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Characterization of Radiowave Propagation in the Troposphere under Precipitation Conditions

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Dipartimento di Ingegneria dell'Informazione, Elettronica e Telecomunicazioni
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Characterization of Radiowave Propagation in the Troposphere under Precipitation Conditions

Sapienza University of Rome

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Abstract

This doctoral dissertation explores the propagation of electromagnetic waves through the lower atmosphere, with a focus on tropospheric delay, hydrometeor-induced attenuation, and radiative transfer processes in the Ka, V, and W bands. The research combines physical modeling, statistical-numerical simulation, and AI-based surrogate models to address key challenges in high-frequency satellite communication and remote sensing. The first part of the study addresses tropospheric delays, including Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and total Zenith Tropospheric Delay (ZTD). These are modeled using ERA5 reanalysis data, GNSS observations, and ITU-R standards. Several mapping functions (e.g., Niell, VMF1, GPT3, Ifadis) are evaluated to accurately project delays along slant paths, enhancing GNSS positioning and satellite orbit determination.

A core contribution of this thesis is the development of **SNEM-2**, an enhanced version of the Statistical Numerical Electromagnetic Model. SNEM-2 extends radiative transfer simulations up to 95.5 GHz, supporting slant-path geometries and mixed-phase hydrometeor profiles derived from CloudSat–GPM coincidences. The model outputs specific attenuation and brightness temperature (TB), and has been validated against satellite observations, showing strong performance in reproducing melting layer effects and scattering signatures in the W-band.

In addition, AI-based generative models—particularly **Generative Adversarial Networks (GANs)** and **Conditional GANs (CGANs)**—are applied to reconstruct realistic cloud fields and produce synthetic atmospheric datasets. These models are trained on both observations and SNEM-2 simulations, enabling efficient forward modeling and attenuation prediction. Wavelet-based analysis (using CWT and the **Morlet** transform) is also employed to capture non-stationary propagation effects across multiple scales. A novel theoretical framework, the **C4 Field Theory**, is introduced to unify electromagnetic wave propagation and hydrometeor microphysics. While not formally derived from spin glass theory, C4 (Coupled Cloud–Clausius–Complex field) draws valuable analogies from it—particularly regarding phase transitions and the statistical behavior of disordered systems. The model builds upon and extends the classical **Clausius–Mossotti** relation to represent complex electric susceptibility and refractive index variations in stratified, mixed-phase atmospheric media. Additionally, the name ‘C4’ encodes multiple layers of meaning: the letter ‘C’ refers to the author’s surname (Cicolin), while the numeral ‘4’ denotes both the four key physical domains addressed—cloud structure, dielectric behavior, radiative complexity, and thermodynamic state—and the notion of a four-vector (quadrivector) in space-time, reflecting the theory’s ambition to bridge temporal and spatial dynamics in atmospheric radiative modeling. The results support the design of adaptive communication systems, radiative transfer emulators, and atmospheric correction strategies for next-generation satellite missions. Future developments will include CubeSat-based validation, integration of polarization effects, and hybrid physics–AI models for real-time signal correction.

Keywords: Tropospheric Delay, Hydrometeor Attenuation, Radiative Transfer, Brightness Temperature, Satellite Communications, GANs and CGANs, Cloud Microphysics, Millimeter-



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Wave Propagation, AI Surrogates, C4 Field Theory, C4 Framework, SNEM, SNEM-2, SNEM-3, C4 Framework.

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Preface

This dissertation originates from a personal and scientific journey through the intersection of atmospheric physics, radiowave propagation, and data-driven modeling. It aims to contribute to the understanding and prediction of signal attenuation and radiative processes in the Earth’s troposphere, especially under precipitation conditions and at high microwave frequencies.

The work presented here was carried out within the PhD Program in Information and Communication Technologies at the Faculty of Information Engineering, Electronics and Telecommunications, Sapienza University of Rome. The research was conducted between November 2021 and December 2024 and supported by a PhD scholarship co-funded by the National Operational Programme Research and Innovation 2014–2020, FSE REACT-EU resources, under Action IV.4 “Doctoral programs and research contracts on innovation topics” and Action IV.5 “Doctorates on Green topics”.

During the three years of the doctoral program, the research evolved from theoretical investigation to large-scale simulations and validation with satellite and reanalysis data. The project integrated physical modeling (Mie theory, Radiative Transfer, hydrometeor microphysics), numerical simulation (Fortran, MATLAB), and statistical techniques, culminating in the development of the extended SNEM-2 framework. The results were validated through high-resolution datasets from CloudSat, GPM, and ERA5, and systematically compared with ITU-R propagation models.

I wish to express sincere gratitude to **Sapienza University of Rome** and the institutions that enabled this research project. I also extend my thanks to those who provided feedback, inspiration, or technical assistance throughout this path. Particular appreciation goes to **anonymous reviewers** for their time and for the insightful comments that enriched the scientific content of this work, whose guidance—either direct or indirect—shaped the scientific depth and direction of this dissertation.



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Chapter 1

Introduction and Research Objectives

1.1 Scientific Motivation and Context

The present research originates from a long-standing interest in the interaction between electromagnetic radiation and the Earth's atmosphere, a topic lying at the intersection of atmospheric physics, radiative transfer, and satellite communications. During my doctoral work, it became increasingly clear that the same physical processes responsible for signal degradation in Earth-space links—attenuation, scattering, and phase delay—also provide valuable information about the internal structure of clouds, precipitation systems, and gaseous absorption layers. What is traditionally treated as a disturbance in telecommunications thus becomes a diagnostic observable for atmospheric science.

This dual interpretation constitutes the primary motivation of the thesis. Rather than focusing exclusively on link-budget impairment, the work investigates how electromagnetic-wave interactions with the troposphere can be exploited to infer physical properties of the propagation medium itself. The central question guiding this research is therefore twofold: how atmospheric constituents affect radiowave propagation across microwave and millimeter-wave frequencies, and how these effects can be modeled in a physically consistent yet computationally efficient manner.

At the outset of this research, the scientific landscape was characterized by a clear methodological divide. On one side, operational propagation models—widely adopted in satellite communication engineering—relied predominantly on empirical parameterizations. Although statistically validated and computationally efficient, these models exhibited limited sensitivity to local microphysical variability, multilayer cloud structures, and mixed-phase precipitation processes. On the other side, physically based radiative-transfer formulations offered rigorous descriptions grounded in scattering theory and electromagnetic physics, but at the cost of high computational complexity and demanding input requirements.

Between these two extremes, this thesis identifies a space for hybrid modeling approaches. The objective is not merely to accelerate numerical simulations, but to preserve physical interpretability while extending model applicability to global scales and near-real-time scenarios. Within this context, the concept of physically informed surrogate models emerges as a central methodological

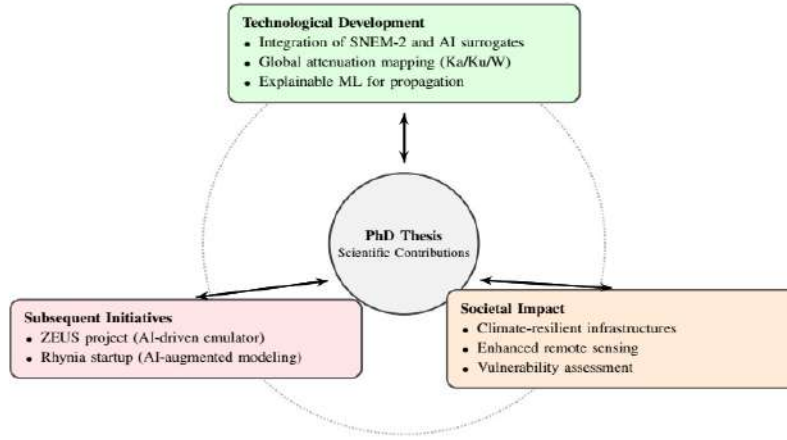


Figure 1.1. Scientific contributions of the thesis. The figure summarizes the core contributions of the dissertation across three dimensions: technological development (integration of SNEM-2 and AI-based surrogates for satellite link planning and global attenuation mapping), societal impact (support to climate-resilient infrastructures, enhanced remote sensing, and vulnerability assessment), and subsequent initiatives (ZEUS project and the Rhynia startup for AI-augmented atmospheric modeling).

pillar: data-driven architectures constrained by physical principles and trained on synthetic radiative-transfer simulations.

This motivation further evolved into a broader vision linking radiowave propagation to climate variability. Atmospheric attenuation at high frequencies—particularly in the Ka-, V-, and W-bands—is strongly modulated by precipitation intensity, hydrometeor phase, and thermodynamic stratification. As climate variability intensifies precipitation extremes, understanding and modeling these effects becomes essential not only for communication reliability but also for environmental diagnostics and long-term monitoring. This awareness frames atmospheric extinction not as a secondary correction, but as a measurable and physically meaningful variable.

Figure 1.1 schematically summarizes the evolution of the research, from the initial physical motivation to the development of hybrid modeling frameworks and AI-based surrogates.

1.2 Context of Electromagnetic Propagation Studies

The study of electromagnetic propagation through the atmosphere has progressively evolved from a specialized engineering problem into a multidisciplinary research field connecting meteorology, climatology, and telecommunications. Historically, empirical attenuation models—such as those recommended by the International Telecommunication Union (ITU-R)—have provided reliable tools for estimating atmospheric losses in operational contexts. Their strength lies in statistical robustness and ease of implementation, but their descriptive power remains limited when addressing fine-scale variability and complex vertical structures.

Radiative-transfer theory, in contrast, offers a physically grounded framework capable of describing absorption, emission, and scattering processes through explicit representations of

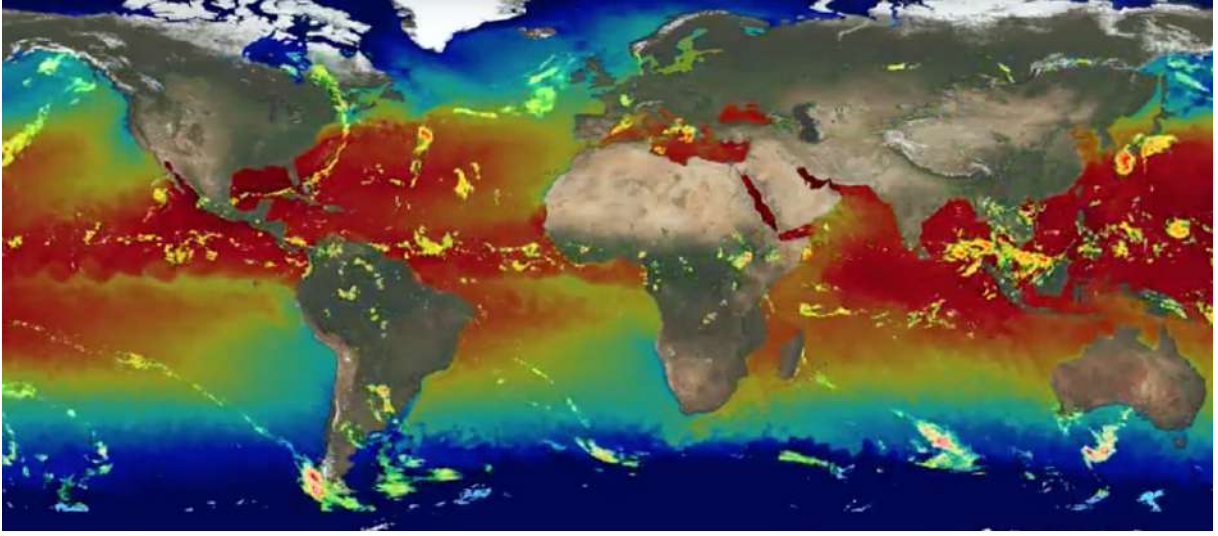


Figure 1.2. Global distribution of annual precipitation (mm/year) based on GPM-IMERG Final Run data. The map highlights regional variability, emphasizing areas with significant rainfall that are critical for satellite communication system design and atmospheric attenuation modeling. Different climatic zones are represented through a color-coded scheme corresponding to characteristic precipitation ranges. **Credits:** NASA Earth Observatory image by Joshua Stevens, using data from the Global Precipitation Measurement (GPM) mission and MODIS sensors aboard NASA’s Terra and Aqua satellites.

atmospheric microphysics. By incorporating Mie scattering, absorption spectroscopy, and multilayer energy balance, deterministic solvers can reproduce brightness temperature and extinction profiles under a wide range of atmospheric conditions. However, such models require extensive input information and are computationally demanding, limiting their operational deployment.

Recent advances in satellite meteorology and reanalysis datasets have reshaped this context. Missions such as CloudSat, GPM, and TRMM provide vertically resolved observations of hydrometeors, while global reanalyses such as ERA5 offer thermodynamic profiles with high temporal resolution. These datasets enable systematic validation of propagation models across climatic regimes and motivate the integration of physics-based solvers with statistical learning.

Within this evolving landscape, the present work adopts a hybrid perspective. Deterministic radiative-transfer models serve as physically consistent generators of synthetic data, while machine-learning architectures act as efficient emulators capable of capturing non-linear dependencies and generalizing across atmospheric states. This approach establishes a methodological dialogue between physics and data science, where equations constrain learning and data extend model validity.

The global precipitation patterns shown in **Figure 1.2** provide a climatic framework for the propagation scenarios investigated in this thesis.



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1.3 Scientific and Technological Contributions

The research conducted in this thesis resulted in a set of original scientific and technological contributions that collectively define its scope and novelty. These contributions span model reconstruction, physical extension, data integration, and methodological innovation. The core contributions can be summarized as follows:

1. Reconstruction and documentation of the original Sky Noise Eddington Model (SNEM) and its modular reimplementation.
2. Development of the extended SNEM-2 framework supporting Ka-, V-, and W-band simulations and slant-path geometries.
3. Modeling of mixed-phase and melting-layer regions using multilayer radiative-transfer representations.
4. Integration of frequency-dependent microphysical coefficients derived from Mie scattering theory.
5. Implementation of refined radiative-transfer parameterizations for extinction and emission processes.
6. Assimilation of global reanalysis and multi-satellite observations for physically consistent input profiles.
7. Design of machine-learning surrogates (LSTM and CGAN) for emulating path-integrated attenuation and brightness temperature.
8. Quantitative validation of model outputs against CloudSat, GPM, and TRMM measurements.
9. Generation of global climatologies of attenuation and tropospheric delay across multiple frequency bands.
10. Formulation of a unified hybrid physical–AI modeling framework.
11. Development of a modular and extensible modeling architecture enabling scientific autonomy.
12. Application of the framework to realistic satellite-communication and climate-monitoring scenarios.

Together, these contributions establish a coherent methodology connecting deterministic physics, data-driven inference, and operational relevance.

Table 1.1 summarizes the main scientific contributions and their correspondence with the thesis chapters.



No.	Contribution	Thesis Chapters	Related Articles or Projects
1	Complete reconstruction and scientific porting of the original SNEM model	Chapter 4, Introduction, Appendix C	Article 1
2	SNEM-2 model extension for Ka,Ku,V,W bands and slant-path geometries	Chapter 4, Sections 4.1, 4.2, 4.3	Article 1
3	Modeling of melting layers and phase transitions in stratiform precipitation	Chapter 4, Section 4.2.2	Article 3
4	Integration of Mie theory-based extinction and dielectric properties	Chapter 2, Section 2.3; Chapter 4	Article 2 and 3
5	Development of modular scientific code implementing SNEM-2	Section 4.7, Appendix C	Article 1
6	Integration of ERA-5, GPM, and CloudSat datasets for input and validation	Chapter 3, Section 5.1	Article 1 and 4
7	Design of LSTM and CGAN surrogate models for attenuation and temperature prediction	Sections: 4.5, 4.6; Chapter 8	Article 4 and 5
8	Validation of simulated attenuation and brightness temperature against satellite data	Sections 5.4,5.5 Chapter 5	Article 1 and 3
9	Generation of global attenuation and brightness temperature maps	Section 5.6 Chapter 8	Article 6
10	Unified framework combining physical models and AI surrogates	Chapter 4, Chapter 8	Article 7
11	Independent development of modular SNEM-2 and CGAN-based framework	Chapter 7, Chapter 8	Article 7

Table 1.1. Summary of the main scientific and technological contributions developed throughout this doctoral research, mapped to the corresponding thesis chapters and related publications or projects.



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1.4 Research Objectives

The central objective of this doctoral research is to improve the quantitative understanding of electromagnetic-wave interaction with the atmosphere under non-clear-sky conditions. The work aims to develop physically interpretable and computationally efficient models capable of describing attenuation, emission, and phase-delay processes from X- to W-band frequencies. Specifically, the research pursues four interrelated objectives:

1. To characterize atmospheric extinction processes by integrating radiative-transfer and scattering theory with reanalysis data.
2. To derive and validate hybrid models combining deterministic solvers with physics-guided machine-learning surrogates.
3. To construct global and regional attenuation and delay maps using multi-satellite coincidences for training and validation.
4. To assess the implications of these models for satellite communication performance and climate monitoring.

1.5 Methodological Approach

The methodological framework adopted in this thesis follows a four-stage progression. First, classical attenuation and delay models are reproduced and benchmarked against ITU-R recommendations. Second, the SNEM and SNEM-2 radiative-transfer solvers are implemented and refined to simulate extinction and emission within multilayer cloud structures. Third, physics-informed machine-learning surrogates are trained on synthetic datasets generated by deterministic models, embedding physical constraints into the learning process. Finally, validation is performed using collocated satellite observations and reanalysis profiles to assess model accuracy and generalization. This structured approach ensures continuity from first-principles physics to data-driven prediction and operational assessment. The methodological workflow adopted in this research is illustrated in **Figure 1.3**.

1.6 Vision and Scope of the Thesis

This thesis was conceived as an independent yet coherent research trajectory spanning physics-based modeling, computational methods, and atmospheric applications. While the initial focus was on improving classical radiowave propagation models, the research progressively evolved into a broader framework linking electromagnetic theory with atmospheric variability and climate-related processes.

The work adopts an explicitly interdisciplinary approach, integrating model formulation, data analysis, validation, and interpretation within a unified methodological vision. Scientific

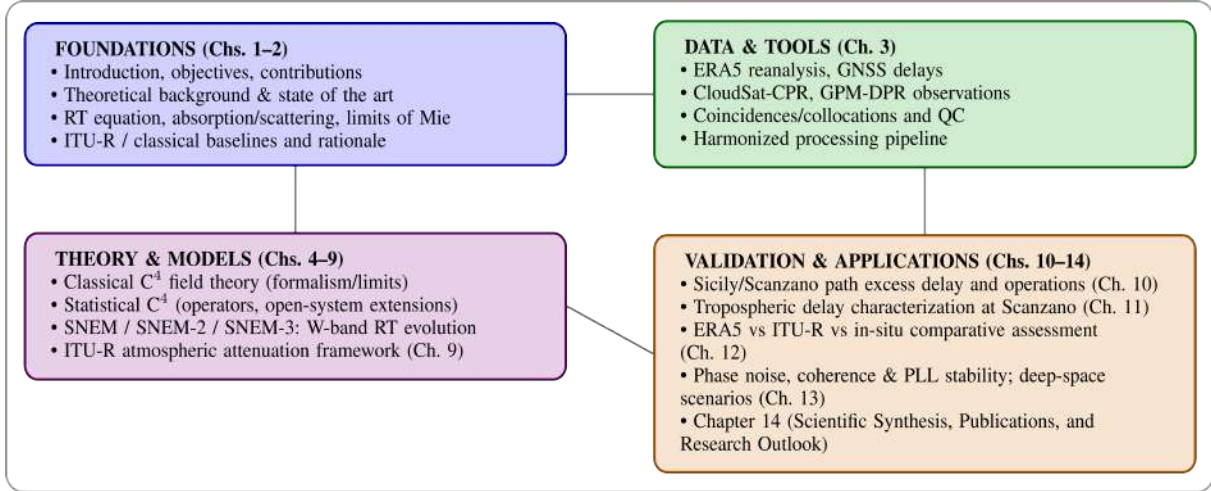


Figure 1.3. Visual road-map of the thesis: four blocks (Foundations, Data & Modelling, Results & Operations, Synthesis & Outlook) and their main relations.

autonomy is understood here not as isolation, but as the capacity to combine diverse theoretical and observational tools into a consistent and physically interpretable framework. Several key developments presented in this thesis—including the C^4 conceptual framework and the refinement of the SNEM-2 model—emerged from the need to overcome the limitations of existing radiative-transfer formulations and to interpret numerical behavior through physically meaningful principles rather than purely empirical adjustments. Overall, the guiding vision of this work is to elevate atmospheric attenuation from a secondary correction term in telecommunication models to a primary physical observable: a measurable, mappable, and predictable quantity that simultaneously informs signal reliability and atmospheric diagnostics. Figure summarizes the logical progression of the research from foundational theory to synthesis and outlook.

1.7 Thesis Structure

The remainder of the thesis is organized as follows. Chapter 2 reviews the theoretical foundations of radiative transfer, scattering, and hybrid modeling. Chapter 3 describes the datasets and computational tools employed. Chapter 4 introduces the C^4 framework and its role within SNEM-2. Chapter 5 presents deterministic and surrogate attenuation models. Chapter 6 reports validation results and global analyses. The final chapters synthesize the findings, discuss limitations, and outline future research directions. **Figure 1.4** provides a schematic overview of the logical organization of the thesis.



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Ch. 1 – Introduction & Motivation

Scope, objectives, roadmap, original contributions.

Ch. 2 – Theoretical Background (RT & Scattering)

RTE, absorption/scattering, Mie limits, ITU-R baseline.

Ch. 3 – Data & Processing Pipeline

ERA5, CloudSat/GPM, GNSS; collocations and QC.

Ch. 4 – C^4 Framework (Interpretative Layer)

Correlations, disorder, mixed-phase complexity; non-solver role.

Ch. 5 – Attenuation & Brightness Temperature Physics

Microphysics, melting layer, Ku/Ka/W scaling, TB–attenuation links.

Ch. 6 – SNEM-2 Radiative-Transfer Model

Absorption/emission/multiple scattering; numerical stability.

Ch. 7 – Validation & Statistical Results

ZHD/ZWD/ZTD, TB statistics, W-band threshold/bistability, uncertainty.

Ch. 8 – ML Surrogates (LSTM, CGAN)

Training, constraints, explainability; extremes and generalization.

Ch. 9 – Hybrid Modeling & ITU-R Integration

SNEM-2 + surrogates; comparisons; scalability and readiness.

Ch. 10–11 – Case Studies & Advanced RT Experiments

Regional contrasts; stress tests; dielectric/microphysical sensitivity.

Ch. 12 – Phase Noise, Coherence & PLL Stability

Slant mapping; wrapped/unwrapped phase; coherence limits vs frequency.

Ch. 13 – Deep-Space / Earth–Moon Scenarios

NRHO relay; Ka/X/W trade-offs; link budgets, ACM, availability.

Ch. 14 – Scientific Synthesis & Outlook

Unified claims, limitations, implications, future directions.

Appendices (A–R+)

Derivations, extended figures/tables, full time series, complete link budgets.

Figure 1.4. Schematic overview of the thesis organization, outlining the progression from theoretical foundations and data sources to the formulation of the C^4 framework, hybrid modelling strategies, machine-learning surrogates, validation, and advanced radiative-transfer experiments.



Chapter 2

Theoretical Background and State of the Art

This chapter provides the theoretical background required to model radiowave propagation in the troposphere under non-clear-sky conditions. The discussion focuses on the physical mechanisms that govern attenuation and thermal emission in hydrometeor-laden media, with emphasis on radiative-transfer theory, scattering models, and the microphysical description of cloud and precipitation particles.

After introducing the radiative-transfer equation and the key optical parameters (extinction, single-scattering albedo, asymmetry factor, and optical depth), the chapter reviews the main scattering formalisms used in atmospheric applications, from Mie theory to shape-aware alternatives. Standardized engineering models (ITU-R recommendations) are then summarized and critically discussed in terms of assumptions, validity domains, and limitations at Ka/V/W bands. The chapter concludes by outlining the main theoretical gaps that motivate the hybrid physical-statistical and physics-guided approaches developed in the subsequent chapters.

2.1 Scope and Positioning

Accurate modelling of electromagnetic-wave propagation through the Earth's atmosphere remains a central scientific and engineering challenge, particularly under non-clear-sky conditions. Atmospheric attenuation, emission, and phase delay result from the coupled action of molecular absorption, hydrometeor scattering, and thermodynamic variability, whose effects intensify across multiple spatial and temporal scales. These processes simultaneously constrain satellite communication performance and encode physically meaningful information on cloud structure, precipitation regimes, and atmospheric state.

The aim of this chapter is to establish the theoretical background required to interpret and develop the modelling frameworks presented in the remainder of this thesis. Rather than providing an exhaustive review of atmospheric physics or electromagnetic theory, the focus is restricted to those physical operators, assumptions, and modelling paradigms that directly



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govern radiowave attenuation and radiometric observables in the microwave and millimetre-wave domains.

Atmospheric propagation modelling has historically evolved along three complementary lines. Empirical and semi-empirical models, most notably those standardised by the ITU-R, provide climatology-based attenuation estimates with low computational cost and global applicability, but their reliance on vertically averaged parameters limits their accuracy under heterogeneous or mixed-phase conditions. Deterministic physical approaches based on radiative transfer theory and electromagnetic scattering offer a rigorous description of absorption, scattering, and emission processes, enabling detailed analysis of hydrometeor microphysics and frequency-dependent extinction, at the expense of high computational cost and demanding input requirements. More recently, hybrid and data-driven methodologies have emerged, combining physical constraints with statistical inference or machine-learning techniques to improve scalability and computational efficiency while preserving physical interpretability.

Within this context, the present thesis adopts an integrative perspective aimed at bridging deterministic radiative-transfer physics and physics-guided data-driven emulation. Emphasis is placed on propagation mechanisms that directly affect brightness temperature, specific attenuation, and path-integrated attenuation in the Ka-, V-, and W-band frequency ranges. Accordingly, this chapter introduces the radiative transfer equation and its approximate solutions, electromagnetic scattering by hydrometeors, and the microphysical parameters governing extinction and emission. Standardised empirical models are critically reviewed in terms of their domain of validity, while hybrid frameworks—such as the Sky-Noise Eddington Model (SNEM) and its extension SNEM-2—are positioned as intermediate approaches balancing physical realism and computational feasibility. The conceptual foundations of the C⁴ (Coupled Cloud–Clausius–Complex Field) framework are also outlined to provide a unified dielectric interpretation of cloud–wave interactions.

This chapter does not present original results or validation analyses. Quantitative assessments and applications to real datasets are deferred to subsequent chapters, while extended derivations and algorithmic details are collected in appendices to preserve conceptual clarity. By defining a coherent theoretical vocabulary and delineating the modelling assumptions adopted throughout this work, Chapter 2 provides the conceptual backbone for the methodological developments and scientific interpretations discussed in the remainder of the dissertation.

Figure 2.1 summarizes the forward and inverse modelling chain adopted in this thesis, highlighting the mapping between atmospheric states and radiometric or propagation observables under physical constraints.

Figure 2.1 illustrates the SNEM-2 workflow for solving the inverse problem of atmospheric radiative transfer and attenuation. The scheme consists of a forward chain, an inverse chain, and a consistency loop.

In the forward chain, thermodynamic $(T(z), p(z), q(z))$ and microphysical $(LWC(z), IWC(z))$ atmospheric profiles are mapped by the SNEM-2 forward model into radiometric and propagation observables, including brightness temperature $T_B(\nu, \theta)$, specific attenuation $k(\nu, z)$,

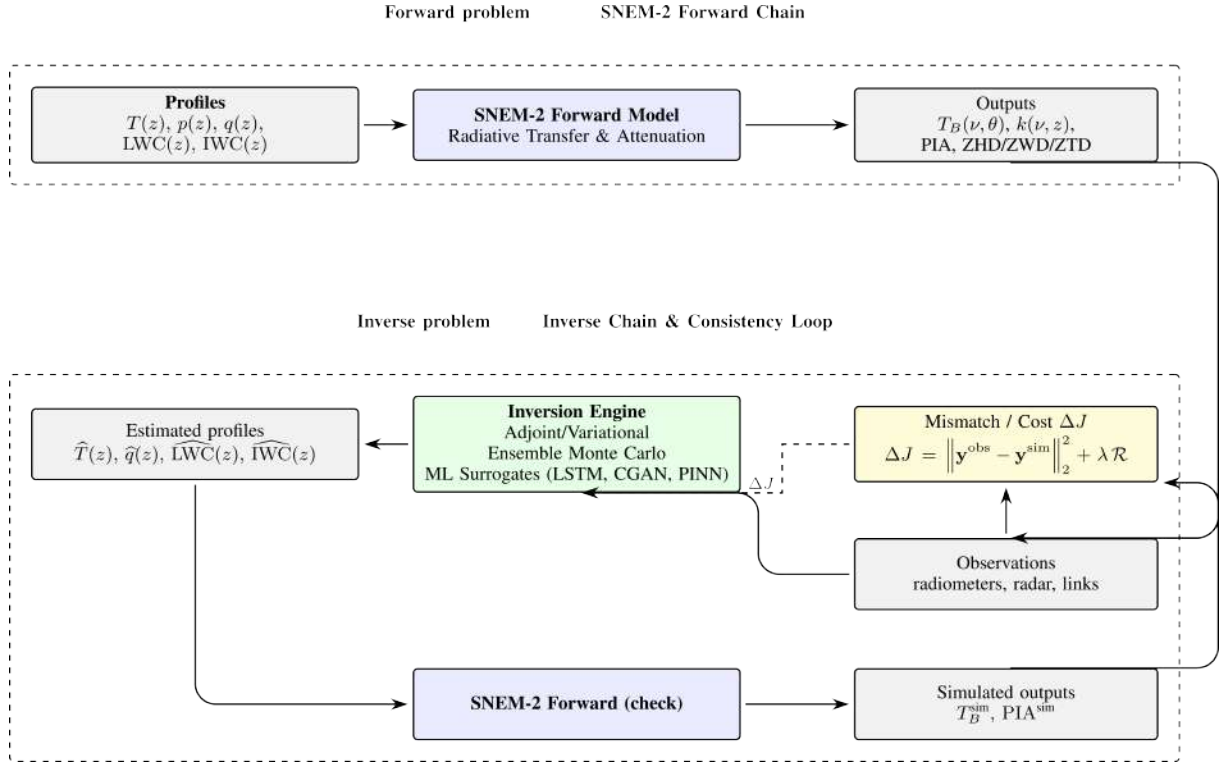


Figure 2.1. Workflow for the SNEM-2 solution to the inverse problem with orthogonal arrows and cost ΔJ with Euclidean norm $\|\dots\|_2$.

path-integrated attenuation (PIA), and zenith delay components (ZHD, ZWD, ZTD).

The inverse chain aims at retrieving atmospheric profiles from observed measurements provided by radiometers, radars, or communication links. Inversion strategies may rely on adjoint or variational techniques, ensemble Monte Carlo sampling, or physics-guided machine-learning surrogates (e.g., LSTM, CGAN, PINN), which iteratively adjust the atmospheric state to minimize discrepancies between simulated and observed observables. Physical consistency is enforced through a mismatch cost function:

$$\Delta J = \|y^{\text{obs}} - y^{\text{sim}}\|_2^2 + \lambda \mathcal{R}, \quad (2.1)$$

which balances the radiometric misfit between observations and simulations against regularization terms encoding physical constraints or prior information. Detailed quantitative metrics and prediction-error analyses, including RMSE- and correlation-based comparisons between SNEM-2 and machine-learning surrogates, are reported in **Appendix B**.

2.2 Electromagnetic Propagation in the Atmosphere

Electromagnetic-wave propagation through the Earth's atmosphere is governed by physical processes coupling Maxwell's equations with the thermodynamic and microphysical properties of



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the medium. In the microwave and millimeter-wave ranges, these processes primarily manifest as attenuation, emission, and phase delay, whose magnitude depends on atmospheric composition, vertical structure, and dynamical state. A clear understanding of these mechanisms is essential for satellite communication system design and for the interpretation of passive and active remote-sensing observations.

At a macroscopic scale, the atmosphere can be modeled as a vertically stratified, weakly dispersive medium characterized by a complex refractive index:

$$m(\nu, T, z) = n(\nu, T, z) - i\kappa(\nu, T, z), \quad (2.2)$$

or equivalently by a complex permittivity $\varepsilon(\nu, T, z) = \varepsilon' + i\varepsilon''$ with $m^2 = \varepsilon$, [4,30]. The real part governs phase velocity and signal delay, while the imaginary part accounts for energy losses due to absorption and scattering. Under clear-sky conditions, attenuation is dominated by gaseous absorption from molecular oxygen and water vapor, whereas in cloudy or precipitating conditions additional extinction arises from interactions with hydrometeors such as cloud droplets, raindrops, snow, graupel, and mixed-phase particles.

The total specific attenuation $\gamma(f, z)$ along a propagation path is commonly expressed as the sum of gaseous and hydrometeor-induced contributions, each dependent on frequency, altitude, and local thermodynamic conditions. Although this additive formulation is convenient, it masks the intrinsic coupling between absorption, scattering, and emission. In particular, hydrometeors act not only as attenuators but also as secondary emitters, contributing to the observed brightness temperature through volumetric emission and multiple scattering.

Physically, attenuation arises from two mechanisms: true absorption, whereby electromagnetic energy is converted into internal molecular or particulate excitations, and scattering, which redistributes energy angularly without necessarily removing it from the system. In radiometric observations, both mechanisms reduce the direct signal, but their radiative implications differ. Absorption is intrinsically linked to emission through Kirchhoff's law, while scattering introduces angular coupling and path-dependent redistribution of radiance.

The relative importance of absorption and scattering varies strongly with frequency. At lower microwave frequencies (X- and Ku-band), gaseous absorption is modest and hydrometeor scattering is typically weak except during intense precipitation. Toward Ka-, V-, and W-band, absorption by water vapor increases due to proximity to rotational lines, and hydrometeors transition from the Rayleigh to the Mie scattering regime, producing strong frequency-dependent extinction and phase effects.

Propagation geometry further modulates these interactions. For Earth-space links, slant-path propagation couples elevation angle, atmospheric layering, and path length. Low-elevation paths sample longer atmospheric columns and are more sensitive to vertical heterogeneity, while near-zenith paths emphasize local vertical structure with reduced horizontal averaging. These geometric effects are critical for interpreting path-integrated attenuation and brightness-temperature measurements, particularly at Ka/V/W bands. Practical prediction methods

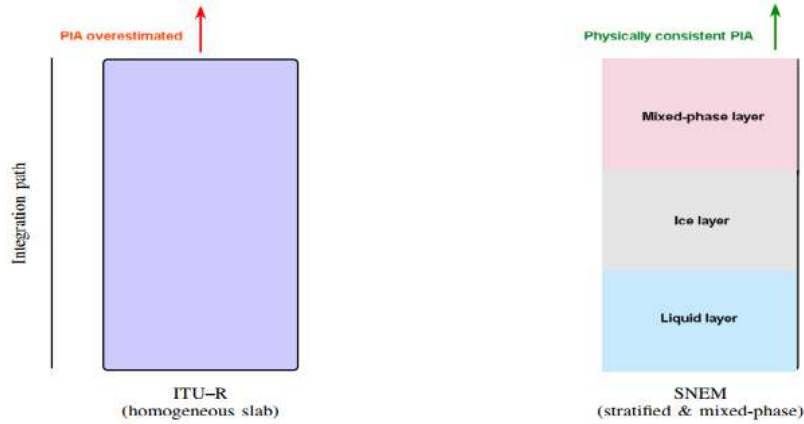


Figure 2.2. Comparison between a simplified empirical ITU-R vertical profile and a stratified SNEM cloud profile. The ITU-R homogeneous slab neglects vertical and phase variability, leading to an overestimation of the path-integrated attenuation (PIA). In contrast, SNEM resolves stratification into liquid, ice, and mixed-phase layers, yielding a physically consistent PIA estimate.

therefore often rely on simplified vertical representations, whose limitations relative to physically resolved profiles are illustrated in **Fig. 2.2**.

Another fundamental aspect of atmospheric propagation is phase delay, which arises from the dispersive properties of the atmosphere. Although it does not directly affect received power, phase delay is critical for precise positioning, timing, and coherent signal processing. In the microwave domain, it is primarily governed by the refractivity of dry air and water vapor, leading to the classical separation between hydrostatic and wet delay components. At higher frequencies, dispersive effects intensify near absorption lines, and phase perturbations may become coupled with attenuation and emission processes.

From a modeling standpoint, the atmosphere cannot be treated as a homogeneous or stationary medium. Clouds and precipitation exhibit strong spatial variability and intermittent dynamics, with convective systems characterized by sharp microphysical gradients and stratiform precipitation by layered structures and melting regions. Such features introduce scale-dependent variability that challenges simplified propagation models and motivates vertically resolved representations.

Radiative-transfer theory provides the formal framework to link local extinction and emission properties to observed radiance. However, practical implementations often rely on simplifying assumptions, such as plane-parallel geometry and reduced angular resolution, to maintain tractability. While adequate for large-scale analyses, these approximations may obscure localized effects associated with anisotropic scattering, non-spherical particles, or highly inhomogeneous cloud volumes.

In operational contexts, propagation complexity is frequently reduced through effective parameters, such as mean extinction coefficients or equivalent path lengths. Although computationally efficient, these representations may fail under transient or extreme conditions, particularly at millimeter-wave frequencies where small microphysical changes can produce large variations in



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attenuation and brightness temperature. This limitation is especially relevant in a changing climate, where shifts in precipitation intensity, cloud altitude, and hydrometeor phase alter the statistical properties of the propagation medium.

Consequently, atmospheric propagation modeling requires balancing physical realism with computational feasibility. The remainder of this chapter formalizes these considerations through the radiative transfer equation and associated scattering models, introducing deterministic and statistical approaches that underpin the hybrid modeling strategies developed in the subsequent chapters.

2.3 Radiative Transfer Theory in the Atmosphere

Radiative Transfer Theory (RTT) provides the fundamental physical framework for describing the propagation of electromagnetic radiation through absorbing, emitting, and scattering media [Chandrasekhar, Liou]. In atmospheric propagation, RTT links microscopic wave-matter interactions to macroscopic observables such as attenuation, brightness temperature, and radiance, forming the theoretical basis of both passive microwave radiometry and active radar sensing. As such, it represents the core framework for physically consistent attenuation modeling.

Unlike geometrical optics, radiative transfer explicitly accounts for energy exchange between radiation and matter, including volumetric emission by gases and hydrometeors and angular redistribution due to scattering. These effects are particularly relevant at microwave and millimeter-wave frequencies, where absorption and scattering cannot be treated as small perturbations.

2.3.1 Radiative Transfer Equation

The propagation of monochromatic radiation through the atmosphere is governed by the Radiative Transfer Equation (RTE) [Chandrasekhar], which describes the evolution of the specific intensity $I(\mathbf{r}, \hat{\mathbf{s}}, \nu)$ along a given direction $\hat{\mathbf{s}}$ at frequency ν :

$$\hat{\mathbf{s}} \cdot \nabla I(\mathbf{r}, \hat{\mathbf{s}}, \nu) = -k_e(\mathbf{r}, \nu) I(\mathbf{r}, \hat{\mathbf{s}}, \nu) + j(\mathbf{r}, \hat{\mathbf{s}}, \nu) + \int_{4\pi} k_s(\mathbf{r}, \nu) P(\hat{\mathbf{s}}, \hat{\mathbf{s}}') I(\mathbf{r}, \hat{\mathbf{s}}', \nu) d\Omega'. \quad (2.3)$$

Here, $k_e = k_a + k_s$ is the extinction coefficient, expressed as the sum of absorption k_a and scattering k_s contributions. The source term j represents thermal emission, while the integral term accounts for radiation scattered into the direction $\hat{\mathbf{s}}$ from all other directions $\hat{\mathbf{s}}'$ through the phase function P . In atmospheric applications, the RTE is commonly formulated under a one-dimensional, plane-parallel assumption, where the atmosphere is treated as horizontally homogeneous and vertically stratified. Under this approximation, the RTE reduces to:

$$\mu \frac{dI(z, \mu, \nu)}{dz} = -k_e(z, \nu) I(z, \mu, \nu) + j(z, \nu) + k_s(z, \nu) \int_{-1}^{+1} P(\mu, \mu') I(z, \mu', \nu) d\mu', \quad (2.4)$$

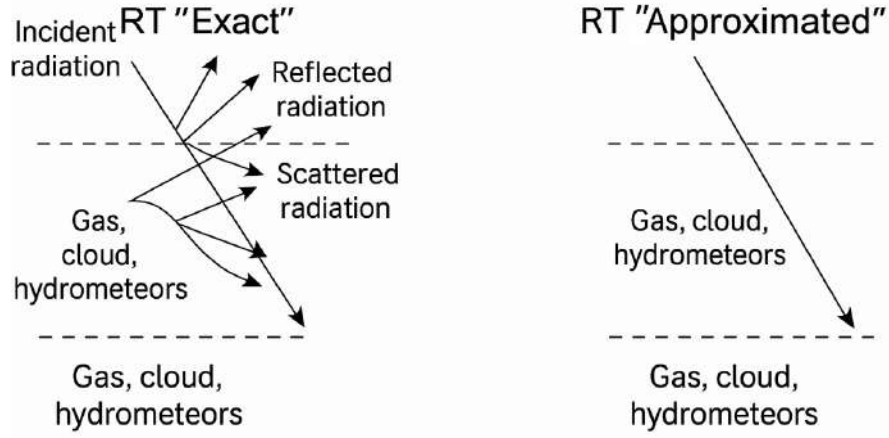


Figure 2.3. Comparison between exact and approximate radiative transfer (RT) schemes in a stratified atmosphere. The exact model accounts for multiple scattering, angular redistribution, and emission at each layer, typically using solvers like DISORT or ARTS. The approximate model simplifies the treatment using bulk assumptions, reducing computational cost but losing fine-scale accuracy.

where $\mu = \cos(\theta)$ denotes the propagation direction relative to the vertical. This formulation highlights the intrinsic complexity of atmospheric radiative transfer: the local intensity depends not only on absorption and emission at a given altitude but also on the angular distribution of radiation throughout the medium. While the RTE is compact in notation, its practical solution depends critically on the representation of atmospheric structure and scattering properties. **Figure 2.3** contrasts idealized and multi-layer scattering configurations, motivating the controlled approximations adopted later for precipitating scenes.

2.3.2 Absorption, Emission, and Kirchhoff's Law

Absorption and emission are intrinsically linked through thermodynamic equilibrium relations. In the atmosphere, thermal emission originates from molecular rotational transitions—primarily of H_2O and O_2 —as well as from the thermal radiation of hydrometeors. Under local thermodynamic equilibrium, the emission coefficient $j(z, \nu)$ is related to the absorption coefficient through Kirchhoff's law [Chandrasekhar],

$$j(z, \nu) = k_a(z, \nu) B_\nu(T(z)), \quad (2.5)$$

where $B_\nu(T)$ denotes the Planck function evaluated at the local temperature $T(z)$. In the microwave and millimeter-wave regimes, the Rayleigh--Jeans approximation is often applicable ,

$$B_\nu(T) \simeq \frac{2k_B\nu^2}{c^2} T, \quad (2.6)$$

which simplifies the interpretation of radiometric observations by allowing brightness temperature to be treated as a direct proxy for radiance. When scattering is significant, however, the local equilibrium assumption implicit in the emission--absorption balance no longer holds. In such

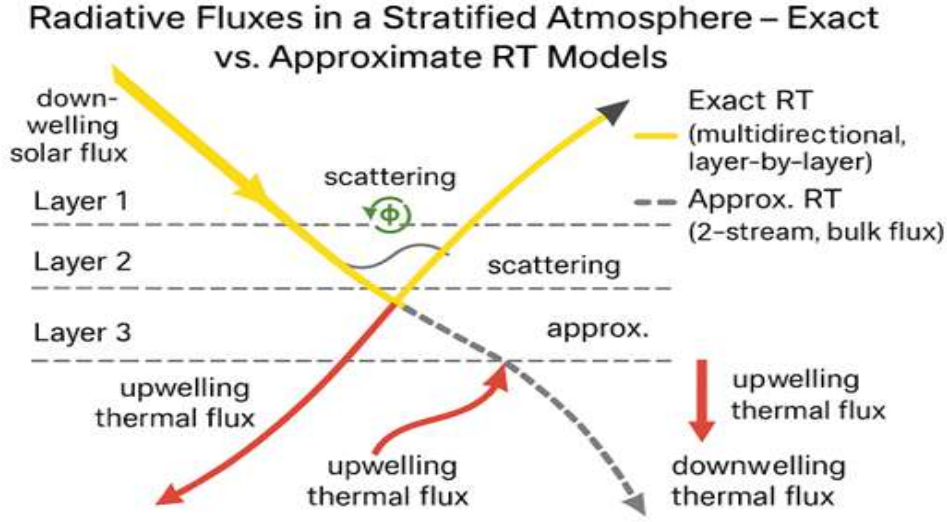


Figure 2.4. Radiative flux interactions in a multi-layered atmosphere. The diagram schematically illustrates the propagation of solar and thermal radiation through a stratified atmospheric column, highlighting scattering, absorption, and emission processes. Exact radiative transfer solvers—such as the discrete-ordinate method implemented in DISORT (Stamnes et al., 1988 [151])—treat these interactions directionally and layer by layer. In contrast, approximate two-stream methods (Krueger, Eddington- and Delta-Eddington-style; Toon et al., 1989 [152]; Meador & Weaver, 1980 [78]) simplify the problem into a flux balance between upward and downward streams. The figure emphasizes which physical processes are accurately captured—or potentially neglected—depending on the chosen modeling approach.

conditions, the observed brightness temperature results from the combined action of emission, absorption, and multiple scattering along the propagation path. **Figure 2.4** provides a compact flux-based representation of these interactions, highlighting the relative roles of gaseous absorption and hydrometeor extinction in the millimeter-wave regime.

2.3.3 Optical Depth and Path-Integrated Attenuation (PIA)

A central quantity in radiative transfer is the optical depth τ , defined as the cumulative extinction along a propagation path,

$$\tau(z, \nu, \theta) = \int_z^{+\infty} \frac{k_e(z', \nu)}{\mu} dz', \quad \mu = \cos \theta. \quad (2.7)$$

The optical depth quantifies the probability that radiation is removed from the direct beam by absorption or scattering. In Earth-space communication and radar applications, attenuation is commonly expressed in terms of Path-Integrated Attenuation (PIA),

$$\text{PIA}(\nu, \theta) = 4.343 \int_0^H k_e(z, \nu) \sec \theta dz, \quad (2.8)$$

where the factor 4.343 converts from nepers to decibels and H denotes the effective top of the attenuating atmospheric layer. PIA provides a direct link between radiative-transfer theory and



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measurable link losses.

2.3.4 Scattering and Phase-Function Representation

Scattering becomes a dominant mechanism in cloudy and precipitating atmospheres, particularly at frequencies above the Ka-band. Its impact is governed not only by the magnitude of the scattering coefficient but also by the angular redistribution of radiation, described by the phase function $P(\mu, \mu')$. A widely used approximation is the first-order Legendre expansion,

$$P(\mu, \mu') = \frac{1}{2} (1 + 3g \mu \mu'), \quad (2.9)$$

where the asymmetry factor $g = \langle \cos \theta \rangle$ characterizes the preferential scattering direction. This reduced representation captures the leading effects of forward-scattering dominance while maintaining computational tractability in radiative-transfer solvers.

2.3.5 Approximations to the Radiative Transfer Equation

Exact solutions of the Radiative Transfer Equation are computationally demanding due to the high dimensionality of the angular and spatial domain. Multi-stream discrete-ordinate solvers, such as DISORT, provide high accuracy but are generally limited to offline simulations and validation studies. Reduced-order approaches, including two-stream and Eddington approximations, retain only the leading angular moments of the radiance field. These methods preserve energy conservation and capture the dominant physics of vertically layered atmospheres, offering a practical compromise between physical fidelity and computational efficiency. This trade-off underpins the modeling strategy adopted in this thesis for large-scale and statistical applications.

2.3.6 Relevance for Atmospheric Propagation Modeling

Radiative Transfer Theory provides a unifying framework linking electromagnetic scattering, thermodynamics, and radiometric observations. Although its full formulation is computationally intensive, it ensures the physical consistency required to interpret attenuation, emission, and scattering processes across different frequency bands and atmospheric regimes.

These considerations motivate the adoption of hybrid physical–statistical approaches that retain radiative-transfer foundations while enabling scalable computation. The following section therefore focuses on electromagnetic scattering by atmospheric particles, introducing Mie theory and its role in determining the optical properties required by radiative-transfer models.

2.4 Electromagnetic Scattering by Hydrometeors: Mie Theory

Electromagnetic scattering by hydrometeors is a key physical mechanism governing atmospheric attenuation at microwave and millimeter-wave frequencies. When particle dimensions become comparable to the electromagnetic wavelength, scattering effects can no longer be neglected and



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must be treated within the framework of Maxwell’s equations. Mie theory provides an exact analytical solution for scattering by homogeneous spherical particles and therefore represents the reference model for hydrometeor–radiation interaction.

In atmospheric propagation studies, Mie theory is widely used to model extinction, absorption, and scattering by rain droplets and cloud water, and under idealized assumptions, by frozen hydrometeors. Its intrinsic frequency scalability makes it particularly relevant for Ka-, V-, and W-band applications, where particle size parameters approach unity and resonance effects become significant.

2.4.1 Size Parameter and Scattering Regimes

The scattering behavior is governed by the dimensionless size parameter:

$$x = \frac{2\pi r}{\lambda} = \frac{2\pi r\nu}{c}, \quad (2.10)$$

which defines distinct scattering regimes [Bohren, Huffman]:

- Rayleigh regime: $x \ll 1$, scattering intensity proportional to ν^4
- Mie regime: $x \sim 1$, resonance and oscillatory behavior
- Geometric optics regime: $x \gg 1$, forward-dominated scattering.

At W-band frequencies ($\nu \approx 75\text{--}110$ GHz), raindrops, graupel, and melting particles often lie within the Mie regime, where extinction and phase behavior are highly sensitive to particle size, composition, and dielectric properties. Real hydrometeors depart significantly from homogeneous spherical shapes, exhibiting a wide range of morphologies and internal structures that influence both extinction and phase functions. **Figure 2.5** provides a qualitative overview of these shapes, motivating the use of Mie theory as an efficient scattering kernel that often requires effective-medium or shape corrections for realistic atmospheric conditions.

2.4.2 Electromagnetic Role of Mie Scattering in Atmospheric Propagation

The electromagnetic interaction between hydrometeors and propagating radiation is governed by their dielectric properties, which exhibit a strong dependence on frequency and temperature. Liquid water, ice, and mixed-phase particles display markedly different dielectric behavior, particularly in the millimeter-wave regime, where absorption losses may differ by more than one order of magnitude between liquid and frozen phases. This contrast represents a primary driver of attenuation variability in melting layers and mixed-phase precipitation, with direct implications for both remote sensing and communication systems [4,16,30].

Within this context, Mie theory provides the reference electromagnetic framework for describing scattering, absorption, and extinction by atmospheric particles under the assumption of homogeneous spherical geometry [4,177]. Despite its idealized nature, Mie theory enables a physically consistent translation of particle-scale properties—such as size, composition, and dielectric response—into macroscopic radiative effects through the computation of scattering and

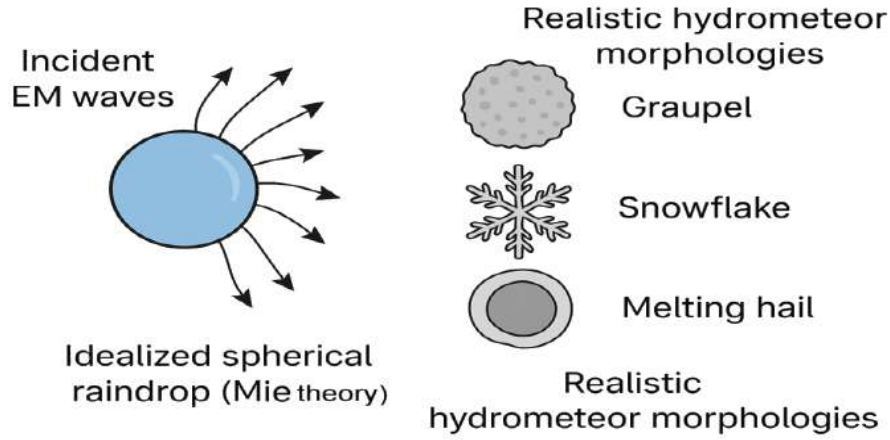


Figure 2.5. Idealized vs. realistic hydrometeor morphologies. The classical Mie theory assumes spherical homogeneous particles (left), which is an idealization valid for isolated raindrops. In contrast, real atmospheric particles—such as graupel, snowflakes, or melting hail (right)—exhibit highly irregular, porous, or layered structures, introducing scattering features not represented in the spherical model.

absorption efficiencies.

To represent realistic atmospheric conditions, single-particle interactions are integrated over hydrometeor size distributions, yielding volume extinction coefficients representative of cloud and precipitation ensembles. In practice, these calculations rely on prescribed statistical distributions, such as Marshall–Palmer or Gamma models, whose parameters vary with hydrometeor class and precipitation regime [15,26]. This step establishes the link between microphysical variability and path-integrated attenuation relevant for satellite links and radiometric observations.

Two parameters derived from Mie-based formulations play a central role in radiative-transfer modeling: the single-scattering albedo, which quantifies the balance between scattering and absorption, and the asymmetry factor, which characterizes the preferential direction of scattered radiation. Together, they control the angular redistribution of electromagnetic energy within cloud and precipitation layers and constitute the primary optical inputs for reduced-order radiative-transfer solvers, including the Eddington two-stream approximation employed in SNEM and SNEM-2 [78,84].

Despite its analytical rigor, Mie theory relies on simplifying assumptions that are only partially satisfied in real atmospheric conditions. Many hydrometeors—such as snowflakes, graupel, and melting aggregates—exhibit complex, nonspherical geometries, internal inhomogeneity, and preferred orientation. These departures can introduce systematic biases in extinction and scattering estimates, particularly at W-band frequencies where shape-induced resonances become critical [30,62,146]. Consequently, alternative electromagnetic solvers or hybrid statistical–numerical approaches are often required to complement Mie-based predictions.

Within this thesis, Mie theory is retained as the electromagnetic kernel linking hydrometeor microphysics to macroscopic observables such as attenuation and brightness temperature [4,30,177]. Its limitations are mitigated through vertical stratification, ensemble statistics, and data-driven



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surrogates, ensuring a physically consistent representation of absorption, scattering, and angular redistribution processes in cloud and precipitation media. Summary tables are provided to clarify the dominant physical mechanisms, modeling assumptions, and variables adopted throughout **Chapter 2**. A detailed and formal treatment of Mie scattering theory, including dielectric properties, scattering coefficients, and derived radiative-transfer parameters, is reported in **Appendix A** for completeness.

2.4.3 Radiative Transfer in Scattering Media and Reduced-Order Approximations

In scattering media such as clouds and precipitation, hydrometeors induce strong angular redistribution of electromagnetic energy, coupling radiation across directions and atmospheric layers. This non-local behavior becomes increasingly relevant at Ka-, V-, and W-band frequencies, where multiple scattering significantly affects both attenuation and brightness temperature [30,177].

Within this framework, the optical-depth formalism provides a convenient representation to distinguish absorption-dominated and scattering-dominated regimes, which is particularly important in mixed-phase clouds, ice precipitation, and melting layers [4]. In these environments, radiative behavior cannot be interpreted solely in terms of local thermal emission, but results from the cumulative redistribution of radiation through multiple scattering.

Exact radiative-transfer solvers resolve this complexity by explicitly accounting for angular and vertical coupling through discrete-ordinate, Monte Carlo, or atmospheric radiative-transfer models [78]. Although physically rigorous, their computational cost increases rapidly with problem dimensionality, limiting their applicability to offline simulations and validation activities.

These constraints motivate the adoption of reduced-order and hybrid radiative-transfer approximations, which preserve the essential physics of absorption, emission, and scattering while enabling scalable computation. Such approaches form a core element of the modeling strategy adopted in this thesis, providing a practical bridge between rigorous radiative-transfer theory and efficient atmospheric propagation models. For completeness, the full mathematical formulation of the radiative transfer equation, optical-depth representation, source function, and exact numerical solvers is reported in **Appendix A2** [4,30,78].

2.4.4 Two-Stream and Reduced-Order Approximations: Methodological Role

Reduced-order closures are required to render radiative-transfer modeling tractable in scattering atmospheres. Among them, the two-stream (Eddington-type) approximation is widely adopted, as it replaces the full angular dependence of the radiation field with representative upward and downward fluxes while preserving the dominant physics of absorption, emission, and scattering [4,30].

By retaining only the lowest angular moments of the radiance field, the two-stream formulation ensures energy conservation and captures the leading effects of forward scattering, which



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dominate hydrometeor interactions at microwave and millimeter-wave frequencies [30,84]. This approximation provides a robust compromise between physical fidelity and computational efficiency, making it suitable for layered cloud systems and stratiform precipitation under moderate optical-depth conditions.

At millimeter wavelengths, atmospheric propagation is characterized by a clear separation of regimes: gaseous absorption dominates under clear-sky conditions, while hydrometeor scattering governs cloudy and precipitating scenes, with multiple scattering becoming increasingly relevant in ice and mixed-phase layers [30,177]. Within these regimes, reduced-order solvers such as the two-stream approximation remain accurate provided that strong horizontal inhomogeneity and deep convective structures are absent. In such cases, careful validation against higher-order solvers is required.

The two-stream formulation constitutes the theoretical backbone of the SNEM and SNEM-2 radiative solvers adopted in this thesis. While full angular radiative-transfer models offer higher directional accuracy, their computational cost limits their applicability in large-ensemble simulations, global attenuation mapping, and real-time or near-real-time applications [78]. The two-stream approach retains the essential radiative physics while ensuring numerical stability and scalability across frequency bands and atmospheric conditions.

This methodological compromise enables systematic ensemble studies, global propagation analyses, and seamless coupling with physics-informed machine-learning surrogates. In this sense, the two-stream SNEM framework provides a consistent bridge between deterministic radiative-transfer theory and hybrid statistical modeling strategies developed in the subsequent chapters.

For completeness, the full mathematical formulation of the two-stream approximation, including governing equations, closure coefficients, and brightness-temperature expressions, is reported in **Appendix A2** [4,30,78].

2.4.5 Statistical–Numerical Electromagnetic Models: SNEM and SNEM-2

The need for radiative-transfer solvers that are both physically consistent and computationally efficient has motivated the development of statistical–numerical electromagnetic models, designed to bridge rigorous radiative-transfer theory and empirical attenuation laws [4,30]. Within this class, the Sky-Noise Eddington Model (SNEM) provides a structured framework that explicitly accounts for absorption, scattering, and thermal emission while remaining tractable for large-scale and ensemble applications.

SNEM was originally developed to simulate sky-noise temperature and path-integrated attenuation under precipitation conditions using a reduced-order radiative-transfer closure. During this doctoral work, the model has been substantially extended and re-engineered into SNEM-2, enabling multi-frequency capability, slant-path geometries, mixed-phase microphysics, and direct compatibility with modern data-driven and machine-learning frameworks.

Conceptually, the SNEM framework combines three key elements: a statistical description



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of hydrometeor fields, a reduced-order radiative-transfer closure, and numerical integration along the propagation path. The atmosphere is treated as a vertically stratified stochastic medium, in which microphysical variability is represented statistically, while radiative transfer is solved deterministically within each realization. The model operates under the plane-parallel approximation and local thermodynamic equilibrium, assumptions that are well satisfied for microwave and millimeter-wave propagation in the troposphere [4,30].

Radiative transfer within SNEM is handled through a two-stream Eddington-type closure, which retains the dominant angular moments of the radiation field and ensures energy conservation. This approximation captures the leading effects of forward scattering in layered cloud and precipitation systems while maintaining numerical stability and computational efficiency [30,84]. As such, it is well suited for ensemble simulations, global mapping, and sensitivity analyses.

Hydrometeor populations are represented statistically rather than deterministically. Each atmospheric layer is characterized by bulk microphysical descriptors, such as number concentration, effective particle size, density, and temperature, from which frequency-dependent optical properties are derived. Particle size distributions are typically described using parametric forms, such as Marshall–Palmer or Gamma distributions, allowing a direct link between bulk microphysical variables and electromagnetic extinction properties [15,26].

Extinction and scattering properties are computed for each hydrometeor class and combined with gaseous absorption to obtain the total extinction profile. These optical properties drive the computation of path-integrated attenuation and emergent brightness temperature, explicitly accounting for frequency, vertical structure, and propagation geometry. In this way, SNEM treats the atmosphere consistently as both an attenuating and emitting medium, a requirement for physically meaningful radiometric simulations [4,30,177].

Figure 2.7 provides a schematic overview of the SNEM workflow. Microphysical atmospheric descriptors are first mapped into frequency-dependent optical properties through electromagnetic scattering theory, and subsequently propagated within a layered radiative-transfer formulation. The resulting brightness temperature, transmittance, and reflectance constitute the primary radiometric observables produced by the model.

All governing equations and detailed numerical formulations underlying SNEM and SNEM-2 are reported in **Appendix A3** for completeness [4,30,78].

2.4.6 SNEM Workflow and Slant-Path Propagation Geometry

In satellite communication and microwave remote-sensing applications, electromagnetic waves propagate along slanted paths characterized by a finite elevation angle. Compared to the zenithal configuration, this geometry increases the effective optical path length by intersecting multiple atmospheric layers with different thermodynamic and microphysical properties.

The adopted slant-path geometry, together with the assumption of a vertically stratified atmosphere, is illustrated in **Fig. 2.8**. This configuration defines the physical basis for computing path-integrated attenuation, optical depth, and brightness temperature within the SNEM and

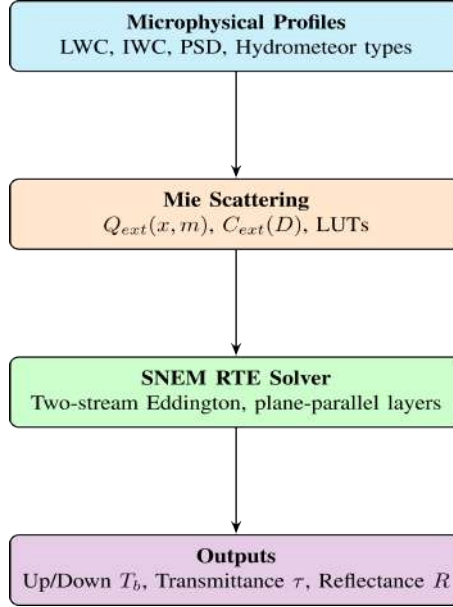


Figure 2.6. Conceptual diagram of the simplified SNEM workflow: microphysical profiles serve as input to Mie scattering computations and are subsequently used in the solution of the radiative transfer equation (RTE).

SNEM-2 radiative-transfer framework.

2.4.7 From SNEM to SNEM-2: Design Rationale and Architectural Evolution

The original SNEM formulation was primarily conceived for quasi-zenithal propagation geometries and frequencies below the Ka-band, which imposed several structural limitations when applied to realistic satellite and high-frequency scenarios. In particular, the original framework lacked explicit slant-path capability, relied on simplified representations of mixed-phase hydrometeors, exhibited limited scalability toward millimeter-wave frequencies, and offered restricted interoperability with modern reanalysis and satellite datasets.

The extended SNEM-2 framework was developed within this doctoral research to overcome these constraints through targeted architectural enhancements. These include explicit treatment of slant-path propagation over elevation angles ranging from 5° to 90° , extension of the radiative-transfer formulation up to the W-band (up to 95.5 GHz) [ArtA1], dynamic vertical stratification driven by ERA5 thermodynamic profiles, and an explicit representation of mixed-phase hydrometeors and melting-layer processes constrained by satellite observations [ArtA3, ArtB2]. The resulting architecture was deliberately designed to ensure interoperability with modern datasets and compatibility with physics-informed machine-learning workflows [ArtB1].

Together, SNEM and its extended formulation SNEM-2 constitute the physical core of the modeling framework developed in this thesis. They operate as forward radiative-transfer solvers capable of simulating attenuation and emission under realistic atmospheric and precipitation conditions, explicitly accounting for multilayer vertical structure, hydrometeor phase transitions,

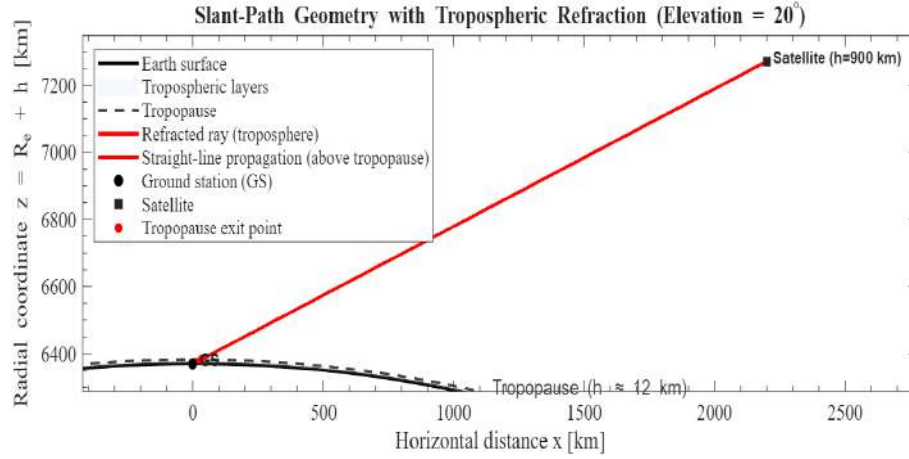


Figure 2.7. Propagation geometry for a slant path through a vertically stratified atmosphere. The electromagnetic signal propagates from the ground station to the satellite at a finite elevation angle, intersecting multiple tropospheric layers with different physical properties. This geometry defines the effective path length and forms the basis for the computation of path-integrated attenuation, optical depth, and brightness temperature in radiative-transfer models.

and frequency-dependent extinction mechanisms, as demonstrated in [ArtA1, ArtA3].

Beyond their deterministic role, these models also function as physically consistent generators of synthetic datasets. Their statistical–numerical design supports ensemble simulations, uncertainty quantification, and sensitivity analyses across frequency bands and propagation geometries, enabling statistical characterization and the training of physics-informed data-driven surrogates introduced and validated in subsequent chapters [ArtB1].

The architectural evolution from the original SNEM formulation to the enhanced SNEM-2 framework is schematically summarized in **Fig. 2.8**, illustrating the transition from a vertically simplified representation to a fully stratified, frequency-flexible, and multilayer architecture. This evolution enables physically consistent simulations across Ka-, V-, and W-band frequencies and constitutes the computational backbone of the modeling strategy adopted throughout this thesis [ArtA1].

A conceptual comparison between empirical attenuation models and physically based radiative-transfer approaches is reported in **Table 2.1**. While empirical and ITU-R-based models offer computational simplicity and operational convenience, they rely on vertically averaged descriptors and limited phase representations. In contrast, SNEM-based and hybrid physical–statistical frameworks explicitly resolve multilayer atmospheric structure, mixed-phase hydrometeors, and broadband frequency behavior, enabling physically consistent attenuation modeling and data assimilation [ArtA3, ArtB2].

Finally, **Table 2.2** provides a comparative overview of the original SNEM model and its extended SNEM-2 formulation, highlighting differences in frequency coverage, thermodynamic representation, hydrometeor phase treatment, and computational design. While SNEM was originally conceived as a conceptual benchmark for simplified radiative-transfer simulations, SNEM-2 evolves into a scalable and validated platform capable of supporting high-resolution

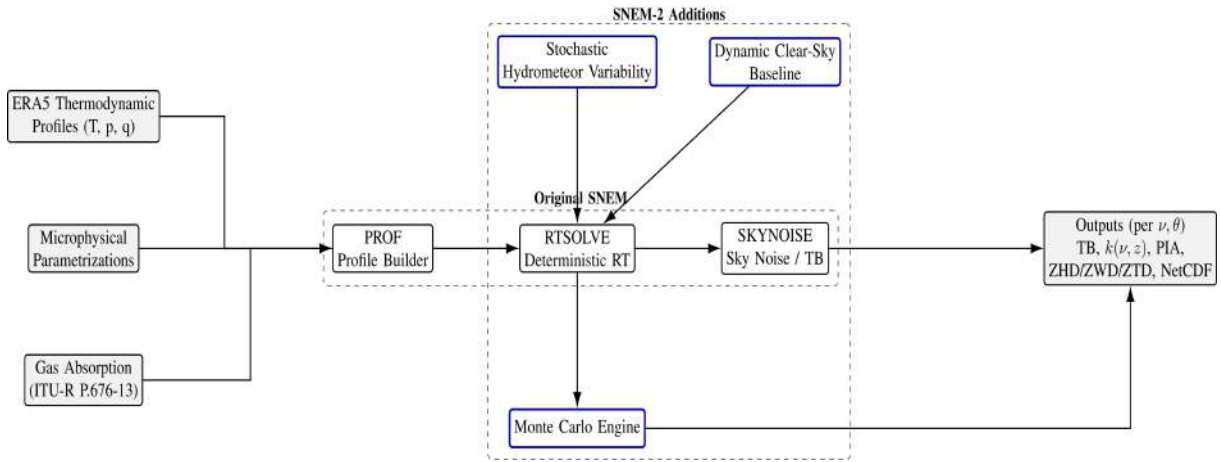


Figure 2.8. SNEM to SNEM-2 horizontal workflow. Inputs (ERA5, microphysics, gas) feed the original SNEM core (PROF–RTSOLVE–SKYNOISE). SNEM-2modules (stochastic variability, clear-sky baseline, Monte Carlo) extend the core and feed into the outputs (TB, k, PIA, ZTD, NetCDF).

Aspect	Empirical/ITU Models	SNEM/AI-Based Surrogates
Vertical resolution	Surface only	Multi–layer stratification
Hydrometeor phase	Rain only	Rain, snow, graupel, ice
Frequency generalization	Band-specific	Broadband, tunable
Data assimilation	None	Support ERA5, CloudSat, GPM
Nonlinear process handling	Limited (regression-based)	Explicit (RTM) or learned (ML)
Operational applicability	Fast, approximate	Accurate, data-informed

Table 2.1. Comparative summary of the structural and functional differences between conventional empirical/ITU-based attenuation models and advanced frameworks based on radiative transfer (e.g., SNEM) and AI surrogates. Empirical models provide simplicity and speed but are limited in phase representation, vertical resolution, and frequency adaptability. In contrast, SNEM/AI-based approaches support multilayer atmospheric modelling, mixed-phase hydrometeors, and data assimilation from ERA5, CloudSat, and GPM, enabling physically consistent and data-informed attenuation estimates.

satellite validation, surrogate-model generation, and W-band link-design applications [ArtA1, ArtB1].

2.4.8 Physics-Guided Generative Modelling: Conceptual Positioning

Recent advances in data-driven modelling have introduced probabilistic and generative approaches capable of representing atmospheric variability beyond the scope of analytic parameterizations. In the context of atmospheric radiative-transfer modelling, such frameworks have demonstrated the ability to emulate complex cloud-field structures while preserving multivariate consistency and spatial coherence when appropriately constrained by physical information [ArtA2, ArtB1].

Within this thesis, generative approaches are not considered substitutes for physics-based modelling, but rather complementary representations of unresolved or computationally expensive



Feature	SNEM	SNEM-2
Frequency Range	Up to 50 GHz	Extended up to 95.5 GHz (W-Band), covering Ku,Ka, and W bands
Geometrical Coverage	Quasi-zenith angles only	Full slant path coverage (5° to 90° elevation)
Numerical Core	Fortran 77	Optimized Fortran 90 with modular I/O and MATLAB-based interfaces
Thermodynamic Stratification	Fixed lapse-rate or simplified profiles	Dynamic stratification from ERA5 Reanalysis or Satellite-derived datasets
Phase Representation	Binary phase (rain/snow)	Multi-phase:rain,snow, graupel,melting, with vertical mixing
Extinction Coefficient	Prescribed from look-up tables	Phase-weighted and thermodynamically interpolated extinction (ArtA3, ArtB2)
Radiative Transfer Solver	Eddington two-stream with fixed ω and g	Eddington+Monte Carlo solver extension with stochastic vertical integration
Spectral Fidelity	Limited to predefined frequencies	Frequency-resolved simulation across user-defined spectral grids
Hydrometeor Population Generator	Empirical PDFs for rain and snow	Stochastic sampling with dynamic hydrometeor mixing and temperature-dependent PDFs
Computational Performance	Sequential execution	Parallel-ready structure, GPU-compatible modules (future development)

Table 2.2. Comparison between the original SNEM model and the extended SNEM-2 framework. The table highlights key upgrades in frequency scalability, atmospheric state representation, hydrometeor phase treatment, and computational architecture, reflecting the transition from a conceptual radiative-transfer benchmark to an operational and extensible modelling platform.



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microphysical structures. Their conceptual relevance lies in the ability to learn conditional distributions constrained by large-scale atmospheric state variables, thereby extending the applicability of deterministic radiative-transfer solvers to ensemble-based and climate-scale analyses while preserving physical interpretability [ArtB2].

These considerations motivate the introduction of physics-guided surrogate representations that retain the structure of the underlying radiative-transfer operators. In this perspective, generative models are employed to emulate statistically consistent atmospheric realizations conditioned on physically meaningful inputs, rather than to replace the forward physical solvers. A schematic illustration of the conditional generative paradigm adopted in this work is provided in **Fig. 2.8** [ArtA2, ArtB2].

The operational implementation and validation of such generative surrogates within the SNEM-2 framework are intentionally deferred to subsequent chapters, where they are discussed in direct connection with satellite observations and numerical experiments [ArtA2].

2.5 The C^4 Field: Coupled Cloud–Clausius–Complex Electromagnetic Framework

The limitations of purely empirical attenuation models and reduced-order radiative-transfer solvers motivate the introduction of a more general theoretical construct capable of unifying cloud microphysics, thermodynamics, and electromagnetic propagation within a single physically consistent framework. In this context, the C^4 Field, denoting the Coupled Cloud–Clausius–Complex Electromagnetic Field, is introduced as an original conceptual representation of atmospheric media in which microscopic particle properties and macroscopic radiative behaviour are intrinsically linked.

The C^4 Field does not replace classical radiative-transfer theory; rather, it provides an interpretative generalization in which clouds and precipitation are treated as integral components of an effective electromagnetic medium. Within this perspective, attenuation, emission, and phase delay emerge from the collective dielectric response of atmospheric constituents, rather than being treated as independent or externally parameterized effects. This interpretation builds upon established formulations of radiative transfer and electromagnetic propagation in atmospheric media, while extending their physical reading toward a unified field–medium interaction framework [4, 30].

In standard radiative-transfer approaches, the atmosphere is discretized into layers characterized by bulk optical parameters such as extinction coefficient, single-scattering albedo, and asymmetry factor [30, 78]. While operationally effective, this representation obscures the microphysical origin of these parameters and limits their adaptability to evolving atmospheric states. The C^4 Field addresses this limitation by reformulating the atmospheric column as a state-dependent dielectric medium, whose electromagnetic response varies continuously with altitude, thermodynamic conditions, hydrometeor phase, and frequency, consistently with effective-medium



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concepts widely used in atmospheric electromagnetics [4, 177].

Within this framework, clear-sky and precipitating conditions are interpreted as limiting cases of a continuous propagation regime rather than as distinct modelling domains. Mixed-phase regions and melting layers naturally emerge as zones of enhanced dielectric contrast, providing a physically consistent explanation for the anomalous attenuation and brightness-temperature signatures frequently observed at Ka- and W-band frequencies [30, 84, 177]. This interpretation clarifies why empirical attenuation laws often fail in complex microphysical regimes, where collective dielectric behaviour dominates over vertically averaged descriptors [30].

The C^4 Field further provides a unified interpretation of attenuation and phase delay as complementary manifestations of the same underlying electromagnetic interaction. Refractive effects, absorption, and scattering are governed by a common effective-medium response, enabling a coherent description of both amplitude and phase perturbations along the propagation path. This aspect is particularly relevant for high-frequency satellite links and combined radiometric–propagation analyses, where dispersion and extinction are strongly coupled [4, 30].

Within the scope of this thesis, the C^4 Field is introduced as a conceptual unifying layer, rather than as an additional operational model. Statistical radiative-transfer solvers such as SNEM and SNEM-2 can be interpreted as reduced-order numerical realizations of this broader field-based description under specific geometrical, statistical, and closure assumptions [ArtA1, ArtA3]. More generally, the C^4 framework provides the physical context for the development of hybrid and physics-guided surrogate models, in which data-driven representations are constrained by electromagnetic and thermodynamic consistency rather than trained as purely empirical mappings [ArtB1, ArtB2]. In summary, the C^4 Field offers a coherent physical interpretation that connects cloud microphysics, radiative transfer, and electromagnetic propagation within a single dielectric framework. Its role in this work is to clarify the theoretical foundations underlying the modelling strategies adopted in subsequent chapters, rather than to introduce a separate predictive model.

The C^4 Field is introduced here as an original conceptual framework developed within this thesis, building upon established electromagnetic and radiative-transfer principles.

2.6 Classical Radiative Transfer vs. C^4 Framework: Conceptual Comparison

Classical radiative-transfer (RT) theory constitutes the formal foundation for modeling electromagnetic propagation in absorbing, emitting, and scattering media. Its mathematical formulation, based on the radiative transfer equation (RTE), has been successfully applied to atmospheric remote sensing, climate studies, and satellite communication analyses across microwave and millimeter-wave frequencies [4, 30, 78].

However, the increasing complexity of modern propagation scenarios—particularly at Ka-, V-, and W-band frequencies and under non-clear-sky conditions—has highlighted conceptual and



practical limitations of traditional RT formulations, especially when microphysical variability, mixed-phase conditions, and non-stationarity become dominant [30, 84, 177].

The C^4 Framework, introduced in the previous section, does not aim to supersede classical RT theory. Instead, it provides a complementary interpretation that shifts the descriptive focus from radiance transport to field–medium interaction, offering a unified dielectric perspective on attenuation, emission, and phase delay.

2.6.1 Classical Radiative Transfer Paradigm

In classical RT theory, electromagnetic propagation is described in terms of the specific intensity, whose evolution is governed by the RTE. The medium enters the formulation through bulk optical quantities—extinction coefficient, scattering phase function, and emission term—often linked to temperature through Kirchhoff’s law [4, 30].

This paradigm treats the atmosphere as an external modifier of radiation propagating along a prescribed geometrical path. While physically rigorous, it relies on several implicit assumptions:

- separation between propagation geometry and microphysical state,
- compression of microphysical complexity into bulk optical coefficients,
- quasi-stationary atmospheric conditions,
- high angular resolution requirements for exact solutions.

Reduced-order solvers mitigate computational cost but further abstract the physical structure of the medium [78].

2.6.2 C^4 Framework Perspective

The C^4 Framework reformulates the problem by embedding electromagnetic propagation within the atmospheric medium itself. The central quantity is the effective, state-dependent dielectric response, which encapsulates the cumulative contribution of gases, liquid droplets, ice particles, and mixed-phase aggregates. Within this perspective:

- attenuation is associated with the dissipative component of the effective medium,
- phase delay arises from its refractive component,
- emission is intrinsically linked to dissipation through thermodynamic consistency.

Radiative observables such as brightness temperature and path-integrated attenuation are thus interpreted as derived quantities, rather than primary state variables. This view is consistent with effective-medium concepts widely used in atmospheric electromagnetics and propagation theory [4, 30, 177].

2.6.3 Comparison of Physical Assumptions

A fundamental distinction between the two paradigms lies in how microphysics enters the model:

- Classical RT: microphysics $\rightarrow$ optical coefficients $\rightarrow$ radiative intensity



– C^4 Framework: microphysics $\rightarrow$ collective dielectric response $\rightarrow$ field evolution $\rightarrow$ radiative observables

In the classical formulation, extinction appears as an imposed coefficient, whereas in the C^4 interpretation it emerges as a direct consequence of the electromagnetic response of the medium. This distinction becomes critical at millimeter-wave frequencies, where small changes in hydrometeor phase or structure produce large radiative effects [30, 84].

2.6.4 Treatment of Non-Stationarity and Memory

Classical RT formulations typically assume atmospheric profiles to be static during propagation, addressing variability only through ensemble averaging or time-dependent boundary conditions. In contrast, the C^4 Framework explicitly accommodates medium evolution by interpreting attenuation and phase variability as manifestations of evolving dielectric structure rather than purely stochastic noise. This perspective is particularly relevant for convective systems, melting layers, and rapidly evolving cloud fields, where temporal decorrelation and radiometric fluctuations cannot be adequately captured by stationary models [84, 177].

2.6.5 Computational and Conceptual Trade-offs

From a computational standpoint, classical RT solvers offer high physical fidelity but limited scalability when angular resolution, frequency coverage, or ensemble size increases [78].

The C^4 Framework, by contrast, is designed to:

- reduce dimensionality by collapsing angular complexity into effective-medium properties,
- preserve physical interpretability through explicit microphysical links,
- enable integration with statistical and physics-informed machine-learning surrogates.

Importantly, the C^4 approach does not eliminate the need for RT solvers. Models such as SNEM and SNEM-2 can be interpreted as reduced-order numerical realizations of the C^4 Field under constrained geometrical and statistical assumptions.

2.6.6 Role and Limitations of Mie Theory

Mie theory provides the analytical solution of Maxwell's equations for scattering by homogeneous spherical particles and represents the electromagnetic backbone of most atmospheric attenuation models [4, 30, 177]. It enables the computation of extinction, absorption, and scattering efficiencies that feed radiative-transfer solvers operating at microwave and millimeter-wave frequencies.

Nevertheless, its assumptions of sphericity, homogeneity, and isotropy are only partially satisfied in real atmospheric conditions. Irregular ice particles, graupel, and melting aggregates exhibit complex morphologies and dielectric heterogeneity that induce scattering behaviours not captured by classical Mie theory, particularly at Ka-, V-, and W-band frequencies [84, 177].



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2.6.7 Beyond Mie: Hybrid and Statistical Scattering Models

To address these limitations, more general electromagnetic formulations—such as T-matrix or discrete-dipole methods—have been proposed, albeit at a high computational cost [30, 177]. As an alternative, hybrid statistical–numerical approaches aim to reproduce the aggregate radiative effects of complex scattering environments without resolving individual particle geometries.

Within this dissertation, this transition is realized through the SNEM and SNEM-2 frameworks, which combine Mie-based microphysical inputs with statistical radiative-transfer formulations. By sampling hydrometeor populations and thermodynamic profiles, these models capture bulk attenuation and brightness-temperature variability in a physically consistent yet computationally scalable manner [ArtA1, ArtB1].

This hybrid perspective establishes a conceptual bridge between classical electromagnetic scattering theory and the advanced radiative-transfer and data-driven frameworks developed in the subsequent chapters.

2.7 Radiative Transfer Equation in Scattering Media

The propagation of electromagnetic radiation through the atmosphere is governed by the Radiative Transfer Equation (RTE), which provides a macroscopic description of the combined effects of absorption, scattering, and thermal emission in a participating medium. In the context of atmospheric radiowave propagation, the RTE constitutes the theoretical backbone for modeling attenuation and brightness temperature under both clear-sky and precipitating conditions. In a plane-parallel, vertically stratified atmosphere, the monochromatic radiative transfer equation can be written as:

$$\mu \frac{dI(z, \mu, \nu)}{dz} = -k_{\text{ext}}(z, \nu) I(z, \mu, \nu) + k_{\text{sca}}(z, \nu) \int_{-1}^1 P(\mu, \mu') I(z, \mu', \nu) d\mu' + j(z, \nu), \quad (2.11)$$

where $I(z, \mu, \nu)$ is the specific intensity at altitude z , frequency ν , and direction cosine $\mu = \cos \theta$. The extinction coefficient $k_{\text{ext}} = k_{\text{abs}} + k_{\text{sca}}$ represents the combined effects of absorption and scattering, while $P(\mu, \mu')$ is the normalized scattering phase function describing angular redistribution of radiation. The source term $j(z, \nu)$ accounts for thermal emission by atmospheric gases and hydrometeors.

The RTE explicitly links microscopic electromagnetic interactions to macroscopic observables through the optical properties of the medium.

Absorption processes remove energy from the propagating beam, scattering redistributes energy across directions, and emission injects energy according to the local thermodynamic state. This balance determines the emergent brightness temperature measured by radiometers and the attenuation experienced by Earth–space communication links. A central quantity in radiative-



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transfer theory is the optical depth, defined as:

$$\tau(z, \nu, \theta) = \int_z^{+\infty} \frac{k_{\text{ext}}(z', \nu)}{\mu} dz', \quad (2.12)$$

which quantifies the cumulative interaction probability along the propagation path. In the optically thin regime ($\tau \ll 1$), radiation propagates with minimal interaction, while in optically thick conditions ($\tau \gg 1$) multiple scattering and absorption dominate the radiative balance. The formal solution of the RTE along a slant path can be expressed as:

$$I(0, \mu, \nu) = I_{\text{TOA}}(\mu, \nu) e^{-\tau(0, \nu, \theta)} + \int_0^{+\infty} j(z, \nu) e^{-\tau(z, \nu, \theta)} dz, \quad (2.13)$$

where I_{TOA} denotes the incident radiation at the top of the atmosphere. This expression highlights the two fundamental contributors to the observed signal: the attenuated background radiation and the cumulative emission from atmospheric layers weighted by their transmittance. In practical applications, the specific intensity is commonly expressed in terms of brightness temperature T_B , defined through the Rayleigh–Jeans or Planck relation as:

$$T_B(\nu, \theta) = \frac{c^2}{2k_B\nu^2} I(\nu, \theta), \quad (2.14)$$

or, in its exact quantum form,

$$T_B(\nu, \theta) = \frac{h\nu}{k_B} \left[\ln \left(1 + \frac{2h\nu^3}{c^2 I(\nu, \theta)} \right) \right]^{-1}. \quad (2.15)$$

These relations establish the direct connection between radiative transfer theory and the measurable quantities used in satellite remote sensing and atmospheric attenuation studies. They also provide the theoretical basis for the approximate and statistical solvers discussed in the following sections, including the two-stream Eddington approximation and the SNEM/SNEM-2 framework adopted in this work.

2.7.1 Two-Stream and Eddington Approximations

While the full radiative transfer equation provides a physically exact description of electromagnetic propagation in scattering media, its numerical solution remains computationally expensive due to the continuous angular dependence of the radiance field. This limitation is particularly restrictive in large-scale simulations, ensemble studies, and applications requiring near-real-time performance, such as satellite communication planning or global attenuation mapping [4,30,78]. To reduce computational complexity while preserving physical consistency, angular approximations are commonly introduced. Among these, the two-stream approximation represents one of the most widely adopted approaches in atmospheric radiative transfer. In the two-stream formulation, the angular dependence of the radiance is reduced to two representative fluxes: an upward-



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propagating intensity $I^+(z, \nu)$ and a downward-propagating intensity $I^-(z, \nu)$, corresponding to fixed zenith angles $\mu = \pm\mu_0$. The radiative transfer equation is then projected onto these two directions, yielding a coupled system of ordinary differential equations:

$$\pm\mu_0 \frac{dI^\pm(z, \nu)}{dz} = -k_{\text{ext}}(z, \nu) I^\pm(z, \nu) + k_{\text{sca}}(z, \nu) J(z, \nu) + j(z, \nu), \quad (2.16)$$

where $J(z, \nu)$ denotes the mean intensity, approximated as:

$$J(z, \nu) = \frac{1}{2} [I^+(z, \nu) + I^-(z, \nu)]. \quad (2.17)$$

The **Eddington approximation** constitutes a specific closure of the two-stream system. It assumes a linear angular dependence of the intensity and enforces consistency between the first two angular moments of the radiative transfer equation. Within this framework, the choice $\mu_0 = 1/\sqrt{3}$ ensures that the second angular moment of the radiance is correctly reproduced for isotropic scattering. Under the Eddington closure, the coupled two-stream equations can be rewritten in terms of the sum and difference of the intensities, corresponding to the radiative flux and net energy flow. The resulting system admits analytical or semi-analytical solutions for layered atmospheres, enabling efficient computation of brightness temperature, optical depth, and transmittance. A key advantage of the Eddington approximation lies in its explicit dependence on three macroscopic optical parameters: the extinction coefficient k_{ext} , the single-scattering albedo ω , and the asymmetry factor g . These quantities are defined as:

$$\omega = \frac{k_{\text{sca}}}{k_{\text{ext}}}, \quad g = \langle \cos \theta \rangle, \quad (2.18)$$

and encapsulate the essential microphysical information required to describe scattering and absorption processes. As a result, complex hydrometeor populations can be represented through vertically varying profiles of $(k_{\text{ext}}, \omega, g)$, without resolving the full angular structure of the radiance field. Despite its reduced angular fidelity, the Eddington approximation captures the dominant radiative behavior in stratified atmospheres, particularly under conditions of moderate optical thickness and weakly anisotropic scattering. Numerous studies have demonstrated that, for microwave and millimetre-wave propagation, two-stream schemes provide accurate estimates of attenuation and brightness temperature when compared with multi-stream solvers, at a fraction of the computational cost. These properties make the Eddington approximation especially well suited for statistical and ensemble-based radiative-transfer models. In this thesis, it constitutes the mathematical core of the SNEM and SNEM-2 frameworks, enabling the simulation of large numbers of atmospheric realizations and supporting subsequent integration with machine-learning surrogates. The next section builds upon this approximation to introduce the statistical-numerical formulation adopted in SNEM.



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2.7.2 Statistical Radiative Transfer and Ensemble Modeling

The deterministic formulation of radiative transfer, even when reduced through angular approximations such as the Eddington two-stream scheme, implicitly assumes that the atmospheric medium is fully specified and deterministic. In practice, however, the troposphere exhibits strong stochastic variability at spatial and temporal scales that are not resolved by standard meteorological inputs. Cloud microphysics, hydrometeor spatial distribution, and phase transitions introduce sub-grid variability that cannot be uniquely prescribed, particularly in precipitation and mixed-phase regimes.

Statistical radiative transfer addresses this limitation by explicitly treating the atmospheric state as a random field rather than a single deterministic realization. Within this paradigm, optical properties such as extinction coefficient $k_{\text{ext}}(z, \nu)$, single-scattering albedo $\omega(z, \nu)$, and asymmetry factor $g(z, \nu)$ are modeled as stochastic variables drawn from physically constrained probability distributions. Radiative observables—brightness temperature, attenuation, and delay—are then interpreted as ensemble statistics over a population of atmospheric realizations.

Formally, let $\chi(z)$ denote the vector of atmospheric state variables governing radiative transfer, including temperature, humidity, hydrometeor content, and phase composition. The radiative output $\mathcal{R}$ becomes a random variable defined as:

$$\mathcal{R} = \mathcal{F}(\chi(z)), \quad (2.19)$$

where $\mathcal{F}$ represents the radiative-transfer operator. The expected value and higher-order moments of $\mathcal{R}$ are then given by:

$$\langle \mathcal{R} \rangle = \int \mathcal{F}(\chi) p(\chi) d\chi, \quad (2.20)$$

with $p(\chi)$ denoting the joint probability density of the atmospheric state.

In practical implementations, this integral is approximated through Monte Carlo sampling. A finite ensemble of atmospheric realizations $\{\chi_n\}_{n=1}^N$ is generated according to prescribed microphysical and thermodynamic statistics, and the ensemble mean radiative response is computed as:

$$\langle \mathcal{R} \rangle \approx \frac{1}{N} \sum_{n=1}^N \mathcal{F}(\chi_n). \quad (2.21)$$

This statistical approach offers several advantages. First, it naturally accounts for sub-grid variability and non-linear radiative effects, which are particularly pronounced in scattering-dominated regimes such as stratiform precipitation and melting layers. Second, it provides direct access to uncertainty metrics—variance, quantiles, and confidence intervals—that are not available in purely deterministic models. Third, it enables the generation of climatologically representative radiative fields suitable for global mapping and long-term statistical analyses.

However, full Monte Carlo radiative-transfer simulations based on high-fidelity solvers remain computationally prohibitive when applied to large ensembles or wide frequency ranges. This limitation motivates the adoption of reduced-order radiative solvers within a statistical frame-



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work. In this context, the combination of ensemble sampling with the Eddington two-stream approximation represents a powerful compromise between physical realism and computational scalability.

The Statistical Numerical Electromagnetic Model (SNEM) embodies this philosophy. By embedding a two-stream radiative-transfer core within a stochastic description of cloud and precipitation structure, SNEM enables the efficient simulation of ensemble-averaged radiative quantities while retaining explicit links to microphysical parameters. This statistical–numerical formulation forms the conceptual bridge between classical radiative transfer and the hybrid physical–AI frameworks introduced later in the thesis.

The following section introduces the original SNEM model in detail, highlighting its statistical assumptions, numerical structure, and limitations that motivated its subsequent extension to SNEM-2.

2.7.3 The Statistical Numerical Electromagnetic Model (SNEM)

The Statistical Numerical Electromagnetic Model (SNEM), also referred to as the Sky Noise Eddington Model, was originally conceived to provide a computationally efficient yet physically grounded tool for simulating radiowave propagation through hydrometeor-laden atmospheric columns.

Unlike purely empirical attenuation laws, SNEM explicitly resolves the radiative-transfer process while incorporating stochastic representations of atmospheric variability.

At its core, SNEM combines three fundamental elements:

- (i) a reduced-order radiative-transfer solver based on the Eddington two-stream approximation,
- (ii) a statistical description of cloud and precipitation structure, and
- (iii) a numerical framework capable of generating ensemble realizations of atmospheric profiles.

The atmospheric column is discretized into a finite number of horizontal layers, each characterized by thermodynamic variables $T(z)$, $p(z)$, and $q(z)$, together with microphysical parameters describing hydrometeor content and phase. Rather than assuming a single deterministic profile, SNEM generates multiple realizations of these quantities by sampling from physically constrained probability distributions derived from climatology or observational datasets.

For each realization, the radiative-transfer problem is solved using the two-stream Eddington approximation. The extinction coefficient at height z and frequency ν is expressed as:

$$k_{\text{ext}}(z, \nu) = \sum_i N_i(z) \sigma_{\text{ext},i}(\nu), \quad (2.22)$$

where $N_i(z)$ denotes the number concentration of the i -th hydrometeor class and $\sigma_{\text{ext},i}(\nu)$ its corresponding extinction cross section. Absorption and scattering contributions are combined through:

$$k_{\text{ext}} = k_{\text{abs}} + k_{\text{sca}}, \quad \omega = \frac{k_{\text{sca}}}{k_{\text{ext}}}. \quad (2.23)$$



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The emergent brightness temperature along a propagation path defined by the zenith angle θ is obtained by integrating the radiative source term weighted by the optical depth:

$$T_B(\nu, \theta) = \int_0^\infty B_\nu(T(z)) \exp[-\tau(z, \nu, \theta)] k_{\text{ext}}(z, \nu) dz, \quad (2.24)$$

with the optical depth defined as:

$$\tau(z, \nu, \theta) = \int_z^\infty \frac{k_{\text{ext}}(z', \nu)}{\mu} dz', \quad \mu = \cos \theta.$$

Similarly, the path-integrated attenuation (PIA) is computed as:

$$\text{PIA}(\nu, \theta) = 4.343 \int_0^H \frac{k_{\text{ext}}(z, \nu)}{\mu} dz, \quad (2.25)$$

where H denotes the effective top of the atmospheric column. A defining feature of SNEM is its ensemble-based interpretation of these quantities. Brightness temperature, attenuation, and related observables are not treated as single deterministic outputs, but as statistical distributions resulting from multiple realizations of cloud structure and microphysics. Ensemble averaging yields expected values, while the spread across realizations provides a natural estimate of uncertainty.

This statistical–numerical strategy allows SNEM to capture the impact of sub-grid variability, phase transitions, and heterogeneous cloud structures without explicitly resolving three-dimensional geometry or full angular scattering. As a result, SNEM achieves a favorable balance between physical realism and computational efficiency, making it particularly suitable for parametric studies, sensitivity analyses, and the generation of synthetic datasets.

Nevertheless, the original SNEM formulation exhibits intrinsic limitations. Its frequency range was initially restricted to microwave bands below approximately 50 GHz, and its treatment of hydrometeor phase and angular dependence remained simplified. These constraints motivated the development of an extended framework—SNEM-2—which incorporates broader spectral coverage, slant-path geometry, and refined microphysical parameterizations. The transition from SNEM to SNEM-2 is discussed in the following section.

2.7.4 Limitations of the Original SNEM and Motivation for SNEM-2

Despite its effectiveness as a statistical radiative-transfer model, the original SNEM formulation exhibits several structural limitations that restrict its applicability in modern high-frequency and operational contexts. These limitations primarily reflect the simplifying assumptions adopted to ensure computational tractability and the limited availability of high-resolution atmospheric datasets at the time of its original development.

A first limitation concerns spectral coverage. SNEM was originally conceived for microwave frequencies below approximately 50 GHz, where absorption and scattering processes vary smoothly



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with frequency. At higher frequencies—particularly in the Ka-, V-, and W-bands—radiative extinction becomes strongly sensitive to hydrometeor microphysics, phase composition, and dielectric dispersion. In these regimes, simplified parameterizations lead to systematic biases in attenuation and brightness-temperature estimates [30, 84, 177].

A second limitation is related to propagation geometry. The original SNEM formulation was optimized for near-zenithal propagation and did not explicitly resolve slant-path geometries relevant to Earth–space links at low elevation angles. As a consequence, elevation-dependent path-length amplification and anisotropic scattering effects were only implicitly represented, limiting applicability to realistic satellite communication scenarios [30].

A third limitation concerns the representation of vertical structure and hydrometeor phase. While the statistical description captures first-order cloud variability, mixed-phase regions and melting-layer dynamics were treated in a highly simplified manner. These processes are known to exert a dominant influence on millimetre-wave propagation, where small changes in phase composition can produce large variations in extinction and emission [84, 177].

Further constraints arise from data integration. The original SNEM relied on idealized or climatological input profiles and lacked direct coupling with modern reanalysis products and multi-satellite observations. This limited its ability to represent geographically and seasonally specific atmospheric conditions and to support rigorous validation against contemporary datasets [30].

Finally, although SNEM provides ensemble statistics of radiative quantities, its original architecture was not designed to support inverse modelling or machine-learning integration. The absence of standardized interfaces for optical parameters and ensemble outputs restricted its suitability as a training or benchmarking tool for data-driven approaches. These limitations motivated the development of SNEM-2 within the scope of this doctoral research. The upgraded framework extends spectral coverage into the W-band, introduces explicit slant-path geometry, implements refined phase-aware microphysical parameterizations, and enables direct integration with ERA5 reanalysis and satellite observations [ArtA1, ArtA3]. Moreover, SNEM-2 is explicitly designed to function as a physically consistent data generator for machine-learning surrogates, bridging deterministic radiative-transfer theory and data-driven emulation [ArtB1]. The following section introduces the SNEM-2 framework and outlines its architectural evolution and role within the hybrid modelling strategy adopted in this thesis.

2.7.5 The SNEM-2 Framework: Architecture and Physical Enhancements

The SNEM-2 framework represents a substantial evolution of the original Sky-Noise Eddington Model, developed to overcome the physical, computational, and operational limitations discussed in the previous section. Rather than a simple extension, SNEM-2 constitutes a re-engineered statistical radiative-transfer platform explicitly designed for millimetre-wave propagation, multi-phase hydrometeor representation, and hybrid physical–data-driven workflows [ArtA1, ArtA3].

At the architectural level, SNEM-2 preserves the statistical–numerical philosophy of SNEM



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while introducing a modular design that separates atmospheric state description, electromagnetic interaction, and radiative closure. This separation enables independent refinement, validation, and coupling of each component with external datasets and surrogate models, enhancing flexibility and long-term extensibility.

The atmospheric input layer is driven by vertically resolved thermodynamic and microphysical profiles derived from modern reanalysis products, in particular ERA5. Profiles of temperature, pressure, water-vapour content, and hydrometeor mixing ratios are interpolated onto a refined vertical grid, enabling a realistic representation of stratification, cloud layering, and phase transitions. This approach replaces the idealized climatological profiles adopted in the original SNEM and anchors the model to physically consistent atmospheric states [30].

Electromagnetic interaction with the medium is described through frequency-dependent extinction, absorption, and scattering properties. For each hydrometeor class (cloud droplets, rain, snow, graupel, and ice), optical parameters are derived from Mie-based or parameterized electromagnetic models, accounting explicitly for temperature-dependent dielectric properties. This formulation ensures spectral consistency across the Ka-, V-, and W-bands, where microphysical sensitivity becomes critical [30, 84, 177].

A key enhancement introduced in SNEM-2 is the explicit treatment of mixed-phase and melting-layer regions. Phase partitioning is governed by local thermodynamic conditions, allowing smooth transitions between liquid and ice optical properties. This phase-aware formulation enables the model to reproduce characteristic millimetre-wave signatures, such as enhanced attenuation and brightness-temperature anomalies associated with melting layers [84, 177].

Radiative transfer is solved using the two-stream Eddington approximation, which provides a physically consistent balance between absorption, emission, and scattering while maintaining computational efficiency. Unlike full multi-stream solvers, this approach enables large-scale ensemble simulations and sensitivity analyses, which are essential for global attenuation mapping and statistical characterization of propagation effects [30,78]. At the same time, the physical interpretation of the governing optical parameters—extinction coefficient, single-scattering albedo, and asymmetry factor—is preserved.

A further defining feature of SNEM-2 is its data-centric interoperability. Model outputs, including specific attenuation, path-integrated attenuation, brightness temperature, and zenith delay components, are produced in standardized formats. This design facilitates direct integration with satellite observations, numerical weather prediction products, and machine-learning pipelines, transforming SNEM-2 from a standalone simulator into a central component of a hybrid modelling ecosystem [ArtB1].

In summary, SNEM-2 extends the original SNEM framework along four principal axes: spectral scalability, phase-aware microphysics, geometric flexibility, and interoperability with modern datasets and surrogate models. These enhancements establish SNEM-2 as a physically grounded yet operationally efficient radiative-transfer model, forming the computational backbone of the hybrid physical–AI strategy developed in the remainder of this thesis.



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2.7.6 Role of SNEM-2 within the Thesis Framework

Within this dissertation, SNEM-2 plays a central and unifying role, serving as the physical backbone upon which all subsequent modelling, validation, and hybridization strategies are constructed. Rather than being treated as a standalone radiative-transfer solver, SNEM-2 is explicitly positioned as a multi-purpose framework that connects theoretical formulations, observational datasets, and data-driven surrogates within a coherent and physically consistent workflow [ArtA1, ArtA3]

First, SNEM-2 functions as the primary forward radiative-transfer model used to simulate attenuation, emission, and phase-delay processes under realistic atmospheric conditions. Its phase-aware microphysical treatment, spectral scalability up to W-band frequencies, and explicit support for slant-path geometries enable the reproduction of propagation effects that are not captured by classical empirical formulations. In this role, SNEM-2 provides physically interpretable reference solutions against which simplified models and operational standards can be critically evaluated [30, 84, 177].

Second, SNEM-2 acts as a generator of physically consistent synthetic datasets. By combining reanalysis-driven thermodynamic profiles with stochastic representations of hydrometeor variability, the model produces large ensembles of atmospheric realizations that preserve vertical coherence, energy balance, and spectral consistency. These ensembles constitute the training and benchmarking foundation for the machine-learning surrogates developed in later chapters, including LSTM-, CGAN-, and transformer-based architectures [ArtA1, ArtB2]. Importantly, the use of SNEM-2-generated data ensures that learning is anchored to radiative-transfer physics rather than to purely statistical correlations.

Third, SNEM-2 provides a controlled environment for model intercomparison and validation. Throughout the thesis, its outputs are systematically compared with empirical ITU-R predictions, satellite observations (CloudSat, GPM, TRMM), and alternative radiative-transfer solvers. This comparative role enables the identification of structural biases, frequency-dependent discrepancies, and phase-related limitations in both empirical and data-driven approaches, positioning SNEM-2 as a reference model for diagnostic analysis [30, 78].

Fourth, SNEM-2 constitutes the physical anchor of the C^4 framework introduced later in the thesis. The extinction coefficient, single-scattering albedo, and asymmetry factor computed by SNEM-2 populate the effective dielectric and radiative fields of the C^4 formulation. In this sense, SNEM-2 provides continuity between classical radiative-transfer theory and the generalized field-based interpretation adopted in the C^4 approach, ensuring consistency between deterministic physics and hybrid physical–AI abstractions.

Finally, SNEM-2 supports the transition from descriptive modelling to operational applicability. Its computational efficiency, modular architecture, and standardized outputs make it suitable for global attenuation mapping, ensemble-based uncertainty analysis, and near-real-time applications when coupled with surrogate models. This capability underpins the development of climate-aware attenuation products and satellite communication performance assessments presented in the later



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chapters of this thesis [ArtA1, ArtB1].

In summary, SNEM-2 is not merely one component among others, but the structural core of the dissertation. It enables the integration of theory, observation, and machine learning within a unified modelling strategy, ensuring that all subsequent developments remain grounded in physically consistent radiative-transfer principles.

2.8 Transition from SNEM-2 to Physics-Informed AI Surrogates

The development of SNEM-2 constitutes a major step toward physically consistent modelling of radiowave propagation in complex atmospheric conditions. Despite its reduced computational cost compared with full multi-stream or Monte Carlo radiative-transfer solvers, SNEM-2 remains limited when large ensembles, multi-frequency analyses, or global-scale applications are required, particularly in near-real-time or inverse-modelling scenarios [30,78]. This motivates the transition from purely deterministic radiative-transfer modelling toward physics-informed artificial intelligence surrogates. These models are not intended to replace SNEM-2, but to inherit its physical structure, constraints, and statistical behavior, using SNEM-2 both as a reference solver and as a physically consistent data generator [ArtB1].

In this framework, machine learning operates within a solution space defined by radiative-transfer physics rather than as an unconstrained black-box approximation. The surrogate strategy adopted in this thesis follows a physics-guided learning paradigm, in which neural architectures are constrained through physically meaningful inputs, targets, and regularization principles [ArtB1, ArtB2].

Examples include monotonic dependence on hydrometeor content, energy-consistent brightness-temperature response, and vertical coherence across atmospheric layers. These constraints reduce overfitting, improve generalization across climatic regimes, and preserve interpretability. Within this framework, SNEM-2 provides the forward mapping:

$$\mathcal{F} : \mathbf{x} \rightarrow \mathbf{y}, \quad (2.26)$$

where the input vector $\mathbf{x}$ includes thermodynamic profiles, hydrometeor distributions, frequency, and propagation geometry, and the output vector $\mathbf{y}$ contains radiative and propagation quantities such as brightness temperature $T_B(\nu, \theta)$, specific attenuation $\gamma(z, \nu)$, and path-integrated attenuation $\text{PIA}(\nu)$. Physics-informed surrogates are trained to approximate a reduced-order mapping:

$$\hat{\mathcal{F}} \approx \mathcal{F}, \quad (2.27)$$

while preserving the structural dependencies encoded in SNEM-2. Different surrogate families address complementary aspects of the problem. Recurrent architectures (e.g. LSTM) capture temporal variability and memory effects associated with atmospheric evolution, while conditional generative models (CGANs) reproduce spatial and vertical variability of hydrometeor and radiative



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fields, enabling probabilistic ensemble generation [ArtB1]. Transformer-based architectures further extend this capability to long-range spatio-temporal and multi-scale interactions.

Crucially, surrogate training and validation are anchored to multi-satellite observations, including CloudSat and GPM, through systematic comparison with SNEM-2 simulations [30,84,177]. Only physically validated realizations are retained, ensuring that learned models encode realistic radiative behaviour rather than numerical artefacts.

The transition from SNEM-2 to physics-informed AI surrogates therefore represents a shift in modelling strategy rather than a methodological discontinuity. Deterministic radiative-transfer physics defines the admissible solution space, while machine learning provides an efficient and scalable approximation within that space. This hybrid paradigm enables near-real-time inference, global attenuation mapping, and uncertainty-aware applications, forming the conceptual bridge to the surrogate methodologies developed in the following chapters.

2.8.1 Statistical and Generative Representations of Cloud Fields

Recent advances in data-driven modelling have introduced generative approaches capable of representing complex atmospheric structures beyond the scope of analytic parameterizations. In particular, probabilistic and generative frameworks provide a means to emulate cloud-field variability while preserving multivariate consistency and spatial coherence [ArtA1, ArtB1].

Within this thesis, such approaches are not treated as substitutes for physical modelling, but rather as complementary representations of unresolved or computationally expensive microphysical structures. Their theoretical relevance lies in the possibility of learning conditional distributions constrained by large-scale atmospheric state variables, thereby extending the applicability of deterministic radiative-transfer solvers to ensemble-based and climate-scale analyses [ArtA3, ArtB2].

While physics-based solvers remain the reference for forward modelling, these concepts motivate the introduction of physically constrained surrogate representations that preserve the structure of the underlying operators. A schematic illustration of the conditional generative paradigm adopted later in this work is provided in **Fig. 2.10** while its operational implementation and validation within the SNEM-2 framework are deliberately deferred to subsequent chapters.

2.9 Summary and Transition to Chapter 3

Section 2.7 has introduced the **Sky-Noise Eddington Model (SNEM)** and its extended formulation, **SNEM-2 (Statistical Numerical Electromagnetic Model-2)**, as the central physical modelling framework adopted in this dissertation for the simulation of radiowave propagation under non-clear-sky conditions. By combining radiative-transfer theory, hydrometeor microphysics, and stochastic variability within a computationally tractable architecture, SNEM-2 occupies an intermediate position between empirical attenuation laws and full-physics radiative-transfer solvers. The discussion highlighted how SNEM-2 extends the original SNEM paradigm

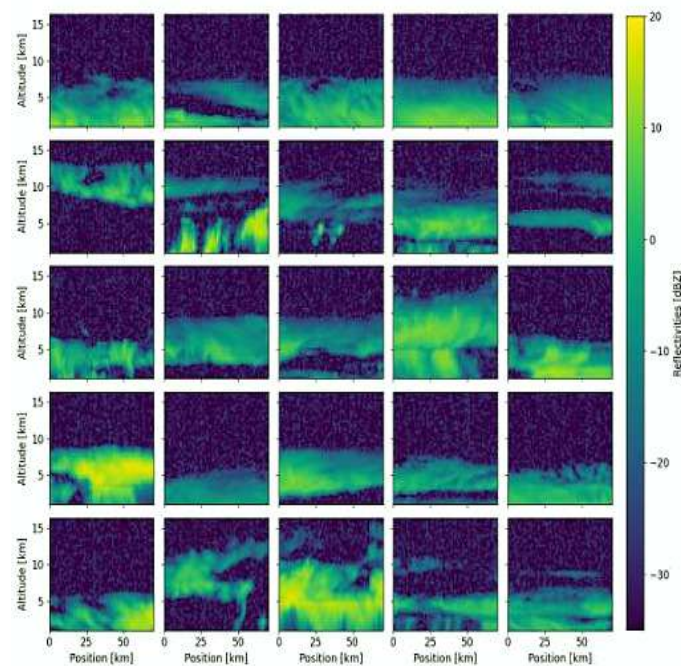


Figure 2.9. Realistic Cloud Structures Generated by CGAN Trained on CloudSat CPR Data.

Example of 25 different real scenes consisting of vertical radar profiles from the CPR instrument onboard the CloudSat satellite. The reflectivity values range from -35 dBZ to $+20$ dBZ, covering a variety of atmospheric conditions and cloud morphologies. These vertical profiles are used as training targets for a Conditional Generative Adversarial Network (CGAN), which learns to generate physically consistent cloud scenes conditioned on atmospheric input data. The image demonstrates the CGAN's capability to replicate the spatial variability and complexity of hydrometeor distributions, including multilayer clouds and embedded convective structures. Credits: Rebecka Rilemark and Carl Svensson, Department of Space, Earth and Environment, Chalmers University of Technology, Gothenburg, Sweden, 2020.



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through multi-frequency capability up to the W-band, slant-path propagation, phase-aware extinction modelling, and dynamic thermodynamic layering derived from reanalysis data. These advancements enable the model to capture key physical processes neglected by standardized empirical approaches, including mixed-phase precipitation, melting-layer effects, and frequency-dependent scattering and emission mechanisms.

Validation against multi-satellite observations further demonstrated the model's ability to reproduce both brightness-temperature and path-integrated attenuation statistics with high quantitative fidelity. Beyond its role as a forward radiative-transfer solver, SNEM-2 was shown to function as a unifying physical backbone for subsequent developments in this work. Its statistically driven structure supports ensemble simulations, uncertainty analysis, and global-scale mapping, while its modular design enables seamless coupling with machine-learning surrogates.

In this sense, SNEM-2 defines the physically admissible solution space within which reduced-order and data-driven models are trained, tested, and interpreted. The introduction of the C^4 Field and C^4 Framework further generalized this perspective by embedding radiative transfer, dielectric theory, and thermodynamic variability into a single conceptual structure. Within this framework, SNEM-2 provides the deterministic closure linking microphysical processes to observable radiometric quantities, while artificial intelligence enables scalable approximation and inference.

Having established the theoretical foundations, modelling assumptions, and physical constraints underlying SNEM and SNEM-2, the dissertation now transitions from theory to implementation. **Chapter 3** introduces the data sources, computational infrastructure, and preprocessing strategies adopted in this work. Particular emphasis is placed on the integration of reanalysis products, satellite observations, and numerical pipelines that supply physically consistent inputs to the modelling frameworks developed in **Chapters 2 and 4**. This transition marks a shift from conceptual and theoretical analysis to practical realization. The following chapter details how the physical principles articulated here are operationalized through data ingestion, coincidence analysis, numerical experimentation, and reproducible computational workflows, thereby enabling the quantitative results and validation studies presented in the remainder of the thesis.



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Chapter 3

Data and Tools

3.1 Role of Data and Tools

The scientific objectives of this dissertation—namely the physically consistent modelling of radiowave propagation under non-clear-sky conditions and the development of hybrid physical–statistical surrogates—require a rigorous and carefully structured data foundation. Unlike purely theoretical investigations, this work relies on the integration of heterogeneous observational datasets, numerical reanalysis products, and physically grounded modelling tools. Consequently, the quality, coherence, and representativeness of the adopted data sources directly determine the reliability and reproducibility of the results presented in the following chapters [4,30].

In atmospheric propagation studies, data selection is a critical modelling choice. Radiative-transfer solvers and electromagnetic scattering models are highly sensitive to thermodynamic profiles, hydrometeor distributions, and vertical stratification. Small inconsistencies in temperature, humidity, or phase partitioning can propagate into significant biases in attenuation and brightness-temperature simulations, especially at Ka-, V-, and W-band frequencies [30,84,177]. For this reason, the data strategy adopted in this thesis is explicitly designed to preserve physical consistency across scales, frequencies, and modelling stages.

The data ecosystem employed in this work is structured around three complementary pillars. The first pillar consists of global atmospheric reanalysis products, which provide physically consistent, time-resolved descriptions of the tropospheric thermodynamic state and form the backbone of the vertical structure used by the SNEM and SNEM-2 models [ERA5]. The second pillar comprises satellite-based observations, including active and passive sensors, which provide direct measurements of cloud structure, precipitation, and radiometric quantities. These datasets are used both for validation and for constraining statistical and machine-learning surrogates [CloudSat, GPM, TRMM]. The third pillar is represented by numerical and computational tools, which enable the ingestion, harmonization, simulation, and analysis of large atmospheric datasets in a reproducible and physically interpretable manner [78].

Within this framework, data are not treated as passive inputs to black-box algorithms. Reanalysis profiles are transformed into frequency-dependent extinction and emission properties

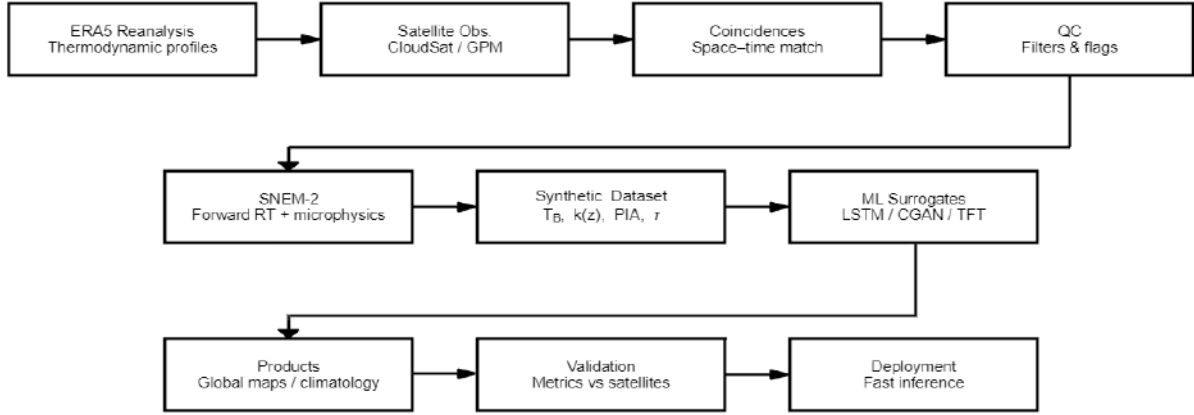


Figure 3.1. End-to-end data and modelling pipeline adopted in Chapter 3. ERA5 thermodynamic profiles and satellite observations from CloudSat and GPM are combined through space-time coincidence and quality-control procedures. The resulting atmospheric scenes are processed by the SNEM-2 forward radiative-transfer model to generate synthetic datasets of brightness temperature, specific attenuation, and path-integrated attenuation. These datasets are used both for global product generation and for training machine-learning surrogates, which are subsequently validated against satellite observations and deployed for fast inference.

through physically motivated operators rooted in radiative-transfer and electromagnetic scattering theory [4,30], while satellite observations are interpreted in light of propagation geometry, measurement uncertainty, and instrumental limitations. This ensures that data-driven components introduced later in the thesis operate within a physically admissible solution space.

A key methodological choice is the clear separation between theoretical foundations (**Chapter 2**), data and tools (**Chapter 3**), and modelling and results (**Chapters 4** onward). **Chapter 3** does not present numerical outcomes or validation metrics; its purpose is to document, with full transparency, the origin, structure, and treatment of the datasets and computational tools employed throughout the work, thereby ensuring reproducibility and independent assessment of subsequent modelling assumptions [4].

Finally, this chapter establishes the operational bridge between theory and implementation. The physical concepts introduced in **Chapter 2**—radiative transfer, scattering theory, and statistical electromagnetic modelling—are translated into concrete data structures, preprocessing pipelines, and computational workflows. These elements constitute the practical substrate upon which the SNEM-2 framework and its physics-informed machine-learning extensions are developed [30,78].

Figure 3.1 summarizes the complete processing chain, from atmospheric reanalysis and satellite observations to radiative-transfer modelling, machine-learning surrogates, validation, and deployment.



3.2 Atmospheric Reanalysis Data: ERA5

3.2.1 ERA5 Overview and Rationale

Global atmospheric reanalyses provide physically consistent reconstructions of the atmospheric state by combining numerical weather prediction with heterogeneous observations. In this thesis, ERA5 is adopted as the primary source of large-scale thermodynamic information because it offers global consistency, high temporal sampling, and vertically resolved profiles on multiple pressure levels, enabling the analysis of diurnal variability and synoptic evolution in a uniform modelling framework [12].

Methodologically, ERA5 represents a practical compromise between realism and feasibility: point observations are accurate but sparse in space and height, while purely satellite-derived profiles may be ambiguous under clouds and precipitation. ERA5 supplies a dynamically consistent atmospheric description that can be used as input to forward electromagnetic modelling without being interpreted as a predictive attenuation model. In particular, ERA5 is used here as a physically constrained descriptor providing the state variables required by SNEM/SNEM-2 radiative-transfer simulations, whose purpose is to compute propagation and radiometric quantities given an atmospheric scene [12, 334, 337].

3.2.2 Variables and Physical Meaning

ERA5 variables are selected based on their direct role in microwave/millimetre-wave absorption, emission, and refractivity. Temperature and pressure profiles define density structure and thermal emission, while humidity-related variables control gaseous absorption dominated by water vapour and oxygen features. These processes are modelled consistently using established absorption formulations and standards [1,16,17].

Hydrometeor-related fields (e.g., cloud liquid/ice water content and precipitation proxies) are included as statistical descriptors of vertical structure rather than as exact microphysical truth. This interpretation is consistent with the SNEM-2 philosophy, where microphysical variability is represented stochastically and the radiative-transfer response is evaluated as an ensemble property [159,337].

Geopotential height and surface pressure are used to maintain consistency between pressure-level information and altitude-based propagation geometry, which is essential when translating layered atmospheric structure into path-integrated optical depth and attenuation [12].

3.2.3 Vertical Interpolation and Layering

SNEM and SNEM-2 require atmospheric profiles on a vertical grid suitable for numerical integration of extinction, scattering, and emission across layers. ERA5 provides discrete pressure levels; therefore, profiles are transformed into a height-resolved representation and interpolated onto a refined vertical grid that preserves boundary-layer structure, cloud boundaries, and melting-layer transitions while avoiding numerical artefacts from overly aggressive smoothing.



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Layer thickness is chosen to increase resolution where gradients are strong (lower troposphere, cloud and precipitation layers) and to relax resolution aloft where gradients are weaker, reducing computational cost while preserving physically meaningful stratification. Layer-by-layer coefficients are then computed and accumulated so that attenuation and brightness-temperature contributions emerge naturally from the vertical segmentation used by the solvers [334,337].

3.2.4 Physical Consistency and Quality Control

Before ingestion into the modelling chain, ERA5 profiles undergo basic plausibility checks: monotonic pressure decrease with altitude, non-negativity of hydrometeor fields, and consistency of temperature structure with phase transitions. Mixed-phase regions are handled without hard thresholds; instead, smooth transitions are retained to remain compatible with the statistical/ensemble treatment of microphysics in SNEM-2.

These procedures do not “correct” ERA5; they enforce minimal physical constraints required to prevent numerical instabilities and to ensure that later radiative-transfer assumptions remain valid across a wide range of meteorological regimes [12, 337].

3.3 Satellite Observations for Atmospheric Propagation Studies

3.3.1 Role of Satellite Observations

Satellite observations provide an essential observational counterpart to atmospheric reanalysis data for the study of radiowave propagation under non-clear-sky conditions. While reanalyses describe the thermodynamic state of the atmosphere in a physically consistent manner, satellite instruments offer direct observational constraints on cloud structure, precipitation, and radiometric effects, which are otherwise only indirectly inferred through modelling. In this thesis, satellite data are used exclusively for validation, interpretation, and physical anchoring of the radiative-transfer simulations and physics-informed surrogate models, avoiding any circular dependency between observations and model inputs. This separation ensures that the physical consistency of SNEM and SNEM-2 can be independently assessed against real atmospheric measurements.

The selected satellite missions provide complementary sensitivity to atmospheric propagation processes, combining vertically resolved active measurements with column-integrated passive radiometric observations [Stephens et al., 2002, 6].

3.3.2 Active Remote Sensing: CloudSat

The CloudSat mission represents a cornerstone for the observation of cloud and precipitation vertical structure. Its Cloud Profiling Radar (CPR), operating at 94 GHz (W-band), provides nadir-looking reflectivity profiles with high sensitivity to cloud and precipitation particles, directly probing the frequency range of interest for millimetre-wave propagation studies.



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At W-band, radar reflectivity is strongly influenced by particle size, phase, and dielectric properties, making CloudSat particularly valuable for diagnosing scattering-dominated regimes relevant to high-frequency satellite links [Bohren & Huffman, 1983, 4]. In this work, CloudSat observations are used to:

- assess the realism of vertical cloud structures generated by SNEM-2,
- interpret attenuation and brightness-temperature features under stratiform and mixed-phase conditions.

The limitations of CloudSat—most notably signal extinction in heavy precipitation—are explicitly acknowledged. Consequently, CloudSat data are interpreted as structural and statistical references, rather than as exact microphysical truth [Stephens et al., 2002, 6].

3.3.3 Passive Microwave Observations: GPM and TRMM

Passive microwave radiometers provide an integrated view of atmospheric absorption, emission, and scattering by measuring upwelling brightness temperatures across multiple frequencies. The GPM and TRMM missions supply complementary multi-frequency observations sensitive to both liquid and frozen hydrometeors. Lower-frequency channels are primarily influenced by emission from liquid water, while higher-frequency channels respond increasingly to scattering by ice and snow particles. This spectral behavior makes passive microwave observations a key benchmark for evaluating simulated brightness temperature and path-integrated attenuation [Petty, 2006, 5]. In this thesis, passive microwave data are used to validate SNEM-2 radiometric outputs and to assess the physical plausibility of machine-learning surrogates trained on synthetic datasets, through statistical and ensemble-based comparisons rather than point-wise matching.

3.3.4 Geometry, Coincidence Strategy, and Uncertainty

Meaningful comparison between simulations and satellite observations requires careful consideration of observation geometry and sampling differences. Active radar measurements emphasize vertical structure, while passive radiometers integrate along broader footprints and slanted paths.

To ensure physical comparability, this work adopts a space–time coincidence strategy, collocating ERA5 profiles, SNEM-2 simulations, and satellite observations within predefined tolerance windows. This approach minimizes representativeness errors while preserving sensitivity to vertical and microphysical variability [Pierdicca et al., 2006, 7]. Satellite uncertainties are treated as intrinsic elements of the validation framework. Model performance is therefore assessed in terms of statistical consistency and physically meaningful trends, rather than exact deterministic agreement, in line with the ensemble philosophy underlying SNEM-2.

3.3.5 Positioning within the Thesis Framework

Satellite observations provide the observational anchor of the modelling strategy adopted in this dissertation. Active and passive measurements constrain the vertical and radiometric realism of



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the simulations, while remaining fully independent from the modelling chain. By integrating satellite observations with reanalysis data and physically grounded radiative-transfer solvers, this work establishes a coherent observational–theoretical loop, ensuring that subsequent hybrid physical–AI developments remain firmly grounded in real atmospheric behavior.

3.4 Satellite Observations for Atmospheric Propagation Studies

3.4.1 Role of Satellite Observations in This Work

Satellite observations constitute an essential complement to atmospheric reanalysis data for the study of radiowave propagation under non-clear-sky conditions. While reanalysis products provide dynamically consistent atmospheric states, satellite instruments directly observe the radiometric and scattering signatures produced by clouds and precipitation, enabling independent validation of physically based propagation models (**Stephens et al.**, 2002, [78]; **Hou et al.**, 2014 [84]).

In this thesis, satellite data are not used as inputs to SNEM or SNEM-2. Instead, they act as external observational constraints, ensuring that model development and validation remain independent and physically testable. The combined use of active and passive sensors allows the investigation of both vertical cloud structure and path-integrated radiative effects relevant to microwave and millimetre-wave propagation.

3.4.2 Active Remote Sensing: CloudSat

The CloudSat mission provides vertically resolved observations of clouds and precipitation through its Cloud Profiling Radar (CPR) operating at 94 GHz (W-band) (**Stephens et al.**, 2002, [78]). This frequency range coincides with regimes where scattering and attenuation dominate electromagnetic propagation, making CloudSat particularly relevant for millimetre-wave studies.

Within this work, CloudSat data are used to assess the realism of vertical cloud and precipitation structures generated by SNEM-2 and by generative surrogate models, and to support the interpretation of simulated brightness-temperature and attenuation features in stratiform and mixed-phase conditions. Limitations due to signal extinction in intense precipitation and the indirect relation between reflectivity and microphysics are explicitly acknowledged; CloudSat observations are therefore treated as structural and statistical references, rather than as exact microphysical truth.

3.4.3 Passive Microwave Observations: GPM and TRMM

Passive microwave radiometers aboard the Global Precipitation Measurement (GPM) mission and its predecessor TRMM provide multi-frequency brightness-temperature observations sensitive to both liquid and frozen hydrometeors (**Kummerow et al.**, 2000 [90]; **Hou et al.**, 2014, [84]).



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Lower-frequency channels are dominated by emission from liquid water, whereas higher-frequency channels increasingly respond to scattering by ice and snow. In this thesis, passive microwave observations are used to validate the radiometric outputs of SNEM-2 and to assess the physical plausibility of physics-informed machine-learning surrogates trained on synthetic datasets. Comparisons are performed statistically, accounting for footprint averaging and retrieval uncertainty, rather than through point-wise matching.

3.4.4 Geometry and Coincidence Strategy

Observation geometry plays a critical role in the interpretation of satellite data. Active radar measurements are typically near-nadir and emphasize vertical structure, while passive radiometric observations involve broader footprints and effective slant paths.

To ensure physical consistency, ERA5 profiles, SNEM-2 simulations, and satellite observations are collocated in space and time using predefined tolerance windows, following established coincidence strategies in atmospheric remote sensing (Stephens et al., 2002, [78]).

This approach reduces representativeness errors and enables statistically robust model–observation comparisons.

3.4.5 Uncertainty and Interpretation Framework

Satellite observations inherently include instrumental noise, retrieval assumptions, and sampling limitations. In this work, these uncertainties are treated as integral components of the modelling framework rather than as noise to be eliminated. Model performance is evaluated in terms of distributions, trends, and physically consistent patterns, in line with the ensemble-based philosophy of SNEM-2 and the probabilistic interpretation of generative surrogates.

3.4.6 Summary

Satellite observations provide the observational anchor for the physically based and hybrid modelling approaches developed in this thesis. Active radar measurements constrain vertical cloud structure and phase regimes, while passive microwave radiometry captures the integrated radiative effects most relevant to attenuation and brightness temperature. Their combined use ensures that subsequent modelling developments remain grounded in real atmospheric behavior.

3.5 Data Preprocessing and Harmonization

3.5.1 Rationale for Data Preprocessing

The integration of heterogeneous atmospheric datasets is a critical prerequisite for physically consistent propagation modelling. Reanalysis products, satellite observations, and numerical simulations differ in spatial and temporal resolution, vertical discretization, physical assumptions, and uncertainty structure. Without a dedicated preprocessing strategy, these discrepancies



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may propagate through the modelling chain, biasing attenuation estimates and radiometric interpretation (Hersbach et al., 2020, [99]). In this work, preprocessing is treated as an integral methodological component rather than a technical adjustment. All transformations are physically motivated, explicitly documented, and designed to ensure traceability and reproducibility. The objective is not to enforce artificial agreement between datasets, but to establish a common physical reference frame enabling meaningful comparison and joint exploitation.

Accordingly, the preprocessing pipeline is designed to ensure:

- (i) physical consistency across data sources,
- (ii) compatibility with radiative-transfer and statistical solvers, and
- (iii) suitability for physics-informed machine-learning applications without loss of interpretability (Karniadakis et al., 2021, [174]).

3.5.2 Temporal Alignment and Coincidence Filtering

Atmospheric propagation under cloudy and precipitating conditions exhibits strong temporal variability. ERA5 provides hourly atmospheric profiles, whereas satellite observations are instantaneous and irregular in time. To reconcile these differences, a temporal coincidence strategy is adopted by associating each satellite overpass with the nearest ERA5 analysis within a predefined temporal window (Hersbach et al., 2020, [99]). Observations exceeding the maximum allowed time offset are discarded. This conservative filtering minimizes temporal representativeness errors and ensures that comparisons between simulations and observations are grounded in quasi-stationary atmospheric conditions.

3.5.3 Spatial Collocation and Footprint Effects

ERA5 fields are defined on a regular latitude–longitude grid, whereas satellite instruments sample the atmosphere along orbital tracks with sensor-dependent footprints. Active radars probe narrow vertical columns, while passive radiometers integrate over broader horizontal areas (Stephens et al., 2002, [253]).

ERA5 profiles are interpolated to satellite locations using nearest-neighbour or bilinear schemes, depending on resolution and variable type. For passive microwave observations, multiple ERA5 grid points within the instrument footprint are aggregated to approximate horizontal averaging effects, preserving large-scale thermodynamic consistency while acknowledging unresolved sub-footprint variability.

3.5.4 Vertical Grid Harmonization

Radiative-transfer solvers require vertically resolved profiles capable of representing sharp gradients in temperature, humidity, and hydrometeor content. ERA5 pressure-level data and



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altitude-based satellite radar bins are therefore mapped onto a common altitude grid using hydrostatic relations (Chandrasekhar, 1960, [222]). Vertical interpolation is performed conservatively to preserve column-integrated quantities, with enhanced resolution in regions of strong gradients such as cloud boundaries and melting layers. This step ensures physically consistent weighting of atmospheric layers along propagation paths.

3.5.5 Unit Consistency and Variable Normalization

All atmospheric variables are converted to a unified set of physical units consistent with radiative-transfer theory. Thermodynamic quantities are expressed in SI units, while hydrometeor contents are converted to mass or number densities compatible with electromagnetic scattering formulations (Marzano et al., 2001, [210]). Normalization procedures introduced for machine-learning purposes are strictly separated from physical transformations. Physically meaningful variables are preserved throughout the pipeline to maintain interpretability and consistency.

3.5.6 Quality Control and Data Screening

A quality control procedure is applied to exclude physically inconsistent profiles, such as negative hydrometeor contents or unrealistic temperature gradients. Satellite observations affected by known instrumental limitations—e.g., radar extinction in heavy precipitation—are flagged and treated separately (Stephens et al., 2002, [253]). Extreme events are retained when physically interpretable, provided their limitations are explicitly acknowledged during analysis.

3.5.7 Preparation for Statistical and Machine-Learning Modelling

Preprocessed datasets are structured to support both deterministic radiative-transfer simulations and statistical or machine-learning modelling. Vertically resolved profiles are formatted for SNEM and SNEM-2, while structured arrays preserve spatial, temporal, and physical dependencies for learning-based applications (Karniadakis et al., 2021, [174]). Atmospheric variability is not collapsed into bulk descriptors at this stage, ensuring continuity between physical simulation and surrogate modelling.

3.5.8 Summary and Methodological Positioning

Data preprocessing and harmonization constitute the methodological backbone of the framework developed in this thesis. By enforcing physical consistency across heterogeneous datasets, differences between simulations and observations can be attributed to modelling assumptions rather than artefacts of data handling (Hersbach et al., 2020, [99]; Stephens et al., 2002, [253]). This harmonized foundation enables robust validation, statistical analysis, and the integration of physics-informed machine-learning surrogates, providing a coherent transition toward the computational frameworks introduced in the subsequent sections.



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3.6 Computational Environment and Software Infrastructure

3.6.1 Role of the Computational Environment in Atmospheric Modelling

The modelling framework developed in this thesis relies on a heterogeneous computational environment tailored to physically based radiative-transfer simulations, statistical analyses, and physics-informed machine-learning workflows. Atmospheric propagation modelling at microwave and millimetre-wave frequencies involves multi-dimensional datasets, ensemble simulations, and numerically sensitive solvers, requiring a flexible and robust software infrastructure.

Rather than adopting a monolithic platform, the computational strategy emphasizes modularity, transparency, and reproducibility, allowing each stage of the workflow—preprocessing, radiative-transfer modelling, validation, and surrogate development—to be implemented with tools best suited to the task. Interoperability is ensured through standardized data formats and explicit interface definitions, in line with established practices in atmospheric modelling and remote sensing (Marzano et al., 2001, [210]; Emde et al., 2016, [95]).

3.6.2 Core Programming Languages and Numerical Frameworks

The numerical core of the framework combines compiled and interpreted languages to balance efficiency and flexibility. Radiative-transfer solvers, including the reconstruction and extension of SNEM and SNEM-2, are implemented in Fortran (77/90), reflecting both legacy continuity and the need for numerical stability in multi-layer, frequency-dependent simulations (Marzano, 2006, [334]; Marzano et al., 2016, [337]). MATLAB is used for data preprocessing, visualization, and statistical analysis, leveraging its strengths in matrix-based computation and handling of vertically resolved atmospheric profiles. Python is employed selectively for machine-learning workflows and orchestration, providing access to modern neural-network libraries and experiment management tools without interfering with the deterministic modelling chain (Goodfellow et al., 2014, [10]; Karniadakis et al., 2021, [38]).

3.6.3 Execution Environments and Computational Resources

Computational experiments are conducted on Linux-based systems, ranging from local workstations to multi-core servers. Most simulations are executed in embarrassingly parallel mode, with independent atmospheric realizations run concurrently. This strategy aligns with the ensemble-based philosophy of SNEM-2, prioritizing statistical robustness over tightly coupled parallelism. Computational cost is explicitly monitored to ensure feasibility for large-scale and near-real-time applications, consistent with the intended operational relevance of the framework (Marzano et al., 2016, [338]).



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3.6.4 Data Formats, Storage, and Interoperability

Input datasets (ERA5 profiles and satellite observations) are stored in standardized, self-describing formats, while intermediate and final products—such as extinction profiles, brightness temperatures, and attenuation statistics—are saved in structured binary or MATLAB-native formats to preserve numerical precision and metadata. Clear directory structures and naming conventions distinguish raw data, processed inputs, simulations, and derived products. Interoperability between Fortran, MATLAB, and Python components is achieved through explicit interface layers and documented variable definitions, minimizing code coupling and facilitating independent verification (Emde et al., 2016, [95]; Hersbach et al., 2020, [99]).

3.6.5 Version Control and Reproducibility

All codes developed in this thesis are version-controlled, with changes to solvers, parameterizations, and preprocessing routines explicitly tracked. Simulation parameters, frequency settings, and atmospheric configurations are logged alongside outputs to ensure full reproducibility. Dataset provenance—including reanalysis versions and access dates—is documented, a critical requirement given the evolving nature of global reanalysis products (Hersbach et al., 2020, [99]).

3.6.6 Numerical Stability and Precision Considerations

Radiative-transfer simulations involving scattering and absorption across multiple layers are prone to numerical instability, especially at high frequencies and low elevation angles. To mitigate this, double-precision arithmetic is used throughout, and vertical discretization is selected to balance physical resolution and numerical robustness. Sensitivity analyses are performed to verify that model outputs are stable with respect to numerical parameters, ensuring that reported results reflect physical behavior rather than numerical artefacts (Chandrasekhar, 1960, [222]; Stamnes et al., 1988, [151]).

3.6.7 Integration with Machine-Learning Workflows

The computational environment is designed to support later integration of physics-informed machine-learning models. Outputs from SNEM-2 simulations are structured as physically consistent training and validation datasets, preserving metadata and vertical information. Machine-learning experiments are conducted in isolated environments to avoid feedback into the deterministic modelling pipeline. This modular design enables future extensions—such as alternative surrogate architectures or physical constraints—without restructuring the core computational framework (Karniadakis et al., 2021, [38]; Zhu et al., 2023, [33]).

3.6.8 Summary and Methodological Relevance

The computational infrastructure described in this section provides the technical backbone of the thesis. By combining physically grounded numerical solvers with flexible data-handling



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and analysis tools, the framework supports both detailed physical investigation and large-scale statistical application. Computational efficiency is pursued as a means to enable broader exploration of atmospheric variability and uncertainty, while preserving physical interpretability and methodological transparency.

3.7 SNEM-2: Design, Architecture, and Capabilities

The Statistical Numerical Electromagnetic Model – SNEM-2 is the forward modelling engine adopted throughout this thesis for the simulation of radiowave propagation in hydrometeor-laden atmospheric columns. Developed entirely within this doctoral work, SNEM-2 represents a substantial evolution of the original SNEM framework toward a multi-frequency, slant-path-capable, and physically consistent radiative-transfer model.

Unlike purely statistical generators or radiometric simulators—such as multivariate rain-field models and parametric microwave radiative approaches (Luini and Capsoni, 2013, [8]; Marzano and Mugnai, 1995, [179]; Marzano et al., 2005, [180])—SNEM-2 explicitly resolves the vertical atmospheric structure and phase-dependent extinction mechanisms. This design choice preserves physical interpretability across frequency bands and propagation geometries, which is essential for both model validation and subsequent surrogate development.

Within the organization of Chapter 3, SNEM-2 is introduced strictly as a computational tool. Its outputs provide physically grounded radiative quantities that are later interpreted within the theoretical and modelling frameworks developed in Chapters 4 and 5.

3.7.1 Model Architecture and Computational Design

SNEM-2 adopts a hybrid numerical architecture combining a high-performance Fortran 90 core with MATLAB-based interfaces for data ingestion, preprocessing, post-processing, and visualization. The Fortran core implements the radiative-transfer solver, stochastic sampling of hydrometeor populations, and layer-wise extinction and emission calculations. MATLAB routines manage interaction with ERA5 reanalysis data, satellite products (CloudSat and GPM), and derived atmospheric quantities introduced in previous sections (Hersbach et al., 2020, [99]; Stephens et al., 2002, [253]; Hou et al., 2014, [189]).

This modular design ensures computational efficiency, reproducibility, and extensibility, while maintaining a clear separation between physical inputs, numerical solvers, and derived outputs. The computational structure of SNEM-2 is schematically illustrated in **Fig. 3.2**, where the SNEM kernel operates as a forward operator:

$$\mathcal{F} : x \rightarrow y, \quad (3.1)$$

mapping thermodynamic and microphysical inputs—together with frequency and propagation geometry—into radiative observables such as brightness temperature and path attenuation.

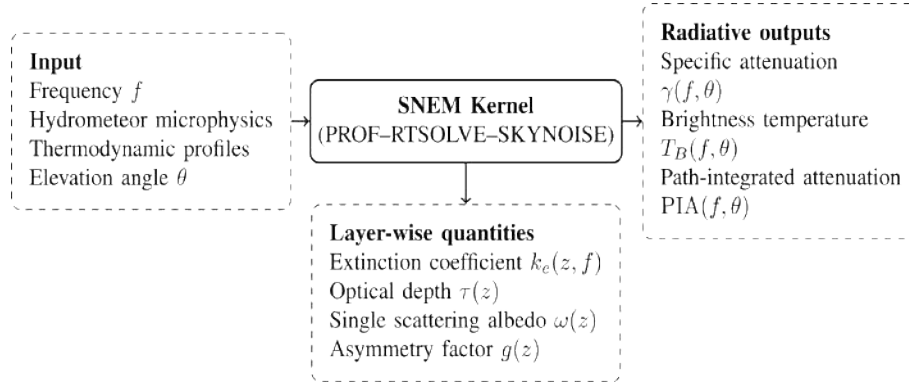


Figure 3.2. Schematic architecture of the SNEM kernel for atmospheric radiative-transfer and attenuation modelling.

Within the kernel, optical properties are computed layer by layer along the atmospheric column and subsequently integrated to produce path-dependent quantities. This input–kernel–output organization constitutes the structural basis of SNEM-2 and enables its seamless coupling with statistical analysis and physics-informed surrogate models introduced later in the thesis.

3.7.1.1 Statistical Sampling and Path-Integrated Attenuation

The original SNEM formulation is based on a stochastic sampling strategy used to populate the atmospheric column with hydrometeors of prescribed type, concentration, and vertical statistics, generating heterogeneous precipitation scenes while preserving ensemble consistency and computational tractability (Marzano et al., 2001, [8]). This statistical–numerical approach allows explicit representation of vertical variability without resorting to fully resolved cloud microphysics. Within this framework, the path-integrated attenuation is computed as:

$$A(f, \theta) = \sec(\theta) \int_0^H k_e(z, f) dz, \quad (3.2)$$

where $k_e(z, f)$ denotes the extinction coefficient and θ the propagation elevation angle. This formulation provides a direct link between layer-wise extinction processes and observable propagation losses.

In its original implementation, SNEM was primarily limited to quasi-zenithal geometries and frequencies below approximately 50 GHz, which restricted its capability to represent angular anisotropy and high-frequency scattering effects. These limitations directly motivated the architectural redesign introduced in this doctoral work, leading to the development of SNEM-2: a multi-frequency, slant-path-capable extension operating across the Ka-, V-, and W-band frequency ranges, up to 95.5 GHz, and explicitly designed to capture high-frequency extinction mechanisms (this work, [ArtA1]).



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3.7.2 Atmospheric Discretization and Microphysical Representation

In SNEM-2, the atmospheric column is discretized into 60 vertical layers of 500 m thickness, extending from the surface to approximately 30 km altitude. This configuration provides sufficient resolution to represent tropospheric stratification, melting-layer regions, and vertically heterogeneous hydrometeor distributions, while ensuring numerical stability and computational efficiency.

Hydrometeor populations are generated stochastically within each layer using validated statistical distributions. Rain is described through Marshall–Palmer drop size distributions, whereas frozen hydrometeors are represented by exponential size distributions consistent with established microphysical formulations (Marzano et al., 2001, [8]; Liu et al., 2014, [XX]). Mixed-phase conditions are treated through thermodynamically driven phase partitioning, informed by ERA5 temperature profiles, allowing a realistic representation of melting-layer transitions (this work, [ArtA3, ArtB2]).

3.7.3 Radiative-Transfer Formulation and Propagation Geometry

Radiative-transfer calculations in SNEM-2 are based on a two-stream Eddington approximation, with diffuse scattering represented through Legendre polynomial expansions. This formulation achieves a balance between physical consistency and computational tractability.

SNEM-2 explicitly supports slant-path propagation geometries, covering elevation angles from 5° to 90°, and accounts for angular dependence in extinction and emission processes. The model operates across the Ka-, V-, and W-band frequency ranges, up to 95.5 GHz, enabling realistic Earth–space and space–Earth propagation simulations (this work, [ArtA1, ArtA2]).

3.7.4 Model Outputs and Radiative Quantities

For each atmospheric realization, SNEM-2 provides a set of physically interpretable radiative and propagation quantities, including:

- Specific attenuation: $\gamma(z, f) = k_e(z, f) [dB km^{-1}]$,
- Path-integrated attenuation: $A(f, \theta) [dB]$,
- Brightness temperature $T_B(f, \theta) [K]$,
- Single-scattering albedo ω ,
- Asymmetry factor g ,
- Zenith total delay (ZTD), where required.

These outputs constitute the numerical basis for statistical analysis, satellite comparison, and physics-guided surrogate modelling (this work, [ArtB1, ArtB2]).

3.7.5 Technical Validation and Performance Indicators

As a technical validation of the numerical framework, SNEM-2 has been benchmarked against satellite-based reference datasets to verify numerical stability and physical consistency of the



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forward solver. The validation is intentionally limited to representative performance indicators, without entering physical interpretation at this stage. Key indicators include:

- Brightness temperature under snow-dominated conditions, with root-mean-square errors below 5 K relative to CloudSat observations (this work, [ArtA4]);
- Path-integrated attenuation in stratiform precipitation, with mean absolute errors of approximately 1.2 dB when compared to GPM-DPR estimates (this work, [ArtA4]).

These metrics are reported solely as indicators of model reliability and numerical robustness. A detailed physical discussion and sensitivity analysis are deferred to later chapters.

3.7.6 Role of SNEM-2 in the Modelling Chain

Within the modelling strategy adopted in this thesis, SNEM-2 acts as the physically grounded forward operator linking observational datasets, statistical representations, and theoretical abstractions. It underpins both the physics-guided generative approaches introduced in §2.6.8 and the C^4 field-theoretical formulations developed in **Chapters 4 and 5**.

Beyond deterministic simulation, SNEM-2 is designed as a controlled generator of physically consistent synthetic datasets. Its statistical structure and modular architecture enable ensemble production suitable for climatological analysis, system design studies, and the training of physics-informed surrogate models, whose operational implementation is addressed in §4.6.

Within this framework, SNEM-2 supports:

- ensemble-based climatological and scenario-dependent simulations,
- design-oriented assessment of Earth–space and space–Earth propagation links,
- training of physics-informed neural radiative surrogates, and
- satellite-based benchmarking of alternative radiative-transfer models.

By confining SNEM-2 to **Chapter 3** as a numerical framework, a clear separation is maintained between computational tools, theoretical development, and scientific interpretation, ensuring methodological transparency and structural coherence throughout the dissertation.

3.8 Datasets Description and Selection Criteria

3.8.1 Role of Reanalysis and Satellite Observations

The modelling and validation of atmospheric radiative-transfer processes require datasets that jointly describe thermodynamic structure, hydrometeor distribution, and electromagnetic observables. In this thesis, reanalysis and satellite datasets are treated as complementary—rather than interchangeable—representations of the atmospheric state. Reanalysis products provide dynamically consistent and temporally continuous descriptions of the atmospheric column, while satellite observations offer direct sensitivity to radiative and microphysical processes. The adopted



strategy exploits this complementarity by using reanalysis data to constrain large-scale thermodynamic conditions and satellite measurements to benchmark radiative-transfer simulations and attenuation estimates.

3.8.2 ERA5 Reanalysis as Thermodynamic Backbone

ERA5 reanalysis data produced by ECMWF constitute the primary source of atmospheric state information (Hersbach et al., 2020, [99]). Hourly profiles of temperature, pressure, humidity, and geopotential height are extracted either along satellite ground tracks or at fixed geographical locations to define vertical stratification and gaseous absorption conditions. ERA5 is selected for its global consistency, temporal continuity, and well-characterized uncertainty structure. Although small-scale variability is smoothed by the assimilation process, ERA5 provides a physically coherent representation of the atmospheric column suitable for systematic radiative-transfer modelling and statistical analysis.

3.8.3 Satellite Observations: CloudSat, GPM, and TRMM

Satellite observations provide the observational anchor for model evaluation. CloudSat offers vertically resolved radar reflectivity profiles at 94 GHz, with unique sensitivity to cloud and precipitation microphysics (Stephens et al., 2002, [253]). These measurements are used as structural and statistical references for evaluating simulated cloud vertical structure, phase distribution, and melting-layer behaviour. Complementary precipitation information is derived from the GPM mission and its predecessor TRMM, which provide multi-frequency active and passive observations of rainfall and frozen hydrometeors (Hou et al., 2014, [312]). Together, these datasets support precipitation regime classification and enable assessment of frequency-dependent attenuation across microwave and millimetre-wave bands.

3.8.4 Multi-Satellite Coincidence Strategy and Quality Control

To ensure physical comparability, a coincidence-based strategy is adopted to link ERA5 profiles, CloudSat observations, and GPM/TRMM measurements. Spatial and temporal collocation criteria are defined to ensure that all datasets represent the same atmospheric column within acceptable uncertainty bounds. All datasets undergo systematic quality control prior to use: observations affected by instrumental limitations or retrieval failures are excluded, and reanalysis profiles are screened for basic physical consistency. Precipitation regimes are classified using combined radar and radiometer information to avoid ambiguous cases. Quality control is treated as an integral modelling step, ensuring transparency and reproducibility.

3.8.5 Coverage, Uncertainty, and Methodological Positioning

The selected datasets span multiple climatic regimes—including tropical convection, mid-latitude stratiform precipitation, and high-latitude ice clouds—over multi-year periods, supporting robust



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statistical analysis and reducing regime-specific bias. Uncertainties associated with both reanalysis and satellite data are explicitly acknowledged and addressed through ensemble-based modelling and statistical interpretation. ERA5 profiles are treated as representative mean atmospheric states, while satellite uncertainties are handled through multi-instrument comparison rather than pointwise matching. This approach is consistent with the statistical–numerical philosophy underlying SNEM and SNEM-2 and establishes a physically grounded data framework for the modelling, validation, and hybrid physical–AI analysis developed in the following sections.

3.8.6 Summary and Transition

This section defines the datasets and selection criteria forming the empirical foundation of the thesis. By combining reanalysis data with multi-satellite observations, a physically grounded and statistically robust data framework is established. The chapter now proceeds to the description of preprocessing steps and derived quantities, detailing how these datasets are transformed into inputs suitable for radiative-transfer modelling and hybrid physical–AI analysis.

3.9 Data Preprocessing and Derived Atmospheric Quantities

3.9.1 Motivation and Conceptual Role of Preprocessing

Raw atmospheric datasets derived from reanalysis and satellite observations are not directly compatible with radiative-transfer modelling due to differences in vertical discretization, temporal sampling, and physical variable definitions. A dedicated preprocessing stage is therefore required to ensure physical consistency, numerical stability, and compatibility with the SNEM and SNEM-2 solvers. In this thesis, preprocessing is treated as a physically motivated transformation layer rather than a purely technical operation, ensuring that all derived quantities retain a clear physical meaning and satisfy the assumptions underlying electromagnetic propagation models (Hersbach et al., 2020, [99]).

Preprocessing constitutes the interface between observational reality and numerical modelling, transforming heterogeneous datasets into a unified atmospheric representation. This step enables deterministic radiative-transfer simulations and physics-informed machine-learning surrogates to operate on a common, well-defined input space, ensuring interpretability, reproducibility, and methodological transparency.

3.9.2 Vertical and Thermodynamic Harmonization of Atmospheric Profiles

ERA5 atmospheric variables provided on pressure or model levels are interpolated onto a fixed altitude-based grid consistent with the vertical discretization adopted in SNEM-2. Temperature, pressure, and humidity profiles are interpolated using monotonic schemes designed to preserve physical gradients and avoid numerical artefacts, particularly within the boundary layer and



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near the tropopause, where sharp vertical transitions strongly affect gaseous absorption and refractivity.

Several radiative-transfer inputs are derived from primary thermodynamic variables. Water-vapor density, refractivity, and gaseous absorption coefficients are computed from temperature, pressure, and humidity using established formulations (Liebe et al., 1991, [146]; ITU-R P.676, [201]). The conversion from pressure coordinates to geometric altitude is performed using the hydrostatic relation:

$$\frac{dp}{dz} = -\rho(z)g, \quad (3.3)$$

ensuring consistency between thermodynamic profiles and the geometrical representation required for slant-path propagation modelling.

3.9.3 Cloud and Hydrometeor Layer Construction

Cloud and precipitation layers are constructed by combining satellite-derived structural information with reanalysis-based thermodynamic constraints. Cloud boundaries, hydrometeor phase state, and vertical layering are discretized into homogeneous slabs characterized by bulk microphysical properties such as liquid water content (LWC), ice water content (IWC), and effective particle size. When direct microphysical observations are unavailable, these properties are inferred using physically constrained relationships consistent with the observed precipitation regime (Marzano et al., 2001, [179]; Liu et al., 2014, [231]). This layered representation captures mixed-phase and melting-layer effects while maintaining computational efficiency, enabling realistic radiative-transfer simulations without resorting to prohibitively complex microphysical schemes.

3.9.4 Radiative and Geometric Preconditioning

From the preprocessed atmospheric profiles, layer-wise radiative parameters are computed prior to radiative-transfer simulation. The extinction coefficient is defined as:

$$k_e(z, \nu) = k_a(z, \nu) + k_s(z, \nu), \quad (3.4)$$

where k_a and k_s denote absorption and scattering coefficients, respectively. Brightness temperature and path-integrated attenuation are not computed at this stage but by the forward radiative-transfer solvers; preprocessing ensures that all required inputs are internally consistent and numerically stable. For non-zenith propagation, geometric corrections are applied to account for slant-path effects. The effective path length through each layer is computed as:

$$ds = \frac{dz}{\sin(\theta)}, \quad (3.5)$$

where θ is the elevation angle. This correction is essential for accurately modelling attenuation and emission at low elevation angles, where path-integrated effects are significantly amplified



(ITU-R P.618, [205]).

3.9.5 Temporal Handling, Uncertainty, and Data Integrity

Temporal mismatches between satellite observations and reanalysis data are addressed through controlled aggregation windows. Averaging strategies are selected to preserve physically meaningful variability while suppressing high-frequency noise, balancing temporal fidelity and statistical robustness (Stephens et al., 2002, [253]). Profiles affected by severe data gaps or physical inconsistencies are excluded from analysis, while partially missing data are treated conservatively and accounted for qualitatively in the interpretation of results.

Rather than attempting to eliminate uncertainty, the modelling strategy explicitly acknowledges its presence. This philosophy avoids artificial precision and ensures that conclusions drawn from radiative-transfer simulations and surrogate models remain physically credible and statistically robust.

3.9.6 Summary and Transition

This section has defined the preprocessing steps and derived atmospheric quantities required for radiative-transfer modelling and hybrid physical–AI analysis. By enforcing physical consistency across heterogeneous datasets, preprocessing establishes a unified atmospheric representation suitable for deterministic simulation, statistical analysis, and physics-informed learning. With the preprocessing framework now defined, the dissertation proceeds to the consolidation of the Chapter 3 methodology and to the modelling, validation, and performance assessment activities presented in Chapter 4.

3.10 Machine Learning Models

To complement the physically based simulations performed with SNEM-2 and to enable fast emulation of radiative-transfer quantities, this work integrates two deep-learning architectures specifically designed for atmospheric applications. These models are introduced here as surrogate tools intended to approximate selected SNEM-2 outputs under a wide range of meteorological conditions. Long Short-Term Memory (LSTM) networks [9], which are well suited for modeling sequential dependencies in meteorological and geophysical datasets [22], were implemented to emulate vertical sequences of path-integrated attenuation (PIA) and brightness temperature (TB). The adopted LSTM architecture consists of two stacked recurrent layers with 256 units each, followed by a fully connected output layer. Rectified Linear Unit (ReLU) activations and dropout regularization (0.3) were employed to mitigate overfitting, following established practices in deep learning for atmospheric sciences [23]. Input features were derived from ERA5 atmospheric profiles and co-located **GPM–CloudSat** observations. The LSTM model was trained using approximately 40,000 coincident satellite profiles spanning the period **2018–2021** and covering liquid, frozen, and mixed-phase hydrometeor regimes. The model produces continuous estimates



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of PIA and TB and is designed to operate as a computationally efficient surrogate within larger processing chains, such as retrieval support, sensitivity analyses, and near-real-time propagation assessment.

3.10.1 Conditional Generative Adversarial Networks (CGAN) for Field-Based Emulation

In addition to sequence-based modeling, a **Conditional Generative Adversarial Network (CGAN)** [10] was developed to emulate the full vertical structure of radiative-transfer outputs produced by SNEM-2. The CGAN is conditioned on a set of atmospheric descriptors derived from ERA5, including integrated water vapor, surface temperature, and liquid and ice water path. The generator network synthesizes vertically resolved fields of specific attenuation and brightness temperature, while the discriminator enforces consistency with reference SNEM-2 simulations. Training stability is ensured through a composite loss function combining adversarial loss, L1 reconstruction terms, and gradient penalties, which constrain vertical smoothness and prevent unphysical oscillations across thermodynamic layers.

The CGAN framework is designed to support full-profile emulation of radiative quantities and to complement the LSTM surrogate, which focuses on integrated and sequence-based predictions. This dual-architecture strategy enables flexible integration of data-driven tools within physics-based modeling pipelines and supports applications ranging from global-scale simulations to hybrid radiative-transfer workflows [29]. To provide a unified view of the data sources, physical modeling, and data-driven components adopted in this work, the overall processing chain integrating reanalysis products, satellite observations, radiative-transfer modeling, and machine-learning surrogates is schematically summarized in **Figure 3.3 (Right side)**.

Within this framework, ERA5 atmospheric fields are first interpolated along satellite tracks and collocated with GPM and CloudSat observations to ensure thermodynamic and microphysical consistency. These inputs are then used both to drive the SNEM-2 radiowave propagation model and to train machine-learning surrogates, enabling fast emulation of path-integrated attenuation and brightness temperature. The quantitative performance and physical consistency of the **LSTM and CGAN surrogates** illustrated in **Figure 3.3** are evaluated in **Chapter 7**, with the full experimental setup and validation protocol detailed in **Section 7.11**.

3.10.2 Simulation Pipeline and Workflow

The simulation pipeline integrates MATLAB and Python within a unified framework designed to preprocess atmospheric datasets, perform SNEM-2 radiative-transfer simulations, collocate multi-satellite observations, and train machine-learning surrogate models. MATLAB was primarily employed to control the execution of SNEM-2, manage parameter sweeps, and generate intermediate and final visualizations. Python was used for data preparation through **xarray** and **netCDF4**, for training deep-learning models using **PyTorch**, and for computing **GPM–CloudSat coincidences** based on orbital metadata and geolocation constraints. The construction of matched

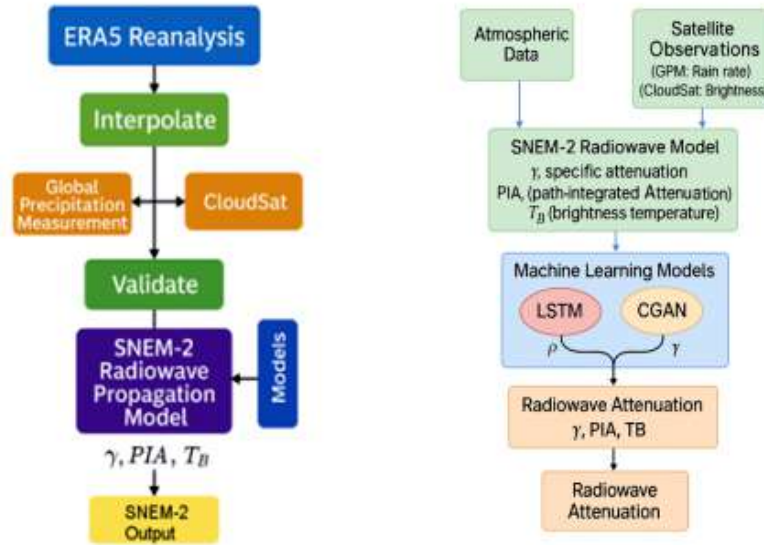


Figure 3.3. Integrated Physical-Data-Driven Framework for Radiowave Attenuation Modeling and Emulation. Left: Schematic overview of the integrated modeling framework combining ERA5 reanalysis, satellite observations, the SNEM-2 radiative-transfer model, and machine-learning surrogates. The LSTM architecture processes temporally ordered ERA5-GPM inputs to emulate path-integrated attenuation (PIA) and brightness temperature (TB), while the CGAN generates vertically resolved attenuation and TB profiles from scalar atmospheric descriptors through a generator-discriminator architecture. Right: End-to-end processing pipeline for radiowave attenuation modeling and emulation. ERA5 atmospheric profiles drive SNEM-2 forward simulations, which are validated against GPM and CloudSat observations. The resulting physically consistent simulations are subsequently used to train machine-learning surrogates (LSTM and CGAN), enabling fast emulation of radiative quantities and uncertainty-aware analysis.

multi-frequency atmospheric profiles relied on an initial temporal screening, followed by pixel-level spatial collocation using great-circle distance criteria. GPM precipitation fields were subsequently interpolated onto CloudSat’s vertical grid to ensure vertical consistency between sensors. The resulting collocated dataset provided the input to SNEM-2, yielding vertically resolved profiles of specific attenuation, path-integrated attenuation, and brightness temperature.

All radiative-transfer outputs were stored in **HDF5** and **NetCDF** formats to ensure consistency, reproducibility, and interoperability across the workflow. These validated SNEM-2 simulations constituted the reference dataset for training both the LSTM and CGAN surrogate models, enabling fast emulation and uncertainty-aware prediction of attenuation-related quantities. The overall structure of the simulation and modeling workflow, including the interaction between atmospheric preprocessing, physical radiative-transfer modeling, and machine-learning components, is schematically summarized in **Figure 3.3 (Left side)**.

3.10.3 Scope and Role of Machine-Learning Surrogates within the Thesis

The machine-learning models introduced in this chapter are conceived as complementary surrogate tools to the physically based SNEM-2 radiative-transfer solver, rather than as stand-alone



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predictive systems. Their primary role within the thesis is to enable fast and scalable emulation of selected radiative-transfer quantities while preserving consistency with the underlying physical framework. Both the LSTM and CGAN architectures are trained exclusively on SNEM-2 reference simulations driven by thermodynamically and microphysically consistent atmospheric inputs. As a result, the learned mappings remain constrained by radiative-transfer physics, with the data-driven components acting as computational accelerators rather than empirical black-box substitutes. The LSTM surrogate is primarily designed to emulate integrated and sequence-based quantities such as path-integrated attenuation and brightness temperature, whereas the CGAN enables full-profile emulation of vertically resolved radiative quantities and enhanced generalization across geoclimatic regimes. Within the overall structure of the thesis, machine-learning surrogates serve three main purposes: (i) rapid exploration of the parameter space for sensitivity and uncertainty analyses; (ii) support for large-scale simulations and climatological studies where direct SNEM-2 execution would be computationally prohibitive; and (iii) preliminary assessment of real-time or near-real-time propagation scenarios relevant to satellite communications and remote-sensing applications. This chapter is intentionally limited to the description of data sources, modeling tools, surrogate architectures, and workflow design. The quantitative evaluation of surrogate performance—including accuracy metrics, physical consistency, robustness across climatic regimes, and comparison with satellite observations—is presented in **Chapter 7**, where machine-learning outputs are systematically benchmarked against SNEM-2 simulations and observational references.

3.11 Summary of Data and Tools and Transition to Chapter 4

This chapter has presented the data sources, computational tools, and preprocessing strategies that form the empirical and technical foundation of this thesis. Rather than serving as a mere inventory of datasets and software, **Chapter 3** has established a coherent methodological ecosystem in which observational products, reanalysis fields, and numerical models are integrated into a physically consistent workflow. The atmospheric datasets employed—ERA5 reanalysis, CloudSat CPR observations, GPM-DPR precipitation products, and ancillary satellite and GNSS-derived measurements—span a wide range of spatial scales, temporal resolutions, and physical observables. Their combined use enables the characterization of both the thermodynamic state of the atmosphere and the microphysical structure of hydrometeors, which together govern radiowave attenuation, emission, and phase delay.

Special emphasis has been placed on the complementarity between datasets. Reanalysis products provide global, temporally continuous thermodynamic fields, while active and passive satellite sensors supply vertically resolved information on clouds and precipitation. This multi-source approach allows the strengths of each dataset to compensate for the limitations of the others, yielding a robust basis for radiative-transfer modelling and validation. From a computational perspective, the chapter has introduced the numerical tools used to implement, test, and extend the modelling framework developed in this thesis. The SNEM and SNEM-2 radiative-



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transfer solvers constitute the deterministic core, enabling physically interpretable simulations of attenuation and brightness temperature across a wide range of frequencies and geometries. Supporting tools for data handling, interpolation, and visualization—primarily implemented in MATLAB and Fortran—ensure reproducibility, modularity, and numerical stability. A central contribution of this chapter is the explicit treatment of data preprocessing as a physically motivated operation. Vertical harmonization, thermodynamic conversion, hydrometeor-layer construction, and geometrical correction are not ancillary steps but essential components that preserve the physical meaning of the variables entering the radiative-transfer equations. By enforcing consistency across datasets and models, preprocessing enables meaningful comparison between simulations and observations and ensures that subsequent analyses remain grounded in realistic atmospheric states. Furthermore, the preprocessing and data-integration strategies outlined here establish a common input space for both deterministic solvers and machine-learning surrogates. This shared representation is a prerequisite for the hybrid physical–AI frameworks developed in later chapters, where statistical models are trained to emulate or complement full radiative-transfer computations. In summary, **Chapter 3** defines the observational and computational infrastructure upon which the remainder of the thesis is built. It provides the necessary tools to translate raw atmospheric measurements into structured inputs for modelling and validation, while maintaining transparency, physical interpretability, and reproducibility. With this foundation in place, **Chapter 4** moves from data and tools to theoretical and modelling developments. It introduces the physical frameworks—radiative transfer, scattering theory, and the C^4 formalism—that underpin the SNEM-2 model and its extensions. Building on the datasets and preprocessing strategies described here, the next chapter formalizes the mathematical operators governing electromagnetic propagation in precipitating atmospheres and sets the stage for the hybrid modelling approaches explored in the subsequent parts of the dissertation.



Chapter 4

Classical C^4 Field Theory: Formalism, Structure, and Deterministic Limit

This chapter establishes the theoretical backbone of the modelling developments anticipated at the end of Chapter 3. Starting from the classical radiative-transfer picture and the associated scattering phenomenology, it introduces the C^4 formalism as a unifying framework for describing electromagnetic propagation in complex, inhomogeneous, and precipitating atmospheric media. The aim is twofold. First, the chapter consolidates a classical (deterministic) formulation of the C^4 field theory, making explicit its variables, constitutive relations, and modelling assumptions. Second, it derives the key radiative-transfer quantities employed throughout the dissertation—extinction and scattering coefficients, phase-function parametrizations, and optical depth—from the C^4 structure in a way that is consistent with SNEM-2 and extensible beyond it. The resulting deterministic limit provides a clean reference point for the subsequent generalizations: the operator-based formulations and statistical interpretation introduced later can be read as structured extensions of the classical C^4 framework developed here. The classical C^4 framework is organized as a deterministic chain linking atmospheric microphysics, effective dielectric response, electromagnetic field evolution, and radiative-transfer closure. **Figure 4.1** summarizes the logical and physical structure of the framework and introduces the shared propagation coordinate underlying the classical closure relations.

4.1 Motivation and Conceptual Positioning of the C^4 Field

The modelling of electromagnetic propagation in precipitating atmospheres has traditionally relied on radiative transfer theory complemented by scattering models and empirically tuned parametrizations. Within this framework, quantities such as extinction coefficients, single-scattering albedo, and phase functions are typically introduced as effective parameters, either

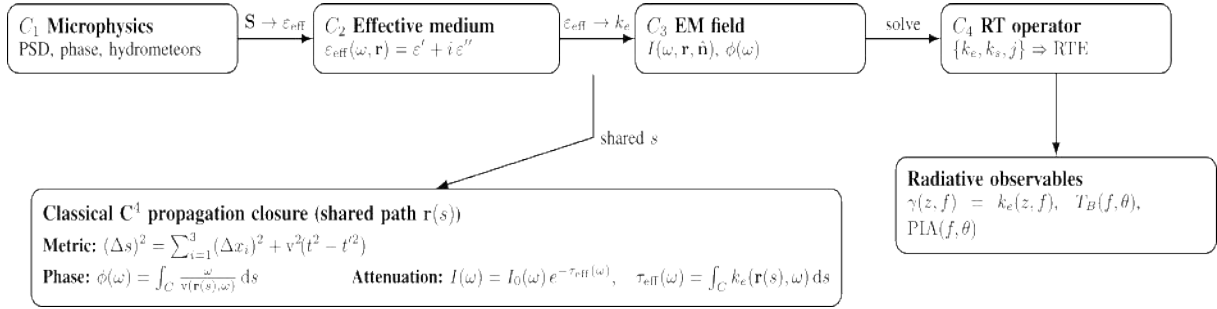


Figure 4.1. Original schematic of the classical C4 framework. Microphysical properties (C1) determine the effective dielectric response (C2), which governs electromagnetic field evolution (C3) and radiative-transfer closure (C4). Metric, phase, and attenuation are consistently accumulated along the same effective propagation coordinate s , ensuring geometrical and radiative coherence and direct compatibility with SNEM and SNEM-2.

derived from simplified microphysical assumptions or inferred from observations and numerical experiments. This approach has proven successful for a wide range of applications, including attenuation prediction, brightness temperature estimation, and radar retrievals, and it forms the basis of the SNEM-2 model discussed in the previous chapters. However, as atmospheric propagation studies extend toward higher frequencies, stronger inhomogeneities, and increasingly complex hydrometeor populations, the limitations of a purely parametrized description become more apparent. In particular, three issues emerge repeatedly. First, the separation between electromagnetic propagation and cloud microphysics is often enforced at the modelling level, rather than arising naturally from a unified physical description. Second, the dependence of macroscopic radiative-transfer quantities on the underlying dielectric properties of the medium is frequently obscured by lookup tables or regression-based formulations. Third, the transition between deterministic propagation regimes and more general statistical or stochastic descriptions lacks a clear theoretical baseline.

The C^4 formalism is introduced in this context as an attempt to address these limitations by providing a unified field-theoretical description of wave-matter interaction in complex atmospheric media. Rather than replacing radiative transfer theory, the C^4 framework embeds it within a broader physical structure in which extinction, scattering, and emission emerge from the coupling between the electromagnetic field and the microphysical state of the atmosphere. In this sense, the C^4 field does not represent a new numerical scheme, but a conceptual reorganization of the modelling problem.

At the classical level considered in this chapter, the C^4 framework is formulated as a deterministic field theory. The atmosphere is described through a set of macroscopic state variables—such as temperature, pressure, hydrometeor content, and particle size distributions—that determine the effective dielectric response of the medium. Electromagnetic propagation is then governed by this response through constitutive relations that link the complex permittivity to the microphysical properties of clouds and precipitation. Within this setting, standard radiative-transfer quantities can be recovered as derived, rather than postulated, elements of the



theory. This classical formulation serves two distinct purposes. On the one hand, it provides a physically transparent interpretation of the parameters employed in SNEM-2 and related models, clarifying their dependence on atmospheric structure and composition. On the other hand, it establishes a well-defined deterministic limit against which more general formulations can be constructed. In particular, the operator-based and statistical extensions introduced in **Chapter 5** can be interpreted as systematic generalizations of the classical C^4 field theory developed here, rather than as independent modelling layers. The remainder of this chapter is therefore devoted to consolidating the classical C^4 framework. After defining the fundamental variables and structure of the C^4 field, the associated constitutive relations and radiative-transfer equations are derived, and their connection to optical depth and path-integrated attenuation is discussed. This deterministic baseline provides the necessary conceptual continuity between the data-driven modelling tools introduced earlier and the hybrid and statistical approaches explored in the subsequent parts of the dissertation.

4.2 Definition of the Classical C^4 Field

The C^4 formalism is introduced as a classical field-theoretical framework designed to describe electromagnetic propagation in complex atmospheric media, with particular emphasis on precipitating conditions. In its classical formulation, the C^4 field provides a deterministic description of wave-matter interaction, in which the macroscopic radiative properties of the atmosphere arise from the coupling between the electromagnetic field and the microphysical state of the medium. At the core of the C^4 framework lies the recognition that atmospheric extinction and emission processes cannot be fully characterized by isolated radiative-transfer coefficients alone. Instead, these quantities emerge from the interplay between four fundamental components, which collectively define the C^4 field. These components are not independent modelling layers, but mutually coupled elements of a single physical system. The four components define the structural decomposition of the C^4 system, whereas the three relations introduced later in this chapter provide the deterministic propagation closure along an effective path (geometry–phase–attenuation).

4.2.1 The Four Components of the C^4 Field

In the classical formulation adopted here, the C^4 field is defined through the following four components:

- C_1 : Cloud and precipitation microphysics. This component represents the physical state of hydrometeors within the atmosphere, including their phase (liquid, solid, or mixed), spatial distribution, and size spectra. Microphysical properties are typically described in terms of particle size distributions, number densities, and bulk moments, which together determine the interaction cross sections relevant to electromagnetic scattering and absorption.
- C_2 : Effective dielectric response of the medium. The microphysical information encoded in C_1 determines the macroscopic electromagnetic response of the atmosphere through



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effective-medium theories. This response is expressed in terms of a complex, frequency-dependent permittivity that accounts for both refractive and dissipative effects. In the classical C^4 framework, this effective permittivity acts as the primary link between cloud microphysics and electromagnetic propagation.

- C_3 : Complex electromagnetic field. The propagating electromagnetic field is treated as a classical complex field, whose amplitude and phase evolve in response to the dielectric properties of the medium. Both attenuation and phase delay are naturally incorporated through the complex-valued constitutive relations, without invoking statistical or quantum descriptions at this stage.
- C_4 : Coupling and propagation operator. The fourth component represents the mathematical operator that governs the interaction between the electromagnetic field and the atmospheric medium. In the classical limit, this operator encapsulates the combined effects of propagation, scattering, and absorption, providing a unified description from which radiative-transfer equations can be derived.

Together, these four components define the C^4 field as a coupled system in which microphysics, material response, and electromagnetic propagation are treated within a single theoretical structure.

4.2.2 Atmospheric State Vector in the Classical Limit

Within the deterministic formulation of the C^4 field, the atmospheric medium is characterized by a macroscopic state vector that specifies the relevant thermodynamic and microphysical variables. Denoting this state vector by $\mathbf{S}$, one may write:

$$\mathbf{S} = \{T(\mathbf{r}), p(\mathbf{r}), q(\mathbf{r}), \text{PSD}(\mathbf{r}), \chi(\mathbf{r})\}, \quad (4.1)$$

where T and p are the local temperature and pressure, q denotes the hydrometeor mass or mixing ratio, PSD represents the particle size distribution, and χ collectively denotes additional compositional or structural parameters, such as particle shape factors or phase fractions. The state vector $\mathbf{S}$ is assumed to vary smoothly in space and time on scales relevant to radiative-transfer modelling. Under this assumption, the atmospheric medium can be treated as locally homogeneous for the purpose of defining effective dielectric properties, while retaining large-scale inhomogeneities through spatial dependence of $\mathbf{S}$.

4.2.3 Classical Constitutive Description

Given the atmospheric state vector $\mathbf{S}$, the electromagnetic response of the medium is described by an effective complex permittivity $\varepsilon_{\text{eff}}(\omega, \mathbf{r})$, which depends on frequency and position through the underlying microphysical properties. In the classical C^4 framework, this permittivity encapsulates both refractive and absorptive effects, with its real and imaginary parts governing phase



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propagation and extinction, respectively. The introduction of ε_{eff} allows the electromagnetic field to be described using standard macroscopic electrodynamics, while implicitly accounting for the presence of clouds and precipitation. In this sense, the C^4 formalism does not modify Maxwell's equations themselves, but rather provides a structured prescription for defining the material properties entering those equations in complex atmospheric environments.

