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ScienceDirect

Procedia Structural Integrity 44 (2023) 1688–1695

Structural Integrity Procedia

www.elsevier.com/locate/procedia

XIX ANIDIS Conference, Seismic Engineering in Italy

Machine-learning-enhanced variable-angle truss model to predict the shear capacity of RC elements with transverse reinforcement

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Abstract

This contribution presents a numerical model for the shear capacity prediction of reinforced concrete (RC) elements with transverse reinforcement. The proposed model originates from one of the most popular mechanical models adopted in building codes, namely the variable-angle truss model. Starting from the formulation proposed in the Eurocode 2, two empirical coefficients governing the concrete contribution (i.e., the shear capacity ascribed to crushing of compressed struts) are adjusted and enriched through machine learning, in such a way to improve the predictive efficiency of the model against experimental results. More specifically, genetic programming is used to derive closed-form expressions of the two corrective coefficients, thus facilitating the use of this model for practical purposes. The proposed expressions are validated by comparison with a wide set of experimental results collected from the literature concerning RC beams and columns failing in shear under both monotonic and cyclic loading conditions, respectively. It is demonstrated that the proposed formulation, thanks to the two novel corrective coefficients, not only attains higher accuracy than the original Eurocode 2 formulation, but also outperforms many other existing design code provisions while preserving a sound mechanical basis.

Keywords: Reinforced concrete beams; Reinforced concrete columns; Design code; Genetic programming; Machine learning; Reinforced concrete; Shear capacity; Variable-angle truss model; Eurocode.

1. Introduction

The prediction of the shear capacity of reinforced concrete (RC) elements with transverse reinforcement is a critical topic to which several studies have been devoted over the last four decades (ASCE-ACI Committee 445, 1998). This is motivated by the fact that existing RC structures are often provided with transverse reinforcement much lower than

10.1016/j.prostr.2023.01.216

that recommended in current design codes, and thus frequently exhibit a shear-dominated failure. In order to perform an accurate vulnerability assessment of such structures, the development of reliable, unbiased, and precise numerical formulations capable of predicting the actual shear strength of RC elements with stirrups is of utmost importance. Some formulations proposed in the past years within international codes were found to be overly conservative compared to experimental findings and are often characterized by excessive dispersion (Cladera and Marí 2007; De Domenico and Ricciardi 2020). Recently, the use of machine learning techniques has been also exploited to obtain more accurate pure data-driven predictions of the shear capacity (Azadi Kakavand 2021; Quaranta et al. 2020).

Instead of adopting a pure data-driven approach to derive new empirical capacity equations as proposed by ongoing studies (Fiore et al. 2016; Feng et al. 2021), the strategy devised in this contribution employs machine learning tools for enhancing one of the most popular mechanical models adopted in international codes for the shear capacity prediction of RC elements with stirrups, i.e., the variable-angle truss model (European Committee for Standardization 2004). Specifically, genetic programming is used to calibrate two coefficients ruling the concrete contribution in such a capacity model to increase its accuracy (Quaranta et al. 2022). In this way, the mechanical basis of the resisting mechanism is preserved, but the correctness of the final predictions is improved thanks to such hybridization. The effectiveness and potentials of the proposed formulation are demonstrated by comparison with experimental results collected from a large database including RC beams and columns failing in shear under monotonic and cyclic loading, respectively. A comparative analysis is also made for a large set of expressions from reference building codes to show that the proposed unified shear capacity equation leads to more accurate outcomes and, ultimately, to prove that it is suitable for practical design applications.

