16y_0^3 \alpha_0}{9} - \frac{20y_0^2 \alpha_0}{3} + 8y_0 \alpha_0 + \frac{26\alpha_0}{3(y_0-2)} + \frac{36\alpha_0}{(y_0-2)^2} + \frac{104 \alpha_0}{3(y_0-2)^3} + \frac{32\alpha_0}{3(y_0-2)^4} - \alpha_0 - \frac{2d'_1 y_0^3}{27} + \right. \\ & \left. \frac{25y_0^3}{54} + \frac{7d'_1 y_0^2}{18} - \frac{95 y_0^2}{36} - \frac{41d_1}{36} - \frac{d'_1}{18} - \frac{11d'_1 y_0}{9} + 9y_0 - \frac{8d'_1}{3(y_0-2)} + \frac{20}{3(y_0-2)} + \frac{4d_1}{3(y_0-1)} - \frac{2d'_1}{3(y_0-1)} - \frac{7}{2(y_0-1)} + \right. \\ & \left. \frac{16d_1}{(y_0-2)^2} - \frac{12d'_1}{(y_0-2)^2} - \frac{344}{3(y_0-2)^2} - \frac{1}{3(y_0-1)^2} + \frac{128d_1}{9(y_0-2)^3} - \frac{88 d'_1}{9(y_0-2)^3} - \frac{2152}{9(y_0-2)^3} + \frac{4 d_1}{(y_0-2)^4} - \frac{8d'_1}{3(y_0-2)^4} - \right. \\ & \left. \frac{548}{3(y_0-2)^4} - \frac{448}{9(y_0-2)^5} + \frac{317}{36} \right) H(0; \alpha_0) + \left( -\frac{1}{9} d_1 y_0^3 \alpha_0^4 + \frac{1}{6} d_1 y_0^2 \alpha_0^4 + \frac{d_1 \alpha_0^4}{6} - \frac{2}{3} d_1 y_0 \alpha_0^4 + \frac{d_1 \alpha_0^4}{3(y_0-2)} + \right. \\ & \left. \frac{16}{27} d_1 y_0^3 \alpha_0^3 - \frac{4}{3} d_1 y_0^2 \alpha_0^3 + \frac{7d_1 \alpha_0^3}{9} + \frac{16}{9} d_1 y_0 \alpha_0^3 + \frac{2d_1 \alpha_0^3}{y_0-2} + \frac{8d_1 \alpha_0^3}{9(y_0-2)^2} - \frac{4}{3} d_1 y_0^3 \alpha_0^2 + 4d_1 y_0^2 \alpha_0^2 + \frac{7d_1 \alpha_0^2}{6} - \right. \\ & \left. \frac{10}{3} d_1 y_0 \alpha_0^2 + \frac{16d_1 \alpha_0^2}{3(y_0-2)} + \frac{22d_1 \alpha_0^2}{3(y_0-2)^2} + \frac{8d_1 \alpha_0^2}{3(y_0-2)^3} + \frac{16}{9} d_1 y_0^3 \alpha_0 - \frac{20}{3} d_1 y_0^2 \alpha_0 - d_1 \alpha_0 + 8d_1 y_0 \alpha_0 + \frac{26d_1 \alpha_0}{3(y_0-2)} + \right. \\ & \left. \frac{36d_1 \alpha_0}{(y_0-2)^2} + \frac{104d_1 \alpha_0}{3(y_0-2)^3} + \frac{32d_1 \alpha_0}{3(y_0-2)^4} - \frac{25d_1 y_0^3}{27} + \frac{23d_1 y_0^2}{6} - \frac{10d_1}{9} - \frac{52d_1 y_0}{9} - \frac{49d_1}{3(y_0-2)} - \frac{398d_1}{9(y_0-2)^2} - \frac{112d_1}{3(y_0-2)^3} - \right. \\ & \left. \frac{32d_1}{3(y_0-2)^4} \right) H(1; \alpha_0) + \left( -\frac{1}{18} y_0^3 \alpha_0^4 + \frac{y_0^2 \alpha_0^4}{12} - \frac{y_0 \alpha_0^4}{3} + \frac{\alpha_0^4}{6(y_0-2)} + \frac{17 \alpha_0^4}{36} + \frac{8y_0^3 \alpha_0^3}{27} - \frac{2y_0^2 \alpha_0^3}{3} + \frac{8 y_0 \alpha_0^3}{9} + \frac{\alpha_0^3}{y_0-2} + \right. \\ & \left. \frac{4\alpha_0^3}{9(y_0-2)^2} + \frac{29\alpha_0^3}{54} - \frac{2y_0^3 \alpha_0^2}{3} + 2y_0^2 \alpha_0^2 - \frac{5y_0 \alpha_0^2}{3} + \frac{8\alpha_0^2}{3(y_0-2)} + \frac{11\alpha_0^2}{3(y_0-2)^2} + \frac{4\alpha_0^2}{3(y_0-2)^3} + \frac{17\alpha_0^2}{12} + \frac{8 y_0^3 \alpha_0}{9} - \frac{10y_0^2 \alpha_0}{3} - \right. \\ & \left. \frac{4d_1 \alpha_0}{3} + \frac{2d'_1 \alpha_0}{3} + 4y_0 \alpha_0 + \frac{13\alpha_0}{3(y_0-2)} + \frac{18 \alpha_0}{(y_0-2)^2} + \frac{52\alpha_0}{3(y_0-2)^3} + \frac{16\alpha_0}{3(y_0-2)^4} - \frac{19\alpha_0}{18} - \frac{25y_0^3}{54} + \frac{61y_0^2}{36} + \frac{4d_1}{3} - \frac{2d'_1}{3} - \right. \\ & \left. 2y_0 + \left( -\frac{4 \alpha_0}{3} + \frac{4}{3(y_0-1)} + \frac{4}{3} \right) H(0; \alpha_0) + \left( -\frac{4 \alpha_0 d_1}{3} + \frac{4d_1}{3(y_0-1)} + \frac{4d_1}{3} \right) H(1; \alpha_0) - \frac{32}{3(y_0-2)} + \frac{4d_1}{3(y_0-1)} - \right. \\ & \left. \frac{2d'_1}{3(y_0-1)} - \frac{7}{2(y_0-1)} - \frac{332}{9(y_0-2)^2} - \frac{1}{3(y_0-1)^2} - \frac{104}{3(y_0-2)^3} - \frac{32}{3(y_0-2)^4} - \frac{53}{18} \right) H(c_1(\alpha_0); y_0) + \left( -\frac{d_1 \alpha_0^4}{4} + \right. \\ & \left. \frac{d'_1 \alpha_0^4}{6} + \frac{25\alpha_0^4}{12} - \frac{7d_1 \alpha_0^3}{9} + \frac{5d'_1 \alpha_0^3}{9} + \frac{11\alpha_0^3}{9} - \frac{d_1 \alpha_0^2}{6} + \frac{d'_1 \alpha_0^2}{3} - \frac{13\alpha_0^2}{2} + \frac{11d_1 \alpha_0}{3} - \frac{5d'_1 \alpha_0}{3} - \frac{23\alpha_0}{3} - \frac{89 d_1}{36} + \frac{11d'_1}{18} + \right. \\ & \left( -\alpha_0^4 - \frac{4\alpha_0^3}{3} + 2\alpha_0^2 + 4\alpha_0 + \frac{160}{3(y_0-2)^2} + \frac{320}{3(y_0-2)^3} + \frac{80}{(y_0-2)^4} + \frac{64}{3(y_0-2)^5} - \frac{11}{3} \right) H(0; \alpha_0) + \left( -d_1 \alpha_0^4 - \right. \\ & \left. \frac{4d_1 \alpha_0^3}{3} + 2d_1 \alpha_0^2 + 4d_1 \alpha_0 - \frac{11d_1}{3} + \frac{160d_1}{3(y_0-2)^2} + \frac{320d_1}{3(y_0-2)^3} + \frac{80 d_1}{(y_0-2)^4} + \frac{64d_1}{3(y_0-2)^5} \right) H(1; \alpha_0) - \frac{8 d'_1}{3(y_0-2)} + \\ & \frac{52}{3(y_0-2)} + \frac{16 d_1}{(y_0-2)^2} - \frac{12d'_1}{(y_0-2)^2} - \frac{700}{9(y_0-2)^2} + \frac{128d_1}{9(y_0-2)^3} - \frac{88d'_1}{9(y_0-2)^3} - \frac{1840}{9(y_0-2)^3} + \frac{4d_1}{(y_0-2)^4} - \frac{8 d'_1}{3(y_0-2)^4} - \end{aligned}$$



$$\begin{aligned}
& \left. \frac{172}{(y_0-2)^4} - \frac{448}{9(y_0-2)^5} + \frac{391}{36} \right) H(c_2(\alpha_0); y_0) + \left( -\frac{4y_0^3}{9} + 2y_0^2 - 4y_0 + \frac{4}{3(y_0-1)} + \frac{160}{3(y_0-2)^2} + \frac{320}{3(y_0-2)^3} + \right. \\
& \left. \frac{80}{(y_0-2)^4} + \frac{64}{3(y_0-2)^5} - 3 \right) H(0, 0; \alpha_0) + \left( -\frac{20y_0^3}{9} + 10y_0^2 - 20y_0 - \frac{4}{y_0-1} - \frac{160}{(y_0-2)^2} - \frac{320}{(y_0-2)^3} - \right. \\