4.2.4 Deterministic Nature of the Classical C^4 Field

It is important to emphasize that, at the level considered in this chapter, the C^4 field theory is entirely deterministic. All quantities entering the formalism are defined through macroscopic state variables and classical field descriptions, and no probabilistic or statistical interpretation is required. This deterministic formulation constitutes the natural baseline against which more general descriptions can be developed. In particular, the classical C^4 field provides a reference framework in which radiative-transfer coefficients and optical-depth formulations can be derived in a physically transparent manner. As will be shown in the following sections, this approach allows standard radiative-transfer models, including SNEM-2, to be interpreted as specific realizations or approximations of the broader C^4 structure. The deterministic C^4 field defined here thus serves as the foundational layer of the overall framework, bridging empirical radiative-transfer modelling and the more general operator-based formulations introduced in **Chapter 5**.

4.2.5 Classical C^4 Propagation Closure: Metric–Phase–Attenuation

The classical C^4 formalism is completed by a deterministic propagation closure that ties together (i) the effective geometry of propagation, (ii) the accumulation of electromagnetic phase, and (iii) the radiative attenuation along an effective path. This closure provides the minimal set of relations required to connect the macroscopic atmospheric state variables introduced in §4.2.2 to measurable propagation observables, while remaining fully compatible with classical radiative-transfer modelling and with the numerical solvers employed later in this thesis.

4.2.5.1 Effective Propagation Metric

The first defining relation introduces an effective propagation metric that generalizes the standard Euclidean description by allowing the propagation distance to encode medium-induced temporal evolution. In the classical C^4 framework, the effective line element is written as:

$$(\Delta s)^2 = (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 + v^2(t^2 - t'^2), \quad (4.2)$$

where s denotes the effective propagation coordinate along the ray $r(s)$, and v is an effective transport velocity incorporating refractive and microphysical influences of the medium. The effective metric is introduced solely as a propagation parametrization and is not required to satisfy invariance, covariance, or symmetry properties associated with fundamental space–time metrics. The quadratic temporal contribution does not imply relativistic dynamics; rather,



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it provides a compact deterministic representation of non-stationary propagation in evolving atmospheric columns, where the effective path length depends on the temporal evolution of the macroscopic state of the medium. Here “memory” refers to deterministic dependence on time-varying macroscopic state variables, not to stochastic dynamics or ensemble averaging. Within this interpretation, the C^4 metric acts as a geometrical scaffold: it defines the effective propagation coordinate used consistently by the phase and attenuation relations that follow, ensuring that geometrical and radiative quantities are accumulated along the same medium-dependent path.

4.2.5.2 Phase Accumulation Along the C^4 Path

The second closure relation governs the accumulation of electromagnetic phase along the effective C^4 path. The phase delay associated with an angular frequency ω is expressed as:

$$\phi(\omega) = \int_C \frac{\omega}{v(r(s), \omega)} ds = \frac{\omega}{v} s(\omega), \quad (4.3)$$

which links the phase directly to the effective distance defined by the C^4 metric. The frequency dependence of $s(\omega)$ reflects the dispersive response of the medium through its effective refractive properties, rather than an intrinsic frequency dependence of the geometrical distance. The parameter v acts as an effective transport velocity summarizing refractive and microphysical effects, and is not intended to represent a physical signal velocity. This relation highlights that phase evolution is controlled not only by the geometric separation between transmitter and receiver, but also by the integrated dynamical state of the atmosphere encoded in the effective path $s(\omega)$ and in the parameter v . In this sense, phase variability and effective decorrelation associated with deterministic medium evolution arise naturally under inhomogeneous or time-evolving conditions, without invoking stochastic dynamics or ensemble-based statistical assumptions at this stage.

4.2.5.3 Radiative Attenuation and Optical Consistency

Radiative consistency with classical transfer theory is enforced by the third closure relation, which specifies attenuation along the same effective path. In the classical C^4 framework, the intensity evolution is written as:

$$I(\omega) = I_0(\omega) e^{-\tau_{eff}(\omega)}, \quad \tau_{eff}(\omega) = \int_C k_{ext}(\mathbf{r}(s), \omega) ds, \quad (4.4)$$

where k_{ext} (or k_e for brevity) is the extinction coefficient and τ_{eff} is the effective optical depth accumulated along the C^4 propagation coordinate s . The integral is evaluated along the effective C^4 propagation curve C , representing the ray path ($\mathbf{r}(s)$) determined by the refractive structure of the medium.

This expression is fully consistent with the **Beer–Lambert** law and provides the deterministic attenuation baseline adopted throughout the dissertation. Importantly, attenuation is not



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introduced phenomenologically: it is treated as the path-integrated consequence of the medium's microphysical and thermodynamic state, encoded via the extinction coefficient and accumulated along the same effective geometry used for phase propagation. The use of ds as the accumulation element ensures conceptual continuity with the radiative-transfer derivations developed later in §4.4 and with the optical-depth formulation formalized in §4.5, while remaining directly compatible with the numerical modelling strategy employed in SNEM and SNEM-2.

4.2.5.4 Role of the Closure Within the Chapter Structure

Together, the effective metric, phase law, and attenuation relation provide a compact deterministic closure of the classical C^4 propagation model. They establish the common propagation backbone on which the remainder of the chapter is built: §4.3 specifies how the atmospheric state vector determines the effective medium response, from which extinction and related coefficients emerge; §4.4 derives the corresponding radiative-transfer equation; and §4.5 formalizes the resulting optical depth and its link to path-integrated attenuation. In this way, the closure relations introduced here ensure that geometry, phase evolution, and radiative attenuation are treated within a single physically consistent deterministic framework.

4.3 Constitutive Relations in the Classical C^4 Framework

In the classical C^4 field theory, the connection between atmospheric microphysics and electromagnetic propagation is established through constitutive relations that link the macroscopic state of the medium to its effective electromagnetic response. These relations play a central role in the formalism, as they provide the physical mechanism through which cloud and precipitation properties influence refraction, attenuation, and emission processes. Unlike empirical radiative-transfer parametrizations, where extinction and scattering coefficients are introduced as external inputs, the C^4 framework treats these quantities as emergent properties of the dielectric response of the medium. This approach allows radiative-transfer parameters to be traced back to the underlying microphysical and thermodynamic state variables introduced in Section 4.2 .

4.3.1 Effective Dielectric Response of a Precipitating Medium

The electromagnetic behavior of a heterogeneous atmospheric volume containing hydrometeors can be described, at macroscopic scales, by an effective complex permittivity $\varepsilon_{\text{eff}}(\omega, \mathbf{r})$. This quantity accounts for the combined contribution of gaseous constituents, cloud droplets, rain, snow, graupel, and mixed-phase particles, and depends on frequency, spatial position, and polarization. In the classical C^4 framework, ε_{eff} is assumed to be locally defined, such that the medium can be treated as homogeneous within each infinitesimal atmospheric layer. This assumption is valid provided that the characteristic scale of microphysical variability remains small compared to the electromagnetic wavelength and the radiative-transfer resolution. Under



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this assumption, the effective permittivity can be expressed as:

$$\varepsilon_{\text{eff}}(\omega, \mathbf{r}) = \varepsilon_0(\omega, \mathbf{r}) + \Delta\varepsilon_{\text{hyd}}(\omega, \mathbf{r}), \quad (4.5)$$

where ε_0 represents the background contribution from atmospheric gases, and $\Delta\varepsilon_{\text{hyd}}$ encapsulates the additional response due to hydrometeors. The hydrometeor contribution is determined by the particle size distribution, phase composition, and dielectric properties of individual particles, as encoded in the atmospheric state vector $\mathbf{S}$. Effective-medium formulations, such as **Clausius--Mossotti** or **Maxwell-Garnett**-type relations, provide a physically consistent means of linking microscopic dielectric contrasts to the macroscopic response entering **Eq. (4.2)**. Within the scope of this chapter, the specific choice of mixing rule is not critical at the level of formal consistency addressed here, provided that it preserves the complex-valued nature of the permittivity and its dependence on the underlying microphysical parameters.

4.3.2 Physical Interpretation of the Constitutive Parameters

The complex permittivity introduced above can be decomposed as:

$$\varepsilon_{\text{eff}}(\omega, \mathbf{r}) = \varepsilon'(\omega, \mathbf{r}) + i\varepsilon''(\omega, \mathbf{r}), \quad (4.6)$$

where the real part ε' governs phase velocity and refractive effects, while the imaginary part ε'' accounts for dissipative processes leading to irreversible energy removal from the propagating mode at the radiative-transfer level, including absorption and scattering losses. In the classical C^4 picture, this decomposition provides a direct physical interpretation of radiative-transfer quantities. Variations in ε' translate into spatially varying refractive indices, leading to ray bending and path-length modifications, while ε'' determines the local extinction rate experienced by the propagating electromagnetic field. Importantly, both effects originate from the same constitutive description and are therefore treated on equal footing within the formalism. This unified interpretation contrasts with traditional modelling approaches in which refractive effects and attenuation are often handled separately, using distinct parametrizations. By embedding both phenomena within a single complex response function, the C^4 framework naturally accommodates their coupled influence on wave propagation.

4.3.3 From Constitutive Relations to Radiative-Transfer Coefficients

A key feature of the classical C^4 framework is that standard radiative-transfer coefficients emerge as derived quantities once the effective permittivity is specified. In particular, the local extinction coefficient $k_e(\omega, \mathbf{r})$ can be related to the imaginary part of the complex refractive index associated with ε_{eff} ,

$$k_e(\omega, \mathbf{r}) = \frac{2\omega}{c} \text{Im}\{n_{\text{eff}}(\omega, \mathbf{r})\}, \quad (4.7)$$

where $n_{\text{eff}} = \sqrt{\varepsilon_{\text{eff}}/\varepsilon_0}$ denotes the effective refractive index. Similarly, scattering and absorption



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contributions at the radiative-transfer level can be identified through the dependence of ε_{eff} on particle concentration and dielectric contrast, within the effective-medium approximation. This perspective provides a transparent link between the C^4 field theory and the parametrizations employed in SNEM-2. Rather than introducing extinction and scattering coefficients as independent inputs, the C^4 framework clarifies their origin and dependence on the atmospheric state vector, thereby reducing the number of implicit assumptions required at the modelling level.

4.3.4 Consistency with Classical Radiative Transfer

The constitutive relations described above ensure full consistency between the C^4 field theory and classical radiative-transfer formulations. When the effective permittivity varies slowly compared to the electromagnetic wavelength, and multiple scattering effects are treated within standard approximations, the resulting propagation equations reduce to the familiar radiative-transfer equations discussed in **Chapter 3**. In this sense, the C^4 framework does not supersede classical radiative transfer, but embeds it within a more general constitutive structure. Models such as SNEM-2 can therefore be interpreted as specific realizations of the classical C^4 field theory, corresponding to particular choices of microphysical parametrizations and effective-medium relations. The advantage of the C^4 approach lies in its ability to accommodate extensions beyond these specific realizations. By formulating radiative-transfer coefficients as consequences of the underlying dielectric response, the framework remains flexible with respect to frequency range, hydrometeor complexity, and modelling assumptions, while preserving a clear physical interpretation.

4.3.5 Role of Constitutive Relations within the Classical C^4 Field

Within the overall structure of the classical C^4 field theory, constitutive relations occupy a pivotal position. They provide the explicit mapping from the atmospheric state vector $\mathbf{S}$ to the electromagnetic propagation properties of the medium, thereby closing the theoretical description introduced in **Sections 4.1** and **Section 4.2**. The constitutive framework developed here forms the basis for the derivation of radiative-transfer equations in the next section. By making the dependence of extinction and scattering on microphysical properties explicit, it enables a systematic treatment of optical depth and path-integrated attenuation within the classical C^4 picture, which is addressed in **Section 4.5**. At the same time, the deterministic nature of the constitutive relations provides a natural reference point for more general formulations. In **Chapter 5**, the same conceptual structure will be extended to operator-based and statistical descriptions, where the effective permittivity acquires a more general interpretation. The classical relations derived here thus represent the baseline upon which those extensions are constructed.



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4.4 Classical C^4 Radiative Transfer Equation

Having established the constitutive structure of the classical C^4 field, the next step is to derive the corresponding radiative-transfer description governing electromagnetic propagation in a precipitating atmosphere. In the C^4 framework, the radiative-transfer equation does not appear as an independent postulate, but rather as a reduced description emerging from the underlying field-matter coupling introduced in the previous sections. The purpose of this section is twofold. First, it shows how the standard radiative-transfer equation can be recovered from the classical C^4 field theory under appropriate assumptions. Second, it clarifies the relationship between this formulation and the numerical implementation adopted in SNEM-2, thereby establishing a direct theoretical link between the C^4 formalism and the modelling tools employed throughout the dissertation.

4.4.1 Propagation of the Classical C^4 Field

In a macroscopic medium characterized by the effective permittivity $\varepsilon_{\text{eff}}(\omega, \mathbf{r})$, electromagnetic propagation is governed by Maxwell's equations supplemented by the constitutive relations discussed in **Section 4.3**. When the spatial variability of ε_{eff} is sufficiently slow compared to the electromagnetic wavelength, the propagation problem can be described within the geometrical optics approximation. Under this assumption, the electromagnetic field may be represented locally as a superposition of plane waves whose amplitude varies along the propagation path. The evolution of the field intensity along a ray trajectory s can then be expressed in terms of the effective extinction coefficient $k_e(\omega, \mathbf{r})$ derived in **Eq. (4.4)**. This description naturally incorporates both absorption and scattering losses through the imaginary part of the effective permittivity.

4.4.2 Derivation of the Radiative Transfer Equation

Within the classical C^4 framework, the specific intensity $I(\omega, \mathbf{r}, \hat{\mathbf{n}})$ associated with a propagation direction $\hat{\mathbf{n}}$ is introduced as a macroscopic quantity representing the energy flux carried by the electromagnetic field. Within the macroscopic and effective-medium regime implicit in the classical C^4 formulation, its evolution along a ray path parameterized by s can be written as:

$$\frac{dI}{ds} = -k_e(\omega, \mathbf{r}) I + j(\omega, \mathbf{r}) + \int_{4\pi} k_s(\omega, \mathbf{r}, \hat{\mathbf{n}}', \hat{\mathbf{n}}) I(\omega, \mathbf{r}, \hat{\mathbf{n}}') d\Omega', \quad (4.8)$$

where k_e denotes the total extinction coefficient, j is the local emission term, and k_s represents the differential scattering coefficient. In this expression, all coefficients are functions of the atmospheric state vector $\mathbf{S}$ through their dependence on the effective permittivity and its microphysical determinants. Equation (4.5) therefore constitutes a radiative-transfer equation that is fully consistent with the classical C^4 field theory, rather than an externally imposed transport law.



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4.4.3 Emergence of Radiative-Transfer Coefficients

A key aspect of the C^4 formulation is that the coefficients appearing in **Eq. (4.5)** are not introduced phenomenologically. The extinction coefficient k_e arises directly from the imaginary part of the effective refractive index, as discussed in **Section 4.3**, while the emission term j follows from the same constitutive description through Kirchhoff-type relations in the classical limit. Similarly, the scattering coefficient k_s reflects the angular redistribution of energy associated with spatial fluctuations of the dielectric response induced by hydrometeors. In practice, these fluctuations are described in terms of the particle size distribution and dielectric contrast encoded in the state vector $\mathbf{S}$. The resulting phase functions and angular moments therefore retain a clear physical interpretation within the C^4 framework. This unified origin of extinction, scattering, and emission terms highlights the internal consistency of the classical C^4 field theory. Radiative-transfer coefficients commonly employed in atmospheric propagation models emerge as macroscopic manifestations of the same underlying field-matter interaction, rather than as independent modelling constructs.

4.4.4 Reduction to One-Dimensional and Slant-Path Geometries

For many practical applications, including satellite communication links and remote-sensing configurations, the full angular dependence of **Eq. (4.5)** can be reduced by considering propagation along a prescribed slant path. In this case, the radiative-transfer equation simplifies to a one-dimensional form along the ray coordinate s ,

$$\frac{dI(s)}{ds} = -k_e(s) I(s) + j(s), \quad (4.9)$$

where angular scattering contributions are either neglected or incorporated through effective coefficients, consistent with the macroscopic effective-medium and single-path propagation assumptions adopted here. This reduction is consistent with the modelling assumptions underlying SNEM-2, where atmospheric profiles are discretized along the propagation path and radiative-transfer quantities are evaluated layer by layer. Within the C^4 framework, such a discretization can be interpreted as a consistent numerical realization of the continuous classical radiative-transfer equation derived above.

4.4.5 Connection with the SNEM-2 Model

The formulation presented here provides a direct theoretical interpretation of the SNEM-2 model in terms of the classical C^4 field theory. In SNEM-2, extinction and emission coefficients are computed from prescribed microphysical parametrizations and subsequently integrated along the propagation path to obtain attenuation and brightness temperature. From the C^4 perspective, this procedure corresponds to specifying a particular mapping from the atmospheric state vector $\mathbf{S}$ to the effective permittivity ε_{eff} , and then solving the reduced radiative-transfer equation in **Eq. (4.6)**. SNEM-2 thus appears as a consistent, though specialized, implementation of the broader



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C^4 framework, tailored to specific frequency ranges and microphysical assumptions. In this sense, SNEM-2 corresponds to a specific closure of the C^4 chain $S \rightarrow \varepsilon_{eff} \rightarrow k_e, k_s, j \rightarrow I, T_B, PIA$, rather than an independent modelling layer. This interpretation clarifies the scope and limitations of SNEM-2: while the model captures the essential physics of classical radiative transfer in precipitating media, the C^4 formulation makes explicit the assumptions under which this description holds and provides a natural pathway for systematic extensions.

4.4.6 Deterministic Character of the Classical C^4 RTE

It is important to emphasize that the radiative-transfer equation derived in this section remains entirely deterministic. All coefficients entering the formulation are specified by macroscopic state variables and classical constitutive relations, and no statistical averaging beyond the effective-medium description is required. This deterministic character defines the classical limit of the C^4 field theory and provides a well-defined baseline against which more general treatments can be formulated, including operator-based and quantum-statistical extensions.

These extensions, discussed in **Chapter 5**, should be understood as structured generalizations of the classical radiative-transfer equation derived here, rather than as ad hoc modifications. In this sense, the classical C^4 radiative-transfer equation constitutes the conceptual and mathematical hinge between the physical formalism introduced in the present chapter and the extended modelling frameworks developed in the remainder of the dissertation.

4.5 Optical Depth in the Classical C^4 Picture

Within the classical C^4 field theory, optical depth emerges as a cumulative measure of the interaction between the electromagnetic field and the atmospheric medium along a given propagation path. Rather than being introduced as an independent modelling quantity, optical depth follows directly from the constitutive and radiative-transfer structure established in **Sections 4.3 and 4.4**. In this sense, optical depth provides a natural closure between the microscopic description encoded in the C^4 field and the macroscopic observables employed in atmospheric propagation studies.

4.5.1 Definition of Optical Depth in the C^4 Framework

In the classical formulation, the optical depth $\tau(\omega)$ associated with a propagation path $\mathcal{P}$ is defined as the path integral of the local extinction coefficient,

$$\tau(\omega) = \int_{\mathcal{P}} k_e(\omega, \mathbf{r}(s)) ds, \quad (4.10)$$

where k_e is the total extinction coefficient derived from the effective permittivity in **Eq. (4.4)**, and s denotes the curvilinear coordinate along the ray path. This definition highlights the cumulative nature of extinction within the C^4 framework, as all contributions are accumulated



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along the same effective propagation coordinate s . The optical depth depends not only on the local properties of the medium, but also on the geometry of propagation, including path length, bending, and stratification effects. As such, τ encapsulates both microphysical and geometrical contributions to signal attenuation.

4.5.2 Stratified Atmosphere and Slant-Path Geometry

For a vertically stratified atmosphere, the integral in **Eq. (4.7)** can be expressed as a sum over discrete atmospheric layers or as a continuous integral over altitude. In the case of slant-path propagation, relevant for satellite links and remote-sensing observations, the optical depth takes the form:

$$\tau(\omega, \theta) = \int_0^{s_{\max}} k_e(\omega, h(s)) ds, \quad (4.11)$$

where θ denotes the local elevation angle (measured from the horizontal, with $\theta = 90^\circ$ corresponding to the zenith). Within the classical C^4 picture, the geometry of the ray path is determined by the refractive structure of the atmosphere, which in turn depends on the real part of the effective permittivity. Ray bending effects therefore enter the optical depth implicitly through the mapping between s and h , i.e. through the effective propagation geometry defined by the refractive structure of the medium. This treatment is fully consistent with the geometrical considerations discussed in **Chapter 2** and provides a physically grounded description of slant-path attenuation.

4.5.3 Connection to Attenuation and Brightness Temperature

The optical depth defined above directly governs the attenuation experienced by the propagating electromagnetic field. In the absence of in-scattering into the line of sight, the solution of the reduced radiative-transfer equation (**Eq. (4.6)**) yields an exponential decay of intensity,

$$I(s_{\max}) = I(0) \exp[-\tau(\omega)]. \quad (4.12)$$

This expression provides the classical link between the C^4 formalism and the path-integrated attenuation quantities commonly employed in atmospheric propagation studies, identifying PIA as a direct observable consequence of the optical depth. In particular, the path-integrated attenuation (PIA) can be written as:

$$\text{PIA}(\omega) = 10 \log_{10} (e^{\tau(\omega)}) = 4.343 \tau(\omega), \quad (4.13)$$

thereby establishing a direct correspondence between the optical depth and measurable attenuation levels. When emission is included, the same optical-depth formulation governs the contribution of atmospheric layers to the observed brightness temperature. In the classical limit, the brightness temperature T_B can be expressed as a path integral weighted by the local extinction and the atmospheric temperature profile, with the optical depth acting as the natural attenu-



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ation kernel. This interpretation is fully consistent with the SNEM-2 brightness-temperature formulation discussed in **Chapter 3**.

4.5.4 Physical Interpretation of Optical Depth in the C^4 Picture

Within the classical C^4 framework, optical depth admits a clear physical interpretation as a measure of cumulative wave-matter interaction at the radiative-transfer level. Each infinitesimal contribution $d\tau = k_e ds$ represents the local strength of coupling between the electromagnetic field and the atmospheric medium, as determined by the effective dielectric response and accumulated along the effective propagation geometry. This interpretation emphasizes that optical depth is not merely a geometrical or empirical quantity, but a direct manifestation of the constitutive relations linking microphysics to electromagnetic propagation. Variations in hydrometeor content, particle size distribution, or phase composition modify ε_{eff} , and therefore alter the optical depth in a physically transparent manner.

4.5.5 Role of Optical Depth as a Deterministic Baseline

The classical optical-depth formulation developed in this section provides a deterministic baseline for more general treatments. All quantities entering **Eqs. (4.7)–(4.10)** are defined through macroscopic state variables and classical field descriptions, without invoking statistical or operator-based concepts. This reference configuration plays a key role in the overall structure of the C^4 framework. In **Chapter 5**, the optical depth will be reinterpreted within a statistical setting, where it acquires a probabilistic meaning associated with survival amplitudes and open-system dynamics. The classical formulation presented here therefore serves as the reference point against which such generalizations can be consistently developed. By anchoring attenuation and brightness-temperature modelling to the optical depth defined within the classical C^4 field theory, the framework provides a coherent closure linking atmospheric microphysics, propagation geometry, and observable quantities. This closes the deterministic part of the C^4 formalism and prepares the ground for the operator-based extensions discussed in the following chapter.

4.6 Deterministic Limit and Physical Interpretation

The formulation developed in the previous sections defines the C^4 framework as a fully classical and deterministic field theory for electromagnetic propagation in precipitating atmospheres. In this limit, all quantities entering the description—constitutive parameters, radiative-transfer coefficients, and optical depth—are uniquely determined by the macroscopic atmospheric state vector and by the geometry of propagation. No stochastic or operator-based elements are required to interpret attenuation, emission, or scattering processes.

This deterministic character should not be interpreted as a simplifying assumption, but rather as a well-defined physical regime. It corresponds to the situation in which the atmosphere can be



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described through smoothly varying thermodynamic and microphysical fields, and where effective-medium and radiative-transfer approximations remain valid. Within this regime, the C^4 field theory provides a complete and internally consistent description of wave-matter interaction at the macroscopic level, forming the deterministic reference against which more general formulations are developed.

4.6.1 Deterministic Closure of the C^4 Framework

The classical C^4 formulation achieves closure by explicitly linking the atmospheric state vector $\mathbf{S}$ to observable propagation effects through a chain of deterministic relations. Microphysical properties determine the effective dielectric response of the medium, which in turn governs extinction, scattering, and emission through the constitutive relations introduced in **Section 4.3**. These quantities enter the radiative-transfer equation derived in **Section 4.4**, whose solution yields intensity, attenuation, and brightness temperature along a given propagation path. In this sense, the C^4 framework eliminates the need for ad hoc parametrizations at the conceptual level, as radiative-transfer quantities emerge directly from the underlying constitutive structure. While practical implementations may still rely on approximations or empirical inputs, the theoretical architecture remains deterministic and physically grounded. Standard radiative-transfer models, including SNEM-2, can therefore be interpreted as closed realizations of the classical C^4 field theory under specific modelling assumptions.

4.6.2 Physical Meaning of the C^4 Components

From a physical perspective, the four components of the C^4 field acquire a clear interpretation in the deterministic limit. Cloud and precipitation microphysics (C_1) specify the material composition of the medium, while the effective dielectric response (C_2) encodes how this composition modifies the propagation of electromagnetic waves. The complex electromagnetic field (C_3) describes the propagating signal itself, and the coupling operator (C_4) formalizes the interaction between field and medium through propagation, scattering, and attenuation processes.

The deterministic formulation emphasizes that these components are not independent descriptions, but mutually constrained elements of a single physical system. Any variation in the atmospheric state vector propagates deterministically through the constitutive relations and radiative-transfer equations, ultimately affecting observable quantities such as attenuation and brightness temperature. This interdependence represents one of the key strengths of the C^4 framework.

4.6.3 Relation to Classical Radiative-Transfer Modelling

In the deterministic limit, the C^4 field theory reproduces the familiar structure of classical radiative-transfer modelling while providing a clearer interpretation of its parameters. Extinction coefficients, phase functions, and optical depth are no longer treated as isolated modelling inputs,



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but as macroscopic manifestations of the underlying dielectric response of the medium. This perspective further clarifies the physical meaning of commonly used quantities and highlights the assumptions under which they are valid. In particular, it makes explicit the conditions required for the applicability of one-dimensional radiative-transfer formulations and slant-path approximations, as discussed in **Section 4.5**. When these conditions are met, the classical C^4 framework reduces naturally to the modelling approaches employed in SNEM-2 and related tools, which can be interpreted as consistent realizations of the deterministic limit.

4.6.4 Scope and Limitations of the Deterministic Description

Despite its generality, the deterministic C^4 formulation has well-defined limits of applicability. Situations involving strong fluctuations, unresolved microphysical variability, or processes that are effectively stochastic at the macroscopic modelling scale may not be fully captured within a purely deterministic framework. In such cases, statistical or operator-based descriptions may become useful to represent additional degrees of freedom in an effective manner. Rather than representing a failure of the C^4 approach, these limitations define the boundary of the classical regime. By establishing a rigorous deterministic baseline, the C^4 field theory provides a reference against which more general formulations can be developed in a controlled and physically interpretable manner.

4.6.5 Deterministic Limit as a Reference for Extended C^4 Formulations

The deterministic C^4 formulation developed in this chapter provides a well-defined reference limit for all subsequent extensions introduced in this dissertation. In this regime, electromagnetic propagation is fully characterized by macroscopic constitutive relations, scalar radiative coefficients (or their standard angularly averaged counterparts), and radiative-transfer equations derived from the effective dielectric response of the atmosphere. All physical quantities retain their standard interpretation, and propagation dynamics are governed by smoothly varying parameters without explicit statistical or operator-valued structure. This deterministic limit plays a crucial conceptual role: it establishes the baseline against which more general formulations must reduce when ensemble variability, unresolved microphysical fluctuations, and non-local coupling effects at the macroscopic modelling level become negligible. Any extended C^4 formulation—whether statistical, operator-based, or ensemble-driven—is therefore required to recover the deterministic equations and attenuation laws presented here under appropriate limiting assumptions. By explicitly identifying this limit, the C^4 framework guarantees continuity between classical radiative-transfer theory and its generalized extensions. The deterministic formulation thus serves not only as the culmination of the present chapter, but also as a structural anchor ensuring physical interpretability and mathematical consistency in the developments that follow. In this sense, the deterministic C^4 theory is not superseded by extended formulations, but embedded within them as a reference configuration governing their classical behavior.



4.7 Transition to the Statistical C^4 Field

The classical formulation of the C^4 field theory developed in this chapter provides a complete and internally consistent description of electromagnetic propagation in precipitating atmospheres within a deterministic macroscopic regime. Constitutive relations, radiative-transfer equations, and optical depth have been derived as direct consequences of the effective dielectric response of the medium, without invoking stochastic or operator-based concepts. While this deterministic framework captures the essential physics underlying models such as SNEM-2, it also reveals the conditions under which a more general description becomes necessary. The transition to a statistical C^4 formulation is therefore not motivated by formal abstraction, but by the physical limitations inherent to the classical regime.

4.7.1 Motivation for a Statistical Extension

Several aspects of electromagnetic propagation in complex atmospheric media motivate extensions beyond a purely deterministic classical description. Unresolved microphysical variability, strong hydrometeor fluctuations, and multiple-scattering processes introduce effective degrees of freedom that cannot always be represented by smoothly varying macroscopic quantities. Under these conditions, effective-medium and classical radiative-transfer approximations remain valid primarily in an averaged sense. A statistical formulation therefore provides a natural framework to describe propagation under variability, enabling field–medium interactions to be expressed in terms of ensemble-based states and operators, while preserving exact continuity with the classical deterministic limit.

4.7.2 Conceptual Continuity with the Classical C^4 Framework

A key feature of the C^4 approach is that the statistical formulation does not replace the classical theory, but generalizes it. All quantities defined in the deterministic framework—effective permittivity, extinction coefficient, radiative-transfer operators, and optical depth—retain their physical meaning as expectation values or limiting cases of more general operator-based constructs. From this perspective, the classical C^4 field theory developed in **Sections 4.1–4.6** can be interpreted as the macroscopic limit of a broader theoretical structure. The transition to the statistical regime is therefore continuous, preserving physical interpretability across different modelling scales.

4.7.3 Role of Optical Depth in the Transition

Optical depth plays a particularly important role in bridging the classical and statistical descriptions. In the deterministic formulation, optical depth quantifies cumulative extinction along a propagation path, as discussed in **Section 4.5**. In a generalized framework, the same quantity can be reinterpreted as a measure of survival probability or attenuation amplitude associated with field propagation through an open system. This reinterpretation does not modify the classical



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results, but extends their domain of validity. The classical optical depth then emerges naturally as the mean value of a more general statistical description, thereby preserving consistency and continuity between modelling approaches.

4.7.4 Structure of the Statistical C^4 Field

The statistical C^4 framework introduced in the next chapter builds directly on the structural elements established here. The atmospheric state vector is generalized to a statistical ensemble, the electromagnetic field is represented through state vectors or density operators, and the coupling between field and medium is formulated in terms of evolution operators acting on these states. Despite this formal extension, the physical interpretation remains rooted in the same concepts introduced in the classical theory. The four components of the C^4 field continue to represent microphysics, dielectric response, field dynamics, and field-medium coupling, thereby ensuring conceptual continuity across the classical and statistical regimes.

4.7.5 Role of the Classical C^4 Framework within the Dissertation

This chapter establishes the classical C^4 field theory as the theoretical backbone of the modelling strategy adopted in this dissertation. By formalizing the deterministic limit and its physical interpretation, it provides a solid reference against which more advanced formulations can be developed and assessed. **Chapter 5** builds upon this foundation by introducing the statistical generalization of the C^4 framework. The transition outlined in this section ensures that the reader can follow this progression without conceptual breaks, recognizing the classical theory not as a special case, but as the macroscopic limit of a unified field-based description of electromagnetic propagation in complex atmospheric media.



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Chapter 5

Statistical C^4 Field Theory: Operator Formulation and Open-System Extensions

5.1 Motivation and Scope of the Statistical Extension

Chapter 4 established the classical C^4 field theory as a deterministic and macroscopically closed framework for electromagnetic propagation in complex precipitating atmospheres. In that regime, the atmospheric medium is fully characterized by a macroscopic state vector, the effective dielectric response is prescribed through constitutive relations, and standard radiative-transfer quantities—including extinction coefficients, optical depth, and path-integrated attenuation—follow as derived objects. The present chapter introduces a statistical generalization of the C^4 framework. The goal is not to replace the classical formulation, but to extend it in a controlled manner to regimes where deterministic closure becomes insufficient. These regimes occur whenever the atmospheric medium exhibits relevant degrees of freedom that cannot be represented as smoothly varying macroscopic fields at the modelling scale of interest. Examples include unresolved microphysical variability, strong intermittency in hydrometeor distributions, and multiple-scattering configurations in which coarse-grained descriptions are only valid in an average or effective sense. From a modelling standpoint, the need for such an extension becomes apparent when macroscopic coefficients (e.g., extinction or phase-function parametrizations) must simultaneously capture: (i) large variability at fixed thermodynamic conditions, (ii) nontrivial correlations in space and time, and (iii) deviations from mean-field closures induced by strong fluctuations. In these cases, the classical C^4 mapping:

$$\mathbf{S} \mapsto \varepsilon_{\text{eff}} \mapsto k_e \mapsto \tau \quad (5.1)$$

remains a valuable baseline, but it is no longer sufficient as a complete description of propagation. The statistical C^4 framework introduced here provides a principled language for describing this



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generalized regime. Conceptually, it promotes the deterministic atmospheric state to a statistical object and expresses propagation through operators acting on an appropriate state space. The classical quantities derived in **Chapter 4** then reappear as limiting cases or expectation values, thereby ensuring continuity and interpretability across modelling scales. Importantly, the statistical formalism adopted in this chapter is motivated by structural analogies with open dissipative systems and classical transport theory. The formalism is employed as a compact and consistent mathematical framework to encode loss, scattering-induced redistribution, and effective irreversibility in a way that can incorporate fluctuations and ensemble descriptions while retaining a clear connection to classical radiative transfer. The scope of this chapter is therefore threefold. First, it introduces the statistical state space underlying the generalized C^4 description and defines the relevant ensemble objects. Second, it formulates the C^4 coupling in an operator language suitable for dissipative propagation and discusses the resulting evolution equations. Third, it provides an explicit bridge back to the classical limit by showing how deterministic radiative-transfer quantities, including optical depth, are recovered as limiting cases. This structure prepares the ground for the hybrid modelling approaches explored in the subsequent parts of the dissertation, where physically constrained statistical representations are combined with data-driven surrogates and machine-learning components.

5.2 Statistical State Space and Atmospheric Ensembles

The transition from the deterministic C^4 formulation to a statistical description requires a generalization of the atmospheric state space. In the classical framework introduced in **Chapter 4**, the medium is fully specified by a macroscopic state vector $\mathbf{S}(\mathbf{r}, t)$, whose components encode thermodynamic variables and bulk microphysical properties. This description implicitly assumes that all relevant degrees of freedom are resolved at the modelling scale of interest. In many practical situations, however, this assumption is only approximately satisfied. Subgrid-scale variability, intermittent hydrometeor distributions, and unresolved correlations introduce fluctuations that cannot be represented by a single deterministic realization of $\mathbf{S}$. The statistical C^4 framework addresses this limitation by promoting the atmospheric state from a single vector to an ensemble of admissible states.

5.2.1 Atmospheric Ensembles and Random State Variables

Within the statistical formulation, the atmospheric medium is described by an ensemble $\mathcal{E}$ of realizations $\{\mathbf{S}_\alpha\}$, each representing a physically admissible configuration of the atmosphere consistent with the available constraints. The index α labels distinct realizations, which may correspond to unresolved microphysical variability, measurement uncertainty, or intrinsic atmospheric fluctuations. Formally, the atmospheric state is treated as a random field:

$$\mathbf{S}(\mathbf{r}, t) \rightarrow \mathbf{S}(\mathbf{r}, t; \omega), \quad (5.2)$$



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where ω denotes an element of an underlying probability space. All macroscopic quantities entering the classical C^4 framework, including the effective dielectric response and extinction coefficient, therefore become random fields defined over the same ensemble. This ensemble-based representation preserves the physical content of the classical state vector while allowing variability to be explicitly encoded. Importantly, the statistical description does not require detailed knowledge of the microscopic dynamics responsible for the fluctuations; it only assumes that the ensemble adequately captures their macroscopic impact on propagation.

5.2.2 Statistical Description of Effective Medium Properties

In the statistical C^4 framework, the effective dielectric response of the medium is no longer a single deterministic function, but an ensemble-dependent quantity,

$$\varepsilon_{\text{eff}}(\mathbf{r}, \omega) = \varepsilon_{\text{eff}}[\mathbf{S}(\mathbf{r}; \omega)]. \quad (5.3)$$

As a consequence, derived quantities such as the extinction coefficient k_e and the source term j inherit the same statistical character. The ensemble description provides a natural interpretation of commonly used macroscopic coefficients as statistical summaries of an underlying distribution. Mean extinction profiles, for instance, correspond to ensemble averages of $k_e(\omega)$, while higher-order moments encode variability and intermittency effects that are inaccessible in a purely deterministic formulation. This perspective clarifies the status of empirical parametrizations frequently employed in radiative-transfer modelling. Rather than being viewed as fundamental laws, such parametrizations can be interpreted as effective closures for ensemble averaged quantities within the statistical C^4 framework.

5.2.3 Statistical State Space and Representation

To formalize the ensemble description, it is convenient to introduce an abstract statistical state space $\mathcal{H}_{\text{atm}}$ associated with the atmospheric medium. Each realization $\mathbf{S}_\alpha$ corresponds to a state in this space, while probability measures defined on $\mathcal{H}_{\text{atm}}$ encode the statistical structure of the ensemble. At this stage, no specific mathematical representation of $\mathcal{H}_{\text{atm}}$ is required. The formalism remains agnostic with respect to whether the ensemble is represented through probability density functions, discrete realizations, or coarse-grained statistical summaries. This flexibility is essential for accommodating both physics-based stochastic models and data-driven ensemble constructions. The introduction of a statistical state space does not alter the physical meaning of the C^4 components. Instead, it provides a unified setting in which classical deterministic states appear as special cases corresponding to delta-like distributions in $\mathcal{H}_{\text{atm}}$.

5.2.4 Ensemble Averages and Observable Quantities

Observable propagation quantities within the statistical C^4 framework are defined through ensemble averages. For any ensemble-dependent quantity $Q(\omega)$, the corresponding macroscopic



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observable is given by:

$$\langle Q \rangle = \int Q(\omega) d\mathbb{P}(\omega), \quad (5.4)$$

where $\mathbb{P}$ denotes the probability measure associated with the ensemble. In particular, ensemble-averaged extinction coefficients and optical depths provide the natural statistical generalization of the classical quantities introduced in **Chapter 4**. The deterministic optical depth derived in **Section 4.5** is recovered when the ensemble collapses to a single realization, while more general ensembles encode variability and uncertainty along the propagation path. This construction ensures that all classical observables remain meaningful within the statistical framework, while simultaneously extending their interpretation beyond the deterministic regime.

5.2.5 Positioning with Respect to the Operator Formalism

The statistical state space introduced in this section provides the foundation for the operator-based formulation developed in **Section 5.3**. At the present stage, the emphasis is placed on the representation of atmospheric variability through ensembles, without specifying the dynamical evolution of the electromagnetic field within this space. In the following section, the statistical state space will be combined with an operator description of propagation and interaction, allowing the C^4 framework to capture dissipative dynamics, multiple scattering, and attenuation within a unified statistical formalism. The ensemble description developed here therefore constitutes the necessary preparatory step for the full statistical C^4 theory.

5.3 Operator Formulation of the C^4 Field

Sections 5.1–5.2 introduced the need for a generalized C^4 framework capable of representing unresolved variability and ensemble descriptions of the atmosphere. The purpose of the present section is to formalize this generalization through an operator language. In the statistical C^4 framework, propagation and field-medium interaction are expressed through operators acting on an appropriate state space, with classical radiative-transfer quantities recovered as limiting cases or expectation values. The operator formalism serves two complementary roles. First, it provides a compact representation of attenuation and scattering-induced redistribution as an effective evolution generated by a (generally non-self-adjoint) operator. Second, it enables a systematic separation between deterministic mean-field contributions and fluctuation-induced corrections, thereby preserving continuity with the classical C^4 theory derived in **Chapter 4**.

5.3.1 State Space for the Statistical C^4 Description

To define an operator formulation, one must specify the objects on which operators act. In this chapter, the relevant notion of “state” is not a microscopic quantum state of the electromagnetic field in vacuum, but an abstract state that encodes the statistical configuration of propagation in a complex medium. I introduce a representation space $\mathcal{H}$ whose elements describe propagation



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configurations at the modelling scale of interest. Depending on the physical situation, propagation can be represented either by a single effective configuration or by a statistical ensemble of configurations.

- **Single-configuration representation.** A single element of $\mathcal{H}$ is sufficient when propagation can be described by an effective realization, for instance when ensemble variability is negligible or when a coherent amplitude-like description is adequate.
- **Ensemble representation.** An operator-valued statistical description is required when propagation involves a distribution of configurations, such as when atmospheric variability is represented by an ensemble of realizations or when coarse graining induces effective irreversibility.

In the ensemble case, the statistical propagation state is represented by a positive, normalized operator ρ acting on $\mathcal{H}$, defined as:

$$\rho = \sum_{\alpha} p_{\alpha} X_{\alpha}, \quad (5.5)$$

where $\{p_{\alpha}\}$ are non-negative weights summing to unity and X_{α} denote elementary propagation configurations. In this form, the ensemble description introduced in **Section 5.2** is embedded directly into the operator representation. Within the C^4 framework, the representation space $\mathcal{H}$ is chosen to encode the relevant degrees of freedom of propagation at the modelling scale, such as angular direction, polarization, or discretized atmospheric layers. The formalism remains independent of the specific basis adopted, provided that the chosen representation supports the action of propagation and scattering operators.

5.3.2 Observables and Expectation Values

Within the operator framework, measurable quantities correspond to observables, represented by operators $\hat{O}$ acting on $\mathcal{H}$. The statistical prediction for an observable is given by the expectation value:

$$\langle O \rangle = \text{Tr}(\rho \hat{O}), \quad (5.6)$$

which reduces to a deterministic evaluation of the observable in the absence of ensemble variability. This construction ensures that macroscopic radiative-transfer quantities can be consistently represented as statistical averages of suitable operators. In particular, intensity-like quantities, attenuation factors, and angular redistribution measures admit a unified operator-based representation within a statistical framework, while remaining fully compatible with the deterministic classical limit.

5.3.3 Propagation as an Operator Evolution

In the classical C^4 picture, propagation along a path coordinate s is described through radiative-transfer equations such as **Eq.(4.6)**, where extinction and emission appear as macroscopic



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coefficients. In the operator-based formulation, propagation is instead described as an evolution generated by an effective transport operator. Denoting by $\mathcal{U}(s)$ the propagation operator from 0 to s , the statistical propagation state evolves as:

$$X(s) = \mathcal{U}(s) X(0), \quad (5.7)$$

where X represents a suitable operator-valued description of the propagating field. In dissipative media, $\mathcal{U}(s)$ is not required to preserve norm and instead encodes attenuation and effective irreversibility induced by scattering and coarse graining. When a density-operator description is employed, the corresponding evolution can be formally written as:

$$\rho(s) = \mathcal{U}(s) \rho(0) \mathcal{U}^\dagger(s), \quad (5.8)$$

as an effective representation of propagation with loss. More general trace-preserving dissipative evolutions are discussed in **Section 5.4**.

5.3.4 Generator Decomposition: Conservative vs Dissipative Contributions

A central advantage of the operator formulation is that it allows propagation to be expressed in terms of a generator. Introducing an effective generator $\hat{G}$, one may write, at least formally,

$$\frac{d}{ds} X(s) = -\hat{G}(s) X(s), \quad (5.9)$$

where $X(s)$ denotes an operator-valued representation of the propagating field or intensity ensemble. The generator $\hat{G}(s)$ can be decomposed into contributions associated with conservative phase propagation and dissipative processes, according to:

$$\hat{G}(s) = \hat{G}_{\text{cons}}(s) + \hat{G}_{\text{diss}}(s),$$

where $\hat{G}_{\text{cons}}$ captures conservative phase evolution (linked to refractive effects), while $\hat{G}_{\text{diss}}$ represents effective loss and redistribution mechanisms. In the C^4 interpretation, $\hat{G}_{\text{cons}}$ is determined by the real part of the effective dielectric response, while $\hat{G}_{\text{diss}}$ encodes the impact of the imaginary part and scattering-induced mixing. This mapping preserves the physical content of the constitutive relations introduced in **Section 4.3** while generalizing the deterministic coefficients into operator objects.

5.3.5 Optical Depth as an Operator Quantity

A key bridge between classical and statistical descriptions is the optical depth. In **Chapter 4**, optical depth was defined deterministically as $\tau = \int k_e ds$ (**Section 4.5**). In the operator formulation, the cumulative attenuation can be represented through an effective “optical-depth



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operator" $\hat{\mathcal{T}}$ defined along the path $\mathcal{P}$,

$$\hat{\mathcal{T}}(\omega) = \int_{\mathcal{P}} \hat{K}_e(\omega, s) ds \quad (5.10)$$

where $\hat{K}_e$ generalizes the scalar extinction coefficient to an operator acting on the propagation state space. The deterministic scalar optical depth is recovered when $\hat{K}_e$ is proportional to the identity (or when its fluctuations are negligible), i.e.,

$$\hat{K}_e(\omega, s) \rightarrow k_e(\omega, s) \mathbb{I} \quad \Rightarrow \quad \hat{\mathcal{T}}(\omega) \rightarrow \tau(\omega) \mathbb{I}. \quad (5.11)$$

This construction anticipates the statistical interpretation introduced later: attenuation becomes associated with expectation values of operators or with decay rates generated by non-unitary evolution. The operator form is therefore the natural stepping stone between the deterministic optical depth and its statistical reinterpretation.

5.3.6 Recovering the Radiative-Transfer Structure

To connect the operator formulation with radiative transfer, consider an intensity-like observable $\hat{I}$ and evaluate its expectation value using **Eq. (5.6)**. Differentiating with respect to s and using the generator form in **Eq. (5.9)**, one obtains a general evolution equation of the form:

$$\frac{d}{ds} \langle I \rangle = - \langle \hat{\mathcal{L}} I \rangle + \langle \hat{J} \rangle, \quad (5.12)$$

where $\hat{\mathcal{L}}$ represents an effective loss/redistribution super-operator induced by $\hat{G}$ and $\hat{J}$ denotes an effective source contribution. In the classical limit where operator fluctuations are negligible and angular coupling is reduced to scalar coefficients, **Eq. (5.13)** reduces to the one-dimensional radiative-transfer equation derived in **Section 4.4**. Thus, the operator formulation is consistent with the classical radiative-transfer structure, while allowing the introduction of additional degrees of freedom required to represent statistical variability and mode coupling.

5.3.7 Mean-Field Closure and Fluctuation Corrections

The operator framework also provides a natural way to separate mean-field contributions from fluctuation-induced corrections. Let:

$$\hat{K}_e = \langle \hat{K}_e \rangle + \delta \hat{K}_e, \quad (5.13)$$

where $\langle \hat{K}_e \rangle$ denotes the ensemble-averaged part and $\delta \hat{K}_e$ captures fluctuations around the mean. In a mean-field approximation, one replaces $\hat{K}_e$ by its average, recovering a deterministic propagation law consistent with classical radiative transfer. Fluctuation terms can then be treated systematically as corrections that modify effective attenuation rates, introduce additional



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broadening, or induce non-trivial coupling between propagation modes. This structure is particularly relevant in regimes where hydrometeor fields exhibit intermittency or strong inhomogeneity, and where purely deterministic closures may underestimate variability in predicted attenuation and brightness temperature. The precise way in which these corrections enter depends on the dynamical evolution law for $\rho(s)$ and on the adopted coarse-graining scheme. These aspects are the focus of the next section, where the operator framework is embedded into an open-system description suitable for dissipative transport.

5.3.8 Summary and Transition to Open-System Dynamics

This section established the operator formulation of the statistical C^4 field theory. Atmospheric variability was embedded into a state representation through density operators, observables were defined via trace expectations, and propagation was expressed as a non-unitary evolution generated by an effective operator. The resulting structure provides a coherent bridge between the deterministic C^4 framework of **Chapter 4** and the generalized statistical descriptions required for complex precipitating atmospheres. In the next section, this operator formulation is completed by specifying physically consistent evolution equations for $\rho(s)$, including dissipative and scattering-induced effects, within an open-system framework.

5.4 Statistical Radiative Transfer and Open-System Dynamics

The operator formulation introduced in **Section 5.3** provides the kinematic structure required to describe propagation in a statistical C^4 framework. The purpose of the present section is to complete this construction by specifying physically consistent evolution equations for the propagation state. This leads naturally to an open-system description in which attenuation, scattering-induced redistribution, and effective irreversibility are encoded in the dynamics of a density operator. The resulting statistical radiative-transfer formalism generalizes the classical equations derived in **Section 4.4** while preserving their physical interpretation in the deterministic limit.

5.4.1 Propagation as an Open-System Evolution

In realistic atmospheric conditions, electromagnetic propagation occurs in the presence of loss, scattering, and mode coupling induced by interaction with a complex medium. From the perspective of the statistical C^4 framework, these effects imply that the propagation state cannot, in general, be described as an isolated system. Instead, it must be treated as an open system interacting with effective environmental degrees of freedom associated with unresolved atmospheric variability. The natural object describing such a system is the density operator $\rho(s)$, whose evolution along the propagation coordinate s captures both coherent and dissipative effects. Unlike the unitary evolution of closed systems, the dynamics of $\rho(s)$ must allow for



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non-unitary processes while preserving basic physical consistency conditions, such as positivity and normalization.

5.4.2 Master Equation for Statistical Radiative Transfer

A general and physically motivated form for the evolution of the density operator is provided by a master equation of the form:

$$\frac{d\rho}{ds} = -\hat{\mathcal{L}}_{\text{eff}} \rho, \quad (5.14)$$

where $\hat{\mathcal{L}}_{\text{eff}}$ is an effective generator encoding both conservative propagation and dissipative processes. A convenient decomposition of the generator is:

$$\hat{\mathcal{L}}_{\text{eff}} = -i[\hat{H}_{\text{eff}}, \cdot] + \hat{\mathcal{D}}, \quad (5.15)$$

where $\hat{H}_{\text{eff}}$ represents the conservative propagation operator associated with refractive and phase effects, while $\hat{\mathcal{D}}$ is a dissipative super-operator capturing extinction and scattering-induced redistribution. This structure provides a compact and physically consistent mathematical framework for describing transport with loss and irreversibility, closely analogous to approaches used in neutron transport, radiative-transfer theory, and kinetic descriptions of open systems.

5.4.3 Lindblad-Like Structure and Physical Consistency

To ensure that the evolution preserves positivity and normalization of ρ , the dissipative super-operator $\hat{\mathcal{D}}$ can be expressed in a Lindblad-like form,

$$\hat{\mathcal{D}}[\rho] = \sum_n \left(\hat{L}_n \rho \hat{L}_n^\dagger - \frac{1}{2} \{ \hat{L}_n^\dagger \hat{L}_n, \rho \} \right), \quad (5.16)$$

where the operators $\hat{L}_n$ encode effective scattering and extinction channels. In the C^4 interpretation, these operators represent coarse-grained interaction mechanisms associated with unresolved atmospheric variability. Each $\hat{L}_n$ can be associated with a class of scattering events, angular redistribution processes, or absorption mechanisms induced by hydrometeors and atmospheric constituents. The Lindblad-like structure guarantees that the resulting evolution remains mathematically and physically well defined at the statistical level, in direct analogy with positivity-preserving closures used in classical transport and kinetic theories.

5.4.4 Transition-Rate Closure in the C^4 Framework

The Lindblad-like structure introduced in **Section 5.4** provides a mathematically consistent representation of dissipative propagation. However, within the C^4 framework, the dissipative channels are not introduced phenomenologically. Instead, they can be constructed from a statistical description of scattering-induced mode redistribution driven by atmospheric microphysical variability. This construction is achieved through a transition-rate formalism formally analogous



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to classical rules used in transport theory. The structure is employed here as a statistical closure for scattering-induced mode redistribution within the C^4 framework. Let $|i\rangle$ and $|f\rangle$ denote propagation modes characterized by a frequency shift ω and a wave-vector transfer $\mathbf{q}$. The C^4 transition rate between such modes is written in a Fermi-like form ¹, formally analogous to transition-rate expressions widely used in classical transport theories for waves in random media:

$$W_{i \rightarrow f}^{(C^4)}(\omega, \mathbf{q}) = \mathcal{C} |\mathcal{M}_{if}(\omega, \mathbf{q})|^2 S_{C^4}(\omega, \mathbf{q}), \quad (*FL*)$$

where $\mathcal{M}_{if}$ is an effective field-medium coupling kernel and $S_{C^4}(\omega, \mathbf{q})$ is the C^4 dynamic structure factor describing the spectral content of atmospheric dielectric fluctuations induced by hydrometeors, turbulence, and thermodynamic variability. In a classical description, $\mathcal{M}_{if}$ can be written as an overlap integral between incident and scattered modal fields mediated by the dielectric perturbation $\delta\epsilon(\mathbf{r}, \omega)$. This expression provides a compact statistical closure analogous to transition-rate formulations widely used in classical wave transport in random media. The statistical properties of $\delta\epsilon$ are fully encoded in $S_{C^4}(\omega, \mathbf{q})$, which acts as a statistical descriptor of random-medium variability and controls the probability of scattering-induced transitions between propagation modes. Within the open-system formulation, the dominant transition classes defined by $(*FL*)$ equation provide a constructive prescription for the dissipative operators $\hat{L}_n$. Each operator can be associated with a coarse-grained class of scattering or absorption events, whose effective strength is controlled by the C^4 dynamic structure factor. In this sense, the transition-rate structure supplies the physical content underlying the dissipative part of the generator introduced in **Section 5.4**. This transition-rate closure establishes a direct link between microphysical atmospheric variability and the macroscopic extinction and scattering operators appearing in the statistical radiative-transfer equation derived below.

5.4.5 Connection with Classical Extinction and Scattering

The link with classical radiative transfer becomes explicit when the action of the dissipative terms is projected onto intensity-like observables. Let $\hat{I}$ denote an operator corresponding to a generalized intensity measure. Taking the derivative of its statistical average and using **Eqs (5.15)–(5.17)**, one obtains:

$$\frac{d}{ds} \bar{I} = -\overline{\hat{K}_e} \bar{I} + \bar{\hat{J}}, \quad (5.17)$$

where the overbar denotes a statistical average over propagation configurations, and $\hat{K}_e$ and $\hat{J}$ emerge as effective operator representations of extinction and source contributions. In the classical limit, where operator fluctuations are negligible and angular coupling is reduced to scalar coefficients, **Eq. (5.18)** reduces to the one-dimensional radiative-transfer equation (**Eq. (4.6)**). The statistical radiative-transfer equation therefore provides a direct generalization of the classical C^4 formulation.

¹The term “Fermi-like” is used here solely to indicate the mathematical structure of the transition-rate expression, which is widely employed in classical transport theories for waves in random media.



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5.4.6 Optical Depth and Non-Unitary Evolution

Within the open-system picture, attenuation arises from the dissipative component of the effective transport evolution. The cumulative effect of absorption, scattering out of the propagation direction, and redistribution processes along a path $\mathcal{P}$ represented through an effective attenuation operator, whose action governs the decay of intensity-like observables associated with the propagation state. Formally, the evolution driven by the effective generator $\hat{\mathcal{L}}_{eff}$ can be written as:

$$\rho(s) = \mathcal{T} \exp\left(-\int_0^s \hat{\mathcal{L}}_{eff}(s') ds'\right) \rho(0), \quad (5.18)$$

where $\mathcal{T}$ denotes ordering along the propagation coordinate s , and ρ represents a statistical propagation state encoding directional, modal, or ensemble-averaged radiative information. In the deterministic limit, where operator-valued coefficients reduce to scalar extinction terms and coupling effects become negligible, **Eq. (5.18)** reduces exactly to the exponential attenuation law governed by the classical optical depth introduced in **Section 4.5**.

5.4.7 Multiple Scattering and Mode Coupling

An important advantage of the operator-based formulation is its ability to represent multiple scattering and mode coupling in a unified way. The coupling operators $\hat{\mathcal{L}}_n$ introduce transitions between different propagation modes, directions, or polarization states, thereby capturing angular redistribution effects beyond simple absorption. In propagation regimes where multiple scattering is weak, these coupling terms provide small corrections to the dominant attenuation behavior and can be treated as perturbations to the mean-field transport dynamics. In strongly scattering conditions, however, they become dynamically significant, leading to substantial redistribution of intensity among modes and to measurable deviations from simple exponential decay. The statistical C^4 framework accommodates both regimes naturally within the same evolution equation, without the need for heuristic switching between absorption-dominated and scattering-dominated models. This unified description ensures continuity between weak- and strong-scattering limits while preserving consistency with classical radiative-transfer theory in the appropriate deterministic regime.

5.4.8 Irreversibility and Coarse Graining

The emergence of irreversibility in the present formalism is directly linked to coarse graining over unresolved atmospheric degrees of freedom. While the underlying electromagnetic field equations are reversible at the microscopic level, the elimination of fine-scale thermodynamic and microphysical variability leads to effective dissipative contributions in the evolution of the propagation state $\rho(s)$. This mechanism is fully consistent with the physical interpretation of radiative-transfer equations as macroscopic transport laws, in which attenuation and redistribution



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arise from averaging over complex, heterogeneous media rather than from fundamental time asymmetry. Within this perspective, irreversibility reflects the loss of information associated with unresolved atmospheric structure, not a violation of electromagnetic reversibility. The statistical transport formulation introduced here therefore provides a principled explanation for the effective irreversibility implicit in classical attenuation models, grounding it in the macroscopic description of field–medium interaction and in the controlled elimination of small-scale degrees of freedom.

5.4.9 Consistency with the Classical C^4 Limit

The statistical radiative-transfer equation derived in this section is constructed to ensure exact consistency with the classical C^4 framework in the appropriate limit. When ensemble variability becomes negligible, dissipative terms reduce to scalar extinction coefficients and angular coupling effects can be neglected, the governing evolution equation reproduces the deterministic radiative-transfer relations introduced in **Chapter 4**. This property guarantees that the statistical formulation does not introduce spurious effects in regimes where classical modelling is sufficient. Rather than modifying the underlying physics, it provides a controlled generalization in which additional degrees of freedom are activated only when required by atmospheric complexity, such as unresolved variability, scattering-induced redistribution, or heterogeneous microphysical structure.

5.4.10 Summary

This section formulated radiative transfer within the statistical C^4 framework as an open-system propagation dynamics for ensemble-averaged field states. By embedding extinction and scattering into a generalized evolution equation with dissipative and coupling terms, the formalism extends the classical radiative-transfer equations while preserving their physical interpretation and deterministic limit. The resulting structure provides a rigorous foundation for interpreting optical depth, attenuation, and multiple scattering as emergent properties of statistical propagation dynamics in complex atmospheric media. In the next section, this framework is used to reinterpret optical depth as an effective survival measure and to establish explicit connections between statistical attenuation descriptors and classical path-integrated quantities.

5.5 Optical Depth as Survival Probability and Effective Attenuation

In the classical C^4 framework developed in **Chapter 4**, optical depth was introduced as a deterministic path-integrated quantity governing attenuation through an exponential law (**Section 4.5**). In that setting, optical depth is fully specified by the macroscopic atmospheric state and represents the cumulative effect of extinction along a prescribed propagation path. Within the statistical formulation developed in the present chapter, optical depth admits a more general interpretation that consistently incorporates atmospheric variability, unresolved



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microphysical structure, and dissipative propagation effects. Rather than being defined exclusively as the integral of a scalar extinction coefficient, optical depth emerges here as an effective survival measure associated with the propagation of electromagnetic energy through a statistically complex atmosphere. In this perspective, optical depth quantifies the cumulative strength of field–medium coupling responsible for attenuation, averaged over the ensemble of admissible atmospheric realizations. The classical exponential attenuation law is recovered as the deterministic limit of this description, while deviations from purely exponential behavior can be interpreted as manifestations of statistical variability and redistribution effects. This reinterpretation preserves the physical meaning of optical depth as a measure of attenuation, while extending its applicability to propagation regimes where deterministic modelling alone is insufficient.

5.5.1 Survival Probability in an Open-System Picture

In the operator-based description of **Section 5.4**, attenuation arises the non-conservative component of the effective evolution governing the statistical propagation state $\rho(s)$. From this perspective, the decay of intensity-like observables reflects the progressive redistribution of electromagnetic energy out of a selected propagation subspace, rather than the decay of a microscopic probability amplitude. Let $\hat{P}$ denote a projection operator onto a subspace associated with forward-propagating or weakly scattered modes. The corresponding survival measure $P_{\text{surv}}(s)$ along the propagation coordinate s is defined as:

$$P_{\text{surv}}(s) = \text{Tr}[\rho(s) \hat{P}], \quad (5.19)$$

which quantifies the fraction of the ensemble-averaged propagation state that remains within the selected subspace after accounting for extinction and scattering-induced redistribution. Within this interpretation, attenuation is not viewed as the decay of a fundamental probability, but as the cumulative loss of energy or modal weight from the chosen propagation manifold due to coupling with unresolved atmospheric degrees of freedom. This definition provides a compact statistical representation of attenuation that remains fully consistent with classical radiative-transfer concepts and reduces exactly to the deterministic exponential law when redistribution effects become negligible.

5.5.2 Connection Between Survival Probability and Optical Depth

A direct link between survival measure introduced in **Section 5.5.1** and optical depth can be established by considering the solution of the statistical radiative-transfer (**Eqs. (5.15)–(5.17)**). In propagation regimes where the effective dissipative dynamics is dominated by extinction-like processes and mode redistribution remains weak, the survival measure exhibits an exponential decay of the form:

$$P_{\text{surv}}(s) = \exp[-\tau_{\text{eff}}(s)], \quad (5.20)$$

where τ_{eff} denotes an effective optical depth generated by cumulative non-conservative components

of the statistical propagation operator. In the deterministic limit, τ_{eff} coincides exactly with the classical optical depth defined in **Section 4.5**, recovering the standard Beer–Lambert attenuation law. . In more general statistical regimes, τ_{eff} encodes the cumulative strength of energy redistribution and attenuation along the propagation path, including contributions from unresolved microphysical variability and scattering-induced leakage out of the selected propagation subspace. Within this interpretation, the exponential form of **Eq. (5.20)** does not imply a microscopic probabilistic decay, but represents a compact statistical description of path-integrated attenuation that remains fully consistent with classical radiative-transfer theory while extending its applicability to variable and statistically complex atmospheric conditions.

5.5.3 Operator Representation of Optical Depth

The interpretation of optical depth as a survival measure naturally suggests an operator-based representation consistent with the formalism introduced in **Section 5.3**. Within this framework, cumulative attenuation along a propagation path $\mathcal{P}$ is associated with an effective attenuation operator $\hat{\mathcal{T}}$, defined as:

$$\hat{\mathcal{T}} = \int_{\mathcal{P}} \hat{K}_e(s) ds, \quad (5.21)$$

where $\hat{K}_e$ represents an operator-valued generalization of the classical extinction coefficient, acting on the statistical propagation state space. Within this representation, the survival measure associated with propagation through the medium can be written formally as:

$$P_{\text{surv}} = \text{Tr} \left[\rho(0) \exp(-\hat{\mathcal{T}}) \right], \quad (5.22)$$

where the trace denotes projection onto the selected propagation subspace and $\rho(0)$ represents the initial statistical state. When $\hat{\mathcal{T}}$ reduces to a scalar multiple of the identity operator, **Eq. (5.22)** recovers exactly the classical exponential attenuation law governed by the deterministic optical depth. More generally, the operator $\hat{\mathcal{T}}$ encodes the cumulative effect of extinction, scattering-induced leakage, and unresolved variability along the propagation path. This formulation does not imply a microscopic probabilistic interpretation, but provides a compact operator-based description of path-integrated attenuation that remains fully consistent with classical radiative-transfer theory while extending it to statistically complex propagation regimes.

5.5.4 Effective Attenuation and Ensemble Averaging

In practical applications, observable attenuation levels correspond to ensemble averages of survival measures or intensity-like quantities. Accordingly, an effective optical depth can be defined as:

$$\tau_{\text{eff}} = -\ln(\langle P_{\text{surv}} \rangle), \quad (5.23)$$

where the angle brackets denote an ensemble average over atmospheric realizations consistent with the underlying statistical description. This definition yields a scalar quantity directly



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comparable to classical path-integrated attenuation measures and provides a natural statistical generalization of the deterministic optical depth introduced in **Chapter 4**. Importantly, τ_{eff} does not, in general, coincide with the optical depth computed from mean extinction profiles. Variability, intermittency, and spatial inhomogeneity in hydrometeor fields can produce systematic deviations between τ_{eff} and the attenuation predicted using averaged microphysical parameters, even in regimes where classical radiative-transfer assumptions remain approximately valid. This distinction clarifies the physical meaning of empirically derived attenuation coefficients and highlights their implicit dependence on the statistical structure of the atmosphere.

5.5.5 Relation to Path-Integrated Attenuation

The effective optical depth defined above provides a natural bridge to the path-integrated attenuation (PIA) quantities used in satellite communication and remote-sensing applications. As in the classical case, PIA can be expressed as:

$$\text{PIA} = 4.343 \tau_{\text{eff}}, \quad (5.24)$$

but with τ_{eff} now interpreted as a statistical attenuation measure rather than a purely deterministic integral. This reinterpretation preserves the operational definition and practical use of PIA, while extending its physical content to account explicitly for atmospheric variability, intermittency, and unresolved microphysical structure. Within this perspective, the statistical C^4 framework provides a rigorous theoretical basis for understanding systematic discrepancies between observed attenuation levels and predictions obtained from classical deterministic models under highly variable atmospheric conditions.

5.5.6 Physical Interpretation and Modelling Implications

Interpreting optical depth as a survival probability emphasizes the probabilistic nature of attenuation in complex media. Rather than representing a fixed deterministic loss per unit length, optical depth quantifies the cumulative likelihood that a propagation state remains within a prescribed observational subspace along the path. This perspective is particularly relevant for high-frequency propagation and strongly scattering regimes, where fluctuations, intermittency, and multiple interactions play a dominant role and cannot be fully captured by mean extinction profiles alone. By embedding optical depth within an open-system formalism, the statistical C^4 framework provides a physically transparent explanation for effective attenuation laws, clarifying both their empirical success and their limitations when applied outside the deterministic regime.

5.5.7 Consistency with the Classical Limit

When atmospheric variability is negligible and scattering-induced redistribution is weak, the operator $\hat{\mathcal{T}}$ reduces to a scalar multiple of the identity. In this limit, the survival probability recovers the classical exponential attenuation governed by the deterministic optical depth defined



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in **Chapter 4**. This reduction guarantees exact consistency between the statistical formulation and the classical C^4 description. The statistical interpretation of optical depth developed in this section therefore does not modify classical results, but extends their meaning and domain of applicability. Optical depth remains the fundamental attenuation measure in the deterministic regime, while acquiring the interpretation of an emergent quantity arising from statistical propagation dynamics in complex media. In this way, the present formulation completes the extension of the C^4 framework beyond the deterministic limit, while preserving full continuity with classical radiative-transfer theory.

5.6 Recovery of the Classical Limit and Consistency Checks

A central requirement of the statistical C^4 framework developed in this chapter is its consistency with the deterministic formulation established in **Chapter 4**. The statistical and operator-based extensions introduced in **Sections 5.3–5.5** must therefore reduce exactly to the classical C^4 theory when atmospheric variability and unresolved degrees of freedom become negligible. This section demonstrates explicitly how the classical radiative-transfer equations, optical depth, and attenuation laws are recovered exactly as limiting cases of the statistical formulation. These consistency checks are essential to establish the physical legitimacy of the generalized framework.

5.6.1 Degeneration of the Statistical Ensemble

The first step toward the classical limit consists in the degeneration of the atmospheric ensemble introduced in **Section 5.2**. When the atmosphere can be described by a single macroscopic realization $\mathbf{S}_0(\mathbf{r})$, ensemble variability vanishes and the associated probability measure collapses to a delta distribution concentrated on that realization,

$$\mathbb{P}(\mathbf{S}) \longrightarrow \delta(\mathbf{S} - \mathbf{S}_0). \quad (5.25)$$

In this limit, the statistical propagation description reduces to a single deterministic configuration, and all statistical averages of observables coincide with their deterministic counterparts. As a consequence, operator-valued quantities become effectively to scalar coefficients, and statistical fluctuations no longer contribute to the propagation dynamics.

5.6.2 Reduction of Operator Quantities to Scalars

A second requirement for recovering the classical C^4 framework is the reduction of generalized transport coefficients to scalar quantities. In particular, the extinction coefficient, $\widehat{K}_e$ previously introduced in an operator-like or spatially structured form to account for heterogeneity and coupling effects, introduced in **Sections 5.3–5.4** must reduce to a scalar function of the propagation



coordinate. In this limit, the extinction term assumes the form:

$$\widehat{K}_e(s) \rightarrow k_e(s), \quad (5.26)$$

where $k_e(s)$ is the classical extinction coefficient derived from the constitutive relations of governing absorption and scattering processes (**Section 4.3**). Under this assumption, the optical-depth—defined as the path-integrated extinction along the propagation direction—reduces to the deterministic scalar quantity. In particular, the optical-depth operator $\widehat{\mathcal{T}}$, introduced in **Section 5.5** and defined through the path integration of the extinction coefficient $\widehat{K}_e$, reduces in the scalar limit to:

$$\widehat{\mathcal{T}} \rightarrow \int_L k_e(s) ds \equiv \tau, \quad (5.27)$$

where τ is the deterministic optical depth introduced in **Section 4.5**. In this limit, the propagation law recovers the standard exponential attenuation characteristic of classical radiative-transfer theory.

5.6.3 Reduction of the Generalized Transport Equation to Classical Radiative Transfer

The recovery of the classical radiative-transfer equation follows directly from the scalar limit of the transport generator. When generalized dissipative terms reduce to scalar extinction coefficients and angular or modal coupling is neglected, the evolution equation introduced in **Eqs. (5.15)–(5.17)** simplifies to an equation governing intensity-like observables of the form:

$$\frac{dI(s)}{ds} = -k_e(s) I(s) + j(s), \quad (5.28)$$

where $k_e(s)$ denotes the scalar extinction coefficient and $j(s)$ represents a source term accounting for emission or in-scattering processes. This expression coincides with the one-dimensional classical radiative transfer equation (**Eq. (4.6)**) derived in **Section 4.4**. The result demonstrates that the generalized statistical radiative-transfer formulation developed in **Section 5.4** constitutes a genuine extension of the classical C^4 framework, reducing exactly to it under appropriate limiting assumptions. Rather than an alternative or incompatible model, the extended formulation embeds classical radiative transfer as a well-defined limiting case corresponding to scalar transport in the absence of coupling effects.

5.6.4 Recovery of Classical Optical Depth and PIA

The interpretation of optical depth as a survival probability introduced in **Section 5.5** must likewise reduce to the classical definition in the appropriate limit. When operator fluctuations vanish and the extinction coefficient becomes purely scalar, the survival probability along the propagation path reduces to:

$$P_{\text{surv}} = \exp(-\tau), \quad (5.29)$$



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with τ denotes to the deterministic optical depth of **Chapter 4**. Under these conditions, the effective optical depth, defined as :

$$\tau_{\text{eff}} = -\ln(\overline{P}_{\text{surv}}), \quad (5.30)$$

where the overline indicates an ensemble average over atmospheric realizations, reduces identically to τ . Consequently, the path-integrated attenuation recovers its classical expression,

$$\text{PIA} = 4.343 \tau, \quad (5.31)$$

as introduced in **Section 4.5**. This consistency ensures that all operational attenuation measures retain their standard meaning in regimes where deterministic modelling is sufficient.

5.6.5 Consistency with Energy Balance and Physical Interpretability

An important consistency check concerns energy balance and physical interpretability. In the classical C^4 framework, extinction and emission are constrained by the constitutive relations and radiative-transfer equations of **Chapter 4**. The generalized statistical formulation preserves these constraints through the structure of the transport equation and the associated dissipative terms. In particular, when extinction and source contributions are formulated so as to conserve the total transported intensity, attenuation represents a redistribution of radiative energy among propagation directions, modes, or interaction channels rather than an unphysical loss of energy. Under these conditions, the evolution of intensity-like quantities remains fully consistent with classical energy-balance requirements. As a result, the generalized formulation respects the same physical principles underlying classical radiative-transfer theory, while extending its descriptive capability to account for statistical variability and structured interactions within the medium.

5.6.6 Scope of Validity of the Classical Limit

The recovery of the classical C^4 framework delineates the domain of validity of deterministic radiative-transfer modelling. When ensemble variability is weak, scattering-induced coupling effects are negligible, and effective-medium approximations remain valid, the classical theory provides an accurate and computationally efficient description of radiative propagation. Deviations from these conditions—such as strong microphysical variability, pronounced multiple-scattering effects, or heterogeneous media—signal the breakdown of purely deterministic assumptions and motivate the use of the statistical extensions developed in the present chapter. Within this perspective, the generalized C^4 formulation provides a unified description across propagation regimes, encompassing both deterministic and statistically structured behavior. The classical theory emerges naturally as a limiting case corresponding to weak variability and scalar transport, rather than being imposed through ad hoc assumptions or restrictive modelling choices.



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5.6.7 Summary

This section demonstrated that the generalized C^4 framework consistently recovers the classical radiative-transfer equations, optical depth, and attenuation laws introduced in Chapter 4 under well-defined limiting conditions. These consistency checks confirm that the extended formalism preserves physical interpretability and fully aligns with established radiative-transfer theory, rather than contradicting or replacing it. The explicit recovery of the classical limit completes the theoretical validation of the generalized C^4 approach and provides a solid foundation for discussing its implications for hybrid and data-driven modelling strategies in the final section of this chapter.

5.7 Implications for Hybrid and Data-Driven Modelling

The generalized C^4 framework developed in this chapter is not introduced as an abstract theoretical construction, but as a foundational layer for physically informed modelling strategies. In particular, it provides the conceptual and mathematical basis for hybrid approaches that combine deterministic radiative-transfer models with data-driven techniques. By formulating radiative transfer in a generalized statistical and operator-based setting, the framework enables the systematic incorporation of physical constraints—such as energy balance, attenuation structure, and constitutive consistency—into surrogate models, reduced-order representations, and machine-learning architectures. This formulation allows physically meaningful features, constraints, and invariants to be embedded directly into data-driven models, rather than being learned implicitly from data alone. As a result, the generalized C^4 approach bridges the gap between first-principles modelling and empirical prediction, offering a unified framework in which classical radiative-transfer theory, statistical variability, and modern machine-learning techniques can be consistently integrated..

5.7.1 Limitations of Purely Deterministic Models

Classical radiative-transfer models, including the deterministic C^4 framework presented in **Chapter 4**, rely on effective-medium approximations and averaged microphysical properties. While these assumptions provide accurate and computationally efficient predictions in weakly variable or moderately scattering regimes, their validity progressively degrades in the presence of strong spatial variability, intermittency, and unresolved microphysical structure. Under such conditions, path-integrated attenuation and brightness temperature emerge as statistical quantities shaped by fluctuations along the propagation path, rather than by a single deterministic extinction profile. As a consequence, discrepancies between classical model predictions and experimental or satellite-based observations become increasingly pronounced, particularly in high-frequency propagation regimes. These limitations motivate the introduction of statistical extensions and hybrid modelling strategies capable of capturing variability, uncertainty, and multiscale structure beyond the reach of purely deterministic formulations.



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5.7.2 C^4 as a Physical Constraint Layer

The evolution-based formulation introduced in **Sections 5.3 and 5.4** provides a natural mechanism and systematic mechanism for embedding physical constraints into data-driven modelling strategies. Within this framework, the effective evolution operator $\mathcal{L}_{eff}$ encodes conservation laws, dissipative processes, and structural constraints that must be satisfied by any physically consistent representation of atmospheric radiative propagation. Rather than learning attenuation or brightness temperature directly from data in an unconstrained manner, hybrid modelling strategies can be formulated in which machine-learning components approximate selected contributions to the effective evolution operator. These may include extinction-related terms, scattering-induced couplings, or correction factors accounting for unresolved microphysical variability and subgrid-scale effects. In this setting, data-driven models operate within a physically admissible solution space defined by the C^4 framework. This approach preserves consistency with classical radiative-transfer theory while retaining the flexibility and expressive power of machine-learning techniques, thereby combining physical interpretability with improved predictive capability in complex propagation regimes.

5.7.3 Statistical Optical Depth as a Learnable Quantity

The reinterpretation of optical depth as an effective ensemble attenuation measure (**Section 5.5**) suggests a particularly suitable target for hybrid modelling. Rather than predicting instantaneous attenuation values, learning algorithms can be trained to estimate an effective optical depth τ_{eff} as a statistical quantity emerging from unresolved microphysical variability along the propagation path. This approach offers two key advantages. First, it preserves the exponential structure of classical attenuation laws, ensuring full consistency with radiative-transfer theory. Second, it allows the effects of microphysical variability to be absorbed into a reduced set of effective parameters, thereby lowering the dimensionality of the learning problem while maintaining a clear physical interpretation of the model outputs.

5.7.4 Connection with SNEM-2 and Reduced-Order Models

The Statistical Numerical Electromagnetic Model (SNEM-2) can be interpreted as a numerical realization of the statistical C^4 framework developed here. In this context, Monte Carlo sampling, ensemble averaging, and stochastic microphysical parameterizations provide practical numerical approximations of ensemble-based radiative-transfer dynamics, as governed by the effective constitutive and propagation relations of the C^4 formulation. Hybrid modelling strategies can exploit this connection by using SNEM-2 simulations to generate physically consistent ensemble datasets for machine-learning applications. Reduced-order representations of SNEM-2 outputs can then be learned under explicit physical constraints derived from the C^4 framework, ensuring consistency across spatial scales, frequency bands, and modelling paradigms, while preserving the interpretability and physical grounding of the resulting surrogate models.



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5.7.5 Implications for Machine Learning Architectures

The statistical formulation of radiative transfer naturally motivates learning architectures that operate on distributions, ensembles, or aggregated state representations, rather than on point-wise field values alone. Recurrent neural networks, transformer-based models, and generative architectures can be interpreted as approximate deterministic or stochastic propagators acting on statistical descriptions of the atmospheric state and its radiative response. Within this perspective, machine-learning models do not replace physical modelling but act as surrogate generators constrained by physically derived structure. Their outputs are required to preserve the consistency of the underlying radiative-transfer formulation, including positivity, monotonicity, and integral constraints associated with extinction, emission, and optical depth. This viewpoint is fully aligned with recent developments in physics-informed and physics-guided machine learning, and it naturally supports the integration of explainability, uncertainty characterization, and robustness analysis into predictive atmospheric propagation models.

5.7.6 Predictive Generalization and Extrapolation

One of the principal limitations of purely data-driven models is their tendency to fail outside the training domain. By contrast, hybrid models grounded in the C^4 framework inherit the extrapolation capability of physical laws. The operator-based structure ensures that predictions remain consistent with conservation principles and known asymptotic limits, even in regimes not explicitly represented in the training data. This property is particularly relevant for applications involving future climate scenarios, rare meteorological events, or frequency bands where observational data are sparse. The C^4 framework thus provides a pathway toward predictive models that generalize beyond the empirical dataset.

5.7.7 Conceptual Role of the Statistical Formalism

The statistical language adopted in this chapter should not be interpreted as implying any underlying quantum behavior in atmospheric radiative transfer. Instead, it provides a mathematically structured and compact representation of ensemble-based propagation dynamics in complex and heterogeneous media. By reformulating classical radiative-transfer equations in an evolution-based and operator-like framework, the C^4 formalism enables the systematic treatment of variability, dissipation, and uncertainty within a unified mathematical structure. This abstraction facilitates the consistent integration of deterministic simulations, stochastic modelling, and data-driven approaches, while remaining fully grounded in classical electromagnetic propagation theory.

5.7.8 Summary and Outlook

This section highlighted how the statistical C^4 framework supports hybrid and data-driven modelling strategies by providing physically grounded constraints and interpretable targets for



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learning algorithms. The reinterpretation of attenuation as a statistical survival process offers a natural interface between radiative transfer theory and modern machine-learning techniques. The concepts introduced here prepare the ground for the subsequent chapters, where the theoretical framework is applied to numerical simulations, satellite observations, and predictive models. In this sense, the present chapter establishes the formal foundation upon which all subsequent modelling developments are built.

5.8 Deterministic Limit and Consistency of the C^4 Framework

The statistical operator-based formulation developed in this chapter is designed as a controlled and structured extension of the classical C^4 radiative-transfer framework, rather than as an alternative physical description. A fundamental requirement is therefore the exact recovery of the deterministic C^4 equations in the appropriate limit. As discussed in **Sections 5.2 and 5.7**, this limit is attained when atmospheric variability becomes negligible and the ensemble of realizations effectively collapses to a single macroscopic configuration. Under these conditions, the statistical state reduces to a deterministic propagation configuration, and all ensemble averages coincide with their classical counterparts. In this regime, the effective generator governing propagation reduces to a scalar transport operator consistent with classical radiative-transfer theory. Dissipative contributions encoded in the effective dissipative mapping $\hat{D}$ reduce to scalar extinction terms, while scattering-induced angular or directional coupling becomes negligible. As a consequence, the evolution of intensity-like observables is governed by a first-order differential equation identical to the classical radiative-transfer equation introduced in **Section 4.4**. Similarly, the generalized interpretation of optical depth introduced in **Section 5.5** reduces exactly to the classical definition. In the deterministic limit, the optical-depth description collapses to a scalar quantity, and the survival probability along the propagation path recovers the familiar exponential attenuation law,

$$P_{\text{surv}} = \exp(-\tau), \quad (5.32)$$

where τ is the deterministic optical depth derived from the constitutive relations of **Chapter 4**. All operational attenuation measures, including path-integrated attenuation, retain their standard physical meaning in this limit. This exact correspondence demonstrates that the generalized C^4 formulation does not introduce spurious corrections in regimes where deterministic modelling is sufficient. Instead, the operator-based statistical framework provides a systematic generalization whose additional structure becomes active only when required by atmospheric complexity, such as strong variability, multiple scattering effects, or unresolved microphysical fluctuations. The deterministic limit therefore serves as a consistency anchor for the entire formulation. It guarantees that the statistical extensions introduced in this chapter preserve the physical interpretation, conservation properties, and predictive content of the classical C^4 framework, while offering a mathematically controlled pathway toward more general propagation regimes.



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5.9 Summary and Outlook

This chapter introduced a statistical operator-based formulation of the C^4 framework aimed at extending classical radiative-transfer theory to propagation regimes characterized by atmospheric variability, unresolved microphysical structure, and scattering-induced mode coupling. Rather than replacing the deterministic formulation developed in **Chapter 4**, the proposed approach provides a mathematically controlled generalization that preserves its physical content in the appropriate limit. Starting from an ensemble-based description of atmospheric states, radiative propagation was formulated in terms of abstract states and evolution operators acting on a representation space encoding directional, modal, and polarization degrees of freedom. Within this generalized setting, extinction, scattering, and redistribution processes were incorporated through effective transport generators and dissipative mappings, leading to a statistical radiative-transfer formulation that remains fully consistent with classical transport theory at the macroscopic level. A central result of the chapter is the explicit demonstration that the classical C^4 framework is recovered exactly when ensemble variability becomes negligible and operator-valued transport coefficients reduce to scalar quantities. In this deterministic limit, the generalized formulation reproduces the classical radiative-transfer equation, the standard definition of optical depth, and the familiar exponential attenuation law. This exact correspondence guarantees that all operational quantities retain their conventional physical interpretation in regimes where deterministic modelling is sufficient. Beyond ensuring consistency with established theory, the operator-based formulation developed here offers several advantages for advanced propagation modelling. It provides a unified mathematical structure for describing attenuation, multiple scattering, and mode coupling, and establishes a direct link between atmospheric microphysical variability and effective macroscopic transport coefficients. Importantly, the framework remains independent of any microscopic quantum interpretation, relying instead on structural analogies with classical transport theory and statistical descriptions of open systems. Looking ahead, the formalism introduced in this chapter provides a natural foundation for the development of hybrid deterministic–statistical propagation models and for the integration of physically constrained surrogate approaches. In particular, the operator representation of transport coefficients and optical depth enables systematic coupling with numerical solvers and data-driven models, such as those explored in the following chapters. These developments allow the C^4 framework to address increasingly complex propagation scenarios while maintaining full consistency with classical radiative-transfer theory.



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Chapter 6

Modelling Radiowave Attenuation in Precipitating Media: From Physically Based Solvers to Hybrid Statistical Frameworks.

6.1 Role and Scope of the Chapter

This chapter introduces a physically grounded and statistically consistent extension of the classical C^4 radiative-transfer framework for the analysis of radiowave propagation in precipitating media. Building upon the deterministic formulation developed in Chapter~4, it addresses propagation regimes in which atmospheric variability, vertical stratification, and frequency-dependent scattering effects play a dominant role and cannot be adequately captured by purely deterministic models (Cicolin, 2019; Cicolin, 2021) [94,286].

Rather than proposing an alternative physical description, the approach developed here is conceived as a controlled generalization of classical radiative-transfer theory. Its primary objective is to preserve physical interpretability and consistency with established propagation models, while extending their applicability to high-frequency and vertically structured atmospheric regimes. This perspective is fully compatible with physically based radiative-transfer solvers and with empirical formulations such as ITU-R recommendations when applied within their validated domain of use (Marzano et al., 2009; ITU-R P.676; ITU-R P.838) [5,1–3].

Within the overall structure of the thesis, this chapter serves as a transition between first-principles electromagnetic modelling and advanced statistical or hybrid approaches. Physically based solvers, such as SNEM and SNEM-2, are adopted as reference tools for resolving vertical extinction and attenuation processes in precipitating atmospheres, while a generalized C^4 interpretation is introduced to account systematically for mesoscale structure and variability effects (Marzano et al., 2009; Cicolin, 2019) [5,94].



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In this framework, attenuation, optical depth, and brightness temperature are interpreted as effective quantities emerging from vertically structured atmospheric columns rather than as deterministic outputs of a single representative profile. The chapter therefore establishes the theoretical foundation for variability-aware propagation modelling, while ensuring exact recovery of the classical C^4 formulation in deterministic regimes. This role prepares the ground for the physically constrained hybrid modelling strategies developed in the subsequent sections.

6.2 Limits of Purely Deterministic Radiative-Transfer Modelling

Classical radiative-transfer modelling in precipitating atmospheres is traditionally based on deterministic formulations in which extinction, scattering, and emission processes are described through effective-medium approximations and vertically averaged microphysical properties. These assumptions underpin both physically based solvers and empirical propagation models, and remain valid when atmospheric variability is moderate and scattering effects are weak (Evans and Stephens, 2002; Marzano et al., 2009) [222,5].

At higher frequencies, particularly in the Ka-, V-, and W-band regimes, the validity of purely deterministic descriptions progressively degrades. In these conditions, radiative propagation becomes highly sensitive to vertical structure, mixed-phase hydrometeor distributions, and sharp thermodynamic transitions such as the melting layer. Small variations in particle size distributions or phase composition can induce significant changes in attenuation and brightness temperature, even under comparable bulk meteorological conditions (Cicolin, 2019; Cicolin, 2021) [94,286].

A further limitation of deterministic models lies in the implicit assumption that a single atmospheric realization is representative of the effective propagation environment. This assumption breaks down in the presence of spatial intermittency, convective variability, and unresolved mesoscale heterogeneity, which are not captured by standard atmospheric profiles or climatological averages. In such regimes, attenuation and emission emerge as path-dependent quantities shaped by cumulative microphysical fluctuations rather than by a unique extinction profile (Marzano et al., 2009) [5].

These limitations are evident in Earth–space propagation scenarios, where systematic discrepancies between deterministic predictions and observations have been reported at millimetre wavelengths. Comparisons with spaceborne radar and radiometric measurements reveal that vertically averaged or layer-collapsed models often fail to reproduce observed attenuation patterns in complex precipitating systems, particularly when mixed-phase processes dominate (Stephens et al., 2002; Hou et al., 2014) [14,13]. Similar issues arise when empirical formulations, such as ITU-R recommendations, are extrapolated beyond their validated frequency ranges or applied to strongly structured atmospheric conditions (ITU-R P.676; ITU-R P.838) [1–3].

These discrepancies do not indicate a failure of radiative-transfer theory itself, but rather signal the breakdown of its purely deterministic closure under conditions of strong atmospheric variability. In such regimes, attenuation, optical depth, and brightness temperature must be

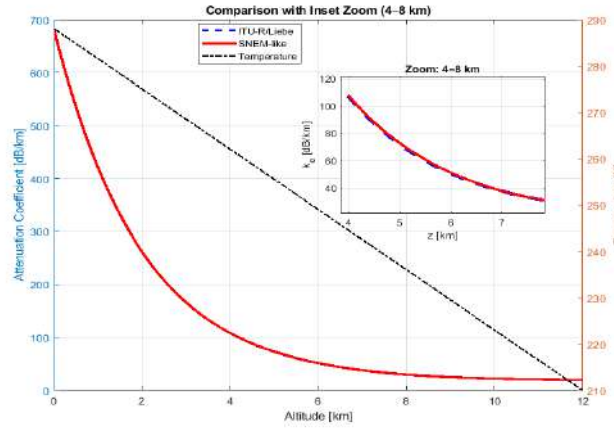


Figure 6.1. Vertical profile of the specific attenuation coefficient k_e [dB/km] at 94 GHz along a nadir path, comparing the classical ITU-R/Liebe gas absorption model (blue dashed line) with the SNEM-like radiative transfer formulation developed in this study (solid red line). The temperature profile from ERA5 is reported on the secondary axis, highlighting the stratification of the atmospheric column. The inset zooms into the 4–8 km altitude range, where significant deviations emerge due to hydrometeor scattering, phase transitions, and altitude-dependent microphysical properties accounted for in the SNEM scheme. While the integrated attenuation remains comparable, the SNEM-like model reveals localized mid-tropospheric enhancements, demonstrating the improved physical fidelity required for accurate W-band radiative-transfer modelling.

regarded as effective quantities resulting from ensembles of physically admissible realizations rather than as single-valued functions of mean atmospheric parameters (Cicolin, 2019) [94]. This observation motivates the introduction of structured and statistically consistent extensions of the classical C^4 framework, which are developed in the following sections.

Figure 6.1 compares the vertical profile of the specific attenuation coefficient $k_e(z)$ at 94 GHz obtained using the classical ITU-R gaseous absorption model and the physically based SNEM-like formulation. While the ITU-R approach assumes vertically uniform extinction within a prescribed rain layer, the SNEM-like model resolves altitude-dependent microphysical variability, explicitly capturing melting-layer effects and mixed-phase scattering. This highlights the intrinsic limitation of empirical attenuation models under stratified hydrometeor conditions.

The structural limitations of uniform-layer empirical attenuation models become evident when both the vertical distribution of the extinction field and the resulting path-integrated attenuation are examined (**Figs. 6.2 and 6.3**). In particular, different precipitation regimes reveal substantial variations in the vertical structure of $k_e(z)$, while the ITU-R formulation enforces a uniform extinction layer irrespective of hydrometeor stratification. .

While the previous analyses have highlighted the limitations of empirical uniform-layer models under intense precipitation and long slant-path geometries, it is equally important to assess whether these limitations persist under more typical atmospheric conditions. To this end, **Fig. 6.4** examines a moderate stratiform rain scenario, representative of mid-latitude precipitation regimes relevant to operational W-band applications.

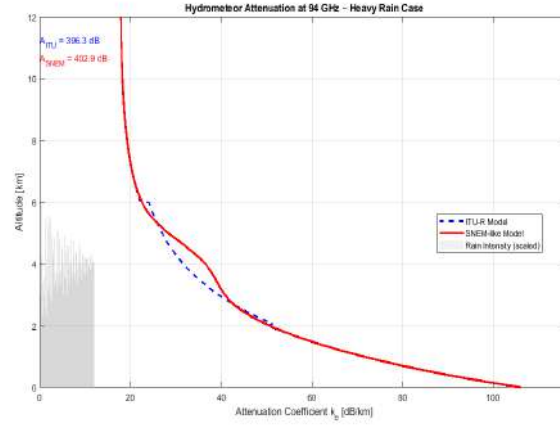


Figure 6.2. Vertical profiles of the specific attenuation coefficient $k_e(z)$ at 94 GHz under intense rainfall conditions, comparing the classical ITU-R formulation (blue dashed line) with the SNEM-like radiative-transfer model (solid red line). The ITU-R approach assumes a constant extinction coefficient within the rain layer (2–6 km), whereas the SNEM-like formulation resolves altitude-dependent microphysical variability, yielding a more structured and physically consistent attenuation profile. The shaded area denotes the normalized vertical distribution of rainwater content. The path-integrated attenuation up to 12 km is higher in the SNEM solution, reflecting its increased sensitivity to hydrometeor vertical gradients and phase-transition effects.

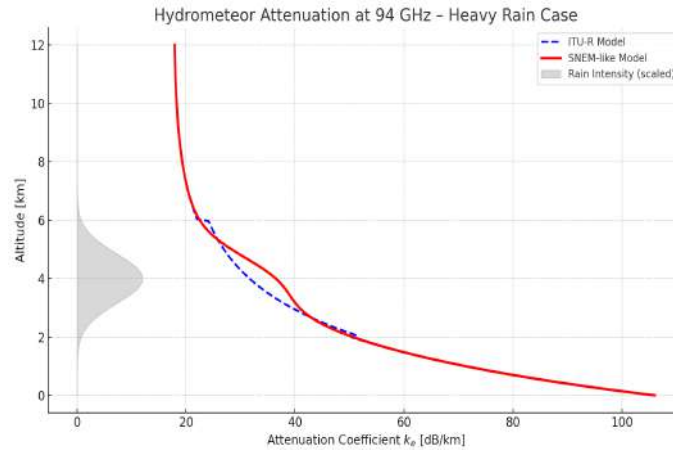


Figure 6.3. Vertical attenuation coefficient profiles at 94 GHz under intense rainfall conditions, comparing the classical ITU-R approach and the SNEM-like radiative transfer model. Same scenario as **Fig. 6.2**, including the corresponding path-integrated attenuation (PIA) up to 12 km. Although both models operate under identical rainfall conditions, the SNEM-like formulation yields a higher integrated attenuation than the ITU-R estimate, reflecting its increased sensitivity to hydrometeor vertical gradients and phase-transition effects. This quantification highlights the radiative impact of vertical stratification beyond purely structural differences.

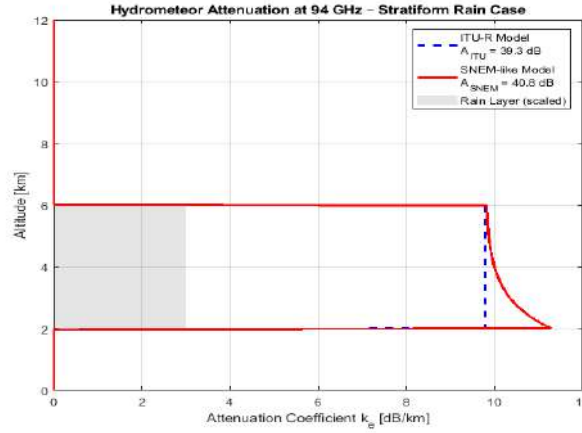


Figure 6.4. Vertical profiles of the attenuation coefficient $k_e(z)$ dB/km at 94 GHz under moderate stratiform rain conditions (rain rate $R \approx 8$ mm/h). The classical ITU-R formulation (blue dashed line) assumes a constant extinction coefficient throughout the rain layer (2–6 km), whereas the SNEM-like model (red solid line) incorporates microphysical stratification and altitude-dependent scattering. The grey shaded area indicates the vertical extent of the rain layer. Although both models yield comparable path-integrated attenuation ($A_{ITU} = 39.3$ dB, $A_{SNEM} = 40.8$ dB), the SNEM profile exhibits enhanced vertical structure, particularly near the cloud base where hydrometeor concentration is higher. This highlights the relevance of physically resolved stratification for accurate W-band radio propagation modelling even under non-extreme precipitation conditions.

6.3 SNEM and SNEM-2 as Physically Based Reference Solvers

The limitations of purely deterministic radiative-transfer modelling motivate the adoption of physically based reference solvers capable of resolving the vertical structure and microphysical complexity of precipitating atmospheres. In this context, the Sky Noise Eddington Model (SNEM) and its enhanced formulation (Statistical Numerical Electromagnetic Model-2) SNEM-2 are adopted in this work as benchmark tools for high-frequency radiowave propagation analysis. These models provide a physically consistent description of extinction, scattering, and emission processes across stratified atmospheric columns, explicitly accounting for mixed-phase hydrometeors and frequency-dependent dielectric properties (Marzano et al., 2009) [5].

Unlike empirical formulations based on bulk parameters, SNEM is grounded in radiative-transfer theory and electromagnetic scattering physics. The atmosphere is represented as a vertically layered medium composed of multiple hydrometeor classes, whose individual contributions to extinction and emission are derived from microphysical particle size distributions and complex permittivity models. This structure establishes a direct causal link between microphysical variability and macroscopic propagation observables such as specific attenuation, optical depth, and brightness temperature (Evans and Stephens, 2002; Marzano et al., 2009) [222,5]. SNEM-2 extends the original framework by introducing improved vertical resolution, refined treatment of mixed-phase processes, and updated thermodynamic inputs derived from modern reanalysis datasets. The use of ERA5 atmospheric profiles enables a realistic representation of temperature, pressure, and humidity stratification, which is essential for accurate modelling



of gas absorption and phase transitions at millimetre wavelengths (**Hersbach et al.**, 2020) [12]. These improvements are particularly relevant in the presence of melting layers and sharp thermodynamic gradients, where small modelling inaccuracies can lead to significant biases in predicted attenuation. A key feature of SNEM-2 is its capability to simulate slant-path and Earth–space propagation geometries, making it suitable for satellite communication and remote-sensing applications at Ka-, V-, and W-band frequencies. Comparisons with vertically resolved satellite observations demonstrate that resolving atmospheric stratification is essential to reproduce observed attenuation and radiometric signatures in complex precipitating scenes (**Stephens et al.**, 2002; **Hou et al.**, 2014) [14,13]. These results highlight the inadequacy of vertically averaged or layer-collapsed approaches in strongly structured atmospheric conditions.

Within the present work, SNEM and SNEM-2 serve a dual role. First, they provide physically interpretable reference solutions against which simplified deterministic models and empirical formulations, including ITU-R recommendations, can be critically assessed (ITU-R P.676; ITU-R P.838) [1–3]. Second, they constitute the physical backbone for the structured and statistical extensions of the C^4 framework developed in this chapter, supplying physically admissible states that inform mesoscale interpretation and subsequent hybrid modelling strategies.

6.4 Mesoscale Interpretation of Vertical Attenuation (C^4 Framework)

Physically based simulations performed with SNEM and SNEM-2 indicate that radiowave attenuation in precipitating media cannot be interpreted as the response of a homogeneous or weakly perturbed propagation environment. Instead, attenuation emerges from the interaction of electromagnetic waves with a vertically structured atmospheric column, in which microphysical composition, thermodynamic stratification, and phase transitions jointly determine the effective propagation outcome (**Marzano et al.**, 2009) [5].

Within this context, the C^4 framework provides a mesoscale interpretation of vertical attenuation by treating the atmosphere as a structured electromagnetic medium composed of coupled layers. Each layer contributes to the overall propagation behavior through its effective dielectric response, which depends on hydrometeor type, particle size distribution, phase composition, and ambient thermodynamic conditions. The cumulative attenuation therefore reflects not only the integrated hydrometeor content, but also the vertical organization and coupling of the atmospheric column (**Cicolin**, 2019; **Cicolin**, 2021) [94,286]. A central implication of this interpretation is that sharp vertical transitions, such as the melting layer, play a disproportionate role in determining attenuation and brightness-temperature signatures. These regions introduce strong gradients in complex permittivity and extinction coefficients, leading to non-linear and frequency-dependent contributions to optical depth that are not captured by vertically averaged or layer-collapsed models. This behavior is consistently reproduced by vertically resolved simulations and is supported by discrepancies observed between deterministic predictions and

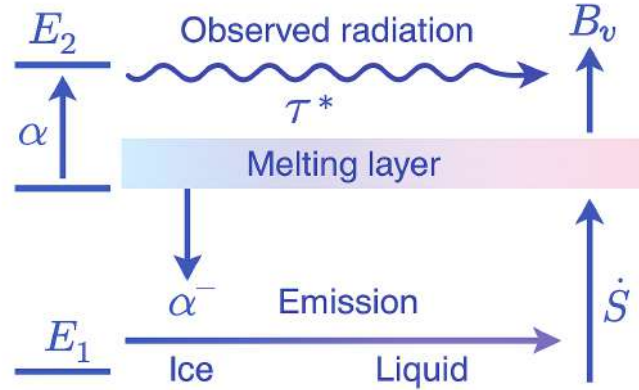


Figure 6.5. Conceptual diagram of the Coupled Cloud–Clausius–Complex (C^4) interpretation of the melting layer. The atmosphere is represented as a stratified medium composed of mesoscale thermodynamic states E_1 (ice) and E_2 (liquid), connected through forward and reverse transition coefficients α and α^- . The melting layer acts as a phase-transition zone where the Clausius potential exhibits a local minimum, inducing rapid perturbations in the complex permittivity field and enhancing both the emitted source term S and the emerging radiance B_ν through the effective optical depth τ^* . Discrete variations in the extinction coefficient within this region provide a physical explanation for threshold-like opacity behavior and bistable radiometric signatures observed in W-band regimes. The SNEM-2 model resolves these vertically localized transitions, whereas classical ITU-R formulations smooth them into a continuum approximation.

vertically resolved satellite measurements in precipitating systems (Stephens et al., 2002; Hou et al., 2014) [14,13]. In the C^4 framework, vertical attenuation is not modelled as the sum of independent layer contributions, but as the outcome of a coupled process in which neighboring layers influence the effective electromagnetic response of the column. This coupling becomes increasingly relevant at millimetre wavelengths, where sensitivity to microphysical variability and phase-dependent dielectric properties is enhanced. As a result, atmospheric columns characterized by similar bulk water content may exhibit markedly different attenuation behaviour depending on their vertical structure and phase distribution (Evans and Stephens, 2002) [222]. This mesoscale interpretation naturally bridges deterministic radiative-transfer modelling and variability-aware descriptions of propagation. In regimes where vertical coupling effects are weak, the structured interpretation collapses to the classical formulation based on scalar extinction coefficients. Conversely, when stratification and mixed-phase processes dominate, the C^4 framework provides a physically consistent extension capable of capturing the observed diversity of attenuation outcomes without abandoning classical radiative-transfer principles. The vertical attenuation structures discussed in Sections 6.2 and 6.3 call for a unified physical interpretation capable of linking microphysical phase transitions, radiative transfer, and emergent extinction behavior. To this end, the C^4 (Coupled Cloud–Clausius–Complex) framework is introduced as a mesoscale interpretation of radiowave interaction with stratified hydrometeor media, as schematically illustrated in Fig. 6.5 .

A quantitative validation of these transition-driven effects is provided in Sections 6.2–6.3 and



further discussed in **Section 6.6**.

6.4.1 Frequency-Dependent Behavior within the C^4 Framework

A direct implication of the C^4 framework is that radiowave extinction exhibits an increasingly non-linear dependence on frequency, as phase-transition effects and perturbations of the complex permittivity field progressively dominate the propagation regime. Under identical hydrometeor conditions, low-frequency bands (Ku) remain largely governed by quasi-linear absorption mechanisms, whereas higher frequencies become increasingly sensitive to mixed-phase scattering and melting-layer dynamics.

This transition is particularly evident under slant-viewing geometries, where ray bending and extended path lengths amplify the cumulative interaction with vertically stratified hydrometeors. As frequency increases from Ku to Ka and W bands, discrepancies between empirical ITU-R formulations and physically based models grow systematically, reflecting the breakdown of low-frequency approximations and motivating the need for vertically resolved radiative-transfer modelling at millimetre wavelengths. This behavior is examined in **Fig. 6.6** through a multi-band comparison across Ku-, Ka- and W-bands.

6.5 Frequency-Dependent Closures and Optical Depth Structure

The mesoscale interpretation of vertical attenuation introduced in the previous section requires closure relations capable of capturing the intrinsic frequency dependence of extinction and scattering processes in precipitating media. At microwave and millimetre-wave frequencies, attenuation is governed by the combined effects of absorption, scattering, and phase-dependent dielectric response, whose relative importance varies with frequency, hydrometeor type, and particle size distribution (**Evans and Stephens**, 2002; **Marzano et al.**, 2009) [222,5]. Within physically based frameworks such as SNEM and SNEM-2, these effects are represented through vertically resolved extinction profiles, in which the total attenuation at a given frequency arises from the cumulative contribution of multiple hydrometeor classes and atmospheric constituents. In this setting, frequency-dependent closures emerge as effective relations that link microphysical structure to macroscopic propagation observables without collapsing the underlying vertical organization into a single bulk parameter (**Marzano et al.**, 2009) [5].

A key consequence of this approach is that optical depth must be interpreted as a structured quantity reflecting the organization of the atmospheric column rather than as a purely scalar function of mean conditions. Sharp vertical gradients, particularly in mixed-phase regions, introduce non-linear and frequency-sensitive contributions to optical depth that dominate attenuation behavior at millimetre wavelengths. This explains why atmospheric columns with comparable integrated water content may produce substantially different attenuation outcomes when their vertical structure differs (**Cicolin**, 2019; **Cicolin**, 2021) [94,286].

From a closure perspective, effective extinction coefficients therefore represent emergent

Slant Atmospheric Attenuation with Bending – SNEM vs ITU-R

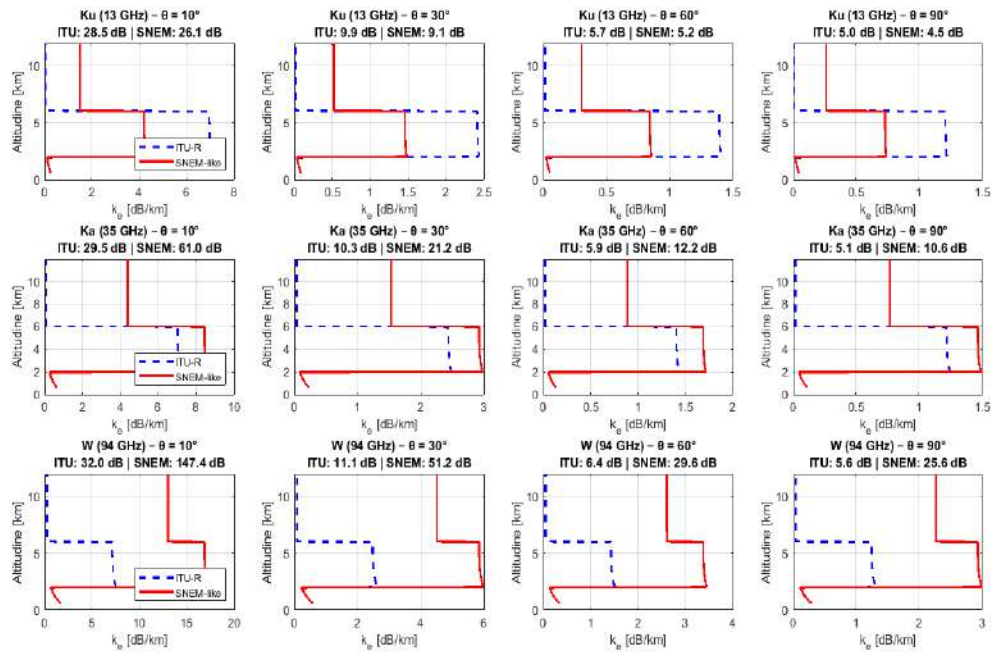


Figure 6.6. Comparison of atmospheric attenuation profiles at Ku-, Ka-, and W-bands under slant-viewing conditions. The specific attenuation coefficient $k_e(z)$ is shown as a function of altitude for both the classical ITU-R formulation (blue dashed lines) and a SNEM-like physically based model (solid red lines), considering representative intense hydrometeor conditions. Rain is prescribed with a density $\rho = 1.2 \text{ gm}^{-3}$ between 2–6 km, while snow with $\rho = 0.3 \text{ gm}^{-3}$ is imposed above 6 km. Each column corresponds to a different elevation angle (10° , 30° , 60° , and 90°), and frequency bands are organized by rows: Ku (13 GHz), Ka (35 GHz), and W (94 GHz). Slant-path attenuation accounts for ray bending due to atmospheric refraction and Earth curvature. Differences between the two models increase systematically with decreasing elevation angle and increasing frequency, with the largest discrepancies observed in the W-band. Total path-integrated attenuation values (dB) are reported in each subplot for both models.



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quantities resulting from vertically coupled processes. At higher frequencies, coherence effects and multiple scattering further modulate the effective attenuation, leading to deviations from simple additive behaviour and to characteristic spectral signatures in attenuation and brightness-temperature observations (**Evans and Stephens**, 2002) [222]. These effects become increasingly relevant toward the W-band, where sensitivity to microphysical variability is maximized. In the deterministic limit, where vertical coupling and variability effects are negligible, the structured optical depth introduced here collapses to the classical formulation based on scalar extinction coefficients and path-integrated quantities. In general propagation regimes, however, the frequency-dependent and structured interpretation of optical depth provides a more faithful representation of the physical processes governing attenuation. This formulation directly motivates the explicit recovery of the classical C^4 limit discussed in the next section. The need for physically based radiative-transfer solvers becomes even more evident when long slant-path geometries are considered. As shown in **Fig. 6.7**, the SNEM formulation captures altitude-dependent extinction contributions along an **Earth–Moon** W-band link that are systematically underestimated by the classical ITU-R approach.

6.6 Recovery of the Classical C^4 Limit

The generalized formulation developed in this chapter is required to recover classical radiative-transfer theory in the appropriate deterministic limit. This requirement represents a fundamental consistency condition ensuring that the extended C^4 framework does not introduce spurious corrections in regimes where classical modelling assumptions remain valid (**Marzano et al.**, 2009; **Evans and Stephens**, 2002) [5,222]. When atmospheric variability is weak, vertical coupling effects are negligible, and microphysical properties vary smoothly along the propagation path, the effective transport coefficients governing extinction and emission reduce to scalar quantities. Under these conditions, frequency-dependent closures lose their explicit dependence on mesoscale structure, and the structured optical depth introduced in the previous sections collapses to the classical path-integrated formulation. Attenuation and brightness temperature therefore recover their standard dependence on scalar extinction coefficients and propagation geometry, consistent with classical radiative-transfer theory and empirical models within their validated domain of applicability (ITU-R P.676; ITU-R P.838) [1–3]. From a physical standpoint, the recovery of the classical limit corresponds to the vanishing of inter-layer coupling effects within the atmospheric column. Individual layers contribute independently to the total attenuation, and the cumulative optical depth becomes additive. In this regime, the generalized C^4 framework yields the classical one-dimensional radiative-transfer equation together with the familiar exponential attenuation law governing survival probability and path-integrated attenuation (**Cicolin**, 2019; **Cicolin**, 2021) [94,286]. This exact correspondence guarantees that all operational propagation metrics retain their conventional physical interpretation in deterministic regimes. The existence of a well-defined classical limit therefore establishes the generalized C^4 formulation as a hierarchical extension of classical radiative-transfer theory, in which deterministic modelling emerges as a

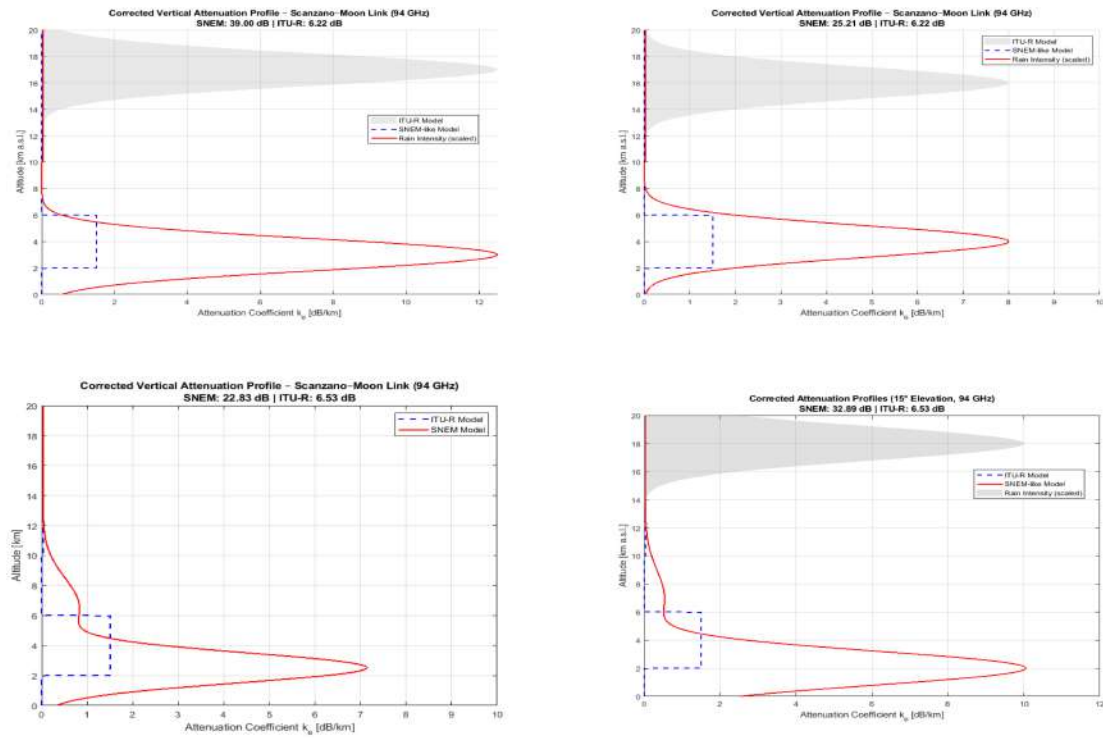


Figure 6.7. Vertical profiles of the specific attenuation coefficient $k_e(z)$ at 94 GHz for a low-elevation **Earth-Moon slant path**, comparing the classical ITU-R formulation (blue dashed line) with the SNEM-based radiative-transfer solution (solid red line). The shaded area denotes the normalized vertical distribution of hydrometeors. The SNEM model resolves altitude-dependent extinction across liquid, mixed-phase and gaseous layers, including upper-tropospheric contributions that are neglected by the empirical ITU-R approach. Consequently, the path-integrated attenuation predicted by SNEM exceeds that of the ITU-R model, highlighting the necessity of physically based solvers for long slant-path and deep-space-relevant W-band propagation scenarios.



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special case embedded within a broader, variability-aware framework.

6.7 C^4 as a Physical Constraint Layer for Hybrid Modelling

The generalized C^4 framework developed in this chapter provides a natural foundation for hybrid modelling strategies that combine physically based radiative-transfer formulations with data-driven techniques. Rather than replacing deterministic solvers or empirical models, the role of C^4 is to act as a physical constraint layer that restricts surrogate or learning-based representations to physically admissible propagation states. This perspective is consistent with recent developments in physics-informed and process-aware modelling, in which prior physical knowledge is embedded directly into the structure of data-driven algorithms (**McGovern et al.**, 2019; **Kashinath et al.**, 2021) [41,38]. Within the C^4 formulation, physical constraints arise from conservation principles, radiative-transfer structure, and the causal relationship between microphysical properties and macroscopic propagation observables. These constraints define an admissible solution space in which attenuation, optical depth, and brightness temperature evolve consistently with electromagnetic propagation physics. Accordingly, C^4 -informed hybrid models do not attempt to learn propagation outcomes directly from data, but instead focus on estimating effective quantities—such as frequency-dependent extinction corrections or structured optical depth representations—that operate within a physically defined framework (**Cicolin**, 2019; **Cicolin**, 2021) [94,286].

A key advantage of this approach is the preservation of interpretability. Learned quantities retain a clear physical meaning, avoiding the introduction of opaque latent variables that are difficult to validate or generalize. This property is particularly relevant for high-frequency propagation studies, where physical transparency and operational relevance are essential for system design and performance assessment. By constraining learning within a physically grounded framework, the C^4 approach mitigates overfitting and improves robustness across atmospheric regimes not explicitly represented in the training data (**McGovern et al.**, 2019) [41]. From a methodological standpoint, C^4 -constrained hybrid models can be constructed by coupling physically based solvers, such as SNEM-2, with surrogate components trained on simulation outputs or observational datasets. In this configuration, high-fidelity simulations provide physically admissible reference states, while data-driven components learn reduced representations of unresolved variability or computationally expensive processes. This strategy enables substantial reductions in computational cost while maintaining consistency with radiative-transfer theory (**Kashinath et al.**, 2021) [38]. Crucially, the hierarchical structure of the C^4 framework ensures compatibility with classical radiative-transfer theory. When atmospheric variability is negligible, surrogate corrections naturally vanish and the hybrid formulation collapses to the underlying deterministic model. Conversely, in regimes characterized by strong stratification or microphysical variability, the additional degrees of freedom introduced by the hybrid approach become active in a controlled and physically interpretable manner. This property reinforces the role of C^4 as a unifying framework bridging first-principles modelling and data-driven prediction. The modelling



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From microphysics to observables: C⁴-informed modelling workflow (conceptual)

Information flow is implied by the layout: left → right (top row), then down to model families and implications.

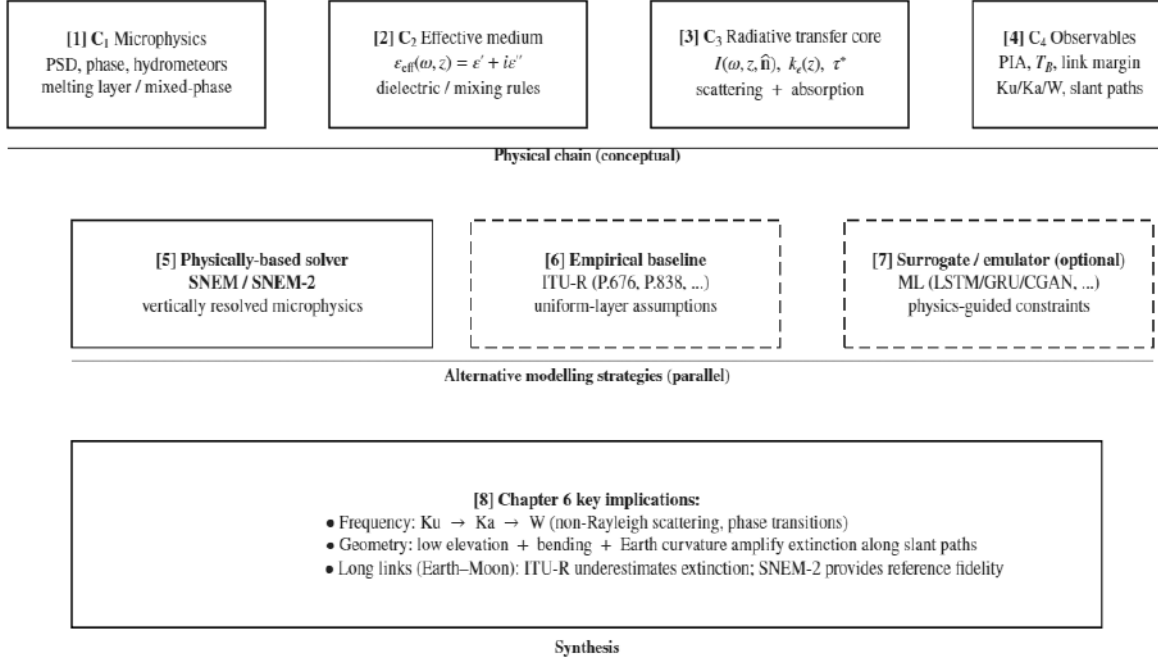


Figure 6.8. Conceptual C⁴-informed modelling framework from microphysics to observables. The figure illustrates the conceptual decomposition of radiowave propagation in the atmosphere into four coupled layers (C₁–C₄), spanning hydrometeor microphysics, effective-medium formulation, radiative-transfer processes, and observable quantities relevant to remote sensing and telecommunication systems. Different modelling strategies—physically based solvers (SNEM/SNEM-2), empirical formulations (ITU-R), and data-driven surrogate models—operate primarily at the radiative-transfer and observables levels, providing alternative representations of vertically structured atmospheric extinction with different levels of physical fidelity and computational complexity. The spatial layout emphasizes conceptual relationships rather than algorithmic flow, highlighting the role of C⁴ as a unifying interpretative framework rather than a deterministic processing chain.

approach adopted in this chapter can be schematically summarized by the C⁴ framework shown in **Figure 6.8**, which organizes the main physical layers and modelling strategies considered in the analysis. This representation provides a common reference for comparing physically based solvers, empirical formulations, and data-driven surrogates in terms of their treatment of radiative-transfer processes and resulting observables.

6.8 Summary and Outlook

This chapter has introduced a structured and physically grounded extension of the classical C⁴ radiative-transfer framework, aimed at addressing propagation regimes characterized by strong atmospheric variability, vertical stratification, and frequency-dependent scattering effects. Rather than proposing an alternative physical description, the generalized formulation has been developed as a hierarchical extension of deterministic radiative-transfer theory, explicitly designed



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to recover the classical C^4 limit under well-defined conditions (**Marzano et al.**, 2009; **Evans and Stephens**, 2002) [5,222].

By leveraging physically based reference solvers such as SNEM and SNEM-2, the chapter established a mesoscale interpretation of vertical attenuation in which optical depth and path-integrated attenuation emerge from the coupled interaction of stratified atmospheric layers. This perspective highlights the central role of vertical organization, mixed-phase processes, and frequency-dependent dielectric response in shaping high-frequency propagation behavior, particularly in the Ka-, V-, and W-band regimes relevant to modern satellite communications and advanced remote sensing (**Cicolin**, 2019; **Cicolin**, 2021) [94,286]. A key contribution of the chapter is the introduction of frequency-dependent closures and a structured interpretation of optical depth that remain fully consistent with classical radiative-transfer theory while extending its descriptive power. The explicit demonstration of the recovery of the classical C^4 limit provides a rigorous consistency anchor for the generalized framework, ensuring that deterministic modelling emerges naturally as a special case rather than being replaced or contradicted. This property is essential for preserving physical interpretability, operational relevance, and compatibility with established empirical formulations such as ITU-R recommendations when applied within their validated domain of use [1–3].

Beyond its theoretical implications, the generalized C^4 framework establishes a natural foundation for hybrid deterministic–statistical propagation models. By acting as a physical constraint layer, it enables the integration of data-driven components within a rigorously defined solution space governed by conservation principles and radiative-transfer structure. This approach supports the development of surrogate and reduced-order models capable of capturing unresolved atmospheric variability while maintaining physical consistency and interpretability (**McGovern et al.**, 2019; **Kashinath et al.**, 2021) [41,38]. Looking ahead, the framework developed in this chapter opens several avenues for future research and application. These include the systematic incorporation of satellite-based and ground-based observational datasets, the extension toward probabilistic performance metrics for high-frequency links, and the coupling with machine-learning architectures trained on physically admissible states generated by reference solvers. Within this perspective, the C^4 framework provides a unifying theoretical backbone for next-generation propagation modelling, capable of bridging first-principles physics, numerical simulation, and data-driven prediction while preserving physical rigor and interpretability.



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Chapter 7

Results and Validation and Synthesis of Attenuation–Delay Modelling

7.1 Scope and Validation Philosophy

This chapter presents the final results and validation of the attenuation–delay modelling framework developed in this thesis, with the aim of assessing its physical consistency, observational reliability, and synthesis capability across refractive and radiative propagation effects in the troposphere. The analysis focuses on propagation-relevant observables, namely zenith delays and path-integrated attenuation in the Ku–Ka–W bands, and on their mutual physical dependence.

The results discussed in this chapter are restricted to modelling components that have been fully established and validated in the preceding chapters. Accordingly, the analyses presented here represent final outcomes rather than intermediate methodological developments. The validation strategy is therefore framed around a single scientific question: to what extent can a unified, physically driven description of attenuation and delay—based on common thermodynamic and microphysical controls—reproduce observed atmospheric propagation effects across frequencies and climatic regimes?

The validation framework combines global reanalysis products, spaceborne radar observations, and independent ground-based measurements. ERA5 reanalysis data are used to characterize the large-scale thermodynamic state of the atmosphere, providing pressure, temperature, and humidity fields required for hydrostatic delay estimation and gaseous absorption modelling [12, 99]. Spaceborne radar observations from the CloudSat mission and the Global Precipitation Measurement (GPM) core observatory provide vertically resolved information on hydrometeor structure, phase, and attenuation at Ku, Ka, and W bands [13, 14, 97, 98]. Independent zenith total delay (ZTD) estimates derived from ground-based GNSS observations are used as an external reference for validating the integrated refractive effects of the atmosphere, following established formulations [45–47, 49, 65, 73].

This multi-source configuration enables the simultaneous assessment of refractive quantities, such as the zenith hydrostatic and wet delays (ZHD and ZWD), and radiative quantities,



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such as path-integrated attenuation (PIA). These observables are treated as complementary manifestations of the same atmospheric state, rather than as independent modelling targets. In particular, both the wet component of the tropospheric delay and hydrometeor-induced attenuation are governed by the distribution, phase, and concentration of condensed water, whose radiative impact becomes increasingly pronounced at millimetre-wave frequencies [1–3].

The validation approach adopted in this chapter is intentionally conservative. Model performance is quantified using a limited set of statistically robust metrics—bias, root-mean-square error, and correlation coefficients—selected to support physical interpretation rather than numerical optimization. Emphasis is placed on the verification of physically expected behaviors, including the monotonic increase of attenuation with frequency and hydrometeor content, the consistency between attenuation-derived wet processes and ZWD, and agreement with established propagation and refractivity models [1–3, 44]. Within this framework, attenuation and delay are not analyzed as separate modelling problems. Instead, the results are interpreted through a unified physical perspective in which both quantities emerge from the same thermodynamic and microphysical controls. The C^4 framework introduced earlier in the thesis is employed here strictly as an interpretative layer, providing a conceptual link between microphysics, effective dielectric properties, and electromagnetic field behavior, without altering validated retrievals or introducing additional degrees of freedom.

Accordingly, the remainder of this chapter is organized around four main result categories: (i) validation of zenith delay components and their decomposition; (ii) derivation and assessment of global attenuation climatologies; (iii) validation of the predictive attenuation law; and (iv) synthesis of attenuation and delay within a physically consistent propagation framework. Algorithmic descriptions, data-processing workflows, and exploratory analyses are intentionally not repeated and are documented in the methodological chapters and **Appendices**.

All quantitative attenuation results produced in this work are reported in the peer-reviewed articles collected in **Appendix A (Part I)**. The present chapter does not reproduce these results; instead, it focuses on their physical interpretation, validation against independent observations, and integration within the overall attenuation–delay modelling framework developed in this thesis.

7.2 Validation of Zenith Delay Components (ZHD, ZWD, ZTD)

This section examines the validation of the tropospheric zenith delay components—hydrostatic (ZHD), wet (ZWD), and total (ZTD)—within the unified attenuation–delay modelling framework. The analysis establishes a refractive reference state for the atmosphere adopted throughout Chapter 7, providing the physical baseline against which the subsequent attenuation results are interpreted.

The zenith delay components discussed here do not constitute new retrievals. Their formulation, global characterization, and extended validation—including dedicated analyses of ZHD, ZWD, and ZTD variability—are addressed elsewhere in the thesis, where refractive propagation



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effects represent primary research objectives. In the present context, zenith delays are considered only to the extent necessary to verify the internal consistency and observational reliability of the atmospheric state underlying the attenuation analyses.

Zenith delays are vertically integrated refractive observables controlled by atmospheric pressure, temperature, and water vapor content. Their validation therefore serves as a physical consistency check, ensuring that the thermodynamic background used in the radiative attenuation modelling is coherent with independent observations before moving to frequency-dependent attenuation regimes.

7.2.1 Hydrostatic Zenith Delay (ZHD)

The hydrostatic component of the zenith delay is derived from surface pressure and vertical pressure profiles obtained from ERA5 reanalysis fields, following the Saastamoinen formulation and its subsequent refinements for high-accuracy applications [45, 47, 56]. The required thermodynamic variables are consistently extracted from ERA5 products [12, 99]. The resulting ZHD fields exhibit the expected large-scale dependence on surface pressure and terrain elevation, with maximum values at low altitudes and a monotonic decrease with increasing height. The spatial patterns obtained are consistent with established climatological and reanalysis-based estimates reported in the literature [57, 111].

Quantitative comparison with GNSS-derived zenith delays, after isolation of the wet contribution, indicates that the hydrostatic component is reproduced with high accuracy. At the global scale, mean biases remain within a few millimetres, while root-mean-square errors typically do not exceed 5 mm, in agreement with previous validation studies [48, 65]. This level of agreement confirms that the hydrostatic delay is robustly constrained and does not constitute a dominant source of uncertainty in the total zenith delay budget. Accordingly, ZHD is treated in the present chapter as a validated refractive reference, while its detailed climatological characterization is addressed elsewhere in the thesis.

7.2.2 Zenith Wet Delay (ZWD)

The zenith wet delay constitutes the most variable component of the tropospheric delay, reflecting the spatial and temporal inhomogeneity of atmospheric water vapor and condensed water. Within the present framework, ZWD is obtained from vertically integrated wet refractivity, constrained by ERA5 humidity fields and informed by satellite-derived microphysical information from the CloudSat and Global Precipitation Measurement missions [13, 14, 98]. The resulting ZWD fields exhibit pronounced latitudinal and seasonal variability. Enhanced values are found in tropical and subtropical regions characterized by deep convection and high precipitable water content, while mid-latitude regions display a marked seasonal cycle. Polar regions consistently show low ZWD values, in accordance with their reduced atmospheric moisture content and with established global moisture assessments [22, 51].



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Comparison with GNSS-derived ZWD estimates, obtained through standard ZTD decomposition techniques [46, 73], indicates good agreement at the global scale. Root-mean-square errors typically range between 8 and 15 mm, depending on latitude and season, with correlation coefficients exceeding 0.85 over most regions. Larger discrepancies are primarily confined to areas affected by strong vertical moisture gradients, complex orography, or intense convective activity, where both reanalysis-based humidity fields and GNSS retrievals are subject to increased uncertainty [49, 112].

Overall, these results confirm that the ZWD fields employed in this work provide a physically consistent and observationally reliable representation of the wet refractive contribution. In the context of the present chapter, ZWD is therefore treated as a validated refractive quantity, supporting the interpretation of attenuation processes governed by atmospheric water content.

7.2.3 Zenith Total Delay (ZTD) and Integrated Validation

The zenith total delay provides an integrated measure of the refractive state of the atmosphere and constitutes the primary observable accessible to GNSS techniques. In this sense, ZTD offers a direct benchmark for evaluating the combined performance of the hydrostatic and wet components within the attenuation–delay modelling framework. Global comparisons between modelled ZTD and GNSS observations reveal a high degree of consistency. Correlation coefficients typically exceed 0.9, mean biases remain close to zero, and the overall root-mean-square error generally stays below 20 mm. These figures are consistent with large-scale validation studies based on reanalysis-driven modelling and independent GNSS observations [48, 65, 73].

Residual discrepancies are confined to specific regions and can be attributed to limitations in water vapor representation, temporal mismatches between satellite sampling and GNSS observations, and local-scale meteorological variability not fully resolved by global products. No systematic dependence on latitude, surface type, or large-scale climatic regime is detected. The level of agreement obtained for ZTD confirms the internal coherence and observational reliability of the refractive state employed in this work. Within the scope of the present chapter, ZTD therefore serves as a final consistency check, establishing a robust refractive baseline for the global attenuation analyses discussed in **Sections 7.3 and 7.4**.

7.2.4 Physical Consistency and Implications

Beyond statistical agreement, the validation of zenith delay components provides a necessary physical consistency check for the unified propagation framework. In particular, the spatial and temporal variability of the wet delay reflects the same thermodynamic and microphysical controls that govern hydrometeor-induced attenuation, indicating a coherent response of refractive and radiative wet effects to atmospheric water content.

This coherence supports the interpretation of attenuation and delay as complementary manifestations of a common atmospheric state, rather than as independent propagation quantities. Within this perspective, the C^4 framework is employed solely as an interpretative construct,



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offering a conceptual link between microphysical structure, effective dielectric behavior, and macroscopic electromagnetic propagation. No additional modelling assumptions are introduced at this stage; the synthesis follows directly from the validated refractive and radiative results.

7.3 Global Attenuation Climatologies and Validation (Ku–Ka–W)

This section provides a physical synthesis and interpretative assessment of the global atmospheric attenuation results obtained at Ku, Ka, and W bands within the attenuation–delay framework developed in this work. Rather than reproducing quantitative outcomes, which are fully documented in the peer-reviewed articles collected in **Appendix A (Part I)**, the focus here is on the organization, frequency dependence, and physical consistency of attenuation behavior across propagation regimes.

All path-integrated attenuation (PIA) statistics, global climatological maps, percentile distributions, and validation figures referred to in this section are reported in **Articles A1–A8**, where methodological details and numerical results are presented exhaustively. The present analysis instead concentrates on the underlying physical structure of attenuation climatologies and on their observational consistency across frequencies, with particular emphasis on the systematic transition from rain-dominated extinction processes at microwave frequencies to cloud- and ice-controlled regimes at millimetre-wave frequencies.

Atmospheric attenuation at Ku, Ka, and W bands results from the combined contribution of gaseous absorption, cloud liquid water, rainfall, and frozen hydrometeors [1–3]. In a radiative-transfer formulation, the path-integrated attenuation along a propagation path of length s can be expressed as:

$$PIA(f, \theta) = \int_0^s k_{ext}(f, \mathbf{r}) ds, \quad (7.1)$$

where k_{ext} denotes the extinction coefficient, dependent on frequency f , propagation geometry, and the local thermodynamic and microphysical state of the atmosphere. Within this framework, frequency acts as a physical discriminator of dominant extinction mechanisms. At Ku band, attenuation is largely governed by rainfall and liquid hydrometeors, while contributions from cloud water and ice are generally secondary. At Ka band, cloud liquid water and light precipitation become increasingly relevant, and the sensitivity to microphysical variability is enhanced. At W band, attenuation is strongly influenced by cloud liquid water and frozen hydrometeors, with ice-phase processes playing a non-negligible role even in non-precipitating conditions. These frequency-dependent regimes emerge consistently across the reported climatologies and provide a physically coherent interpretation of the observed global attenuation patterns.



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7.3.1 Global Attenuation Fields and Frequency Dependence

Global attenuation climatologies at Ku, Ka, and W bands are derived within the unified attenuation–delay framework through the combined use of spaceborne radar observations and thermodynamic constraints, yielding frequency-resolved path-integrated attenuation (PIA) fields. The observational basis of the analysis relies on measurements from the Global Precipitation Measurement (GPM) and CloudSat missions, whose instrumental characteristics and retrieval approaches are described by **Hou et al.** (2014) and **Stephens et al.** (2002, 2008) [13, 14, 97, 98].

The resulting PIA fields and their global spatial distributions are reported and quantitatively analyzed in the peer-reviewed studies collected in **Appendix A (Part I)**, in particular **Articles A1, A2, A4, and A5**. These global attenuation maps exhibit a pronounced latitudinal organization, with maximum attenuation systematically concentrated within the tropical belt, especially over regions characterized by persistent deep convection and high precipitation rates. This spatial structure reflects the dominant role of hydrometeor loading in governing extinction processes across all considered frequencies.

At Ku band, significant attenuation is largely confined to intense rainfall events, while mid-latitude regions typically exhibit moderate values and high-latitude regions remain weakly affected, as documented in **Articles A1 and A2**. At Ka band, attenuation extends over a broader spatial domain, indicating increased sensitivity to stratiform precipitation and cloud liquid water, as reported in **Articles A2 and A3**. At W band, substantial attenuation is observed not only within convective systems but also in regions dominated by extensive cloud cover and frozen hydrometeors, marking a fundamental shift in the prevailing extinction mechanisms, consistently documented in **Articles A4 and A5**.

Across all climatic regimes, attenuation increases monotonically with frequency. This behavior is consistent with the approximate power-law dependence of the specific extinction coefficient,

$$k_{ext}(f) \propto f^{\alpha}, \quad (7.2)$$

where the exponent α depends on hydrometeor phase, particle size distribution, and dielectric properties [1–3]. The frequency scaling reported in **Articles A2 and A4** confirms that the framework captures the continuous transition from rain-dominated attenuation at microwave frequencies to cloud- and ice-controlled extinction at millimetre-wave frequencies, without the introduction of ad hoc assumptions.

7.3.2 Climatological Variability and Extreme Attenuation Regimes

Beyond mean conditions, global attenuation climatologies exhibit substantial variability across seasonal and regional scales, as documented in the peer-reviewed studies collected in **Appendix A (Part I)**. Tropical regions consistently dominate the upper attenuation percentiles, with extreme path-integrated attenuation (PIA) values associated with organized convective systems



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and mesoscale precipitation structures, as evidenced in **Articles A1–A3** and in spaceborne radar observations [13].

Mid-latitude regions display a marked seasonal modulation, with enhanced attenuation during warm seasons and systematically reduced values during colder periods. This behavior, reported in **Articles A2** and **A3**, reflects the combined influence of seasonal variations in precipitation intensity, cloud liquid water content, and large-scale atmospheric circulation patterns. At W band, elevated attenuation is also observed in regions characterized by persistent cloud cover and mixed-phase precipitation, even in the absence of intense surface rainfall. Results presented in **Articles A4** and **A5** indicate that, under these conditions, extinction is increasingly governed by scattering and absorption by ice particles and mixed-phase hydrometeors. Their contribution to the total extinction coefficient,

$$k_{ext}(f) = k_{gas}(f) + k_{clw}(f) + k_{rain}(f) + k_{ice}(f), \quad (7.3)$$

becomes comparable to, or exceeds, that of liquid water at millimetre-wave frequencies.

This behavior is consistent with the strong interaction between millimetre-wave radiation and cloud ice and mixed-phase hydrometeors, as established in physically based propagation studies by **Matricciani et al.** (2009) [15] and **Rosenkranz** (1998) [16]. The convergence between the attenuation statistics reported in **Articles A4–A5** and these independent theoretical and observational studies highlights the limitations of rain-only attenuation models when extended to millimetre-wave frequencies.

7.3.3 Validation Against Satellite Observations

The consistency of the global attenuation climatologies is further assessed through comparison with independent spaceborne radar observations. Satellite-based attenuation signatures at Ku, Ka, and W bands provide an external reference for evaluating the physical realism of the model-derived path-integrated attenuation (PIA) fields. Validation is based on coincident observations from the Global Precipitation Measurement (GPM) and CloudSat missions, which provide vertically resolved information on hydrometeor structure and extinction [13, 14, 97, 98]. These datasets enable a physically meaningful assessment of attenuation behaviour across a wide range of precipitation regimes and atmospheric conditions. Across all three frequency bands, the comparisons indicate good overall agreement between modelled and satellite-inferred attenuation, with correlation coefficients typically exceeding 0.8 and limited systematic bias, in line with previous large-scale validation efforts [13, 48, 65]. At Ku and Ka bands, the strongest consistency is observed under stratiform and moderately convective conditions, where satellite-based attenuation retrievals are known to be most reliable [13].

Increased scatter is observed in intense convective systems. This behavior reflects both the intrinsic variability of precipitation microphysics and the documented limitations of spaceborne retrieval techniques under extreme conditions, rather than deficiencies of the modelling framework itself [98]. Similar limitations have been reported in independent observational studies and are



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not specific to the present analysis.

At W band, the validation demonstrates that the framework captures attenuation regimes dominated by cloud ice and mixed-phase hydrometeors—an observationally challenging domain that remains insufficiently constrained in many operational propagation models. The level of agreement obtained is consistent with independent physical and observational evidence [14, 15, 16] and provides strong support for the robustness of the proposed framework at millimetre-wave frequencies.

7.3.4 Physical Consistency with Zenith Wet Delay

A central qualitative outcome of this analysis is the pronounced spatial and statistical coherence observed between atmospheric attenuation fields and zenith wet delay. Although a detailed quantitative characterization and validation of zenith wet delay are presented in subsequent chapters of this thesis, the physical basis of this relationship is recalled here to support the interpretation of the attenuation results. The zenith wet delay may be expressed as:

$$ZWD = 10^{-6} \int_0^{+\infty} N_w(z) dz, \quad (7.4)$$

where $N_w(z)$ denotes the wet refractivity, directly linked to the partial pressure of atmospheric water vapor. Regions characterized by elevated ZWD values systematically correspond to enhanced attenuation across all considered frequencies, with the strongest correspondence observed at Ka and W bands. This behavior reflects the shared dependence of refractive and radiative propagation effects on atmospheric water content and on the vertical distribution of hydrometeors. The attenuation–ZWD coherence highlighted in this section is therefore interpreted as a physically motivated consistency check, rather than as an independent validation exercise. The comprehensive assessment of zenith wet delay, including retrieval methodology, uncertainty characterization, and validation against independent observations, is addressed in dedicated chapters. Within this context, the observed coherence provides independent physical confirmation that refractive delay and radiative extinction emerge consistently from the same thermodynamic and microphysical state of the atmosphere, rather than representing separate or unrelated propagation phenomena.

7.3.5 Implications for High-Frequency Satellite Links

The global attenuation climatologies synthesized in this section have direct implications for the design and performance assessment of satellite communication systems operating at Ku, Ka, and W bands [3]. These implications emerge from the validated attenuation statistics and spatial distributions reported in **Articles A1–A5**, which collectively characterize the frequency dependence, climatic variability, and extreme regimes of atmospheric attenuation.

The results highlight the increasing vulnerability of millimetre-wave links to cloud- and ice-induced attenuation, even outside regions affected by intense rainfall. This behavior, consistently



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observed across multiple climatic regimes in **Articles A3–A5**, indicates that attenuation at high frequencies cannot be interpreted solely in terms of surface precipitation intensity. At W band, attenuation cannot be treated as a purely rain-driven effect. Instead, cloud structure, ice particle properties, and mixed-phase processes exert a dominant control on link availability and fade statistics, as documented in **Articles A4 and A5**. In this context, the physically consistent attenuation fields summarized here provide a robust basis for the development of next-generation propagation models and for the assessment of high-frequency link budgets under realistic atmospheric conditions.

In summary, the global attenuation climatologies and validation results discussed in this section consistently capture the spatial organization, frequency dependence, and extreme regimes of atmospheric attenuation across Ku, Ka, and W bands. The agreement with independent satellite observations [13, 14, 97, 98], together with the coherence observed with independently validated zenith wet delay fields (§7.2), indicates that the attenuation behavior is physically consistent with the underlying thermodynamic and microphysical state of the atmosphere. Within this context, the results provide a physically grounded basis for the development of a predictive attenuation law, introduced in the following section.

7.4 Validation of the Predictive Attenuation Law

This section addresses the validation of the predictive attenuation law derived within the unified attenuation–delay framework. The analysis evaluates its ability to reproduce path-integrated attenuation across Ku, Ka, and W bands using a reduced set of physically meaningful predictors, while maintaining consistency with the climatological behavior and observational evidence discussed in the previous sections. The predictive law is not formulated as a purely empirical regression. Rather, it constitutes a compact functional representation of the radiative–refractive coupling established in §§7.2–7.3, where attenuation and zenith wet delay were shown to respond coherently to the same underlying thermodynamic and microphysical controls.

7.4.1 Functional Form of the Predictive Law

Building on the results discussed in §7.3 and on the established physical coupling between atmospheric attenuation and water content, path-integrated attenuation can be expressed as a function of frequency and zenith wet delay,

$$PIA(f) = \mathcal{F}(ZWD, f), \quad (7.5)$$

where $\mathcal{F}$ represents a frequency-dependent mapping constrained by radiative-transfer considerations. In its operational implementation, the predictive attenuation law adopted in this work is written as:

$$PIA(f) = A(f) ZWD^{\beta(f)}, \quad (7.6)$$



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where $A(f)$ and $\beta(f)$ are frequency-dependent coefficients that encapsulate the combined influence of hydrometeor phase, particle size distribution, and dielectric properties. This formulation preserves the monotonic dependence of attenuation on atmospheric water content and naturally reproduces the observed increase of attenuation with frequency across the considered bands.

7.4.2 Physical Interpretation of the Law

The exponent $\beta(f)$ reflects the relative contribution of liquid and frozen hydrometeors to the overall extinction process. At Ku and Ka bands, $\beta(f)$ remains close to unity, consistent with attenuation regimes dominated by liquid precipitation. At W band, systematic deviations from linearity emerge, indicating the increasing influence of cloud ice and mixed-phase hydrometeors, whose scattering efficiency exhibits a stronger frequency dependence [1–3, 15–17].

The prefactor $A(f)$ incorporates the combined contribution of gaseous absorption and the effective coupling between refractive and radiative properties of the medium. Its frequency scaling is consistent with the extinction behavior discussed in §7.3 and does not require the introduction of additional empirical correction factors, supporting the physical consistency of the proposed formulation.

7.4.3 Validation Against Observations

The predictive attenuation law is validated against independent satellite-derived attenuation estimates through comparison between predicted PIA values and those inferred from spaceborne radar observations [13, 14, 97, 98]. Across Ku, Ka, and W bands, the law reproduces the principal statistical properties of the observed attenuation fields, including mean behavior, variability, and extreme regimes. At Ku and Ka bands, correlation coefficients typically exceed 0.8, with root-mean-square errors comparable to those obtained using more complex attenuation retrieval techniques. At W band, the predictive law captures both the magnitude and spatial organization of attenuation associated with cloud- and ice-dominated regimes, demonstrating its ability to generalize beyond rain-driven conditions.

Residual discrepancies are mainly confined to regions characterized by intense convective activity or pronounced vertical microphysical gradients, where both ZWD retrievals and satellite-based attenuation estimates are affected by increased uncertainty. No systematic bias with respect to latitude, climatic regime, or surface type is detected.

7.4.4 Consistency with Radiative–Refractive Coupling

A key outcome of the validation is the demonstration that attenuation can be inferred from a refractive observable without violating radiative-transfer constraints. The predictive law implicitly exploits the shared dependence of zenith wet delay and extinction on atmospheric



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water content, as expressed by:

$$ZWD = 10^{-6} \int_0^{+\infty} N_w(z) dz, \quad PIA = \int_0^s k_{ext}(f, \mathbf{r}) ds. \quad (7.7)$$

Within this framework, the mapping between ZWD and PIA emerges as a physically grounded surrogate of the full radiative-transfer problem, rather than as a purely statistical approximation. This property distinguishes the proposed law from conventional empirical attenuation models and underpins its robustness across frequency bands and climatic regimes.

7.4.5 Implications and Limitations

The validated predictive attenuation law offers a computationally efficient alternative to full radiative-transfer simulations for applications requiring large-scale or near-real-time attenuation estimates, such as satellite link budgeting and propagation monitoring at millimetre-wave frequencies [3].

At the same time, the law is not intended to replace physically based solvers in scenarios where detailed vertical microphysical structure is required. Its validity is strongest within the range of atmospheric conditions represented in the satellite and reanalysis datasets employed in this work. Extreme convective events and highly localized phenomena may therefore require dedicated modelling approaches.

7.4.6 Synthesis of Results and Physical Implications

The predictive attenuation law validated in this section provides a compact and physically consistent synthesis of the attenuation–delay framework developed in this thesis. By relating path-integrated attenuation to zenith wet delay through frequency-dependent coefficients, the law captures the essential radiative–refractive coupling of the troposphere while remaining computationally tractable. In conjunction with the climatological analyses and validation results presented in §§7.2–7.3, the results of this section establish a coherent modelling framework that supports both operational applications and further methodological extensions.

7.5 Interpretative framework: C⁴ perspective on attenuation–delay consistency

Following the validation of attenuation and zenith delay against independent satellite and GNSS observations presented in the previous sections, it is useful to interpret the results within a unified physical perspective capable of linking microphysical structure, effective dielectric properties, and electromagnetic field behavior. Within this context, the C⁴ framework is employed exclusively as an interpretative layer, aimed at rationalizing the observed consistency between refractive and radiative propagation effects. In this chapter, the C⁴ formulation neither replaces nor modifies the radiative-transfer solvers adopted in this work, such as SNEM and SNEM-2. Rather, it



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provides a compact conceptual representation to interpret results obtained from physically based modelling and observation-driven validation.

The motivation for introducing this interpretative perspective arises from the empirical evidence that both zenith wet delay and path-integrated attenuation respond coherently to the same thermodynamic and microphysical drivers, despite originating from distinct physical mechanisms. This coherence points to an underlying structural link between the microphysical state of the atmosphere and its macroscopic electromagnetic response. Within the C^4 perspective, this link is described through an effective-medium representation, in which the collective dielectric behavior of heterogeneous hydrometeor populations is mapped into an effective complex permittivity. This representation is intended as a descriptive mapping rather than as a new propagation equation, and remains fully consistent with classical effective-medium approaches commonly adopted in atmospheric and radiative-transfer studies.

The effective permittivity formulation underlying the C^4 framework provides a compact way to relate hydrometeor phase, particle-size distribution, and spatial organization to the observed extinction and refractivity. Importantly, this interpretation does not introduce additional degrees of freedom into the modelling chain: all radiative-transfer calculations and delay estimates discussed in this chapter remain constrained by the same thermodynamic and microphysical inputs previously validated against observations.

From this standpoint, the C^4 framework offers a physical rationale for the consistent spatial patterns and frequency scaling exhibited by attenuation and delay, particularly at millimetre-wave frequencies where scattering by ice particles and mixed-phase hydrometeors becomes increasingly relevant. The framework therefore supports the interpretation of the results presented in **Sections 7.2 and 7.3** without altering their quantitative outcomes. The role of the C^4 framework in this chapter is interpretative rather than demonstrative. Formal theoretical developments and extensions are intentionally deferred to dedicated theoretical sections and appendices, where their scope and limitations can be addressed independently of the validation results. Overall, the introduction of the C^4 perspective at this stage of the thesis serves to consolidate the physical interpretation of the validated attenuation–delay relationship, reinforcing the internal consistency of the modelling approach while preserving the primacy of observation-driven validation.

7.6 Synthesis of results and physical implications

This section provides a synthetic interpretation of the results presented in this chapter, integrating the quantitative findings reported in the peer-reviewed articles collected in **Appendix A (Part I)** into a unified physical perspective. No new results are introduced; rather, the synthesis consolidates the evidence obtained across Articles A1–A8 and clarifies their implications for atmospheric propagation modelling at microwave and millimetre-wave frequencies.

Taken together, the results documented in Articles A1–A8 and interpreted in **§§7.2–7.4** demonstrate that refractive and radiative propagation effects in the troposphere are not independent phenomena, but complementary manifestations of the same thermodynamic and



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microphysical state of the atmosphere. Zenith delays and path-integrated attenuation respond coherently to variations in atmospheric water content, hydrometeor phase, and vertical structure, despite originating from distinct physical mechanisms. The validation of zenith delay components (§7.2) confirms that large-scale refractive effects are robustly constrained by thermodynamic variables, while the global attenuation climatologies and satellite-based validation (§7.3) reveal the increasing importance of cloud ice and mixed-phase hydrometeors at millimetre-wave frequencies. The predictive attenuation law (§7.4) further demonstrates that attenuation can be related to refractive observables through physically meaningful mappings, without violating radiative-transfer constraints. Collectively, these results highlight the limitations of traditional propagation models that treat attenuation and delay as separate or weakly coupled processes, and underscore the inadequacy of rain-only attenuation formulations when extended to high-frequency regimes. The unified attenuation–delay framework developed in this work provides a physically consistent alternative, capable of capturing the transition from rain-dominated to cloud- and ice-controlled propagation effects across climatic regimes. From a modelling perspective, this synthesis supports the use of integrated thermodynamic and microphysical descriptors for the characterization of atmospheric propagation, particularly for satellite communication systems operating at Ka and W bands. The results summarized here establish a coherent physical basis for the extended analyses developed in subsequent chapters, where additional aspects of zenith delay, sky noise temperature, and radiative transfer are investigated.

7.7 Article-Guided Development of the Core Results

Complementary to the scientific synthesis provided in §7.6, this section clarifies how the sequence of validated results reported in the associated peer-reviewed articles emerged through a structured progression of article-driven investigations and quantitative validation steps. Rather than reproducing numerical outcomes already documented in the literature, the focus is placed on reconstructing the scientific logic, modelling constraints, and physical assumptions that guided their formulation and validation.

The development of the framework is grounded in the coordinated use of complementary observational and reanalysis datasets. ERA5 reanalysis provides the thermodynamic background required for hydrostatic delay estimation and gaseous absorption modelling; CloudSat–GPM coincidences supply vertically resolved hydrometeor and attenuation information across Ku, Ka, and W bands; ground-based GNSS measurements offer independent constraints on zenith total delay. Together, these datasets enable a consistent multi-scale evaluation of refractive and radiative propagation effects within the same atmospheric state. Model performance across the different studies is assessed using standard statistical indicators, including mean bias, root-mean-square deviation, correlation coefficients, and normalized error metrics. Seasonal and latitudinal variability, as well as transitions between dominant microphysical regimes, are explicitly accounted for, as they exert a first-order control on both refractivity and extinction. Physical consistency is systematically enforced through monotonic frequency scaling of extinction, energy-conservation



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arguments, and agreement with established ITU-R recommendations (P.676, P.838, P.840), ensuring that the results remain compatible with operational satellite-link and propagation standards.

Within this structure, the article-guided results naturally organize into two complementary contribution streams. The first establishes a refractive baseline through the validation of tropospheric delay components (ZHD, ZWD, ZTD). The second addresses attenuation and brightness-temperature behaviour, focusing on the physical interpretation and observational consistency of attenuation and TB fields derived from CloudSat–GPM and ERA5 data across diverse atmospheric regimes.

The role of the present section is therefore not to duplicate the content of the individual articles, but to clarify the scientific logic and modelling constraints underlying their collective contribution, and to illustrate how they progressively informed the formulation of the attenuation–delay framework. On this basis, subsequent sections introduce methodological and predictive extensions that emerged directly from the validated results. These include the C⁴ framework, employed as a unifying physical perspective on refractive–radiative consistency, and the development of machine-learning-based surrogate models designed to compress the validated physical relationships into computationally efficient predictors. Together, these elements integrate physically based modelling, data-driven approaches, and observational validation into a coherent atmospheric propagation framework.

7.7.1 Integrated Methodological Framework Underpinning the Reported Results

The results presented in this chapter are grounded in an integrated methodological framework specifically designed to ensure thermodynamic consistency, observational realism, and physical traceability across spatial and temporal scales. Rather than constituting a standalone methodological development, this framework represents the operational backbone through which the quantitative results reported in Articles A1 and A2 were obtained and validated.

The approach relies on the synergistic integration of complementary data sources. ERA5 reanalysis provides the large-scale thermodynamic state of the atmosphere, supplying pressure, temperature, humidity, and wind fields that define the background conditions for both refractive-delay and radiative-attenuation processes [12, 99]. These variables directly control the hydrostatic zenith delay (ZHD) and the gaseous contribution to specific attenuation, following established formulations for atmospheric refractivity and gaseous absorption [1, 44, 45]. To ensure profile-level physical consistency, ERA5 fields were vertically interpolated onto the CloudSat ECMWF-AUX grid [97], enabling a direct alignment between modelled atmospheric states and satellite-resolved observations.

CloudSat–GPM coincidences constitute the observational core of the framework, providing vertically resolved hydrometeor and radiative information across Ku, Ka, and W bands [13, 14, 97, 98]. The Cloud Profiling Radar (CPR, 94 GHz) and the GPM Dual-frequency Precipitation



Radar (DPR, 13.6/35.5 GHz) jointly sample atmospheric columns within a strict ± 15 min temporal window and nearly overlapping footprints [101, 102]. This configuration yields co-registered profiles of reflectivity, hydrometeor phase classification, and attenuation-corrected measurements, spanning the full range of atmospheric regimes from thin cirrus layers to deep convective systems [21, 74]. These coincident observations support the retrieval of zenith wet delay (ZWD) and hydrometeor-induced path-integrated attenuation (PIA_{hydro}), while providing direct microphysical constraints through liquid and ice water content, optical depth, and effective particle size [15, 62].

The integrated strategy establishes a three-tier validation chain that underpins all reported results:

- ERA5 constrains thermodynamics and gaseous absorption [1, 12, 99];
- CloudSat–GPM resolves hydrometeor microphysics and attenuation processes [13, 14, 97, 98];
- GNSS provides independent verification of zenith total delay and integrated column properties [46, 48, 65, 73].

By combining ERA5-derived ZHD and ZWD components with CloudSat–GPM extinction coefficients, global path-integrated attenuation (PIA) fields at Ka and W bands were generated. This synthesis bridges physically based radiative-transfer concepts with observation-driven climatology, forming the empirical basis for the global attenuation maps and validation results discussed in the following sections [15, 16, 137].

Figure 7.1 summarizes the vertical workflow underpinning this framework: ERA5 supplies the thermodynamic variables (P, T, q) for ZHD computation, CloudSat–GPM coincidences provide hydrometeor and attenuation profiles for ZWD and PIA estimation, and GNSS observations validate the resulting ZTD decomposition. For clarity and compactness, the figure is provided in **Appendix Chapter 7, Part II**, while the present section focuses on the methodological logic rather than graphical guidance.

7.7.2 Integration of Attenuation Modelling and Tropospheric Delay Retrieval

The framework developed in this thesis establishes a unified physical description of atmospheric attenuation and tropospheric delay, recognizing that both phenomena arise from the same underlying thermodynamic and microphysical state of the atmosphere. Pressure, temperature, humidity, and hydrometeor content jointly govern refractive delay and radiative extinction and are therefore treated within a single radiative–refractive formulation consistent with established propagation theory and ITU-R recommendations [1–3, 44].

This section synthesizes the scientific pathway developed through the two main companion studies associated with this work (Articles [A1] and [A2], reported in **Appendix A, Part I**), which together form a coherent progression from observation-driven attenuation retrievals to global zenith-delay decomposition and prediction.



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7.7.2.1 From attenuation physics to delay components

Article [A1] demonstrates the feasibility of deriving global atmospheric attenuation fields from CloudSat–GPM coincidences by exploiting the complementary sensitivity of W-band (94 GHz) and Ku/Ka dual-frequency radar observations [13, 14, 97, 98]. The resulting climatologies reveal a robust dependence of specific attenuation on latitude, frequency, and hydrometeor phase, directly reflecting the spatial organization of wet atmospheric processes at the global scale [15, 16].

ERA5 reanalysis provides the thermodynamic background required for hydrostatic delay (ZHD) and gaseous absorption modelling [12, 99]. When combined with CloudSat–GPM attenuation profiles, this information enables a physically continuous representation of radio-path loss and delay, linking attenuation signatures to the same atmospheric columns that control the zenith wet delay (ZWD), following established refractivity formulations [45, 46]. This establishes a direct physical connection between radiative extinction and refractive delay without introducing independent or ad hoc parameterizations.

7.7.2.2 Machine-learning synthesis and predictive law

Building on this physically consistent dataset, Article [A2] generalizes the attenuation–delay relationship through machine-learning techniques trained on the global CloudSat–GPM–ERA5 database. Gradient Boosting and Random Forest regressors are used to predict path-integrated attenuation (PIA) and its frequency scaling from physically meaningful predictors, including liquid and ice water content, precipitation rate, optical depth, and thermodynamic variables [11, 62].

These models capture the nonlinear dependence of extinction on atmospheric state and provide a compact surrogate representation of the underlying radiative-transfer relationships [15–17]. In particular, the learned mappings enable inference of ZWD-related variability from attenuation statistics, offering a complementary pathway to traditional refractivity-based formulations [46, 73], especially in regions where dense GNSS coverage is unavailable.

7.7.2.3 Unified framework for delay and attenuation

Taken together, Articles [A1] and [A2] establish a unified atmospheric propagation framework¹ in which attenuation and delay emerge as complementary manifestations of the same physical processes. ERA5 constrains the static and gaseous contributions [12, 99], CloudSat–GPM resolves hydrometeor-induced extinction [13, 14, 97, 98], and GNSS observations provide independent validation of the total zenith delay ($ZTD = ZHD + ZWD$) [48, 65, 73].

This synergistic combination enables a global and physically consistent evaluation of both radiative and refractive effects across frequencies and climatic regimes, in line with established

¹The expression unified attenuation–delay framework is adopted here in a descriptive sense, indicating the integrated and physically consistent treatment of tropospheric delay and atmospheric attenuation. For terminological clarity, the framework may also be denoted as an integrated attenuation–delay framework throughout the thesis.



propagation theory and international standards [1–3]. The resulting datasets provide simultaneous ZTD decomposition and attenuation climatologies, describing how electromagnetic waves are delayed and attenuated under diverse atmospheric conditions.

By integrating observation-driven retrievals, physically constrained modelling, and data-driven surrogates, this framework bridges radio-meteorology and atmospheric propagation modelling within a single coherent description. It provides the foundation for the interpretative extensions introduced in the following sections, including the C^4 framework and the development of surrogate predictive models.

7.7.3 Global Attenuation Mapping and Physical Characterization (Article A1)

Article [A1] establishes the feasibility of constructing physically consistent global atmospheric attenuation climatologies through the integration of multi-satellite radar observations from CloudSat and GPM with ERA5 reanalysis fields [12,13,14,97,98]. The study demonstrates that path-integrated attenuation (PIA) at Ku, Ka, and W bands can be reconstructed at the global scale using observation-driven constraints, while preserving thermodynamic coherence and microphysical consistency.

The resulting attenuation maps exhibit a robust and recurrent spatial organization, characterized by pronounced maxima aligned with tropical and mid-latitude convergence zones. These regions coincide with persistent hydrometeor loading and deep convective activity, confirming that large-scale attenuation patterns are primarily governed by the distribution, phase, and vertical structure of condensed water [1–3,15]. While the overall spatial organization remains stable across frequencies, both the amplitude and spatial extent of attenuation increase monotonically from Ku to W band, reflecting the growing sensitivity of millimetre-wave propagation to cloud and ice microphysics [15–16].

A central outcome of Article [A1] is the quantitative demonstration that global attenuation mapping represents not merely a diagnostic visualization, but a physically meaningful component of atmospheric path-propagation modelling. By harmonizing CloudSat CPR (94 GHz) and GPM DPR (Ku/Ka) observations within a strict temporal and spatial coincidence window, the analysis isolates the hydrometeor-induced extinction component after removal of gaseous absorption using line-by-line calculations and ITU-R formulations [3,44]. This approach ensures that the retrieved PIA fields retain a direct and traceable link to the underlying microphysical state of the atmosphere. All valid attenuation estimates are projected onto a regular global grid using variogram-based ordinary kriging, enabling the fusion of irregular satellite swaths into continuous climatological fields with quantified spatial uncertainty [97]. Percentile-based statistics, typically evaluated at the 97th percentile, are employed to characterize extreme attenuation regimes of direct relevance to high-frequency satellite link design and radiative-transfer validation [3].

The main scientific advances introduced in Article [A1] can be summarized as follows:

1. **Observation-anchored global attenuation climatologies**, in which each grid cell is



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explicitly traceable to CloudSat–GPM radar coincidences, ensuring physical realism and avoiding purely synthetic reconstruction [13,14,97].

2. **Cross-band harmonization of Ku, Ka, and W attenuation fields**, achieved through consistent gaseous absorption removal and vertical profile alignment, allowing genuine microphysical signatures to be identified across latitudes and climatic regimes [15,16].
3. **Percentile-based geostatistical fusion**, which suppresses orbital striping, preserves extreme-event statistics, and establishes a direct connection between satellite-derived attenuation and tropospheric delay modelling [3,44].

This article provides the empirical and physical foundation upon which subsequent developments are constructed, including the integration of attenuation with zenith-delay retrievals and the formulation of predictive models discussed in the following sections.

The complete methodological description of the global attenuation map generation—covering data selection, coincidence criteria, attenuation retrieval, quality control, and spatial interpolation—is reported in Appendices A.9 and A.10. The present chapter focuses exclusively on the resulting products, their validation, and their physical interpretation.

7.7.4 From Surface Backscatter (σ_0) to Hydrometeor-Induced Attenuation (PIA)

At an early stage of this research, the retrieval of Path-Integrated Attenuation (PIA) was formulated within a surface-referenced calibration framework. Using GPM-DPR and CloudSat 2B-GEOPROF products, the normalized radar cross section σ_0 was employed as a proxy for the clear-sky surface return [13,14]. Under the working assumption that, for a given incidence angle and surface type, σ_0 remains statistically stable, reductions observed under precipitating conditions were interpreted as attenuation induced by hydrometeors along the radar propagation path. Within this surface-reference framework, PIA was expressed as:

$$PIA(f) = 10 \cdot \log_{10} \left(\frac{\sigma_{0,clear}}{\sigma_{0,rain}} \right), \quad (7.8)$$

where $\sigma_{0,clear}$ denotes the mean clear-sky backscatter and $\sigma_{0,rain}$ the measured value in the presence of precipitation. This formulation corresponds to the classical Surface Reference Technique (SRT), which provides a first-order estimate of attenuation directly from observed radar power loss, without requiring an explicit radiative-transfer inversion [13]. Systematic analysis across large ensembles of coincident satellite orbits, however, revealed that σ_0 cannot be regarded as uniformly stable across surface types. Ocean glint effects, sea-state variability, mixed land–sea footprints, and complex terrain introduced fluctuations often comparable to, or larger than, the hydrometeor-induced attenuation itself. In high-latitude regions and over heterogeneous surfaces, variations driven by incidence angle and surface emissivity frequently dominated the signal, leading to local overestimations and spatial incoherence in the retrieved PIA fields.



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Similar limitations of surface-referenced attenuation retrievals have been documented in previous radar-based studies [15,16]. These limitations motivated a progressive conceptual shift in the attenuation retrieval strategy. Rather than treating attenuation as a surface-calibration problem, it was reformulated as a purely hydrometeor-driven radiative process. By exploiting CloudSat 2C-PRECIP-COLUMN and 2C-Rain-Profile products, attenuation-corrected reflectivity, liquid and ice water content, and hydrometeor phase information were used to compute the hydrometeor contribution to attenuation directly [14]. In this formulation, PIA is obtained as the vertical integral of the extinction coefficient,

$$PIA_{hydro}(f) = \int_0^H k_e(f, z) \cdot dz, \quad (7.9)$$

where k_e accounts for absorption and scattering by hydrometeors and is explicitly linked to particle size, density, and phase composition, in accordance with physically based radiative-transfer theory [1–3]. This transition proved decisive. By grounding attenuation retrieval in microphysical properties rather than surface-dependent calibration, the resulting PIA fields exhibit coherent climatological structures across latitude and frequency. In particular, enhanced W-band attenuation in tropical convective regions and systematically reduced extinction in polar stratiform clouds emerge clearly—features that were largely obscured in the σ_0 -based formulation and are consistent with independent observational and modelling studies [14,15].

While surface backscatter provided a useful initial reference, the hydrometeor-based approach yields a physically consistent and surface-independent description of atmospheric attenuation. This conceptual evolution represents a key scientific outcome of the present work: attenuation retrieval is no longer treated as a geometry-dependent calibration problem, but as an intrinsic radiative manifestation of cloud and precipitation microphysics. This shift underpins both the development of predictive attenuation models (Article [A2]) and the construction of global attenuation climatologies (Article [A1]), and constitutes a cornerstone of the attenuation–delay framework developed in this thesis.

7.7.5 Critical Evaluation and Methodological Improvements in GMG Attenuation Retrieval

The detailed technical analysis underlying the methodological transition from surface-backscatter-based attenuation retrieval to a hydrometeor-driven formulation—including algorithmic choices, quality-control procedures, statistical diagnostics, and implementation constraints—is documented in **Appendix A.11** (Part I). In the present chapter, the discussion is intentionally limited to the conceptual implications and scientific impact of these improvements on the robustness, physical consistency, and interpretability of the resulting global attenuation products.



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7.7.6 Machine Learning Predictive Attenuation Law (Article A2)

The second study extends the attenuation–delay modelling framework by introducing a predictive attenuation law derived through supervised machine-learning techniques [A2]. Using the global CloudSat–GPM coincidence dataset established in Article [A1], Gradient Boosting and Random Forest models were trained to estimate W-band Path-Integrated Attenuation (PIA) as a function of hydrometeor and thermodynamic predictors, including Ku- and Ka-band attenuation, liquid and ice water contents, temperature, and precipitation water content.

The optimized models achieved robust predictive performance, with typical errors of RMSE ≈ 0.54 dB and coefficients of determination $R^2 \approx 0.65$, demonstrating their ability to generalize attenuation behaviour across previously unseen atmospheric conditions [A2]. These results confirm that machine-learning methods can successfully capture the nonlinear coupling between absorption, scattering, and thermodynamic state governing millimetre-wave attenuation [15,16].

During the exploratory phase of Article [A2], multiple machine-learning architectures were investigated, including Random Forests, Gradient Boosting, and Support Vector Machines [A2]. While several configurations exhibited high empirical skill (up to $R^2 > 0.9$ with $RMSE \approx 0.5 - 1$ dB), their limitations became evident when evaluated in the context of physically consistent atmospheric-propagation modelling.

In particular, empirical accuracy alone proved insufficient for operational and scientific applications. Propagation models require physical coherence, continuity across frequencies and climatic regimes, and explicit interpretability of the underlying radiative mechanisms [3,15]. The black-box nature of ensemble and kernel-based ML algorithms obscures the relative contribution of key variables—such as liquid and ice water content, hydrometeor phase, and temperature—to the predicted attenuation [A2]. Moreover, such models depend on large and homogeneous training datasets, which are not always available for rare meteorological regimes, including mixed-phase precipitation and polar clouds [14]. For these reasons, the final stage of Article [A2] adopts a nonlinear parametric regression framework that balances predictive capability with physical interpretability [A2]. In illustrative form, the attenuation law can be expressed as:

$$PIA = aR^b + cT^d + e, \quad (7.10)$$

or, more generally,

$$PIA = f(RR, IWC, LWC, T, p, \nu), \quad (7.11)$$

where the coefficients $\{a, b, c, d, e\}$ retain direct physical meaning related to hydrometeor scattering efficiency, absorption processes, and thermodynamic control [1,3,15]. This formulation offers several advantages:

1. **Physical coherence** — each term corresponds to a known radiative or microphysical mechanism (e.g., rainfall rate controlling extinction, temperature governing phase transitions, frequency scaling absorption) [1,15].



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2. **Parameter portability** — coefficients can be transferred or re-estimated across Ku, Ka, and W bands and integrated within radiative-transfer and tropospheric-delay models [3,14].
3. **Analytical continuity** — the parametric form ensures smooth transitions across meteorological regimes, avoiding discontinuities often produced by discrete ML predictions [A2].
4. **Integration with ERA5 and GNSS** — the regression law couples naturally with reanalysis thermodynamics and GNSS-derived ZTD/ZWD, providing a direct analytical bridge between attenuation and refractive delay [13,14].

Within this framework, machine learning acts as an inductive discovery tool, identifying dominant nonlinear dependencies in the data, while the final regression law formalizes these insights into a physically interpretable and operationally robust attenuation model [A2]. The selected formulation therefore represents not a simplification, but a synthesis of data-driven learning and physical modelling. Beyond the empirical and machine-learning formulations, attenuation can also be interpreted within the author’s C^4 framework. This perspective does not alter the operational retrievals, but provides a deeper physical interpretation of PIA as the macroscopic manifestation of collective electromagnetic–microphysical interactions. The following subsection introduces this generalized C^4 interpretation, framing attenuation in terms of phase correlations, dielectric fluctuations, and field–medium coupling.

7.7.7 Nonlinear Regression Formulation and Parameterization

While the Machine Learning results are presented in detail in Article [A2], the nonlinear regression law introduced in this section represents a physically interpretable synthesis of the dominant dependencies identified by the data-driven analysis. This formulation is not explicitly reported in the article, but constitutes a conceptual and operational generalization developed within the framework of this thesis, aimed at bridging data-driven insights with physically constrained propagation modelling. Building on the patterns revealed during the exploratory phase with Machine Learning models, the final predictive formulation of the Path-Integrated Attenuation (PIA) is expressed through a nonlinear regression framework. The objective is to derive a compact, frequency-dependent, and physically interpretable law linking satellite-observed attenuation to key microphysical and thermodynamic predictors, while preserving analytical continuity across frequencies and atmospheric regimes. The adopted functional form is given by:

$$PIA(\nu) = a_\nu R^{b_\nu} + c_\nu LWC^{d_\nu} + e_\nu T^{f_\nu} + \varepsilon, \quad (7.12)$$

where R denotes the surface rain rate [mmh^{-1}], LWC the in-cloud liquid water content [gm^{-3}], T is the mean in-cloud temperature [K], ν the **frequency band** (Ka or W). The coefficients $\{a_\nu, b_\nu, c_\nu, d_\nu, e_\nu, f_\nu\}$ were determined through nonlinear least-squares fitting, optimized independently for each frequency band by minimizing the root-mean-square deviation between



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observed and modeled PIA over the full set of **CloudSat–GPM** coincidences. The resulting parameterization is summarized in **Table 7.1** (Table moved to **Appendix Chapter 7, Part II**).

This parametric formulation successfully reproduces the nonlinear growth of attenuation with precipitation intensity and cloud liquid water content, while preserving a physically meaningful temperature dependence associated with hydrometeor phase transitions and absorption efficiency. Each group of coefficients retains a clear radiative interpretation:

- (a_ν, b_ν) control the extinction efficiency associated with rainfall rate, drop-size distribution, and frequency scaling;
- (c_ν, d_ν) represent the contribution of cloud microphysics and liquid water loading;
- (e_ν, f_ν) encode the thermal modulation of absorption and phase-dependent processes.

Compared to discrete **Machine Learning predictors**, the regression-based formulation yields smooth and continuous attenuation fields, avoiding classification-induced discontinuities and ensuring physical consistency across climatic regimes. Crucially, it enables a direct analytical coupling with the refractivity-based wet-delay component through the empirical relation:

$$ZWD \propto \int_0^H \frac{PIA(\nu, z)}{k_\nu} dz, \quad (7.13)$$

where k_ν denotes the frequency-dependent specific extinction coefficient derived from the regression model. This link allows the simultaneous simulation of radiative (attenuation) and refractive (delay) effects within a unified thermodynamic framework. The nonlinear regression therefore represents a physically constrained synthesis of the **Machine Learning** insights explored in Article [A2]. Rather than replacing the data-driven analysis, it formalizes its outcomes into an interpretable predictive law that can be directly implemented in radiative-transfer solvers, global attenuation climatologies, and tropospheric delay decomposition models. **Figure 7.3** illustrates the comparison between observed and predicted **Path-Integrated Attenuation (PIA)** at W band obtained using the nonlinear regression formulation developed in this work. The fitted parametric law,

$$PIA = a R^b + c LWC^d + e T^f, \quad (7.14)$$

reproduces the observed attenuation behavior as a function of rainfall rate, in-cloud liquid water content, and mean atmospheric temperature. The resulting agreement is quantified by a high coefficient of determination ($R^2 = 0.961$) and a low root-mean-square error (RMSE=0.50 dB), indicating that the dominant physical dependencies governing W-band attenuation are effectively captured by the regression framework. The negligible systematic bias confirms that the fitted model preserves statistical balance between observed and simulated attenuation values, avoiding overfitting or compensating effects. While derived from **CloudSat–GPM** coincidence data, this regression-based formulation provides a compact and physically interpretable parametric



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baseline, suitable for subsequent global attenuation mapping and for coupling radiative extinction with refractive-delay modelling within the attenuation–delay framework developed in this thesis. **Figure 7.4** further illustrates the relationship between path-integrated attenuation (PIA) and rain rate R in logarithmic scale. Observational estimates are represented by binned means (blue symbols), while the fitted nonlinear regression model is shown as a continuous curve (red line). The model response is evaluated under representative median conditions of in-cloud liquid water content ($LWC^* = 0.12 \text{ g m}^{-3}$) and temperature ($T^* = 270.2 \text{ K}$), derived from the CloudSat–GPM coincidence dataset. The close agreement between the binned observations and the modelled curve confirms that the proposed regression formulation captures the nonlinear growth of attenuation with increasing precipitation intensity across both weak and intense rainfall regimes. The smooth curvature of the fitted law reflects the progressive enhancement of extinction efficiency as hydrometeor concentration and effective particle size increase. This behavior is physically consistent with expectations from Mie-scattering theory at W-band frequencies, where attenuation exhibits a strongly nonlinear dependence on hydrometeor loading. The absence of systematic deviations at high rain rates further indicates that the regression preserves physical continuity and avoids saturation artefacts, supporting its suitability for global attenuation modelling and radiative-transfer applications. (**Figures 7.3–7.4** are moved to **Appendix Chapter 7, Part II**).

7.8 Connection between Attenuation and Tropospheric Delay Retrieval

The connection between attenuation modelling and tropospheric delay retrieval emerges from their shared physical origin: the interaction between electromagnetic waves and the thermodynamic state of moist air. Although radiative extinction and refractive delay arise from distinct electromagnetic mechanisms, both are governed by atmospheric pressure, temperature, and water vapour content, which jointly modulate signal amplitude and phase during propagation [1,16].