2. Brief review of code-conforming shear capacity equations

The first truss model was proposed by Ritter and Mörsch in the early 1900s. Considering the conservativeness later observed in the predictions of this model, two modifications were developed over the years to improve the accuracy: 1) the additive approach, wherein the truss contribution, with compression struts inclined at 45°, is accompanied by an additional concrete contribution (generally having empirical nature) (e.g., ACI 318 Building Code 2019 and pre-standard version of the Eurocode 2 1991); 2) the variable-angle truss model, in which the compression diagonals are inclined of angles generally less than 45° (e.g., Model Code 90 1993, the Eurocode 2 2004 and other national building codes in Germany and Italy). The "variable" inclination is introduced to inherently account for some physical phenomena occurring during shear failure of RC members with stirrups, such as aggregate interlock, dowel forces and residual tensile stress, that indeed produce a strut rotation crossing adjacent cracks (Walraven et al. 2013).

All these mechanical models are based on pure equilibrium conditions (and the theory of plasticity) without any explicit consideration for compatibility conditions. An alternative group of mechanics-based models determines the strut inclination angle by incorporating compatibility equations and material constitutive relationships in addition to equilibrium equations, e.g., the modified compression field theory (MCFT) (Vecchio and Collins 1986; Bentz et al. 2006). The MCFT inspired the development of simplified code expressions incorporated into the AASHTO standards (2012) and the Canadian Building Code A23.3-04 (2004). Finally, the Model Code 2010 (2013) presents various levels of approximation, including a fixed-angle truss model (in which the compression diagonals inclination is less than 45°) without concrete contribution and an additive approach with concrete contribution calculated through compatibility conditions, similarly to the simplified MCFT (Sigrist et al. 2013).

3. Improvement of the variable-angle truss model via machine learning

An overview of the existing formulations from various technical codes for practice highlights that the shear capacity of RC members is based on different mechanical models, involving various sets of parameters governing the overall resisting mechanism. Evidently, the accuracy of these models strongly relies on the way such parameters are computed. In almost all models, the concrete contribution is governed by a resisting mechanism, but some underlying parameters have an empirical (or partly empirical) basis and were calibrated upon experimental data.

Several models from the literature adopted a pure data-driven methodology, in which the adjective "pure" refers to the fact that the resulting capacity equation was not correlated to any resisting mechanism, but it was uniquely based on data (e.g., Mansour et al. 2004; Naderpour and Mirrashid 2020). This approach would likely lead to more accurate predictions; however, a shear capacity model that is based on mechanical principles is also a desired objective because, in such a model, the governing coefficients have a clear mechanical role, and it is possible to extend the model for other cases not originally considered in the calibration procedure. Based on this motivation, a hybrid novel approach has is proposed, in which machine learning does not replace a mechanics-based model but is exploited to improve the predictive accuracy of the capacity equations through the data-driven definition of some corrective parameters. The adopted "gray-box" framework is, therefore, quite different from alternative white box (i.e., fully mechanics-based) and black box (i.e., pure data-driven) modeling techniques proposed in the literature.

The starting point of the proposed model is the Eurocode 2 variable-angle truss model, which is enriched and improved by two corrective coefficients whose expressions are identified by means of genetic programming (GP), to better match experimental data. The governing equations of the proposed variable-angle truss model with machine-learning-calibrated coefficients are expressed as follows:

$$V = min\{V_{Rs}, V_{Rc}\},\ \ V_{Rs} = \frac{A_{sw}}{s} z f_{ysw} cot\theta,\ \ V_{Rc} = \eta_c b_w z \eta f_c \frac{cot\theta}{1+cot^2\theta} \tag{1}$$

where V_{Rs} and V_{Rc} are the shear capacity ascribed to two failure mechanisms, namely the yielding of the steel transverse reinforcement and the crushing of the concrete struts, respectively, A_{sw} is the cross-sectional area of the transverse reinforcement, s is the spacing of the stirrups, z is the inner lever arm (with $z = 0.9d$, d being the effective depth of the cross section), f_{ysw} is the yield strength of the shear reinforcement, θ is the angle between the concrete compression struts and the longitudinal axis of the RC member, b_w is the (minimum) width of the concrete cross section and f_c is the concrete (cylinder) compressive strength. The terms η and η_c are the two tuning coefficients to be determined through the machine learning approach. The expressions in (1), apart from the coefficients η and η_c, coincide with those of the EC2 model, and they can be obtained starting from the idealization of the variable-angle truss resistant mechanism. More specifically, η and η_c are introduced in the proposed formulation as substitutes of ν and α_c governing the concrete contribution in the original EC2 (2004) formulation. The coefficient η, like ν, represents a strut efficiency factor, whereas the term η_c incorporates not only the effect of applied compressive stress on the resulting shear capacity (like α_c) but also the effect of cyclic loading.