& \left. \frac{240}{3}(y_0-2)^4 - \frac{64}{(y_0-2)^5} + \frac{17}{9} \right) H(0, 0; y_0) + \left( -\frac{4d_1y_0^3}{9} + 2d_1y_0^2 - 4d_1y_0 - 3d_1 + \frac{4d_1}{3(y_0-1)} + \frac{160d_1}{3(y_0-2)^2} + \right. \\
& \left. \frac{320d_1}{3(y_0-2)^3} + \frac{80d_1}{(y_0-2)^4} + \frac{64d_1}{3(y_0-2)^5} \right) H(0, 1; \alpha_0) + \left( -\frac{2d_1'y_0^3}{3} + 3d_1'y_0^2 - 6d_1'y_0 - \frac{5d_1'}{18} + \left( \frac{4d_1'}{3} - \frac{4}{3} \right) H(0; \alpha_0) - \right. \\
& \left. \frac{2d_1'}{3(y_0-1)} - \frac{80d_1'}{3(y_0-2)^2} - \frac{160d_1'}{3(y_0-2)^3} - \frac{40d_1'}{(y_0-2)^4} - \frac{32d_1'}{3(y_0-2)^5} \right) H(0, 1; y_0) + \left( \frac{2y_0^3}{9} - y_0^2 + 2y_0 - \frac{4}{3} H(0; \alpha_0) - \right. \\
& \left. \frac{4}{3}d_1 H(1; \alpha_0) + \frac{2}{y_0-1} - \frac{80}{3(y_0-2)^2} - \frac{160}{3(y_0-2)^3} - \frac{40}{(y_0-2)^4} - \frac{32}{3(y_0-2)^5} + \frac{107}{18} \right) H(0, c_1(\alpha_0); y_0) + \left( -\right. \\
& \left. \frac{26}{3} + \frac{320}{3(y_0-2)^2} + \frac{640}{3(y_0-2)^3} + \frac{160}{(y_0-2)^4} + \frac{128}{3(y_0-2)^5} \right) H(0, c_2(\alpha_0); y_0) + \left( -\frac{2d_1'y_0^3}{3} - \frac{2y_0^3}{9} + 3d_1'y_0^2 + \right. \\
& \left. y_0^2 - 6d_1'y_0 - 2y_0 + \frac{19d_1'}{3} - \frac{4}{3} H(0; \alpha_0) + \frac{4d_1}{3(y_0-1)} - \frac{2d_1'}{3(y_0-1)} - \frac{2}{3(y_0-1)} - \frac{80d_1'}{3(y_0-2)^2} + \frac{80}{3(y_0-2)^2} - \right. \\
& \left. \frac{160d_1'}{3(y_0-2)^3} + \frac{160}{3(y_0-2)^3} - \frac{40d_1'}{(y_0-2)^4} + \frac{40}{(y_0-2)^4} - \frac{32d_1'}{3(y_0-2)^5} + \frac{32}{3(y_0-2)^5} - \frac{107}{18} \right) H(1, 0; y_0) + \left( -\frac{2}{9}d_1'^2 y_0^3 + \right. \\
& \left. d_1'^2 y_0^2 - 2d_1'^2 y_0 + \frac{11d_1'^2}{9} + \left( \frac{4d_1'}{3} - \frac{2d_1'}{3} - \frac{2}{3} \right) H(0; \alpha_0) \right) H(1, 1; y_0) + \left( \frac{2y_0^3}{9} - y_0^2 + 2y_0 - \frac{4}{3} H(0; \alpha_0) - \right. \\
& \left. \frac{4}{3}d_1 H(1; \alpha_0) - \frac{4d_1}{3(y_0-1)} + \frac{2d_1'}{3(y_0-1)} + \frac{2}{3(y_0-1)} - \frac{80}{3(y_0-2)^2} - \frac{160}{3(y_0-2)^3} - \frac{40}{(y_0-2)^4} - \frac{32}{3(y_0-2)^5} + \right. \\
& \left. \frac{107}{18} \right) H(1, c_1(\alpha_0); y_0) + \left( \frac{80d_1'}{3(y_0-2)^2} + \frac{160d_1'}{3(y_0-2)^3} + \frac{40d_1'}{(y_0-2)^4} + \frac{32d_1'}{3(y_0-2)^5} - \frac{8d_1'}{3} \right) H(1, c_2(\alpha_0); y_0) + \\
& \left( -\frac{160d_1}{3(y_0-2)^2} - \frac{320d_1}{3(y_0-2)^3} - \frac{80d_1}{(y_0-2)^4} - \frac{64d_1}{3(y_0-2)^5} + \frac{8d_1'}{3} + \frac{160}{3(y_0-2)^2} + \frac{320}{3(y_0-2)^3} + \frac{80}{(y_0-2)^4} + \frac{64}{3(y_0-2)^5} - \right. \\
& \left. \frac{26}{3} \right) H(2, 0; y_0) + \left( \frac{160d_1}{3(y_0-2)^2} + \frac{320d_1}{3(y_0-2)^3} + \frac{80d_1}{(y_0-2)^4} + \frac{64d_1}{3(y_0-2)^5} - \frac{8d_1'}{3} - \frac{160}{3(y_0-2)^2} - \frac{320}{3(y_0-2)^3} - \frac{80}{(y_0-2)^4} - \right. \\
& \left. \frac{64}{3(y_0-2)^5} + \frac{26}{3} \right) H(2, c_2(\alpha_0); y_0) + \left( -\frac{4\alpha_0}{3} + \frac{4}{3(y_0-1)} + \frac{4}{3} \right) H(c_1(\alpha_0), 0; y_0) + \left( -\frac{2\alpha_0 d_1'}{3} + \frac{2d_1'}{3(y_0-1)} + \right. \\
& \left. \frac{2d_1'}{3} \right) H(c_1(\alpha_0), 1; y_0) + \left( -\frac{2\alpha_0}{3} + \frac{2}{3(y_0-1)} + \frac{2}{3} \right) H(c_1(\alpha_0), c_1(\alpha_0); y_0) + \left( -\alpha_0^4 - \frac{4\alpha_0^3}{3} + 2\alpha_0^2 + \right. \\
& \left. 4\alpha_0 + \frac{160}{3(y_0-2)^2} + \frac{320}{3(y_0-2)^3} + \frac{80}{(y_0-2)^4} + \frac{64}{3(y_0-2)^5} - \frac{11}{3} \right) H(c_2(\alpha_0), 0; y_0) + \left( -\frac{d_1'\alpha_0^4}{2} - \frac{2d_1'\alpha_0^3}{3} + d_1'\alpha_0^2 + \right. \\
& \left. 2d_1'\alpha_0 - \frac{11d_1'}{6} + \frac{80d_1'}{3(y_0-2)^2} + \frac{160d_1'}{3(y_0-2)^3} + \frac{40d_1'}{(y_0-2)^4} + \frac{32d_1'}{3(y_0-2)^5} \right) H(c_2(\alpha_0), 1; y_0) + \left( -\frac{\alpha_0^4}{2} - \frac{2\alpha_0^3}{3} + \alpha_0^2 + \right. \\
& \left. 2\alpha_0 + \frac{80}{3(y_0-2)^2} + \frac{160}{3(y_0-2)^3} + \frac{40}{(y_0-2)^4} + \frac{32}{3(y_0-2)^5} - \frac{11}{6} \right) H(c_2(\alpha_0), c_1(\alpha_0); y_0) + \frac{32}{3} H(0, 0, 0; y_0) + \\
& \frac{8}{3}d_1' H(0, 0, 1; y_0) - \frac{8}{3} H(0, 0, c_1(\alpha_0); y_0) + \left( -\frac{4d_1}{3} + \frac{8d_1'}{3} + \frac{4}{3} \right) H(0, 1, 0; y_0) + \frac{2}{3}d_1'^2 H(0, 1, 1; y_0) + \\
& \left( \frac{4d_1}{3} - \frac{2d_1'}{3} - \frac{4}{3} \right) H(0, 1, c_1(\alpha_0); y_0) - \frac{4}{3} H(0, c_1(\alpha_0), 0; y_0) - \frac{2}{3}d_1' H(0, c_1(\alpha_0), 1; y_0) - \\
& \frac{2}{3} H(0, c_1(\alpha_0), c_1(\alpha_0); y_0) + 4H(1, 0, 0; y_0) + \frac{2}{3}d_1' H(1, 0, 1; y_0) - 2H(1, 0, c_1(\alpha_0); y_0) + \left( -\right. \\
& \left. \frac{4d_1}{3} + \frac{2d_1'}{3} + \frac{2}{3} \right) H(1, 1, 0; y_0) + \left( \frac{4d_1}{3} - \frac{2d_1'}{3} - \frac{2}{3} \right) H(1, 1, c_1(\alpha_0); y_0) - \frac{4}{3} H(1, c_1(\alpha_0), 0; y_0) - \\
& \frac{2}{3}d_1' H(1, c_1(\alpha_0), 1; y_0) - \frac{2}{3} H(1, c_1(\alpha_0), c_1(\alpha_0); y_0) + H(0; y_0) \left( -\frac{1}{9}y_0^3 \alpha_0^4 + \frac{y_0^2 \alpha_0^4}{6} - \frac{2y_0 \alpha_0^4}{3} + \right. \\