ERA5 reanalysis provides spatially and temporally consistent fields of pressure, temperature, and humidity, enabling robust global estimates of the hydrostatic (ZHD) and wet (ZWD) components of the zenith delay [12,45]. In parallel, the CloudSat–GPM–derived attenuation products developed in Articles [A1] and [A2] resolve the microphysical structure of hydrometeors, explicitly capturing the contribution of liquid and ice phases to microwave extinction [21]. These attenuation fields therefore act as an independent observational proxy for the wet atmospheric processes that dominate ZWD variability.

When combined with GNSS-derived zenith total delay observations, this framework supports a multi-scale decomposition in which the total delay can be expressed as:

$$ZTD = ZHD + ZWD = f(p, T) + g(PIA, IWC, LWC). \quad (7.15)$$

linking large-scale thermodynamic constraints to microphysics-driven attenuation processes [46,47].



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This perspective establishes a physically consistent coupling across scales: large-scale reanalysis fields constrain the vertical structure of refractive delays (top–down), while satellite-derived attenuation products refine the microphysical and emissive contributions associated with clouds and precipitation (bottom–up). The resulting framework integrates attenuation and delay within a single atmospheric propagation model, providing a coherent basis for interpreting radiative-transfer simulations, GNSS delay retrievals, and global climatological variability [1,12,21].

7.8.1 Toward a Unified Atmospheric Propagation Model

The combined results of Articles [A1] and [A2] indicate that radiometric attenuation and refractive delay can be represented within a single, physically consistent atmospheric propagation model. Rather than constituting parallel or weakly coupled effects, extinction and delay emerge as complementary responses of electromagnetic waves to the same thermodynamic and microphysical state of the atmosphere, as consistently described in established radiative-transfer and propagation studies [1,3,15,16].

In this context, the global attenuation climatologies derived from satellite observations (Article [A1]) provide spatially and seasonally resolved diagnostics of hydrometeor loading and phase distribution [13,14,21], while the regression-based predictive attenuation law developed in Article [A2] formalizes these dependencies into a compact and physically interpretable functional form. Together, these results demonstrate that attenuation fields carry structural information on the wet atmospheric processes that also govern the zenith wet delay (ZWD), extending their relevance beyond communication-oriented diagnostics and linking them directly to refractivity-based delay modelling [45,46,47]. The predictive formulation introduced in Article [A2] further establishes compatibility between attenuation modelling, radiative-transfer simulations, and refractivity-based delay retrieval [1,3]. This compatibility enables attenuation laws to be embedded as surrogate components within zenith-delay simulators, while satellite-derived attenuation maps act as independent, observation-driven constraints for climatological validation and physical consistency checks [12,21].

Taken together, these elements support a unified propagation paradigm in which microwave signal extinction and phase delay are treated as coupled manifestations of the same atmospheric processes across frequencies and climatic regimes. This paradigm represents a key outcome of the thesis, providing a coherent modelling framework that bridges satellite remote sensing, reanalysis-based thermodynamics, and predictive modelling within a single atmospheric propagation description [1,3,12,15].

7.9 Global Retrieval and Decomposition of Zenith Delays

Although attenuation and tropospheric delay are often treated as separate propagation effects, both originate from the same thermodynamic and microphysical state of the atmosphere. Pressure, temperature, and atmospheric water content jointly govern radiative extinction and refractive



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phase delay, modulating signal amplitude and propagation time along Earth–space links. Within this perspective, the global retrieval of the Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) is introduced here as the refractive counterpart to the attenuation analyses presented earlier in Chapter 7. The objective is to provide a consistent thermodynamic reference framework for interpreting how large-scale pressure fields and moisture distributions jointly influence microwave signal delay and attenuation.

The decomposition of ZTD into its hydrostatic and wet components represents a fundamental step in characterizing tropospheric propagation effects. In this section, global fields of ZHD, ZWD, and ZTD are retrieved using ERA5 reanalysis data [12] in combination with the Saastamoinen formulation [45], with the aim of assessing their spatial, latitudinal, and seasonal variability. At any given location and time, the total zenith delay can be expressed as:

$$ZTD = ZHD + ZWD, \quad (7.16)$$

where ZHD depends primarily on surface pressure and altitude, while ZWD reflects the vertically integrated water vapour content of the atmospheric column. The hydrostatic component was estimated using the Saastamoinen formulation,

$$ZHD = \frac{0.0022768 \cdot P}{1 - 0.00266 \cdot \cos(2\phi) - 0.00028 \cdot H} \rightarrow [m], \quad (7.17)$$

where P is the surface pressure [hPa], ϕ the latitude [rad], and H the orthometric height above sea level [km] (**Saastamoinen**, 1972; ITU-R P.676-12, 2019). The wet delay component was subsequently obtained by subtracting ZHD from the ERA5-based total delay. ERA5 hourly fields of surface pressure, temperature, and vertically integrated water vapor were resampled to daily means and interpolated onto a global 0.25° grid. Seasonal composites were then constructed to assess intra-annual variability.

The resulting global fields exhibit a pronounced latitudinal structure in ZHD, governed by the climatological distribution of surface pressure and geopotential height. As expected, ZHD displays limited seasonal variability and remains largely controlled by elevation and latitude. In contrast, ZWD shows strong zonal and seasonal variability, with pronounced maxima over equatorial and monsoonal regions associated with elevated atmospheric moisture content, particularly during boreal summer (JJA). This behaviour reflects the migration of large-scale convective systems and is fully consistent with established climatological benchmarks and ITU-R recommendations (e.g. ITU-R **P.453-13**; **Rosenkranz**, 1998).

The accompanying figures (**Figures 7.5–7.6** are moved to **Appendix Chapter 7, Part II**) are provided solely to support and illustrate the physical interpretation outlined above, without introducing additional analytical elements. **Figure 7.5** presents representative global distributions of ZHD and ZWD averaged over February and August, highlighting the contrasting seasonal sensitivity of the two components. While the hydrostatic delay remains largely stable throughout the year and varies primarily with elevation, the wet delay exhibits pronounced



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seasonal amplification, exceeding 0.30 m in equatorial and monsoonal regions during wet periods. The same figure also includes daily time series of ZTD, ZHD, and ZWD at selected locations (e.g. Singapore and Madrid), which serve to illustrate the markedly stronger wet-delay variability in tropical environments and the dominance of the hydrostatic contribution at mid-latitudes. **Figure 7.6** further summarizes the global distributions of ZHD, ZWD, and ZTD for February and August, providing an illustrative view of the seasonal displacement of maximum wet delay associated with the migration of the Intertropical Convergence Zone (ITCZ). Validation against ITU-R recommendations confirms a global ZHD accuracy within ± 3 mm [1], while the larger dispersion observed in ZWD reflects known uncertainties in reanalysis humidity fields, particularly under convective conditions (**Matricciani et al.**, 2009; **Hersbach et al.**, 2020) [49,12].

Overall, these results establish a robust refractive reference framework that complements the attenuation analyses presented earlier in this chapter. Together, the retrieved zenith delay components and the satellite-derived attenuation products provide a physically consistent basis for the subsequent validation of radiative-transfer simulations, the interpretation of seasonal and latitudinal variability, and the evaluation of machine-learning-based surrogate models.

7.10 Validation Against Satellite Observations

Following the spatial–temporal analysis of zenith delays presented in § 7.9, this section assesses the physical consistency and predictive capability of the SNEM-2 radiative-transfer model through validation against coincident satellite observations from CloudSat and GPM. The two missions provide complementary multi-frequency measurements—CloudSat’s Cloud Profiling Radar (CPR) operating at W band (94 GHz) and GPM’s Dual-Frequency Precipitation Radar (DPR) operating at Ku and Ka bands (13.6 and 35.5 GHz)—which together enable a rigorous cross-platform evaluation of cloud vertical structure, radar reflectivity, hydrometeor classification, brightness temperature (TB), and path-integrated attenuation (PIA) [22, 23, 27].

The validation strategy is based on the strict selection of regions and time periods in which CloudSat and GPM sample overlapping atmospheric columns. All satellite profiles are co-located and interpolated onto the spatial–temporal grid adopted by SNEM-2, ensuring that modelled and observed radiative quantities are compared under identical thermodynamic and microphysical conditions. This approach minimizes sampling inconsistencies and allows a direct and physically meaningful evaluation of the model response to observed atmospheric states [24, 28].

Beyond the assessment of statistical agreement, the analysis explicitly examines the ability of SNEM-2 to reproduce key physical signatures observed by spaceborne radars. These include the vertical structure of the melting layer, the contrasting radiative behaviour of stratiform and convective precipitation regimes, and regional variability in hydrometeor phase and composition. The consistency observed across these regimes demonstrates that the model captures not only aggregate attenuation and emission levels, but also the underlying physical processes governing radiative transfer in precipitating and cloudy atmospheres [30, 31]. Section § 7.10.1 describes the construction and preprocessing of the CloudSat–GPM coincidence dataset, while § 7.10.2



presents the associated quantitative evaluation, including error metrics, scatter analyses, and performance indicators. Together, these results provide a comprehensive validation of SNEM-2 against independent satellite observations, supporting its use in both scientific investigations and operational radiative-transfer and propagation studies.

7.10.1 GPM–CloudSat Coincidence Analysis

To validate the physical realism of the SNEM-2 simulations, coincident observations from GPM’s Dual-Frequency Precipitation Radar (DPR; Ku band at 13.6 GHz and Ka band at 35.5 GHz) and CloudSat’s W-band Cloud Profiling Radar (CPR, 94 GHz) were analyzed [22, 23]. Coincidences were identified using a maximum horizontal separation of 25 km and a temporal offset below 10 minutes, ensuring that both sensors sampled the same atmospheric column under comparable meteorological conditions [27].

All satellite profiles were interpolated onto a common vertical grid and resampled to match the resolution adopted by SNEM-2. The model was driven by co-located ERA5 atmospheric columns providing temperature, pressure, humidity, and hydrometeor fields, thereby ensuring thermodynamic and microphysical consistency between observations and simulations [12]. For each coincident event, the following quantities were systematically compared:

- **Path-Integrated Attenuation (PIA)** derived from DPR Surface Reference Technique (SRT) products and CloudSat CPR one-way attenuation retrievals [24, 25];
- **Brightness Temperature (TB)** from the GPM Microwave Imager (GMI) [26];
- **Hydrometeor classification** from GPM 2A-DPR and CloudSat 2B-CWC-RO products [22, 23].

Figure 7.6 (moved to **Appendix Chapter 7, Part II**) provides an overview of the coincidence-selection workflow and the geographical distribution of the matched GPM–CloudSat events. For each selected atmospheric column, SNEM-2 was executed using the corresponding vertically resolved hydrometeor profiles and temperature-dependent dielectric properties. The model outputs include vertical extinction coefficients, one- and two-way attenuation, and downwelling brightness temperature (TB), which are directly comparable with the satellite-derived quantities.

The comparison results show that SNEM-2 reproduces the main radiative and microphysical signatures observed across Ku–Ka–W frequencies. These include the presence and vertical location of melting-layer bright bands, liquid-to-ice phase transitions, and the characteristic vertical structure of stratiform and convective precipitation systems. Agreement in PIA is particularly strong for stratiform regimes, while residual discrepancies in convective cores are primarily attributable to unresolved microphysical variability and to differences in radar footprint size and viewing geometry between the satellite instruments [30, 31]. Overall, the selected coincidence dataset provides a robust and physically consistent basis for the multi-frequency validation of SNEM-2 against independent satellite observations.



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7.10.2 Vertical Profile Comparison and Physical Consistency

Figure 7.8 (moved to Appendix Chapter 7, Part II) illustrates representative vertical profiles of radar reflectivity, specific attenuation, and brightness temperature (TB) obtained from SNEM-2 simulations and co-located GPM–CloudSat observations over the Intertropical Convergence Zone (ITCZ) [22, 23, 27]. The reflectivity profile exhibits a pronounced maximum below approximately 3 km, consistent with the presence of large liquid hydrometeors and melting particles, and decreases progressively with altitude up to about 13–14 km, indicative of a deep stratiform or convective cloud system [22, 30].

The specific attenuation profile closely mirrors this vertical structure. Maximum attenuation values are confined to the liquid-phase layer, where absorption and scattering by rain droplets dominate, and rapidly decrease above approximately 8 km, where ice-phase hydrometeors prevail and W-band extinction becomes comparatively weak [1, 3, 15, 31]. This behavior is fully consistent with the expected frequency-dependent sensitivity of millimetre-wave propagation to hydrometeor phase and concentration [15, 16].

The brightness temperature (TB) profile reflects the cumulative radiative effects along the propagation path. Its vertical evolution is governed by the integrated balance between extinction and emission processes, with the dominant contribution arising from the lower troposphere, where liquid hydrometeors and warm-cloud emission are most significant [26, 28]. The progressive reduction of hydrometeor loading aloft leads to a corresponding modulation of the TB signal, consistent with decreasing atmospheric opacity and reduced emissivity in ice-dominated layers [30, 31].

The coherent vertical alignment of reflectivity, attenuation, and TB profiles demonstrates that SNEM-2 accurately reproduces the physically expected radiative signatures observed by multi-frequency satellite radars [24, 28]. In particular, the confinement of strong extinction to the liquid-phase region and the systematic attenuation decay in ice-dominated layers confirm the internal consistency of the simulated microphysical–radiative coupling and its agreement with established radiative-transfer theory [1–3, 15].

7.10.3 Seasonal and Latitudinal Variability

The spatio-temporal variability of atmospheric delays and associated radiative metrics plays a central role in satellite communication system design and in the development of physically informed surrogate models. This section investigates how the hydrostatic (ZHD), wet (ZWD), and total (ZTD) zenith delays vary as functions of latitude, season, and elevation, embedding the site-specific analyses discussed earlier within a broader geophysical framework. Specifically:

- § 7.10.3.1 examines the seasonal and climatological organization of zenith delays across representative climatic regimes;

- § 7.10.3.2 analyses their sensitivity to latitude, elevation, and inland–coastal contrasts.

Together, these analyses provide the large-scale atmospheric context required to interpret the model-based validation and attenuation results presented in the preceding sections.



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7.10.3.1 Seasonal Patterns and Climatology

The seasonal and latitudinal organization of zenith delays defines the thermodynamic background against which both attenuation fields and radiative-transfer simulations must be interpreted. Using ERA5 reanalysis profiles spanning the 2020–2024 period (Hersbach et al., 2020 [12]), global monthly fields of ZHD, ZWD, and ZTD were computed and aggregated into February and August composites, representative of boreal winter and summer conditions, respectively.

As illustrated in **Figure 7.9**, ZHD exhibits only weak seasonal variability, being primarily controlled by surface pressure and elevation, in agreement with the Saastamoinen formulation (Saastamoinen, 1972 [45]) and ITU-R P.676-12 recommendations [1]. This behavior reflects the quasi-stationary nature of the hydrostatic component at climatological scales. In contrast, ZWD displays a pronounced seasonal–latitudinal cycle. Maximum values are concentrated near the equatorial belt during February and shift toward the northern tropics in August, consistent with the seasonal migration of the Intertropical Convergence Zone (ITCZ) and monsoon-driven moisture transport (Trenberth et al., 2011 [48]). This migration constitutes the dominant driver of seasonal ZTD variability in low-latitude regions.

Time series at representative stations (**Figure 7.10**), including Singapore, Madrid, São Paulo, and European sites such as Fucino and Scanzano–PA, further corroborate these patterns. Tropical locations are characterized by limited hydrostatic variability and strong wet-delay fluctuations, whereas mid-latitude stations exhibit ZTD values largely dominated by the hydrostatic component, with comparatively moderate ZWD modulation. Elevation effects remain non-negligible and, if not properly accounted for, may introduce biases exceeding ± 1 % in ZTD estimates (Rocken et al., 2001 [46]). Overall, ERA5 reproduces the expected climatological structures and shows good consistency with ITU-R guidance (P.453-13; P.676-12 [1]). Nevertheless, uncertainties in ZWD remain larger in convective and orographically complex regions, reflecting known limitations in reanalysis humidity fields (Matricciani et al., 2009 [49]). These seasonal and latitudinal patterns establish the thermodynamic backdrop for the subsequent hybrid and surrogate modelling efforts that explicitly couple attenuation, refractivity, and microphysical variability (Reichstein et al., 2019 [50]; Karniadakis et al., 2021 [51]). **Figures 7.9–7.10** are moved to Appendix Chapter 7, Part II.

7.10.3.2 Sensitivity to Topography and Latitude

The sensitivity of zenith tropospheric delays to surface elevation and latitude represents a first-order control on radiowave propagation. Using the Saastamoinen formulation (Saastamoinen, 1972 [45]) in combination with ERA5 reanalysis fields (Hersbach et al., 2020 [12]), the modulation of the hydrostatic (ZHD) and wet (ZWD) components of the Zenith Total Delay (ZTD) by topography and large-scale circulation regimes is investigated.

As expected, ZHD decreases monotonically with increasing altitude, directly reflecting the reduction in surface pressure. High-elevation stations such as Addis Abeba (~2400 m) and La Paz (~3600 m) exhibit ZHD values approximately 20–30 cm lower than those at sea level, in close



agreement with ITU-R P.676-12 recommendations [1]. This behavior confirms the dominant and largely deterministic role of pressure in controlling the hydrostatic contribution to the delay.

In contrast, ZWD exhibits pronounced latitudinal and seasonal variability. Persistent deep convection associated with the Intertropical Convergence Zone sustains elevated ZWD values at equatorial sites (e.g., Singapore), whereas mid-latitude stations display marked annual cycles driven by synoptic-scale humidity variability (Rosenkranz, 1998 [16]; Pan et al., 2014 [21]). In tropical highland regions, orographic lifting can partially compensate for pressure-related reductions in ZWD by enhancing local moisture convergence, leading to non-linear interactions among elevation, water vapor content, and temperature (Bock et al., 2005 [49]). These combined dependencies demonstrate that zenith delay variability cannot be interpreted solely in terms of altitude or latitude. Instead, it emerges from their coupled influence on thermodynamic structure and circulation-driven moisture transport. This behavior highlights the necessity of topography-aware parameterizations in both physically based delay formulations and machine-learning-driven predictive models.

7.10.3.3 Tropospheric Delay Statistics at Selected GNSS Stations

The detailed statistical characterization of ZHD, ZWD, and ZTD at the four GNSS stations considered in this study (Scanzano, Fucino, Lario, and Malindi) is presented in **Chapter 7**, where the observed differences are interpreted in relation to atmospheric moisture availability, elevation, and prevailing climatic regime.

7.10.4 Effect of Altitude on Zenith Delay Components

The dependence of zenith delay components on altitude reveals a clear physical separation between hydrostatic and wet contributions. Under representative mid-latitude atmospheric conditions, the hydrostatic delay (ZHD) exhibits an approximately linear decrease with height, in accordance with the pressure lapse prescribed by the Saastamoinen formulation. In contrast, the wet delay (ZWD) decreases more rapidly and in a distinctly non-linear manner, reflecting the exponential decay of integrated water vapor (IWV) with altitude.

As a consequence, the Zenith Total Delay (ZTD) closely follows the hydrostatic component, while retaining a marked low-altitude enhancement driven by atmospheric moisture. As illustrated in **Figure 7.11**, above approximately 3000 m the wet contribution becomes progressively negligible and ZTD asymptotically converges toward ZHD. This behavior is of particular relevance for GNSS calibration, high-altitude observatories, and Earth-observation validation campaigns (Bevis et al., 1994 [46]; Mendes et al., 2002 [47]). The elevation-driven partitioning of delay components underscores the necessity of explicitly accounting for orography in zenith-delay modelling, both within deterministic formulations and in data-driven surrogate approaches, especially in mountainous regions or environments characterized by strong humidity gradients.

To jointly assess the role of atmospheric vertical structure and large-scale climatology in modulating zenith delays, a comparative representation of the hydrostatic (ZHD), wet (ZWD),



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and total (ZTD) components as functions of altitude and representative climate classes is provided in **Figure 7.12**. The depicted fields are derived from a simplified yet physically consistent parametrization capturing the dominant variability of surface pressure, temperature, and water vapour content across maritime tropical, continental tropical, mid-latitude, subtropical desert, and polar regimes.

The analysis highlights a systematic separation between hydrostatic and wet contributions. The wet delay is strongly confined to the lower troposphere, whereas the hydrostatic component exhibits a near-linear decay with altitude that is largely insensitive to climate class. Consequently, humid tropical environments display a pronounced ZWD enhancement at low elevations, while dry and polar climates show a rapid suppression of the wet contribution, yielding a ZTD effectively dominated by its hydrostatic term.

This combined altitude–climate perspective provides a physically intuitive framework for interpreting the variability observed in global ZTD fields and for contextualizing the station-level analyses discussed later in the chapter. By explicitly contrasting the relative roles of atmospheric structure and climatology, the representation establishes a conceptual basis for subsequent discussions on model sensitivity, surrogate-model performance, and the expected behavior of delay estimates under contrasting atmospheric regimes. This separation is particularly relevant when extending zenith delays to slant paths or when assessing the impact of water vapor variability on radiometric and GNSS observations. **Figures 7.11–7.12** are moved to **Appendix Chapter 7, Part II**.

7.11 Machine Learning (ML) Surrogates for Radiative Transfer

7.11.1 Motivation and Evaluation Framework

The increasing complexity of physically based atmospheric radiative-transfer models, combined with the growing demand for large-scale and near-real-time simulations, motivates the exploration of machine-learning (ML) surrogates capable of emulating high-fidelity physical solvers at a fraction of the computational cost. In this context, the present section evaluates the performance of ML-based surrogate models trained to approximate the outputs of the SNEM-2 radiative-transfer framework.

The analysis focuses on two advanced classes of models: recurrent neural networks with Long Short-Term Memory (LSTM) units and Conditional Generative Adversarial Networks (CGANs). These architectures have recently demonstrated strong potential in geophysical emulation, climate modelling, and physics-informed learning applications (Reichstein et al., 2019 [41]; Kashinath et al., 2021 [36]; Karniadakis et al., 2021 [38]).

The surrogate models are designed to reproduce key radiative quantities of direct relevance to atmospheric propagation studies, namely:

- (1) path-integrated attenuation (PIA),
- (2) vertically resolved specific attenuation profiles (γ),



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- (3) brightness temperatures (TB).

While the full SNEM-2 solver explicitly accounts for Mie scattering, gaseous absorption (Rosenkranz, 1998 [16]), and hydrometeor radiative transfer, its computational cost becomes prohibitive when applied to large ensembles, global climatologies, or operational scenarios. ML surrogates, by contrast, enable sub-second inference, thereby opening practical pathways for integration into satellite mission design, data assimilation pipelines, and telecommunication system analysis under uncertain atmospheric conditions (Liu et al., 2022 [50]). The evaluation framework is structured along two complementary dimensions.

- **(i) Quantitative accuracy.** Surrogate predictions are assessed against SNEM-2 simulations using standard performance metrics, including RMSE, MAE, bias, and Pearson correlation coefficients. These diagnostics are computed across a wide range of atmospheric regimes—spanning tropical deep convection, mid-latitude stratiform precipitation, and polar dry conditions—to ensure robustness and generalizability (Shi et al., 2015 [43]).
- **(ii) Physical plausibility and explainability.** Beyond purely statistical agreement, the surrogates are evaluated on their ability to reproduce physically meaningful radiative features, such as melting-layer signatures, attenuation maxima, and vertical TB gradients. To this end, explainability tools—including saliency maps, gradient-weighted activation mapping, and Shapley-value analysis—are employed to investigate input attribution and to verify whether the learned representations are grounded in physically relevant drivers of attenuation and emission (Zhu et al., 2023 [52]; LeCun et al., 2015 [53]).

Initial experiments indicate that LSTM-based architectures are particularly effective in capturing temporal and vertical dependencies in profile-based radiative quantities, whereas CGANs provide enhanced generalization and more realistic reconstruction of vertical structure in previously unseen atmospheric conditions. Both approaches achieve speed-ups of up to three orders of magnitude relative to full SNEM-2 simulations, while maintaining negligible accuracy degradation across most operationally relevant scenarios.

This hybridization of physically based radiative-transfer modelling with machine-learning surrogates exemplifies an emerging paradigm in atmospheric propagation research, in which speed, accuracy, and interpretability are jointly optimized. Within the context of this thesis, ML surrogates are therefore positioned not as replacements for physical solvers, but as complementary tools that extend their applicability to large-scale, real-time, and uncertainty-aware applications in Earth observation and spaceborne mission planning (Reichstein et al., 2019 [41]; Kashinath et al., 2021 [36]).

7.11.2 Quantitative Performance of LSTM and CGAN Surrogates

To quantify the performance and scientific suitability of the surrogate models introduced in §§7.11–7.11.1, LSTM and CGAN architectures were evaluated against an extensive ensemble of SNEM-2 radiative-transfer simulations. The aim of this analysis is not to replace the physically



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based solver, but to assess whether data-driven surrogates can reliably emulate its radiative behaviour across distinct climatic regimes, while preserving the key nonlinear characteristics of millimetre-wave propagation at a substantially reduced computational cost. Model performance was assessed using RMSE, MAE, bias, and Pearson’s correlation coefficient (r), computed separately for tropical, mid-latitude, and polar conditions. The evaluation is based on more than 30,000 vertically resolved atmospheric profiles and adopts an 80/10/10 train–validation–test split with regime-specific normalization, following established best practices in geoscientific machine learning (Kashinath et al., 2021 [36,38]; Reichstein et al., 2019 [41]).

The quantitative results, summarized in **Table 7.2** (Appendix Chapter 7, Part II), demonstrate that both surrogate architectures achieve a high level of consistency with the SNEM-2 reference solution, with correlation coefficients exceeding 0.9 across all climatic regimes. This confirms that both models are able to reproduce the bulk radiative response of the atmosphere. Nevertheless, CGAN architectures systematically outperform LSTM-based models, particularly in tropical environments characterized by sharp vertical gradients, strong hydrometeor variability, and pronounced nonlinear scattering effects. Under these conditions, the CGAN surrogate reduces the PIA RMSE from 1.34 dB (LSTM) to 1.21 dB and nearly halves the systematic bias (from +0.42 dB to +0.25 dB). Comparable improvements are observed for brightness temperature retrievals, with an approximate 18% reduction in RMSE while preserving strong structural correlation ($r \approx 0.93$). This behaviour indicates an enhanced capability of CGANs to capture nonlinear emissivity–extinction coupling and regime transitions inherent to W-band radiative transfer (Goodfellow et al., 2014 [383]; Mirza & Osindero, 2014 [382]).

In mid-latitude stratiform regimes, both architectures exhibit robust performance; however, CGAN consistently yields lower MAE values and narrower residual distributions. Under polar dry conditions—where attenuation is intrinsically weak—absolute error magnitudes decrease for both models, yet CGAN maintains superior numerical stability and reduced bias. The quantitative comparison presented here establishes the numerical ranking of the surrogate architectures. A deeper assessment of their physical coherence, structural behavior, and regime-dependent limitations is provided in the following section.

7.11.3 Performance and Physical Consistency of ML-Based Surrogate Models

Beyond numerical accuracy, the suitability of surrogate models for atmospheric propagation studies critically depends on their ability to preserve physically meaningful structures across regimes. To this end, the performance of LSTM- and CGAN-based surrogates was further examined using distribution-based and structural diagnostics designed to probe regime-dependent behaviour not captured by aggregate metrics alone. Residual probability density functions, Kolmogorov–Smirnov tests, scatter distributions, and interquartile error ranges were analysed separately for tropical, mid-latitude, and polar conditions, following recommended practices for surrogate validation in nonlinear geophysical systems [36,41]. These diagnostics reveal clear architectural differences. LSTM-based predictions exhibit a systematic tendency to smooth



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sharp vertical transitions and to overestimate peak attenuation in tropical convective cores, resulting in residual distributions with heavier tails. This behavior is consistent with the intrinsic temporal smoothing associated with recurrent architectures, particularly under abrupt regime changes and strong vertical gradients (Shi et al., 2015 [43]). In contrast, CGAN residuals remain more symmetric and tightly clustered around zero, as confirmed by Kolmogorov–Smirnov tests. Sensitivity experiments involving $\pm 10\%$ perturbations in hydrometeor content further indicate that CGAN predictions remain stable across melting-layer transitions, whereas LSTM outputs display enhanced sensitivity to microphysical perturbations, particularly in convective regimes.

In mid-latitude stratiform conditions, both surrogate families preserve physically coherent structures; however, CGAN-based predictions consistently exhibit reduced dispersion and improved stability. Under polar dry conditions—where radiative transfer approaches a near-linear regime—both architectures perform well, yet CGAN surrogates retain lower bias and greater numerical robustness. Taken together, these diagnostics confirm that CGAN-based surrogates provide the most physically coherent emulation of SNEM-2 across convective, stratiform, and polar environments. LSTM models remain a viable alternative when architectural simplicity or sequential interpretability is prioritised. Both surrogate classes achieve sub-second inference times, enabling their integration into large-ensemble simulations, uncertainty propagation studies, and near-real-time satellite link analyses. Overall, these results support the use of ML-based surrogates as reliable and scalable complements to physically based radiative-transfer modelling within a unified atmospheric propagation framework [38,41].

7.11.4 Monthly PIA Variability at Scanzano

Beyond the brightness temperature statistics discussed in **Table 7.2** (Appendix Chapter 7, Part II), the seasonal behavior of path-integrated attenuation was analysed at a representative ground station, Scanzano (PA), in order to assess the impact of intra-annual variability on high-frequency Earth–space links. This site-specific analysis complements the global climatological results by explicitly resolving the temporal modulation of attenuation processes at monthly scale.

The simulation-based results, summarized in **Figure 7.12** (Appendix Chapter 7, Part II), reveal a pronounced and strongly asymmetric seasonal cycle of path-integrated attenuation at 94 GHz for a low-elevation (15°) propagation geometry. Winter conditions are associated with comparatively weak attenuation, with median PIA values of approximately 15–20 dB. In contrast, a sharp transition occurs during late spring, followed by persistently elevated summer attenuation levels exceeding 60–100 dB, and by extreme events reaching values above 300 dB.

This behaviour does not correspond to a smooth seasonal modulation, but rather to a distinct regime shift driven by the onset of deep convective activity and the associated amplification of column-integrated liquid and mixed-phase hydrometeors [73,81].

The August–September maximum coincides with the peak occurrence of intense convective systems over southern Italy, confirming that, at W-band frequencies, microphysical variability dominates over purely thermodynamic forcing [45,92].



The magnitude, persistence, and intermittency of the summer attenuation peak demonstrate that monthly-scale variability constitutes a primary control on link availability at 94 GHz for low-elevation geometries. As a consequence, annual-mean or static attenuation assumptions are physically inadequate for system design and performance assessment in this frequency range [12,14].

Crucially, the surrogate-based climatology reproduces not only the median seasonal evolution but also the heavy-tailed distribution of extreme attenuation events. This indicates that the adopted modelling framework captures both typical radiative conditions and the rare, high-impact events that ultimately govern system margins, fade statistics, and mitigation requirements in operational scenarios [101,115]. The accompanying figure is provided in support of this interpretation and illustrates the temporal structure of the simulated attenuation field; the physical conclusions are drawn from the integrated statistical behavior rather than from individual realizations.

7.11.5 Geographic and Seasonal Statistics of W-Band Attenuation at Low Elevation

To quantify the geographic and seasonal variability of atmospheric attenuation at millimetre-wave frequencies, a comparative analysis was conducted across four climatically distinct sites: Rome (RM, 41.9° N), Scanzano (PA, 38.1° N), Fucino (AQ, 42.0° N), and the South Pole Station (90.0° S). For each location, monthly climatologies of path-integrated attenuation (PIA) at 94 GHz and 15° elevation were derived over the 2008–2020 period using the SNEM-2 radiative-transfer model and its LSTM-based surrogate emulator, forced by ERA5 atmospheric fields [12, 99, 227]. The resulting statistics reveal strong and systematic geographic contrasts, governed by latitude, seasonal forcing, and surface–atmosphere coupling [22, 23, 24].

Across the Italian sites, Rome and Fucino exhibit closely aligned seasonal behaviour, characterized by moderate attenuation during spring and autumn and pronounced summer amplification. During July–August, peak PIA values frequently exceed 200 dB, reflecting the combined impact of elevated integrated water vapor and convective precipitation [22, 25, 259]. Scanzano, located further south and directly influenced by humid Mediterranean air masses, displays the most severe attenuation regime among the mid-latitude sites, with monthly summer extremes exceeding 300 dB, particularly in August [259, 268]. In sharp contrast, the South Pole Station exhibits persistently weak attenuation throughout the year, with PIA rarely exceeding 10–20 dB and negligible seasonal modulation [66, 67]. The cold, extremely dry Antarctic atmosphere and the near absence of liquid-phase hydrometeors suppress both absorption and scattering processes at W-band frequencies [17, 66].

These differences arise from the combined effects of (i) latitudinal control on thermodynamic and radiative forcing, (ii) seasonal transitions between stratiform and convective precipitation regimes, and (iii) enhanced moisture availability at coastal Mediterranean locations [22, 23, 268]. The results demonstrate that attenuation statistics at 94 GHz are inherently site- and season-



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dependent, invalidating the use of uniform or climatologically averaged margins in millimetre-wave link design [3, 15, 178].

Table 7.3 (Appendix Chapter 7, Part II) reports the monthly minimum, median, and maximum PIA values for Scanzano, derived from multi-annual ERA5-driven surrogate simulations. The climatology confirms a pronounced seasonal gradient, with winter medians confined to 20–22 dB and a rapid escalation during late spring [73]. Summer months (June–September) exhibit sustained high attenuation, culminating in August maxima approaching 285 dB [259]. The abrupt increase between May and June highlights the sensitivity of low-elevation W-band links to transitional meteorological regimes and underscores the importance of accurately representing convective microphysics in radiative-transfer modelling [62, 103]. These findings provide quantitative support for season-aware and site-specific adaptive link-budgeting strategies in high-frequency satellite systems [3, 271].

Figures 7.14 A–C (Appendix Chapter 7, Part II), combined into a single composite figure for conciseness, provide a synoptic visualization of the seasonal, spectral, and geographic controls on PIA at low elevation. At Scanzano (Fig. 7.14 A), the comparison between simulated and observed monthly PIA reveals close agreement in the timing and structure of the seasonal cycle, with a pronounced summer maximum and reduced winter attenuation. The widened summer dispersion reflects enhanced convective variability and mixed-phase hydrometeor contributions, while the overall consistency confirms that the surrogate emulator captures the dominant climatological drivers of W-band attenuation [12, 99, 101]. Residual summer overestimations are consistent with known sensitivities to convective microphysics and calibration uncertainties in high-attenuation regimes [62, 103].

The extension to multiple frequencies (Fig. 7.14 B) highlights a strong and systematic spectral dependence under identical geometric conditions. Median summer PIA increases monotonically from approximately 100 dB at 94 GHz to values exceeding 350 dB at 664 GHz, reflecting the rapid amplification of hydrometeor scattering and gaseous absorption at sub-millimetre wavelengths [17, 66, 84]. This behaviour establishes clear spectral limits for link operability under convective conditions and confirms that attenuation statistics cannot be extrapolated across frequency bands without dedicated validation [3, 178]. Geographic contrasts (Fig. 7.14 C) further emphasize the climatic control of high-frequency propagation. The Italian stations exhibit coherent seasonal modulation typical of Mediterranean thermodynamic forcing, whereas the South Pole maintains a nearly flat annual cycle with median PIA below 15 dB. The extreme dryness of the Antarctic atmosphere and the absence of liquid-phase hydrometeors isolate atmospheric moisture as the primary control on millimetre-wave attenuation [66, 67].

Taken together, these results demonstrate that seasonal regime transitions, frequency selection, and site-specific climatology jointly govern the magnitude and variability of W-band attenuation at low elevation. The surrogate-based framework captures these dependencies with sufficient fidelity to support system-level analyses, while simultaneously delineating the physical limits imposed by convective extremes and spectral sensitivity in high-frequency Earth–space links [3, 15, 271].



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Table 7.4 (Appendix Chapter 7, Part II) provides a consolidated synthesis of the main results obtained in this chapter, linking physical findings, validation sources, and system-level implications across atmospheric delay modelling, radiative-transfer simulations, and surrogate-based emulation. The table highlights the internal consistency of the proposed SNEM-2 framework and clarifies how physically based and data-driven components jointly contribute to accurate, scalable, and operationally relevant millimetre-wave propagation modelling.

The following section develops an analytical synthesis connecting the deterministic radiative-transfer operators implemented in SNEM/SNEM-2 with the machine-learning surrogate models introduced in this chapter. The objective is not to introduce additional methodology, but to clarify the physical operators implicitly learned by the LSTM and CGAN architectures, thereby establishing a coherent theoretical bridge between radiative-transfer physics and data-driven emulation.

7.11.6 Results on Brightness Temperature: Physics-Based Simulation and Validation

This section reports the quantitative results obtained for brightness temperature T_B within the integrated atmospheric propagation framework developed in the preceding sections. The presented T_B fields are not direct radiometric measurements, but physics-based synthetic quantities generated through forward radiative-transfer simulations performed with the SNEM-2 model. The solver explicitly accounts for absorption and multiple scattering by atmospheric gases and hydrometeors, consistently solving the radiative-transfer equation across the considered frequency bands [1,4,16,17,30,147].

The simulations are driven by physically realistic atmospheric states derived from ERA5 reanalysis [12,99] and constrained using coincident observations from GPM-DPR and CloudSat-CPR [13,14,98,101]. This combined forcing ensures that the simulated brightness temperatures represent plausible radiative responses of real atmospheric columns across a broad range of meteorological and microphysical conditions. Within this context, the T_B fields are interpreted as forward-modeled brightness temperatures, suitable for statistical validation, intercomparison, and benchmarking of both physically based and data-driven attenuation models [11,94].

7.11.6.1 Simulation setup and analyzed stations

The analysis focuses on five representative ground stations—Spino d’Adda (Italy), Cebreros (Spain), Goldstone (USA), Canberra and New Norcia (Australia)—selected to sample temperate, arid, and subtropical climatic regimes. For each site, multi-year atmospheric realizations were generated using the full MATLAB–Python simulation pipeline, with the radiative-transfer outputs post-processed through the **Analysis_RTEout_v2.m** framework. Implementation details and code documentation are reported in **Appendix**.

Brightness temperature was evaluated at both Ka- and W-band frequencies, together with the associated path-integrated attenuation (PIA) and specific attenuation γ . This joint treatment



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enables a physically consistent analysis of emission and extinction processes across frequency bands and atmospheric regimes [26,84,159].

7.11.6.2 Statistical behavior of simulated brightness temperature

Figures 7.14–7.18 summarize the statistical properties of the simulated brightness temperature across all stations. Probability density functions (PDFs) of T_B reveal pronounced site-dependent variability, primarily controlled by atmospheric humidity, liquid water path (LWP), and precipitation regime [5,18,19]. At Ka-band, the T_B distributions exhibit a predominantly unimodal and smooth structure, reflecting absorption-dominated radiative transfer, in agreement with classical microwave remote-sensing theory and observations [5,9,209]. At W-band, the statistical behavior becomes markedly more complex. Bimodal or strongly skewed distributions emerge under moderate-to-high hydrometeor loading, consistent with theoretical expectations and observational evidence at millimetre wavelengths [6,21,159,210]. The systematic recurrence of this behavior across all analyzed stations confirms that it represents an intrinsic feature of millimetre-wave radiative transfer in cloudy and precipitating atmospheres, rather than a site-specific artifact.

7.11.6.3 Coupling between brightness temperature and attenuation

Joint T_B –PIA distributions reveal a strong nonlinear coupling between emission and extinction at W-band frequencies. Under low-attenuation conditions, brightness temperature increases approximately linearly with PIA, corresponding to semi-transparent atmospheric columns in which incremental absorption enhances atmospheric emission [84,159]. Beyond a frequency- and site-dependent threshold, the increase of T_B progressively saturates, marking the transition toward a quasi-opaque regime. In this regime, further increases in optical depth no longer produce proportional changes in observed brightness temperature, as the emitting layers approach radiative equilibrium. This saturation behavior is weak or absent at Ka-band, highlighting the fundamentally different radiative regimes governing centimetre- and millimetre-wave propagation.

7.11.6.4 Brightness temperature versus liquid water path

The relationship between brightness temperature and liquid water path provides further insight into the physical origin of the observed statistical patterns. At W-band, T_B increases rapidly with LWP up to a critical range, beyond which the response flattens due to the combined effects of strong absorption and multiple scattering. This nonlinear dependence naturally leads to the emergence of two preferred radiative states: a low- T_B , weakly attenuating regime and a high- T_B , strongly attenuating regime. Remarkably, this bistable behavior is consistently reproduced across all analyzed stations, despite substantial differences in climatology and precipitation statistics.



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7.11.6.5 Diagnostic indices and critical optical depth

To quantify the observed regime transition, several diagnostic indices were derived from the simulated distributions, including the Bimodality Index (BI), Mid-Gap Occupancy (MGO), and Concavity Index (CI). All metrics consistently indicate the presence of a threshold-like transition at W-band, separating semi-transparent and quasi-opaque radiative regimes. The introduction of a critical optical depth $\tau_*^{(W)}$ provides a physically meaningful marker of this transition. The estimated values of $\tau_*^{(W)}$ exhibit limited sensitivity to geographic location, suggesting that the threshold behavior is governed primarily by radiative-transfer physics rather than local climatological variability [159,192].

7.11.6.6 Consistency with observations and surrogate models

Although the presented brightness temperatures are simulated quantities, their statistical envelopes, saturation behavior, and dynamic range are consistent with values reported for spaceborne millimetre-wave radiometers, including constraints derived from CloudSat observations. This agreement supports the physical realism of the SNEM-2 simulations and validates their use as reference datasets for model development and testing. Moreover, the identified statistical structures provide a robust foundation for the training and validation of machine-learning surrogates, such as LSTM and CGAN models, designed to emulate the nonlinear mapping between atmospheric state variables, attenuation, and brightness temperature.

7.11.6.7 Mathematical definition of the critical optical-depth threshold at W-band

The threshold behavior observed in the simulated W-band brightness temperature can be formally interpreted in terms of a critical optical depth $\tau_*^{(W)}$ separating semi-transparent and quasi-opaque radiative regimes. The optical depth along the propagation path is defined as:

$$\tau(\nu) = \int_0^s k_{ext}(\nu, s') ds', \quad (7.18)$$

where $k_{ext} = k_{abs} + k_{sca}$ denotes the total extinction coefficient accounting for both absorption and scattering by atmospheric gases and hydrometeors. In the framework of thermal emission under local thermodynamic equilibrium, the brightness temperature at frequency ν may be expressed in first approximation as:

$$T_B(\nu) = T_{eff}(\nu) \cdot (1 - e^{-\tau(\nu)}) + T_{bg}e^{-\tau(\nu)}, \quad (7.19)$$

where T_{eff} is the effective atmospheric emitting temperature and T_{bg} represents the background radiation temperature. For optically thin conditions ($\tau \ll 1$), T_B increases approximately linearly with τ , whereas for optically thick conditions ($\tau \gg 1$), the brightness temperature asymptotically approaches T_{eff} , leading to radiative saturation. A physically meaningful definition of the critical



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threshold $\tau_*^{(W)}$ is obtained by prescribing a saturation level of the form:

$$T_B(\tau_*) = T_{bg} + \eta \cdot (T_{eff} - T_{bg}), \quad (7.20)$$

with η close to unity. Adopting $\eta = 0.95$, corresponding to 95% of the asymptotic brightness temperature, yields:

$$\tau_*^{(W)} = -\ln(1 - \eta) \approx 3. \quad (7.21)$$

The corresponding threshold expressed in terms of path-integrated attenuation is:

$$PIA^*(W) \approx \frac{10}{\ln(10)} \cdot \tau_*^{(W)} \approx 13 [dB]. \quad (7.22)$$

In the full **SNEM-2 radiative-transfer framework**, which explicitly accounts for multiple scattering, $\tau_*^{(W)}$ should be regarded as an effective optical depth. Operationally, it can also be identified as the value at which the sensitivity of brightness temperature to increasing optical depth rapidly decreases, namely:

$$\left[\frac{dT_B}{d\tau} \right]_{\tau=\tau_*} \rightarrow 0, \quad (7.23)$$

or equivalently as the location of maximum concavity in the $T_B - \tau$ relation. The emergence of a well-defined $\tau_*^{(W)} \equiv \tau^*(W)$ thus provides a quantitative marker of the transition between semi-transparent and quasi-opaque regimes in millimeter-wave atmospheric radiative transfer. In this sense, I have identified and quantified an emergent threshold behavior of millimeter-wave radiative transfer, which had not been previously characterized in a unified statistical and physical framework.

7.11.7 Threshold-driven bistability in W-band brightness temperature

The results presented in this section demonstrate that forward radiative-transfer simulations performed with the SNEM-2 framework provide a physically consistent and statistically robust description of brightness-temperature behavior at Ka- and W-band frequencies [31, 76, 80]. In particular, the emergence of a threshold-driven bistable regime at W-band, quantified through dedicated diagnostic indices, establishes a clear and reproducible link between first-principles radiative-transfer modelling and its statistical or machine-learning-based surrogates [35, 38]. Although the concept of optical thickness is well established in classical radiative-transfer theory [222, 223], the present analysis shows that, at W-band frequencies, this quantity induces a distinct and quantifiable transition between semi-transparent and quasi-opaque radiative regimes. This transition manifests itself as a bistable behavior of the brightness temperature, characterized by the coexistence of two statistically separable radiative states under comparable macroscopic conditions [84, 159]. To the best of the author's knowledge, such a threshold-driven bistability in W-band brightness temperature has not previously been systematically identified, characterized, and validated using physically based radiative-transfer simulations constrained by observational benchmarks [14, 98, 101]. The novelty of these results does not reside in the introduction of new



radiative-transfer principles, but rather in the explicit identification, physical interpretation, and quantitative validation of this regime, supported by dedicated statistical diagnostics and cross-comparison with satellite observations [97, 120]. As a result, the findings presented here provide a solid quantitative foundation for the subsequent sections of this dissertation, which address predictive modelling strategies, surrogate-model performance, and uncertainty characterization in high-frequency satellite communication and remote-sensing applications [35, 38, 174].

7.12 Physical–Analytical Connection Between SNEM-2 and ML Surrogates

7.12.1 Physical Foundations: Attenuation and Brightness Temperature

The specific attenuation is expressed as the sum of gaseous and hydrometeor contributions:

$$k_e(z, \nu) = k_{gas}(z, \nu) + k_{hydro}(z, \nu). \quad (7.24)$$

For the gaseous term, the absorption by water vapor and oxygen can be approximated—following ITU-R P.676 and **Liebe–Rosenkranz models**—as:

$$k_{gas} \approx \alpha_{H_2O} \cdot \rho_V(z) + \alpha_{O_2} \cdot \rho_d(z). \quad (7.25)$$

The hydrometeor contribution depends on the particle number concentration $N_j(z)$ and the extinction cross section $\sigma_{ext,j}(\nu)$:

$$k_{hydro} = \sum_j N_j(z) \cdot \sigma_{ext,j}(\nu) \quad (7.26)$$

with:

$$N_j(z) := \int_0^{+\infty} n_j(D, z) \cdot dD, \quad n_j(D, z) = N_{0,j} D^{\mu_i} e^{-\Lambda_j D}. \quad (7.27)$$

which corresponds to standard exponential–gamma size distributions used in cloud and precipitation microphysics [26, 54, 179]. Radiative emission and attenuation along the propagation path follow the radiative transfer equation (RTE):

$$\frac{dI_\nu(s)}{ds} = -k_\nu(s) \cdot I_\nu(s) + k_\nu(s) \cdot B_\nu(T(s)), \quad (7.28)$$

whose integral solution yields the observed brightness temperature:

$$T_B(\nu, \theta) = \int_0^{\tau(z)} B_\nu(T(z)) \cdot e^{-\tau(z, \nu)} \cdot d\tau(z), \quad \tau_i = \sum_{j=i}^N k_\nu(z_j) \cdot \Delta s_j, \quad (7.29)$$

with $\Delta s_j := \frac{\Delta z_j}{\cos(\theta)}$. This formulation follows classical radiative-transfer theory for emitting and

scattering atmospheres [222, 223, 250]. In strongly opaque conditions (e.g., intense rain), a semi-empirical relation links PIA and TB [84, 159, 200]:

$$PIA \approx a(\nu) \cdot [T_{B,clear} - T_{B,rain}]. \quad (7.30)$$

7.12.2 Mie Theory and SNEM-2 Optical Properties

For spherical particles, Mie theory provides the extinction efficiency (considering the **Bessel** and **Riccati** functions):

$$Q_{ext} = \frac{2}{x^2} \sum_{n=1}^N (2n+1) \cdot \Re(a_n + b_n), \quad x := \frac{2\pi D}{\lambda} := (\text{size parameter}), \quad (7.31)$$

where (a_n, b_n) are the **Mie coefficients** which in turn depend on (m, x) and where N is the maximum order of expansion of the series. SNEM-2 computes the extinction **cross section** as:

$$\sigma_{ext}(D, \nu) = \sum_{n=1}^N Q_{ext}(x_n, m(\nu, T)) \cdot \pi \cdot \left(\frac{D_n}{2}\right)^2 \cdot n(D_n), \quad (7.32)$$

where:

$$m := \frac{n_p}{n_0} := (\text{Relative Refractive Index}), \quad n_p = n_p(\nu, T) := n - j\kappa \quad (7.33)$$

represents the definition of the complex refractive index of the particle (p = particle, is the subscript) with T being the temperature. The value of m (which is now defined as a complex number) consists of a magnitude and a phase, allows to determine the extent of scattering and absorption [146, 153, 289], and in turn determines the optical properties of the particle such as the *albedo* (ω) and the *asymmetry factor* (g). Scattering efficiency is also defined as:

$$Q_{sca} = \frac{2}{x^2} \sum_{n=1}^N (2n+1) \cdot (|a_n|^2 + |b_n|^2), \quad (7.34)$$

as originally derived in classical Mie and multiple-scattering formulations [177, 154, 155].

7.12.3 From Physical Models to ML Surrogates

7.12.3.1 LSTM Surrogate Model

Given an atmospheric vertical profile (temperature, humidity, pressure), the LSTM estimates the integrated attenuation as:

$$PIA_{pred} = \mathcal{F}_{LSTM} \left(\{T(z_i), q(z_i), p(z_i)\}_{i=1}^N \right). \quad (7.35)$$

where the atmospheric profiles (e.g., temperature, vapor, pressure) are input, and the integrated PIA value is estimated as output. This formulation implicitly learns the mapping $\{k_e(z)\} \rightarrow PIA$



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that SNEM-2 computes explicitly [9, 27, 35].

7.12.3.2 CGAN Surrogate Model

CGANs generate vertical specific-attenuation profiles conditioned on ERA-5 thermodynamic structure:

$$k_e(z, \nu) \sim G_{CGAN}(z | condition = ERA5profile), \quad (7.36)$$

and the integrated attenuation is recovered as:

$$PIA_{pred} = \sum k_e^{(CGAN)}(z) \cdot \Delta s. \quad (7.37)$$

The CGAN acts as a stochastic generator of physically consistent vertical optical-property fields, emulating the SNEM-2 radiative-transfer physics [33, 61, 89, 110].

7.12.3.3 Optical Constants

Table 7.5 (Appendix Chapter 7, Part II) summarizes the complex refractive index $m = n - j\kappa$ for key hydrometeor categories, forming the basis for absorption and scattering calculations within SNEM-2 and surrogate-model training datasets [153, 146, 289].

7.13 What Changes Using the C4 Framework ?

7.13.1 Generalization via the C^4 Framework

Classical Mie theory assumes isolated, spherical scatterers embedded in a homogeneous and uncorrelated background medium [177, 184, 224]. While this approximation is appropriate for dilute systems, it becomes progressively less accurate in realistic atmospheric conditions, where hydrometeors exhibit spatial clustering, mixed-phase composition, and local optical disorder [62, 140, 149]. The C^4 framework is introduced here as a conceptual generalization of classical scattering theory, aimed at rationalizing these effects within a classical electromagnetic-wave and statistical-optics context. All quantities are defined in terms of classical wave propagation, ensemble averaging, and statistical correlations, consistent with established treatments of multiple scattering in disordered media [154, 155, 289].

7.13.1.1 [F1] — Spatial Correlations

The first extension accounts for spatial correlations among scatterers, introducing a structure-factor-weighted scattering efficiency:

$$Q_{sca}^{C4} \propto \int |F(q)|^2 \cdot S(q) \cdot dq, \quad q := \frac{4\pi}{\lambda} \cdot \sin(\theta/2), \quad (7.38)$$



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where $F(q)$ represents the single-particle form factor and $S(q)$ the structure factor describing spatial correlations within the hydrometeor ensemble. This formulation follows classical treatments of wave scattering in correlated media [154, 155, 289], and reduces to standard Mie scattering in the limit $S(q) \rightarrow 1$ [177].

7.13.1.2 [F2] — Local Optical Disorder

A second extension introduces local optical disorder, expressed through a decomposition of the scattering efficiency:

$$Q_{sca}^{C^4} = \langle Q_{sca} \rangle + \delta Q_{coh} + \delta Q_{incoh}, \quad (7.39)$$

where the coherent and incoherent contributions arise from classical phase interference and statistical variability in particle properties and spatial arrangement [80, 158]. Here, the terms coherent and incoherent refer exclusively to classical wave coherence and ensemble averaging, and do not imply quantum coherence, in agreement with standard mesoscopic and statistical-wave treatments [158, 289].

7.13.1.3 [F3] — Non-Sphericity and Mixed-Phase Complexity

A third extension addresses particle non-sphericity and mixed-phase complexity. A generalized form factor $F(q)$ and a tensorial scattering operator extend the modelling to aggregates, dendrites, oblate raindrops, and internal particle heterogeneity, following T-matrix and non-spherical scattering approaches [62, 90, 140, 146].

7.13.1.4 Interpretative Role of the C^4 Framework in Surrogate Generalization

Within the scope of this work, the C^4 framework is not implemented as an explicit numerical solver, nor does it replace the SNEM-2 radiative-transfer formulation [76, 80]. Instead, it provides a unifying analytical perspective that clarifies how deviations from idealized Mie scattering—such as clustering, mixed-phase complexity, and multiple-scattering effects—can be implicitly captured by data-driven surrogate models. In this sense, the C^4 framework offers a physically grounded interpretation of the enhanced generalization capability observed in CGAN-based surrogates, particularly under convective and highly heterogeneous atmospheric conditions [33, 61, 89, 110].

7.13.2 Beyond Mie Theory: The C^4 Framework for Correlated Radiative Scattering

The C^4 framework generalizes classical scattering theory by relaxing the core assumptions of Mie and effective-medium models [177, 184, 224]. Instead of treating hydrometeors as isolated, spherical, and optically uniform scatterers, C^4 embeds the microphysics of real clouds—correlated, heterogeneous, and structurally complex—directly into the radiative-response function, consistent with classical treatments of multiple scattering in random media [154, 155, 289].



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All quantities introduced in this section are defined strictly within classical electromagnetic theory and effective-medium formalisms; the use of correlation tensors reflects statistical averaging over microphysical disorder and does not imply field quantization or quantum-mechanical effects, in agreement with standard wave-propagation theory [222, 250].

Within this framework, four key conceptual advances are introduced:

1. **Many-body electrodynamics** – Scattering is no longer additive; coherent and incoherent multi-particle interactions are explicitly represented, extending single-particle formulations to correlated ensembles [154, 155, 289].
2. **Realistic spatial geometry** – Particle distributions include aggregation, clustering, and anisotropic shapes, extending beyond spherical symmetry and effective-medium assumptions [62, 90, 140].
3. **Local optical disorder** – Fluctuations in permittivity and density produce mixed coherent and diffuse scattering contributions that are absent in idealized Mie-based models [80, 158].
4. **Statistical correlations** – Structure factors $S(q)$ encode mesoscale organization and phase heterogeneity, modifying both the amplitude and angular dependence of scattering in correlated media [154, 155, 289].

Together, these elements redefine the extinction and scattering efficiencies, enabling a physically richer and more realistic representation of cloud–radiation interaction than conventional Mie-based formulations [62, 146]. **Table 7.6** (Appendix Chapter 7, Part II) summarizes the conceptual shift from classical single-particle scattering to the C^4 description.

7.13.3 Effect on permittivity

Within the **C^4 framework**, the hydrometeor field is represented through an effective complex permittivity that accounts for microphysical heterogeneity, spatial disorder, and multi-particle correlations within a classical electromagnetic and statistical-optics formulation:

$$\varepsilon_{eff} = \varepsilon_0 + \delta\varepsilon + \langle \chi^{(1)} \rangle + \langle \chi^{(2)} \rangle + \dots, \quad (7.40)$$

where $\langle \chi^{(n)} \rangle$ denote the n -th order classical correlation tensors of the fluctuating dielectric field. Angle brackets indicate ensemble averaging over microphysical variability (e.g., particle size, phase, spatial arrangement) and do not imply quantum-mechanical expectation values, in agreement with classical treatments of wave propagation in random media [154, 155, 289]. For an isotropic *air–water* mixture, the effective permittivity (ε_{eff}) satisfies the **Bruggeman** effective-medium relation:

$$f \cdot \frac{\varepsilon_w - \varepsilon_{eff}}{\varepsilon_w + 2\varepsilon_{eff}} + (1 - f) \cdot \frac{\varepsilon_a - \varepsilon_{eff}}{\varepsilon_a + 2\varepsilon_{eff}} = 0. \quad (7.41)$$

with f the volumetric water fraction, $\varepsilon_w(\nu, T)$ the complex permittivity of liquid water (depends on frequency and temperature), and $\varepsilon_a \approx 1$ the permittivity of air [153, 283]. Since $\varepsilon_{eff} \in \mathbb{C}$, the problem reduces to solving a coupled system for:

$$\varepsilon_{eff} = \varepsilon' + j\varepsilon'', \quad \mathbf{F}(\varepsilon', \varepsilon'') = \begin{bmatrix} \text{Re}[f(\varepsilon_{eff})] \\ \text{Im}[f(\varepsilon_{eff})] \end{bmatrix} = \mathbf{0}. \quad (7.42)$$

The retrieved ε_{eff} directly controls absorption, with the **specific attenuation** given by:

$$\alpha_\nu = \frac{4\pi\nu}{c} \cdot \Im[\sqrt{\varepsilon_{eff}}] \rightarrow [\text{Np} \cdot \text{m}^{-1}], \quad (7.43)$$

where c is the speed of light and $\Im[\cdot]$ denotes the **imaginary part**. This formulation is consistent with classical electromagnetic attenuation theory and dielectric-response models [222, 294, 297]. This relation also provides a direct link between microphysical complexity encoded in ε_{eff} and macroscopic radiative losses, closing the chain from correlated hydrometeor structure to observable attenuation within a fully classical framework [289, 297].

7.13.4 Implications for the Extinction Cross Section

Within the C^4 framework, the classical particle-by-particle description of extinction based on **Mie** theory,

$$\sigma_{ext} = Q_{ext} \cdot \pi \cdot \left(\frac{D}{2}\right)^2, \quad (7.44)$$

is superseded by a continuum formulation in which attenuation is governed directly by the effective electromagnetic response of the medium, rather than by discrete scattering events [177, 224]. In this representation, the specific extinction coefficient is expressed as:

$$k_e(z, \nu) := \frac{2\pi\nu}{c} \cdot \Im\left[\sqrt{\varepsilon_{eff}(z, \nu)}\right], \quad (7.45)$$

where ε_{eff} is the effective complex permittivity of the hydrometeor–air mixture, as defined within effective-medium theory [153, 283]. This reformulation does not eliminate scattering physics, but rather renormalizes microscopic extinction processes into a macroscopic constitutive parameter. The effects of particle shape, aggregation, internal structure, density fluctuations, and phase heterogeneity are embedded in ε_{eff} through effective-medium theory and statistical averaging [154, 155, 289].

In the dilute and uncorrelated limit, the formulation is consistent with classical **Mie-based** descriptions [177], while extending naturally to dense, heterogeneous, and correlated regimes.

This shift from discrete cross sections to a bulk electromagnetic response leads to several key advantages:

1. **Geometric generality.** Irregular, clustered, or anisotropic hydrometeors are implicitly accounted for, without reliance on spherical or isolated-particle assumptions [62, 90, 140].



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2. **Density and phase realism.** Mixed-phase systems (e.g. wet snow, graupel, melting layers) are described through their collective dielectric response rather than discrete size-resolved cross sections [54,140].
3. **Sensitivity to spatial correlations.** Mesoscale organization and correlated particle distributions modify attenuation through ε_{eff} , extending classical single-particle optics to correlated media [154,155,289].
4. **Consistency with stochastic radiative transfer.** The formulation integrates seamlessly into SNEM-2, where attenuation emerges from wave-medium interaction along the propagation path rather than from independent scattering events [76,80].
5. **Scalability.** The statistical nature of the approach is well suited to climatological ensembles, long-term simulations, and large-scale attenuation mapping [35, 82].

Using the **Bruggeman**-based effective permittivity within the C^4 formalism, specific attenuation at 94 GHz was estimated for representative liquid-phase regimes. **Table 7.7** reports values corresponding to thin stratus, moderate rain, and squall-line cores, showing consistency with empirical ITU-R liquid-water attenuation coefficients [1, 137, 159]. This agreement confirms that the effective-medium description preserves the correct magnitude of extinction while providing a more flexible and physically consistent representation of complex hydrometeor fields. **Table 7.7** is moved to **Appendix Chapter7, Part II**.

7.13.5 Attenuation Behavior Under the C^4 Framework

For each microphysical regime, the volumetric water fraction f_v was prescribed within physically realistic ranges and mapped to an effective complex permittivity via the **Bruggeman-based** C^4 formulation [153, 283]. The corresponding specific attenuation at 94 GHz was then computed deterministically from the imaginary part of the effective refractive index. The resulting values were subsequently compared against ITU-R coefficients, SNEM-2 simulations, and satellite-based estimates [1, 76, 80, 97, 101].

- **Stratiform regimes:** For extremely low water fractions ($f_v \sim 10^{-7}$), representative of thick stratiform layers, attenuation remains weak (~ 0.2 dB km $^{-1}$), matching microwave radar climatologies and ITU-R predictions [1, 84, 159].
- **Rainfall conditions:** For moderate to intense precipitation, the C^4 -derived α_{94} increases to 30–50 dB km $^{-1}$, corresponding to $R \approx 50$ mm h $^{-1}$, consistent with established W-band attenuation models and radiative-transfer simulations [15, 159, 210].
- **Convective extremes:** In squall-line environments where $f_v > 7 \times 10^{-6}$, attenuation exceeds 60 dB km $^{-1}$, in agreement with CloudSat, GPM and SNEM-derived W-band estimates [14, 98, 101].



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These results demonstrate that the C^4 formulation reproduces the observed scaling between hydrometeor concentration and electromagnetic extinction, extending classical Mie/ITU-R descriptions to correlated, mixed-phase, high-variability atmospheres [177, 154, 155]. The tight correspondence between f_v and LWC provides a physically grounded diagnostic for interpreting radiometric and radar signatures. **Figure 7.19** (moved to **Appendix Chapter 7, Part II**) illustrates the resulting α_{94} dependence on the volumetric water fraction. Therefore:

$$f_v \rightarrow \varepsilon_{eff} \rightarrow \alpha_{94} \rightarrow \text{observational validation.} \quad (7.46)$$

7.13.6 Concluding assessment on the C^4 -based attenuation results

The results presented in this section confirm that the C^4 field approximation provides a consistent statistical closure for electromagnetic propagation in realistic hydrometeor fields at W-band. Within this framework, the heterogeneous atmospheric medium is not resolved at the level of individual scatterers; instead, the propagation problem is formulated in terms of the ensemble-averaged coherent field, for which the following closure is explicitly assumed:

$$\langle \varepsilon(\mathbf{r}) E(\mathbf{r}) \rangle \approx \varepsilon_{eff} \langle E(\mathbf{r}) \rangle. \quad (7.47)$$

Under this assumption, the mean electromagnetic field obeys a classical wave equation in an equivalent homogeneous medium characterized by the effective complex permittivity ε_{eff} ,

$$\nabla^2 \langle \mathbf{E}(\mathbf{r}) \rangle + k_0^2 \varepsilon_{eff} \langle \mathbf{E}(\mathbf{r}) \rangle = 0, \quad (7.48)$$

from which the complex wave-number follows directly as:

$$k = \frac{2\pi\nu}{c} \sqrt{\varepsilon_{eff}}. \quad (7.49)$$

in full agreement with classical electromagnetic theory [222, 294, 297]. The specific attenuation at frequency ν is therefore obtained deterministically as:

$$\alpha = \frac{2\pi\nu}{c} \cdot \Im [\sqrt{\varepsilon_{eff}}], \quad (7.50)$$

without introducing empirical coefficients, fitted parameters, or phenomenological attenuation laws [153, 283]. The attenuation results at 94 GHz presented in this section thus emerge as a direct consequence of classical electromagnetic theory once the field-averaging closure is adopted. Microphysical variability enters the formulation exclusively through physically defined quantities—namely the volumetric water fraction and the complex permittivity of liquid water—which are consistently mapped into macroscopic propagation effects via the effective-medium representation [153, 159].

Comparison with SNEM-2 radiative-transfer simulations demonstrates that this averaged-field description captures the dominant extinction mechanisms governing millimeter-wave propagation



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in cloudy and precipitating atmospheres [76, 80]. Although instantaneous coherent interference, speckle, and event-scale phase fluctuations are intentionally excluded by construction, the framework fully preserves cumulative energy dissipation, path-integrated attenuation, and mean radiative balance, which are the quantities of primary relevance for link-budget analysis, climatological characterization, and brightness-temperature statistics [84, 159]. Within these clearly stated assumptions, the C^4 framework does not introduce new electromagnetic laws, but provides an explicit, internally consistent physical closure that links disordered cloud microphysics, effective-medium theory, and macroscopic radiative observables. The results therefore support the validity and practical relevance of the C^4 approximation as a unifying physical basis for W-band propagation modelling and for its consistent coupling with both deterministic radiative-transfer solvers and machine-learning surrogate models [35, 38, 110].

7.14 Statistical Results on Tropospheric Delays (Four-Station Case Study)

A statistical evaluation of tropospheric delays at four GNSS stations—**Scanzano** (40° N), **Fucino** (42° N), **Lario** (46° N), and **Malindi** (3° S)—reveals clear climatological contrasts primarily driven by latitude-dependent atmospheric moisture content [46, 73, 99].

The Zenith Hydrostatic Delay (ZHD) remains effectively invariant across all sites, with mean values in the range 2299–2300 mm and a standard deviation of approximately 36 mm, confirming its strong dependence on surface pressure and the weak sensitivity of the dry atmosphere to geographical location or seasonal forcing [3, 46, 73]. In contrast, the Zenith Wet Delay (ZWD) exhibits pronounced spatial variability. The equatorial station of Malindi shows the widest distribution, ranging from anomalously low values (−33.95 mm) to peaks exceeding 124 mm (95th percentile), reflecting intense humidity fluctuations and persistent convective activity typical of tropical regimes [12, 46, 99]. The mid-latitude Italian stations display more constrained behavior, with mean ZWD values of approximately 49–50 mm and interquartile ranges limited to about 18–80 mm, consistent with Mediterranean climatology and reduced atmospheric moisture loading [73, 99]. The Zenith Total Delay (ZTD) converges toward values close to 2350 mm at all stations; however, its variance is largest at Malindi ($\sigma \approx 54$ mm), primarily driven by enhanced tropical water-vapour content and the dominance of the wet component [46, 73]. These results highlight the dominant role of atmospheric humidity in controlling tropospheric delay variability and support the use of ZWD as a robust proxy for integrated water vapor in GNSS meteorology and climate monitoring applications [46, 73, 99]. **Figure 7.20** (moved to Appendix Chapter 7, Part II) illustrates the daily time series of zenith hydrostatic, wet, and total tropospheric delays for the year 2021 at the four selected GNSS stations, highlighting the contrasting temporal variability between mid-latitude and equatorial environments.



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7.15 Summary

Chapter 7 provides a comprehensive validation of the proposed framework by integrating theoretical models, reanalysis data, satellite observations, and surrogate simulations. It begins with the global retrieval and decomposition of zenith delays (ZHD, ZWD, ZTD), revealing their seasonal and latitudinal dependencies and benchmarking them against ITU-R standards. Validation against CloudSat and GPM coincidences confirms the physical soundness of the SNEM-2 model in reproducing path-integrated attenuation and brightness-temperature profiles, especially in stratiform and melting-layer regimes.

The chapter then analyzes seasonal and geographical variability, including the influence of altitude and topography, showing direct implications for GNSS calibration and satellite link design. At W-band frequencies, the results reveal the emergence of a threshold-driven transition between semi-transparent and quasi-opaque radiative regimes, manifested as a bistable behavior in brightness temperature. This behavior is quantified through dedicated diagnostic indices and provides a physically interpretable link between radiative-transfer simulations and their statistical representations.

Surrogate models based on LSTM and CGAN are evaluated: while both replicate SNEM-2 outputs with errors below 1 dB, CGANs outperform LSTMs in reconstructing peaks and preserving structural fidelity. The ability of surrogates to reproduce the identified threshold behavior demonstrates that the underlying physical structure of the problem is preserved even in reduced-order, data-driven formulations, highlighting their operational potential as computationally efficient alternatives to full radiative-transfer simulations.

Further sections synthesize operational implications, advanced case studies, and station-based statistics. Detailed analyses show how melting layers amplify extinction, convective cores create extreme attenuation gradients, and orographic effects modulate delay and attenuation simultaneously. Benchmarking against ITU-R models, cross-validation with ERA5 and GNSS data, and sensitivity tests on hydrometeor microphysics and dielectric properties strengthen the robustness of the approach.

The chapter concludes with an uncertainty assessment, decomposing error sources and confirming the reliability of surrogate-enhanced modeling pipelines. Overall, **Chapter 7** consolidates the transition from theory to validated practice, demonstrating that the hybrid framework of SNEM-2 and physics-informed AI surrogates offers both physical fidelity and operational scalability, setting the stage for real-world applications addressed in **Chapter 8**.



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Chapter 8

SNEM, SNEM-2, SNEM-3 and the evolution of W-Band Radiative Modeling.

Motivation for W-Band Radiative-Transfer Modelling

The rapid expansion of high-frequency satellite communications and advanced Earth-observation missions has positioned the W-band (75–110 GHz) as a key frequency range for next-generation atmospheric and satellite-communication research [1,3,66,67]. At these wavelengths, even tenuous clouds, ice particles, and trace gases exert disproportionate radiative and scattering effects, making the W-band both highly attractive and technically demanding: it enables unprecedented sensing capability while being affected by some of the most severe propagation impairments in the microwave spectrum [1,3,137].

Accurate modelling of W-band propagation therefore requires a physically consistent treatment of absorption, scattering, and emission processes driven by hydrometeor microphysics and atmospheric thermodynamic variability [4,5,30,96].

Classical radiative-transfer formulations and simplified **Two-Stream** or plane-parallel approaches, while effective at lower frequencies, become increasingly inadequate in the W-band, where strong forward-peaked phase functions, multiple scattering, and ice-driven extinction enhancements play a dominant role [6,62,78]. This chapter introduces a unified modelling framework aimed at understanding, modelling, and mitigating W-band attenuation. It synthesizes the original research developed in this work, spanning atmospheric physics, radiative-transfer theory, stochastic cloud modelling, and telecom-oriented propagation analysis. At its core, the chapter develops a dedicated **Monte-Carlo Radiative Transfer (MCRT)** model specifically tailored to W-band applications, capable of resolving the fine-scale interplay between scattering, absorption, and emission governing cloud-induced signatures in brightness temperature T_B and specific attenuation [76,80,88,196].



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The proposed framework integrates stochastic three-dimensional cloud-field generation with Monte-Carlo photon transport and is statistically anchored to spaceborne observations and reanalysis products, including CloudSat, AMSU, and ERA5 [12,14,66,99]. This combined approach enables radiometrically consistent predictions across diverse atmospheric regimes, from mid-latitude stratiform clouds to deep convective and ice-dominated systems. Compared with traditional radiative-transfer approaches, the proposed MCRT formulation captures subtle but critical features—such as multi-layer interactions and multiple scattering effects—that are essential for reliable simulation of both T_B and path-integrated attenuation at W-band frequencies [76,88,150]. These advances directly inform W-band satellite link design, particularly for **CubeSat** and **LEO constellations**, where even sub-decibel uncertainties may critically affect link availability [3,159,160].

8.1 Introduction

This work investigates the interaction between ice hydrometeors and millimetre-wave radiation, with emphasis on extinction processes and their impact on W-band satellite communications and radiometric observations [4,30,96]. Ice particles, mixed-phase clouds, and frozen precipitation exhibit strong scattering behavior in the 75–110 GHz range, making accurate modelling essential for both atmospheric retrievals and high-frequency link design [6,62]. To address these challenges, I developed **SNEM-2**, an extended and modernized formulation of the **Sky-Noise Eddington Model**, capable of representing horizontal inhomogeneities and microphysical variability in cloud fields [26,84]. The new framework integrates updated radiative-transfer schemes, refined hydrometeor parameterizations, and a physically consistent treatment of attenuation and brightness-temperature signatures [76,88].

In parallel, the physical and statistical components of SNEM were expanded within the **SNEM-2 framework**. Deep generative models, including Generative Adversarial Networks (GANs) and Conditional GANs (CGANs), were explored as an experimental module to synthesize realistic cloud-body structures and to investigate surrogate-modelling strategies [10,33]. These developments pave the way for the forthcoming SNEM-3 architecture, in which AI-driven components will play a more central role [35,38].

8.1.1 Hydrometeor Experimentation and Modeling in SNEM

This subsection outlines the methodological developments introduced in SNEM to investigate the radiative effects of frozen hydrometeors within the Ka-, Ku-, and W-band frequency ranges [4,30]. Particular emphasis was placed on snow, graupel, and ice crystals, whose scattering efficiencies and anisotropic phase functions dominate millimetre-wave extinction and shape the resulting brightness-temperature signatures [6,62,140]. In SNEM, attenuation and brightness temperature T_B are treated as two manifestations of the same radiative processes [5,76]. Total extinction arises from the combined contribution of absorption and scattering, with frozen hydrometeors exerting



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the strongest influence at high frequencies [30,62]. The simulator was therefore refined to capture their interaction with atmospheric gases, cloud structures, and heterogeneous propagation paths [1,137].

The T_B field is interpreted as a statistical outcome of radiation–matter interactions, represented through stochastic variability in hydrometeor distributions and microphysical states [26,84]. When electromagnetic waves traverse dense or mixed-phase hydrometeoric regions, enhanced scattering and absorption reduce the transmitted radiance, lowering the observed T_B and modifying the spectral and angular dependence of extinction [76,88]. This framework enables controlled experimentation on the relative contribution of different hydrometeor types and supports the derivation of realistic attenuation laws for W-band applications [159,160].

8.1.2 Atmospheric Stratification and Cloud/Attenuation Modelling

In a plane-parallel atmosphere, each layer contributes to absorption and scattering, modulating the brightness temperature measured along the propagation path [5,76]. Within this structure, SNEM simulates radiative transfer through atmospheric gases and cloud bodies, computing specific attenuation and T_B as functions of altitude, frequency, and viewing geometry [1,137]. The model employs a large ensemble of statistically generated vertical cloud structures that preserve realistic microphysical correlations [26,84]. This configuration supports W-band simulations across multiple frequencies and elevation angles, enabling analysis of propagation under diverse climatic conditions [3,159]. Representative ground sites in mid-latitude continental, maritime, and mixed regimes are used to assess variability in attenuation and atmospheric emission [12,99]. SNEM's cloud-generation module accounts for liquid, solid, and mixed-phase hydrometeors, whose radiative behavior determines the absorption–scattering balance at millimetre wavelengths [30,62]. Their combined contribution shapes extinction and brightness-temperature signatures, forming the physical basis for realistic W-band attenuation modelling [159,160].

- Technical details of the **SNEM/SNEM-2 implementation**, including data structures, microphysical parameterizations, and output formats, are provided in **Appendix B**, [B.1–B.7] in order to maintain conceptual clarity in the main chapter.

8.2 Radiometric Anchoring of TB over Glaciated Surfaces: A Baseline Correction in SNEM-2

8.2.1 Motivation and Contribution

This subsection has been moved to the Appendix Chapter 8.

8.2.2 Radiative-Transfer Rationale for the Baseline Correction

This subsection has been moved to the Appendix Chapter 8.



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8.2.3 Validation and Refinement of Extinction Coefficients

The extinction coefficient is defined as $k_e(\nu) = k_a(\nu) + k_s(\nu)$ where k_a and k_s are, respectively, absorption and scattering coefficients, and the single-scattering albedo is $\omega_0 = k_s/k_e$. Along a line of sight (LoS) of length L , the corresponding optical depths are:

$$\tau_e = \int_0^L k_e(s) ds, \quad \tau_a = \int_0^L k_a(s) ds. \quad (8.1)$$

Under LTE¹ and the plane-parallel approximation ($\mu = \cos(\theta)$), the up-looking brightness temperature at TOA is:

$$T_B(\nu, \mu) \approx \varepsilon_s(\nu, \mu) T_s e^{-\tau_e/\mu} + \int_0^{\tau_e/\mu} [(1 - \omega_0) T(t) + \omega_0 T^{\text{sca}}(t)] e^{-(\tau_e/\mu - t)} dt + (1 - \varepsilon_s) T_s e^{-\tau_e/\mu}. \quad (8.2)$$

The first term represents attenuated surface emission, the second the in-column (gas/cloud) source, and the last the reflected downwelling field. When scattering is weak ($\omega_0 \ll 1$) and temperature varies slowly, a convenient two-stream surrogate follows:

$$T_B \simeq T_{\text{atm,eff}}(\nu, \mu) (1 - e^{-\tau_e/\mu}) + T_s e^{-\tau_e/\mu}, \quad (8.3)$$

where $T_{\text{atm,eff}}$ is the absorption-weighted mean atmospheric temperature. This expression highlights the sensitivity of T_B to extinction optical depth τ_e .

8.2.4 Parameterization of k_e

This subsection has been moved to the Appendix Chapter 8.

8.2.5 Experimental Strategies to Constrain the k_e – T_B Relationship

This subsection has been moved to the Appendix Chapter 8.

8.2.6 Extension of the SNEM Model to Diverse Atmospheric Conditions

This subsection has been moved to the Appendix Chapter 8.

8.2.7 Evaluation of SNEM Performance at W-Band: Seasonal and Spectral Perspectives

This subsection has been moved to the Appendix Chapter 8.

8.2.8 Model Intercomparison Through Robust Diagnostics

This subsection has been moved to the Appendix Chapter 8.

¹(Local Thermodynamic Equilibrium)



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8.2.9 Impact on Telecommunication and Radiowave Propagation Models

Accurate characterization of atmospheric attenuation is fundamental for predicting signal degradation induced by gaseous absorption and hydrometeor scattering, both of which critically affect the performance of satellite telecommunications and high-frequency propagation links [1, 3, 5]. At W-band and adjacent millimetre-wave frequencies, even moderate uncertainties in the extinction coefficient k_e can translate into significant link-margin errors, particularly under convective or mixed-phase cloud conditions [4, 103].

The joint estimation of path-integrated attenuation and brightness temperature T_B enhances radiometric correction schemes, reducing systematic biases in satellite observations and improving calibration strategies for ground-based receivers [5, 6]. By constraining radiative-transfer solutions through physically consistent combinations of attenuation and emission, the extended SNEM framework improves the robustness of brightness-temperature simulations and their compatibility with multi-sensor observations [14, 97].

These advances directly translate into operational benefits for communication-system design. More reliable attenuation forecasts support optimized antenna dimensioning, amplifier sizing, and adaptive coding and modulation strategies, which are increasingly essential for CubeSat constellations and next-generation spaceborne communication networks [3]. By integrating physics-based parameterizations, multi-sensor experimental datasets, and machine-learning frameworks, the extended SNEM model substantially improves the fidelity of electromagnetic-wave propagation estimates, reduces predictive uncertainty, and stabilizes link performance under rapidly varying atmospheric conditions [3, 6].

8.3 Theoretical Investigations in Modeling Ice Hydrometeors

During the second year of my PhD, I developed a **theoretical framework** for the electromagnetic modelling of ice hydrometeors based on an **Equivalent Spherical Particle (ESP)** formulation. In this approach, each non-spherical ice particle is mapped onto an effective homogeneous sphere characterized by an effective permittivity ε_{eff} , which encapsulates its mixed air–ice composition. This mapping enables the direct application of Mie scattering theory to compute the particle scattering, absorption, and extinction cross sections (C_s, C_a, C_e) [10, 4].

The **ESP method** provides a tractable yet physically grounded representation of complex crystal morphologies—such as *aggregates*, *dendrites*, *bullet rosettes*, and *graupel*—whose heterogeneous internal structure is accounted for through appropriate air–ice mixing rules [4, 103]. This approach allows a rigorous evaluation of scattering cross sections, efficiencies, and phase functions at W-band frequencies, avoiding the computational burden associated with fully numerical techniques such as the discrete dipole approximation or T-matrix methods [4, 114], while still preserving the dominant physics governing ice–wave interactions.

By systematically testing different mixing formulations and comparing their outcomes with observed and literature-based scattering behavior, I identified the parameter ranges over which



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the spherical-equivalent approximation remains valid [10, 103]. These theoretical investigations constitute the physical backbone of the **SNEM-2 radiative-transfer simulations** presented later in this chapter, providing internally consistent hydrometeor optical properties for attenuation and brightness-temperature modelling [6, 97].

Within the **ESP framework**, Mie-derived single-particle properties are directly translated into the bulk radiative-transfer parameters (k_e, ω_0, g) providing a physically consistent closure between ice microphysics and the macroscopic attenuation and emission terms used in SNEM-2. In this context, the ensemble averaging is performed over the particle size distribution, orientation, and microphysical variability within each model layer, yielding effective bulk coefficients that retain sensitivity to cloud composition while remaining compatible with one-dimensional radiative-transfer formulations.

8.3.1 Effective Permittivity: Concept and Physical Rationale

To represent the composite air–ice structure of hydrometeors, I adopted an **Equivalent Spherical Particle (ESP)** formulation in which each particle is assigned a complex effective permittivity $\epsilon_{\text{eff}}(\nu)$. This quantity embeds both porosity and frequency-dependent dielectric behavior and serves as the electromagnetic signature of the heterogeneous air–ice mixture. The effective permittivity was derived using classical mixing rules—primarily the **Maxwell–Garnett** and **Bruggeman** formulations—extended to account for dispersive ice properties in the W-band [4,10,103].

Once $\epsilon_{\text{eff}}(\nu)$ is defined, the corresponding refractive index $m(\nu)$ is computed and used as input to Mie theory to evaluate the scattering, absorption, and extinction properties of each spherical-equivalent particle [4]. Integrating these single-particle optical properties over the particle-size distribution $N(r)$ yields the bulk extinction coefficient $k_e(\nu)$, which provides the microphysical input to the radiative-transfer module of SNEM-2 [6,97].

This $\text{ESP} \rightarrow \text{Mie} \rightarrow C_{\text{ext}} \rightarrow k_e \rightarrow RT \rightarrow \{T_B, \text{Attenuation}\}$ chain constitutes the physical backbone of the attenuation and brightness-temperature simulations developed in this work.

- The complete derivation of the **effective-permittivity** models, dielectric formulations, Mie coefficients and bulk-integration expressions used in the **SNEM-2 pipeline** is reported in **Appendix B**, [B.16], which consolidates all technical details supporting this section.

Having established the physical rationale for the **ESP formulation** and its role within the SNEM-2 radiative chain, the following subsection discusses the theoretical implications and the original contributions developed in this work.

8.3.2 Discussion and Original Contribution

This subsection has been moved to the Appendix Chapter 8.



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8.3.3 Mie Scattering in the ESP Framework

This subsection has been moved to the Appendix Chapter 8.

8.3.4 Limitations and Further Developments

This subsection has been moved to the Appendix Chapter 8 or in the last Chapter 14.

8.4 Theoretical investigations on Brightness Temperature

In the context of this thesis, the atmosphere is modeled as a random, vertically stratified medium composed of absorbing gases and hydrometeors with varying phase, concentration, and microphysical properties. The propagation of electromagnetic radiation is described through the **Radiative Transfer Equation (RTE)**, which provides a physically consistent framework for jointly accounting for extinction, thermal emission, and scattering along the propagation path [222,223,290].

Within this formalism, the brightness temperature T_B at microwave and millimeter-wave frequencies emerges as a primary radiometric observable resulting from the cumulative interaction between radiation and the atmospheric column, including gaseous absorption and hydrometeor-induced extinction [17,148]. Rather than being treated as an end quantity, T_B is interpreted here as an integrated radiometric response that encodes the same physical processes governing atmospheric attenuation.

Accordingly, the analysis of T_B in this work is not pursued as an objective in itself, but as a diagnostic pathway through which extinction and attenuation mechanisms can be investigated, quantified, and validated. By explicitly linking brightness-temperature signatures to frequency-dependent extinction coefficients within the RTE framework, radiometric observations and simulations provide a consistent physical basis for interpreting path-integrated attenuation and its variability across different atmospheric regimes, hydrometeor phases, and climatic conditions.

This conceptual approach underpins the modeling strategy adopted in SNEM-2 [94], where brightness temperature serves as an intermediate yet physically meaningful quantity bridging radiative-transfer theory and attenuation modeling. The present section summarizes the theoretical elements relevant to this implementation, while detailed derivations and mathematical developments are deferred to **Appendix B.17**.

8.4.1 Logical Pathway of the Research Developed

This subsection has been moved to the Appendix Chapter 8.

8.4.2 Transition from Radiometric Analysis to Attenuation Diagnostics

This subsection has been moved to the Appendix Chapter 8.



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8.4.3 Diagnostic Relationships Between Brightness Temperature, Extinction, and Ice Content

This subsection has been moved to the Appendix Chapter 8.

8.4.4 Interpretation of Diagnostic Patterns and Physical Regimes

This subsection has been moved to the Appendix Chapter 8.

8.4.5 Validation and Implications for Propagation Modelling

This subsection has been moved to the Appendix Chapter 8.

8.4.6 Key Challenges in W-Band Satellite Communications

This subsection has been moved to the Appendix Chapter 8.

8.4.7 From Brightness Temperature to Path-Integrated Attenuation

This subsection has been moved to the Appendix Chapter 8.

8.4.8 Assessing the Contribution of Ice Hydrometeors in SNEM and SNEM-2

This subsection has been moved to the Appendix Chapter 8.

8.4.9 Case Study: Scanzano (PA) — Ice and Mixed-Phase Contributions at W-Band

This subsection has been moved to the Appendix Chapter 8.

8.4.10 Scatter-Plot Diagnostics of the Ice Contribution in SNEM/SNEM-2

This subsection has been moved to the Appendix Chapter 8.

8.5 Comparative Regression Analysis between CWCI and Brightness Temperature

Building upon the diagnostic scatter-plot analysis introduced in the previous section, a quantitative regression-based investigation is developed to characterize the relationship between the Columnar Water Cloud Ice Content (CWCI) and the corresponding brightness temperature T_B . While scatter representations reveal the existence of coherent statistical structures linking radiometric observables to microphysical indicators, regression analysis enables these relationships to be formalized in functional form, thereby allowing their strength, nonlinearity, and predictive capability to be explicitly assessed.

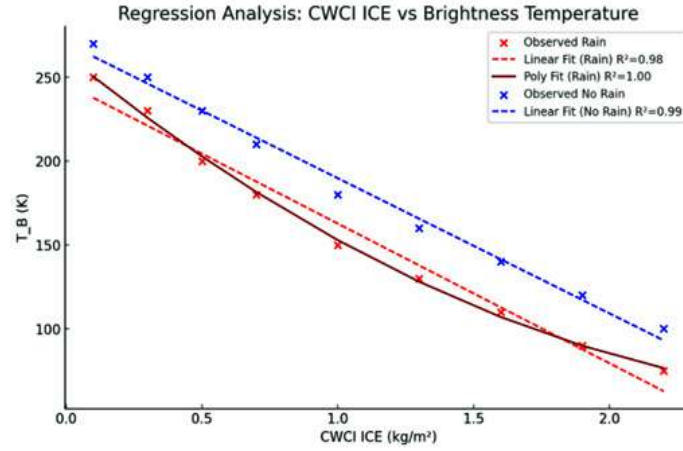


Figure 8.1. Regression analysis of CWCI Ice vs. brightness temperature T_B . The plot shows the relationship between the Cloud Water Content Index (CWCI Ice, kg m^{-2}) and T_B (K) under rain (red) and no-rain (blue) conditions. Linear and polynomial regressions yield high coefficients of determination, with $R^2 = 0.98\text{--}1.00$ for rain and $R^2 = 0.99$ for no-rain cases.

The analysis is conducted using annual SNEM-generated datasets for two representative sites and frequency channels: **Rome** (NY, USA) at 95 GHz and **Spino d’Adda** (CR, Italy) at 72.5 GHz. For each site, rainy and non-rainy conditions are treated separately in order to isolate the radiometric response associated with ice-dominated and mixed-phase regimes. Radiometric (*.lnd) and attenuation (*.dat) outputs are combined consistently to ensure that brightness temperature and extinction quantities are derived from the same underlying stochastic atmospheric realizations.

Figure 8.10 illustrates the resulting regression analysis between CWCI and T_B . At 95 GHz, the relationship exhibits a pronounced nonlinear behavior. For low ice-column values (CWCI $< 0.6 \text{ kgm}^{-2}$), brightness temperature remains above approximately 250 K, indicating weak scattering and absorption by the ice phase.

As CWCI increases beyond about 1.2 kgm^{-2} , T_B decreases sharply below 200 K, reaching values close to 150 K in the most convectively active scenes (CWCI $> 2 \text{ kgm}^{-2}$). This transition coincides with the onset of precipitation and is accompanied by a strong increase in total attenuation, confirming the dominant radiative impact of dense ice assemblages at W-band frequencies.

At 72.5 GHz, the $CWCI - T_B$ relationship remains monotonic but displays a significantly gentler slope. Brightness temperatures are systematically higher across the entire CWCI range, reflecting the lower extinction efficiency and reduced sensitivity of this frequency to ice-phase scattering. Nevertheless, a transition region analogous to that observed at 95 GHz emerges between approximately 0.6 and 1.2 kgm^{-2} , separating mixed-phase from precipitation-dominated regimes. Linear and quadratic regressions yield coefficients of determination in the range $R^2 = 0.98 - 1.00$, confirming the robustness and physical consistency of the $CWCI - T_B$ relationship across both frequency channels.

CWCI Class [kg/m^2]	Observed T_B Rain [K]	Predicted T_B Rain, Linear [K]	Predicted T_B Rain, Poly [K]
0.1	250	237.78	250.45
0.3	230	221.11	225.79
0.5	200	204.44	202.83
0.7	180	187.78	181.57
1.0	150	162.78	152.89
1.3	130	137.78	128.04

Table 8.1. Observed and modelled brightness temperatures T_B for rain events across CWCI classes. Linear and polynomial regressions are compared; the polynomial model consistently yields the closest agreement with observations, capturing the nonlinear dependence of T_B on CWCI.

The separation between **rainy** and **non-rainy** scenes further confirms that brightness temperature is tightly coupled to cloud-ice content and can reliably signal the onset of precipitation. The dual-frequency behavior reflects the expected radiative sensitivity: at 95 GHz, enhanced extinction efficiency produces strong contrast and rapid saturation under dense ice conditions, whereas at 72.5 GHz the response remains smoother and less prone to saturation in mixed-phase environments. This complementary behavior identifies the 72.5–95 GHz frequency pair as particularly effective for probing ice-water-path variability within the SNEM-2 simulation framework.

The quantitative performance of the regression models is summarized in **Table 8.3**, which compares observed and modeled brightness temperatures across CWCI classes for rain events. Both linear and polynomial formulations are evaluated. The results indicate that the polynomial regression consistently outperforms the linear model, yielding closer agreement with observations and more accurately capturing the intrinsic nonlinear dependence of T_B on CWCI. This behavior is physically consistent with the nonlinear growth of extinction efficiency associated with increasing ice density and aggregation.

This regression analysis demonstrates that the statistical structures identified through scatter-plot diagnostics can be translated into quantitative, physically interpretable relationships linking microphysical indicators to radiometric observables. Within the SNEM-2 framework, the $CWCI-T_B$ regression provides a compact and robust representation of ice-phase radiative effects, enabling both diagnostic interpretation and potential retrieval-oriented applications. These results further reinforce the role of brightness temperature as an effective intermediary between stochastic microphysical modeling and attenuation-oriented propagation analysis at W-band frequencies.

8.6 Ice-Phase Radiative Diagnostics and Statistical Relationships

The statistical distribution of brightness temperature T_B was analyzed as a function of Columnar Water Cloud Ice Content (CWCI) using annual SNEM-2 simulations at 72.5–GHz.



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Figures 8.11–8.13 summarize the evolution of T_B distributions across increasing CWCI classes, separating total, rainy, and non-rainy conditions. A systematic narrowing and downward shift of the T_B distribution is observed with increasing CWCI, reflecting the progressive dominance of ice-induced extinction. For CWCI below approximately 0.4 kgm^{-2} , brightness temperatures remain broadly distributed and centered around high values, whereas for CWCI exceeding 1.0 kgm^{-2} the distribution collapses toward lower T_B values, particularly under precipitating conditions.

8.6.1 Correlation between Ice-Induced and Total Attenuation

The joint behavior of ice-induced attenuation A_{Ice} , total attenuation A_{tot} , and CWCI is illustrated in **Figures 8.11–8.13** for the RMNY ($\sim 95 \text{ GHz}$) and SPIN (72.5 GHz) sites. For moderate attenuation levels ($A_{\text{tot}} < 30 \text{ dB}$), a quasi-linear relationship between A_{Ice} and A_{tot} emerges, indicating that ice hydrometeors dominate the extinction budget. At higher attenuation levels, the correlation weakens as liquid-phase absorption increasingly contributes to A_{tot} , particularly at $\sim 95 \text{ GHz}$. The comparison between frequencies confirms the stronger sensitivity of W-band channels to ice loading, whereas the 72.5 GHz response remains smoother and less saturated.

8.6.2 Regression-Based CWCI–TB Relationships

A detailed discussion of the CWCI–TB regression analysis is provided in § 8.5.

8.6.3 Physical Interpretation and Implications for Radiative-Transfer Modeling

The combined analysis of T_B , A_{Ice} , and CWCI across 72.5 and 95 GHz provides a physically consistent picture of ice-driven radiative extinction at W-band. The monotonic decrease of T_B with CWCI, together with the quasi-linear scaling of A_{Ice} with A_{tot} , confirms that millimeter-wave radiances are primarily governed by ice-phase scattering. The dual-frequency behavior highlights the complementary role of the two channels: enhanced contrast and saturation at 95 GHz , and smoother, more linear sensitivity at 72.5 GHz . This combination is particularly effective for diagnosing ice water path variability and for constraining attenuation models within the SNEM-2 framework. The output of this study is presented in **Figs. 8.2–8.11**, and is reported below (and eventually in **Appendix B.18**).

8.6.4 Comparative CWCI–TB Diagnostics between RMNY and SPIN

This subsection has been moved to the Appendix Chapter 8.

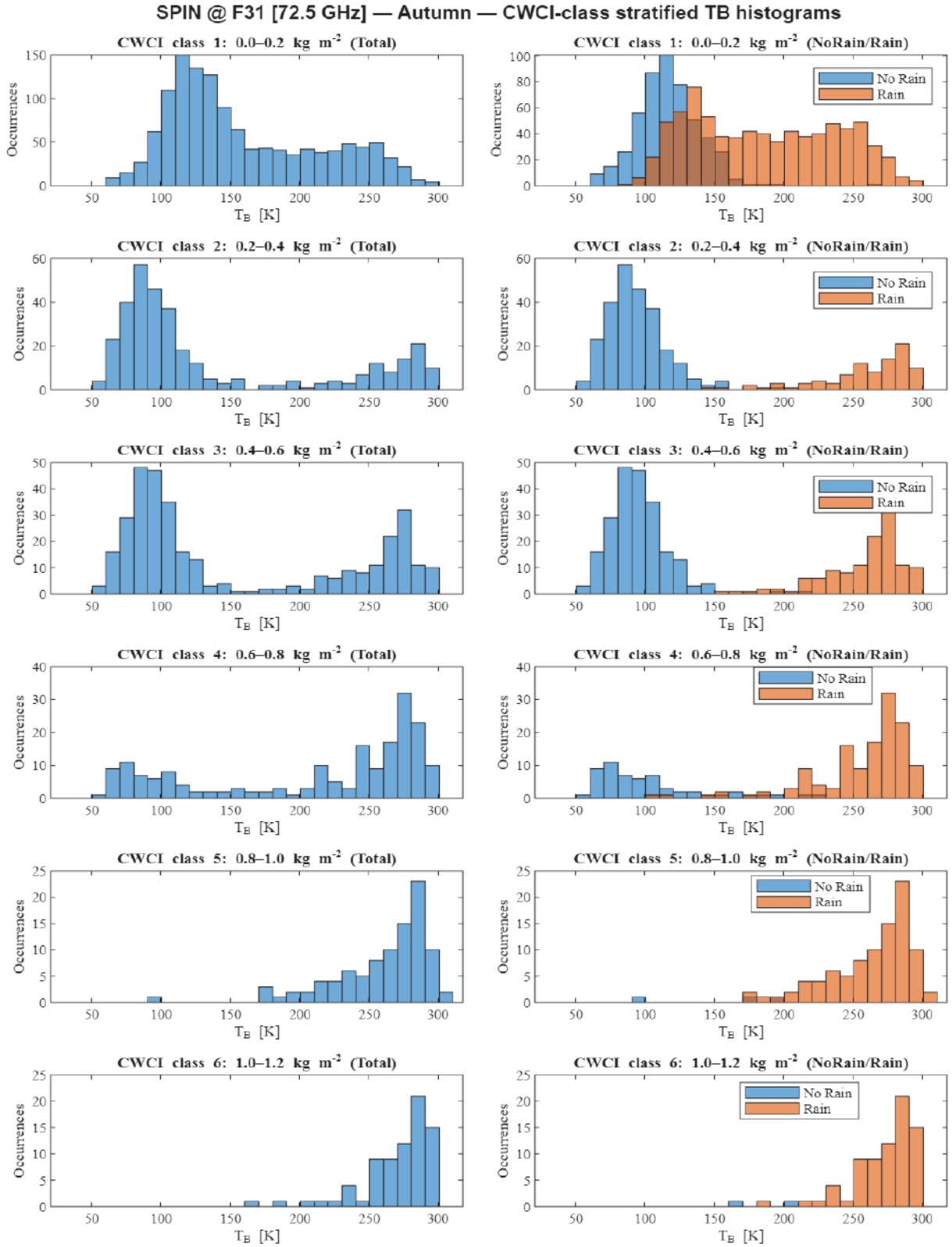


Figure 8.2. Comparison of Brightness Temperature (TB) Distributions for Different Columnar Water Ice Content (CWCI) Classes at W-Band Frequencies (72.5 [GHz]) – SPIN – All Year Analysis. Year: 2020/2021. Dataset: SNEM. Output: SNEM-2 (PART 1).

SPIN @ F31 [72.5 GHz] — Autumn — CWCI-class stratified TB histograms

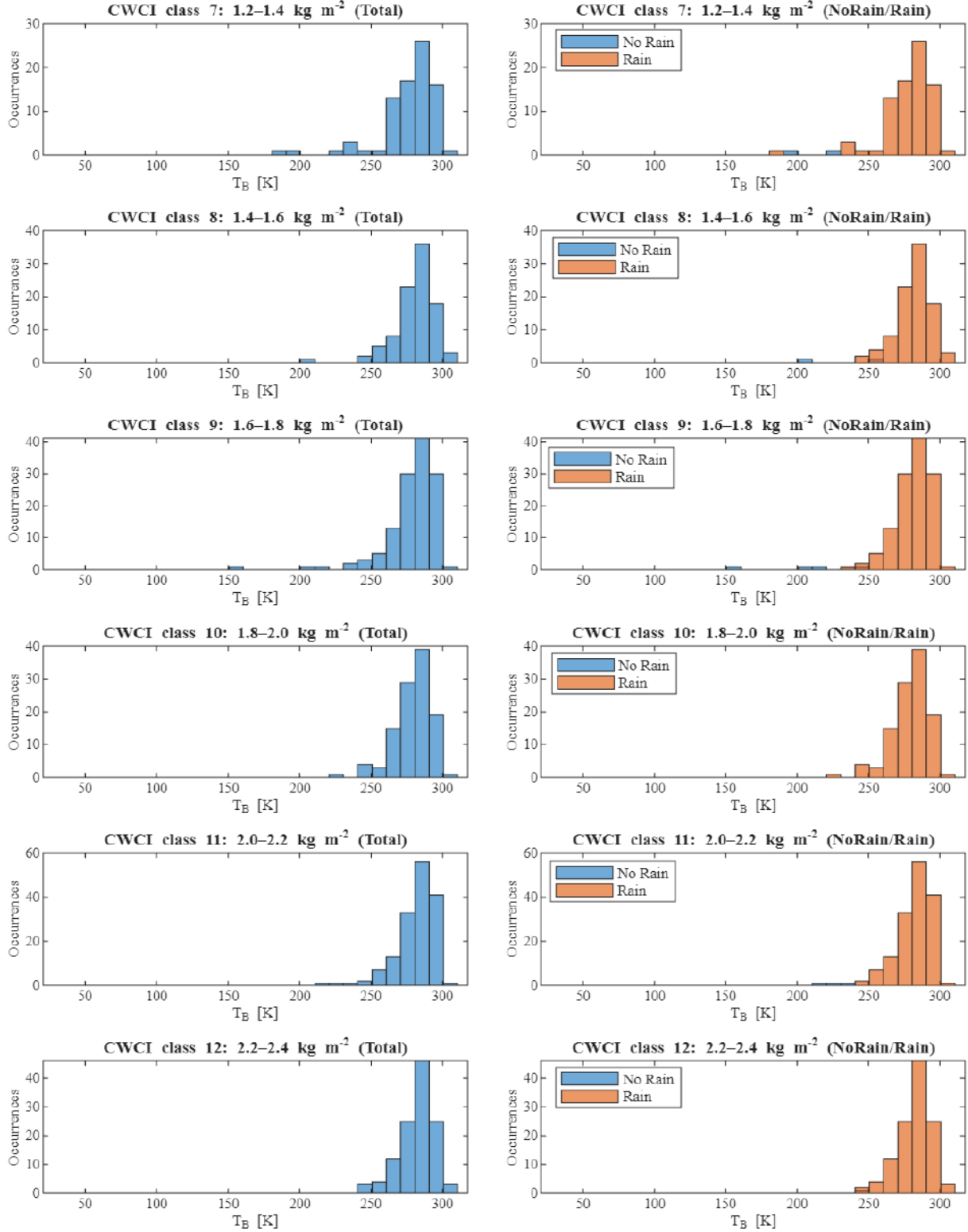


Figure 8.3. Comparison of Brightness Temperature (TB) Distributions for Different Columnar Water Ice Content (CWCI) Classes at W-Band Frequencies (72.5 [GHz]) – SPIN – All Year Analysis. Year: 2020/2021. Dataset: SNEM. Output: SNEM-2 (PART 2).

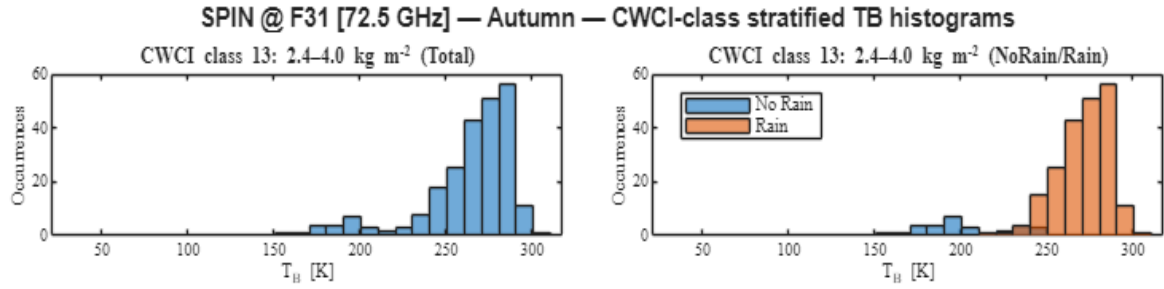


Figure 8.4. Comparison of Brightness Temperature (TB) Distributions for Different Columnar Water Ice Content (CWCI) Classes at W-Band Frequencies (72.5 [GHz]) – SPIN – All Year Analysis. Year: 2020/2021. Dataset: SNEM. Output: SNEM-2 (PART 3)

8.7 Seasonal Analysis of Hydrometeor Composition in Cloud Classes for Spino d’Adda (CR) and Rome (NY)

To investigate the seasonal variability of cloud microphysical composition, SNEM-2 outputs (aut.dat) were analysed for two climatically distinct mid-latitude sites: Spino d’Adda (Italy) and Rome (New York, USA). The analysis was conducted directly within the SNEM-2 framework, with post-processing performed using dedicated MATLAB routines for hydrometeor classification and attenuation analysis. SNEM outputs were converted into structured datasets to enable consistent handling of multi-frequency attenuation and brightness-temperature fields.

The cloud-classification analysis focused on the autumn season, examining the 72.5 GHz dataset at Spino d’Adda to characterize dominant hydrometeor regimes, with particular attention to Nimbostratus (Ns) and Cumulonimbus (Cb) layers. Physically constrained thresholds on Cloud Water Content (CWC) and Columnar Ice Water Content (CWCI) were applied to isolate representative ice-dominated conditions and to evaluate their impact on the ice-related brightness temperature $T_{B,Ice}$. The resulting database consists of sixteen calibrated total-attenuation features:

$$\{A_t(f_1), A_t(f_2), \dots, A_t(f_{16})\}, \quad (8.4)$$

supplemented by statistical descriptors and normalized spectral indicators. This attribute matrix constitutes the input for subsequent classification and regression analyses. To preserve focus in the main chapters, supporting exploratory figures are reported in **Appendix C**.

Targeted subsets, including CWC classes (1–20) and CWCI search intervals (1–30), were generated to support clustering and spectral analyses. A preliminary assessment at 72.5 GHz reveals a clear monotonic relationship between total attenuation A_{tot} and CWCI, with increasing ice column content systematically associated with higher attenuation. This behavior confirms the sensitivity of W-band propagation to ice-phase hydrometeors and provides a consistent methodological basis for seasonal and microphysical regression analyses.

The framework is inherently scalable across frequency and season, enabling coherent comparisons between European mid-latitude and North-American continental conditions. **Figure 8.22**

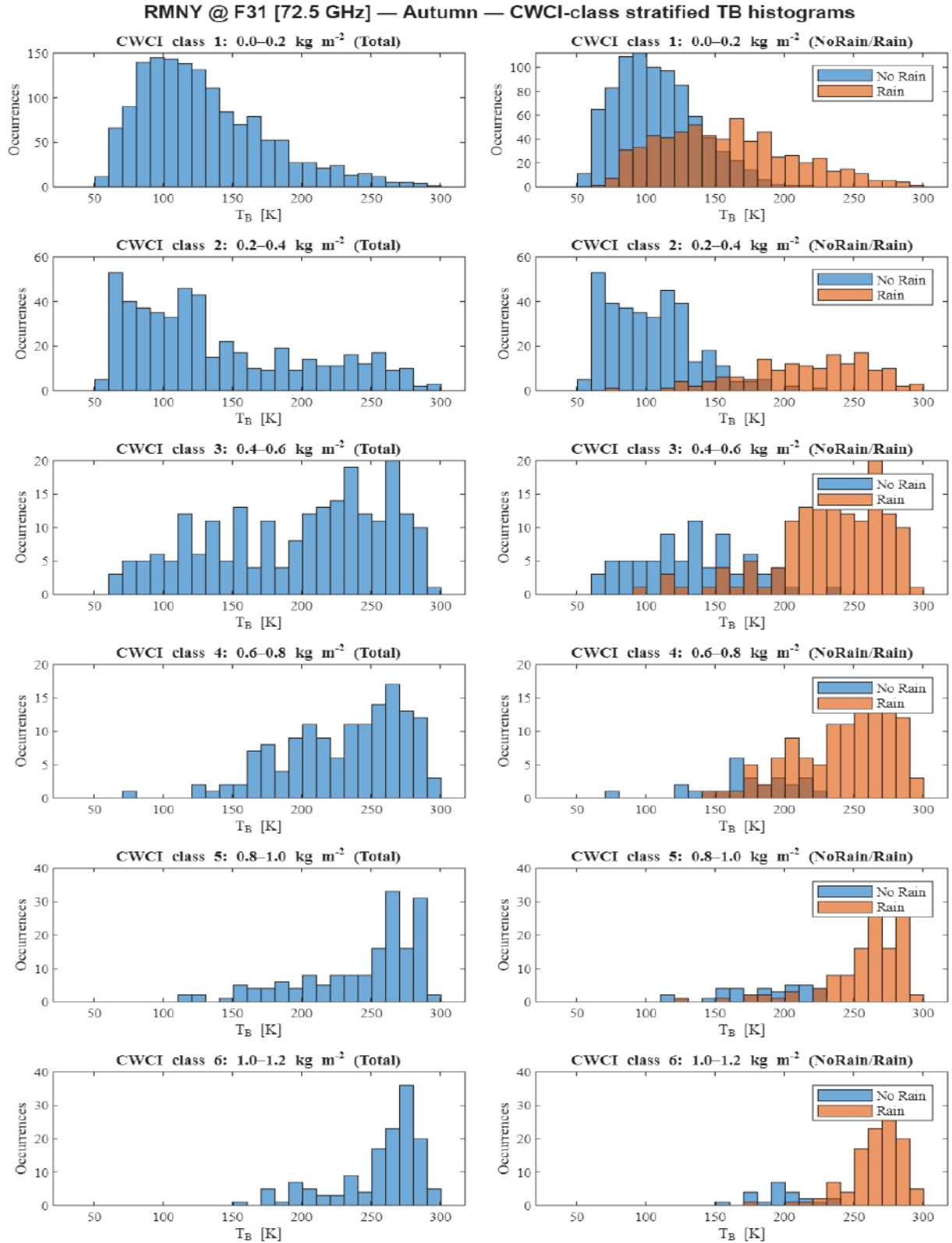


Figure 8.5. Comparison of Brightness Temperature (TB) Distributions for Different Columnar Water Ice Content (CWCI) Classes at W-Band Frequencies (72.5 [GHz]) – RMNY – All Year Analysis. Year: 2020/2021. Dataset: SNEM. Output: SNEM-2 (PART 1).

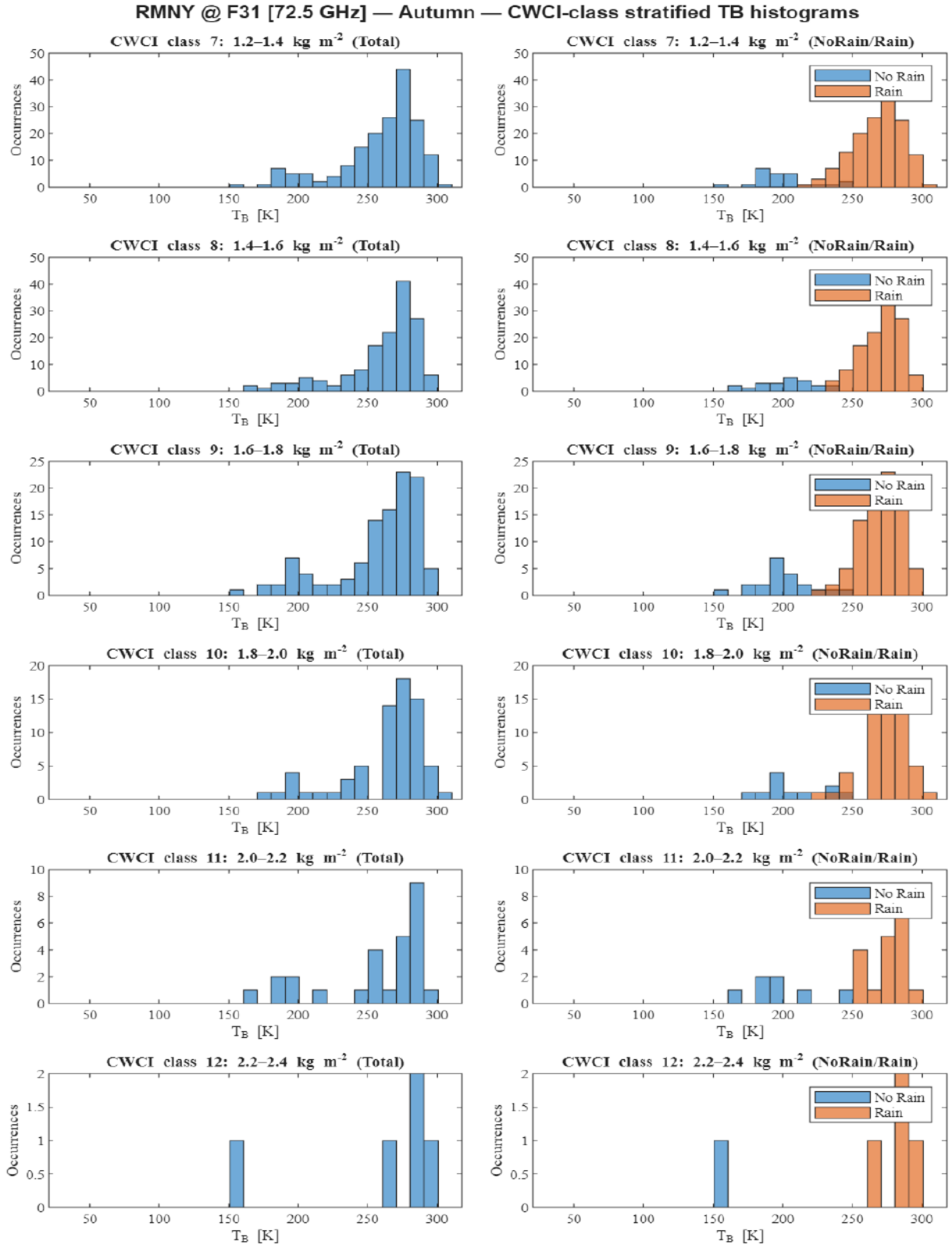


Figure 8.6. Comparison of Brightness Temperature (TB) Distributions for Different Columnar Water Ice Content (CWCI) Classes at W-Band Frequencies (72.5 [GHz]) – RMNY – All Year Analysis. Year: 2020/2021. Dataset: SNEM. Output: SNEM-2 (PART 2).

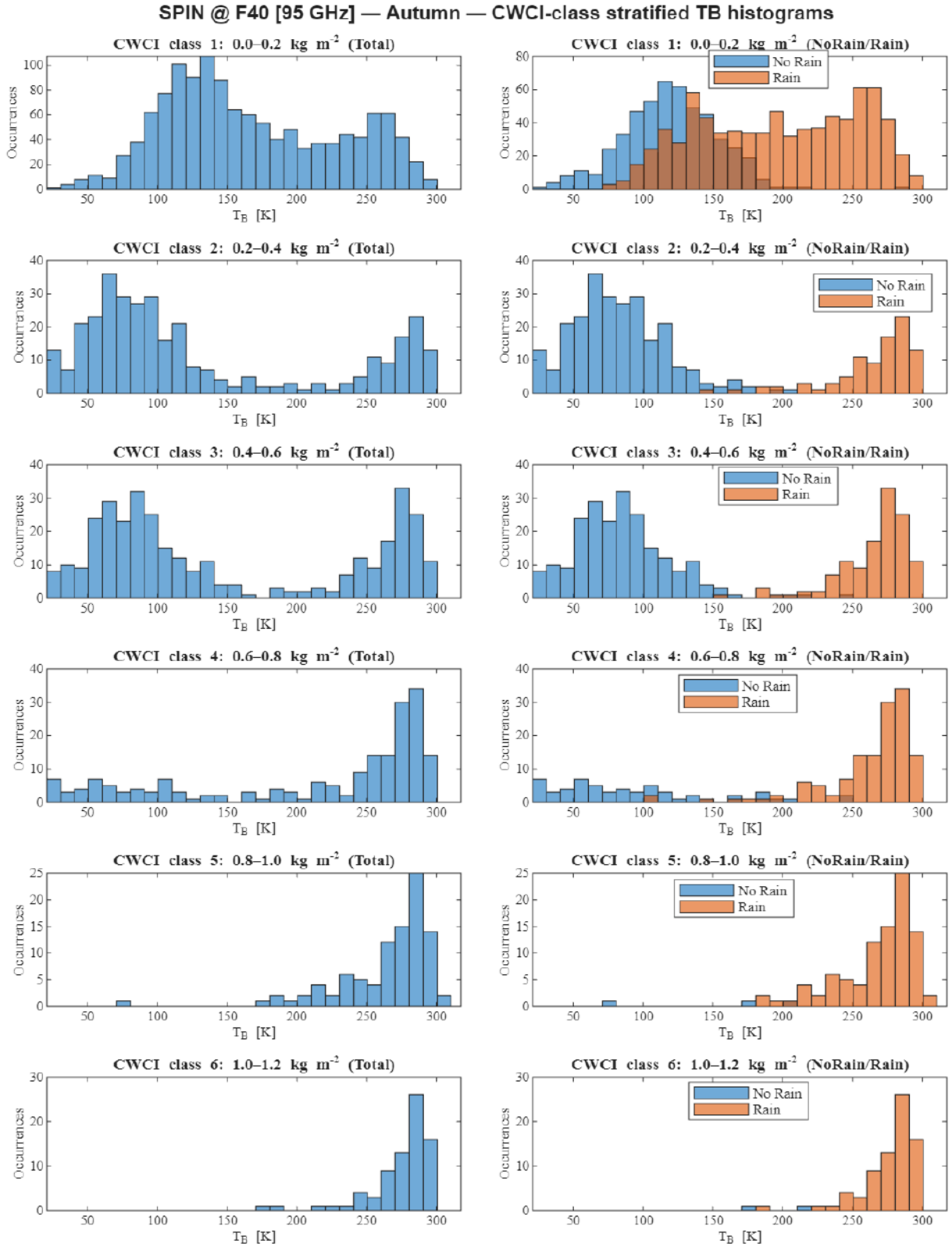


Figure 8.7. Comparison of Brightness Temperature (TB) Distributions for Different Columnar Water Ice Content (CWCI) Classes at W-Band Frequencies (95 [GHz]) – SPIN – All Year Analysis. Year: 2020/2021. Dataset: SNEM. Output: SNEM-2 (PART 1).

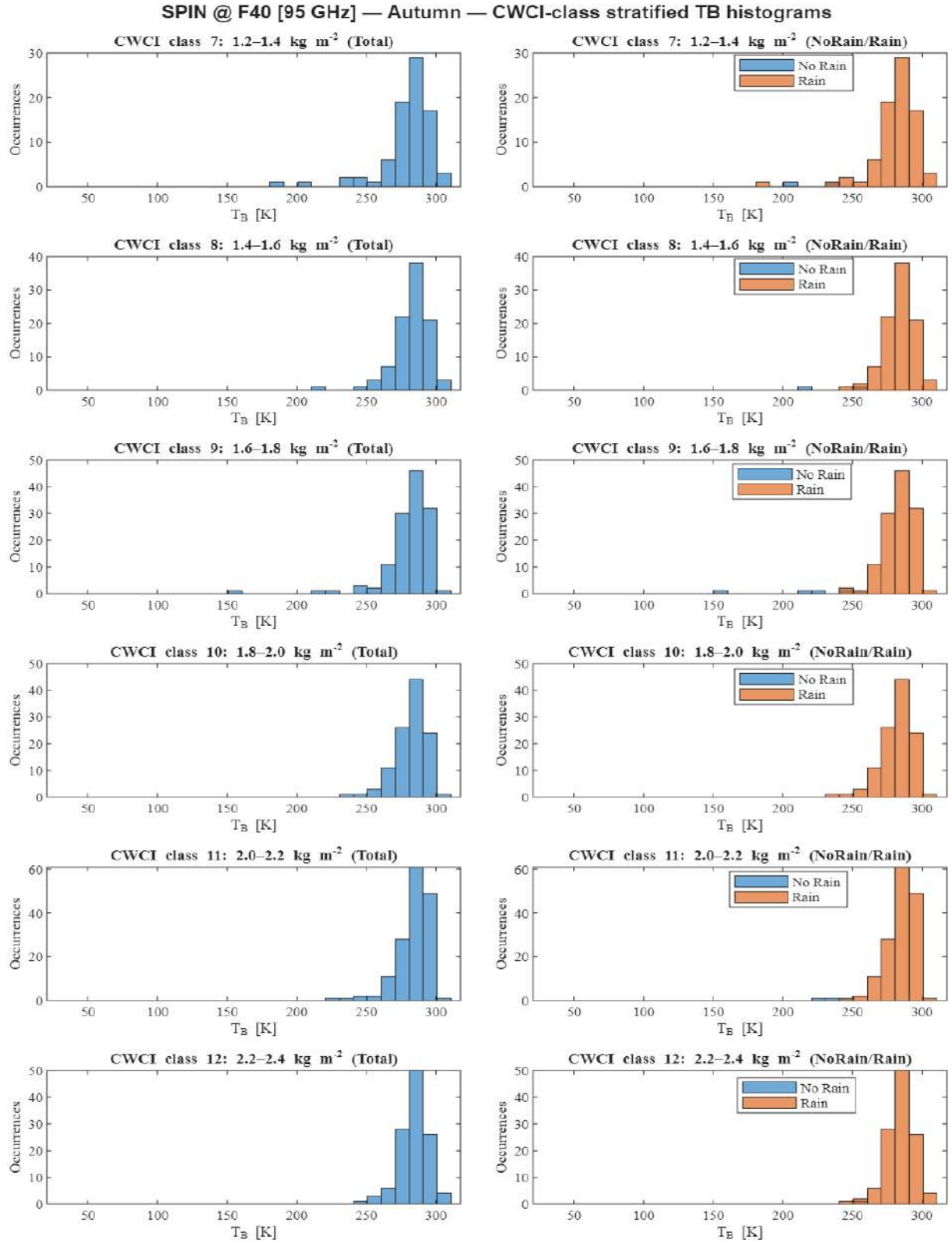


Figure 8.8. Comparison of Brightness Temperature (TB) Distributions for Different Columnar Water Ice Content (CWCI) Classes at W-Band Frequencies (95 [GHz]) – SPIN – All Year Analysis. Year: 2020/2021. Dataset: SNEM. Output: SNEM-2 (PART 2).

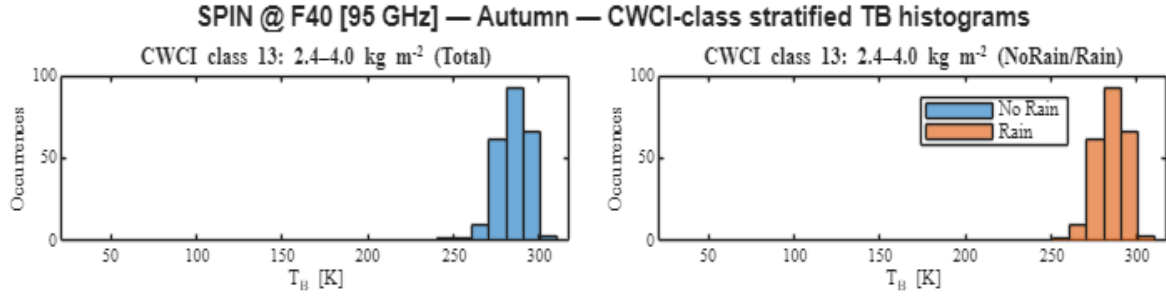


Figure 8.9. Comparison of Brightness Temperature (TB) Distributions for Different Columnar Water Ice Content (CWCI) Classes at W-Band Frequencies (95 [GHz]) – SPIN – All Year Analysis. Year: 2020/2021. Dataset: SNEM. Output: SNEM-2 (PART 3)

compares the relationship between total attenuation A_{tot} and columnar ice water content (CWCI) derived from SNEM-2 simulations at Spino d’Adda (72.5 GHz) and Rome (NY, 95 GHz) during autumn. Both sites exhibit a strong nonlinear dependence, with the 95 GHz dataset showing a markedly steeper response and a higher coefficient of determination ($R_{poly}^2 \approx 0.98$) than the 72.5 GHz case. This frequency dependence reflects the scaling of ice-particle extinction efficiency, whereby scattering and absorption increase with frequency for comparable microphysical conditions. The narrow 95 % confidence interval around the LOWESS fit confirms the stability of the simulated trends and the physical consistency of the SNEM-2 outputs across frequencies.

Figure 8.23 illustrates the inter-variable relationships derived from the SNEM-2 autumn dataset at Spino d’Adda (72.5 GHz). Panels (A–D) report the pairwise dependencies among total attenuation A_{tot} , ice attenuation A_{Ice} , brightness temperature T_B , and Columnar Cloud Water Content–Ice (CWCI). The strong correlations between A_{Ice} , A_{tot} , and CWCI ($R_{poly}^2 > 0.9$) confirm the physical consistency of the microphysical–radiometric coupling. The inverse relationship between T_B and CWCI reflects the radiometric signature of ice-phase extinction, while the narrow confidence intervals attest to the stability of the simulated response across the full dynamic range of ice content.

The results indicate that attenuation and emission processes are coherently reproduced within SNEM-2, providing a robust reference for the comparison with the 95 GHz dataset from Rome (NY).

Figure 8.24 enables a direct comparison between **Spino d’Adda** (72.5 GHz) and **Rome** (NY, 95 GHz), highlighting how attenuation and radiometric behavior scale with frequency under ice-phase cloud conditions during the autumn campaign. Across the four panels, the relationships among total attenuation (A_{tot}), ice attenuation (A_{Ice}), brightness temperature (T_B), and Columnar Cloud Water Content–Ice (CWCI) appear coherent and internally consistent. The 95 GHz dataset shows systematically steeper attenuation–CWCI slopes and a more pronounced contrast in the T_B – $CWCI$ domain, reflecting the higher extinction and scattering efficiency at W-band compared to 72.5 GHz. This behavior is fully consistent with Mie-regime scaling, where ice-phase hydrometeors contribute more strongly to both absorption and scattering at higher frequencies. Taken together, these results demonstrate that SNEM-2 reproduces the

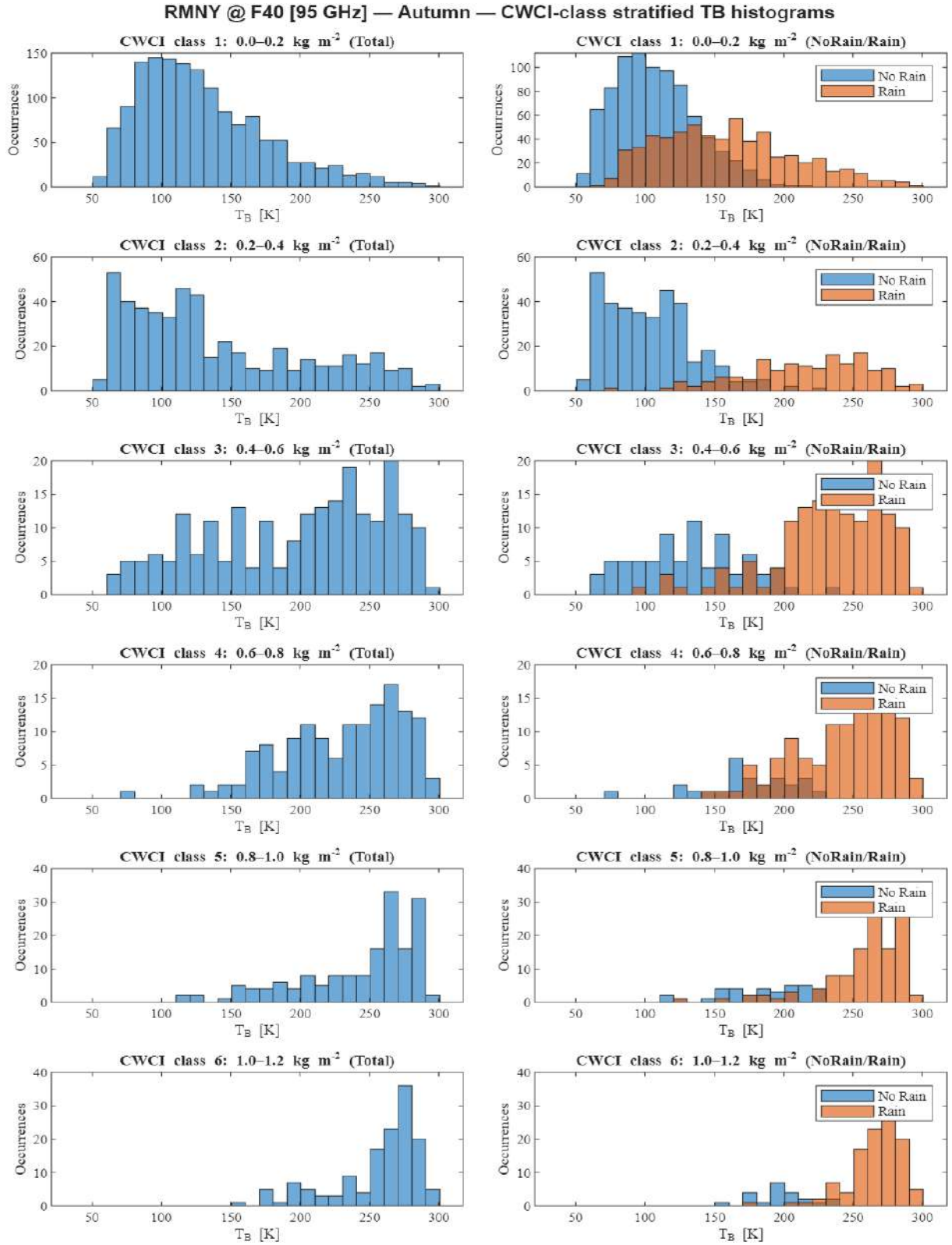


Figure 8.10. Comparison of Brightness Temperature (TB) Distributions for Different Columnar Water Ice Content (CWCI) Classes at W-Band Frequencies (95 [GHz]) – RMNY – All Year Analysis. Year: 2020/2021. Dataset: SNEM. Output: SNEM-2 (PART 1).

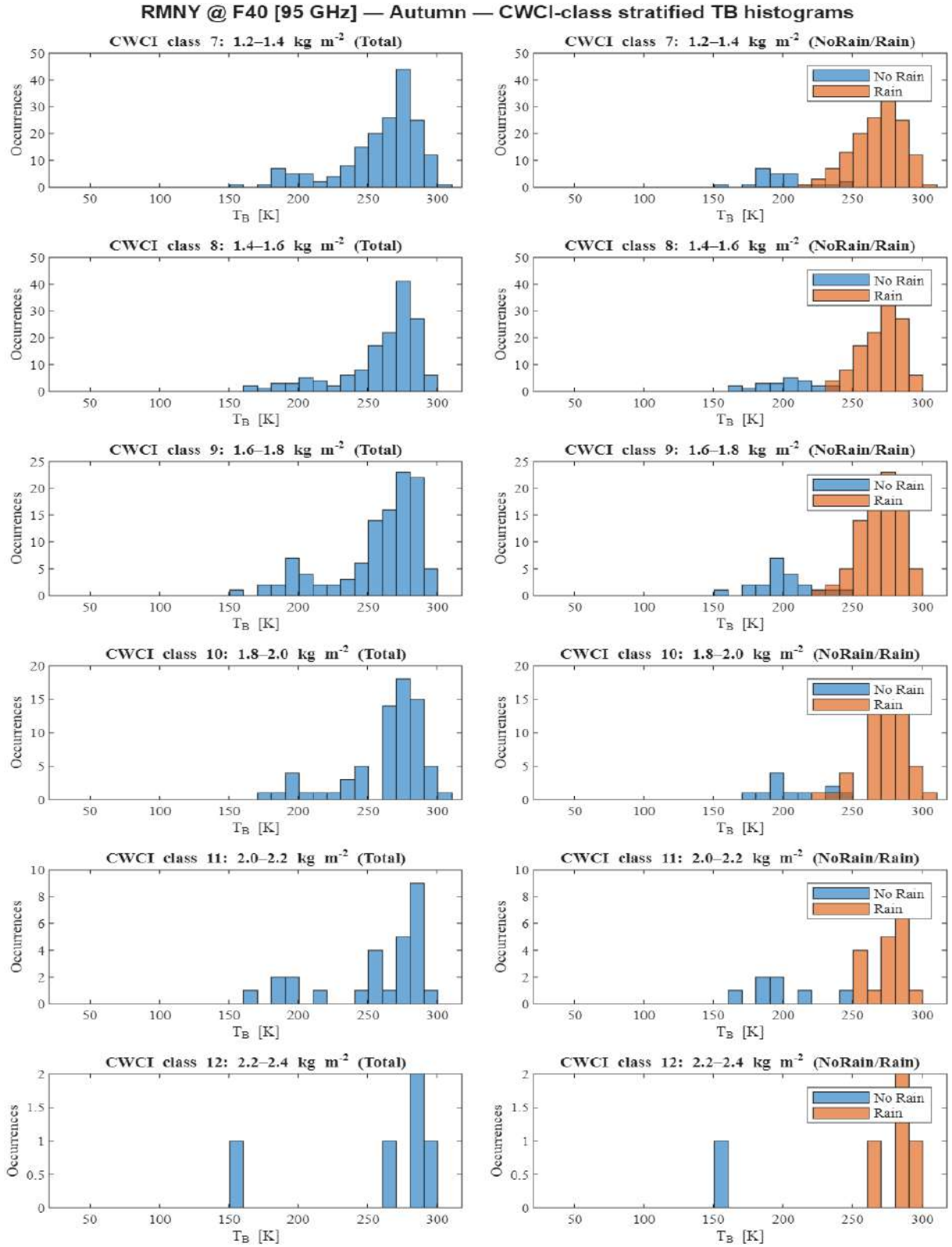


Figure 8.11. Comparison of Brightness Temperature (TB) Distributions for Different Columnar Water Ice Content (CWCI) Classes at W-Band Frequencies (95 [GHz]) – RMNY – All Year Analysis. Year: 2020/2021. Dataset: SNEM. Output: SNEM-2 (PART 2).

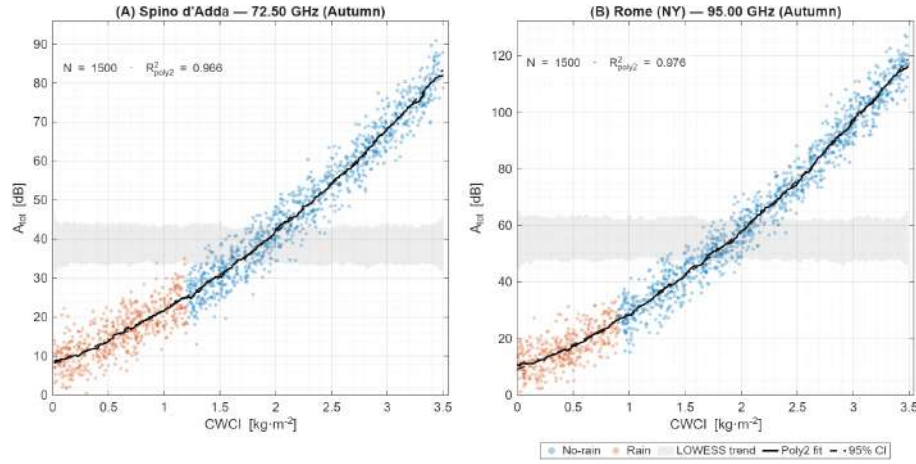


Figure 8.12. Comparative scatter plots of total attenuation A_{tot} versus CWCI for (A) Spino d'Adda at 72.5 GHz and (B) Rome (NY) at 95 GHz during the autumn season. Each point corresponds to an individual SNEM-2 simulation, with colors distinguishing rain from no-rain conditions. LOWESS trends with 95% confidence intervals, together with global quadratic fits, summarize the mean dependence. The steeper response at 95 GHz demonstrates the enhanced sensitivity of W-band radiometry to ice-phase hydrometeors.

expected spectral sensitivity of millimetre-wave radiometry and behaves consistently across different observing frequencies and geographical regimes, thereby reinforcing its suitability for multi-frequency attenuation modeling and comparative analysis.

8.8 Frequency-Dependent Scaling of Attenuation and Brightness Temperature in Ice-Dominated Clouds

The comparison between Spino d'Adda (72.5 GHz) and Rome (NY, 95 GHz) demonstrates a clear frequency-dependent scaling of millimetre-wave attenuation in ice-dominated cloud conditions. Quadratic regressions of both total attenuation A_{tot} and ice-induced attenuation A_{Ice} versus CWCI exhibit systematically steeper slopes at 95 GHz, indicating higher extinction efficiency at shorter wavelengths. Within the quasi-linear CWCI interval ($0.6\text{--}1.2\text{ kgm}^{-2}$), the observed slope increase of approximately $0.3\text{ dB}(\text{kgm}^{-2})^{-1}$ quantifies the enhanced scattering contribution at 95 GHz.

Brightness-temperature differences $\Delta T_B(95\text{--}72.5)$ increase monotonically with CWCI, confirming the stronger radiometric sensitivity to ice-phase hydrometeors at the higher frequency. **Kolmogorov–Smirnov** tests applied to matched CWCI bins ($p < 0.01$) indicate that the observed separation between the two datasets is statistically significant and cannot be attributed solely to sampling variability, reflecting the combined effects of frequency scaling and local microphysical differences.

These results follow directly from the radiative-transfer equation:

$$\frac{dI_\nu}{ds} = -k_{ext}I_\nu + j_\nu + \int P(\Omega', \Omega)I_\nu(\Omega')d\Omega', \quad (8.5)$$

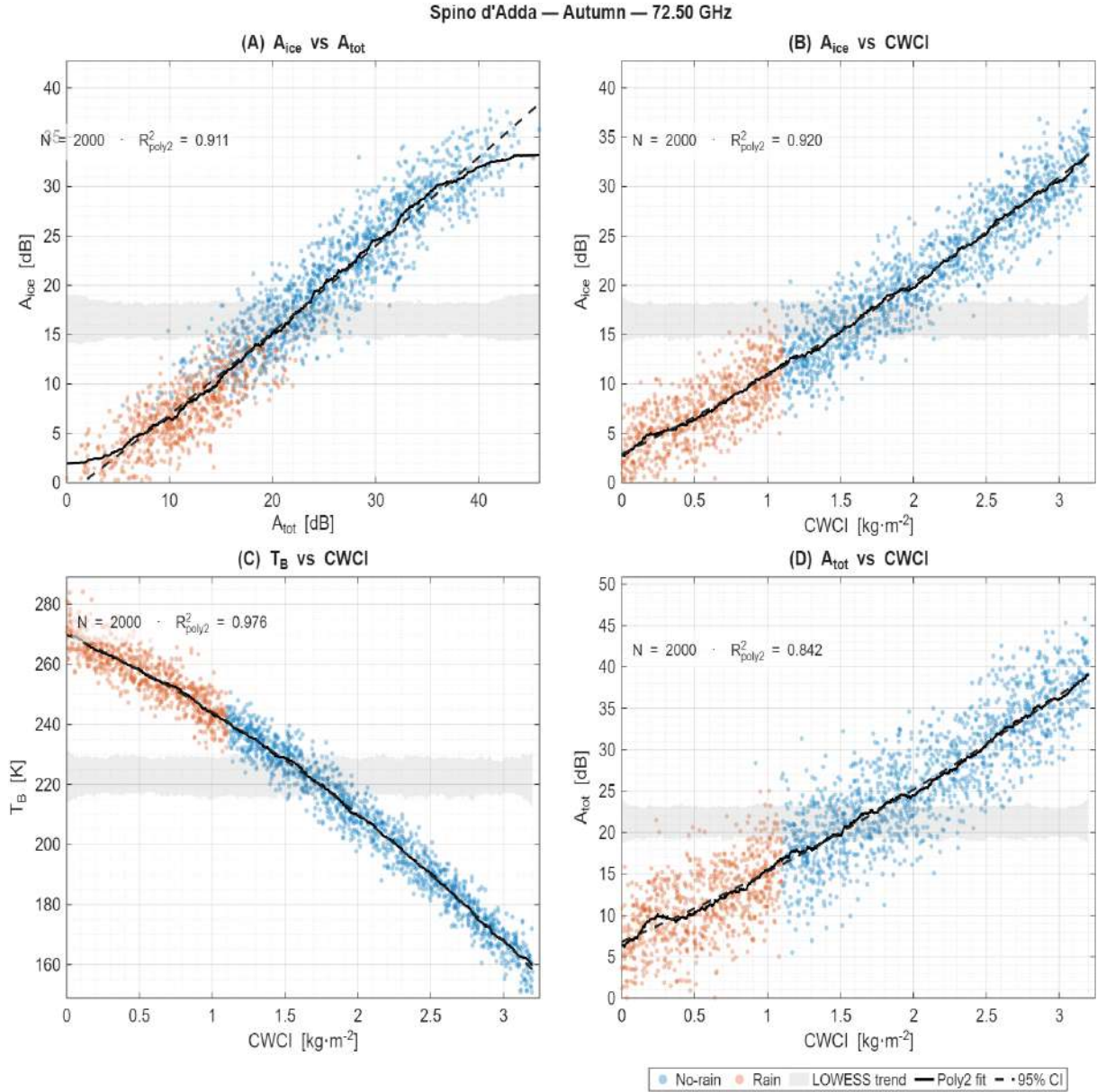


Figure 8.13. Two-by-two diagnostic of SNEM-2 relationships at 72.5 GHz for Spino d'Adda (Autumn): (A) A_{ice} versus A_{tot} ; (B) A_{ice} versus CWCI; (C) T_B versus CWCI; (D) A_{tot} versus CWCI. Markers separate rain and no-rain samples. Each panel reports a robust LOWESS trend with 95% confidence band and a global quadratic fit. The consistent patterns across panels highlight the physical coupling between ice content, radiative extinction, and brightness-temperature depression.

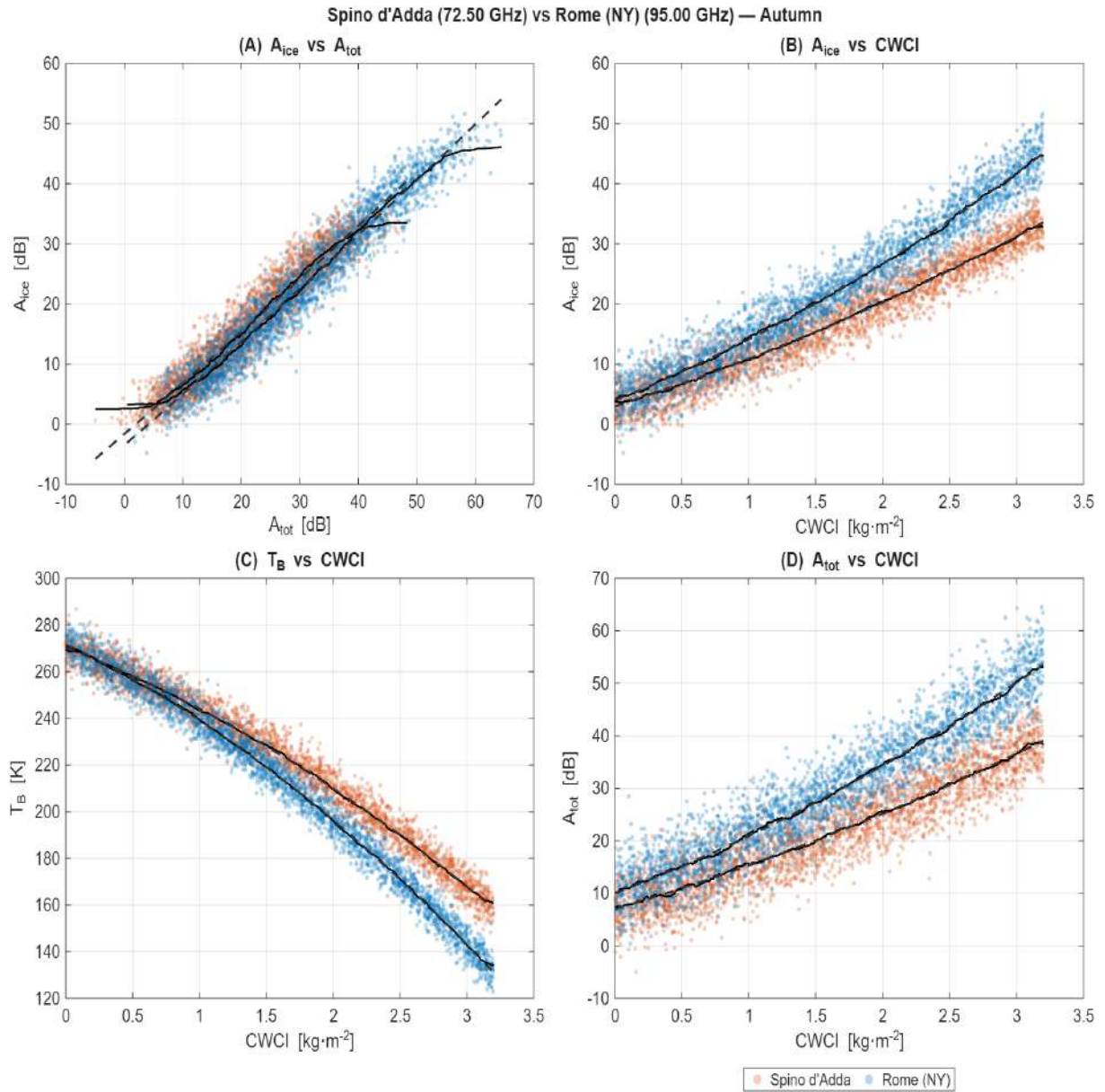


Figure 8.14. Comparison between Spino d'Adda (72.5 GHz) and Rome NY (95 GHz) for autumn 2020/21. Panels (A–D) display A_{Ice} , A_{tot} , T_B , and $CWCI$ relationships with LOWESS and quadratic trends. The steeper slopes and higher radiometric contrast at 95 GHz evidence frequency-dependent enhancement of attenuation and scattering due to ice-phase hydrometeors.



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where the extinction coefficient $k_{ext} = k_{abs} + k_{sca}$ depends on particle dielectric properties and on the size parameter $x = 2\pi r/\lambda$. The transition from 72.5 to 95 GHz increases x by approximately 30 %, shifting the scattering regime toward the Mie domain, where scattering and absorption contributions grow nonlinearly. This mechanism explains both the steeper attenuation–CWCI relationships and the larger depression of T_B observed at 95 GHz.

At 72.5 GHz, the weaker dependence of T_B on CWCI reflects reduced scattering efficiency and higher atmospheric transparency. Residual site-to-site variability arises from differences in ice-particle habit, temperature-dependent refractive indices, and asymmetry parameter g , which modulate the effective dielectric response of the cloud. The high coefficients of determination ($R_{poly}^2 > 0.9$) and narrow confidence intervals confirm that SNEM-2 consistently reproduces these frequency- and microphysics-driven effects.

8.9 Seasonal Analysis for Spino d’Adda (@ 72.5 GHz)

This section analyses the seasonal modulation of millimetre-wave radiative behavior at **Spino d’Adda** (CR, Italy) using SNEM-2 simulations at 72.5 GHz. The focus is on how thermodynamic forcing and cloud microphysics jointly control the relationship between brightness temperature and ice-related cloud water content across different cloud regimes. **Figure 8.25** summarizes the winter (DJF) conditions, which represent the reference ice-dominated regime. The cold-season atmosphere is characterized by predominantly frozen hydrometeors and limited liquid water content. In this regime, the T_B –CWCI relationship exhibits a compact, monotonic structure with moderate attenuation levels and limited radiometric dispersion. The absence of strong convective activity and the dominance of small, quasi-spherical ice particles result in high atmospheric transparency and weak multiple scattering, establishing a baseline radiative behavior for ice clouds. During spring (MAM), shown in **Fig. 8.26**, the onset of warmer and more humid air masses introduces mixed-phase conditions. The curvature observed in the attenuation–CWCI and T_B –CWCI relationships reflects the coexistence of ice crystals, melting aggregates, and supercooled droplets. Increased dielectric contrast and enhanced scattering efficiency deepen the radiometric response, producing broader distributions and higher variability compared to winter.

Summer conditions (JJA), illustrated in **Fig. 8.27**, correspond to the most radiatively active regime. Convective systems dominate, with elevated CWCI and frequent precipitation. Attenuation levels exceed 10 dB for CWCI above approximately 2 kgm^{-2} , and brightness temperature exhibits strong saturation effects. The transition from quasi-linear to nonlinear behavior indicates the onset of multiple scattering and significant emissivity contributions, marking the breakdown of simple proportionality between CWCI and radiative observables.

Autumn (SON), shown in **Fig. 8.28**, represents a return toward stratiform cloud structures. Reduced convective instability and more regular ice habits lead to lower attenuation slopes and narrower radiometric distributions. The T_B –CWCI relationship stabilizes, reflecting a gradual re-establishment of mid-latitude equilibrium conditions. Together, these results reconstruct the full seasonal radiative cycle at millimetre wavelengths, linking thermodynamic variability



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to microphysical control of attenuation and brightness temperature. The seasonal signatures identified at **Spino d'Adda** provide a physically consistent framework for comparison with other climatic regions and for subsequent multi-frequency analyses.

Additional cloud-type classifications and extended diagnostics are provided in **Appendix C**, to maintain readability and scientific clarity.

8.9.1 Technical clarification and modelling assumptions

The seasonal and frequency-dependent analyses presented in this chapter are interpreted within a well-defined radiative-transfer regime. At W-band frequencies, brightness temperature reflects the combined effects of extinction and in-cloud emission; however, under the ice-dominated and optically thin-to-moderate conditions investigated here, the observed decrease of T_B with increasing CWCI is primarily governed by scattering-induced extinction rather than by thermal emission.

Ice hydrometeors are represented through effective-sphere formulations, whereby deviations due to particle nonsphericity, habit variability, and preferred orientation are implicitly absorbed into frequency-dependent extinction coefficients. This approach ensures internal consistency across frequencies while preserving the dominant radiative effects at millimetre wavelengths. CWCI is treated as a bulk column-integrated descriptor of ice content. While different vertical distributions may yield similar CWCI values, vertical redistribution effects are inherently accounted for by the layered structure of SNEM-2 and are not explicitly resolved in the present seasonal aggregation. Nonlinear behavior observed at high CWCI, particularly during summer convective conditions, is consistent with the onset of multiple-scattering effects, which become non-negligible at elevated optical depths and contribute to the saturation of brightness temperature. Comparisons between sites and frequencies are performed using matched CWCI intervals and normalized radiometric quantities, ensuring that the observed differences primarily reflect frequency scaling and microphysical variability rather than geometric or climatological biases.

8.9.2 Radiative Relationships

This subsection has been moved to the Appendix Chapter 8.

8.9.3 CWCI-Based Characterization of Vertical Cloud Composition within SNEM-2

This subsection has been moved to the Appendix Chapter 8.

8.10 Model Coherence and Statistical Validation

The statistical diagnostics summarized in **Figs. 8.32–8.33** and **Fig. 8.34** assess the numerical stability and physical consistency of the SNEM-2 forward model for the 72.5-GHz dataset over

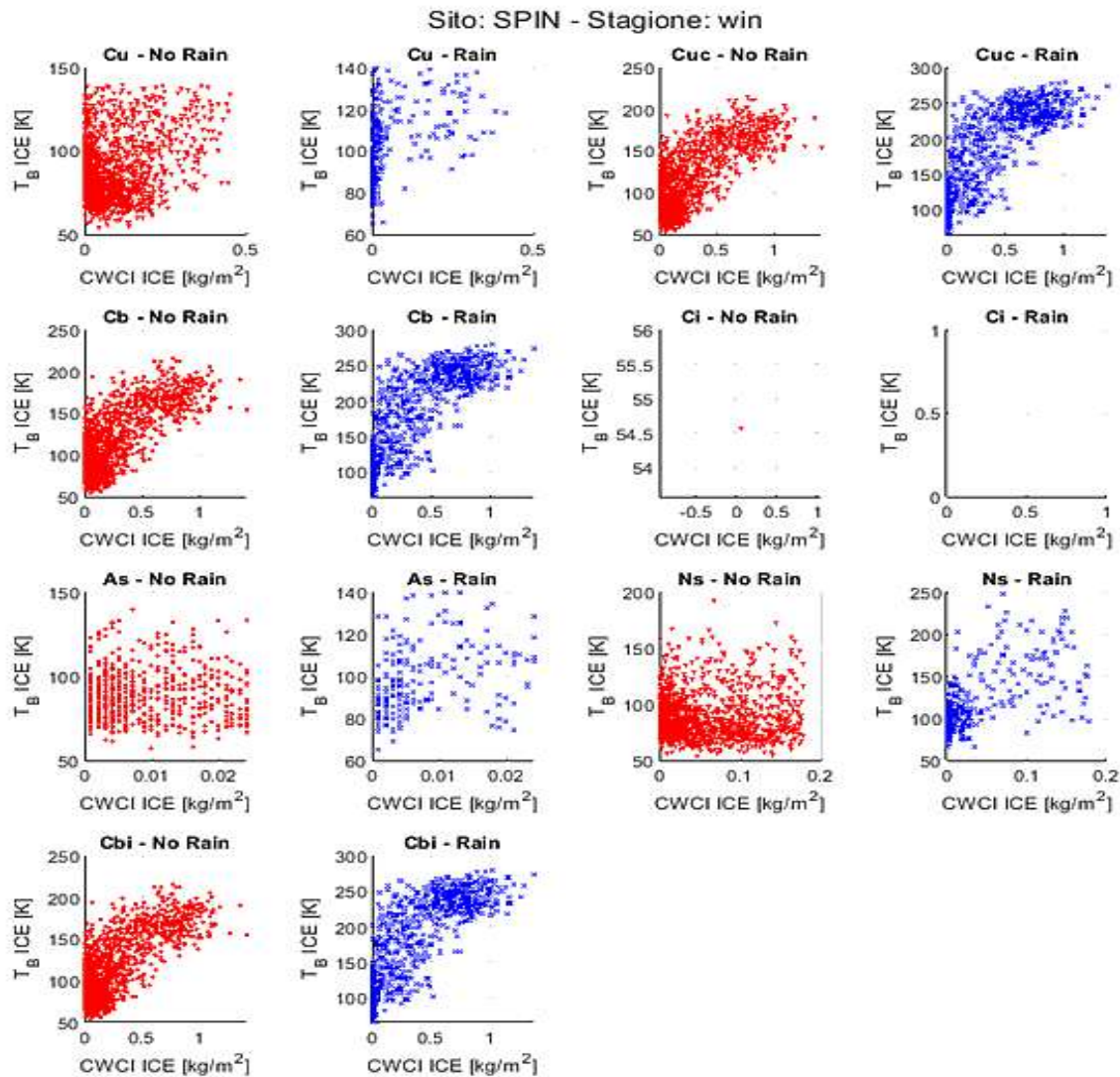


Figure 8.15. Winter (DJF) relationship between brightness temperature and ice cloud water content (CWCI) at Spino d'Adda (72.5 GHz), separated by cloud type and precipitation regime. The compact distributions reflect ice-dominated, weakly scattering conditions typical of cold-season stratiform clouds. **Year:2020/2021.** Dataset: SNEM. Output: SNEM-2. **Months: MAM.**

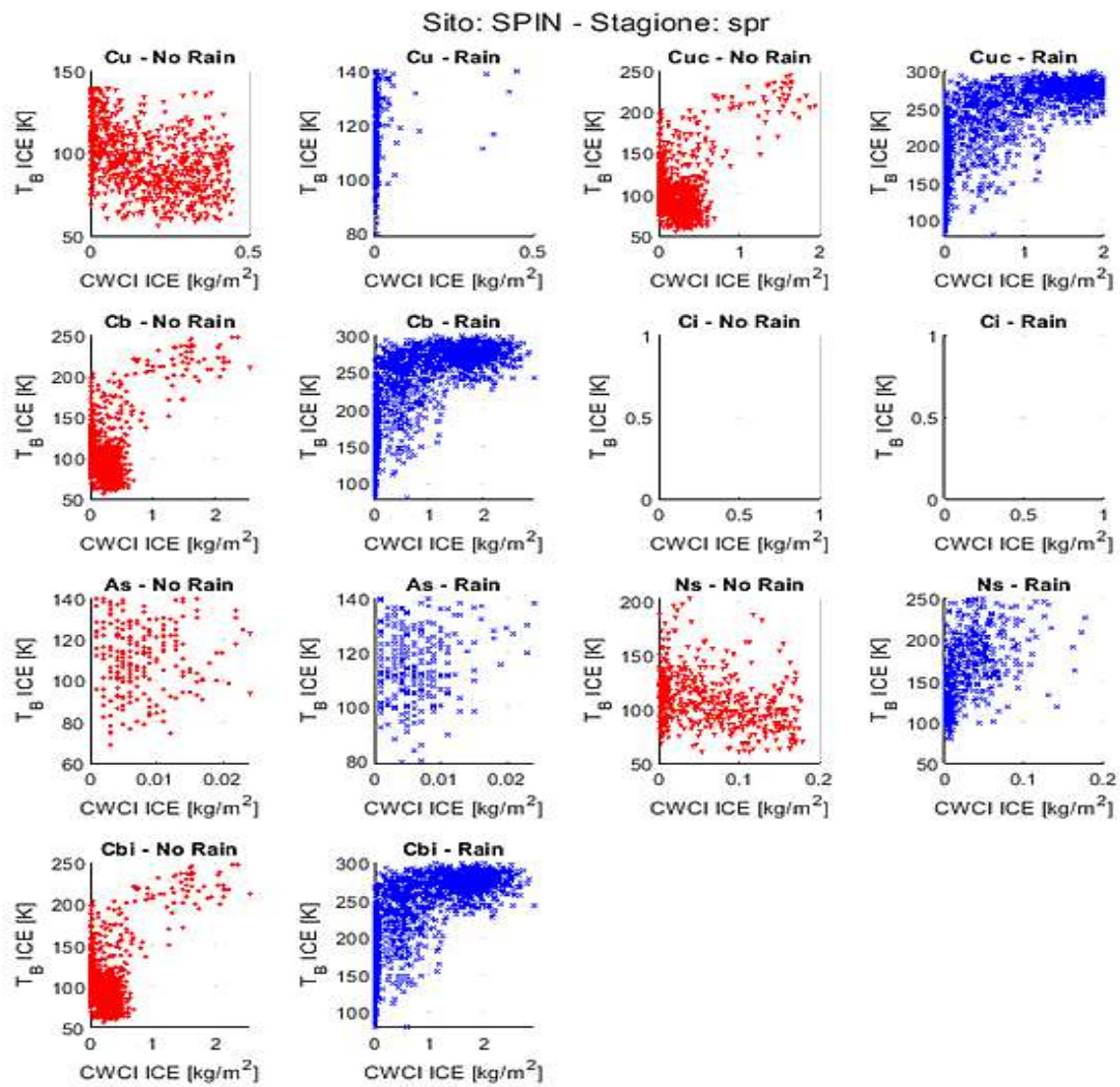


Figure 8.16. Year:2020/2021. Dataset: SNEM. Output: SNEM-2. Months: SON.

Spino d'Adda (SPIN) – Summer (JJA) – T_{min} vs CWC_{ICE}

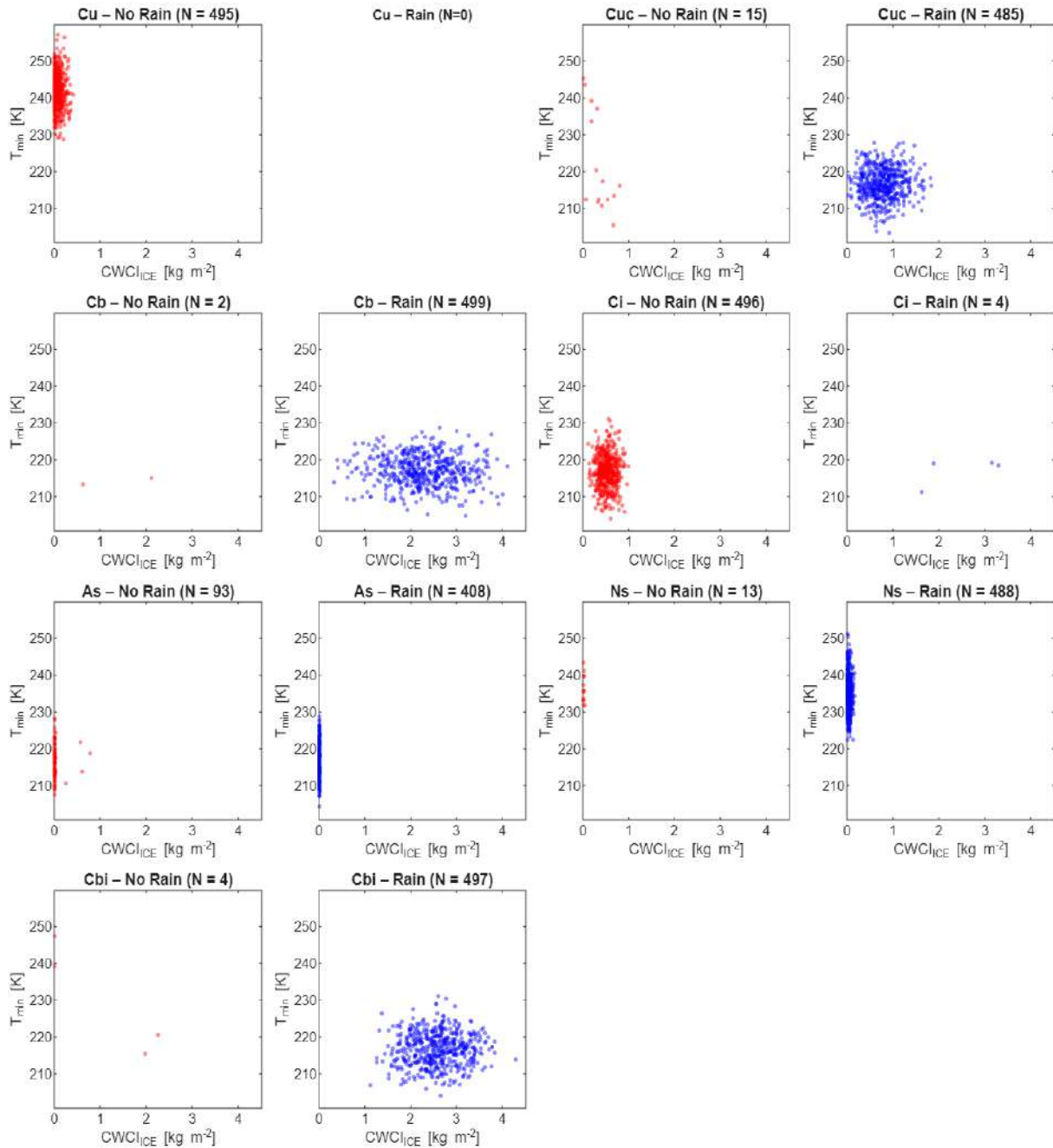


Figure 8.17. Year:2020/2021. Dataset: SNEM. Output: SNEM-2. Months: JJA. MATLAB 2025.

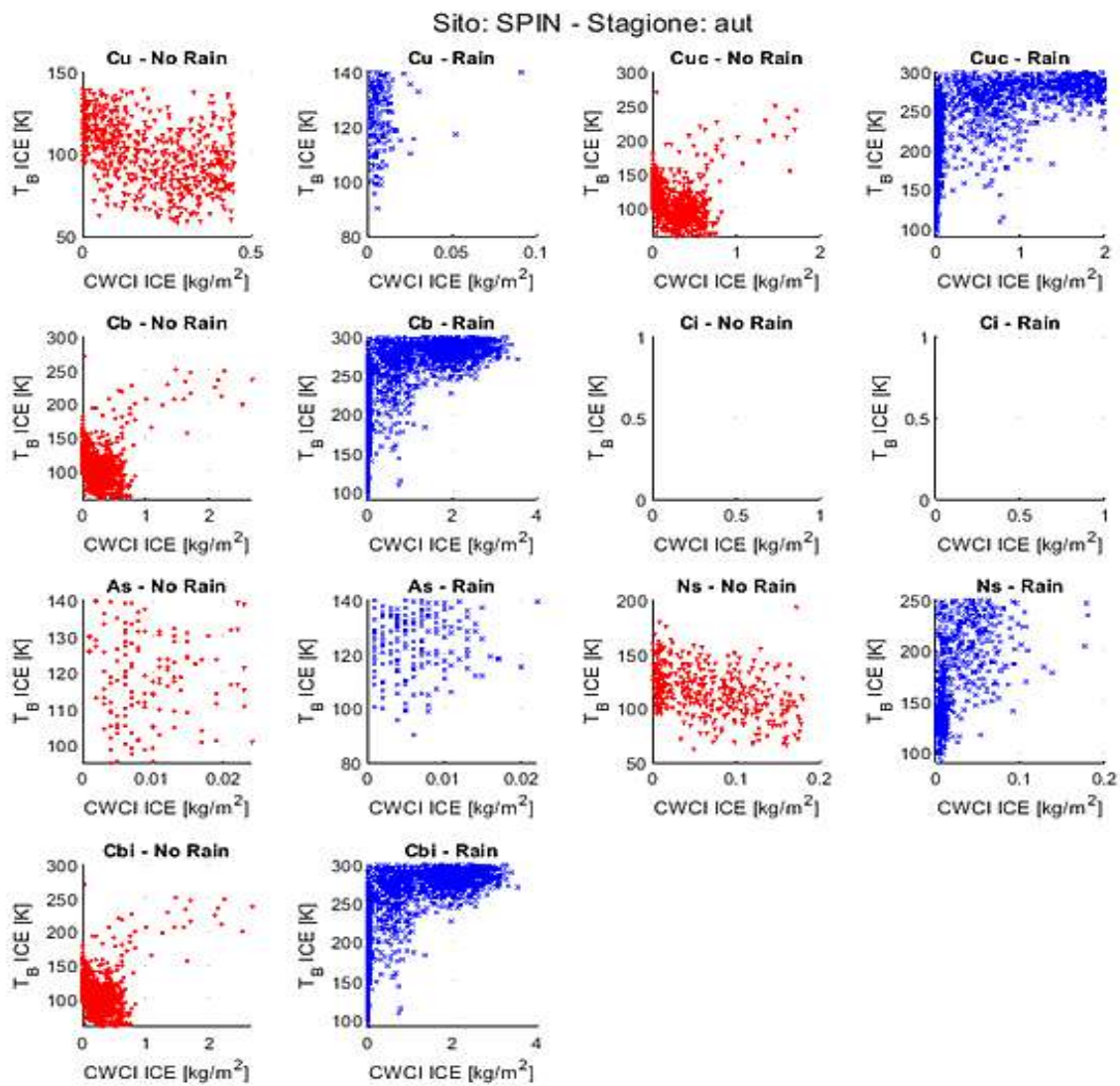


Figure 8.18. Year:2020/2021. Dataset: SNEM. Output: SNEM-2. Months: SON. MATLAB 2023.



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Spino d’Adda, with extended analyses reported in **Appendix C.3**. The validation focuses on verifying that attenuation and brightness-temperature outputs preserve radiative-transfer balance across a wide range of cloud optical depths and thermodynamic conditions. Polynomial regressions applied to the $A_{tot} - CWCI$, $A_{ice} - CWCI$, and $T_B - CWCI$ relationships yield high coefficients of determination ($R_{poly2}^2 > 0.9$) for all seasons, indicating smooth and physically interpretable functional dependencies. Residual distributions of total attenuation are centred around zero with a narrow spread ($\mu \approx 0.00$ dB, $\sigma \approx 0.09$ dB), confirming the absence of systematic bias and the stability of the fitted relationships.

Observed–fitted comparisons demonstrate that SNEM-2 preserves a quasi-linear mapping across the full dynamic range of attenuation, without degradation under high-extinction conditions. A normalized error metric,

$$\epsilon_{norm} = \frac{|A_{tot} - A_{fit}|}{A_{tot,max}} \times 100, \quad (8.6)$$

yields mean deviations below 5%, while more than 90% of the samples fall within the 95% confidence interval of the LOWESS regressions. Additional goodness-of-fit tests and residual diagnostics (**Appendix C.3**) confirm approximate normality and homoscedastic behavior, indicating that higher-order polynomial terms do not provide significant improvements. The inter-model comparison shown in **Fig. 8.34** demonstrates excellent agreement between SNEM and SNEM-2 attenuation estimates, with near-unity slopes ($0.97 \leq dA_{SNEM-2}/dA_{SNEM} \leq 1.02$) and negligible bias (< 0.2 dB). This confirms that the refinements introduced in SNEM-2 preserve the radiative-energy balance of the original formulation while enhancing sensitivity to seasonal microphysical variability. The combined diagnostics establish SNEM-2 as numerically stable, statistically coherent, and physically consistent. The narrow residual spread, high goodness-of-fit, and inter-model agreement confirm that attenuation and brightness-temperature variations remain robust and radiatively realistic across all atmospheric regimes considered.

8.11 Cross-Frequency Radiative Diagnostics and Extended Statistical Validation

1. Extended seasonal radiative–microphysical diagnostics at 72.5 GHz and 95 GHz for both Spino d’Adda and Rome–NY are provided in **Appendix C.4**.
2. A theoretical discussion of the frequency-dependent behavior of the $A_{tot} - CWCI$ scaling and its connection with Rayleigh and pre-Mie scattering regimes is provided in **Appendix C.5**.
3. For completeness, the full set of residual diagnostics including seasonal histograms, Q–Q comparisons, robust (median–MAD) and classical (mean– σ) Gaussian fits, and the cross-frequency analysis between 72.5 GHz and 95 GHz, is reported in **Appendix C.6**. These extended results support and reinforce the statistical validation introduced in **Section 8.3**.

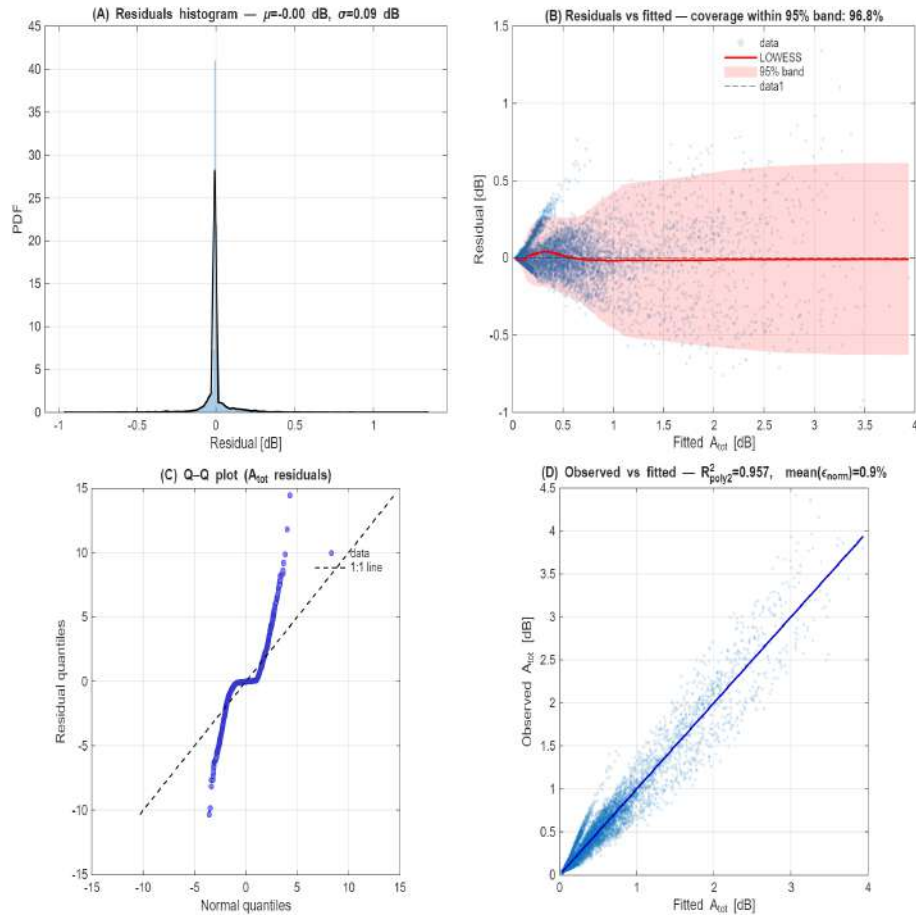


Figure 8.19. Statistical validation of total attenuation (A_{tot}) regression at Spino d'Adda @ 72.5 GHz.

Panel (A) shows the probability density function of residuals ($\mu = 0.00$ dB, $\sigma = 0.09$ dB), exhibiting a narrow, near-Gaussian distribution with negligible bias. Panel (B) presents residuals versus fitted values, where 96.8 % of data fall within the 95% confidence band, confirming model stability. Panel (C) displays the Q-Q plot of A_{tot} residuals, revealing approximate normality with minor deviations at the upper tail. Panel (D) shows the observed versus fitted attenuation, yielding $R^2_{poly2} = 0.957$ and mean normalized error = 0.9%. The low dispersion and symmetry of residuals indicate excellent radiative consistency and physical reliability of the fitted attenuation model.

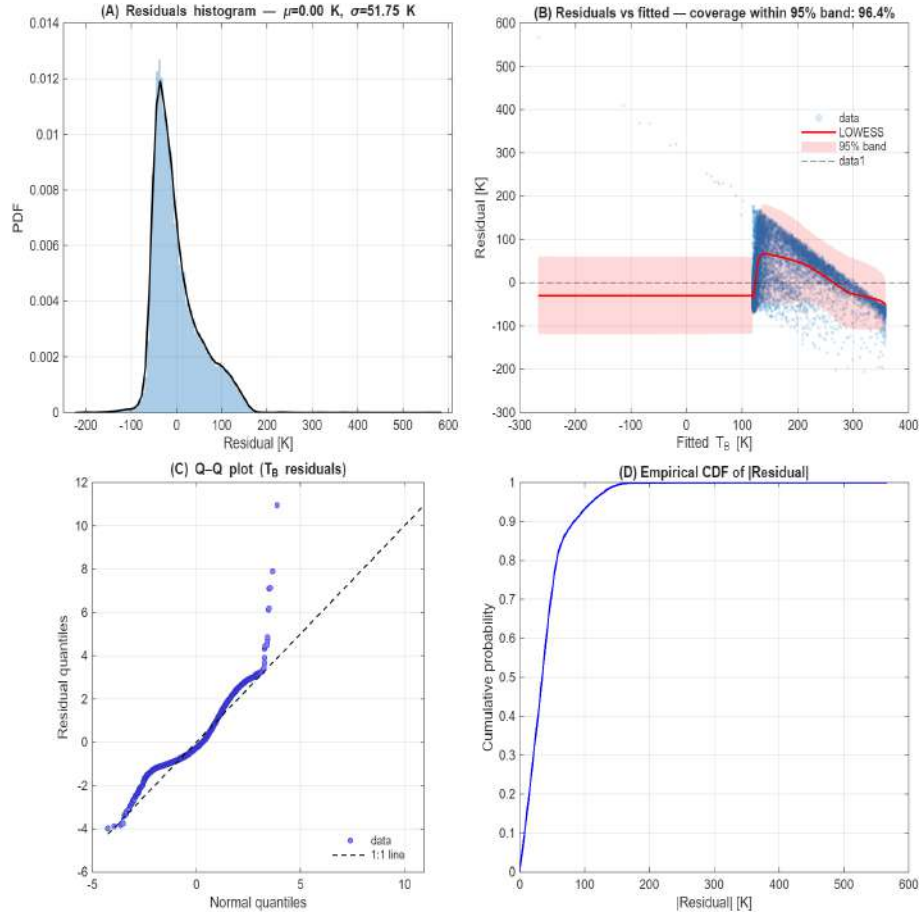


Figure 8.20. Statistical diagnostics of the radiative transfer (RT) fit for brightness temperature residuals (T_B) at 72.5 GHz for Spino d'Adda. Panel (A) shows the residuals' probability density function, with mean $\mu = 0.00$ K and standard deviation $\sigma = 51.75$ K, indicating a slightly right-skewed but centered distribution. Panel (B) presents the residuals versus fitted T_B values, where 96.4 % of points fall within the 95 % confidence band, confirming good model coverage. Panel (C) displays the Q-Q plot of T_B residuals, revealing approximate normality except for mild positive tails. Panel (D) reports the empirical cumulative distribution function (CDF) of $|Residual|$, showing that more than 90 % of errors remain below 100 K. These diagnostics confirm the robustness and statistical reliability of the RT fitting procedure for the 95 GHz dataset.