The methodology pursued in this paper aims at finding the best tuning (corrective) coefficients η and η_c to match a series of experimental data collected from the literature. The determination of the best expression for η is carried out for each sample of the database (training) under the assumption $\eta_c = 1$, i.e., by selectively excluding the effect of axial compressive stress (only RC beams are considered) and that of cyclic loading (only monotonic loading is considered). In mathematical terms, the optimal value for η is obtained for each sample of the database by solving the following constrained optimization problem:

$$\min_{\eta} \frac{|V_{num}(\eta|\eta_c = 1) - V_{exp}|}{V_{exp}} \qquad s.t. \begin{cases} \eta_{min} \le \eta \le \eta_{max} \\ \theta_{min} \le \theta \le \theta_{max} \end{cases} \tag{2}$$

where $V_{num}(\eta|\eta_c = 1)$ represents the numerical prediction of the shear capacity through Eq. (1) depending on the coefficient η and assuming $\eta_c = 1$. Based on Eq. (2), the optimal value of η is sought so that the numerical shear capacity approximates the corresponding experimental result in the best possible manner, while satisfying certain constraints given by η_{min} and η_{max}, θ_{min} and θ_{max}. Once the optimal value of η is determined for each sample of the database, namely η_{opt}, a machine learning technique is adopted to obtain an analytical expression depending on some explanatory variables $\boldsymbol{\vartheta}$, namely $\eta = f(\boldsymbol{\vartheta})$, such that its predictions fit the retrieved optimal values η_{opt}. Similarly, the determination of the best expression for η_c is carried out for each sample of the database including RC columns under cyclic loading (thus including the effects of compressive stress and cyclic loading) assuming the previously obtained expression for η, namely $\eta = f(\boldsymbol{\vartheta})$. This requires the solution of the following constrained optimization problem:

$$\min_{\eta_c} \frac{|V_{num}(\eta_c|\eta = f(\boldsymbol{\vartheta})) - V_{exp}|}{V_{exp}} \qquad s.t. \begin{cases} \eta_{c,min} \le \eta_c \le \eta_{c,max} \\ \theta_{min} \le \theta \le \theta_{max} \end{cases} \tag{3}$$

where $V_{num}(\eta_c|\eta = f(\boldsymbol{\vartheta}))$ represents the numerical prediction of the shear capacity through Eq. (1) depending on the coefficient η_c. Based on Eq. (3), the optimal value of η_c is sought so that the numerical shear capacity approaches, in the best possible manner, the corresponding experimental results, while satisfying certain constraints given by $\eta_{c,min}$ and $\eta_{c,max}$, θ_{min} and θ_{max}. Once the optimal value of η_c is obtained for each sample of the database, namely $\eta_{c,opt}$, a machine learning technique is adopted to derive an analytical expression depending on some explanatory variables $\boldsymbol{\varphi}$, namely $\eta_c = g(\boldsymbol{\varphi})$, such that its predictions fit the retrieved optimal values $\eta_{c,opt}$.