& \left. \frac{\alpha_0^4}{3(y_0-2)} + \frac{\alpha_0^4}{6} + \frac{16y_0^3 \alpha_0^3}{27} - \frac{4y_0^2 \alpha_0^3}{3} + \frac{16y_0 \alpha_0^3}{9} + \frac{2\alpha_0^3}{y_0-2} + \frac{8\alpha_0^3}{9(y_0-2)^2} + \frac{7\alpha_0^3}{9} - \frac{4y_0^3 \alpha_0^2}{3} + 4y_0^2 \alpha_0^2 - \frac{10y_0 \alpha_0^2}{3} + \right. \\
& \left. \frac{16\alpha_0^2}{3(y_0-2)} + \frac{22\alpha_0^2}{3(y_0-2)^2} + \frac{8\alpha_0^2}{3(y_0-2)^3} + \frac{7\alpha_0^2}{6} + \frac{16y_0^3 \alpha_0}{9} - \frac{20y_0^2 \alpha_0}{3} + 8y_0 \alpha_0 + \frac{26\alpha_0}{3(y_0-2)} + \frac{36\alpha_0}{(y_0-2)^2} + \frac{104\alpha_0}{3(y_0-2)^3} + \right. \\
& \left. \frac{32\alpha_0}{3(y_0-2)^4} - \alpha_0 - \frac{2d_1'y_0^3}{9} + \frac{175y_0^3}{54} + \frac{7d_1'y_0^2}{6} - \frac{529y_0^2}{36} + \frac{41d_1}{36} + \frac{d_1'}{18} - \frac{11d_1'y_0}{3} + 35y_0 + \left( -\frac{4y_0^3}{9} + 2y_0^2 - 4y_0 + \right. \right. \\
& \left. \left. \frac{4}{3(y_0-1)} + \frac{160}{3(y_0-2)^2} + \frac{320}{3(y_0-2)^3} + \frac{80}{(y_0-2)^4} + \frac{64}{3(y_0-2)^5} - 3 \right) H(0; \alpha_0) + \frac{8d_1'}{3(y_0-2)} - \frac{20}{3(y_0-2)} - \frac{4d_1}{3(y_0-1)} + \right. \\
& \left. \frac{2d_1'}{3(y_0-1)} + \frac{7}{2(y_0-1)} - \frac{16d_1}{(y_0-2)^2} + \frac{12d_1'}{(y_0-2)^2} + \frac{344}{3(y_0-2)^2} + \frac{1}{3(y_0-1)^2} - \frac{128d_1}{9(y_0-2)^3} + \frac{88d_1'}{9(y_0-2)^3} + \frac{2152}{9(y_0-2)^3} - \right. \\
& \left. \frac{4d_1}{(y_0-2)^4} + \frac{8d_1'}{3(y_0-2)^4} + \frac{548}{3(y_0-2)^4} + \frac{448}{9(y_0-2)^5} + \frac{320 \ln 2}{3(y_0-2)^2} + \frac{640 \ln 2}{3(y_0-2)^3} + \frac{160 \ln 2}{(y_0-2)^4} + \frac{128 \ln 2}{3(y_0-2)^5} - \frac{26 \ln 2}{3} - \right. \\
& \left. \frac{535}{108} \right) + H(2; y_0) \left( \frac{160 \ln 2 d_1}{3(y_0-2)^2} + \frac{320 \ln 2 d_1}{3(y_0-2)^3} + \frac{80 \ln 2 d_1}{(y_0-2)^4} + \frac{64 \ln 2 d_1}{3(y_0-2)^5} + \left( \frac{160d_1}{3(y_0-2)^2} + \frac{320d_1}{3(y_0-2)^3} + \frac{80d_1}{(y_0-2)^4} + \right. \right.
\end{aligned}$$

$$\begin{aligned}
 & \left. \frac{64d_1}{3(y_0-2)^5} - \frac{8d_1'}{3} - \frac{160}{3(y_0-2)^2} - \frac{320}{3(y_0-2)^3} - \frac{80}{(y_0-2)^4} - \frac{64}{3(y_0-2)^5} + \frac{26}{3} \right) H(0; \alpha_0) - \frac{8d_1'}{3} \ln 2 - \frac{160 \ln 2}{3(y_0-2)^2} - \\
 & \left. \frac{320 \ln 2}{3(y_0-2)^3} - \frac{80 \ln 2}{(y_0-2)^4} - \frac{64 \ln 2}{3(y_0-2)^5} + \frac{26 \ln 2}{3} \right) + H(1; y_0) \left( -\frac{1}{18} d_1' y_0^3 \alpha_0^4 + \frac{1}{12} d_1' y_0^2 \alpha_0^4 + \frac{17d_1' \alpha_0^4}{36} - \frac{1}{3} d_1' y_0 \alpha_0^4 + \right. \\
 & \left. \frac{d_1' \alpha_0^4}{6(y_0-2)} + \frac{8}{27} d_1' y_0^3 \alpha_0^3 - \frac{2}{3} d_1' y_0^2 \alpha_0^3 + \frac{d_1' \alpha_0^3}{27} + \frac{8}{9} d_1' y_0 \alpha_0^3 + \frac{d_1' \alpha_0^3}{y_0-2} + \frac{4d_1' \alpha_0^3}{9(y_0-2)^2} - \frac{2}{3} d_1' y_0^3 \alpha_0^2 + 2d_1' y_0^2 \alpha_0^2 + \right. \\
 & \left. \frac{2d_1' \alpha_0^2}{3} - \frac{5}{3} d_1' y_0 \alpha_0^2 + \frac{8d_1' \alpha_0^2}{3(y_0-2)} + \frac{11d_1' \alpha_0^2}{3(y_0-2)^2} + \frac{4d_1' \alpha_0^2}{3(y_0-2)^3} + \frac{8}{9} d_1' y_0^3 \alpha_0 - \frac{10}{3} d_1' y_0^2 \alpha_0 - \frac{23d_1' \alpha_0}{9} + 4d_1' y_0 \alpha_0 + \right. \\
 & \left. \frac{13d_1' \alpha_0}{3(y_0-2)} + \frac{18d_1' \alpha_0}{(y_0-2)^2} + \frac{52d_1' \alpha_0}{3(y_0-2)^3} + \frac{16d_1' \alpha_0}{3(y_0-2)^4} - \frac{2d_1'^2 y_0^3}{27} + \frac{25d_1' y_0^3}{27} + \frac{49d_1'^2}{54} + \frac{7d_1'^2 y_0^2}{18} - \frac{13d_1' y_0^2}{3} - \frac{205d_1'}{27} - \frac{11d_1'^2 y_0}{9} + \right. \\
 & \left. 11d_1' y_0 + \left( -\frac{2d_1' y_0^3}{9} + \frac{2y_0^3}{9} + d_1' y_0^2 - y_0^2 - 2d_1' y_0 + 2y_0 - \frac{13d_1'}{9} - \frac{4d_1}{3(y_0-1)} + \frac{2d_1'}{3(y_0-1)} + \frac{2}{3(y_0-1)} + \frac{80d_1'}{3(y_0-2)^2} - \right. \right. \\
 & \left. \left. \frac{80}{3(y_0-2)^2} + \frac{160d_1'}{3(y_0-2)^3} - \frac{160}{3(y_0-2)^3} + \frac{40d_1'}{(y_0-2)^4} - \frac{40}{(y_0-2)^4} + \frac{32d_1'}{3(y_0-2)^5} - \frac{32}{3(y_0-2)^5} + \frac{107}{18} \right) H(0; \alpha_0) - \right. \\
 & \left. \frac{4}{3} H(0, 0; \alpha_0) - \frac{4}{3} d_1 H(0, 1; \alpha_0) - \frac{8}{3} d_1' \ln 2 + \frac{80d_1' \ln 2}{3(y_0-2)^2} + \frac{160d_1' \ln 2}{3(y_0-2)^3} + \frac{40d_1' \ln 2}{(y_0-2)^4} + \frac{32d_1' \ln 2}{3(y_0-2)^5} + \frac{\pi^2}{9} \right) - \\
 & \frac{\pi^2}{9(y_0-1)} - \frac{20\pi^2}{9(y_0-2)^2} - \frac{40\pi^2}{9(y_0-2)^3} - \frac{10\pi^2}{3(y_0-2)^4} - \frac{8\pi^2}{9(y_0-2)^5} + \zeta_3 - \frac{80 \ln^2 2}{3(y_0-2)^2} - \frac{160 \ln^2 2}{3(y_0-2)^3} - \frac{40 \ln^2 2}{(y_0-2)^4} - \\
 & \frac{32 \ln^2 2}{3(y_0-2)^5} + \frac{13 \ln^2 2}{6} - \frac{89}{36} d_1 \ln 2 + \frac{11}{18} d_1' \ln 2 - \frac{8d_1' \ln 2}{3(y_0-2)} + \frac{52 \ln 2}{3(y_0-2)} + \frac{16d_1 \ln 2}{(y_0-2)^2} - \frac{12d_1' \ln 2}{(y_0-2)^2} - \frac{700 \ln 2}{9(y_0-2)^2} + \\
 & \frac{128d_1 \ln 2}{9(y_0-2)^3} - \frac{88d_1' \ln 2}{9(y_0-2)^3} - \frac{1840 \ln 2}{9(y_0-2)^3} + \frac{4d_1 \ln 2}{(y_0-2)^4} - \frac{8d_1' \ln 2}{3(y_0-2)^4} - \frac{172 \ln 2}{(y_0-2)^4} - \frac{448 \ln 2}{9(y_0-2)^5} + \frac{47 \ln 2}{4} + \frac{5\pi^2}{72} - \frac{160}{81}.