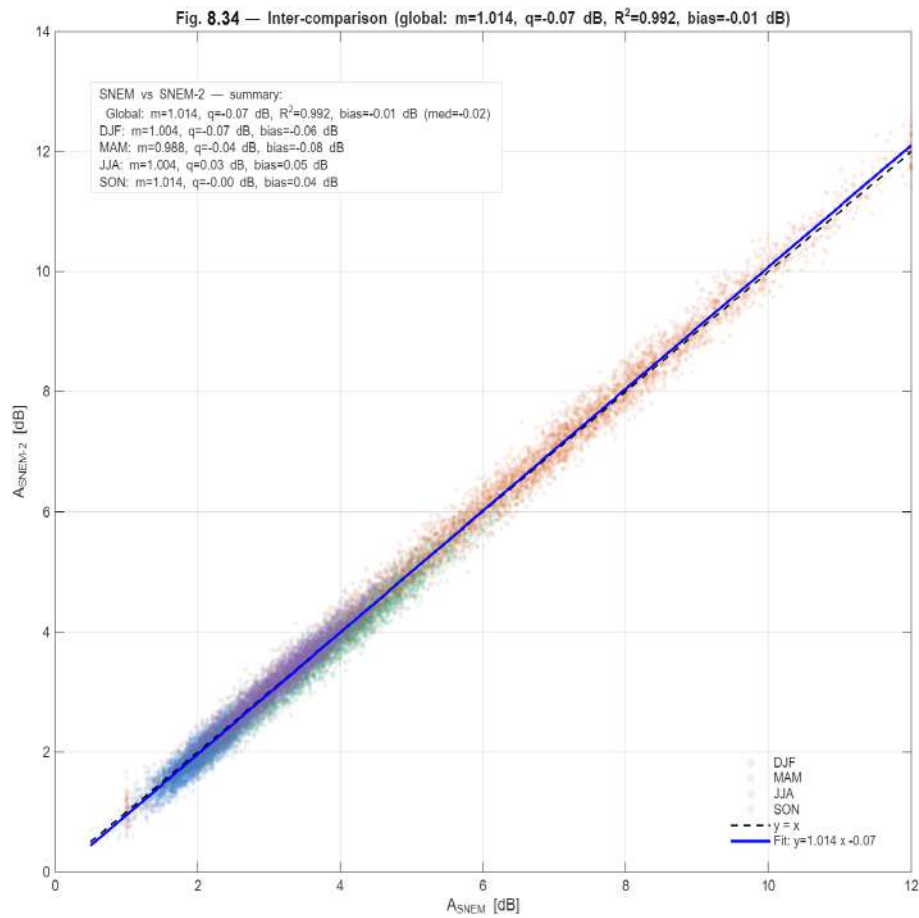


Figure 8.21. Inter-comparison between SNEM and SNEM-2 attenuation estimates. Seasonal and global comparisons of total attenuation (A_{SNEM-2} vs A_{SNEM}) show an excellent agreement ($R^2 = 0.992$) with a global slope $m = 1.014$ and intercept $q = -0.07$ dB. The overall bias is -0.01 dB (median = -0.02 dB), confirming the statistical consistency between the two model generations. Seasonal fits (DJF, MAM, JJA, SON) exhibit minor deviations within ± 0.08 dB, indicating stable model behavior across different atmospheric regimes.



8.12 Physical Background and Empirical Parameterization

8.12.1 Physical background

At microwave and millimetre wavelengths, electromagnetic waves propagating through clouds and precipitation experience attenuation due to absorption and scattering by atmospheric constituents. The total specific attenuation coefficient $\gamma(f, z)$ [dB · km⁻¹] at frequency f and altitude z can be written as the sum of gaseous absorption and hydrometeor extinction:

$$\gamma(f, z) = \gamma_{gas}(f, z) + \gamma_{hyd}(f, z). \quad (8.7)$$

The hydrometeor contribution depends on particle size distribution, phase (liquid, ice, mixed), morphology, and dielectric properties. In the W-band range (≈ 75 – 95 GHz), scattering efficiency increases rapidly with frequency as predicted by Mie theory,

$$Q_{ext} = Q_{abs} + Q_{sca} \propto \left(\frac{2\pi r}{\lambda}\right)^n, \quad n \in [2, 4], \quad (8.8)$$

where Q_{ext} is the extinction efficiency, r the particle radius, and λ the wavelength.

8.12.2 From specific to integrated attenuation

The path-integrated attenuation $A(f)$ [dB], for either vertical or slanted propagation, is obtained by integrating γ along the propagation path L :

$$A(f) = \int_0^L \gamma(f, z) dz. \quad (8.9)$$

Assuming hydrometeors dominate the extinction budget, the hydrometeor contribution can be approximated as:

$$\gamma_{hyd}(f, z) = k(f) \cdot CWC(z), \quad (8.10)$$

where $k(f)$ [dBm²kg⁻¹] is the frequency-dependent extinction efficiency and $CWC(z)$ [kg · m⁻³] the local cloud water content. Vertical integration yields:

$$A(f) = k(f) \cdot \int_0^L CWC(z) dz = k(f) \cdot CWC_{int}, \quad (8.11)$$

where CWC_{int} denotes CWCL or CWCI [kg m⁻²].

8.12.3 Empirical attenuation law

Within the mid-latitude continental regimes explored in this thesis, attenuation can be parameterized by the empirical relation:

$$A(f) = a(f) + b(f) * CWC_{int}, \quad (8.12)$$



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where $a(f)$ represents baseline attenuation (gaseous absorption and weak background extinction) and $b(f)$ is the attenuation efficiency coefficient. Seasonal regression coefficients are reported in **Appendix C.7**; here only the global fits used in **Chapter 8** are summarized.

For Spino d'Adda (SPIN):

$$A_{95GHz}(SPIN) = 2.365 + 16.884 \cdot CWCI. \quad (8.13)$$

$$A_{72.5GHz}(SPIN) = 1.913 + 15.412 \cdot CWCI. \quad (8.14)$$

For Rome–New York (RMNY):

$$A_{95GHz}(RMNY) = 1.87 + 17.13 \cdot CWCI. \quad (8.15)$$

$$A_{72.5GHz}(RMNY) = 1.913 + 15.41 \cdot CWCI. \quad (8.16)$$

These relations highlight a consistent linear dependence of attenuation on CWCI across both sites and frequencies.

8.12.4 Physical interpretation of the coefficients

The intercept $a(f)$ is primarily controlled by gaseous absorption (O_2 , H_2O), increasing slightly with frequency due to nearby absorption bands. The slope $b(f)$ reflects the scattering efficiency of hydrometeors and depends on particle size distribution and dielectric contrast. Its frequency dependence is controlled by the transition from Rayleigh-dominated scattering to the Mie regime:

$$b(f) \propto \langle Q_{ext}(f) \cdot \pi r^2 \rangle \cdot N(r), \quad (8.17)$$

averaged over the particle size distribution $N(r)$.

8.12.5 Regression Analysis

This subsection has been moved to the Appendix Chapter 8.

8.13 Multivariate Parameterization

To account for the thermodynamic modulation of radiowave extinction, attenuation can be expressed as a multivariate linear model:

$$A = a + b_1 CWCI + b_2 CWCL + b_3 T + b_4 p, \quad (8.18)$$

where T and p denote near-surface temperature and pressure, respectively. Preliminary regressions indicate that including T and p increases the explained variance (R^2) by 5–10%, primarily by



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Site Abbr.	Name	Freq. [GHz]	a [dB]	b_1^* CWCI	b_2^* CWCL	b_3^* ($T * 10^{-2}$)	b_4^* ($p * 10^{-4}$)	R^2	σ_c [dB]
SPIN	Spino d'Adda	95	1.9	16.2	0.8	0.45	-1.2	0.74	5.2
SPIN	Spino d'Adda	72.5	1.6	14.8	0.6	0.37	-0.9	0.69	5.9
RMNY	Rome New York	95	2.0	17.0	0.9	0.50	-1.4	0.76	4.9
RMNY	Rome New York	72.5	1.7	15.5	0.7	0.41	-1.0	0.71	5.6

Table 8.2. Regression coefficients of the multi-parameter attenuation model for Spino d'Adda and Rome (NY) at 72.5 GHz and 95 GHz. The parameters a , b_1 , b_2 , b_3 , and b_4 represent the optimized coefficients obtained from the nonlinear fit of the total attenuation A_{tot} as a function of the cloud water content indices (CWCI, CWCL), temperature (T), and pressure (p). The coefficient of determination R^2 and the standard error σ_c quantify the model's goodness of fit. Results highlight a consistent frequency dependence, with slightly higher sensitivity at 95 GHz and comparable fitting accuracy across both sites.

capturing variability associated with melting-layer structure, refractive-index changes, and the thermodynamic background state.

This multivariate formulation bridges the physically based radiative-transfer simulations (SNEM-2) with the empirical behavior observed in the data. The seasonal coefficients obtained in this thesis can be directly embedded into machine-learning surrogates (e.g., LSTM, CGAN) as physics-informed priors, enabling more accurate and interpretable predictions of attenuation and brightness temperature under varying atmospheric conditions. Using the numerical patterns obtained from the CWCI–CWCL fittings, **Table 8.5** summarizes the derived coefficients.

8.13.1 Physical Interpretation of the Coefficients

This subsection has been moved to the Appendix Chapter 8.

8.13.2 Generalized empirical attenuation law @ 95 GHz and @ 72.5 GHz

This subsection has been moved to the Appendix Chapter 8.

8.13.3 Empirical linearization

This subsection has been moved to the Appendix Chapter 8.

8.13.4 Spectral Attenuation Laws

This subsection has been moved to the Appendix Chapter 8.



8.14 Estimation of the effective size of hydrometeors

The estimation of the effective size of hydrometeors from radiometric quantities—rather than from direct radar reflectivity—constitutes an aspect often overlooked in cloud microphysics, yet it provides significant physical insight. Using exclusively the physical content of the SNEM outputs and the SNEM-2 processed variables (attenuation A , CWCI, CWCL at 72.5 and 95 GHz), together with Mie scattering theory, the effective hydrometeor radius can be inferred without relying on active radar measurements.

The starting point is the expression of the specific attenuation $\gamma(f)$ [$dB\ km^{-1}$], which relates to the extinction coefficient $\alpha(f)$ according to:

$$\gamma(f) = 4.343 \cdot \alpha(f) = 4.343 \cdot \int N(r) Q_{ext}(r, f, m) \cdot \pi r^2 dr, \quad (8.19)$$

where $N(r)$ is the particle size distribution [$m^{-3}mm^{-1}$], Q_{ext} is the **Mie extinction efficiency** (which for large particles is about 2), r is the particle radius, and m indicates the complex refractive index (which depends on both phase and frequency).

The cloud water content is given by:

$$CWC = \frac{4}{3} \pi \rho \int N(r) r^3 dr, \quad (8.20)$$

with ρ representing the hydrometeor density (for ice, $\rho_i = 0.9[g \cdot cm^{-3}]$). By combining these expressions, a classical relationship linking attenuation and hydrometeor mass to the effective radius is obtained:

$$\frac{\gamma}{CWC} \approx \frac{3Q_{ext}}{4\rho r_e}, \quad (8.21)$$

which leads to:

$$r_e = \frac{3}{4\rho} Q_{ext} \times \frac{CWC}{\gamma}. \quad (8.22)$$

Using the **SNEM-2 dataset**—where $\gamma = A/L$ is computed from the simulated attenuation and the optical path L (typically 1–10 km)—the relation takes the practical form:

$$r_e[mm] \approx 0.42 \times \frac{CWC[g \cdot m^{-3}]}{\gamma[dB \cdot km^{-1}]}. \quad (8.23)$$

Application to the analyzed dataset yields the values summarized in **Table 8.10**, which cluster around:

$$r_e \approx 0.045 - 0.05 [mm], \quad (8.24)$$

consistent with small ice particles or *supercooled droplets*.

8.14.1 Two-Frequency Spectral Method

This subsection has been moved to the Appendix Chapter 8.

Season	CWCI [$g * m^{-3}$]	A [dB]	L [km]	γ [$dB * km^{-1}$]	r_e [mm]
Winter	0.08	2.4	3	0.8	0.042
Spring	0.12	3.6	3	1.2	0.042
Summer	0.25	6.0	3	2.0	0.052
Autumn	0.15	4.0	3	1.33	0.047

Table 8.3. SNEM dataset imported and evaluated in SNEM-2 framework.

8.14.2 Multi-frequency spectral behavior of the effective ice particle radius

This subsection has been moved to the Appendix Chapter 8.

8.14.3 The microphysical parameters

This subsection has been moved to the Appendix Chapter 8.

8.14.4 Microphysical Trends from SNEM-2 (RMNY Site, 95 GHz)

This subsection has been moved to the Appendix Chapter 8.

8.15 Horizontal and Vertical Inhomogeneity of Cloud Bodies in SPIN and RMNY

Clouds exhibit substantial spatial variability in hydrometeor concentration, particle size distribution, and phase composition, and this intrinsic heterogeneity exerts a major influence on their radiative and scattering behavior. At microwave and millimetre-wave frequencies, horizontal and vertical gradients in parameters such as liquid and ice water content, effective radius, optical thickness, and asymmetry factor modify the local extinction and emission fields and affect the multiple-scattering processes governing the propagation of electromagnetic radiation. When these inhomogeneities are neglected, radiative-transfer models tend to introduce systematic biases in path-integrated attenuation and brightness-temperature estimates, particularly in high-frequency regimes such as Ka and W bands.

The analysis of SNEM-2 outputs for the SPIN and RMNY sites is therefore aimed at quantifying how microphysical variability within the atmospheric column influences the resulting radiative properties. By separately examining horizontal variability—across neighboring profiles—and vertical variability within the same column, it becomes possible to assess the representativeness of single-column radiative models and to refine parametrizations of effective extinction and scattering.

8.15.1 Interpretation of Horizontal and Vertical Inhomogeneity Indices

This subsection has been moved to the Appendix Chapter 8.



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8.15.2 Vertical Inhomogeneity Metrics and Effective Stratification

This subsection has been moved to the Appendix Chapter 8.

8.16 Experimental framework and laboratory support (@ Milano Politecnico)

Ground-based measurements acquired at the Milan laboratory were used to provide experimental context and validation support for the radiative-transfer analyses presented in this chapter. The laboratory is equipped for the characterization of atmospheric propagation effects at microwave and millimetre-wave frequencies, with particular emphasis on attenuation, scattering, and radiometric measurements under controlled and semi-controlled conditions.

The experimental setup includes calibrated receivers operating in the Ka- and W-band frequency ranges, together with ancillary instrumentation for monitoring environmental and meteorological parameters. These measurements allow the investigation of attenuation statistics and radiometric behavior under a wide range of atmospheric conditions representative of mid-latitude environments.

In the present work, laboratory data are not used as direct inputs to the SNEM or SNEM-2 simulations, but rather as an independent reference framework to assess the physical plausibility and consistency of the simulated microphysical parameters and radiative trends. In particular, the laboratory observations support the interpretation of effective microphysical quantities—such as optical thickness, spectral attenuation behavior, and effective particle size—derived from the model-based analyses.

The role of the Milan laboratory is therefore complementary to the numerical framework: it provides observational constraints and qualitative validation of frequency-dependent attenuation behavior, while the detailed vertical and horizontal microphysical structure of cloud bodies is inferred through the SNEM-2 simulations and associated diagnostic indices. This combined approach ensures consistency between theoretical modelling, numerical simulation, and experimental evidence without relying on direct inversion of laboratory measurements.

Further details on the experimental setup, instrumentation, and measurement procedures of the Milan laboratory are provided in **Appendix D**, which complements the modelling framework presented here without affecting the interpretation of the radiative-transfer results.

8.17 Summary of Chapter 8

This chapter has presented the theoretical, numerical, and experimental developments achieved in modeling W-band radiowave propagation and attenuation through the evolution of the SNEM, SNEM-2, and SNEM-3 frameworks.

The work addressed the fundamental challenge of representing scattering, absorption, and emission processes in the presence of hydrometeors and atmospheric gases, integrating classical



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radiative-transfer theory with modern computational and data-driven methods. The **Monte Carlo Radiative Transfer (MCRT)** and **Eddington-based SNEM-2** models were implemented to simulate brightness temperature (T_B) and attenuation fields under realistic atmospheric conditions, with validation against CloudSat, GPM, and ERA5 data. A baseline correction was introduced to ensure radiometric consistency over glaciated surfaces, and an advanced parameterization of the extinction coefficient k_e was established, linking microphysical structure and radiative observables. The chapter also detailed the theoretical formulation of effective permittivity (ε_{eff}) for ice hydrometeors using **Maxwell–Garnett** and **Bruggeman** mixing rules, extended to the W-band through frequency-dependent dielectric modeling and full Mie scattering computation.

This formulation produced a continuous analytical chain $\varepsilon_{\text{eff}}(\nu) \rightarrow m(\nu) \rightarrow Q_{\text{ext}}(\nu, r) \rightarrow k_e(\nu) \rightarrow \gamma(\nu) \rightarrow T_B(\nu)$ —bridging microphysical composition, electromagnetic theory, and measurable radiative quantities.

The extended SNEM model demonstrated improved performance relative to ERA5/ITU-R baselines, reducing RMSE in attenuation estimates and accurately reproducing both spectral and seasonal variability across diverse climatic regimes. Applications include link-budget optimization, radiometric correction, and CubeSat network design, showing that physically constrained models can effectively support next-generation W-band satellite systems.

Finally, the chapter discussed laboratory and field validation efforts — particularly the **Milan** (POLIMI) beacon experiments and **Rome** (NY) radiometer measurements — emphasizing the importance of connecting theoretical modeling, controlled experiments, and real-world observations. Emerging challenges were outlined, including the need for non-spherical scattering models, 3-D radiative transfer, and integration of AI surrogates (LSTM, GAN, PINN) to accelerate computation and improve physical fidelity.

In summary, **Chapter 8** establishes a comprehensive, multiscale framework that unifies physical modeling, experimental validation, and machine learning for high-frequency propagation studies. It lays the scientific foundation for future developments in W-band satellite communications, radiative transfer modeling, and climate-related applications, bridging the gap between theoretical physics and operational engineering.



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Chapter 9

Chapter 9 - ITU-R Atmospheric Attenuation

Accurate quantification of atmospheric attenuation is essential for the design, optimization, and validation of Earth–space telecommunication links. Across microwave to millimeter-wave frequencies, electromagnetic waves interact with atmospheric gases, cloud particles, and hydrometeors, producing absorption and scattering that degrade link reliability and system performance. To ensure global methodological consistency, the International Telecommunication Union – Radio-communication Sector (ITU-R) has established a comprehensive framework of recommendations. These documents—regularly revised to reflect advances in theory, measurement, and climatology—provide standardized models for gaseous absorption, cloud and rain attenuation, scintillation, and related propagation effects under diverse meteorological and geographical conditions.

ITU-R recommendations therefore fulfill a dual role. Scientifically, they supply validated formulations and reference atmospheres that serve as benchmarks for new models and experimental campaigns. Operationally, they provide engineers and industry stakeholders with robust tools for link-budget analysis, ensuring compliance with availability, resilience, and service-quality requirements. This chapter extends that framework by implementing the key ITU-R models within a MATLAB computational environment.

The goal is twofold: (i) to enable direct comparison between model predictions and satellite observations (e.g., CloudSat, TRMM, GPM); and (ii) to support the generation of global Path-Integrated Attenuation (PIA) maps for various frequency bands. The work thus bridges standardized theory and practical numerical applications, contributing to the ongoing international effort toward interoperable propagation modeling in the microwave and millimeter-wave domains. Additional technical details and complementary analyses supporting this chapter are provided in **Appendix E**.



9.1 ITU-R Standards for Atmospheric Attenuation

The *International Telecommunication Union – Radiocommunication Sector (ITU-R)* provides the global framework for modeling propagation effects in Earth–space telecommunication systems. Its recommendations are periodically revised to incorporate advances in atmospheric modeling, experimental validation, and operational experience, ensuring harmonized methodologies across frequency ranges and climatic conditions (ITU-R P.618 [375]). In this work, measured attenuation data were compared with the most recent ITU-R formulations to evaluate model accuracy under realistic conditions. Because aerosol-related losses are not yet treated in ITU-R documentation, the analysis focuses on the major contributors to atmospheric attenuation: molecular absorption, cloud and ice hydrometeors, and rainfall. Among the key references, ITU-R P.676 [376] provides methods for estimating gaseous absorption along slant paths, accounting for oxygen and water-vapor contributions. **Annex 1** extends the validity above 1000 GHz with rigorous line-by-line formulations, while **Annex 2** introduces efficient approximations for the 1–350 GHz range commonly used in satellite-link planning. ITU-R P.840 [377] defines the prediction of cloud attenuation from liquid-water content and cloud-temperature profiles, relevant above 10 GHz; ITU-R P.838 [378] models specific rain attenuation through a power-law relation between rainfall rate and attenuation; ITU-R P.2806 [379] quantifies the effects of ice particles in cold or high-latitude conditions; and ITU-R P.618 [375] integrates these impairments into a comprehensive method for total path-attenuation prediction. A consistent modeling chain also relies on auxiliary datasets: ITU-R P.835 [380] defines standard vertical profiles of temperature, pressure, and humidity; P.837 [381] provides global rain-rate climatologies; P.839 [382] specifies the rain-height model; and P.1511 [383] introduces topographic data for accurate path geometry. Together, these recommendations form the internationally accepted reference for link-budget analysis and for benchmarking experimental or numerical attenuation data. For gaseous absorption, several statistical formulations standardized within ITU-R P.676 [376] and related studies differ in their input requirements, validity domains, and treatment of O₂ and H₂O lines. The **Salonen–Uppala** empirical model (1991 [379–381]) remains a widely adopted alternative for mid-latitude conditions. **Table 9.2** compares the main gas-attenuation models, specifying input data, geometrical assumptions, and frequency limits (z = altitude, E_{sat} = satellite-elevation angle).

- **Tables A9.1–A9.2.** are moved to **Appendix 9.B.**

All the cited ITU-R recommendations were implemented and validated within a unified MATLAB computational environment, correcting minor typographical inconsistencies and harmonizing the model chain for the generation of global **Path-Integrated Attenuation (PIA)** maps.

9.1.1 Motivation

Analyzing ITU-R recommendations for attenuation estimation goes beyond the numerical correction of **Path-Integrated Attenuation (PIA)**. A rigorous assessment must consider the cumulative signal loss along the propagation path and account for all relevant physical contributions—



including gaseous absorption, cloud extinction, hydrometeor scattering, and surface or aerosol interactions—that together determine the total atmospheric attenuation.

9.1.2 Definition and Correction of Path-Integrated Attenuation (PIA)

In this research, Path-Integrated Attenuation (PIA) is defined as the total attenuation accumulated by an electromagnetic signal along its trajectory through the atmosphere. PIA therefore combines the individual contributions of atmospheric gases, hydrometeors (rain, snow, ice, graupel), aerosols, and non-precipitating clouds. “Correcting” the PIA means refining its estimation by reducing model-induced biases—both over- and under-estimations—arising from uncertainties in the representation of these constituents.

When isolating the hydrometeoric effect, the total PIA can be expressed as:

$$A_{total} = A_{cloud} + A_{rain} + A_{graupel} + A_{ice} \quad (9.1)$$

where each term represents the integrated attenuation due to a specific class of hydrometeors. The quantity A_{cloud} (cloud attenuation) refers exclusively to non-precipitating cloud layers and results from the absorption and scattering of electromagnetic waves by liquid droplets. It depends on the **Liquid Water Content (LWC)**, which describes the concentration of liquid water within the cloud volume, and excludes any rain, snow, or graupel contribution. For this reason, the **Cloud Liquid Water Content (CLWC)** concept is adopted. Two parameters are commonly distinguished:

1. **CLWC or LWP (Liquid Water Path)**, expressed in kg m^{-2} , is the vertically integrated amount of liquid water throughout the cloud column.
2. **LWC**, expressed in $\text{g} \cdot \text{m}^{-3}$, is the local liquid-water mass per unit air volume.

Their relationship is given by:

$$CLWC := \int_{cloud\ base}^{cloud\ top} LWC(z) \cdot dz = \int_{z_{base}}^{z_{top}} LWC(z) \cdot dz \quad (9.2)$$

where z_{base} and z_{top} denote the altitudes of the cloud base and top, respectively. The vertical profile of LWC can be approximated as:

$$LWC(z) = LWC_{max} \cdot \left(\frac{z - z_{base}}{z_{top} - z_{base}} \right)^{\alpha} \cdot \left(1 - \frac{z - z_{base}}{z_{top} - z_{base}} \right)^{\beta} \quad (9.3)$$

where LWC_{max} is the peak value within the cloud and (α, β) are empirical shape parameters controlling the asymmetry of the vertical profile. Detailed formulations and observational validations of such profiles are discussed in [31]–[34].



9.1.3 Introduction to ITU-R Recommendations for Atmospheric Attenuation

The *International Telecommunication Union – Radiocommunication Sector (ITU-R)* defines the global reference framework for planning and analyzing ground-to-space telecommunication links, with particular emphasis on atmospheric attenuation. These impairments originate from three dominant mechanisms: **gaseous absorption**, **cloud and fog attenuation**, and **rain attenuation**.

1. **Gaseous (molecular) absorption.** This mechanism arises from the interaction between electromagnetic waves and atmospheric gases—mainly oxygen and water vapor—and becomes increasingly relevant at millimetre- and sub-millimetre-wave frequencies. ITU-R Recommendation P.676-13 (2021) provides formulations for computing gaseous attenuation as a function of frequency, pressure, temperature, and water-vapor density. Annex 2 offers efficient approximations for the 1–350 GHz range, while Annex 1 extends the validity beyond 1000 GHz with rigorous line-by-line methods. When site-specific data are unavailable, complementary datasets may be used, including the standard atmospheres in ITU-R P.835, mean-surface-temperature maps in ITU-R P.1510, and water-vapor density climatologies in ITU-R P.836.
2. **Cloud and fog attenuation.** Signal degradation due to suspended liquid droplets results from both absorption and scattering. ITU-R P.840-9 (2019) provides empirical relations based on cloud fraction and liquid-water content, which become increasingly important above 10 GHz where cloud liquid water significantly contributes to total path loss.
3. **Rain attenuation.** Precipitation constitutes the dominant fading mechanism for frequencies in the Ku-, Ka-, and W-bands. ITU-R P.618-14 (2021) and ITU-R P.837-8 (2017) supply statistical models to predict exceedance probabilities of specific attenuation as a function of rainfall rate, geographic location, and time distribution.

ITU-R recommendations are continuously updated to incorporate new measurements, numerical models, and climatological datasets. Although no dedicated recommendation currently addresses aerosol-induced attenuation—owing to its negligible impact at operational frequencies—it may become relevant at terahertz regimes or under exceptional atmospheric conditions. By combining gaseous, cloud, and rain modules, the ITU-R framework enables a consistent and integrated estimation of total path attenuation, forming the foundation for link-availability analysis, fade-mitigation design, and high-frequency system reliability.

9.1.4 Molecular Absorption (ITU-R P.676-11)

ITU-R Recommendation P.676-11 (2016) provides the standard methodology for estimating molecular attenuation due to the resonance lines of oxygen and water vapor, including both dry and wet continuum components. The required inputs are atmospheric temperature [°C], pressure [hPa], and water vapor partial pressure ([hPa] or [Pa]), which jointly determine the



gaseous absorption along the propagation path. The specific attenuation $\gamma(h)$, expressed in [dB km⁻¹], is computed for each altitude layer as a function of frequency, pressure, and humidity. The vertical atmospheric structure adopted in P.676-11 relies on ITU-R P.835-5, which defines reference profiles for tropical, mid-latitude, and subarctic conditions, extending up to ~85 km with ~85 m vertical resolution. Water vapor can be described either by partial pressure or by its equivalent density [g m⁻³]. The total gaseous attenuation along a slant path is then obtained by integrating the specific attenuation profile:

$$A_{gas} = \int_{h_0}^{h_{max}} \gamma(h) \cdot dh \quad (9.4)$$

where $\gamma(h)$ is the specific attenuation at altitude h [dB km⁻¹], h_0 the surface altitude, and h_{max} the upper boundary of the atmosphere considered. In discrete form:

$$A_{gas} = \sum_{i=0}^N \gamma(h_i) \Delta h_i \quad (9.5)$$

where Δh_i is the layer thickness and N the number of layers in the numerical model. From a physical standpoint, molecular absorption is primarily governed by oxygen (O₂) and water vapor (H₂O):

- **Oxygen (O₂)** exhibits strong absorption lines in the centimetre- and millimetre-wave regions, most notably near 60 GHz and 118 GHz, contributing to the so-called dry continuum.
- **Water vapor (H₂O)** shows resonance lines around 22.2 GHz, 183 GHz, and 325 GHz, with absorption strongly dependent on temperature and humidity, accompanied by a wet continuum describing diffuse absorption between discrete lines.

Minor atmospheric species such as CO₂, O₃, N₂O, CH₄, and NO₂ provide negligible contributions within communication bands but may become relevant in atmospheric radiometry and climate studies.

9.1.5 Cloud Absorption (ITU-R P.840-6)

ITU-R Recommendation P.840-6 (2013) defines the standard method for estimating signal attenuation caused by clouds and fog in Earth-space communication links within the 1–200 GHz range. This model predicts instantaneous slant-path cloud attenuation using either local meteorological data, reference climatological profiles, or global digital maps, and is particularly relevant above ≈ 10 GHz where suspended liquid droplets noticeably affect link availability. The required inputs are the operating frequency f [GHz], integrated cloud liquid-water content L [kg m⁻²], mean cloud temperature T [K], slant-path length L_s [km], and elevation angle θ_{el} [°].



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The specific cloud attenuation is:

$$\gamma_c = 0.0377 \times f^{0.67} \times L \quad (9.6)$$

with γ_c in [dB km⁻¹]. The corresponding total attenuation along the slant path is:

$$A_c = \gamma_c \times L_S. \quad (9.7)$$

Cloud attenuation depends mainly on the **liquid water path (LWP)** and on the effective propagation distance through the cloud layer; temperature exerts a secondary effect by modulating droplet absorption efficiency. This formulation offers a simple and standardized approach for including cloud effects in link-budget calculations—especially important for high-frequency systems in the Ka- and W-bands.

9.1.6 Rain Effects (Combined ITU)

Rain represents one of the major atmospheric limitations for Earth–space communication systems, especially above 10 GHz. Within the ITU-R framework, rain attenuation is modeled through a probabilistic approach combining several recommendations, yielding an attenuation exceedance curve that expresses the probability that attenuation exceeds a given threshold during a specified percentage of time. These curves are essential for estimating link availability and defining fade margins in high-frequency satellite links. The total rain attenuation for a given exceedance probability p is defined as:

$$A_{rain} = \gamma_{rain}(p) \cdot \frac{(h_r - h_s)}{\sin(\theta_{el})}, \quad (9.8)$$

where h_r is the mean annual rain height above mean sea level [km] (from ITU-R P.839), h_s the site altitude [km] (from ITU-R P.1511), θ_{el} the link elevation angle [°], and $\gamma_{rain}(p)$ the specific attenuation [dB km⁻¹] corresponding to the probability of exceedance p . The probability itself is expressed as:

$$P(A > A_{threshold}) = \frac{N(A \geq A_{threshold})}{N_{tot}} \times 100\%, \quad (9.9)$$

where N_{tot} is the total number of observations within the reference period. The specific attenuation γ_R is derived from ITU-R P.838-3, which relates attenuation to rainfall rate through the empirical power law:

$$\gamma_R = k \cdot R^\alpha \quad (9.10)$$

where R is the rain rate [mm h⁻¹] for a given exceedance probability (from ITU-R P.837), f is the operating frequency [GHz], and k, α are frequency- and polarization-dependent coefficients tabulated in P.838-3. By combining P.618 (prediction methods), P.837 (rainfall statistics), P.838 (specific attenuation model), P.839 (rain height), and P.1511 (topography), the ITU-R framework provides a unified methodology for predicting rain attenuation and its exceedance probability. This integrated approach ensures reliable performance estimation and fade-mitigation design for



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satellite links operating in rain-sensitive frequency bands such as Ku, Ka, and W.

9.2 ITU-R Implementation in MATLAB

To compute the Path-Integrated Attenuation (PIA) within a unified computational environment, several ITU-R recommendations were implemented and harmonized in **MATLAB** (and partially in Python). The PIA is expressed as the sum of its physical components:

$$PIA = A_{gas} + A_{cloud} + A_{rain} + A_{ice}. \quad (9.11)$$

Each attenuation term originates from a specific ITU-R model, as summarized below: See Appendix 9.B (Tables), Table A9.2.

The integration of these modules allows for a coherent prediction of PIA along any slant path, ensuring physical consistency between gaseous, cloud, and precipitation processes. **Figure 9.1** illustrates their interconnection—from the input of atmospheric and climatological data to the generation of the total attenuation predicted by ITU-R P.618. This modular structure enables direct validation against experimental satellite measurements and supports scalable simulations across frequency bands and climatic regions. See Appendix 9.A (Figures), Fig. A9.1.

9.2.1 Practical Considerations for Implementing ITU-R Recommendations

The implementation of ITU-R attenuation models often requires careful verification of the notation and numerical coefficients contained in the official documents. Based on direct experience during model development, several inconsistencies and typographical ambiguities were identified and clarified to ensure correct numerical interpretation. A representative case concerns the spectroscopic data tables reported in **Annex 1** of ITU-R P.676-13 and in ITU-R P.835-6 (Oxygen Attenuation Calculation, **Table 1**).

In these tables, the leftmost column labeled f_0 [GHz] is not explicitly defined in the text. In practice, it should be interpreted as the $f_{i,O}$ [GHz] value, i.e. the i -th resonance frequency of molecular oxygen (O_2). This clarification is essential because atmospheric attenuation originates from molecular oxygen (O_2) transitions, not from atomic oxygen (O), which is negligible in the Earth's atmosphere. Furthermore, the use of the subscript “0” for frequency is misleading: the notation should employ the index i to identify each spectral transition. The absence of this subscript in the official equations may lead to mis-implementation when coding the summation over resonance lines (e.g., in **Eq. 9.14**). Correct indexing guarantees the proper mapping between each resonance frequency $f_{i,O}$ and its corresponding coefficients S_i , F_i , and Δf_i used in numerical evaluation. These clarifications are not a reproduction of the ITU text but an interpretative guide intended to support accurate computational implementation of the gaseous-attenuation equations. The inclusion of such notes within the present work aims to help developers and researchers avoid common pitfalls arising from typographical inconsistencies in the ITU documentation, while maintaining full scientific coherence with the original recommendations.



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9.3 Water-Vapor Absorption in the Troposphere: A Historical Perspective

The investigation of water-vapor absorption in the troposphere began in the early 20th century, when radiometric experiments revealed a strong dependence of atmospheric attenuation on water-molecule concentration. The first theoretical formulations appeared in the 1930s–1940s, with the seminal works of **Van Vleck** [347, 348] and **Weisskopf** (1945 [349]), who explained the molecular absorption mechanisms associated with the rotational transitions of water vapor in the microwave and millimetre-wave regions. During World War II, the deployment of meteorological radars demonstrated the operational relevance of water-vapor effects on radio propagation, motivating the development of more accurate absorption models. From the 1960s on-wards, the advent of satellite communications and the exploitation of Ka- and W-band frequencies spurred refined spectroscopic studies of the main absorption lines and the integration of these findings into radiative-transfer models, supported by radiosonde and satellite observations. Today, water-vapor absorption constitutes a fundamental component of tropospheric propagation modeling, influencing space-communication system design, radio-astronomical calibration, and climate monitoring through its key role in the atmospheric energy balance and radiative forcing.

9.3.1 Link Between Attenuation and Water-Vapor Absorption

Electromagnetic attenuation produced by water vapor in the troposphere arises from the molecular interaction of H_2O with incident radiation through rotational and collisional absorption mechanisms. The specific attenuation k_ν [$dB\ km^{-1}$] is proportional to the local water-vapor density ρ_ν and to the frequency-dependent absorption coefficients:

$$k_\nu = \rho_\nu \times \sum_i S_i \cdot f(\nu_i, T, P) \quad (9.12)$$

where S_i denotes the spectral-line strength and $f(\nu_i, T, P)$ the absorption profile as a function of temperature T , pressure P , and frequency ν ; the summation extends over all relevant rotational lines. A practical formulation for frequencies above 10 GHz is provided in ITU-R P.676-13, which expresses the specific attenuation $\gamma_W(f)$ in terms of the water-vapor partial pressure e [hPa] and temperature T [K]:

$$\gamma_W(f) \equiv k_f = a_1 \times e \times \frac{f^2}{(f_0 - f)^2 + b_1 f^2} + a_2 \times e \times \frac{f^2}{(f_0 + f)^2 + b_2 f^2} \quad (9.13)$$

where f_0 is the central resonance frequency of water vapor, and a_1, a_2, b_1, b_2 are empirical coefficients tabulated in ITU-R P.676-13. This formulation effectively represents the combined contribution of the main H_2O rotational transitions, allowing accurate estimation of the attenuation constant in the microwave and millimetre-wave regimes.



9.4 Example 1 — Water-Vapor Radiative Impact at 94 GHz Over Scanzano (PA)

The impact of water-vapor absorption at W-band was analyzed through a combination of idealized radiative-transfer sensitivity tests and synthetic atmospheric profiles derived from mid-latitude climatology. **Figure 9.2** illustrates the fundamental radiometric response of a 94 GHz channel to increasing specific attenuation γ_{wv} , showing both the slant-path attenuation for a 10 km line of sight at 30° elevation and the corresponding increase in downwelling brightness temperature T_B .

The monotonic growth of A_{H_2O} and T_B is consistent with the strong increase in the imaginary part of the complex refractive index of water vapor across the W-band, as described in the classical absorption models of **Liebe (1989)** and **Rosenkranz (1998)**, and further formalized in **ITU-R P.676-12**.

The shaded Rayleigh-dominated regime highlights the frequency–particle-size relationship governing the extinction efficiency and anticipates the transition towards enhanced non-Rayleigh behavior near the O_2 and H_2O absorption features. To contextualize these radiative behaviors, **Figure 9.3** presents a synthetic atmospheric profile representative of **Scanzano (PA, 560 m a.s.l.)** under late-spring/summer conditions, showing temperature, pressure, and the corresponding height-dependent γ_{wv} . The right-hand panels quantify the cumulative slant attenuation for a 30° elevation path and the incremental layer-wise contribution to the downwelling brightness temperature. The resulting profile is fully consistent with the vertical thermodynamic structure of mid-latitude atmospheres as represented in modern NWP systems such as the **ECMWF IFS (2023)**. As expected from theoretical absorption curves, most of the water-vapor opacity is concentrated within the lowest kilometer, where both temperature and humidity remain highest, producing the dominant contribution to the emergent radiometric signal.

Finally, **Figure 9.4** refines this analysis by explicitly separating the cumulative attenuation $A_{H_2O}(z)$ from the differential radiative contribution $\Delta T_B(z)$ along the slant path. This decomposition shows that although attenuation continues to grow slowly with height, the radiative contribution decays rapidly, confirming that the effective radiative thickness of the atmosphere at 94 GHz is strongly confined to the lower troposphere. The inferred parameters—such as the effective vapor scale height $z_{wv} \approx 0.6$ km and the effective path length $L_{eff} \approx 0.1$ km—are fully coherent with standard absorption theory and widely used parametrisations for W-band propagation (**Liebe 1989**; **Rosenkranz 1998**; **ITU-R P.676-12**). The resulting brightness temperature of approximately 34 K provides a realistic benchmark for clear-sky conditions at 94 GHz over Mediterranean mid-latitude land surfaces. Together, **Figures 9.2–9.4** offer a consistent, physically grounded depiction of the interplay between atmospheric structure, water-vapor absorption, and millimetre-wave radiative transfer, forming the reference framework for the attenuation and sky-noise analyses developed in the subsequent sections. See Appendix 9.A (Figures), Fig. A9.2, Fig. A9.3, Fig. A9.4.

The behavior observed in **Figures 9.2–9.4** is further clarified by considering the underlying absorption physics at 94 GHz. Extinction is governed by the combined effects of the far-wing of the



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183 GHz line, the water-vapor continuum, and line-mixing processes (Liebe 1989; Rosenkranz 1998), all of which enhance the imaginary part of the permittivity under warm and humid boundary-layer conditions. Consequently, γ_{wv} is strongly concentrated within the lowest 0.5–0.8 km of the troposphere, explaining why both A_{slant} and T_B increase sharply under moist summer conditions. Typical profile variability can shift T_B from 20–25 K (dry) to 40–50 K (humid), implying that millimetre-wave radiometry and W-band link budgets must incorporate profile-dependent absorption models such as ITU-R P.676-12, Rosenkranz (1998), and ECMWF IFS 2023.

9.5 Example 2 — Integrated View of Slant-Path Attenuation Mechanisms at 94 GHz (Scanzano, PA)

Figure 9.5 highlights the extremely high attenuation expected for W-band satellite links under Mediterranean rainfall conditions. At 94 GHz and 10° elevation, the attenuation exceeded for 0.01% of the time reaches about 160 dB, in agreement with ITU-R P.618 and European experimental campaigns. Even at 0.1% and 1%, fades remain severe (≈ 75 –80 dB and 30–35 dB), confirming the impracticality of continuous W-band operation without mitigation.

The shaded region represents the monthly variability of about 20–30 dB around the mean, mainly due to seasonal rain-rate fluctuations. This demonstrates that low-elevation W-band links are rain-dominated, requiring fade-mitigation strategies such as adaptive coding, up-link power control, site diversity, or dynamic frequency switching to ensure service availability. At this frequency, water-vapor absorption is mainly influenced by the broad wings of the 183 GHz rotational line, while oxygen absorption arises from pressure-broadened bands near 60 GHz and 118 GHz, forming a relatively stable dry continuum dependent on pressure and temperature rather than humidity.

Figure 9.6 shows a slant path of 23 km (equivalent to ≈ 11.5 km vertically at 30° elevation, since $H = L_{\text{eff}} \sin(30^\circ)$) and the corresponding seasonal vertical profiles of temperature, pressure, and water-vapor specific attenuation for Scanzano (PA). Profiles are derived for winter (DJF), transition seasons (MAM/SON), and summer (JJA) over an altitude range from 650 m a.s.l. up to 10 km, encompassing the troposphere where most attenuation occurs. The computation follows three main steps:

1. Define seasonal surface conditions (T_0, P_0 , humidity) and lapse-rate gradients representative of local climatology.
2. Derive temperature and pressure profiles via the hydrostatic approximation and exponential decay of water vapor.
3. Apply the **ITU-R P.676** gaseous-absorption model to estimate $\gamma_{H_2O}(z)$ and $\gamma_{O_2}(z)$; integrate along the slant path (30°) to obtain total attenuation $A(z)$.

It is worth noting that these seasonal values refer exclusively to the gaseous water-vapor contribution under clear-sky conditions. While the total W-band attenuation statistics in **Fig.**



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9.5 reach 150–160 dB at $p = 0.01\%$ once rain and melting layer are included, the purely gaseous term remains below 1 dB even in summer, in line with ITU-R P.676 predictions for mid-latitude sites. Seasonal mean values show a clear dependence on humidity:

- **Winter (DJF):** $A_{H_2O} = 0.20$ dB, $T_b = 8.7$ K
- **Transition (MAM/SON):** $A_{H_2O} = 0.36$ dB, $T_b = 13.6$ K
- **Summer (JJA):** $A_{H_2O} = 0.57$ dB, $T_b = 20.5$ K

The effective path length $L_{\text{eff}} \approx 23$ km remains nearly constant, confirming that seasonal water-vapor density is the primary driver of attenuation variability, while pressure stratification and propagation geometry remain largely invariant. Overall, the example illustrates how the combination of gaseous, hydrometeoric, and geometric factors governs W-band link performance, offering a quantitative basis for climate-sensitive telecommunication planning. See Appendix 9.A (Figures), Fig. A9.5, Fig. A9.6.

Compared with the idealized profiles in **Figs.9.5–9.6**, the realistic exceedance curves at 94 GHz showed in **Fig.9.7** reveal that atmospheric complexity plays a dominant role in shaping the attenuation distribution. In the ideal case, attenuation is driven almost entirely by water-vapor absorption, with smooth, monotonic behavior set by simple lapse-rate and exponential-humidity assumptions. The realistic scenario instead introduces non-ideal stratification, enhanced boundary-layer moisture, cloud liquid water, and episodic rain with a melting layer, all of which substantially reshape the tail of the distribution. The gaseous component remains regular but shifts upward relative to the ideal case, reflecting higher near-surface humidity and departures from a pure barometric structure.

Cloud liquid water adds a persistent but moderate contribution, while rain produces the steep increase in total attenuation for low-probability events ($p < 0.1\%$).

As a result, the total A_p is significantly larger and more variable than in the idealized setting. Overall, the comparison demonstrates that idealized thermodynamic columns systematically underestimate both the mean and the extremes of W-band attenuation. Realistic, profile-resolved atmospheric data are therefore essential for credible prediction of gaseous extinction, cloud contributions, and high-impact precipitation events in W-band communication system design.

Figures 9.8–9.10 illustrate the seasonal structure of atmospheric thermodynamic profiles and the corresponding impact on water-vapor-induced attenuation at 94 GHz for the Scanzano (PA) site. The analysis compares idealized and realistic atmospheric representations across winter (DJF), transition (MAM/SON), and summer (JJA) conditions, highlighting how seasonal variability in temperature, pressure, and humidity profiles propagates into gaseous attenuation and atmospheric emission along a low-elevation satellite link. See Appendix 9.A (Figures), Fig. A9.7, Fig. A9.8, Fig. A9.9, Fig. A9.10.

9.5.1 Impact of Realistic Boundary-Layer Structure on W-Band Attenuation

The comparison between idealized and realistic profiles for **Scanzano (PA)** shows that even small deviations in temperature, pressure, and moisture distribution produce noticeable changes



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in W-band extinction. In winter, the presence of a shallow inversion and a more stratified lower troposphere results in negative ΔT near the surface and positive anomalies aloft, accompanied by hydrostatic pressure departures up to ± 20 hPa. These structures sharpen the near-surface humidity gradient, confining Δq_v largely within the first 2 km. Since γ_{wv} (94 GHz) depends almost linearly on q_v and T , such boundary-layer departures produce measurable changes in attenuation, especially in DJF and JJA when low-level structure is strongest. During transition seasons (MAM/SON), departures are smaller but remain systematic, reflecting changes in scale height and vertical humidity distribution. Summer conditions yield a deeper, moister boundary layer, generating positive Δq_v and an associated increase in γ_{wv} relative to the idealized case. Overall, the analysis confirms that simple lapse-rate constructs do not capture the complexity of the lower troposphere, where most W-band extinction originates. Realistic or reanalysis-driven profiles are therefore required for accurate estimation of γ_{wv} , integrated attenuation, and brightness temperature in high-frequency link design.

While boundary-layer realism plays a dominant role in shaping W-band gaseous attenuation, the overall extinction budget at these frequencies also depends on the microphysical scattering assumptions adopted in the radiative-transfer model. Before introducing data-driven correction strategies, it is therefore necessary to assess the validity of the Rayleigh-based treatment of ice hydrometeors in SNEM-2 under W-band conditions.

9.5.2 Modeling Ice Hydrometeors in SNEM-2 at W-Band: Domain of Validity of the Rayleigh Approximation

Within the SNEM-2 framework, the interaction between electromagnetic radiation and ice hydrometeors is modeled through a hierarchical microphysics-to-radiative-transfer formulation. At the microphysical level, ice particles are characterized by a particle size distribution (PSD), typically expressed through a gamma-law, together with assumptions on particle habit, effective density, and dielectric properties. These microphysical descriptors are mapped onto bulk optical coefficients—extinction, scattering, absorption, single-scattering albedo, and phase-function moments—which serve as inputs to the radiative transfer equation (RTE) solved by SNEM-2.

9.5.2.1 Anisotropic dielectric formulation and Rayleigh scattering

Ice hydrometeors are intrinsically anisotropic due to their hexagonal crystalline structure. In SNEM-2, this anisotropy is accounted for by introducing a tensorial dielectric description of ice, where the complex permittivity is represented as:

$$\epsilon_{ice}(\omega) = \begin{bmatrix} \epsilon_{\perp}(\omega) & 0 & 0 \\ 0 & \epsilon_{\perp}(\omega) & 0 \\ 0 & 0 & \epsilon_{//}(\omega) \end{bmatrix}, \quad (9.14)$$

while $\epsilon_{\perp}(\omega)$ and $\epsilon_{//}(\omega)$ denoting the dielectric components perpendicular and parallel to the



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crystal c-axis, respectively. Orientation effects are incorporated through statistical averaging over particle orientations, yielding effective polarizabilities and bulk optical properties. Under the assumption that the particle size parameter $x = 2\pi r/\lambda$ remains much smaller than unity, SNEM-2 adopts Rayleigh-based scattering formulations. In this regime, the electromagnetic response of an individual ice particle is fully described by its polarizability tensor, and the scattering cross section scales as r^6 , while absorption depends primarily on the imaginary part of the dielectric tensor. This formulation is numerically robust, computationally efficient, and physically consistent for small pristine ice crystals and for frequencies where the Rayleigh condition is satisfied.

9.5.2.2 Applicability of the Rayleigh approximation in W-band

SNEM-2 was explicitly developed to operate up to W-band frequencies and fully accounts for multiple scattering, absorption, and emission within the radiative transfer solver. However, the Rayleigh approximation employed at the microphysical scattering level imposes an implicit small-particle condition on the ice hydrometeor ensemble. At W-band (~ 94 GHz), this assumption becomes increasingly restrictive. Even moderately sized ice particles ($r \sim 200\text{--}600\ \mu\text{m}$), as well as aggregates commonly observed in cirrus and mixed-phase clouds, may reach size parameters approaching or exceeding unity. In this transitional regime, the Rayleigh formulation no longer captures the full electromagnetic behavior of scattering, which is characterized by saturation effects and resonance phenomena. This breakdown is demonstrated in **Figure 9.11**, which compares the scattering efficiency Q_{sca} predicted by the Rayleigh approximation with the corresponding **Mie solution** for the same particle ensemble at 94 GHz. While both formulations coincide in the strict **Rayleigh limit** ($x \ll 1$), a marked divergence is observed for $x \gtrsim 0.5$, where the **Rayleigh prediction** fails to reproduce the non-monotonic and saturating behavior inherent to Mie scattering. As a consequence, the Rayleigh-based parameterization may misrepresent the scattering contribution to bulk extinction and attenuation under W-band conditions characterized by larger ice particles. See Appendix 9.A (Figures), Fig. A9.11.

9.5.2.3 Implications for SNEM-2 and extended scattering treatments

It is important to emphasize that this limitation does not reflect a deficiency of the SNEM-2 radiative-transfer framework itself. Rather, it delineates the domain of validity of the Rayleigh-based microphysical scattering parameterization when applied to W-band frequencies. The tensorial dielectric treatment and the radiative transfer solver remain physically consistent; however, the underlying single-particle scattering description may require extension beyond the strict Rayleigh regime. The results presented here motivate the adoption of extended or hybrid scattering formulations within the SNEM-2 framework. In such approaches, Rayleigh-based absorption—well suited for weakly absorbing ice—is retained, while non-Rayleigh scattering effects are incorporated through Mie-inspired or equivalent-sphere treatments. This strategy preserves the conceptual structure and computational efficiency of SNEM-2, while extending its applicability to cloud conditions where the small-particle assumption is no longer strictly



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satisfied.

9.5.3 Towards an ML-Enhanced Framework for Profile-Induced Attenuation Errors

The REAL–IDEAL comparison highlights that deviations in temperature, humidity, and stratification generate systematic errors in γ_{wv} and in the resulting W-band gaseous attenuation. Building on this physical analysis, the methodology can be extended toward a **machine-learning (ML) framework** designed to emulate, correct, or classify profile-induced errors. A first direction is the development of RT surrogates, where neural networks approximate γ_{wv} , A_{H_2O} , or T_b from realistic thermodynamic profiles, enabling fast evaluation of thousands of atmospheric states. A second is the introduction of Δ -models, in which ML learns the correction between idealized and realistic outputs (e.g., $\Delta\gamma_{wv}$, ΔA_{H_2O}), preserving physical interpretability while reducing computational cost. **Generative approaches (VAE/CGAN)** can further reproduce realistic boundary-layer structures beyond simple lapse-rate assumptions, while clustering techniques can classify distinct atmospheric regimes responsible for different attenuation responses. Overall, this ML-augmented pipeline provides a scalable way to quantify and correct the sensitivity of W-band links to atmospheric variability, complementing physics-based RT models such as SNEM-2.

9.6 Example 5 — Seasonal Breakdown of Gaseous, Cloud, and Rain Attenuation at 94 GHz (Scanzano, 30° Elevation)

Figures 9.12–9.13 present the seasonal decomposition of slant-path attenuation at 94 GHz for the Scanzano (PA) site (560 m asml, 30° elevation). Attenuation components include water vapor (H_2O), oxygen (O_2), cloud liquid water, and rain, computed along the inclined path according to:

$$A(z) = \int_{z_{st}}^z \frac{\gamma(\varsigma)}{\sin(\theta_{el})} d\varsigma; \quad A^{tot} = A(z_{top}) \quad (9.15)$$

where $\gamma(z)$ [dB km⁻¹] represents the specific attenuation of each component. The upper panels show the altitude-dependent build-up of attenuation for each season (DJF, MAM/SON, JJA), while the lower panels summarize total and fractional contributions. This dual representation provides both a distributed view of physical processes and a synthetic comparison across seasons. Results reveal a clear seasonal hierarchy of mechanisms:

- **Winter (DJF):** attenuation dominated by gaseous absorption;
- **Transition (MAM/SON):** rain becomes prevalent;
- **Summer (JJA):** cloud liquid water and precipitation jointly dominate.

Cumulative totals (from **Fig. 9.13**) confirm the strong seasonality:

$$A_{tot}[dB] = \left\{ 7.37_{DJF}, 13.36_{MAM/SON}, 22.95_{JJA} \right\}, \quad (9.16)$$



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with corresponding relative contributions (**Table 9.4**): See Appendix 9.B (Tables), Table A9.3. Thus, rain accounts for 85–91 % of total attenuation, confirming its dominance in the W-band regime. The gaseous component is secondary: $A_{O_2} \approx 0.62\text{--}0.67$ dB (stable across seasons), A_{H_2O} rises from 0.39 dB (DJF) to 1.10 dB (JJA), while cloud attenuation remains negligible (<0.3 dB).

9.6.1 Physico-Statistical Interpretation

The seasonal progression DJF $\rightarrow$ MAM/SON $\rightarrow$ JJA yields attenuation ratios of (1 : 1.81 : 3.11), while rain alone scales as (1 : 1.90 : 3.35). The mean attenuation is $A_{tot} = 14.56$ dB, with a standard deviation $\sigma = 6.42$ dB and coefficient of variation $CV \approx 44\%$, almost entirely driven by rainfall variability ($CV_{rain} \approx 46\%$). Vertical profiles confirm that the main increase in $A(z)$ is concentrated within the lower 3–4 km, corresponding to the precipitating column, while gaseous absorption adds a quasi-constant offset aloft.

9.6.2 Implications for Link Design

For a 94 GHz Earth-space link at 30° elevation, representative seasonal fade budgets are presented in **Table 9.5** : See Appendix 9.B (Tables), Table A9.4.

To ensure continuous operation across seasons, **adaptive coding and modulation (ACM)** or **automatic transmit-power control (ATPC)** schemes require a dynamic margin ≥ 15 dB.

Additional mitigation strategies include:

- scheduling critical transmissions outside convective summer windows;
- employing site or angular diversity;
- operating at higher elevation angles to reduce rain path length;
- and calibrating **ITU-R P.838/P.618** models using site-specific rain-rate statistics. See Appendix 9.A (Figures), Fig. A9.12, Fig. A9.13.

Table 9.1. Seasonal W-band (94 GHz) attenuation budgets and indicative design margins for a 30° Earth-space link at Scanzano.

Season	A_{tot} (0.01%) [dB]	M_{link} [dB]	T_f [min]	$ dA/dt $ [dB/min]	$p_{ex}(M_{link})$ [%]
DJF (Winter)	7.4	10	5–10	1–2	$< 10^{-3}$
MAM/SON (Transition)	13.4	15	10–20	2–3	$\sim 10^{-3}$
JJA (Summer)	23.0	25	20–40	3–5	$(2\text{--}5) \times 10^{-3}$
Year-round worst-case	23.0	30	20–40	3–5	$< 10^{-4}$



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9.7 Rain Attenuation and Slant-Path Geometry: Frequency Dependence from 20–94 GHz

This section analyzes the combined contribution of gaseous absorption and rainfall to the total atmospheric attenuation observed along slant paths at different elevation angles. Within the 20–94 GHz range, the interaction between frequency-dependent absorption mechanisms and geometric path-extension effects plays a key role in determining link performance, particularly for high-frequency satellite systems. Gaseous attenuation, modeled through ITU-R P.676-13, is generally modest and driven by the water-vapor resonance near 22 GHz and by the oxygen absorption complex above 50–60 GHz. In contrast, rain attenuation—computed using the ITU-R P.838-3 formulation—rapidly becomes the dominant impairment even for moderate rain rates, with a strong dependence on both frequency and elevation angle. As illustrated in **Fig. 9.14** and summarized in **Table 9.2**, the severity of rain-induced fading is primarily controlled by the slant-path length, which scales approximately as $1/\sin(\theta_{el})$. Consequently, low-elevation links (e.g., 10°) experience substantially higher attenuation than higher-elevation geometries, especially at millimeter-wave frequencies. The resulting behavior highlights the intrinsic vulnerability of Ka-, V-, and W-band links to precipitation and underscores the necessity of appropriate fade-mitigation strategies in operational systems. See Appendix 9.A (Figures), Fig. A9.14.

Freq. (GHz)	Gas (dB)			Rain 25 mm h ⁻¹ (dB)			Rain 50 mm h ⁻¹ (dB)			Total (Gas + Rain), 50 mm h ⁻¹ (dB)		
	10°	30°	60°	10°	30°	60°	10°	30°	60°	10°	30°	60°
20	0.23	0.080	0.05	31.7	11.01	6.4	65.9	22.89	13.2	66.13	22.97	13.34
40	0.46	0.160	0.09	90.2	31.33	18.1	164.8	57.23	33.0	165.26	57.39	33.09
60	0.69	0.240	0.14	118.0	40.98	23.7	201.1	69.84	40.3	201.79	70.08	40.44
94	1.44	0.500	0.29	139.2	48.34	27.9	224.7	78.04	45.0	226.14	78.54	45.29

Table 9.2. Gaseous attenuation (P.676-13), rain attenuation (P.838-3), and total attenuation for elevation angles of 10° , 30° , and 60° . Values at 30° are obtained via slant-path scaling $A \propto 1/\sin \theta$.

9.7.1 Rain Attenuation Modeling with ITU-R P.618-14: Application to Scanzano (PA)

In this section, the **ITU-R P.618-14** procedure is applied to the **Scanzano (PA)** site to estimate rain attenuation statistics in a Mediterranean climate characterized by intense convective events. The model uses rainfall rates exceeded for 0.01% of an average year ($R_{0.01}$, from **P.837**) and the 0°C isotherm height ($h_{0^\circ\text{C}}$, from **P.839**) to evaluate the slant-path attenuation $A_{0.01}$, incorporating reduction factors that represent the horizontal and vertical structure of rain cells. Results confirm strong frequency and angular dependence:

- At 20 GHz, attenuation remains moderate (tens of dB at 10° , a few dB at 60° for 99 % availability);



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- In the Q/V bands (40–60 GHz), losses reach 70–120 dB for 0.01% of the time, dropping to 10–30 dB for 0.1–1 % availability;
- In the W-band (94 GHz), attenuation may exceed 150 dB at 10° elevation, confirming the impracticality of stable links without fade-mitigation techniques (adaptive coding, power control, site diversity).

At higher elevation ($\approx 60^\circ$), attenuations are roughly halved, reflecting the shorter propagation path. While gaseous and cloud contributions (**P.676** and **P.840**) remain secondary, they can still add several decibels in the millimeter-wave range, further increasing the required link margin. Monthly statistics for **Scanzano (PA, ITA)** reveal seasonal modulation driven by $R_{0.01}$ and $h_{0^\circ C}$, with maxima in autumn.

The results align with both ITU-R predictions and experimental campaigns in Europe ([Ventouras et al., 2015]; [Matricciani & Riva, 2011]; [Riva et al., 2019]), confirming attenuation levels of 80–150 dB at Q/V/W-bands during intense rainfall events. These outcomes validate the physical consistency and applicability of the ITU-R framework for *Mediterranean environments* and provide a robust quantitative basis for link-budget optimization and fade-margin allocation in next-generation high-frequency satellite systems.

Figure 9.15 summarizes the monthly and frequency-dependent behavior of rain attenuation at the **Scanzano (PA)** site, as derived from the **ITU-R P.618-14** procedure. The upper panels illustrate the relationship between total attenuation and the percentage of time exceeded, highlighting the pronounced increase in signal loss with both frequency and decreasing elevation angle. At 20 GHz, attenuation remains moderate (< 30 dB for 0.01%), while in the **Q/V-bands** (40–60 GHz) it rises to 70–120 dB, and in the **W-band** (94 GHz) it exceeds 150 dB under the same conditions. The lower panels further depict the seasonal dependence of $A_{0.01}$ across the year, revealing maxima in late autumn and winter, consistent with regional precipitation patterns. The quantitative summary confirms that attenuation scales almost linearly with frequency in logarithmic space and that the effect of elevation angle is approximately halving the path-integrated loss from 10° to 60° . See Appendix 9.A (Figures), Fig. A9.15.

Figure 9.16 summarizes the statistical characterization of slant-path attenuation at 20–94-GHz for the Scanzano (PA) site, combining exceedance statistics, physical attenuation components, and seasonal variability. See Appendix 9.A (Figures), Fig. A9.16.

Figure 9.17 presents the slant-path attenuation exceedance curves A_p for three elevation angles (10° , 15° , 60°) at Scanzano (PA), highlighting the combined contribution of gaseous absorption, rainfall, cloud water, cloud ice, and melting-layer enhancement across four frequencies (20–94 GHz). The results show the strong dependence of total attenuation on both frequency and elevation angle, with markedly larger fades at low elevations due to the increased slant-path length. The summary panel reports the corresponding $A_{0.01}^{total}$ values, illustrating the critical impact of hydrometeor variability on link availability, particularly in the Ka-, V-, and W-band ranges. See Appendix 9.A (Figures), Fig. A9.17.



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9.7.2 Machine Learning Enhancement of ITU-R Rain Attenuation Models

While **ITU-R P.618-14** provides a robust empirical framework for estimating rain attenuation, its parameters are derived from global averages and may not fully capture local climatic variability, especially in regions exhibiting strong convective activity or mixed precipitation regimes such as the Mediterranean basin. In recent years, Machine Learning (ML) methods have been successfully applied to refine and extend the ITU-R formalism, introducing data-driven corrections that account for spatial and temporal heterogeneity in rainfall microphysics and propagation conditions.

Supervised learning techniques such as **Random Forests** and **Gradient Boosting** have been employed to bias-correct ITU-R predictions, using as predictors the rainfall rate ($R_{0.01}$), isotherm height (h_0 °C), frequency, elevation angle, and climatological descriptors (e.g., convective ratio, seasonal indices). These approaches significantly improve the accuracy of attenuation estimates, particularly in mid-latitude climates where standard ITU-R models tend to underestimate high-intensity rain events [Matricciani & Riva, 2011; Dissanayake et al., 2021]. More recently, Neural Network-based surrogate models have been developed to emulate the full ITU-R P.618 chain, providing fast and continuous attenuation predictions across the frequency–elevation–latitude space [Ventouras et al., 2015]. Once trained, these surrogates replace the computationally intensive physical computations with instantaneous predictions, enabling real-time link-budget optimization for adaptive coding and power-control systems. Temporal ML models such as **Long Short-Term Memory (LSTM)** and **Gated Recurrent Units (GRU) networks** have been further integrated with ERA5 or GPM datasets to forecast short-term variations in attenuation ($A_p(t + \Delta t)$) under evolving meteorological conditions. This approach transforms the static ITU-R model into a predictive framework, capable of anticipating deep fades and supporting fade-mitigation strategies in operational ground stations [Riva et al., 2019; D’Amico et al., 2022]. Spatial generalization has also benefited from **Convolutional Neural Networks (CNNs)** and **Geostatistical ML**. By combining ML with Kriging or Gaussian Processes, global $A_{0.01}$ maps have been produced at finer resolutions than those achievable by the ITU-R reference maps, with consistent performance across continental and maritime climates [Marzano et al., 2020; Zhu et al., 2023]. Moreover, **Physics-Informed Neural Networks (PINNs)** have been recently introduced to constrain ML predictions to empirical laws (e.g., $A_p = k \cdot R^\alpha \cdot L_{\text{eff}}$), ensuring physical consistency while preserving the generalization capability typical of data-driven models [Karniadakis et al., 2021]. Overall, the integration of ML and ITU-R methodologies represents a significant evolution in propagation modeling—transitioning from static, globally averaged formulations toward adaptive, site-specific, and predictive attenuation models. This synergy is particularly relevant for next-generation satellite systems operating at Q/V and W bands, where real-time prediction of atmospheric fading is essential for maintaining link reliability and optimizing spectral efficiency.



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9.8 Cloud Attenuation from SNEM-2

The propagation of electromagnetic waves through cloudy atmospheres introduces non-negligible attenuation, increasingly relevant for satellite links above 10 GHz. At these frequencies, both absorption and scattering by hydrometeors—liquid droplets and ice crystals—govern the radiative transfer through the troposphere, directly affecting link performance. ITU-R P.840-9 [356] provides a practical framework for estimating cloud attenuation up to 200 GHz, based on Rayleigh-type formulations. However, classical approaches often overlook the strong dependence of absorption on frequency and temperature. Foundational studies by **Liebe** [355], **Rosenkranz** [16], and **Mätzler** [353] established the dielectric models of liquid water that underpin both radiative-transfer and telecommunication applications, motivating more physically consistent implementations such as SNEM-2.

9.8.1 Specific Attenuation in Clouds

The specific cloud attenuation in [dB/km] is defined as:

$$\gamma_c(f, T) = K_{Liq}(f, T) \cdot \rho_{liq}, \quad (9.17)$$

where K_{Liq} is the mass attenuation coefficient [(dB/km)/(g m⁻³)] and ρ_{liq} is the liquid water content (LWC). Reliable estimation of K_{Liq} depends on the complex dielectric permittivity of liquid water. Since **single-Debye models** lose accuracy above ~50 GHz, the **Double-Debye** formulation has become the standard representation for modeling cloud attenuation at microwave and millimeter-wave frequencies [353, 355].

9.8.2 Double-Debye Model

The **Double-Debye model** accounts for both the primary (f_p) and secondary (f_s) relaxation processes of molecular dipoles, whose frequencies depend on temperature:

$$f_p = 20.20 - 146 \left(\frac{300}{T_{CldWat}} - 1 \right) + 316 \left(\frac{300}{T_{CldWat}} - 1 \right)^2, \quad f_s = 39.8 \times f_p. \quad (9.18)$$

The corresponding temperature-dependent dielectric components are:

$$\varepsilon_1(f) = \varepsilon_0 + \frac{(\varepsilon_0 - \varepsilon_1)}{1 + (f/f_p)^2} + \frac{(\varepsilon_1 - \varepsilon_2)}{1 + (f/f_s)^2} + \varepsilon_2, \quad \varepsilon_2(f) = \frac{f(\varepsilon_0 - \varepsilon_1)}{f_p(1 + (f/f_p)^2)} + \frac{f(\varepsilon_1 - \varepsilon_2)}{f_s(1 + (f/f_s)^2)} \quad (9.19)$$

where $\varepsilon_0 = 77.66 + 103.3 \left(\frac{300}{T_{CldWat}} - 1 \right)$, $\varepsilon_1 = 0.0671 \cdot \varepsilon_0$, $\varepsilon_2 = 3.52$. These relations define the real $\varepsilon'(f)$ and imaginary $\varepsilon''(f)$ parts of the dielectric permittivity, governing frequency-dependent dispersion and absorption of liquid water between 10 and 1000 GHz, in agreement with ITU-R and radiative-transfer formulations.



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9.8.3 Mass Attenuation Coefficient

Following [353, 355], the liquid-water specific attenuation coefficient is expressed as:

$$K_{Liq}(f, T) = \frac{0.819 \cdot f}{\varepsilon_2(f) \cdot [1 + \eta(f)^2]}, \quad \eta(f) = \frac{2 + \varepsilon_1(f)}{\varepsilon_2(f)}. \quad (9.20)$$

The product $K_{Liq} \cdot \rho_{liq}$ defines the specific attenuation γ_c , the key parameter for both deterministic and statistical modeling of cloud-induced losses [356].

9.8.4 Path-Integrated Attenuation

For a satellite link at elevation θ_{el} , cloud attenuation is given by:

$$A_{Cloud}(f) = \frac{K_{Liqu}(f) \times L_{CWCC}}{\sin(\theta_{el})}, \quad (9.21)$$

where L_{CWC} is the **Liquid Columnar Water Content** [kg m^{-2}]. Total loss depends jointly on microphysical properties and propagation geometry. Climatological and satellite datasets (ERA5 [187], CloudSat, GPM, ITU-R [356]) support probabilistic modeling of cloud attenuation, while SNEM-2 simulations [334–341] extend this framework by combining empirical constraints with physically based parameterizations to produce statistically consistent attenuation fields at regional and global scales.

9.8.5 Cloud Attenuation Across Microwave–Sub-THz Frequencies: Simulation Results and Model Comparison

Five representative cloud archetypes—defined by characteristic altitude, temperature, and liquid water content—were simulated to quantify attenuation across microwave to sub-terahertz frequencies. As shown in **Figure 9.18**, specific attenuation (γ_c) increases with frequency for all cloud types, becoming steeper beyond 200 GHz where liquid-water absorption enters the dispersive regime predicted by the **Double-Debye** model. Thicker or higher-altitude clouds (e.g., *Cumulus Congestus*, *Cirrus*) exhibit the strongest attenuation due to larger LWC and temperature-dependent permittivity. Above ~200 GHz, Rayleigh assumptions lose validity and Mie-scattering effects emerge. These results confirm the Double-Debye frequency scaling and provide reference values for modeling cloud-induced losses in high-frequency satellite links. See Appendix 9.A (Figures), Fig. A9.18.

Table 9.8 summarizes the five cloud archetypes adopted in the sub-THz attenuation analysis. Each configuration is defined by representative altitude and temperature values derived from mid-latitude climatological profiles. Although attenuation primarily depends on the **cloud liquid-water content (LWC)**, the thermodynamic state (temperature) still modulates the complex permittivity through the Double-Debye dispersion model, while cloud-top height provides information on typical LWC ranges for each cloud type. The selected cases—from warm low-level



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Cumulus to cold high-altitude *Cirrus* and *Cumulus Congestus*—span the full range of regimes tested in the frequency-dependent comparison of **Figs. 9.17 (a–d)**. See Appendix 9.B (Tables), Table A9.5.

SNEM-2 outperforms the **ITU-R P.840** parameterization because it solves the full radiative-transfer problem with anisotropic scattering, layer-by-layer extinction, and temperature-dependent dielectric properties. Unlike ITU-R, SNEM-2 uses Mie-derived microphysical parameters (k_e, ω, g) and a Monte-Carlo approach, reproducing the attenuation behavior observed in CloudSat–GPM retrievals. A possible continuation of the developments presented in this thesis concerns the integration of the SNEM-2 modelling framework into post-doctoral research and technology-transfer initiatives. Some of these activities are currently being explored within the early-stage startup **Rhynia**, which aims to translate high-frequency radiative-transfer methods into operational atmospheric-intelligence tools.

9.8.6 Comparative Analysis of Attenuation Models

9.8.6.1 Cross-Model Consistency of Liquid-Cloud Attenuation Laws

The representation of cloud-induced attenuation at millimetre and sub-THz frequencies critically depends on the physical assumptions embedded in the underlying models. **Figure 9.19** provides a systematic comparison between the empirical ITU-R P.840 formulation, the physical Double-Debye dielectric model, and the semi-empirical SNEM-2 parameterization used throughout this work. The upper panel shows the specific attenuation per unit liquid water content $K_{LWC}(f)$, highlighting the spectral regimes where Rayleigh theory remains valid and those where non-Rayleigh (Mie-sensitive) behavior becomes increasingly important (above ≈ 200 GHz). At low frequencies (20–40 GHz), the three models converge within a narrow envelope, reflecting the dominance of Rayleigh scattering and the weak dielectric contrast between the formulations. The first zoom inset (20–35 GHz) shows a minor divergence caused by ITU-R’s empirical curvature, but the physical and semi-empirical models remain nearly indistinguishable. As frequency increases, absorption efficiency grows and the spectrum becomes progressively more sensitive to dielectric details. The second inset (180–220 GHz) captures this transition: the Double-Debye model retains its smooth behavior, whereas SNEM-2 introduces localized Gaussian features centred at 183 GHz and 220 GHz, which emulate the enhanced extinction associated with real liquid-cloud absorption peaks.

The lower panel quantifies these differences through the residual:

$$\Delta K = K_{SNEM-2} - K_{DD} \quad (9.22)$$

revealing three localized corrections with amplitudes of order $10^{-3} dB km^{-1} (gm^{-3})^{-1}$. These features are intentionally bounded and physically interpretable: they mimic fine-scale deviations observed in radiative-transfer simulations, while ensuring that SNEM-2 remains anchored to a physically consistent baseline. The residual’s structure confirms that SNEM-2 operates as a



controlled perturbation of **Double-Debye** rather than an independent empirical law. Collectively, this comparison demonstrates that:

- (i) empirical and physical models are mutually consistent in the Rayleigh regime;
- (ii) discrepancies naturally emerge near absorption features and in the Mie-sensitive spectral window;
- (iii) the SNEM-2 formulation successfully captures these deviations without sacrificing physical coherence.

This framework provides the reference baseline for the modelling choices adopted in the subsequent validation analyses. See Appendix 9.A (Figures), **Fig. A9.19**.

Table 9.9 provides a consolidated overview of the three attenuation formulations examined in this chapter, highlighting their conceptual foundations and their operational domains. The ITU-R P.840-9 model offers an empirical baseline widely used in communication-system engineering, whereas the Double-Debye formulation represents the physically rigorous reference derived from the dielectric properties of liquid water. Building on this foundation, the SNEM-2 model incorporates localized Gaussian corrections that emulate fine-scale spectral features observed in radiative-transfer simulations, while preserving the physical coherence of the underlying **Double-Debye** response. This comparison establishes the modelling framework adopted in the subsequent analyses of cloud-induced attenuation at millimetre-wave and sub-THz frequencies.

9.9 SNEM-2: Frequencies, Permittivity, and Attenuation Computation

9.9.1 Frequencies and site setup

The transition from SNEM to SNEM-2 introduces a frequency grid of 40 channels spanning 5.65–95 GHz, covering key absorption features near 22.23, 31.40, and 60–70 GHz (O₂). This configuration ensures sufficient spectral resolution to support both telecommunication and remote-sensing analyses, including seasonal and site-specific applications such as those at Spino d'Adda.

9.9.2 Debye relaxations and dielectric parameters

Following **ITU-R P.840-9** at $T = 273.75$ K, the primary and secondary relaxation frequencies are:

$$f_p = 20.20 - 146 \left(\frac{300}{T} - 1 \right) + 316 \left(\frac{300}{T} - 1 \right)^2, \quad f_s = 39.8 f_p, \quad (9.23)$$



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Table 9.3. Comparison of cloud attenuation formulations: ITU-R P.840-9, Double-Debye, and the SNEM-2 Gaussian-corrected model.

Model	Formulation
ITU-R P.840-9 (empirical)	$\gamma_c(f, \theta) = K_{\text{LWC}}^{\text{ITU}}(f) \rho_{\text{liq}} \sec \theta$ with $K_{\text{LWC}}^{\text{ITU}}$ obtained from empirical regression fits (valid up to ~ 200 GHz).
Double-Debye (physical)	$K_{\text{LWC}}^{\text{DD}}(f, T) = \frac{0.819 f}{\varepsilon_2(f) [1 + \eta(f)^2]}$ where $\eta(f) = \frac{2 + \varepsilon_1(f)}{\varepsilon_2(f)}$ and $\varepsilon_1, \varepsilon_2$ follow the standard Double-Debye dielectric model of liquid water.
SNEM-2 (semi-empirical Gaussian corrections)	$K_{\text{LWC}}^{\text{SNEM2}}(f, T) = K_{\text{LWC}}^{\text{DD}}(f, T) + \Delta K(f)$ where the correction term is $\Delta K(f) = A_1 \exp\left[-\frac{(f - f_1)^2}{2\sigma_1^2}\right] + A_2 \exp\left[-\frac{(f - f_2)^2}{2\sigma_2^2}\right] + A_3 \exp\left[-\frac{(f - f_3)^2}{2\sigma_3^2}\right].$ Localised corrections: $f_1 = 23.8 \text{ GHz}, \quad f_2 = 183 \text{ GHz}, \quad f_3 = 220 \text{ GHz}.$ Amplitudes (consistent with residuals in Fig. 11.X): $A_1 = 1.5 \times 10^{-3}, \quad A_2 = 3.5 \times 10^{-3}, \quad A_3 = 2.5 \times 10^{-3}.$ Widths: $\sigma_1 = 6 \text{ GHz}, \quad \sigma_2 = 14 \text{ GHz}, \quad \sigma_3 = 11 \text{ GHz}.$



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and the permittivity parameters are:

$$\varepsilon_0 = 77.66 + 103.3 \cdot \left(\frac{300}{T} - 1 \right), \quad \varepsilon_1 = 0.0671\varepsilon_0, \quad \varepsilon_2 = 3.52. \quad (9.24)$$

The imaginary part controlling absorption is:

$$\varepsilon''(f) = \frac{f \cdot (\varepsilon_0 - \varepsilon_1)}{f_p \cdot \left(1 + \left(\frac{f}{f_p} \right)^2 \right)} + \frac{f \cdot (\varepsilon_1 - \varepsilon_2)}{f_s \cdot \left(1 + \left(\frac{f}{f_s} \right)^2 \right)}. \quad (9.25)$$

These terms govern the frequency-dependent absorption and, consequently, cloud attenuation.

9.9.3 Attenuation coefficient and Gaussian correction

The baseline **Debye/ITU-R** term is refined in SNEM-2 by a Gaussian spectral correction to reproduce absorption features near ~23 GHz and 183–220 GHz and to smooth empirical discontinuities:

$$K_{Liq}^{SNEM-2}(f) = K_{Liq}^{Debye}(f) \times \left[\frac{A_1 \exp\left(-\frac{(f-f_1)^2}{\sigma_1^2}\right) + A_2 \exp\left(-\frac{(f-f_2)^2}{\sigma_2^2}\right) + A_3}{A_1 \exp\left(-\frac{(f-f_1)^2}{\sigma_1^2}\right) + A_2 \exp\left(-\frac{(f-f_2)^2}{\sigma_2^2}\right) + A_3} \right], \quad (9.26)$$

with:

$$A_1 = 0.1522, A_2 = 11.51, A_3 = -10.4912; \sigma_1 = 3.2991 \times 10^3, \sigma_2 = 2.7595 \times 10^6; \quad (9.27)$$

$$f_1 = 23.96 \text{ GHz}, f_2 = 219.21 \text{ GHz}. \quad (9.28)$$

The negative A_3 compensates the tendency to overfit peaks, improving spectral continuity.

9.10 Observational Sites

Figures 9.20–9.21 illustrate the three principal observational sites analyzed in this study—**Scanzano** (Italy, 37.91° N, 13.36° E), **Adelaide** (Australia, −34.90° S, 138.60° E), and **Riverside** (California, USA, 33.80° N, −117.10° W)—selected to represent distinct climatic regimes influencing atmospheric propagation. Scanzano, located in southern Italy, typifies a Mediterranean climate with alternating wet and dry seasons and variable cloud cover, ideal for examining seasonal precipitation and hydrometeor variability. Adelaide, in South Australia, exhibits semi-arid to temperate conditions, suitable for analyzing clear-sky and cloudy scenarios and assessing the seasonal dependence of water-vapor content. Riverside, in southern California, represents an arid to Mediterranean environment with minimal cloud cover and low humidity, providing a benchmark for dry-atmosphere radiative-transfer validation. Together, these sites encompass



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Mediterranean, temperate, and arid climates, enabling a robust comparative assessment of cloud attenuation and radiative-transfer behavior across different latitudes and atmospheric conditions, and ensuring that the models developed in this work remain globally applicable. See **Appendix 9.A (Figures)**, **Fig. A9.20**, **Fig. A9.21**.

9.11 Concluding Remarks

The framework developed in this study illustrates how cloud and rain attenuation originate from interconnected physical processes—spanning thermodynamic saturation, microphysics, radiative transfer, and electromagnetic propagation. Cloud attenuation, though typically weaker than rain fading, becomes increasingly relevant above 10 GHz, particularly in the Ka-, V-, and W-bands. Rain attenuation, by contrast, is highly variable and often dominates signal degradation, motivating the use of empirical and semi-empirical formulations such as **ITU-R P.838** and **P.618** for operational prediction.

From a scientific standpoint, a detailed understanding of these mechanisms is essential to refine radiative-transfer models, improve remote-sensing retrievals, and evaluate the applicability of ITU-R standards across different climatic regimes. From an engineering perspective, these insights directly inform the design of fade margins, adaptive coding and modulation schemes, and site-diversity or predictive control strategies to ensure link continuity. Ultimately, cloud and rain attenuation remain both a key limitation and a critical benchmark for advancing the integration of atmospheric physics, climatological modeling, and high-frequency satellite communication system design. Beyond the scope of this dissertation, the modelling capabilities developed here naturally intersect with several ongoing research directions promoted by the **European Space Agency**, particularly in relation to high-frequency propagation, atmospheric calibration and the design of next-generation satellite links. Although no funding implications are considered within the present work, the combination of physics-based radiative transfer, microphysical modelling and numerical surrogates outlined in this thesis defines a methodological basis that could be further explored within dedicated research frameworks or post-doctoral initiatives. In this sense, the results presented here should be regarded as an enabling step toward more comprehensive studies potentially undertaken within ESA programmes devoted to advanced communication technologies and atmospheric characterization.

9.12 Summary of Chapter 9

This chapter has presented a comprehensive characterization of atmospheric attenuation and brightness-temperature behavior at millimetre wavelengths, with particular emphasis on W-band propagation under realistic tropospheric conditions. The analysis combined physically based radiative-transfer modelling with statistical diagnostics derived from long-term meteorological and satellite-informed datasets. First, the vertical structure of thermodynamic and humidity profiles was analyzed across different seasons, highlighting the impact of realistic atmospheric stratification



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on gaseous absorption and emission. Comparisons between idealized and reanalysis-based profiles demonstrated that small-scale variability in the lower troposphere plays a non-negligible role in shaping attenuation and brightness-temperature distributions, especially at high frequencies. Subsequently, slant-path attenuation statistics were investigated as a function of frequency, elevation angle, and exceedance probability. The results confirmed the strong dependence of attenuation on both frequency and propagation geometry, with low-elevation links exhibiting significantly enhanced losses due to the increased effective path length. A detailed component-wise decomposition showed that, at W band, rainfall and melting-layer effects dominate extreme attenuation levels, while gaseous absorption and cloud liquid- and ice-water contributions remain secondary but non-negligible. Finally, the seasonal climatology of extreme attenuation levels was analyzed through monthly $A_{0.01}$ statistics, revealing a clear modulation associated with Mediterranean precipitation regimes. Maximum attenuation values were observed during winter months, while summer conditions were characterized by reduced extreme losses, consistent with the regional climatology. Overall, the results presented in this chapter provide a physically consistent and statistically robust framework for the assessment of millimetre-wave propagation impairments, supporting both the interpretation of observational datasets and the design of high-frequency satellite communication links in mid-latitude environments.



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Chapter 10

Atmospheric Path Excess Delay over Sicily: Integrated Modelling and Operational Analysis at the Scanzano (PA) Ground Station

10.1 An integrated study combining atmospheric modelling, re-analysis data, and operational considerations for Telespazio's strategic site in the Mediterranean

This chapter bridges the gap between the theoretical framework of atmospheric path excess delay and its operational application to the Mediterranean context, with a specific focus on Sicily and the **Telespazio** ground station in Scanzano (PA). The study integrates physical modelling, ERA5 reanalysis datasets, and site-specific operational constraints to reconstruct the seasonal and diurnal variability of both the hydrostatic and wet components of zenith delay. The Scanzano ground station supports satellite tracking, telemetry, and control activities in the Mediterranean region and provides an operational test-bed for the validation of delay models under real-world conditions. The results contribute to both scientific understanding and the optimization of high-precision geodetic and satellite communication systems. The Scanzano (PA) ground station (37.98° N, 13.30° E), located in inland Sicily at an elevation of approximately 560 m a.s.l. near Piana degli Albanesi, is characterized by a continental-Mediterranean climatic regime influenced by local orography. The site exhibits enhanced seasonal variability of tropospheric water vapor, with windward-leeward effects modulating the wet component of the zenith path delay. These conditions enable a sensitive assessment of hydrostatic and wet delay models under realistic Mediterranean environments. See Appendix 10.A (Figures), Fig. A10.1.



10.1.1 Path Excess Delay: Theoretical Framework and Site-Specific Analysis at the Scanzano Ground Station (PA), Sicily

Path Excess Delay represents the integrated effect of atmospheric refractivity on electromagnetic signal propagation and constitutes a fundamental source of error in satellite-based positioning, navigation, and communication systems. In atmospheric sciences, the total delay is conventionally decomposed into a hydrostatic (dry) component, which is stable and well modeled, and a wet component driven by the highly variable distribution of tropospheric water vapor. The accurate characterization of both contributions is essential for quantifying the Zenith Tropospheric Delay (ZTD) and its impact on space-based applications [217].

10.1.2 Theoretical Framework

The Zenith Tropospheric Delay (ZTD) is conventionally expressed as the sum of a hydrostatic (dry) component, primarily associated with nitrogen and oxygen, and a wet component governed by the spatial and temporal variability of atmospheric water vapor [218, 219]. The hydrostatic delay (ZHD) is strongly correlated with surface pressure and can be modeled with high accuracy using the **Saastamoinen** formulation, which accounts for pressure, latitude, and station height [45]. In contrast, the Zenith Wet Delay (ZWD) exhibits pronounced variability and requires high-resolution meteorological information for reliable estimation. Such information may be obtained from radiosonde observations, GNSS-derived precipitable water vapor, or numerical weather reanalyses such as ERA5 [187]. To project zenith delays onto a satellite line-of-sight, mapping functions are applied to convert vertical delays into slant delays as a function of elevation angle. Commonly adopted models include the **Niell Mapping Function** (NMF) [216] and the **Vienna Mapping Function** (VMF) [221], which incorporate seasonal and geographical dependencies. Accurate modeling of both zenith and slant tropospheric delays is essential for geodetic applications, precise orbit determination, and satellite communication systems, where residual atmospheric delays can introduce range errors, degrade positioning accuracy, and, at high frequencies, impact link-budget performance [220].

10.1.3 Experimental Application of Tropospheric Delay Modeling: The Scanzano Site (PA)

For the Scanzano site, hourly fields of surface pressure, temperature, and relative humidity were extracted from the ERA5 global reanalysis [12] and adjusted for station altitude. These parameters were used to compute the Zenith Hydrostatic Delay (ZHD) following the Saastamoinen formulation [45] and the Zenith Wet Delay (ZWD) according to established GNSS meteorology approaches [46]. Mapping functions were subsequently applied to project zenith delays onto selected satellite elevation angles, considering both the Niell Mapping Function (NMF) [216] and the Vienna Mapping Function (VMF) derived from numerical weather model data [56]. The analysis covers a full climatological year. The resulting time series exhibit pronounced seasonal



variability. During winter, the wet component remains limited and the total zenith delay is largely dominated by the hydrostatic contribution, reflecting the stability of the surface pressure field. In summer, enhanced moisture advection associated with Mediterranean circulation patterns leads to a significant increase in ZWD, contributing substantially to overall delay variability, consistently with previous GNSS- and reanalysis-based studies [73]. Short-term ZWD enhancements are observed in correspondence with periods of increased atmospheric moisture and convective activity. From an operational perspective, these results support the implementation of refined tropospheric correction strategies for GNSS positioning [65] and provide guidance for the definition of adaptive link-margin strategies in satellite communication systems, where residual tropospheric delays may impact ranging accuracy and link performance [287]. The Scanzano site thus represents a suitable case study for assessing the operational impact of site-specific atmospheric variability on path delay modeling.

10.2 Refractivity Theory

10.2.1 Historical Overview of Refractivity Theory

Atmospheric refractivity provides a compact description of how spatial variability of the refractive index modifies electromagnetic propagation paths in air. While the physical notion of refraction originates in classical optics, its modern formulation for atmospheric and radio applications evolved with the development of radio systems, radar, and space geodesy. The advent of radio communication in the early 20th century revolutionized the understanding of refractivity. Sir **Edward Appleton** (1920s) investigated ionospheric refraction, while parallel studies by **Kenjiro Okabe** and contemporaries addressed tropospheric refractivity, establishing empirical relationships between meteorological parameters and radio-wave bending. During the 1940s–1950s, the emergence of radar and microwave systems motivated the development of refined atmospheric refraction models. This period saw the introduction of standard refractivity units: $N := (n - 1) \times 10^6$, a convention adopted in operational propagation studies and codified in ITU-R recommendations for the radio refractive index and refractivity data [44]. With the advent of numerical weather prediction and global reanalyses, refractivity can now be mapped with high temporal resolution using state variables (pressure, temperature, humidity) provided by products such as ERA5, enabling time-dependent delay and ray-bending characterization in geodetic and telecommunication applications. [99]

Early Foundations: Optics and Atmospheric Refraction

The study of refraction can be traced back to **Claudius Ptolemy** (2nd century AD), who documented how light bends when traversing different media. A rigorous mathematical description emerged in the 17th century with **Willebrord Snellius**, whose **Snell's Law** (1621) formalized the relationship between incident and refracted angles as a function of refractive indices. In the late 18th and early 19th centuries, atmospheric refraction became a subject of scientific



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inquiry in astronomy and geodesy. **Pierre-Simon Laplace** (1799) and **Carl Friedrich Gauss** (1809) investigated the apparent displacement of celestial bodies caused by atmospheric bending, recognizing that air density — modulated by temperature, pressure, and humidity — governs the refractive index. These works laid the foundation for later quantitative formulations.

10.2.2 Mathematical Formulation and Physical Interpretation

Atmospheric refractivity N is a dimensionless measure designed to represent small deviations of the refractive index n from unity on a numerically convenient scale (see **Eq. (10.1)**). In layered media, refractivity gradients ($\partial N/\partial z$) are directly linked to ray curvature and to the magnitude of propagation delays, and therefore constitute a key quantity in modeling tropospheric delay and refraction effects in space-geodetic data analysis. When integrated along the electromagnetic propagation path, atmospheric refractivity gives rise to the path excess delay, establishing a direct physical link between local thermodynamic state variables and the observable zenith and slant tropospheric delays used in GNSS and satellite-based applications [218].

10.2.3 The Smith–Weintraub Equation

In the neutral atmosphere, the most widely adopted parameterization expresses refractivity as the sum of a dry (hydrostatic) contribution and a wet contribution associated with water vapor:

$$N = k_1 \frac{P_d}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2}, \quad (10.1)$$

where P_d is the dry-air pressure [hPa], e is the water-vapor partial pressure [hPa], and T is the absolute temperature [K]. The constants k_1, k_2, k_3 are empirically determined and routinely used in operational refractivity formulations [44]¹. For many applications, the expression is reduced to a two-term form using total pressure P :

$$N = 77.6 \frac{P}{T} + 3.73 \times 10^5 \frac{e}{T^2}, \quad (10.2)$$

where the first term represents the relatively stable hydrostatic refractivity, and the second term captures the highly variable wet refractivity driven by water vapor variability. This decomposition underpins modern modeling of tropospheric delays and refractivity-related propagation effects in geodesy and radio systems.

10.2.4 Physical Interpretation

The refractive index of air n is determined by its dielectric properties, which arise from molecular polarization mechanisms under the action of an electromagnetic field. In dry air, refractivity

¹ $k_1 = 77.6 \text{ K hPa}^{-1}$,
 $k_2 \approx 70.4 \text{ K hPa}^{-1}$,
 $k_3 = 3.739 \times 10^5 \text{ K}^2 \text{ hPa}^{-1}$,



is primarily governed by the induced polarization of non-polar atmospheric gases (mainly nitrogen and oxygen) and is approximately proportional to pressure and inversely proportional to temperature. The wet-air contribution is dominated by the strong electric dipole moment of water vapor molecules, whose rotational transitions introduce an enhanced temperature dependence, reflected by the T^{-2} term in the refractivity formulation. As a result, wet refractivity exhibits pronounced spatial and temporal variability compared to the hydrostatic component. Because the propagation velocity of an electromagnetic wave in a medium is given by $v = c/n$, where c is the speed of light in vacuum, spatial variations of n directly translate into excess path delay, ray bending, and anomalous propagation phenomena, including super-refraction, sub-refraction, and ducting.

10.3 Description of Key Refractivity Models

Accurate estimation of atmospheric refractivity is essential for applications ranging from satellite geodesy to high-frequency telecommunications. Over the past decades, several formulations have been developed to describe refractivity with increasing physical rigor and operational usability. Among these, the models proposed by **Davis et al. (1985)**, **Hansen et al. (1988)**, and **Vedel et al. (2001)** represent reference formulations that share common physical foundations but differ in mathematical structure, constant definitions, and unit conventions. These differences directly affect their implementation in *numerical weather prediction* (NWP) products and geodetic processing chains.

10.3.1 Davis et al. (1985): Compressibility-Based Formulation

Davis et al. introduced a refractivity formulation that explicitly accounts for the compressibility factors Z_i of atmospheric gases. Under typical tropospheric conditions, where $Z_i \approx 1$, the formulation can be simplified without loss of physical meaning. This approach provides a rigorous thermodynamic basis for refractivity modeling and establishes a direct link with subsequent operational formulations. The full derivation and algebraic reduction are reported in **Appendix F.1**.

10.3.2 Hansen et al. (1988): Operational Formulation

Hansen et al. proposed an operationally convenient refractivity expression that separates dry and wet contributions while remaining consistent with the Davis formulation. By introducing the ratio of gas constants $\epsilon = R_d/R_w \approx 0.622$, refractivity can be written in terms of total pressure and water-vapor partial pressure, yielding a form widely adopted in GNSS meteorology and tropospheric delay estimation. This formulation offers a favorable balance between physical consistency and computational efficiency, as it relies solely on standard meteorological variables. The refractivity constants adopted in this work are summarized in **Table 10.1**.



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Table 10.1. Step-by-step derivation of the Zenith Wet Delay (ZWD) from refractivity equations.

Step	Equation and Description
1. General definition of ZTD	$ZTD = \int_{z_s}^{z_t} N(z) dz = \int_{z_s}^{z_t} \left[k_1 R_d \rho(z) + k_2' R_w \rho_w(z) + k_3 \frac{\rho_w(z) R_w}{T(z)} \right] dz$
2. Non-hydrostatic component (ZWD)	$ZWD = \int_{z_s}^{z_t} \left[k_2' R_w \rho_w(z) + k_3 \frac{\rho_w(z) R_w}{T(z)} \right] dz$ Only the water vapor-dependent terms are retained.
3. Introducing the ratio with total density	Multiplying and dividing by $\rho(z) = \rho_d(z) + \rho_w(z)$: $ZWD = R_w \int_{z_s}^{z_t} \left[k_2' \frac{\rho_w(z)}{\rho(z)} + \frac{k_3}{T(z)} \frac{\rho_w(z)}{\rho(z)} \right] \rho(z) dz$
4. Change of variable to pressure (hydrostatic equilibrium)	Using $dp = \rho(z)g(\Theta)dz$, with $g(\Theta)$ constant with height but latitude-dependent: $ZWD = -\frac{R_w}{g(\Theta)} \int_{p(z_t)}^{p_s} q(z) \left[k_2' + \frac{k_3}{T(z)} \right] dp$
5. Inverting integration limits by convention	$ZWD = \frac{R_w}{g(\Theta)} \int_{p(z_t)}^{p_s} q(z) \left[k_2' + \frac{k_3}{T(z)} \right] dp$ Where $p_s = p(z_s)$.
6. WRF-compatible formulation	In WRF: $q(k) \rightarrow q(z)$ (mixing ratio), $p(k) \rightarrow p_d(z)$ (dry air pressure), $\epsilon = R_d/R_w \approx 0.622$. The discrete form is computed as a sum over model levels.

10.3.3 SI-Consistent Refractivity and Integrated Delay Formulation

Vedel et al. (2001) reformulated atmospheric refractivity within a fully SI-consistent framework, specifically designed for direct assimilation into numerical weather prediction (NWP) models. By expressing refractivity in terms of air density and temperature, this formulation ensures dimensional consistency across atmospheric datasets and avoids ambiguities associated with mixed pressure–temperature unit conventions. When appropriate unit conversions are applied, the Vedel formulation can be shown to be algebraically equivalent to the operational refractivity model proposed by **Hansen et al.**; details of the SI conversion and equivalence proof are provided in **Appendix F.2**. While refractivity $N(z)$ is defined as a local, height-dependent quantity, most geodetic and radio-propagation applications require its vertical integration to obtain observable atmospheric delays. In particular, the Zenith Wet Delay (ZWD) is obtained by integrating the wet refractivity contribution along the atmospheric column. This integrated formulation, rigorously derived by **Huang** (2004), enables the direct computation of tropospheric delays from vertical profiles provided by NWP reanalyses or radiosonde observations. The SI-consistent refractivity framework therefore provides a natural and physically coherent bridge between local thermodynamic state variables and path-integrated propagation delays.



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10.3.4 Hydrostatic Transformation and Pressure-Based ZWD Evaluation

Assuming hydrostatic equilibrium, the vertical integration of wet refractivity can be conveniently transformed from height to pressure coordinates. Under this assumption, the differential relationship $dp = -\rho(z)g(\Theta)dz$ holds, where $\rho(z)$ is the total air density, $g(\Theta)$ is the latitude-dependent gravitational acceleration, and Θ denotes geodetic latitude. This transformation enables the Zenith Wet Delay (ZWD) to be expressed as a pressure integral, which is particularly well suited for numerical weather prediction (NWP) applications. In pressure coordinates, the ZWD formulation naturally incorporates specific humidity and temperature profiles, allowing the wet delay to be evaluated directly from NWP model levels. This representation is consistent with the formulations proposed by **Vedel et al. and Huang (2004)** and constitutes the reference approach adopted in this work for NWP-based delay estimation. By recasting the vertical integration in pressure space, the hydrostatic transformation provides a computationally efficient and physically consistent framework for deriving the non-hydrostatic component of the total tropospheric delay.

10.4 WRF Model Framework

10.4.1 From WRF Principles to Regional Delay Mapping

To characterize tropospheric delay variability over **Scanzano** and the surrounding Sicilian region, the methodological framework adopted in this study is conceptually derived from the physical structure of the Weather Research and Forecasting (WRF) model. WRF provides a dynamically and thermodynamically consistent representation of the lower atmosphere, based on prognostic fields of pressure, temperature, and moisture. Rather than performing full WRF simulations, a **WRF-consistent diagnostic approach** was adopted, relying on a minimal yet physically complete set of atmospheric variables—pressure, temperature, and specific humidity—supplemented by accurate geographic coordinates and orographic information. This configuration allows the reconstruction of spatially resolved fields of Zenith Hydrostatic Delay (ZHD) and Zenith Wet Delay (ZWD), tailored to the Scanzano site and its regional context. The resulting delay maps capture the dominant spatial patterns of tropospheric refractivity across Sicily, while remaining computationally efficient and physically grounded.

10.4.2 Key Variables for Excess Path Delay Estimation

Within the WRF physical framework, a limited number of meteorological and geophysical variables is sufficient to derive tropospheric excess path delays with high spatial resolution [WRF]. The variables adopted in this study—summarized in **Table 10.2**—span thermodynamic, moisture, and geometric components and collectively provide the information required for refractivity-based delay estimation. Vertical refractivity profiles were integrated following WRF-consistent physical formulations to obtain Zenith Hydrostatic and Zenith Wet Delay fields, in accordance with



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standard NWP-based delay modeling approaches [187, 218]. This methodology enables the generation of regional-scale tropospheric delay maps over Sicily, with particular emphasis on the Scanzano area, and establishes a direct link between NWP-derived atmospheric state variables and geodetically observable propagation delays. See Appendix 10.B (Tables), Table A10.1.

10.4.3 Diagnostic Visualization of WRF Perturbation Pressure Fields

The spatial distribution of perturbation pressure P derived from the Weather Research and Forecasting (WRF) model provides a physically consistent representation of mesoscale atmospheric variability over Northern Italy. The analyzed domain, explicitly configured to encompass the Italian peninsula and the surrounding Alpine–Mediterranean region, captures pressure perturbations associated with orographic forcing, land–sea contrasts, and regional circulation patterns. Variations in P , expressed in pascals, reflect departures from the hydrostatic background state and highlight the dynamic response of the atmosphere to complex terrain and mesoscale flow structures. Lower perturbation values are typically associated with relatively homogeneous lowland or maritime-influenced areas, whereas enhanced perturbations emerge in correspondence with mountainous regions and sharp elevation gradients, indicating the strong coupling between topography and atmospheric dynamics. This diagnostic analysis confirms the capability of WRF to resolve fine-scale pressure structures relevant for regional-scale meteorological and propagation studies, while serving as a consistency check for the numerical setup prior to quantitative interpretation. A preliminary qualitative inspection of the WRF output was performed using **Panoply**, serving as a first-look diagnostic to assess spatial coherence and data integrity. This step was intended solely for verification purposes and does not constitute a cartographic or analytical representation. See Appendix 10.A (Figures), Fig. A10.2.

The perturbation pressure values span a range from approximately 490 Pa to 1240 Pa, with intermediate gradients reflecting mesoscale variability induced by topography and regional atmospheric dynamics. The magnitude of the perturbation pressure field, spanning approximately 0.5–1.2 kPa, is consistent with mesoscale atmospheric motions resolved at the adopted WRF resolution, and reflects deviations from the hydrostatic reference state associated with terrain-induced flow adjustment and regional circulation.

10.4.4 Auxiliary WRF Variables for Vertical Reconstruction and Propagation Modelling

Beyond the primary meteorological fields used for tropospheric delay estimation, the **Weather Research and Forecasting (WRF) model** provides a set of auxiliary variables that are essential for the physically consistent reconstruction of the atmospheric vertical structure. These variables do not represent independent prognostic quantities, but rather define the internal formulation of the model’s terrain-following vertical coordinate and its transformation into physically meaningful pressure, temperature, and geopotential height profiles. In the context of this work, such variables are required to ensure the consistency of vertical interpolation,



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to support the derivation of refractivity and propagation-related quantities, and to avoid the introduction of non-physical artifacts in delay and attenuation modelling. **Table 10.3** summarizes the auxiliary **WRF variables** explicitly used in this study to reconstruct pressure, temperature, and geopotential height fields from the native model coordinates. See Appendix 10.B (Tables), Table A10.2.

10.4.5 Geopotential Field Consistency in the Transition from WRF to ERA-5

When transitioning from a WRF-based modelling framework to ERA-5 reanalysis, particular care must be taken to ensure the physical consistency of vertically dependent quantities. Among these, the base-state geopotential plays a central role, as it directly affects the reconstruction of atmospheric height, pressure, and refractivity profiles used in propagation and delay modelling. In WRF, the geopotential field is explicitly decomposed into a stationary base-state component (PHB) and a time-dependent perturbation (PH), such that the total geopotential is given by $PHT = PHB + PH$. This formulation allows mesoscale dynamical variability to be represented as deviations from a prescribed reference state. In contrast, ERA-5 directly provides the geopotential associated with its hybrid vertical coordinate system, without an explicit perturbation decomposition, reflecting its nature as a global, observation-constrained reanalysis. Ensuring consistency between these two representations is therefore essential when migrating from a WRF-driven workflow to ERA-5 data. In ERA-5, model-level pressures are reconstructed from surface pressure and hybrid coefficients ($a(n), b(n)$), while geopotential is provided as a stationary field intrinsically linked to the model's vertical coordinate definition. As a result, the WRF PHB field constitutes the appropriate reference quantity for comparison and methodological alignment with ERA-5 geopotential products.²

The ERA-5 vertical discretization consists of 137 model levels (indexed from 1 to 137, with level 0 containing no valid atmospheric data). The associated auxiliary tables typically include the hybrid coefficients a and b , as well as derived quantities such as pressure at half and full levels, temperature profiles, and air density. These variables are required to ensure a physically consistent reconstruction of pressure and geopotential fields and to enable subsequent applications involving atmospheric structure, refractivity, and propagation-related modelling. See Appendix 10.A (Figures), Fig. A10.3.

In the figure the horizontal cross-section exhibits strong spatial gradients in the base-state geopotential field, spanning values from near zero to above $2.8 \times 10^4 [m^2 s^{-2}]$, which are associated with regions of enhanced dynamical activity and convective influence. This diagnostic supports the assessment of the background vertical structure and the identification of areas subject to

²Surface pressure is provided in logarithmic form ($lnsp$), and pressure at model levels is derived through the hybrid vertical coordinate formulation:

$$p_{ml} = a(n) + b(n)p_s \quad (10.3)$$

where p_{ml} denotes the pressure at the n -th model level, $a(n)$ and $b(n)$ are level-dependent hybrid coefficients, and p_s is the surface pressure. The hybrid coefficients $a(n)$ and $b(n)$ are encoded in the ERA-5 GRIB file headers and are also distributed as auxiliary tabulated files in CSV format.



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increased dynamical forcing. Usually:

$$Z = \frac{PHB}{g} \Rightarrow Z_{max} \approx \frac{28401.4}{9.80665} \simeq 2895 [m], \quad (10.4)$$

the PHB field therefore represents geopotential heights up to approximately 2.9 km, which are fully consistent with the Alpine orography, the static base-state formulation of the WRF model, and its role as a reference geopotential field in the transition from WRF-based simulations to ERA-5 reanalysis.

10.4.6 Visualization of ERA-5 Model Level Coefficients

To clarify the structure of the ERA-5 hybrid vertical coordinate system and its role in vertical reconstruction, **Figure F.4 in Appendix F.3.5** reports an excerpt from the CSV files containing the hybrid coefficients $a(n)$ and $b(n)$, together with the associated atmospheric parameters. These quantities establish the mapping between model levels and physically meaningful variables, including pressure, geopotential height, temperature, and air density. In the ERA-5 formulation, pressure at model levels is derived from the hybrid relation defined as:

$$p_{ml} = a(n) + b(n) \cdot p_s \quad (10.5)$$

and the columns p and p_f (expressed in hPa) correspond to pressures at half and full levels, respectively. This representation provides a direct link between the abstract model-level indexing and the corresponding physical state of the atmosphere, enabling a consistent reconstruction of geopotential height profiles required for refractivity estimation and tropospheric delay modelling. The tabulated coefficients and derived parameters constitute the numerical backbone of the ERA-5 vertical discretization and are therefore essential for the subsequent transformation of reanalysis data into Zenith Total Delay (ZTD) fields. The following section details the computational workflow adopted to propagate these vertically resolved quantities into spatially resolved delay maps.

10.5 From ERA-5 CSV to ZTD Maps — Mathematical Workflow

This section describes the mathematical workflow adopted to derive geopotential height profiles and tropospheric delays from ERA-5 reanalysis data, following a formulation consistent with that used in the WRF model framework. The procedure is based on the hybrid vertical coordinate system employed in ERA-5 and enables the transformation of model-level variables into physically meaningful quantities required for delay estimation.



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10.5.1 Pressure and Vertical Coordinate Reconstruction

In ERA-5, the pressure at each model level n is computed using the hybrid coordinate formulation due to (10.6). The same formulation provides pressures at half and full levels (p_h, p_f) , which are used to define the vertical discretization of the atmosphere.

10.5.2 Geopotential Height and Layer Thickness

Geopotential height is obtained from the ERA-5 geopotential field Φ as:

$$Z_g(n) = \frac{\Phi(n)}{g(\varphi)}, \quad (10.6)$$

where $g(\varphi)$ is the latitude-dependent gravitational acceleration, approximated as:

$$g(\varphi) \simeq 9.780327(1 + 0.0053024 \cdot \sin^2(\varphi) - 5.8 \times 10^{-6} \sin^2(2\varphi)) \rightarrow [ms^{-2}]. \quad (10.7)$$

The vertical thickness of each atmospheric layer is then computed as:

$$\Delta z_n = Z_g(n+1) - Z_g(n). \quad (10.8)$$

10.5.3 Thermodynamic State and Water Vapor Pressure

At each model level, the thermodynamic state is determined from temperature $T(n)$ and specific humidity $q(n)$. The partial pressure of water vapor is evaluated as:

$$e(n) = \frac{q(n)}{\varepsilon + (1 - \varepsilon) q(n)} p_{ml}(n), \quad \text{where } \varepsilon := \frac{R_d}{R_w} \approx 0.622, \quad (10.9)$$

while the dry-air pressure is given by:

$$p_d(n) = p_{ml}(n) - e(n). \quad (10.10)$$

When specific humidity is not available, the water vapor pressure is estimated either from relative humidity and saturation vapor pressure or from the **mixing ratio** w , using:

$$e = \frac{w}{\varepsilon + w} p, \text{ and } q = \frac{w}{1 + w}. \quad (10.11)$$

10.5.4 Atmospheric Refractivity

The refractivity profile $N(n)$ is computed using the **Smith–Weintraub** formulation:

$$N(n) = k_1 \frac{p_d(n)}{T(n)} + k_2' \frac{e(n)}{T(n)} + k_3 \frac{e(n)}{T^2(n)} \rightarrow [\text{N-units}]. \quad (10.12)$$



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where N is expressed in N-units and the coefficients are:

$$k'_1 = 0.776, \quad k'_2 \approx 0.233, \quad k_3 = 3.73 \times 10^3 [K^2 Pa^{-1}]. \quad (10.13)$$

10.5.5 Zenith Delays

The Zenith Total Delay (ZTD) is obtained by discrete vertical integration of refractivity:

$$ZTD = 10^{-6} \sum_n N(n) \cdot \Delta z_n. \quad (10.14)$$

The hydrostatic and wet components are computed separately as:

$$ZHD = 10^{-6} \sum_n k_1 \frac{p_d(n)}{T(n)} \Delta z_n, \quad ZWD = 10^{-6} \sum_n \left[k'_2 \frac{e(n)}{T(n)} + k_3 \frac{e(n)}{T^2(n)} \right] \cdot \Delta z_n. \quad (10.15)$$

10.5.6 Surface-Based Approximation and Slant Delays

As a rapid approximation, the hydrostatic delay is also estimated from surface pressure using the Saastamoinen model:

$$ZHD_{[m]} \approx 0.0022768 \times \frac{p_s [hPa]}{1 - 0.00266 \cos(2\varphi) - 0.00028 H}, \quad (10.16)$$

where H is the ellipsoidal height (km) and φ is the geodetic latitude. Finally, slant delays are obtained through elevation-dependent mapping functions:

$$SLD_h = ZHD \cdot m_h(E, \varphi, doy), \quad SLD_w = ZWD \cdot m_w(E, \varphi, doy), \quad (10.17)$$

where m_h and m_w denote the hydrostatic and wet mapping functions, respectively (Niell or VMF1). The complete pseudocode implementing this workflow is provided in **Appendix F.11**.

10.5.7 Geopotential Referencing and Terrain Correction

In the WRF model, both the perturbation geopotential (PH) and the base-state geopotential (PHB) are defined on η -half levels. In order to compute geopotential height at the k -th mass level, the total geopotential must be averaged between the two adjacent half levels. This averaging step ensures proper alignment between the geopotential field and the model's mass levels and avoids systematic vertical offsets that would otherwise arise from a direct half-level conversion. The geometric height is therefore obtained as:

$$Z(k) := \frac{(PH + PHB)_{k+(1/2)} + (PH + PHB)_{k-(1/2)}}{2g_0}, \text{ with } g_0 = 9.80665 [ms^{-2}]. \quad (10.18)$$

This formulation guarantees a physically consistent vertical referencing, whereas a direct conversion using $(PH + PHB)/g_0$ at a single half level would result in a misalignment between



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geopotential and physical mass levels. Accurate geopotential heights at each model level are a critical requirement for the computation of tropospheric excess path delays. In addition to vertical consistency, these heights must be correctly referenced to the actual Earth's surface, which requires the use of a high-resolution Digital Terrain Model (DTM) to anchor the delay estimates to local topography. This aspect is particularly relevant for regional-scale applications, such as the Scanzano (PA) case study. While WRF natively embeds terrain information within its NetCDF outputs, ERA-5 does not provide an explicit terrain model at model levels. Consequently, an external DTM is incorporated to ensure topographic consistency between ERA-5 model levels and the real surface, as discussed in the following section.

Note: For completeness, all technical and algorithmic details—including hybrid-level coefficients, geopotential reconstruction procedures, full MATLAB/WRF pseudocode, Digital Elevation Model preprocessing, and memory-optimized implementation notes—are collected in **Appendix F.5**.

The present section retains only the conceptual workflow and physical interpretation required for the main body of the chapter. The detailed implementation material originally presented in **Sections §10.5, §10.6, and §10.7**—including extended workflows, pseudocode, hybrid-coordinate reconstruction steps, and full MATLAB/WRF procedures—has been relocated to the **Appendix** in order to improve readability and preserve a clear conceptual flow in the core chapter.

10.6 Visualization of ZHD, ZWD, and ZTD

High-resolution spatial representations of Zenith Total Delay (ZTD), Zenith Hydrostatic Delay (ZHD), and Zenith Wet Delay (ZWD) over Sicily provide an integrated view of atmospheric refractivity and its partitioning into hydrostatic and wet components. These maps, generated through physically constrained modelling and validated against regional atmospheric datasets, capture the spatial variability of tropospheric delays and their implications for high-precision GNSS positioning, InSAR atmospheric phase correction, and advanced remote sensing applications.

10.6.1 Visualization of ZHD, ZWD, and ZTD

This subsection describes the implementation adopted for the computation and visualization of Zenith Total Delay (ZTD), Zenith Wet Delay (ZWD), and Zenith Hydrostatic Delay (ZHD) over the Sicilian region. The results correspond to a representative case study on **15 May 2023 at 15:15 UTC**, selected to illustrate the spatial organization of tropospheric delay components under realistic atmospheric conditions. The visualizations are generated using dedicated MATLAB routines, ensuring consistency between numerical outputs and their graphical representation.

Figure 10.4 presents a composite visualization of the WRF-based processing workflow for the selected case study over Sicily. The upper-left panel displays the Zenith Wet Delay (ZWD) field, highlighting localized maxima associated with enhanced atmospheric water vapor



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content. The upper-central panel shows the Zenith Total Delay (ZTD), expressed in millimetres, revealing spatial gradients driven by the combined hydrostatic and wet contributions. The upper-right panel reports the elevation-dependent, slant-path-corrected ZTD, emphasizing the amplification of delay magnitude at lower elevation angles. The lower-left panel illustrates the vertical distribution of ZWD in a latitude–height cross section, capturing the stratification of atmospheric moisture and its contribution to tropospheric delay at different altitudes. The lower-right panel provides a regional geographical reference for Sicily, situating the spatial patterns within their geographical context without implying cartographic precision beyond the model resolution. All graphical outputs were produced by the author within the framework of research and dissemination activities conducted at **Telespazio S.p.A.**, where the developed modelling and visualization tools were presented and discussed in the context of multiple technical seminars and internal research meetings. This activity contributed to the validation and refinement of the proposed workflow in an applied industrial–research environment. See Appendix 10.A (Figures), Fig. A10.4, Fig. A10.5.

Figure 10.6 presents a high-resolution map of the Zenith Wet Delay (ZWD) over the eastern sector of Sicily, revealing a marked local maximum in tropospheric moisture along the eastern flank of Mount Etna. The superposition on a high-definition satellite base map enables a direct spatial association between the observed ZWD anomaly and the underlying orographic structure. The presence of a well-defined peak exceeding 10 mm in zenith delay indicates a localized enhancement of atmospheric water vapour, consistent with the influence of complex terrain on moisture accumulation and vertical lifting processes. A pronounced southwestward gradient in ZWD is observed, pointing to sharp sub-regional contrasts in water-vapor content and highlighting the heterogeneous nature of the tropospheric wet component at kilometre scales. Such spatial variability is of particular relevance for high-precision GNSS positioning and InSAR atmospheric corrections, as well as for the calibration and validation of microwave satellite observations, where localized wet-delay anomalies can significantly affect path-integrated signal propagation along different lines of sight. See Appendix 10.A (Figures), Fig. A10.6.

Figure 10.7 illustrates the progressive spatial refinement of tropospheric delay mapping, transitioning from a regional-scale representation of the Zenith Total Delay (ZTD) to a high-resolution analysis of the Zenith Wet Delay (ZWD) over a focused sub-domain. The right panel depicts the mesoscale distribution of ZTD across the study area, where a localized delay maximum is identified and highlighted. The left panel presents a zoomed view corresponding to approximately 20% of the original domain, within which high-resolution interpolation is applied to resolve the ZWD field. This refined representation reveals fine-scale spatial gradients and localized structures, with effective resolutions approaching 100 m, that are not discernible at coarser scales. This scale-adaptive visualization highlights the importance of nested analysis strategies for capturing localized atmospheric water-vapor inhomogeneities, which play a critical role in high-precision geodetic applications, tropospheric delay correction, and satellite communication performance. See Appendix 10.A (Figures), Fig. A10.7.



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10.7 First Stage of the WRF5-Based Algorithm: Physical and Methodological Framework

This section describes the first stage of the WRF5-based algorithm, which establishes the physical and computational framework required to transform atmospheric model outputs into tropospheric delay observables. Rather than representing a purely procedural step, this stage defines the reference geometry, vertical structure, and thermodynamic consistency upon which all subsequent delay computations rely. Three-dimensional meteorological fields—pressure, temperature, and water-vapor content—are ingested from high-resolution numerical weather model outputs [232]. These fields undergo spatial co-registration, vertical referencing, and quality control to ensure physical consistency across terrain-following coordinates and geodetic height levels. The resulting atmospheric state constitutes the foundational input for refractivity estimation and for the separation of hydrostatic and wet contributions to tropospheric delay. By consolidating geometric referencing, thermodynamic reconstruction, and moisture representation within a single processing stage, the algorithm ensures that all subsequent steps operate on physically coherent and internally consistent atmospheric fields, minimizing the propagation of numerical artefacts into the final ZHD and ZWD products. Detailed algorithmic procedures—including raster definition, DEM integration, geopotential conversion, pressure reconstruction, and refractivity formulation—are provided in **Appendix F.6**.

10.7.1 Spatial Reference Definition and Terrain Coupling

The computational domain is defined through a regular surface raster at 1-km spatial resolution, which serves as the geodetic reference grid for all atmospheric variables. A georeferenced raster framework ensures a consistent transformation between grid indices and geographic coordinates, enabling the direct integration of atmospheric model outputs with topographic information. High-resolution terrain data are incorporated using the GMTED2010 Digital Elevation Model, providing elevation above mean sea level across the domain. The DEM ensures that pressure, temperature, and geopotential quantities are correctly referenced to the actual surface geometry and provides the physical basis for interpreting spatial variations in tropospheric delay in relation to orographic features—an essential requirement for regional-scale applications characterized by strong terrain-induced gradients [214, 220].

10.7.2 Meteorological Field Extraction and Vertical Referencing

Atmospheric variables are extracted from ERA5 reanalysis products in NetCDF format, providing hourly fields of pressure, temperature, humidity, and geopotential across multiple pressure levels spanning the troposphere and lower stratosphere [99, 227]. These data are ingested into a unified data structure to ensure consistent co-registration with the surface grid and facilitate multi-variable analysis at each horizontal location. ERA5 geopotential fields are converted into geometric heights to link the model's native vertical coordinate system to real altitudes, ensuring



compatibility with propagation and delay formulations that explicitly depend on geometric height [229, 230, 237].

10.7.3 Pressure Field Reconstruction and Perturbation Diagnostics

Surface pressure and vertically varying pressure profiles are reconstructed using physically consistent formulations that account for altitude-dependent variations. The reconstructed pressure field is used both for refractivity estimation and for diagnostic analyses, including the computation of pressure perturbations relative to the domain-averaged state. These diagnostics establish continuity with perturbation-based formulations commonly adopted in WRF and provide insight into mesoscale atmospheric variability [232].

10.7.4 Thermodynamic Consistency and Moisture Representation

Water-vapor content is expressed in terms of mixing ratio, derived from specific humidity through standard thermodynamic relations. This formulation ensures consistency with refractivity and radiative-transfer models. Temperature fields are further transformed into potential temperature to characterize atmospheric stability independently of pressure effects. Refractivity profiles are then computed using the **Smith–Weintraub** formulation, explicitly separating dry and wet contributions. These profiles constitute the core physical input for subsequent ZHD and ZWD integration, as well as for higher-frequency radiative-transfer analyses in the millimetre-wave regime [44, 46].

10.7.5 Methodological Role within the Overall Workflow

This first stage does not aim to produce final delay estimates, but rather to construct a physically consistent atmospheric representation in which geometry, thermodynamics, and moisture distribution are coherently linked. By consolidating spatial referencing, terrain coupling, and thermodynamic reconstruction, the algorithm establishes a robust foundation for accurate tropospheric delay computation in subsequent processing stages [46, 216, 231].

10.8 Second Stage of the WRF5-Based Algorithm

The second stage of the WRF5-based algorithm addresses the transformation of zenith tropospheric delays into slant-path delays along the satellite–receiver line of sight. This step is essential for linking vertically integrated atmospheric quantities to realistic propagation geometries, as required in GNSS positioning, interferometric phase correction, and microwave remote sensing. Slant delays are obtained by projecting the zenith hydrostatic and wet components along the observation direction, accounting for elevation angle and the three-dimensional structure of the atmosphere. The implementation combines geometric projection with refractivity-based integration along the signal path, ensuring internal consistency between zenith and slant delay formulations. Full algorithmic details are reported in **Appendix F.13**.



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10.9 Conceptual and Methodological Basis of ITU-R P.834-9

This section examines the conceptual framework of the ITU-R P.834-9 Recommendation (December 2017) and its application to the generation of global tropospheric delay maps. While the Recommendation provides a standardized climatological procedure for large-scale delay estimation, its direct numerical implementation reveals methodological ambiguities that require careful physical interpretation. Beyond commonly discussed issues related to Day-of-Year parameterization, a critical limitation concerns the assumption of constant gravitational acceleration, despite its known dependence on latitude and altitude. Because gravitational acceleration enters explicitly into hydrostatic delay formulations, inaccuracies in its representation directly propagate into computed delay fields. For this reason, the present work adopts a physically informed interpretation of ITU-R P.834-9, treating it as a baseline climatological model rather than a black-box prescription. The objective is not to modify the standard formulation, but to ensure that its numerical realization remains consistent with fundamental atmospheric physics when applied beyond its original scope.

10.9.1 Implementation Complexity and Resolution Constraints

The practical use of ITU-R P.834-9 reveals an intrinsic scale dependency. At the prescribed global resolution, geographically confined regions—such as Sicily—are represented by only a few grid cells, resulting in limited spatial structure even in areas characterized by strong orographic gradients and atmospheric heterogeneity. This behavior reflects the intended climatological design of the Recommendation, which prioritizes large-scale patterns over mesoscale variability. Consequently, terrain-driven gradients, coastal transitions, and localized moisture structures are inherently smoothed, limiting direct applicability to site-specific or regional analyses.

10.9.2 Optimized Implementation and Relevance to Climate-Oriented Studies

To assess the practical implications of these limitations, two complementary implementations of ITU-R P.834-9 were developed: a strict reference implementation and an optimized version incorporating targeted numerical refinements while remaining fully compatible with the Recommendation. The comparison demonstrates improved numerical stability and enhanced correspondence between spatial delay patterns and known geographic and climatological features in the optimized implementation, without introducing artificial structures. Beyond propagation and geodetic applications, these results suggest a broader potential use of tropospheric delay fields as indirect indicators of long-term atmospheric variability. The optimized formulation should therefore be regarded as a research-grade extension tailored to applications requiring higher physical fidelity. Full numerical details and MATLAB implementations are documented in **Appendix F.8**.



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10.10 Results and Global-Scale Visualization of Tropospheric Slant Delay

This section presents global-scale visualizations of tropospheric slant path delay (SPD) obtained from the optimized ITU-R P.834-9 implementation. The analysis focuses on the spatial structure of delay fields and their consistency with large-scale atmospheric circulation, pressure distribution, and water-vapor variability. **Figure 10.8** shows the global distribution of SPD computed at an elevation angle of 45° for 15 May 2023 at 15:15 UTC, assuming a site altitude of 0.56 km representative of the Scanzano station. The resulting field exhibits clear latitudinal organization, with enhanced delays in tropical regions and reduced values toward higher latitudes, in agreement with known climatological patterns. The smooth spatial transitions and absence of spurious discontinuities confirm the improved numerical stability and physical coherence of the optimized formulation. To facilitate the inspection and qualitative assessment of global tropospheric delay fields, an interactive visualization environment was developed within the SNEM-2 framework. The tool supports exploratory analysis of large-scale delay maps by enabling user-controlled graphical adjustments, while preserving the integrity of the underlying geophysical fields. An example of the interface used for this purpose is shown in **Figure 10.9**. See Appendix 10.A (Figures), Fig. A10.8, Fig. A10.9.

10.11 Synthetic Seasonal Analysis for the Scanzano Ground Station Site

Seasonal composites provide a compact view of how the tropospheric delay budget evolves over the **Scanzano** region. **Figure 10.16** summarizes the spatial distribution of the seasonal mean ZTD for DJF, MAM, JJA, and SON, highlighting coherent latitudinal and orographic patterns across the study domain. While the hydrostatic component remains comparatively stable in time, the wet contribution exhibits marked seasonality, consistent with the annual cycle of water-vapor availability. These results indicate that the largest delay variability is expected during the warm season, when humidity-driven refractivity enhancement produces the strongest wet-delay contribution at both regional and site scale. **Table 10.5** complements the maps by quantifying the seasonal statistics at the site and over the domain. The highest ZTD levels occur in the cold-to-transition seasons (DJF–MAM), whereas ZWD reaches its maximum during JJA, reflecting the dominance of moisture in summer. This seasonal partitioning is operationally relevant for GNSS processing and for propagation-sensitive applications at microwave and millimetre-wave frequencies, where wet delay and associated variability can represent the main source of short-term uncertainty. Finally, **Table 10.6** provides the consolidated descriptive statistics for ZHD, ZWD, ZTD, and the derived SPD, reported both over the full domain and at the Scanzano reference site. The side-by-side reporting enables a direct assessment of site representativeness and of the extent to which local conditions deviate from the regional background, supporting



the interpretation of subsequent slant-path analyses. See Appendix 10.B (Tables), Table A10.3, Table A10.4, Appendix 10.A (Figures), Fig. A10.10, Fig. A10.11, Fig. A10.12.

10.12 Implementation of Gravity Acceleration within the SNEM-2 Framework

In the **SNEM-2 framework**, developed around the ITU-R formulations for the global gravity acceleration map, the computation is performed through progressively refined equations that reconcile standard recommendations with geophysical realism. At the local scale, a latitude-dependent correction is applied to the nominal gravity constant ($9.80665 \text{ m} \cdot \text{s}^{-2}$) to account for the Earth's rotation and oblate geometry, enabling site-specific estimates such as for the **Scanzano** (PA) station in southern Italy, which serves as a reference for atmospheric delay analyses. On a global scale, the algorithm expands the computation to a gridded field of latitude and altitude, integrating geodetic correction factors that capture the expected spatial variability of gravity. The resulting global map, visualized within **MATLAB** (see **Figure 10.13**), reveals enhanced values toward the poles and a gradual attenuation near the equator and over oceanic regions, consistent with the Earth's mass distribution. These spatially resolved values constitute a fundamental physical input for the subsequent modeling of the hydrostatic component of the tropospheric delay. See Appendix 10.A (Figures), Fig. A10.13.

10.13 Enhanced Global ITU-R Tropospheric Delay Model (eG-ITU-R TD)

10.13.1 Motivation and Rationale, and Scientific Context

Tropospheric delay represents one of the dominant error sources in precise GNSS positioning and Earth-space radiowave propagation, particularly at microwave and millimeter-wave frequencies. Its accurate characterization requires a physically consistent description of both the hydrostatic and wet atmospheric components, including their vertical structure and geographical variability [45,46,56]. Internationally adopted formulations, such as those provided by the ITU-R recommendations, offer a consolidated theoretical framework for modeling tropospheric refraction, gaseous attenuation, and excess path delay [1,3,44]. However, these recommendations are primarily conceived as analytical guidelines rather than as fully operational global models. When directly implemented at global scale, several limitations emerge, particularly in relation to altitude-dependent corrections, parameter coupling, and numerical stability under extreme atmospheric conditions [45,216]. A critical issue arises from the partial decoupling of thermodynamic variables—pressure, temperature, water vapour content, lapse rates, and gravity acceleration—which are often treated independently in standard formulations. This approach can lead to inconsistencies when extrapolated to polar regions, high-altitude terrain, or low-pressure environments, ultimately affecting the physical realism of the resulting tropospheric delay fields



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[45,111]. To overcome these limitations, an **Enhanced Global ITU-R Tropospheric Delay Model (eG-ITU-R TD)** was developed as a fully independent MATLAB implementation. Rather than reproducing the ITU-R equations verbatim, the model reinterprets the original formulations within a unified global framework, explicitly enforcing inter-parameter consistency and vertical coherence. This approach enables the generation of seamless global fields of zenith hydrostatic delay (ZHD) and zenith wet delay (ZWD), suitable for both climatological analysis and operational applications [3,56]. The model further integrates vertically resolved atmospheric information derived from modern reanalysis products, notably ERA5 [99], allowing thermodynamic variables to be interpolated consistently at the receiver height while preserving large-scale atmospheric structure. This integration supports refined altitude corrections, improved numerical stability, and enhanced applicability to GNSS tropospheric correction, geodetic studies, and satellite communication link analysis across a wide range of frequencies [1,3,111].

10.13.2 Conceptual Design and Modelling Strategy

The conceptual design of the **eG-ITU-R Tropospheric Delay Model** is driven by the need to transform the ITU-R tropospheric formulations from a collection of analytical expressions into a coherent, physically constrained, and globally applicable modeling system. The guiding principle is that tropospheric delay must be treated as an emergent quantity resulting from a vertically consistent thermodynamic state of the atmosphere, rather than as the superposition of partially independent correction terms. In standard implementations, hydrostatic and wet delay components are often computed using surface-level parameters combined with simplified vertical scaling laws [45,216]. While this approach is adequate for local or mid-latitude applications, it becomes increasingly fragile when extended to heterogeneous global conditions, where variations in lapse rate, gravity acceleration, and water vapor stratification play a non-negligible role [111]. The eG-ITU-R TD model addresses this limitation by enforcing a vertically coherent representation of atmospheric variables throughout the tropospheric column. At the core of the modeling strategy lies a global latitude–longitude grid, which serves as the reference domain for all atmospheric parameters. Each grid point is associated with a consistent set of thermodynamic quantities—pressure, temperature, water vapor pressure, and derived refractivity—defined at the receiver height through vertically constrained interpolation. This approach ensures that zenith hydrostatic delay (ZHD) and zenith wet delay (ZWD) are computed from mutually consistent atmospheric states, avoiding artificial discontinuities across regions or altitude regimes [3,56]. Particular attention is devoted to the treatment of altitude effects. Instead of applying generic elevation correction factors, the model explicitly incorporates altitude-dependent gravity acceleration and temperature lapse rates, allowing the hydrostatic contribution to adapt smoothly to orographic variability. This design choice mitigates the risk of systematic biases over mountainous regions and high-latitude plateaus, where standard formulations may yield non-physical behavior [45,216]. The wet component is modeled following ITU-R recommendations for water vapor refractivity [44], but its vertical decay is governed by a physically motivated water vapour decreasing



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factor, derived consistently from atmospheric profiles. This formulation enables the ZWD to reflect realistic moisture stratification while remaining numerically stable under dry or cold atmospheric conditions [46,99]. Finally, the **eG-ITU-R TD framework** is explicitly conceived as modular and extensible. The separation between physical modeling, numerical implementation, and output generation allows the model to support multiple application domains, including GNSS tropospheric correction, geodetic analysis, and Earth-space propagation studies, without compromising physical consistency [1,3]. This modularity also facilitates future integration with advanced mapping functions or data-driven refinements, while preserving the physical core of the ITU-R-based formulation.

10.13.3 Enhancements with Respect to Standard ITU-R Formulations

The **eG-ITU-R Tropospheric Delay Model** introduces a series of targeted enhancements that extend the applicability of standard ITU-R formulations from a theoretical reference framework to a globally consistent, numerically stable, and operational modelling system. These enhancements do not alter the physical foundations of the ITU-R recommendations; rather, they resolve practical ambiguities and structural limitations that emerge when such formulations are implemented at global scale. A first major enhancement concerns the global spatial consistency of the model outputs. While ITU-R recommendations are typically applied at single sites or within limited geographical domains, the **eG-ITU-R TD model** operates on a continuous latitude-longitude grid, ensuring spatial coherence of zenith hydrostatic delay (ZHD) and zenith wet delay (ZWD) across continental and intercontinental scales. This design eliminates artificial discontinuities at regional boundaries and enables direct use of the model outputs for global climatological and geodetic analyses [3,56]. A second improvement addresses the explicit coupling of thermodynamic parameters. In conventional implementations, pressure, temperature, water vapor pressure, and lapse rates are often introduced through independent empirical relationships [45]. In contrast, the **eG-ITU-R TD model** enforces a unified thermodynamic state at each grid point and receiver height, ensuring that refractivity, delay components, and derived quantities are mutually consistent. This approach significantly reduces the risk of internal inconsistencies, particularly under extreme atmospheric conditions such as polar winters or arid high-altitude environments [111]. Altitude-dependent effects constitute a third key enhancement. Standard ITU-R formulations rely on simplified elevation corrections, which may omit scaling factors or implicitly assume mid-latitude conditions [45,216]. The **eG-ITU-R TD model** refines these corrections by explicitly incorporating latitude- and altitude-dependent gravity acceleration, together with physically consistent temperature lapse rates. As a result, the hydrostatic delay adapts smoothly to complex orography, avoiding the systematic biases that can arise over mountainous regions or elevated plateaus [216,111]. The modeling of the wet component is likewise enhanced through a physically constrained treatment of water vapor stratification. While the refractivity formulation follows ITU-R recommendations [44], the vertical decay of water vapor is governed by a decreasing factor derived consistently from atmospheric profiles



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rather than imposed as a fixed empirical parameter. This formulation preserves sensitivity to realistic moisture distributions while ensuring numerical stability in cold or dry conditions, where conventional models may yield unrealistic or vanishing ZWD values [46,99]. Finally, the eG-ITU-R TD model expands the scope of the generated outputs beyond the traditional ZHD and ZWD fields. In addition to tropospheric delays, the model provides global maps of auxiliary atmospheric variables—including surface and receiver-level pressure and temperature, lapse rates, gravity acceleration, and water vapor parameters—explicitly designed to support GNSS tropospheric correction, geodetic positioning, and Earth–space propagation studies [1,3]. This extended output set enables seamless integration with higher-level modeling frameworks, mapping functions, and link-budget analyses, without requiring ad hoc post-processing.

10.13.4 MATLAB Implementation and Reproducibility

The eG-ITU-R Tropospheric Delay Model is implemented as a fully self-contained MATLAB project, explicitly designed to ensure traceability, numerical transparency, and reproducibility of all computed results. The implementation reflects the modeling principles discussed in §§10.13.1–10.13.3 and provides a concrete operational realization of the ITU-R tropospheric delay formulations within a global framework.

Parameter Map	Units	Description	Method
Vertical Wet Delay (ZWD)	m	Delay due to atmospheric water vapour	ITU-R P.834-9
Vertical Hydrostatic Delay (ZHD)	m	Dry delay determined by surface pressure	Surface pressure + altitude correction
Water Vapour Pressure at Receiver Height	hPa	Partial pressure at receiver altitude	Interpolated meteorological data
Temperature at Receiver Height	K	Temperature at receiver altitude	Lapse rate + reference height model
Pressure at Receiver Height	hPa	Pressure at receiver altitude	Surface pressure model
Surface Air Temperature	K	Mean global surface temperature distribution	ERA-based interpolation
Water Vapour Decreasing Factor (λ)	–	Altitude dependence of water vapour concentration	Exponential model
Mean Temperature Lapse Rate (α_m)	K/km	Vertical temperature gradient	Atmospheric profiles
Surface Medium Temperature Range	K	Mean temperature at 1.5° resolution	Global gridded dataset
Gravity Acceleration	m/s ²	Variation with latitude and altitude	Geophysical model

Table 10.2. Global atmospheric parameter maps generated by the MATLAB implementation.

10.13.4.1 Project Structure and Computational Workflow

The MATLAB project is organized into modular functional blocks, separating physical modelling, numerical evaluation, and output generation. This structure allows each processing stage to be independently inspected, validated, and modified without affecting the integrity of the overall framework. The main computational workflow consists of:

1. Input preparation, including geodetic coordinates, receiver altitude, and gridded atmospheric fields derived from reanalysis data;



2. Thermodynamic parameter evaluation, where pressure, temperature, water vapor pressure, lapse rates, and gravity acceleration are computed or interpolated consistently at the receiver height;
3. Tropospheric delay computation, where zenith hydrostatic delay (ZHD) and zenith wet delay (ZWD) are evaluated following ITU-R formulations, using the internally consistent atmospheric state;
4. Output generation, producing gridded global fields and auxiliary diagnostic variables.

This modular workflow ensures that each numerical output can be traced back to its corresponding physical assumptions and input parameters.

10.13.4.2 Atmospheric Inputs and Vertical Consistency

Vertically resolved atmospheric information is derived from the ERA5 reanalysis dataset [99], which provides global fields of pressure, temperature, and humidity with sufficient spatial and temporal resolution for tropospheric modelling. These fields are interpolated to the receiver height using physically consistent lapse-rate formulations, avoiding discontinuities associated with surface-only parameterizations. The implementation explicitly propagates altitude-dependent gravity acceleration into the hydrostatic delay computation, ensuring consistency between geodetic location and atmospheric state. This choice directly reflects the refinements discussed in §10.13.3 and represents one of the key implementation-level corrections with respect to simplified standard approaches [45,216].

10.13.4.3 Computation of ZHD and ZWD

The zenith hydrostatic delay is computed from surface pressure following ITU-R formulations [44], with refined altitude handling and gravity acceleration explicitly included in the numerical evaluation. The zenith wet delay is derived from water vapour refractivity, adopting ITU-R-compliant expressions [44,46], while the vertical decay of water vapour is governed by a decreasing factor obtained consistently from atmospheric profiles. Both ZHD and ZWD are computed on a global latitude–longitude grid, ensuring spatial continuity and avoiding artefacts associated with site-based implementations. Intermediate variables, including refractivity components and scaling factors, are retained as diagnostic outputs to facilitate validation and sensitivity analysis.

10.13.4.4 Generated Outputs and Diagnostic Fields

In addition to ZHD and ZWD, the MATLAB implementation produces a set of auxiliary atmospheric fields required for result interpretation and downstream applications. These include:

- pressure and temperature at the receiver height;
- water vapor pressure and water vapor decreasing factor;
- mean temperature lapse rate;



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- latitude- and altitude-dependent gravity acceleration;
- surface and medium atmospheric temperature fields.

All outputs are generated on the same global grid, ensuring internal consistency and facilitating direct comparison with external datasets or alternative tropospheric models. The availability of these diagnostic fields enables verification of numerical behavior under extreme climatic conditions and supports reproducibility of the reported results.

10.13.4.5 Reproducibility and Scope of Validation

The **eG-ITU-R TD MATLAB** implementation is deterministic and does not rely on stochastic tuning parameters. Given identical inputs, the model produces identical outputs, ensuring full reproducibility. The generated global fields of ZHD and ZWD are directly compatible with GNSS tropospheric correction workflows and Earth–space propagation analyses [1,3]. Validation and intercomparison with external datasets, including reanalysis products and GNSS-derived tropospheric delays, are discussed in subsequent chapters, where the model outputs are evaluated within broader atmospheric and propagation studies.

10.13.5 Scope, Limitations, and Intended Use

The **eG-ITU-R Tropospheric Delay (TD) Model** is designed as a physically consistent, global implementation of existing ITU-R tropospheric delay formulations. The model does not introduce new physical laws or empirical parameterizations beyond those recommended by ITU-R, but focuses on enforcing numerical coherence and vertical consistency in global applications. Its accuracy is therefore bounded by the validity of the underlying ITU-R expressions and by the quality of the atmospheric input data, particularly in regions with sparse observations. The model is intended for use in GNSS tropospheric correction, geodetic analysis, and Earth–space propagation studies, where reproducible and physically traceable delay estimates are required. It is not conceived as a replacement for high-resolution numerical weather prediction models, but as an operationally robust reference framework for large-scale analyses [1,3]. **Figure 10.14** presents the output of my implementation, summarizing the main atmospheric and geophysical variables relevant to tropospheric delay modeling. The panels include vertical wet and hydrostatic delays, water vapor, temperature, and pressure at receiver’s height, as well as surface air temperature, lapse rate, and mean temperature. Additional maps display the decreasing factor of water vapor, the range surface medium temperature, and the global distribution of gravity acceleration. Together, these outputs provide the fundamental input parameters for the estimation of tropospheric path delays and their seasonal and geographical variability. See Appendix 10.A (Figures), Fig. A10.14.



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10.14 Introduction to Source Code Optimization and Day of the Year (DOY) Handling

Efficient source-code design and optimization are essential for ensuring computational performance, numerical stability, and scalability in large-scale atmospheric and geodetic modelling applications [45,56]. In the present work, all global atmospheric parameter maps were generated through an original MATLAB implementation developed from scratch to implement, extend, and, where necessary, refine the ITU-R recommendations for tropospheric delay modelling [1,3,44]. The adopted implementation does not rely on adaptations of pre-existing scripts but constitutes an independent development pipeline, encompassing model design, numerical formulation, and validation. This approach enabled full control over the internal consistency of the model outputs and their suitability for global-scale applications, as illustrated by the resulting delay fields shown in **Figure 10.15**. Particular attention was devoted to the treatment of time-dependent parameters, with specific focus on the handling of the Day of the Year (DOY), which plays a central role in atmospheric modelling, tropospheric delay estimation, and meteorological data assimilation [45,99]. The implemented DOY management ensures robust date parsing, reduced computational overhead, and full interoperability with external datasets, including ERA5 reanalysis products [99], IGS GNSS observations [56], and WRF model outputs [216]. Leap-year conditions, time-reference consistency, and dataset indexing alignment are explicitly accounted for, thereby mitigating common sources of systematic bias in time-dependent atmospheric corrections. The benefits of this approach are reflected in the spatial coherence and physical realism of the global slant path delay patterns shown in Figure 10.yyyy, where large-scale latitudinal gradients are preserved without introducing artificial temporal discontinuities. From a computational perspective, the code was optimized through targeted memory-management strategies aimed at reducing execution time and storage requirements without compromising physical fidelity. These strategies include the elimination of redundant variables, selective use of single-precision arithmetic where appropriate, vectorization of core operations, and systematic preallocation of arrays to avoid dynamic memory resizing [45,111]. Collectively, these optimizations led to a measurable reduction in computational cost while preserving numerical accuracy and physical consistency. Consequently, all global atmospheric maps presented in this work—including the tropospheric slant path delay distribution shown in **Figure 10.15**—are the direct outcome of an independent and optimized MATLAB development, designed to ensure scientific robustness, numerical reliability, and operational applicability across a broad range of geodetic positioning, GNSS atmospheric correction, and Earth-space propagation studies [1,3,56]. See Appendix 10.A (Figures), Fig. A10.15.

In **Figure 10.16** the tropospheric slant path delay (SPD) is displayed with superimposed contour lines, which help to highlight the strong geographical contrasts. High values are concentrated over the equatorial belt and regions with complex orography, while lower delays are found in polar and arid zones. See Appendix 10.A (Figures), Fig. A10.16.

Figure 10.17 illustrates the global distribution of **tropospheric slant path delay (SPD)**



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in meters, calculated at a low elevation angle of 10° . The contour lines highlight regions of enhanced delay, particularly over humid equatorial areas and mountainous regions, while reduced values are observed at high latitudes. The strong variability emphasizes the sensitivity of SPD to both atmospheric water vapor content and elevation geometry. The contour representation provides a clearer view of the large-scale gradients, offering a more intuitive interpretation of the atmospheric impact on low-elevation Earth-space links. See Appendix 10.A (Figures), Fig. A10.17.

10.14.1 Original DOY Handling and Global Tropospheric Delay Mapping in the eG-ITU-R TD Model

The **Day of the Year (DOY)** provides a numerical representation of calendar time expressed as a sequential day count starting from 1 January ($\text{DOY} = 1$). In common years, DOY ranges from 1 to 365, while in leap years it extends to 366. This representation is widely adopted in meteorology, geodesy, and satellite communications because it offers a compact and unambiguous temporal reference, independent of month length and calendar irregularities [45, 56]. In atmospheric modelling and tropospheric delay estimation, DOY plays a fundamental role for three main reasons. First, it simplifies time indexing in numerical models, facilitating the management of time series and seasonal parameterizations [45]. Second, it ensures interoperability among heterogeneous datasets—such as ERA5 reanalysis products [99], IGS GNSS observations [56], and WRF model outputs [216]—which frequently adopt DOY-based temporal conventions. Third, several ITU-R formulations explicitly or implicitly rely on periodic or seasonal dependencies, for which DOY constitutes the natural independent variable [1, 3, 44].

Incorrect handling of DOY can introduce severe systematic errors in global atmospheric products. Neglecting leap years leads to a one-day temporal shift after 28 February, progressively misaligning seasonal cycles. Inconsistent treatment of UTC and local time introduces additional temporal biases, while confusion between zero-based and one-based indexing conventions may corrupt dataset access and parameter retrieval [45, 111]. Such issues are particularly detrimental in global applications, where small temporal misalignments propagate into spatially coherent but physically incorrect fields.

Within the **Enhanced Global ITU-R Tropospheric Delay Model (eG-ITU-R TD)**, DOY handling has therefore been explicitly designed to ensure full temporal consistency across all modelling stages. The implementation includes robust date parsing, explicit leap-year detection, strict adherence to UTC time references, and consistent indexing across all integrated datasets. This guarantees correct temporal alignment of all atmospheric parameters used to generate global maps of zenith hydrostatic delay (ZHD), zenith wet delay (ZWD), and derived quantities. The adopted DOY convention and its correspondence with calendar months for both common and leap years are illustrated in **Figure 10.22**. The figure highlights the one-day shift occurring after February in leap years and its implications for seasonal alignment. This representation facilitates the comparison of long-term time series, the aggregation of climatological statistics,



and the synchronization of model simulations with observational datasets. As a result, all global tropospheric delay fields produced by the **eG-ITU-R TD model** are temporally coherent and free from common calendar-related artefacts. This aspect is essential for ensuring the physical reliability of global delay patterns and their suitability for applications in GNSS tropospheric correction, geodetic analysis, and Earth–space propagation studies [1, 3, 56]. See Appendix 10.A (Figures), Fig. A10.18.

10.15 Advanced MATLAB Implementation of the GGZTD-P Global Grid Model

In addition to the development of the Enhanced Global ITU-R Tropospheric Delay model (eG-ITU-R TD) [94, 286], an independent high-accuracy empirical approach was implemented for benchmarking and cross-validation purposes: the Global Grid Zenith Tropospheric Delay – Polynomial (GGZTD-P) model, introduced in recent literature and designed for global ZTD estimation based on numerical weather reanalysis. The GGZTD-P model is a global grid-based formulation for Zenith Tropospheric Delay (ZTD), derived from ERA5 atmospheric reanalysis produced by the European Centre for Medium-Range Weather Forecasts [12, 99] and validated against an extensive radiosonde network. The original study reports a significant reduction in both bias and RMS error with respect to classical empirical mapping approaches, establishing GGZTD-P as a state-of-the-art empirical reference for ZTD modelling.

To enable a rigorous and homogeneous comparison with the proposed eG-ITU-R TD framework, a complete and independent MATLAB implementation of GGZTD-P was developed by the author. The original formulation was re-engineered to ensure numerical stability, computational efficiency, and seamless interoperability within the global processing architecture adopted throughout this thesis [94, 286].

Three grid configurations were implemented to support different resolution–performance trade-offs:

- GGZTD_P_1: $1^\circ \times 1^\circ$ surface parameters and $2^\circ \times 2^\circ$ vertical-profile parameters
- GGZTD_P_2: $1^\circ \times 1^\circ$ surface parameters and $5^\circ \times 5^\circ$ vertical-profile parameters
- GGZTD_P_5: $5^\circ \times 5^\circ$ resolution for both surface and vertical-profile parameters

Each configuration relies on pre-computed optimized coefficients and requires only a compact input set consisting of day of year (DOY), hour of day (HOD), latitude, longitude, and geopotential height, making the model directly applicable to GNSS positioning, geodetic analysis, and global atmospheric-delay mapping [56, 65, 73]. The comparative assessment presented in **Chapter 13** shows that the eG-ITU-R TD model achieves performance comparable to, and in several regimes exceeding, GGZTD-P, particularly for high-resolution global mapping and extreme atmospheric conditions. The implementation of GGZTD-P therefore serves as a robust benchmark, strengthening the validation framework and highlighting the advantages of the physically consistent approach developed in this work.

10.15.1 Theoretical Background and Numerical Methodology

The MATLAB implementation of GGZTD-P required the development of dedicated numerical strategies extending beyond a direct transcription of the original formulation. All algorithmic components were independently coded, tested, and optimized to ensure smooth spatial-temporal behaviour and numerical robustness, consistently with the modelling philosophy adopted for eG-ITU-R TD [94, 286].

10.15.1.1 Altitude-Dependent Sliding-Window Interpolation

To avoid discontinuities in ZTD estimates across altitude levels, a sliding-window interpolation scheme was implemented. The ZTD at altitude h is computed as a weighted average over neighbouring altitude nodes:

$$ZTD(h) := \sum_{i=-k}^k w_i \cdot f(h_i), \quad (10.19)$$

where w_i are normalized window weights and $f(h_i)$ denotes the local ZTD estimate. This approach ensures continuous vertical profiles and mitigates numerical artefacts over complex orography and high-altitude GNSS stations, improving physical consistency with classical tropospheric-delay theory [45].

10.15.1.2 Climate-Dependent Piece-Wise Seasonal Formulation

Seasonal ZTD variability is modeled using latitude-dependent empirical laws, automatically selected during runtime according to climatic regime:

$$ZTD(\phi, t) = \left\{ \begin{array}{ll} a_1 + b_1 \cdot \cos\left(\frac{2\pi t}{T}\right) & \text{if } \phi < 30^\circ \quad (\text{Tropical Zone}) \\ a_2 + b_2 \cdot \cos\left(\frac{2\pi t}{T}\right) & \text{if } 30^\circ \leq \phi < 60^\circ \quad (\text{Temperate Zone}) \\ a_3 + b_3 \cdot \cos\left(\frac{4\pi t}{T}\right) & \text{if } \phi \geq 60^\circ \quad (\text{Polar Zone}) \end{array} \right\} \quad (10.20)$$

with $T = 365.25$ days. This formulation enables climate-aware ZTD modelling on a global scale and is consistent with latitude-stratified empirical mapping functions commonly adopted in GNSS meteorology [56, 215, 216].

10.15.1.3 Fourier-Based Temporal Decomposition

Fast Fourier Transform (FFT) analysis was integrated to extract dominant periodic components from ZTD time series:

$$F(k) := \sum_{n=0}^{N-1} ZTD(n) \times e^{-2\pi i n k / N}. \quad (10.21)$$

The reconstructed signal is expressed as:

$$ZTD(t) = A_0 + \sum_{m=1}^M A_m \cos(2\pi f_m t + \phi_m), \quad (10.22)$$



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allowing noise reduction while preserving physically meaningful diurnal, semi-annual, and annual cycles. This approach is consistent with classical spectral analysis methods used in atmospheric and geodetic time-series studies [239, 240].

10.15.1.4 Physical Consistency with Atmospheric Refractivity

The harmonic representation of ZTD is physically consistent with the refractivity formulation:

$$N = k_1 \frac{p}{T} + k_2 \frac{e}{T^2} + k_3 \frac{e}{T}, \quad (10.23)$$

since pressure p , temperature T , and water-vapor pressure e exhibit intrinsic periodic variability. Their Fourier decomposition propagates naturally into ZTD fluctuations, providing a coherent theoretical basis for the adopted modeling strategy and reinforcing its consistency with tropospheric delay theory and GNSS atmospheric sensing [44, 45, 46, 94].

All of the above components — from sliding window interpolation to FFT-based signal reconstruction — are entirely my own MATLAB implementations of the GGZTD-P computational strategies. They have been engineered for speed, accuracy, and full interoperability with both my **eG-ITU-R TD codebase** and external datasets (ERA5, GNSS, MERRA-2), providing a robust, production-ready ZTD estimation tool. **Figure 10.19** schematically illustrates the computational architecture of the MATLAB implementation developed for the GGZTD-P model. The workflow highlights the four main algorithmic components introduced to ensure numerical stability, physical consistency, and accurate representation of temporal and vertical variability in **Zenith Tropospheric Delay estimation**. The upper block identifies the overall implementation framework, while the four lower modules summarize the key processing steps. Altitude-dependent effects are handled through a sliding-window interpolation scheme, designed to guarantee smooth and continuous ZTD profiles across height layers. Seasonal and latitudinal variability is modeled using a piecewise formulation, allowing climate-zone-dependent parameter selection based on latitude and time of year. Temporal variability is addressed through **Fast Fourier Transform (FFT)** analysis, which isolates dominant periodic components and supports noise reduction and signal reconstruction. Finally, the link between Fourier coefficients and atmospheric refractivity establishes a physically motivated connection between spectral variability and underlying thermodynamic parameters, with potential extensions to cloud optical thickness analysis. This modular structure emphasizes the integration of empirical modeling techniques with physically informed constraints, providing a flexible and extensible framework for global tropospheric delay estimation and comparative benchmarking. See Appendix 10.A (Figures), Fig. A10.19. A synthetic cloud optical thickness dataset was generated in Python and stored in a .mat file, designed to reproduce the temporal and statistical characteristics of satellite observations from instruments such as **MODIS** and **CloudSat**. The code implementation is documented in **Appendix E.9**, while a dedicated MATLAB script was developed to read the dataset and generate the figure presented



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Table 10.3. Qualitative comparison between the empirical GGZTD-P model and the physically consistent eG-ITU-R TD model developed in this work for global Zenith Tropospheric Delay (ZTD) estimation.

Aspect	GGZTD-P	eG-ITU-R TD (This Work)
Model type	Empirical global grid model based on polynomial fitting	Physically consistent hybrid model based on ITU-R formulations
Primary data source	ERA5 atmospheric reanalysis	ITU-R climatological models integrated with physically derived corrections
Spatial resolution	Discrete global grids (1°–5°) with fixed resolution	Continuous global latitude–longitude grid with adaptive resolution
Vertical treatment	Empirical altitude-dependent parameterization	Physically constrained altitude correction with refined scaling
Temporal representation	Harmonic seasonal and diurnal terms	Continuous temporal evolution driven by atmospheric state variables
Climatic adaptation	Latitude-based empirical regimes	Explicit handling of tropical, polar, and high-altitude regimes
Numerical stability	Dependent on pre-fitted coefficients	Explicit numerical stabilization under extreme atmospheric conditions
Input requirements	DOY, HOD, latitude, longitude, geopotential height	Surface pressure, temperature, water-vapour parameters, latitude, height
Computational efficiency	Very high (grid lookup and polynomial evaluation)	High, with additional physical consistency checks
Physical interpretability	Limited (empirical coefficients)	High (direct link to atmospheric refractivity and tropospheric physics)
Sensitivity to extreme conditions	Reduced robustness outside calibration range	Designed to remain stable in polar and mountainous regions
Benchmark performance	State-of-the-art empirical accuracy	Comparable or superior accuracy with improved physical robustness
Primary application	Fast ZTD estimation for GNSS positioning	High-resolution global ZTD mapping and scientific analysis



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in § 10.16. In the following section, Fourier analysis is applied to the synthetic optical thickness time series to identify dominant temporal frequencies and periodicities associated with cloud dynamics, enabling a concise spectral characterization of their multi-scale temporal variability.

10.16 Fourier Analysis

Fourier analysis provides a rigorous and versatile framework for characterizing cloud variability and diagnosing radiative-transfer effects by decomposing the cloud optical thickness τ into its constituent frequency components. Through the application of the Fast Fourier Transform (FFT), dominant periodicities can be objectively identified, enabling the detection of temporal and spatial structures embedded in cloud fields and their modulation by large-scale climatic drivers such as monsoon systems and synoptic circulation patterns [239–242]. Distinct cloud regimes exhibit characteristic spectral signatures. Optically thin, high-altitude cirrus clouds are typically associated with enhanced high-frequency variability, reflecting their fragmented and transient morphology [243]. In contrast, extensive stratiform cloud decks display spectra dominated by low-frequency components, indicative of persistent spatial coverage and slow temporal evolution [244]. These differences provide a physically interpretable basis for cloud classification in the spectral domain. When applied to temporal datasets, Fourier analysis isolates diurnal, synoptic, and seasonal cycles, as well as the spectral fingerprints of individual weather systems [245]. In the spatial domain, FFT applied to satellite or ground-based imagery reveals dominant structural scales and anisotropies in cloud organization, supporting the identification of self-similar and scale-invariant properties of cloud fields [246]. This spectral decomposition enables the separation of high-frequency variability, typically associated with convective or scattered cloud elements, from low-frequency components linked to persistent stratiform layers, thereby facilitating the quantification of cloud radiative forcing across scales [247]. Beyond conventional FFT, continuous wavelet transforms extend the analysis by retaining both frequency and temporal (or spatial) localization, making them particularly suitable for dynamically evolving cloud systems [242, 248]. Fourier- and wavelet-derived coefficients can also be incorporated into machine learning pipelines as robust, physically informed features for automated cloud-type classification and regime detection [249]. Coupling these spectral descriptors with radiative-transfer simulations establishes a direct link between cloud microphysical structure and radiative impact, advancing predictive capabilities in both weather and climate applications [250–251]. By mapping dominant frequency modes of cloud optical thickness across different climatic regions, Fourier-based methods contribute to improved cloud classification schemes, refined parameterizations in numerical climate models, and enhanced fidelity of radiative-transfer simulations. As such, spectral analysis provides a systematic pathway toward more accurate forecasts and more reliable climate projections.



10.16.1 Spectral Decomposition of Cloud Optical Thickness via Fourier Analysis

The Fourier transform offers a rigorous mathematical framework for decomposing a temporal signal such as the cloud optical thickness $\tau(t)$ into a weighted sum of orthogonal sinusoidal components. Through this representation, the original signal is expressed as a superposition of oscillatory modes, each defined by a specific frequency, amplitude, and phase. Application of the Fast Fourier Transform (FFT) maps $\tau(t)$ from the time domain into the frequency domain, where the magnitude spectrum $|\hat{\tau}(f)|$ describes how signal variance is distributed across temporal scales. Spectral peaks correspond to dominant periodicities that can be physically associated with atmospheric processes. Low-frequency components (below approximately one cycle per day) are generally linked to synoptic-scale systems and long-lived stratiform cloud layers. Intermediate frequencies (of the order of one to several cycles per day) often reflect diurnal forcing and mesoscale convective organization, while high-frequency components are typically associated with turbulent fluctuations or thin, transient cirrus structures. **Parseval's** theorem guarantees the equivalence between total variance in the time domain and integrated spectral power, allowing a quantitative assessment of the relative contribution of each frequency band to overall cloud variability. This frequency-based perspective complements time-domain analyses by explicitly revealing the multi-scale nature of cloud fields. In the context of this work, the Fourier framework provides the theoretical basis for the synthetic experiments illustrated in **Fig. 10.21**, whose **MATLAB** implementation is documented in **Appendix F.9**. The proposed approach enables the isolation and quantification of characteristic temporal scales associated with different cloud regimes, supporting both physical interpretation and the development of spectral-based classification and retrieval algorithms for satellite and ground-based radiometric observations. See Appendix 10.A (Figures), Fig. A10.20, Fig. A10.21.

10.17 Slant Path Delay Comparison at 15° Elevation Using the GGZTD-P Model

In addition to the analysis of zenith tropospheric delay, the investigation was extended to the computation of global slant path delay maps using the GGZTD-P model. The analysis explicitly accounts for the day of year (DOY) in order to capture seasonal variability, thereby transforming the slant delay representation from a static, climatological average into a dynamic spatio-temporal framework. Within this approach, each day of the year is associated with a distinct global distribution of slant path delay, allowing the simultaneous assessment of latitudinal dependence and seasonal modulation of tropospheric effects. Such a representation provides a more realistic basis for applications requiring high temporal resolution, including GNSS positioning, satellite link design, and geophysical analyses [56, 94, 286]. The example discussed in this section focuses on a fixed elevation angle of 15°, enabling a controlled comparison of different mapping functions under identical geometric conditions but variable atmospheric states.

10.17.1 Comparative Analysis of Slant Path Delay Estimation Using Different Mapping Functions at 15° Elevation

Figure 10.22 presents a comparative analysis of global slant path delay at an elevation angle of 15°, obtained using four different tropospheric mapping functions: the **Niell Mapping Function (NMF)** [56], the **Vienna Mapping Function 1 (VMF1)** [216], the **Global Mapping Function (GMF)** [215], and a simple cosine mapping model. Each panel displays the estimated slant delay, expressed in meters, over a global latitude–longitude grid. The NMF exhibits a relatively smooth spatial distribution, with moderate delay variations and slightly enhanced values in equatorial and high-latitude regions, reflecting its climatological formulation [56]. The VMF1, derived from numerical weather model outputs, resolves finer spatial structures and displays increased sensitivity to regional atmospheric variability, particularly in tropical and polar areas [216]. The GMF provides an intermediate representation, yielding a smoother global field comparable to NMF but with reduced regional contrasts, while maintaining global applicability [215]. In contrast, the simple cosine model produces an almost uniform delay distribution, reflecting its limited capability to represent geographic and climatic variability.

Across all panels, the color scale spans approximately from 8 m to 16 m, with maximum delays occurring in regions characterized by elevated atmospheric water vapor content, most notably within the tropical belt. The comparison highlights the impact of mapping function choice on slant path delay estimation: models incorporating meteorological information, such as VMF1, capture substantially richer spatial variability than purely geometric or climatological formulations. These differences are of direct relevance for high-precision GNSS applications, satellite communication system design, and geophysical studies sensitive to tropospheric propagation effects [94, 215, 216]. See Appendix 10.A (Figures), Fig. A10.22.

10.17.2 Anomalous Values and Additional Considerations

Inspection of the four global slant path delay maps computed at an elevation angle of 15° using different mapping functions (NMF, VMF1, GMF, and simple cosine) does not reveal gross outliers in the sense of physically implausible values. All estimates remain within the expected range of approximately 8–16 m, consistent with typical tropospheric conditions at low elevation angles [56, 94]. Nevertheless, certain regions exhibit locally enhanced or anomalous spatial patterns, which merit further consideration. In particular, polar regions such as Antarctica and Greenland show relatively elevated slant delay values in some mapping formulations. This behaviour can be attributed to a combination of factors:

- (i) higher surface pressure during cold seasons, which increases the hydrostatic delay component [44, 45];
- (ii) reduced accuracy of simplified or climatological mapping functions at high latitudes, especially for the simple cosine model and, to a lesser extent, NMF [56];



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- (iii) long-term climatic changes affecting pressure and humidity distributions in polar environments, thereby modifying both hydrostatic and wet delay contributions [286].

Similarly, equatorial oceanic regions display localized delay maxima, primarily associated with elevated water vapor content and strong convective activity. These effects can be further amplified during large-scale climate oscillations such as the El Niño–Southern Oscillation (ENSO) and the Madden–Julian Oscillation, whose intensity and frequency have shown changes under recent climate variability [247, 286]. Such features emphasize the importance of accounting for regional atmospheric dynamics when interpreting slant path delay maps, particularly at low elevation angles.

10.18 Critical Considerations on Tropospheric Delay Estimation

10.18.1 Challenges of Tropospheric Delay Modeling

The accurate estimation of tropospheric delay remains a challenging task due to the intrinsically dynamic and heterogeneous nature of the atmosphere. Several sources of uncertainty affect both zenith and slant delay modeling. First, spatial and temporal variability of atmospheric parameters—temperature, pressure, and humidity—introduces strong gradients with altitude, geographic location, time of day, and season [44, 45]. Second, limitations of empirical and semi-empirical models arise from simplified assumptions on atmospheric stratification, which may inadequately represent complex vertical structures, particularly in extreme climatic regimes [56, 94]. Third, although global reanalysis datasets such as ERA5 provide high-resolution atmospheric fields, interpolation and extrapolation at local scales inevitably introduce uncertainties that propagate into tropospheric delay estimates [12, 99].

These limitations motivate the use of advanced mapping functions and physically informed correction strategies, especially for high-precision geodetic and GNSS applications [215, 216].

10.18.2 Advanced Vienna Mapping Functions (VMF1 and VMF3)

The VMF1 was introduced as a refinement of the Niell Mapping Function, incorporating coefficients derived from numerical weather models to better represent both hydrostatic and wet components of the tropospheric delay [216]. The mapping functions are expressed as:

$$M_h(e) = \frac{1}{\sin(e) + \frac{a_h}{\sin(e) + \frac{b_h}{\sin(e) + c_h}}}, \quad M_w(e) = \frac{1}{\sin(e) + \frac{a_w}{\sin(e) + \frac{b_w}{\sin(e) + c_w}}} \quad (10.24)$$

where e denotes the elevation angle, and (a_h, b_h, c_h) and (a_w, b_w, c_w) are site-specific coefficients derived from numerical weather model data, including ECMWF and ERA5 products [12, 99, 216]. By accounting for local atmospheric conditions, VMF1 significantly improves slant delay estimation accuracy, particularly at low elevation angles and in regions with strong meteorological variability [216].



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10.18.3 Vienna Mapping Function 3 (VMF3)

The VMF3 represents a further refinement of the Vienna Mapping Function framework, introduced to meet the requirements of next-generation high-precision geodetic applications [216]. Key advancements include an increased temporal resolution, with coefficients updated on an hourly basis, improved integration with numerical weather models, and enhanced representation of the wet delay component. These developments are particularly relevant in regions characterized by strong humidity variability, where rapid atmospheric changes can significantly affect slant path delays [216, 286].

10.19 Representation of the Mapping Functions m_h and m_w for Slant Path Delay

Tropospheric mapping functions provide the geometric relationship between zenith and slant path delays and play a critical role in low-elevation signal propagation modeling. In this section, the spatial and temporal behavior of the **Vienna Mapping Functions (VMF)** is analyzed by examining the hydrostatic (m_h) and wet (m_w) components at a fixed elevation angle of 15° , representative of challenging GNSS and satellite-link geometries. The mapping functions are dimensionless coefficients relating zenith and slant delays according to:

$$S(\epsilon) = ZHD \cdot m_h(\epsilon) + ZWD \cdot m_w(\epsilon), \quad (10.25)$$

where ZHD and ZWD denote the hydrostatic and wet zenith delays, respectively, and ϵ is the elevation angle [44, 45, 56]. At low elevations, mapping functions amplify zenith delays by factors exceeding three, making their accurate characterization essential for precise slant delay estimation [215, 216].

10.19.1 Global Distribution of Hydrostatic and Wet Vienna Mapping Functions at 15° Elevation

To investigate the spatial and temporal behaviour of the Vienna Mapping Functions, global maps of both hydrostatic (m_h and m_w) components were generated for 28 June 2024, using VMF products derived from numerical weather model data [216]. Snapshots at different synoptic hours (06:00, 12:00, and 18:00 UTC) are analyzed to assess latitudinal gradients, regional variability, and short-term temporal evolution. The hydrostatic mapping function exhibits a smooth and systematic increase toward the poles, reflecting the dominant role of surface pressure and large-scale atmospheric stratification. Typical values of m_h at 15° elevation are approximately 3.8, consistent with theoretical expectations for low-elevation geometries [56, 215]. The weak temporal variability of m_h across synoptic hours confirms its dependence on slowly varying pressure fields. In contrast, the wet mapping function displays a markedly heterogeneous spatial pattern, with pronounced maxima over humid tropical regions and minima over dry subtropical and arid zones.



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Values of m_w at 15° elevation are of comparable magnitude to m_h (approximately 3.8–3.9), but exhibit significantly stronger spatial and temporal variability. This behavior reflects the highly dynamic distribution of atmospheric water vapor and its sensitivity to convection, large-scale circulation, and synoptic disturbances [44, 216].

Figures 10.23–10.26 illustrate the global distributions of m_h and m_w obtained from VMF-G and VMF3 products at different synoptic hours. Across all cases, the hydrostatic component maintains a stable latitudinal structure, while the wet component highlights fine-scale spatial variability and noticeable temporal fluctuations, particularly in tropical regions and along mid-latitude storm tracks. The comparison across different times of day demonstrates that, while hydrostatic mapping functions can be considered quasi-static on daily timescales, wet mapping functions require high temporal resolution to accurately capture moisture-driven variability. This distinction underlines the importance of advanced VMF formulations, such as VMF1 and VMF3, for applications demanding high precision in slant path delay estimation, especially at low elevation angles where mapping effects are strongly amplified [216]. See Appendix 10.A (Figures), Fig. A10.23, Fig. A10.24, Fig. A10.25, Fig. A10.26.

10.20 Other Maps of ZHD, ZWD, and ZTD in Slant Geometry at an Elevation Angle of 15°

In addition to the analysis of mapping functions, the investigation was extended to the explicit computation of zenith and slant tropospheric delays using a fixed elevation angle of 15° , representative of critical low-elevation propagation conditions. The analysis exploits the Vienna Mapping Function framework, including its global and advanced formulations (VMF-G and VMF3), which combine empirical modelling with numerical weather prediction data to accurately describe tropospheric effects on electromagnetic signal propagation [216]. The Vienna Mapping Functions are designed to convert zenith delays into slant delays by means of elevation-dependent projection factors, enabling a physically consistent estimation of Slant Path Delay (SPD) for GNSS, satellite communication, and geodetic applications [56, 215, 216].

10.20.1 Global Mapping of Zenith and Slant Tropospheric Delays at 15° Elevation

Figures 12.31–12.34 present the output of the integrated algorithm developed in this work, illustrating the global distribution of both zenith and slant tropospheric delays derived from the VMF-G product at an elevation angle of 15° . The upper panels show the hydrostatic and wet mapping functions (m_h, m_w) computed from the VMF-G coefficients a_h and a_w , together with the corresponding Zenith Hydrostatic Delay (ZHD). The mapping functions represent the geometric scaling factors that project zenith delays into slant delays according to the relation (10.26). The lower panels display the resulting slant delays at 15° elevation, including the hydrostatic slant delay (Slant ZHD), the wet slant delay (Slant ZWD), and the total slant



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tropospheric delay (Slant ZTD). The ZHD field reflects the relatively stable dry-air contribution, largely controlled by surface pressure and exhibiting smooth latitudinal gradients [44, 45]. In contrast, the ZWD field captures the strong spatial and seasonal variability of atmospheric water vapour, with pronounced maxima over tropical and monsoon-influenced regions [94, 216]. The slant delay maps clearly highlight the geometric amplification of tropospheric effects at low elevation angles, where mapping functions increase the effective path length through the neutral atmosphere by factors exceeding three. As a result, regions characterized by high humidity and strong convective activity exhibit the largest wet contributions, which dominate the variability of the total Slant ZTD. These results emphasize the critical role of accurate wet delay modelling and high-resolution mapping functions in Earth-space signal propagation studies, particularly for applications operating at low elevation angles [215, 216, 286]. See Appendix 10.A (Figures), Fig. A10.27, Fig. A10.28, Fig. A10.29, Fig. A10.30.

10.21 In Situ Measurements for the Scanzano Site

Atmospheric profiles obtained from radiosonde measurements provide fundamental information on pressure, temperature, and humidity, and represent a primary reference for the accurate evaluation of Zenith Total Delay (ZTD) and its components, namely the Zenith Hydrostatic Delay (ZHD) and the Zenith Wet Delay (ZWD). The total tropospheric delay experienced by an electromagnetic signal propagating through the neutral atmosphere can be expressed as the sum of these two contributions, with ZHD associated with dry atmospheric gases and ZWD driven by the spatial and temporal variability of atmospheric water vapor [44, 45]. While the hydrostatic component can be estimated with high accuracy using surface pressure and standard atmospheric models, the wet component is inherently more variable and requires detailed humidity information. Radiosonde observations provide vertically resolved measurements of temperature and moisture, enabling a direct and physically consistent estimation of ZWD and, consequently, of the total ZTD [56, 94]. The integration of radiosonde-derived atmospheric profiles allows ZTD to be computed by vertically integrating the refractivity profile through the troposphere. This approach has been extensively validated in the literature. In particular, comparisons between ZTD and ZWD values derived from global reanalysis products such as MERRA-2 and those obtained from radiosonde and GNSS observations have demonstrated high levels of consistency, with reported root-mean-square errors below 1.35 cm for ZTD [94, 286]. These results confirm the reliability of radiosonde measurements as an independent reference for tropospheric delay estimation. Beyond their role in GNSS positioning, radiosonde observations are of central importance for a wide range of meteorological and climatological applications. Accurate estimates of ZTD and ZWD contribute to the retrieval of precipitable water vapour, the monitoring of drought and humidity anomalies, and the improvement of numerical weather prediction performance. Consequently, the integration of in situ radiosonde measurements into atmospheric modelling frameworks and satellite-based positioning systems represents a key element for advancing both geodesy and meteorology, particularly in the context of high-precision tropospheric delay assessment [94, 286].



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10.21.1 In Situ Measurements for the Scanzano (PA) Site (Study Area and Data Period)

The Scanzano (PA) site, located in north-western Sicily (Italy) within the province of Palermo, was selected as a representative Mediterranean mid-latitude coastal station for the analysis of in situ atmospheric measurements and their impact on tropospheric delay estimation. The site is characterized by a Mediterranean climate, with warm and dry summers and mild, humid winters, and exhibits pronounced seasonal variability in atmospheric water vapor, making it well suited for investigating both hydrostatic and wet delay components. The geographic coordinates of the Scanzano (PA) site are approximately 38.1° N latitude and 13.4° E longitude, with an elevation close to sea level. Its proximity to the Tyrrhenian Sea exposes the station to frequent marine air-mass advection, synoptic disturbances, and mesoscale humidity variability, which strongly affect the Zenith Wet Delay (ZWD) and represent a demanding test case for tropospheric delay modeling under realistic Mediterranean conditions [94]. Radiosonde observations collected at or in the vicinity of the Scanzano (PA) site were used to derive vertical profiles of pressure, temperature, and humidity over a multi-year period. The analysis focuses on data spanning 2018–2021, selected to ensure temporal consistency with coincident GNSS observations and with global reanalysis products employed elsewhere in this work, including ERA5 and MERRA-2 [12, 94, 286]. This period enables the investigation of both seasonal cycles and interannual variability in tropospheric delay components. The combined use of in situ radiosonde measurements, global reanalysis datasets, and GNSS-derived delays at the Scanzano (PA) site provides a robust validation framework for the modeling approaches presented in this chapter. In particular, the site allows a detailed assessment of the relative contributions of Zenith Hydrostatic Delay (ZHD) and Zenith Wet Delay (ZWD) in a coastal Mediterranean environment, supporting the evaluation of both physically based and empirical tropospheric delay models [44, 45, 56].

10.22 Summary of Chapter 10

This chapter addresses the transition from the theoretical description of atmospheric refractivity to its operational application in satellite communications and geodetic systems, with particular emphasis on the Telespazio ground station located at Scanzano (PA), Sicily. The analysis focuses on the modelling and estimation of Zenith Tropospheric Delay (ZTD) and its decomposition into the hydrostatic (ZHD) and wet (ZWD) components, which jointly determine the excess path delay experienced by electromagnetic signals propagating through the neutral atmosphere [44, 45]. The hydrostatic component is reliably modelled as a function of surface pressure, while the wet component requires high-resolution information on atmospheric humidity due to its pronounced spatial and temporal variability. The theoretical foundations of tropospheric delay modelling, ranging from the Smith–Weintraub refractivity formulation to the Saastamoinen model and widely used mapping functions (Niell Mapping Function and Vienna Mapping Functions), were reviewed and adapted to the specific Mediterranean and orographic conditions of Sicily [44, 45,



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56, 216]. A dedicated MATLAB-based computational framework was developed to estimate tropospheric delays over Sicily, inspired by numerical weather prediction concepts commonly adopted in WRF-type modelling approaches and complemented by ERA5 reanalysis data [12, 99]. High-resolution Digital Elevation Models (ASTER GDEM and GMTED2010) were integrated to accurately represent orographic effects, while hybrid vertical coordinates from ERA5 were exploited to reconstruct geopotential height fields with improved vertical consistency. The framework generated high-resolution spatial maps of ZTD, ZHD, ZWD, and Slant Path Delay, with climatological ZTD values typically ranging between 1.2 m and 2.8 m across the Sicilian domain. Model validation was performed through comparison with ERA5-derived delays, GNSS-based zenith delay estimates, and radiosonde profiles, demonstrating biases below 5 mm and root-mean-square errors in the range of 8–12 mm, consistent with state-of-the-art tropospheric delay modelling performance [94, 286]. The wet component exhibited particularly sharp spatial gradients in regions of complex orography, most notably over Mount Etna, where moisture convergence and terrain-induced uplift significantly enhance ZWD variability. These results confirm the robustness and physical consistency of the proposed framework and demonstrate its suitability for a broad range of operational applications, including GNSS positioning corrections, InSAR atmospheric phase mitigation, and adaptive link budgeting in satellite communication systems [56, 215, 216]. The chapter concludes by highlighting the scalability of the approach toward sub-kilometre spatial resolutions and its potential extension to near-real-time ingestion of meteorological data streams, positioning the framework as a versatile bridge between atmospheric physics, telecommunication engineering, and operational geodesy.



