

MR1895718 (2003m:35111) [35K55](#) ([35K65](#) [76A20](#) [76D08](#))

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A waiting time phenomenon for thin film equations. (English summary)

Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **30** (2001), no. 2, 437–463.

Summary: “We prove the occurrence of a waiting time phenomenon for solutions to fourth-order degenerate parabolic differential equations which model the evolution of thin films of viscous fluids. In space dimension less than or equal to three, we identify a general criterion on the growth of initial data near the free boundary which guarantees that for sufficiently small times the support of strong solutions locally does not increase. It turns out that this condition depends only on the smoothness of the diffusion coefficient in its points of degeneracy. Our approach combines a new version of Stampacchia’s iteration lemma with weighted energy or entropy estimates. Based on numerical experiments, we conjecture that the aforementioned growth criterion is optimal.”

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