The functional relationships f and g to be identified by GP are searched within the class of function sets including standard arithmetic operators only, so that the resulting expressions are suitable for practical design purposes. Lower and upper bounds for η and η_c in Eq. (2) and (3), respectively, are assumed equal to $\eta_{\min} = 1.001\omega_{sw}$ (where ω_{sw} is the mechanical ratio of transverse reinforcement), $\eta_{\max} = 1.0$, $\eta_{c,\min} = 0.3$ and $\eta_{c,\max} = 3.0$. On the other hand, $\theta_{\max} = 45°$ in agreement with EC2 (2004), since this upper bound is motivated by mechanical considerations and is also supported by experimental data (Biskinis et al. 2004). With regard to the lower bound $\theta_{\min}$, Biskinis et al. (2004) pointed out that it can be lower than the value of 21.8° recommended in the EC2 formulation, as also suggested by other truss resisting mechanisms (Colajanni et al. 2014; De Domenico 2021); in this paper, it is assumed $\theta_{\min} = 11.31°$ (i.e., $1 \leq \cot\theta \leq 5$), in line with other truss models from the literature (De Domenico and Ricciardi 2019).

A wide database of RC beams (Mansour et al. 2004; Zhang et al. 2016; Reineck et al. 2014, 2017) and RC columns (NEES ACI 369 rectangular column database compiled by Ghannoum et al. 2015, with recent extensions by Azadi Kakavand et al. 2019 in the PRJ-2526 database) was collected from the literature. Excluding replications of samples among different databases and specimens not failing in shear, additional filters were applied to ensure consistency with the underlying hypotheses of the truss resisting mechanism considered in this work, namely: 1) shear span-to-effective depth ratio $a/d \geq 2.2$; 2) mechanical ratio of transverse reinforcement $\omega_{sw} \leq 0.25$; 3) applied compressive stress ratio $\sigma_c/f_c \leq 0.50$. After excluding replications and incorporating filters, the database includes 373 RC beams and 119 RC columns.

In the GP approach, dimensional and non-dimensional variables are taken into account in the derivation of the final expressions of $\eta = f(\boldsymbol{\vartheta})$ or $\eta_c = g(\boldsymbol{\varphi})$, i.e., $\boldsymbol{\vartheta} = \{f_c, b_w/d, a/d, \omega_{sw}\}$ and $\boldsymbol{\varphi} = \{f_c, b_w/d, a/d, \omega_{sw}, \sigma_c/f_c, \mu\}$, where μ is the displacement ductility demand appearing in the case of RC columns tested under cyclic loading. The entire database is divided into training (80% of the samples) and testing (20% of the samples) datasets, and the final expressions of the corrective factors $\eta = f(\boldsymbol{\vartheta})$ or $\eta_c = g(\boldsymbol{\varphi})$ are:

$$\eta = 0.12 + \frac{3.9\,(1+b_w/d)}{8.2-0.08f_c+(f_c-0.08)(b_w/d)} \qquad (with\ f_c\ in\ [MPa]) \tag{4}$$

$$\eta_c = \begin{cases} 1 & for\ beams, monotonic\ loading \\ 0.37 + 3.8\,\dfrac{0.30+0.75(\sigma_c/f_c)-1.39(a/d)\omega_{sw}}{a/d}\left(1+15\,\dfrac{\sigma_c/f_c}{\mu^2}\right) & for\ columns, cyclic\ loading \end{cases} \tag{5}$$

which are valid under the constraints $0.1 \leq \eta \leq 1.0$, $1/3 \leq \eta_c \leq 2.6$. These expressions are plotted (in terms of some of the parameters) in Fig. 1. It can be noted that the corrective coefficient η decreases with increasing f_c, in line with the inverse relationship existing between ν and f_c in the original EC2 formulation, and generally decreases as the b_w/d ratio increases. This result may be justified, from a mechanical standpoint, by the fact that the effective compressive stress of compression struts decreases as the flexural inertia of the concrete diagonals decreases, an aspect that is not incorporated in the coefficient ν of the EC2 formulation. On the other hand, the corrective coefficient η_c increases with increasing compressive stresses σ_c/f_c, similar to the α_c expression of the original EC2 formulation, and decreases with increasing μ, ω_{sw} and a/d, which are three factors that are not involved in the original EC2 formulation.