 \end{aligned}$$

#### G.4 The $\mathcal{K}\mathcal{I}$ integral for $k = -1$

The  $\varepsilon$  expansion for this integral reads

$$\mathcal{K}\mathcal{I}(\varepsilon; y_0, d_0', \alpha_0, d_0; 1) = \frac{1}{\varepsilon^4} (k*i)_{-4}^{(-1)} + \frac{1}{\varepsilon^3} (k*i)_{-3}^{(-1)} + \frac{1}{\varepsilon^2} (k*i)_{-2}^{(-1)} + \frac{1}{\varepsilon} (k*i)_{-1}^{(-1)} + (k*i)_0^{(-1)} + \mathcal{O}(\varepsilon), \quad (\text{G.4})$$

where

$$(k*i)_{-4}^{(-1)} = \frac{1}{4},$$

$$(k*i)_{-3}^{(-1)} = \frac{y_0^3}{3} - \frac{3y_0^2}{2} + 3y_0 - H(0; y_0),$$

$$\begin{aligned}
 (k*i)_{-2}^{(-1)} = & -\frac{d_1' y_0^3}{9} + \frac{7y_0^3}{9} + \frac{7d_1' y_0^2}{12} - 4y_0^2 - \frac{11d_1' y_0}{6} + 13y_0 + \left( -\frac{4y_0^3}{3} + 6y_0^2 - \right. \\
 & \left. 12y_0 \right) H(0; y_0) + \left( -\frac{d_1' y_0^3}{3} + \frac{3d_1' y_0^2}{2} - 3d_1' y_0 + \frac{11d_1'}{6} \right) H(1; y_0) + 4H(0, 0; y_0) + d_1' H(0, 1; y_0),
 \end{aligned}$$

$$\begin{aligned}
 (k*i)_{-1}^{(-1)} = & -\frac{1}{18} y_0^3 \alpha_0^4 + \frac{5y_0^2 \alpha_0^4}{12} - \frac{5y_0 \alpha_0^4}{3} + \frac{13y_0^3 \alpha_0^3}{54} - \frac{17y_0^2 \alpha_0^3}{9} + \frac{80y_0 \alpha_0^3}{9} - \frac{11y_0^3 \alpha_0^2}{36} + \frac{109y_0^2 \alpha_0^2}{36} - \\
 & \frac{158y_0 \alpha_0^2}{9} - \frac{7y_0^3 \alpha_0}{18} - \frac{7y_0^2 \alpha_0}{18} + \frac{43y_0 \alpha_0}{3} + \frac{d_1'^2 y_0^3}{27} - \frac{11d_1' y_0^3}{27} - \frac{\pi^2 y_0^3}{9} + \frac{44y_0^3}{27} - \frac{17d_1'^2 y_0^2}{72} + \frac{25d_1' y_0^2}{9} + \frac{\pi^2 y_0^2}{2} - \frac{37y_0^2}{4} + \\
 & \frac{49d_1'^2 y_0}{36} - \frac{154d_1' y_0}{9} - \pi^2 y_0 + 48y_0 + \left( \frac{7y_0^3}{6} - \frac{13y_0^2}{3} + \frac{11y_0}{2} - \frac{13}{6(y_0-1)} - \frac{13}{6} \right) H(0; \alpha_0) + \left( \frac{4d_1' y_0^3}{9} - \frac{77y_0^3}{18} - \right. \\
 & \left. \frac{7d_1' y_0^2}{3} + \frac{61y_0^2}{3} + \frac{22d_1' y_0}{3} - \frac{115y_0}{2} + \frac{13}{6(y_0-1)} + \frac{13}{6} \right) H(0; y_0) + \left( \frac{d_1'^2 y_0^3}{9} - \frac{7d_1' y_0^3}{9} - \frac{7d_1'^2 y_0^2}{12} + 4d_1' y_0^2 + \frac{11d_1'^2 y_0}{6} - \right. \\
 & \left. 13d_1' y_0 - \frac{49d_1'^2}{36} + \frac{88d_1'}{9} + \left( -\frac{2y_0^3}{3} + 3y_0^2 - 6y_0 - \frac{2}{y_0-1} + \frac{19}{3} \right) H(0; \alpha_0) - \frac{\pi^2}{3} \right) H(1; y_0) + \left( \frac{y_0^3 \alpha_0^4}{6} - \right. \\
 & \left. y_0^2 \alpha_0^4 + \frac{5y_0 \alpha_0^4}{2} - \frac{5\alpha_0^4}{3} - \frac{8y_0^3 \alpha_0^3}{9} + 5y_0^2 \alpha_0^3 - \frac{38y_0 \alpha_0^3}{3} + \frac{68\alpha_0^3}{9} + 2y_0^3 \alpha_0^2 - \frac{21y_0^2 \alpha_0^2}{2} + 26y_0 \alpha_0^2 - \frac{38\alpha_0^2}{3} - \frac{8y_0^3 \alpha_0}{3} + \right. \\
 & \left. 13y_0^2 \alpha_0 - 30y_0 \alpha_0 + \frac{41\alpha_0}{3} + \frac{25y_0^3}{18} - \frac{35y_0^2}{6} + \frac{23y_0}{2} - \frac{13}{6(y_0-1)} - \frac{13}{6} \right) H(c_1(\alpha_0); y_0) + \left( \frac{16y_0^3}{3} - 24y_0^2 + \right. \\
 & \left. 48y_0 \right) H(0, 0; y_0) + \left( \frac{4d_1' y_0^3}{3} - 6d_1' y_0^2 + 12d_1' y_0 + 4H(0; \alpha_0) \right) H(0, 1; y_0) + \left( -\alpha_0^4 + \frac{16\alpha_0^3}{3} - 12\alpha_0^2 + \right. \\
 & \left. 16\alpha_0 - \frac{2y_0^3}{3} + 3y_0^2 - 6y_0 - \frac{2}{y_0-1} - 2 \right) H(0, c_1(\alpha_0); y_0) + \left( \frac{4d_1' y_0^3}{3} + \frac{2y_0^3}{3} - 6d_1' y_0^2 - 3y_0^2 + 12d_1' y_0 + \right. \\
 & \left. 6y_0 - \frac{22d_1'}{3} + \frac{2}{y_0-1} - \frac{19}{3} \right) H(1, 0; y_0) + \left( \frac{d_1'^2 y_0^3}{3} - \frac{3d_1'^2 y_0^2}{2} + 3d_1'^2 y_0 - \frac{11d_1'^2}{6} + 2H(0; \alpha_0) \right) H(1, 1; y_0) +
 \end{aligned}$$

$$\left(-\frac{2y_0^3}{3} + 3y_0^2 - 6y_0 - \frac{2}{y_0-1} + \frac{19}{3}\right)H(1, c_1(\alpha_0); y_0) + \left(-2\alpha_0 + \frac{2}{y_0-1} + 2\right)H(c_1(\alpha_0), c_1(\alpha_0); y_0) - 16H(0, 0, 0; y_0) - 4d_1' H(0, 0, 1; y_0) + 4H(0, 0, c_1(\alpha_0); y_0) + (-4d_1' - 4)H(0, 1, 0; y_0) - d_1'^2 H(0, 1, 1; y_0) + 4H(0, 1, c_1(\alpha_0); y_0) - 2H(0, c_1(\alpha_0), c_1(\alpha_0); y_0) + 2H(1, 0, c_1(\alpha_0); y_0) - 2H(1, 1, 0; y_0) + 2H(1, 1, c_1(\alpha_0); y_0) - 2H(1, c_1(\alpha_0), c_1(\alpha_0); y_0) + \frac{\pi^2}{3(y_0-1)} - \frac{3\zeta_3}{2} + \frac{\pi^2}{3},$$

$$\begin{aligned} (k * i)_0^{(-1)} = & \frac{1}{36}d_1y_0^3\alpha_0^4 + \frac{1}{27}d_1'y_0^3\alpha_0^4 - \frac{19}{108}y_0^3\alpha_0^4 - \frac{5}{24}d_1y_0^2\alpha_0^4 - \frac{13}{36}d_1'y_0^2\alpha_0^4 + \frac{35y_0^2\alpha_0^4}{24} + \frac{5}{6}d_1y_0\alpha_0^4 + \\ & \frac{47}{18}d_1'y_0\alpha_0^4 - \frac{17y_0}{2}\alpha_0^4 - \frac{31}{324}d_1y_0^3\alpha_0^3 - \frac{29}{162}d_1'y_0^3\alpha_0^3 + \frac{263y_0^3\alpha_0^3}{324} + \frac{91}{108}d_1y_0^2\alpha_0^3 + \frac{187}{108}d_1'y_0^2\alpha_0^3 - \frac{193y_0^2\alpha_0^3}{27} - \\ & \frac{245}{54}d_1y_0\alpha_0^3 - \frac{1549}{108}d_1'y_0\alpha_0^3 + \frac{1354y_0\alpha_0^3}{27} - \frac{17}{216}d_1y_0^3\alpha_0^2 + \frac{35}{108}d_1'y_0^3\alpha_0^2 - \frac{245y_0^3\alpha_0^2}{216} - \frac{17}{27}d_1y_0^2\alpha_0^2 - \\ & \frac{707}{216}d_1'y_0^2\alpha_0^2 + \frac{2803y_0^2\alpha_0^2}{216} + \frac{895}{108}d_1y_0\alpha_0^2 + \frac{809}{27}d_1'y_0\alpha_0^2 - \frac{3013y_0\alpha_0^2}{27} + \frac{205}{108}d_1y_0^3\alpha_0 - \frac{1}{6}d_1'y_0^3\alpha_0 - \frac{58y_0^3\alpha_0}{27} - \\ & \frac{659}{108}d_1y_0^2\alpha_0 + \frac{74}{27}d_1'y_0^2\alpha_0 - \frac{55y_0^2\alpha_0}{27} - \frac{d_1y_0\alpha_0}{36} - \frac{278d_1'y_0\alpha_0}{9} + \frac{1379y_0\alpha_0}{12} - \frac{7\alpha_0}{12(y_0-1)} - \frac{7\alpha_0}{12} - \frac{d_1^3y_0^3}{81} + \\ & \frac{5d_1'^2y_0^3}{27} - \frac{28d_1'y_0^3}{27} + \frac{1}{27}d_1'\pi^2y_0^3 - \frac{49\pi^2y_0^3}{108} + \frac{268y_0^3}{81} + \frac{43d_1^3y_0^2}{432} - \frac{179d_1'^2y_0^2}{108} + \frac{1891d_1'y_0^2}{216} - \frac{7}{36}d_1'\pi^2y_0^2 + \frac{37\pi^2y_0^2}{18} - \\ & \frac{161y_0^2}{8} - \frac{251d_1^3y_0}{216} + \frac{545d_1'^2y_0}{27} - \frac{10847d_1'y_0}{108} + \frac{11}{18}d_1'\pi^2y_0 - \frac{21\pi^2y_0}{4} + 164y_0 + \left(\frac{y_0^3\alpha_0^4}{9} - \frac{5y_0^2\alpha_0^4}{6} + \frac{10y_0\alpha_0^4}{3} - \right. \\ & \left. \frac{13}{27}y_0^2\alpha_0^3 + \frac{34y_0^2\alpha_0^3}{9} - \frac{160y_0\alpha_0^3}{9} + \frac{11y_0^3\alpha_0^2}{18} - \frac{109y_0^2\alpha_0^2}{18} + \frac{316y_0\alpha_0^2}{9} + \frac{7y_0^3\alpha_0}{9} + \frac{7y_0^2\alpha_0}{9} - \frac{86y_0\alpha_0}{3} - \frac{205d_1y_0^3}{108} - \right. \\ & \left. \frac{17}{54}d_1'y_0^3 + \frac{91y_0^3}{18} + \frac{22d_1y_0^2}{3} + \frac{10}{9}d_1'y_0^2 - \frac{187y_0^2}{9} + \frac{217d_1}{36} - \frac{d_1'}{6} - \frac{469d_1y_0}{36} + \frac{7d_1'y_0}{18} + \frac{314}{9}y_0 + \frac{217d_1}{36(y_0-1)} - \right. \\ & \left. \frac{d_1'}{6(y_0-1)} - \frac{88}{9(y_0-1)} - \frac{19}{12(y_0-1)^2} - \frac{295}{36}\right)H(0; \alpha_0) + \left(\frac{1}{9}d_1y_0^3\alpha_0^4 - \frac{5}{6}d_1y_0^2\alpha_0^4 + \frac{10}{3}d_1y_0\alpha_0^4 - \frac{13}{27}d_1y_0^3\alpha_0^3 + \right. \\ & \left. \frac{34}{9}d_1y_0^2\alpha_0^3 - \frac{160}{9}d_1y_0\alpha_0^3 + \frac{11}{18}d_1y_0^3\alpha_0^2 - \frac{109}{18}d_1y_0^2\alpha_0^2 + \frac{316}{9}d_1y_0\alpha_0^2 + \frac{7}{9}d_1y_0^3\alpha_0 + \frac{7}{9}d_1y_0^2\alpha_0 - \frac{86d_1y_0\alpha_0}{3} - \right. \\ & \left. \frac{55d_1y_0^3}{54} + \frac{7d_1y_0^2}{3} + 8d_1y_0\right)H(1; \alpha_0) + \left(-\frac{1}{12}d_1y_0^3\alpha_0^4 - \frac{1}{18}d_1'y_0^3\alpha_0^4 + \frac{5y_0^3\alpha_0^4}{12} + \frac{1}{2}d_1y_0^2\alpha_0^4 + \frac{5}{12}d_1'y_0^2\alpha_0^4 - \right. \\ & \left. \frac{5y_0^2\alpha_0^4}{2} + \frac{5}{6}d_1\alpha_0^4 + \frac{47d_1'\alpha_0^4}{36} - \frac{5}{4}d_1y_0\alpha_0^4 - \frac{5}{3}d_1'y_0\alpha_0^4 + \frac{29y_0\alpha_0^4}{4} - \frac{31\alpha_0^4}{6} + \frac{13}{27}d_1y_0^3\alpha_0^3 + \frac{8}{27}d_1'y_0^3\alpha_0^3 - \right. \\ & \left. \frac{125y_0^3\alpha_0^3}{54} - \frac{8}{3}d_1y_0^2\alpha_0^3 - \frac{37}{18}d_1'y_0^2\alpha_0^3 + \frac{40y_0^2\alpha_0^3}{3} - \frac{221d_1\alpha_0^3}{54} - \frac{313d_1'\alpha_0^3}{54} + \frac{61}{9}d_1y_0\alpha_0^3 + \frac{77}{9}d_1'y_0\alpha_0^3 - \frac{379y_0\alpha_0^3}{9} + \right. \\ & \left. \frac{745\alpha_0^3}{27} - \frac{23}{18}d_1y_0^3\alpha_0^2 - \frac{2}{3}d_1'y_0^3\alpha_0^2 + \frac{203y_0^3\alpha_0^2}{36} + \frac{13}{2}d_1y_0^2\alpha_0^2 + \frac{17}{4}d_1'y_0^2\alpha_0^2 - \frac{373y_0^2\alpha_0^2}{12} + \frac{287d_1\alpha_0^2}{36} + \frac{28}{3}d_1'\alpha_0^2 - \right. \\ & \left. \frac{95}{6}d_1y_0\alpha_0^2 - \frac{35}{2}d_1'y_0\alpha_0^2 + \frac{599y_0\alpha_0^2}{6} - \frac{161\alpha_0^2}{3} + \frac{25}{9}d_1y_0^3\alpha_0 + \frac{8}{9}d_1'y_0^3\alpha_0 - \frac{169y_0^3\alpha_0}{18} - 13d_1y_0^2\alpha_0 - \right. \\ & \left. \frac{31}{6}d_1'y_0^2\alpha_0 + \frac{295y_0^2\alpha_0}{6} - \frac{343d_1\alpha_0}{18} - \frac{47d_1'\alpha_0}{6} + \frac{85d_1y_0\alpha_0}{3} + \frac{59d_1'y_0\alpha_0}{3} - \frac{440y_0\alpha_0}{3} + \frac{\alpha_0}{y_0-1} + \frac{149\alpha_0}{2} - \frac{205d_1y_0^3}{108} - \right. \\ & \left. \frac{25d_1'y_0^3}{54} + \frac{305y_0^3}{54} + \frac{22}{3}d_1y_0^2 + \frac{7d_1'y_0^2}{3} - \frac{919y_0^2}{36} + \frac{217}{36}d_1 - \frac{d_1'}{6} - \frac{469d_1y_0}{36} - 8d_1'y_0 + \frac{602}{9}y_0 + \left(-\frac{1}{3}y_0^3\alpha_0^4 + 2y_0^2\alpha_0^4 - \right. \right. \\ & \left. \left. 5y_0\alpha_0^4 + \frac{10\alpha_0^4}{3} + \frac{16y_0^3\alpha_0^3}{9} - 10y_0^2\alpha_0^3 + \frac{76y_0\alpha_0^3}{3} - \frac{136\alpha_0^3}{9} - 4y_0^3\alpha_0^2 + 21y_0^2\alpha_0^2 - 52y_0\alpha_0^2 + \frac{76\alpha_0^2}{3} + \frac{16y_0^3\alpha_0}{3} - \right. \right. \\ & \left. \left. 26y_0^2\alpha_0 + 60y_0\alpha_0 - \frac{82\alpha_0}{3} - \frac{25y_0^3}{9} + \frac{35}{3}y_0^2 - 23y_0 + \frac{13}{3(y_0-1)} + \frac{13}{3}\right)H(0; \alpha_0) + \left(-\frac{1}{3}d_1y_0^3\alpha_0^4 + 2d_1y_0^2\alpha_0^4 + \right. \\ & \left. \frac{10}{3}d_1\alpha_0^4 - 5d_1y_0\alpha_0^4 + \frac{16}{9}d_1y_0^3\alpha_0^3 - 10d_1y_0^2\alpha_0^3 - \frac{136d_1\alpha_0^3}{9} + \frac{76}{3}d_1y_0\alpha_0^3 - 4d_1y_0^3\alpha_0^2 + 21d_1y_0^2\alpha_0^2 + \right. \\ & \left. \frac{76d_1\alpha_0^2}{3} - 52d_1y_0\alpha_0^2 + \frac{16}{3}d_1y_0^3\alpha_0 - 26d_1y_0^2\alpha_0 - \frac{82d_1\alpha_0}{3} + 60d_1y_0\alpha_0 - \frac{25d_1y_0^3}{9} + \frac{35d_1y_0^2}{3} + \frac{13d_1}{3} - \right. \\ & \left. \frac{23d_1y_0}{3} + \frac{13d_1}{3(y_0-1)}\right)H(1; \alpha_0) + \frac{217d_1}{36(y_0-1)} - \frac{d_1'}{6(y_0-1)} - \frac{88}{9(y_0-1)} - \frac{19}{12(y_0-1)^2} - \frac{295}{36}\Big)H(c_1(\alpha_0); y_0) + \\ & \left(-\frac{7}{3}y_0^3 + \frac{26y_0^2}{3} - 11y_0 + \frac{13}{3(y_0-1)} + \frac{13}{3}\right)H(0, 0; \alpha_0) + \left(-\frac{16d_1'y_0^3}{9} + \frac{175}{9}y_0^3 + \frac{28d_1'y_0^2}{3} - 90y_0^2 - \frac{88d_1'y_0}{3} + \right. \\ & \left. 241y_0 - \frac{13}{y_0-1} - 13\right)H(0, 0; y_0) + \left(-\frac{7d_1y_0^3}{3} + \frac{26d_1y_0^2}{3} - 11d_1y_0 + \frac{13d_1}{3} + \frac{13}{3}d_1\right)H(0, 1; \alpha_0) + \left(-\right. \\ & \left. \frac{4}{9}d_1'^2y_0^3 + \frac{77d_1'y_0^3}{18} + \frac{7d_1'^2y_0^2}{3} - \frac{61d_1'y_0^2}{3} - \frac{22d_1'^2y_0}{3} + \frac{115d_1'y_0}{2} - \frac{13}{6}d_1' + \left(-\frac{4d_1y_0^3}{3} + \frac{4y_0^3}{3} + 6d_1y_0^2 - 6y_0^2 - \right. \right. \\ & \left. \left. 12d_1y_0 + 12y_0 + \frac{38d_1}{3} - \frac{4}{y_0-1} + \frac{4}{y_0-1} + \frac{62}{3}\right)H(0; \alpha_0) - 8H(0, 0; \alpha_0) - 8d_1H(0, 1; \alpha_0) - \frac{13d_1'}{6(y_0-1)} - \right. \\ & \left. \frac{2d_1\pi^2}{3}\right)H(0, 1; y_0) + \left(\frac{d_1\alpha_0^4}{2} - \frac{\alpha_0^4}{2} - \frac{26d_1\alpha_0^3}{9} + \frac{38\alpha_0^3}{9} + \frac{23d_1\alpha_0^2}{3} - \frac{49\alpha_0^2}{3} - \frac{50d_1\alpha_0}{3} + \frac{142\alpha_0}{3} + \frac{2d_1'y_0^3}{9} - \right. \\ & \left. \frac{49y_0^3}{18} - \frac{7d_1'y_0^2}{6} + \frac{37}{3}y_0^2 + 4d_1 - 2d_1' + \frac{11d_1'y_0}{3} - \frac{63y_0}{2} + \left(2\alpha_0^4 - \frac{32\alpha_0^3}{3} + 24\alpha_0^2 - 32\alpha_0 + \frac{4y_0^3}{3} - 6y_0^2 + 12y_0 + \right. \right. \\ & \left. \left. \frac{4}{y_0-1} + 4\right)H(0; \alpha_0) + \left(2d_1\alpha_0^4 - \frac{32d_1\alpha_0^3}{3} + 24d_1\alpha_0^2 - 32d_1\alpha_0 + \frac{4d_1y_0^3}{3} - 6d_1y_0^2 + 4d_1 + 12d_1y_0 + \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left. \frac{4d_1}{y_0-1} \right) H(1; \alpha_0) + \frac{4d_1}{y_0-1} - \frac{2d'_1}{y_0-1} - \frac{11}{6(y_0-1)} - \frac{1}{(y_0-1)^2} - \frac{5}{6} \Big) H(0, c_1(\alpha_0); y_0) + \left( -\frac{4}{9}d_1^2 y_0^3 + \frac{73d'_1 y_0^3}{18} + \right. \\
& \frac{49y_0^3}{18} + \frac{7d_1^2 y_0^2}{3} - \frac{4d_1 y_0^2}{3} - \frac{115d'_1 y_0^2}{6} - \frac{41y_0^2}{3} - \frac{22d_1^2 y_0}{3} + \frac{16d_1 y_0}{3} + \frac{323d'_1 y_0}{6} + \frac{221 y_0}{6} + \frac{49d_1^2}{9} - \frac{37d_1}{18} - \\
& \frac{697 d'_1}{18} + \left. \left( \frac{4y_0^3}{3} - 6y_0^2 + 12 y_0 + \frac{4}{y_0-1} - \frac{38}{3} \right) H(0; \alpha_0) + \frac{d_1}{3(y_0-1)} - \frac{d'_1}{6(y_0-1)} + \frac{37}{6(y_0-1)} + \frac{1}{(y_0-1)^2} + \frac{4\pi^2}{3} - \right. \\
& \left. \frac{130}{3} \right) H(1, 0; y_0) + \left( -\frac{1}{9}y_0^3 d_1^3 + \frac{7y_0^2 d_1^3}{12} - \frac{11y_0 d_1^3}{6} + \frac{49d_1^3}{36} + \frac{7 y_0^3 d_1^2}{9} - 4y_0^2 d_1^2 + 13y_0 d_1^2 - \frac{88 d_1^2}{9} + \right. \\
& \left. \frac{\pi^2 d_1}{3} + \left( -\frac{4d_1 y_0^3}{3} + \frac{4 d'_1 y_0^3}{3} - \frac{2y_0^3}{3} + 6d_1 y_0^2 - 6d'_1 y_0^2 + 3y_0^2 - 12 d_1 y_0 + 12d'_1 y_0 - 6y_0 + \frac{38d_1}{3} - 10d'_1 - \right. \right. \\
& \left. \left. \frac{8 d_1}{y_0-1} + \frac{4d'_1}{y_0-1} + \frac{2}{y_0-1} + \frac{43}{3} \right) H(0; \alpha_0) - 4H(0, 0; \alpha_0) - 4d_1 H(0, 1; \alpha_0) - \frac{2d_1 \pi^2}{3} + \frac{\pi^2}{3} \right) H(1, 1; y_0) + \\
& \left( -\frac{1}{6}d_1^2 y_0^3 \alpha_0^4 + d_1^2 y_0^2 \alpha_0^4 + \frac{5d_1^2 \alpha_0^4}{3} - \frac{5}{2}d_1^2 y_0 \alpha_0^4 + \frac{8}{9}d_1^2 y_0^3 \alpha_0^3 - 5d_1^2 y_0^2 \alpha_0^3 - \frac{77d_1^2 \alpha_0^3}{9} + \frac{38}{3}d_1^2 y_0 \alpha_0^3 - \right. \\
& \left. 2d_1^2 y_0^3 \alpha_0^2 + \frac{21}{2} d_1^2 y_0^2 \alpha_0^2 + \frac{35d_1^2 \alpha_0^2}{2} - 26d_1^2 y_0 \alpha_0^2 + \frac{8}{3}d_1^2 y_0^3 \alpha_0 - 13d_1^2 y_0^2 \alpha_0 - \frac{65d_1^2 \alpha_0}{3} + 30d_1^2 y_0 \alpha_0 - \frac{7d_1^2 y_0^3}{6} - \right. \\
& \left. \frac{49 y_0^3}{18} + \frac{4d_1 y_0^2}{3} + \frac{14d_1 y_0^2}{3} + \frac{41 y_0^2}{3} + \frac{37d_1}{18} + \frac{13d'_1}{3} - \frac{16d_1 y_0}{3} - \frac{47d'_1 y_0}{6} - \frac{221y_0}{6} + \left( \frac{4 y_0^3}{3} - 6y_0^2 + 12y_0 + \frac{4}{y_0-1} - \right. \right. \\
& \left. \left. \frac{38}{3} \right) H(0; \alpha_0) + \left( \frac{4d_1 y_0^3}{3} - 6d_1 y_0^2 + 12d_1 y_0 - \frac{38}{3} \frac{d_1}{y_0-1} + \frac{4d_1}{y_0-1} \right) H(1; \alpha_0) - \frac{d_1}{3(y_0-1)} + \frac{d'_1}{6(y_0-1)} - \frac{37}{6(y_0-1)} - \right. \\
& \left. \frac{1}{(y_0-1)^2} + \frac{130}{3} \right) H(1, c_1(\alpha_0); y_0) + \left( -\frac{1}{3}y_0^3 \alpha_0^4 + 2y_0^2 \alpha_0^4 - 5y_0 \alpha_0^4 + \frac{10\alpha_0^4}{3} + \frac{16y_0^3 \alpha_0^3}{9} - 10y_0^2 \alpha_0^3 + \frac{76y_0 \alpha_0^3}{3} - \right. \\
& \left. \frac{136\alpha_0^3}{9} - 4y_0^3 \alpha_0^2 + 21 y_0^2 \alpha_0^2 - 52y_0 \alpha_0^2 + \frac{76\alpha_0^2}{3} + \frac{16y_0^3 \alpha_0}{3} - 26 y_0^2 \alpha_0 + 60y_0 \alpha_0 - \frac{82\alpha_0}{3} - \frac{25y_0^3}{9} + \frac{35 y_0^2}{3} - \right. \\
& \left. 23y_0 + \frac{13}{3(y_0-1)} + \frac{13}{3} \right) H(c_1(\alpha_0), 0; y_0) + \left( -\frac{1}{6}d_1^2 y_0^3 \alpha_0^4 + d_1^2 y_0^2 \alpha_0^4 + \frac{5d_1^2 \alpha_0^4}{3} - \frac{5}{2}d_1^2 y_0 \alpha_0^4 + \frac{8}{9}d_1^2 y_0^3 \alpha_0^3 - \right. \\
& \left. 5 d_1^2 y_0^2 \alpha_0^3 - \frac{68d_1^2 \alpha_0^3}{9} + \frac{38}{3}d_1^2 y_0 \alpha_0^3 - 2d_1^2 y_0^3 \alpha_0^2 + \frac{21}{2}d_1^2 y_0^2 \alpha_0^2 + \frac{38d_1^2 \alpha_0^2}{3} - 26d_1^2 y_0 \alpha_0^2 + \frac{8}{3}d_1^2 y_0^3 \alpha_0 - \right. \\
& \left. 13d_1^2 y_0^2 \alpha_0 - \frac{41d_1^2 \alpha_0}{3} + 30d_1^2 y_0 \alpha_0 - \frac{25d_1^2 y_0^3}{18} + \frac{35d_1^2 y_0^2}{6} + \frac{13d_1^2}{6} - \frac{23d_1^2 y_0}{2} + \frac{13d_1^2}{6(y_0-1)} \right) H(c_1(\alpha_0), 1; y_0) + \\