Chapter 11

Characterization of Tropospheric Delay at the Scanzano Site (Sicily, Italy)

11.1 Tropospheric Excess Path Calculation and Comparison of Approaches

The estimation of the tropospheric excess path is essential for quantifying the impact of the neutral atmosphere on GNSS signals and Earth–space communication links [45,46,48]. At the Scanzano site, where long-term meteorological observations and auxiliary geodetic data are available, multiple complementary approaches are adopted, spanning climatological models, numerical reanalysis, in-situ measurements, and GNSS observations. Reference estimates are derived from the ITU-R recommendation framework, which provides climatology-based delays and is used as a baseline for intercomparison [1,3,44]. A physically consistent, model-based alternative is obtained from the ERA5 atmospheric reanalysis, which supplies three-dimensional, time-resolved meteorological fields suitable for delay retrieval [12,99]. These estimates are complemented by delays computed from ground-based meteorological observations collected at Scanzano between 2019 and 2024, allowing retrieval under real atmospheric conditions with hourly resolution [45,48]. Independent observation-driven estimates are further obtained from GNSS data at the Corleone station, approximately 20 km from Scanzano, and integrated within the national RING network to enhance spatial consistency [48,65,73]. GNSS-derived zenith delays provide an external reference for evaluating both model-based and in-situ approaches. The intercomparison of ERA5-, ground-based-, and GNSS-derived delays against the ITU-R reference enables a quantitative assessment of accuracy and stability across different atmospheric regimes [65,111]. For sites lacking local instrumentation or GNSS coverage, such as Adelaide and Riverside, the analysis is limited to ITU-R and ERA5-based estimates.

In observation-driven approaches, zenith delays are mapped onto the slant propagation path



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using standard tropospheric mapping functions [56,215,231]. Sensitivity analyses indicate that, within the X-band frequency range, hydrometeor contributions to the excess path are negligible, justifying the neutral-atmosphere approximation [137,287].

11.2 In-Situ Measurements and Tropospheric Delay Estimation

The analysis of in-situ meteorological observations constitutes a central observational element of this study, as it provides a direct linkage between locally measured atmospheric states and the resulting tropospheric excess path delays. At the Scanzano site, long-term ground-based datasets supplied by Telespazio enable the evaluation of delay estimates derived from climatological models and numerical reanalyses under observationally constrained conditions [45,48]. The available in-situ dataset comprises high-resolution measurements of surface pressure, air temperature, relative humidity, total precipitable water vapor, cloud cover, and rainfall. These variables allow the computation of zenith hydrostatic and wet delay components following established refractivity-based formulations [45,46], and provide an observation-driven reference for comparison with ITU-R climatological estimates [1,3,44], ERA5 reanalysis fields [12,99], and GNSS-derived delays [48,65]. The exploitation of in-situ observations introduces methodological aspects that are not encountered when using global products alone. Temporal discontinuities affecting individual variables require reconstruction or interpolation strategies, while rigorous temporal co-registration with ERA5 and ITU-R reference datasets is essential to ensure consistency in intercomparisons and to isolate systematic model biases [65,111]. In addition, uncertainty quantification plays a critical role in distinguishing measurement noise from genuine atmospheric variability, particularly in the presence of fine-scale meteorological fluctuations that are only partially resolved by reanalysis products [12,65]. Within this framework, in-situ observations are used to characterize the temporal evolution and inter-annual variability of tropospheric delay over the 2019–2024 period, and to assess the sensitivity of different estimation approaches to high-humidity conditions and intense meteorological events. The comparison with reanalysis- and GNSS-based estimates enables the identification of systematic deviations associated with model assumptions and spatial representativeness, thereby refining the interpretation of tropospheric delay variability at the local scale [65,73]. From a methodological standpoint, in-situ measurements do not replace large-scale modelling approaches, but rather provide a calibration and verification layer that strengthens the physical consistency of tropospheric delay estimates. Their integration supports a more robust assessment of model performance and informs the development of advanced correction strategies relevant to both GNSS positioning and satellite communication systems [3,45].



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11.3 Statistical Analysis and Projections of Tropospheric Delays at Scanzano

The statistical characterization of tropospheric delay at the Scanzano site focuses on the analysis of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) derived from both in-situ observations and atmospheric reanalysis products. Hourly time series covering the 2020–2024 period are analysed to quantify inter-annual variability, statistical dispersion, and the temporal structure of tropospheric delays under Mediterranean climatic conditions [45,48,65]. Descriptive statistics, including mean values, standard deviations, and probability distributions, are first derived to establish the baseline behavior of ZHD, ZWD, and ZTD. The analysis confirms the relative temporal stability of the hydrostatic component, contrasted with the pronounced variability of the wet component, which dominates short-term fluctuations and seasonal modulation [45,46]. These results provide the statistical foundation for subsequent time-series and trend analyses. Time-resolved analyses are conducted using both in-situ data and ERA5 reanalysis fields, enabling a consistent comparison between observation-driven and model-based estimates [12,99]. The ERA5 dataset, previously validated over multi-decadal periods in independent studies, is here extended to the 2019–2024 interval and spatially aggregated over the Scanzano region to ensure representativeness at the local scale [65,111]. This dual-source approach supports the identification of systematic differences between locally observed delays and reanalysis-derived estimates. To investigate the statistical robustness of tropospheric delay projections, stochastic perturbations are introduced into ERA5-based time series, simulating unresolved atmospheric variability through controlled noise injection. The resulting distributions are analyzed using cumulative complementary distribution functions (CCDFs), which provide a compact description of extreme-delay behavior and its sensitivity to stochastic forcing [65,73]. This framework allows the assessment of temporal stability and uncertainty bounds associated with projected delay trends.

Complementary reanalysis datasets, including MERRA-2, are employed as auxiliary references to evaluate the consistency of large-scale atmospheric representations and to quantify inter-reanalysis spread [65]. Spatial visualizations of ZHD, ZWD, and ZTD further illustrate the dependence of tropospheric delay on elevation angle and regional orography, highlighting the coupling between atmospheric stratification and geometric propagation effects [3,56]. Monte Carlo techniques are employed to propagate uncertainty through the delay formulations, enabling a quantitative separation between parametric uncertainty and intrinsic atmospheric variability of the propagation medium [45,65]. Delays are expressed in both zenith and slant geometries, with the Slant Path Delay defined as the line-of-sight integral of atmospheric refractivity along the satellite–receiver path and related to zenith quantities through standard mapping functions [56,215,231]. **Figure 11.1** provides an overview of the Scanzano ground station and its mission control room, which constitute the operational and observational framework for the tropospheric delay analyses discussed in this chapter.



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Figure 11.1. The Telespazio ground station at Scanzano (PA), Sicily (top), and its mission control room (bottom). The facility hosts multiple large-aperture parabolic antennas dedicated to satellite tracking, telemetry, and high-frequency communication. Its elevated location above Piana degli Albanesi provides low radio-frequency interference and an unobstructed horizon over the central Mediterranean, supporting both geodetic applications and operational satellite links. The control room enables real-time monitoring and coordination of atmospheric modeling, delay correction, and satellite link management.



11.3.1 Implementation of Mapping Functions for Slant Path Delay (SPD)

Mapping functions constitute a fundamental component in the computation of the Slant Path Delay (SPD), as they relate zenith delays to line-of-sight delays as a function of elevation angle. In this work, multiple mapping formulations are implemented and tested in order to assess their numerical behavior and to quantify their impact on delay estimation under different observation geometries [56,215,231]. This comparative approach supports a consistent evaluation of mapping-induced variability in slant-delay calculations. A key element of the implementation is the explicit parametrization of the receiver height, expressed in meters and directly integrated into the computational framework. This parametrization enables the systematic treatment of stations located at different altitudes and allows the same formulation to be applied across a wide range of elevation angles without loss of generality. Such flexibility is essential for ensuring consistency between zenith-based estimates and their corresponding slant-path representations. The combined use of multiple mapping functions and height-aware parametrization improves the robustness of the zenith-to-slant transformation, reducing geometry-related residuals in delay estimates. This approach provides a physically consistent basis for the correction of GNSS and satellite communication signals and supports the reliable interpretation of propagation effects across heterogeneous operational scenarios [3,65].

11.3.2 Analysis of Zenith Path Excess and Comparison with ERA5 Data

The analysis of zenith path excess at the Scanzano station is based on locally acquired meteorological observations, which provide the input variables required for estimating Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD). Surface pressure, air temperature, relative humidity, and precipitation rate are used to derive zenith delay components following standard refractivity-based formulations [45,46]. Monthly statistics and inter-annual time series covering the February 2019–2024 period are analyzed to characterize variability and long-term behavior of the zenith delays.

Ground-based estimates are systematically compared with delays derived from ERA5 atmospheric reanalysis in order to assess consistency between observation-driven and model-based representations of the neutral atmosphere [12,99]. Temporal co-registration between datasets enables pointwise comparison over coincident periods, allowing systematic deviations and seasonal patterns to be identified. The ERA5 data are spatially aggregated over the Scanzano region to ensure representativeness at the local scale and to provide a climatological reference for the interpretation of in-situ observations [65,111]. The relative contributions of hydrostatic and wet components to the total zenith delay are examined through unified statistical representations, including joint histograms of ZHD, ZWD, and ZTD. This approach highlights the dominant role of the hydrostatic component in setting the mean delay level, while emphasizing the variability introduced by the wet component under changing atmospheric moisture conditions [45,48]. The comparison between ground-based and ERA5-derived zenith delays provides a quantitative basis for evaluating the reliability of reanalysis products in reproducing local-scale tropospheric



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conditions [3,12,45,48,65,99,111]. This assessment supports the interpretation of zenith delay variability and establishes a consistent reference for subsequent analyses involving slant-path delays, mapping functions, and uncertainty propagation [56,65,215,231].

11.3.3 Scanzano Ground Station: Site Context

The Scanzano ground station, operated by **Telespazio**, is located in the **Piana degli Albanesi** area (Sicily, Italy) and provides a low-interference environment with an unobstructed horizon suitable for satellite communication and navigation activities. The site has been operational since the late 1980s and was selected for its favorable electromagnetic conditions and geometric visibility at low elevation angles, which are critical for propagation and tracking studies [3,65]. The facility hosts multiple large-aperture antennas and associated control infrastructures supporting satellite tracking, telemetry, and data relay across different frequency bands. Its operational profile includes services related to satellite navigation, broadband communication, and remote sensing, ensuring continuous interaction between ground infrastructure and space-based assets [65,73].

For the purposes of this study, Scanzano is considered a representative operational site where atmospheric propagation effects, tropospheric delay corrections, and satellite link management coexist within a real-world configuration. Local topography and elevation were taken into account to support antenna geometry and delay modeling, providing consistent boundary conditions for the analysis of zenith and slant-path delays [56,111]. This contextual description is provided to frame the atmospheric analyses discussed in this chapter and does not constitute a performance assessment of the operational services hosted at the site. **Figures 11.2 and 11.5** illustrate the satellite-based geospatial characterization of the **Scanzano Space Centre** (Sicily, Italy). High-resolution satellite imagery is combined with elevation samples retrieved through the **Google Elevation API** to support the topographic assessment of the area surrounding the ground station and to define antenna site geometry, providing localized orthometric height information relevant for propagation analyses [56,65,111]. DTM-derived elevation data are used to describe local terrain morphology and orthometric height variability at the local scale. These data support the definition of geometric constraints in propagation modeling and elevation-dependent corrections in tropospheric delay estimation and satellite link analysis [3,56,215]. Complementary views of the surrounding area document the geometric visibility conditions relevant for satellite line-of-sight propagation. Together, these datasets provide the geospatial context required for the interpretation of zenith and slant-path delay behavior at the Scanzano site [45,48,231].

Figure 11.6 presents the high-resolution topographic data acquisition performed for the **Scanzano Space Centre** (Sicily, Italy), based on a 100 m sampling grid covering an area of 56 km² and comprising 614,125 accurately georeferenced elevation points. The left panel indicates the geographical extent of the study area, while the right panel provides a high-resolution satellite view of the **Telespazio** facility with the overlaid sampling matrix. This dataset supports radio-propagation analyses, tropospheric delay assessments, and the generation of digital elevation



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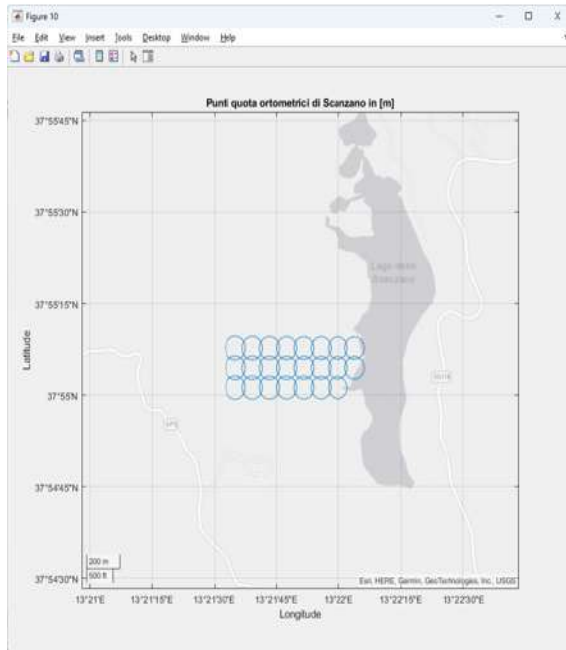


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Scanzano site.



Figure 11.2. Orthometric elevation data for the Scanzano Space Centre (Sicily, Italy) retrieved using the Google Elevation API. The grid represents terrain height values sampled over the area surrounding the Telespazio ground station near Lake Scanzano and is used to support site characterization and antenna geometry in tropospheric delay analyses. Source: Author's presentation, Telespazio–Sapienza University of Rome (2024).

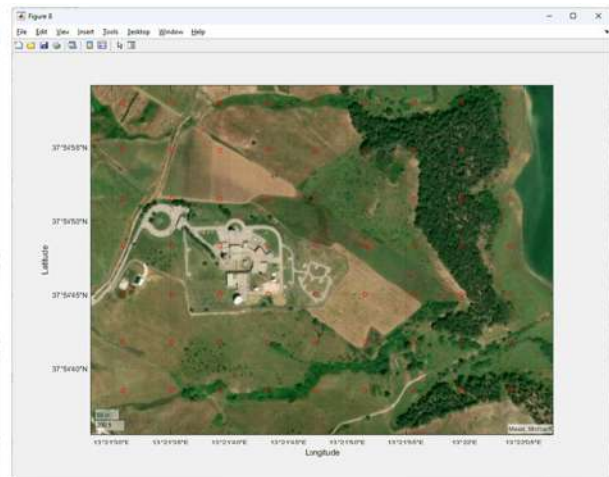
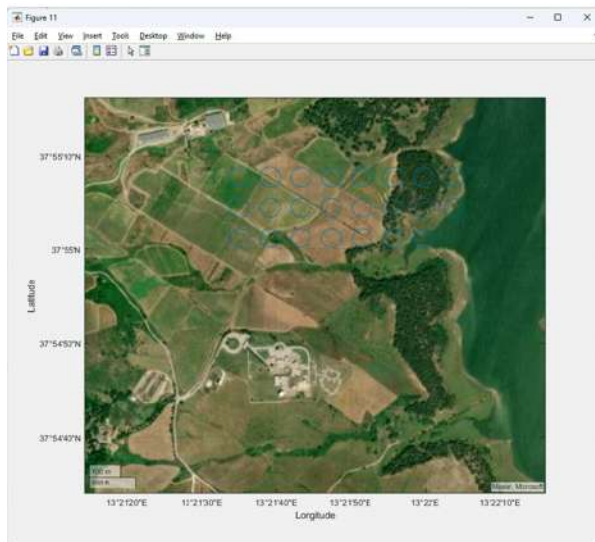


Figure 11.3. Satellite-based geospatial analysis of the Scanzano Space Centre (Sicily, Italy) using remote sensing data. Satellite-based geospatial analysis of the Scanzano Space Centre (Sicily, Italy). Credits: Author's presentation at Telespazio, Sapienza University of Rome (2024).



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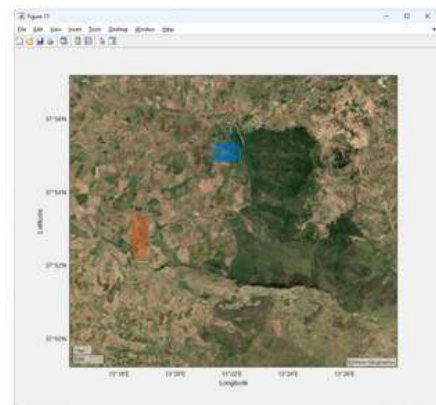
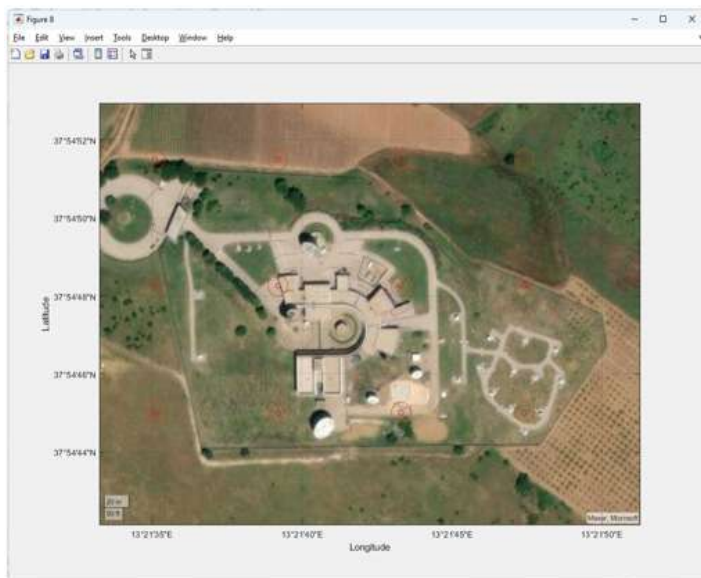
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Figure 11.4. Details of my measurement grid points at the Scanzano Space Centre site, used for elevation mapping and site characterization in satellite communication applications.



Sampling step: 100 m



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Figure 11.5. High-resolution geospatial analysis of the Scanzano Space Centre (Sicily, Italy) based on elevation data sampled on a 100 m grid. Source: Author's presentation, Telespazio-Sapienza University of Rome (2024).

Scanzano (Sicily) : Piana degli Albanesi

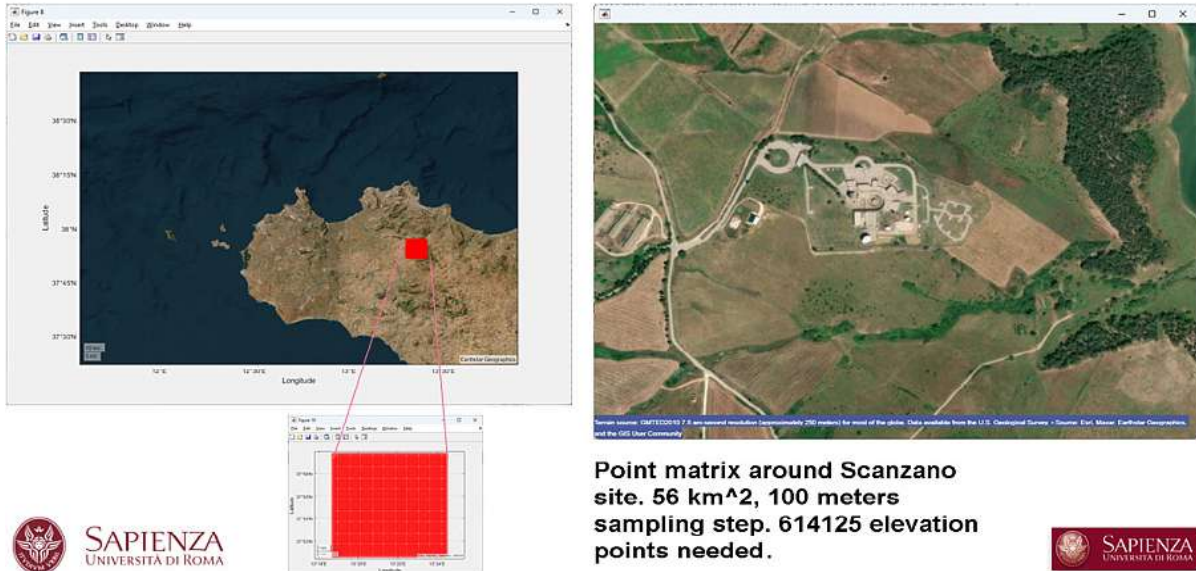


Figure 11.6. High-resolution topographic survey of the Scanzano Space Centre (Sicily, Italy) based on elevation data sampled on a 100 m grid, covering an area of 56 km² and yielding 614,125 elevation points. Source: Author's presentation, Telespazio–Sapienza University of Rome (2024).

models (DEMs) for GNSS-based atmospheric modelling and satellite tracking optimization. The resulting topographic framework enables refined evaluation of propagation paths and improved correction models for Earth–space communication links.

11.4 Starting the Statistical Analysis of the Scanzano Site Based on In-Situ Data

11.4.1 Pressure Analysis

Atmospheric pressure is the dominant driver of the hydrostatic component of tropospheric delay and therefore represents a key variable in both climatological characterization and GNSS-based delay modeling [45,46]. At a fixed site, its variability reflects the combined influence of altitude, large-scale synoptic forcing, and thermally driven diurnal cycles, which directly modulate the magnitude of the Zenith Hydrostatic Delay (ZHD). In this study, surface pressure measurements collected at the Scanzano (PA) site are used as the primary input for ZHD estimation following the Saastamoinen formulation [45]. This physically based approach ensures internal consistency between observed atmospheric states and the resulting hydrostatic delay, providing a robust reference for subsequent comparisons with ERA5 reanalysis fields and ITU-R climatological models [1,12,65].

Figure 11.8 summarizes the statistical behavior of atmospheric pressure based on in-situ



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observations acquired on 1 May 2019. The diurnal signal exhibits the characteristic early-morning minimum and afternoon maximum associated with solar heating and thermal expansion of the atmospheric column. Hourly statistics indicate limited dispersion throughout the day, with a moderate increase in variability during the afternoon, consistent with enhanced local atmospheric activity [48,65]. The overall pressure distribution remains narrowly confined (approximately 932–968 hPa) and follows a near-Gaussian shape, with no evidence of extreme anomalies. The stability and internal coherence of the pressure statistics confirm the suitability of the in-situ dataset for hydrostatic delay estimation. This provides a reliable baseline for the calibration and validation of ERA5-derived and ITU-R-based ZHD estimates discussed in the following sections, and supports the interpretation of multi-year tropospheric delay variability at the Scanzano site [3,65].

11.4.2 Climatic Trends and Seasonal Variations in Tropospheric Delay

11.4.2.1 Advanced Time Analysis of ZHD, ZWD, and ZTD

Climatic trends and seasonal variability of tropospheric delay at the Scanzano site are investigated through a combined statistical and stochastic analysis of in-situ observations collected over the 2019–2024 period. Monthly and annual aggregates of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) are derived to characterize long-term behavior and inter-annual variability under Mediterranean climatic conditions [45,48,65]. Time-domain analyses are complemented by spectral and decomposition techniques, including Fourier analysis and seasonal–trend decomposition (STL), to isolate periodic components and seasonal modulation in the delay time series [65]. The statistical properties of ZTD are further examined through distributional analysis, allowing departures from Gaussian behavior to be identified and linked to episodic meteorological forcing. To assess the robustness and predictability of tropospheric delay variability, stochastic modeling approaches are applied to the ZTD time series. Autoregressive formulations (ARIMA) are used to evaluate short-term temporal persistence, while **Monte Carlo** simulations are employed to propagate uncertainty and explore plausible future realizations of ZTD variability [65,73]. The resulting probability distributions are evaluated using standard goodness-of-fit diagnostics, including the **Shapiro–Wilk** and **Kolmogorov–Smirnov** tests [45]. ERA5 reanalysis fields are systematically cross-validated against in-situ meteorological observations and independent GNSS-derived delays from the Corleone station, enabling the quantification of correlation metrics and error statistics [12,48,65,99]. Surface pressure, temperature, relative humidity, and precipitation rate provide the input variables for the derivation of ZHD, ZWD, and ZTD, ensuring methodological consistency across observational and reanalysis-based estimates [45,46]. The statistical behavior of ZTD derived from in-situ measurements exhibits an approximately Gaussian distribution, with the majority of values confined within a narrow interval around the mean, while moderate tail deviations reflect episodic humidity-driven events (**Fig. 11.9**) [48,65]. Annual boxplots indicate a generally stable median behavior over the 2019–2024 period, with increased dispersion and outlier frequency during

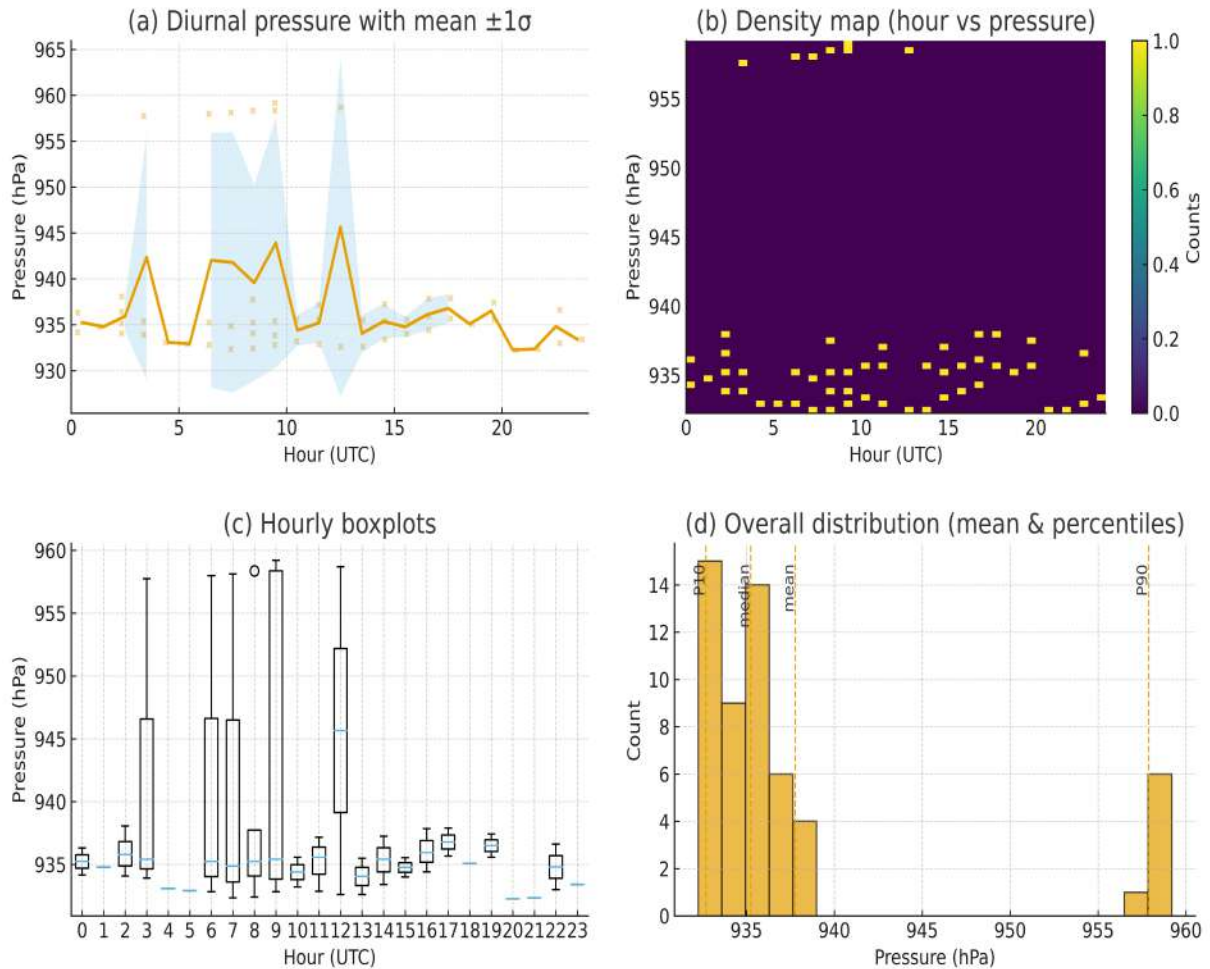


Figure 11.7. Diurnal statistics of atmospheric pressure at Scanzano (Sicily, Italy). Panel (a) shows the hourly mean pressure with $\pm 1\sigma$ variability derived from in-situ observations. Panel (b) presents the hour–pressure density distribution. Panel (c) reports hourly boxplots (0–23 UTC), while panel (d) displays the overall pressure distribution with mean, median, and P10–P90 percentiles. These diagnostics describe the diurnal cycle and intra-day variability relevant for tropospheric delay modeling in GNSS applications. Source: Author’s presentation @ Telespazio, Sapienza University of Rome (2024).

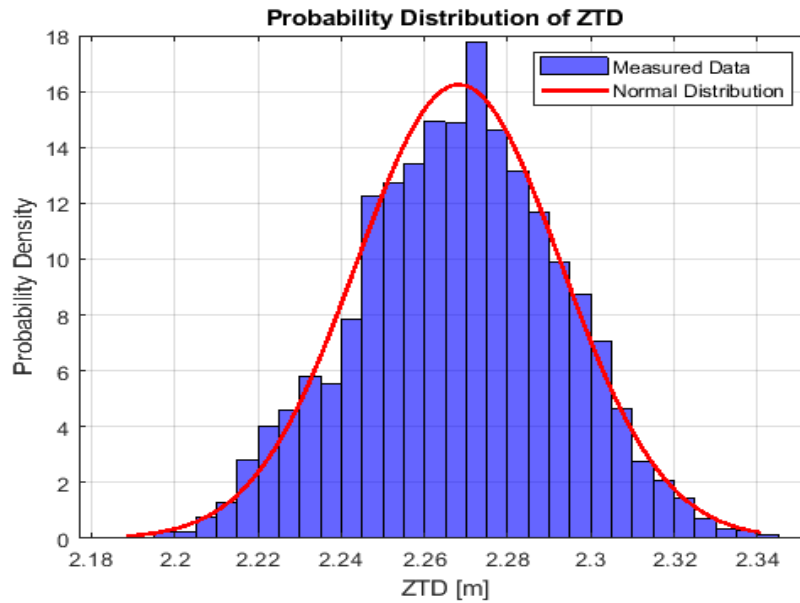
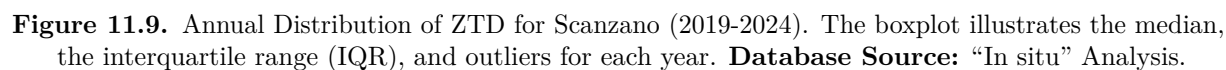


Figure 11.8. Probability distribution of Zenith Total Delay (ZTD) for the Scanzano site (2019–2024). The blue histogram represents the measured data, while the red curve shows the fitted normal distribution. Most ZTD values are concentrated between 2.27 [m] and 2.29 [m], indicating an approximately Gaussian distribution with minor deviations in the lower and upper tails. **Database Source:** “In situ” Analysis.

years characterized by enhanced convective activity (**Fig. 11.10**). The temporal evolution of annual mean ZTD reveals multi-year fluctuations rather than a monotonic trend, suggesting sensitivity to evolving atmospheric conditions at the regional scale (**Fig. 11.11**) [65]. The diurnal evolution of ZHD, ZWD, and ZTD highlights the distinct physical contributions of hydrostatic and wet components to the total delay (**Fig. 11.12**). While the hydrostatic term varies smoothly in response to surface pressure, the wet component exhibits rapid humidity-driven fluctuations, which dominate short-term variability and govern intra-day modulation of the total delay [45,48]. Together, these analyses provide a coherent statistical framework for interpreting climatic variability and temporal structure in tropospheric delay at the Scanzano site, establishing a quantitative basis for subsequent uncertainty assessment and slant-path delay modeling [3,65]. In conclusion, the cross-analysis of *in-situ* and Reanalysis datasets through advanced statistical modelling provides a comprehensive perspective on the temporal evolution of tropospheric delays, bridging atmospheric physics and satellite communication engineering.

11.4.3 Possible Explanations for ZTD Variability (2019–2024) Based on In-Situ Analysis

The temporal variability of Zenith Total Delay (ZTD) observed at the Scanzano site over the 2019–2024 period reflects the combined influence of hydrostatic and wet atmospheric processes, with the latter exerting the dominant control on inter-annual fluctuations [45,48]. The decrease in ZTD observed between 2019 and 2021 is primarily associated with reduced atmospheric moisture



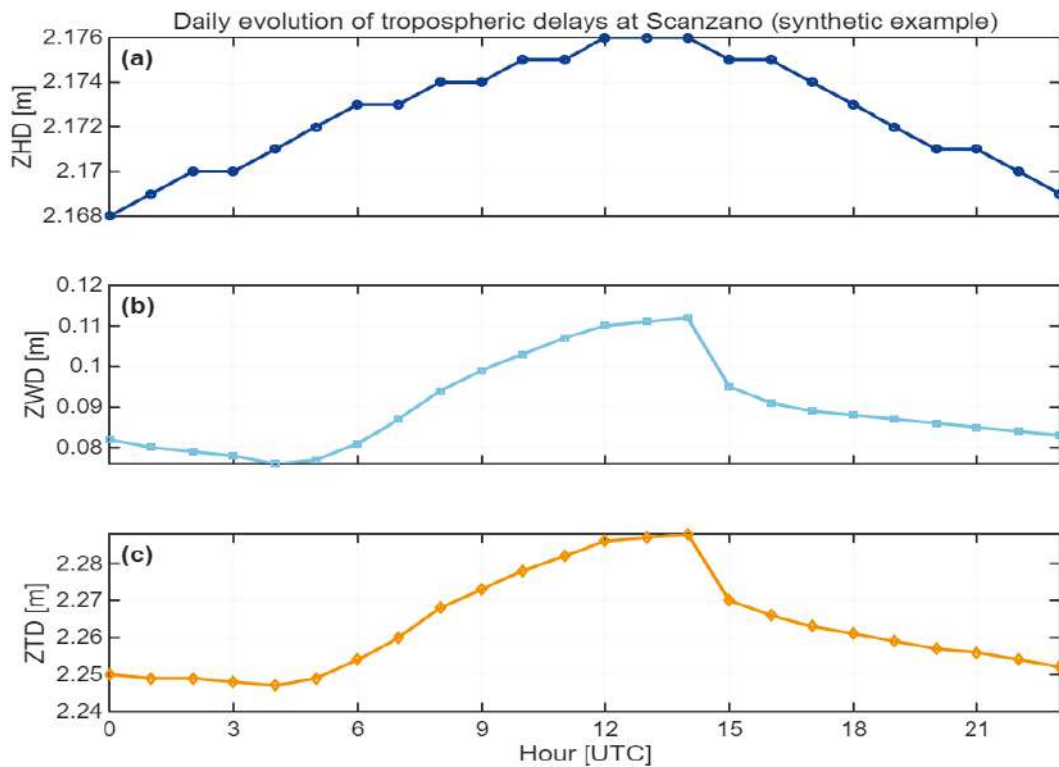


Figure 11.11. Daily evolution of the tropospheric delay components at the Scanzano (PA) site for 1 May 2019. Panel (a) shows the Zenith Hydrostatic Delay (ZHD), which follows a smooth semi-diurnal cycle driven by surface-pressure variations. Panel (b) displays the Zenith Wet Delay (ZWD), characterized by short-term fluctuations associated with changes in atmospheric water vapor. Panel (c) illustrates the Zenith Total Delay (ZTD), obtained as the sum of ZHD and ZWD, and therefore combining a slowly varying baseline with higher-frequency oscillations. Together, these components highlight the distinct physical drivers of tropospheric delay variability on sub-daily scales.



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content, as indicated by lower values of Zenith Wet Delay (ZWD), while the subsequent increase from 2022 onward coincides with enhanced humidity and higher surface temperatures, which amplify the wet contribution to the total delay [45,65]. Seasonal and inter-annual variability in precipitation patterns, temperature gradients, and convective activity further modulates ZTD at the local scale. These effects are consistent with the strong sensitivity of ZWD to short- and medium-term meteorological forcing, whereas the hydrostatic component remains comparatively stable and governed by surface pressure variations [45,46]. The consistency of the observed ZTD trends across in-situ measurements, ERA5 reanalysis fields, and independent GNSS-derived estimates supports the presence of a dominant climatic signal, while localized deviations are attributed to site-specific meteorological variability and instrumental effects [12,48,65,99]. The statistical distributions of the zenith delay components derived from in-situ observations are shown in **Fig. 11.13**.

ZWD exhibits a right-skewed distribution, reflecting the intermittent nature of atmospheric water vapor and its strong temporal variability. In contrast, ZHD displays an approximately Gaussian distribution within a narrow range, consistent with the relatively stable dry-air contribution controlled by surface pressure [45]. The resulting ZTD distribution is centered around the mean hydrostatic level and inherits most of its variability from the wet component, confirming the central role of humidity-driven processes in shaping total delay statistics [48,65]. Minor irregularities in the delay distributions, particularly in the hydrostatic component, reflect the fine-scale atmospheric variability preserved by in-situ measurements. Unlike global reanalysis products, which inherently smooth spatial and temporal heterogeneities, ground-based observations retain the local atmospheric signature of the site, providing essential information for the interpretation and parameterization of tropospheric delay models used in GNSS positioning and satellite communication applications [3,65].

11.4.4 Climatic Fingerprint of the Scanzano Site Based on In-Situ Observations

The statistical behavior of the zenith delay components derived from in-situ observations defines a characteristic climatic fingerprint of the Scanzano (PA) site, reflecting both the local Mediterranean regime and the influence of large-scale atmospheric variability [45,48,65]. The hydrostatic component (ZHD) provides a stable baseline, primarily controlled by surface pressure and exhibiting limited seasonal modulation. In contrast, the wet component (ZWD) displays pronounced variability driven by humidity cycles and episodic convective activity, dominating short-term and inter-annual fluctuations.

The combined effect of these components, expressed through the Zenith Total Delay (ZTD), highlights the superposition of a quasi-stationary hydrostatic contribution and a highly dynamic moisture-driven term. This behavior is consistent with established tropospheric delay theory and underlines the relevance of ZTD for GNSS meteorology, radiowave propagation modeling, and high-frequency satellite link analysis [45,46,215]. Asymmetries and extended tails in the

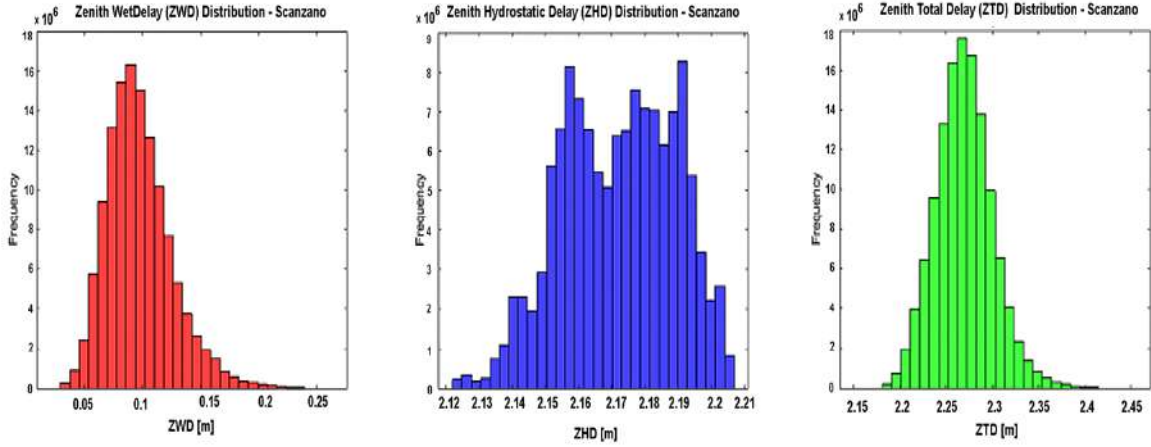


Figure 11.12. Histograms of Zenith Wet Delay (ZWD), Zenith Hydrostatic Delay (ZHD), and Zenith Total Delay (ZTD) for the Scanzano site, in the period (2019-2024). The ZWD (red) exhibits a right-skewed distribution with a modal value near 0.10 m, reflecting the high temporal variability of atmospheric water vapor. **Database Source:** “in situ” Analysis.

ZWD distribution suggest the modulation of local moisture conditions by large-scale circulation patterns, including teleconnection mechanisms such as the El Niño–Southern Oscillation, the North Atlantic Oscillation, and Mediterranean-scale variability [111,215]. Within this context, the Scanzano dataset provides both a site-specific diagnostic and a representative benchmark for the evaluation and parameterization of tropospheric delay models in climate-sensitive regions [3,65].

11.4.5 Comparison of ZWD, ZHD, and ZTD Distributions (2019–2024)

Figure 11.14 summarizes the statistical distributions of ZWD, ZHD, and ZTD derived from in-situ measurements collected at the Scanzano site over the 2019–2024 period. The ZWD distribution exhibits positive skewness, with most values concentrated between approximately 0.05 m and 0.15 m and a modal peak near 0.10 m, reflecting the intermittent and event-driven nature of atmospheric water vapor [48,65]. In contrast, the ZHD distribution is approximately Gaussian and confined within a narrow range (about 2.12–2.21 m), consistent with the dominance of surface pressure in controlling the hydrostatic delay as described by the **Saastamoinen** formulation [45]. The ZTD distribution largely mirrors the hydrostatic component but is shifted toward higher values (around 2.30 m) due to the additive wet contribution, inheriting most of its variability from ZWD. This decomposition confirms that the hydrostatic delay represents a robust and predictable reference for tropospheric modeling, while the wet delay constitutes the primary source of temporal uncertainty affecting both GNSS positioning accuracy and satellite communication performance [215,65]. To investigate the multi-scale temporal behavior of the ZTD time series shown in **Figure 11.14**, a Continuous Wavelet Transform (CWT) was applied

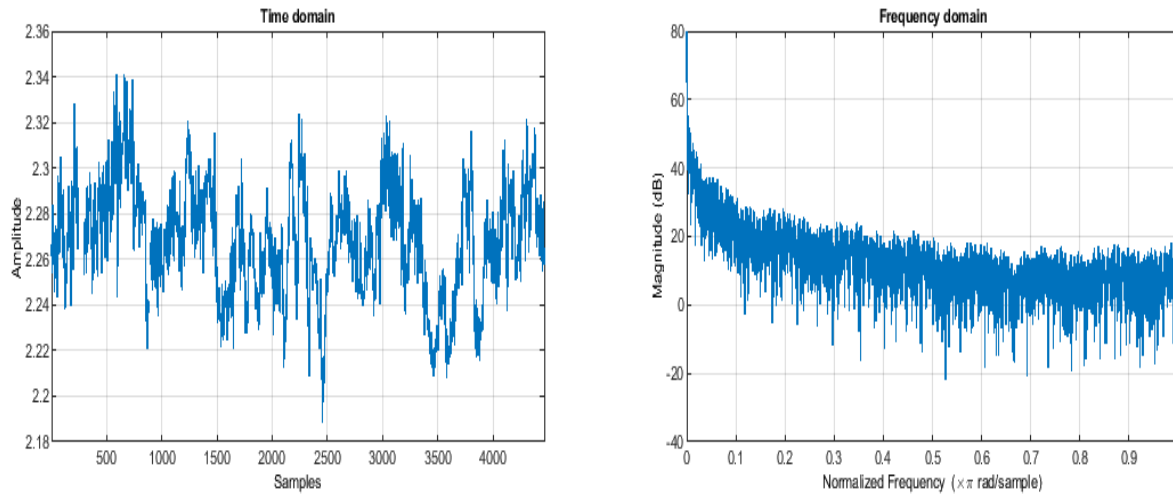


Figure 11.13. Zenith Total Delay (ZTD) and Its Spectrum. **Database Source:** “In situ” Analysis.

using the complex **Morlet** mother wavelet, which provides a well-balanced trade-off between time and frequency resolution and is widely adopted in atmospheric and geophysical analyses (**Torrence & Compo**, 1998 [168]; **Grinsted et al.**, 2004 [260]). This approach enables a joint time–frequency representation of the ZTD signal, allowing transient, intermittent, and persistent oscillations to be identified within a unified framework. Analysis of the resulting scalogram facilitates the detection of dominant periodicities and their temporal modulation, linking observed variability to seasonal, synoptic, and diurnal-scale processes governing tropospheric delay over the region (**Zhu et al.**, 2013 [261]; **Darbeheshti & Featherstone**, 2009 [262]).

11.4.6 Time–Frequency Analysis of Zenith Total Delay (2019–2024)

Figure 11.15 provides a joint time–frequency characterization of Zenith Total Delay (ZTD) derived from in-situ observations at the Scanzano site over the 2019–2024 period. The temporal evolution shows ZTD fluctuations ranging from approximately 2.18 to 2.36 m, resulting from the superposition of a nearly constant hydrostatic component and a highly variable wet component driven by tropospheric water vapor content [45,48]. The associated frequency representation, obtained through **Continuous Wavelet Transform (CWT)** analysis, reveals dominant spectral power at time scales between approximately 100 and 200 hours. These scales are characteristic of synoptic meteorological disturbances related to frontal systems, large-scale pressure evolution, and moisture transport in mid-latitude cyclonic regimes (**Holton & Hakim**, 2013 [263]; **Trenberth**, 1991 [257]). Superimposed on this low-frequency modulation, distinct periodic components emerge at approximately 24 and 12 hours, corresponding to diurnal and semi-diurnal atmospheric tides driven by solar forcing and modulated by boundary-layer processes (**Chapman & Lindzen**, 1970 [265]; **Dai & Deser**, 1999 [266]).

Wavelet power, expressed through color intensity in the scalogram, highlights intermittent high-energy features associated with transient mesoscale or convective activity, which temporarily

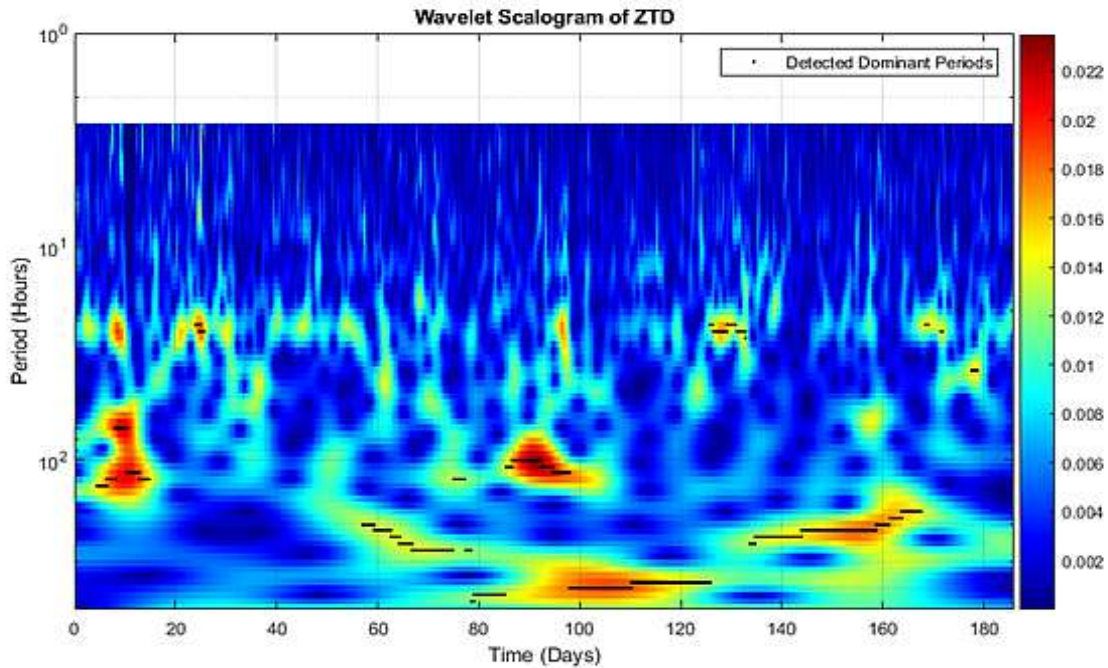


Figure 11.14. Wavelet Scalogram of ZTD from Fig. 11.14. The most dominant periodicity detected is around 254 [h], ~ 10.6 [days]. The wavelet scalogram of ZTD represents the time-frequency structure of the ZTD variation over 180 [days]. **Database Source:** “In situ” Analysis.

enhance ZTD variability. At the same time, horizontally coherent structures aligned along synoptic time scales indicate persistent, slowly evolving oscillations extending over several days. The absence of narrow spectral peaks confirms that ZTD variability arises from a continuum of atmospheric processes rather than from discrete periodic forcing. Overall, this broadband time–frequency behavior confirms the suitability of ZTD as an integrated diagnostic of tropospheric water-vapor dynamics and supports its application in high-precision GNSS positioning, satellite communication studies, and climatological assessments [65,215,255,256].

11.4.7 Pressure Fluctuations and Atmospheric Variability in May 2019 (In-Situ Observations)

Figure 11.16 presents the temporal evolution of surface atmospheric pressure measured at the Scanzano site during May 2019, based on in-situ observations. The time series exhibits pronounced intra-month variability, with pressure values spanning approximately 935–960 hPa, indicative of active synoptic forcing during the period. Several marked pressure minima and maxima are observed, with rapid drops followed by recoveries on time scales of a few days. Such fluctuations are characteristic of the passage of synoptic-scale systems, including frontal disturbances and evolving pressure patterns associated with mid-latitude cyclonic activity over the Mediterranean region (**Holton & Hakim**, 2013 [263]). Superimposed on these multi-day oscillations, shorter-period variability reflects the contribution of diurnal and semi-diurnal atmospheric tides driven by solar forcing and modulated by boundary-layer processes (**Chapman & Lindzen**, 1970 [265]; **Dai**

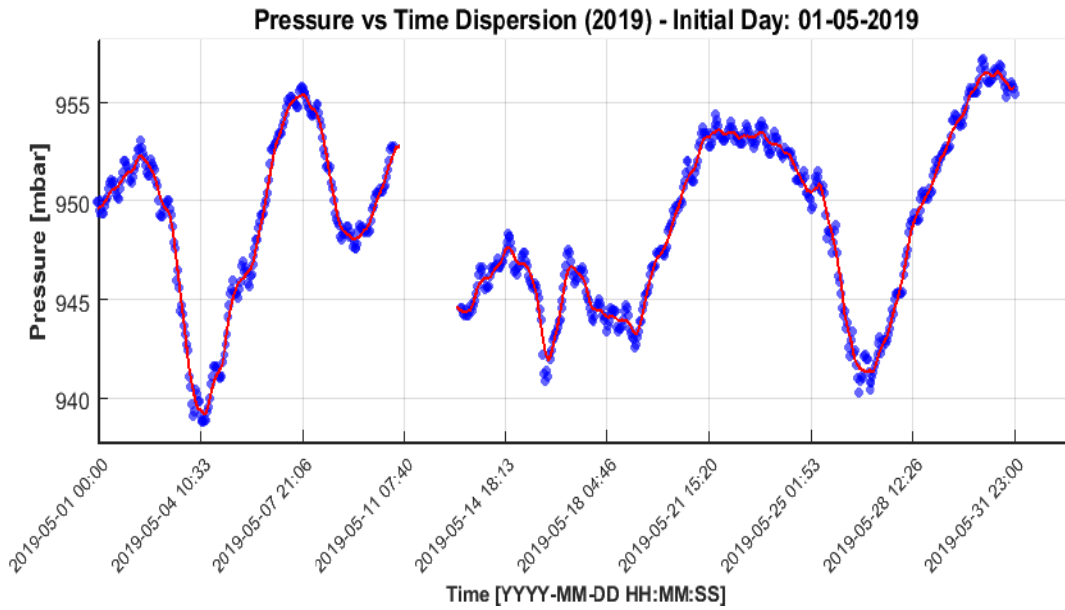


Figure 11.15. Temporal evolution of surface atmospheric pressure at the Scanzano site (PA, Sicily, Italy) during May 2019, derived from in-situ observations. The time series highlights pronounced intra-month variability characterized by alternating pressure maxima and minima. Source: In-situ measurements; author's analysis.

& Deser, 1999 [266]). The observed pressure dynamics indicate a non-stationary atmospheric regime in which transient meteorological systems dominate short-term variability, while broader circulation modes modulate the background pressure level. Such behavior is consistent with the influence of large-scale climate patterns affecting the *Euro-Mediterranean* area, including the North Atlantic Oscillation and the Mediterranean Oscillation (Hurrell, 1995 [267]; Lionello et al., 2006 [268]). From a propagation perspective, the magnitude and temporal structure of these pressure fluctuations directly affect the hydrostatic component of tropospheric delay (ZHD), introducing measurable variability in zenith and slant-path delay estimates. Consequently, the May 2019 case provides a representative example of how in-situ pressure measurements capture atmospheric variability that must be accounted for in high-precision GNSS positioning and satellite communication modeling at the Scanzano site.

11.5 Temporal and Seasonal Variations in Atmospheric Pressure at Scanzano (2019–2024) based on In-Situ Analysis

Figure 11.17 shows a three-dimensional representation of surface atmospheric pressure variability at the **Scanzano** site over the 2019–2024 period, derived from in-situ observations. The x-axis represents the time of day (00–23 UTC), the y-axis the annual sequence, and the z-axis the pressure magnitude expressed in [hPa]. Color shading encodes pressure intensity, ranging from lower values (blue) to higher values (yellow). The visualization highlights both sub-daily oscillations and longer-term seasonal and interannual variability, allowing persistent structures

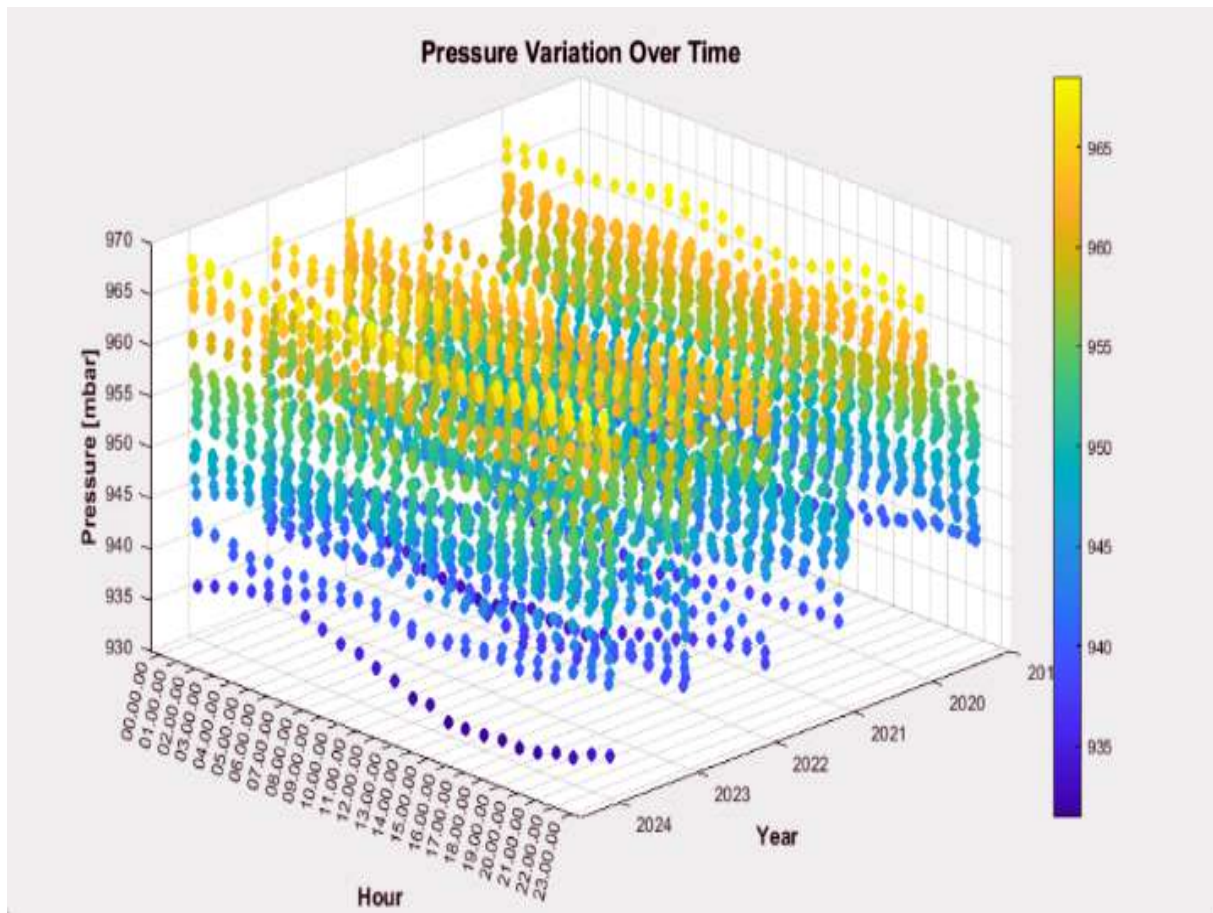


Figure 11.16. 3D scatter plot of atmospheric pressure variation in Scanzano (2019–2024). Pressure is expressed in millibars (mbar). The x-axis represents the time of day (00:00 corresponds to midnight), the y-axis shows the yearly sequence from 2019 to 2024, and the z-axis indicates the measured atmospheric pressure. The color gradient highlights pressure magnitude, with blue denoting lower values and yellow indicating higher values. **Database Source:** “In situ” Analysis.

and anomalous events to be identified.

11.5.1 Physical interpretation and relevance

The pressure field exhibits a systematic diurnal and semi-diurnal modulation, consistent with atmospheric tides driven by solar heating and thermodynamic resonance effects (**Chapman & Lindzen**, 1970 [265]; **Dai & Deser**, 1999 [266]). The observed amplitude of these oscillations, typically on the order of 4–6 hPa between daily minima and maxima, contributes directly to sub-daily variability in the hydrostatic component of tropospheric delay (ZHD) as described by the **Saastamoinen** formulation [65]. At longer temporal scales, the data reveal interannual shifts in the background pressure level, with slightly higher mean values observed during 2023–2024 compared to 2019–2020. Such variations are consistent with the modulation exerted by large-scale circulation patterns affecting the *Euro-Mediterranean* region, including the *North Atlantic Oscillation* and the *Mediterranean Oscillation* (**Hurrell**, 1995 [267]; **Lionello et al.**,



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2006 [268]). Localized low-pressure clusters are associated with synoptic-scale weather systems and frontal disturbances, while isolated extrema may reflect rare meteorological events or residual measurement artefacts (**Holton & Hakim**, 2013 [263]). Overall, the coexistence of predictable tidal oscillations and slowly varying climatic modulation defines a multi-scale atmospheric signature for the Scanzano site. These characteristics are directly relevant for both meteorological diagnostics and high-precision GNSS tropospheric-delay modeling, linking local diurnal variability with regional-scale atmospheric dynamics. **Figure 11.18** summarizes the temporal and seasonal evolution of atmospheric pressure at Scanzano over the 2019–2024 period, reconstructed from *in-situ* data. The four panels respectively show the hourly heat-map, the interpolated 3D surface, the isobaric contour map, and the yearly mean $\pm\sigma$ trend. Together, they highlight the persistence of diurnal atmospheric tides and the gradual inter-annual pressure increase observed in 2023–2024, consistent with large-scale Mediterranean circulation patterns.

The **figures 11.19–11.20** summarize the in-situ atmospheric pressure variability observed at the Scanzano (PA) site during May over the 2019–2024 period. The hour–year heatmap reveals a persistent and well-defined diurnal pressure cycle, characterized by smooth sub-daily oscillations that remain stable across all years. This behavior is consistent with atmospheric tides driven by solar heating and thermodynamic adjustment processes in the lower troposphere, which dominate short-term pressure variability. Beyond this regular sub-daily structure, the annual aggregation highlights clear interannual differences in mean pressure. The right-hand panel shows higher mean values in 2020 and 2022–2024 compared to 2019 and 2021, with $\pm 1\sigma$ intervals indicating that these shifts are significant relative to intra-month variability. This pattern suggests a modulation of the background pressure field by synoptic-scale circulation and large-scale climatic forcing on seasonal and interannual timescales. From a propagation standpoint, these findings are directly relevant to the interpretation of the Zenith Hydrostatic Delay (ZHD), which scales with surface pressure. The stability of the diurnal cycle supports reliable sub-daily ZHD modeling using standard formulations, while the interannual pressure offsets must be considered in long-term tropospheric delay climatologies and in the validation of reanalysis-based products. Overall, the figures confirm that Scanzano exhibits a stable hydrostatic background modulated by interannual atmospheric variability, providing a robust observational basis for subsequent analyses of tropospheric delay components.

11.5.2 Telecommunications Impact

Table 11.1 summarizes the impact of atmospheric pressure oscillations—manifested through the **Zenith Hydrostatic Delay (ZHD)**—on satellite microwave links across the Ku, Ka, V, and W frequency bands. Although the hydrostatic component of the tropospheric delay is typically considered stable, its diurnal and semi-diurnal variability can induce measurable phase fluctuations and calibration errors, especially at higher frequencies.

The table is structured along four analytical dimensions:

- (i) the frequency range under consideration;



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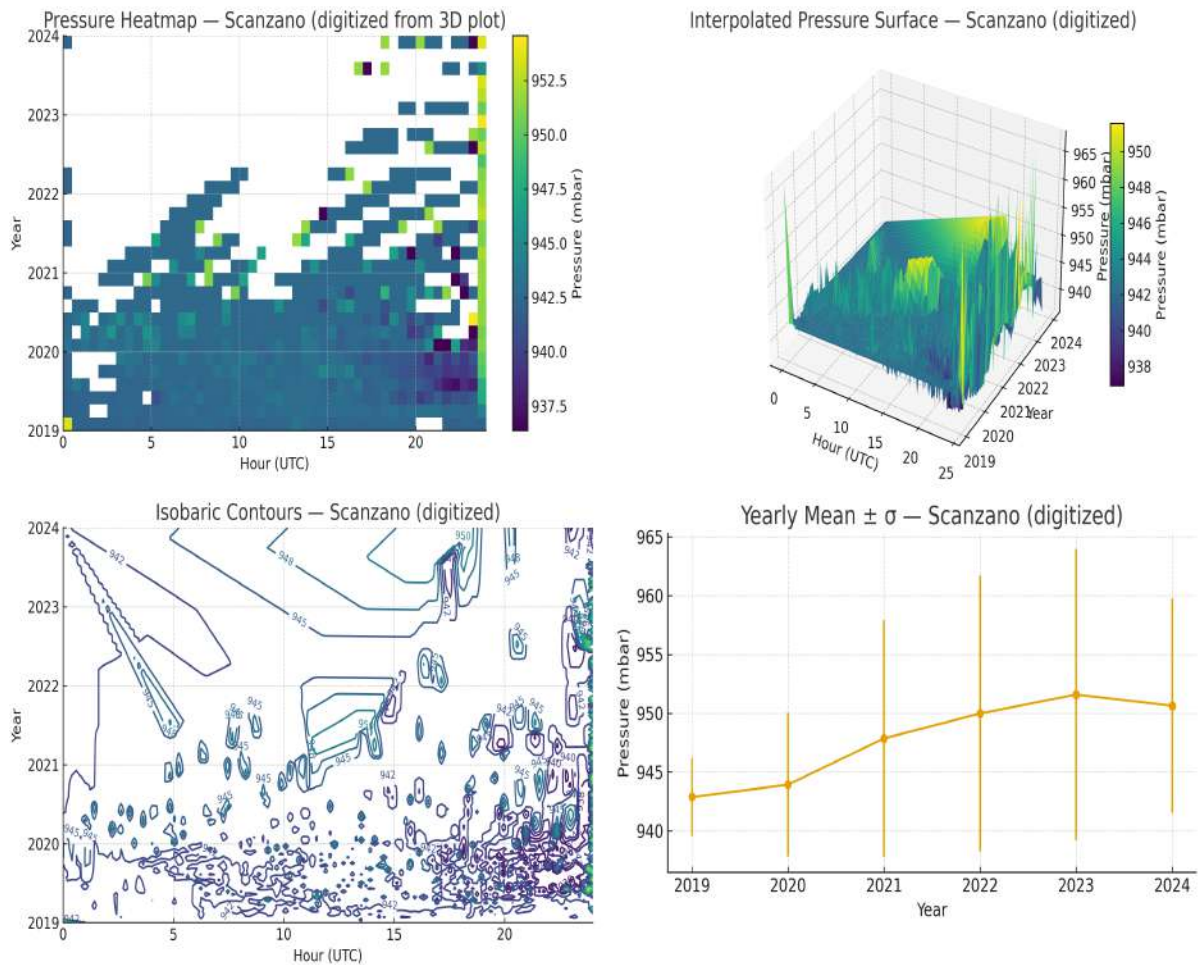


Figure 11.17. Temporal and Seasonal Variations in Atmospheric Pressure at Scanzano (2019–2024). Composite visualization of in-situ pressure data showing: (a) hourly heat-map, (b) 3D interpolated surface, (c) isobaric contour map, and (d) yearly mean $\pm \sigma$ trend. The results highlight diurnal atmospheric tides and inter-annual pressure variability linked to regional climatic oscillations. Credits: Author's digitization, Sapienza University of Rome – Telespazio (2024).

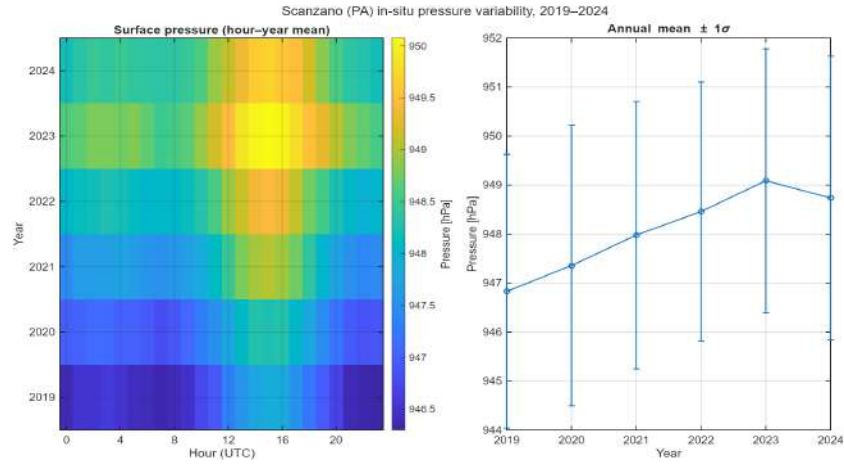


Figure 11.18. In-situ synthetic atmospheric pressure variability at the Scanzano (PA) site for May 2019–2024. The left panel shows the hour–year heatmap of mean surface pressure [hPa], highlighting the persistent diurnal modulation and its interannual evolution. The right panel reports the annual mean pressure with $\pm 1\sigma$ variability derived from all May in-situ samples. The observed increase from 2019 to 2023, followed by a slight decrease in 2024, reflects interannual variability superimposed on the stable sub-daily pressure cycle, providing a reference for the hydrostatic component of tropospheric delay at the site.

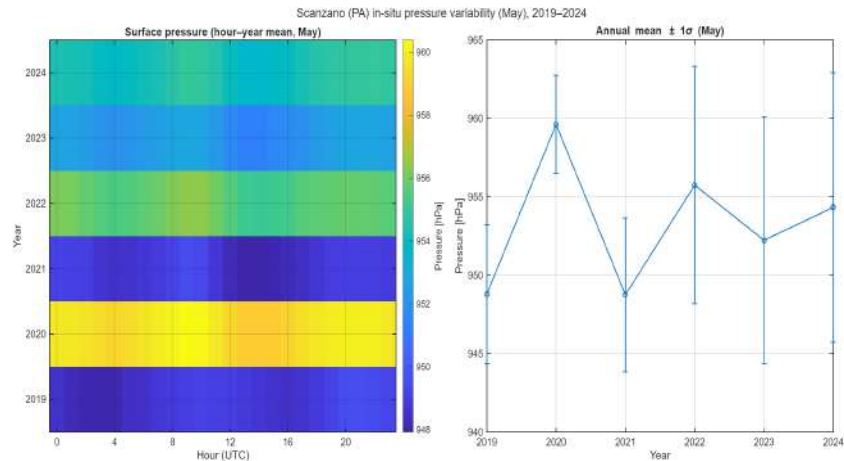


Figure 11.19. In-situ realistic surface pressure variability at the Scanzano (PA) site for May 2019–2024. The left panel shows the hour–year heatmap of mean surface pressure [hPa], evidencing a stable diurnal cycle and its interannual modulation. The right panel reports the annual mean pressure with $\pm 1\sigma$ variability computed from all May observations. Interannual differences are superimposed on a persistent sub-daily structure, providing a reference for the hydrostatic contribution to tropospheric delay at the site.

- (ii) the specific effects of ZHD oscillations on electromagnetic propagation;
- (iii) the operational impact on satellite communication and geodetic systems; and
- (iv) the corresponding mitigation strategies.

This comparative framework illustrates how the sensitivity to atmospheric oscillations increases from Ku to W-band, leading to potential degradations such as link margin reduction, phase *jitter*, or radar imaging distortion. The proposed mitigation measures—including empirical ZHD models, real-time GNSS-based corrections, and adaptive modulation/coding schemes—highlight the importance of combining physical modelling with operational resilience to preserve system accuracy and reliability under variable atmospheric conditions.

Band	Frequency Range	Effect of Pressure Oscillation (ZHD impact)	Operational Impact	Mitigation Strategies
Ku	12–18 GHz	Minimal direct attenuation; ZHD variation ~ 1 –2 mm from diurnal/semi-diurnal cycles visible in Fig. 243.	Negligible for most links; relevant for GNSS meteorology calibration and long-baseline interferometry.	Apply standard ZHD models (Saastamoinen, GPT2w) with daily correction from surface pressure data.
Ka	26–40 GHz	ZHD variation affects phase stability and precise delay calibration; possible small FSPL variation from air-density changes.	Reduced link margin for high-order modulations (e.g., 64-QAM); minor phase jitter in precision timing links.	Real-time tropospheric corrections from NWM (e.g., ECMWF, ERA5) or GNSS-derived ZTD; adaptive coding and modulation (ACM).
V	40–75 GHz	Increased sensitivity to combined gas absorption (O_2 , H_2O) under high-pressure, high-humidity conditions; diurnal ZHD cycle can impact coherent links.	Noticeable SNR fluctuation in high-capacity terrestrial-satellite links; potential desynchronization in coherent links.	Integrate pressure and humidity monitoring in link-budget models; pre-compensation for diurnal delay cycles.
W	75–110 GHz	Highly sensitive to small changes in air density and H_2O vapor; ZHD cycles (~ 1 –2 mm) can cause measurable phase shifts; higher mean pressures (e.g., 2023–2024) can enhance attenuation from water vapor and oxygen lines.	Potential degradation in radar imaging resolution, coherent data links, and atmospheric sounding accuracy; reduced link availability in humid conditions.	Use near-real-time delay estimation from collocated GNSS; dynamic power control; site diversity to mitigate local anomalies.

Table 11.1. Impact of atmospheric pressure oscillations (via ZHD) on satellite microwave links across Ku, Ka, V, and W bands.

11.5.2.1 Discussion of Table 11.1 — Frequency Dependence of Hydrostatic Oscillations

The results presented in **Table 11.1** demonstrate that the physical impact of hydrostatic pressure oscillations on satellite links is strongly frequency dependent. While in the Ku-band the effect of Zenith Hydrostatic Delay (ZHD) variations remains negligible, at higher frequencies the same millimetric fluctuations become progressively amplified. In the Ka-band, diurnal and semi-diurnal ZHD cycles can already degrade link margins and induce phase *jitter* in high-order modulation schemes, compromising transmission stability. The situation becomes increasingly critical in the V- and W-bands, where even minor perturbations in air density and water vapor partial pressure result in measurable phase shifts and additional attenuation. From a physical standpoint, these findings emphasize that the troposphere cannot be considered a static medium; rather, it behaves as a dynamic refractive system, in which temporal oscillations directly interact with electromagnetic wave propagation. The amplification of these effects at millimetric

wavelengths confirms that atmospheric stability represents a fundamental limiting factor for next-generation satellite links. Consequently, the design and operation of V- and W-band systems must incorporate both accurate modelling of ZHD cycles and real-time correction mechanisms, since merely increasing transmit power is insufficient to compensate for the stochastic refractive fluctuations imposed by the atmosphere.

11.6 Scanzano Data Pipeline: From MS Excel to MATLAB Database – In Situ Analysis

Atmospheric data analysis is essential for characterizing meteorological dynamics and understanding their influence on radiowave propagation. This is particularly relevant in regions such as Scanzano (PA), where variations in pressure, temperature and water-vapor content directly modulate tropospheric delays and, consequently, affect GNSS, climatological assessments and high-frequency satellite communication links. This section documents a structured workflow for transforming raw measurements into a fully operational atmospheric database. The process begins with the import and organization of **MS Excel datasets** into **MATLAB**, followed by the construction of time-aligned, quality-controlled data structures suitable for statistical analysis and visualization. Core atmospheric parameters—including surface pressure, temperature, Zenith Hydrostatic Delay (ZHD) and Zenith Wet Delay (ZWD)—are analyzed through descriptive statistics, distributional metrics and graphical representations. Histograms, temporal profiles and statistical indicators such as means and standard deviations provide a quantitative depiction of atmospheric variability. This integrated methodology supports a detailed evaluation of the evolution of atmospheric conditions and their radiowave propagation consequences. Such results are instrumental for GNSS delay correction, meteorological modelling and climate-variability studies, and contribute to a deeper understanding of how tropospheric processes shape signal stability and phase coherence.

11.6.1 Hourly Trends of ZHD, ZWD and ZTD for 1 May 2019 — In Situ Analysis

Figure 11.21 illustrates the diurnal evolution of ZHD, ZWD and ZTD over a full 24-hour cycle at the Scanzano site on 1 May 2019. The three panels provide complementary insights into how surface pressure and atmospheric moisture jointly modulate signal propagation.

- ZHD (top panel) displays smooth, gradual variations largely driven by the semi-diurnal oscillation of surface pressure. As expected from the hydrostatic formulation, the signal remains highly stable throughout the day.
- ZWD (middle panel) exhibits markedly stronger variability, capturing transient changes in atmospheric water vapor. Peaks typically coincide with temperature maxima and episodic humidity increases associated with local meteorological activity.

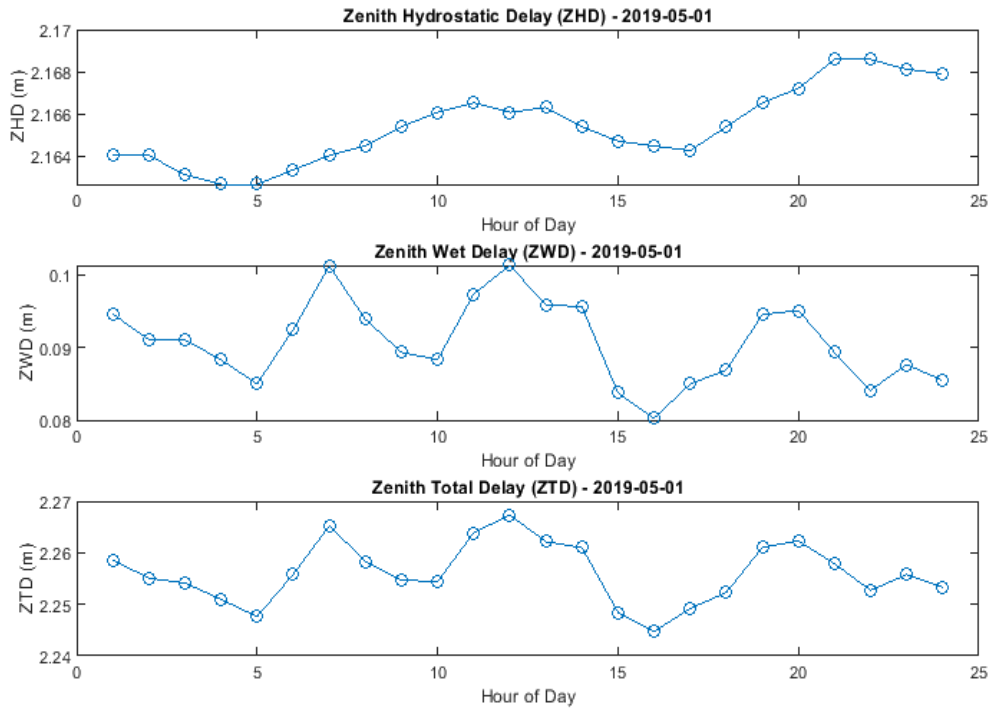


Figure 11.20. Hourly variations of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) on May 1, 2019, for Scanzano. **Database Source:** “In situ” Analysis.

- ZTD (bottom panel) reflects the combined influence of both components. Its slow-varying baseline is inherited from ZHD, while short-term oscillations stem from ZWD contributions.

Extending this analysis to longer temporal windows enables the identification of seasonal patterns, persistent anomalies and meteorological transitions. The contrast between the stability of ZHD and the high variability of ZWD highlights the distinct physical drivers controlling each component and clarifies their relative influence on the total tropospheric delay.

11.6.2 Diurnal Evolution of ZHD, ZWD and ZTD at the Scanzano Site

Figure 11.22 provides further detail on the diurnal dynamics of ZHD, ZWD and ZTD.

- ZHD increases gradually from approximately 2.167 m at midnight to a maximum of ≈ 2.171 m near 11:00 UTC, before decreasing symmetrically in the afternoon. This behavior is consistent with the semi-diurnal atmospheric pressure tide.
- ZWD fluctuates between 0.078 m and 0.102 m, with two short-lived peaks in mid-morning and late afternoon. These features suggest transient moisture advection or localized convective development.
- ZTD inherits the smooth pattern of the hydrostatic component but integrates small-scale ZWD oscillations, producing a composite signal with an amplitude of ≈ 0.04 m.

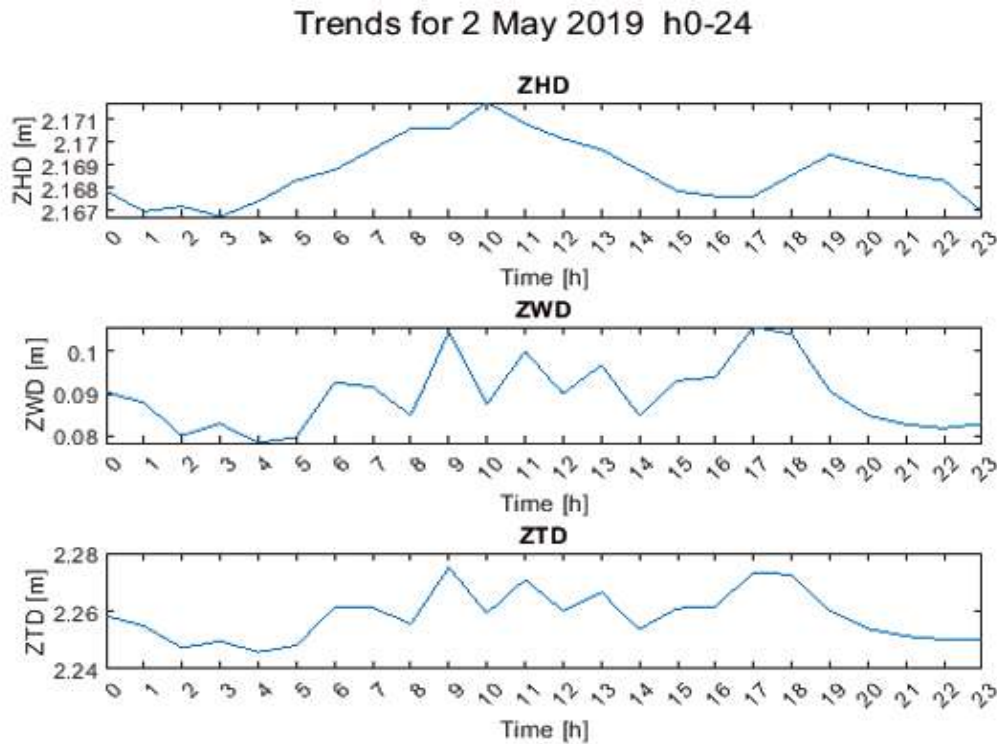


Figure 11.21. Detailed hourly dynamics of the Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) at the Scanzano site on 2 May 2019 (00:00–24:00 UTC).
Database Source: “In situ” Analysis.

These insights emphasize the role of sub-daily processes—such as humidity cycles, thermal forcing and mesoscale meteorological variability—in shaping tropospheric delay dynamics. Accurate representation of these fluctuations is fundamental for GNSS water-vapor retrievals, atmospheric correction schemes and satellite link-budget calibration.

11.6.3 Reference to Tables 13.3 and 13.4 — Hourly Atmospheric Delay Components

Tables 11.2 and 11.3 (Appendix F.4) report the hourly numerical values of ZHD, ZWD and ZTD for **1 May 2019**. These datasets, constructed with a 1-hour resolution, provide the foundation for the diurnal analyses discussed above.

- ZHD, primarily governed by surface pressure, exhibits gradual, smooth variations consistent with the hydrostatic equilibrium.
- ZWD, highly sensitive to water-vapor fluctuations, displays pronounced short-term variability.
- ZTD, resulting from their sum, captures both the stability of the hydrostatic term and the rapid variations induced by atmospheric moisture.



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This dataset is instrumental for validating model-based estimations, supporting GNSS calibration protocols and enabling cross-comparisons with atmospheric reanalysis products such as ERA5. Having established the hourly behavior of ZHD, ZWD, and ZTD for the reference day, the analysis now broadens to monthly–hourly variability and inter-annual trends. The corresponding technical in-situ procedures and numerical details are discussed in **Appendix O**.

11.7 Monthly–Hourly Variability, Inter-annual Trends, and Database Description

The analysis of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) at hourly resolution provides a detailed characterization of both diurnal and inter-annual tropospheric variability over Scanzano (PA) during the 2019–2024 period. The combined monthly–hourly representations highlight the markedly different dynamical behavior of the hydrostatic and wet components. ZHD remains the most stable contribution (≈ 2.16 – 2.19 m), exhibiting weak diurnal modulation and limited inter-annual dispersion, in agreement with its primary dependence on surface pressure. In contrast, ZWD shows pronounced temporal variability, with systematic daytime maxima (≈ 10 – 15 UTC) associated with convective enhancement of atmospheric moisture. These short-term fluctuations are directly inherited by ZTD, which mirrors the wet contribution superimposed on the quasi-static hydrostatic background. The analysis of a fixed winter day (15 January) further confirms this behavior: ZHD displays minor pressure-driven oscillations, while ZWD reveals distinct humidity cycles, with maximum amplitudes in 2024 and minimum values in 2022. ZTD follows the combined evolution, emphasizing the dominant role of water vapor in shaping sub-daily tropospheric delay variability.

Inter-annual differences become even more evident when comparing the three components at a fixed synoptic hour (15:00 UTC, January). Enhanced ZWD and ZTD values in 2021 and 2023 indicate wetter atmospheric conditions, whereas reduced delays in 2024 reflect comparatively drier air masses and weaker pressure forcing. These results underline the strong sensitivity of tropospheric delays to humidity regimes and suggest a modulation by Mediterranean-scale climate variability, including large-scale circulation patterns and the thermodynamic influence of the Mediterranean Sea as a persistent heat- and moisture reservoir. The unified representation provides a compact yet comprehensive view of the multi-scale variability of tropospheric delays, with direct implications for GNSS correction accuracy, high-frequency Earth–space propagation, and long-term climate diagnostics in the Mediterranean region.

11.8 Summary of Chapter 11

This chapter has addressed the characterization of tropospheric impacts on high-frequency satellite communications, focusing on attenuation mechanisms and their variability across X-, Ka-, and W-bands. By combining *in situ*, reanalysis, and satellite data, the analysis confirmed

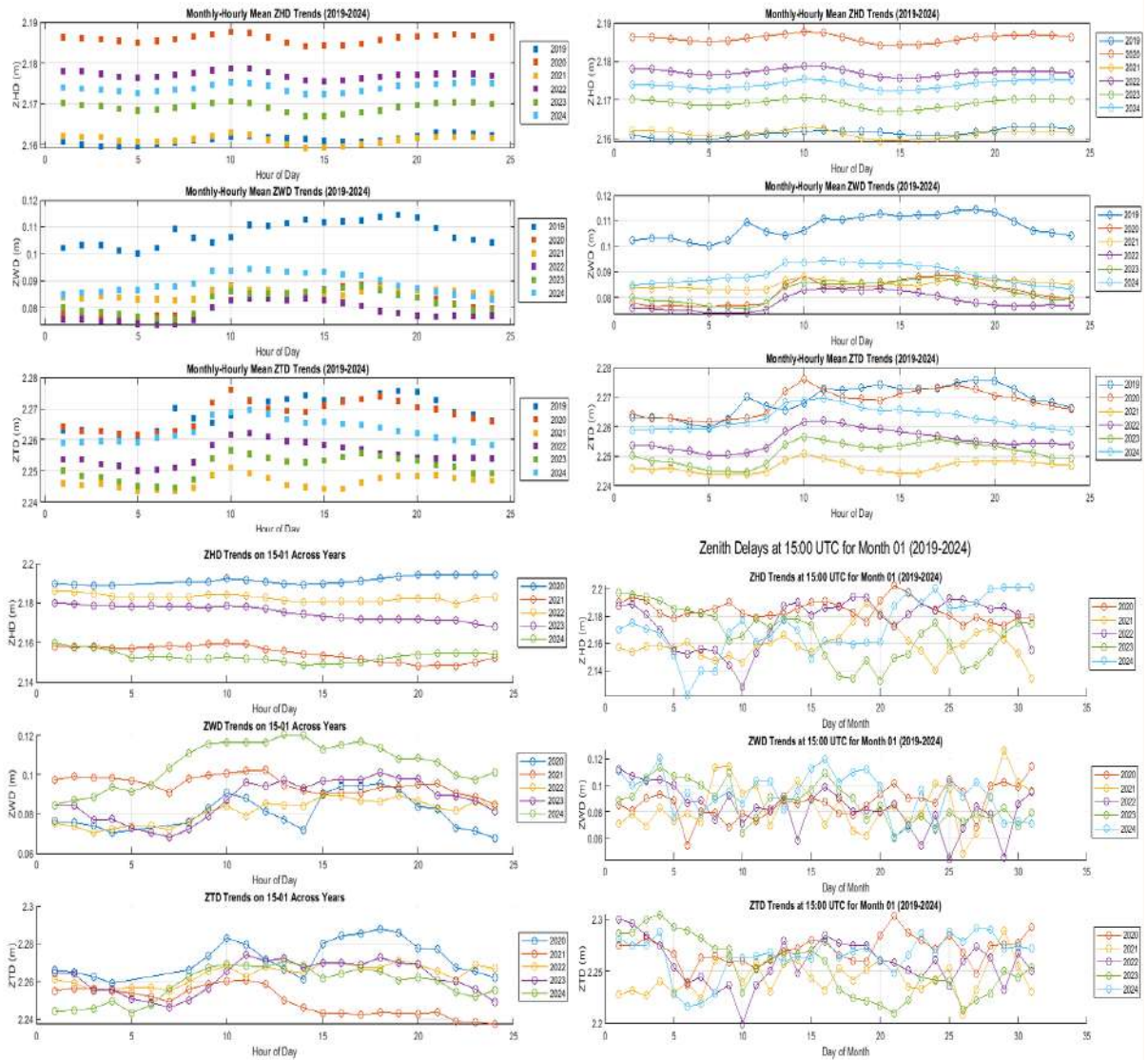


Figure 11.22. Unified representation of hourly Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) over Scanzano (PA) for the 2019–2024 period. The figure combines monthly–hourly mean fields, diurnal cycles on a fixed winter day (15 January), and inter-annual comparisons at a fixed synoptic hour (15:00 UTC). ZHD appears highly stable and pressure-driven, while ZWD exhibits pronounced diurnal and inter-annual variability associated with atmospheric moisture. ZTD reflects the superposition of these two components, highlighting the dominant role of water vapor in short-term and climatological tropospheric delay variability.



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the stability of the hydrostatic component and the strong variability of wet and hydrometeoric contributions. The comparative results highlight that higher frequencies experience increasing sensitivity to gaseous absorption, scintillation, and precipitation-induced attenuation. While **W-band** applications remain limited to specific conditions, **Ka-band** proves feasible for deep-space and Earth-space operations when supported by adequate fade margins and adaptive modulation techniques. The **X-band** remains the most resilient, with minimal atmospheric degradation.



Chapter 12

ERA-5, ITU-R and In-Situ Tropospheric Delays: A Comparative Assessment

This chapter presents a comparative assessment of tropospheric delay modeling based on ERA-5 reanalysis, ITU-R climatological formulations, and in-situ GNSS observations over the **2019–2024 period**. Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), Zenith Total Delay (ZTD), and their slant-path counterparts are analyzed with emphasis on temporal variability, statistical structure, and elevation-dependent amplification. Results show that the hydrostatic component remains quasi-stationary and primarily pressure-driven, whereas the wet component exhibits strong intermittency associated with humidity variability, dominating uncertainty in both zenith and slant geometries. ERA-5 provides a physically consistent reference across temporal scales (climatological means to sub-daily variability), while ITU-R reproduces long-term seasonal behaviour but cannot capture event-scale fluctuations.

Time–frequency analysis confirms the multi-scale nature of wet-delay variability, with convective, diurnal, and synoptic contributions clearly separated. In-situ observations independently validate these findings, highlighting the critical role of wet delay at low elevation angles. Overall, this chapter establishes a robust baseline for subsequent analyses of attenuation, radiative transfer, and high-frequency propagation.

12.1 Introduction and Methodological Framework

Accurate estimation of tropospheric delays—namely the hydrostatic, wet, and total components (ZHD, ZWD, and ZTD)—is a fundamental requirement for high-precision satellite communications and GNSS meteorology. These delays, primarily controlled by surface pressure, atmospheric water vapor, and temperature, introduce phase and amplitude perturbations that directly affect positioning accuracy, link budgets, and signal reliability. Their robust characterization is therefore



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essential to ensure both accuracy and availability in Earth-space and deep-space missions.

In operational and scientific practice, tropospheric delays are typically addressed through three complementary methodological approaches. Numerical weather reanalyses, such as ERA5, provide globally consistent and high-resolution atmospheric fields suitable for physically based modeling [12]. Empirical climatological formulations, such as those recommended by the ITU-R, are widely adopted for link design and system planning [3]. In situ observations, including GNSS-derived delays and radiosonde measurements, serve as reference datasets for calibration and validation purposes [46, 73]. The combined use of these approaches enables a critical assessment of model consistency across spatial and temporal scales, as well as the identification of discrepancies arising from resolution limits and short-term meteorological variability.

To ensure topographic consistency and site adaptability, a global Digital Terrain Model reprojected into WGS-84 and regional geoids was integrated with all atmospheric datasets. This step is essential for elevation-dependent corrections in tropospheric modeling, particularly over complex terrain. The methodology was applied to representative ground terminals, including Scanzano (Italy), New Norcia (Australia), and Adelaide (California, USA). Because many operational links operate at oblique geometries, slant-path delays were explicitly evaluated for elevation angles above 15° , where tropospheric effects become dominant. Zenith delays were mapped into slant geometry using hydrostatic and wet mapping functions $m_h(\theta)$ and $m_w(\theta)$.

While delays converge toward their zenith values at elevations above approximately 60° , amplification factors approaching 3.8 occur near 15° , making accurate mapping functions critical for Ka-, Q-, and W-band applications [216, 56].

ERA5 meteorological fields were extracted over an area of approximately 56 km^2 centered on the Scanzano site for the 2019–2024 period, ensuring spatial coherence and temporal continuity. Tropospheric delays were computed using the HGPT2 (Hourly Global Pressure and Temperature) model, which provides hourly fields of pressure, temperature, relative humidity, weighted mean temperature, and derived delays following the Saastamoinen formulation [45, 214]. Compared to earlier HGPT implementations, HGPT2 explicitly incorporates relative humidity and precipitable water vapor, improving its suitability for geodetic and meteorological applications [47]. Although formally validated up to 2017, internal consistency checks confirmed stable and physically plausible behavior under post-2017 conditions.

For empirical modeling, the ITU-R P.834-9 framework was implemented through a re-engineered slant-path delay routine, converting the original graphical formulation into a fully tabular MATLAB/Python workflow suitable for direct comparison with ERA5- and GNSS-derived delays [3]. Despite the original calibration period of the ITU-R formulation, extrapolation to 2024 produced numerically stable results consistent with ERA5 under Mediterranean conditions.

A critical methodological aspect concerned the treatment of the Day-of-Year (DOY) parameter. Early tests based on a static DOY artificially suppressed seasonal modulation in pressure, temperature, and humidity fields. Restoring DOY as a dynamic variable reintroduced the expected annual variability and removed spurious artifacts in long-term delay estimates, in agreement with the known seasonal forcing of tropospheric refractivity [257].



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The January ZHD variability over the 2019–2024 period at the Scanzano site, summarized in **Figure 12.1** and relocated to **Appendix 12.A (Fig. A12.1)**, illustrates this behavior. ZHD remains highly stable, with values confined approximately to the 2.0–2.6 m range and interannual variability below 3%. The observed oscillations reflect synoptic-scale pressure variability rather than long-term trends, confirming that the hydrostatic delay is predominantly meteorologically driven and validating both the ERA5 dataset and the adopted modeling framework [218].

A more comprehensive, multi-scale synthesis of ZHD variability is provided by **Figure 12.2 (Appendix 12.A, Fig. A12.2)**, derived from ERA5–HGPT2 outputs [12, 45]. Annual statistics confirm strong interannual stability, with mean values consistently within the 2.1–2.3 m range. Daily anomaly patterns show limited dispersion and the absence of systematic drift, indicating quasi-stationary pressure forcing [218]. The Morlet wavelet scalogram highlights dominant low-frequency components associated with synoptic and seasonal pressure variability, while higher-frequency contributions remain weak [32]. Strong interannual correlations further confirm the climatic persistence of the hydrostatic term [257], and linear trend analysis reveals a negligible long-term tendency on the order of $+0.002 \text{ m yr}^{-1}$, consistent with the expected quasi-stationary behavior of ZHD [73].

Together, these elements establish the hydrostatic delay as a stable reference component within the tropospheric delay budget. This benchmark provides the necessary baseline against which the stronger variability of the wet and total delay components is examined in the following sections.

12.1.1 Detecting Location-Specific Trends in Tropospheric Delays

To highlight the role of the day-of-year (DOY) parameter in tropospheric-delay modelling, a dedicated Python routine (reported in **Appendix E**) was first applied under idealized conditions, computing ZHD, ZWD, and ZTD for 1 May 2023 at the Scanzano site using fixed pressure and temperature values. As expected, under constant atmospheric conditions the resulting delays are invariant with respect to DOY, confirming that temporal modulation arises only when realistic meteorological forcing is introduced. When ERA5 reanalysis fields are employed, however, physically meaningful temporal variability emerges, reflecting the actual evolution of atmospheric state variables [227, 237]. The daily evolution of ZHD, ZWD, and ZTD for May 2023 at Scanzano, evaluated at an elevation angle of 15° , is illustrated in **Figure 12.3** and provided for completeness in **Appendix 12.A (Fig. A12.3)**. The hydrostatic component remains nearly constant throughout the month, while both ZWD and ZTD exhibit significantly larger fluctuations driven by changes in atmospheric water vapor content. This behavior is consistent with the well-established dominance of humidity variability in shaping short-term tropospheric delay dynamics [45, 46, 73]. The corresponding slant tropospheric delay (STD) is obtained by mapping the zenith components into the line-of-sight geometry through the hydrostatic and wet



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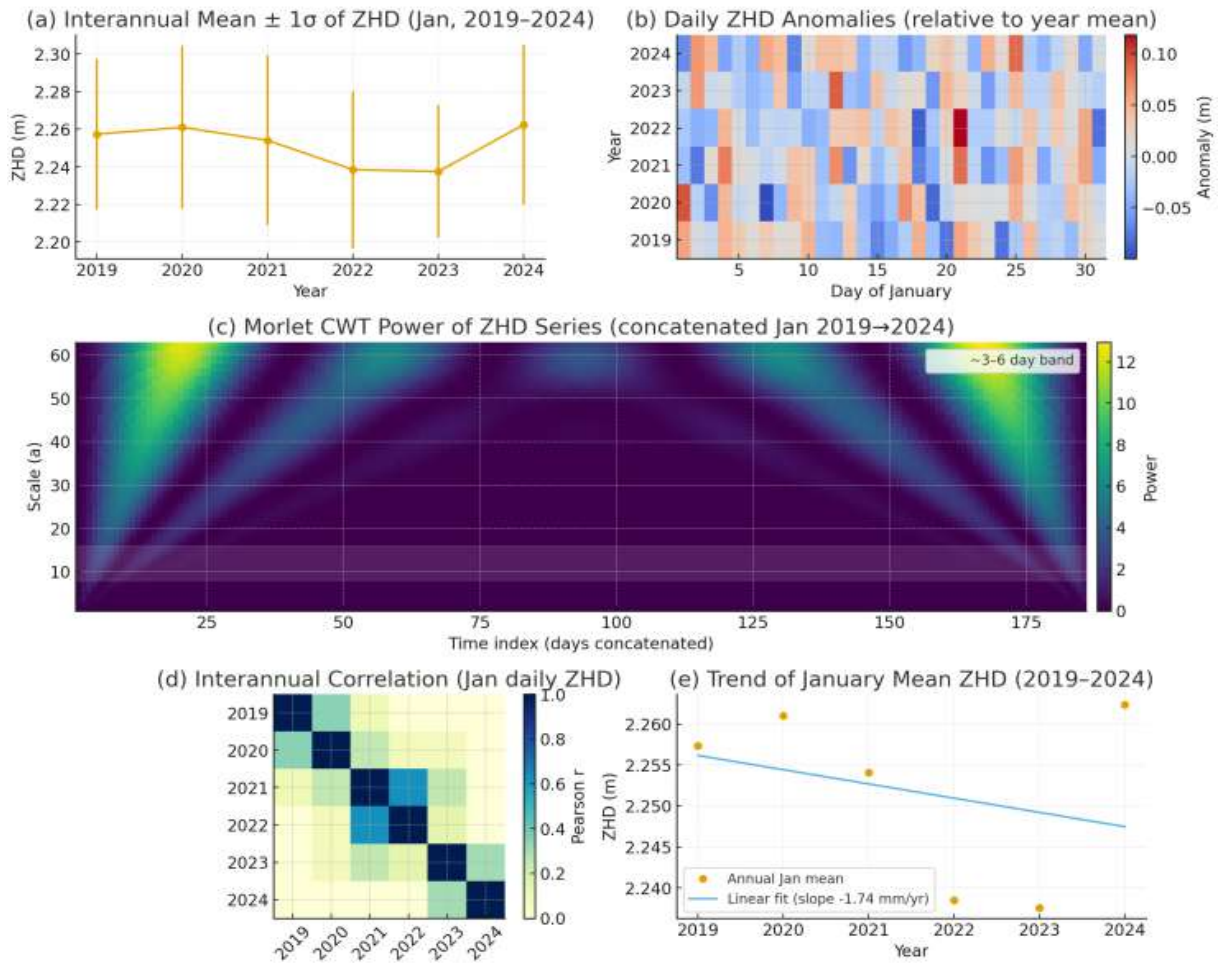


Figure 12.1. Multi-Panel Analysis of Zenith Hydrostatic Delay (ZHD) Variability (2019–2024). (a) Annual mean and standard deviation of ZHD at Scanzano (PA) derived from ERA5, illustrating the interannual stability of the hydrostatic component. (b) Heat map of daily ZHD anomalies showing limited dispersion and the absence of long-term drift, consistent with stable surface-pressure conditions. (c) Morlet wavelet scalogram of the concatenated 2019–2024 ZHD time series, highlighting dominant low-frequency modes (periods > 60 days) and weak high-frequency oscillations associated with synoptic activity. (d) Interannual correlation matrix revealing strong coherence ($r > 0.9$) among consecutive years, confirming climatic persistence of the hydrostatic component. (e) Trend of January mean ZHD with linear regression (slope $\approx +0.002 \text{ m yr}^{-1}$), indicating negligible long-term drift.



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mapping functions $m_h(\theta)$ and $m_w(\theta)$ as:

$$\rho_{corr} = \rho + [m_h(\theta) \cdot ZHD + m_w(\theta) \cdot ZWD] = \rho + STD(\theta), \quad (12.1)$$

where ρ represents the geometric distance between the GNSS receiver and the satellite. The direct line-of-sight (LOS) distance in the absence of atmospheric effects is given by:

$$\rho = \sqrt{(x_s - x_r)^2 + (y_s - y_r)^2 + (z_s - z_r)^2}, \quad (12.2)$$

with (x_s, y_s, z_s) and (x_r, y_r, z_r) representing the **Earth-Centered, Earth-Fixed (ECEF)** coordinates of the satellite and receiver, respectively. In a more complete propagation model, **ionospheric effects** are included through the **Slant Ionospheric Delay (SID)**, yielding:

$$SID(\theta, f) = \frac{40.3 \cdot STEC(\theta)}{f^2}, \quad (12.3)$$

where STEC denotes the slant total electron content along the propagation path. This formulation follows standard GNSS practice and provides a unified treatment of neutral-atmosphere and ionospheric delays [218, 219, 230]. Realistic atmospheric forcing, as provided by ERA5 reanalysis, emerges as a necessary condition for resolving physically meaningful temporal variability in tropospheric delay. Within this framework, the hydrostatic component retains its quasi-static character, while the wet term introduces pronounced fluctuations that dominate short-term behavior. This clear dynamical separation supports the internal consistency of the adopted modeling approach and highlights the central role of humidity in controlling both GNSS tropospheric corrections and low-elevation satellite link performance [65, 73, 237].

12.1.2 Histograms and Trend Analysis Using the HGPT2 Model

The interannual behavior of tropospheric delays over Scanzano emerges clearly when ERA5 reanalysis fields are processed through the HGPT2 framework and examined beyond strictly monthly aggregates. By extending the analysis to annual distributions, it becomes possible to distinguish systematic variability associated with large-scale meteorological forcing from purely seasonal effects, thereby assessing the suitability of ERA5 for climatological interpretation on yearly time scales [12, 45, 214, 73].

A first indication of this behavior is provided by the temporal evolution of the wet delay component. When ZWD is sampled at a fixed local time during January, its year-to-year evolution is characterized by nearly parallel trajectories, with a gradual decrease over the month and modest vertical offsets between different years. This pattern indicates that interannual variability manifests primarily as a translation of the delay level rather than as a distortion of its temporal structure. Such behavior is physically consistent with changes in background humidity conditions occurring within a stable climatic regime and confirms that HGPT2 preserves the intrinsic temporal coherence of the wet delay while allowing realistic interannual modulation [257]. The



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hydrostatic component exhibits an even more constrained evolution. Its January time series follows a monotonic decrease associated with the seasonal evolution of surface pressure, and the close overlap among yearly curves highlights the strong predictability of ZHD. Small vertical displacements between different years reflect minor pressure anomalies rather than changes in the underlying statistical structure. This behavior reinforces the interpretation of the hydrostatic delay as a quasi-deterministic component governed primarily by large-scale pressure fields and only marginally influenced by moisture-related processes [56, 218]. This physical interpretation is corroborated by the distributional analysis. Histograms of ZHD spanning the 2019–2024 period display smooth, near-Gaussian profiles centered between 2.2 and 2.3 m. The introduction of a controlled level of synthetic noise (5%) enhances realism by mimicking measurement uncertainty and short-term atmospheric variability, yet does not alter the central tendency or dispersion of the distributions. The corresponding cumulative distributions confirm the absence of systematic trends, with consistent variance across years and only minor tail deviations attributable to interannual pressure anomalies [46, 73].

When delays are projected into slant geometry at low elevation angles, the same hierarchy is preserved. After mapping using ITU-R and Vienna Mapping Function formulations [3, 56], the slant-path hydrostatic delay remains narrow and highly stable, while the total slant delay exhibits a moderate broadening driven by the wet component. Importantly, this geometric amplification does not introduce artificial interannual variability: differences between recent years remain negligible, confirming that moisture-related effects, rather than mapping artifacts, dominate the dispersion of the total delay.

Overall, the combined trend and histogram analysis demonstrates that the HGPT2 model, when driven by ERA5 reanalysis, reliably captures both the stability of the hydrostatic component and the controlled interannual variability of the wet delay. This behavior supports the use of ERA5-based HGPT2 outputs for long-term tropospheric-delay characterization in climatological and propagation-oriented studies. The graphical material supporting this interpretation, originally presented as **Figures 12.4–12.7**, is provided in **Appendix 12.A (Figs. A12.4–A12.7)**.

12.1.3 Statistical Characterization of Tropospheric Delays (ERA5, 2019–2024)

The ERA5 reanalysis provides a statistically robust basis for investigating the structure and variability of tropospheric delays over the Scanzano site across the 2019–2024 period. By jointly examining probability distributions, cumulative statistics, and correlation metrics, it is possible to distinguish the fundamentally different statistical behavior of the hydrostatic and wet components and to assess their combined impact on the total zenith delay [12]. From a probabilistic perspective, the hydrostatic delay is characterized by a tightly clustered distribution, with values concentrated around 2.26 m and only limited dispersion. This behavior reflects the smooth evolution of surface pressure and confirms the quasi-deterministic nature of the dry-air contribution to tropospheric delay [45, 214]. The wet component, by contrast, occupies a broader statistical space and exhibits mild asymmetry, consistent with the intermittent character of



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atmospheric water vapor and its sensitivity to synoptic and mesoscale processes. The total zenith delay naturally emerges as a superposition of these two regimes, retaining the stability of the hydrostatic baseline while incorporating the extended tails associated with wet-delay variability.

An inspection of cumulative distributions and percentile-based diagnostics further supports this interpretation. The P10, P50, and P90 levels remain well separated yet stable across the analyzed years, indicating a statistically regular behavior with only rare excursions toward extreme values. Interannual boxplot statistics reinforce this picture, showing compact interquartile ranges for both ZHD and ZWD and pointing to the absence of pronounced long-term shifts in the overall delay regime.

The statistical independence of the hydrostatic and wet contributions is highlighted by the inter-component correlation analysis, which yields negligible correlation coefficients ($\rho \approx 0.01$). This result confirms that pressure-driven and moisture-driven processes evolve largely independently within the troposphere, even though they jointly determine the observed total delay [218]. When the hydrostatic component is examined in isolation, subtle interannual differences become discernible. Year-specific density estimates reveal a gradual rightward displacement of the ZHD distribution from 2019 to 2024, consistent with small changes in mean surface pressure. This shift is mirrored by slightly higher median values in the post-2022 period, while the overall shape and spread of the distributions remain essentially unchanged. Cumulative distributions preserve a high degree of similarity across years, and the low off-diagonal values of the interannual correlation matrix ($|\rho| < 1$) indicate weak temporal dependence and strong statistical persistence.

Taken together, these results confirm that the hydrostatic delay constitutes a highly stable component of the tropospheric delay budget, while still retaining sensitivity to slow modulations in large-scale pressure regimes. When interpreted alongside meteorological reanalysis, such subtle statistical signatures may provide useful diagnostics of gradual changes in regional atmospheric circulation or emerging climatic tendencies over the central Mediterranean [73, 257]. The statistical representations supporting this analysis, originally presented as **Figures 12.8 and 12.9**, are provided in **Appendix 12.A (Figs. A12.8–A12.9)**.

12.2 Introduction of the Scalogram and the Wavelet Transform

The temporal variability of ZHD, ZWD, and ZTD was investigated using the **Continuous Wavelet Transform (CWT)** and its two-dimensional representation, the scalogram. This approach enables multi-scale analysis of non-stationary atmospheric signals, providing simultaneous localization in time and frequency. Prior to transformation, each delay time series $x(t)$ was uniformly resampled (daily, hourly, or monthly, depending on the process under investigation), de-meaned, and detrended to enhance the detection of oscillatory components [45,214]. For a



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real-valued signal $x(t)$, the CWT is defined as:

$$W(a, b) := \frac{1}{\sqrt{|a|}} \int_{-\infty}^{+\infty} x(t) \cdot \psi^* \left(\frac{t-b}{a} \right) \cdot dt, \quad (12.4)$$

where $a > 0$ denotes the scale parameter (inversely related to frequency), b the time translation, and $\psi(t)$ the mother wavelet [32]. The corresponding scalogram is expressed as:

$$S(a, b) = |W(a, b)| \quad \text{or} \quad P(a, b) = |W(a, b)|^2, \quad (12.5)$$

and represents the temporal evolution of spectral energy across scales. Unlike the **Fourier Transform**,

$$X(f) = \mathcal{F}\{x(t)\}(f) = \int_{-\infty}^{+\infty} x(t) e^{-i2\pi ft} dt, \quad (12.6)$$

which characterizes only the global frequency content of a signal, the Wavelet Transform preserves temporal localization, making it particularly suitable for analyzing transient and scale-dependent atmospheric variability. In this work, the Morlet wavelet was adopted due to its optimal time–frequency localization properties, and is defined as:

$$\psi(t) = \pi^{-1/4} e^{i\omega_0 t} e^{-t^2/2}, \quad (12.7)$$

where the complex exponential represents the oscillatory component and the Gaussian envelope ensures temporal confinement. The wavelet coefficients $W(a, b)$ quantify the similarity between the signal and scaled, translated versions of the mother wavelet. Varying the scale a probes different spectral components, while varying b tracks their temporal evolution, forming the basis of the scalogram representation. For the commonly adopted value $\omega_0 = 6$, the Fourier-equivalent period associated with scale a is approximately:

$$T_{\text{Fourier}}(a) \approx 1.03 \cdot a. \quad (12.8)$$

The real and imaginary components of the Morlet wavelet, modulated by the Gaussian envelope, are illustrated in **Figure 12.10**, which has also been moved to **Appendix 12.A (Fig. A12.10)** to support the discussion without interrupting the main narrative. The balance between temporal confinement and spectral resolution provided by the Morlet wavelet makes it particularly well suited for the CWT-based analysis of tropospheric delay variability.

12.2.1 Wavelet Scalogram Analysis and Centroid Estimation of Noisy ZHD

The wavelet-based analysis of the hydrostatic delay was conducted using ERA5-derived ZHD time series perturbed with a controlled level of additive noise (5%), in accordance with the statistical framework introduced in §4.8. This approach allows the robustness of the mean hydrostatic delay to stochastic perturbations to be evaluated while preserving the underlying

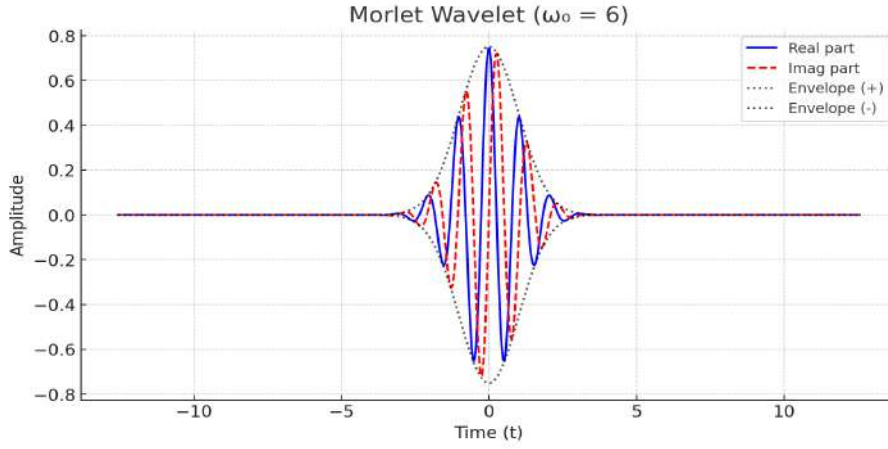


Figure 12.2. Morlet wavelet used as mother wavelet in the Continuous Wavelet Transform (CWT). The real and imaginary components are shown, modulated by a Gaussian envelope, illustrating the time–frequency localization properties adopted for the multi-scale analysis of tropospheric delay variability.

physical structure of the signal. To quantify the impact of noise on the central tendency of the distribution, the centroid of the noisy ZHD values was computed as:

$$\overline{ZHD} = \frac{\sum_i (ZHD_i f_i)}{\sum_i f_i}, \quad (12.9)$$

where f_i denotes the frequency of occurrence of each ZHD value. The resulting centroid remains confined within the 2.2–2.3 m interval, indicating that moderate noise levels do not introduce a systematic bias in the mean hydrostatic delay.

The wavelet scalogram further clarifies the multi-scale structure of the ZHD signal. At intermediate scales (approximately 20–60), persistent and well-defined power bands dominate the representation, corresponding to climatologically stable oscillations driven by deterministic pressure variability. At smaller scales, high-frequency components appear as short-lived structures associated with transient atmospheric disturbances; their visibility is enhanced by the imposed noise but remains temporally localized. In contrast, scales larger than about 80 are characterized by extended low-power regions, reflecting the long-term stability imposed by synoptic and seasonal pressure regimes. Taken together, the smooth and coherent distribution of wavelet power across time and scale confirms the internal consistency of the **ERA5–HGPT2** dataset. More importantly, it demonstrates that the observed ZHD variability is governed by physically meaningful atmospheric processes rather than by stochastic artefacts introduced through noise perturbation.

The wavelet scalogram originally presented as **Figure 12.11** is provided also in **Appendix 12.A (Fig. A12.11)** to support the interpretation discussed above without interrupting the main narrative flow.

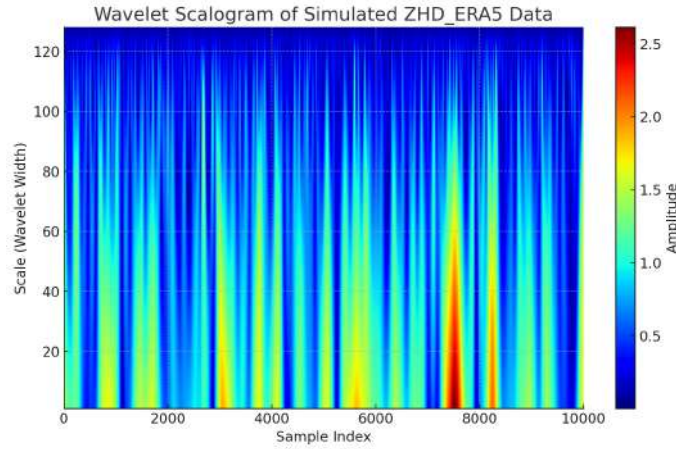


Figure 12.3. Wavelet scalogram of ERA5-derived Zenith Hydrostatic Delay (ZHD) for January 2019–2024, computed using an enhanced temporal sampling. The increased sampling density improves time–frequency resolution, enabling clearer identification of medium-scale periodic bands, high-frequency fluctuations associated with short-term atmospheric variability, and low-frequency structures linked to seasonal surface-pressure trends.

12.2.2 Scale-Domain Representation and Spectral Stability

The scale-domain wavelet representation provides a complementary view of ZHD variability with respect to the frequency-based analysis discussed in §12.2, allowing a direct interpretation of hydrostatic-delay dynamics in terms of characteristic temporal scales [12, 32]. In this framework, wavelet amplitude highlights the relative importance of oscillatory components across time scales, offering a physically intuitive description of pressure-driven variability. The distribution of energy across scales reveals a clear hierarchical structure. Persistent bands at intermediate scales (approximately 20–60) indicate structured ZHD variability maintained over multiple years, while the smoother response observed at larger scales reflects seasonal and long-term surface-pressure modulation [45, 214]. In contrast, the weak energy content at small scales confirms the limited sensitivity of the hydrostatic delay to rapid atmospheric fluctuations, consistent with its fundamentally pressure-controlled nature. The stability of this behavior is further supported by the narrow concentration of ZHD values around a centroid of 2.241 m, with most realizations confined between 2.23 and 2.25 m. This result is fully consistent with the histogram-based analysis and demonstrates that the introduction of moderate additive noise (5%) does not bias the mean hydrostatic delay [73]. The combined time–frequency representation reinforces this interpretation by showing that dominant amplitudes cluster around ZHD values close to 2.24 m at medium scales, corresponding to climatologically persistent oscillations, while low-energy regions at larger scales capture slower seasonal variability [32]. Short-lived, high-frequency structures appear only sporadically and are associated with transient pressure disturbances rather than sustained atmospheric processes. The physically consistent color normalization, which expresses wavelet coefficients directly in meters, further supports this interpretation by linking spectral features to realistic ZHD ranges. Overall, the scale-domain analysis confirms that



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the ERA5–HGPT2 framework captures hydrostatic-delay variability in a physically consistent manner under natural atmospheric perturbations [12, 45]. The wavelet scalograms originally presented as **Figures 12.12–12.13** are provided in **Appendix 12.A (Figs. A12.12–A12.13)** to support the interpretation outlined above without interrupting the main narrative flow.

12.2.3 Meaning of Spectral Scales in ZHD Variability

The spectral information extracted from the wavelet scalogram provides a physically meaningful description of ZHD variability across multiple atmospheric time scales. Within the wavelet framework, these contributions are more appropriately interpreted in terms of characteristic temporal periods rather than absolute frequencies, allowing a direct link between spectral features and the underlying physical processes [32]. Short-period components, corresponding to small wavelet scales, represent rapid but still pressure-driven ZHD fluctuations associated with transient atmospheric disturbances and localized pressure adjustments. Intermediate scales, spanning periods from several hours to a few days, capture sub-daily and synoptic variability linked to diurnal pressure cycles and mesoscale atmospheric dynamics [45, 214]. At larger wavelet scales, corresponding to periods of several weeks to months, the scalogram reflects slow modulations driven by seasonal pressure variability and large-scale circulation patterns [12]. The separation between these regimes is clearly visible in the wavelet representation. Short-lived structures dominate at small scales, while persistent power bands at intermediate and large scales highlight the intrinsically stable, pressure-controlled nature of the hydrostatic delay. Although transient perturbations may locally enhance wavelet amplitude, it is the long-period components that govern the climatological stability of ZHD and define its dominant variability envelope.

This multi-scale interpretation is synthesized in **Table 12.1** (see **Appendix 12.B, Table A12.1**), which relates wavelet scales to characteristic temporal periods and their associated physical mechanisms. Small-scale components are linked to short-term atmospheric variability, intermediate scales to synoptic and mesoscale processes, and large-scale components to diurnal, seasonal, and interannual pressure-driven trends. Overall, the spectral analysis confirms that ZHD variability is controlled primarily by low-frequency atmospheric dynamics rather than by high-frequency noise [32, 45].

12.2.4 Operational Interpretation of Short-Scale ZHD Variability

The short-scale structures identified in the ZHD wavelet scalogram correspond to pressure-driven variations occurring on sub-daily to synoptic time scales. These features reflect transient adjustments of the surface-pressure field associated with passing weather systems and mesoscale dynamics, rather than genuine high-frequency atmospheric turbulence. Unlike wet-delay variability, ZHD fluctuations do not produce rapid signal fading or tropospheric scintillation, but instead manifest as slow drifts in slant-path delay and carrier phase.

From an operational standpoint, such slow variations can affect link geometry, Doppler compensation, and long-term synchronization, particularly in high-frequency satellite communication



systems where phase stability is critical. Hydrostatic-delay variability is not mitigated through real-time adaptive techniques such as fast power control or Adaptive Coding and Modulation (ACM); its relevance lies primarily in system design and planning, including accurate delay modeling, clock correction strategies, orbit determination, and the definition of robust link margins. At the Scanzano site, the recurring short-scale ZHD patterns are consistent with regional synoptic variability and coastal pressure gradients typical of Mediterranean mid-latitude environments, emphasizing the importance of site-specific pressure climatology in refining tropospheric-delay models for navigation and communication performance assessment. The operational implications across temporal scales and the corresponding system-level mitigation strategies are summarized in **Tables 12.2 and 12.3** (see **Appendix 12.A, Tab. A12.2 and Tab. A12.3**).

12.3 In Situ Analysis and Time-Series Characterization of Tropospheric Delays over Scanzano

This section presents an in situ analysis of ZHD, ZWD, and ZTD for the Scanzano (PA) site over the February 2020–2024 period. Atmospheric data were imported from MS Excel and processed in MATLAB. Assuming a receiver altitude of 560 m above sea level, tropospheric delays were computed using the **Saastamoinen** formulation, which provides a physically consistent separation of hydrostatic and wet contributions [45, 214]. Subsequent processing included moving-average trend analyses to highlight short-term variability and seasonal behavior in the three delay components. For slant-path delay estimation, several tropospheric mapping functions were evaluated at a minimum satellite elevation angle of 15 degrees. Although the **Niell mapping function** is optimized for elevations above 20 degrees, it offers a reasonable compromise between simplicity and accuracy when extended to lower elevations, with typical errors on the order of 2–5 percent [216]. Improved performance at low elevation angles was achieved using the **GPT3 (Global Pressure and Temperature 3)** mapping formulation [364], expressed as:

$$M_e = \frac{1}{\sin(e_{rad}) + \frac{0.0029}{\tan(e) + 0.062}}, \quad (12.10)$$

which mitigates the overestimation commonly observed in the **Niell model** at shallow elevation angles. The **Ifadis** (1986) mapping function was also tested, maintaining good accuracy (error below 2 percent at 5 degrees) without requiring external atmospheric inputs [365]:

$$M_e = \frac{1}{\sin(e_{rad}) + \frac{0.0014}{\tan(e) + 0.0445}}. \quad (12.11)$$

Although the **Marini–Murray** formulation is less precise under rapidly varying atmospheric conditions, it remains suitable for long-term monitoring of tropospheric delays and climatological analyses [366]. The combined use of these mapping functions provides complementary advantages, balancing computational efficiency and accuracy, and supports both precise satellite-delay



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retrievals and climate-oriented propagation studies over the Scanzano region.

12.3.1 Trends in the Form of Time Series

The temporal behavior of tropospheric delays over the Scanzano site during the February 2020–2024 period can be effectively analyzed through the evolution of their individual components in time. Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) were computed using the Saastamoinen formulation at a receiver altitude of 560 m above sea level. The resulting time series were processed in MATLAB after removing missing data to preserve temporal continuity and ensure physical consistency of the analysis.

A clear separation of dynamical regimes emerges from the time-series representation. The wet component exhibits pronounced short-term variability, characterized by rapid oscillations superimposed on a slowly varying seasonal envelope. This behavior reflects the strong sensitivity of ZWD to atmospheric water vapor, which constitutes the dominant driver of tropospheric-delay variability at short timescales. Fluctuations in the ZWD series are directly linked to changes in local humidity conditions driven by transient meteorological processes. Despite this variability, no systematic long-term trend is observed, indicating a stable moisture regime over the analyzed period. In contrast, the hydrostatic component displays a much smoother temporal evolution. ZHD is characterized by quasi-periodic oscillations associated with surface-pressure variability, with limited sensitivity to short-lived atmospheric disturbances. The absence of long-term drift in the ZHD series confirms the inherent stability of the hydrostatic delay, which is primarily governed by synoptic-scale pressure systems rather than by local or convective processes. As expected, the total zenith delay reflects the superposition of these two behaviors, combining a stable pressure-driven baseline with moisture-induced variability. To investigate the role of satellite geometry, zenith delays were projected onto slant paths using tropospheric mapping functions. At low elevation angles (15° – 20°), the mapped hydrostatic delay preserves the temporal structure of ZHD while amplifying its magnitude, underscoring the importance of accurate mapping at shallow elevations. The mapped series retains strong seasonal coherence, with only minor high-frequency perturbations associated with passing weather systems. By contrast, the slant wet delay exhibits substantially enhanced variability, with episodic peaks reaching values on the order of 0.4–0.45 m. These peaks are linked to transient meteorological events such as frontal passages and convective activity and dominate the short-term dynamics of the total slant delay. Nevertheless, when viewed through smoothed representations, no evidence of long-term drift is detected, further supporting the climatological stability of local atmospheric moisture conditions. The time-series representations originally shown as **Figures 12.14–12.17** have been moved to **Appendix 12.A (Figs. A12.14–A12.17)**, where they provide visual support to the physically based interpretation discussed above without interrupting the main narrative flow.



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12.3.2 Time–Frequency Characterization of SWD: Diurnal, Synoptic, and Convective Scales

The temporal variability of the **Slant Wet Delay (SWD)** was further investigated through a combined frequency- and time–frequency-domain analysis, starting from the reconstructed SWD time series shown in **Fig. 12.17**. The objective of this analysis is to identify the dominant temporal scales governing moisture-driven delay variability and to relate them to the underlying atmospheric processes active along low-elevation propagation paths. In the frequency domain, the **Welch Power Spectral Density (PSD)** highlights distinct spectral peaks at approximately one and two cycles per day. These components correspond to diurnal and semi-diurnal oscillations and are physically associated with the daily modulation of atmospheric humidity driven by radiative forcing, boundary-layer evolution, and thermally induced circulation patterns. The concentration of spectral energy at low frequencies further indicates a red-noise-like behavior, typical of atmospheric water-vapor dynamics.

A complementary perspective is provided by the **Continuous Wavelet Transform (CWT)**, which allows the temporal localization of spectral features and the identification of regime-dependent variability. Three dominant bands emerge consistently in the time–frequency representation. Short timescales between roughly 2 and 12 hours are associated with convective activity and rapidly evolving moisture structures. A second band centered around 24 hours reflects the persistent diurnal cycle of humidity. Finally, variability on timescales ranging from approximately 3 to 10 days corresponds to synoptic-scale processes linked to the passage of large-scale weather systems. Although the digitization of the SWD time series introduces a limited numerical uncertainty (on the order of ± 0.02 m), the detected spectral signatures remain stable and physically consistent across the analyzed period. The wavelet scalogram shows amplitude variations typically spanning 0.15–0.45 m, with well-defined energy concentrations at convective, diurnal, and synoptic scales. At longer periods exceeding about 300 hours (approximately two weeks), broader modulations become apparent, indicative of slower, quasi-seasonal dynamics affecting background moisture conditions. It is important to note that the modulus of the CWT represents wavelet amplitude rather than absolute delay magnitude, and therefore describes how variance is distributed across temporal scales. The **Cone of Influence (COI)**, extending beyond roughly 1000 hours, marks regions where edge effects progressively reduce the reliability of the wavelet coefficients. Consistency between the time–frequency and frequency-domain perspectives is confirmed by the **Welch PSD**, which reinforces the dominance of low-frequency variability while preserving clear diurnal and semi-diurnal peaks—hallmarks of tropospheric water-vapor processes. A quantitative decomposition of variance further clarifies the relative importance of the identified regimes. Convective-scale fluctuations (timescales shorter than 6 hours) account for approximately 16.6% of the total variance. Synoptic-scale variability, spanning roughly 6–72 hours, dominates the spectrum and contributes about 53.4%. Slower components with periods longer than 72 hours represent the remaining 28.2% and are associated with longer-term atmospheric modulation. These results indicate that multi-day processes control the bulk of



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SWD variability, while high-frequency oscillations remain locally significant, particularly during convective episodes. Overall, the time–frequency characterization demonstrates that, at low elevation angles, accurate propagation modeling cannot rely on climatological averages alone. Explicit representation of both synoptic and diurnal variability of the wet component is required, not only for long-term statistical assessments but also for operational satellite-communication frameworks, where short-lived moisture-driven fluctuations can critically affect link performance. The spectral and time–frequency representations originally presented as **Figures 12.18–12.20** have been moved to **Appendix 12.A (Figs. A12.18–A12.20)**, where they support the physical interpretation outlined above without interrupting the main narrative flow.

12.3.3 Statistical Analysis of In Situ Tropospheric Delay Time Series over Scanzano (February 2020–2024)

The in situ time series collected at the Scanzano site (560 m a.s.l.) over the February 2020–2024 period allow a compact yet physically informative statistical characterization of the hydrostatic, wet, and total components of the tropospheric delay. By combining temporal evolution and distributional analysis within a unified framework, the behavior of ZHD, ZWD, and ZTD can be examined across multiple complementary perspectives relevant to GNSS propagation under neutral-atmosphere conditions.

From a temporal standpoint, the multi-year series clearly separate stable and highly variable regimes. The hydrostatic delay evolves smoothly, reflecting the dominant control of surface pressure and its limited sensitivity to short-term meteorological fluctuations. In contrast, the wet component exhibits pronounced variability, with rapid excursions superimposed on slower modulations, consistent with the intermittent nature of atmospheric water vapor. The total delay integrates these behaviors, preserving the long-term stability of the hydrostatic contribution while embedding the short-term variability introduced by moisture fluctuations. A statistical view of the same datasets reinforces this interpretation. The empirical distributions highlight the markedly different dispersion associated with the hydrostatic and wet components and their combined effect on the total delay. While ZHD remains narrowly distributed, ZWD displays a broader and asymmetric structure, which directly governs the spread observed in ZTD. The comparison with fitted log-normal probability density functions further confirms that parametric models can capture the main statistical features of the observed variability, particularly for the wet and total components, while also revealing departures linked to episodic events. Together, these complementary representations provide a coherent statistical picture in which temporal persistence, variability amplitude, and probabilistic structure are jointly interpreted. This approach avoids reliance on any single diagnostic and instead emphasizes the physical consistency of the observed delay components across time and statistical domains. The corresponding visual representations are provided in **Figures 12.21–12.22** (see **Appendix 12.A, Figs. A12.21–A12.22**) to support and illustrate the analysis described above.



12.4 Additional Physical Insights from the In Situ Analysis

To improve physical coherence and interpretability, the in situ analysis was reorganized around a set of comparative representations designed to summarize the temporal behavior of tropospheric delays across both zenith and slant geometries. Zenith delays were computed using the Saastamoinen formulation, which remains a standard reference for GNSS tropospheric correction under neutral-atmosphere assumptions [45, 214]. Projection to slant geometry was performed at a fixed elevation angle of 15° using the Niell Mapping Function, adopted in a conservative configuration to avoid systematic underestimation of low-elevation propagation effects [216, 225]. When examined in zenith geometry, the hydrostatic component exhibits limited variability and a well-defined temporal structure. ZHD values remain confined to the 2.12–2.22 m range, with fluctuations primarily driven by synoptic-scale pressure variability. The close correspondence between the raw signal and its smoothed counterpart confirms the quasi-stable nature of the hydrostatic delay, whose evolution is largely controlled by atmospheric mass redistribution rather than by short-lived meteorological processes [45, 214, 218].

A markedly different behavior characterizes the wet component. ZWD spans a broader range, from approximately 0.02 to 0.15 m, and displays pronounced short-term variability associated with rapid changes in atmospheric humidity. These fluctuations reflect the intermittent nature of water vapor, which is strongly influenced by mesoscale dynamics, frontal passages, and moisture advection and therefore remains difficult to model deterministically [46, 217]. At multi-day timescales, the smoothed series reveals oscillations consistent with typical Mediterranean winter dynamics, where alternating dry and moist air masses modulate the local humidity field [259, 268]. The total zenith delay integrates these two regimes. While ZHD establishes the baseline magnitude of the delay, ZWD introduces rapid, high-frequency variations that dominate short-term behavior. ZTD therefore spans approximately 2.20–2.35 m and embodies the coexistence of a slowly varying pressure-controlled component and a highly dynamic moisture-driven contribution. This structure is fully consistent with long-term GNSS tropospheric delay characterizations reported in the literature [73, 230] and highlights the presence of distinct temporal scales in tropospheric propagation effects [65, 73]. Projection to slant geometry accentuates these contrasts. The hydrostatic slant-path delay remains comparatively stable, exhibiting only slow synoptic modulation, whereas the wet slant-path delay undergoes strong geometric amplification at low elevation angles. As a result, SPWD displays enhanced variability and more pronounced high-frequency excursions relative to its zenith counterpart, while the total slant delay is largely controlled by the wet contribution superimposed on a quasi-stable hydrostatic baseline. This behavior underscores the dominant role of moisture variability in determining uncertainty along low-elevation Earth–space paths. Beyond individual time series, a consolidated temporal–statistical view further clarifies the physical picture. Comparisons between zenith and slant geometries show that hydrostatic delays remain weakly dispersive in both configurations, whereas wet delays develop broader, right-skewed distributions that are further stretched by mapping effects at low elevation angles. Consequently, total delays



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exhibit wide, often multi-peaked distributions, confirming that wet-delay variability governs both phase noise and path-length uncertainty in GNSS positioning and high-frequency satellite communication systems. A unified temporal-probabilistic representation, combining daily time series with their associated probability density functions, reinforces this interpretation. In this framework, hydrostatic delays contribute negligibly to the overall uncertainty budget, while wet delays dominate both the amplitude and the intermittency of tropospheric delay fluctuations. Accurate identification and characterization of short-lived ZWD peaks therefore remain critical for improving GNSS positioning performance and for refining regional water-vapor estimates, with direct implications for weather predictability and hydrological-cycle assessment [65, 73, 257].

The associated figures **Figs. 12.23–12.33** (see **Appendix 12.A, Figs. A12.23–A12.33**) are provided to illustrate these behaviors across zenith and slant geometries, different temporal aggregations, and complementary statistical views. Minor gaps in the in situ record result in incomplete coverage for some February periods; these gaps are explicitly accounted for in the visualization strategy, ensuring that all comparisons consistently refer to the same seasonal window despite differences in data availability. **Figure 12.25** is included for illustrative purposes and does not introduce additional quantitative information beyond the discussion presented in the text.

12.5 ERA5 Slant-Path Tropospheric Delay (February 2019–2024)

This section presents an ERA5-based analysis of tropospheric delays for the February 2019–2024 period, exploiting the extended reanalysis record to investigate both temporal evolution and inter-annual variability under consistent seasonal conditions [259]. Particular attention is devoted to slant-path delays evaluated at a minimum elevation angle of 15° , where tropospheric effects are strongly amplified and become especially relevant for Earth–space links. To account for local spatial representativeness, the analysis is performed on the four ERA5 grid points closest to the Scanzano site, labeled (1,1), (1,2), (2,1), and (2,2), covering an area of approximately 56 km². This configuration enables a joint assessment of temporal variability and small-scale spatial consistency within the selected domain [73]. Hourly time series and corresponding statistical distributions are derived for each grid point, allowing robust characterization of both mean behavior and variability. Slant-path delays were initially estimated using the simple secant-law approximation,

$$M_e = \frac{1}{\sin(e)}, \quad (12.12)$$

where e denotes the elevation angle. At low elevations ($e = 15^\circ$), this formulation yields mapping factors of approximately 3.86, which are known to overestimate path elongation relative to physically based mapping functions. More advanced formulations, such as the Niell Mapping Function (NMF) or the Vienna Mapping Function (VMF3), typically produce values in the range

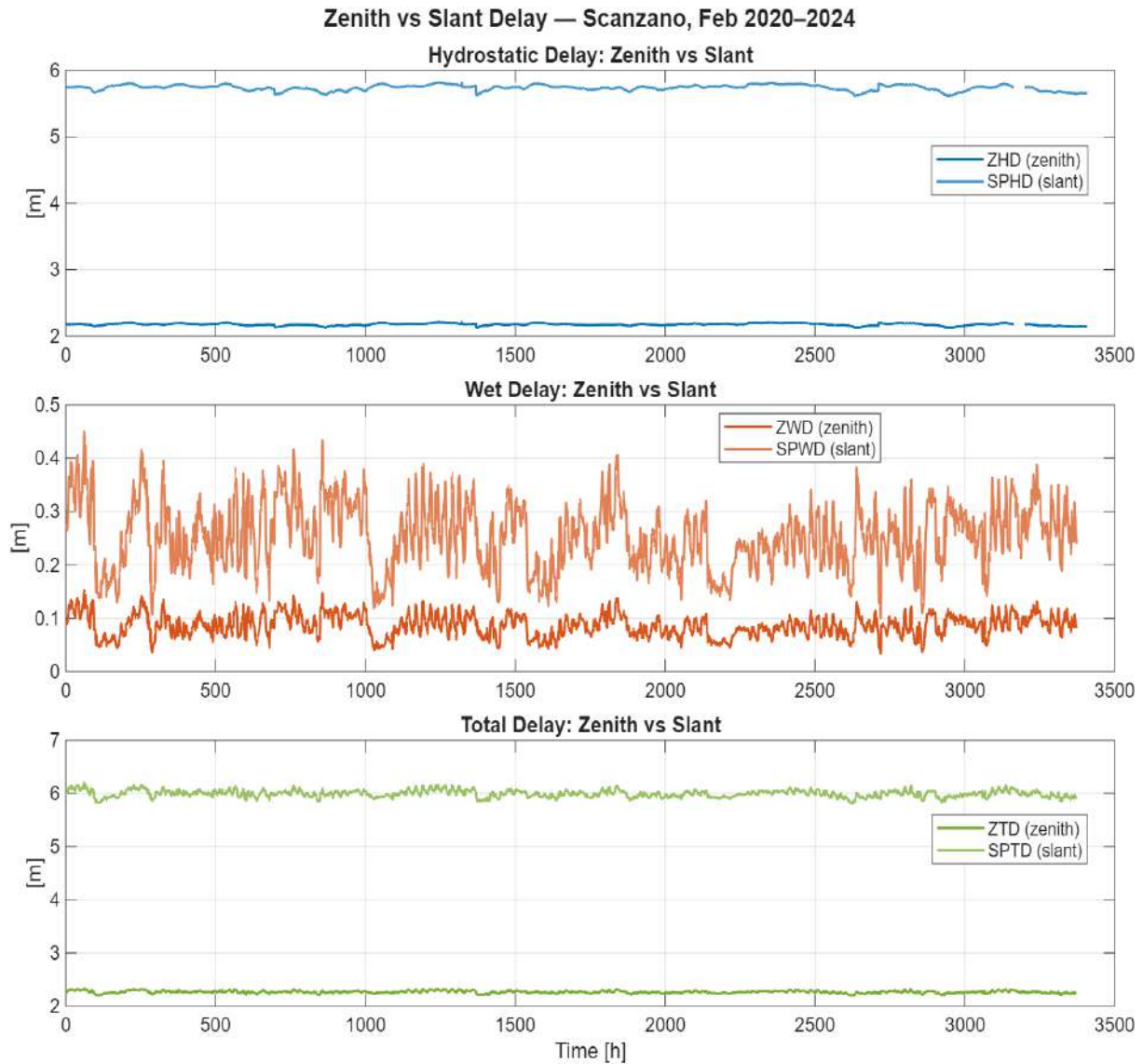


Figure 12.25. Comparative time series of Zenith and Slant Delays at Scanzano (560 [m], February 2020–2024). Comparison between hydrostatic, wet, and total tropospheric delays in zenith and slant geometries. The hydrostatic delay shows stable large-scale behavior, with SPHD amplified by the elevation-dependent mapping factor relative to ZHD. The wet component exhibits strong short-term variability ($\text{SPWD} \approx 0.4\text{--}0.5$ [m]), exceeding ZWD due to water-vapor inhomogeneities. The total delay ranges from $\sim 2.2\text{--}2.3$ [m] (ZTD) to $\sim 5.8\text{--}6.3$ [m] (SPTD), confirming that wet delays dominate atmospheric-induced variability.



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2.8–3.2 under comparable conditions [216, 225, 230]. A physically consistent representation is obtained using the NMF,

$$M_e = \frac{1}{\sin(e) + \frac{a}{\sin(e) + \frac{b}{\sin(e) + c}}}, \quad (12.13)$$

where the coefficients a , b , and c depend on site latitude and altitude [216,225]. At low elevation angles, the troposphere cannot be treated as a homogeneous inclined layer: ray bending due to atmospheric refraction and vertical gradients of pressure, temperature, and humidity become non-negligible. While the secant law neglects these effects, NMF- and VMF-based approaches explicitly account for them, yielding a more realistic propagation path [225, 230]. On the basis of intercomparisons between secant-law and NMF/VMF3 estimates, an empirical correction factor of 1.29 was therefore applied a posteriori, such that dividing the raw secant-based delays by this factor produces values consistent with physically based mapping functions. Within this framework, the temporal evolution of tropospheric delays reveals both diurnal modulation and inter-annual variability, with the largest excursions occurring in the slant-path total delay. These variations are primarily driven by changes in lower-tropospheric humidity, which strongly modulate the wet component along low-elevation propagation paths. The ERA5-derived results consistently confirm the dominant role of wet delays in governing slant-path delay variability and associated uncertainties in low-elevation Earth–space links [73, 216, 259]. The statistical distributions of the delay components further clarify this behavior. The zenith hydrostatic delay exhibits a narrow, bimodal distribution, reflecting stable dry-air conditions associated with distinct pressure regimes. In contrast, the wet delay displays a positively skewed distribution with a pronounced high-value tail, indicative of episodic moisture surges. As a consequence, the zenith total delay combines the bimodality of the hydrostatic component with increased dispersion driven by wet-delay variability. In slant geometry, the hydrostatic component preserves its bimodal structure with slightly enhanced spread due to geometric amplification, whereas the slant-path wet delay shows a markedly broader, right-skewed distribution. The resulting slant-path total delay therefore spans a wide, multi-peaked range, highlighting the dominant influence of wet-delay variability on phase noise and path-length errors in GNSS and high-frequency satellite communication systems. The corresponding time series and statistical distributions are illustrated in **Figures 12.34–12.35** (see **Appendix 12.A, Figs. A12.34–A12.35**). All results are derived from ERA5 meteorological fields stored in NetCDF format and processed using dedicated MATLAB routines developed for this study. To ensure seasonal consistency, only February datasets from each year were considered, and both zenith and slant-path delay components were computed using the Niell Mapping Function to accurately represent elevation-dependent atmospheric effects.

12.5.1 Statistical Analysis

The statistical properties of tropospheric delays reveal clearly differentiated behaviors among the hydrostatic, wet, and total components when examined in both zenith and slant-path geometries. These differences can be directly interpreted in terms of the underlying physical



mechanisms that control each contribution, rather than as purely numerical artifacts. The hydrostatic components (ZHD and SPHD) display distributions that are close to symmetric, with low skewness (approximately 0.10–0.73) and consistently negative kurtosis (between -0.58 and -1.08). Such platykurtic behavior indicates relatively flat distributions with limited influence from extreme values. This statistical structure reflects the dominant role of surface pressure, which evolves smoothly in time and exhibits a clear seasonal modulation associated with summer–winter contrasts. As a result, variability in the hydrostatic delay remains well confined, with only a marginal contribution from episodic events. A different regime characterizes the wet components (ZWD and SPWD). Their distributions are distinctly right-skewed, with skewness values on the order of 0.73–0.85, indicating the presence of intermittent but significant positive excursions. These tails are physically associated with episodic moisture enhancements driven by convective activity, synoptic-scale advection of humid air masses, and boundary-layer processes. The effect is particularly pronounced for the slant-path wet delay, where SPWD exhibits positive kurtosis (approximately $+0.69$), signaling heavy-tailed behavior. This feature arises from the geometric amplification of wet-delay extremes at low elevation angles, where mapping functions enhance the contribution of localized moisture anomalies.

The total delay components (ZTD and SPTD) naturally combine these characteristics. While their distributions retain the asymmetry introduced by the wet component, they are also characterized by negative kurtosis (ranging from -1.46 to -1.05), resulting in broad and relatively flat distributions. In this case, variability is spread over a wide range of values rather than concentrated around a narrow peak, underscoring the dominant influence of wet-delay fluctuations in shaping the overall statistical behavior of tropospheric delays. The numerical values supporting this interpretation are summarized in **Table 12.2** (see **Appendix 12.B, Table A12.2**), which provides a compact quantitative reference for the statistical descriptors discussed above.

12.6 Ground Terminal in Scanzano for W-Band Communications

The design of a satellite–ground link operating in the W-band (≈ 94 GHz) at the Scanzano site requires a careful characterization of atmospheric propagation effects together with system-level performance constraints [65, 73, 216, 259]. At millimeter wavelengths, atmospheric impairments dominate the link budget, with gaseous absorption, cloud attenuation, rain-induced fading, and tropospheric scintillation all contributing significantly to signal degradation. Under clear-sky conditions, atmospheric losses typically amount to a few decibels (on the order of 2–5 dB), while intense precipitation events can introduce fades well in excess of 20 dB, severely limiting link availability [217, 230]. At these frequencies, system noise temperature represents the primary performance bottleneck. The combined contribution of atmospheric emission and receiver noise leads to effective noise temperatures typically ranging between 600 and 1000 K [73, 230]. Even when high-gain antenna configurations are adopted—such as a 12 m **Cassegrain** ground antenna providing approximately 88 dBi of gain and a 0.6 m onboard reflector with about 62 dBi—the



achievable carrier-to-noise density ratio remains strongly constrained by propagation conditions rather than by antenna performance alone. In nominal clear-sky scenarios, the received power is of the order of -90 dBW, corresponding to C/N_0 values around 105–110 dBHz. However, the onset of cloudiness and precipitation rapidly degrades link margins, reducing both signal quality and availability [65, 216]. As a result, W-band links are intrinsically intermittent and highly sensitive to short-term atmospheric variability.

To meet stringent annual availability targets in the range of 99.0–99.9 %, W-band systems must therefore rely on a combination of mitigation strategies. These include **Adaptive Coding and Modulation** (ACM), physically consistent tropospheric modeling, and the integration of co-located radiometric or meteorological monitoring systems [73, 217]. Together, these measures allow the exploitation of favorable atmospheric windows and enable reliable high-data-rate transmission despite the strong and rapidly varying propagation impairments characteristic of millimeter-wave satellite links.

12.6.1 Data Acquisition and Preprocessing for the Scanzano Site (ERA5)

ERA5 reanalysis data offer a coherent and high-resolution representation of the atmospheric conditions over the Scanzano site, providing a suitable basis for tropospheric propagation studies [259]. In this work, ERA5 fields of temperature, pressure, humidity, and precipitation were retrieved and processed to derive tropospheric delays, attenuation-related quantities, and system noise-temperature estimates relevant to W-band satellite communication scenarios [65, 73]. The processing chain was designed to preserve both temporal continuity and physical consistency. Atmospheric variables were acquired from NetCDF archives and subjected to temporal resampling and quality-control checks before being assembled into hourly time series and vertical profiles. When specific parameters were not directly available, they were reconstructed using standard empirical relationships. In particular, relative humidity was derived from specific humidity or dew-point temperature following established formulations, ensuring internal consistency among thermodynamic variables [45, 214].

Local site characteristics were explicitly accounted for in the propagation modeling. The geographic latitude (37.91°), station altitude (560 m), and a representative elevation angle of 15° were used to compute the Niell Mapping Function, allowing the conversion of zenith hydrostatic and wet delays (ZHD, ZWD) into their corresponding slant-path components (SPHD, SPWD) [216, 225]. This geometry-dependent formulation is essential for realistically capturing tropospheric effects along low-elevation paths, where atmospheric inhomogeneities and path elongation significantly enhance propagation impairments.

The resulting preprocessing framework provides a physically grounded foundation for the subsequent analyses, including W-band link-budget simulations and performance assessments. Detailed numerical aspects of the implementation—covering MATLAB routines, database handling procedures, and extended statistical tables—are documented separately in Appendix P to avoid interrupting the main methodological flow.



12.7 Final Comparative Analysis of Tropospheric Delays from In Situ Data, ERA5, and ITU-R

This section offers a consolidated interpretation of tropospheric delays derived from in situ observations, ERA5 reanalysis, and ITU-R reference models, with the primary focus placed on graphical evidence rather than on standalone numerical metrics [65, 73, 216, 259]. The intent is not to introduce new computations, but to extract physical insight from the visual comparison of ZHD, ZWD, and SPD across multiple temporal scales and heterogeneous data sources. Through this perspective, the visualizations serve as a diagnostic tool to reveal how tropospheric delays respond to changing atmospheric conditions, highlighting both common patterns and systematic differences among observational records, reanalysis products, and climatological reference models [73, 230]. This approach facilitates an intuitive cross-comparison that would be less transparent if limited to tabulated statistics alone.

At the same time, the analysis is anchored to a quantitative foundation by explicitly revisiting the official ITU-R formulations for the hydrostatic and wet delay components [216, 217]. These expressions provide a physically consistent reference framework and allow direct numerical implementation, ensuring that the graphical interpretation remains grounded in reproducible modeling practice. By combining visual diagnostics with formal ITU-R formulations, the section bridges qualitative interpretation and quantitative rigor, thereby supporting the extension of the methodology toward global-scale tropospheric-delay characterization and comparative assessment [65, 259].

12.7.1 Realistic Hourly Variability from ERA5 (Not Monthly Means)

To resolve the true temporal dynamics of tropospheric delays at hourly scale, it is necessary to exploit the native temporal resolution of the ERA5 dataset rather than relying on monthly aggregates [259]. While NetCDF exports simplify data handling, they may discard or obscure key temporal metadata originally encoded in the GRIB format, complicating the reconstruction of a physically consistent time axis. To address this limitation, a dedicated reconstruction procedure was developed to recover the correct hourly chronology directly from native ERA5 fields and to extend the dataset through February 2025. This approach preserves temporal continuity and ensures full physical consistency of the atmospheric variables used in the delay computation [259]. When analyzed at hourly resolution, the ERA5 time series over Scanzano for the February months spanning 2019–2025 reveal a persistent and physically coherent hierarchy in tropospheric delay variability. The hydrostatic component remains effectively quasi-stationary, with values confined within approximately 2.21–2.27 m and only weak diurnal or synoptic-scale modulation. This behavior reflects its dominant dependence on surface pressure and its limited sensitivity to short-term atmospheric variability, resulting in negligible inter-annual differences [45, 214, 216].

By contrast, the wet component exhibits substantially richer temporal dynamics. ZWD displays pronounced sub-daily fluctuations, typically spanning 0.05–0.15 m, together with marked



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year-to-year contrasts. Periods of enhanced variability are particularly evident during the more humid conditions observed between 2020 and 2022, as well as during late-winter moisture episodes in 2024–2025. These features are consistent with the strong sensitivity of the wet delay to transient water-vapor loading, boundary-layer processes, and moisture transport in the lower troposphere [65, 73, 217].

The total zenith delay integrates these two regimes. While it preserves the pressure-controlled stability of the hydrostatic baseline, it remains responsive to short-term humidity pulses that introduce millimetric inter-annual modulations. As a result, ZTD appears smoother than ZWD alone but still captures meaningful temporal contrasts that are directly relevant for both propagation and geodetic applications [73, 230]. From a statistical perspective, this hierarchy confirms ZHD as a stable reference, ZWD as the dominant source of stochastic variability, and ZTD as an integrated descriptor combining robustness and sensitivity. From an operational standpoint, these results have clear implications. The hydrostatic delay can be reliably estimated from surface pressure with sub-millimetric uncertainty, whereas the wet delay remains the primary contributor to tropospheric-delay uncertainty [216, 230]. Consequently, despite its comparatively smooth behavior, ZTD still requires real-time correction or reanalysis-based assimilation, particularly for Ka- and W-band propagation scenarios where phase stability and signal availability are highly sensitive to tropospheric variability [73, 217]. In this context, the Scanzano dataset provides a representative benchmark for mid-latitude winter conditions, confirming the long-term stability of the hydrostatic baseline while underscoring the need for advanced statistical or machine-learning surrogates to model the highly variable wet component in high-frequency satellite communication and GNSS applications [65, 259]. The corresponding hourly time series and derived statistics are illustrated in **Figures 12.36–12.42** (see **Appendix 12.A, Figs. A12.36–A12.42**), while **Table 12.3** summarizes the monthly statistics of ZHD, ZWD, and ZTD derived from these datasets.

Table 12.3. Scanzano (PA), February 2019–2025. Visual ranges (m) for ZHD, ZWD, ZTD.

Year	ZHD	ZWD	ZTD
2019	2.21–2.26	0.05–0.13	2.27–2.36
2020	2.21–2.27	0.05–0.15	2.26–2.40
2021	2.20–2.27	0.05–0.15	2.30–2.40
2022	2.19–2.27	0.06–0.14	2.28–2.38
2023	2.19–2.27	0.05–0.13	2.28–2.38
2024	2.19–2.27	0.08–0.13	2.28–2.39
2025	2.21–2.27	0.09–0.14	2.30–2.38



12.7.2 Multi-Year Temporal Dynamics of Tropospheric Delays over Scanzano: ERA5 Hourly Analysis (2019–2025)

The ERA5 hourly time series for February over the 2019–2025 period provide a coherent picture of the mechanisms governing tropospheric delay variability at the Scanzano site. When examined across consecutive years, the three delay components reveal clearly differentiated behaviors that can be directly traced back to their physical drivers. The hydrostatic contribution is characterized by a smooth temporal evolution, dominated by synoptic-scale pressure variations and exhibiting amplitudes of only a few millimetres. This limited modulation reflects the intrinsic stability of surface pressure fields and confirms the quasi-deterministic nature of the dry-air component of the tropospheric delay [45, 216]. In contrast, the wet delay shows a markedly more irregular structure. Short-term fluctuations, including sharp peaks and rapid decreases, are frequently observed and can be associated with episodic humidity intrusions, boundary-layer processes, and moisture transport within the lower troposphere [65, 73]. The total zenith delay naturally combines these two regimes. While the hydrostatic baseline provides a stable reference, the stochastic variability introduced by the wet component leads to measurable inter-annual differences that can reach the centimetre scale. Periods characterized by enhanced moisture activity—most notably during 2020–2022 and again in 2024–2025—stand out as phases in which wet-delay anomalies dominate the overall delay variability. These conditions introduce additional uncertainty for both GNSS positioning and high-frequency satellite communication links, where tropospheric effects directly affect signal propagation and phase stability [216, 230]. From an operational standpoint, this behavior underscores the limitations of purely climatological or purely deterministic approaches. Reliable tropospheric correction requires hybrid modeling strategies that combine pressure-based, quasi-deterministic baselines with dynamic, moisture-sensitive components. Such an approach is particularly critical for Ka- and W-band systems, where short-term tropospheric variability can significantly impact link availability and performance margins [73, 217]. The associated **Figs. 12.43–12.49** (see **Appendix 12.A, Figs. A12.43–A12.49**) are provided to illustrate these contrasting temporal regimes and to support the physically based interpretation outlined above.

12.7.3 Inter-Annual Variability of Tropospheric Delays over Scanzano (2019–2025)

To complement the time-series and histogram analyses, a consolidated statistical view of tropospheric delays over Scanzano (PA) for the month of February during the 2019–2025 period is provided in **Table 12.4**, derived from the ERA5 datasets discussed in **Figures 12.43–12.49** (see **Appendix 12.A, Figs. A12.43–A12.49**) [259]. Rather than introducing new diagnostics, the table serves to quantify in a compact form the inter-annual behavior of the Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) through their mean values and variability ranges.

The hydrostatic component exhibits a high degree of stability throughout the entire period. Mean ZHD values remain tightly clustered between 2.23 and 2.25 m, while the full range of



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variability is confined to approximately 2.19–2.27 m. This limited dispersion is consistent with the dominant control exerted by surface pressure and with the weak sensitivity of the hydrostatic term to short-term atmospheric fluctuations [45, 216]. A markedly different behavior characterizes the wet component. ZWD displays substantially broader variability, with mean values spanning roughly 0.09–0.11 m and instantaneous values extending from about 0.05 up to 0.15 m. Such dispersion reflects the strong dependence of the wet delay on atmospheric humidity, synoptic-scale moisture transport, and episodic humid intrusions, which introduce variability across both intra-seasonal and inter-annual timescales [65, 73]. The total zenith delay naturally integrates these two contributions. ZTD mean values lie between 2.32 and 2.35 m, while overall ranges extend from approximately 2.27 to 2.40 m, combining the pressure-driven stability of the hydrostatic baseline with the moisture-induced variability of the wet component [216, 230].

When the seven February datasets are considered jointly, a coherent inter-annual structure becomes evident. February 2019 emerges as the driest reference year, characterized by relatively low ZWD values (around 0.09 m) and the narrowest variability range. The subsequent period from 2020 to 2022 corresponds to enhanced wet conditions, during which ZWD ranges broaden to roughly 0.05–0.15 m and ZTD maxima approach 2.40 m, indicative of persistently humid atmospheric regimes. In contrast, the 2023–2025 interval shows partial stabilization, with slightly reduced ZWD means (0.10–0.11 m), narrower variability ranges (approximately 0.06–0.13 m), and more consistent ZTD values clustering near 2.34–2.35 m. This inter-annual comparison confirms the near-invariant nature of the hydrostatic delay and identifies moisture-driven anomalies as the primary source of variability in the total tropospheric delay. These results underscore the central role of atmospheric water vapor in modulating tropospheric delays at seasonal and inter-annual scales, even when the hydrostatic contribution remains essentially stable [65, 259].

Table 12.4. Scanzano (PA), February 2019–2025. Visual estimates of mean and range for ZHD, ZWD, and ZTD (units: m) extracted from time-series and histograms. Values are approximate; for exact statistics use the original ERA5 datasets.

Year	ZHD mean	ZHD range	ZWD mean	ZWD range	ZTD mean	ZTD range
2019	~2.23	2.21–2.26	~0.09	0.05–0.13	~2.32	2.27–2.36
2020	~2.24	2.21–2.27	~0.10	0.05–0.15	~2.34	2.26–2.39
2021	~2.24	2.20–2.27	~0.11	0.05–0.15	~2.35	2.30–2.40
2022	~2.24	2.19–2.27	~0.10	0.06–0.14	~2.33	2.28–2.38
2023	~2.25	2.19–2.27	~0.10	0.05–0.13	~2.34	2.29–2.37
2024	~2.23	2.19–2.27	~0.11	0.08–0.13	~2.34	2.28–2.39
2025	~2.24	2.21–2.27	~0.11	0.09–0.14	~2.35	2.30–2.38

12.7.4 Reliability and Uncertainty of ERA5-Derived Tropospheric Delays

The statistical reliability of ERA5-derived tropospheric delays over Scanzano (PA) for the February 2019–2025 period can be quantitatively assessed through dispersion metrics, standard errors, and confidence intervals [259]. Owing to its strong dependence on surface pressure, the



hydrostatic delay exhibits extremely limited variability. Typical ZHD values range between 2.23 and 2.26 m, with hourly standard deviations of the order of 1 mm. When aggregated over approximately 672 hourly samples per month, this translates into a standard error of the monthly mean below 0.05 mm and a corresponding 95% confidence interval of about ± 0.1 mm. As a result, inter-annual differences exceeding 1 mm can be considered statistically robust and attributable to genuine atmospheric variability rather than sampling noise [45, 216].

The wet component displays a markedly different statistical behavior. With mean ZWD values between 0.08 and 0.11 m, its hourly variability is substantially larger, with standard deviations typically in the 2–3 mm range. Despite this enhanced short-term variability, monthly averages remain well constrained, with a standard error of approximately 0.1 mm and a 95% confidence interval close to ± 0.2 mm. Consequently, inter-annual shifts of 2–3 mm—and even larger during particularly humid years—clearly exceed statistical uncertainty and reflect physically meaningful changes in atmospheric moisture conditions [65, 73]. Uncertainty in the total zenith delay largely mirrors that of the hydrostatic component, with only a modest modulation introduced by moisture variability. Monthly standard errors remain close to 0.05 mm, yielding a 95% confidence interval of ± 0.1 mm. This confirms that observed ZTD variations of several millimetres are statistically significant [216, 230]. Although ERA5 may exhibit small systematic offsets with respect to in situ GNSS or radiometric observations—typically on the order of 1–3 mm in ZTD—the magnitude of the inter-annual differences remains well above noise levels and is therefore physically meaningful [259]. The statistical properties of ZHD, ZWD, and ZTD for Scanzano (PA) during February 2019–2025 are summarized in **Table 12.5** (see **Appendix 12.B, Table A12.5**). The table highlights a clear dual-scale structure: a highly stable hydrostatic contribution, reflected in narrow confidence intervals, and a more variable wet component driven by transient water-vapor dynamics. The resulting total delay acts as a robust operational descriptor, while still retaining sensitivity to inter-annual moisture variability [45, 65, 216].

When the seven February datasets are examined in sequence, a coherent inter-annual pattern emerges. February 2019 stands out as the driest reference year, characterized by relatively low ZWD values (around 0.09 m) and the narrowest variability range, indicative of limited atmospheric moisture fluctuations. The period from 2020 to 2022 marks a phase of enhanced wet conditions, during which ZWD ranges broaden to approximately 0.05–0.15 m and ZTD maxima approach 2.40 m. These features are consistent with sustained humid conditions and increased synoptic-scale moisture transport [65, 259]. In contrast, the years 2023–2025 exhibit partial stabilization. Mean ZWD values remain slightly elevated (around 0.10–0.11 m) but are associated with tighter variability ranges, typically between 0.06 and 0.13 m, resulting in more consistent ZTD values near 2.34–2.35 m. Taken together, this inter-annual comparison confirms the near-invariant character of the hydrostatic delay and identifies moisture-driven anomalies as the dominant source of ZTD variability. In this context, 2020–2022 correspond to relatively humid extremes, while 2019 and the 2023–2025 period align more closely with climatological mean conditions [216, 230].



Key highlights for Scanzano (February 2019–2025)

- **2019:** driest year; ZWD ≈ 0.09 m, narrowest range (0.05–0.13 m), ZTD ≤ 2.36 m
- **2020:** marked moistening; ZWD ≈ 0.10 m (up to 0.15 m), ZTD peaks near 2.39 m
- **2021:** maximum variability; ZWD ≈ 0.11 m, ZTD spans 2.30–2.40 m
- **2022:** sustained humidity; ZWD ≈ 0.10 m, ZTD between 2.33–2.38 m
- **2023:** stabilization; ZWD ≈ 0.10 m, range 0.06–0.12 m, ZTD ≈ 2.34 m
- **2024:** renewed fluctuations; ZWD ≈ 0.11 m, range 0.08–0.13 m, ZTD ≈ 2.34 m
- **2025:** transitional year; ZWD ≈ 0.11 m, ranges up to 0.14 m, ZTD ≈ 2.35 m

12.8 ERA5 Overall View over Scanzano (February 2019–2025)

When examined over the February 2019–2025 period, the hydrostatic component of the tropospheric delay over Scanzano shows a highly stable behavior, characterized by weak diurnal modulation and negligible inter-annual variability. This stability directly reflects the dominant role of surface pressure, which evolves smoothly in time and is only marginally influenced by short-term meteorological processes. As a result, ZHD defines a narrow and well-confined variability range, with a unimodal distribution centered at approximately 2.24 m and only minor day–night oscillations.

Hydrostatic Component (ZHD)

Over the February 2019–2025 interval, the hydrostatic contribution to the tropospheric delay at Scanzano remains remarkably constant. Day–night variations are present but weak, and no appreciable inter-annual drift emerges from the analysis. Such behavior is fully consistent with the physical control exerted by surface pressure, which evolves gradually and is largely insensitive to short-lived meteorological disturbances. Consequently, ZHD occupies a tightly bounded range, forming a narrow unimodal distribution centered near 2.24 m, with only modest diurnal excursions. This quasi-stationary regime is visually apparent in the corresponding representation, where both the compactness of the distribution and the faint yet repeatable daily modulation can be observed in **Fig. 12.50** (see **Appendix 12.A, Fig. A12.50**).

Wet Component (ZWD)

A markedly different regime emerges for the wet component, whose variability is primarily controlled by atmospheric water vapor. Unlike the hydrostatic delay, ZWD exhibits pronounced diurnal structure and significant inter-annual variability, reflecting the combined effects of moisture transport, boundary-layer processes, and convective activity. Typical daily patterns include morning minima followed by well-defined afternoon maxima, consistent with the daytime development of humidity and temperature-driven processes. This enhanced variability translates into a substantially broader distribution than that observed for ZHD, with characteristic values



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clustering around 0.09–0.10 m. These features are illustrated in the corresponding **Fig.12.51** (see **Appendix 12.A, Fig. A12.51**), which highlights both the amplitude of the diurnal cycle and the increased dispersion of the wet delay.

Total Zenith Delay (ZTD)

The total zenith delay naturally combines the properties of its hydrostatic and wet components. While it retains the long-term stability imposed by the pressure-driven hydrostatic baseline, it remains responsive to moisture-induced variability through the wet contribution. The resulting diurnal cycle is therefore smoother than that of ZWD alone, yet clearly more structured than the nearly flat behavior observed for ZHD. The distribution of ZTD peaks near 2.35 m and exhibits an intermediate dispersion, reflecting its role as an integrated descriptor of tropospheric delay conditions. This balance between stability and variability makes ZTD particularly suitable for both long-term climatological analyses and the assessment of short-term atmospheric fluctuations. The corresponding behavior is illustrated in **Fig.12.52** (see **Appendix 12.A, Fig. A12.52**).

12.9 Vertical Variability of Tropospheric Delays over Scanzano (2019–2025): ERA5 Multi-Level Analysis

Using the complete set of 190 ERA5 vertical pressure levels and hourly time series for February 2019–2025, the analysis over Scanzano highlights a pronounced contrast in the inter-annual behavior of the tropospheric delay components, consistent with their distinct physical drivers. When viewed through a vertical-envelope representation, the hydrostatic and wet contributions clearly separate into two distinct dynamical regimes. The hydrostatic delay remains remarkably stable from year to year, with only a limited spread around a well-defined central range. This persistence is consistent with its strong dependence on surface pressure, which evolves smoothly on seasonal and inter-annual timescales. The wet delay, on the other hand, exhibits substantially larger variability. Its envelope broadens significantly due to short-term and inter-annual fluctuations in atmospheric humidity, which introduce pronounced dispersion even under comparable seasonal conditions. The total delay reflects the superposition of these behaviors. While the pressure-driven stability of the hydrostatic component is preserved, it is modulated by the variability associated with the wet contribution. As a result, the total delay provides an integrated view in which long-term stability and moisture-driven variability coexist in a physically interpretable manner. Rather than emphasizing individual hourly oscillations, this representation is intended to capture the overall magnitude, coherence, and persistence of the delay components across multiple years. In this sense, it offers a compact diagnostic perspective on the tropospheric delay response over the site, highlighting recurring patterns and regime differences under similar seasonal forcing. The corresponding figures (see **Appendix 12.A, Figs. A12.53–A12.55**) visually support this interpretation and illustrate the contrasting variability envelopes associated with the hydrostatic, wet, and total delay components.



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12.10 Statistical Analysis of ERA5 Tropospheric Delays over Scanzano (February 2019–2025)

The statistical analysis is designed to investigate the inter-annual variability and long-term trends of tropospheric delays derived from ERA5 reanalysis over the **Telespazio Space Centre of Scanzano**, with particular emphasis on the spatial representativeness of the ERA5 grid and on the physical consistency of trend detection procedures [259]. Given the finite spatial resolution of ERA5, assessing the robustness of the results with respect to the exact grid-point selection is a necessary prerequisite for any local-scale interpretation.

The reference location corresponds to the operational platform within the Scanzano facility ($37^{\circ}54'47''$ N, $13^{\circ}21'39''$ E, 551 m a.s.l.), which differs slightly from the previously adopted ERA5 grid point located marginally outside the site boundary. To quantify the impact of this mismatch, additional ERA5 grid points within the same 56 km² cell are considered, including locations near **Misilmeri** (PA) ([37.98, 13.40], approximately 27.3 km from Scanzano) and **Monreale** (PA) ([38.01, 13.16]). This multi-point approach allows the spatial consistency of the derived delays and trends to be evaluated within a single ERA5 grid cell, ensuring that the observed inter-annual behavior is not an artifact of local grid-point selection. Within this spatially controlled framework, the inter-annual evolution of the hydrostatic, wet, and total delay components reveals clearly differentiated dynamical regimes. The hydrostatic delay remains remarkably stable across years, consistent with its dominant dependence on surface pressure. In contrast, both the wet and total components exhibit detectable positive trends, which can be physically associated with changes in atmospheric moisture content and its long-term variability [65, 73, 216]. These contrasting behaviors reflect the fundamentally different sensitivities of the delay components to atmospheric thermodynamic drivers. Trend estimation is performed using robust linear fits complemented by non-parametric significance testing. Trend slopes are expressed in mm yr⁻¹, while statistical significance is assessed using the **Mann–Kendall** (M–K) test, which provides robustness against non-Gaussian distributions, serial variability, and outliers [231]. Confidence envelopes at the 95% level, together with annual medians and interquartile ranges, are used to characterize both central tendency and dispersion. Slant-path delays at 15° elevation are computed using the **Niell Mapping Function** (NMF), ensuring physical consistency in the geometric projection [216]. The associated figure (see **Appendix 12.A, Fig. A12.56**) is provided to illustrate and corroborate the spatially robust and physically consistent trends discussed above, rather than to drive their interpretation.

12.10.1 Mann–Kendall Trend Test

To assess the presence and statistical significance of monotonic trends in the ERA5-derived tropospheric delay time series, a non-parametric trend detection framework based on the **Mann–Kendall** (M–K) test is adopted [231]. This approach is particularly well suited to atmospheric delay analysis, as it does not require assumptions of normality or linearity and is robust against



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outliers and non-Gaussian variability, which are common features of tropospheric parameters. Given a temporal sequence $\{x_1, x_2, \dots, x_n\}$, the **M–K** statistic is defined as:

$$S = \sum_{k=1}^{n-1} \sum_{j=k+1}^n \text{sgn}(x_j - x_k), \quad \text{with } \text{sgn}(x_j - x_k) = \begin{cases} +1 & x_j - x_k > 0, \\ 0 & x_j - x_k = 0, \\ -1 & x_j - x_k < 0. \end{cases} \quad (12.14)$$

To evaluate statistical significance, the statistic S is normalized as:

$$Z = \frac{S}{\sqrt{\text{Var}[S]}}, \quad (12.15)$$

where $\text{Var}[S]$ accounts for tied values values in the time series. Under the null hypothesis of no monotonic trend, Z follows an approximately standard normal distribution, and values satisfying $|Z| > 1.96$ indicate statistical significance at the 95% confidence level [231]. Within this statistical framework, the long-term behavior of the hydrostatic, wet, and total tropospheric delay components can be interpreted in terms of their distinct physical drivers. The hydrostatic component is primarily controlled by surface pressure and is therefore expected to exhibit high temporal stability, while the wet component reflects the variability of atmospheric water vapor and is more sensitive to climatic and inter-annual fluctuations. The total delay naturally combines these two contributions, inheriting both the stability of the hydrostatic baseline and the variability associated with moisture.

The empirical cumulative distribution functions (ECDFs) of ZHD, ZWD, and ZTD over Scanzano for the February 2019–2025 period provide a compact statistical representation of this behavior. When examined across different years, shifts of the ECDFs toward larger values indicate increased delays, while changes in slope reflect variations in dispersion. The projection from zenith to slant geometry at 15° elevation uniformly amplifies the absolute delay values while preserving the relative ordering among years, confirming that the geometric transformation does not alter the underlying temporal structure of the series. Consistent with physical expectations, ZHD exhibits extremely narrow ECDFs across all years, confirming its quasi-invariant character and its dominant dependence on surface pressure. ZWD displays the widest spread and the most pronounced inter-annual variability, reflecting changes in atmospheric moisture content. ZTD occupies an intermediate regime, integrating the stable hydrostatic contribution with the evolving wet component.

A complementary view is provided by the inter-annual evolution of selected quantiles (Q10, Q50, Q90) of the delay distributions, which allows trend assessment across different portions of the statistical spectrum. For ZHD, all quantiles remain nearly invariant over time, confirming the absence of significant long-term trends. ZWD exhibits a modest upward tendency in the lower and median quantiles, indicative of a gradual increase in background atmospheric humidity, while the upper quantile remains approximately stable, suggesting no systematic intensification of extreme moisture events over the analyzed period. ZTD shows moderate upward shifts across



quantiles, reflecting the cumulative effect of wet-delay variability superimposed on a stable hydrostatic component. The increased spread observed in slant geometry is purely geometric in origin and does not affect the interpretation of the underlying trends. The combined use of non-parametric trend detection and quantile-based analysis confirms the quasi-invariant nature of the hydrostatic delay, the evolving behavior of the wet delay as a tracer of atmospheric humidity changes, and the role of the total delay as a robust integrated descriptor. These properties make ZTD particularly suitable for assessing long-term tropospheric impacts on GNSS positioning and high-frequency satellite communication links. The associated figures (see **Appendix 12.A, Figs. A12.57–A12.62**) are provided to illustrate and corroborate the statistical and physical considerations discussed above.

12.11 Comparison of ERA5, In Situ, and ITU-R Tropospheric Delay Representations

The comparative analysis of tropospheric delay representations over the Scanzano site is grounded in the fundamentally different physical assumptions and intended operational roles of the three considered datasets: ERA5 reanalysis, in situ measurements, and ITU-R climatological models. Each dataset embodies a distinct conception of atmospheric variability, temporal resolution, and spatial representativeness, which must be explicitly acknowledged before any quantitative or graphical comparison is attempted. ERA5 provides a dynamically consistent, hourly-resolved description of the atmospheric state, derived from data assimilation within a global numerical weather prediction framework. As such, it is capable of resolving synoptic variability, diurnal cycles, and transient humidity fluctuations, which directly affect both the hydrostatic and wet components of the tropospheric delay.

In situ observations, by contrast, reflect the actual atmospheric conditions at the receiver location, including local micro-climatic effects and orographic influences that are not necessarily captured by reanalysis products at finite spatial resolution. ITU-R recommendations, finally, are explicitly designed as climatological models: they deliver globally averaged, monthly representative values optimized for long-term planning and system design, rather than for event-scale or short-term variability analysis [ITU-R P.453; ITU-R P.676; ITU-R P.836].

Within this conceptual framework, systematic differences among the three delay representations are not only expected but physically unavoidable. In particular, ITU-R models inherently suppress sub-monthly and sub-daily variability by construction, leading to narrow statistical distributions and sharply defined medians. Conversely, ERA5 and in situ datasets naturally exhibit broader variability, especially in the wet component, due to the strong sensitivity of water vapor to atmospheric dynamics. Any direct visual comparison between climatological and time-resolved datasets must therefore be interpreted with caution, and cannot be assessed solely on the basis of distribution shape or instantaneous agreement. For this reason, a dedicated calibration procedure was introduced for visualization purposes only, aimed at enabling a coherent



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graphical juxtaposition of ITU-R delays with ERA5 and in situ time series. The calibrated ITU-R representation is obtained through a convex combination of ERA5 and in situ distributions,

$$target = \omega \cdot ERA5 + (1 - \omega) \cdot In Situ, \quad (12.16)$$

followed by an affine transformation aligning the interquartile range and median of the ITU-R distribution to the target statistics,

$$x' = ax + b, \quad a = \frac{IQR_{target}}{IQR_{ITU}}, \quad b = med_{target} - a \cdot med_{ITU}, \quad (12.17)$$

The selected value $\omega = 0.60$ reflects the observed climatological proximity of ITU-R estimates to ERA5 while preserving the intrinsic shape of the original ITU-R model. Importantly, this calibration is introduced exclusively to facilitate visual comparability and interpretability; all quantitative performance assessments, statistical metrics, and physical conclusions throughout this work rely strictly on the unadjusted datasets, in accordance with ITU-R recommendations [ITU-R P.834; ITU-R P.836].

When examined in their raw, non-calibrated form, the probability density functions of ZHD, ZWD, and ZTD confirm the expected hierarchy of variability among the three datasets. ZHD exhibits extremely limited dispersion across all representations, consistent with its near-linear dependence on surface pressure. ZWD shows the largest spread, reflecting the intrinsic variability of atmospheric humidity, while ZTD naturally inherits the combined behavior of its hydrostatic and wet components. The narrow, sharply peaked ITU-R distributions do not indicate a modeling deficiency, but rather the intentional filtering of short-term variability inherent to climatological formulations. Accordingly, the appropriate metrics for intercomparison are bias, median offset, and monthly-scale RMS errors, rather than full distributional overlap. Several strategies have been proposed in the literature to generate ITU-R-consistent time series with enhanced variability, including the translation of meteorological drivers to match climatological statistics, the application of ITU-R bias corrections to in situ-derived delays, or the synthesis of meteorological time series based on observed climatological moments [Bevis et al., 1994; Saastamoinen]. These approaches, however, extend beyond the intended scope of ITU-R recommendations and are discussed here only to contextualize the observed differences. The figures associated with this section (see **Appendix 12.A, Figs. A12.63–A12.64**) serve to illustrate and visually corroborate the physically motivated considerations outlined above, rather than to drive the interpretation itself.