4. Validation with experimental results and comparison with other code-based models

Considering that the developed model represents an improvement of a code-based formulation (i.e., the EC2 truss model), the accuracy of the proposed shear capacity equation is evaluated by comparison with shear strength expressions from alternative technical codes proposed by international organisms, national/federal regulatory agencies or standardization bodies, namely: 1) original EC2 formulation (EC2); 2) Model Code 2010 (MC); 3) Italian Technical Code 2018 (NTC); 4) ACI 318 Building Code (ACI); 5) provisions from the American Association of State Highway and Transportation Officials (AASHTO); 6) New Zealand Standards for concrete structures (NZS); 7) Codes for Design of Concrete Structures (China) GB50010-2010 (GB); 8) Japan Society of Civil Engineering Guidelines for Concrete no. 15 (JSCE).

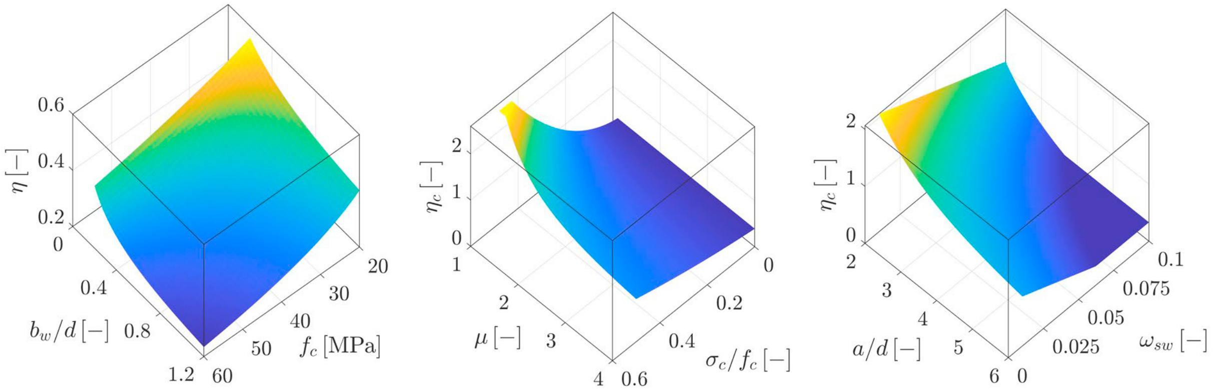

Fig. 1. Parametric study of the proposed machine-learning-based formulation for the new parameters ruling the concrete contribution.

Fig. 2 illustrates the experimental shear strength versus the shear strength numerically predicted by the proposed capacity equations. The same scale is adopted for the two axes, so that the perfect agreement between the two sets of data is obtained along the 45° line. A good agreement is observed for both RC beams under monotonic loading and RC columns under cyclic loading, with most of the numerical-to-experimental shear strength ratios falling close to the unity, with a mean ratio equal to 1.0034 (1.0324) and a coefficient of variation (CoV) equal to 32.45% (28.07%) for RC beams (columns).