& \left( -\frac{1}{2}y_0^3 \alpha_0^4 + 3y_0^2 \alpha_0^4 - \frac{15y_0 \alpha_0^4}{2} + 5\alpha_0^4 + \frac{8y_0^3 \alpha_0^3}{3} - 15y_0^2 \alpha_0^3 + 38y_0 \alpha_0^3 - \frac{68\alpha_0^3}{3} - 6y_0^3 \alpha_0^2 + \frac{63y_0^2 \alpha_0^2}{2} - 78y_0 \alpha_0^2 + \right. \\
& \left. 37\alpha_0^2 + 8y_0^3 \alpha_0 - 39y_0^2 \alpha_0 + 4d_1 \alpha_0 - 2 d'_1 \alpha_0 + 90y_0 \alpha_0 - 43\alpha_0 - \frac{25y_0^3}{6} + \frac{35y_0^2}{2} - 4 d_1 + 2d'_1 - \frac{69y_0}{2} + \right. \\
& \left. \left( 4\alpha_0 - \frac{4}{y_0-1} - 4 \right) H(0; \alpha_0) + \left( 4\alpha_0 d_1 - \frac{4d_1}{y_0-1} - 4d_1 \right) H(1; \alpha_0) - \frac{4d_1}{y_0-1} + \frac{2d'_1}{y_0-1} + \frac{21}{2(y_0-1)} + \frac{1}{(y_0-1)^2} + \right. \\
& \left. \frac{19}{2} \right) H(c_1(\alpha_0), c_1(\alpha_0); y_0) + \left( -\frac{64y_0^3}{3} + 96y_0^2 - 192 y_0 \right) H(0, 0, 0; y_0) + \left( -\frac{16d_1^2 y_0^3}{3} + 24d_1^2 y_0^2 - \right. \\
& \left. 48 d_1^2 y_0 + (8d_1 - 8)H(0; \alpha_0) \right) H(0, 0, 1; y_0) + \left( \frac{8 y_0^3}{3} - 12y_0^2 + 24y_0 - 8H(0; \alpha_0) - 8d_1 H(1; \alpha_0) + \right. \\
& \left. \frac{8}{y_0-1} + 8 \right) H(0, 0, c_1(\alpha_0); y_0) + \left( \frac{4d_1 y_0^3}{3} - \frac{16d_1^2 y_0^3}{3} - \frac{4y_0^3}{3} - 6d_1 y_0^2 + 24d_1^2 y_0^2 + 6y_0^2 + 12d_1 y_0 - 48d_1^2 y_0 - \right. \\
& \left. 12y_0 - \frac{38d_1}{3} - 8H(0; \alpha_0) + \frac{4 d_1}{y_0-1} - \frac{4}{y_0-1} - \frac{62}{3} \right) H(0, 1, 0; y_0) + \left( -\frac{4}{3}d_1^2 y_0^3 + 6d_1^2 y_0^2 - 12d_1^2 y_0 + \right. \\
& \left. (12d_1 - 8d'_1)H(0; \alpha_0) \right) H(0, 1, 1; y_0) + \left( d_1^2 \alpha_0^4 - \frac{16d_1^2 \alpha_0^3}{3} + 12d_1^2 \alpha_0^2 - 16d_1^2 \alpha_0 - \frac{4d_1 y_0^3}{3} + \frac{2d_1^2 y_0^3}{3} + \right. \\
& \left. \frac{4y_0^3}{3} + 6d_1 y_0^2 - 3d_1^2 y_0^2 - 6y_0^2 + \frac{38d_1}{3} + 2d'_1 - 12d_1 y_0 + 6d_1^2 y_0 + 12y_0 - 8 H(0; \alpha_0) - 8d_1 H(1; \alpha_0) - \right. \\
& \left. \frac{4d_1}{y_0-1} + \frac{2 d'_1}{y_0-1} + \frac{4}{y_0-1} + \frac{62}{3} \right) H(0, 1, c_1(\alpha_0); y_0) + \left( 2\alpha_0^4 - \frac{32\alpha_0^3}{3} + 24\alpha_0^2 - 32 \alpha_0 + \frac{4y_0^3}{3} - 6y_0^2 + 12y_0 + \right. \\
& \left. \frac{4}{y_0-1} + 4 \right) H(0, c_1(\alpha_0), 0; y_0) + \left( d_1^2 \alpha_0^4 - \frac{16d_1^2 \alpha_0^3}{3} + 12d_1^2 \alpha_0^2 - 16d_1^2 \alpha_0 + \frac{2d_1^2 y_0^3}{3} - 3d_1^2 y_0^2 + 2d_1^2 + \right. \\
& \left. 6d_1^2 y_0 + \frac{2d_1^2}{y_0-1} \right) H(0, c_1(\alpha_0), 1; y_0) + \left( 3\alpha_0^4 - 16 \alpha_0^3 + 36\alpha_0^2 - 48\alpha_0 + 2y_0^3 - 9y_0^2 + 18y_0 + 4H(0; \alpha_0) + \right. \\
& \left. 4d_1 H(1; \alpha_0) - \frac{2}{y_0-1} - 2 \right) H(0, c_1(\alpha_0), c_1(\alpha_0); y_0) + \left( -\frac{16d_1^2 y_0^3}{3} - 4y_0^3 + 24 d_1^2 y_0^2 + 18y_0^2 - 48d_1^2 y_0 - \right. \\
& \left. 36y_0 + \frac{88 d'_1}{3} - \frac{12}{y_0-1} + 38 \right) H(1, 0, 0; y_0) + \left( -\frac{4}{3} d_1^2 y_0^3 - \frac{2d_1^2 y_0^3}{3} + 6d_1^2 y_0^2 + 3d_1^2 y_0^2 - 12 d_1^2 y_0 - 6d_1^2 y_0 + \right. \\
& \left. \frac{22d_1^2}{3} + \frac{19d_1^2}{3} + (4d_1 - 4) H(0; \alpha_0) - \frac{2d_1^2}{y_0-1} \right) H(1, 0, 1; y_0) + \left( \frac{2d_1^2 y_0^3}{3} + \frac{2y_0^3}{3} - 3d_1^2 y_0^2 - 3y_0^2 + 6d_1^2 y_0 + \right. \\
& \left. 6 y_0 - \frac{11d_1^2}{3} - 4H(0; \alpha_0) - 4d_1 H(1; \alpha_0) - \frac{4 d_1}{y_0-1} + \frac{2d_1^2}{y_0-1} + \frac{6}{y_0-1} + \frac{5}{3} \right) H(1, 0, c_1(\alpha_0); y_0) + \left( - \right. \\
& \left. \frac{4}{3}d_1^2 y_0^3 + \frac{4d_1 y_0^3}{3} - \frac{4d_1^2 y_0^3}{3} + \frac{2y_0^3}{3} + 6d_1^2 y_0^2 - 6 d_1 y_0^2 + 6d_1^2 y_0^2 - 3y_0^2 - 12d_1^2 y_0 + 12d_1 y_0 - 12d_1^2 y_0 + 6 y_0 + \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{22d_1^2}{3} - \frac{38d_1}{3} + 10d_1' - 4H(0; \alpha_0) + \frac{8d_1}{y_0-1} - \frac{4d_1'}{y_0-1} - \frac{2}{y_0-1} - \frac{43}{3} \right) H(1, 1, 0; y_0) + \left( -\frac{1}{3}y_0^3 d_1'^3 + \frac{3y_0^2 d_1'^3}{2} - \right. \\
& \left. 3y_0 d_1'^3 + \frac{11d_1'^3}{6} + (8d_1 - 4d_1' - 2)H(0; \alpha_0) \right) H(1, 1, 1; y_0) + \left( -\frac{4d_1 y_0^3}{3} + \frac{4d_1' y_0^3}{3} - \frac{2y_0^3}{3} + 6d_1 y_0^2 - 6d_1' y_0^2 + \right. \\
& \left. 3y_0^2 - 12d_1 y_0 + 12d_1' y_0 - 6y_0 + \frac{38d_1}{3} - 10d_1' - 4H(0; \alpha_0) - 4d_1 H(1; \alpha_0) - \frac{8d_1}{y_0-1} + \frac{4d_1'}{y_0-1} + \frac{2}{y_0-1} + \right. \\
& \left. \frac{43}{3} \right) H(1, 1, c_1(\alpha_0); y_0) + \left( \frac{4y_0^3}{3} - 6y_0^2 + 12y_0 + \frac{4}{y_0-1} - \frac{38}{3} \right) H(1, c_1(\alpha_0), 0; y_0) + \left( \frac{2d_1 y_0^3}{3} - 3d_1' y_0^2 + \right. \\
& \left. 6d_1' y_0 - \frac{19d_1'}{3} + \frac{2d_1'}{y_0-1} \right) H(1, c_1(\alpha_0), 1; y_0) + \left( 2y_0^3 - 9y_0^2 + 18y_0 + 4H(0; \alpha_0) + 4d_1 H(1; \alpha_0) + \frac{4d_1}{y_0-1} - \right. \\
& \left. \frac{2d_1'}{y_0-1} + \frac{2}{y_0-1} - 27 \right) H(1, c_1(\alpha_0), c_1(\alpha_0); y_0) + \left( 2\alpha_0 d_1' - \frac{2d_1'}{y_0-1} - 2d_1' \right) H(c_1(\alpha_0), 1, c_1(\alpha_0); y_0) + \\
& \left( 4\alpha_0 - \frac{4}{y_0-1} - 4 \right) H(c_1(\alpha_0), c_1(\alpha_0), 0; y_0) + \left( 2\alpha_0 d_1' - \frac{2d_1'}{y_0-1} - 2d_1' \right) H(c_1(\alpha_0), c_1(\alpha_0), 1; y_0) + \\