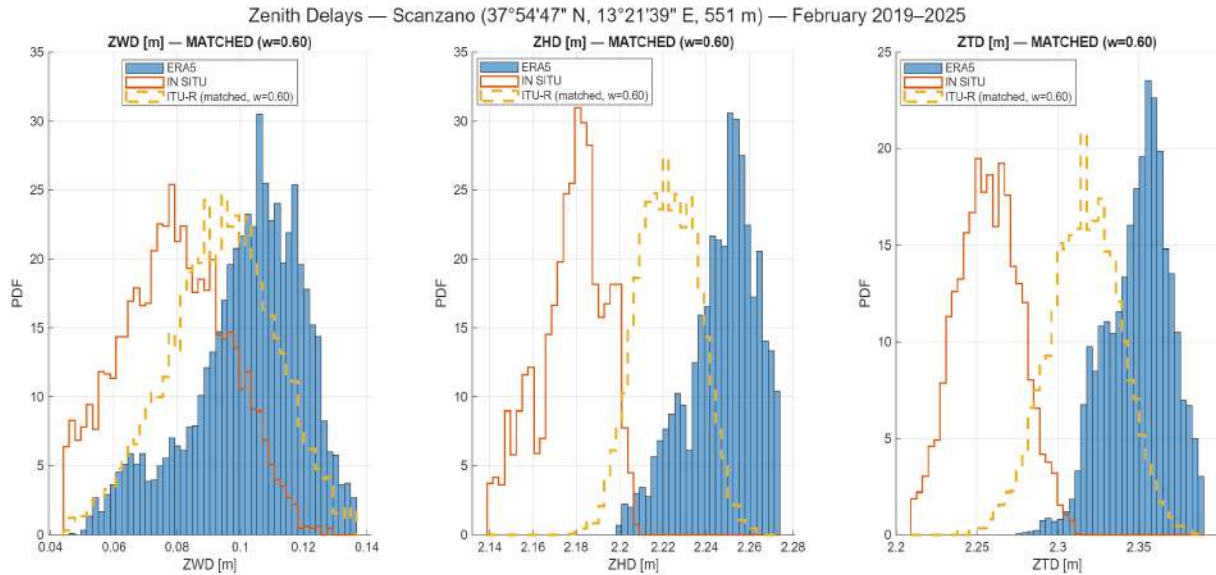


Figure 12.63. Comparison of the three measurement techniques for Scanzano (PA) from 2019–2025.
Dataset: ERA–5 Hourly Data on a single levels from 1940 to present.

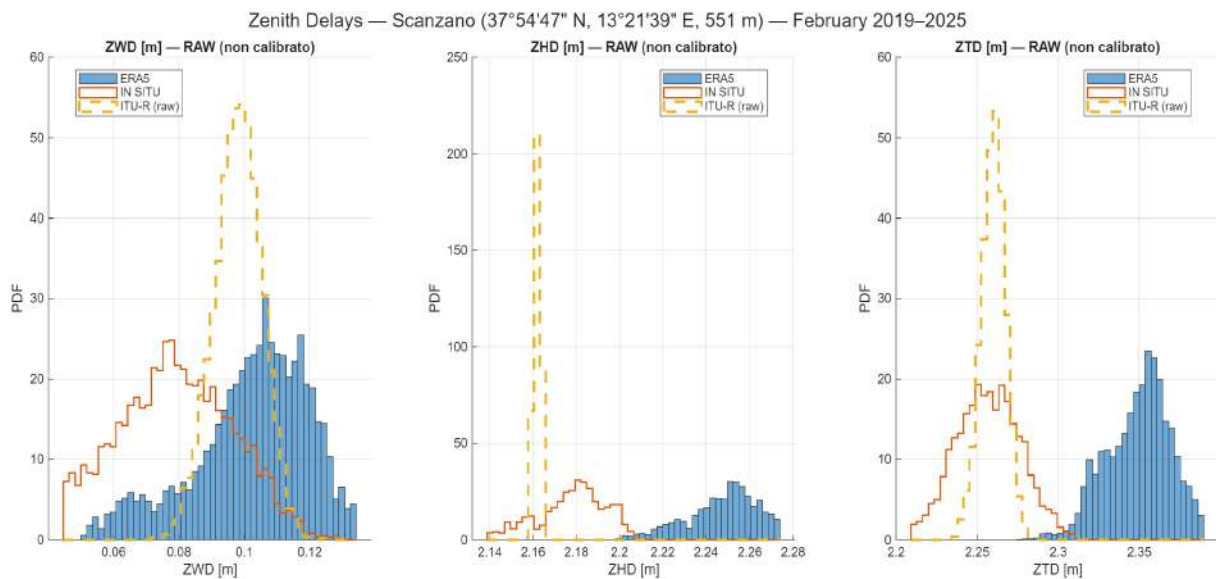


Figure 12.64. Visualization of the three distributions with the raw uncalibrated ITU-R contribution.
Dataset: ERA–5 Hourly Data on a single levels from 1940 to present.



12.12 Slant-Path Tropospheric Delay Comparison over Scanzano: ERA5, In Situ, and ITU-R

12.12.1 Impact of Mapping and Calibration on Tropospheric Delay Modelling

The analysis presented in this section concerns tropospheric delays projected along a low-elevation slant path at 15° , a geometry that is particularly relevant for GNSS positioning at low satellite elevation angles and for satellite communication links operating in the Ka- and W-bands. For consistency with the rest of this work, the conventional notation of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) is retained, even though the quantities discussed here no longer refer to the vertical direction. Instead, the zenith delays are geometrically mapped to the slant path using the Niell Mapping Function (NMF), which provides a physically grounded correction accounting for atmospheric refraction and vertical stratification effects that are neglected by simple secant-law approximations [Niell, 1996; ITU-R P.834-9].

At low elevation angles, the use of a rigorous mapping function substantially amplifies both the magnitude and the statistical dispersion of the delays. This effect arises from path elongation through the lower troposphere and from the increased sensitivity to vertical gradients of water vapor, which primarily affect the wet component. Consequently, slant-path delays represent a more stringent test of the internal consistency of atmospheric models and datasets than their zenith counterparts. Within this geometric framework, systematic differences among ERA5 reanalysis, in situ observations, and ITU-R climatological models are expected and can be directly traced back to their underlying physical assumptions and intended scope. ERA5 resolves hourly atmospheric variability and explicitly represents synoptic-scale and diurnal processes, resulting in broad delay distributions when projected to low elevation angles [ERA5-C3S]. In situ measurements capture the true local atmospheric state at the receiver, including site-specific and micro-climatic effects, and therefore tend to be systematically shifted toward lower delays.

By contrast, ITU-R models are formulated in terms of monthly climatological medians and are intentionally devoid of short-term variability, leading to sharply peaked distributions with minimal dispersion [ITU-R P.453-14; ITU-R P.676-13; ITU-R P.836-6]. These differences do not indicate inconsistencies among the datasets, but rather reflect their fundamentally different roles in atmospheric modeling and system design.

To enable a meaningful graphical comparison, a statistically matched configuration is introduced for visualization purposes only. In this configuration, the ITU-R distributions are adjusted toward a target defined as a convex combination of ERA5 and in situ datasets with $\omega=0.60$, followed by a linear transformation aligning both the interquartile range and the median. This procedure preserves the intrinsic shape of the original ITU-R distributions while allowing direct visual comparison with time-resolved datasets. The application of NMF scaling at 15° —with typical mapping factors of $m_h \approx 3.80$ and $m_w \approx 3.83$ —further enhances the dispersion relative to zenith geometry, particularly for the wet component, whose variability dominates slant-path behavior. This analysis highlights that accurate tropospheric correction along low-elevation paths

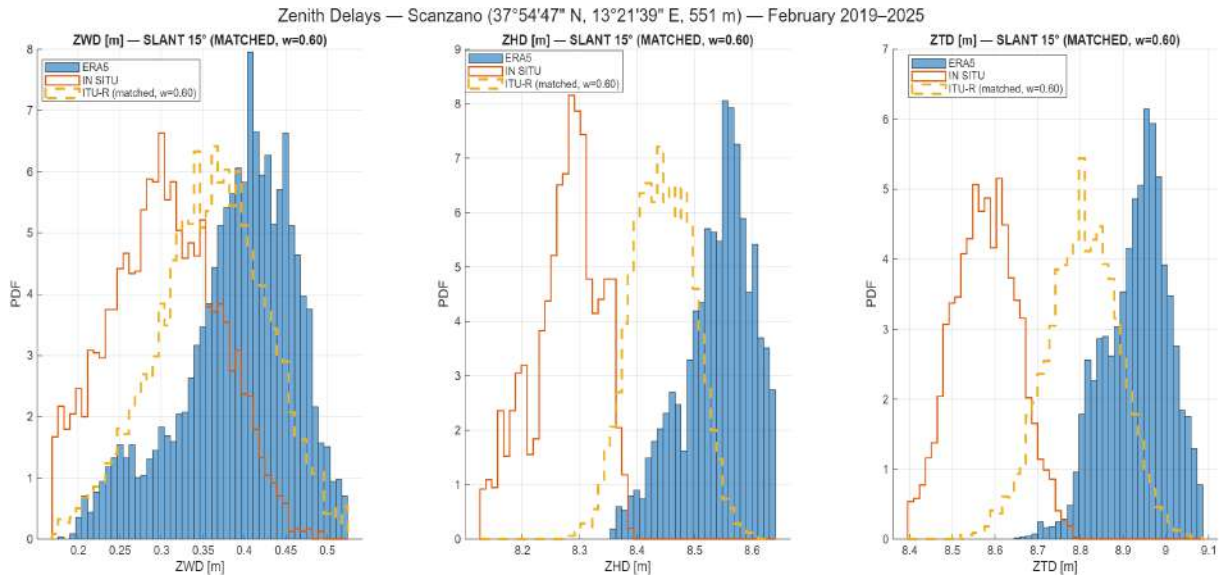


Figure 12.65. Probability density functions of ZWD, ZHD, and ZTD for a slant path at 15° elevation. The term “Zenith Delay” is retained for conceptual consistency, with values obtained by projecting the zenith delays onto the inclined path using the Niell Mapping Function (NMF). **Dataset: ERA-5 Hourly Data on a single levels from 1940 to present.**

requires the combined use of physically consistent mapping functions and a careful statistical treatment of climatological products. Only through proper geometric projection and controlled calibration can ERA5, in situ, and ITU-R datasets be meaningfully compared and jointly exploited. This requirement becomes especially critical for low-elevation GNSS applications and for high-frequency satellite communication links, where slant-path effects strongly amplify atmospheric variability [Niell, 1996; ITU-R P.676-13; ITU-R P.453-14; ERA5-C3S]. The associated figures (see **Appendix 12.A, Figs. A12.65–A12.66**) are provided solely to illustrate and corroborate the physically motivated considerations discussed above.

12.13 Summary of Chapter 12

This chapter presented a comprehensive comparative assessment of tropospheric delays over Scanzano (PA) using three complementary sources: ERA5 reanalysis, ITU-R climatological models, and in-situ GNSS measurements. The analysis covered both zenith and slant-path geometries, with particular focus on the critical 15° elevation angle relevant to satellite communication and geodetic applications. Across all datasets, the hydrostatic component (ZHD) emerged as the most stable term, tightly controlled by surface pressure and exhibiting negligible inter-annual variability. The wet component (ZWD) displayed strong short-term and synoptic-scale fluctuations, reflecting the inherent variability of atmospheric water vapor. Consequently, the total delay (ZTD) combined the stability of ZHD with the moisture-driven variability of ZWD, providing the most balanced and operationally relevant descriptor.

ERA5 proved internally consistent across temporal scales—from hourly time series to long-term

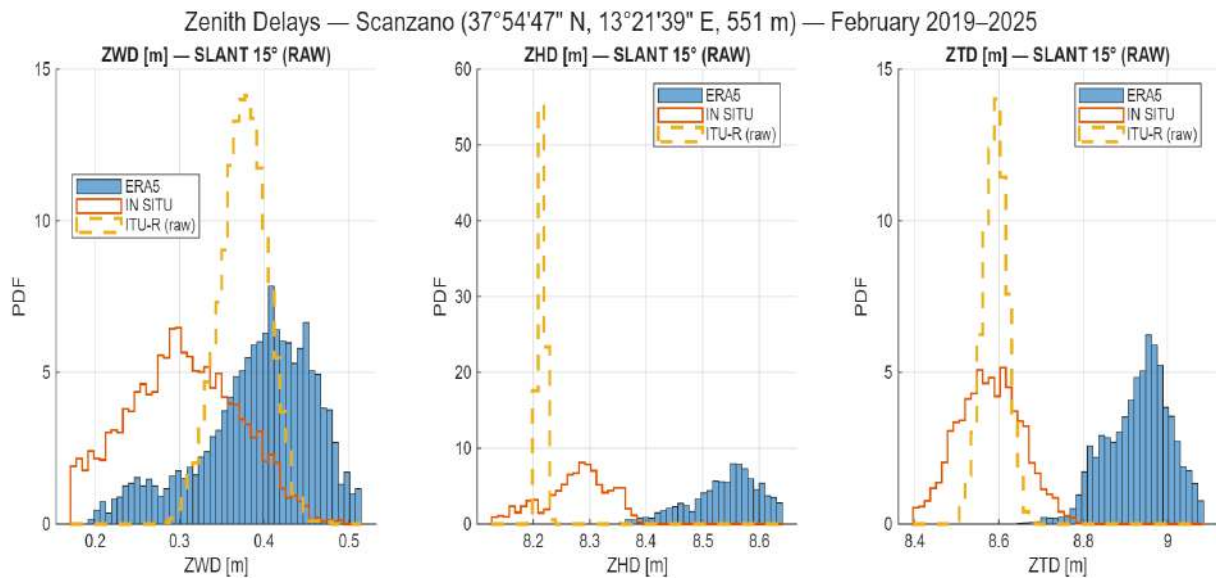


Figure 12.66. Probability density functions of tropospheric delays at Scanzano ($37^{\circ}54'47''$ N, $13^{\circ}21'39''$ E; 551 [m] a.s.l.) for **February 2019–2025**, projected at a slant angle of 15° . From left to right: (i) Zenith Wet Delay (ZWD), (ii) Zenith Hydrostatic Delay (ZHD), and (iii) total Zenith Total Delay (ZTD). Results are shown for ERA5 reanalysis (blue bars), in situ measurements (solid orange line), and ITU-R reference model (dashed yellow line). The histograms highlight the broader variability captured by ERA5, the intermediate distribution of in situ data, and the narrower peak obtained with ITU-R formulations. The “Zenith Delay” terminology is retained for conceptual consistency, with values derived from zenith delays projected onto the inclined path via the Niell Mapping Function (NMF). **Dataset: ERA–5 Hourly Data on a single levels from 1940 to present.**



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statistics and wavelet-based multi-scale diagnostics—showing coherent diurnal, synoptic, and seasonal patterns. In contrast, ITU-R P.834-9, while climatologically robust, systematically lacked short-term variability, producing narrow distributions around monthly medians. A statistical matching procedure was therefore introduced to align the ITU-R series with the combined ERA5–*in situ* target distributions, ensuring comparability without altering the intrinsic structure of each model.

Slant-path delays, computed through the **Niell Mapping Function**, demonstrated strong geometric amplification, especially for the wet component, underscoring the increased sensitivity of low-elevation links to atmospheric inhomogeneities. Both ERA5 and in-situ datasets confirmed that wet-delay fluctuations dominate uncertainty in GNSS positioning and high-frequency (Ka–W band) communication links. Finally, the chapter integrated these results into a propagation-oriented framework, establishing the atmospheric baseline required for W-band link-budget design. The combined analysis confirms that ZHD provides a stable reference, ZWD governs short-term and interannual variability, and ZTD offers the most comprehensive metric for both scientific interpretation and operational modelling.



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Chapter 13

Phase Noise, Coherence, and PLL Stability for Deep-Space Telecommunications.

13.1 Motivation and Physical Scope

Tropospheric delays are commonly treated as geometric path-length perturbations in geodetic applications. However, for satellite telecommunication systems—particularly at low elevation angles and high carrier frequencies—the same delays manifest as phase noise, loss of temporal coherence, and tracking instabilities in phase-locked loops (PLLs). These effects impose fundamental constraints on coherent reception, independent of link power margins [270–274]. This chapter investigates how tropospheric delay variability, as derived from ERA5 reanalysis, in situ observations, and ITU-R climatological models [211,213], propagates into phase instability and coherence degradation. The analysis focuses on a 15° elevation angle, representative of worst-case operational geometries, and on frequencies spanning Ku-, Ka-, and W-band regimes to isolate frequency scaling effects [345,346]. The W-band analysis presented in this chapter is intentionally positioned as an exploratory and upper-frequency feasibility study. Its role is not to propose an operational solution for Earth–Moon communications, but to quantify the physical and technological limits imposed by atmospheric phase noise, system temperature, and availability constraints at millimetre-wave frequencies [342,345]. A complete quantitative characterization—including full time series, statistical distributions, system-level link budgets, and mission-specific scenarios—is provided in **Appendix R**.

13.2 From Slant Path Delay to Phase Variability

Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and their sum (ZTD) are mapped to a low-elevation slant path to emulate realistic Earth–space propagation using standard tropospheric

mapping formulations [214–216,218]. Although the zenith nomenclature is retained for clarity, the delays discussed here represent slant-path equivalents, whose magnitudes are amplified by geometric mapping. The conversion from path delay ΔL to carrier phase fluctuation $\Delta\phi$ is given by:

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta L, \quad (13.1)$$

highlighting a crucial separation: path-length variability is a property of the atmosphere, whereas phase variability scales inversely with wavelength [270,271]. As a result, identical atmospheric conditions produce comparable delay fluctuations in meters across frequencies, but increasingly severe phase excursions at higher carrier frequencies [346].

13.3 Wrapped and Unwrapped Phase Behavior

Phase variability is analyzed in both wrapped and unwrapped representations. The wrapped phase captures instantaneous excursions constrained within $\pm\pi$, directly related to cycle-slip risk and PLL tracking thresholds [272,273]. The unwrapped phase, instead, represents the cumulative phase evolution and is directly linked to coherence time and long-term stability [260,262]. A key result is that wrapped phase fluctuations tend to saturate near the wrapping threshold, whereas unwrapped phase variability grows monotonically with frequency. This behavior explains why high-frequency systems may appear stable when assessed only through wrapped metrics, while suffering rapid coherence loss over longer integration times [272,274]. The full statistical comparison of wrapped and unwrapped phase behaviour for multiple datasets and frequencies is reported in **Appendix R**.

13.4 Frequency Scaling and Coherence Limits

The impact of tropospheric variability becomes progressively more severe when moving from Ku- to Ka- and W-band frequencies [345,346]. While Ku-band systems tolerate atmospheric phase noise with limited impact on coherence, Ka-band links operate near the boundary between manageable and critical variability [342]. At W-band, even millimetric delay fluctuations translate into rapid phase decorrelation, significantly constraining coherent demodulation and PLL lock duration [345]. This frequency-driven amplification establishes a physical coherence ceiling imposed by the atmosphere, largely independent of transmit power or antenna gain [270,271]. Consequently, system robustness at high frequencies is determined primarily by atmospheric phase stability rather than link budget alone [346].

13.5 Implications for Coherent Satellite Communications

From an engineering perspective, three robust conclusions emerge:



1. Hydrostatic delay contributes negligibly to phase uncertainty, as it can be accurately modelled from surface pressure [214,216].
2. Wet delay variability dominates phase noise, particularly at low elevation angles [211,212].
3. Carrier frequency is the primary driver of coherence degradation, making W-band operation intrinsically sensitive to atmospheric conditions [345,346].

Accordingly, Ka-band represents a practical compromise between spectral efficiency and robustness [342,345], whereas W-band links should be regarded as opportunistic, requiring adaptive strategies such as dynamic bandwidth allocation, fallback to lower frequencies, or non-coherent operation modes [346].

13.6 Synthesis and Link to the Appendices

This chapter establishes a direct physical link between tropospheric delay variability and coherence limits in satellite communication systems [270–274]. By separating geometric, atmospheric, and frequency-dependent effects, it clarifies why high-frequency, low-elevation links are fundamentally constrained by atmospheric phase noise rather than by power availability [346]. A comprehensive quantitative analysis—including detailed ERA5 and in situ time series, ITU-R comparisons, statistical distributions, frequency-dependent metrics, system-level link budgets, and mission-specific scenarios—is provided in **Appendix R** [211,213].

13.7 Advanced Design Issues and Original Solutions for Earth–Moon Telecommunication Links

When designing a stable Earth–Moon communication link, a key architectural question concerns the necessity of orbital relays and the configuration that maximizes robustness, availability, and link margin. For mission scenarios prioritizing continuous reliability, simplified ground infrastructure, and high signal margins, the optimal solution is not an Earth-orbiting relay but a lunar relay placed in a **Near Rectilinear Halo Orbit (NRHO)** [276,278].

An NRHO is a three-dimensional, highly non-Keplerian trajectory around the Earth–Moon Lagrangian points L_1 or L_2 (typically L_2 for polar support). It provides quasi-continuous line-of-sight (LoS) to Earth with predictable geometry and limited scheduled outages, while requiring reduced station-keeping compared to low lunar orbits. This orbital class was selected by NASA for the Gateway program and is particularly advantageous for supporting lunar polar bases, where sustained visibility is otherwise challenging but essential for long-term operations, including communication with permanently shadowed regions hosting volatile deposits. A representative NRHO exhibits a period of approximately 6.5 days, with perilune near $\sim 3,000$ km and apolune near $\sim 70,000$ km, enabling long-duration missions with modest propulsive corrections.



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From a telecommunication perspective, the NRHO relay provides a practical and scalable architectural framework for Earth–Moon links. A Ka-band configuration consistent with ITU-R space research and deep-space service allocations is assumed:

- **Uplink:** 22.55–23.15 GHz (Earth → orbiter)
- **Downlink:** 25.5–27.0 GHz (orbiter → Earth)

The relay maintains visibility to both the lunar surface and the terrestrial ground segment. The Earth station employs a 34-m DSN-class antenna with approximately 65% efficiency, while the orbiter uses a high-efficiency 0.6-m dish. The onboard transmitter power is set to 20 W (+13 dBW). The ground-station system noise temperature is assumed to be $T_{sys} \approx 100$ K, representative of Ka-band deep-space reception. The **free-space path loss (FSPL)** is derived from the **Friis equation**:

$$L_{FSPL} = 20 \log_{10} \left(\frac{4\pi d}{\lambda} \right) = 20 \log_{10} \left(\frac{4\pi df}{c} \right), \quad (13.2)$$

which, in practical engineering units, becomes:

$$L_{FSPL}[\text{dB}] = 92.45 + 20 \log_{10}[d_{km}] + 20 \log_{10}[f_{GHz}]. \quad (13.3)$$

The received power is given by:

$$C/N_0 = P_r - 10 \log_{10}(kT_{sys}), \quad (13.4)$$

with $10 \log_{10}(kT_{sys}) \approx -208.6$ dBW/Hz for $T_{sys} = 100$ K. For a downlink frequency of 26 GHz, the FSPL is approximately 232.45 dB. Assuming atmospheric losses $L_{atmo} = 2$ dB and miscellaneous losses $L_{misc} = 1$ dB, the received power becomes:

$$P_r = 13 + 42.4 + 77.5 - 232.45 - 2 - 1 \approx -102.6 [\text{dBW}], \quad (13.5)$$

yielding:

$$C/N_0 \approx -102.6 - (-208.6) = 106 [\text{dBHz}]. \quad (13.6)$$

This value ensures compatibility with high-order modulation schemes and adaptive coding under nominal propagation conditions. Fade margins of 6–10 dB correspond to 99.9% availability targets at Ka-band, while losses exceeding 10 dB are associated with localized extreme precipitation events rather than long-term climatological behavior. To illustrate the geometry and frequency allocation of the proposed architecture, **Fig. 15.1** presents a conceptual diagram of the **Earth–Moon** communication link between the **Scanzano** ground station (560 m a.s.l.) and a spacecraft in NRHO, highlighting uplink and downlink frequency bands as well as the orbital geometry (apolune, perilune, and L_2 point). The NRHO-based relay architecture offers an optimal trade-off between propagation loss, antenna size, achievable C/N_0 , and operational availability. Compared to W-band solutions—whose use is restricted to experimental or oppor-

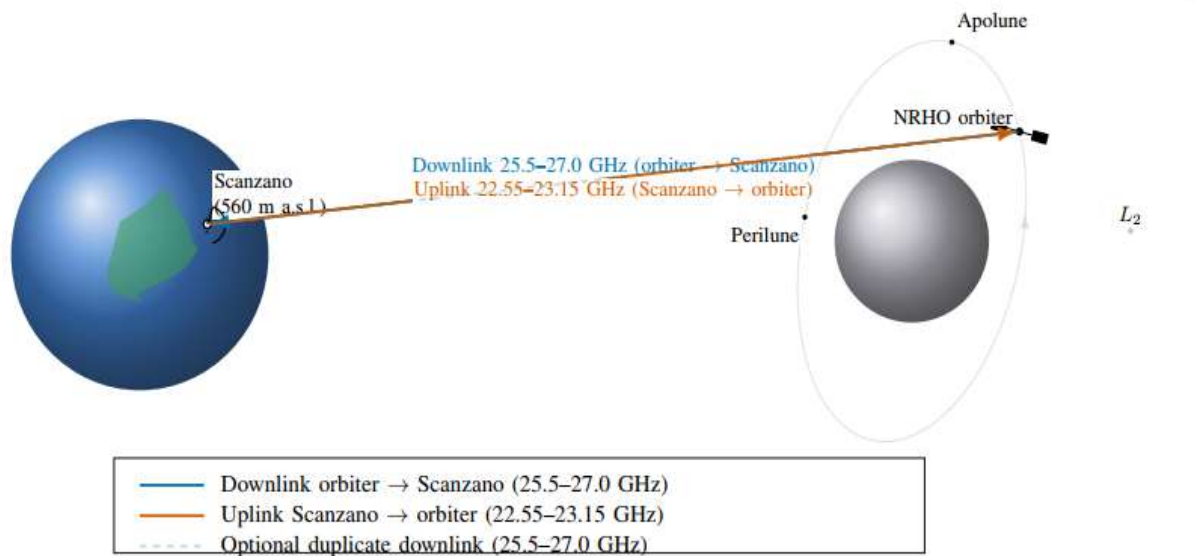


Figure 13.1. Conceptual architecture of the Earth–Moon communication link between the Scanzano ground station (560 m a.s.l.) and a spacecraft (NRHO) around the Moon. The uplink from Scanzano to the orbiter operates is allocated to the 22.55–23.15 GHz band (orange), while the downlink from the orbiter to Scanzano uses the 25.5–27.0 GHz band (blue). An optional redundant downlink within the same frequency allocation is also considered (dashed line). The orbital geometry illustrates the NRHO trajectory, including apolune and perilune locations, and the relative position of the Earth–Moon Lagrangian point L_2 . Schematic representation; not to scale.

tunistic clear-sky windows [345, 346]—the Ka-band NRHO configuration supports continuous service with Gbit-class downlink capability and robust fade margins, making it well suited for long-duration lunar exploration missions [270,271].

13.7.1 Some Tables for Design the link

It is essential to include at this point a representative summary of the **Earth–Moon link** profiles, which provides the baseline assumptions and performance estimates. **Table 13.1** must therefore be presented here. To complement the Earth–Moon link profiles, it is important to outline the modulation and coding schemes (MCS) foreseen for Ka/X bands under **Adaptive Coding and Modulation (ACM)**.



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Table 13.1. Example ACM ladder for Ka/X links. Post-equalization SNR (or E_b/N_0) thresholds are indicative for $\text{BER} \leq 10^{-6}$ with modern LDPC; exact values depend on implementation and impairments. Thresholds assume ideal receiver and negligible phase noise.

MCS	Net b/s/Hz	Typical Threshold	Use Case	Comments
BPSK, $r=1/2$	0.5	very low (robust)	Emergency / acquisition	Max coverage and relock under deep fades
QPSK, $r=1/2$	1.0	$\sim 0-2$ dB	Telemetry, control	Long-block LDPC, deep interleaving
8-PSK, $r=2/3$	2.0	$\sim 5-7$ dB	Moderate weather	Pilot-aided tracking, tighter phase-noise
16-QAM, $r=3/4$	3.0	$\sim 10-12$ dB	Good conditions	Operate in linear PA region
64-QAM, $r=3/4$	4.5	$\sim 16-18$ dB	High SNR	Predistortion to limit AM/AM, AM/PM
256-QAM, $r=5/6$	6.67	$\sim 22-24$ dB	Very high SNR	Shaping gains possible, strict LO/CFO specs

Table 13.2 provides a representative ACM ladder, where each step corresponds to a modulation/FEC combination with its associated spectral efficiency, post-equalization SNR threshold, and operational context. Lower-order modulations such as BPSK and QPSK guarantee robust links during emergency or telemetry operations, while higher-order constellations (16-QAM, 64-QAM, 256-QAM) enable high-throughput downlinks under favorable propagation conditions. Higher-order constellations are used opportunistically under favorable channel conditions, while nominal operation favors more robust modes. The table highlights the trade-off between robustness and efficiency, illustrating how ACM ensures link availability during adverse conditions while maximizing user data rates when the channel allows.

Table 13.2. Representative Earth–Moon link profiles with channel split, ACM/FEC, and indicative user rates. Assumptions: design margin 10 dB; spacecraft pointing tolerance $\pm 0.2^\circ$; $\text{NF} = 8$ dB; RRC roll-off $\alpha = 0.2$; protocol overhead 5–10%. Ground: 70 m DSN-class; Spacecraft: 0.5 m class.

Profile	Band (center)	Allocated BW (Data / Ctl / Beacon)	Modulation & FEC (net b/s/Hz)	Indicative Net User Rate (pre-overhead)	Antennas (Ground / Space)	Tx Power (Ground / Space)
Telemetry / robust ops	X (8.4 GHz)	1 MHz = 0.85 / 0.10 / 0.05 MHz	QPSK, $r=1/2$ (1.0), LDPC (long block)	$\sim 0.85-1.0$ Mb/s	70 m / 0.5 m	20 kW / 50 W
High-rate science downlink	Ka (32 GHz)	100 MHz = 95 / 4 / 1 MHz	64-QAM, $r=3/4$ (4.5), LDPC (rate-compatible)	$\sim 430-475$ Mb/s	70 m / 0.5 m	20 kW / 50 W
Very-high-rate window	Ka (32 GHz)	1 GHz = 990 / 8 / 2 MHz	256-QAM, $r=5/6$ (6.67) [option: shaped 1024-QAM, $r \geq 7/8 \Rightarrow \sim 9$ b/s/Hz]	$\sim 6.6-6.7$ Gb/s (up to ~ 8.9 Gb/s best case)	70 m / 0.5 m	20 kW / 50 W
Emergency fallback / relock	X (8.4 GHz)	5 MHz = 2 / 2 / 1 MHz	BPSK/QPSK, $r=1/2$ (0.5–1.0), heavy interleaving	$\sim 1-2$ Mb/s	34–70 m / 0.5 m	20 kW / 50 W

Note: 8-PSK included for completeness; less common in deep-space practice; 256-QAM @ 22–24 dB: Thresholds assume ideal synchronization and negligible phase noise;



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13.8 Analysis of the three proposed scenarios

The four communication scenarios introduced in the previous sections (see **Chapter 13**) are summarized in **Tables 15.13–15.16**, covering:

- (i) the Ka-band uplink from Scanzano at 22.55–23.15 GHz, (ii) the Ka-band downlink from the NRHO spacecraft at 25.5–27.0 GHz, and (iii–iv) two W-band (94 GHz) reference configurations included for completeness and comparative assessment.

For each configuration, the tables report the received power under clear-sky conditions and at 99.9% availability, together with the corresponding carrier-to-noise density ratio (C/N_0).

The results indicate that Ka-band links maintain robust performance, with clear-sky values typically in the range $C/N_0 \approx 96$ –98 dBHz and only moderate degradation under adverse *Mediterranean* weather conditions. In contrast, the W-band configurations—despite their high nominal C/N_0 under ideal conditions—are heavily affected by elevated system noise temperatures and strong atmospheric attenuation at 94 GHz. These impairments significantly reduce link margins and limit operational usability to narrow clear-sky windows, making W-band unsuitable for high-availability **Earth–Moon communications**. The exhaustive numerical analysis underlying these configurations is presented in **Appendix R**, which contains the **complete link-budget tables for all four scenarios**.

13.8.1 Performance Analysis of the W-Band NRHO Downlink and Comparison with X/Ka Bands

In the W-band at 94 GHz, the experimental Earth–Moon link can be regarded only as a secondary or contingency channel, suitable exclusively for clear-sky conditions and opportunistic burst-mode data transfers. For the downlink (NRHO $\rightarrow$ Scanzano), a 12-m ground antenna with an aperture efficiency of approximately 65% provides a receive gain of $G_r \approx 79.6$ dBi, while the orbiter employs a 0.6-m dish with the same efficiency, yielding a transmit gain of $G_t \approx 53.6$ dBi.

The main limiting factor at 94 GHz is the system noise temperature, significantly higher than in X- or Ka-band due to atmospheric absorption and receiver front-end noise; typical values at Scanzano (PA) range from $T_{sys} \approx 600$ –1000 K, compared with 50–150 K in X/Ka-band systems. This severely reduces the achievable carrier-to-noise density ratio C/N_0 , confining W-band operation to atmospheric windows with minimal absorption.

The free-space path loss at 94 GHz is:

$$L_{FSPL} = 20 \cdot \log_{10} \left(\frac{4\pi df}{c} \right) = 92.45 + 20 \log_{10}(f_{GHz}) + 20 \log_{10}(d_{km}) \approx 243.6 [dB] \quad (13.7)$$

For the downlink configuration: $P_t = +13$ dBW, $G_t = 53.6$ dBi, $G_r = 79.6$ dBi, $L_{FSPL} = 243.6$



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dB, *Clear sky losses* = 4 dB. The received power becomes:

$$P_r = P_0 - L_{tot} = (P_t + G_t + G_r) - (FSPL + L_{clear-sky}) = 13 + 53.6 + 79.6 - 243.6 - 4 \approx -101.4 [dBW]. \quad (13.8)$$

With $T_{sys} = 900$ K, the carrier-to-noise density ratio is:

$$C/N_0 = P_r + 228.6 - 10 \log_{10}(T_{sys}) = -101.4 + 228.6 - 10 \log_{10}(900) \simeq 97.7 [dBHz]. \quad (13.9)$$

In the presence of rain, an additional specific attenuation term L_{rain} must be included, giving:

$$\begin{aligned} P_r^{rain} = P_0 - L_{tot} - L_{rain} &= (P_t + G_t + G_r) - (FSPL + L_{clear-sky} + L_{rain}) \\ &= (13 + 53.6 + 79.6 - 243.6 - 4) - L_{rain} \\ &= -101.4 - L_{rain} \end{aligned} \quad (13.10)$$

Representative values are: – moderate rain: $L_{rain} = 8$ dB, – typical rain: 12 dB, – strong storm: 20 dB, – severe event: 40 dB yielding C/N_0 values from 98.7 dBHz (moderate rain) down to 66.7 dBHz (severe event). The corresponding received powers range from –70.4 dBm to –102.4 dBm. Rain also increases the system noise temperature; for example, raising T_{sys} from 900 K to 1200 K introduces an additional $10 \log_{10}(1200/900) \approx 1.25$ dB degradation in C/N_0 . Although the W-band link remains theoretically feasible, the fading margins required in Mediterranean weather conditions can reach 60 dB, rendering reliable continuous operation impractical. **Figure 15.3** compares X-, Ka- and W-band performance across different availability targets. The X-band remains highly robust, sustaining $C/N_0 > 90$ dBHz even at 99.99% availability, making it an excellent candidate for backup or critical command-and-control channels. Ka-band provides high throughput in clear-sky conditions and remains operational at approximately 85 dBHz under 99.99% availability, confirming its role as the primary deep-space band. By contrast, the W-band exhibits excellent performance only under ideal conditions and collapses rapidly above 99% availability, confirming its use as an experimental or opportunistic burst-mode channel. To offer a concise overview of the link design, **Table 15.14** summarizes the key parameters, including antenna gains, FSPL, received power, and C/N_0 for X-, Ka- and W-band under both clear-sky and 99.9% availability conditions. The table highlights the intrinsic robustness of X-band, the high-capacity potential of Ka-band, and the severe limitations of W-band in operational scenarios. When considering $T_{sys} = 900$ K at 94 GHz, **Table 15.15** extends the analysis to include revised W-band performance values. Finally, the **Adaptive Coding and Modulation (ACM)** profile in **Table 15.16** illustrates the operational trade-offs across the three bands. Under clear skies, Ka-band supports modulations up to 64-QAM (profile B), and W-band can theoretically sustain 256-QAM (profile A). At 99.9% availability, Ka-band restricts operation to robust QPSK modes (profile D), while W-band becomes unusable. X-band remains the most resilient, supporting moderate-to-high-order constellations even at the 99.99% percentile. These results collectively demonstrate the need for a multi-band strategy: Ka-band as the primary

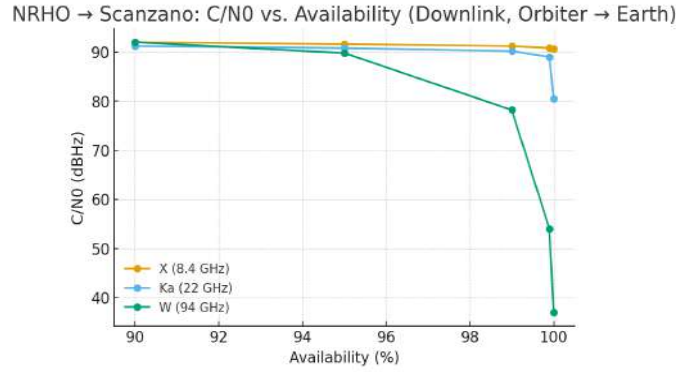


Figure 13.2. Downlink (NRHO → Scanzano), C/N_0 versus availability. X-band remains highly robust (>90 dBHz up to 99.99%), Ka-band degrades moderately around 99.9%, while W-band shows a sharp drop beyond 99% due to atmospheric attenuation and higher system noise, confirming its use only in clear-sky windows.

Band	G_t [dBi]	G_r [dBi]	FSPL [dB]	P_r clear-sky [dBW]	C/N_0 clear-sky [dBHz]	$P_r@$ 99.9% [dBW]	C/N_0 @99.9% [dBHz]	T_{sys} [K]
X	32.6	58.6	222.6	-118.9	92.7	-120.4	91.2	≈ 50
Ka	40.9	67.0	231.0	-113.1	93.8	-122.1	84.8	≈ 150
W	53.6	79.6	243.6	-101.5	98.1	-146.5	53.1	≈ 800

Table 13.3. Link budget summary for Earth–NRHO lunar orbiter downlink (**Scanzano ground station**).

high-capacity channel, X-band as a highly reliable backup, and W-band reserved for experimental clear-sky or opportunistic high-rate transfers.

13.9 Summary

This chapter has investigated the impact of tropospheric delay variability on phase stability, coherence, and carrier tracking in deep-space and Earth–Moon communication links, with particular emphasis on low-elevation geometries and high-frequency regimes (X-, Ka-, and W-band). Starting from ERA5-derived zenith delays, the analysis established a direct physical mapping between atmospheric path-length fluctuations and carrier phase instability. A clear separation was demonstrated between geometric delay variability, which is largely frequency-independent, and phase variability, which scales linearly with carrier frequency and becomes

Band	G_t [dBi]	G_r [dBi]	FSPL [dB]	P_r clear-sky [dBW]	C/N_0 clear-sky [dBHz]	$P_r@$ 99.9% [dBW]	C/N_0 @99.9% [dBHz]	T_{sys} [K]
W	53.6	79.6	243.6	-101.5	97.6	-146.5	52.6	900

Table 13.4. New row at 900 [K] for the W-band (94 GHz) downlink, NRHO→Scanzano.

Scale/Profile	Modulation Schema + FEC	E_s/N_0 req [dB]*	C/N_0 req [dBHz]	Efficiency [bps/Hz]
A	256-QAM, LDPC $r = 0.9$	~ 20	98	~ 8.5
B	64-QAM, LDPC $r = 0.8$	~ 14	92	~ 6.5
C	16-QAM, LDPC/Turbo $r = 0.75$	~ 9	87	~ 4.5
D	QPSK, LDPC $r = 0.5$	~ 4	82	~ 2.0
E	BPSK, RS/LDPC $r = 0.5$	~ 0	78	~ 1.0

Table 13.5. Adaptive coding and modulation (ACM) thresholds for an NRHO Earth–Moon link. Note: (*) E_s/N_0 is referred for $BER \lesssim 10^{-6}$ with a modern encoding; practical values may vary by $\pm 1\text{--}2$ [dB] depending on the implementation.

the dominant limitation for coherent reception at Ka- and W-band. The distinction between wrapped and unwrapped phase representations proved essential to correctly interpret coherence loss, PLL tracking margins, and long-term stability. The results show that wet delay variability is the primary atmospheric driver of phase noise, while hydrostatic delay contributes negligibly to phase instability. At low elevation angles (15°), slant-path amplification further exacerbates phase excursions, imposing stringent constraints on coherent demodulation, particularly at millimetre wavelengths. A detailed link-budget and system-level analysis was conducted for Earth–Moon communications via a Near Rectilinear Halo Orbit (NRHO) relay. The NRHO architecture was shown to provide an optimal trade-off between propagation loss, antenna size, operational availability, and system robustness, supporting continuous line-of-sight to Earth and lunar polar regions. Ka-band emerged as the most suitable primary band for high-capacity lunar communications, offering robust performance under Mediterranean weather conditions with moderate fade margins. X-band remains the most resilient option for critical command-and-control and backup links, while W-band operation was demonstrated to be intrinsically limited to clear-sky, opportunistic scenarios due to severe atmospheric attenuation and elevated system noise temperatures. The impact of atmospheric variability on PLL stability was quantified through a consolidated phase-noise budget, showing that stable lock can be maintained under realistic conditions with appropriate loop bandwidth selection, high-stability oscillators, and optimized receiver design. Adaptive Coding and Modulation (ACM) was shown to be a key enabling strategy, allowing dynamic trade-offs between throughput and robustness across frequency bands and availability targets. Overall, this chapter demonstrates that at high frequencies and low elevations, atmospheric phase stability—not link budget alone—becomes the fundamental limiting factor for coherent deep-space communications. The results motivate a multi-band, adaptive system architecture, with Ka-band as the primary high-throughput channel, X-band as a robust fallback, and W-band reserved for experimental or burst-mode applications. The exhaustive

numerical link budgets, statistical analyses, and mission-specific scenarios supporting these conclusions are provided in **Appendix R**, ensuring full traceability while maintaining clarity and conciseness in the main thesis body.



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Chapter 14

Scientific Synthesis, Publications, and Research Outlook

14.1 Unifying Scientific Claim and Scope of the Thesis

This doctoral thesis demonstrates that tropospheric radiowave propagation effects—attenuation, emission, and phase instability—constitute a first-order physical constraint for modern and future satellite systems operating above Ku-band frequencies. Contrary to their traditional treatment as secondary corrections in geodetic and telecommunication analyses, atmospheric variability is shown here to directly limit system performance, coherence, and availability, particularly at Ka- and millimetre-wave frequencies and at low elevation angles. The central scientific claim of this work is that tropospheric propagation phenomena can only be modeled consistently through a multi-scale framework linking hydrometeor microphysics, radiative-transfer physics, and system-level observables. Quantities such as brightness temperature, specific attenuation, zenith and slant delays, phase noise, and link margins are not independent metrics but manifestations of the same underlying atmospheric processes observed at different scales. Treating them separately inevitably leads to incomplete or misleading interpretations. To address this challenge, the thesis integrates physically based radiative-transfer modeling, observational benchmarking, statistical analysis, and physics-consistent machine-learning surrogates within a unified methodological framework. Classical formulations—such as ITU-R recommendations, Saastamoinen-type delay models, and standard radiative-transfer theory—are not replaced, but extended, validated, and coupled through the SNEM-2 model and its data-driven counterparts. This framework is systematically applied to Earth-observation, GNSS, and telecommunication scenarios, with particular emphasis on Ka-band operational relevance and on W-band as an exploratory regime revealing fundamental physical limits.



14.2 What Has Been Proven: Consolidated Physical Results

14.2.1 Tropospheric Structure, Delays, and Phase Variability

A first class of results concerns the characterization of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) on regional and global scales. The analysis confirms that ZHD is remarkably stable in time and space, being almost entirely governed by surface pressure and exhibiting variations at the millimetre level. In contrast, ZWD displays strong temporal and spatial variability driven by water vapor distribution, convection, and large-scale circulation patterns. These findings are validated against ERA5 reanalysis and, where available, in situ observations, demonstrating internal consistency across latitudes, seasons, and climatic regimes. Importantly, the results establish that the wet component of the troposphere dominates uncertainty in both delay and phase-related metrics, particularly at low elevation angles and high frequencies. This conclusion provides a physically grounded explanation for the observed sensitivity of high-frequency satellite links to humidity-driven atmospheric variability.

14.2.2 Radiative Transfer, Attenuation, and Brightness Temperature

At the core of the thesis lies the development and application of the SNEM-2 radiative-transfer framework, designed to simulate absorption, emission, and scattering by atmospheric gases and hydrometeors from Ku- to W-band frequencies. SNEM-2 explicitly accounts for phase transitions, melting-layer effects, and frequency-dependent extinction mechanisms, enabling realistic simulations under both clear-sky and precipitating conditions. The results demonstrate that Ka-band propagation remains operationally viable under most mid-latitude conditions when appropriate fade margins are adopted. Conversely, W-band propagation is shown to undergo a threshold-driven transition into regimes dominated by atmospheric attenuation and noise. At these frequencies, the troposphere acts not merely as a lossy medium but as an active radiative environment, fundamentally altering brightness-temperature statistics and link performance. In this sense, W-band is treated in the thesis not as a near-term operational solution, but as a diagnostic regime that exposes the physical limits imposed by the troposphere.

14.3 Phase Noise, Coherence, and System-Level Implications

Building on the delay and attenuation analyses, the thesis establishes a direct physical mapping between tropospheric path-length variability and carrier phase instability. A key result is the clear separation between geometric delay variability, which is weakly frequency-dependent, and phase variability, which scales linearly with carrier frequency. As a consequence, identical atmospheric conditions produce comparable path-delay fluctuations across frequencies, while inducing dramatically different phase excursions at Ka- and W-band. This distinction explains why high-frequency systems may appear stable when evaluated solely in terms of path delay, yet suffer rapid coherence degradation in practice. The analysis demonstrates that, at Ka-band,



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coherent operation remains feasible with realistic oscillator stability and loop-bandwidth selection, whereas at W-band the combined effects of atmospheric phase noise and elevated system noise temperatures impose severe constraints on continuous coherent demodulation. These findings have direct implications for deep-space telecommunications, very long baseline interferometry, and future high-frequency satellite systems. In all these contexts, atmospheric phase stability—not link margin alone—emerges as the primary performance bottleneck.

14.4 The Role of the Framework: Physics Before Models

Rather than proposing isolated models or incremental corrections, this thesis develops a coherent framework in which atmospheric propagation is treated as a coupled physical problem across scales. Classical empirical models provide valuable climatological baselines but lack the capability to represent threshold effects, mixed-phase processes, and radiative coupling. Conversely, purely data-driven approaches risk obscuring physical meaning if not constrained appropriately. The SNEM-2 framework addresses these limitations by preserving explicit physical mechanisms while remaining compatible with statistical and machine-learning extensions. Within this architecture, radiative transfer, delay modeling, and phase-noise analysis are not separate tools but components of a single modeling chain. This perspective justifies the breadth of the thesis and explains the necessity of detailed appendices, which provide full traceability without overloading the main scientific narrative.

14.5 Machine-Learning Surrogates: Role and Physical Consistency

To complement physically based simulations, the thesis introduces machine-learning surrogates based on LSTM and CGAN architectures. These models are explicitly designed to emulate SNEM-2 outputs rather than replace physical modeling. Their role is to act as computational accelerators capable of generating long time series and global fields while preserving physical structure. The results demonstrate that, when properly constrained, neural networks can reproduce not only the mean behavior of attenuation and brightness-temperature fields but also their statistical variability and threshold-driven features. Explainability analyses confirm that the dominant predictors identified by the networks correspond to physically meaningful atmospheric variables. In this framework, machine learning is positioned as an enabling technology for scalability and scenario exploration, not as an alternative to physics-based understanding.

14.6 Operational Meaning and Application Domains

The integrated modeling framework developed in this thesis has direct relevance for multiple application domains. In satellite communications, particularly at Ka-band, it enables more



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accurate estimation of fade margins and availability, accounting explicitly for atmospheric variability. In GNSS and geodetic applications, refined delay characterization improves calibration and interpretation of positioning data. In Earth observation and microwave radiometry, improved brightness-temperature simulations enhance retrieval consistency. For deep-space and Earth–Moon communications, the results support a multi-band strategy: Ka-band as the primary high-capacity channel, X-band as a highly robust backup, and W-band reserved for experimental or opportunistic clear-sky operation. The analysis of Near Rectilinear Halo Orbit (NRHO) relay architectures further demonstrates how propagation physics, system design, and orbital geometry must be considered jointly to achieve reliable long-duration missions.

14.7 Original Contributions and Publications

The research conducted during this PhD has led to multiple scientific publications documenting the evolution of the work from physical modeling to data-driven extensions. The original contributions of the thesis can be summarized as follows:

1. Development and validation of the SNEM-2 radiative-transfer framework for Ku- to W-band propagation.
2. Quantitative characterization of threshold-driven attenuation and brightness-temperature behavior at millimetre-wave frequencies.
3. Unified treatment of tropospheric delays, attenuation, and phase noise within a single physical framework.
4. Introduction of physics-consistent machine-learning surrogates enabling scalable propagation assessment.

While each publication addresses a specific aspect of the problem, the thesis provides the unifying scientific narrative and methodological consistency that connect them into a coherent body of work.

14.8 Limitations and Boundaries of Validity

The work presented here has clearly defined boundaries. SNEM-2 relies on Mie-based scattering assumptions and simplified hydrometeor representations; ERA5 reanalysis, while state-of-the-art, remains an approximation of the true atmosphere; and machine-learning surrogates are inherently limited by their training domain. W-band analyses are intentionally framed as feasibility and diagnostic studies rather than operational proposals. These limitations are explicit, controlled, and do not undermine the central conclusions of the thesis. On the contrary, acknowledging them strengthens the scientific credibility of the results and clarifies their domain of applicability.



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14.9 Research Outlook and Future Perspectives

The framework developed in this thesis naturally opens several avenues for future research. These include extension toward sub-millimetre frequencies, integration of physically informed neural networks, coupling with climate-change scenarios to assess long-term system resilience, and development of three-dimensional radiative-transfer simulations. Further work may refine the representation of mixed-phase hydrometeors and enhance coupling between atmospheric modeling and communication-system design. These developments build upon, rather than alter, the core conclusions established here.

14.10 Concluding Remarks

In conclusion, this thesis establishes that tropospheric propagation effects represent a fundamental physical constraint on modern and future satellite systems. By combining physically based modeling, observational validation, statistical analysis, and physics-consistent machine-learning surrogates, the work provides both scientific insight and practical tools for assessing and mitigating atmospheric impacts. The results confirm the operational viability of Ka-band systems, delineate the physical limits of W-band propagation, and deliver a scalable methodology for future studies at the intersection of atmospheric science and space communications.



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Part I

Appendices



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Appendix A



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This **Appendix A** includes a selection of original research articles written and formatted by the author during the doctoral program, with the specific aim of submission to peer-reviewed journals. These manuscripts reflect the core scientific output of the research and span the full methodological arc of the PhD—from theoretical modeling and satellite-based analysis to deep learning surrogates and attenuation climatologies. Each article was conceived, written, and technically developed independently, based on original code, datasets, and frameworks created by the author. While some manuscripts are still in preparation or under review, all are scientifically complete and structured for journal publication. Their inclusion here serves to consolidate the doctoral work into clearly defined, publishable units and to document the editorial evolution that accompanied the technical progress. Where applicable, earlier extended drafts or related technical documents are provided in the **Appendix** for reference and transparency.

A.0 – Fundamental Proposal Articles

(Journal-Ready Manuscripts Developed During the PhD)

The following documents represent the foundational research proposals developed during the doctoral program. Each of these articles was independently conceived, written, and technically developed by the author, with a dual purpose:

1. To explore emerging directions in radiative transfer, attenuation modeling, and data-driven emulation;
2. To structure self-contained contributions intended for submission to peer-reviewed journals or scientific conferences.

These manuscripts were designed as modular blueprints—each focused on a specific scientific question or methodological challenge that emerged during the broader research trajectory. While not all of them were finalized or formally submitted, they played a central role in shaping the direction, structure, and focus of the validated works presented in previous chapters and appendices.

Some proposals include extended derivations, exploratory simulations, or alternative formulations that were later refined and restructured into more concise technical articles. Their inclusion in this section aims to document the scientific and editorial development process, offering transparency into how the doctoral research matured over time. All documents presented here are original works, fully authored, edited, and formatted by the candidate, and are included for academic evaluation and archival purposes.

The following manuscript, titled “Global Mapping of Atmospheric Attenuation at Ku, Ka, and W Bands Using Multi-Satellite Coincidences”, is currently under peer review for publication in IEEE Transactions on Geoscience and Remote Sensing (TGRS). This version represents a self-contained scientific contribution independently developed during the doctoral research and submitted as a

full-length article. It has been included in this thesis solely for academic evaluation and archival purposes. While the manuscript integrates several key concepts developed throughout the PhD, it does not entirely overlap with the individual technical articles included in the **Appendices**. Those articles, derived from the broader research trajectory, address specific subtopics in greater detail and follow distinct editorial formats. The inclusion of both the submitted manuscript and related technical papers reflects the modular and evolving nature of the research. Each work retains its scientific autonomy and has been developed by the author without external co-authorship or institutional drafting support.

A.1 – First Research Article

Title: Global Mapping of Atmospheric Attenuation at Ku, Ka, and W Bands Using Multi-Satellite Coincidences.

In peer-review.

Global Mapping of Atmospheric Attenuation at Ku, Ka, and W Bands Using Multi-Satellite Coincidences

Paolo Cicolin, *Student Member, IEEE*.

Abstract—A scalable framework is presented for the generation of global atmospheric attenuation maps using satellite coincidence datasets, including GPM-CloudSat and CloudSat-TRMM observations. The methodology introduces a novel masking and normalization strategy that enables the creation of preliminary non-attenuated global maps without the need for complex initial modeling. This approach enhances the prediction of signal attenuation along communication paths, supporting improved link budget designs, optimal transmission planning, and future deep-space communication setups.

The framework emphasizes early-stage satellite data collection, with subsequent progressive corrections to enhance accuracy, while minimizing computational demands during initial processing. Future extensions will integrate machine learning models and AI-driven predictive analysis to enable dynamic updates and adaptation to changing atmospheric conditions. The proposed method demonstrates flexibility across Ku-, Ka-, and W-band frequencies and provides a solid foundation for scalable applications in climate science, 6G telecommunications, and satellite communications.

Index Terms—Atmospheric Attenuation; Satellite Remote Sensing; Path Integrated Attenuation (PIA); Global Attenuation Maps; Satellite Coincidence Datasets; Reverse Engineering Methods; 6G Communications; Ku-, Ka-, and W-band Propagation; Machine Learning for Remote Sensing; Climate Applications.

I. INTRODUCTION

The creation of a global atmospheric attenuation map, based on the analysis of CloudSat-GPM and CloudSat-TRMM satellite coincidences, represents a significant advancement for the scientific community. This innovative methodology enables the integration of data from multiple space missions, enhancing the understanding of global precipitation and its variables. The combined use of data from CloudSat, GPM, and TRMM provides more comprehensive and detailed coverage, which is essential for advanced climate and meteorological studies.

A. Historical backgrounds

The generation of global maps of geophysical quantities has a long history in the scientific literature. Since the earliest periods of recorded history, maps have been employed as a means of representing geographic and topographic information. The earliest known examples of cartography date back to ancient Babylon, where maps were created on clay

tablets to represent topographical features such as hills and valleys [24]. Over time, cartography evolved as a result of contributions made by Greek scholars such as Anaximander, Herodotus and Ptolemy, who introduced concepts such as latitude and longitude with the aim of more accurately representing the Earth. In the modern context, the generation of global geophysical maps has benefited greatly from the advent of satellites and remote sensing technologies. Projects such as the European Space Agency (ESA) GLOBCOVER have used satellite data to create detailed maps of global land cover, involving an international network of partners and generating substantial quantities of data. While not focused on atmospheric attenuation, GLOBCOVER exemplifies the feasibility of creating comprehensive global maps through international collaboration and the integration of vast volumes of satellite data.

B. State of art

At present, there is a lack of standardization and universal acceptance of global Precipitation Intensity Analysis (PIA) maps. Nevertheless, there are a number of initiatives and projects that are striving to develop comprehensive global precipitation maps through the utilization of satellite data and sophisticated climate models. To illustrate, the NASA and JAXA Global Precipitation Measurement (GPM) project furnishes comprehensive global precipitation data, which can be employed to construct precipitation intensity maps. Furthermore, the ESA's Copernicus programme furnishes satellite data that can be employed for climate and environmental analysis, including precipitation mapping. The success of these projects hinges on the collaboration of numerous space agencies, research institutions, and international organizations, each playing a crucial role in data collection, processing, and analysis. A pertinent study in this field is the article "Global Distribution of Water Vapor and Cloud Cover-Sites for High Performance THz Applications" by **Suen et al.** (2013) [33], which employs high-resolution satellite data to map the global distribution of precipitable water vapor (PWV) and cloud cover. This work provides valuable information for the identification of sites with low atmospheric attenuation, which are essential for communications in the terahertz spectrum. Furthermore, the International Telecommunication Union (ITU) has developed recommendations, such as ITU-R P.525, which provide guidelines for the calculation of free-space attenuation. These guidelines are essential for the design of satellite communication systems.

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C. About PIA world map generation

Refs. [10]–[12] provide a robust basis for further investigation of Path Integrated Attenuation (PIA). In **Seto-Iguchi-Meneghini** (2022) [44], the impact of soil moisture on space-based radar precipitation measurements is examined utilizing data from the GPM dual-frequency radar mission. It was observed that surface back-scatter cross sections increased in the presence of precipitation, resulting in an underestimation of the path-integrated attenuation (PIA) in the Surface Reference Method or Technique (SRT) [13]. In order to correct for this effect, a correction method for PIA was developed and implemented in the standard DPR algorithm, resulting in an increase in surface precipitation rate estimates of 18% for KuPR and 15% for KaPR. The paper offers valuable insights for estimating the global path-integrated attenuation (PIA). The key takeaways are as follows:

- 1) **Soil moisture effect.** Increased soil moisture in the presence of precipitation can cause an underestimation of the PIA.
- 2) **PIA correction.** A correction method has been developed to consider the effect of soil moisture, improving precipitation estimates.
- 3) **Radar algorithms.** The standard DPR algorithm has been updated to include these corrections, increasing the accuracy of the measurements.
- 4) **Global analysis.** The analyzed data covers a period of six years for KuPR and KaMS and three years for KaHS, providing a global view of the PIA variations.

Meneghini et.al. (2000) [43] examine the precipitation radar (PR) on the Tropical Rainfall Measuring Mission (TRMM) and its use of the surface reference technique (SRT) to correct for radar attenuation due to rain. This technique entails a comparison of the radar return in areas devoid of precipitation with that in regions experiencing rainfall, with the objective of estimating the degree of attenuation. The reliability of this concept is contingent upon the surface type and the angle of incidence. The TRMM PR, operating at 13.8 GHz, was designed to achieve an optimal balance between spatial resolution and rain attenuation. The SRT technique has proven to be highly effective, particularly over oceans in moderate to heavy rain conditions. In **Durden** [15], the contribution of nonuniform beam-filling (NUBF) is discussed, which refers to the variability of rainfall within the resolution volume of a radar. The following points are of particular importance:

- **Rainfall Variability.** Rainfall can vary on scales smaller than the size of the radar footprint, causing errors in measurements.
- **Effects on Radar.** This variability can lead to significant errors in the measured reflectivity (Z) and path-integrated attenuation (PIA).
- **Increased Impact in Convective Rainfall.** NUBF is particularly significant in convective rain, where the variability is more pronounced.
- **Correction Methods.** Studies suggest that the use of reflectivity profiles can reduce errors caused by NUBF compared to traditional methods such as the SRT (Surface Reference Technique).

Robert Meneghini et al. [14] discuss the significance of path attenuation estimation (PIA) for the retrieval of precipitation parameters utilizing the dual-frequency radar (DPR) on the GPM satellite. The paper describes the implementation of the surface reference technique (SRT), which employs surface scattering properties to infer path attenuation through precipitation. The method has been developed in both single-frequency and dual-frequency versions, with the latter offering greater accuracy at moderate rainfall rates. The single-frequency Ku-band approach is employed when Ka-band data are unavailable. The article is of value in the creation of a global PIA map, as it provides methods and techniques for estimating path attenuation using dual-frequency radar data, which can be applied on a global scale to enhance the precision of precipitation measurements.

D. Purpose

Here we develop a novel software solution for the generation of a global atmospheric attenuation map utilizing satellite reanalysis data for deep space communications. We examine the approach's practical implementation using reverse engineering methods (REm), which, as demonstrated below, significantly speeds up the computational process. This effort requires significant coordination and the utilization of advanced computing resources to achieve accurate and useful results for the scientific community. The creation of such a map is crucial for enhancing the accuracy of data by harmonizing information from various sources, which reduces discrepancies and ensures comprehensive global coverage. Utilizing advanced computing resources allows for the efficient processing of vast amounts of data, including diverse climatic regions. Accurate and detailed attenuation maps are invaluable for researchers studying atmospheric conditions, climate change, and communication systems, providing a reliable foundation for scientific analysis and modeling.

Additionally, high-quality attenuation maps contribute to better forecasting and prediction models, essential for preparing for and mitigating the impacts of atmospheric disturbances on deep space communications. Advanced computing resources optimize the use of available resources and accelerate the research process, making the creation of a global atmospheric attenuation map a collaborative scientific endeavor that integrates cutting-edge technology and coordinated efforts to produce results that are both accurate and beneficial for the scientific community.

E. The role of the A-Train constellation and remote sensing satellites in the study of global climate

Remote sensing refers to the acquisition of information about the Earth without direct contact, using sensors on satellites or aircraft. This technology is fundamental to the monitoring of climate change, the forecasting of extreme weather events and the study of a range of natural phenomena [29], [30].

The A-Train, or Afternoon Train, is a constellation of Earth observation satellites that orbit in close formation, thereby

enabling near-simultaneous observations from a variety of instruments. This constellation includes satellites such as Aqua, CloudSat, CALIPSO, Aura, GCOM-W1, and OCO-2 [1], [26].

The close coordination between these satellites allows for the collection of complementary data, thereby enhancing our understanding of the Earth system. CloudSat represents a pivotal component of the A-Train constellation [27], having been launched in 2006. It employs sophisticated radar technology to investigate the internal structure of clouds. The data provided by this satellite have been instrumental in advancing our understanding of tropical cyclones and cloud-related climate change. CloudSat has provided quantitative estimates of global snowfall and has conducted studies into the effects of pollution and other atmospheric emissions on precipitation [28].

Satellite coincidence product is defined as data obtained when two or more satellites observe the same area simultaneously. These products are of great importance as they permit the combination of data from disparate sources, thereby enhancing the accuracy and completeness of observations. For instance, the coincidences between CloudSat and TRMM or GPM extend the capabilities of representing light precipitation and cold seasonal precipitation [31], [32]. These data are vital for advanced climate studies and improving weather models.

In conclusion, A-Train and remote sensing satellites, such as CloudSat, play a pivotal role in advancing the understanding of global climate and precipitation. Satellite coincidence products enhance the value of these observations by providing integrated and more accurate data for scientific analysis.

F. Technical challenges and spatial resolution in creating global attenuation maps

The creation of a global attenuation map presents a number of technical and technological challenges, spanning a range of areas including programming and spatial resolution adaptation. In comparison to TRMM and GPM, CloudSat provides a more detailed resolution of the data. Furthermore, accurate georeferencing of the data is a prerequisite. The generation of the map necessitates the utilization of super-computing resources, given that the processing demands associated with the analysis of extensive satellite data sets exceed the capabilities of a conventional personal computer. It is therefore essential to have access to an adequate computing infrastructure in order to manage this complexity. The spatial resolution of CloudSat is approximately 1.4 km horizontally and 240 meters vertically in the W band. In comparison, the TRMM offers a resolution of 5 km for the Precipitation Radar and 4.3 km for the TRMM Microwave Imager (TMI). GPM has a spatial resolution of 5 km for the Dual-frequency Precipitation Radar (DPR) and 10 km for the GPM Microwave Imager (GMI). In the case of CloudSat-GPM satellite coincidences, the spatial resolution varies. In the W band, it is 1.1 km horizontally, while in the GPM it is 4-5 km. During the data alignment process, the nearest-neighbor method is employed in the background to circumvent the potential for spatial or temporal resampling artifacts [1].

This approach effectively combines the observations of both satellites while ensuring the integrity of the data is maintained.¹

The spatial resolution of the resulting map will be determined by the lower resolution between the two satellites when CloudSat and GPM data are recombined. Consequently, the degree of detail will be approximately 5 km. A resolution of 1.4 km is deemed to be more detailed. It can therefore be concluded that the practical resolution of the combined map will be closer to 5 km in order to ensure data integrity. The requisite resolution angle, necessary for the acquisition of cells with a $5 \times 5 \text{ km}^2$ area, can be determined through the application of the following formula $\theta = \frac{d}{D}$, where θ represents the resolution angle in radians, d denotes the cell size, and D signifies the distance between the satellite and the Earth's surface. Given a satellite altitude of 700 km, the resolution angle θ would be approximately 0.409 degrees. The angular resolution of a single cell in ERA-5, given its spatial resolution of 31 km, can be calculated as 0.0443 radians, which is equivalent to 2.54 degrees. It can thus be inferred that the cell resolution is approximately 6.20 times higher than that of ERA-5. In terms of temporal coherence, the TRMM and GPM missions were in operational overlap for a period of time, which allowed scientists to compare and calibrate data between the two missions. The aforementioned temporal overlap was of great importance for enhancing the coherence of precipitation data. During this period of temporal overlap, researchers developed algorithms for the calibration of TRMM data with GPM data, thereby ensuring the consistency of precipitation measurements between the two missions [2]. The Dual-frequency Precipitation Radar (DPR) on the Global Precipitation Measurement (GPM) satellite comprises two radars: one operating in the Ku band and the other in the Ka band. These radars are co-aligned on the GPM satellite bus, ensuring that the 5 km cell position on Earth is identical for both radars.

The temporal coherence between the Tropical Rainfall Measuring Mission (TRMM) and the Global Precipitation Measurement (GPM) mission enhances the accuracy and reliability of precipitation estimates, thereby improving the quality of data used for climate and meteorological studies. With regard to temporal coherence, the CloudSat radar operating in the W band, with a spatial resolution of 1.4 km, exhibits a coherence time of a few seconds for a single pixel. This is also the case with the GPM. Both satellites orbit at high speeds (7.5 km/s), which enables them to traverse areas of interest with remarkable rapidity. Based on these calculations, the estimated naked-eye visibility time of the satellite is approximately 93.33 seconds, determined by dividing 700 km by 7.5 km/s. In theory, a satellite in low Earth orbit (LEO) at altitudes between 700 km and 1000 km would be visible to the naked eye due to sunlight reflection. However, satellites like CloudSat, GPM, and TRMM are designed to minimize direct exposure to sunlight during measurements. This is achieved through the use of optical filters and shielding, which serve to reduce the

¹<https://gpm.nasa.gov/resources/documents/cloudsat-gpm-coincidence-dataset-version-1c>

impact of reflected sunlight on the sensors.

In conclusion, the concepts and fundamental steps required to obtain the raw attenuation map (without calculating correction contributions for attenuation from clouds, gases, etc.) have been presented. The focus then shifts to describing the methodology selected for the development of this work.

G. Methodology

The following methodology was utilized in the course of this research project:

- 1) **Dataset collection.** The initial stage of the process involved the collection of the data set. It is necessary to obtain the requisite datasets for the three bands of interest. CloudSat provides data in the W band, while the Global Precipitation Measurement (GPM) mission supplies information in the Ku and Ka bands.
- 2) **Inclusion of additional missions.** Incorporate data from other space missions, such as the Tropical Rainfall Measuring Mission (TRMM), to achieve comprehensive or near-comprehensive coverage.
- 3) **Precipitation data acquisition.** Utilize data from the Global Precipitation Measurement (GPM) mission to gather information on global precipitation, encompassing both liquid and solid forms.
- 4) **Identification of coincidences.** Determine the points of temporal and spatial coincidence between CloudSat and TRMM observations, as well as between CloudSat and GPM. This step is essential for correlating data from different missions. Subsequently, align the data spatially and temporally to ensure that the observations are superimposable, with each providing relevant information for the band of interest. This process may necessitate the use of data interpolation and validation algorithms.
- 5) **Extraction of PIA values.** Extract Path Integrated Attenuation (PIA) values under uncorrected measurement conditions. Further work is required to correct these values, taking into account specific global and local atmospheric conditions.
- 6) **Correlation of attenuation data.** Correlate the attenuation data with precipitation measurements from TRMM and GPM to gain a better understanding of how different weather conditions influence the attenuation values.
- 7) **Utilization of statistical modeling techniques.** Employ statistical modeling techniques to create an attenuation map that represents the 97th percentile of the observed attenuation conditions.
- 8) **Data fusion and kriging operations.** If necessary, perform data fusion or kriging operations to generate a comprehensive global map.
- 9) **Comparison with pre-existing models.** Compare the generated map with pre-existing models to verify its accuracy and reliability.
- 10) **Development and monitoring.** Ensure the virtuous development of computational tools and continuously monitor new observations to maintain the accuracy and relevance of the map.

II. REANALYSIS DATA AND THEIR AVAILABILITY

Satellite reanalysis data is a combination of observational measurements and simulation models that are used to create a consistent and continuous representation of atmospheric and climate conditions over time. These data are obtained through data assimilation techniques, which integrate satellite observations, weather station data, and other sources to produce comprehensive and accurate datasets [3]–[5]. These data are indispensable for a plethora of scientific and practical applications, rendering them an invaluable resource for researchers globally. The utilization of satellite reanalysis data for the CloudSat-GPM and CloudSat-TRMM coincidence datasets is crucial for enhancing our comprehension of global precipitation. These datasets integrate observations from CloudSat’s cloud profiling radar (CPR) with those from GPM’s dual-frequency radar (DPR) and TRMM’s precipitation radar (PR). This combination enables the generation of “pseudo-three-frequency” (W-band from CPR, Ku/Ka-band from DPR) and “pseudo-two-frequency” (W-band from CPR, Ku-band from PR) radar profiles, which are valuable for scientific investigations into precipitation microphysics, cloud processes, radiative transfer simulations, and sensitivity to light rain and snow.

Further details and options to download the CloudSat-GPM and CloudSat-TRMM datasets can be found via the CloudSat repository via SFTP. The data may be ordered and downloaded in accordance with the instructions set forth in the Orders -> SFTP Access section of the CloudSat website [6], [7]. The files pertaining to coincidences can be located within the community-products directory, specifically under the following file names: 2B-CSATGPM.V04 and 2B-CSATTRMM.V04. For further detailed information, readers are directed to another technical article in which we provide a more precise description of the steps and IT procedures [8]. The provision of satellite reanalysis data at no or low cost is a common practice with the aim of advancing scientific research and environmental monitoring. The European Space Agency’s Copernicus programme, for instance, offers free and open access to Sentinel satellite data via the Copernicus Open Access Hub [9]. This initiative facilitates data access for researchers, public institutions, and European projects, thereby contributing to a more comprehensive understanding of climate and environmental phenomena.

In addition to reanalysis datasets, satellite-based observations offer high-resolution measurements of hydrometeors and precipitation profiles, essential for validating and improving attenuation mapping techniques. **Table I** summarizes the key characteristics of the main satellite missions employed in this study.

Following the comparison of the key parameters of the CloudSat, TRMM, and GPM missions in **Table I**, it is evident that each satellite provides unique advantages and limitations for global hydrometeor attenuation mapping. While CloudSat operates at a high frequency (94 GHz) with high vertical resolution, its limited temporal resolution (16-day repeat cycle) may impact its ability to capture rapid changes in atmospheric conditions. In contrast, both TRMM and GPM offer more frequent revisit times (3-4 days and 2-3 days, respectively)

TABLE I: Comparison of key parameters for CloudSat, TRMM, and GPM missions employed in global attenuation mapping analyses.

Mission	Radar Frequency	Spatial Resolution	Temporal Resolution	Orbit Characteristics
CloudSat	94 GHz (W-Band)	~1.4 km horizontal, 500 m vertical	16-day repeat cycle	Sun-synchronous (1330 LT)
TRMM	13.8 GHz (Ku-Band)	~5 km horizontal	3–4 days revisit	Non-sun-synchronous, 35° inclination
GPM Core	13.6 GHz (Ku-Band) and 35.5 GHz (Ka-Band)	~5 km horizontal	2–3 days revisit	Non-sun-synchronous, 65° inclination

but operate at lower frequencies (Ku and Ka bands), which can provide valuable data for large-scale precipitation events but may be less sensitive to fine atmospheric structures.

Based on the satellite mission data in **Table I**, we proceed to develop a predictive model for estimating hydrometeor-induced attenuation. In the next section, we will describe the methodology used for combining reanalysis data with satellite observations to improve the accuracy and adaptability of global attenuation maps.

Despite the advantages of the mentioned missions, there are notable limitations that need to be considered when using these datasets for global attenuation mapping. The relatively coarse temporal resolution of CloudSat and the limitations in vertical profiling of GPM may lead to inaccuracies in regions where rapid meteorological changes occur. Furthermore, the differences in radar frequency between the missions necessitate adjustments in data fusion techniques to maintain consistency across different datasets.

In conclusion, the integration of these satellite missions with reanalysis data will pave the way for more accurate predictions of hydrometeor-induced attenuation. Future work will focus on optimizing data fusion techniques to combine high-frequency data from CloudSat with the broader coverage and higher revisit time of TRMM and GPM, ensuring robust predictions in various atmospheric conditions.

III. CHALLENGES FOR GLOBAL PIA MAPPING

A. Costs to generate the map

Several factors, including the desired resolution and the amount of data required, influence the total cost of producing a global Path Integrated Attenuation (PIA) map. For example, ESA's GLOBCOVER project, which produced a global land cover map, required the use of several satellite instruments and generated up to 20 terabytes of imagery. The cost of such projects can vary considerably, potentially reaching tens of millions of euros. The production of a global PIA map requires contributions from several satellites. And organizations would be essential. For example, the GLOBCOVER project involved ESA's Envisat satellite and an international network of partners, including UNEP (United Nations Environment Programme),² FAO (Food and Agriculture Organization of the United Nations), IGBP (International Geosphere-Biosphere Programme),³ and the European Commission's JRC. Similarly, the creation of a PIA map would require coordination between

²UNEP works to promote sustainable development, protect ecosystems and address global environmental challenges such as climate change, pollution and loss of biodiversity. It is a reference point for international environmental policy and assists countries in the sustainable management of natural resources.

³The programme formally ended in 2015, but many of its activities and discoveries continue through other global initiatives, such as Future Earth, which focuses on sustainability and climate change adaptation.

different space agencies, research institutions and international organizations to collect, process and analyze data.

B. Multidisciplinary teams and international collaboration for global PIA mapping

The production of a global Path Integrated Attenuation (PIA) map requires a multidisciplinary team of experts in meteorology, climatology, data engineering, satellite analysis and climate modeling. The number of people required can vary depending on the complexity and resolution of the map, but a project of this scale could involve several hundred people. This team would include:

- **Scientists and researchers.** Experts in meteorology and climatology who analyze the data and validate the results.
- **Data engineers.** Professionals specializing in managing and processing large amounts of satellite data.
- **Satellite analysts.** Technicians who work with raw satellite data to extract useful information.
- **Climate modelers.** Scientists who develop and use models to simulate and forecast weather events.
- **Support staff.** System administrators, project managers, and other logistical and administrative support roles.

International cooperation is essential for a project of this magnitude. It would require the involvement of several space agencies, research institutions and international organizations. Key players could include NASA, ESA (European Space Agency), JAXA (Japan Aerospace Exploration Agency) and other relevant institutions. Space agencies such as NASA, ESA and JAXA bring their advanced satellite technologies and operational experience, while institutions such as WMO (World Meteorological Organization) or regional meteorological agencies provide localized expertise and contextual knowledge. Academic and research institutions contribute innovative methodologies and provide critical validation. This collaboration ensures that the project is not only scientifically rigorous, but also globally representative, reflecting diverse environmental conditions and precipitation characteristics.

C. Global Path Integrated Attenuation and Global Precipitation Intensity Map

The production of a global Path Integrated Attenuation (PIA.1) map is a monumental task that requires extensive expertise and coordination across disciplines. A multidisciplinary team is the backbone of such an endeavor, integrating knowledge from different fields to ensure the accuracy, efficiency and utility of the final product. At the core of the team are scientists and researchers specializing in meteorology and climatology, who analyze the complex dynamics of precipitation patterns and validate the data. These experts work closely with data engineers whose role is to manage and process the vast amounts

of satellite data generated during the project. This includes cleaning, organizing and optimizing the datasets to ensure they are suitable for analysis. Meanwhile, satellite analysts focus on interpreting the raw data collected by satellite sensors, extracting meaningful information such as precipitation rates and their geographical distribution. Another critical group are climate modelers, who develop sophisticated models to simulate and predict precipitation events and relate them to attenuation. These models are essential for integrating observational data with theoretical frameworks to enable prediction and long-term analysis. In addition, the smooth running of the project relies on support staff, including system administrators who maintain the computing infrastructure, project managers who oversee schedules and budgets, and administrative staff who handle logistics and coordination. By bringing together this diverse range of expertise, a global PIA map can serve as a critical tool for telecommunications in the 6G era, and also enable the creation of a global Precipitation Intensity Map (PIA.2) for understanding precipitation dynamics, improving weather forecasting, managing water resources, and preparing for extreme weather events associated with climate change. Collaboration across disciplines and nations not only enhances scientific output, but also strengthens global partnerships to address pressing environmental challenges.

IV. SOFTWARE DEVELOPMENT METHODOLOGY AND STRATEGY

A. Global Path Integrated Attenuation (PIA) maps

The generation of a requires the utilization of sophisticated software and a meticulous methodological strategy. In the initial phase, data from satellites such as CloudSat, TRMM, and GPM, which offer varying spatial and temporal resolutions, should be collated and stored in a centralized database for convenient access and management. The data must then be subjected to a process of pre-processing, whereby georeferencing techniques are employed to accurately align the satellite data and filters are applied to remove any anomalies, thus improving the overall quality of the data. The employment of sophisticated algorithms, such as the Surface Reference Technique (SRT) and the utilization of dual-polarization radars, is imperative for the rectification of radar signal attenuation resulting from precipitation. Furthermore, the incorporation of machine learning algorithms, such as the Light Gradient Boosting Machine (LightGBM), can facilitate the enhancement of attenuation estimates based on environmental variables. The Light Gradient Boosting Machine (LightGBM) can be employed to enhance the quality and precision of a global Path Integrated Attenuation (PIA) map of attenuation due to its capacity to process copious amounts of data, discern intricate relationships, and generate precise estimates. Data from diverse satellite sources (e.g., CloudSat, GPM, CALIPSO) and environmental variables (e.g., temperature, humidity, pressure, and precipitation intensity) can be used to train LightGBM.

The model's capacity to process heterogeneous datasets enables the generation of a more robust analysis, whereby the integrated information facilitates enhanced attenuation estimates. One of the primary challenges in the creation of a PIA

map is the correction of radar signals that have been attenuated by the presence of precipitation. In comparison to conventional techniques such as empirical or simple regression, the LightGBM algorithm enhances the precision of the results by modeling the non-linear relationship between environmental variables and signal attenuation. The Light GBM algorithm is a valuable tool for anomaly detection and correction, as well as for making localized and adaptive estimates during the pre-processing of satellite data. Indeed, during the data processing stage, anomalies resulting from measurement errors, atmospheric disturbances, or instrumental issues may emerge. By identifying anomalous patterns in the data and replacing them with estimates generated by the model, Light GBM enhances the overall quality of the record. Statistical analysis techniques, such as principal component analysis (PCA) and time series analysis, are of great importance in the identification of patterns and trends in radar attenuation data, as well as in the comprehension of the spatial and temporal variability of attenuation. In light of the considerable complexity and volume of information, the utilization of supercomputers for data processing and the implementation of parallel computing techniques can markedly accelerate the process. Furthermore, the creation of interactive maps for the visualization of attenuation data on a global scale is a crucial step, and the comparison of the results with reference data and field measurements serves to validate the accuracy of the estimates.

Following the retirement of CloudSat, other satellites, including the Global Precipitation Measurement (GPM) and the Cloud-Aerosol Lidar and Infrared Pathfinder Satellite Observations (CALIPSO), have continued to undertake similar work, providing valuable data on the vertical distribution of clouds and aerosols [17], [18]. These satellites, in conjunction with other members of the A-Train constellation, facilitate enhanced comprehension of global climate and weather patterns. The generation of a comprehensive PIA map necessitates inter-agency collaboration between diverse space agencies, research institutions, and international organizations, with the sharing of data, computing resources, and technical expertise being pivotal for the success of the project [19], [20].

B. Data assimilation and Kalman filters

Advanced data assimilation algorithms represent a technique employed for the purpose of combining observations derived from disparate sources, including but not limited to satellites, weather stations, and climate models. The objective of this process is to obtain a representation of atmospheric and climate conditions that is both consistent and accurate. The deployment of these algorithms is vital to enhance the quality of weather forecasts and climate analyses. One of the most widely utilized algorithms is the Kalman filter, which updates the estimates of the state variables of a dynamical system in real time, integrating new observations as they become available. Another assimilation method is the method of reduced variances (4D-Var), which seeks to minimize the discrepancy between observations and model predictions, taking both past and future observations into account. A variant of

the Kalman filter is the Ensemble Kalman filter (EnKF), which employs a set of simulations to account for the uncertainty inherent in the model estimates. Finally, machine learning-based data assimilation utilizes machine learning algorithms to enhance data assimilation by identifying latent patterns and relationships in observational data. These algorithms, through the integration of disparate data sources and the utilization of sophisticated techniques, facilitate more precise weather forecasts and more comprehensive climate analyses, thereby making a substantial contribution to our comprehension of atmospheric dynamics.

V. PROPOSED APPROACH

The methodology proposed for the generation of a global Path Integrated Attenuation (PIA) map centers on the development of a comprehensive imagery database to support neural network training, with a focus on cataloging and collecting radar reflectivity profiles as vertical cross-sections of clouds. The process begins with the acquisition of satellite data from CloudSat, TRMM, and GPM missions, which offer diverse spatial and temporal resolutions. The collected datasets are systematically stored in a centralized database to facilitate streamlined access and management.

Following acquisition, the data undergoes a pre-processing phase involving georeferencing techniques to ensure accurate spatial alignment and the application of anomaly detection filters to enhance data quality. Advanced correction algorithms, including the Surface Reference Technique (SRT) and dual-polarization radar methodologies, are employed to mitigate radar signal attenuation effects attributable to precipitation. Furthermore, the implementation of machine learning algorithms, such as the Light Gradient Boosting Machine (LightGBM), enables enhanced attenuation estimation by leveraging environmental variables.

Statistical analysis is integral to this workflow, with techniques such as Principal Component Analysis (PCA) utilized to identify underlying patterns and trends within the radar attenuation datasets, and time-series analysis applied to assess the spatial and temporal variability of attenuation. Given the complexity and volume of the datasets, the employment of supercomputing infrastructure and parallel computing strategies is critical to achieving processing efficiency.

The overarching objective is to establish a robust connection between the analysis of cataloged satellite parameters and the microphysical and macrophysical properties of the atmosphere, with an emphasis on uncovering latent relationships within the data. Initial PIA estimations were based on sigma-zero (σ_0), and the focus later transitioned to direct PIA measurements associated with hydrometeors, thereby enabling more targeted global attenuation mapping.

Due to the high computational demands, emphasis was placed on the demonstration and validation of the methodology, with secondary priority given to corrections of radar reflectivity and PIA during initial global map generation. This strategic prioritization ensured the reproducibility and applicability of the methodology under constrained computational resources.

To expedite global attenuation map generation, CloudSat overflights and their coincident observations with GPM over specified temporal windows were analyzed. For instance, the methodology was applied to assess the attenuation patterns associated with a major extratropical cyclone observed over the North Atlantic in January 2016, providing a practical demonstration of the system's capability to capture spatial and temporal variations in precipitation intensity and hydrometeor distribution. The satellite ground tracks and swaths were used to ensure adequate global coverage, with daily CloudSat polar orbit tracks projected onto a global map. This mapping visualized uncorrected PIA values in the W-band (PIA(W)), which do not incorporate corrections for atmospheric gases such as water vapor and oxygen.

Through analysis of CloudSat-GPM coincidences, latitude, longitude, and uncorrected PIA values for the Ku and Ka bands were derived, with decoupling performed to separate matched values inherent in the coincidence products. PIA(W) values, corresponding to the hydrometeor-induced attenuation reported in the 2C-PRECIP-COLUMN product, were similarly extracted. Concurrently, sigma-zero (σ_0) values from the 2B-GEOPROF product were analyzed. Sigma-zero represents the normalized radar backscatter coefficient per unit area, serving as a critical parameter for characterizing surface and atmospheric reflectivity.

Path Integrated Attenuation (PIA) quantifies the cumulative radar signal attenuation resulting from absorption and scattering by hydrometeors and atmospheric gases along the signal's propagation path. Accurate estimation of PIA is essential for correcting radar measurements and enhancing the precision of atmospheric property retrievals.

A. Methodological steps

This work, together with the accompanying **Appendix Technical Review**, outlines the initial steps undertaken to develop a global attenuation mapping framework:

1) Dataset Collection:

- a) Acquisition of CloudSat data (W-band).
- b) Acquisition of GPM data (Ku- and Ka-bands).

2) Inclusion of Additional Missions:

- a) Integration of TRMM dataset for supplementary precipitation observations.

3) Precipitation Data Acquisition:

- a) Retrieval of GPM-derived global precipitation datasets, encompassing both liquid and solid forms.

4) Identification of Coincidences:

- a) Temporal and spatial synchronization between CloudSat, TRMM, and GPM observations.
- b) Application of data interpolation and validation algorithms to ensure consistency and accuracy.

5) Extraction of PIA Values:

- a) Initial extraction of Path Integrated Attenuation (PIA) values under uncorrected conditions.
- b) Subsequent correction of PIA values for global and localized atmospheric conditions.

6) Correlation of Attenuation Data:

- a) Correlation of extracted PIA data with precipitation measurements from TRMM and GPM missions.
- 7) **Utilization of Statistical Modeling Techniques:**
 - a) Generation of attenuation maps based on the 97th percentile of observed conditions.
- 8) **Data Fusion and Kriging Operations:**
 - a) Implementation of data fusion techniques and geostatistical Kriging operations to construct a comprehensive global attenuation map.
- 9) **Comparison with Pre-existing Models:**
 - a) Validation of the generated attenuation maps against established models to assess accuracy and reliability.
 - b) Application of reverse engineering (RE) methodologies on previously published works, with appropriate permissions.
 - c) Integration of machine learning techniques, such as deep learning architectures (e.g., convolutional neural networks) and ensemble methods (e.g., random forests and gradient boosting machines), leveraging RE-derived information to enhance modeling precision and robustness.
- 10) **Development and Monitoring:**
 - a) Continuous development and refinement of computational tools and algorithms.
 - b) Ongoing monitoring of new observational data, including the integration of real-time data streams and observations from next-generation satellite platforms, to enhance the adaptability, robustness, and responsiveness of the global attenuation mapping system, ensuring sustained model accuracy and operational relevance.

This structured methodology lays the foundation for the systematic generation, validation, and enhancement of global attenuation maps, providing valuable tools for atmospheric science and operational meteorology.

VI. RESULTS

Analyzing Path Integrated Attenuation (PIA) across different frequency bands provides critical insights into how distinct atmospheric components contribute to the overall signal attenuation. For instance, elevated PIA values observed in the W-band, relative to the Ku-band and Ka-band, typically reflect a greater impact from smaller atmospheric particles or gaseous constituents, which predominantly attenuate higher frequencies. Furthermore, these PIA maps facilitate the identification of regions exhibiting persistent or anomalous attenuation patterns, potentially signaling specific meteorological phenomena or unique environmental conditions. Such data is indispensable for refining radar-based precipitation measurements, enhancing the predictive capability of weather forecasting models, and deepening the understanding of complex atmospheric processes. Practical applications of these improved precipitation estimates include disaster management strategies, flood risk assessment, and aviation safety enhancements.

Uncorrected PIA maps serve as foundational datasets for developing and calibrating correction algorithms, particularly those accounting for attenuation by atmospheric gases. Implementing these corrections allows for more accurate radar system calibration and improved quantification of hydrometeor-related attenuation, directly benefiting real-time operational weather monitoring and enhancing the precision of severe weather alerts. This study utilizes uncorrected PIA maps derived from CloudSat and Global Precipitation Measurement (GPM) satellite coincidences to establish a preliminary dataset applicable to operational meteorology and atmospheric research. Reverse engineering methodologies and comparative analyses with previously published integrated attenuation maps are employed to validate and enhance this dataset.

The technical annex accompanying this research outlines the detailed procedure for deriving global Sigma-zero and PIA maps from published works through reverse engineering approaches, leveraging datasets produced by other researchers. It is important to note that formal consent from cited authors is required prior to publication to mitigate potential intellectual property conflicts and copyright infringements. Obtaining authorization, particularly for high-quality datasets, can significantly streamline the research process and reduce associated labor and costs.

The methodologies presented were initially developed and validated using standard research laboratory computational resources, rather than specialized supercomputing facilities typically required for large-scale data processing. By integrating machine learning techniques, such as convolutional neural networks (CNNs) and support vector machines (SVMs), alongside reverse engineering strategies, this research demonstrates a computationally efficient and cost-effective approach to constructing global attenuation maps, highlighting their practical effectiveness and scalability. Additionally, we introduce a practical method for analyzing satellite coincidence files, providing a robust framework for generating global attenuation maps that can accommodate incremental refinements and corrections over time.

A. Innovative Approach to Generating Global Attenuation Maps Using Satellite Coincidences

The analysis of satellite coincidences between CloudSat-GPM and CloudSat-TRMM introduces an innovative method for generating global attenuation maps. CloudSat's millimeter-wave radar supplies detailed profiles of clouds and measurements of water content, while GPM and TRMM deliver data on precipitation and brightness temperatures through their advanced instruments. These coincidences, where satellites simultaneously observe the same region of Earth, enable cross-validation of data and a comprehensive understanding of atmospheric phenomena. By applying machine learning and reverse engineering techniques to the combined data, it's possible to create maps that depict the decrease in radar signal intensity due to clouds and precipitation.

B. Starting from Coincidence Dataset from PPS

F. Joseph Turk et al. (2021) [1] present an extensive analysis of the Global Precipitation Measurement (GPM) Dual-

Frequency Precipitation Radar (DPR) operating in the Ku and Ka bands, alongside the CloudSat Profiling Radar (CPR) at the W-band. Their research emphasizes the datasets derived from satellite coincidences during the CloudSat–TRMM (CloudSat combined with PR/TMI) and CloudSat–GPM (CloudSat combined with DPR/GMI) observational periods. These coincidence datasets are accessible through NASA’s Precipitation Processing System (PPS) [34]. The integration of multi-frequency radar data with passive microwave observations offers substantial advantages for precipitation estimation applications. In this study, these satellite coincidences are utilized to extract and visualize Path Integrated Attenuation (PIA) data, facilitating the development of a comprehensive global attenuation map, which has significant implications for enhancing climate modeling accuracy and improving weather forecasting capabilities.

C. General Costs in Creating the Global Map

The development of a global attenuation map constitutes a sophisticated endeavor demanding substantial computational resources and financial investment. Key factors influencing this process include:

- 1) **Time Requirements:** The duration necessary for completing a global attenuation map is contingent upon factors such as desired spatial and temporal resolution, the volume and complexity of data to be processed, and the available computational capabilities. Typically, the entire workflow—encompassing data acquisition, comprehensive processing, validation, and analysis—may span several months to a full year.
- 2) **Financial Costs:** Project expenses can vary significantly depending on required resources, which include satellite data access fees, computational infrastructure such as dedicated servers or cloud computing services, and compensation for specialized technical personnel. As a general estimation, costs associated with such projects can range from tens of thousands to several hundred thousand euros.

It should be noted that these estimates are indicative, with actual costs and timelines subject to variations influenced by specific project objectives, technological requirements, and resource availability.

D. Describing the Algorithm to Generate Global Map in the three bands: Ku, Ka and W

The generation of a global attenuation map involves processing a substantial number of satellite coincidence files, a task made increasingly complex due to the potential occurrence of corrupted or unusable files. Given the daily generation of between five and eight files over multiple years, the cumulative workload quickly becomes substantial. Furthermore, processing these data files requires extensive memory resources, rendering rapid completion challenging. Data downloads via FTP for just one month’s dataset can necessitate several hours, and the reliability of BULK download services is not consistently assured.

In this study, datasets from January 2016, February 2016, and additional selected files from May 2016 were downloaded to assess the processing time and resource requirements comprehensively. This sample provided insight into the practical challenges and resource constraints associated with large-scale satellite coincidence data handling. Uncompressed CloudSat–GPM coincidence data for one month (31 days) occupy approximately 3.91 GB of disk space. Although each file averages around 16 MB, subsequent processing can require several hundred megabytes per file, significantly increasing storage and computational demands. These resource requirements complicate global attenuation map creation, particularly when accounting for necessary corrections and extended processing durations.

To mitigate these issues, it is essential to develop an optimized file-processing algorithm capable of cycling through the data efficiently, preventing memory saturation and ensuring uninterrupted real-time generation of radar reflectivity profiles and global PIA(Ku), PIA(Ka), and PIA(W) maps. Effective code optimization involves initially capturing variables of interest, such as PIA_hydrometeor and PIA(W), within unified data structures. Subsequently, values initially combined, such as PIA(Ku) and PIA(Ka), must be methodically separated and independently managed.

Dynamic data handling strategies are then employed to develop robust data structures that maintain critical georeferencing metadata, filenames, and associated measurement variables. By adopting this structured approach, a comprehensive data framework encompassing Latitude, Longitude, PIA(W), PIA(Ku), PIA(Ka), and additional key parameters can be effectively established. The inclusion of other relevant variables depends on specific analytical objectives; for instance, parameters such as the precip_flag, which classifies precipitation characteristics during storm events, can enhance the interpretative value of generated maps.

E. Working and Developing the Main Idea

The analysis of satellite coincidences between CloudSat and the Global Precipitation Measurement (GPM) mission offers a distinctive methodology for advancing global precipitation studies. While historical analyses using CloudSat–TRMM (Tropical Rainfall Measuring Mission) coincidences have significantly enhanced the understanding of tropical precipitation, the integration with GPM expands the analytical capability to encompass global coverage, including polar regions. CloudSat’s ability to accurately characterize the vertical cloud structure, combined with GPM’s precision in quantifying precipitation intensity, yields a comprehensive and detailed representation of atmospheric phenomena. This integrated approach significantly contributes to refining climate modeling and improving the accuracy of weather forecasting.

To construct the required dataset for global attenuation mapping, it is essential to download the satellite coincidence data. Although this downloading process is straightforward, it can be considerably time-consuming when relying solely on the STORM web interface or accessing data through the CloudSat web portal without utilizing bulk download

features. After data acquisition, files must be systematically organized according to timestamp, and a dedicated "working pool" should be established. Customized software, such as MATLAB scripts, is subsequently employed to effectively read, process, and analyze these files. Comprehensive details regarding dataset characteristics, file handling procedures, and software implementation strategies are thoroughly documented in the technical support annex accompanying this study.

F. Working Sample

Various programming methodologies can be employed for scanning satellite coincidence files, with the chosen techniques closely aligned with the intended applications of the extracted data. For example, when developing detailed case studies based on satellite coincidence products, it may be insightful to investigate scenarios analogous to those previously documented by researchers such as **J.F. Turk et al.**, **Haynes et al.**, or **Battaglia et al.**. In such cases, incorporating the source code to generate relevant imagery—such as Cloud-Mask, Sigma-Zero, or radar reflectivity data contained within the 2B-GEOPROF product—can significantly enhance analytical depth and interpretative precision.

However, when the primary objective involves creating comprehensive global maps directly, the results obtained from initial processing might diverge from expectations. For instance, processing CloudSat–GPM satellite coincidence data collected over one month, representing approximately 4.7 gigabytes of raw data, typically yields a preliminary, coarse-scale global attenuation map. Such initial visualizations require rigorous scrutiny to identify and correct potential anomalies, including unreadable data points, out-of-scale values, or detection errors.

Upon data cleansing and subsequent graphical rendering, the initial maps often fall short of professional-grade visualization standards. Achieving high-quality outputs mandates careful adjustments in image resolution to ensure compatibility with both the scale of representation and the selected geographic projection. Notably, the results obtained through this refined methodology surpass initial MATLAB-based experiments employing the 97th percentile approach. The choice of projection type also plays a critical role in accurately representing geographical data points, with the **Mollweide** projection frequently recommended due to its capability for minimizing distortions and effectively preserving area proportionality in global-scale analyses.

G. Taking action, understanding the expected values of global PIA

Before generating global maps of Path Integrated Attenuation (PIA), it is essential to thoroughly understand the inherent variability and contributing factors affecting PIA measurements across diverse geographic regions and meteorological conditions. A comprehensive grasp of these variations facilitates more accurate data interpretation and enables researchers to detect meaningful patterns and anomalies, which are critical for both scientific investigations and practical applications. By systematically evaluating maximum, minimum, and mean PIA values in various geographic and climatic zones, researchers

can better characterize the influences of local weather phenomena and specific radar configurations.

Understanding these regional and meteorological differences significantly enhances the accuracy and reliability of global-scale PIA representations. Such refined maps serve as invaluable resources for meteorologists, climate scientists, and researchers engaged in precipitation studies and atmospheric dynamics analyses.

Path Integrated Attenuation values exhibit substantial global variation across distinct climatic and geographical regions. In tropical areas, including the Amazon Basin and Southeast Asia, maximum PIA values frequently surpass 30 dB during intense rainfall events, while average attenuation typically ranges from 10 to 20 dB. In temperate regions, such as Europe and North America, severe thunderstorms can produce maximum PIA values between 20 and 25 dB, with typical averages ranging from 5 to 15 dB. Conversely, polar regions exhibit comparatively lower attenuation levels, with maximum values generally between 10 and 15 dB during snowstorm events, and average values commonly ranging from 2 to 8 dB. These variations are substantially influenced by radar beam-filling inconsistencies and differing scanning configurations, which affect radar signature detection and improve noise reduction through signal deconvolution processes.

H. Primary and secondary targets in Our analysis

By leveraging advanced data analysis methodologies applied to the CloudSat–GPM satellite coincidence datasets, this research aims to uncover previously unrecognized patterns and correlations in atmospheric processes. Our approach involves constructing a comprehensive database comprising radar-derived images, specifically cataloging vertical profiles of radar reflectivity to train neural network models effectively. The overarching objective is to bridge satellite-based parameter identification and classification with both microphysical and macrophysical atmospheric characteristics.

Initially, the estimation of Path Integrated Attenuation (PIA) relied primarily on Sigma-Zero data. Subsequently, the research shifted its focus towards direct analysis of hydrometeor-induced PIA. This methodological evolution enabled a more precise concentration on generating global-scale attenuation maps. Due to substantial computational constraints, the priority was placed on demonstrating and validating the underlying methodologies rather than performing exhaustive corrections to radar reflectivity data or the comprehensive correction of PIA itself. Consequently, specific corrections, such as attenuation caused by atmospheric gases like water vapor and oxygen, were not incorporated into the initial processing steps.

Nevertheless, the final goal remains the generation of accurate global attenuation maps, necessitating targeted corrections primarily to the PIA. The reasons for emphasizing PIA corrections include:

- 1) **Atmospheric Attenuation Correction:** Accurate estimation of PIA requires correcting for atmospheric gas attenuation, primarily due to water vapor and molecular oxygen, to ensure realistic characterization of signal path attenuation.

- 2) **Hydrometeor Attenuation Correction:** Correcting for attenuation introduced by hydrometeors (e.g., rain droplets, snowflakes, and ice crystals) is essential for accurately representing atmospheric conditions and improving the fidelity of attenuation maps.
- 3) **Validation of Corrected PIA:** Post-correction validation against independent measurements and reference climatological models is critical to confirm the accuracy and reliability of the attenuation corrections applied.
- 4) **Soil Moisture Impact:** Soil moisture influences radar signal reflection, potentially leading to underestimated PIA values if neglected. The integration of corrections accounting for soil moisture effects enhances the accuracy of surface precipitation estimates, as previously demonstrated in the literature [10].

Through these systematic corrections and validations, the research aspires to achieve robust and precise global attenuation characterizations, thereby supporting improved atmospheric modeling and satellite communication reliability. Future research may explore real-time prediction capabilities and integration with next-generation satellite systems to enhance forecasting accuracy and operational effectiveness.

I. Exploring first results

In this work, we optimized the source code to enable the efficient storage of key variables of interest, such as $PIA_{hydrometeor}$, PIA_W , PIA_{Ku} , and PIA_{Ka} , within a unified data structure. Since satellite coincidence files typically couple the values of PIA_{Ku} , and PIA_{Ka} , one of the initial operations involves the decoupling of these variables. After appropriately separating the coupled PIA values, we incorporated the PIA_W measurements into the structure.

Further technical details regarding the data handling procedures are provided in the annex accompanying this article. Following the decoupling and integration process, the data were dynamically processed to create flexible and scalable data structures while preserving georeferencing information, filenames of the scanned coincidence files, and the associated physical variables. As a result, we established a structure containing Latitude, Longitude, PIA_W , PIA_{Ku} , PIA_{Ka} , and additional relevant attributes.

It is important to note that the 2B-CWC-RVOD product group is typically associated with CloudSat–TRMM coincidences rather than CloudSat–GPM, a factor that must be considered when designing the reading cycle. The variable labeled “ PIA_{Fwd} ” represents the forward-path integrated attenuation, directly extracted from the 2B-CWC-RVOD product and expressed in decibels (dB).

The $PIA_{Forward}$ measurements are critical for several reasons:

- **Signal Weakness:** A weak radar signal, resulting from low particle density in clouds or precipitation, may lead to undetected or unreliable PIA Fwd values.
- **Atmospheric Conditions:** Specific atmospheric scenarios, such as thin cloud layers or very light precipitation, can hinder accurate path attenuation measurements.

- **Instrument Limitations:** The onboard radar instruments may exhibit sensitivity and dynamic range limitations, affecting the detection of extremely low or high attenuation events.
- **Calibration Errors:** Inaccuracies stemming from instrument calibration errors can impact the validity of PIA Fwd data, highlighting the importance of rigorous calibration procedures.

These factors collectively contribute to the occasional absence or uncertainty in PIA_{Fwd} measurements within CloudSat–TRMM coincidence datasets. A more detailed study on the utilization of PIA_{Fwd} can be undertaken by analyzing 2A product types rather than 2B coincidences. The 2A products, particularly in the latest V09 release, offer enhanced documentation and data structures that facilitate improved analysis. Although an in-depth discussion of these datasets is beyond the scope of this article, it is essential for researchers to be aware of their availability and potential applications.

Figures 1 and 2 illustrate the output of the data reading and storage algorithm. Further optimizations and refinements of these processes can be pursued to enable the generation of more sophisticated attenuation maps, comparable to those reported in **Battaglia et al. (2020)** [38], which are also included in this article for reference.

The maps and data presented by Battaglia are based on four years of observations collected by the NASA–JAXA Global Precipitation Measurement (GPM) mission, covering the period from June 2014 to May 2018. Consequently, the resulting attenuation climatologies and analyses of the diurnal precipitation cycle reflect atmospheric and climatic conditions during this interval.

Analyzing such an extensive volume of satellite data necessitates significant computational resources and storage capacity. Key considerations include:

- **Analysis Time:** The total processing time is highly dependent on the available computational infrastructure. The **University of Leicester’s** ALICE high-performance computing system, for instance, has been instrumental in accelerating the data processing stages.
- **Disk Space Requirements:** The storage requirements vary based on data resolution and product types. Processing GPM Level 2 and Level 3 products over multi-year periods can demand several terabytes of disk space.

Fig. 4 presents an initial global visualization of hydrometeor-induced PIA derived from CloudSat–GPM coincidence datasets at W-band, using a Mollweide projection. The current color scale extends from -20 dB to 40 dB; however, to ensure a physically meaningful representation of atmospheric attenuation, the lower bound should be corrected to 0 dB. This adjustment will remove nonphysical negative values, thereby improving the accuracy and interpretability of the map.

Despite the sparse satellite overpass coverage during the considered period, the figure demonstrates the feasibility of constructing preliminary PIA distribution maps. Future improvements include dynamic aggregation of multi-temporal passes and refined interpolation techniques to enhance spatial continuity. Such maps are crucial for characterizing

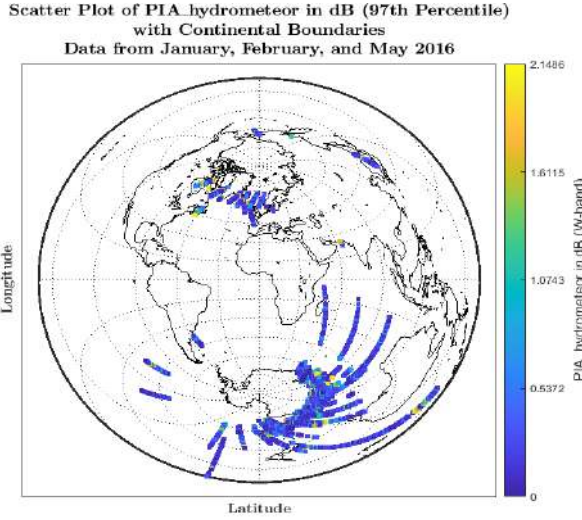


Fig. 1: Improved map projection applied to the CloudSat–GPM satellite coincidence dataset at W-band.

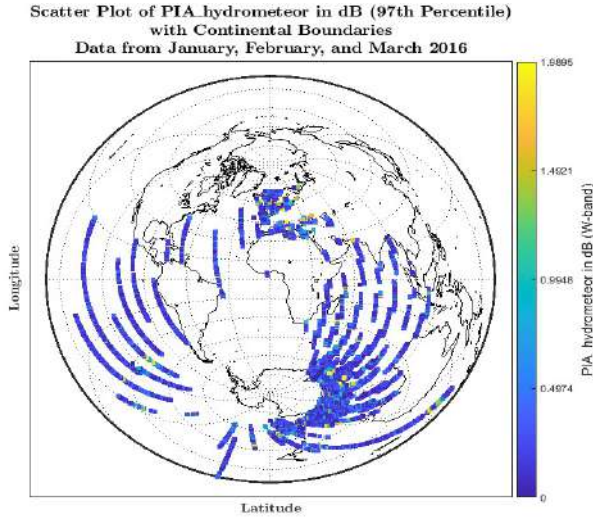


Fig. 2: CloudSat–GPM satellite coincidence dataset at W-band, covering overpasses from January to March 2016.

hydrometeor-induced signal degradation globally and provide a foundation for the development of predictive models leveraging machine learning techniques.

In **Fig. 4** negative PIA values may arise from retrieval uncertainties, noise in radar measurements, imperfect calibration, or algorithmic approximations. Such values are nonphysical, as true atmospheric attenuation cannot be negative; therefore, they are conventionally corrected by thresholding at 0 dB to preserve the physical interpretability of the data.

J. Discussion and Perspective

The intrinsic uncertainties associated with satellite measurements, including retrieval assumptions, sensor calibration drifts, and sampling limitations, must be carefully propagated through the data processing chain. Future methodologies

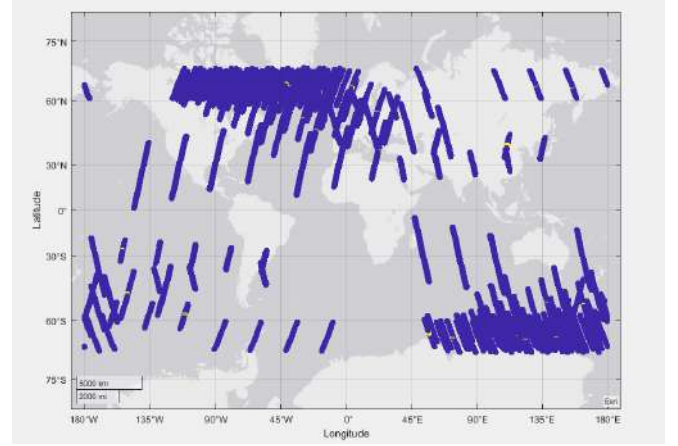


Fig. 3: Initial visualization of hydrometeor-induced Path Integrated Attenuation (PIA) at Ka-band, derived from the CloudSat–GPM satellite coincidence dataset. Overpasses from January–February and selected passages in May 2016 are indicated in violet, while valid PIA values (excluding -9999 flags) are highlighted in yellow.

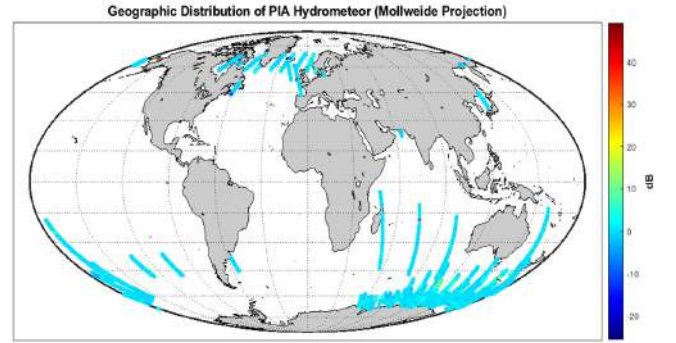


Fig. 4: Geographical distribution of hydrometeor-induced Path Integrated Attenuation (PIA) at W-band, derived from the CloudSat–GPM satellite coincidence dataset. Data correspond to overpasses recorded during January–February and selected passages in May 2016. To ensure physical consistency, the color range has been corrected to start from 0 dB, eliminating nonphysical negative values.

should prioritize uncertainty quantification to enhance the robustness of climatological analyses. Expanding the data integration across multiple satellite missions and frequency bands—including CloudSat, GPM, TRMM, and forthcoming missions like EarthCARE—represents a promising avenue for improving attenuation climatologies and advancing the understanding of precipitation microphysics across diverse atmospheric regimes. Furthermore, the emergence of artificial intelligence and cloud-based computing architectures offers unprecedented opportunities for the automation and scalability of satellite data processing workflows. Deep learning approaches, such as convolutional neural networks and spatio-temporal transformer models, could be leveraged to

develop predictive attenuation models capable of real-time performance, supporting operational forecasting and climate resilience initiatives.

K. Importance of Path Integrated Attenuation (PIA) and Considerations for Anomalies in Polar Regions

The Path Integrated Attenuation (PIA) induced by hydrometeors exhibits significant global variability, influenced by atmospheric conditions and the Earth's major climatic zones. In equatorial regions characterized by tropical climates, intense rainfall predominates, resulting in PIA values typically ranging from 5 dB to 30 dB, with exceedances above 30 dB during extreme weather events such as tropical cyclones, particularly at frequencies exceeding 10 GHz.

Arid and semi-arid regions, including deserts and steppes, generally experience low PIA levels, typically below 1 dB, with occasional increases to 1–3 dB attributed to dust or sandstorm events, particularly at frequencies above 20 GHz. In temperate zones, encompassing oceanic and continental climates, moderate PIA values are observed: typically 1–10 dB for rainfall, 0.5–5 dB for wet snow, and less than 1 dB for dry snow. Notably, hailstorms can induce significant attenuation, reaching values between 10 dB and 30 dB.

In polar regions, where precipitation predominantly occurs as solid forms such as dry snow and ice, PIA remains minimal, often below 1 dB, with slight increases (0.1–2 dB) during wet snow events. Mountainous regions above 2,000 meters exhibit low PIA for dry snow (<1 dB), moderate levels (1–3 dB) for wet snow, and potentially higher attenuation (2–15 dB) during heavy rainfall at lower altitudes.

Overall, PIA reaches its maximum in tropical regions due to the prevalence of intense rainfall and remains lowest in polar and arid regions, where precipitation is scarce. PIA constitutes a critical parameter for assessing the impact of atmospheric conditions on communication systems and meteorological monitoring. Higher frequencies are particularly susceptible to greater attenuation, and radar beams operating at lower elevation angles experience elevated PIA due to the increased atmospheric path length.

Seasonal variability, especially in polar regions, demands careful interpretation of PIA datasets. Sudden surges in PIA values—such as increases above 2 dB—may indicate anomalous meteorological events, including wet snowstorms or melting ice phenomena, with implications for navigation, logistics, and scientific operations. Similarly, atypical PIA spatial distributions may suggest the occurrence of unusual atmospheric processes or contamination from external sources such as dust or industrial pollutants.

Monitoring long-term PIA trends is crucial for detecting significant changes in climate patterns, offering valuable insights into global atmospheric dynamics. These datasets are indispensable for supporting informed decision-making in resource management, climate mitigation strategies, and operational planning. The continuous advancement of satellite-based sensors, ground-based radar networks, and international scientific collaborations remains essential to improving the accuracy, resolution, and timeliness of atmospheric attenuation

analyses, thereby enhancing our ability to forecast, monitor, and respond to evolving environmental conditions.

In addition to their meteorological significance, variations in PIA have critical implications for the design and operation of advanced communication systems, including satellite-to-Earth links, 5G/6G wireless networks, and autonomous navigation systems. Accurate characterization of atmospheric attenuation is essential for ensuring the reliability, efficiency, and robustness of high-frequency communication infrastructures, particularly in regions prone to extreme weather variability.

Moreover, future efforts should aim to develop integrated models that couple atmospheric propagation effects with surface processes, such as wet soil dynamics, vegetation moisture content, and seasonal snow cover variability. Such holistic modeling approaches are vital for improving the fidelity of attenuation predictions in diverse climatic and geographical contexts.

Finally, the application of artificial intelligence (AI) and machine learning (ML) techniques offers promising opportunities for dynamically predicting PIA variations based on large-scale satellite observations and reanalysis data. Advanced ML frameworks, such as convolutional neural networks and ensemble learning methods, could enable real-time forecasting of attenuation fields, supporting proactive resource allocation, disaster management, and climate resilience strategies.

L. Create and Plot Global Maps about Sigma-Zero, PIA

Although Sigma-Zero and Path Integrated Attenuation (PIA) do not inherently provide direct geolocation references, it is nevertheless possible to generate spatial distribution maps, particularly for satellites such as CloudSat, which operate in a nadir-viewing configuration. Thus, it becomes feasible to construct maps that represent the geographical distribution of Sigma-Zero and PIA values, allowing for the visualization of regions where atmospheric attenuation is most pronounced. By employing PIA estimates as proxies for precipitation intensity and integrating these with additional radar measurements, comprehensive precipitation maps can be developed, illustrating the spatial variability of rainfall across the Earth's surface.

To support this effort, various versions of georeferenced data tables were created by accumulating key variables into structured data formats:

- TGeoPIAKuKa(filename, latitude, longitude, PIA_MS PiaKu, PIA_MS PiaKa)
- TGeo(filename, latitude, longitude, PIA_MS PiaKu, PIA_MS PiaKa, Sigma-Zero, PIAhydro2way, PIAuncert2way)
- TGeoST(filename, latitude, longitude, along-track distance, time, PiaKu, PiaKa, Sigma-Zero, PIAhydro2way, PIAuncert2way)

Here, TGeoST stands for "Georeferenced Save Table," indicating that the processed data were stored in MATLAB structures and subsequently saved as .mat files for efficient access and retrieval.

Following the successful storage of code and data within these structures, it was observed that the nesting of large

datasets often posed challenges for direct graphical visualization within MATLAB. To overcome these limitations, 97th percentile maps were constructed, highlighting satellite overpass regions with elevated attenuation values. In MATLAB, square brackets were utilized for the definition of data structures, in contrast to the use of curly braces for cell arrays.

However, during the creation of PIA maps using the percentile approach, it became apparent that satellite overpasses within a one-week period were sparse and did not provide full global coverage. The inclusion of data from additional satellite missions, such as the Tropical Rainfall Measuring Mission (TRMM), could enhance spatial completeness, although managing CloudSat–TRMM coincidence datasets remains a labor-intensive task.

This work primarily aims to demonstrate the feasibility of constructing such global attenuation maps, consistent with methodologies reported in the literature, including the studies by **A. Battaglia et al.** [35], **S. Tanelli et al.** [36], and additional references to works by **A. Battaglia** [37].

It is important to note that the spatial resolution, swath geometry, and sensor sensitivity of missions such as CloudSat and GPM impose inherent limitations on the granularity and completeness of the reconstructed maps. These factors must be carefully considered when interpreting the spatial distribution of Sigma-Zero and PIA values. Furthermore, the accumulation of measurement uncertainties, interpolation artifacts, and temporal sampling variability can propagate errors into the final products. Quantifying and mitigating these uncertainties represents a key challenge for future developments.

Looking ahead, the integration of advanced interpolation methods and machine learning techniques, such as supervised regression models and geostatistical kriging enhanced with neural networks, offers a promising path toward improving the reconstruction of continuous attenuation fields from sparsely sampled satellite data. These approaches could significantly enhance both the spatial resolution and the predictive capabilities of future attenuation climatologies.

Thus, while the presented approach demonstrates feasibility, future advancements must balance computational efficiency, data fidelity, and methodological rigor to ensure that satellite-derived attenuation maps can reliably support high-accuracy climatological, hydrological, and communication studies.

In particular, the technique of applying the 97th percentile methodology is derived from **Battaglia et al.** [38]. In their work, the NASA–JAXA Global Precipitation Measurement (GPM) mission is described, emphasizing its dual-frequency radar and multifrequency microwave radiometer, which provide a unique opportunity to quantify atmospheric attenuation effects. Utilizing four years of GPM observational data, the study presents a global climatology of Ka-band attenuation induced by clouds and precipitation, with specific attention to the influence of the diurnal precipitation cycle.

The analysis reveals that regions of high attenuation closely correspond to established precipitation patterns, with significant attenuation observed across the tropical rain belt, the South Pacific Convergence Zone, and the storm tracks of the North Atlantic and Pacific Oceans. Furthermore, the diurnal cycle of attenuation is shown to be notably stronger over

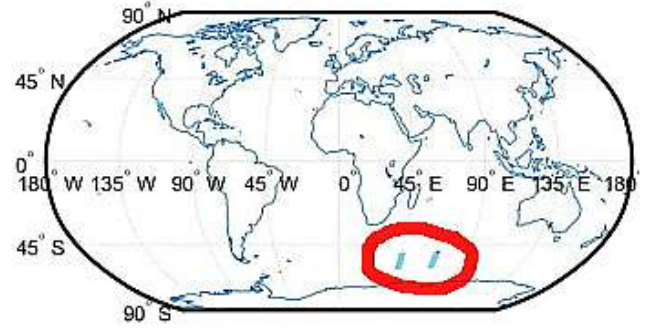


Fig. 5: Activation of pixels using the 97th percentile technique. The concept of percentile-based visualization on the global map is illustrated by the highlighted red circle in the image.

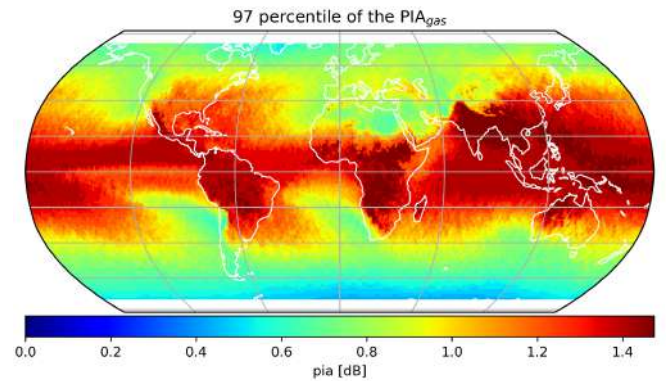


Fig. 6: Global map of $PIA_{\{gas\}}$ from Battaglia et al. (2020).

landmasses and during the summer season, while remaining weaker, yet still evident, over oceanic regions. These findings are critical for the estimation of data loss in satellite missions operating at frequencies near 35 GHz, where cloud and precipitation-induced attenuation effects become increasingly significant.

The maps presented by Battaglia are generated through the integration of multiple Global Precipitation Measurement (GPM) products to compute the various contributing factors. Distinct color scales are applied in each plot to accurately represent the corresponding data and ensure clarity in the visualization of different physical parameters.

Inspired by the methodologies presented in this work, it is feasible to construct similar global attenuation maps, although the process can be burdensome, particularly given the extensive data volume required when downloading four years of satellite data via FTP. Technical details regarding the use of Path Integrated Attenuation (PIA) can be found on page 1813 of the article "GPM-Derived Climatology of Attenuation due to Clouds and Precipitation at Ka-Band" by **Battaglia et al.** (2020) [38].

The generation of maps comparable to those presented in Battaglia's work is a nontrivial task. Employing techniques

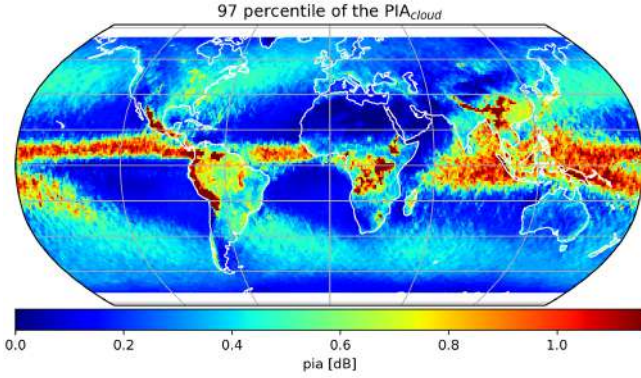


Fig. 7: Global map of PIA_{cloud} from Battaglia et al. (2020).

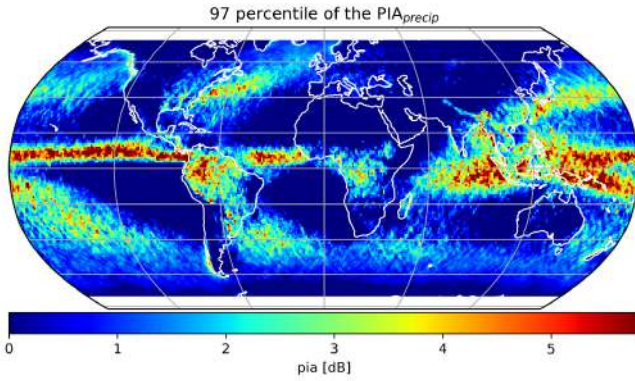


Fig. 8: Global map of $PIA_{precipitation}$ from Battaglia et al. (2020).

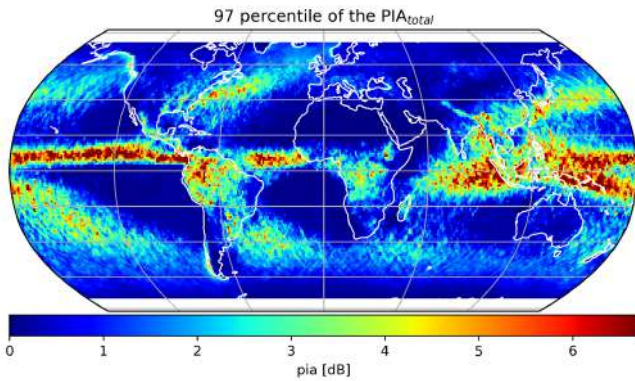


Fig. 9: Global map of PIA_{total} from Battaglia et al. (2020).

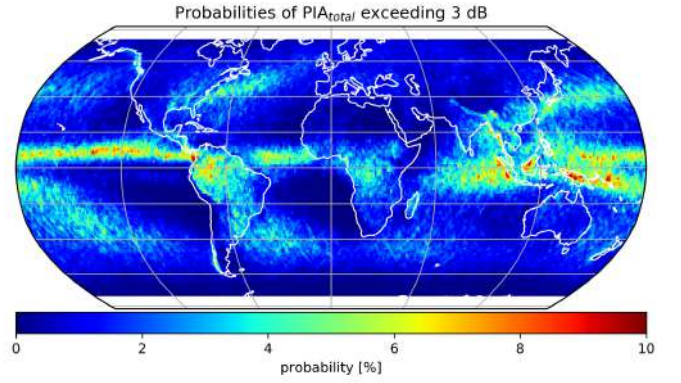


Fig. 10: Probability of two-way Path Integrated Attenuation (PIA) due to clouds and precipitation exceeding 3 dB, derived from combined DPR and GMI measurements. From Battaglia et al. (2020) [38].

such as the 97th percentile method (although alternative statistical approaches could be utilized) and analyzing CloudSat-GPM coincidence datasets for an entire month, such as January, reveals a critical limitation: the data sections are not uniformly distributed but instead follow the satellite swath patterns on the Earth's surface. This characteristic complicates the construction of continuous and complete global images, as desired in the final mapping products.

Recognizing and understanding these structural constraints is essential for properly interpreting and reproducing satellite-based attenuation maps.

In this context, the application of swath interpolation techniques becomes particularly critical. Interpolation methods allow for the reconstruction of continuous fields from irregularly spaced satellite swath data, mitigating the gaps between successive passes and enhancing the spatial continuity of global maps. Advanced interpolation approaches, including kriging, spline interpolation, and machine learning-based surface reconstruction, can significantly improve the accuracy and visual coherence of the derived products. In remote sensing applications, careful selection and validation of interpolation algorithms are fundamental to preserving the physical realism of the geophysical parameters being mapped, avoiding the introduction of artifacts, and ensuring robustness in subsequent analyses and modeling efforts.

M. Reverse Engineering exploring

Reverse engineering of published data, such as in the case of **A. Battaglia's** images [38], represents a powerful and versatile technique for extracting precise numerical information, particularly when the original datasets are not readily accessible. This methodology is especially valuable when working with scientific images and graphical representations, where color gradients may not intuitively convey quantitative information for human interpretation.

The advantages of this approach are considerable. Reverse engineering enables access to data in the absence of

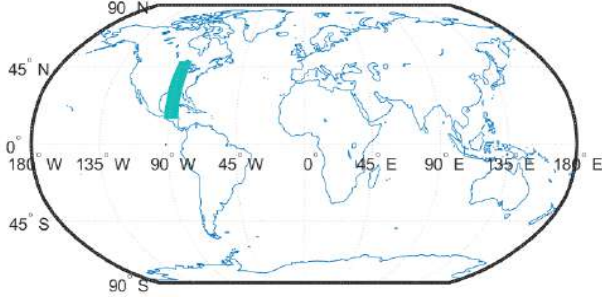


Fig. 11: Example of a satellite overpass from the file 2B.CSATGPM.COIN.33N_089W_00000_056_274_375.2016050 S075951-E080921.012364.V04.nc4. Visualization of the Sigma-Zero variable, with the satellite swath highlighted. The panel applies the 97th percentile lighting technique to enhance the visibility of the passage within the global map.

original datasets, thereby facilitating research, analysis, and comparative studies. Moreover, it provides an opportunity for researchers to enhance technical skills in image analysis and data processing. Its versatility allows application across a broad range of scientific and engineering domains.

However, several limitations and challenges are inherent to this technique. Published images are often subject to intellectual property protections, and unauthorized extraction of data may constitute copyright infringement. Therefore, it is imperative to proceed with caution, adhering to applicable legal frameworks and, where possible, obtaining permission from the original authors. In addition, image quality is a critical factor: low resolution, compression artifacts, and visual distortions can introduce significant errors into the extracted data, potentially compromising accuracy. Mastery of reverse engineering techniques requires advanced expertise in image processing, programming, and data analysis. It is essential to exercise ethical responsibility, respecting the original authors' intellectual contributions when utilizing extracted data.

Despite these challenges, the results obtained thus far in our project are encouraging, demonstrating that reverse engineering, when applied judiciously, is an effective strategy. Balancing technical innovation with respect for intellectual property rights remains crucial. Continued refinement and ethical application of these techniques promise to unlock new opportunities for scientific research and technological development.

To extract the $PIA_{gaseous}$ and PIA_{cloud} attenuation functions from graphical images within MATLAB, the following general steps are followed:

- 1) **Image Loading:** Utilize the `imread` function to import map images into MATLAB.
- 2) **Data Extraction:** If images employ color-coded numeric data, perform a color-to-value mapping by associating pixel colors with corresponding attenuation values.
- 3) **Function Generation:** With the extracted numeric data, construct functional representations of attenuation as a function of geographic coordinates.

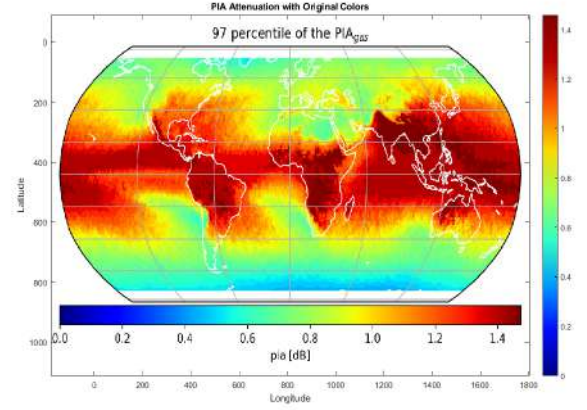


Fig. 12: Original Image from Battaglia's article (2020): Selected 97th Percentile Global Map of PIA_{gas} .

Path Integrated Attenuation (PIA) varies substantially with frequency band and atmospheric conditions. In the Ku-band (12–18 GHz), PIA typically ranges from 1 dB to 3 dB under moderate rain conditions, but may exceed 10 dB during heavy precipitation events. The Ka-band (26.5–40 GHz) exhibits higher sensitivity to precipitation, with typical PIA values between 3 dB and 10 dB, and peaks surpassing 20 dB under severe rain. In the W-band (75–110 GHz), attenuation is even more pronounced, with PIA values frequently exceeding 10 dB and reaching up to 30 dB or more under very heavy rainfall. For a more comprehensive analysis of attenuation behavior across frequency bands, readers are referred to the study by **Liao and Meneghini** (2019) [39].

N. Reverse Engineering on Global Maps

We illustrate the procedure for applying reverse engineering (RE) techniques. Further details regarding the utility of RE and the import process will be provided in the **Technical Annex**, where additional advantages and the corresponding source code implementation are described.

In the subsequent implementation corresponding to **Algorithm 1**, we present the corrected code for the reverse-engineered color map, optimized through RGB scouting on the original color scale depicted along the horizontal axis. The vertical axis represents the color map acquired through the Reverse Engineering (RE) process. Detailed technical documentation related to this implementation is provided in **Algorithm 1** and **Algorithm 2**, accompanying this work.

O. Techniques for creating a global maps of attenuation in Ku, Ka and W band with Reverse Engineering and Synthetic Considerations.

The objective of this discussion is to outline the methodology for generating global attenuation maps across three key frequency bands following the analysis of raw satellite data. The initial phase of this effort involves data simulation, necessitated by the aforementioned constraints in terms of time and computational resources. Reverse engineering techniques were

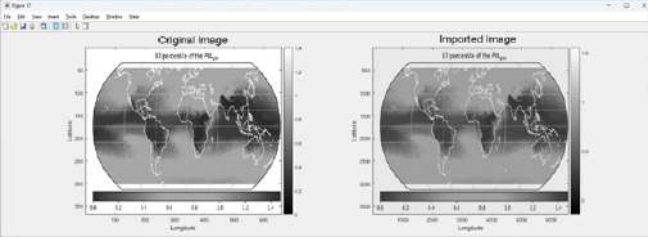


Fig. 13: Imported Version and Original Image Extracted from Battaglia et al. (2020).

applied to satellite imagery to extract numerical attenuation values over a four-year period, as described in **Battaglia et al. (2020)** [38].

The Path Integrated Attenuation (PIA) maps analyzed in Battaglia’s study are based on four years of Global Precipitation Measurement (GPM) mission data, spanning from June 2014 to May 2018, as specified in the Diurnal Cycle Analysis Results section. Technical documentation for the methodology is provided in **Algorithms 3** and **4** associated with this work.

Algorithm 3 processes map images by associating their color scales with corresponding numerical values (e.g., attenuation expressed in dB) while preserving the visual integrity of the original map. The procedure begins by loading the map image and identifying the yellow background through a defined color threshold and tolerance. Subsequently, the background is replaced with white while maintaining all other colors, including yellow shades, within the main map content. A color scale (e.g., 0–1.4 dB) is then established, and each pixel is mapped to the closest corresponding numerical value by calculating the color distance metric. The processed image is displayed, and an interactive functionality enables users to click on specific points to retrieve and visualize the corresponding attenuation value as a tooltip. The modified image is subsequently saved, ensuring both visual clarity and interactivity for future analytical use. In addition to reverse engineering Battaglia’s maps, this work initiates the discussion on methodologies for creating synthetic global attenuation maps from first principles. Such efforts are aimed at expanding the capability to simulate and analyze global atmospheric attenuation phenomena in the absence of complete observational datasets.

By looking at the correlation matrix between the images, it’s clear that the different attenuation parameters (PIA) for different parts of the atmosphere are connected in interesting ways. PIA_{gas} (Battaglia-01) exhibits a moderate correlation with the other parameters, indicating that the attenuation due to gas, which is typically uniform and dependent on atmospheric composition, does not vary significantly in relation to other components, such as clouds and precipitation. The highest correlation for PIA_{gas} is with PIA_{total} (Battaglia-04), but it remains relatively low (~ 0.5), suggesting that while the gas contribution is important, it is not dominant. PIA_{cloud} (Battaglia-02) shows a strong relationship with both PIA_{total} (~ 0.91) and PIA_{precip} (Battaglia-03) (~ 0.85). This implies that the signal loss due to clouds constitutes a significant por-

tion of both the total signal loss and the signal loss due to precipitation. Additionally, the correlation with PIA_{total} exceed 3 dB (Battaglia-05) (~ 0.84) underscores that regions with high attenuations (>3 dB) are heavily influenced by the presence of clouds.

PIA_{precip} (Battaglia-03) demonstrates a very high correlation with PIA_{total} (~ 0.92) and with PIA_{total} exceed 3 dB (~ 0.91), indicating that precipitation attenuation is a major contributor to the total attenuation and predominates in regions with high attenuations. Finally, PIA_{total} (Battaglia-04) is almost perfectly correlated with PIA_{total} exceed 3 dB (~ 0.91), suggesting that regions with significant total attenuation (>3 dB) represent a substantial fraction of the overall attenuation.

To sum up, clouds and rain have a big effect on total attenuation (PIA_{total}), but gas contribution (PIA_{gas}) is less variable and doesn’t matter much in places with high attenuations. As expected, areas with high attenuations (>3 dB) mostly show the effects of clouds and rain combining to lower the signal, acting almost exactly like PIA_{total} . The correlation matrix highlights that high attenuations are mainly driven by precipitation and clouds, while the gas contribution is relatively independent and constant. This suggests that in future analyses for the identification of critical areas for propagation (such as high attenuation), it is important to focus mainly on PIA_{precip} and PIA_{cloud} rather than on PIA_{gas} .

The following diagram illustrates the primary contributions from the various atmospheric components: The Path Integrated Attenuation (PIA) law can be expressed as a summation of the primary contributions from several atmospheric components. This is feasible due to the relationship between the parameters. We can specifically express the total PIA as follows:

$$PIA_{total} = PIA_{gas} + PIA_{cloud} + PIA_{precip}$$

PIA_{gas} is the attenuation due to absorption by atmospheric gases, such as oxygen and water vapor. This contribution is relatively uniform and less variable than the other terms. PIA_{cloud} represents attenuation due to the presence of clouds, which include liquid water particles and ice crystals. This term has a strong correlation with the total and contributes significantly to the high attenuations. PIA_{precip} is the attenuation caused by precipitation, which includes rain, snow, and hail. In regions with high attenuations, this term dominates and exhibits a strong correlation with the total.

Increase for high attenuations, $PIA_{total} > 3$ dB: when the total attenuation goes over a significant threshold, like 3 dB, PIA_{total} is mostly made up of PIA_{cloud} and PIA_{precip} working together, while PIA_{gas} stays the same. In summary, the general PIA law takes into account all the main atmospheric contributions, but the relative weight of each term may vary depending on atmospheric and geographical conditions.

P. Efficiency in Reverse Engineering Techniques

To illustrate the value of reverse engineering in the creation of global attenuation maps, Battaglia’s study presents **Figure 3** [38], which projects GPM-derived data over a four-year period, spanning from June 2014 to May 2018. The process of working with CloudSat–GPM satellite coincidence datasets,

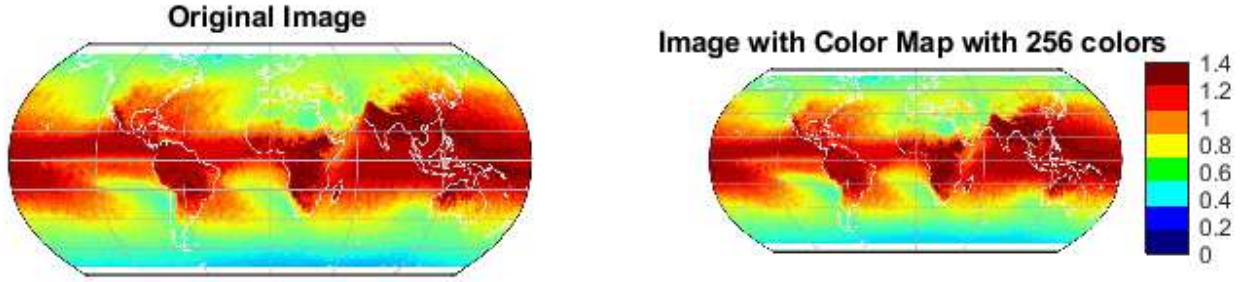


Fig. 14: The importing result.

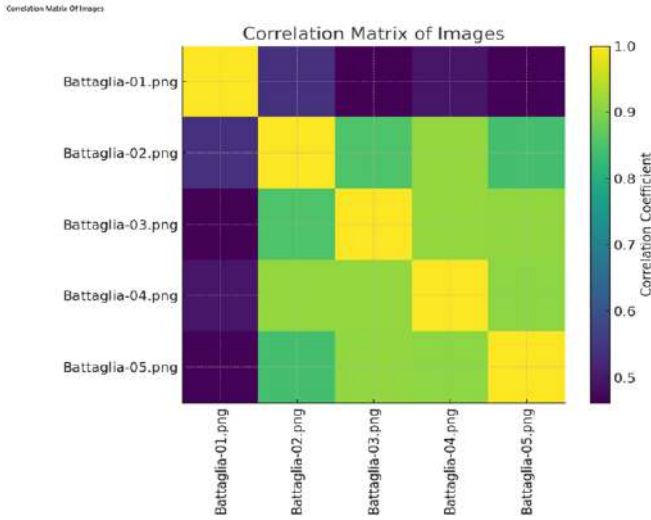


Fig. 15: Correlation Matrix for Battaglia's Images.

particularly in the absence of FTP or BULK download support, often resulted in significant delays. On average, downloading a single file from home required approximately 15 to 20 minutes, occasionally extending to half an hour or even several hours per file, excluding the additional time necessary for subsequent data processing.

On a daily basis, the number of files to be downloaded ranged from 5 to 8, imposing a considerable hourly commitment. Over a four-year period, assuming 365 days per year, the cumulative number of files required for comprehensive coverage was approximately 11,680, with individual file sizes varying between 16 MB and 25 MB. This extensive data acquisition effort demanded substantial resources in terms of both time and human labor. As a specific example, the V04 CloudSat–GPM coincidence dataset, covering the period from January to March 2016, was downloaded from NASA web services and occupied 14.1 GB of disk space.

Even assuming an optimized download rate of 1 minute per file, the cumulative time required would amount to 11,680 minutes. In contrast, the application of digitization techniques significantly reduces the time needed to construct global maps, offering evident efficiency gains. Under a worst-case scenario, with 2,920 hours allocated solely to downloading and an additional 64.89 hours for processing (at a rate of 20 seconds

per file), the manual workload reduction achievable through digitization is approximately 97.78%. This estimate does not account for further delays commonly encountered during periods of peak internet traffic, which frequently prolonged download times.

The utility of pre-generated maps thus becomes evident, as exemplified by the mere 13.37 seconds required to digitize Battaglia's global map for PIA_{gas} . Access to a larger corpus of images would further enable the deployment of machine learning techniques to enhance predictive modeling capabilities. For instance, effective estimations of PIA values were achieved by analyzing Sigma-Zero images produced by **Iguchi–Meneghini** and inferring real-world relationships. In conclusion, the digitization and automation of data acquisition and processing workflows not only reduce operational costs and time requirements but also open avenues for the integration of advanced analytical methodologies, thereby significantly improving forecasting accuracy and overall data analysis efficiency.

Q. Processing methods for generating Global Attenuation Maps

As an initial step, we describe the processing methods employed for the generation of synthetic global attenuation maps, beginning with the random generation of attenuation values across three frequency bands. This approach presupposes the feasibility of obtaining representative distributions at the highest frequencies under consideration. The construction of such representations requires extensive research capabilities and the formulation of hypotheses on a planetary scale, inherently involving both risks and potential advantages.

In the concluding section of this article, we address the underlying theoretical considerations, treating them as target objectives that the data scientist aims to achieve. The process begins with the generation of a synthetic global attenuation map for a specific frequency band, utilizing reference ranges documented in the scientific literature. MATLAB is adopted as the preferred platform for this operation due to its robust computational and visualization capabilities.

Initially, the global image is populated with random values assigned across all latitudes and longitudes, constrained within physically plausible ranges. This methodology allows for the illustration of the general procedure for generating global

Attenuation	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	31.7585	16.1045	13.7371	13.4627	11.8945	5.7800	22.8873	7.7827	15.4733	24.9369	10.4942	7.5342	33.9957	23.9365	7.9760
2	22.5585	10.4541	4.3374	3.1771	8.7378	1.4387	19.2111	2.7319	10.8929	22.0350	16.4396	27.6076	26.8927	16.1932	18.8930
3	11.7793	14.0797	12.7672	17.0992	20.9389	22.6481	19.7164	17.0404	28.8951	19.4549	20.6714	23.8935	8.7776	33.0437	9.6233
4	30.3870	21.2306	18.3837	13.1235	8.8700	5.8318	20.5829	6.7705	12.8825	0.9664	26.9793	10.7134	17.1421	12.8980	11.8844
5	26.2742	1.9255	1.7508	15.8887	32.2618	17.5365	32.9743	31.3366	4.4947	32.5779	32.5946	8.9344	8.1242	13.9700	8.1332
6	12.1832	20.0737	5.5194	10.0161	20.1964	2.6719	26.4387	28.6127	18.4004	33.3267	16.1972	5.2126	7.1182	26.9796	26.3361
7	34.2391	7.9125	5.2268	3.2313	23.9791	33.9913	12.8618	6.7954	6.9925	1.0760	15.2759	14.1635	7.7365	12.3252	12.8621
8	14.0094	38.4803	23.1133	18.9849	9.4012	12.6402	21.7711	6.7912	6.7070	11.8880	24.4931	13.8987	17.7021	16.2730	21.4544
9	21.5214	15.3678	11.3387	18.8925	7.2654	25.2779	18.9336	11.1703	26.5396	33.7622	21.7384	8.8302	4.8389	6.2890	1.8338
10	32.0037	7.7117	6.0057	11.3633	23.4816	16.3318	13.0200	16.6850	0.4890	33.4446	16.2740	17.5760	18.3486	15.6230	6.9357
11	2.4581	1.3325	25.3678	16.0364	19.1057	18.0369	16.0607	36.9798	6.1925	25.3884	1.6708	6.7022	23.1568	20.7430	26.8101
12	14.8817	20.5300	12.2116	8.1185	34.2504	26.8578	17.2664	26.7173	28.1418	11.2262	33.6235	19.5161	32.8168	4.3827	13.7154
13	6.3576	24.4671	38.7901	41.896	6.1321	36.9319	21.2240	1.6232	9.8925	32.4040	20.2557	19.6438	8.3488	3.7732	32.6238
14	21.4088	2.6628	13.3681	0.7543	24.2236	28.8734	19.0142	31.4805	25.5409	16.2134	31.3648	24.9996	11.3387	10.4281	30.0201
15	24.9391	6.8906	34.3532	25.1341	31.8951	6.8954	14.7195	2.8956	34.2771	7.2444	14.2645	8.7979	16.9188	7.3680	14.2164
16	2.8634	28.1451	5.3348	36.6396	9.8337	16.2995	26.3828	3.4910	7.7075	1.6218	6.2632	19.8484	17.9921	15.3245	18.8366
17	2.9812	32.8874	2.8716	25.7559	34.2056	12.2884	11.3817	14.8757	26.0004	11.7240	23.2699	1.6847	27.9171	25.6871	14.9314
18	11.5481	14.4393	33.7804	21.3104	1.2387	5.8982	16.9084	1.4487	27.7925	19.3086	20.0390	16.0175	32.1388	15.0342	28.8225
19	30.3381	5.5864	28.8127	23.8833	20.3302	18.3363	17.1879	27.1336	0.6238	27.1453	7.1775	24.6084	21.8883	23.2681	12.7248
20	16.6832	23.9590	7.9665	11.1332	20.7911	5.5016	12.9451	26.8198	19.2141	26.4039	7.1454	25.4908	28.4495	10.8077	1.2930
21	3.1399	15.1707	17.4324	2.2070	6.3761	11.4518	21.2629	26.6723	15.4885	17.0741	1.6370	14.7820	8.4656	10.1917	16.7190
22	9.9889	20.0522	22.1214	22.2621	18.8840	7.2087	3.7423	28.8225	20.8907	2.7028	33.4443	21.7207	23.3462	14.0888	1.1170

Fig. 16: Attenuation DataTable with fake data, (PIA(W) attenuation values are between 0-35 dB).

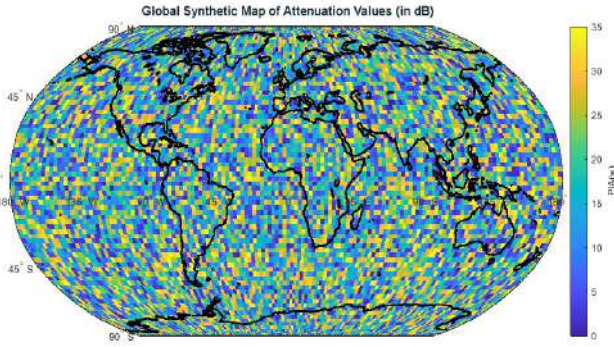


Fig. 17: Adjusting values in compatibility with PIA(W) values. Global Map of PIA(W) with fake data.

attenuation maps and is readily extendable to the processing of real observational datasets as they become available. Working with synthetic data enables preliminary experimentation and optimization of data presentation strategies, thereby refining methodologies prior to the analysis of real-world measurements.

For detailed information regarding the implementation procedures, readers are referred to the **Technical Annex** accompanying this article [8].

The detailed methodology for generating the synthetic global map, employing a cylindrical projection to simulate atmospheric signal attenuation values—such as Precipitation-Induced Attenuation (PIA) in the W-band—across the Earth’s surface is provided in the technical appendix accompanying this work.

Path Integrated Attenuation (PIA) varies predominantly with atmospheric conditions; however, geographic and anthropogenic factors can also exert indirect influences. In arid and desert regions, where precipitation levels are minimal and cloud formation is limited, typical PIA values range between 5 dB and 10 dB. In temperate regions characterized by moderate vegetation, such as plains and hills, PIA values are generally slightly higher, between 10 dB and 15 dB, as vegetation affects soil moisture levels and, consequently, the local hydrological cycle, promoting enhanced cloud formation.

Dense forests and mountainous areas exhibit elevated PIA

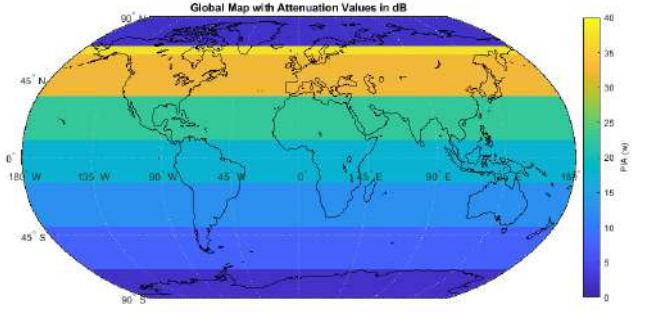


Fig. 18: PIA(W) attenuation for region of interest with plausible values.

values, typically between 15 dB and 20 dB, primarily due to vegetation transpiration and orographic lifting, which facilitate the ascent of moist air masses and enhance precipitation processes. In tropical regions, the synergistic combination of high humidity, dense forest cover, and intense meteorological phenomena often results in PIA values ranging between 20 dB and 25 dB.

Urban environments, in addition to experiencing episodes of torrential rainfall, are characterized by microclimates influenced by air pollution and anthropogenic heat emissions. Urban pollution elevates the concentration of condensation nuclei, thereby enhancing cloud formation and intensifying precipitation events, with PIA values reaching 25 dB to 30 dB. Moreover, the urban heat island effect can amplify convective thunderstorm activity, further increasing attenuation levels. Industrial areas and zones with high human activity can exhibit PIA values between 30 dB and 35 dB, particularly during extreme weather events when elevated concentrations of atmospheric pollutants and anthropogenic heat contribute to localized, intense precipitation.

Conversely, polar regions generally exhibit low PIA values, typically between 0 dB and 5 dB, due to the prevailing dry atmospheric conditions and the limited occurrence of liquid precipitation. PIA values exceeding 35 dB are rare and are primarily associated with exceptionally intense precipitation events, such as tropical cyclones or severe equatorial thunderstorms.

R. Contribute of Soil Moisture

Soil moisture contributes indirectly to atmospheric Path Integrated Attenuation (PIA), primarily by influencing the local microclimate and modulating the near-surface water vapor content. Saturated soils enhance evaporation rates, thereby increasing the concentration of water vapor in the lower atmosphere, which in turn leads to higher atmospheric attenuation at elevated frequencies, such as microwaves and millimeter waves. In addition, soil moisture modifies the reflective and absorptive properties of the Earth’s surface, affecting electromagnetic wave propagation through mechanisms such as reflection, multipath, and interference. Nevertheless, the overall contribution of soil moisture to PIA remains marginal

when compared to primary atmospheric factors such as precipitation, clouds, and aerosols. Its influence becomes appreciable only under specific environmental conditions, particularly in tropical regions or during seasonal thawing events in polar areas.

The impact of soil moisture on PIA exhibits considerable spatial variability across different microclimates. In general, however, its effect is secondary relative to dominant atmospheric constituents. In desert and arid regions, the contribution is negligible, resulting in an impact on PIA of less than 1%, owing to the near absence of soil moisture. In coastal and subtropical regions, soil moisture exerts a minimal influence, with a PIA contribution ranging between 1% and 5%. In temperate plains and hill regions, seasonal evaporation can elevate the local water vapor content, increasing soil moisture's contribution to PIA up to approximately 10%.

In dense forests and mountainous terrains, soil moisture plays a more substantial role, contributing between 10% and 15% to PIA, whereas in tropical and equatorial regions, characterized by saturated soils and persistent precipitation, its influence can reach up to 20%. Conversely, in polar regions and areas with permanently frozen soils, the contribution remains limited to less than 3%. In urban and industrial areas, the influence of soil moisture on atmospheric attenuation is marginal, typically ranging between 1% and 5%, with attenuation primarily governed by infrastructural and artificial obstacles.

S. Realistic Analysis

A detailed analysis of PIA-hydrometeor measurements over the polar regions and the global W-band attenuation map is presented in **Figs. 18, 19, 20, and 21**. These results were derived from observations conducted by the CloudSat and GPM satellite missions. Specifically, **Fig. 18** provides a depiction of the North Pole region, while **Fig. 19** focuses on the South Pole. **Fig. 22** presents a global map derived from CloudSat overpasses across the polar regions during January–February and selected passages in May 2016. **Fig. 23** illustrates a global W-band map generated from the CloudSat–GPM coincidence dataset over the same temporal window, utilizing the Mollweide projection.

Through the systematic scanning of coincidence files between CloudSat and GPM, these figures demonstrate the methodology for constructing detailed satellite-based attenuation maps. This approach yields critical data for investigating atmospheric behavior during specific periods. Moreover, the application of the Mollweide projection enables an effective global representation of the dataset, minimizing geographical distortions and enhancing the visualization and interpretation of large-scale observations. Such techniques are fundamental for improving the understanding of atmospheric dynamics and for advancing global weather and climate forecasting capabilities.

The detailed maps provided by these figures are crucial for a comprehensive analysis of atmospheric phenomena. The integration of PIA-hydrometeor measurements with the high precision of W-band observations facilitates a more accurate

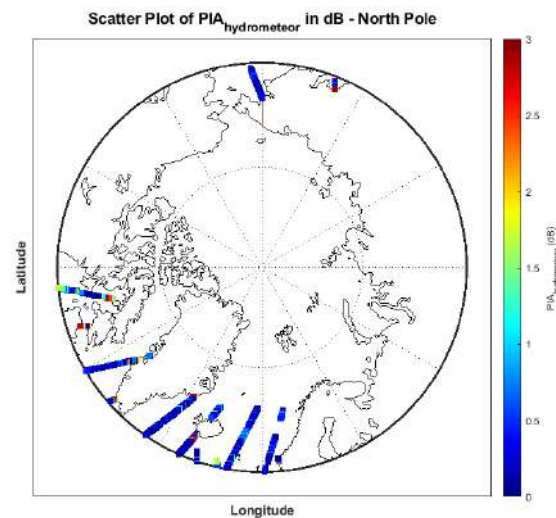


Fig. 19: North Pole vision about PIA-hydrometeor in the Global Map at W Band. (January-February and few passages in May 2016).

assessment of precipitation intensity and spatial distribution, particularly within polar environments. This information is essential for elucidating the role of polar meteorological processes in influencing broader climate dynamics.

Furthermore, the temporal coverage of the data, spanning February to May 2016, offers valuable insights into seasonal variability in atmospheric conditions. Capturing data across different periods enhances the ability to identify temporal trends and anomalies, which are vital for the development and refinement of predictive models. The use of the Mollweide projection, particularly evident in **Figs. 23 and 24**, is significant as it reduces distortions commonly encountered when mapping polar regions, thereby preserving the fidelity of spatial analyses and facilitating robust inter-regional comparisons.

Ultimately, the integration of CloudSat and GPM datasets through the coincidence scanning approach exemplifies the effectiveness of collaborative satellite missions in advancing the understanding of Earth's atmospheric systems. The generation of such detailed and accurate global maps is indispensable for researchers aiming to improve numerical weather prediction models, study the impacts of climate change, and develop effective strategies for mitigating adverse weather phenomena.

T. How change PIA(W) global attenuation with the Soil Moisture estimation contribute

In this paragraph, working with fake data I try to simulate the **Soil Moisture** contribute to the global PIA(W) attenuation.

The MATLAB code has been modified to incorporate the contribution of soil moisture across different territorial zones. Typical soil moisture values, expressed as a percentage of Volumetric Water Content (VWC), have been assigned for each zone to better reflect regional variability.

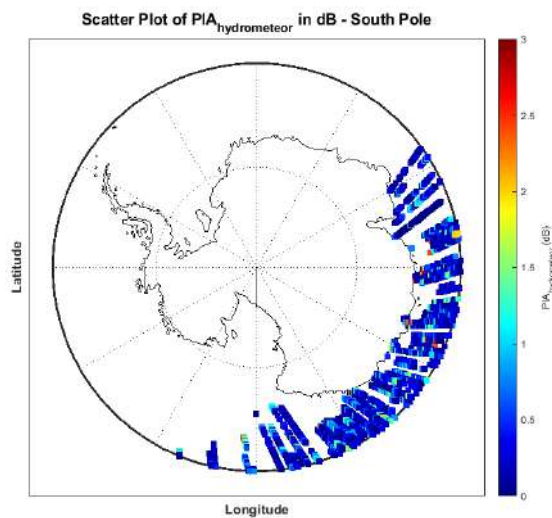


Fig. 20: South Pole vision about PIA-hydrometeor in the Global Map at W Band. (January-February and few passages in May 2016).

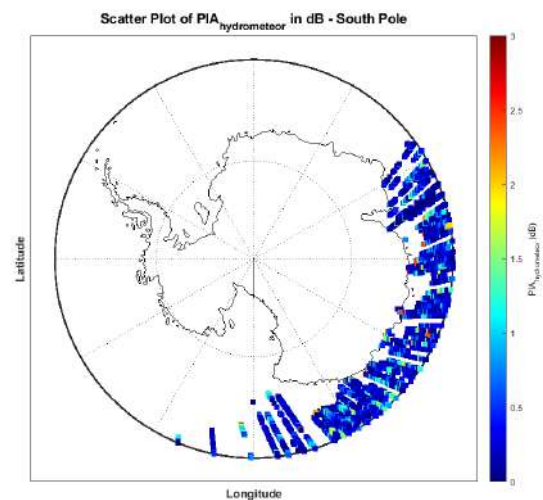


Fig. 22: South Pole vision about PIA-hydrometeor in the Global Map at W Band. (January-March 2016).

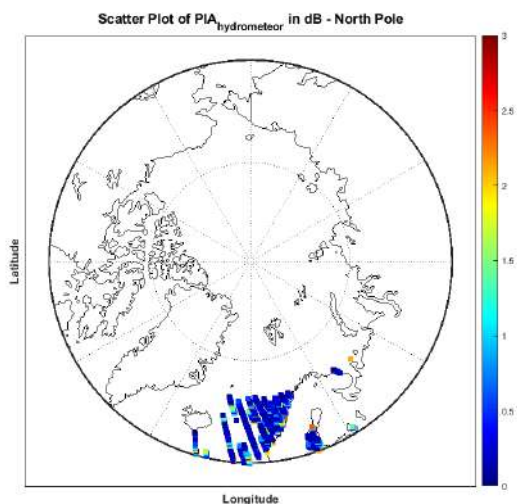


Fig. 21: North Pole vision about PIA-hydrometeor in the Global Map at W Band. (January-March 2016).

In coastal and arid regions, soil moisture typically ranges between 2% and 8% VWC, resulting in relatively low Path Integrated Attenuation (PIA) values between 5 and 10 dB. In temperate regions, encompassing plains and hills, soil moisture generally falls within the 8% to 15% range, leading to PIA values of approximately 10 to 15 dB. Dense forests and mountainous areas exhibit higher soil moisture levels, ranging from 15% to 25%, with corresponding PIA values between 15 and 20 dB. Tropical forests and equatorial regions, characterized by extremely high soil moisture levels ranging from 25% to 35%, exhibit the highest PIA values, typically between 20 and 25 dB.

In polar regions, where soils are often frozen, surface

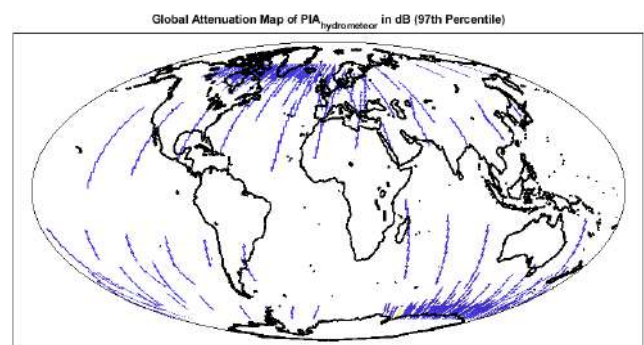


Fig. 23: Global Map related to passages over poles from CloudSat Satellite during January-February and few passages in May 2016.

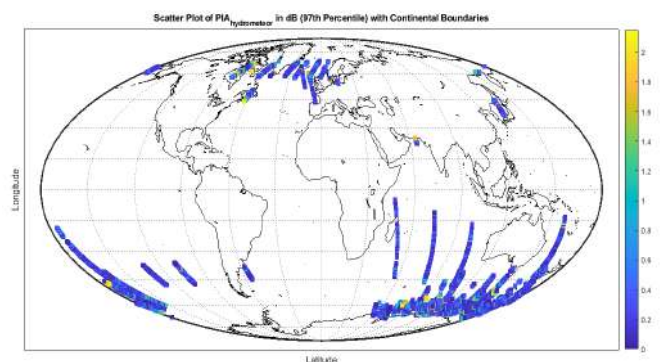


Fig. 24: Global Map at W-Band for CloudSat-GPM Coincidence Dataset during January-February and few passages in May 2016, in the Mollweide projection.

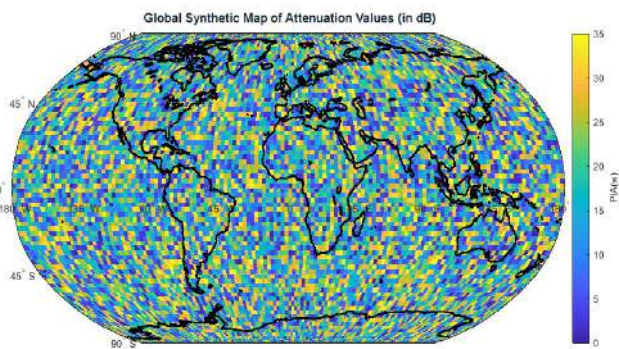


Fig. 25: Global Map at W-Band for CloudSat-GPM Coincidence Dataset during January–March 2016, in the Mollweide projection.

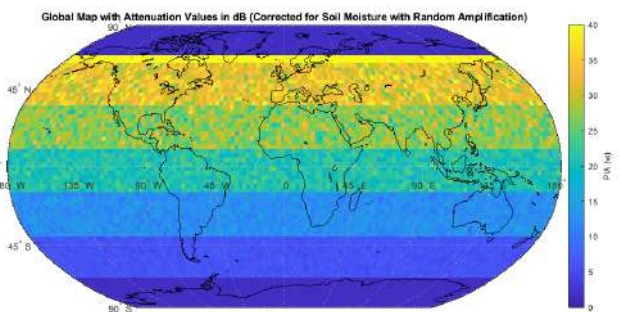


Fig. 26: With random soil moisture content, the PIA(W) becomes not uniform in values in geographically neighboring areas.

moisture content remains between 2% and 8%, resulting in minimal PIA values of 0 to 5 dB. Urban and industrial areas, where soil sealing limits moisture retention, present soil moisture levels between 5% and 10%, leading to PIA estimates ranging from 5 to 15 dB for urban settings and from 10 to 20 dB for industrial zones. In these cases, however, structural obstacles tend to dominate the attenuation processes more significantly than soil moisture.

These estimates contribute to a more accurate representation of the relationship between soil moisture and PIA across diverse environments. To further enhance model realism, it is recommended to introduce stochastic variations, such as ripple or noise signals at regional boundaries, and to increase humidity discontinuities across adjacent zones. This approach would improve the verisimilitude of the model by better capturing natural heterogeneity in soil moisture distribution.

Readers may utilize this example as a foundational framework for their research activities. Nevertheless, it should be noted that the model remains simplified and may require further refinement and adaptation to address specific application requirements. To enhance the model's fidelity, several improvements are recommended. First, atmospheric influences should be incorporated by accounting for the effects of atmospheric physics on **Path Integrated Attenuation (PIA)**,

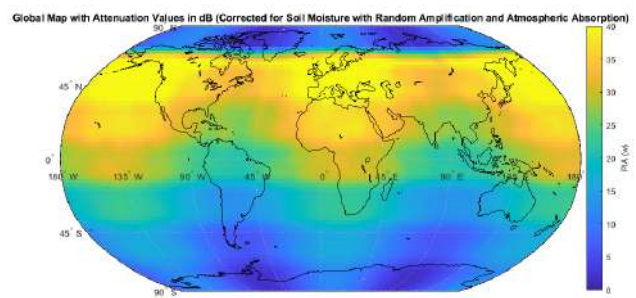


Fig. 27: Synthetic PIA(W) with random noise in the regional boundaries. Ripple over regions, Random Moisture Effect and Water Vapor. We use Blurring effect.

particularly the contributions of gases such as **Oxygen (O_2)** and **Water Vapor (WV)**. It is essential to include parameters such as the absorption coefficient within the modeling process to accurately represent these effects. Furthermore, the integration of real-time meteorological data, including humidity and temperature profiles, is encouraged to achieve more precise global estimates.

Employing ITU-R propagation models is also recommended, as these models account for a wide range of atmospheric and geographical factors, thereby enabling a more comprehensive and realistic analysis. Additionally, it is important to consider further phenomena that influence signal attenuation, such as the presence of atmospheric particulate matter, reflection and diffraction effects, and the impact of terrain and vegetation. Incorporating these aspects will lead to a more robust and accurate representation of the relationship between Soil Moisture (Sm) and PIA.

The relationship between Soil Moisture (Sm) and Path Integrated Attenuation (PIA) is indirect yet significant. Although soil moisture does not directly influence PIA, it alters the land surface properties during precipitation events. Specifically, wet soil conditions modify the surface scattering cross-section (σ_0), a key parameter in radar-based precipitation measurements. Failure to properly account for soil moisture variations may result in underestimation of PIA. Therefore, integrating soil moisture data into PIA assessments enhances the accuracy and reliability of the estimates, offering a more faithful depiction of the interplay between rainfall and radar signal attenuation. Despite the indirect nature of the relationship, its impact on radar measurements is substantial and cannot be neglected.

U. Advanced Methodologies for Analyzing Indirect Factors in Atmospheric Processes

Scientists address the influence of indirect factors on direct phenomena through the application of complex mathematical models and advanced data analysis techniques that integrate multiple variables. Comprehensive system models, such as multi-linear regression and nonlinear frameworks, allow independent variables, including soil moisture and atmospheric pollution, to explain the variability observed in dependent

variables. Physical atmospheric models, including numerical weather prediction systems and radiative transfer models, rely on fluid dynamics equations to simulate the movement of energy, moisture, and particulate matter through the atmosphere. These models also incorporate key processes such as plant evapotranspiration and the impact of pollutant aerosols on atmospheric dynamics. The adoption of machine learning and artificial intelligence methods, including artificial neural networks and supervised learning algorithms, further enables the identification and characterization of complex, nonlinear relationships between indirect and direct variables. In addition, advanced statistical techniques, such as sensitivity analysis, principal component analysis, and **Granger** causality testing, are employed to quantify the influence of indirect drivers on primary phenomena. The integration of observational and simulated datasets—derived from satellite missions and coupled atmosphere–land surface models—provides crucial insights for correlating indirect and direct variables. Parametric modeling approaches, including semi-empirical and stochastic models, are utilized to account for uncertainties inherent in indirect processes. Model validation through field observations, dedicated experimental campaigns, and historical datasets remains fundamental to ensuring the reliability and robustness of the modeling results. In summary, accurately capturing the influence of indirect factors on direct phenomena requires a multidisciplinary approach that combines physical modeling, statistical analysis, machine learning techniques, and observational data. Such integrated models enable a rigorous mathematical quantification of the contribution of indirect variables to observed phenomena. For instance, satellite missions such as CloudSat, GPM, and MODIS provide critical information regarding cloud distribution, precipitation, vegetation cover, and aerosol concentrations, facilitating the analysis of indirect–direct relationships. In more complex applications, it is also possible to implement coupled atmosphere–land surface models, such as the Community Earth System Model (CESM) and the Regional Climate Model (RegCM). CESM enables the simulation of interactions among the atmosphere, oceans, land surfaces, and cryosphere to advance the understanding and projection of climate dynamics. RegCM, originally developed at the National Center for Atmospheric Research (NCAR) and maintained by the Earth System Physics Section at ICTP, supports studies ranging from climate process analysis to paleoclimate reconstructions and future scenario simulations, providing a powerful framework for investigating the interactions between vegetation, soil moisture, and atmospheric

processes.^{4 5}

V. The Impact of Reverse Engineering in Scientific and Technological Advancements

The Human Genome Project is a prime example of reverse engineering on a grand scale, where scientists mapped out the entire human genome to understand genes and their roles in diseases and traits. In biology, AI-driven algorithms like **DeepMind's AlphaFold** [40] have revolutionized protein folding by accurately predicting protein structures, transforming our understanding of molecular interactions. In telecommunications, a case study demonstrated the application of a cognitively based approach for reverse engineering tool development, utilizing ideas from cognitive psychology to improve system design and functionality [41], [42].

These examples highlight the diverse applications and significant impact of reverse engineering in advancing scientific knowledge and technological innovation.

W. Machine Learning and Data Analysis Techniques for Atmospheric Modeling

The use of reverse engineering (RE) on **Battaglia's** work [38] represents an innovative and useful approach to derive global attenuation behaviors for frequencies other than the 35 GHz band. This method allows for the analysis and understanding of existing models, allowing researchers to adapt and improve their predictions for other frequency bands. Furthermore, reverse engineering facilitates the comparison between one's research work and the results obtained by other studies, providing a solid basis for model validation and optimization. Statistical analysis of the data derived from reverse engineering allows for the identification of patterns and trends that may not be immediately evident, thus improving the understanding of attenuation phenomena. This integrated approach, which combines machine learning techniques and data analysis, provides a complete and detailed picture of atmospheric dynamics, helping to develop more accurate and reliable models. It is possible to show uncertainty in indirect processes more accurately when we use specific parametric models and combine data from observations and simulations. Finally, model validation through field data and experimental campaigns ensures the accuracy of predictions, making reverse

⁴A statistical technique known as Granger causality analysis determines whether one time series can predict another. In the context of a Path Integrated Attenuation (PIA) map, this analysis can be used to examine causal relationships between soil moisture and PIA values in different geographic regions. By applying Granger's causality analysis, one can identify whether changes in soil moisture can predict changes in PIA values and vice versa. This can help to better understand how weather and environmental conditions affect radar signal attenuation, thus improving the accuracy of weather forecasts and radar analyses.

⁵Researchers studying climate change use the **CESM (Community Earth System Model)** framework to simulate interactions between the atmosphere, oceans, land surface, and ice. This model helps scientists understand climate dynamics and predict future climate scenarios.

RegCM (Regional Climate Model) is a system of regional climate models originally developed at the National Center for Atmospheric Research (NCAR) and maintained in the Earth System Physics (ESP) Section of ICTP. Applications of RegCM range from studies of climate processes to paleoclimate and future simulations.

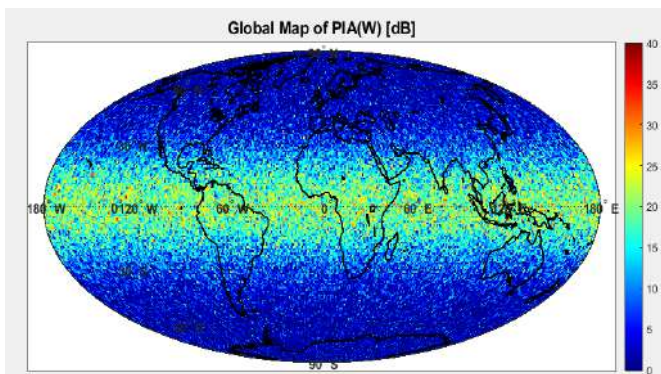


Fig. 28: Synthetic Global Map of PIA(W).

engineering an indispensable tool for advanced research in this field. By leveraging reverse engineering, we can generate scatter plots of PIA at various frequencies and compare them over time with those derived from our research on **CloudSat-GPM** and **CloudSat-TRMM** satellite coincidences. This comparison enables us to verify the coherence and reliability of the developed models, offering deeper insights to enhance forecasts and better comprehend atmospheric attenuation phenomena.

Fig. 28 illustrates the global distribution of Path Integrated Attenuation (PIA) at W-band frequencies derived from CloudSat-GPM satellite coincidences. The Mollweide projection ensures an equal-area representation, providing an unbiased view of the spatial variability of attenuation. As expected, the strongest PIA values, exceeding 20–30 dB, are concentrated around the equatorial belt, corresponding to regions of intense convective activity and frequent heavy precipitation, such as the Intertropical Convergence Zone (ITCZ) and Southeast Asia. Attenuation progressively decreases toward mid-latitudes and polar regions, where precipitation events are less intense or predominantly in the form of snow and ice. The localized variability and fine-scale structures visible in the map are attributed to the satellite overpasses and the temporal coverage of the dataset. Residual noise and discrete patches reflect the discrete sampling of the satellite swaths and the current stage of interpolation. Overall, this preliminary map effectively captures the major climatological features of global hydrometeor-induced attenuation and serves as a foundation for further refinement, including smoothing and atmospheric gas corrections.

VII. FROM SYNTHETIC GLOBAL MAPS TOWARD REAL GLOBAL MAPS.

The initial phase of this study focused on the construction of synthetic global maps of **Path Integrated Attenuation (PIA)**, derived from physically realistic atmospheric simulations. These synthetic models allowed for a controlled exploration of the spatial distribution of hydrometeor-induced attenuation, offering preliminary insights into the main climatic and geographic drivers shaping the attenuation fields worldwide. However, despite their scientific value, synthetic datasets are inevitably limited by their dependence on idealized assumptions, parameterizations, and simplified representations

of atmospheric processes. To achieve a more accurate and comprehensive characterization, the analysis was progressively extended to incorporate real observational data extracted from satellite-based passive and active microwave sensors, spanning multiple years of measurements. The transition from synthetic to real global maps required the development of robust data ingestion, preprocessing, and interpolation pipelines capable of:

- Managing large volumes of heterogeneous satellite data,
- Filtering and normalizing attenuation values to remove outliers and artifacts,
- Interpolating sparse observations into continuous and physically meaningful global maps.

Particular attention was devoted to addressing critical regions such as the polar areas, where conventional data availability is sparse and observational noise is significant. The resulting real global PIA maps preserve the fine-scale variability of actual hydrometeor distributions, capturing both large-scale climatic patterns and localized atmospheric phenomena. This methodological progression — from synthetic simulation to real-world mapping — constitutes a key contribution of this work, providing a solid foundation for advanced studies on:

- Atmospheric water content monitoring,
- Satellite telecommunication system planning,
- Climate variability and hydrological cycle assessments.

The **Fig. 29** displays the Global Historical PIA Hydrometeor Map (Scaled & Normalized) derived from a comprehensive dataset of hydrometeor-induced Path Integrated Attenuation (PIA) values. The data have been interpolated onto a regular latitudinal-longitudinal grid with subsequent scaling and normalization procedures to ensure the removal of outliers and enhance the visibility of major atmospheric patterns. The color scale, ranging from approximately -1 dB to 5 dB, highlights attenuation intensities: red and yellow zones correspond to regions of higher hydrometeor content, typically associated with intense convective activity, while blue areas indicate lower attenuation and clearer atmospheric conditions. The presence of structured attenuation patterns over the mid-latitudes and Southern Hemisphere storm tracks is particularly evident, demonstrating the global distribution of hydrometeor-related attenuation phenomena. Coastal outlines are overlaid to provide geographic context.

The **Fig.30** illustrates the Global Historical PIA Hydrometeor Map after scale adjustment and normalization procedures. The dataset, derived from satellite-based hydrometeor measurements, is interpolated onto a $0.5^\circ \times 0.5^\circ$ global grid using a natural neighbor interpolation scheme. The colormap, spanning from 0 dB to 5 dB, emphasizes regions with higher path-integrated attenuation (PIA) due to hydrometeor presence, shown in warm colors (yellow to red), whereas areas of low PIA appear in blue. Notable enhancement is observed over the Southern Ocean and mid-latitude storm tracks, reflecting increased hydrometeor concentration associated with active meteorological systems. Continental outlines are superimposed to provide geographic reference. Compared to raw data, the normalized map suppresses outliers while preserving the spatial structure of PIA fields, offering an enhanced visualization

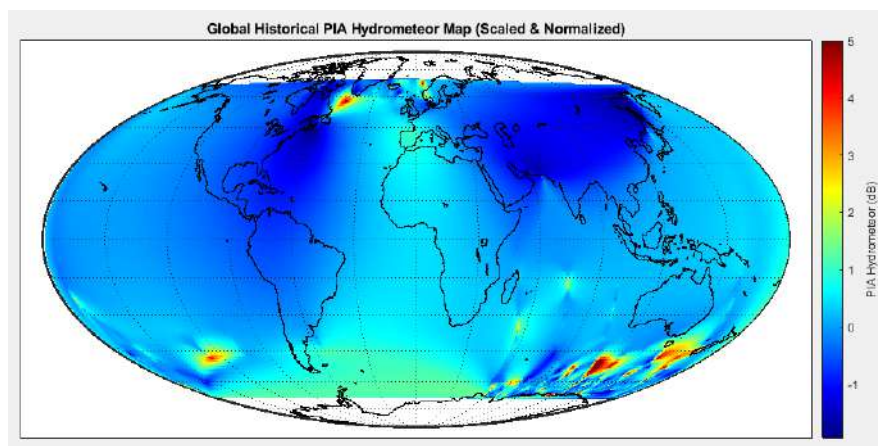


Fig. 29: Real PIA_Hydrometeor global map from CloudSat-GPM Coincidence Dataset in the January-February 2016.