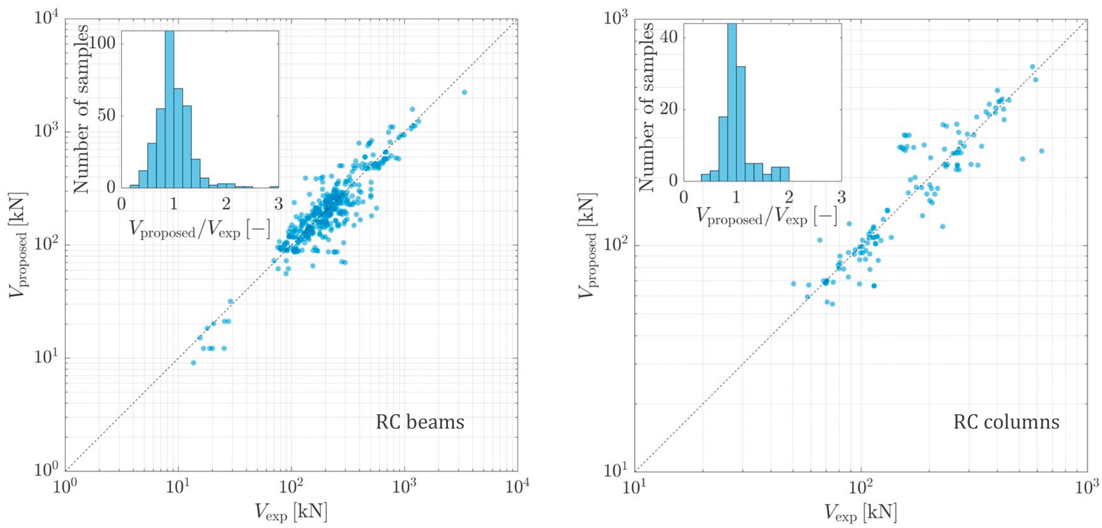

Fig. 2. Comparison between experimental shear strength values and corresponding predictions obtained by means of the proposed capacity equation for RC beams under monotonic loading (left) and RC columns under cyclic loading (right).

Similar plots corresponding to alternative code-based formulations are shown in Fig. 3 and Fig. 4 for RC beams and columns, respectively. It is clearly seen that the predictive accuracy worsens in the other capacity equations. As an example, by considering again the numerical-to-experimental shear strength ratios for the EC2 formulation, such samples are characterized by a mean equal to 0.7926 (1.1816) and a CoV equal to 54.16% (46.12%) for RC beams (columns). These results highlight that the EC2 formulation is generally biased and more dispersed than the proposed model and provides conservative (unconservative) estimates of the shear strength for RC beams (columns). These values, if compared to the previous metrics obtained for the proposed formulation, demonstrate that the machine-learning improvement of the corrective coefficients η and η_c in the proposed formulation remarkably enhances the predictive accuracy of the variable-angle truss model of the EC2.

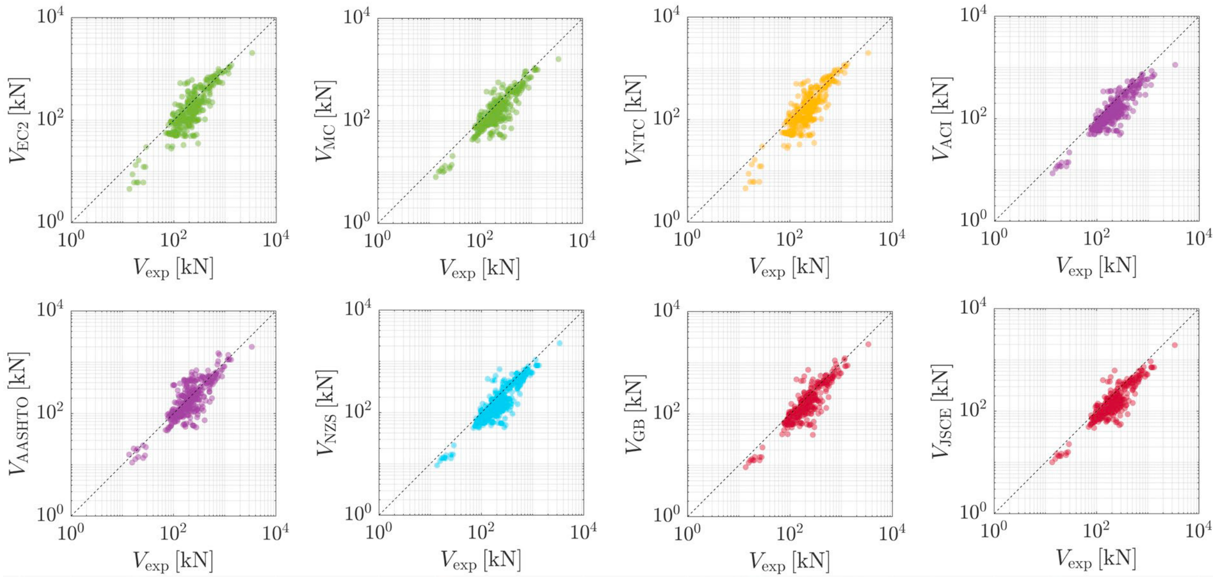

Fig. 3. Experimental vs numerical shear strengths for RC beams obtained using capacity models available in some technical codes and guidelines.