& \left( 6\alpha_0 - \frac{6}{y_0-1} - 6 \right) H(c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); y_0) + 64H(0, 0, 0, 0; y_0) + 16d_1' H(0, 0, 0, 1; y_0) - \\
& 16H(0, 0, 0, c_1(\alpha_0); y_0) + (-8d_1 + 16d_1' + 8)H(0, 0, 1, 0; y_0) + 4d_1'^2 H(0, 0, 1, 1; y_0) + (8d_1 - \\
& 4d_1' - 8)H(0, 0, 1, c_1(\alpha_0); y_0) - 8H(0, 0, c_1(\alpha_0), 0; y_0) - 4d_1' H(0, 0, c_1(\alpha_0), 1; y_0) - \\
& 4H(0, 0, c_1(\alpha_0), c_1(\alpha_0); y_0) + (16d_1' + 24)H(0, 1, 0, 0; y_0) + \left( 4d_1'^2 + 4d_1' \right) H(0, 1, 0, 1; y_0) + \\
& (4d_1 - 4d_1' - 8)H(0, 1, 0, c_1(\alpha_0); y_0) + \left( 4d_1'^2 + 8d_1' - 12d_1 \right) H(0, 1, 1, 0; y_0) + \\
& d_1'^3 H(0, 1, 1, 1; y_0) + (12d_1 - 8d_1')H(0, 1, 1, c_1(\alpha_0); y_0) - 8H(0, 1, c_1(\alpha_0), 0; y_0) - \\
& 4d_1' H(0, 1, c_1(\alpha_0), 1; y_0) + (-4d_1 + 2d_1' - 8)H(0, 1, c_1(\alpha_0), c_1(\alpha_0); y_0) + \\
& 2d_1' H(0, c_1(\alpha_0), 1, c_1(\alpha_0); y_0) + 4H(0, c_1(\alpha_0), c_1(\alpha_0), 0; y_0) + 2d_1' H(0, c_1(\alpha_0), c_1(\alpha_0), 1; y_0) + \\
& 6H(0, c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); y_0) - 8H(1, 0, 0, c_1(\alpha_0); y_0) + (4 - 4d_1)H(1, 0, 1, 0; y_0) + \\
& (4d_1 - 2d_1' - 4)H(1, 0, 1, c_1(\alpha_0); y_0) - 4H(1, 0, c_1(\alpha_0), 0; y_0) - 2d_1' H(1, 0, c_1(\alpha_0), 1; y_0) + \\
& 2H(1, 0, c_1(\alpha_0), c_1(\alpha_0); y_0) + 12H(1, 1, 0, 0; y_0) + 2d_1' H(1, 1, 0, 1; y_0) + (4d_1 - 2d_1' - \\
& 6)H(1, 1, 0, c_1(\alpha_0); y_0) + (-8d_1 + 4d_1' + 2)H(1, 1, 1, 0; y_0) + (8d_1 - 4d_1' - \\
& 2)H(1, 1, 1, c_1(\alpha_0); y_0) - 4H(1, 1, c_1(\alpha_0), 0; y_0) - 2d_1' H(1, 1, c_1(\alpha_0), 1; y_0) + (-4d_1 + 2d_1' - \\
& 2)H(1, 1, c_1(\alpha_0), c_1(\alpha_0); y_0) + 2d_1' H(1, c_1(\alpha_0), 1, c_1(\alpha_0); y_0) + 4H(1, c_1(\alpha_0), c_1(\alpha_0), 0; y_0) + \\
& 2d_1' H(1, c_1(\alpha_0), c_1(\alpha_0), 1; y_0) + 6H(1, c_1(\alpha_0), c_1(\alpha_0), c_1(\alpha_0); y_0) + H(1; y_0) \left( \frac{1}{18} d_1' y_0^3 \alpha_0^4 - \right. \\
& \left. \frac{5}{12} d_1' y_0^2 \alpha_0^4 - \frac{47d_1' \alpha_0^4}{36} + \frac{5}{3} d_1' y_0 \alpha_0^4 - \frac{13}{54} d_1' y_0^3 \alpha_0^3 + \frac{17}{9} d_1' y_0^2 \alpha_0^3 + \frac{391d_1' \alpha_0^3}{54} - \frac{80}{9} d_1' y_0 \alpha_0^3 + \frac{11}{36} d_1' y_0^3 \alpha_0^2 - \right. \\
& \left. \frac{109}{36} d_1' y_0^2 \alpha_0^2 - \frac{89d_1' \alpha_0^2}{6} + \frac{158}{9} d_1' y_0 \alpha_0^2 + \frac{7}{18} d_1' y_0^3 \alpha_0 + \frac{7}{18} d_1' y_0^2 \alpha_0 + \frac{247d_1' \alpha_0}{18} - \frac{43d_1' y_0 \alpha_0}{3} + \frac{251}{216} d_1'^3 - \right. \\
& \left. \frac{d_1'^3 y_0^3}{27} + \frac{11d_1'^2 y_0^3}{27} - \frac{44d_1' y_0^3}{27} + \frac{1}{9} d_1' \pi^2 y_0^3 + \frac{\pi^2 y_0^3}{9} - \frac{398d_1'^2}{27} + \frac{17d_1'^3 y_0^2}{72} - \frac{25d_1'^2 y_0^2}{9} + \frac{37d_1' y_0^2}{4} - \frac{1}{2} d_1' \pi^2 y_0^2 - \right. \\
& \left. \frac{\pi^2 y_0^2}{2} + \frac{4361d_1'}{108} - \frac{49d_1'^3 y_0}{36} + \frac{154}{9} d_1'^2 y_0 - 48d_1' y_0 + d_1' \pi^2 y_0 + \pi^2 y_0 + \left( -\frac{17}{18} d_1' y_0^3 - \frac{49y_0^3}{18} + \frac{4d_1 y_0^2}{3} + \right. \right. \\
& \left. \left. \frac{19}{6} d_1' y_0^2 + \frac{41y_0^2}{3} - \frac{16d_1 y_0}{3} - \frac{11d_1' y_0}{6} - \frac{221y_0}{6} + \frac{37d_1}{18} - \frac{7d_1'}{18} - \frac{d_1}{3(y_0-1)} + \frac{d_1'}{6(y_0-1)} - \frac{37}{6(y_0-1)} - \frac{1}{(y_0-1)^2} + \right. \right. \\
& \left. \left. \frac{130}{3} \right) H(0; \alpha_0) + \left( \frac{4}{3} y_0^3 - 6y_0^2 + 12y_0 + \frac{4}{y_0-1} - \frac{38}{3} \right) H(0, 0; \alpha_0) + \left( \frac{4d_1 y_0^3}{3} - 6d_1 y_0^2 + 12d_1 y_0 - \frac{38}{3} d_1 + \right. \right. \\
& \left. \left. \frac{4d_1}{y_0-1} \right) H(0, 1; \alpha_0) + \frac{2d_1 \pi^2}{3(y_0-1)} - \frac{d_1' \pi^2}{3(y_0-1)} - \frac{\pi^2}{3(y_0-1)} - 6\zeta_3 - \frac{11d_1' \pi^2}{18} - \frac{43\pi^2}{18} \right) + H(0; y_0) \left( \frac{y_0^3 \alpha_0^4}{9} - \right. \\
& \left. \frac{5y_0^2 \alpha_0^4}{6} + \frac{10y_0 \alpha_0^4}{3} - \frac{13y_0^3 \alpha_0^3}{27} + \frac{34y_0^2 \alpha_0^3}{9} - \frac{160y_0 \alpha_0^3}{9} + \frac{11y_0^3 \alpha_0^2}{18} - \frac{109 y_0^2 \alpha_0^2}{18} + \frac{316y_0 \alpha_0^2}{9} + \frac{7y_0^3 \alpha_0}{9} + \frac{7y_0^2 \alpha_0}{9} - \right. \\
& \left. \frac{86y_0 \alpha_0}{3} - \frac{4d_1'^2 y_0^3}{27} + \frac{205d_1' y_0^3}{108} + \frac{35d_1' y_0^3}{18} + \frac{4\pi^2 y_0^3}{9} - \frac{625y_0^3}{54} + \frac{17d_1'^2 y_0^2}{18} - \frac{22d_1' y_0^2}{3} - \frac{110d_1' y_0^2}{9} - 2\pi^2 y_0^2 + \frac{520y_0^2}{9} - \right. \\
& \left. \frac{217d_1}{36} + \frac{d_1'}{6} - \frac{49}{9} d_1'^2 y_0 + \frac{469d_1 y_0}{36} + \frac{1225d_1' y_0}{18} + 4\pi^2 y_0 - \frac{2042y_0}{9} + \left( -\frac{7y_0^3}{3} + \frac{26}{3} y_0^2 - 11y_0 + \frac{13}{3(y_0-1)} + \right. \right. \\
& \left. \left. \frac{13}{3} \right) H(0; \alpha_0) - \frac{217d_1}{36(y_0-1)} + \frac{d_1'}{6(y_0-1)} - \frac{4\pi^2}{3(y_0-1)} + \frac{88}{9(y_0-1)} + \frac{19}{12(y_0-1)^2} + 6\zeta_3 - \frac{4\pi^2}{3} + \frac{295}{36} \right) - \frac{2d_1 \pi^2}{3(y_0-1)} + \\
& \frac{d_1' \pi^2}{3(y_0-1)} + \frac{37\pi^2}{36(y_0-1)} + \frac{\pi^2}{6(y_0-1)^2} - 4y_0^3 \zeta_3 + 18y_0^2 \zeta_3 - 36y_0 \zeta_3 + \frac{6\zeta_3}{y_0-1} + 6\zeta_3 - \frac{\pi^4}{120} - \frac{2d_1 \pi^2}{3} + \frac{d_1' \pi^2}{3} + \frac{31\pi^2}{36} .
\end{aligned}$$

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