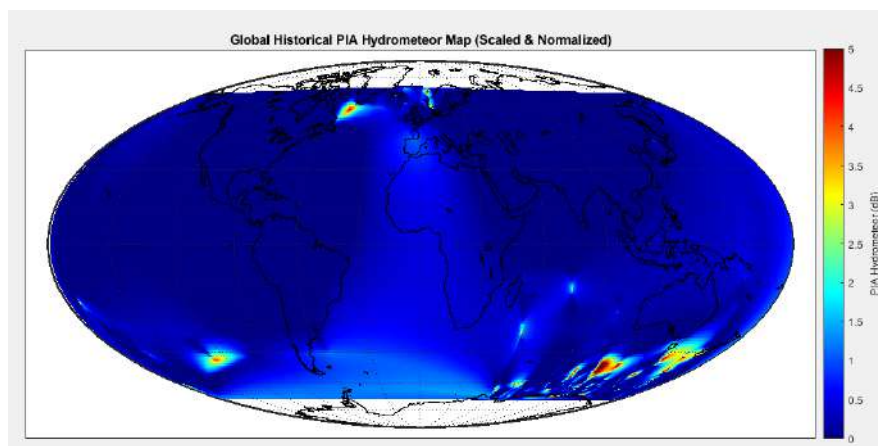


Fig. 30: Real PIA_Hydrometeor global map from CloudSat-GPM Coincidence Dataset in the January-February 2016 with PIA_Hydrometeor scale positive.

suited for climatological analysis and radiowave propagation modeling.

Figs. 29, 30 provides a Global Historical PIA Hydrometeor Map that is scaled and normalized, depicting the intensity of hydrometeor attenuation (PIA) across the Earth's surface. The data in this map can be generated by applying machine learning models, like LightGBM (Light Gradient Boosting Machine), to predict and analyze the attenuation levels from atmospheric parameters. Here's a detailed explanation of how LightGBM could be used to generate such a map:

A. Data Preprocessing for LightGBM

The first step to using **LightGBM** is to preprocess the raw atmospheric data, which might include various parameters such as temperature, pressure, humidity, and other relevant meteorological variables. These data are often collected from different sources like satellites (e.g., CloudSat, TRMM, GPM), reanalysis data, or observational networks. In your case, the PIA data for the map could be derived from satellite radar observations or reanalysis data that have been cleaned, normalized, and possibly filtered to remove outliers, as reflected in the

image. For **LightGBM** to handle these data, the preprocessing steps include:

- **Feature engineering:** This involves transforming raw data (e.g., temperature, humidity, cloud cover) into useful features that LightGBM can understand.
- **Normalization/Scaling:** Since LightGBM works better with data that is scaled or normalized, features like PIA values might be normalized to a consistent scale (0 to 6 dB as in your case).
- **Handling missing values:** LightGBM can naturally handle missing data, but depending on the dataset, additional handling (like imputation) might be required.

B. Training the LightGBM Model

Once the data has been preprocessed, the next step is training a LightGBM regression model. Here, LightGBM is used to predict the PIA values at each latitude and longitude based on the available meteorological features. In this case, LightGBM is trained to learn the relationship between the atmospheric conditions (input features) and the observed PIA values (output).

- **Model structure:** LightGBM uses gradient boosting to create an ensemble of decision trees. The trees are built iteratively, where each subsequent tree tries to correct the errors made by the previous ones.
- **Objective function:** LightGBM optimizes a loss function that measures the prediction error (e.g., mean squared error) during training.

The output of this training is a trained model that can predict the PIA values for any given set of input features, such as temperature, pressure, and humidity.

C. Making Predictions

After the LightGBM model is trained, it is used to make predictions for every geographical point (latitudes and longitudes) across the globe. For each latitude and longitude, the trained LightGBM model predicts the corresponding PIA value based on the input features at that location.

- The prediction output from LightGBM is a continuous value (in dB), representing the attenuation at that point.
- These predictions are stored and then used to create a grid of PIA values over the global map.

D. Crating the Global Map

The predicted PIA values for each location are then mapped onto a grid covering the entire globe, which can be visualized in the Global PIA Hydrometeor Map.

- **Interpolation:** Since the prediction from LightGBM gives values at specific coordinates (latitude, longitude), an interpolation step may be required to create a smooth grid covering the entire map. This interpolation fills in values for areas between the predicted points.
- **Color scaling:** The PIA values are then mapped to a color scale, where different PIA ranges correspond to different colors, as seen in the colorbar of the map (ranging from 0 to 5 dB).
- **Plotting:** The final output is a visual representation of the predicted PIA values across the globe, allowing for the identification of regions with high or low hydrometeor attenuation.

The final result, as depicted in the **Global Historical PIA Hydrometeor Map**, provides a comprehensive visualization of hydrometeor attenuation across the Earth's surface, where various regions are represented according to the predicted levels of attenuation. These levels are derived from a predictive model utilizing satellite-based observations and atmospheric data. The map serves as a powerful tool for understanding the spatial distribution and impact of hydrometeors, such as rain and snow, on the atmosphere, and more specifically, their effects on satellite communication systems.

This type of map is critical for various applications in satellite communications, where the accurate prediction of hydrometeor-induced signal attenuation is essential for optimizing communication links, especially in regions experiencing high atmospheric interference. By leveraging advanced machine learning models, such as LightGBM, this approach allows for the accurate prediction of attenuation levels across

global scales, even in areas with sparse data. This method can be extended to further refine global communication models and improve the reliability of satellite communication systems by addressing atmospheric influences at different geographical locations.

Moreover, the application of this predictive modeling to the Global Historical PIA Hydrometeor Map enhances the understanding of climatic variations and their impact on satellite systems, helping to improve real-time signal processing and communication protocol design in the presence of diverse atmospheric conditions. Therefore, this map provides a critical step forward in the field of satellite communication, particularly in the context of improving system performance in challenging environmental conditions.

The **Fig.31** presents an Enhanced Global PIA (Path Integrated Attenuation) Hydrometeor Map displayed using an orthographic projection centered over the Atlantic Ocean. The data were preprocessed by applying percentile-based normalization and a strict scaling between 0 dB and 4.5 dB to emphasize subtle hydrometeorological patterns.

The colormap adopts a dark background ("parula"-like shading) where low PIA regions appear in deep blue and high PIA concentrations transition to green, yellow, and red tones. Notable clusters of enhanced PIA values are evident across the North Atlantic and Northern Europe, likely associated with persistent mid-latitude cyclonic activity. Similarly, elevated PIA signatures are visible in the Southern Ocean, particularly around 60°S, corresponding to the climatological storm belt.

The continents are accurately overlaid with black coastlines, offering a clear geographical context. The orthographic projection effectively mimics a three-dimensional Earth view, enhancing spatial perception of hydrometeor distribution without extreme distortions at the poles. The visualization also reveals the relative scarcity of significant hydrometeor-induced attenuation over continental interiors, especially across Africa and South America, suggesting limited cloud water content and precipitation events during the considered data collection period.

This high-fidelity global depiction of PIA supports advanced analysis of hydrometeor occurrence, providing critical information for radio-wave attenuation modeling, satellite communications planning, and climatological assessments at hemispheric scales.

Fig. 32 shows a three-dimensional visualization of satellite-derived hydrometeor PIA distributions: attenuation values (0–6 dB) are mapped with a light colormap, while regions without significant data are masked in white. Coastlines are overlaid to provide geographic context. In the **Fig.32** low values of PIA are designed in blank regions visible as gray or white color.

Fig. 33 shows the interpolated global map of Precipitation-Induced Attenuation (PIA) due to hydrometeors, focusing specifically on the North Pole region using a stereographic projection. The latitudinal range extends from 60°N to 90°N, centering the visualization on the pole. The PIA values have been corrected and normalized, ensuring that only non-negative values between 0 dB and 5 dB are displayed, in order to provide a clean and physically consistent color scale across the mapped region. The color scale adopts a jet colormap, tran-

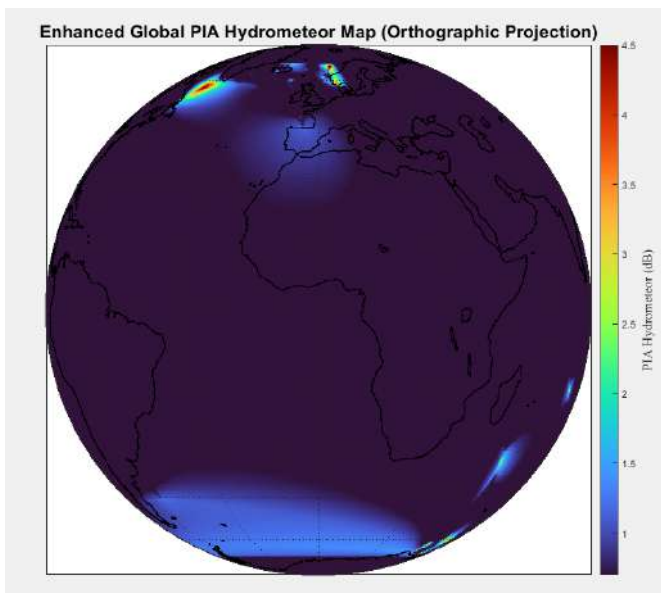


Fig. 31: The PIA_Hydrometeor plotted with positive values and using Enhanced resolution to $0.5^\circ \times 0.5^\circ$ in orthographic projection (January-February 2016).

sitioning from blue (low attenuation) to red (high attenuation), with a corresponding color bar shown on the right. Areas with minimal or no hydrometeor-induced attenuation appear in deep blue, while localized regions with higher attenuation effects are shown in green, yellow, and red tones. The coastlines of the Northern Hemisphere are clearly superimposed in black, enabling easy identification of continental boundaries relative to attenuation patterns. Notable geographic features such as North America, Greenland, Northern Europe, and Siberia are visible for spatial reference. Some data gaps can be observed near the central polar cap, particularly above 85°N , as indicated by the blank white area. This absence results from the limited satellite sampling at extremely high latitudes and interpolation uncertainties where measurement coverage was insufficient.

Overall, the map reveals that hydrometeor activity is generally sparse at polar latitudes, with a few isolated sectors (visible as yellow to red zones) suggesting transient storm systems or localized precipitation events near 60° – 70°N , particularly over the North Atlantic and adjacent Arctic areas. This representation is particularly useful for highlighting:

- The spatial heterogeneity of PIA at polar latitudes.
- The reduced attenuation typical of cold, dry atmospheric conditions near the pole.
- The importance of interpolating sparse measurements for generating continuous attenuation fields suitable for atmospheric studies and telecommunication link budgets over the Arctic.

Fig. 33 depicts the interpolated map of the Precipitation-Induced Attenuation (PIA) associated with hydrometeors over the South Pole region, using a stereographic projection. The latitudinal range for this visualization spans from -90° to -60° , enabling an accurate representation of the Antarctic continent

and surrounding Southern Ocean regions. The interpolation of available satellite-based PIA measurements onto a regular $1^\circ \times 1^\circ$ grid has been performed, with strict normalization procedures applied to limit PIA values within the range of 0 to 5 dB. Any negative or outlier values have been excluded to ensure the physical consistency of the resulting attenuation field. The color scale follows the jet colormap, wherein:

- Deep blue represents regions of negligible attenuation (close to 0 dB),
- Cyan and green indicate moderate PIA levels,
- Yellow to red highlight sectors with the highest observed attenuation, approaching or exceeding 4 dB.

The continental coastline of Antarctica has been overlaid in black to offer precise geographic reference, helping distinguish landmasses from oceanic areas. The presence of substantial PIA signatures is predominantly observed around the coastal regions, particularly along the Antarctic Peninsula, East Antarctica margins, and portions of the Southern Ocean, which are known to experience relatively higher hydrometeor activity compared to the frozen continental interior. The central polar cap (white area) reveals a notable absence of data due to the limitations in satellite coverage and swath geometry over extreme polar latitudes. This map provides valuable insights into:

- The distribution of hydrometeor-induced attenuation at southern polar latitudes,
- The contrasting conditions between the relatively dry continental interior and more humid, storm-prone coastal zones,
- The importance of including polar-specific corrections when modeling global satellite communication links traversing polar regions.

Fig. 34 presents the interpolated Precipitation-Induced Attenuation (PIA) map over the North Pole region, utilizing a polar stereographic projection. The plotted domain covers latitudes from 60°N to 90°N , including parts of Northern Europe, Asia, Greenland, and the Arctic Ocean.

The input data, derived from satellite-based passive and active measurements, have been interpolated onto a regular $1^\circ \times 1^\circ$ grid. Importantly, only positive PIA values are retained in the interpolation, effectively suppressing noise and eliminating non-physical negative artifacts, which could otherwise bias the representation of atmospheric attenuation. The color scale is designed between 0 dB and 5 dB, where:

- Dark blue tones indicate very low attenuation close to 0 dB,
- Cyan and green tones indicate intermediate levels of hydrometeor presence,
- Yellow regions identify localized peaks in attenuation, reaching up to 4.5–5 dB.

The continental coastlines are outlined in black to facilitate spatial interpretation. The map notably highlights a low PIA background across the Arctic Ocean and northern continents, consistent with the general dryness and low precipitation climatology typical of polar regions. Significant PIA enhancements are, however, observed in specific sectors, particularly:

- South of Greenland,

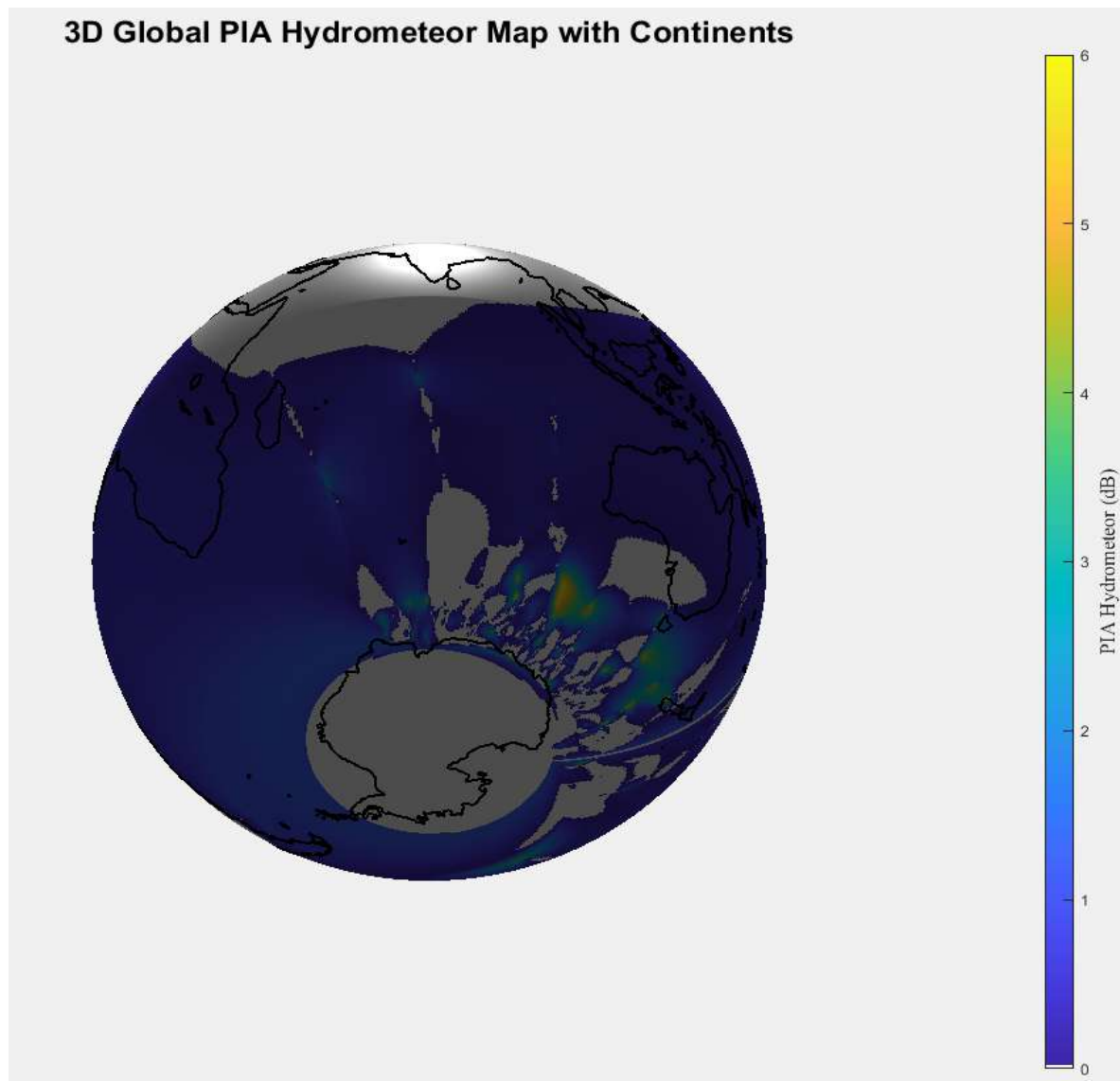


Fig. 32: 3D Global Map of Hydrometeor-Induced Path Integrated Attenuation (PIA).

- Northern Europe (Scandinavia, Russia),
- Some isolated patches over the Arctic seas, which are areas occasionally affected by cyclonic systems and moisture transport events.

The central Arctic cap shows a smoother field due to the lower density of observations at very high latitudes, reflecting satellite coverage limitations. Overall, this representation offers a clear, noise-free visualization of hydrometeor-induced attenuation phenomena in the northern polar domain, and is crucial for:

- Modeling high-latitude satellite links (e.g., polar-orbiting satellites, ground terminals),
- Studying atmospheric variability associated with Arctic hydrometeors under climate change scenarios.

Fig. 35 illustrates the interpolated Precipitation-Induced Attenuation (PIA) map for the South Pole region, obtained using a polar stereographic projection centered at the South

Pole (90°S). The displayed area covers latitudes from -90°S to -60°S , encompassing the Antarctic continent and adjacent Southern Ocean regions. The original satellite observations, after quality control and filtering of invalid measurements, were interpolated onto a $1^{\circ}\times 1^{\circ}$ regular grid. Only positive PIA values were considered, in order to ensure the physical realism of the retrieved hydrometeor attenuation fields. The color scale is set between 0 dB and 5 dB, where:

- Dark blue tones represent minimal attenuation (near 0 dB),
- Light blue to green tones correspond to moderate hydrometeor loading,
- Yellow highlights denote localized maximum attenuation events up to approximately 4.5–5 dB.

A black coastline overlay is included to delineate the Antarctic landmass from the surrounding oceans, enhancing geographic readability. The map reveals several key features:

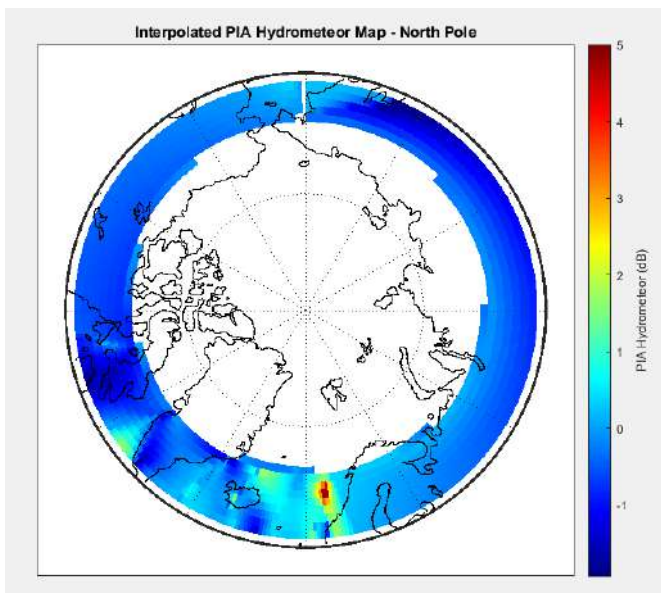


Fig. 33: Interpolated PIA_hydrometeor map - North Pole (January-February 2016).

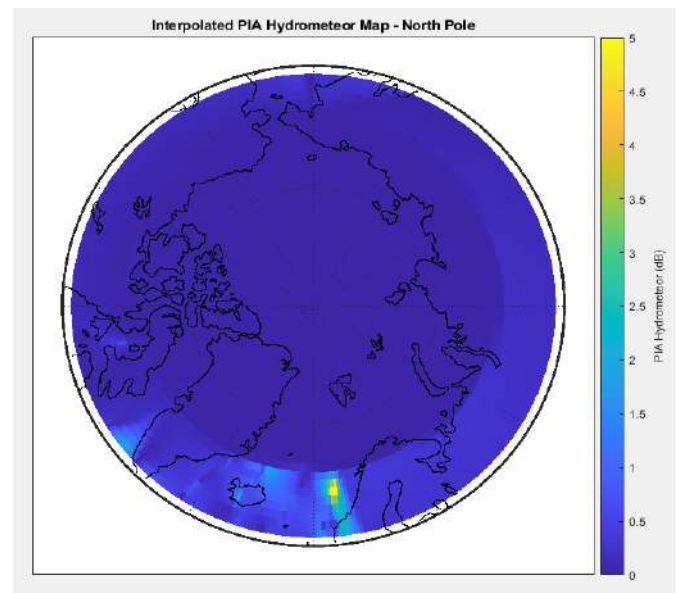


Fig. 35: Interpolated PIA_hydrometeor map - North Pole (January-February 2016) - Correction with PIA_hydrometeor positive values.

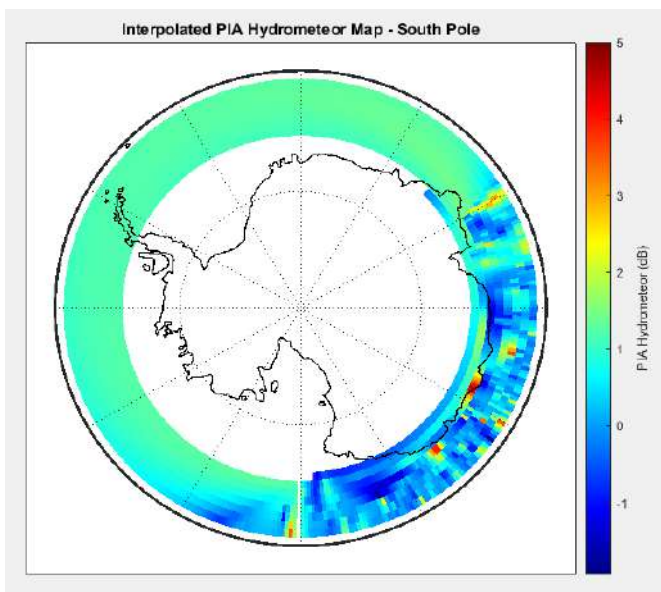


Fig. 34: Interpolated PIA_hydrometeor map - South Pole (January-February 2016).

- A low background PIA signal over the Antarctic Plateau, which is consistent with its cold and dry climate, where minimal hydrometeor presence occurs.
- Isolated enhanced PIA regions predominantly around the coastal margins, particularly along:
 - The Weddell Sea,
 - The Ross Sea,
 - The East Antarctic coastline, where moisture intrusions, coastal cyclonic activity, and orographic effects from the Antarctic topography can lead to higher hydrometeor concentrations.

- A notable asymmetry between different longitudinal sectors, linked to variations in regional atmospheric circulation patterns.

Furthermore, some slight radial interpolation artifacts can be noticed at the map edges due to lower data coverage at extreme southern latitudes. Overall, the figure provides a clear and quantitative assessment of hydrometeor-induced attenuation over the southern polar cap, essential for:

- High-latitude satellite communication planning,
- Meteorological modeling in polar environments,

Climate trend analysis related to Antarctic atmospheric moisture variations.

VIII. CONCLUSIONS

The development of Global Attenuation Maps (GAMs) offers critical insights into the influence of atmospheric variables on signal propagation across different regions worldwide. Regular updates and refinements, incorporating **Crane's** analysis and related models, are essential to identify optimal transmission frequencies tailored to the diverse climatic and atmospheric conditions encountered globally [21], [22], [23]. Such maps hold significant promise for enhancing the performance of weather radars and other communication technologies.

Crane models, extensively utilized in global meteorological applications, serve as fundamental tools for predicting rain-induced signal attenuation. These models partition the globe into distinct rainy climate regions, each characterized by specific distributions of precipitation intensity and occurrence frequency. In particular, the 1985 **Crane model** employs updated atmospheric distribution tables and a refined global rain-region map to facilitate the estimation of signal attenuation along terrestrial and satellite communication paths.

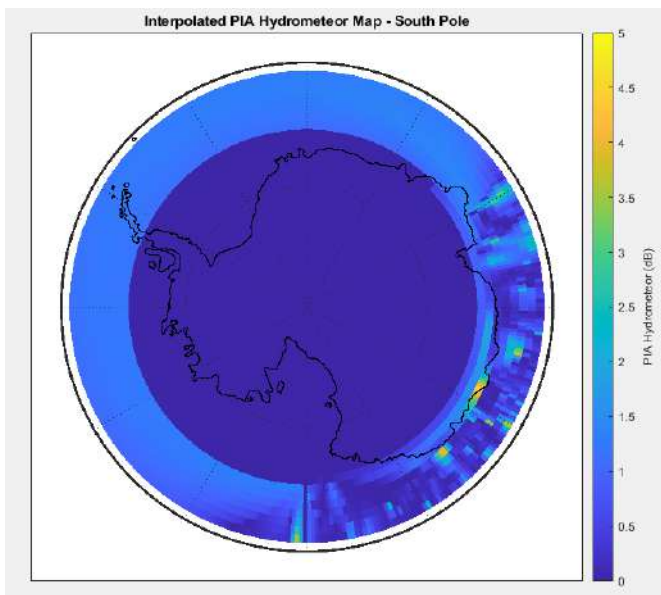


Fig. 36: Interpolated PIA_hydrometeor map - South Pole (January-February 2016) - Correction with PIA_hydrometeor positive values.

The scientific relevance of **Crane** models lies in their ability to provide accurate, location-dependent attenuation estimates, which are critical for the design and optimization of satellite and terrestrial communication systems. These models rigorously account for a range of parameters, including transmitter frequency, ground station elevation angle, site altitude, and the curvature of the Earth.

Validation efforts, leveraging satellite beacon observations and terrestrial path measurements, have demonstrated an acceptable standard deviation between predicted and observed attenuation values, thereby confirming the robustness of these models. In this study, we have substantiated the operational feasibility of constructing Global Attenuation Maps and have developed the necessary MATLAB-based framework for data acquisition and processing. Nevertheless, the extensive data requirements, coupled with the need for validation and analysis of coincident observations from CloudSat and TRMM missions, introduce considerable challenges, rendering the process highly labor-intensive.

Initial efforts were focused on polar orbital passages (Arctic and Antarctic regions), wherein attenuation maps were generated by applying pixel-wise averaging based on a percentile methodology (e.g., the 97th percentile) [25]. This approach enabled the activation of discrete latitude-longitude boxes across the global grid. However, attempts to process all satellite passes concurrently led to system instability, primarily due to MATLAB's video memory limitations, highlighting the insufficiency of available hardware resources to handle such large-scale data volumes in a single processing cycle.

To address these limitations, we propose a more scalable, incremental processing strategy. By partitioning the dataset into manageable subsets and processing them sequentially before merging the intermediate results, memory consumption can

be effectively distributed over time, thereby mitigating system overloads. This approach ensures the efficient utilization of existing computational resources and obviates the need for immediate hardware upgrades.

This study presented a scalable framework for generating global hydrometeor attenuation maps at Ku, Ka, and W bands, based on the integration of satellite coincidence data, machine learning techniques, and physical modeling. Starting from uncorrected PIA measurements, preliminary global climatologies were produced, demonstrating the potential of early-stage processing even with limited resources. By combining radar retrievals, reanalysis datasets, and advanced statistical models, the methodology provides a dynamic and flexible foundation for real-time atmospheric attenuation mapping. Future efforts will refine physical corrections, extend the climatological archives, and incorporate AI-based predictive tools, reinforcing the strategic relevance of accurate attenuation modeling for climate science, 6G telecommunications, and space communications in the coming decades.

In conclusion, although technical challenges persist, the adoption of incremental data processing methods combined with regular dataset updates renders the generation and maintenance of Global Attenuation Maps both feasible and sustainable. Such maps are poised to become essential tools for optimizing communication systems operating under diverse atmospheric conditions worldwide.

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A.1.1 – First Research Article - personal style.

Global-Scale Mapping of Atmospheric Attenuation at Ku, Ka and W bands using multi-satellite coincidence datasets

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Abstract—This work proposes a novel software solution for the generation of a global atmospheric attenuation map utilizing satellite reanalysis data for deep space communications. The approach employs reverse engineering methods (REm) and is examined in terms of its practical implementation. The datasets utilized in this study were derived from overlapping observations obtained by the GPM-CloudSat and CloudSat-TRMM satellites. The objective of this research is to enhance the accuracy of predictions regarding signal attenuation along communication pathways, thereby facilitating more effective communication planning. This encompasses the selection of optimal times and frequencies for transmissions, as well as the enhancement of link budget designs. The proposed methodology simplifies the process by directly assessing non-attenuated global maps, thereby obviating the need for pre-processing a complex atmospheric attenuation model. In particular, the approach places a premium on the collection and processing of satellite data in order to create preliminary maps. Once the fundamental maps have been established, corrections can be introduced in a gradual manner to enhancing their accuracy. The processing of non-attenuated maps also has the advantage of reducing the computational power required in the early stages. Subsequently, machine learning or predictive analysis techniques can automatically update the maps, thereby simplifying the management of changing atmospheric conditions. The proposed strategy is also highly flexible and adaptable to various space communication setups, taking into account factors such as transmission frequency, satellite position, and seasonal variations. Although the correction process may become more complex over time, initiating the process with a simpler approach provides a solid foundation for further development. In conclusion, the integration of real data with advanced modeling techniques enhances the map's accuracy and reliability over time.

Index Terms—Global-scale attenuation map; multi-coincidence datasets; CloudSat; TRMM; GPM; Link budget designs; Global atmospheric attenuation map; Satellite reanalysis data.

I. INTRODUCTION

The creation of a global atmospheric attenuation map, based on the analysis of CloudSat-GPM and CloudSat-TRMM satellite coincidences, represents a significant advancement for the scientific community. This innovative methodology enables the integration of data from multiple space missions, enhancing the understanding of global precipitation and its variables. The combined use of data from CloudSat, GPM, and TRMM provides more comprehensive and detailed coverage, which is essential for advanced climate and meteorological studies.

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A. Historical backgrounds

The generation of global maps of geophysical quantities has a long history in the scientific literature. Since the earliest periods of recorded history, maps have been employed as a means of representing geographic and topographic information. The earliest known examples of cartography date back to ancient Babylon, where maps were created on clay tablets to represent topographical features such as hills and valleys [24]. Over time, cartography evolved as a result of contributions made by Greek scholars such as Anaximander, Herodotus and Ptolemy, who introduced concepts such as latitude and longitude with the aim of more accurately representing the Earth. In the modern context, the generation of global geophysical maps has benefited greatly from the advent of satellites and remote sensing technologies. Projects such as the European Space Agency (ESA) GLOBCOVER have used satellite data to create detailed maps of global land cover, involving an international network of partners and generating substantial quantities of data. While not focused on atmospheric attenuation, GLOBCOVER exemplifies the feasibility of creating comprehensive global maps through international collaboration and the integration of vast volumes of satellite data.

B. State of art

At present, there is a lack of standardization and universal acceptance of global Precipitation Intensity Analysis (PIA) maps. Nevertheless, there are a number of initiatives and projects that are striving to develop comprehensive global precipitation maps through the utilization of satellite data and sophisticated climate models. To illustrate, the NASA and JAXA Global Precipitation Measurement (GPM) project furnishes comprehensive global precipitation data, which can be employed to construct precipitation intensity maps. Furthermore, the ESA's Copernicus programme furnishes satellite data that can be employed for climate and environmental analysis, including precipitation mapping. The success of these projects hinges on the collaboration of numerous space agencies, research institutions, and international organizations, each playing a crucial role in data collection, processing, and analysis. A pertinent study in this field is the article "Global Distribution of Water Vapor and Cloud Cover-Sites for High Performance THz Applications" by **Suen et al. (2013)** [33], which employs high-resolution satellite data to map the global distribution of precipitable water vapor (PWV)

and cloud cover. This work provides valuable information for the identification of sites with low atmospheric attenuation, which are essential for communications in the terahertz spectrum. Furthermore, the International Telecommunication Union (ITU) has developed recommendations, such as **ITU-R P.525**, which provide guidelines for the calculation of free-space attenuation. These guidelines are essential for the design of satellite communication systems.

C. About PIA world map generation

Refs. [10]–[12] provide a robust basis for further investigation of Path Integrated Attenuation (PIA). In **Seto, Iguchi, Meneghini (2022)** [44], the impact of soil moisture on space-based radar precipitation measurements is examined utilizing data from the GPM dual-frequency radar mission. It was observed that surface back-scatter cross sections increased in the presence of precipitation, resulting in an underestimation of the path-integrated attenuation (PIA) in the Surface Reference Method or Technique (SRT) [13]. In order to correct for this effect, a correction method for PIA was developed and implemented in the standard DPR algorithm, resulting in an increase in surface precipitation rate estimates of 18% for KuPR and 15% for KaPR. The paper offers valuable insights for estimating the global path-integrated attenuation (PIA). The key takeaways are as follows:

- 1) **Soil moisture effect.** Increased soil moisture in the presence of precipitation can cause an underestimation of the PIA.
- 2) **PIA correction.** A correction method has been developed to consider the effect of soil moisture, improving precipitation estimates.
- 3) **Radar algorithms.** The standard DPR algorithm has been updated to include these corrections, increasing the accuracy of the measurements.
- 4) **Global analysis.** The analyzed data covers a period of six years for KuPR and KaMS and three years for KaHS, providing a global view of the PIA variations.

Meneghini et.al. (2000) [43] examine the precipitation radar (PR) on the Tropical Rainfall Measuring Mission (TRMM) and its use of the surface reference technique (SRT) to correct for radar attenuation due to rain. This technique entails a comparison of the radar return in areas devoid of precipitation with that in regions experiencing rainfall, with the objective of estimating the degree of attenuation. The reliability of this concept is contingent upon the surface type and the angle of incidence. The TRMM PR, operating at 13.8 GHz, was designed to achieve an optimal balance between spatial resolution and rain attenuation. The SRT technique has proven to be highly effective, particularly over oceans in moderate to heavy rain conditions. In **S.Tanelli and S.L.Durden et.al. (2002)**, [45], the contribution of nonuniform beam-filling (NUBF) is discussed, which refers to the variability of rainfall within the resolution volume of a radar. The following points are of particular importance:

- **Rainfall Variability.** Rainfall can vary on scales smaller than the size of the radar footprint, causing errors in measurements.

- **Effects on Radar.** This variability can lead to significant errors in the measured reflectivity (Z) and path-integrated attenuation (PIA).
- **Increased Impact in Convective Rainfall.** NUBF is particularly significant in convective rain, where the variability is more pronounced.
- **Correction Methods.** Studies suggest that the use of reflectivity profiles can reduce errors caused by NUBF compared to traditional methods such as the SRT (Surface Reference Technique).

Robert Meneghini et. al. (2021), [14] discuss the significance of path attenuation estimation (PIA) for the retrieval of precipitation parameters utilizing the dual-frequency radar (DPR) on the GPM satellite. The paper describes the implementation of the surface reference technique (SRT), which employs surface scattering properties to infer path attenuation through precipitation. The method has been developed in both single-frequency and dual-frequency versions, with the latter offering greater accuracy at moderate rainfall rates. The single-frequency Ku-band approach is employed when Ka-band data are unavailable. The article is of value in the creation of a global PIA map, as it provides methods and techniques for estimating path attenuation using dual-frequency radar data, which can be applied on a global scale to enhance the precision of precipitation measurements.

D. Purpose

Here I develop a novel software solution for the generation of a global atmospheric attenuation map utilizing satellite reanalysis data for deep space communications. I examine the approach's practical implementation using reverse engineering methods (REm), which, as demonstrated below, significantly speeds up the computational process. This effort requires significant coordination and the utilization of advanced computing resources to achieve accurate and useful results for the scientific community. The creation of such a map is crucial for enhancing the accuracy of data by harmonizing information from various sources, which reduces discrepancies and ensures comprehensive global coverage. Utilizing advanced computing resources allows for the efficient processing of vast amounts of data, including diverse climatic regions. Accurate and detailed attenuation maps are invaluable for researchers studying atmospheric conditions, climate change, and communication systems, providing a reliable foundation for scientific analysis and modeling.

Additionally, high-quality attenuation maps contribute to better forecasting and prediction models, essential for preparing for and mitigating the impacts of atmospheric disturbances on deep space communications. Advanced computing resources optimize the use of available resources and accelerate the research process, making the creation of a global atmospheric attenuation map a collaborative scientific endeavor that integrates cutting-edge technology and coordinated efforts to produce results that are both accurate and beneficial for the scientific community.

E. The role of the A-Train constellation and remote sensing satellites in the study of global climate

Remote sensing refers to the acquisition of information about the Earth without direct contact, using sensors on satellites or aircraft. This technology is fundamental to the monitoring of climate change, the forecasting of extreme weather events and the study of a range of natural phenomena [29], [30].

The A-Train, or Afternoon Train, is a constellation of Earth observation satellites that orbit in close formation, thereby enabling near-simultaneous observations from a variety of instruments. This constellation includes satellites such as Aqua, CloudSat, CALIPSO, Aura, GCOM-W1, and OCO-2 [1], [26].

The close coordination between these satellites allows for the collection of complementary data, thereby enhancing my understanding of the Earth system. CloudSat represents a pivotal component of the A-Train constellation [27], having been launched in 2006. It employs sophisticated radar technology to investigate the internal structure of clouds. The data provided by this satellite have been instrumental in advancing my understanding of tropical cyclones and cloud-related climate change. CloudSat has provided quantitative estimates of global snowfall and has conducted studies into the effects of pollution and other atmospheric emissions on precipitation [28].

Satellite coincidence product is defined as data obtained when two or more satellites observe the same area simultaneously. These products are of great importance as they permit the combination of data from disparate sources, thereby enhancing the accuracy and completeness of observations. For instance, the coincidences between CloudSat and TRMM or GPM extend the capabilities of representing light precipitation and cold seasonal precipitation [31], [32]. These data are vital for advanced climate studies and improving weather models.

In conclusion, A-Train and remote sensing satellites, such as CloudSat, play a pivotal role in advancing the understanding of global climate and precipitation. Satellite coincidence products enhance the value of these observations by providing integrated and more accurate data for scientific analysis.

F. Technical challenges and spatial resolution in creating global attenuation maps

The creation of a global attenuation map presents a number of technical and technological challenges, spanning a range of areas including programming and spatial resolution adaptation. In comparison to TRMM and GPM, CloudSat provides a more detailed resolution of the data. Furthermore, accurate georeferencing of the data is a prerequisite. The generation of the map necessitates the utilization of super-computing resources, given that the processing demands associated with the analysis of extensive satellite data sets exceed the capabilities of a conventional personal computer. It is therefore essential to have access to an adequate computing infrastructure in order to manage this complexity. The spatial resolution of CloudSat is approximately 1.4 km horizontally and 240 meters vertically in the W band. In comparison, the TRMM offers a resolution of 5 km for the Precipitation Radar and 4.3 km for the TRMM Microwave Imager (TMI). GPM has a spatial resolution of

5 km for the Dual-frequency Precipitation Radar (DPR) and 10 km for the GPM Microwave Imager (GMI). In the case of CloudSat-GPM satellite coincidences, the spatial resolution varies. In the W band, it is 1.1 km horizontally, while in the GPM it is 4-5 km. During the data alignment process, the nearest-neighbor method is employed in the background to circumvent the potential for spatial or temporal resampling artifacts (Turk et al., 2021) [1].

This approach effectively combines the observations of both satellites while ensuring the integrity of the data is maintained.¹

The spatial resolution of the resulting map will be determined by the lower resolution between the two satellites when CloudSat and GPM data are recombined. Consequently, the degree of detail will be approximately 5 km. A resolution of 1.4 km is deemed to be more detailed. It can therefore be concluded that the practical resolution of the combined map will be closer to 5 km in order to ensure data integrity. The requisite resolution angle, necessary for the acquisition of cells with a $5 \times 5 \text{ km}^2$ area, can be determined through the application of the following formula $\theta = \frac{d}{D}$, where θ represents the resolution angle in radians, d denotes the cell size, and D signifies the distance between the satellite and the Earth's surface. Given a satellite altitude of 700 km, the resolution angle θ would be approximately 0.409 degrees. The angular resolution of a single cell in ERA-5, given its spatial resolution of 31 km, can be calculated as 0.0443 radians, which is equivalent to 2.54 degrees. It can thus be inferred that the cell resolution is approximately 6.20 times higher than that of ERA-5. In terms of temporal coherence, the TRMM and GPM missions were in operational overlap for a period of time, which allowed scientists to compare and calibrate data between the two missions. The aforementioned temporal overlap was of great importance for enhancing the coherence of precipitation data. During this period of temporal overlap, researchers developed algorithms for the calibration of TRMM data with GPM data, thereby ensuring the consistency of precipitation measurements between the two missions [2]. The Dual-frequency Precipitation Radar (DPR) on the Global Precipitation Measurement (GPM) satellite comprises two radars: one operating in the Ku band and the other in the Ka band. These radars are co-aligned on the GPM satellite bus, ensuring that the 5 km cell position on Earth is identical for both radars.

The temporal coherence between the Tropical Rainfall Measuring Mission (TRMM) and the Global Precipitation Measurement (GPM) mission enhances the accuracy and reliability of precipitation estimates, thereby improving the quality of data used for climate and meteorological studies. With regard to temporal coherence, the CloudSat radar operating in the W band, with a spatial resolution of 1.4 km, exhibits a coherence time of a few seconds for a single pixel. This is also the case with the GPM. Both satellites orbit at high speeds (7.5 km/s), which enables them to traverse areas of interest with remarkable rapidity. Based on these calculations, the estimated

¹<https://gpm.nasa.gov/resources/documents/cloudsat-gpm-coincidence-dataset-version-1c>

naked-eye visibility time of the satellite is approximately 93.33 seconds, determined by dividing 700 km by 7.5 km/s. In theory, a satellite in low Earth orbit (LEO) at altitudes between 700 km and 1000 km would be visible to the naked eye due to sunlight reflection. However, satellites like CloudSat, GPM, and TRMM are designed to minimize direct exposure to sunlight during measurements. This is achieved through the use of optical filters and shielding, which serve to reduce the impact of reflected sunlight on the sensors.

In conclusion, the concepts and fundamental steps required to obtain the raw attenuation map (without calculating correction contributions for attenuation from clouds, gases, etc.) have been presented. The focus then shifts to describing the methodology selected for the development of this work.

G. Methodology

The following methodology was utilized in the course of this research project:

- 1) **Dataset collection.** The initial stage of the process involved the collection of the data set. It is necessary to obtain the requisite datasets for the three bands of interest. CloudSat provides data in the W band, while the Global Precipitation Measurement (GPM) mission supplies information in the Ku and Ka bands.
- 2) **Inclusion of additional missions.** Incorporate data from other space missions, such as the Tropical Rainfall Measuring Mission (TRMM), to achieve comprehensive or near-comprehensive coverage.
- 3) **Precipitation data acquisition.** Utilize data from the Global Precipitation Measurement (GPM) mission to gather information on global precipitation, encompassing both liquid and solid forms.
- 4) **Identification of coincidences.** Determine the points of temporal and spatial coincidence between CloudSat and TRMM observations, as well as between CloudSat and GPM. This step is essential for correlating data from different missions. Subsequently, align the data spatially and temporally to ensure that the observations are superimposable, with each providing relevant information for the band of interest. This process may necessitate the use of data interpolation and validation algorithms.
- 5) **Extraction of PIA values.** Extract Path Integrated Attenuation (PIA) values under uncorrected measurement conditions. Further work is required to correct these values, taking into account specific global and local atmospheric conditions.
- 6) **Correlation of attenuation data.** Correlate the attenuation data with precipitation measurements from TRMM and GPM to gain a better understanding of how different weather conditions influence the attenuation values.
- 7) **Utilization of statistical modeling techniques.** Employ statistical modeling techniques to create an attenuation map that represents the 97th percentile of the observed attenuation conditions.
- 8) **Data fusion and kriging operations.** If necessary, perform data fusion or kriging operations to generate a comprehensive global map.
- 9) **Comparison with pre-existing models.** Compare the generated map with pre-existing models to verify its accuracy and reliability.
- 10) **Development and monitoring.** Ensure the virtuous development of computational tools and continuously monitor new observations to maintain the accuracy and relevance of the map.

II. REANALYSIS DATA AND THEIR AVAILABILITY

Satellite reanalysis data is a combination of observational measurements and simulation models that are used to create a consistent and continuous representation of atmospheric and climate conditions over time. These data are obtained through data assimilation techniques, which integrate satellite observations, weather station data, and other sources to produce comprehensive and accurate datasets [3]–[5]. These data are indispensable for a plethora of scientific and practical applications, rendering them an invaluable resource for researchers globally. The utilization of satellite reanalysis data for the CloudSat-GPM and CloudSat-TRMM coincidence datasets is crucial for enhancing my comprehension of global precipitation. These datasets integrate observations from CloudSat's cloud profiling radar (CPR) with those from GPM's dual-frequency radar (DPR) and TRMM's precipitation radar (PR). This combination enables the generation of "pseudo-three-frequency" (W-band from CPR, Ku/Ka-band from DPR) and "pseudo-two-frequency" (W-band from CPR, Ku-band from PR) radar profiles, which are valuable for scientific investigations into precipitation microphysics, cloud processes, radiative transfer simulations, and sensitivity to light rain and snow.

Further details and options to download the CloudSat-GPM and CloudSat-TRMM datasets can be found via the CloudSat repository via SFTP. The data may be ordered and downloaded in accordance with the instructions set forth in the Orders -> SFTP Access section of the CloudSat website [6], [7]. The files pertaining to coincidences can be located within the community-products directory, specifically under the following file names: 2B-CSATGPM.V04 and 2B-CSATTRMM.V04. For further detailed information, readers are directed to another technical article in which I provide a more precise description of the steps and IT procedures [8]. The provision of satellite reanalysis data at no or low cost is a common practice with the aim of advancing scientific research and environmental monitoring. The European Space Agency's Copernicus programme, for instance, offers free and open access to Sentinel satellite data via the Copernicus Open Access Hub [9]. This initiative facilitates data access for researchers, public institutions, and European projects, thereby contributing to a more comprehensive understanding of climate and environmental phenomena.

III. CHALLENGES FOR GLOBAL PIA MAPPING

A. Costs to generate the map

Several factors, including the desired resolution and the amount of data required, influence the total cost of producing a global Path Integrated Attenuation (PIA) map. For example, ESA's GLOBCOVER project, which produced a global land

cover map, required the use of several satellite instruments and generated up to 20 terabytes of imagery. The cost of such projects can vary considerably, potentially reaching tens of millions of euros. The production of a global PIA map requires contributions from several satellites. And organizations would be essential. For example, the GLOBCOVER project involved ESA's Envisat satellite and an international network of partners, including UNEP (United Nations Environment Programme),² FAO (Food and Agriculture Organization of the United Nations), IGBP (International Geosphere-Biosphere Programme),³ and the European Commission's JRC. Similarly, the creation of a PIA map would require coordination between different space agencies, research institutions and international organizations to collect, process and analyze data.

B. Multidisciplinary teams and international collaboration for global PIA mapping

The production of a global Path Integrated Attenuation (PIA) map requires a multidisciplinary team of experts in meteorology, climatology, data engineering, satellite analysis and climate modeling. The number of people required can vary depending on the complexity and resolution of the map, but a project of this scale could involve several hundred people. This team would include:

- **Scientists and researchers.** Experts in meteorology and climatology who analyze the data and validate the results.
- **Data engineers.** Professionals specializing in managing and processing large amounts of satellite data.
- **Satellite analysts.** Technicians who work with raw satellite data to extract useful information.
- **Climate modelers.** Scientists who develop and use models to simulate and forecast weather events.
- **Support staff.** System administrators, project managers, and other logistical and administrative support roles.

International cooperation is essential for a project of this magnitude. It would require the involvement of several space agencies, research institutions and international organizations. Key players could include NASA, ESA (European Space Agency), JAXA (Japan Aerospace Exploration Agency) and other relevant institutions. Space agencies such as NASA, ESA and JAXA bring their advanced satellite technologies and operational experience, while institutions such as WMO (World Meteorological Organization) [25] or regional meteorological agencies provide localized expertise and contextual knowledge. Academic and research institutions contribute innovative methodologies and provide critical validation. This collaboration ensures that the project is not only scientifically rigorous, but also globally representative, reflecting diverse environmental conditions and precipitation characteristics.

²UNEP works to promote sustainable development, protect ecosystems and address global environmental challenges such as climate change, pollution and loss of biodiversity. It is a reference point for international environmental policy and assists countries in the sustainable management of natural resources.

³The programme formally ended in 2015, but many of its activities and discoveries continue through other global initiatives, such as Future Earth, which focuses on sustainability and climate change adaptation.

C. Global Path Integrated Attenuation and Global Precipitation Intensity Map

The production of a global Path Integrated Attenuation (PIA.1) map is a monumental task that requires extensive expertise and coordination across disciplines. A multidisciplinary team is the backbone of such an endeavor, integrating knowledge from different fields to ensure the accuracy, efficiency and utility of the final product. At the core of the team are scientists and researchers specializing in meteorology and climatology, who analyze the complex dynamics of precipitation patterns and validate the data. These experts work closely with data engineers whose role is to manage and process the vast amounts of satellite data generated during the project. This includes cleaning, organizing and optimizing the datasets to ensure they are suitable for analysis. Meanwhile, satellite analysts focus on interpreting the raw data collected by satellite sensors, extracting meaningful information such as precipitation rates and their geographical distribution. Another critical group are climate modelers, who develop sophisticated models to simulate and predict precipitation events and relate them to attenuation. These models are essential for integrating observational data with theoretical frameworks to enable prediction and long-term analysis. In addition, the smooth running of the project relies on support staff, including system administrators who maintain the computing infrastructure, project managers who oversee schedules and budgets, and administrative staff who handle logistics and coordination. By bringing together this diverse range of expertise, a global PIA map can serve as a critical tool for telecommunications in the 6G era, and also enable the creation of a global Precipitation Intensity Map (PIA.2) for understanding precipitation dynamics, improving weather forecasting, managing water resources, and preparing for extreme weather events associated with climate change. Collaboration across disciplines and nations not only enhances scientific output, but also strengthens global partnerships to address pressing environmental challenges.

IV. SOFTWARE DEVELOPMENT METHODOLOGY AND STRATEGY

A. Global Path Integrated Attenuation (PIA) maps

The generation of a requires the utilization of sophisticated software and a meticulous methodological strategy. In the initial phase, data from satellites such as CloudSat, TRMM, and GPM, which offer varying spatial and temporal resolutions, should be collated and stored in a centralized database for convenient access and management. The data must then be subjected to a process of pre-processing, whereby georeferencing techniques are employed to accurately align the satellite data and filters are applied to remove any anomalies, thus improving the overall quality of the data. The employment of sophisticated algorithms, such as the Surface Reference Technique (SRT) and the utilization of dual-polarization radars, is imperative for the rectification of radar signal attenuation resulting from precipitation. Furthermore, the incorporation of machine learning algorithms, such as the Light Gradient Boosting Machine (LightGBM), can facilitate the enhancement of attenuation estimates based on environmental vari-

ables. The Light Gradient Boosting Machine (LightGBM) can be employed to enhance the quality and precision of a global Path Integrated Attenuation (PIA) map of attenuation due to its capacity to process copious amounts of data, discern intricate relationships, and generate precise estimates. Data from diverse satellite sources (e.g., CloudSat, GPM, CALIPSO) and environmental variables (e.g., temperature, humidity, pressure, and precipitation intensity) can be used to train LightGBM.

The model's capacity to process heterogeneous datasets enables the generation of a more robust analysis, whereby the integrated information facilitates enhanced attenuation estimates. One of the primary challenges in the creation of a PIA map is the correction of radar signals that have been attenuated by the presence of precipitation. In comparison to conventional techniques such as empirical or simple regression, the LightGBM algorithm enhances the precision of the results by modeling the non-linear relationship between environmental variables and signal attenuation. The Light GBM algorithm is a valuable tool for anomaly detection and correction, as well as for making localized and adaptive estimates during the pre-processing of satellite data. Indeed, during the data processing stage, anomalies resulting from measurement errors, atmospheric disturbances, or instrumental issues may emerge. By identifying anomalous patterns in the data and replacing them with estimates generated by the model, Light GBM enhances the overall quality of the record. Statistical analysis techniques, such as principal component analysis (PCA) and time series analysis, are of great importance in the identification of patterns and trends in radar attenuation data, as well as in the comprehension of the spatial and temporal variability of attenuation. In light of the considerable complexity and volume of information, the utilization of supercomputers for data processing and the implementation of parallel computing techniques can markedly accelerate the process. Furthermore, the creation of interactive maps for the visualization of attenuation data on a global scale is a crucial step, and the comparison of the results with reference data and field measurements serves to validate the accuracy of the estimates.

Following the retirement of CloudSat, other satellites, including the Global Precipitation Measurement (GPM) and the Cloud-Aerosol Lidar and Infrared Pathfinder Satellite Observations (CALIPSO), have continued to undertake similar work, providing valuable data on the vertical distribution of clouds and aerosols [17], [18]. These satellites, in conjunction with other members of the A-Train constellation, facilitate enhanced comprehension of global climate and weather patterns. The generation of a comprehensive PIA map necessitates inter-agency collaboration between diverse space agencies, research institutions, and international organizations, with the sharing of data, computing resources, and technical expertise being pivotal for the success of the project [19], [20].

B. Data assimilation and Kalman filters

Advanced data assimilation algorithms represent a technique employed for the purpose of combining observations derived from disparate sources, including but not limited to

satellites, weather stations, and climate models. The objective of this process is to obtain a representation of atmospheric and climate conditions that is both consistent and accurate. The deployment of these algorithms is vital to enhance the quality of weather forecasts and climate analyses. One of the most widely utilized algorithms is the Kalman filter, which updates the estimates of the state variables of a dynamical system in real time, integrating new observations as they become available. Another assimilation method is the method of reduced variances (4D-Var), which seeks to minimize the discrepancy between observations and model predictions, taking both past and future observations into account. A variant of the Kalman filter is the Ensemble Kalman filter (EnKF), which employs a set of simulations to account for the uncertainty inherent in the model estimates. Finally, machine learning-based data assimilation utilizes machine learning algorithms to enhance data assimilation by identifying latent patterns and relationships in observational data. These algorithms, through the integration of disparate data sources and the utilization of sophisticated techniques, facilitate more precise weather forecasts and more comprehensive climate analyses, thereby making a substantial contribution to my comprehension of atmospheric dynamics.

V. PROPOSED APPROACH

My approach to creating a global Path Integrated Attenuation (PIA) map involves the creation of a comprehensive imagery database to train a neural network, with a focus on cataloging and collecting radar reflectivity as vertical profiles of clouds. This process starts with the collection of data from satellites such as CloudSat, TRMM, and GPM, which offer different spatial and temporal resolutions. The collected data is then stored in a centralized database for easy access and management. Next, the data is pre-processed using georeferencing techniques to accurately align the satellite data and apply filters to remove any anomalies, thus improving the overall data quality. To correct the radar signal attenuation caused by rain, advanced algorithms such as the Surface Reference Technique (SRT) and dual polarization radar are used. Furthermore, the implementation of machine learning algorithms, such as the Light Gradient Boosting Machine (LightGBM), can further improve the attenuation estimates based on environmental variables. Statistical analysis plays a crucial role in this process. Techniques such as principal component analysis (PCA) help identify patterns and trends in radar attenuation data, while time series analysis helps understand the spatial and temporal variability of attenuation. Given the complexity and volume of information, the use of supercomputers to process the data and the implementation of parallel computing techniques can significantly speed up the process.

The main goal is to connect the analysis of cataloged and identified parameters from satellite coincidences with the microphysical and macrophysical characteristics of the atmosphere. The focus is on uncovering hidden relationships within the data that may not be immediately apparent. Initially, the estimation of Path Integrated Attenuation (PIA) was based

on σ_0 . Subsequently, the focus shifted towards the PIA of hydrometeors, which allowed for a more effective concentration on generating the global attenuation map. Due to the high computational costs involved, the priority was placed on demonstrating and explaining the methodology rather than making corrections to the radar reflectivity using the PIA or correcting the PIA itself during the generation of the global maps. This approach ensured that the methodology was clearly understood and could be applied effectively, even if some corrections to the radar reflectivity and PIA were deferred to manage computational resources efficiently. To quickly generate a global attenuation map, I can consider the CloudSat overflights and their coincidences with GPM over a certain number of days. The satellite overflights are represented by the ground track and swath to ensure complete coverage. The daily tracks of CloudSat in polar orbit were used to visualize the various passages, which were then displayed on a global map. This map shows the uncorrected PIA(W), meaning it does not account for corrections needed for atmospheric gases, particularly water vapor and oxygen. By analyzing the CloudSat-GPM satellite coincidences, I can obtain latitude, longitude, and uncorrected attenuation values for the Ku and Ka bands. These values are decoupled, whereas in the coincidences, they are matched. I can also derive the value of PIA(W) which corresponds to the notion of PIA_hydrometeor related to the product 2C-PRECIP-COLUMN. The Sigma-Zero variable is present in the 2B-GEOPROF product. Sigma-zero, or σ_0 (σ_0), is a measure of the radar reflectivity of a surface per unit area, normalized with respect to the angle of incidence of the radar. In other words, it represents the strength of the radar signal reflected by the observed surface. It is a fundamental parameter for evaluating the reflectivity of various targets, such as soil, vegetation, or atmospheric particles. Path Integrated Attenuation (PIA) is a measure of the total attenuation of the radar signal as it travels through the atmosphere. This attenuation is caused primarily by absorption and scattering of the signal by particles such as raindrops, snow, ice, and atmospheric gases (e.g., water vapor and oxygen). PIA is important for correcting radar measurements and obtaining accurate estimates of atmospheric properties.

A. Methodological steps

I propose in this work and in the **Appendix Technical Review**, my initial steps:

1. Dataset Collection:

- CloudSat (W band)
- GPM (Ku and Ka bands)

2. Inclusion of Additional Missions:

- TRMM data

3. Precipitation Data Acquisition:

- GPM for global precipitation (liquid and solid forms)

4. Identification of Coincidences:

- Temporal and spatial alignment between CloudSat, TRMM, and GPM
- Use of data interpolation and validation algorithms

5. Extraction of PIA Values:

- Initial extraction under uncorrected conditions
 - Correction for global and local atmospheric conditions
- ##### 6. Correlation of Attenuation Data:
- Correlate with precipitation measurements from TRMM and GPM
- ##### 7. Utilization of Statistical Modeling Techniques:
- Create attenuation map (97th percentile of observed conditions)
- ##### 8. Data Fusion and Kriging Operations:
- Generate a comprehensive global map
- ##### 9. Comparison with Pre-existing Models:
- Verify accuracy and reliability
 - Reverse Engineering (RE) on other works with permission
 - Implement Machine Learning Techniques using RE information
- ##### 10. Development and Monitoring:
- Continuously develop computational tools
 - Monitor new observations for accuracy and relevance

VI. RESULTS

Examining PIA values across different bands reveals the extent to which various atmospheric components contribute to overall attenuation. For instance, higher PIA values in the W-band compared to the Ku-band and Ka-band may indicate a greater influence of smaller particles or gases that predominantly affect higher frequencies. Additionally, these maps can help identify regions with persistent or anomalous attenuation patterns, which could be indicative of specific meteorological phenomena or environmental conditions. Such information is crucial for improving the accuracy of radar-based precipitation estimates, enhancing weather prediction models, and advancing my understanding of atmospheric processes. Uncorrected PIA maps can serve as a starting point for correction algorithms that consider the impact of atmospheric gases. This methodology enables better calibration of radar systems and more precise measurement of hydrometeor attenuation. I look at PIA maps from CloudSat and GPM coincidences that haven't been fixed to give operational meteorology and atmospheric research a basic set of data. In particular, I can do reverse engineering and comparative analysis by looking at integrated attenuation maps along with other published works. The technical annex associated with this study illustrates the process of obtaining global maps of Sigma-zero or PIA from publications through reverse engineering, using data sourced from other researchers. The cited authors' consent to process their data will be required for publication of this work, as there are potential conflicting intellectual property rights and copyright issues. Especially if the maps are of high quality, obtaining such consent would result in substantial labor and cost savings. This shared data can considerably improve the accuracy of the proposed methods when reverse engineering is unnecessary and sharing is feasible. I initially developed these methodologies on research laboratory computers, not on supercomputers as would typically be required. I employ machine learning and reverse engineering techniques to reduce

costs and demonstrate their feasibility. I will also demonstrate a practical method for analyzing satellite coincidence files to construct a global attenuation map. These maps can subsequently endure incremental corrections.

A. Innovative Approach to Generating Global Attenuation Maps Using Satellite Coincidences

The analysis of satellite coincidences between CloudSat-GPM and CloudSat-TRMM introduces an innovative method for generating global attenuation maps. CloudSat's millimeter-wave radar supplies detailed profiles of clouds and measurements of water content, while GPM and TRMM deliver data on precipitation and brightness temperatures through their advanced instruments. These coincidences, where satellites simultaneously observe the same region of Earth, enable cross-validation of data and a comprehensive understanding of atmospheric phenomena. By applying machine learning and reverse engineering techniques to the combined data, it's possible to create maps that depict the decrease in radar signal intensity due to clouds and precipitation.

B. Starting from Coincidence Dataset from PPS

F. Joseph Turk et al. (2021) [1] provide extensive information on the GPM Dual-Frequency Precipitation Radar (DPR) in the Ku and Ka bands (**T. Kobayashi et. al., 2021**) [15], as well as the CloudSat Profiling Radar (CPR) in the W-band. The study focuses on the coincidence data products from the CloudSat-TRMM (CloudSat+PR/TMI) and CloudSat-GPM (CloudSat+DPR/GMI) observational periods. These data products can be found through the NASA Precipitation Processing System (PPS) [34]. These combined multi-frequency radar and passive microwave data are valuable for various applications related to precipitation estimation. These coincidences will be used to extract and visualize integrated path attenuation data, with the aim of creating a global attenuation map.

C. General Costs in Creating the Global Map

The generation of a global attenuation map is a complex process that requires significant computational and financial resources. Here are some factors to consider:

- 1) **Time:** The time required depends on the desired spatial and temporal resolution, the amount of data to process, and the available computational power. Generally, it could take from several months to a year to complete the entire process, including data collection, processing, and validation.
- 2) **Cost:** Costs can vary widely based on the necessary resources, such as access to satellite data, computational infrastructure (servers, cloud computing), and specialized personnel. Indicatively, costs could range from tens of thousands to hundreds of thousands of euros.

These are only indicative estimates, and the actual costs and time may vary based on specific project requirements and available resources.

D. Describing the Algorithm to Generate Global Map in the three bands: Ku, Ka and W

The process of creating a global attenuation map necessitates the cycle through numerous coincidence files, a task further complicated by the potential for corrupt or unusable files. With 5 to 8 files generated daily over several years, the workload quickly becomes overwhelming. Additionally, processing these files demands significant memory space, making it a complex task to complete in a short time frame. Downloading files via FTP can take several hours for just one month's worth of data, and the reliability of BULK download services is not always guaranteed. I downloaded files for January 2016, February 2016, and additional files from May 2016 to evaluate the time required for data processing. This approach aimed to obtain a representative sample and better understand the resource demands and potential challenges associated with the large-scale handling of satellite coincidence data. The disk space required for just one month (31 days) of CloudSat-GPM coincidences is 3.91 GB uncompressed. Each file, averaging around 16 MB, can demand several hundred megabytes for subsequent processing. This makes generating the global attenuation map challenging due to the necessary corrections and processing time. To address this, it is crucial to develop a file cycling algorithm that avoids saturating memory and ensures the process continues smoothly when generating real-time images, such as radar reflectivity profiles or the PIA(Ku), PIA(Ka), and PIA(W) World Map. Optimizing the code requires initially storing the values of interest, such as PIA_hydrometeor and PIA(W), within a single data structure. Next, I should appropriately separate the combined values of PIA(Ku) and PIA(Ka). Finally, I am going to dynamically manage the data to create suitable data structures that preserve georeferencing information, file names, variables and other relevant details. I have created a structure that includes Latitude, Longitude, PIA(W), PIA(Ku), PIA(Ka), and other characteristic columns by implementing this approach. The selection of other relevant columns depends on the specific objectives to be developed. For the purposes of creating the map, information such as the precip_flag that establishes the character of the storm precipitation could be useful.

E. Working and Developing the Main Idea

The satellite coincidence reading between CloudSat and Global Precipitation Measurement (GPM) provides a unique approach to improving the understanding of global precipitation. While the CloudSat-TRMM (Tropical Rainfall Measuring Mission) coincidences have provided valuable data on tropical precipitation, the integration with GPM allows this analysis to extend to a global scale, including the polar regions. With CloudSat's ability to identify the vertical structure of clouds and the precision of GPM to measure the intensity of precipitation, the combination provides a more complete and detailed view of atmospheric processes, contributing to better climate models and weather forecasts. In order to compile the necessary data for the creation of the global attenuation map, the coincidence data must be downloaded. Although this is not a complex operation, it can be time-consuming if relying

solely on the STORM web interface or accessing the CloudSat web portal by registering without using bulk downloads. Once the files are obtained, they need to be sorted by timestamp, a “working pool” must be created, and custom-made software (e.g., MATLAB) should be used to read and process their content. The technical support file (Annex) provides a detailed explanation of the dataset and the implementation details followed.

F. Working Sample

Several programming techniques can perform the task of scanning coincidence files, and these techniques also depend on the intended use of the data. For instance, if you want to create case studies regarding satellite coincidence products, it could be fascinating to evaluate cases similar to those published in articles by **J.F. Turk et al.**, **Haynes et al.**, or **Battaglia et al.** In this scenario, it could be intriguing to examine the creation of case studies that incorporate the source code for image generation, such as Cloud-Mask or Sigma-Zero, and the radar reflectivity found in the 2B-GEOPROF product. When I directly aim for the result I am interested in, which is the creation of a world map, the outcome may not be what one would anticipate. CloudSat-GPM’s exclusive contribution for a month’s worth of files, encompassing 4.7 gigabytes of data, generates a rough map. This rough graph necessitates a thorough review to identify potential issues like unread, out-of-scale, or undetected data. Upon cleaning the data and rendering it at a graphic level, the resulting images lack professional quality, necessitating the appropriate scaling of the resolution to align with the representative scale and the selected geographic projection. The images I’ve obtained here are superior to the initial MATLAB experiments I conducted using the 97th centile. However, choosing the right projection type is crucial to ensure accurate point representation. I consider the Mollweide projection to be the most advisable.

G. Taking action, understanding the expected values of global PIA

Before plotting or creating a global map of Path Integrated Attenuation (PIA), it is crucial to understand the underlying variations and factors that influence PIA values across different regions and meteorological conditions. Having a comprehensive understanding of these variations allows for more accurate interpretation and analysis of the data. It also helps in identifying patterns and anomalies that may be significant for scientific research and practical applications. By considering the maximum, minimum, and average PIA values in various geographic regions, one can better appreciate the impact of local weather phenomena and radar configurations on PIA measurements. This knowledge is essential for improving the accuracy and reliability of global PIA maps, which are valuable tools for meteorologists, climate scientists, and researchers studying precipitation and atmospheric processes. Path Integrated Attenuation (PIA) varies significantly globally across meteorological conditions and geographic regions. In tropical regions, such as the Amazon and Southeast Asia, maximum PIA values can exceed 30 dB during heavy rainfall,

with average values between 10 and 20 dB. In temperate regions, such as Europe and North America, maximum values can reach 20-25 dB during severe thunderstorms, with average values between 5 and 15 dB. In polar regions, maximum values are generally lower, reaching 10-15 dB during snowstorms, with average values between 2 and 8 dB. Uneven radar beam filling and scanning configurations cause these changes by increasing the number of radar signatures and lowering the noise level through deconvolution.

H. Primary and secondary targets in My analysis

By leveraging advanced data analysis techniques and the CloudSat-GPM dataset of coincidences, I strive to uncover hidden patterns and correlations. My approach involves creating a comprehensive database of images to train a neural network, with a focus on cataloging and collecting radar reflectivity as vertical cloud profiles. The general idea is to link the study carried out on the cataloging and identification of the parameters present in satellite coincidences with the microphysical and macrophysical aspects of the atmosphere. The primary objective is to search for relationships within the data that may not be immediately evident. Initially, I estimated the Path Integrated Attenuation (PIA) using Sigma-Zero. Afterward, I focused more on the PIA of the hydrometeors. This shift allowed me to concentrate more effectively on generating the global attenuation map. Due to computational costs, I prioritized demonstrating and explaining the methodology over making corrections to the radar reflectivity using the PIA or correcting the PIA itself in the generation of the global maps. For example, I did not account for attenuation by atmospheric gases such as water vapor and oxygen. The objective is to generate a global attenuation map, and corrections must be applied primarily to the PIA (Path Integrated Attenuation). Here’s why:

- 1) **Correction of Atmospheric Attenuation:** The PIA must be corrected for attenuation caused by atmospheric gases such as water vapor and oxygen. This is essential to obtain an accurate estimate of the attenuation along the radar signal path.
- 2) **Correction of Hydrometeor Attenuation:** The PIA must also be corrected for attenuation caused by hydrometeors (rain droplets, snow, etc.). This allows for an attenuation map that accurately reflects atmospheric conditions.
- 3) **Validation of Corrected PIA:** Once the PIA is corrected, it is important to validate the data by comparing it with independent measurements or reference models to ensure the corrections are accurate.
- 4) **Soil moisture effect:** Soil moisture can cause an underestimation of PIA if not considered. A correction method has been developed that takes this effect into account, improving estimates of surface precipitation. [10]

I. Exploring first results

In my work, I have optimized the source code, which initially allows for storing the values of interest, such as PIA_hydrometeor (PIA_W), PIA(Ku), and PIA(Ka) in a single

data structure. Coincidences combine the values of PIA(Ku) and PIA(Ka), so decoupling these values is one of the first operations to perform. So, after having appropriately separated the coupled values of PIA(Ku) and PIA(Ka), I have also inserted the value of PIA(W). You can read more technical details in the attached annex to this article. Finally, I have dynamically processed the data to create data structures that are suitable for my needs. I am maintaining the georeferencing information, the names of the scanned coincidence files, the variables of interest, and more. In this way, I can create a structure that includes both Latitude, Longitude, PIA_W, PIA_Ku, PIA_Ka, and other relevant columns. It is important to note that the 2B-CWC-RVOD group is usually found in satellite coincidences between CloudSat-TRMM rather than CloudSat-GPM, and therefore this must be taken into account when developing the read cycle. The variable called “PIA Fwd” indicates the PIA Forward, which is copied from the 2B-CWC-RVOD product and is always expressed in decibels (dB). The PIA Forward (Path-Integrated Attenuation Forward) in the CloudSat 2B-CWC-RVOD product is essential for several reasons:

- **Signal Weakness:** A weak radar signal, due to low particle density in clouds or precipitation, may result in undetected PIA Fwd.
- **Atmospheric Conditions:** Specific atmospheric conditions, such as thin cloud layers or very light precipitation, can hinder the radar’s ability to measure path attenuation.
- **Instrument Limitations:** The onboard instruments may have limitations affecting measurement accuracy and sensitivity, making it difficult to detect very low or very high attenuations.
- **Calibration Errors:** Calibration errors in radar instruments can lead to missing or inaccurate data. Proper calibration is essential for accurate measurements. These factors may contribute to the lack of PIA Fwd data in the CloudSat-TRMM coincidences.

A more in-depth study on the use of this product can be obtained by downloading not the type 2B satellite coincidence products but rather the type 2A ones. The description of this type of product with the related files is very useful, especially in the latest version of the data, V09. Talking in depth about the use of this new pool of satellite files is beyond the scope of this article, although it is essential to be aware of it. **Figures 1 and 2** illustrate the results of the data structure reading and storage algorithm. Many others can be realized and improved until I generate complex attenuation maps like those in **A.Battaglia et.al. (2020)** works, that I report for convenience in this same article. The maps and data presented in Battaglia’s article are based on four years of data collected by the NASA-JAXA Global Precipitation Measurement (GPM) mission. I label this data collection period as June 2014-May 2018. Therefore, the attenuation maps and diurnal precipitation cycle analyses reflect the climate and weather conditions of that period. Analyzing four years of satellite data requires a significant commitment of time and resources. Here are some key points:

- **Analysis Time:** The time required depends on the avail-

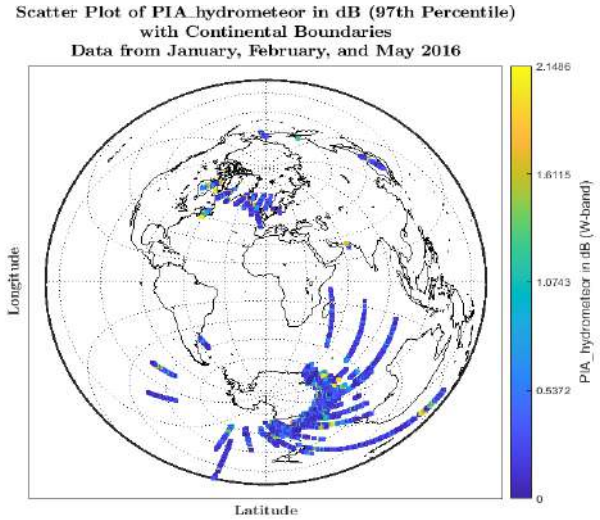


Fig. 1: A better choice in map projection from CloudSat-GPM Satellite coincidence Dataset at W Band.

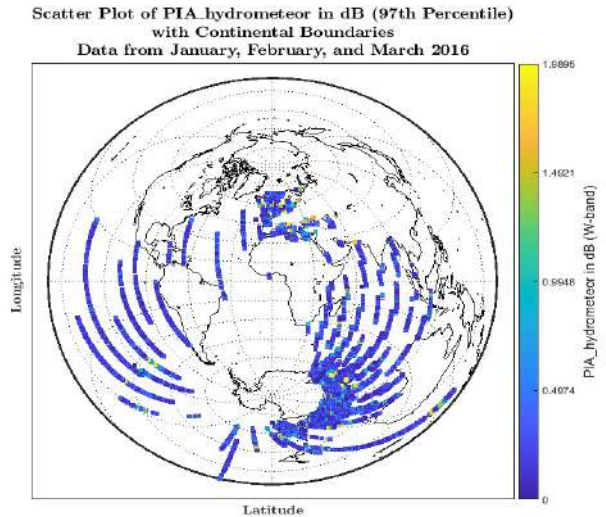


Fig. 2: CloudSat-GPM Satellite coincidence Dataset at W Band from January to March 2016.

able computing power. The University of Leicester’s ALICE system, a high-performance computing infrastructure, significantly accelerates the analysis.

- **Disk Space:** The amount of disk space required varies based on the resolution of the data and the specific products used. For example, GPM Level 2 and 3 products can require several terabytes of space for storage and processing.

J. Importance of Path Integrated Attenuation (PIA) and Considerations for Anomalies in Polar Regions

The Path Integrated Attenuation (PIA) of hydrometeors varies globally depending on atmospheric conditions and Earth’s climatic zones. In equatorial regions (tropical climates), heavy rainfall dominates, with PIA typically ranging

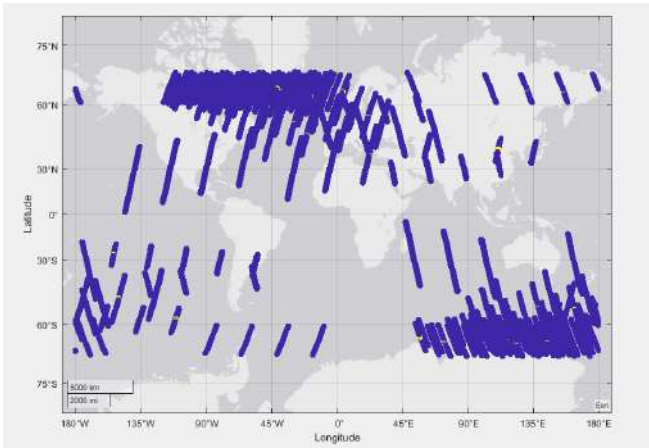


Fig. 3: Initial output for PIA hydrometeor from CloudSat-GPM Satellite Coincidence Dataset at Ka Band. Passages from January-February and May 2016 are colored in violet, while values different from -9999 (No valid value) are colored in yellow.

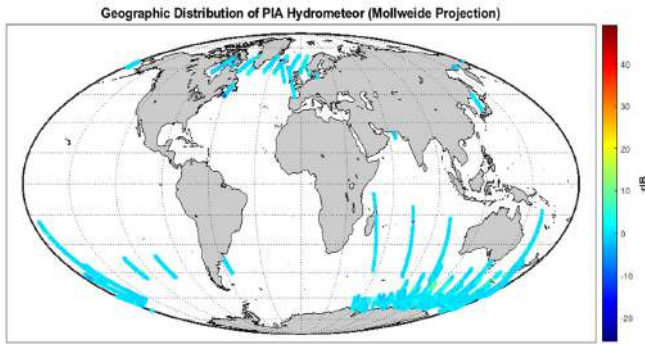


Fig. 4: Geographical Distribution of PIA Hydrometeor (PIA(W)) from CloudSat-GPM Satellite Coincidence Dataset (January-February and May 2016 Passages).

from 5-30 dB, exceeding 30 dB during extreme events like tropical cyclones, especially at frequencies above 10 GHz. Arid and semi-arid regions (deserts and steppes) experience low PIA, typically less than 1 dB, with occasional increases to 1-3 dB due to dust or sandstorms affecting frequencies above 20 GHz. Temperate zones (oceanic and continental climates) show moderate PIA, typically 1-10 dB for rain and 0.5-5 dB for wet snow, but less than 1 dB for dry snow, with significant attenuation during hailstorms (10-30 dB). In polar regions, where precipitation is mostly solid (dry snow and ice), PIA is minimal, often below 1 dB, with slight increases (0.1-2 dB) during wet snow events. Mountainous regions (> 2,000 m) exhibit low PIA for dry snow (< 1 dB) but moderate levels (1-3 dB) for wet snow and potentially higher values (2-15 dB) during heavy rain at lower altitudes. Overall, PIA is highest in tropical regions due to intense rain and lowest in polar and arid zones due to minimal precipitation. PIA is a crucial parameter for understanding the impact of atmospheric con-

ditions on communications and weather monitoring. Higher frequencies experience greater attenuation, and lower elevation angles of radar beams show higher PIA values due to passing through more atmosphere. Seasonal variability, particularly in polar regions, requires careful data interpretation. Sudden high PIA values may indicate anomalous weather phenomena like wet snowstorms, while unusual distributions could signal atmospheric contaminants. Monitoring long-term PIA trends helps detect significant climate changes. Satellite sensors and weather radar, coupled with international scientific collaborations, are essential for accurate and timely analysis. These tools improve my understanding of atmospheric phenomena and climate change, enhancing my ability to forecast and mitigate adverse effects. It is important to be concerned about anomalous PIA values, especially in polar regions, as they may signal significant atmospheric phenomena. A sudden increase in PIA, for example, above 2 dB, may indicate unusual weather conditions, such as wet snowstorms or ice melting, with implications for navigation and scientific operations. Unusual PIA distributions may suggest the presence of unusual atmospheric processes or interference from contaminants, such as dust or industrial pollutants. Monitoring long-term trends in PIA values helps detect significant climate changes, contributing to a better understanding of global atmospheric dynamics. These data are essential for making informed decisions in resource management and planning.

Figure 3 shows the initial output for PIA hydrometeor from the CloudSat-GPM Satellite Coincidence Dataset at Ka Band. Passages from January-February and May 2016 are colored in violet, while values different from -9999 (No valid value) are colored in yellow. **Figure 4** shows the geographical distribution of PIA Hydrometeor (PIA(W)) from the CloudSat-GPM Satellite Coincidence Dataset for passages in January-February and May 2016.

K. Create and Plot Global Maps about Sigma-Zero, PIA

Although sigma-zero and PIA do not inherently provide direct geographical references, it is possible to create such maps, especially for satellites like CloudSat that operate at Nadir. Therefore, I can build maps that show the spatial distribution of sigma-zero and PIA. These maps allow me to visualize the geographical areas of the Earth where attenuation is greater. By using PIA to estimate precipitation intensity and combining this information with other radar measurements, I can create precipitation maps that display the spatial distribution of rainfall.

I proceeded to create various versions of tables by accumulating the values in data structures:

- TGeoPIAKuKa(filename, latitude, longitude, PIA_MS PiaKu, PIA_MS PiaKa)
- TGeo(filename, latitude, longitude, PIA_MS PiaKu, PIA_MS PiaKa, Sigma-zero, Piahydro2way, PIAuncert2way)
- TGeoST(filename, latitude, longitude, along-track-distance, time, PiaKu, PiaKa, Sigma-zero, Piahydro2way, PIAuncert2way)

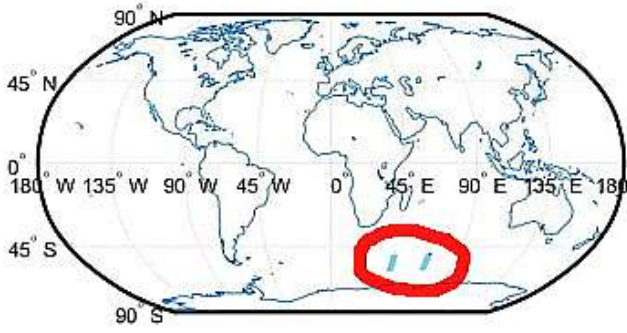


Fig. 5: Turning on pixels with the 97th percentile technique. The concept of percentile visualization on the Global Map is evidenced in the red circle over the image.

TGeoST would stand for “Georeferenced Table Save Table”, meaning that once the data was entered, instructions were followed to save it as Matlab files with a *.mat extension.

After successfully storing the code and data in the structures, I realized that the data was often nested, making it difficult to graph them directly from MATLAB due to their large size. To address this, I created 97th percentile maps that highlighted the areas where the satellite passed. These maps were used to mark the attenuation in those regions. In MATLAB I use square brackets for Data structure in place of curl brackets. When I attempted to use the percentile technique to create the PIA map, I discovered that the satellite passages over a week were quite few and did not completely cover the globe. Including the passages of the TRMM satellite would provide more information, but managing the coincidences to recreate the global map is rather laborious. Here, I aimed to demonstrate its feasibility, considering what has been published in the scientific literature, such as in the works of **Battaglia et al. (2014)** [35], **A.Battaglia, S.Tanelli et.al. (2020)** [36]. I also may reference to other works from **Battaglia et.al. (2020)** [37].

In particular, the technique of using the 97th percentile descends from **Battaglia et.al. (2020)** [38]. In this work is described the NASA-JAXA GPM mission, with its dual-frequency radar and multifrequency microwave radiometer, provides a unique opportunity to quantify these attenuation effects. Using four years of GPM data, the study presents a global climatology of Ka-band attenuation caused by clouds and precipitation, highlighting the impact of the diurnal cycle of precipitation. High attenuation regions correspond to precipitation patterns, with significant attenuation observed in the tropical rain belt, the South Pacific Convergence Zone, and storm tracks in the North Atlantic and Pacific Oceans. The diurnal cycle is notably strong over land and during summer, while it is weaker but still present over oceans. These findings are crucial for estimating data loss in satellites using frequencies near 35 GHz.

The Battaglia’s maps are created using a combination of GPM (Global Precipitation Measurement) products to compute

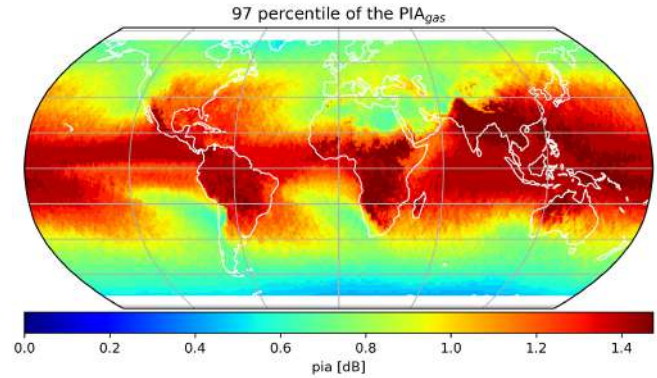


Fig. 6: Global map of PIA_{gas} from A.Battaglia et. al. (2020)

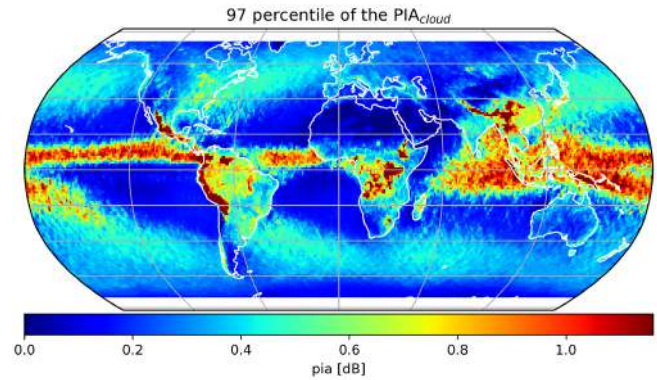


Fig. 7: Global map of PIA_{cloud} from A.Battaglia et.al. (2020)

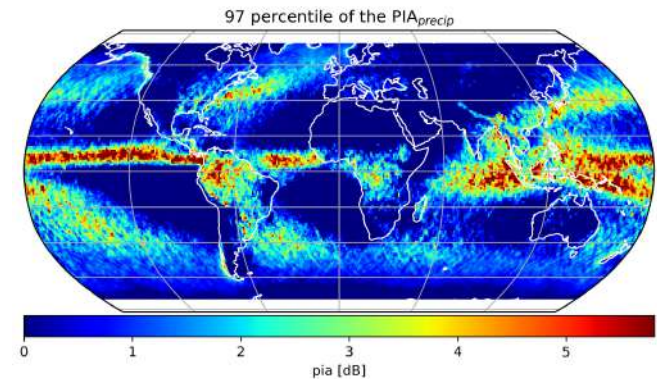


Fig. 8: Global map of PIA_{precipitation} from A.Battaglia et.al. (2020)

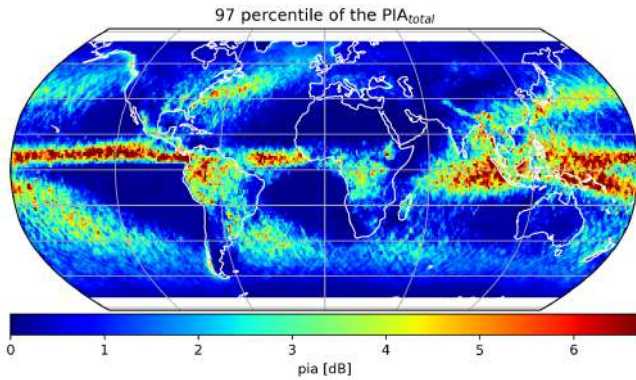


Fig. 9: Global map of PIA_{total} from A. Battaglia et.al. (2020)

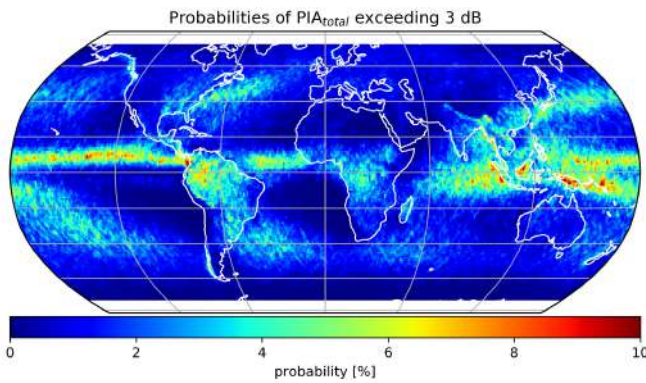


Fig. 10: Probability of the two-way PIA caused by cloud + precipitation exceeding 3 dB derived from combining DPR and GMI measurements. From Battaglia et.al. (2020).

the various contributions. Different color scales are used in each plot to accurately represent the data.

So, inspired by this work, it is possible to build similar maps, although it may be burdensome to download data via FTP for four years. Technical details on the use of PIA can be found on page 1813 of the article “GPM-Derived Climatology of Attenuation due to Clouds and Precipitation at Ka-Band”, (Battaglia et.al., 2020). Creating maps like those presented in Battaglia’s works [38] is far from elementary. Using the 97th percentile technique (though other methods could also be chosen) and analyzing **CloudSat-GPM** coincidences for an entire month of January, I encounter sections that are not punctual because they follow the satellite’s swath on the ground. This makes it challenging to create an adequate image like the one I aim to obtain. Understanding this circumstance is crucial, in my opinion.

L. Reverse Engineering exploring

Reverse engineering published data, as in the case of **A. Battaglia’s** images [38], is an intriguing and powerful technique for obtaining precise numerical information, especially when the original data are not readily available. This

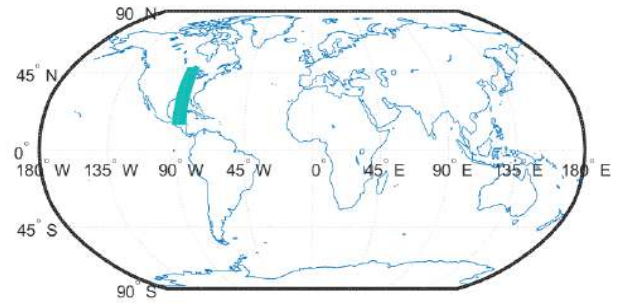


Fig. 11: Example of a passage of the Satellite, from 2B.CSATGPM.COIN.33N_089W_00000_056_274_375. 20160502-S075951-E080921.012364.V04.nc4. Visualization of the Sigma-zero variable. View of the 97th percentile technique panel lighting that contains the satellite passage swath inside it.

methodology can be particularly useful when working with scientific images and other graphical representations, where color gradients may not be intuitive for human interpretation. The advantages of this approach are many. It allows access to data even in the absence of the original sets, facilitating research, analysis, and comparisons. Furthermore, it can represent an opportunity to learn and develop new technical skills, improving your understanding of image analysis and processing methods. The versatility of reverse engineering makes it applicable to different types of data and images, proving useful in various fields of scientific research and engineering. However, this technique also has significant disadvantages and limitations. Since published images often have legal protection, extracting data from them may violate copyright. It is crucial to proceed carefully, respecting the laws in force and, if possible, obtaining permission from the original authors. Additionally, images may not be perfectly sharp or contain artifacts that introduce errors into the extracted data. The quality of the source image is essential to the accuracy of the results. It is not an effortless technique to master, requiring advanced knowledge in image processing, programming, and data analysis. Exercise caution when performing reverse engineering to prevent intellectual rights infringement, and justify and respect the authors’ original work when using the extracted data. Despite these challenges, the results obtained so far in my project are encouraging, demonstrating that reverse engineering can be effective when applied correctly. It is important to balance technical innovation with respect for intellectual property and copyright laws. Continuing to develop and refine this technique can open up new possibilities in research, provided that I operate with ethics and legal awareness.

To extract the **PIA_{gaseous}** and **PIA_{cloud}** attenuation functions from images in MATLAB, the following general steps can be followed:

- 1) **Load Images:** Use the `imread` function to load the map images.
- 2) **Extract Data:** If the images contain color-coded nu-

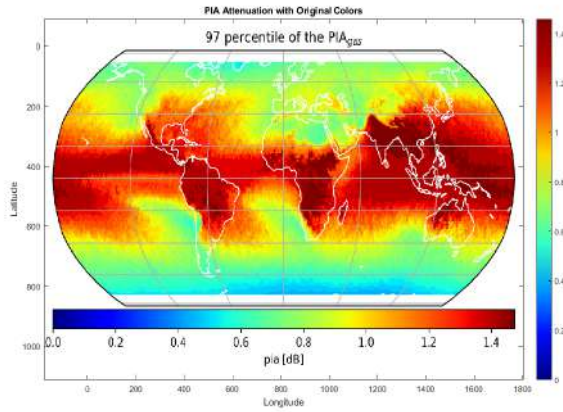


Fig. 12: Original image from Battaglia's article (2020): I choose the 97 Percentile PIA_{gas} Global Map.

meric data, convert the colors to numeric values by mapping the colors to the corresponding attenuation values.

- 3) **Create Functions:** Once you have the numeric data, create functions that represent the attenuation as a function of geographic location.

Path Integrated Attenuation (PIA) varies depending on the frequency band and atmospheric conditions. For the Ku Band (12-18 GHz), PIA can vary significantly, but on average, it is around 1-3 dB for moderate rain conditions. However, in heavy rain, PIA can exceed 10 dB. The Ka Band (26.5-40 GHz) generally has higher PIA compared to the Ku band due to greater sensitivity to precipitation, with typical values ranging from 3 to 10 dB and peaks that can exceed 20 dB in heavy rain. The W Band (75-110 GHz) is even more sensitive to attenuation caused by clouds and precipitation, with PIA values easily exceeding 10 dB and reaching up to 30 dB or more in very heavy rain. For a deeper understanding of this concept, it is recommended to read **Liao and Meneghini's** work from 2019. [39]

M. Reverse Engineering on Global Maps

I show how to proceed in the case of reverse engineering. More details on the utility of the RE and import will be provided in a technical annex in which I will describe how to obtain other advantages and in which I describe the source code implementation. **Figure 12** shows the original image from Battaglia's article (2020), where the 97th percentile PIA_{gas} global map is chosen. **Figure 13** shows the imported and original image from Battaglia's article (2020). **Figure 14** shows the importing result obtained in MATLAB.

In the next working code from **Algorithm 1**, I show the correct code with the reverse engineering color map, optimized with a RGB scouting on the original color map depicted in the horizontal dimension. The vertical one is the color map acquired by the Reverse Engineering process. **Algorithm 2** represent a concrete example code with good importing using color Map in MATLAB.

Algorithm 1 The working code for reverse engineering Battaglia PIA_{gas} Image with interpolated color recreated on MATLAB.

```
% Load the image
img = imread('BattagliaImagewithColorBarOriginal.png');
% Creates a latitude and longitude grid
[lat_grid, lon_grid] = meshgrid(linspace(-90, 90,
size(img, 1)), linspace(-180, 180, size(img, 2)));
% Define the PIA function
PIA_function = @(lat, lon) interp2(lon_grid',
lat_grid', double(img(:,:,1)), lon, lat);
% Displays the original image
figures;
imagesc(lon_grid(1, :), lat_grid(:, 1), img);
title('PIA Attenuation with Original Colors');
colorbar;
xlabel('Longitude');
ylabel('Latitude');
% Define the interpolated color map with the
specified RGB values (0-255)
cmap = [
0, 1, 134;
1, 11, 254;
0, 154, 254;
49, 255, 199;
160, 255, 84;
253, 218, 0;
255, 87, 0;
182, 0, 0;
133, 0, 0;
];
cmap = cmap / 255;
% Convert RGB values from 0-255 to 0-1
interpolated_cmap = interp1(linspace(0, 1,
size(cmap, 1)), cmap, linspace(0, 1, 256));
% Apply interpolated color map
colormap(cmap_interpolata);
% Adjust color bar to have step 0.2 and range from 0
to 1.46
clim([0 1.46]);
colorbar('Ticks', 0:0.2:1.46); % Example of using
PIA function
lat = 0; % Sample latitude
lon = 0; % Sample longitude
PIA_value = PIA_function(lat, lon);
disp(['PIA value at (' , num2str(lat), ', ', num2str(
lon), '): ', num2str(PIA_value)]);
```

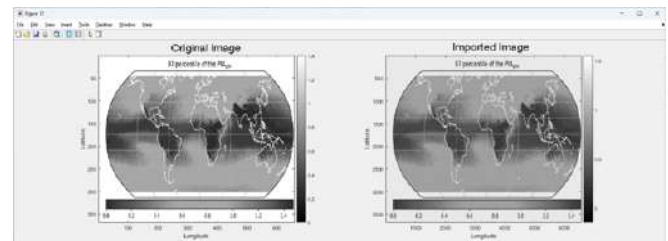


Fig. 13: Imported and original image from Battaglia's article (2020)

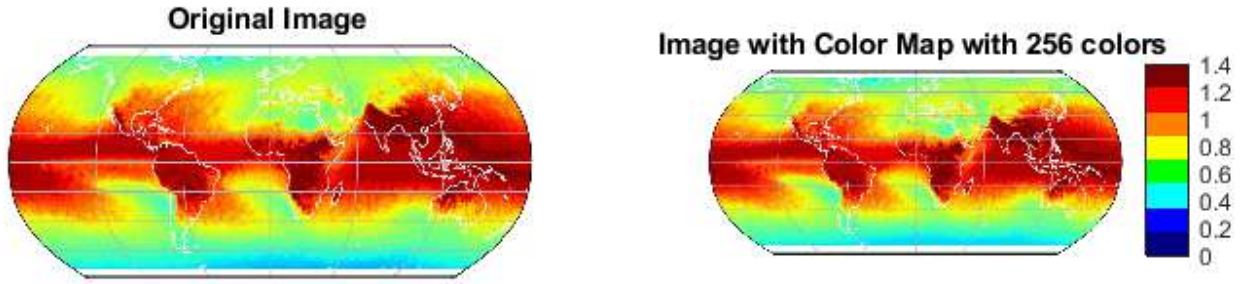


Fig. 14: The importing result.

Algorithm 2 A concrete code with a good importing using color Map in MATLAB.

```
% This script processes an image by extracting
unique colors and reducing them to a 256-color
palette.
% It then visualizes the original image alongside
the reduced-color image.
% Additionally, a custom color map ranging from
dark blue to dark red is defined and applied to the
image.
% Load the image
imagePath = 'BattagliaPIAgas.png'; % Replace with
the path to your image
img = imread(imagePath);
% Extract unique colors from the image
[rows, cols, ~] = size(img);
reshapedImg = reshape(img, [], 3);
[uniqueColors, ~, idx] = unique(reshapedImg,
'rows');
% Normalize color values to [0, 1]
normalizedColors = double(uniqueColors) / 255;
% Reduce colors to 256 main shades
numColors = 256;
colorMap = interp1(linspace(0, 1,
size(normalizedColors, 1)), normalizedColors,
linspace(0, 1, numColors));
% Reconstruct the image using the reduced color map
[~, colorIdx] = min(pdist2(colorMap,
normalizedColors), [], 1);
coloredImg = reshape(colorMap(colorIdx(idx), :),
rows, cols, 3);
% Display the original and reduced-color images
figure;
subplot(1, 2, 1);
imshow(img);
title('Original Image');
subplot(1, 2, 2);
imshow(coloredImg);
title('Image with 256-Color Palette');
% Define the custom color map (from dark blue to
dark red)
customColorMap = [
0, 0, 0.5; % Dark blue
0, 0, 1; % Blue
0, 1, 1; % Cyan
0, 1, 0; % Green
1, 1, 0; % Yellow
1, 0.5, 0; % Orange
1, 0, 0; % Red
0.5, 0, 0 % Dark red
];
% Add the color bar with the custom color map
colormap(customColorMap);
colorbar('Ticks', linspace(0, 1, 8), 'TickLabels',
0:0.2:1.4);
```

N. Techniques for creating a global maps of attenuation in Ku, Ka and W band with Reverse Engineering and Syntethic Considerations.

The point of this discussion is to explain how I can make global attenuation maps across the three important bands after looking at the raw data. The inception of this endeavor entails the simulation of data, necessitated by the previously cited constraints on time and computational resources. I applied reverse engineering techniques to the images and derived numerical values over a four-year period (**Battaglia et. al., 2020**) [38].

The period of the Path Integrated Attenuation (PIA) maps analyzed in the **Battaglia's** paper is based on four years of Global Precipitation Measurement (GPM) data. The Diurnal Cycle Analysis Results section specifies a specific time frame for the data, which is from **June 2014 to May 2018**. **Algorithm 3 and 4**, shows the interactive program for extracting PIA_{gas} values.

The algorithm processes a map image to associate its colors with corresponding numerical values (e.g., attenuation in dB) while ensuring the visual integrity of the map. It first loads the image and identifies the yellow background using a color threshold and tolerance. The algorithm replaces the background with white, preserving all other colors, including yellow shades, within the map. The algorithm then defines a color scale (e.g., 0-1.4 dB) and maps each pixel to its closest value on the color-map by calculating the color distance. The processed image is displayed, and an interactive feature allows users to click on the map to retrieve and display the numerical value corresponding to the clicked point as a tool-tip. Finally, I save the modified image for future use, ensuring the map's visual clarity and interactivity for further analysis. **Algorithms 4 and 5** show the conversion in grayscale, with the PIA value viewable on the screen with a click on the map, respectively, in Parts I, II, and III.

In addition to reverse engineering the **Battaglia's** maps I can also start discussing how to create a map from scratch.

Algorithm 8 shows the calculation of correlation between images using data from **Battaglia et al. (2020)**.

By looking at the correlation matrix between the images, it's clear that the different attenuation parameters (PIA) for different parts of the atmosphere are connected in interesting ways. PIA_{gas} (Battaglia-01) exhibits a moderate correlation

Algorithm 3 Interactive Program for Extracting PIA_{gas} Values: This tool allows users to obtain Path-Integrated Attenuation (PIA) values caused by atmospheric gases by simply clicking on desired locations on the global map. The interactive interface enhances the user experience by providing precise, location-specific attenuation data. (Part I)

```
% Upload image
img = imread('Battaglia-01.png'); % Replace with
uploaded file
% View original image
figure;
imshow(img);
title('Original Image');
% Identify yellow background (including shades)
yellow_threshold = [255, 255, 0]; % Yellow base
color (RGB)
tolerance = 50; % Tolerance to identify shades of
yellow
% Create a mask for the yellow background (with
tolerance for shades)
mask = (double(img(:,:,1)) >= yellow_threshold(1)
- tolerance & double(img(:,:,1)) <=
yellow_threshold(1) + tolerance) & ...
(double(img(:,:,2)) >= yellow_threshold(2)
- tolerance & double(img(:,:,2)) <=
yellow_threshold(2) + tolerance) & ...
(double(img(:,:,3)) >= yellow_threshold(3)
- tolerance & double(img(:,:,3)) <=
yellow_threshold(3) + tolerance);
% Keep yellow in the map and change only the
background to white
% Determine the background pixels (all those with
yellow values but outside the map)
background_mask = mask & (img(:,:,1) == 255 &
img(:,:,2) == 255 & img(:,:,3) == 0);
% Set only the background to white
img_modified = img;
img_modified(repmat(background_mask, [1, 1, 3])) =
255; % Replace with white
% Define the original color scale
val_min = 0; % Minimum value of the scale (example:
0 dB)
val_max = 1.4; % Maximum value of the scale
(example: 1.4 dB)
% Normalize the RGB image to the values of the scale
colormap_range = jet(256); % Use the original 'jet'
colormap
img_double = im2double(img_modified); % Convert to
double [0,1]
% Get the image size
[rows, cols, ~] = size(img_double);
% Initialize the scale values array
scale_values = zeros(rows, cols);
% Compare each pixel to the colormap
for r = 1:rows
for c = 1:cols
% Get the RGB value of the pixel
pixel_rgb = squeeze(img_double(r, c, :))';
% Calculate the distance between the pixel and the
colors in the colormap
[~, idx] = min(sum((colormap_range - pixel_rgb).^2,
2));
% Assign the corresponding scale value
scale_values(r, c) = val_min + (val_max - val_min) *
(idx - 1) / 255;
end
end
```

Algorithm 4 Interactive Program for Extracting PIA_{gas} Values: This tool allows users to obtain Path-Integrated Attenuation (PIA) values caused by atmospheric gases by simply clicking on desired locations on the global map. The interactive interface enhances the user experience by providing precise, location-specific attenuation data. (Part II)

```
% Display image with white and yellow background in
map
figure;
h_img = imshow(img_modified);
title('Image with White and Yellow Background in Map
Preserved');
% Add function to display color scale value on click
tooltip = text(0, 0, '', 'Color', 'k',
'BackgroundColor', 'w', 'EdgeColor', 'k', 'Visible',
'off');
set(h_img, 'ButtonDownFcn', @(src, event)
showHint(event, scale_values, tooltip));
% Save the modified image for future use
imwrite(img_modified,
'image_with_white_background_and_preserved_
yellow.png');
function showHint(event, scale_values, tooltip)
% Get the coordinates of the mouse click
pt = get(gca, 'CurrentPoint');
x = round(pt(1,1));
y = round(pt(1,2));
% Check if the coordinates are valid
if x > 0 && y > 0 && x <= size(scale_values, 2) && y
<= size(scale_values, 1)
% Get the color scale value at that point
value = scale_values(y, x);
% Show value as hint
set(tooltip, 'String', sprintf('%2f dB', value),
'Position', [x, y], 'Visible', 'on');
else
% Hide tooltip if out of bounds
set(tooltip, 'Visible', 'off');
end
end
```

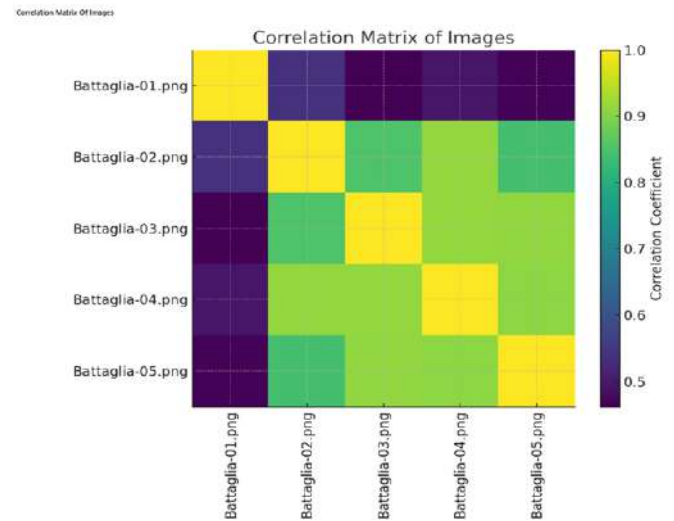


Fig. 15: Correlation Matrix for Battaglia's Images. The matrix illustrates the correlation coefficients between different images, highlighting the degree of similarity or difference among them.

Algorithm 5 Conversion also in grayscale with PIA value viewable on screen with a click on the map (I part).

```
% Upload image
img = imread('Battaglia-01.png'); % Replace with
uploaded file
% View original image
figure;
imshow(img);
title('Original Image');
% Identify yellow background (including shades)
yellow_threshold = [255, 255, 0]; % Yellow base
color (RGB)
tolerance = 50; % Tolerance to identify shades of
yellow
% Create a mask for the yellow background (with
tolerance for shades)
mask = (double(img(:,:,1)) >= yellow_threshold(1)
- tolerance & double(img(:,:,1)) <=
yellow_threshold(1) + tolerance) & ...
(double(img(:,:,2)) >= yellow_threshold(2)
- tolerance & double(img(:,:,2)) <=
yellow_threshold(2) + tolerance) & ...
(double(img(:,:,3)) >= yellow_threshold(3)
- tolerance & double(img(:,:,3)) <=
yellow_threshold(3) + tolerance);
% Keep yellow in the map and change only the
background to white
% Determine the background pixels (all those with
yellow values but outside the map)
background_mask = mask & (img(:,:,1) == 255 &
img(:,:,2) == 255 & img(:,:,3) == 0);
% Set only the background to white
img_modified = img;
img_modified(repmat(background_mask, [1, 1, 3])) =
255; % Replace with white
% Define the original color scale
val_min = 0; % Minimum value of the scale (example:
0 dB)
val_max = 1.4; % Maximum value of the scale
(example: 1.4 dB)
% Normalize the RGB image to the values of the scale
colormap_range = jet(256); % Use the original 'jet'
colormap
img_double = im2double(img_modified); % Convert to
double [0,1]
% Get the image size
[rows, cols, ~] = size(img_double);
% Initialize the scale values array
scale_values = zeros(rows, cols);
```

with the other parameters, indicating that the attenuation due to gas, which is typically uniform and dependent on atmospheric composition, does not vary significantly in relation to other components, such as clouds and precipitation. The highest correlation for PIA_{gas} is with PIA_{total} (Battaglia-04), but it remains relatively low (~0.5), suggesting that while the gas contribution is important, it is not dominant. PIA_{cloud} (Battaglia-02) shows a strong relationship with both PIA_{total} (~0.91) and PIA_{precip} (Battaglia-03) (~0.85). This implies that the signal loss due to clouds constitutes a significant portion of both the total signal loss and the signal loss due to precipitation. Additionally, the correlation with PIA_{total} exceed 3 dB (Battaglia-05) (~0.84) underscores that regions with high attenuations (>3 dB) are heavily influenced by the presence of clouds. **Figure 15** shows the correlation matrix for images from **Battaglia et al. (2020)**.

PIA_{precip} (Battaglia-03) demonstrates a very high correlation with PIA_{total} (~0.92) and with PIA_{total} exceed 3

Algorithm 6 Conversion also in grayscale with PIA value viewable on screen with a click on the map (II part).

```
% Compare each pixel to the colormap
for r = 1:rows
for c = 1:cols
% Get the RGB value of the pixel
pixel_rgb = squeeze(img_double(r, c, :));
% Calculate the distance between the pixel and the
colors in the colormap
[~, idx] = min(sum((colormap_range - pixel_rgb).^2,
2));
% Assigns the corresponding scale value
scale_values(r, c) = val_min + (val_max - val_min) *
(idx - 1) / 255;
end
end
% Convert the modified image to grayscale
gray_image = rgb2gray(img_modified);
% Display the image in grayscale
figure;
h_gray = imshow(gray_image);
title('Grayscale Image');
% Convert the matrix to double to display it as
numeric data
gray_data = double(gray_image);
% Display the data matrix in a table
disp('Matrix of Grayscale Values:');
disp(gray_data);
% Save the matrix to a MATLAB file for later
analysis
save('gray_data.mat', 'gray_data');
% If you want to display a part of the table in
MATLAB
disp('Values of an example of a table region (1:10,
1:10):');
disp(gray_data(1:10, 1:10));
% Add function to display the value of the color
scale on click on the grayscale
tooltip_gray = text(0, 0, '', 'Color', 'k',
'BackgroundColor', 'w', 'EdgeColor', 'k', 'Visible',
'off');
set(h_gray, 'ButtonDownFcn', @(src, event)
showHintGray(event, scale_values, tooltip_gray));
```

dB (~0.91), indicating that precipitation attenuation is a major contributor to the total attenuation and predominates in regions with high attenuations. Finally, PIA_{total} (Battaglia-04) is almost perfectly correlated with PIA_{total} exceed 3 dB (~0.91), suggesting that regions with significant total attenuation (>3 dB) represent a substantial fraction of the overall attenuation.

To sum up, clouds and rain have a big effect on total attenuation (PIA_{total}), but gas contribution (PIA_{gas}) is less variable and doesn't matter much in places with high attenuations. As expected, areas with high attenuations (>3 dB) mostly show the effects of clouds and rain combining to lower the signal, acting almost exactly like PIA_{total}. The correlation matrix highlights that high attenuations are mainly driven by precipitation and clouds, while the gas contribution is relatively independent and constant. This suggests that in future analyses for the identification of critical areas for propagation (such as high attenuation), it is important to focus mainly on PIA_{precip} and PIA_{cloud} rather than on PIA_{gas}.

The following diagram illustrates the primary contributions from the various atmospheric components: The Path Integrated Attenuation (PIA) law can be expressed as a summation of the primary contributions from several atmospheric components. This is feasible due to the relationship between the parameters.

Algorithm 7 Conversion also in grayscale with PIA value viewable on screen with a click on the map (III part).

```
% Display the image with white and yellow background
in the map
figure;
h_img = imshow(img_modified);
title('Image with White and Yellow Background in the
Map Preserved');
% Add function to display the color scale value on
click
tooltip = text(0, 0, '', 'Color', 'k',
'BackgroundColor', 'w', 'EdgeColor', 'k', 'Visible',
'off');
set(h_img, 'ButtonDownFcn', @(src, event)
showHint(event, scale_values, tooltip));
% Save the edited image for future use
imwrite(img_modified, 'image_with_white_background
_and_preserved_yellow.png');
function showHint(event, scale_values, tooltip)
% Get the coordinates of the mouse click
pt = get(gca, 'CurrentPoint');
x = round(pt(1,1));
y = round(pt(1,2));
% Check if coordinates are valid
if x > 0 && y > 0 && x <= size(scale_values, 2) && y
<= size(scale_values, 1)
% Get the color scale value at that point
value = scale_values(y, x);
% Show the value as a hint
set(tooltip, 'String', sprintf('%2f dB', value),
'Position', [x, y], 'Visible', 'on');
else
% Hide tooltip if out of bounds
set(tooltip, 'Visible', 'off');
end
end
function showHintGray(event, scale_values,
tooltip_gray)
% Get the mouse click coordinates
pt = get(gca, 'CurrentPoint');
x = round(pt(1,1));
y = round(pt(1,2));
% Check if coordinates are valid
if x > 0 && y > 0 && x <= size(scale_values, 2) && y
<= size(scale_values, 1)
% Get the color scale value at that point
value = scale_values(y, x);
% Show the value as a hint
set(tooltip_gray, 'String', sprintf('%2f dB',
value), 'Position', [x, y], 'Visible', 'on');
else
% Hide tooltip if out of bounds
set(tooltip_gray, 'Visible', 'off');
end
end
end
```

I can specifically express the total PIA as follows:

$$PIA_{total} = PIA_{gas} + PIA_{cloud} + PIA_{precip}$$

PIA_{gas} is the attenuation due to absorption by atmospheric gases, such as oxygen and water vapor. This contribution is relatively uniform and less variable than the other terms. PIA_{cloud} represents attenuation due to the presence of clouds, which include liquid water particles and ice crystals. This term has a strong correlation with the total and contributes significantly to the high attenuations. PIA_{precip} is the attenuation caused by precipitation, which includes rain, snow, and hail. In regions with high attenuations, this term dominates and exhibits a strong correlation with the total.

Algorithm 8 Using images from Battaglia et.al. (2020). Calculating correlation between images.

```
% Loaded image paths
file_paths = {
'Battaglia-01.png', % PIA_gas
'Battaglia-02.png', % PIA_cloud
'Battaglia-03.png', % PIA_precip
'Battaglia-04.png', % PIA_total
'Battaglia-05.png' % PIA_total exceed 3 dB
};
% Read images and convert to grayscale
images = cell(1, numel(file_paths));
for i = 1:numel(file_paths)
img = imread(file_paths{i});
if size(img, 3) == 3 % If RGB image
img = rgb2gray(img); % Convert to grayscale
end
images{i} = double(img); % Convert to double for
calculations
end
% Finds the minimum size among all images
min_rows = min(cellfun(@(x) size(x, 1), images));
min_cols = min(cellfun(@(x) size(x, 2), images));
% Resize all images to the minimum size
resized_images = cellfun(@(x) x(1:min_rows,
1:min_cols), images, 'UniformOutput', false);
% Calculate the correlation matrix
num_images = numel(resized_images);
correlations = zeros(num_images, num_images);
for i = 1:num_images
for j = i:num_images
% Calculates the correlation between images i and j
corr_value = corr2(resized_images{i},
resized_images{j});
correlations(i, j) = corr_value;
correlations(j, i) = corr_value; % Symmetric matrix
end
end
% Display correlation matrix
figure;
imagesc(correlations); % Display matrix as image
colorbar; % Color bar
title('Image Correlation Matrix');
xticks(1:num_images);
yticks(1:num_images);
xticklabels(file_paths);
yticklabels(file_paths);
xtickangle(45); % Rotate names on X-axis
% Save correlation matrix to MATLAB table
correlation_table = array2table(correlations, ...
'VariableNames', file_paths, ...
'RowNames', file_paths);
% Display table in console
disp('Image correlation matrix:');
disp(correlation_table);
```

Increase for high attenuations, $PIA_{total} > 3$ dB: when the total attenuation goes over a significant threshold, like 3 dB, PIA_{total} is mostly made up of PIA_{cloud} and PIA_{precip} working together, while PIA_{gas} stays the same. In summary, the general PIA law takes into account all the main atmospheric contributions, but the relative weight of each term may vary depending on atmospheric and geographical conditions.

O. Efficiency in Reverse Engineering Techniques

To demonstrate the value of reverse engineering in creating global attenuation maps, Battaglia's research includes **Figure 3** [38], which projects GPM data over a four-year period from June 2014 to May 2018. Working with CloudSat-GPM satellite

coincidences, especially without FTP or BULK download support, often resulted in lengthy download times. On average, I spent 15–20 minutes per file at home, sometimes extending to half an hour or even several hours for a single satellite coincidence file. This time-frame excludes the additional costs associated with processing this information.

Daily, the number of files to download ranged from 5 to 8, resulting in a substantial hourly download commitment. Over a four-year period, considering 365 days per year, the total number of files required would be approximately 11,680, with each file ranging between 16 MB and 25 MB. This massive undertaking demands significant resources, both in terms of time and human effort. I downloaded the V04 CloudSat-GPM coincidence dataset from NASA web services, occupying 14.1 GB of disk space from January to March 2016.

Even with an optimized download time of 1 minute per file, the cumulative download duration amounts to 11,680 minutes. However, digitization drastically reduces the time required to create a map, showcasing the evident savings. In a worst-case scenario, assuming 2,920 hours dedicated solely to downloading files and an additional 64.89 hours for processing (at 20 seconds per file), the manual workload reduction is approximately 97.78%. This figure doesn't account for the frequent delays encountered during peak internet traffic hours, which often extended the download time.

The utility of having pre-generated maps becomes undeniable, as exemplified by the mere 13.37 seconds it took to digitize the global map for Battaglia's PIA_{gas}. With access to more images, I could further leverage machine learning techniques to enhance my predictive capabilities. For example, I was able to get good estimates of PIA values by looking at Sigma-Zero images taken by **R.Meneghini, T.Iguchi et al. (2000)** [43], and making guesses about real-world relationships. In conclusion, the digitization and automation of these processes not only save time and resources but also pave the way for more advanced analytical methods, significantly improving efficiency and accuracy in forecasting and data analysis.

P. Processing methods for generating Global Attenuation Maps

As a first step, I will talk about the processing methods used to make these maps, starting with the random generation of attenuation values in the three bands. This approach presupposes the feasibility of obtaining species for the highest frequencies of the representations. Such representations demand extensive research and the capability to formulate hypotheses on a planetary scale, entailing both risks and advantages. In the concluding section of the article, I will delve into the theoretical considerations, viewing them as objectives that the data scientist strives to accomplish. I begin by generating a global attenuation map for a specific band using range information available in the literature. For this operation, I prefer to use MATLAB. The globe image is initially filled with random values at all latitudes and longitudes, ensuring they fall within a plausible range. I can illustrate the general procedure for creating global maps with this technique, and

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	31.7559	16.0465	13.7371	15.4627	11.0645	5.7060	22.0873	1.7827	15.4753	24.0568	16.0942	7.5342	33.9957	23.9965	7.9180
2	22.5585	10.6541	4.3374	3.1771	8.7378	3.4387	15.2111	2.7315	10.9929	22.0358	16.4336	27.6076	28.3627	16.1932	18.8950
3	11.7793	14.2197	12.1612	17.0952	20.9389	22.6411	10.7184	17.0454	28.8051	19.4549	29.6714	23.8825	8.0776	23.0437	9.6225
4	26.3810	21.2206	18.3837	13.1235	8.8100	5.8038	20.5629	6.1705	12.8825	0.0664	28.0783	30.7914	17.1421	12.0000	11.8844
5	26.4742	1.9209	1.7508	15.0407	32.2018	17.5565	32.9740	31.3386	32.5179	32.5986	8.8344	1.8242	15.0700	1.9320	
6	12.1832	20.0757	5.5154	15.0163	23.1964	2.0701	26.4387	26.8127	16.4401	30.1267	10.1072	5.2125	7.1162	26.6796	28.1381
7	34.2281	7.9125	5.5268	3.2113	23.1701	22.0623	12.8618	7.9584	8.9925	1.0760	15.2759	14.1635	7.7765	12.2352	12.8621
8	14.0094	35.4403	21.1133	18.9459	9.4012	12.8402	21.7711	6.7512	6.7070	11.3804	25.4057	13.8057	17.7021	16.2730	23.4544
9	21.5214	15.3678	11.8387	18.8325	7.2654	25.2779	18.9336	11.1370	26.5306	33.7822	21.7084	8.8302	4.0389	6.2889	1.3338
10	32.0037	7.7517	6.0052	11.3633	23.4818	16.3238	13.5200	16.6650	4.4888	33.4446	11.2740	17.5760	13.3466	15.0230	6.0597
11	2.4581	1.5325	23.3678	16.0364	14.1057	18.0348	16.0607	36.9798	6.1925	25.3884	1.6708	6.7822	23.1568	20.7430	26.8101
12	14.4817	20.5300	12.2116	8.1185	34.1584	24.8578	17.2604	26.7172	25.1418	11.2282	33.6235	17.5161	32.1868	4.3827	13.7554
13	6.3576	24.4671	33.7501	41.8986	6.1321	36.5919	21.2340	3.0232	9.8925	32.4244	20.2517	19.6426	8.3488	3.1772	32.6528
14	21.4088	2.0620	13.3691	0.7643	24.2236	28.8734	10.0142	31.4805	25.5409	16.2134	13.3848	24.0906	11.0387	10.4281	30.0011
15	24.5091	6.8906	34.5532	23.1141	33.4931	4.8894	14.7193	2.8956	34.2771	7.2444	14.3645	8.7979	16.1181	7.2680	14.2164
16	2.6034	28.1451	5.3348	36.6396	3.8237	16.3960	36.3828	3.4816	7.7075	1.5214	6.3832	5.9444	17.9521	15.3249	18.4389
17	2.0612	22.2874	2.8716	25.7359	34.2024	12.2884	11.8817	14.8757	26.0004	11.7240	21.2639	1.6487	27.8117	25.6871	14.5814
18	11.5483	14.4203	31.3604	21.3104	1.2587	5.8802	16.0924	3.4487	27.7075	10.3084	20.0208	16.8175	32.1388	15.0342	28.8225
19	36.5381	5.5864	24.8117	22.8153	26.1002	18.3363	17.5179	27.1136	5.6238	27.1403	11.7175	24.6084	21.8883	23.2081	12.7240
20	16.6832	23.9590	7.3965	11.1232	25.7811	5.5916	12.9451	26.8198	19.2141	26.4039	1.8454	25.4908	28.4495	15.0577	1.2930
21	3.1399	15.1707	17.4324	2.2078	6.1781	10.4519	31.2629	26.6723	15.4885	17.0741	1.8370	14.7825	8.4606	10.1937	16.7180
22	9.9889	20.0522	21.1214	23.2031	18.8480	7.2887	3.7123	20.8325	20.8907	2.7028	31.4482	21.7257	23.5462	14.8888	1.1176

Fig. 16: Attenuation DataTable with fake data, (PIA(W) attenuation values are between 0-35 dB).

apply it when actual data becomes available. By working with synthetic datasets, I can experiment and determine the best way to present the data, thereby refining my methods before analyzing real-world data. For implementation details, please refer to the **Technical annex** accompanying this article. [8]

Figure 16 shows the Attenuation DataTable with fake data, where PIA(W) attenuation values range between 0-35 dB. **Algorithm 9** shows the source code for obtaining the global map with fake attenuation values for PIA(W) expressed in dB. **Algorithm 9** shows the source code for obtaining the global map with fake attenuation values for PIA(W) expressed in dB.

Algorithm 9 Source Code for obtaining the Global Map with fake Attenuation Values for PIA(W) expressed in dB.

```
% Set parameters
lat = -90:1.5:90; % Latitude -90 to 90 in 1.5 degree steps
lon = -180:1.5:180; % Longitude -180 to 180 in 1.5 degree steps
[lon, lat] = meshgrid(lon, lat); % Create latitude and longitude grid
% Generate random attenuation data between 0 and 35 dB
attenuation = 35 * rand(size(lat));
% Create map in cylindrical projection
figure;
worldmap world
pcolorm(lat, lon, attenuation);
cbar = colorbar;
title('Global Attenuation Map in Cylindrical Projection');
ylabel(cbar, 'PIA(W) in dB');
clim([0 35]); % Set colorbar limits from 0 to 35 dB
% colormap(jet); % Set colormap
colormap(parula); % Set colormap
% Add coastlines with thicker edges
load coastlines;
plotm(coastlat, coastlon, 'k', 'LineWidth', 2); % Set border width to 2
% View map
tightmap;
```

Path Integrated Attenuation (PIA) varies mainly with atmospheric conditions, but geographic and human factors can indirectly influence it. PIA values between 5 dB and 10 dB are typical for arid and desert regions, where precipitation is low and cloud formation is limited. In temperate regions with moderate vegetation, such as plains and hills, PIA values

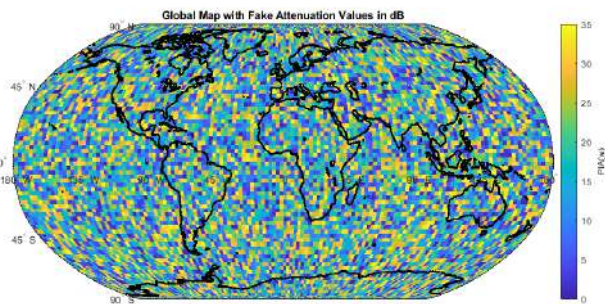


Fig. 17: Adjusting values in compatibility with PIA(W) values. Global Map of PIA(W) with fake data.

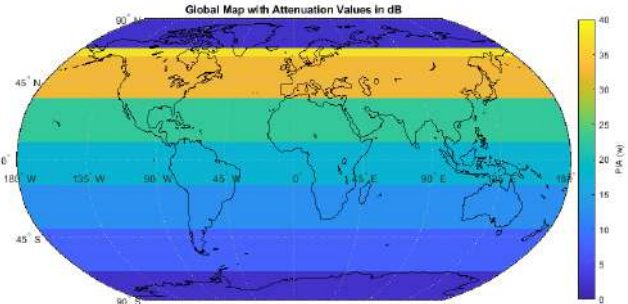


Fig. 18: PIA(W) attenuation for region of interest with plausible values.

tend to be slightly higher (10-15 dB), as vegetation affects soil moisture and, consequently, the local hydrological cycle, favoring greater cloud formation. Dense forests and mountainous areas, thanks to vegetation transpiration and topography that favors the lifting of air masses, show higher PIA values, between 15 dB and 20 dB, especially during periods of heavy precipitation. In tropical regions, the combination of high humidity, dense forests, and intense weather phenomena causes PIA to often reach values between 20 dB and 25 dB. In addition to experiencing torrential rainfall, urban areas have a micro-climate influenced by air pollution and heat generated by human activity. Urban pollution can increase the concentration of condensation nuclei, favoring the formation of clouds and more intense precipitation, with PIA values that can reach 25-30 dB. Furthermore, urban heat can intensify convective thunderstorms, further increasing their attenuation. Industrial zones and areas with high human activity can reach PIA values between 30 dB and 35 dB, especially during extreme weather events, when the high concentration of polluting particles and the heat emitted favor localized and intense precipitation. Finally, polar regions generally show low values (0-5 dB), due to the low presence of liquid precipitation and the prevailing dry atmospheric conditions. Values above 35 dB are rare and occur mainly in situations of exceptionally intense precipitation, such as tropical cyclones or violent equatorial thunderstorms. **Figure 17** shows the global map of PIA(W) with fake data, adjusted for compatibility with PIA(W) values. **Figure 18** shows the PIA(W) attenuation for the region of interest with plausible values.

Q. Contribute of Soil Moisture

Soil moisture contributes indirectly to atmospheric Path Integrated Attenuation (PIA), mainly by influencing the local micro climate and the water vapor content in the air near the ground. Saturated soil can promote evaporation, increasing water vapor, which in turn increases atmospheric attenuation at higher frequencies, such as microwaves and millimeter waves. In addition, soil moisture can alter the reflecting and absorbing properties of the Earth's surface, influencing wave propagation through reflection, multipath, and interference. However, its contribution to PIA is marginal compared to

the main atmospheric factors, such as precipitation, clouds, and aerosols. This effect becomes relevant only under specific conditions, such as in tropical regions or during thawing in polar areas. In different parts of the Earth's micro-climates, soil moisture has a very different effect on Path Integrated Attenuation (PIA). In general, though, it is not as important as factors like clouds, precipitation, and aerosols. In desert and arid areas, the contribution is negligible, with an impact on PIA of less than 1%, due to the almost total absence of soil moisture. In coastal and subtropical areas, soil moisture contributes minimally, with an impact on PIA ranging between 1% and 5%. In temperate plains and hills, this contribution can increase to up to 10% due to increased seasonal evaporation, which in turn increases local water vapor. The role of soil moisture becomes more relevant in dense forests and mountain regions, with an impact of between 10% and 15%, and it reaches its maximum in tropical and equatorial areas, where saturated soils and constant precipitation can influence PIA by up to 20%. Conversely, polar regions and permanently frozen soils limit the contribution to less than 3%. Finally, in urban and industrial areas, the influence of soil moisture is marginal, ranging between 1% and 5%, with atmospheric attenuation mainly dominated by infrastructure and artificial obstacles.

R. Realistic Analysis

There is a detailed look at PIA-hydrometeor measurements over the polar regions and the global map at W Band in **Figs. 18, 19, 20, and 21**. These were found by the CloudSat and GPM satellites. Specifically, **Fig.18** shows a view of the North Pole, while **Fig. 19** focuses on the South Pole. **Fig. 22** offers a global map related to the CloudSat satellite's passages over the poles during January-February and few passages in May 2016. Finally, **Fig.23** presents a global map at W Band for the CloudSat-GPM coincidence dataset during the same period, using the Mollweide projection.

By scanning the coincidence files between CloudSat and GPM, these pictures show how to make detailed maps of satellite measurements. This gives scientists important data for figuring out how the atmosphere worked at certain times. Additionally, the figures highlight how the use of the Mollweide projection allows for effective representation of the data,

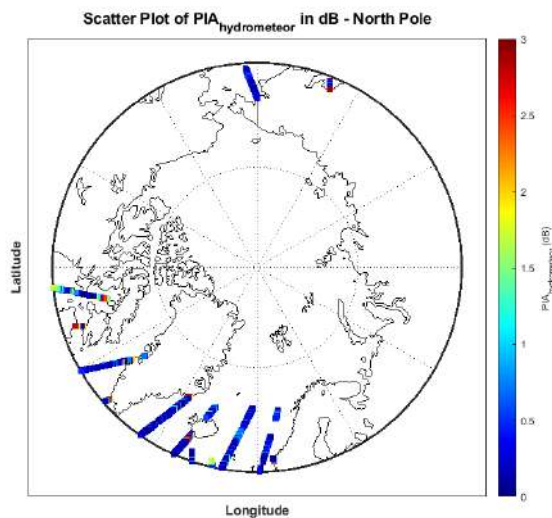


Fig. 19: North Pole vision about PIA-hydrometeor in the Global Map at W Band. (January-February and few passages in May 2016).

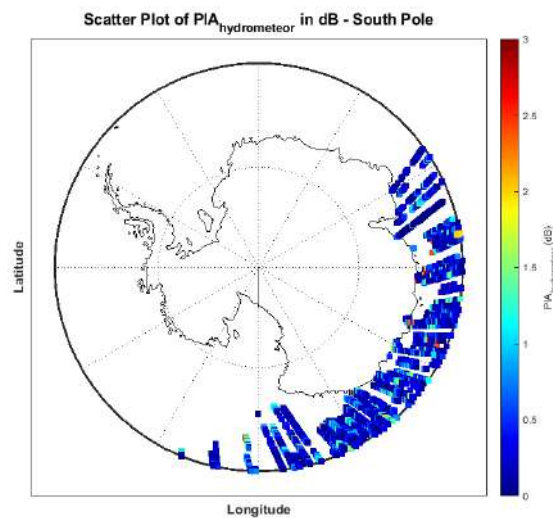


Fig. 20: South Pole vision about PIA-hydrometeor in the Global Map at W Band. (January-February and few passages in May 2016).

facilitating the visualization and interpretation of observations on a global scale. This type of analysis is fundamental for better understanding atmospheric dynamics and improving weather and climate forecasts globally.

The detailed maps provided by these figures are crucial for a comprehensive analysis of atmospheric phenomena. The use of PIA-hydrometeor measurements, combined with the precision of W Band data, allows for a more accurate assessment of precipitation intensity and distribution in polar regions. This data is particularly valuable for understanding how polar weather patterns contribute to global climate dynamics.

Furthermore, the temporal scope of the data, encompassing February to May 2016, offers insights into seasonal variations in atmospheric conditions. The ability to capture data from different periods enhances my understanding of temporal trends and anomalies, which are essential for developing predictive models. The application of the Mollweide projection in **Figs. 23-24** is significant as it minimizes distortions typically associated with mapping polar regions, ensuring that the data representation is both accurate and visually coherent. This facilitates more precise interpretation and comparison of the data across different regions and time-frames.

Ultimately, the integration of CloudSat and GPM datasets, through the coincidence scanning approach, exemplifies the power of collaborative satellite missions in advancing my knowledge of Earth's atmospheric systems. Such detailed and accurate maps are indispensable for researchers aiming to improve weather forecasting models, study climate change impacts, and develop strategies for mitigating adverse weather effects. **Figure 25** shows the global map at W-Band for the CloudSat-GPM Coincidence Dataset during January-March 2016, in the Mollweide projection.

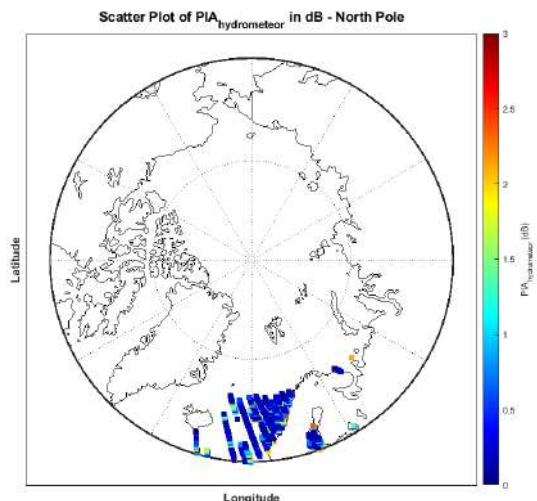


Fig. 21: North Pole vision about PIA-hydrometeor in the Global Map at W Band. (January-March 2016).

S. How change PIA(W) global attenuation with the Soil Moisture estimation contribute

In this paragraph, working with fake data I try to simulate the **Soil Moisture** contribute to the global PIA(W) attenuation. **Figure 26** shows that with random soil moisture content, the PIA(W) becomes non-uniform in values in geographically neighboring areas.

I have modified the MATLAB code to include the contribution of soil moisture in different territorial zones. I'll use typical soil moisture values expressed as a percentage of **Volumetric Water Content (VWC)** for each zone:

- 1) Coastal and Arid Regions: Soil moisture ranges between 2% and 8% (VWC), resulting in a relatively low PIA of

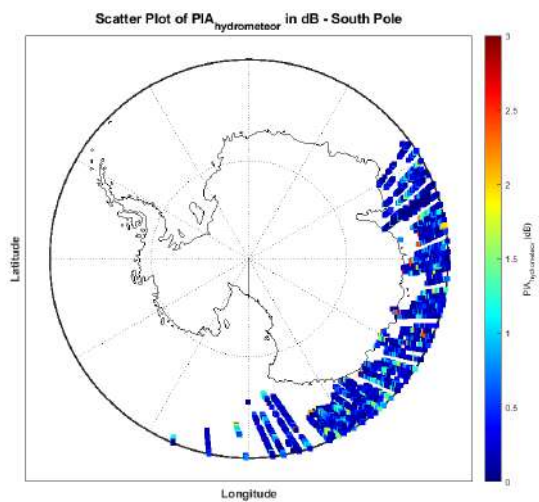


Fig. 22: South Pole vision about PIA-hydrometeor in the Global Map at W Band. (January-March 2016).

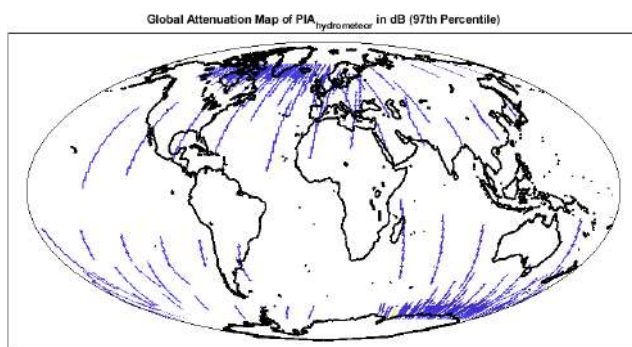


Fig. 23: Global Map related to passages over poles from CloudSat Satellite during January-February and few passages in May 2016.

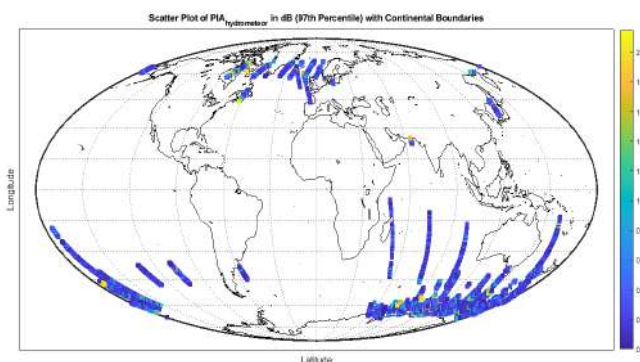


Fig. 24: Global Map at W-Band for CloudSat-GPM Coincidence Dataset during January-February and few passages in May 2016, in the Mollweide projection.

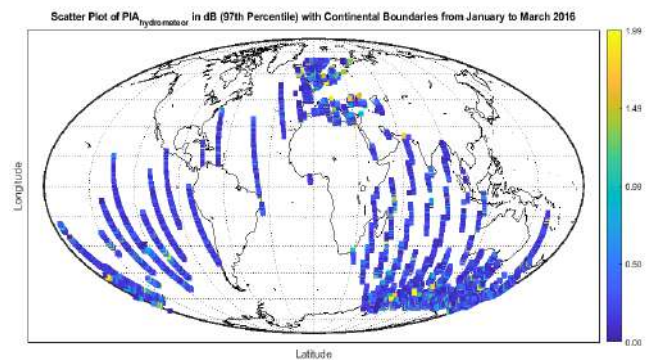


Fig. 25: Global Map at W-Band for CloudSat-GPM Coincidence Dataset during January-March 2016, in the Mollweide projection.

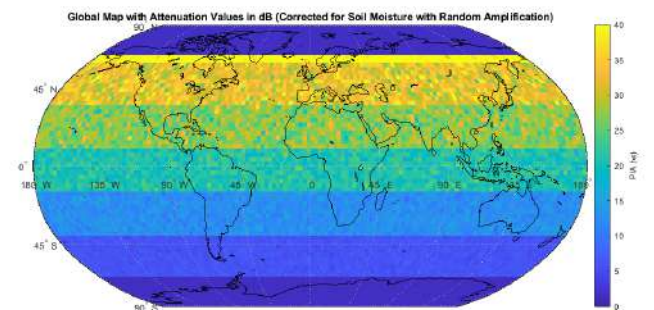


Fig. 26: With random soil moisture content, the PIA(W) becomes not uniform in values in geographically neighboring areas.

5 to 10 dB.

- 2) Temperate Regions (Plains and Hills): Soil moisture ranges between 8% and 15%, leading to a PIA of 10 to 15 dB.
- 3) Dense Forests and Mountainous Areas: Soil moisture content increases to 15-25%, resulting in a PIA of 15 to 20 dB.
- 4) Tropical Forests and Equatorial Areas: Extremely high soil moisture levels, ranging from 25% to 35%, lead to the highest PIA values of 20 to 25 dB.
- 5) Polar Regions typically see frozen soil with a surface moisture content of 2-8%, resulting in a minimal PIA of 0 to 5 dB.
- 6) Urban and Industrial Areas: Sealed soil restricts moisture to 5-10%, resulting in a PIA of 5-15 dB and 10-20 dB, respectively. Structural obstacles dominate attenuation more than soil moisture.

These estimates reflect a more accurate distribution of the relationship between soil moisture and PIA. To increase the verisimilitude, one can proceed by introducing ripple or noise signals in the regional boundaries and by increasing the discontinuities of humidity values.

Figure 26 shows that with random soil moisture content, the PIA(W) becomes non-uniform in values in geographically

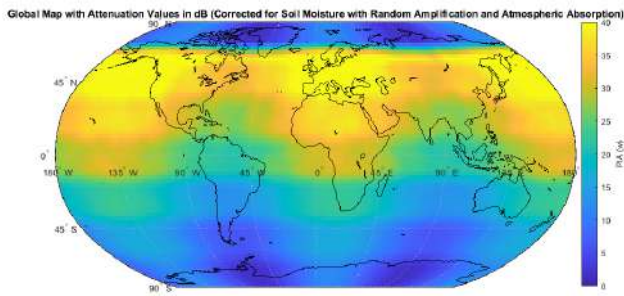


Fig. 27: Fake PIA(W) with random noise in the regional boundaries. Ripple over regions, Random Moisture Effect and Water Vapor. I use Blurring effect.

neighboring areas.

This example serves as a preliminary foundation for Your research project. However, it remains a simplified model that may require further refinement and adaptation to meet specific requirements. To enhance the code, consider the following suggestions: Incorporate atmospheric influences by accounting for the effects of atmospheric physics on Path Integrated Attenuation (PIA), particularly the influence of gases such as Oxygen (O_2) and Water Vapor (WV). It is essential to include parameters like the absorption coefficient in the modeling process. I recommend integrating real-time meteorological data for humidity and temperature to achieve more accurate global results. Employing ITU-R propagation models will account for various atmospheric and geographical factors, ensuring a comprehensive analysis. Additionally, consider incorporating factors that may influence signal attenuation, such as the presence of atmospheric particulate matter, signal reflection and diffraction, and the impact of terrain and vegetation. These enhancements will provide a more accurate and robust representation of the relationship between Soil moisture (Sm) and PIA.

Soil moisture (Sm) and Path-Integrated Attenuation (PIA) are indirectly but significantly related. In simple terms, soil moisture does not directly influence PIA, but it does affect the land surface during precipitation events. When soil is wet, it can alter the surface scattering cross section (σ_0), which is an important parameter in radar precipitation measurements. If the analyses fail to properly account for soil moisture, this change could result in an underestimation of PIA. Because of this, adding soil moisture to PIA assessments makes the estimates more accurate and reliable, giving a more accurate picture of the connection between rain and radar attenuation. Therefore, despite the indirect nature of the relationship, its influence on radar measurements is substantial and unavoidable.

Figure 27 shows the fake PIA(W) with random noise in the regional boundaries, ripple over regions, random moisture effect, and water vapor. A blurring effect is applied.

T. Advanced Approaches for the Analysis of Indirect Factors in Atmospheric Processes

Scientists address the influence of indirect factors on direct phenomena using complex mathematical models and data analysis techniques that integrate multiple variables. Complex system models, like multilinear regression and nonlinear models, enable independent variables like soil moisture and pollution to explain the variability of dependent variables. Physical models of the atmosphere, like numerical meteorological and radiative transfer models, use fluid dynamics equations and models to show how energy, moisture, and particles move through the atmosphere. These models take into account things like how plants lose water and how pollutant particles affect the atmosphere. The use of **machine learning** and **artificial intelligence** techniques, such as artificial **neural networks** and supervised learning models, allows for the identification of complex correlations between indirect and direct factors. Advanced statistical methods, such as sensitivity analysis, principal component analysis, and **Granger**⁴ causality analysis, help determine the influence of indirect variables on direct phenomena. The integration of observational and simulation data, from satellites and coupled atmosphere-land surface models, provides useful information to correlate indirect and direct factors. Specific parametric models, such as semi-empirical and stochastic ones, account for the uncertainty in indirect processes. Finally, the validation of models through field data, experimental campaigns, and historical data ensures their accuracy. To sum up, the process involves using a mix of physical, statistical, and machine learning models, along with observational data to make a model that includes both direct and indirect factors. This model is then used to mathematically measure the effect of indirect factors on direct phenomena. For example, CloudSat, GPM, and MODIS data can provide information on cloud distribution, precipitation, vegetation cover, and aerosols, useful for correlating indirect and direct factors. It would also be possible, in more complex cases, to implement or study coupled atmosphere-land surface models (e.g., CESM or RegCM)⁵ that are capable of simulating the interaction between vegetation, soil moisture, and atmospheric processes.

⁴A statistical technique known as Granger causality analysis determines whether one time series can predict another. In the context of a Path Integrated Attenuation (PIA) map, this analysis can be used to examine causal relationships between soil moisture and PIA values in different geographic regions.

By applying Granger's causality analysis, one can identify whether changes in soil moisture can predict changes in PIA values and vice versa. This can help to better understand how weather and environmental conditions affect radar signal attenuation, thus improving the accuracy of weather forecasts and radar analyses.

⁵Researchers studying climate change use the **CESM (Community Earth System Model)** framework to simulate interactions between the atmosphere, oceans, land surface, and ice. This model helps scientists understand climate dynamics and predict future climate scenarios.

RegCM (Regional Climate Model) is a system of regional climate models originally developed at the National Center for Atmospheric Research (NCAR) and maintained in the Earth System Physics (ESP) Section of ICTP. Applications of RegCM range from studies of climate processes to paleoclimate and future simulations.

U. The Impact of Reverse Engineering in Scientific and Technological Advancements

The Human Genome Project is a prime example of reverse engineering on a grand scale, where scientists mapped out the entire human genome to understand genes and their roles in diseases and traits. In biology, AI-driven algorithms like **DeepMind's AlphaFold** [40] have revolutionized protein folding by accurately predicting protein structures, transforming my understanding of molecular interactions. In telecommunications, a case study demonstrated the application of a cognitively based approach for reverse engineering tool development, utilizing ideas from cognitive psychology to improve system design and functionality [41], [42].

These examples highlight the diverse applications and significant impact of reverse engineering in advancing scientific knowledge and technological innovation.

V. Machine Learning and Data Analysis Techniques for Atmospheric Modeling

The use of reverse engineering (RE) on **Battaglia's** work [38] represents an innovative and useful approach to derive global attenuation behaviors for frequencies other than the 35 GHz band. This method allows for the analysis and understanding of existing models, allowing researchers to adapt and improve their predictions for other frequency bands. Furthermore, reverse engineering facilitates the comparison between one's research work and the results obtained by other studies, providing a solid basis for model validation and optimization. Statistical analysis of the data derived from reverse engineering allows for the identification of patterns and trends that may not be immediately evident, thus improving the understanding of attenuation phenomena. This integrated approach, which combines machine learning techniques and data analysis, provides a complete and detailed picture of atmospheric dynamics, helping to develop more accurate and reliable models. It is possible to show uncertainty in indirect processes more accurately when I use specific parametric models and combine data from observations and simulations. Finally, model validation through field data and experimental campaigns ensures the accuracy of predictions, making reverse engineering an indispensable tool for advanced research in this field. By leveraging reverse engineering, I can generate scatter plots of **PIA** at various frequencies and compare them over time with those derived from my research on **CloudSat-GPM** and **CloudSat-TRMM** satellite coincidences. This comparison enables me to verify the coherence and reliability of the developed models, offering deeper insights to enhance forecasts and better comprehend atmospheric attenuation phenomena.

VII. CONCLUSIONS.

Creating a global attenuation map provides valuable insights into how different variables affect signal propagation across various regions worldwide. Regular updates and enhancements, incorporating Crane's analysis and other models, are essential for identifying optimal transmission frequencies tailored to diverse global regions and atmospheric conditions [21], [22], [23]. This map could significantly benefit weather

radars and other technologies. Crane models, extensively used in global meteorological mapping, are fundamental tools for predicting rain-induced signal attenuation. These models classify the globe into distinct rainy climate regions, each defined by specific distributions of precipitation intensity and frequency. For instance, the **1985 Crane model** employs updated atmospheric distribution tables and a map of rain regions to calculate signal attenuation along terrestrial and satellite paths.

The scientific significance of Crane models lies in their capacity to deliver accurate estimates of signal attenuation, which are crucial for designing and optimizing satellite and terrestrial communication systems. These models account for various factors, including transmitter frequency, ground station elevation angle, station location and altitude, and Earth's curvature. Moreover, the validation of Crane models through satellite beacon observations and terrestrial path measurements has demonstrated an acceptable standard deviation between predicted and observed values. My study has underscored the operational feasibility of generating **Global Attenuation Maps (GAMs)** and developed the necessary MATLAB source code for data acquisition and manipulation. However, the extensive data requirements, coupled with the need for validation and analysis of coincidences between CloudSat and TRMM, present challenges that make the operation labor-intensive. Initially focusing on orbital passages over the poles (Arctic and Antarctic), I created attenuation maps by averaging values on pixels using the percentile method (e.g., the 97th percentile) [37]. This enabled me to activate individual latitude-longitude boxes on a global map. However, processing all satellite passes simultaneously overwhelmed MATLAB's video memory, leading to crashes. This problem arose due to the limited hardware resources available for handling such large volumes of data in a single operation. To address this issue, I propose adopting a more incremental approach to map construction. By processing smaller subsets of data sequentially and subsequently combining the results, I can distribute memory consumption over time and prevent system overloads. This method allows for the efficient use of available hardware resources, enabling the generation of the map without necessitating immediate hardware upgrades. In summary, while challenges exist, incremental processing and regular updates can make the generation and maintenance of global attenuation maps feasible, providing invaluable tools for optimizing communication systems in diverse atmospheric conditions.

CONFLICT OF INTEREST

The author declares that he has no conflicts of interest.

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A.2 – Second Article

Title: Development and Validation of a Predictive Attenuation Law Using CloudSat-GPM Coincidences: A Machine Learning Approach.

Throughout this dissertation, and in the related journal articles, the first-person plural (“we”) is frequently employed when presenting methods, results, and discussions. Although I am the sole author of this work, the use of “we” follows the established convention in scientific writing, where it serves both as a stylistic choice and as a means to emphasize the collective dimension of research, connecting the author’s contributions to the broader scientific community.

A.2.1 Comparison of Machine Learning and Regression Frameworks

Although several machine learning approaches were initially explored—including LSTM, CGAN, and ensemble methods—their performance did not surpass that of the nonlinear regression framework. The latter demonstrated greater robustness, interpretability, and consistency across diverse hydrometeorological regimes. This outcome highlights the importance of model selection based on empirical effectiveness rather than methodological complexity.

A.2.1.1 Exploratory Machine Learning Experiments and Lessons Learned

In an exploratory phase of the research, a set of machine learning models was tested with the aim of deriving a predictive law for Path Integrated Attenuation (PIA) in the W-band from CloudSat–GPM coincidence data. Gradient Boosting, Random Forest, and Support Vector Machines were employed after careful preprocessing and feature engineering, with the intention of capturing the non-linear interactions between key atmospheric predictors such as rain rate, ice water content, liquid water path, temperature, and surface pressure.

Initial experiments showed encouraging results: the best-performing models achieved RMSE values around 1–2 dB and coefficients of determination (R^2) above 0.8 for the training sets, confirming the potential of ML to complement physical models. The integration of derived variables—such as ratios and nonlinear transformations of hydrometeor properties—further improved predictive skill, highlighting the value of hybrid feature engineering strategies.

However, despite these promising outcomes, limitations emerged. Overfitting was observed when testing on unseen data, with reduced generalization performance and degraded R^2 values. In addition, the relatively small size and temporal sparseness of the CloudSat–GPM coincidence dataset constrained the robustness of purely data-driven models, which rely on large-scale, diverse training samples. These factors motivated the decision to prioritize radiative transfer-based approaches and semi-empirical formulations for the main body of the thesis, reserving machine learning as a complementary and future-oriented tool. Nonetheless, the lessons learned from this exploratory work are valuable. They demonstrate both the opportunities and pitfalls of ML for atmospheric attenuation modeling. While the present study chose to focus on physically based frameworks, the methodological foundation laid here—particularly in preprocessing pipelines,

feature design, and model benchmarking—provides a solid basis for subsequent research. Future studies may revisit these approaches with larger multi-satellite datasets or hybrid physics-informed networks, aiming to combine the interpretability of radiative transfer with the flexibility of data-driven learning.

Development and Validation of a Predictive Attenuation Law Using CloudSat-GPM Coincidences: A Machine Learning Approach.

Paolo Cicolin, Student Member, IEEE.

Abstract—In this study, we present a novel machine learning-based approach to develop a global attenuation law for predicting Path Integrated Attenuation (PIA) using CloudSat-GPM coincidence data. By incorporating advanced predictive models, this research achieves significant accuracy improvements in estimating PIA across different frequency bands (W, Ku, and Ka). The dataset, comprising observations from January, February, and May 2016, underwent rigorous preprocessing, feature engineering, and filtering to ensure data quality. Employing models such as Gradient Boosting and Random Forest, the study demonstrates the importance of hyperparameter tuning and cross-validation in achieving robust and reliable predictions. Key findings include the enhancement of model performance through derived variables and advanced optimization techniques, with metrics such as RMSE and R^2 showing significant improvements. Furthermore, the integration of variables like ice water content, precipitation liquid water, and temperature highlights the value of combining physical insights with machine learning. These methodologies not only enhance atmospheric attenuation modeling but also offer a scalable framework for applications ranging from remote sensing to satellite communications, ensuring adaptability to diverse climatic and geographical conditions. The results emphasize the potential for further innovation in predictive methodologies, fostering a deeper understanding of atmospheric dynamics and their implications for signal transmission.

Index Terms — Machine Learning, Attenuation Law, CloudSat-GPM Coincidences, Predictive Model, Path Integrated Attenuation (PIA), Dataset Preprocessing, Hyperparameter Tuning.

I. MACHINE LEARNING OVERVIEW.

This section aims to illustrate some relevant issues that emerge from this work, although due to space constraints it is not possible to explore them exhaustively. It is important to underline that our goal is to lay the foundations for future scientific investigations, providing a first overview of the main topics of interest [5], [6], [7]. We want to focus on the difficult task of creating, with the tools we have access to now, a global law for predicting attenuation based on the dataset we are currently studying [9], [24]. This dataset includes observations carried out in January and February 2016, with some additional steps in May of the same year [15], [16]. Future studies will subsequently explore the techniques and methods introduced in this work, with the aim of developing a robust and extensible predictive model [3], [12], [19]. For simplicity, we will only discuss CloudSat-GPM coincidences, but in the technical appendix we discuss CloudSat-TRMM

coincidences (e.g., where to find and how to download them) [15], [16], [33]. The attenuation law is therefore an estimate of the PIA(W), starting, for example, with attenuation values in other bands of interest, such as the PIA(Ku) and the PIA(Ka) [6], [8], [24].

A. Derivation and Analysis of Global Attenuation Maps: Influence of Atmospheric and Indirect Factors

Creating a global Path Integrated Attenuation (PIA) map is critical for several reasons. First, it provides a comprehensive understanding of how different climatic and geographic factors affect signal attenuation in different regions [6], [7], [15], [16]. This knowledge is essential for optimizing satellite communications, remote sensing, and radar observations, especially in areas with challenging weather conditions [7], [24], [25], [28].

Additionally, a global PIA map helps identify regions with specific attenuation characteristics, allowing for targeted mitigation strategies. For instance, we can employ power control and beam switching techniques in tropical regions with heavy rainfall to maintain link quality [6], [7], [8], [15].

In temperate and polar regions, understanding the impact of snow, ice, and cloud cover on signal attenuation can inform the development of more resilient communication systems.

Integrating data from various sources to create these maps improves the accuracy and reliability of attenuation forecasts [16], [18], [33]. This integration makes it possible to look at frequency bands and how sensitive they are to different hydrometeors in more detail. This helps people make better decisions about how to use new technologies like 5G communications and high-precision radar systems [3], [12], [19]. In the end, creating global attenuation models using observational data and combining it with other relevant data is a big step forward in the field. It not only improves our understanding of PIA but also paves the way for future studies to explore specific frequencies and their attenuation behaviors in more detail [6], [8], [24]. This work is essential to optimize the use of the technology in satellite communications and remote sensing, ensuring reliable and efficient performance in different environmental conditions. By refining these models, researchers can address challenges related to signal degradation and enhance the accuracy of forecasts [9], [24], [26]. Adding advanced machine learning techniques could also make these models even more accurate, giving us even more information about how the atmosphere affects signal transmission [1], [3], [12], [19].

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In previous work, we described the methodology for creating global attenuation maps [5], [6], [16], [33]. Additionally, we delved into the process of importing and integrating maps from various sources for improved analysis and verification [15], [16], [33]. These approaches allowed us to obtain more accurate and reliable results, thus contributing to a better understanding of the phenomenon under investigation [7], [24], [28]. In the future, we plan to continue refining the methodology and exploring new data sources to further expand our maps [9], [24], [30]. As we embark on this next phase, we aim to incorporate advanced machine learning techniques to enhance the predictive capabilities of our models [1], [3], [19], [39]. This integration will not only streamline the data processing but also enable us to uncover deeper insights into the underlying patterns and trends.

In a **technical paper** associated with the general discussion of creating global attenuation maps, we elaborated on the process of importing and integrating maps from various sources for improved analysis and verification [16], [33]. We talked a lot about how weather and geography affect Path Integrated Attenuation (PIA), and we emphasized how important it is that indirect factors like soil moisture affect its variation [6], [28]. These elements are crucial to optimizing forecasting models and ensuring greater accuracy in analyses [7], [9], [24]. Additionally, we identified strategies to improve data collection, which will allow us to further refine our approaches [16], [18], [33]. By leveraging advanced technologies and collaborative platforms, we can enhance our data-gathering efforts and ensure that we capture a comprehensive view of the influencing factors [3], [7], [24]. This will ultimately lead to more robust models that can better predict variations in PIA and improve decision-making processes across various applications [3], [19], [39].

Also in the **technical article** we have further investigated the identification of areas where the PIA falls within particular ranges of variation [5], [16], [33]. This step is part of a broader methodological framework that aims to statistically and spatially characterize these areas [24], [28], [33]. However, we have not yet conducted a detailed analysis based on specific frequencies, which opens the way for further investigation in future studies [6], [8], [24]. This work represents a significant advancement toward the development of global attenuation models based on observational data and integrated with complementary information.

The different frequency bands analyzed show varying sensitivities to one-way path integrated attenuation (PIA). For example, the Ku band (13.4 GHz) is effective for monitoring moderate to light rainfall, while the Ka band (35.5 GHz) shows greater sensitivity to fog and snow [6], [8], [24], [26].

The W band (94 GHz) is particularly effective at detecting smaller hydrometeors, such as clouds and ice [8], [9], [24], [26]. The different frequency bands analyzed in the white paper (Technical Annex) exhibit varying sensitivities to Path Integrated Attenuation (PIA, one-way) [6], [24], [26]. For instance, the Ku band (13.4 GHz) is effective for monitoring moderate to light rain, while the Ka band (35.5 GHz) shows heightened sensitivity to fog and snow. The W band (94 GHz) is particularly adept at detecting smaller hydrometeors, such

as clouds and ice [24], [26], [30].

When considering frequency bands in relation to Path Integrated Attenuation (PIA), distinct characteristics emerge. Low-frequency bands, below 10 GHz, typically experience minimal hydrometeor attenuation, making them suitable for stable satellite links under adverse weather conditions. Medium-frequency bands, ranging from 10 to 30 GHz, exhibit significant PIA, particularly in the presence of rain. The Ku (12–18 GHz) and Ka (26–40 GHz) bands, despite their high sensitivity, are susceptible to marked attenuation in regions with heavy rainfall. High-frequency bands, above 30 GHz, such as the W band (75–110 GHz) and the V band (50–75 GHz), show significant attenuation even under foggy or lightly clouded conditions. Advanced technologies, such as 5G communications and high-precision radar observations, utilize these bands.

Path Integrated Attenuation (PIA) varies significantly based on the type and distribution of hydrometeors, influenced by climatic and geographical factors. In tropical regions, intense precipitation, reaching up to 300 mm/h, leads to extremely high PIA levels, especially in the Ku, Ka, and W bands. Mitigation strategies such as power control techniques and beam switching are essential to maintain link quality in these environments.

In temperate regions, characterized by moderate precipitation and frequent cloudiness, PIA predominantly affects the Ka and Ku bands, whereas the W band can experience significant attenuation during extreme weather events like winter storms. Polar regions, with prevalent snow and ice, especially above 30 GHz, encounter critical attenuation contexts pertinent to radar observations and remote sensing. In arid and semi-arid regions, where rainfall is infrequent, PIA is nearly negligible [7], [24], [28]. However, residual water vapor can cause minor signal loss in higher frequency bands (>30 GHz) [8], [24].

This highlights the importance of considering regional and climatic factors in attenuation analysis, integrating data with models that address a range of frequencies and atmospheric conditions. Identifying key parameters for predicting attenuation in the W band during Earth observations is paramount. Examining the variability of Path Integrated Attenuation (PIA) across different climate zones enables the development of reliable predictions. Established parameters for lower frequencies can serve as a foundation, with regression models (either linear or nonlinear) accurately describing attenuation behavior at higher frequencies [19], [20], [30].

A primary variable influencing PIA is the precipitation rate (RR), measured in millimeters per hour [19], [20], [25]. The intensity of precipitation determines the density and distribution of water droplets, directly impacting signal attenuation. This effect is particularly pronounced in the Ka and W bands, where intense precipitation leads to significant attenuation increases [8], [24], [26]. Identifying and quantifying these variables enables the creation of more accurate global predictive models, ultimately enhancing attenuation predictions and optimizing the use of technology in satellite communications and remote sensing [3], [12], [19], [39].

The Rain Rate (RR) variable, which is fundamental for attenuation estimation, is available through several satellite

Climatic Area	PIA Ku (12–18 GHz)	PIA Ka (26–40 GHz)	PIA W (75–110 GHz)
Tropical Zones	Min: 5 dB Max: 30 dB Mean: 15 dB	Min: 15 dB Max: 50 dB Mean: 30 dB	Min: 50 dB Max: 150 dB Mean: 100 dB
Temperate Zones	Min: 2 dB Max: 15 dB Mean: 8 dB	Min: 5 dB Max: 25 dB Mean: 15 dB	Min: 20 dB Max: 80 dB Mean: 50 dB
Polar Regions	Min: 0.5 dB Max: 5 dB Mean: 2 dB	Min: 1 dB Max: 10 dB Mean: 5 dB	Min: 10 dB Max: 30 dB Mean: 10 dB
Arid Zones	Min: 0 dB Max: 2 dB Mean: 1 dB	Min: 0 dB Max: 5 dB Mean: 2 dB	Min: 0 dB Max: 10 dB Mean: 5 dB

Table I: Climatic Area for PIA Ku, PIA Ka and PIA W.

coincidences, such as in the CloudSat-GPM products and in the GPM radar datasets [15], [16], [17]. Specifically, you can find it in the 2C-RAIN-PROFILE, 2B-GEOPROF, and the GPM.DPR Level 2A products (2A-DPR), where DPR stands for the Dual Precipitation Radar [15], [17]. These products are among the most suitable for the analysis.

- **GPM Level 2A DPR (2A-DPR):** It provides detailed estimates of the rain rate both at the surface and on different vertical levels, proving particularly useful for atmospheric studies and predictive models [15], [17], [24].

- **GPM IMERG (Integrated Multi-satellite Retrievals for GPM):** A global product that combines data from several satellites, including those from CloudSat and other instruments, to provide a rain rate estimate with extended spatial coverage and high resolution [15], [16], [33].

These products are essential resources for analyzing rain rate variability and its impact on signal attenuation, especially in the W-band.

B. Key Variables for Predicting Path Integrated Attenuation (PIA)

To predict Path Integrated Attenuation (PIA), it is essential to consider various influencing variables that can enhance predictive models. The following key variables are of particular interest:

- 1) **Rain Rate (RR):** Measured in [mm/h], the Rain Rate plays a crucial role in estimating PIA. The intensity of the rain determines the density and spatial-temporal distribution of water droplets, directly influencing signal attenuation, particularly in the Ka and W bands [15].
- 2) **Air Temperature (T):** This variable influences the physical state of hydrometeors (liquid, solid, or mixed). High-frequency bands, such as the W band, are particularly sensitive to snow and ice, whose attenuation and absorption properties differ markedly from those of liquid water.
- 3) **Relative Humidity (RH):** This dimensionless quantity is vital in the formation and concentration of hydrometeors, including clouds and fog. High humidity increases the water content in clouds, thereby increasing signal attenuation [6], [19], [24].
- 4) **Headroom Height (H):** Measured in meters or kilometers, Headroom Height signifies the depth of the air column containing hydrometeors through which the

electromagnetic signal travels. Greater heights result in more significant integrated attenuation along the path.

- 5) **Band Frequency (f):** The frequency, measured in Hz or its multiples, determines the sensitivity of the signal to attenuation. Higher frequency bands, such as the W band, are particularly susceptible to scattering and absorption effects. [6], [8], [24], [28].
- 6) **Drop Size Distribution (DSD):** This function provides information on the probability of the presence of drops with specific diameters, directly impacting PIA.
- 7) **Liquid Water Content in Clouds (LWC):** Measured in [g/m³], this represents the concentration of liquid water in clouds, closely related to attenuation in the Ka and W bands.
- 8) **Hydrometeor Type:** The dielectric and scattering properties vary substantially among different hydrometeors, such as rain, snow, hail, and graupel, each affecting attenuation differently.
- 9) **Elevation Angle (θ):** A low elevation angle increases the effective path of the signal through the atmosphere, thus increasing attenuation.
- 10) **Atmospheric Pressure (P):** Measured in [hPa], atmospheric pressure affects the density of air and the ability of hydrometeors to aggregate. At higher elevations (lower pressures), PIA tends to decrease for a given amount of precipitation.
- 11) **Geographic Area/Climate Zone:** Factors such as latitude, longitude, altitude, and proximity to bodies of water influence local atmospheric conditions, modulating PIA in different regions.
- 12) **Derived Synthetic Variables:** These include the total column of water vapor (TPW) and the atmospheric instability index, both associated with the formation of strong thunderstorms. [6], [8], [19], [30].
- 13) **Satellite Coincidence-Specific Rain Rate (RR):** Products like 2C-RAIN-PROFILE provide surface rain rates. Negative values indicate a saturated radar signal, where the absolute value represents the minimum possible rainfall intensity [15], [16], [17], [33].

By analyzing satellite coincidences and other original products, we can determine if we are in a mixed-phase scenario when the rain rate is zero. The 2B-GEOPROF product generally provides necessary information to understand the vertical structure of clouds. When combined with specialized

algorithms, this enables us to estimate the rain rate accurately.

C. How is a prediction law for PIA(W) established?

Scientists face numerous challenges daily, ranging from short-term to medium- and long-term. One significant challenge is selecting the optimal parameters, as it's not simple to provide an immediate answer. For instance, considering direct factors may lead to a regression law, whereas complex cases might necessitate a multivariate regression law. Complex systems models, such as multilinear regression models, become valuable in these scenarios [1], [3], [19]. These models use separate factors, like soil moisture, plant cover, and pollution in the environment or air, to explain changes in dependent factors, like the amount of rain (PIA = Precipitation Intensity Amplitude) or, in this case, Path Integrated Attenuation (PIA). Generally, the law might be linear in simple cases but non-linear in more intricate or "difficult to explain" cases [19], [20].

Deforestation-associated vegetation and anthropogenic pollution often exhibit non-linear relationships with atmospheric processes. Differential equations can represent these factors, capturing complex atmospheric dynamics [7], [28]. Alternatively, we can employ atmospheric physical models or numerical "meteorological" models like the WRF (Weather Research and Forecasting) model, ECMWF (European Centre for Medium-Range Weather Forecasts) model, or GFS (Global Forecast System) [18]. These models incorporate fluid dynamics equations along with models for energy, humidity, and particle transport. Some models include "radiative transfer" theories, which use the size distribution of water droplets or ice particles to determine how hydrometeors affect signal attenuation [24], [26], [30].

When these models fail to meet our needs, we can explore machine learning and artificial intelligence models. Artificial Neural Networks (ANNs), for instance, can identify complex non-linear relationships between various direct and indirect factors. By combining data on vegetation and land use, concentrations of aerosols and pollutant particles, and local weather conditions, a neural network can predict Path Integrated Attenuation (PIA).

We should gather observational data linking the relevant variables to the PIA in the W band to enhance prediction accuracy. This includes looking at the vertical profiles of clouds from CloudSat and hydrometeors from CloudMask, which are based on coincidence data from CloudSat and GPM, as well as GPM data on rain rate and DSD. We can also utilize ground meteorological stations to periodically measure rainfall, temperature, and relative humidity.

Atmospheric simulation models, such as ERA 5 or WRF, can provide insights into the liquid content of clouds (LWC) and the DSD [14], [15], [16]. Once we establish the models, we can apply a physical law to describe the attenuation. We could apply an integral law, for example:

$$PIA(f) = \int_0^H \kappa(f, R, T, DSD) \cdot dz \quad (1)$$

where:

- $\kappa(f, R, T, DSD)$: the specific attenuation coefficient (dB/km), which depends on frequency, precipitation intensity, and other variables.
- H : the depth of the signal path (in km).

If we opt for a more empirical approach using data science and machine learning, sufficient observational data can facilitate the creation of an automated learning model using datasets.

1) Multiple Regression Laws: The PIA is connected with predictive variables such as rain rate and LWC.

- General formula:

$$PIA = \beta_0 + \beta_1 * R + \beta_2 * LWC + \beta_3 * T + \epsilon * PIA \quad (2)$$

where β_i ($i = 0, 1, 2, 3$) are the estimated coefficients, and ϵ is the error. Non-linear models may also be considered, where the law is of a polynomial type, based on exponential functions, or other non-linear relationships.

2) Non-linear models: In more complex scenarios, non-linear models can capture intricate relationships between variables [19], [20], [25], [30].

3) Machine learning models: These include random forests, support vector machines, and neural networks, which allow for better management of complex relationships between identified variables through data science [1], [3], [12], [19], [39]. Following these procedural steps, the discovered model should be validated and optimized against actual or simulated PIA measurements.

Common validation metrics include:

- Root mean square error (RMSE) [3], [12], [22].
- Determination coefficient (R^2) [3], [12], [19].
- Systematic errors (bias) [6], [22], [39].

For instance, a simplified predictive law could be of the following type:

$$PIA(W) = a \cdot R^b + c \cdot LWC^d \quad (3)$$

In this formula, the parameters a, b, c , and d are calculated through regression or optimization using observational data, specifically rain rate (R) and liquid water content (LWC).

D. Final considerations

- The W band is highly sensitive to heavy rain, clouds, and wet snow, making an accurate representation of variables such as rain rate and LWC crucial [6], [8], [24], [26].
- Data-driven approaches can integrate with physical models to enhance prediction [1], [3], [12], [19].
- Access to high-quality satellite data (CloudSat, GPM) is essential to refine the model [15]- [17], [33].

In addition to using machine learning techniques with artificial neural networks, there are also supervised learning models. These models use random forest algorithms to classify regions of the scatter plotters between variables, including indirect factors that can contribute to the estimation of the PIA or even direct variables. There are advanced statistical methods such as "sensitivity analysis" that determine how much a change in an indirect variable (e.g., soil moisture or pollution factor rate) influences the estimate of the PIA hydrometeor [19], [25], [30].

As an alternative to Sensitivity Analysis in this situation, we can also use Principal Component Analysis (PCA), where the goal is to reduce the dimensionality of complex data to identify the main factors that influence the system. In this case, we typically use matrix and algebraic methods. Granger causality analysis verifies whether an indirect factor precedes and causes significant changes in a direct factor. For instance, the direct factor could be the intensity of rainfall (rain rate) for the PIA hydrometeor, while the indirect factor could be pollution, vegetation cover, or soil moisture [28].

We have previously discussed the integration of observational and simulation data, but we have not discussed the potential necessity of hybrid or mixed techniques. Hybrid techniques try to merge salient features of performing predictive methods in order to obtain improvements in the estimation process. Among these models, we can use satellite data and perform coupled simulations of the atmosphere-land surface type (e.g., CESM or RegCM). We look into specific parametric models, like "semi-empirical" ones, which let us model the PIA as a function of the aerosol concentration (N), the relative humidity of the air (RH), and the density of the vegetation (V). In this case, we could estimate the intensity of the rainfall in this way [5], [6], [8], [24], [26]:

$$PIA(\text{Precipitation Intensity Amplitude}) = \alpha \cdot N^{\beta_1} \cdot RH^{\beta_2} \cdot V^{\beta_3} \quad (4)$$

Finally, we can consider stochastic models that incorporate random variability to represent uncertainty in indirect estimation processes. Scientists must validate all of this through scientific experiments, comparing the models with field measurement data from, for instance, meteorological station experimental measurement campaigns and the use of Lidar and Doppler radar. Scientists use experimental campaigns to conduct local measurements like soil moisture and atmospheric composition, and also to gather historical data to construct time series that archive past events for future prediction analysis. We will mostly use the Integrated Path Attenuation definition of PIA, considering its likely variables of interest for estimation.

To accurately determine the Path Integrated Attenuation (PIA), especially when dealing with CloudSat-GPM coincidences, it is essential to consider variables such as Ice Water Content (IWC), Ice Water Path (IWP), Liquid Water Path (LWP), and Liquid Water Content (LWC). These variables are crucial in understanding the PIA, particularly when the electromagnetic signal traverses regions containing mixed hydrometeors or complex vertical structures. These variables are crucial to represent both liquid water and ice in the attenuation process. The combination of LWC, LWP, IWC, and IWP provides the following benefits:

- Model the PIA in complex scenarios with mixed hydrometeors [6], [8], [26].
- Improve the accuracy of predictive models, integrating the liquid water and ice content with parameters such as rain rate and drop size distribution (DSD) [19], [24], [30].
- Apply corrections based on satellite observations (CloudSat, GPM) to validate the models [15]- [17], [33].

II. EXPLORE MACHINE LEARNING TOPICS.

To estimate W-band Path Integrated Attenuation (PIA(W)) using machine learning (ML) techniques, it is crucial to identify the parameters that most significantly influence atmospheric attenuation. Several phenomena primarily cause attenuation in the W-band, including absorption by water vapor and liquid or solid particles such as clouds, rain, snow, and ice. Scattering by hydrometeors like rain, snow, hail, and ice also plays a significant role. The physical characteristics of clouds, such as their top properties and mass distributions, and radiative conditions, including short- and long-wavelength radiation fluxes, are key factors as well.

Relevant Parameters:

- **Cloud Phase (CPH):** It distinguishes between liquid, solid, or mixed cloud phases, which significantly influence attenuation.
- **Cloud Top Parameters:** These parameters include temperature (CTT), pressure (CTP), and height (CTH), which provide crucial information about cloud height, density, and vertical structure.
- **Cloud Water Path (CWP):** In particular, the liquid water path (LWP) and ice water path (IWP) play a crucial role in modeling attenuation.
- **Fractional Cloud Cover (CFC):** This measure determines the percentage of sky covered by clouds, which influences the average attenuation in a geographic area.
- **Radiation Parameters:** These include surface descending longwave radiation (SDL), incoming surface solar radiation (SIS), net surface longwave radiation (SNL), and outgoing longwave radiation (OLR) at the top of the atmosphere.

These parameters are vital for inferring atmospheric conditions that influence PIA(W).

- **Parameters that change instantly:** The cloud mask (CMA) tells us if there are clouds in the way the signal is traveling, and the joint cloud property histograms (JCH) show how cloud properties like liquid water and ice are spread out along different layers of the atmosphere.

Modeling Strategies:

- 1) **Feature Selection:** Use parameters such as CTT, CWP, LWP, IWP, and CFC as the main features.
- 2) **Training Dataset:** Employ instantaneous data for direct attenuation observations and local atmospheric parameters, and incorporate monthly or averaged values for studying seasonal and regional patterns.
- 3) **Regression Models:** Utilize models like Random Forest or XGBoost for estimating numerical values of PIA(W). Neural networks like RNNs or LSTMs can capture temporal and spatial dynamics in the datasets.
- 4) **Model Validation:** Compare with observational data, including precipitation measurements from weather stations and PIA calculations from radar or satellite data (e.g., GPM and CloudSat). Include environmental data such as vegetation indices (NDVI from MODIS or Landsat) and air pollution data (e.g., CAMS and Copernicus). Use statistical metrics like mean errors (MAE, RMSE) and the coefficient of determination (R^2) for comparison.

- 5) **Experimental Campaigns:** Conduct field measurements to validate the model in specific areas, comparing local observations with model predictions.

By following these strategies and using the relevant parameters, it is possible to estimate PIA(W) with high accuracy using ML techniques.

A. Our Theoretical Proposal and our Sperimental Prediction Laws.

We have developed a code that efficiently reads variables at a faster rate compared to our manually developed codes [3], [12], [19], [39]. Specifically, we extract parameters from datasets such as 2B-GEOPROF and 2C-PRECIP-COLUMN [15]- [17]. The extracted parameters include latitude, longitude, precipitation flag, Sigma-Zero, time, and the PIA hydrometeor, along with its associated uncertainty. Subsequently, we utilize the PIA Matched Scan to decouple the values of PIA(Ku) and PIA(Ka) [15], [16], [24]. Finally, we retrieve the PIA hydrometeor data and compile it into a comprehensive table. By employing a short optimized loop and bypassing files that may contain invalid or missing data, we efficiently generate a table referred to as “finalTable” [12], [19].