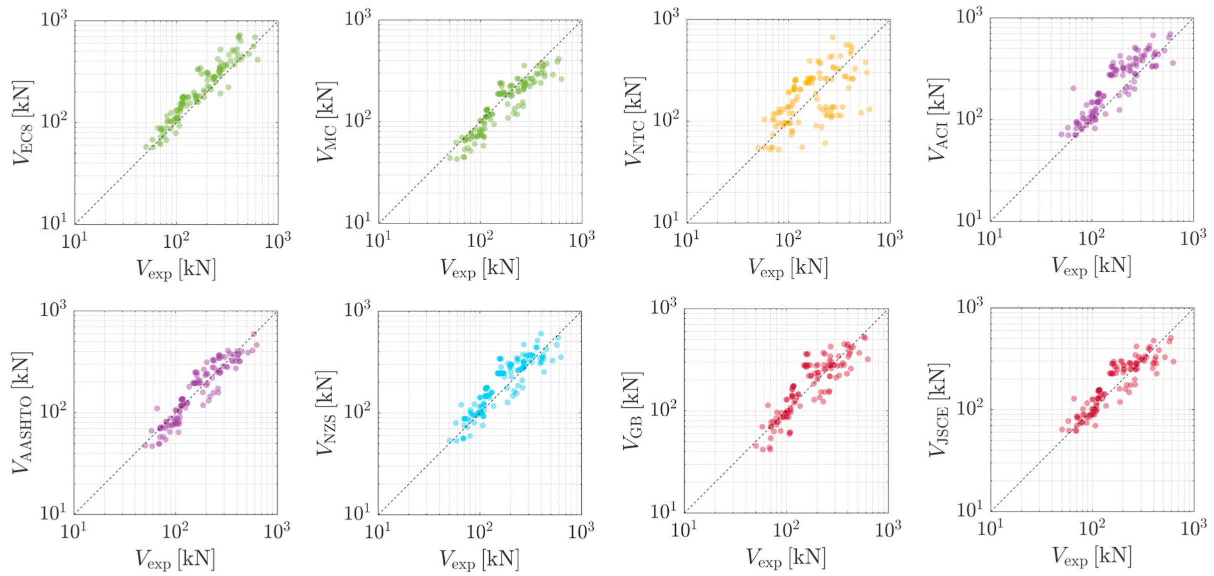

Fig. 4. Experimental vs numerical shear strengths for RC columns obtained using capacity models available in some technical codes and guidelines.

To better infer the predictive accuracy of the proposed model as compared to alternative code-based equations, Fig. 5 reports the mean squared error (MSE) values (obtained as sum of variance and squared bias of the numerical-to-experimental shear strength ratios) for RC beams and columns. In particular, the MSE for RC beams (columns) related to the proposed formulation is 0.10608 (0.085067), which is a relatively low value, whereas that of the original EC2 formulation is 0.22733 (0.33001). Based on this indicator, as well as considering the previous correlation trends and dispersion analysis, the proposed model represents the best equation for predicting the shear strength of RC members with transverse reinforcement among those considered in this study.

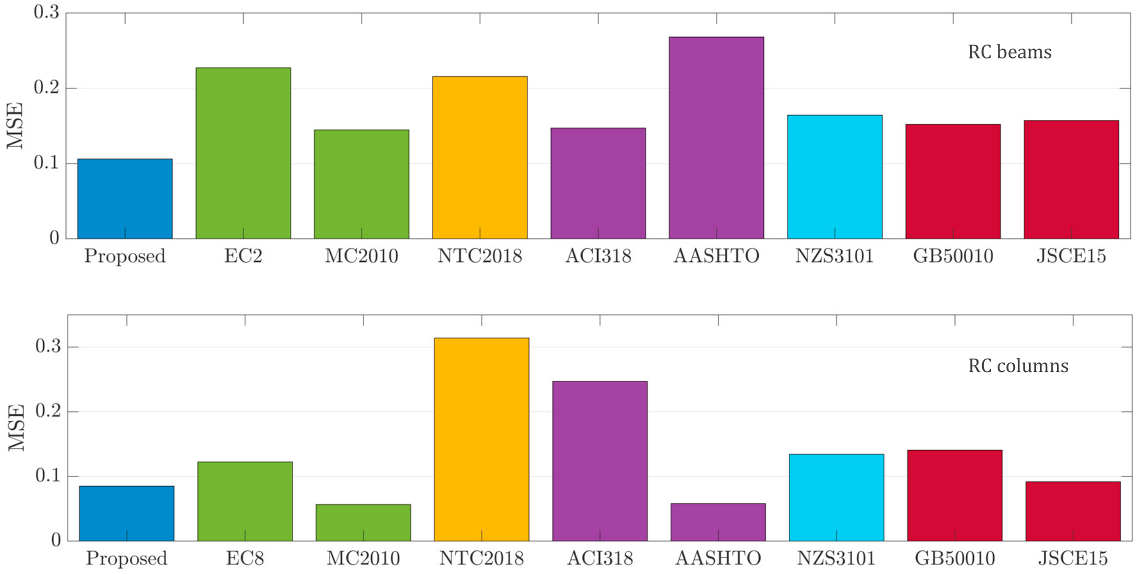

Fig. 5. Mean squared error of the numerical-to-experimental shear-strength ratios for RC beams (top) and columns (bottom).

5. Conclusions

A grey box modelling approach for the shear capacity prediction of RC members with transverse reinforcement has been proposed in this work. The proposed approach is different from alternative black-box modelling (i.e., pure data-driven) techniques and white-box modelling (i.e., mechanics-based) strategies. In the proposed approach, a mechanical model, namely the variable-angle truss model of the EC2 formulation, has been enriched by two corrective coefficients governing the concrete contribution (i.e., the shear capacity ascribed to crushing of compressed struts), whose expressions have been determined through a data-driven technique, namely genetic programming. The proposed model avoids typical overfitting problems of pure data-driven approaches and improves the accuracy of traditional mechanics-based models. The proposed approach has allowed the derivation of relatively compact expressions for two corrective coefficients, called η and η_c, that aims at replacing the coefficients ν and α_c appearing in the concrete contribution of the original EC2 formulation. Although genetic programming has been used to identify data-driven expressions of the corrective parameters, the shear capacity is still ruled by a mechanics-based resisting mechanism. In particular, it has been found that the coefficient η (replacing the efficiency factor ν of the EC2 formulation) depends not only on the compressive strength of concrete f_c, but also on the cross-section shape factor b_w/d, somehow suggesting that the effective concrete compressive strength in the truss model decreases as the flexural inertia of the concrete diagonals cross-section decreases. Moreover, the coefficient η_c (replacing α_c of the EC2 formulation) depends not only on the applied compressive strength on the RC member σ_c/f_c, but also on the displacement ductility demand μ, suggesting that shear strength of RC columns under cycling loading condition decreases as the displacement ductility demand increases, which is consistent with available experimental findings from the literature.

Acknowledgements

The present work is framed within the research project DPC/ReLUIS 2022-2024 – UR RM1 WP11. Giuseppe Quaranta acknowledges the financial support from Sapienza University of Rome (Grant No. RM120172B37F0628).

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