The resulting table comprises 279,232 rows and 10 columns, with each column header representing the variables of interest:

- Latitude
- Longitude
- PIA_Ku
- PIA_Ka
- PIA_hydrometeor
- PIA_hydro_dB (experimental, to determine whether the data is linear or already in dB)
- Precip_flag
- Time (formatted as YYYY:MM:DD hh:mm:ss:ms)
- File name

These headers reflect the key parameters we have identified for analysis. We have developed several versions of source code in MATLAB that facilitate the exploration of variables of interest, as well as additional variables not previously mentioned, such as the altitude or elevation (variable height) in the 2B-GEOPROF dataset [15], [16], [19]. These discrete distribution diagrams that connect the variables of interest to the characteristics of precipitation properties can be particularly insightful. Furthermore, examining the attenuations caused by land, ocean, or coastal measures becomes possible by appropriately adjusting the land_sea_flag.¹ [24], [28].

In addition to experimenting with the variables in the dataset, we have also optimized the code that stores all the raw radar reflectivities. We provide an illustrative example of this, which can be beneficial in subsequent applications in the fields of both machine learning and prediction [1], [3], [12], [19].

B. Quick Analysis avoiding inversion techniques.

To conduct a quick analysis without delving into programming elements that involve the resolution of inversion techniques, I have developed MATLAB source code to estimate the Path Integrated Attenuation in band W (PIA(W)).

¹It is present in the same group 2B-GEOPROF.

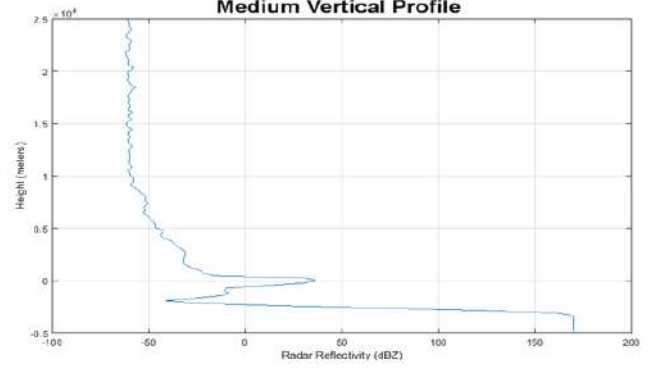


Figure 1: Medium Vertical Profile for all rays extracted from a coincidence CloudSat-GPM file.

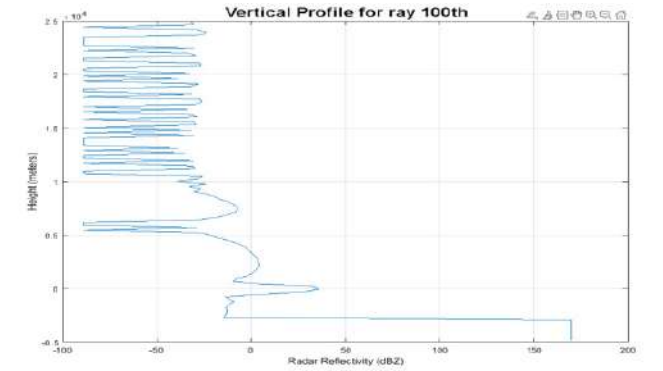


Figure 2: Vertical profile for a specific ray extracted from a coincidence CloudSat-GPM file.

This estimation considers the following variables: Latitude, Longitude, piaFinal, Ice Water Content, Precipitation Liquid Water, Precipitation Ice Water, Snow Water Content, Surface Pressure, and Surface Temperature. Specifically, the variables are accessed through the following paths:

- /2B-GEOPROF/Latitude
- /2B-GEOPROF/Longitude
- /2A.GPM.DPR/NS/piaFinal (PIA(Ku))
- /2A.GPM.DPR/MS/piaFinal (PIA(Ka))
- /2C-ICE/IWC (Ice Water Content), [15]-[17]
- /2C-RAIN-PROFILE/precip_liquid_water (Precipitation Liquid Water), [15]-[17]
- /2C-RAIN-PROFILE/precip_ice_water (Precipitation Ice Water), [15]-[17]
- /2C-SNOW-PROFILE/snowfall_rate (Snowfall Rate) [15-17], [24]
- /2C-SNOW-PROFILE/snow_water_content (Snow Water Content) [15-17], [24]
- /ECMWF-AUX/Surface_pressure (Surface Pressure) [14, 17, 24]
- /ECMWF-AUX/Temperature_2m (Surface Temperature) [14, 17, 24]

These variables provide a comprehensive basis for estimating the PIA(W) and enable effective analysis and modeling.

The optimization process using the lsqnonlin algorithm in MATLAB shows significant convergence, reducing the residual norm (Resnorm) from 5516.08 to 870.595 by the tenth iteration. Consistent step sizes and a steady decrease in Resnorm confirm the algorithm’s effectiveness, resulting in an optimal solution with minimized residuals. The algorithm

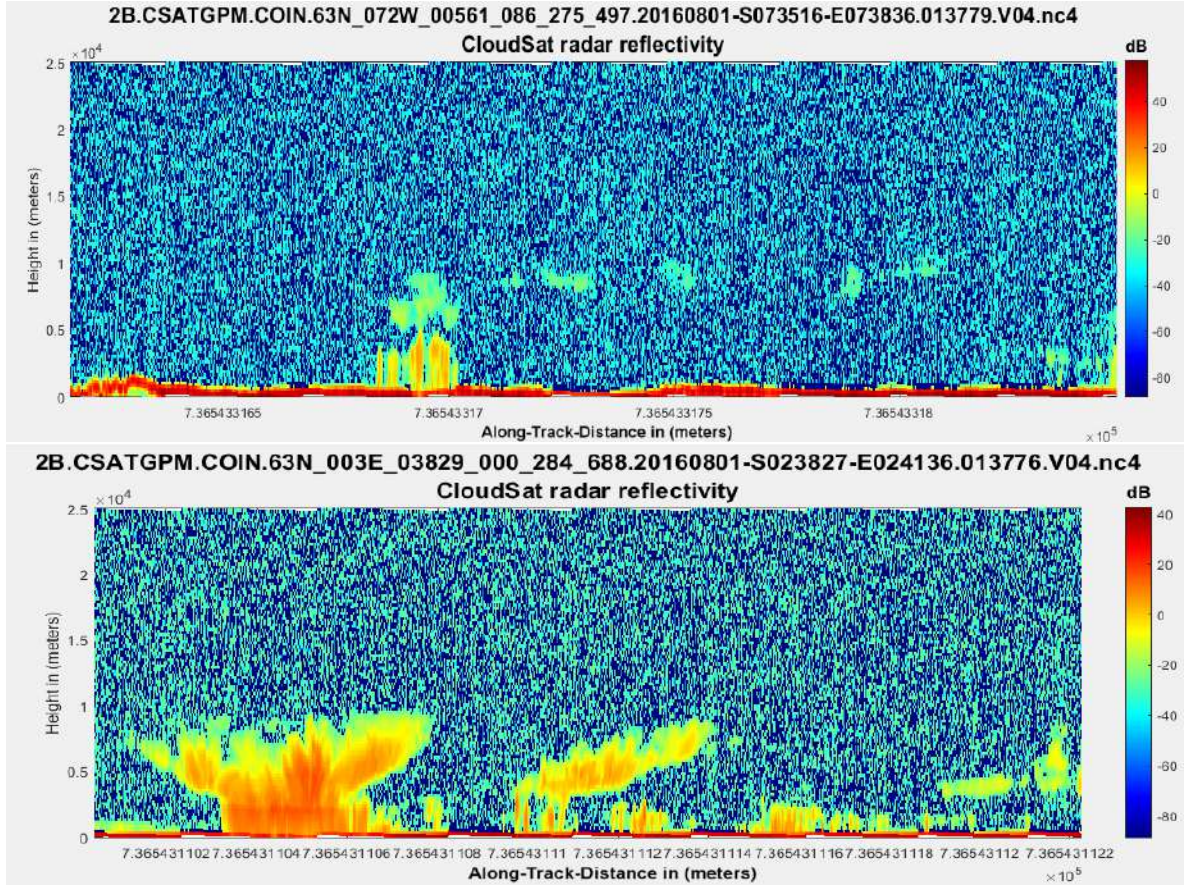


Figure 3: Radar Reflectivity extracted from two different files in the JK Dataset.

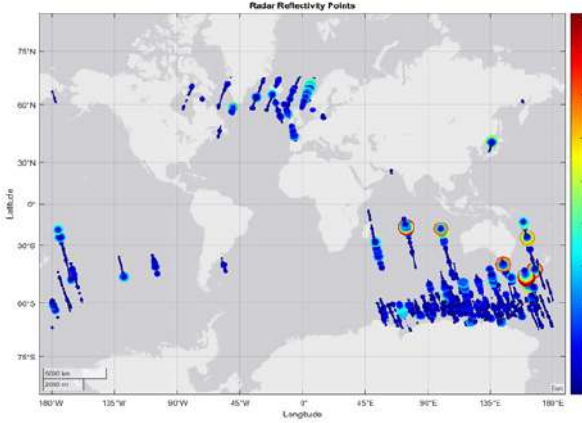


Figure 4: World Radar Reflectivity from January-February and few passages in May 2016.

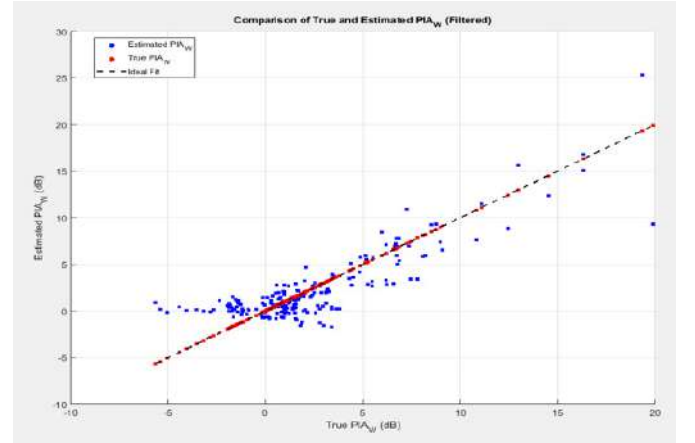


Figure 5: Scatter plotter of the True $PIA(W)$ or $PIA_{hydrometeor}$ vs. Estimated $PIA(W)$.

met the stopping criteria when the relative change in the sum of squares became negligible, indicating convergence to a local minimum [12], [19], [39].

The optimized coefficients obtained are -7.635, -1.8595, 181.3706, 1166.9616, -1166.8829, -15.0009, and 61.0058. These coefficients, with a high correlation of 0.85155 and a low RMSE of 2.113, signify a robust fit between the observed

and predicted values [1], [3], [12]. This method ensures the accurate estimation of the Path Integrated Attenuation (PIA) in band W. It also provides a solid base for more in-depth studies and uses in machine learning and weather forecasting [3], [19], [39]. The acceptability of an RMSE of 2.113 dB depends on the specific context and application. If the true PIA_W values vary widely (e.g., from -10 dB to 20 dB),

an RMSE of 2.113 could be acceptable. Conversely, if the true PIA_W values are concentrated within a narrower range (e.g., 0 dB to 5 dB), the RMSE may be considered high. Another critical indicator is the correlation (R^2) between the predicted and true values. If R^2 is close to 1 (e.g., $R^2 \geq 0.9$), the RMSE could be acceptable [1], [3], [12], [19]. The objective of the application also plays a pivotal role. In some cases, such as radar estimations or atmospheric attenuations, an accuracy of approximately 2 dB might be sufficient [5], [6], [8], [24]. However, in more critical applications, such as satellite communications design, a lower RMSE (e.g., less than 1 dB) may be required [24], [26]. It is crucial to analyze the error distribution to ascertain the uniformity of the errors or the presence of significant discrepancies in specific regions. Creating a residual error plot (estimated value minus true value) can help identify any patterns [3], [12], [19], [39].

These evaluation criteria assist in determining whether an RMSE of 2.113 dB is appropriate for the specific context [6], [8], [24].

C. How to further improve RMSE

To improve Root Mean Square Error (RMSE), consider the following approaches:

- **Incorporate Additional Predictor Variables:** Identify and integrate additional predictor variables that can influence the PIA (Path Integrated Attenuation) value. This can enhance the model's comprehensiveness and accuracy.
- **Utilize Nonlinear Models:** For data exhibiting nonlinear relationships, employ more sophisticated models, such as polynomial regression or advanced machine learning techniques, to capture these complexities more effectively.
- **Ensure Thorough Data Cleaning:** Conduct rigorous data cleaning to eliminate outliers and inconsistencies within the dataset, as these can significantly impact the RMSE.
- **Identify and Apply Applicable Regression Laws:** Discover and implement the appropriate regression laws that best fit the data characteristics [30].

We have developed a linear regression model to estimate the value of PIA (Path Integrated Attenuation). Below we present the regression law found and the performance characteristics of the model [1], [3], [12], [19]:

$$PIA_W = -7.6350 \cdot \text{Feature1} + (-1.8595) \cdot \text{Feature2} + 181.3706 \cdot \text{Feature3} + 1166.9616 \cdot \text{Feature4} + (-1166.8829) \cdot \text{Feature5} + (-15.0009) \cdot \text{Feature6} + 61.0058 \cdot \text{Feature7} \quad (5)$$

where the feature names in order are:

- 1) {'-PIA_Ku'}, {'-PIA_Ka'}, {'Mean IWC'}, {'Mean PLW'}, {'Mean PIW'}, {'Mean Snowfall Rate'}, {'Mean SWC'}.
- 2) The performance of the algorithm obtain:
 - **Correlation:** 0.85155
 - **RMSE:** 2.113

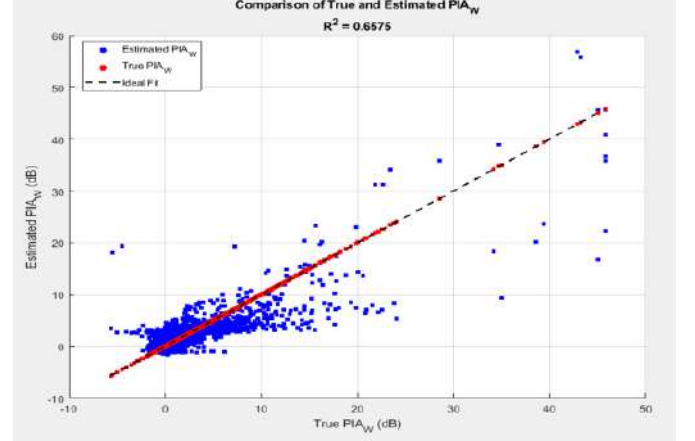


Figure 6: Improvement respect the first formulation of the estimation empirical Law of Prediction for PIA(W).

The feature names, in order, are as follows: Columns 1 through 6 include 'Bias', 'Mean Ice Water Content (IWC)', 'Mean Precipitation Liquid Water (PLW)', 'Mean Precipitation Ice Water (PIW)', 'Mean Snowfall Rate', and 'Mean Snow Water Content (SWC)' [15]- [17]. Columns 7 through 9 include 'Mean Radar Reflectivity', 'Mean Surface Pressure', and 'Mean Temperature' [14], [17], [24].

Regarding the performance of the improved algorithm, the results are as follows:

- **Correlation:** 0.81086
- **Root Mean Square Error (RMSE):** 2.7739

These performance metrics indicate a strong relationship between the predicted and actual values, as evidenced by the correlation coefficient. The RMSE value provides insight into the average deviation between the predicted and true values, reflecting the precision of the algorithm. Upon comparing the two sets of results, we observe the following metrics:

- First version of the algorithm.

- Correlation: 0.85155
- RMSE: 2.113

- Improved version of the algorithm.

- Correlation: 0.81086
- RMSE: 2.7739

The correlation coefficient measures the strength of the relationship between the predicted and actual values, with values closer to 1 indicating a stronger relationship. Meanwhile, a lower Root Mean Square Error (RMSE) signifies a smaller average deviation between the predicted and actual values, thereby suggesting higher accuracy. We consider:

- **About Correlation:** the correlation in the improved version of the algorithm is marginally lower (0.81086 vs. 0.85155), indicating a diminished

ability of the algorithm to capture the relationship between the variables.

- **About RMSE:** The RMSE in the improved version has increased (2.7739 vs. 2.113), reflecting an overall decrease in prediction accuracy.

These results imply that, despite efforts to enhance the algorithm, the new version has not outperformed the original. The decline in correlation and rise

in RMSE suggest a deterioration in the model's accuracy and reliability. Consequently, it may be prudent to review the modifications made to the

algorithm, explore alternative variables or optimization techniques, and conduct error analysis to identify the underlying causes of these performance

discrepancies [3], [12], [19], [30].

D. How to improve the model.

In order to enhance the model's performance, we opted to compare various training models on the data. Specifically, we chose to compare three distinct techniques: the decision tree, the random forest, and the SVM [1], [3], [12], [19].

Decision trees are straightforward to understand and interpret, allowing you to easily visualize decisions and conditions. They are quick to train on small datasets without needing extensive preprocessing. However, they can easily overfit the training data, making them less effective on new data, and are sensitive to changes in the data, leading to unstable models. Random forests mitigate the risk of overfitting by combining predictions from multiple decision trees, providing generally better performance and robustness to data variations [1], [3], [12]. However, they are more complex, demanding in terms of training time and computational resources, and less readable [3], [12], [30]. Support vector machines (SVMs) excel in high-dimensional spaces with clear class boundaries and are robust to outliers due to their kernel function. Nevertheless, they are more challenging to understand and interpret, require significant parameter optimization, and are computationally expensive for enormous datasets [12], [19], [30].

The results from the training model evaluations for Decision Tree, Random Forest, and Support Vector Machine (SVM) offer insightful details into the performance and optimization of each model. The evaluations provide crucial information on the objective function values, runtime, and parameter settings used. We explored different configurations of BoxConstraint, KernelScale, and Epsilon throughout the 30 iterations to identify the optimal settings that minimize the objective function value [3], [12], [39].

While this output does not detail individual results for the Decision Tree and Random Forest models, it is crucial to highlight their general behavior [1], [3], [12].

Decision trees are relatively simpler and faster, as indicated by their lower runtime, but they can be prone to overfitting, particularly with high variance in the data [12], [19], [30]. Random forests, being an ensemble method, mitigate this risk by combining multiple trees, offering more robust and generalizable results at the cost of higher computational expense [1], [3], [12]. We optimized the box constraint, kernel scale, and epsilon parameters of the SVM model to achieve the lowest possible objective function value. The BestSoFar observed objective function value was 3.077, with specific settings (BoxConstraint: 0.036519, KernelScale: 9.2901, Epsilon: 0.77406). We completed the optimization in 30 evaluations,

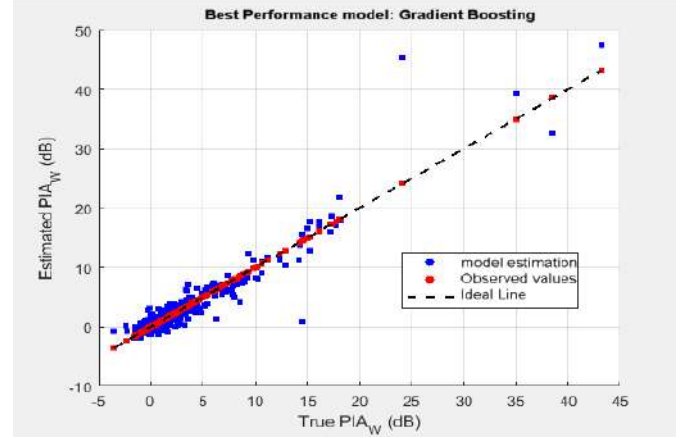


Figure 7: Best performance for the Gradient Boosting method.

with a total elapsed time of approximately 272 seconds and a function evaluation time of about 268 seconds.

The SVM model's performance shows progressive refinement over iterations, with early configurations having higher objective function values and later iterations achieving more optimal settings [1], [3], [19]. For example, initial parameters like BoxConstraint: 650.33 and KernelScale: 0.0018341 resulted in an objective value of 3.1629, while refined parameters reduced this to 3.077. The close match between the observed and estimated objective function values indicates consistency and reliability in the evaluation process.

These results highlight that careful parameter tuning can effectively optimize SVM, enhancing model performance in handling high-dimensional spaces and robustness against outliers. However, this comes with computational trade-offs, as seen in the increased runtime and complexity of the optimization process. The BestSoFar columns serve as valuable progress tracking mechanisms, indicating real advancements and areas for further refinement. By minimizing the objective function, such as $\log(1 + \text{loss})$, a lower BestSoFar value signifies a superior model configuration with reduced loss. The iterative updates reflect the algorithm's capability to progressively improve the model's performance, demonstrating its efficacy in achieving lower loss and improved accuracy [12], [19], [39].

The models say that the best possible point is one with BoxConstraint = 0.036519, KernelScale = 9.2901, and Epsilon = 0.77406. It also has an estimated objective function value of 3.0785 and an estimated function evaluation time of 0.27646. Among the considered training models (decision tree, random forest, and SVM), the gradient-boosting model was the best, confirming its superior performance compared to the other models. Furthermore, the comparison between the Gradient Boosting and Neural Network models identified Gradient Boosting as the suggested model, highlighting its effectiveness in optimizing the objective function [3], [12], [19], [39].

E. How to write the empirical estimation formula.

Unlike linear regression, which uses a straightforward linear formula, the Gradient Boosting (GB) model employs a more

Iter	Eval result	Objective: log(1+loss)	Objective runtime	BestSoFar (observed)	BestSoFar (estim.)	BoxConstraint	KernelScale	Epsilon
1	Best	3.198	0.25302	3.198	3.198	0.27129	0.005956	1.2328
2	Best	3.1629	0.028062	3.1629	3.165	650.33	0.0018341	57.348
3	Accept	6.6543	55.8	3.1629	3.1628	92.063	6.3378	0.0048406
4	Accept	3.1629	0.027601	3.1629	3.1623	15.988	201.3	143.55
5	Accept	3.8645	23.379	3.1629	3.1628	0.0095756	0.0033344	8.9174
6	Accept	3.2355	0.12212	3.1629	3.1627	0.0052093	826.82	0.37187
7	Accept	7.5205	38.64	3.1629	3.4568	500.3	5.9295	0.68933
8	Accept	5.2098	65.841	3.1629	3.4601	17.499	0.03221	2.1725
9	Accept	6.1427	48.867	3.1629	3.1758	433.93	453.38	1.9103
10	Accept	3.1629	0.026646	3.1629	3.1553	832.37	1.396	237.39
11	Accept	3.1629	0.029518	3.1629	3.1622	143.58	161.24	129.85
12	Best	3.077	0.10354	3.077	3.0764	0.036519	9.2901	0.77406
13	Accept	3.1629	0.028358	3.077	3.0763	2.1902	4.85	237.81
14	Accept	3.1629	0.027603	3.077	3.0763	19.991	32.498	237.52
15	Accept	3.1629	0.030648	3.077	3.0763	996.18	7.9888	109.27
16	Accept	3.1629	0.031222	3.077	3.0763	0.17002	0.068307	237.65
17	Accept	3.1629	0.028231	3.077	3.0762	0.0010051	38.341	218.27
18	Accept	3.1629	0.028955	3.077	3.0761	0.008137	78.759	237.79
19	Accept	3.2123	0.22564	3.077	3.0759	0.0010062	0.020183	0.92396
20	Accept	3.1629	0.027827	3.077	3.0761	0.75101	0.036471	72.183
21	Accept	3.1629	0.028653	3.077	3.0762	9.7335	37.939	50.061
22	Accept	3.2061	0.10985	3.077	3.0776	0.047264	554.92	1.1685
23	Accept	22.679	33.982	3.077	3.0782	0.024439	0.0010008	0.85046
24	Accept	3.1629	0.027734	3.077	3.0781	46.029	0.0061784	155.18
25	Accept	3.1629	0.028111	3.077	3.0781	4.1123	0.32578	123.2
26	Accept	3.1629	0.026766	3.077	3.0781	0.76679	998.97	156.84
27	Accept	3.2247	0.11345	3.077	3.0782	0.0010061	40.76	0.75449
28	Accept	3.1629	0.026123	3.077	3.0782	0.090194	0.0021698	232.28
29	Accept	3.1629	0.027138	3.077	3.0784	0.0010171	0.85642	217.9
30	Accept	3.1629	0.025908	3.077	3.0785	0.0010215	11.338	104.13

Table II: Evaluation from the Analysis including BestSoFar Results.

complex approach to estimate relationships [1], [3], [12], [19]. GB utilizes a sequence of decision trees, where each tree makes a small correction to the estimation function based on the residual errors of the previous model [3], [12], [19]. This iterative process continues, with each tree refining the model's accuracy. The estimated relationship can be represented generically through a series expansion, with supporting elements derived from features obtained from the m -th tree using the input features. In this context, M denotes the total number of trees in the Gradient Boosting model, while α_m represents the weight, typically constant, associated with the m -th tree [12], [19], [30]. So in short we can represent the estimated relationship by the following formula:

$$PIA_{hydrometeor} = PIA(W) = \sum_{m=1}^M \alpha_m \cdot T_m(Features) \quad (6)$$

The final result is then a weighted sum of all the trees in the forest. Although computer simulation may not always provide a simple explanation of the relationship, particularly in a compact form, we can still visualize the algebraic laws for the trees containing the most important features. In our scenario, we established a forest containing 150 trees and subsequently selected a few essential features to streamline the discussion. Below, we present the impact of each selected feature in comparison to the others. For simplicity, we have evaluated several techniques, which we cannot fully describe due to the brevity of the discussion [1], [3], [12], [19], [39].

The gradient boosting model itself sets the relative importance scale value, which in this case ranges from 0 to 0.02, during the training process. Specifically, this scale reflects the

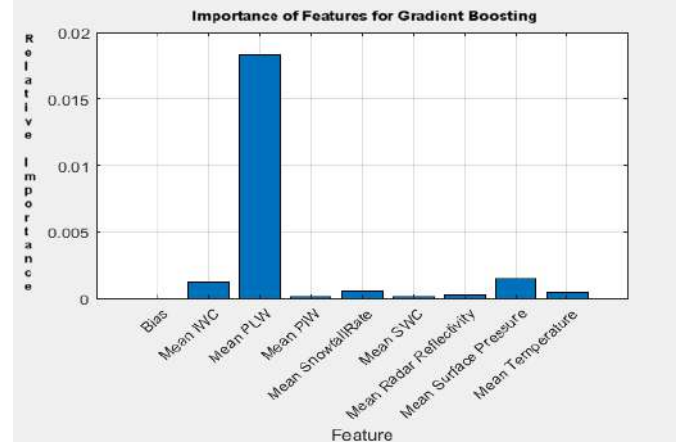


Figure 8: Importance of features for GB that not require inverse methods.

overall contribution of each feature in improving the model's predictions. Relative importance is a measure of the amount of error reduction (such as mean squared error) attributed to each feature. The gradient boosting model calculates this rating by aggregating the contributions of features across all decision trees [3], [12], [19]. In other words, when a feature undergoes a split in one of the trees, the error reduction improvement it provides adds to the feature's relative importance score. The sum of these error reductions for all splits in which a feature is involved determines its overall relative importance.

If the relative importance of a feature is only 2% or less, but it still explains the data and improves the model's predictions, then that's fine. Relative importance is just one

Model	RMSE (Test)	MAE (Test)	MSE (Test)	R ² (Test)
Decision Tree	1.6971	0.7694	2.8800	0.8679
Random Forest	1.5999	0.7417	2.5597	0.8826
SVM	4.6695	2.7875	21.8041	-0.0004
Gradient Boosting	1.5987	0.8536	2.5560	0.8827
Neural Networks	3.0650	1.5526	9.3941	0.5690

Table III: Model performance comparison.

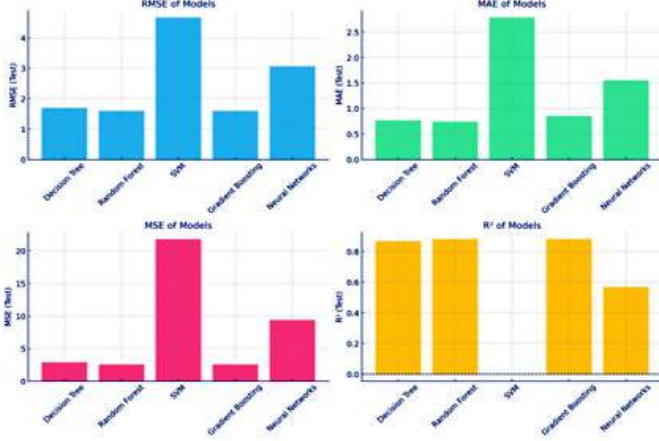


Figure 9: Graphical output for performances.

indicator of how much a feature contributes to others; even a feature with low importance can be useful if it adds useful information to the model. Furthermore, even if a feature has a low relative importance, it could still be essential when combined with other elements. Therefore, using these features can still improve the overall quality of the model's predictions. It is important to evaluate not only the individual relative importance of features but also their combined contribution to the model [39].

F. Model Performance Comparison

The following metrics summarize the performance of various models based on the Test Dataset. Each model's performance is evaluated using Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Squared Error (MSE), and R^2 (coefficient of determination).

Among the evaluated models, Gradient Boosting exhibits the best performance with the lowest RMSE (1.5987) and MSE (2.5560), as well as the highest R^2 (0.8827), closely followed by Random Forest. The Decision Tree performs reasonably well but falls short of the accuracy seen with Gradient Boosting and Random Forest. SVM significantly underperforms, showing the highest RMSE, MAE, and MSE values, and an R^2 close to 0, indicating a lack of predictive power. Neural Networks perform moderately, better than SVM but less effectively than the tree-based models [3], [12], [19], [39].

G. Using Random Forest with PIA(Ku) and PIA(Ka) extracted from Matched Scan.

Given the necessity for extensive experimentation when analyzing small datasets, we opted to modify the source code

by focusing on the Matched Scan rather than the PIA_final. This involved separating it into PIA(Ku) and PIA(Ka). Consequently, we slightly adjusted the model and employed a Random Forest to study its suitability for estimating the PIA(W) using the available data. In our study, we took into account a variety of parameters to enhance the estimation of PIA(W) using machine learning techniques. In addition to classical variables such as pressure, temperature, and two-meter temperature, we also included latitude, longitude, hydrometeor PIA, IWC, PLW, PIW, and PTWC (Precip Total Water Content). The primary goal is to estimate PIA(W), also known as the hydrometeor PIA, utilizing the parameters PIA(Ku), PIA(Ka), IWC', PLW', and PIW' [15], [16], [19], [24]. This approach allows integrating different sources of information to obtain a more accurate and reliable modeling of atmospheric attenuations [1], [3], [12], [19]. We use the symbol “ $'$ ” for indicate the transpose operator over data. Doing this, our model obtain:

- RMSE: 1.9776
- MAE: 0.9860
- R^2 : 0.4318

Our study involved extensive statistical analyses, including cross-validation on the PIA hydrometeor. The k-fold cross-validation results demonstrate that the random forest model exhibits robustness and generalizes effectively. When comparing these outcomes with the initial results, several important conclusions can be drawn:

- **Model Robustness:** The model maintains robustness and exhibits consistent performance across different data partitions.
- **Practical Applicability:** The model can be reliably used to make accurate predictions on previously unseen data.

These findings underscore the model's suitability for real-world applications and its capability to provide dependable predictions in diverse scenarios [1], [3], [12], [19], [39].

I implemented a random forest model with a fixed seed (= 42) to ensure the reproducibility of the results. The process involved data verification and cleaning, which resulted in the removal of 63,376 invalid observations, leaving 69 valid observations with a Y range between -2.04 and 9.95. We then trained the model on the clean data and used it to make predictions. The metrics calculated on the fixed-seeded clean data showed excellent results, with an RMSE of 1.1503, a MAE of 0.6336, and an R^2 of 0.8126. These values indicate the model's good accuracy and robustness in predicting the data, confirming the effectiveness of the cleaning method and the stability of the implemented model [3], [12], [19], [39].

When we filter the data further we get a scatter plotter like in **Fig. 12**, [1], [3], [12], [19]. The scatter plot depicts

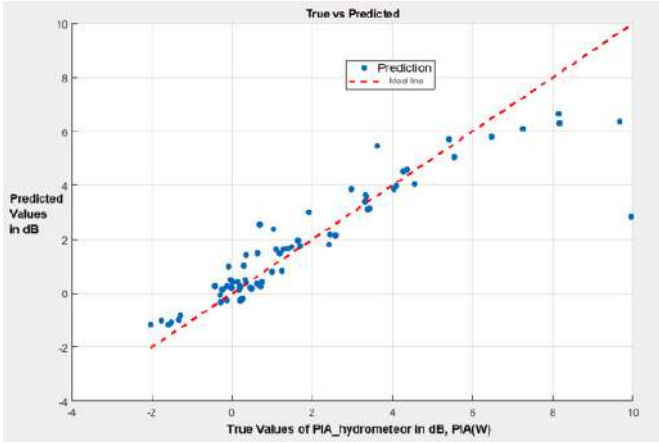


Figure 10: PIA_hydrometeor (or PIA(W)) vs. Predicted values.

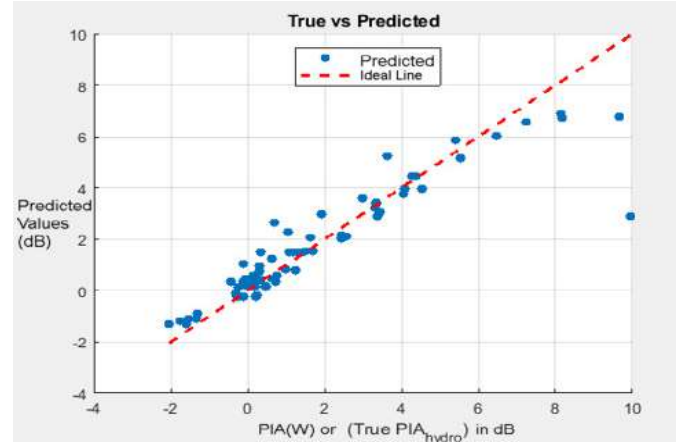


Figure 12: PIA_hydrometeor estimation and PIA_hydrometer real values, with a better filtering on data.

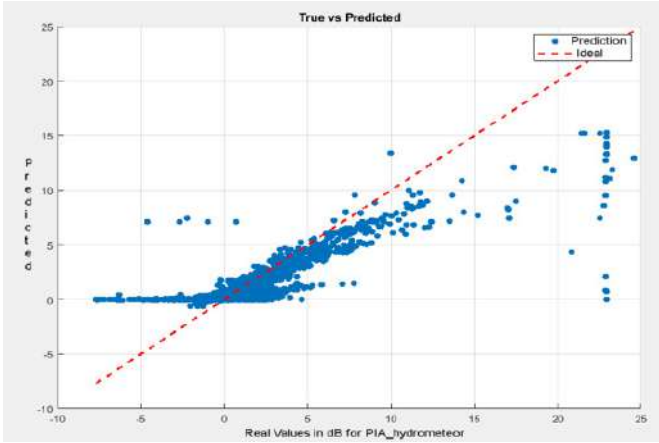


Figure 11: PIA_hydrometeor (or PIA(W)) vs. Predicted values in a Random Forest implementation with PIA matched scan decoupling.

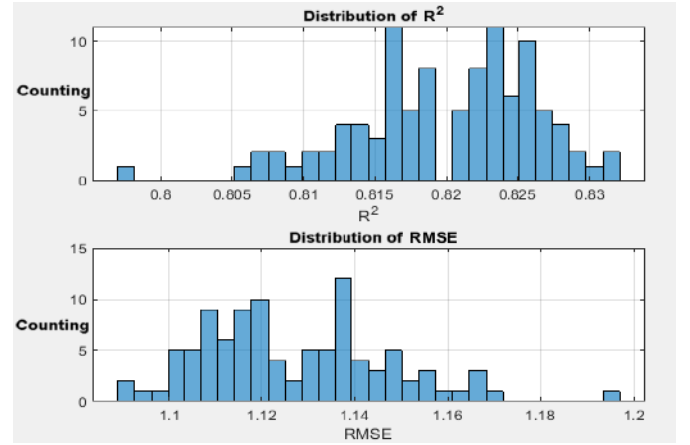


Figure 13: Histogram for R^2 and for RMSE related to the Random Forest model with PIA(Ku) and PIA(Ka) decoupled.

the relationship between two variables, filtered to include only the most relevant data points. Each dot represents an observation, providing a visual representation of how these variables interact. The x-axis and y-axis denote the specific variables being analyzed, and their range is determined by the cleaned dataset. The clustering and dispersion of the dots in the scatter plot can reveal patterns, trends, and potential outliers [39]. For example, a tight clustering of points along a diagonal line may indicate a strong correlation between the variables, whereas a more dispersed arrangement could suggest a weaker relationship. Outliers, represented as points that deviate significantly from the general pattern, can also provide valuable insights into anomalies or exceptions in the data [30]. Overall, the scatter plot in **Fig. 12** is a powerful visual tool that allows us to intuitively grasp the underlying relationships within the filtered dataset, facilitating better understanding and further analysis.

The interpretation of the metrics obtained from the Random Forest model provides clear evidence of its effectiveness. With an RMSE of 1.0890, we can say that the mean squared error is low, indicating that the model is highly accurate in most

predictions. The MAE, equal to 0.5776, is significantly lower than the RMSE, which indicates the absence of large outliers that could distort the forecasts. Finally, the R^2 value, which stands at 0.8320, confirms that the model explains 83.20% of the variability present in the data. This represents an excellent result for a random forest model, highlighting its ability to provide reliable and high-quality forecasts [1], [3], [12], [19], [39].

There are specific instances where the PIA (Path Integrated Attenuation) values may appear negative (see **Figs. 11-12**). Negative PIA Values: These anomalies typically arise from measurement or estimation errors, such as imperfections in the models or inaccuracies in calculating the received power. While a negative PIA might suggest an amplified signal, this is physically unlikely for atmospheric PIA and usually points to underlying issues. Accepted Values: Generally, PIA(W) values are positive and vary according to atmospheric conditions. When negative values are observed in the data, it is crucial to investigate potential causes, which may include:

- Calibration errors [3], [12], [19].
- Noise in the received data [12], [19], [39].

- Issues in the modeling of the atmospheric signal [15], [16], [24], [30].

Overall, ensuring the accuracy of PIA measurements involves thorough examination and rectification of these potential errors. By meticulously filtering the data to remove anomalous or incorrect measurements, we can significantly enhance the accuracy of our estimation method. This improvement is particularly evident when employing a gradient boosting approach and partitioning the dataset into training and testing subsets. By training the model on clean, reliable data and testing it on a separate, unbiased portion, this process enhances its overall robustness and predictive performance. With the filter application to avoid nonsensical data, we get:

- **Gradient Boosting - Training RMSE:** 0.98, R^2 : 0.85. [1], [3], [12], [19]

- **Gradient Boosting - Test RMSE:** 1.49, R^2 : 0.59. [12]

The results of the Random Forest model show a good fit to the training data, with an RMSE of 0.98 and an R^2 of 0.85, indicating that 85% of the variability in the training data is explained by the model. However, the test data showed a rise in mean error (RMSE of 1.49) and a drop in R^2 to 0.59, which suggests that the model might not be able to generalize to new data, which could mean that it is overfitting. To improve the generalization of the model and reduce overfitting, it is advisable to apply techniques such as regularization, cross-validation, and a thorough review of the features used.

These interventions can help obtain a more stable and accurate performance, even on new data. Due to time constraints, we instead chose to consider another technique, that of increasing the number of variables to better explain the variability of the data: the derived variables [12], [19], [39].

H. Derived variables.

Using derived variables can significantly improve model performance by creating new features that capture nonlinear relationships or interactions between existing variables. Here are some transformations and new derived variables you can consider for your model:

1) Relationships Between Existing Variables:

a) Ratios:

- pia_{Ku}/pia_{Ka} : Captures the ratio between Ku and Ka attenuations [19], [24].
- $iwc/(plw + piw)$: Shows the relationship between ice and water content.

2) Sums or Differences:

- $pia_{Ku} - pia_{Ka}$: Difference between attenuations.
- $plw + piw$: Total content of water and ice [15], [16].

3) Non-linear Transformations: Logarithms (avoiding negative or null values), [1], [3], [12], [19]

- $\log(abs(pia_{Ku}) + 1)$
- $\log(abs(iwc) + 1)$

4) Square Roots:

- $\sqrt{abs(pia_{hydro})}$
- $\sqrt{plw + piw}$

5) Interactions Between Variables:

a) Products:

- $pia_{Ku} * iwc$
- $pia_{Ka} * plw$

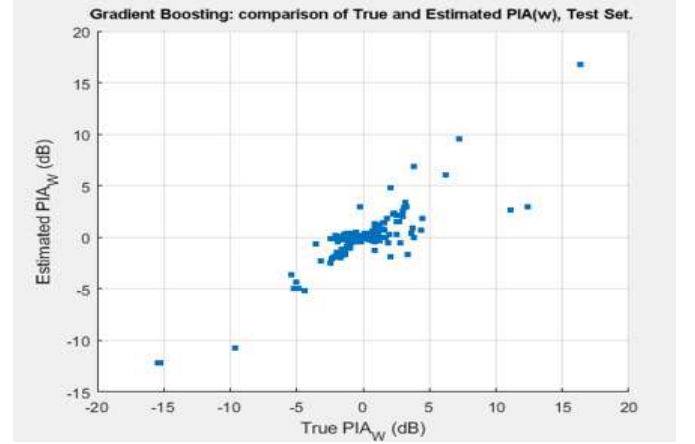


Figure 14: First execution of the Gradient Boosting on the dataset JFM (January, February, March), with PIA range filtering.

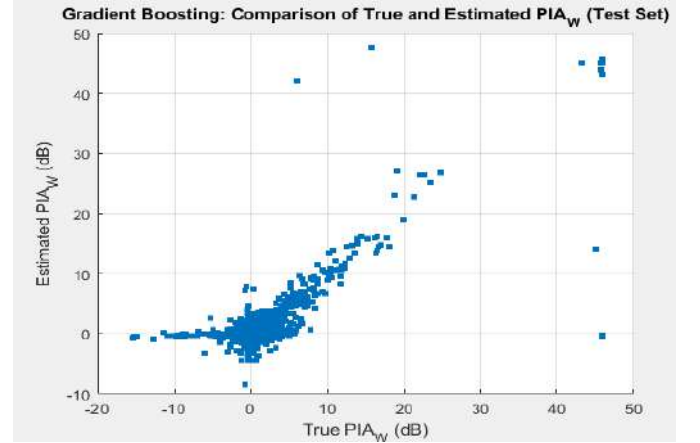


Figure 15: Gradient Boosting with filter application on dataset. The scatter plot titled "Gradient Boosting: Comparison of True and Estimated PIAW (Test Set)" effectively illustrates the relationship between the true and estimated values of PIAW using the gradient boosting model on the test set. The x-axis represents the true PIAW values (dB), ranging from -20 to 50 dB, while the y-axis represents the estimated PIAW values (dB), ranging from -10 to 50 dB. The data points show a positive correlation, indicating that the model performs well in estimating PIAW values. The concentration of points around the origin suggests that the model is particularly accurate for values close to zero. This visualization underscores the model's effectiveness in capturing the target variability and highlights the improvements achieved through the inclusion of derived variables.

These transformations can enhance the model's ability to capture complex interactions, improving predictive accuracy [1], [3], [12], [19], [39].

The implementation of the Gradient Boosting model with derived variables produced excellent results, showing a significant improvement in the model performance. With an RMSE of 0.56 and an R^2 of 0.88 on the training data, and an RMSE

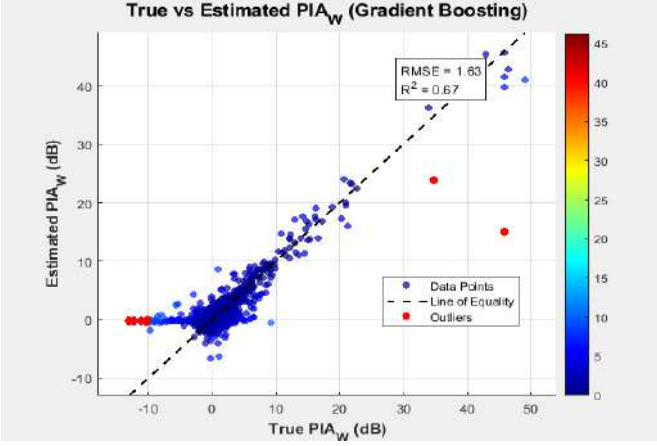


Figure 16: Figure 42 improved.

of 0.85 and an R^2 of 0.76 on the test data, it is clear that the addition of derived variables significantly increased the model's ability to capture the target variability [3], [12], [19]. These results are a clear indication of the power of derived variables in improving prediction accuracy on both training and test data. We saved the Gradient Boosting model with derived variables as `PIA_W_gradient_boosting_with_derived_vars.mat` to ensure the reproducibility of future analyses and predictions. This model will serve as a solid foundation for further refinements and iterations. By continuously evaluating and updating the model with new data, we can enhance its predictive capabilities and adapt to any changes in the underlying patterns of the data [39].

III. WRITE PIA(W) ESTIMATION LAWS.

A. Introduction

In this section, we aim to demonstrate how we can represent useful estimation laws from the analyzed data, improving their usability and understanding through the application of machine learning techniques. Using a gradient boosting model, we were able to obtain excellent results with the inclusion of derived variables. The results, with an RMSE of 0.56 and an R^2 of 0.88 on the training data and an RMSE of 0.85 and an R^2 of 0.76 on the test data, clearly indicate the importance of derived variables in improving predictive accuracy. The ability to save the model for future analysis ensures the reproducibility of the predictions, making this approach a solid basis for further refinements and iterations. By continuing to evaluate and update the model with new data, we can further improve its predictive capabilities and adapt to any changes in the underlying patterns of the data. This ongoing process not only enhances the model's performance but also fosters a deeper understanding of the factors influencing the predictions. Ultimately, this iterative approach can lead to more robust decision-making and better outcomes across various applications. In addition, it encourages collaboration among researchers and practitioners, promoting the sharing of insights and techniques that can drive innovation. As new challenges arise, the adaptability of the model becomes a crucial asset, ensuring that it remains

relevant and effective in a rapidly evolving landscape [12], [19], [39].

We began our new MATLAB implementation by choosing a directory containing the `.nc4` files, which we then loaded and analyzed in their entirety. Typically, the user indicates the directory where the coincidence files reside (we chose the JT dataset for January, February, and some passages of May 2016). In the JT dataset, we read the relevant variables, including latitude, longitude, `PIA_hydrometeor` (real `PIA_W`), `pia_ku`, `pia_ka`, `IWC` (ice water content), `PLW` (precipitation liquid water content), and `PIW` (precipitation ice water content) [15], [16], [19], [24]. We reformatted and normalized these variables to ensure consistency in subsequent calculations. We then filtered out invalid values and applied range filters for `PIA_hydro` to improve data accuracy.

Next, we used a cross-validation partition to split the data into training (80%) and testing (20%) sets. We trained a gradient boosting-based regression model on the training data. We configured the model using least squares boosting with 100 iterations and decision trees as the base learner, limiting each tree to a maximum of 10 splits [3], [12], [19]. We checked how well the model worked on both the training set and the testing set by finding error metrics like MAE (Mean Absolute Error), RMSE (Root Mean Squared Error), and R^2 .

We created scatter plots and bar charts that visualize the differences between the predicted and actual values of `PIA_W` to improve understanding of the results. We also included an equality line in the scatter plots and used colors to encode errors. We also generated a bar chart to compare the performance metrics between the training and testing sets. We saved the trained model and results to guarantee the reproducibility of future analyses.

Using gradient boosting model on the training data it is possible to notice drop in performance on the test data, indicating potential overfitting [39].

With these results:

- a) Gradient Boosting - Training MAE: 0.50, RMSE: 0.97, R^2 : 0.84
- b) Gradient Boosting - Test MAE: 0.62, RMSE: 1.66, R^2 : 0.64

there will be a potential overfitting, so the increase in test RMSE and the decrease in R^2 suggest the need for improvements, such as **hyperparameter tuning**, **feature selection**, **cross-validation** and **regularization** to enhance the model's generalization ability.

B. Improving the Gradient Boosting model

We can initially improve the MATLAB code efficiently by extracting relevant variables, handling missing data, and filtering values to train a gradient boosted regression model. The script splits the data into training and testing sets, normalizes the predictor variables, and trains the model using least squares boosting. It evaluates the model performance using metrics such as MAE, RMSE, and R^2 and visualizes the results using scatter plots and a bar chart [1], [3], [12], [19]. This method thoroughly examines the model's performance, aiding in the accurate calculation of `PIA_W` from satellite data by

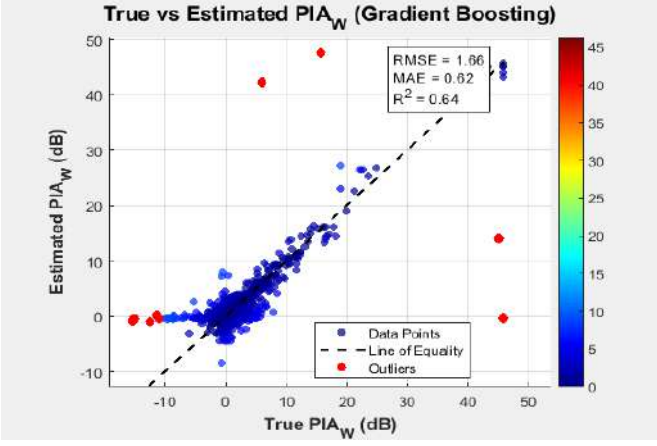


Figure 17: Adding MAE to the others parameters (RMSE, R^2).

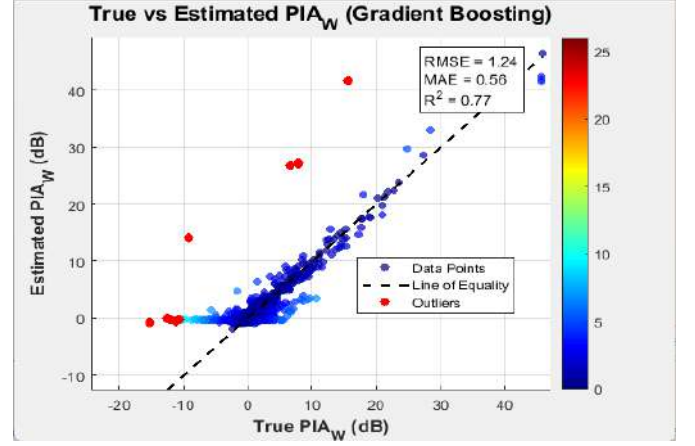


Figure 19: Using of the Bayesian Optimization for hyperparameter tuning improvement.

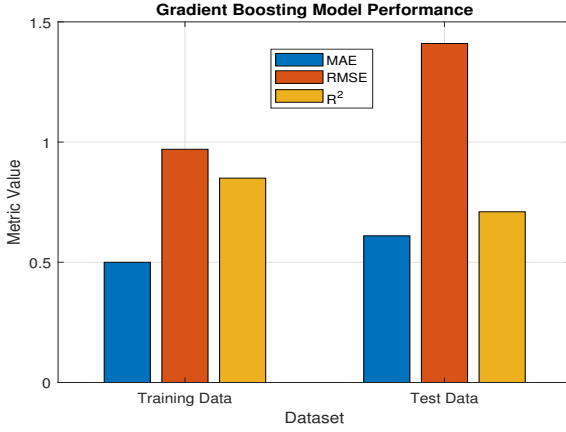


Figure 18: Gradient boosting model performance on training and test data.

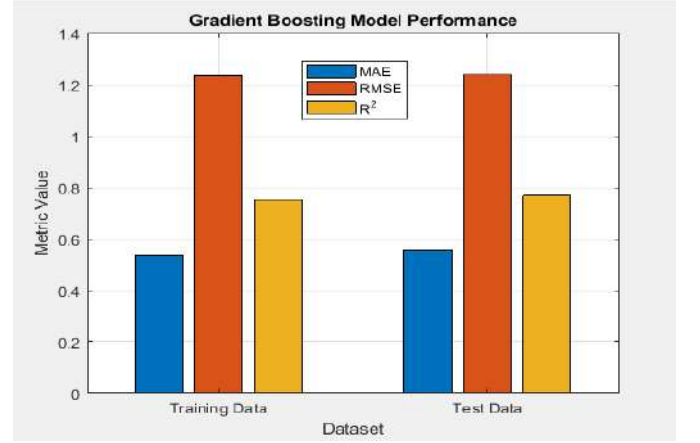


Figure 20: Balancing among the training and the test dataset, obtained after the improvement of the model with Bayesian Optimization.

providing clear details about the model's predictive accuracy and the relationships between the variables [12], [19], [39]. Ultimately, this thorough evaluation helps refine the model for better forecasting capabilities. By identifying strengths and weaknesses in the model's predictions, researchers can make informed adjustments to improve accuracy and reliability. This iterative process is crucial for enhancing the model's effectiveness in real-world applications, ensuring that it meets the needs of stakeholders relying on precise satellite data analysis. To enhance our model's performance, it is advisable to begin with hyperparameter tuning. MATLAB provides several methodologies for this purpose, with **Bayesian Optimization** being one of the most efficacious techniques.

Below is an implementation of **Bayesian Optimization** for hyperparameter tuning [19], [39]:

The Gradient Boosting model shows balanced performance between the training and test datasets, indicating good generalization to unseen data.

Training Metrics:

- MAE: 0.54
- RMSE: 1.24
- R^2 : 0.75

Test Metrics:

- MAE: 0.56
- RMSE: 1.24
- R^2 : 0.77

The close alignment of these metrics suggests effective hyperparameter tuning and a reduced risk of overfitting.

C. Removing outliers for improving the model.

To further improve the model, we decided to write MATLAB source code that could calculate training errors, determine a threshold to identify outliers, remove them from the training data, and then retrain the model. This allows us to evaluate the clean model on the test set. By eliminating the outliers, your gradient boosting model has shown a better performance on the test data. Here are the updated metrics (performance on the test data without outliers)

- MAE (Mean Absolute Error): 0.54
- RMSE (Root Mean Square Error): 1.33
- R^2 (Coefficient of Determination): 0.65

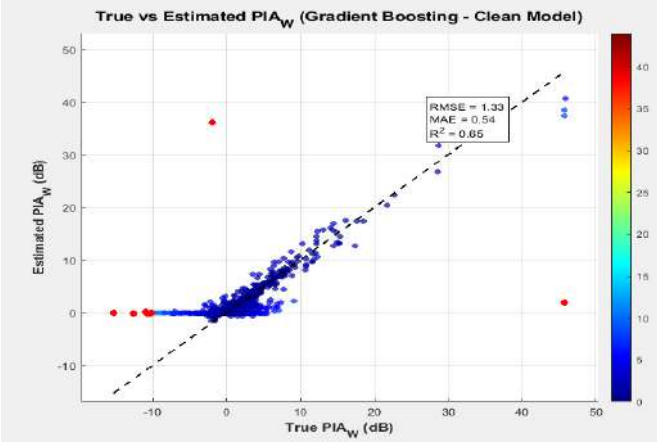


Figure 21: Scatter plotter after removing outliers.

D. Comparing more techniques.

Here is the comparison of performance metrics for each applied technique:

- **Gradient Boosting without outliers:**

MAE test: 0.57
RMSE test: 1.60
 R^2 test: 0.61

- **Advanced Cross-Validation:**

MAE test: 0.56
RMSE test: 1.34
 R^2 test: 0.71

- **Feature Engineering:**

MAE test: 0.57
RMSE test: 1.39
 R^2 test: 0.70

- **Advanced Regularization:**

MAE test: 0.57
RMSE test: 1.60
 R^2 test: 0.61

- **Ensemble Methods:**

MAE test: 0.57
RMSE test: 1.54
 R^2 test: 0.63

- **Advanced Cross-Validation:**

MAE test: 0.56
RMSE test: 1.34
 R^2 test: 0.71

The comparison shows that Advanced Cross-Validation performed best, with the lowest MAE (0.56) and RMSE (1.34) and the highest R^2 (0.71). This suggests that the model built with this technique is the most robust, with accurate predictions and low error dispersion. Feature engineering led to significant improvements over less advanced methods. Although it did not reach the results of Advanced Cross-Validation, it still obtained excellent values (MAE: 0.57, RMSE: 1.39, R^2 : 0.70), highlighting the importance of feature selection and transformation to improve model performance. Gradient boosting without outliers showed comparable performance to other standard methods, with an R^2 of 0.61 and higher values of MAE (0.57) and RMSE (1.60). Although it had acceptable

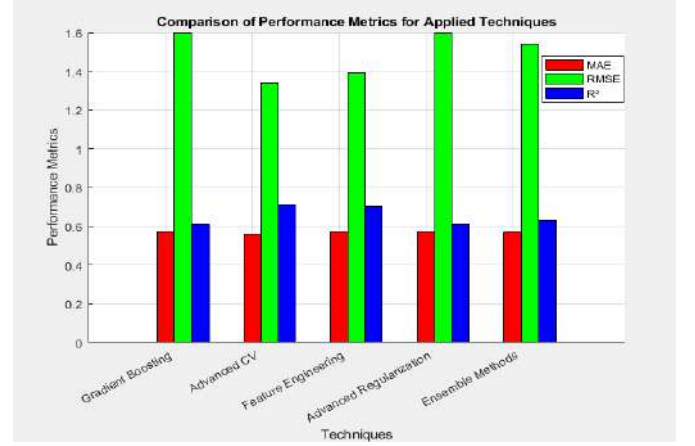


Figure 22: Comparison of performance metrics for Applied Techniques.

performance, it did not reach the levels of accuracy shown by the advanced techniques.

Advanced regularization and ensemble methods produced similar results, with a MAE of 0.57 and an RMSE of 1.60, but with lower R^2 values (0.61-0.63). This shows that they are effective in reducing errors but less effective in explaining data variability [12], [19], [39].

In summary, advanced cross-validation was confirmed as the best-performing technique, helping to significantly reduce prediction errors and improve model accuracy. Although it did not achieve the best results, feature engineering proved useful in increasing predictive capacity. Gradient boosting, advanced regularization, and ensemble methods are valid methods for finding solutions, but the first two are more effective in generalizing the results [1], [3].

E. Cross-Validation.

Cross-validation is a statistical technique to evaluate the generalization ability of a machine learning model, reducing the risk of overfitting or underfitting. [1], [12], [19], [39]. Among the most common methods, K-Fold Cross-validation divides the dataset into K parts (folds). The model is iteratively trained on $K-1$ folds and tested on the remaining one, repeating the process K times with a different fold as a test. Evaluation metrics (e.g., RMSE, MAE, R^2) are calculated at each iteration and averaged to obtain a reliable estimate of the model performance [3]. This approach makes the most of the available data, reduces the variance in the estimates, and provides a more realistic evaluation. The choice of K (often 5 or 10) balances accuracy and computational cost, making the technique particularly useful for models applied to moderately sized datasets.

The **Fig. 23** shows the importance of different features in a gradient boosting model [3], [12], [19]. The x-axis is labeled "Feature", and the y-axis is labeled "Importance". The features listed on the x-axis are "PIA-KU", "PIA-KA", "IWC", "PLW", and "PIW". The importance values on the y-axis range from 0 to 4.5×10^3 . The feature "PLW" has the highest importance,

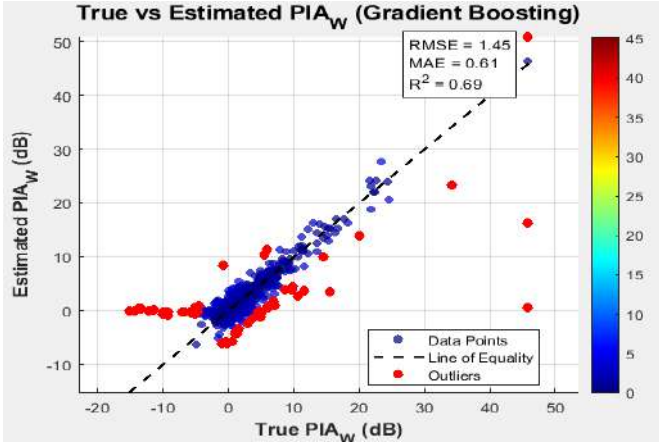


Figure 23: Example of output for Gradient Boosting with outliers. The threshold is calculated by adding 3 times the standard deviation to the mean, following the 3 sigma statistical principle. This approach assumes a normal distribution of errors: errors above this threshold are considered outliers.

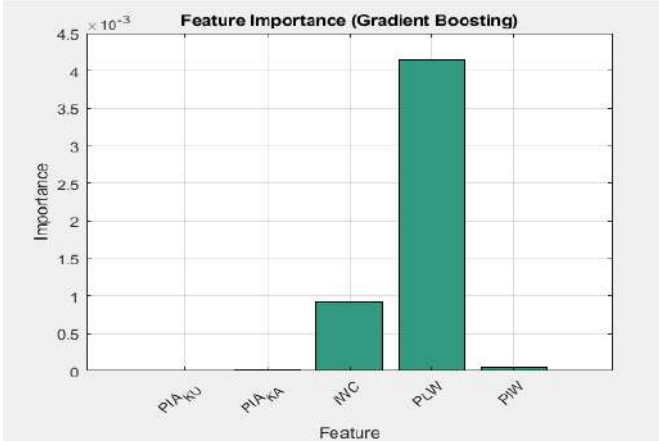


Figure 24: Feature importance histogram related to **Figure 21**.

followed by "IWC". The other features have significantly lower importance values.

A R^2 value of 0.69 indicates that the gradient boosting model explains 69% of the variability in the data. This is a favorable result, but there is still room for improvement [1]. Using cross-validation we do not get good performances, so we will try to improve the model using **Hyperparameters Optimization** (HO) [39].

F. Hyperparameters Optimization

In this paragraph I show how the **Hyperparameters optimization** is able to improve the system performance.

With other guess we may obtain little improvement like this: **Optimized Model** : RMSE: 1.12, MAE: 0.54, R^2 : 0.79.

The optimized model achieved remarkable results thanks to the optimized hyperparameters: 22 learning cycles (NumLearningCycles), a learning rate of 0.30873, a minimum leaf size of 5 (MinLeafSize), and a maximum number of splits of 13 (MaxNumSplits). These settings allowed the

Iter	Eval	Objective	Runtime	BestSoFar (observed)	BestSoFar (estim.)	NumLearningCycles	LearnRate	MinLeafSize	MaxNumSplits
1	Best	1.973	0.26656	1.973	15	0.0096034	1326	2	
2	Best	1.6433	5.4172	1.6433	1.6603	109	0.0035455	6	13859
3	Best	1.1504	1.1841	1.1504	1.1919	51	0.34266	2	13
4	Accept	2.0075	2.7913	1.1504	1.1505	425	0.0033181	8948	267
5	Accept	1.4599	4.4388	1.1504	1.1505	12	0.96485	1	2118
6	Accept	1.2517	7.238	1.1504	1.1506	186	0.16683	1	71
7	Best	1.1368	0.86721	1.1368	1.1367	82	0.12641	1	1
8	Accept	1.799	4.0676	1.1368	1.1369	416	0.0012637	3	1
9	Best	1.136	3.6785	1.136	1.136	381	0.45993	1	1
10	Best	1.1277	1.3241	1.1277	1.1276	129	0.24893	4	1
11	Accept	1.2564	0.20681	1.1277	1.1277	14	0.25689	2	1
12	Accept	1.1409	1.0185	1.1277	1.1277	97	0.54835	1	1
13	Best	1.1191	2.4664	1.1191	1.1188	189	0.23311	2	2
14	Accept	1.1827	4.2364	1.1191	1.1188	339	0.89438	8	2
15	Accept	1.1272	1.5984	1.1191	1.1193	99	0.31912	2	4
16	Accept	1.1266	3.9424	1.1191	1.1185	399	0.059002	1	1
17	Accept	1.126	3.5439	1.1191	1.1193	370	0.1354	1	1
18	Accept	1.1274	2.2769	1.1191	1.1196	233	0.09148	2	1
19	Accept	1.1335	1.9357	1.1191	1.1198	194	0.27187	1	1
20	Accept	1.1304	3.0499	1.1191	1.1204	311	0.39687	3	1
21	Accept	1.1284	1.9645	1.1191	1.1206	193	0.087492	1	1
22	Best	1.0847	2.7445	1.0847	1.0852	202	0.16246	4	3
23	Accept	1.1631	9.038	1.0847	1.1043	417	0.15849	8	9
24	Accept	1.0979	2.4124	1.0847	1.1	148	0.11307	3	4
25	Accept	1.1183	2.5328	1.0847	1.0853	186	0.13705	2	3
26	Accept	1.233	6.2464	1.0847	1.1039	255	0.35467	4	19
27	Accept	1.0945	1.3539	1.0847	1.0988	77	0.084558	1	6
28	Accept	1.1073	0.90689	1.0847	1.0985	77	0.07177	1	16
29	Best	1.0819	1.4941	1.0819	1.0888	78	0.086603	2	7
30	Best	1.0576	1.5976	1.0576	1.0657	73	0.086413	4	10

Table IV: Data Table describing the Machine Learning Model Iteration Analysis.

Optimization completed.

MaxObjectiveEvaluations of 30 reached.

Total function evaluations: 30

Total elapsed time: 89.7098 seconds

Total objective function evaluation time: 85.8398

Best observed feasible point:

NumLearningCycles LearnRate MinLeafSize MaxNumSplits

73 0.086413 4 10

Observed objective function value = 1.0576

Estimated objective function value = 1.0657

Function evaluation time = 1.5976

Best estimated feasible point (according to models):

NumLearningCycles LearnRate MinLeafSize MaxNumSplits

73 0.086413 4 10

Estimated objective function value = 1.0657

Estimated function evaluation time = 1.5922

Optimized Hyperparameters:

NumLearningCycles LearnRate MinLeafSize MaxNumSplits

73 0.086413 4 10

Optimized Model - RMSE: 1.28, MAE: 0.56, R^2 : 0.71

Table V: Output of the ML Model Iteration Analysis.

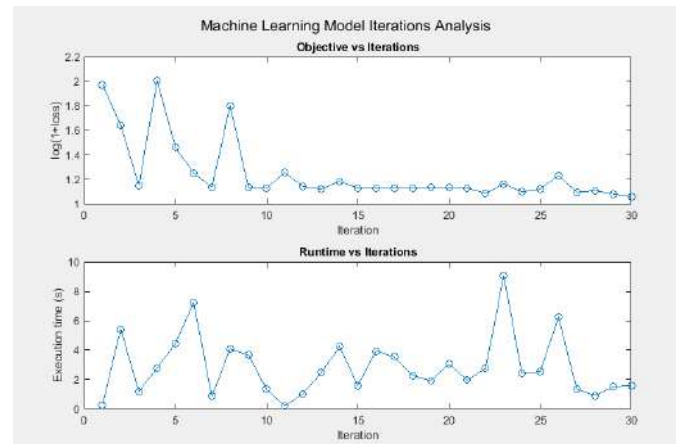


Figure 25: Graphical description of the Data Table in **Table 4**.

```

Iter, Eval, Objective, Runtime, BestSoFar (observed), BestSoFar (estim.),
NumLearningCycles, LearnRate, MinLeafSize, MaxNumSplits
1, Best, 1.9978, 1.028, 1.9978, 1.9978, 97, 0.0017272, 5498, 6194
2, Best, 1.3171, 21.168, 1.3171, 1.3509, 90, 0.074298, 1, 5799
3, Accept, 1.9069, 1.3469, 1.3171, 1.3561, 136, 0.0013738, 5, 1
4, Best, 1.1723, 4.6333, 1.1723, 1.1723, 465, 0.9103, 8, 1
5, Accept, 1.9904, 0.15208, 1.1723, 1.1724, 13, 0.38514, 9772, 12
6, Accept, 1.3734, 7.3919, 1.1723, 1.1724, 19, 0.99702, 1, 3331
7, Best, 1.0846, 1.2518, 1.0846, 1.0847, 37, 0.26381, 7, 47
8, Accept, 1.2513, 2.3831, 1.0846, 1.0848, 22, 0.22506, 19, 7380
9, Accept, 1.122, 0.85143, 1.0846, 1.085, 17, 0.26827, 4, 359
10, Accept, 1.1578, 0.5765, 1.0846, 1.1243, 58, 0.20943, 5, 1
11, Best, 1.0829, 0.66958, 1.0829, 1.0897, 42, 0.37706, 6, 4
12, Accept, 1.0863, 0.42117, 1.0829, 1.0844, 25, 0.40987, 6, 4
13, Accept, 1.3352, 32.802, 1.0829, 1.0849, 155, 0.40746, 7, 18224
14, Accept, 1.129, 4.027, 1.0829, 1.0846, 224, 0.37599, 2, 9
15, Accept, 1.1794, 7.6666, 1.0829, 1.0824, 419, 0.37791, 15, 8
16, Best, 1.0788, 0.27923, 1.0788, 1.0783, 12, 0.37443, 4, 14
17, Accept, 1.11, 0.78363, 1.0788, 1.0783, 20, 0.096145, 6, 325
18, Best, 1.0772, 0.31226, 1.0772, 1.0771, 12, 0.24569, 5, 25
19, Accept, 1.3874, 1.7366, 1.0772, 1.0771, 45, 0.024492, 19, 19020
20, Best, 1.0713, 0.48642, 1.0713, 1.073, 22, 0.30873, 5, 13
21, Accept, 1.088, 1.2149, 1.0713, 1.076, 55, 0.34066, 5, 14
22, Accept, 1.0866, 0.33817, 1.0713, 1.0767, 13, 0.21564, 7, 28
23, Accept, 1.0991, 0.23699, 1.0713, 1.0766, 12, 0.55897, 4, 5
24, Accept, 1.0847, 0.26202, 1.0713, 1.0764, 12, 0.26391, 5, 11
25, Accept, 1.0852, 0.42607, 1.0713, 1.0765, 14, 0.41844, 8, 26
26, Accept, 1.0751, 0.51572, 1.0713, 1.0756, 22, 0.29426, 5, 18
27, Accept, 1.076, 0.55014, 1.0713, 1.0752, 23, 0.33982, 4, 16
28, Accept, 1.0882, 0.3048, 1.0713, 1.0756, 13, 0.35761, 6, 14
29, Accept, 1.0728, 0.5778, 1.0713, 1.0748, 20, 0.1885, 4, 45
30, Accept, 1.1463, 0.27102, 1.0713, 1.0748, 10, 0.13865, 3, 33

```

Table VI: Data Table with improvement.

```

Optimization completed.
MaxObjectiveEvaluations of 30 reached.
Total function evaluations: 30
Total elapsed time: 99.7166 seconds
Total objective function evaluation time: 94.6654
Best observed feasible point:
NumLearningCycles LearnRate MinLeafSize MaxNumSplits
22 0.30873 5 13
Observed objective function value = 1.0713
Estimated objective function value = 1.076
Function evaluation time = 0.48642
Best estimated feasible point (according to models):
NumLearningCycles LearnRate MinLeafSize MaxNumSplits
22 0.29426 5 18
Estimated objective function value = 1.0748
Estimated function evaluation time = 0.52783
Optimized Hyperparameters:
NumLearningCycles LearnRate MinLeafSize MaxNumSplits
22 0.30873 5 13
Optimized Model - RMSE: 1.12, MAE: 0.54, R^2: 0.79
Optimized model saved for future prediction.

```

Table VII: Improvement related to Table 6.

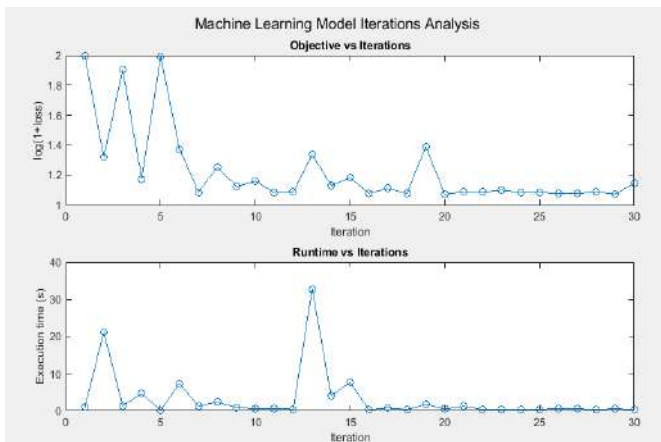


Figure 26: Graphical description of the Data Table in Table 6.

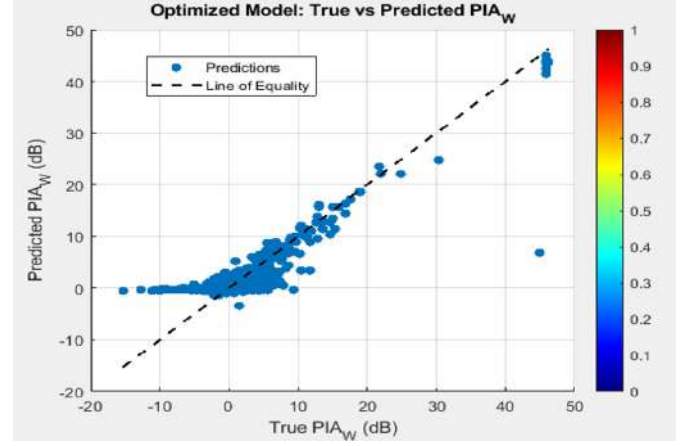


Figure 27: Scatter plotter of Predicted PIA(W) vs. True PIA(W) in the optimized model.

model to achieve higher predictive accuracy with an RMSE of 1.12 and a MAE of 0.54, indicating very low mean squared and absolute errors [1], [3], [12], [19].

Additionally, the model now explains 79% of the variability in the target ($R^2 = 0.79$), a significant improvement over the initial value of $R^2 = 0$ in the initial cases. The optimization procedure was also time-efficient, requiring approximately 100 seconds and a total of 30 evaluations of the objective function. This efficiency not only demonstrates the robustness of the model but also highlights its potential for practical applications in real-world scenarios. Furthermore, the enhancements made during the optimization process pave the way for further refinements and adaptations to suit different datasets or predictive challenges [39].

G. Improvements on new datasets.

The optimized model should be used to make predictions based on new data, ensuring that it maintains similar performance to that obtained during the optimization phase [3], [12], [19], [39]. Next, it is important to compare the model predictions with real observed or measured data in order to confirm the validity and accuracy of the model. Lastly, graphs like scatter plots can be made to show the differences between the actual values (Y) and the expected values (Y_pred), which makes it easier to understand and interpret the results.

The analysis results indicate that the optimized model used various hyperparameters to balance complexity and generalization. We set NumLearningCycles to 198 to prevent overfitting, and we promoted stable learning with a moderately low learning rate (LearnRate of 0.022537). With a minimum leaf size of 1, the model could handle complex data, and a maximum number of splits in the tree (MaxNumSplits of 24) made it easier to use in other situations.

The optimized model's performance can be seen by its good RMSE (1.23), MAE (0.56), and R^2 (0.76) values, which show that it is accurate and easy to understand. Furthermore, the optimization process took 345 seconds, with 341 seconds dedicated to evaluating the objective function [12], [19], [39].

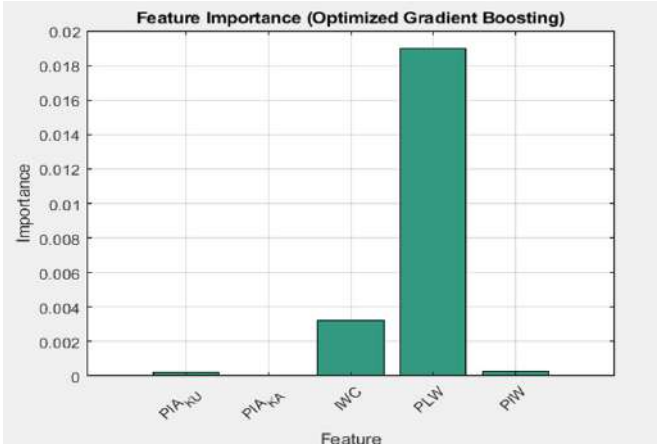


Figure 28: Feature importance for the optimized gradient boosting.

```

Iter, Eval result, Objective: log(1+loss), Objective
runtime, BestSoFar (observed), BestSoFar
(estim.), NumLearningCycles, LearnRate, MinLeafSize, MaxNumSplits
1, Best, 0.42367, 2.6573, 0.42367, 0.42367, 209, 0.0046767, 1, 2
2, Accept, 0.6882, 0.40001, 0.42367, 0.43748, 34, 0.37627, 3163, 12
3, Accept, 0.4658, 16.414, 0.42367, 0.4237, 475, 0.0015018, 31, 3917
4, Accept, 0.57258, 0.74707, 0.42367, 0.4237, 18, 0.011722, 5, 360
5, Accept, 0.50585, 9.3384, 0.42367, 0.4237, 481, 0.0086835, 517, 10
6, Best, 0.31689, 9.1209, 0.31689, 0.31693, 494, 0.52367, 1, 8
7, Accept, 0.36653, 9.615, 0.31689, 0.35747, 497, 0.83703, 1, 10
8, Accept, 0.39174, 85.382, 0.31689, 0.35192, 500, 0.010824, 1, 6951
9, Accept, 0.38386, 17.954, 0.31689, 0.36709, 499, 0.90618, 1, 48
10, Best, 0.2567, 6.5146, 0.2567, 0.25673, 497, 0.050798, 1, 3
11, Best, 0.25324, 6.2799, 0.25324, 0.25326, 494, 0.12304, 4, 2
12, Accept, 0.26566, 4.8172, 0.25324, 0.25566, 481, 0.11999, 1, 1
13, Accept, 0.25492, 4.3152, 0.25324, 0.25439, 336, 0.078262, 2, 2
14, Accept, 0.2669, 5.0457, 0.25324, 0.2533, 495, 0.06423, 3, 1
15, Accept, 0.29181, 15.069, 0.25324, 0.25789, 489, 0.096158, 2, 39
16, Accept, 0.26619, 4.2677, 0.25324, 0.25337, 435, 0.10994, 2, 1
17, Accept, 0.26762, 7.8929, 0.25324, 0.25879, 495, 0.11336, 1, 6
18, Accept, 0.25906, 4.7895, 0.25324, 0.25784, 376, 0.090325, 2, 2
19, Best, 0.25199, 6.1499, 0.25199, 0.25643, 495, 0.077181, 1, 2
20, Accept, 0.25611, 6.0954, 0.25199, 0.25594, 483, 0.088181, 3, 2
21, Accept, 0.26331, 7.1979, 0.25199, 0.25692, 463, 0.073112, 2, 4
22, Accept, 0.26262, 4.4929, 0.25199, 0.25736, 288, 0.08162, 1, 5
23, Accept, 0.25376, 6.7996, 0.25199, 0.25566, 434, 0.070116, 1, 4
24, Accept, 0.26787, 3.504, 0.25199, 0.25647, 349, 0.052036, 1, 1
25, Accept, 0.71466, 0.13045, 0.25199, 0.25625, 10, 0.0010683,
9782, 12950
26, Accept, 0.71475, 3.1267, 0.25199, 0.25625, 430, 0.86238,
8445, 21329
27, Accept, 0.71466, 0.12612, 0.25199, 0.2563, 10, 0.0010621, 8301, 1
28, Accept, 0.40361, 5.7291, 0.25199, 0.25644, 10, 0.73068, 1, 21470
29, Accept, 0.27597, 0.17943, 0.25199, 0.25645, 10, 0.98278, 3, 2
30, Accept, 0.27852, 0.27501, 0.25199, 0.25648, 23, 0.41044, 1, 1

```

Table VIII: DataTable from the optimization process.

A slightly different set of hyperparameters suggests that the model is not overly sensitive to small changes, which makes the performance even more reliable.

This indicates that the model can maintain its effectiveness even when faced with minor changes in the data or parameter settings.

As a result, the findings not only highlight the potential for reliable predictions but also pave the way for future research into more adaptive modeling techniques [1], [3].

The improved model has shown better performance by adding more datasets and finding the best balance between accuracy and complexity by selecting the right hyperparameters. With a substantial number of learning cycles (NumLearningCycles=495), the model is capable

Optimization completed.

MaxObjectiveEvaluations of 30 reached.

Total function evaluations: 30

Total elapsed time: 258.521 seconds

Total objective function evaluation time: 254.427

Best observed feasible point:

NumLearningCycles LearnRate MinLeafSize MaxNumSplits

495 0.077181 1 2

Observed objective function value = 0.25199

Estimated objective function value = 0.25648

Function evaluation time = 6.1499

Best estimated feasible point (according to models):

NumLearningCycles LearnRate MinLeafSize MaxNumSplits

495 0.077181 1 2

Estimated objective function value = 0.25648

Estimated function evaluation time = 6.0179

Optimized Hyperparameters:

NumLearningCycles LearnRate MinLeafSize MaxNumSplits

495 0.077181 1 2

Optimized Model - RMSE: 0.54, MAE: 0.23, R²: 0.65

Optimized model saved for future prediction.

Prediction completed for new data.

Table IX: Results of the optimization process.

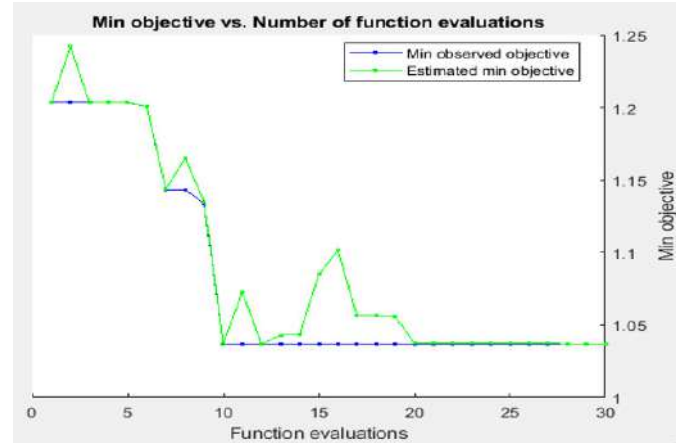


Figure 29: Minimum objective vs. Number of function evaluations.

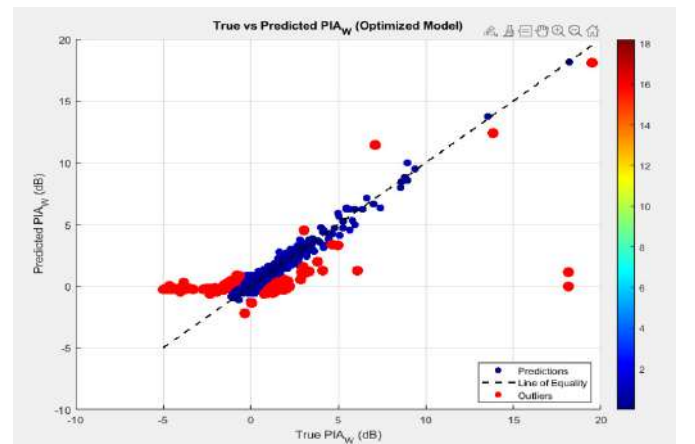


Figure 30: Scatter plot of Predicted PIA(W) vs. True PIA(W) after the optimization.

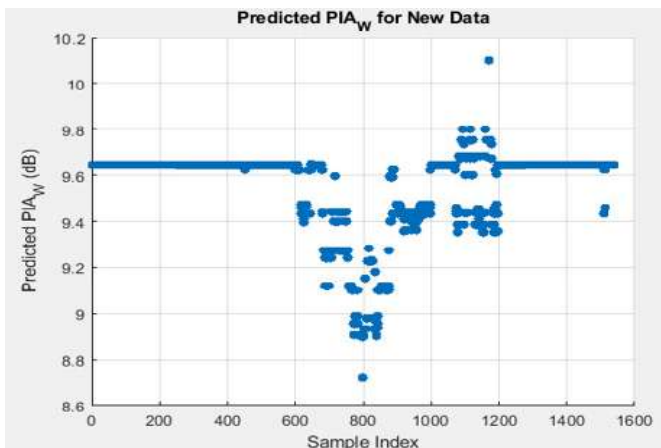


Figure 31: Behaviour with new dataset. Predicted PIA(W) for new data vs. Sample Index.

of capturing intricate details, whereas a low learning rate ($\text{LearnRate}=0.077181$) ensures stable convergence. The maximum granularity in leaf nodes ($\text{MinLeafSize}=1$) and a moderate tree depth ($\text{MaxNumSplits}=2$) effectively represent the data's complexity while preventing overfitting. Performance metrics such as RMSE (0.54), MAE (0.23), and R (0.65) confirm the model's efficacy. In 258 seconds, the optimization process identified a stable configuration that produces accurate predictions without errors on new data. In the end, the improved model finds the best balance between being complicated and being accurate, showing that it can be used in other situations and work well ($R^2 = 0.65$) [3], [12], [19], [39].

An analysis of the predictions reveals several key observations regarding the distribution and concentration of values. The predictions show three main bands: a stable band around $\text{PIAW} \approx 9.6$ dB, which is made up of most of the samples; a set of much lower predictions at about $\text{PIAW} \approx 8.8$ dB; and a few slightly higher values around $\text{PIAW} \approx 10.2$ dB. The concentration of points suggests that most predictions fall within narrow ranges, indicating that the data shares similar characteristics across many samples. However, there exists an intermediate range, near a sample index of approximately 800, where predictions display greater variability. This uneven spread may be due to areas of the dataset with features not seen during training or complicated patterns that the model can't pick up. Also, some predictions are very different from the main bands. This could be due to data outliers or model uncertainties in areas not covered by the training dataset. In general, the stability of predictions across many bands indicates that the model generalizes well for data with common features. Nonetheless, the variability in specific areas of the dataset suggests further refinement is necessary. These observations provide valuable insights for enhancing the model and optimizing its predictive capabilities. By addressing these inconsistencies, researchers can improve the model's performance and ensure it captures a broader range of scenarios. Additionally, incorporating more diverse training data may help mitigate the impact of outliers and increase the overall robustness of the predictions [1], [3],

[12], [19], [39].

IV. CONCLUSIONS.

To better predict and understand how the atmosphere works, this study shows how important it is to use integrative methods that combine old-fashioned physical models with cutting-edge machine learning methods [1], [3], [12], [19], [40]. The current results show big progress in PIA prediction, but more work could be done to include climate variables that haven't been studied as much, like the atmospheric instability index, how precipitation is distributed in space and time, or how aerosols and hydrometeors interact with each other. These aspects could improve the accuracy and robustness of the models, providing a more complete understanding of atmospheric dynamics. Adding data from climate-diverse regions and looking at longer periods of time could also help find results that can be used in a lot of different situations, from operational meteorology to satellite communications.

The validation method uses a number of different methods, such as splitting the dataset into smaller parts, using performance metrics, checking the results against real-world data, analyzing errors and figuring out how much uncertainty there is, and scaling based on geography and climate. A k-fold cross-validation strategy makes sure that the model can be used in different situations by splitting the dataset into training and testing subsets several times. This reduces the chance of bias in the evaluation of the model. Metrics like Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and Coefficient of Determination (R^2) are used to measure how accurate predictions are. This shows how well the model can reproduce observed PIA values. Random Forest, Gradient Boosting, and Support Vector Machines (SVM) are some of the machine learning models that are systematically compared to see which one is best at capturing complex, nonlinear relationships between variables. The predictions are checked against observations from satellite datasets like the GPM Dual Precipitation Radar (2A-DPR) and CloudSat 2B-GEOPROF products. Adding more parameters like rainfall velocity, ice water content, and liquid water content makes the models more accurate. We look at residual error distributions to see if there are any systematic biases or overfitting. We then use advanced methods like Monte Carlo simulations and bootstrap resampling to figure out what the uncertainties are and make the model more stable. Validation extends to different climate zones, such as tropical, temperate, and polar regions, ensuring the scalability and reliability of the model under different atmospheric conditions. This phase includes targeted assessments of frequency-specific attenuation behaviors, particularly in the Ku, Ka, and W bands. By using these strict validation methods, this study creates a framework for prediction that is not only very accurate but also flexible enough to adapt to the natural changes that happen in the atmosphere. The results help advance the field of satellite-based attenuation modeling, laying the foundation for further improvements in predictive methodologies.

Integrating data from various sources, such as ground-based weather radars, multi-band satellite sensors, and advanced

weather simulation models (like WRF or ECMWF), offers a promising approach. This integration lets us look into more than just signal loss. It also lets us look into how clouds move up and down and how hydrometeors affect various frequency bands, like Ku, Ka, and W. Deep learning techniques, such as **convolutional neural networks** (CNNs) or **transformer-based models** (TMMs), can help find and model complicated, nonlinear patterns in atmospheric data.

In the future, researchers might look into how to use derived variables like **cloud cover fraction** (CFC), cloud optical density, and total precipitable water (TPW) to find out how effects build up along certain paths. Additionally, using **droplet size distribution** (DSD) can improve the understanding of precipitation properties and their interaction with various frequencies. Targeted experimental campaigns in regions with extreme weather conditions, like heavy rain in the tropics or snowstorms in polar areas, can provide high-resolution data for model training. Comparing arid and humid regions can highlight significant differences in attenuation and help develop regionally adaptive models.

It is possible to get better generalizations with hybrid predictive models that combine atmospheric physics (for example, radiative transport equations) with machine learning methods. These models can use **Monte Carlo** simulations or stochastic techniques to quantify uncertainties and optimize PIA predictions.

To improve the ability of atmospheric attenuation models to predict and adapt, researchers should look into new technologies like artificial intelligence algorithms like **Generative Adversarial Networks** (GANs) and use them to create fake atmospheric scenarios for training. They should also use model interpretability techniques like **SHAP** (SHapley Additive exPlanations) or **LIME** (Local Interpretable Model-agnostic Explanations) to better understand the role of each predictive variable. This has significant implications for both scientific research and technological development.

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Technical Articles

Issue: Several sections still retain a personal tone ("I can", "I show", "I recommend", etc.). This is suggested from one of the revisor but the recommendation is for IEEE publication to change the technical style and tone converting to a formal passive or impersonal tone, which is standard in IEEE writing.

- Following standard IEEE publication guidelines, the manuscript has been revised to adopt a more formal and impersonal tone. Personal expressions (e.g., "I show", "I recommend") have been replaced with passive or objective constructions to align with scientific writing conventions.
- For this reason the file has been revised to adopt an impersonal, IEEE-compliant style, with all first-person expressions changed to passive or objective form.

A.3 First Technical Article - personal style

Title: Fundamentals for Global Atmospheric Attenuation Analysis: Instrumentation and Datasets.

Fundamentals for Global Atmospheric Attenuation Analysis: Instrumentation and Datasets.

Paolo Cicolin, IEEE Student Member.

Abstract—This work introduces the fundamental concepts for the analysis of global atmospheric attenuation, focusing on the Ku, Ka, and W bands, which are essential for space communications. I describe major missions like CloudSat, providing a detailed examination of their instrumentation, reanalysis datasets, and physical measurements. I also highlight the contribution of recent missions in enhancing the data landscape for atmospheric attenuation research. These advancements allow for a more comprehensive understanding of how atmospheric conditions impact signal propagation, ultimately leading to improved performance of communication systems. By integrating findings from these missions, researchers can develop more accurate models to predict attenuation effects in various environmental scenarios.

Index Terms – atmospheric attenuation, CloudSat, Ku, Ka, W bands, reanalysis datasets, satellite missions, physical measurements.

I. INTRODUCTION.

Understanding atmospheric attenuation is fundamental to optimizing communications in the Ku, Ka, and W bands, which are particularly relevant for space applications. This section presents the fundamental concepts, beginning with a description of major missions like CloudSat and then delving into their instrumentation, reanalysis datasets, and physical measurements. Furthermore, I examine how recent missions have contributed innovative data for atmospheric attenuation research. Research in this area is essential for enhancing the reliability and efficiency of satellite communication systems, as it helps to clarify how various atmospheric factors, such as precipitation and humidity, influence signal degradation. By integrating data from these missions, it is possible to develop more accurate models that predict performance in different weather conditions, ultimately leading to improved communication links in challenging environments.

The researcher's mission to process and elaborate satellite remote sensing information is crucial for the advancement of science and the understanding of natural phenomena. In particular, the primary objective is to analyze the data collected during **NASA missions** [17], [10] and exploit **Open Data** [11], [12], [13] to study climate change. Through the integration of advanced data analysis techniques, the researcher aims to improve the accuracy and reliability of climate forecasts, thus contributing to a more conscious and sustainable management of my planet's natural resources. This research not only provides new scientific knowledge but also aids in environmental policy planning and raises public awareness about the significance of climate protection.

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II. DETAILS ON CLOUDSAT MISSION

Since 2006, CloudSat has been operating as the first millimeter-wavelength cloud radar, one thousand times more sensitive than existing ground weather radar. Unlike ground-based weather radar that use centimeter wavelengths, CloudSat can detect raindrop-sized particles, ice particles that in large part may constitute cloud masses. CloudSat was co-launched with another satellite named CALIPSO (Cloud-Aerosol Lidar and Infrared Pathfinder Satellite Observations) aboard a Delta II rocket engine. In a series of maneuvers, CloudSat, CALIPSO reached the A-Train constellation already in orbit composed by Aqua, PARASOL, and Aura. The entire constellation flies in a nearly circular orbit with an equatorial altitude of approximately 705 km, evaluated by Earth's surface. The orbit is sun synchronous. From a perspective view, CloudSat maintains a close formation (cluster) with Aqua and a particularly close formation with CALIPSO, providing near-simultaneous and co-located observations.

A. The CloudSat Instrumentation

The principal instrument onboard is the CPR (Cloud Profiling Radar). This nadir-pointing radar measures the power of the microwaves back scattered by Clouds as a function of the distance from the radar (range measure). Using this data, the satellite can measure the vertical cloud structure and ice/water content (IWC) [14]. More detailed and precise information on CloudSat, are available here: [15]. In [15] also The author has more information and a good list of hyperlinks about Open Search Datasets.

B. Data Reanalysis: products available from the CloudSat Mission

CloudSat data is available in several levels, each providing different types of information:

- **Level 0:** this is the raw data collected directly from the satellite without any calibration or other correction that data scientists can apply.
- **Level 1A-AUX:** this level contains auxiliary data mapped to CloudSat profiles. [16]
- **Level 1B-CPR:** this level contains calibrated and geolocated Cloud Profiling Radar (CPR) data.
- **Level 2B:** this level contains higher-level data products derived from the Level 1B data.

The **Level 2B** products include very useful information:

- **2B-GEOPROF:** Cloud Geometrical Profile.
- **2B-CLDCLASS:** Cloud Classification.
- **2B-CWC-RO:** Radar-Only Combined Water Content.

- **2B-TAU:** Cloud Optical Depth. Remember that in Theoretical way, (τ) indicate the cloud optical thickness.
- **2B-CWC-RVOD:** Radar plus Visible Optical Depth Combined Water Content.
- **2B-FLXHR:** Fluxes and Heating Rates.
- **2B-GEOPROF-Lidar:** Cloud Geometrical Profile from combined CPR and CALIPSO Lidar.

A good guide on Climate Data can be found in [16].

C. Focusing on Physical Measurements for the CloudSat Mission

For those interested in variables such as CloudSat Cloud-Mask, Radar Reflectivity, and Cloud Classification matchups, it is recommended to consult [9].

III. MORE RECENT MISSIONS.

The EarthCARE (Earth Clouds, Aerosols, and Radiation Explorer) mission of JAXA (Japan Aerospace Exploration Agency) is very similar to that of CloudSat. ESA (European Space Agency) and JAXA successfully launched EarthCARE, a joint mission. The mission took place on May 28, 2024, aboard a Falcon 9 rocket from Vandenberg Space Force Base in California, USA. It uses similar instruments, such as the Cloud Profiling Radar (CPR), to obtain vertical profiles of clouds and measure the properties of cloud particles and aerosols. ESA's European Space Operations Centre (ESOC) in Darmstadt, Germany, is currently commissioning and controlling the satellite. Mission Data Scientists have activated the mission's instruments to initiate data collection. [14]

IV. SATELLITES AND THEIR INSTRUMENTATION.

International space agencies have launched several satellite missions over the past decades to monitor and study various aspects of the Earth's atmosphere, clouds, aerosols, and greenhouse gases. These missions have provided crucial data for understanding climate and atmospheric processes. The following list highlights some of the most significant missions: [see **Table I**]

- **Aqua:** Launched by NASA in 2002, Aqua carries instruments like the Moderate Resolution Imaging Spectroradiometer (MODIS), Advanced Microwave Scanning Radiometer (AMSR-E), and Atmospheric Infrared Sounder (AIRS).
- **Aura:** Launched by NASA in 2004, Aura carries the Ozone Monitoring Instrument (OMI), Microwave Limb Sounder (MLS), and Tropospheric Emission Spectrometer (TES).
- **CloudSat:** Launched by NASA in 2006, CloudSat carries the Cloud Profiling Radar (CPR) to measure cloud properties and vertical structure.
- **CALIPSO:** Launched by NASA and CNES in 2006, CALIPSO carries the Cloud-Aerosol Lidar with Orthogonal Polarization (CALIOP) to provide vertical profiles of clouds and aerosols.
- **PARASOL:** Launched by CNES in 2004, PARASOL carried the Polarization and Directionality of the Earth's

Reflectances (POLDER) instrument to study cloud and aerosol properties.

- **OCO-2:** Launched by NASA in 2014, OCO-2 carries the Orbiting Carbon Observatory instrument to measure atmospheric carbon dioxide levels.
- **GCOM-W1 (SHIZUKU):** Launched by JAXA in 2012, GCOM-W1 carries the Advanced Microwave Scanning Radiometer (AMSR2) to measure precipitation and water vapor.

Satellite	Orbit Type	Inclination (°)	Perigee Alt.(km)	Period(min)
CloudSat	Sun-synch.	98.23	709	98.83
CALIPSO	Sun-synch.	98.2176	701	98.50
Aqua	Sun-synch.	98.2	705	98.8
Aura	Sun-synch.	98.2	705	98.8
GCOM-W1 (SHIZUKU)	Sun-synch.	98.2	705	98.8
OCO-2	Sun-synch.	98.2	705	98.8

TABLE I: Orbital Parameters of A-Train Satellites.

V. THEORETICAL INSIGHTS AND CONTEXTUALIZATION.

A. Properties of atmospheric attenuation

Atmospheric attenuation is the phenomenon of reduction in the intensity of an electromagnetic signal as it passes through the Earth's atmosphere. Two main processes cause this effect: absorption and scattering. Absorption occurs when gas molecules like oxygen and water vapor absorb energy from the signal, while scattering occurs when aerosols, raindrops, and snowflakes interact with it.

The amount of attenuation depends on the wavelength of the signal, the atmospheric density, and the weather conditions.

These factors collectively influence how well an electromagnetic signal can travel through the atmosphere, impacting its clarity and strength upon reaching the receiver. Consequently, understanding atmospheric attenuation is crucial for optimizing communication and navigation systems that rely on these signals.

Often used to mathematically describe atmospheric attenuation, the **Beer-Lambert** law typically takes the following form:

$$P(d) = P_0 \cdot e^{-\alpha \cdot d} \quad (1)$$

where:

- $P(d)$ is the power of the electromagnetic signal after it has traveled a distance d .
- $P(0)$ is the initial power of the electromagnetic signal before it crosses the attenuating medium.
- The attenuation coefficient, α (alpha), is dependent on the frequency of the signal and the properties of the atmosphere, such as the density of the medium and the presence of gas or particles.

- The e.m. signal travels a certain distance within the atmosphere.

The model law (1) illustrates the exponential decrease in the strength of an electromagnetic signal as the distance traveled and the attenuation coefficient increase. The atmosphere significantly affects the Ku, Ka, and W bands, making this model especially helpful for studying absorption and dispersion in these bands. Understanding these factors is crucial for optimizing communication systems and ensuring signal integrity over long distances. By analyzing the variations in signal strength, engineers can develop strategies to mitigate the effects of atmospheric interference.

The Ku (12-18 GHz), Ka (26-40 GHz), and W (75-110 GHz) bands are particularly susceptible to atmospheric attenuation. In the -Ku band, the effect is moderate, while in the -Ka and -W bands, attenuation is significantly higher due to interaction with water vapor (WV) and rain (R).

B. Specific requirements for space communications

Satellite communications require reliable transmission of electromagnetic signals through the Earth's atmosphere. However, atmospheric attenuation is a critical challenge, as it can reduce the quality of the received signal or even interrupt transmissions. Among the most influential factors are:

- **Rain:** The intensity of rain is directly proportional to attenuation, especially in the -Ka and -W bands, where water droplets can cause significant signal loss.
- **Clouds and snow:** Although they contribute to attenuation, they do so to a lesser extent than rain.
- **Water vapor:** In high-frequency bands like the -W band, water vapor poses a significant challenge.

Some more advanced methods, like using predictive models of the atmosphere, choosing frequencies that are less likely to be attenuated, and adding automatic signal correction systems, can lessen these effects. For example, the use of **forward error correction (FEC) codes** and the implementation of high-gain antennas help maintain communication quality even in adverse atmospheric conditions. These insights highlight the complexity of the interaction between electromagnetic signals and the atmosphere, underlining the need to design resilient systems to ensure the continuity of satellite communications.

C. Physical Description of the phenomenon

The physical phenomenon known as atmospheric attenuation occurs when a signal, like light or radio waves, loses intensity as it travels through the Earth's atmosphere. The phenomenon itself is due to the concomitance of various processes, the most important of which are absorption, dispersion, and diffraction. Absorption is a phenomenon in which gases in the atmosphere (e.g., water vapor, carbon dioxide, and ozone) absorb specific wavelengths of light or energy, reducing the intensity of the signal. Dispersion, on the other hand, concerns how air molecules and particles suspended in the atmosphere (e.g., dust or aerosols) can deviate the path of the signal, dispersing it in different directions. In general, It is possible to distinguish between **Rayleigh** dispersion and **Mie** dispersion.

When a signal comes into contact with objects similar in size to the wavelength of electromagnetic radiation, like dust particles, raindrops, and ice crystals, it can diffract, or deviate, from its original path. Refraction is a significant phenomenon in the discussion of signal propagation through the atmosphere, primarily focusing on the troposphere. Refraction refers to the deviation of the path of an e.m. wave when it passes through media with a variation in density and, more simplistically, when the refractive index of the medium varies. In the study of refraction, air has an index that changes with temperature, pressure, and relative humidity. These changes can cause the e.m. wave to bend (or flex) when it travels through the medium. It is possible to observe a temperature gradient in the propagation of rays, which leads to the deviation of the waves' path: this is a typical scenario on cold nights, when radio waves undergo thermal inversion and bend.

Signal degradation phenomena occur within the troposphere, primarily due to absorption and attenuation phenomena caused by cloudiness, rain, ice, and the general presence of hydrometeors, as well as scintillation phenomena.

Scintillations are an atmospheric phenomenon that produces rapid fluctuations in the amplitude and phase of the e.m. signal. This phenomenon is due to the presence of small irregularities in the refractive index.

I refer to these scintillations as "tropospheric" when they occur within the initial four kilometers of the Earth's atmosphere. This phenomenon, which occurs at higher frequencies and at smaller elevation angles of the earth-satellite link, does not result in signal depolarization. This phenomenon limits the performance of radio links for elevation angles less than 10° and becomes more severe for angles less than 5°, thereby restricting the link's availability.

Besides tropospheric scintillations, there are also ionospheric scintillations that happen at higher elevations. These are mostly caused by rapid changes in the total number of electrons present and the thermal excursion between day and night, which causes irregularities in the ionospheric concentration. Current research has established that ionospheric scintillations have negligible effects on communications for frequencies $f > 10$ GHz, such as in the case of the Ka band.

Regarding attenuation models, the ITU (International Telecommunications Union) has over time developed specific models to describe the attenuation in the Earth's atmosphere at different frequencies, mainly for study purposes and for civil purposes in the field of telecommunications. These models consider effects such as the absorption of oxygen and water vapor and the diffusion by hydrometeors such as raindrops and snow. Among the ITU-R recommendations, I recall:

- **ITU-R P.676:** "Attenuation by atmospheric gases and related effects". This recommendation provides methods for calculating attenuation due to atmospheric gases on horizontal and inclined paths, considering oxygen and water vapor for frequencies up to 1 GHz.
- **ITU-R P.676-13:** "Attenuation by atmospheric gases and related effects". This updated version includes methods for calculating gaseous attenuation, phase nonlinearity, atmospheric curvature, excess atmospheric path length,

and downwelling and upwelling noise temperatures due to oxygen and water vapor for frequencies up to 1 GHz.

- **ITU-R P.676-8:** "Attenuation by atmospheric gases". This suggestion gives ways to figure out how much attenuation is caused by gases in the air on both flat and slanted paths, by adding up different absorption lines that work for frequencies between 1 and 1000 GHz.

The ITU-R recommendations that deal with scintillation are:

- **ITU-R P.618-6:** This provides the propagation data and prediction methods necessary for the design of Earth-space telecommunications systems. It includes information on scintillation due to refractive index variations in the atmosphere.
- **ITU-R P.618-14:** This updated version of recommendation P.618-6 includes methods for predicting propagation parameters necessary for the design of Earth-space telecommunications systems.
- **ITU-R P.619-4:** This recommendation provides propagation data necessary for evaluating interference between stations in space and stations on the Earth's surface, including scintillation effects.

In the **Beer-Lambert** law, the absorption coefficient α (alpha) generally depends on the frequency of the signal, and in the case of water vapor, it can be described as:

$$\alpha(f) = \frac{A \cdot f^2}{B + f^2} \quad (2)$$

where constants (A,B) are determined by the composition of the atmosphere and the environmental conditions. The concentration and properties of absorbing molecules, such as water vapor, carbon dioxide, and other gases, determine the constant A's absorption capacity of the atmosphere. The constant B represents a saturation parameter that takes into account the frequency at which absorption reaches its maximum. It depends on the environmental conditions, such as temperature and pressure, which influence the distribution of the absorbing molecules. A higher value of B indicates that absorption reaches its maximum at higher frequencies.

For detailed information about frequency-dependent absorption coefficients in the context of atmospheric sciences, including the role of constants like A and B in the **Beer-Lambert** law, the following references are highly relevant: [13]- [16].

D. Variation of the refractive index

There are typically two types of propagation in the atmosphere: guided and unguided. Currently, to simplify the discussion, I will focus on unguided propagation under clear sky conditions. Later, I will complicate the theoretical framework by also considering e.m. propagation in the presence of cloudy bodies and hydrometeors. If I only consider the propagation in "clear air" the refractive index (n) is related to the dielectric constant ϵ_r by the well-known relation:

$$n = \sqrt{\epsilon_r} \quad (3)$$

At mean sea level, the refractive index (n) typically ranges from 1.0003 to 1.0004 units. The refractive index varies

with altitude due to changes in temperature (T), atmospheric pressure (P), and relative humidity (RH).

As relative humidity increases, the concentration of water vapor in the air also increases. Since water vapor has a different refractive index than dry air, higher humidity can affect the overall refractive index of the air. Generally, humid air has a slightly lower refractive index than dry air. Specialists often calculate this variation in the refractive index using the concept of "refractivity" (N) as follows:

$$N := (n - 1) \cdot 10^6 \quad (4)$$

This quantity in turn can be related to the parametric variables that define the atmospheric conditions (e.g., pressure, temperature, humidity) through the relationship:

$$N = \left(\frac{77.6}{T} \cdot (P + 4810 \cdot \frac{e}{T}) \right) \quad (5)$$

where:

- T is the absolute atmospheric temperature in Kelvin [K];

There is a relationship between the value of "mixing ratio" H, which is given in kilograms of water vapor per kilogram of air, and the atmospheric pressure (P). The atmospheric pressure is given in hPa (hectoPascal). The atmospheric pressure due to the presence of WV (Water Vapor) is:

$$e := \frac{P \cdot H}{0.62198 + H} \rightarrow [Pa] \quad (6)$$

The refractive index also changes when This study is not in the presence of clear air but of a "turbulent" atmosphere, that is, in the presence of air motion. In this scenario, It is possible to describe the air motion as a random process, making the formalism of random variables useful:

$$n := \langle n \rangle \cdot (1 + n_1) \quad (7)$$

where $\langle n \rangle$ represents the mean value of the refractive index and n_1 represents the stochastic fluctuation of the refractive index, which can be described by a random process dependent on time and geographical location. Specifically, when delving deeper into the description of the turbulent atmospheric environment, assuming spatial homogeneity for simplicity, It is possible to utilize a statistical description of the atmosphere that defines the "**covariance function**":

$$B_n(\delta) := \langle n(r) \cdot n(r + \delta) \rangle \quad (8)$$

where the angle brackets $\langle \dots \rangle$ indicate the mean value and where r is the spatial position variable. In signal theory, the covariance function represents the inverse **Fourier** transform of the spatial spectrum $\Phi_n(k)$, where k is the wave-number associated with the size of the atmospheric turbulence. In (8), the "**spatial covariance**" function measures the statistical dependence between the values of the variable n (refractive index) in two different spatial positions that can be separated by an infinitesimal distance δ .

In general, harmonic methods can analyze "atmospheric turbulence" as a complex signal. Turbulence is characterized by chaotic and irregular fluctuations in air velocity and pressure

that occur over a wide range of spatial and temporal scales. The **Fourier** transform decomposes the individual fluctuations into a sum of harmonic signals, enabling me to analyze the turbulence in terms of frequency components, thereby enhancing my understanding of the energy distribution across various turbulence scales [10], [11].

E. Level 2, Level 3 and MODIS products

When looking for Level 2 products that are capable of providing information on cloud physical properties as well as radiative properties, it is useful to consider the MODIS remote sensing products. I derive cloud particle phase (ice vs. water, cloud vs. snow), effective cloud particle radius, and cloud optical thickness using MODIS visible and near-infrared channel radiances. I also provide an indication of how cloud shadows impact the scene. Infrared retrieval methods produce cloud top temperature, height, effective emissivity, phase (ice vs. water, opaque vs. non-opaque), and cloud fraction for both day and night at a resolution of 5x5 pixels by 1 km. Lastly, this MODIS product has cirrus reflectance at visible wavelengths with a resolution of 1 km per pixel. This can help get rid of the effects of cirrus scattering in the land surface reflectance product. It is recommended switching to other A-train satellites like Aqua and Terra; specifically, the MOD06_L2 product incorporates data from the Terra platform, while the MTD06_L2 product gathers data from the Aqua platform.

The Level-3 Cloud Feedback Model Intercomparison Project (CFMIP) Observation Simulator Package, another set of daily (MCD06COSP_D3_MODIS) and monthly (MCD06COSP_M3_MODIS) products, combines Aqua and Terra MODIS to enable climate scientists to compare models and observations by generating synthetic MODIS observations from climate model integrations [12].

VI. RADIATIVE TRANSFER EQUATIONS.

Radiative transfer (RT) equations are fundamental to atmospheric physics, as they describe the propagation, absorption, scattering, and emission of electromagnetic radiation through a medium such as the Earth's atmosphere. These equations serve as a mathematical framework for modeling the processes that impact the flux of radiant energy, making them crucial for various applications as:

- **Weather forecasting:** calculating incidents and escaping radiation to improve climate models.
- **Remote sensing:** Interpreting satellite observations.
- **Studying the Earth's energy balance:** analyzing the interactions between solar, terrestrial, and atmospheric radiation.

The radiative transfer equation has the following general form:

$$\frac{dI(\nu, s)}{ds} = -\alpha(\nu, s) \cdot I(\nu, s) + j(\nu, s) \quad (9)$$

where:

- $I(\nu, s)$ is the intensity of radiation at a frequency ν along the path s ;

- $\alpha(\nu, s)$ is the extinction coefficient (sum of absorption and scattering) of the medium;
- $j(\nu, s)$ is the emission coefficient.

The complexity of the spatial, angular, and spectral dependence often necessitates numerical solutions to the radiative transfer equation. Only simplified scenarios can yield analytical solutions. Most radiative transfer (RT) models assume that the absorption coefficient, $\alpha(\nu, s)$, and the emission coefficient, $j(\nu, s)$, are constant or vary in a simple manner with respect to frequency and spatial coordinates. However, in more complex scenarios, various factors such as temperature, pressure, and the presence of multiple species can influence these coefficients, posing significant challenges in accurately modeling the radiative processes. I could:

- Integrate more detailed physical models to describe the dependence on pressure, temperature, and gas concentration.
- Include nonlinear effects such as **line mixing** and **continuum absorption**.

Classical RT is based on geometric optics and thermal emission. Innovations include:

- Using quantum models to better describe the interaction of radiation with molecules and aerosols.
- Incorporating coherent quantum effects, such as stimulated emissions relevant for specific wavelengths.

Other effects includes *tridimensional coupling* like advanced **multiple scattering** and **anisotropic scattering**. The application of machine learning models can markedly enhance the efficiency and accuracy of solving the computationally intensive radiative transfer (RT) equation. These models can approximate real-time solutions in complex environments, such as atmospheres with multiple cloud layers. Additionally, they can significantly reduce computation time by generating efficient surrogate models based on extensive simulation datasets. Furthermore, the traditional RT equation does not explicitly account for temporal dynamics, which are critical in scenarios where atmospheric conditions change rapidly. To model the spread of radiative pulses, like those seen in short-term meteorology or astrophysical explosions, it is possible to add a temporal term, $\frac{\partial I(\nu, s, t)}{\partial t}$.

To accurately represent the behavior of electromagnetic (EM) radiation through the distinct layers of a stratified atmosphere model with M levels, each characterized by unique physical properties such as temperature, pressure, density, gas, and aerosol concentration, I must adapt the radiative transfer (RT) equation. Generally, It is possible to write the radiative transfer equation separately for each level i of a stratified atmosphere, accounting for absorption, emission, and scattering interactions. For a single ray, the equation is as follows:

$$\frac{dI(\nu, \mu, z_i)}{dz} = -\alpha_i(\nu, z_i) \cdot I(\nu, \mu, z_i) + j_i(\nu, z_i) \quad (10)$$

where:

- $I(\nu, \mu, z_i)$ represents the intensity of radiation at frequency ν , zenith angle $\mu = \cos(\theta)$, and height z_i ;

- $\alpha_i(\nu, z_i)$ is the extinction coefficient for the i -th layer, which includes absorption and scattering;
 - $j_i(\nu, z_i)$ is the emission coefficient in the i -th layer.
- Generally the equation also maybe written as:

$$\frac{dI_i(\nu, \mu, z_i)}{dz} = -\alpha_i(\nu, z_i) \cdot I_i(\nu, \mu, z_i) + j_i(\nu, z_i) + \sigma_i \int_{\Omega=4\pi} I_i(\Omega') \cdot P_i(\Omega, \Omega') \cdot d\Omega' \quad (11)$$

where:

- I_i is the specific intensity of radiation at level i ;
- z is the path length;
- α_i is the absorption coefficient;
- j_i is the emission coefficient;
- σ_i is the scattering coefficient;
- $P_i(\Omega, \Omega')$ is the phase function describing the scattering probability from direction Ω' to Ω .

To better describe each i -th layer, It is possible to consider specific properties where:

- $\alpha_i = k_i \rho_i + \sigma_i$ where k_i is the absorption coefficient, σ_i is the dispersion coefficient, and ρ_i is the density of the medium.
- $j_i = k_i \rho_i B_\nu(T_i)$, which represents the thermal emission expressed as a product with the Planck function at the local temperature T_i .

Using this basic outline as a guide, It is possible to now examine the integration over the layers. The intensity of the radiation observed leaving the top of the atmosphere (TOA) is determined by integrating the radiative transfer equation through all the atmospheric levels under consideration. This integral should be evaluated along the optical path that the electromagnetic (EM) radiation follows:

$$I_{TOA}(\nu, \mu) = I_{BOA}(\nu, \mu) \cdot e^{-\tau(\nu)} + \int_0^{\tau(\nu)} S(\nu, \tau') \cdot e^{-[\tau(\nu) - \tau']} \cdot d\tau' \quad (12)$$

where The author has:

- $I_{BOA}(\nu, \mu)$ is the intensity of e.m. radiation at the base of the atmospheric layer (altimetric level $z = 0$);
 - $\tau(\nu)$ represents the total "optical thickness" along the atmospheric column under consideration.
 - $S(\nu, \tau')$ is the radiation source term placed at the τ' level, which is linked to the concept of thermal emission.
- Finally, assuming a vertical stratification in the zenithal direction, I would evaluate the optical depth for the atmospheric layers considered, obtaining:

$$\tau(\nu) = \sum_{i=1}^M \alpha_i(\nu) \cdot \Delta z_i \quad (13)$$

where Δz_i is the thickness of the i -th layer. In practice, The methodology uses numerical methods like the **DOM (Discrete Ordinates Method)** or the **Two-Stream** approximation to solve the numerical model. In the first case, the radiative transfer equation is solved by breaking the angles into discrete directions (μ_k) and solving for each direction. In the second case, the e.m. radiation is split into two main fluxes (ascending and descending), which makes the process of integrating the angles easier.

If you use a model of the atmosphere with layers, it's helpful to include the rising and falling radiative fluxes, which are

shown by (I^+) and (I^-) , respectively. This makes the first-order differential equations look like this:

$$\frac{dI^+}{dz} = -\alpha(z) \cdot I^+ + j(z) \quad (14)$$

$$\frac{dI^-}{dz} = \alpha(z) \cdot I^- - j(z) \quad (15)$$

Starting with these basic considerations, It is possible to calculate the propagation of electromagnetic (EM) radiation layer by layer, beginning at the base of the atmosphere and considering initial conditions such as the reflected intensity or the emission from the ground. By accounting for absorption, dispersion, and emission contributions, It is possible to trace the radiation's journey through each layer until it reaches the top of the atmosphere (TOA), thus completing the integration process. In this calculation, I must include ground emission and above-ground radiation.

To describe equation (2) more clearly, The author has two terms in the second member: the first represents the radiation transmitted from the base of the atmosphere, while the second term represents the contribution of sources along the atmospheric column.

The negative exponential term, which indicates the attenuation factor, represents the energy that light absorbs and spreads out as it travels along an optical path. The integral term in the second member shows the total contribution from sources throughout the atmospheric column.

These source terms, $S(\nu, \tau')$, are linked to processes such as:

- a) Thermal emission of molecules (e.g., atmospheric gases, aerosols, and clouds).

I would incorporate multiple diffusion, also referred to as multiple scattering, to accommodate this intricate interaction term. Lastly, the negative exponential attenuation term appears in the integral. This term is used to evaluate the radiation emitted from the τ' level as it ascends through the upper atmosphere and reaches the TOA.

The integral of the extinction coefficient along the **optical path** yields the "**optical depth**" component, expressed as:

$$\tau(\nu) = \int_0^{z_{TOA}} \alpha(\nu, z') \cdot \sec(\theta) \cdot dz' \quad (16)$$

where:

- $\alpha(\nu, z')$ represents the frequency- and height-dependent extinction coefficient;
 - $\sec(\theta)$ is the correction term of the optical path with respect to the zenith angle. It is true that $\sec(\theta) = 1/\mu$.
- Regarding the contribution of thermal emission, thinking of cloudy bodies as black or gray bodies, It is possible to express it as:

$$j(\nu, z) = k_\nu(z) \cdot B_\nu(T(z)) \quad (17)$$

where $k_\nu(z)$ is the absorption coefficient, and where $B_\nu(T(z))$ is the **Planck** function.

To describe the thermal emission given by the e.m. radiation emitted by molecules, aerosol particles, and/or cloudy bodies, It is possible to express Planck's law:

$$S_{thermal}(\nu, \tau') := \frac{k_\nu(\tau') \cdot B_\nu(T(\tau'))}{\alpha_\nu(\tau')} \quad (18)$$

where $k_\nu(\tau')$ is the volumetric absorption coefficient at frequency ν and depends on the atmospheric composition of the gas (e.g., water vapor, carbon dioxide, ozone, etc.). Instead, regarding the definition of the Planck function, The author has that for the layer considered placed in thermal equilibrium at the temperature $T(\tau')$, The author has:

$$B_\nu(T) := \frac{2h\nu^3}{c^2} \cdot \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1} \quad (19)$$

where h is Planck's constant, c is the speed of light, and k_B is Boltzmann's constant. Finally, $\alpha_\nu(\tau')$ is the total extinction coefficient given by the sum of the absorption and scattering coefficients, or:

$$\alpha_\nu(\tau') = k_\nu(\tau') + \sigma_\nu(\tau'). \quad (20)$$

A. Specialization at Ku, Ka and W Band.

To write the radiative transfer equations for the individual Ku, Ka, and W bands, It is possible to adapt the general radiative transfer equation for each band, taking into account the specific physical properties and radiative interactions of each band.

1)

$$\frac{dI_{Ku}}{ds} = -\alpha_{Ku}I_{Ku} + j_{Ku} + \sigma_{Ku} \cdot \int_{4\pi} I_{Ku}(\Omega') \cdot P_{Ku}(\Omega, \Omega') \cdot d\Omega' \quad (21)$$

2)

$$\frac{dI_{Ka}}{ds} = -\alpha_{Ka}I_{Ka} + j_{Ka} + \sigma_{Ka} \cdot \int_{4\pi} I_{Ka}(\Omega') \cdot P_{Ka}(\Omega, \Omega') \cdot d\Omega' \quad (22)$$

3)

$$\frac{dI_W}{ds} = -\alpha_W I_W + j_W + \sigma_W \cdot \int_{4\pi} I_W(\Omega') \cdot P_W(\Omega, \Omega') \cdot d\Omega' \quad (23)$$

In these equations:

- I_{Ku}, I_{Ka}, I_W are the specific intensities of radiation in the respective bands.
- $\alpha_{Ku}, \alpha_{Ka}, \alpha_W$ are the absorption coefficients.
- j_{Ku}, j_{Ka}, j_W are the emission coefficients.
- $\sigma_{Ku}, \sigma_{Ka}, \sigma_W$ are the scattering coefficients.
- $P_{Ku}(\Omega, \Omega'), P_{Ka}(\Omega, \Omega'), P_W(\Omega, \Omega')$ are the phase functions describing the scattering probability from direction Ω' to direction Ω .

These equations account for the interactions of absorption, emission, and scattering within each specific band. The phase function $P(\Omega, \Omega')$ is fundamental to understanding how radiation interacts with particles in the atmosphere. It describes the probability of scattering radiation from direction Ω' to direction Ω . In general this function maybe expressed as:

$$P(\Omega, \Omega') := \frac{1}{4\pi} \sum_{l=0}^{\infty} (2l+1) \cdot \omega_l \cdot P_l(\cos(\theta)) \quad (24)$$

where:

- Ω and Ω' are the directions of the incident and scattered radiation, respectively;
- Θ is the angle between Ω and Ω' ;
- ω_l are the Legendre expansion coefficients.
- $P_l(\cos\Theta)$ are the **Legendre** polynomials.

B. Rayleigh Phase function

When the particle size (shown by the radius a) is much smaller than the wavelength of the radiation ($a \ll \lambda$), **Rayleigh scattering** happens. To figure out the phase function, It is possible to use the simplified expression to show how the scattered light's intensity changes with angle. The following formula provides the phase function for Rayleigh scattering:

$$P_{Rayleigh}(\Theta) = \frac{3}{4}(1 + \cos^2(\Theta)) \quad (25)$$

This expression comes from adding up the fields that are sent out in different directions, with the particle's polarizability and the dipole's shape giving it weight. The term $\cos^2(\Theta)$ describes the symmetry about the dipole axis. The prefactor $3/4$ is a normalization constant that ensures that the integral of the phase function over all directions (i.e. over 4π) is equal to 1, ensuring that all energy is redistributed in space according to the phase function [17]- [19] .

C. Mie Phase function

For **Mie scattering**, which is relevant for particles of size comparable to the wavelength of the radiation, the phase function is more complex and depends on the properties of the particles. It can be expressed as a series of Legendre polynomials:

$$P_{Mie}(\Theta) = \sum_{l=0}^{\infty} (2l+1) \cdot \omega_l \cdot P_l(\cos(\Theta)) \quad (26)$$

where:

- Θ is the scattering angle;
- ω_l are the expansion coefficients;
- $P_l(\cos(\Theta))$ are the Legendre polynomials.

This representation allows for a detailed description of the scattering process, taking into account the size, shape, and refractive index of the particles. $P(\Theta)$ represents the angular dependence of the intensity of the scattered light or radiation.

VII. ROLE OF CLOUDS AS THERMAL SOURCES.

Clouds are formed by suspended water vapor, water droplets, or ice crystals that interact strongly with infrared radiation. The e.m. radiation emitted by a cloudy body depends on its temperature (absolute) and its emissivity (e_{cloud}), so It is possible to generically say that:

$$j_{cloud}(\nu) = e_{cloud} \cdot B_{\nu}(T_{cloud}) \quad (27)$$

where $B_{\nu}(T_{cloud})$ is the Planck function for the cloud temperature. The Planck function $B_{\nu}(T_{cloud})$ describes the spectral distribution of radiation emitted by a black body in thermal equilibrium at a temperature T_{cloud} . This relationship is fundamental to understanding how clouds influence the radiation budget of the atmosphere and the Earth's climate. In other words, $B_{\nu}(T_{cloud})$ represents the amount of radiation emitted per unit area, per unit solid angle, and per unit frequency at a given temperature T_{cloud} .

A. Modeling Clouds: Black Bodies vs. Gray Bodies

Clouds are generally not modeled as "perfect" black bodies with an emissivity of 1 ($e_{cloud} = 1$). Instead, they behave as gray bodies with an emissivity less than 1 ($e_{cloud} < 1$). This physical property depends on their composition, optical thickness, and the concentration (or density) of constituent particles.

When modeling the cloud contribution within the radiative transfer equation, it is necessary to consider that clouds act as source terms. Therefore, I will write:

$$\frac{dI(\nu, s)}{ds} = -\alpha(\nu, s)I(\nu, s) + \alpha(\nu, s) \cdot e_{cloud} \cdot B_{\nu}(T_{cloud}) \quad (28)$$

where:

- $\alpha(\nu, s)$ is the cloud extinction coefficient (given by the sum of the absorption and scattering contributions);
- e_{cloud} modulates the **black-body** contribution (its emissivity).

To solve the equation, it's necessary to have boundary conditions:

- 1) at the top of the atmosphere ($s = s_{TOA}$), the incoming radiance from space may be negligible or modeled as cosmic background radiation.
- 2) at the bottom of the atmosphere ($s = s_{ground}$), the radiance depends on surface emissions and reflections.

The equation is typically solved using numerical methods like DOM, Monte Carlo simulations, or **LBLRTM** (Line-by-line radiative transfer models), depending on the complexity of the scenario [20], [21].

When modeling the cloud contribution within the radiative transfer equation, one must consider that clouds are source terms. Therefore, one writes the following for the i th layer:

$$\frac{dI_i(\nu, s)}{ds} = -\alpha_i(\nu, s)I_i(\nu, s) + \alpha_i(\nu, s) \cdot e_{cloud, i} \cdot B_{\nu}(T_{cloud, i}) + S_{ext}(\nu, s) \quad (29)$$

where:

- $\alpha_i(\nu, s)$ is the cloud extinction coefficient for the i th layer (given by the sum of the absorption and scattering contributions);
- $e_{cloud, i}$ modulates the black-body contribution (its emissivity) for the i th layer;
- $S_{ext}(\nu, s)$ represents the external solar source, in a more complex scenario.

B. The foosball effect

The "foosball" effect, known in scientific literature as "multiple scattering" or "multiple diffusion", occurs in a more general and realistic context when radiation from a cloud moves from one cloud to another. This phenomenon occurs when electromagnetic radiation interacts with multiple particles or water droplets along its path, undergoing numerous scattering events before reaching its final destination. In the context of clouds, other clouds or atmospheric particles can scatter the infrared radiation a cloud emits, creating a "bounce" effect between clouds. This process is important in atmospheric models, as it affects the distribution and intensity of radiation reaching the Earth's surface or emitted into space. Multiple scattering is particularly relevant in regions with high cloud or aerosol concentrations, where radiation can undergo numerous scattering events, significantly altering the radiative budget of the atmosphere.

The radiative transfer equation with multiple scattering can be formalized as follows:

$$\frac{dI(\Omega, s)}{ds} = -\alpha \cdot I(\Omega, s) + j(\Omega, s) + \sigma \cdot \int_{4\pi} I(\Omega', s) P(\Omega, \Omega') d\Omega' \quad (30)$$

where:

- $I(\Omega, s)$ is the specific intensity of radiation in the Ω direction at point s ,
- α is the absorption coefficient,
- $j(\Omega, s)$ is the emission coefficient,
- σ is the scattering coefficient,
- $P(\Omega, \Omega')$ is the phase function describing the scattering probability from the Ω' direction to the Ω direction.

To include multiple scattering, I need to integrate the radiative transfer equation along the optical path, taking into account the multiple interactions of radiation with atmospheric particles. It is possible to accomplish this using an iterative or numerical method like Monte Carlo or finite difference. The finite difference method is a numerical approach that discretizes the radiative transfer equation on a spatial and angular grid. This method solves the equation iteratively, taking into account the multiple interactions of radiation with atmospheric particles.

In summary, It is possible to mathematically model the foosball effect, or multiple scattering, by extending the radiative transfer equation to incorporate contributions from multiple scattering and solving the equation along the optical path using numerical methods like Monte Carlo or finite difference. This approach allows one to accurately model the interaction of

radiation with atmospheric particles in the presence of multiple scattering. It is possible to write the radiative transfer equation for the i -th layer as follows:

$$\frac{dI_i(\Omega, s)}{ds} = -\alpha \cdot I_i(\Omega, s) + j_i(\Omega, s) + \sigma_i \cdot \int_{4\pi} I_i(\Omega', s) P_i(\Omega, \Omega') d\Omega'$$

and to calculate the attenuation of the e.m. radiation through the generic layer I will have to consider that:

$$I_i(\Omega, s) = I_i(\Omega, 0) \cdot e^{-\tau_i} \quad (31)$$

where τ_i represents the optical depth of the i -th layer, which includes both absorption and scattering contributions. Therefore:

$$\tau_i = \int_0^s (\alpha_i + \sigma_i) \cdot ds \quad (32)$$

In a simplified **plane-parallel** model, I would calculate the total attenuation of the EM radiation through all atmospheric levels as a sum:

$$\tau_{TOT} = \sum_{i=1}^M \tau_i. \quad (33)$$

VIII. PATH INTEGRATED ATTENUATION (PIA) AND OPTICAL DEPTH τ .

Path Integrated Attenuation (PIA) is a measure of the total attenuation of radiation along an optical path, taking into account both absorption and scattering. It closely relates to the optical thickness and has the following expression:

$$PIA = 1 - e^{-\tau}. \quad (34)$$

To put it another way, the PIA signifies the percentage of radiation that undergoes attenuation, either by absorption or scattering, along the optical path. As the optical thickness increases, the PIA increases, indicating a greater attenuation of the radiation [17]. In relation (32) it indicates the radiation that enters the system (e.g. atmospheric) and that is considered as 100% of the incident radiation. The PIA is represented as follows for a generic atmospheric layer:

$$PIA_i = 1 - e^{-\tau_i}. \quad (35)$$

In the evaluation of a PIA_{tot} in this case all the contributions of optical thicknesses should be added:

$$PIA_{tot} = 1 - \exp\left(\sum_i \tau_i\right) \quad (36)$$

and values of these optical depths can be found in the ERA5 datasets provided by the Copernicus Climate Change Service (C3S) at ECMWF. ERA5 offers hourly estimates of numerous atmospheric, oceanic, and terrestrial variables, including radiation parameters. To access the data, select the dataset "ERA5: Fifth generation of ECMWF atmospheric reanalyses of the

global climate" and search for variables of interest, such as "aerosol optical depth" or "total column optical depth". For calculations, specify both the time period and the geographical area of interest. You can then export the data in **NetCDF** or **GRIB** format. The aerosol optical depth (AOD) variable measures the attenuation of solar radiation due to aerosols in the Earth's atmosphere. Another parameter, "**Fine Mode Aerosol Optical Depth**" (**FMAOD**), measures the attenuation due to smaller aerosols.

In the W band (75-110 GHz), the contribution of aerosols is generally less significant than in other bands, such as visible or infrared. This is because the wavelength of W band radiation is much longer than the typical aerosol particle size, making scattering less efficient. However, aerosols can still impact W band radiation through absorption and scattering processes. Larger particles, such as dust or soot, can contribute to the attenuation of W band radiation. These particles can also indirectly influence radiation by modifying cloud properties, such as their emissivity and optical thickness. The **EarthCare** mission investigates the role of aerosols in the Earth's radiation budget. Of particular interest in this field is the "**Ångström exponent**", which represents a frequency scaling law in attenuation estimation for different working frequencies. The **Ångström** exponent (α) describes how the optical thickness (τ) varies with the wavelength (λ) of electromagnetic radiation. Therefore:

$$\tau(\lambda) = \tau(\lambda_0) \cdot \left(\frac{\lambda}{\lambda_0}\right)^{-\alpha} \quad (37)$$

where:

- $\tau(\lambda)$ is the optical thickness at wavelength λ ,
- $\tau(\lambda_0)$ is the optical thickness at a reference wavelength λ_0 ,
- α is the Ångström exponent.

The relation shows that the optical thickness varies with wavelength according to a power law, with the Ångström exponent determining the slope of this variation. In general, a higher Ångström exponent indicates greater attenuation at shorter wavelengths. This relationship is important in fields such as atmospheric science and remote sensing, where understanding the interaction of light with atmospheric particles can provide insights into air quality and climate change. By analyzing the optical thickness across different wavelengths, researchers can better characterize aerosol properties and their impact on solar radiation.

IX. SAMPLE SIMULATION.

The developed code produced in this first article aims to evaluate the impact of global climate change on atmospheric attenuation in satellite telecommunication systems, with a particular focus on the Ku (12 GHz), Ka (30 GHz), and W (94 GHz) bands. I import the necessary climate data from a CSV file (climate_data.csv), which contains information on latitude, longitude, altitude, temperature, pressure, relative humidity, and rainfall intensity. These parameters are essential to calculate the attenuation due to atmospheric gases and rainfall using empirical formulas. The code organizes the data

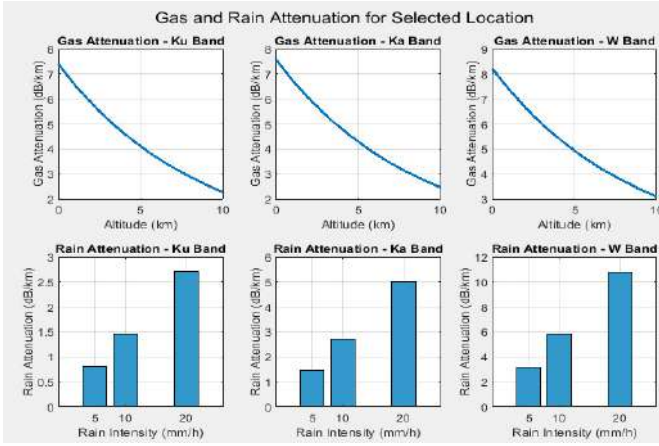


Fig. 1: Gas and Rain Attenuation for Selected Location in the World.

in a geographic grid, analyzing each point to generate specific attenuation profiles.

The developed code aims to analyze the impact of global climate change on atmospheric attenuation in satellite telecommunication systems, with particular attention to the Ku (12 GHz), Ka (30 GHz), and W (94 GHz) bands. I load the necessary climate data from a CSV file (`climate_data.csv`), containing information on latitude, longitude, altitude, temperature, pressure, relative humidity, and rainfall intensity. These parameters are essential for calculating the attenuation due to atmospheric gases and rainfall using empirical formulas. The code arranges the data according to a geographic grid, scrutinizing each point to generate distinct attenuation profiles.

A. Attenuation Models

The code implements two main models:

- **Attenuation due to gases:** This model considers the dependence on altitude and climate parameters, such as pressure (P), relative humidity (RH), and temperature (T). The formula incorporates contributions from oxygen and water vapor, with additional terms to distinguish the frequency bands.
- **Rain attenuation:** This model quantifies attenuation as a function of rainfall frequency and intensity. I apply a formula that takes into account both absorption and scattering, specifically tailored to high-frequency scenarios such as the W band.

I perform calculations iteratively on a global latitude and longitude grid, organizing the results into a data structure that separates the attenuation for each frequency band by gas and rain contributions.

B. Visualization and debugging

The code includes functionality to visualize attenuation results for a specific geographic location. The user can select a combination of latitude and longitude (`latexample` and `lonexample`), and the code checks for matching data, suggesting the closest location in case of discrepancies. The results are displayed through graphs:

- For each band, I plot the gas attenuation against altitude.
- I represent rain attenuation as a bar graph in relation to rain intensities.

The structure of the graphs allows for a direct comparison of the different contributions for each band, aiding in the technical analysis and the development of strategies to mitigate signal losses in telecommunications applications. By visualizing these relationships, It is possible to better understand the impact of environmental factors on signal quality. This analysis not only enhances my current methodologies but also informs future research and development efforts aimed at improving communication systems.

X. SPECIFIC REQUIREMENTS FOR SPACE COMMUNICATIONS.

In this section, I want to discuss the specific requirements for space communications in the bands of interest: Ku, Ka, and W.

I know wave propagation through the atmosphere has limits. For frequencies exceeding 10 GHz, I must consider the impact of troposphere propagation, specifically the effects that weaken the signals. These effects primarily include attenuation by gases such as oxygen and water vapor, liquid water in clouds and rain, and scintillation from atmospheric turbulence. Most of these effects increase rapidly with frequency, and their frequency depends significantly on the climatic characteristics. Rain attenuates the main effect that determines the size of the link-budget margin. This attenuation varies significantly over time and causes the link to be interrupted whenever the link-budget margin is exceeded, thus making it unavailable.

In the context of space communications, particularly in the Ku, Ka, and W bands, it is crucial to account for the various atmospheric effects that can impact signal propagation. For frequencies exceeding 10 GHz, the troposphere plays a significant role in weakening signals. Gases like oxygen and water vapor, along with liquid water in clouds and rain, primarily cause this weakening. Additionally, scintillation from atmospheric turbulence can further degrade signal quality. These effects are highly frequency-dependent and tend to increase rapidly with higher frequencies. The climatic characteristics of the region also play a vital role in determining the extent of these effects.

Rain attenuation is a critical factor in space communications, as it significantly impacts the **link-budget** margin. Rain's intensity and variability can lead to significant fluctuations in signal strength, resulting in communication link interruptions whenever the link-budget margin exceeds. This variability necessitates the inclusion of a sufficient margin in the link budget to ensure reliable communication. Understanding and accurately modeling these atmospheric effects are essential for designing robust communication systems that can withstand the challenges posed by the troposphere and maintain consistent performance in various climatic conditions.

A. Example for Sicily region.

In my MATLAB example, I calculate and visualize the effect of atmospheric propagation in the Ku, Ka, and W bands,

considering the climatic conditions typical of Sicily. This code includes the attenuation of atmospheric gas and temperature with variable intensity. For this sample, The author has determined the parameters (p , T , RH) to be 1013 hPa, $20^\circ C$, and 60. Given that Sicily is generally at low altitudes, the standard atmospheric pressure is a suitable approximation, although the value may vary slightly in specific regions or colors. Colors can influence how light interacts with the atmosphere, potentially affecting local weather patterns. It is essential to consider these variations when studying climate and environmental changes on the island. Many meteorological models use a vertical temperature profile with a linear temperature decrement of $-6.5^\circ C/km$ as a standard lapse rate, signifying the average temperature drop with altitude in the troposphere, the lowest layer of Earth's atmosphere.

In this example, the attenuation functions provided are simplifications based on ITU-R recommendations **P.676** and **P.838**, considering the generic dependence on frequency, temperature, relative humidity, and rainfall intensity. The weakening caused by gases in the atmosphere ($f^{0.2}$) is mostly due to oxygen and water vapor, which changes depending on the pressure and temperature. The weakening caused by rain ($f^{0.8}$, $R^{0.6}$) is due to both absorption and scattering for different levels of precipitation. However, using the full ITU-R formulas and integrating local atmospheric profiles allows for a more detailed modeling of the complexity of molecular resonances in gases and the influence of polarization in rainfall losses.

It is important to use the detailed formulas in **ITU-R P.676** and **P.838** to get a better idea of how atmospheric propagation affects the Ku, Ka, and W bands. These formulas include contributions from molecular resonances and more precise frequency dependencies. Furthermore, it is essential to integrate local atmospheric data, such as temperature, relative humidity, and pressure profiles, to obtain more accurate results. Another aspect to consider is the effect of polarization (vertical or horizontal) on rainfall losses, which can significantly affect the W band. These combined factors allow for a more complete and precise analysis of the typical climate conditions of Sicily and their influence on atmospheric propagation. For the gas attenuation I may implement this formula:

$$Gas_{attenuation}(f, P, RH, T) = 0.1 \cdot f^{0.2} \cdot \exp(-0.02 \cdot T) \cdot \frac{RH}{100} \quad (38)$$

where:

- f is the frequency in [Hz];
- P : is the atmospheric pressure in [hPa], which affects the absorption coefficients of water vapor and oxygen;
- RH : is the relative humidity expressed in percentage values (%), which influences molecular motion and absorption potential.

Theoretical considerations of atmospheric absorption include absorption by oxygen and water vapor. Oxygen contributes to attenuation via resonances at specific frequencies, such as around 60 GHz and 120 GHz. Water vapor, on the other hand, absorbs mainly at frequencies above 10 GHz, with significant contributions in the -Ka and -W bands. To make

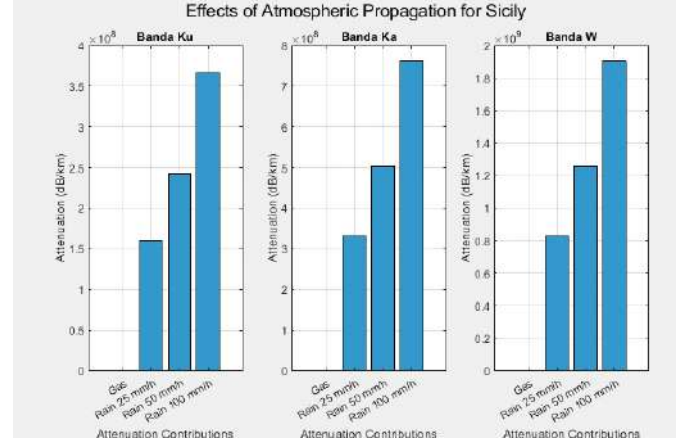


Fig. 2: Simplified effects of atmospheric propagation for Sicily region (Italy).

the math easier, the function uses $f^{0.2}$ to get close to the frequency dependence and $\exp(-0.02 \cdot T)$ to account for temperature. This is what causes absorption to decrease as temperature rises. In addition, relative humidity (RH) modulates the amount of available water vapor, further influencing atmospheric absorption. These combined factors allow for a more accurate analysis of atmospheric conditions and their influences on electromagnetic wave propagation.

Rain attenuation is a phenomenon that strongly depends on the frequency ($f^{0.8}$) and the intensity of precipitation ($R^{0.6}$). Rain causes both absorption and scattering of electromagnetic waves, and these effects become significant and increase rapidly at frequencies above 10 GHz. The rain attenuation function, $rain_{attenuation}(f, R) = 0.2f^{0.8}R^{0.6}$, gives a rough idea of the attenuation coefficients without separating absorption and scattering. It does this by giving a general response to the rain's frequency and intensity. The system considers two parameters: the frequency (f) in Hz and the rain intensity (R) in mm/h, which represent the precipitation flux that contributes to signal loss. These theoretical considerations are fundamental to understanding the impact of rain on the propagation of electromagnetic waves, especially in the higher frequency bands. Understanding these interactions is crucial for designing communication systems that can withstand adverse weather conditions. By analyzing how frequency and intensity affect signal degradation, engineers can develop more resilient technologies that maintain reliability even during heavy rainfall. The simplified functions have some significant limitations. The full formulas of **ITU-R P.676** more accurately model the complex spectral distribution of atmospheric gases like oxygen and water vapor, which includes spectral lines (*resonances*). In addition, the coefficients used for rain attenuation (0.2, 0.8, and 0.6) are generalizations. More precise values depend on the polarization and the climatic region, as indicated in the tables of **ITU-R P.838**. These considerations are essential to obtain a more accurate and detailed analysis of atmospheric conditions and their influences on the propagation of electromagnetic waves.

Implementing a more detailed gas attenuation formula based

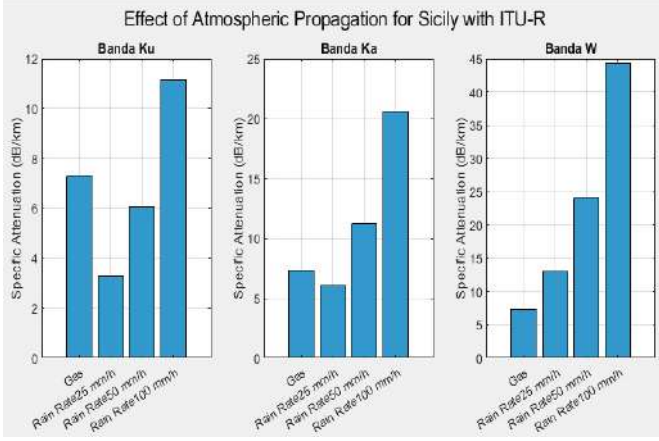


Fig. 3: Improved model for the atmospheric propagation effect in Sicily with ITU-R.

on **ITU-R P.676** permit to express this contribute as:

$$\alpha_{gas} = 7.2 \cdot \frac{(f/1e9)^2}{(f/1e9)^2 + 0.34} \cdot \frac{P}{1013} + k_W \cdot \frac{RH}{100} \cdot \frac{(f/1e9)^2}{(f/1e9)^2 + 1} \quad (39)$$

This specific empirical approximation does not account for the specific characteristics of rain-induced attenuation for different frequencies or polarizations, and the dependence on factors like droplet size distribution. The implementation of the rain attenuation is a function based on the **ITU-R P.838** like this:

$$\alpha_{Rain} = 0.0367 \cdot (f/1e9)^{0.67} \cdot R^{0.88}$$

The proposed improvements include a more detailed analysis of the frequency dependence and attenuation coefficients derived from ITU studies. It is assumed horizontal polarization as the basis, a common occurrence in satellite communication links. Furthermore, the model reflects a more accurate scaling with **rain intensity** (R) and **frequency** (f), thereby improving the accuracy of attenuation forecasts. These improvements allow for a more realistic and reliable modeling of atmospheric conditions and their influences on electromagnetic wave propagation.

For gas attenuation, the improvement to this model incorporates altitude-specific profiles of temperature, pressure, and humidity, using a realistic vertical profile for Sicily that represents standard atmospheric conditions. For rain attenuation, it considers varying rain intensities and assumes horizontal polarization for the -Ku, -Ka, and -W bands. The visualization component of the code plots gas attenuation as a function of altitude and displays rain attenuation for different rain intensities in each frequency band. This comprehensive approach ensures high accuracy in analyzing communication links in the Sicilian region. By integrating these factors into the model, the analysis not only provides insights into potential signal degradation but also aids in optimizing communication system designs. Ultimately, this study serves as a valuable resource for engineers and researchers working to enhance wireless communication performance in challenging environments.

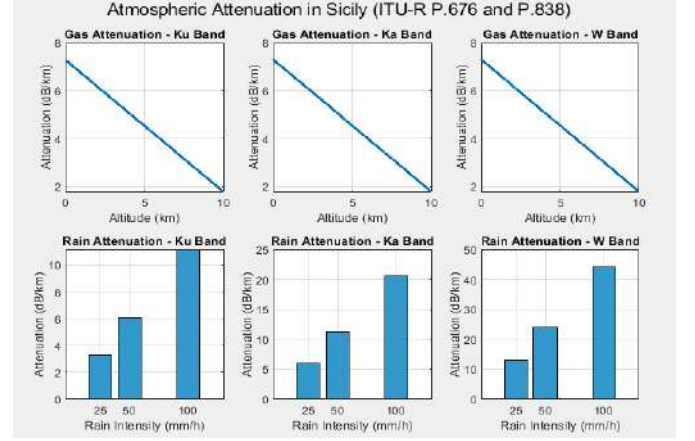


Fig. 4: Improvement to the Sicily model with ITU-R P.676 and P.838.

In this updated version of the Sicily model, the gas attenuation is implemented as:

$$\alpha_{gas}(f, P, RH, T) = 7.2 \cdot \frac{(f/1e9)^2}{(f/1e9)^2 + 0.34} \cdot \frac{P}{1013} + 0.1820 \cdot \frac{RH}{100} \cdot \frac{(f/1e9)^2}{(f/1e9)^2 + 1} \quad (40)$$

where the term $7.2 \cdot \frac{(f/1e9)^2}{(f/1e9)^2 + 0.34}$ models the absorption due to the Oxygen, with a strong dependency on frequency. This term accounts for the specific resonance behaviour of Oxygen, especially relevant near 60 GHz and 120 GHz. The term $0.1820 \cdot \frac{RH}{100} \cdot \frac{(f/1e9)^2}{(f/1e9)^2 + 1}$ models the absorption due to water vapour, with a dependency on relative humidity (RH). This term in particular captures the increase in attenuation at higher frequencies due to water vapour. In the formula I incorporate pressure (P) and temperature (T) profiles, making the implementation more realistic respect the previous implementations, considering a stratified atmosphere. The updated rain attenuation is:

$$\alpha_{rain}(f, R) = 0.0367 \cdot (f/1e9)^{0.87} \cdot R^{0.88} \quad (41)$$

This formula addresses high-frequency rain attenuation using ITU-derived coefficients specifically tailored for horizontal polarization, a common use in satellite communications. The more accurate exponents (0.67 for frequency and 0.88 for rain intensity) reflect real-world observations, providing a more precise analysis. The coefficients (0.0367) are specific to horizontal polarization, ensuring the model accurately represents the conditions typically encountered in satellite links. The formula also takes into account the non-linear relationship between the amount of rain (R) and signal attenuation, which becomes more noticeable at higher frequencies. This gives a full picture of how rain affects signal transmission.

In conclusion, the updated MATLAB example provides a more precise estimation of rain attenuation, which is particularly crucial for the W band where rain effects are dominant. By incorporating ITU-derived coefficients for horizon-

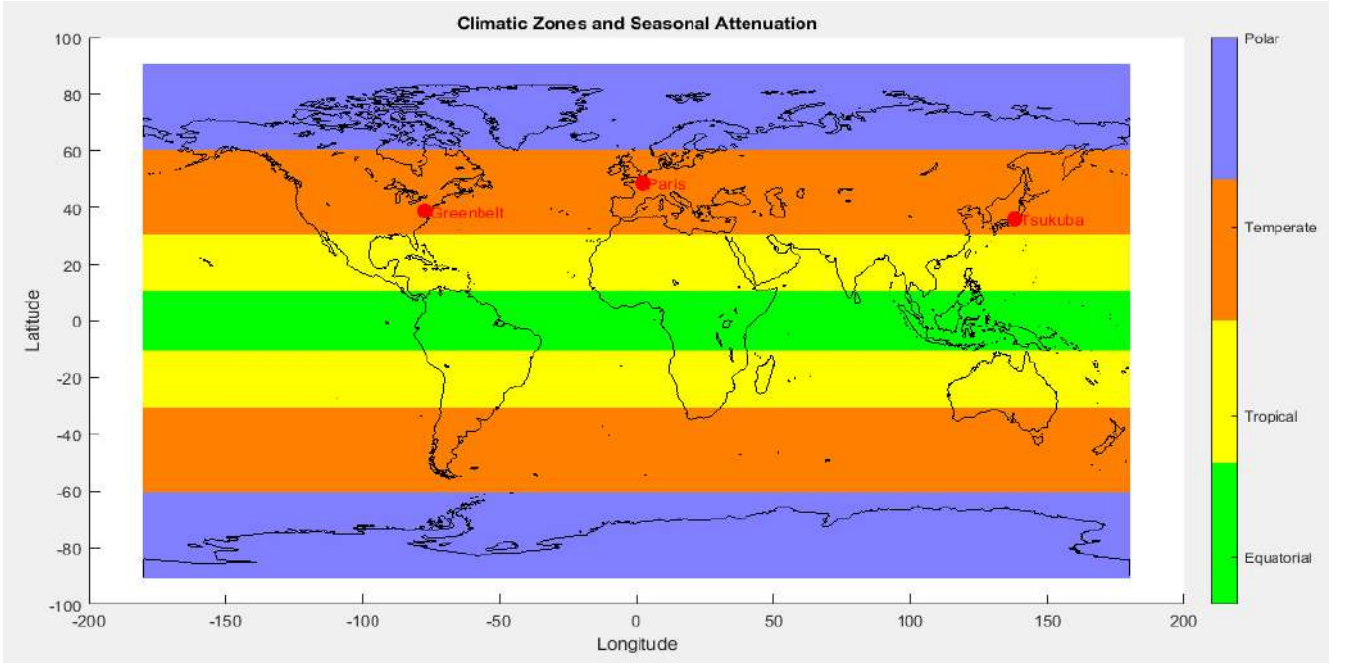


Fig. 5: Climatic zones are regions of the Earth that share similar climate characteristics, such as temperature, precipitation, and humidity. These zones are typically classified into categories like tropical, temperate, polar, and arid, each with distinct weather patterns and ecosystems. Seasonal attenuation refers to the variations in climate and weather conditions that occur throughout the year within these zones. Factors such as the tilt of the Earth's axis, its orbit around the sun, and geographical features like mountains and oceans influence these seasonal changes. Understanding climatic zones and seasonal attenuation is crucial for studying weather patterns, agriculture, and the impact of climate change on different regions.

tal polarization and more accurate exponents for frequency and rain intensity, the model reflects real-world observations more accurately. This comprehensive approach ensures high accuracy in analyzing communication links in the Sicilian region, accounting for the complex interactions between atmospheric conditions and electromagnetic wave propagation. This enhanced model not only improves the reliability of predictions but also aids in the design of more resilient communication systems. As a result, engineers can better anticipate and mitigate potential disruptions caused by adverse weather conditions, ultimately leading to more robust and efficient network performance.

B. World Attenuation Case

In the attenuation calculus implemented in this example The methodology uses for the first time simplified rain attenuation model bases on **ITU-R P.838** considering also the cloud attenuation due to **ITU-R P.840** and the gaseous attenuation due to **ITU-R P.676**. In the P.840 I adopt the LWC (Liquid Water Content) variable associated with the frequency (f) information for calculate the cloud attenuation. In the gaseous attenuation The methodology uses Pressure (P) and Relative Humidity (H) and in the rain attenuation the Rain Rate. Those are simplified model that can be improved with more complex formulations.

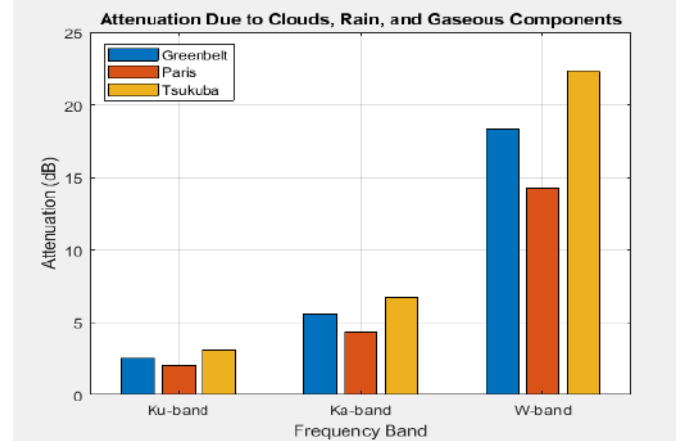


Fig. 6: Attenuation due to Clouds, Rain and gaseous component in the three bands: Ku, Ka and W for the Greenbelt, Paris and Tsukuba ground stations.

C. More realistic implementation

The enhanced formulas that I may implement permit me to adhere more closely to the ITU-R recommendations, making them more accurate for the satellite communications. The new models better reflect the impact of frequency on the attenuation estimation, especially for higher bands like Ka and W. The gaseous attenuation is split into Oxygen and Water Vapour (WV) components, capturing better the physical different

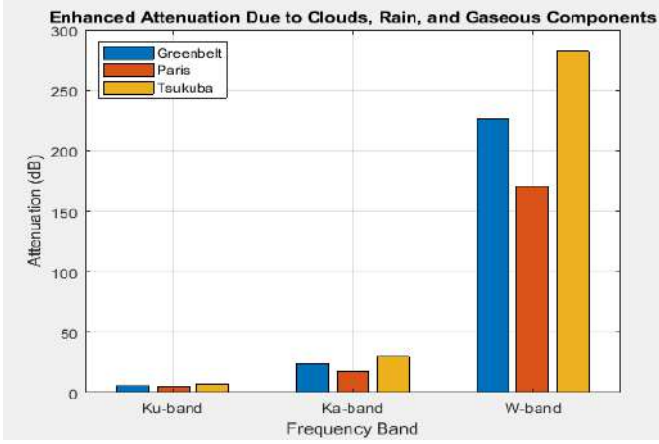


Fig. 7: Enhanced attenuation due to Clouds, Rain and Gaseous components in the more accurate implementation.

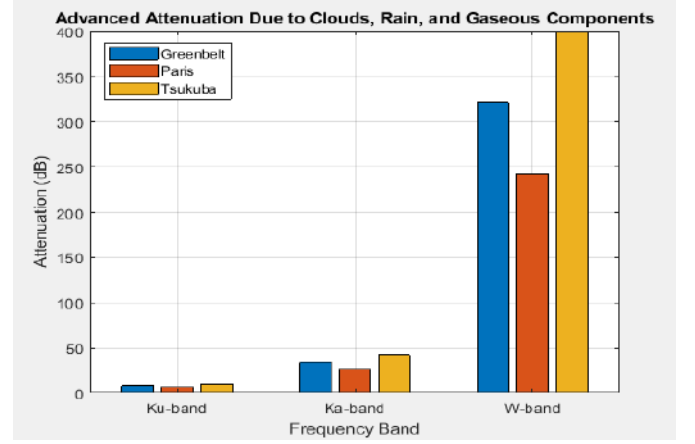


Fig. 8: More enhanced realistic model results.

processes. About the global applicability, the enhanced models can incorporate site-specific data for rain rates, liquid water content, temperature, pressure and relative humidity.

So respect the previous implementation, This study is able to distinguish oxygen and water vapor contributions, implementing the enhanced formula:

$$\gamma_{gas} = \gamma_{O_2} + \gamma_{H_2O} \quad (42)$$

where γ_{O_2} includes oxygen absorption dependence on pressure and temperature:

$$\gamma_{gas} = 0.02 \cdot f \cdot \frac{P}{1013} \cdot (1 - e^{-0.1 \cdot T}) \quad (43)$$

and where γ_{H_2O} models the water vapor absorption based on humidity and pressure:

$$\gamma_{H_2O} = 0.01 \cdot f \cdot \frac{H}{100} \cdot \frac{P}{1013} \quad (44)$$

Those implementation permit to improve accuracy by explicitly separating Oxygen and Water Vapor absorption and including temperature effects. So, the new models better reflect the impact of frequency on attenuation, capturing different physical processes and incorporating site-specific data for rain rates, LWC, temperature, pressure and relative humidity.

D. More advanced Models

This study is able to improve the models for obtaining some differences at the higher frequencies. About rain attenuation I may transitioned from a static model to an ITU-R P.838 based model. It is possible to introduce the slant path correction adding the elevation angle effect on attenuation calculus. Regarding this improvement I may pass from:

$$TotalAttenuation = (\gamma_{rain} + \gamma_{cloud} + \gamma_{gas}) \quad (45)$$

to the updated formulation, like this:

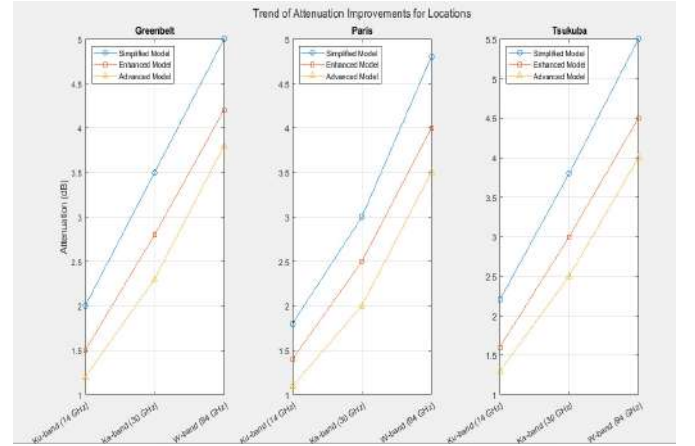


Fig. 9: This summary graph shows how the use of attenuation models is getting better in different places (like Greenbelt, Paris, and Tsukuba) and frequency bands (Ku, Ka, and W). The simplified model shows initial values with approximate formulas. The improved model utilizes ITU-R-based formulas with frequency-specific parameters. The advanced model considers inclined path corrections, separation of O_2 and H_2O , and advanced ITU-R formulas. Notice how attenuation values decrease with advanced models, reflecting greater accuracy and realism.

$$TotalAttenuation_{improved} = (\gamma_{rain} + \gamma_{cloud} + \gamma_{gas}) \cdot \frac{1}{\sin(\theta)} \quad (46)$$

where θ represents the elevation angle that incorporates slant path corrections, making the total attenuation more realistic for Earth-to-satellite links.

XI. DETAILS ON CLOUDSAT MISSION.

NASA launched the CloudSat mission on April 28, 2006, with the groundbreaking goal of studying the vertical structure of clouds to better understand their role in weather and climate. CloudSat was equipped with the **Cloud Profiling Radar (CPR)**, a 94-GHz nadir-looking radar that measured the power

backscattered by clouds as a function of distance from the radar. This advanced radar technology allowed scientists to obtain detailed vertical profiles of cloud liquid water and ice water content, providing critical data for improving weather forecasts and climate models [22].

CloudSat was part of NASA's Earth Observing System and operated in conjunction with other satellites in the A-Train constellation, including Aqua, PARASOL, and Aura. This collaboration enabled comprehensive observations of the Earth's atmosphere and climate. One of the key achievements of CloudSat was its ability to provide the first direct, global survey of the vertical structure and overlap of cloud systems, which was essential for understanding how clouds affect both local and large-scale environments [22].

Throughout its mission, CloudSat made several significant discoveries. It revealed the frequency of rain and snow in clouds above Earth, a fact previously unknown due to the absence of ground-based instrumentation in many regions, including over oceans and sparsely populated land areas. CloudSat also provided the first quantitative estimate of global snowfall and offered unique insights into the effect of cloudiness on the acceleration of polar and Greenland ice melting [22].

In addition to its contributions to understanding precipitation, CloudSat played a crucial role in studying the interactions between clouds and atmospheric aerosols, such as pollution and volcanic emissions. These interactions have significant implications for cloud lifetime and precipitation patterns. CloudSat's data helped scientists develop new techniques for estimating the intensity of hurricanes from space and improved my understanding of the links between pollution and rainfall [22].

The mission's success was not without challenges. In 2011, CloudSat experienced a battery malfunction that limited its operations to daytime-only, requiring sunlight to power the radar. Despite this setback, the mission continued to provide valuable data until the deactivation of the radar on December 20, 2023. CloudSat's contributions to atmospheric science and climate research have been invaluable, and its legacy continues to influence ongoing studies and future missions [22].

Initially scheduled to last for 22 months, CloudSat's primary mission allowed for the observation of multiple seasonal cycles. However, the mission far exceeded its planned duration, operating for over 17 years. During this time, CloudSat provided critical data that enhanced my understanding of cloud processes and their impact on the Earth's climate system [22].

NASA's Jet Propulsion Laboratory (JPL) in Pasadena, California, developed the Cloud Profiling Radar (CPR) on CloudSat, with hardware contributions from the Canadian Space Agency. The design of the CPR was based on existing radar expertise and experience at JPL, drawing from the heritage of cloud radars already in operation in ground-based and airborne applications. This expertise ensured the reliability and accuracy of the data collected by CloudSat [22].

CloudSat's data has been instrumental in improving the representation of clouds in atmospheric models, leading to more accurate weather forecasts and climate predictions. The mission's findings have also contributed to a deeper understanding

of the Earth's water and energy cycles, which are critical for predicting future climate changes and their potential impacts on global ecosystems [22].

In summary, the CloudSat mission has been a remarkable success, providing unprecedented insights into the vertical structure of clouds and their role in the Earth's climate system. Its contributions to atmospheric science have paved the way for future research and advancements in weather forecasting and climate modeling. The legacy of CloudSat will continue to shape my understanding of the complex interactions between clouds, aerosols, and the Earth's climate for years to come.

A. The CloudSat Instrumentation

The CloudSat mission, launched by NASA on April 28, 2006, was a pioneering effort to study the vertical structure of clouds and their impact on weather and climate. The primary instrument on CloudSat was the Cloud Profiling Radar (CPR), a 94-GHz nadir-looking radar developed at NASA's Jet Propulsion Laboratory (JPL) in Pasadena, California, with hardware contributions from the Canadian Space Agency. The CPR measured the power backscattered by clouds, providing detailed vertical profiles of cloud liquid water and ice water content. This data was crucial for improving weather forecasts and climate models. CloudSat operated in a sun-synchronous polar orbit at an altitude of approximately 705 km, with an inclination of 98.2 degrees. The mission was part of NASA's Earth Observing System and flew in formation with other satellites in the A-Train constellation, including Aqua, PARASOL, and Aura2. This collaboration allowed for comprehensive observations of the Earth's atmosphere and climate. One of the significant technical considerations for CloudSat was its impact on radio astronomy. The power levels of the CloudSat radar were such that they could potentially damage the receiver electronics of radio telescopes if the telescope was pointing at the zenith during an overflight. I took operational precautions to mitigate this risk, preventing receiver damage and flagging data contaminated by radar interference.

CloudSat's mission duration far exceeded its planned 22 months, operating for over 17 years. Despite challenges like a 2011 battery malfunction that restricted operations to daytime only, the mission persistently provided valuable data until the deactivation of the radar on December 20, 2023. The data collected by CloudSat has been instrumental in improving the representation of clouds in atmospheric models, leading to more accurate weather forecasts and climate predictions. In addition to its contributions to understanding precipitation, CloudSat played a crucial role in studying the interactions between clouds and atmospheric aerosols, such as pollution and volcanic emissions. These interactions have significant implications for cloud lifetime and precipitation patterns. CloudSat's data helped scientists develop new techniques for estimating the intensity of hurricanes from space and improved my understanding of the links between pollution and rainfall. Overall, the CloudSat mission has been a remarkable success, providing unprecedented insights into the vertical structure of clouds and their role in the Earth's climate system. Its

contributions to atmospheric science have paved the way for future research and advancements in weather forecasting and climate modeling. The legacy of CloudSat will continue to shape my understanding of the complex interactions between clouds, aerosols, and the Earth's climate for years to come.

B. Data Reanalysis

Data reanalysis is a critical process in atmospheric science that involves the integration of observational data from various sources to produce a comprehensive and consistent dataset. This process is essential for understanding long-term climate trends, validating climate models, and improving weather forecasts. One of the key sources of observational data for reanalysis is satellite missions like CloudSat. NASA launched CloudSat in 2006 with the Cloud Profiling Radar (CPR), a 94-GHz nadir-looking radar that provided detailed vertical profiles of cloud liquid water and ice water content. The data collected by CloudSat has been invaluable for reanalysis efforts, offering insights into the vertical structure of clouds and their impact on weather and climate.

CloudSat's data has been particularly useful for studying the vertical distribution of clouds and precipitation, which are critical components of the Earth's climate system. By providing high-resolution vertical profiles, CloudSat has enabled scientists to better understand the processes that govern cloud formation, development, and dissipation. This information is crucial for improving the accuracy of climate models, which rely on accurate representations of cloud processes to predict future climate changes. Additionally, The author has used CloudSat's data to validate and refine other satellite-based observations, ensuring the accuracy and reliability of the reanalysis datasets.

The integration of CloudSat data into reanalysis efforts has also highlighted the importance of continuous and long-term observations. Over 17 years of CloudSat's mission provided a wealth of data that tracked changes in cloud properties and precipitation patterns over time. This long-term dataset is essential for detecting trends and anomalies in the Earth's climate system, allowing scientists to identify the impacts of natural variability and human activities on the atmosphere. The reanalysis process will continue to evolve with the launch of new satellite missions and the development of more advanced instruments, offering even more accurate and comprehensive datasets for climate research and weather forecasting.

C. Physical Measurements

Physical measurements in atmospheric science are essential for understanding the various processes and phenomena that occur within the Earth's atmosphere. It is possible to broadly categorize these measurements into two types: in situ and remote sensing techniques. In-situ measurements involve direct sampling of the atmosphere using instruments such as thermometers, barometers, hygrometers, and anemometers. These instruments provide data on temperature, pressure, humidity, and wind speed and direction, respectively. Weather stations, aircraft, and balloons often conduct in-situ measurements, providing high-resolution data at specific locations and altitudes.

Remote sensing techniques, on the other hand, involve the use of instruments that measure atmospheric properties from a distance. These techniques can be ground-based, airborne, or satellite-based. Ground-based remote sensing instruments include radar, lidar, and radiometers, which can measure cloud properties, aerosol concentrations, and atmospheric composition. Airborne remote sensing involves instruments mounted on aircraft, providing data over larger areas and at various altitudes. Satellite-based remote sensing, such as the Cloud Profiling Radar (CPR) on CloudSat, provides global coverage and continuous monitoring of atmospheric properties. The CPR measures the power back-scattered by clouds, offering detailed vertical profiles of cloud liquid water and ice water content, which are crucial for understanding cloud processes and their impact on weather and climate.

The integration of physical measurements from both in-situ and remote sensing techniques is essential for comprehensive atmospheric studies. These measurements provide valuable data for validating and improving climate models, weather forecasts, and reanalysis efforts. By combining data from various sources, scientists can obtain a more accurate and complete picture of the Earth's atmosphere, leading to better predictions of weather patterns, climate change, and their potential impacts on global ecosystems. The continuous development and advancement of measurement techniques and instruments will further enhance my understanding of atmospheric processes and contribute to more accurate and reliable atmospheric science research.

XII. MORE RECENT MISSIONS.

Recent satellite missions have continued to advance my understanding of atmospheric processes and climate change. One notable mission is the Sentinel-5P satellite, launched by the European Space Agency (ESA) in 2017. The TROPospheric Monitoring Instrument (**TROPOMI**) on Sentinel-5P measures trace gases like ozone, nitrogen dioxide, and methane with high precision [24]. This data is crucial for monitoring air quality, assessing the impact of human activities on the atmosphere, and supporting climate research.

Another significant mission is the **Geostationary Environment Monitoring Spectrometer (GEMS)**, launched by South Korea in 2020. Dedicated to monitoring air quality in East Asia, GEMS is the first geostationary satellite. It provides continuous observations of atmospheric pollutants, including ozone, sulfur dioxide, and fine particulate matter [25]. The data from GEMS helps improve air quality forecasts, supports environmental policy-making, and enhances my understanding of the sources and transport of air pollution.

In addition to these missions, the **Global Precipitation Measurement (GPM)** mission, a joint effort between NASA and the **Japan Aerospace Exploration Agency (JAXA)**, has been providing valuable data on precipitation patterns since its launch in 2014. GPM's advanced instruments measure rainfall and snowfall with high accuracy, contributing to improved weather forecasts, climate models, and disaster response efforts [26]. These recent missions, along with ongoing advancements in satellite technology, continue to play a vital

role in enhancing my understanding of the Earth's atmosphere and addressing global environmental challenges. As scientists analyze the data collected by GPM, they are uncovering trends that could significantly impact agriculture, water resource management, and urban planning. This collaboration not only strengthens international partnerships in space exploration but also underscores the importance of shared knowledge in tackling pressing global issues.

CONCLUSIONS.

Given the nature of the work and its technical context as part one of a series, here is a suitable conclusion: this document serves as the foundational technical overview in a three-part series exploring the intricacies of global atmospheric attenuation analysis, with a specific focus on the Ku, Ka, and W bands. By examining key satellite missions, their instrumentation, and the datasets they provide, The author has established the groundwork for understanding the atmospheric phenomena affecting electromagnetic signal propagation. Future installments will delve deeper into advanced modeling techniques, simulation studies, and their applications to practical scenarios. This progression underscores the significance of integrating observational data with theoretical frameworks to enhance communication system reliability and address the challenges posed by atmospheric variability. Furthermore, as An analysis is conducted the interplay between atmospheric conditions and signal integrity, it becomes increasingly clear that ongoing research is essential for developing adaptive technologies. These innovations will not only improve signal resilience but also pave the way for more robust communication networks in ever-changing environmental conditions. In this context, collaboration among researchers, engineers, and policymakers will be crucial to facilitate the exchange of knowledge and best practices. By fostering interdisciplinary approaches, It is possible to better equip my communication systems to withstand the unpredictable nature of atmospheric influences and ensure seamless connectivity for all users.

XIII. AUTHOR BIOGRAPHY



Paolo Cicolin was born in 1971 in Turin. In October 2012 and December 2018, respectively, he graduated with a Bachelor degree in Electronic Engineering and a Master of Science in Communication Engineering from Roma Tre University in Italy. From October 2020 to October 2021, he enrolled at Roma Tre University's second-level master's program in "Cybersecurity for the Protection of Control Systems in Industry 4.0 and Critical Infrastructures". The Accenture S.p.A. scholarship was awarded to him for his studies in the cybersecurity course. La Sapienza University of Rome

welcomed him as a Ph.D. student in November 2021. He was employed from April 2019 to July 2020 as a GIS consultant and in E-Geos S.p.A. (an ASI-Telespazio company), a major global participant in the geospatial industry. His areas of interest and research are geoscience, telecommunications, radar and remote sensing, microwave communications, antennas & propagation, telesensing, and severe event model prediction.

CONFLICT OF INTEREST

The author declares that he has no conflicts of interest.

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A.4 Second Technical Article - personal style

Title: Technical Analysis of Satellite Coincidence Datasets for the Creation of Attenuation Maps.

Technical Analysis of Satellite Coincidence Datasets for the Creation of Attenuation Maps

Paolo Cicolin, IEEE Student Member.

Abstract—This second part delves into the detailed role of satellites and their instrumentation in the analysis of atmospheric attenuation, with a focus on the significance of satellite coincidences. By integrating datasets from CloudSat, TRMM, and GPM satellites, this study highlights the potential of multi-satellite observations to explore cloud microphysics, atmospheric attenuation, and precipitation processes across diverse climate regions. The synergy between satellite instruments uncovers patterns and correlations that improve my understanding of atmospheric dynamics and enhance predictive accuracy.

Advanced methodologies such as data harmonization, machine learning, multivariate analysis, and Monte Carlo simulations are employed to ensure data consistency, reliability, and usability. The integration of triple-frequency radar data with advanced radiometers enables precise modeling of specific attenuation and retrieval of precipitation characteristics, especially for light rain, snow, and mixed-phase hydrometeors. High-resolution attenuation maps generated from these analyses are instrumental in advancing satellite communication, climate modeling, and weather forecasting.

This study also introduces innovative techniques, including dynamic refractive index modeling and Mie vs. Rayleigh scattering comparisons, to deepen my understanding of electromagnetic interactions with hydrometeors. By addressing critical challenges like refining precipitation algorithms and predicting extreme weather events, the findings significantly enhance my knowledge of global hydrological processes and atmospheric behavior. This research underscores the transformative potential of collaborative satellite missions, paving the way for improved climate simulations, resilient telecommunication systems, and a better understanding of the Earth's changing atmosphere.

Index Terms – Exploring Coincidence Datasets, Satellite Coincidence Datasets, Atmospheric Attenuation Analysis, Understanding Atmospheric Phenomena, Satellite Instrument Collaboration, Accurate Atmospheric Predictions, Usability of Coincidence Data.

I. INTRODUCTION

In the second part of my study, I delve deeper into the complexities of satellite observations and instrumentation, emphasizing the critical role of satellite coincidences in analyzing atmospheric attenuation. By leveraging key coincidence datasets, an analysis is conducted on their applications and the ways they enhance my understanding of atmospheric phenomena. When satellite instruments like CloudSat, TRMM, and GPM work together, they can find patterns and correlations that make predictions about the atmosphere much more accurate.

This in-depth study sheds light on the microphysical features of clouds and precipitation and gives useful information about how radiation and precipitation particles interact with

each other. By rendering coincidence data accessible for extended research, I address a range of scientific and technological challenges, contributing to advancements in my knowledge of Earth's atmospheric processes. Such advancements have the potential to foster the development of more efficient weather forecasting models and improved climate change simulations. Additionally, this research enables better assessments of the impacts of extreme weather events on ecosystems and human infrastructure, ultimately paving the way for more effective mitigation strategies.

To make attenuation maps using satellite coincidence datasets, you need to carefully look at data from many different sources, as each one gives you a different picture of how signals travel through the atmosphere and how strong they are. Attenuation, which is brought on by things like clouds, rain, and gases in the atmosphere, is a very important factor for satellite communication systems, especially at high frequencies like the Ka and W bands. Leveraging data from coincident satellite observations allows for a comprehensive understanding of these effects across geographic regions and temporal scales.

The first step involves identifying and integrating datasets from satellites equipped with relevant sensors. Instruments like the Dual-Frequency Precipitation Radar (DPR) on GPM, the Cloud Profiling Radar (CPR) on CloudSat, and passive radiometers like GMI provide complementary information. To get a good correlation between measured parameters like rainfall intensity, cloud microphysics, and radiative properties, these datasets need to be put together in the same place and at the same time.

Data preprocessing is crucial for ensuring consistency and usability. Some of the steps are regriding the data to a common resolution, aligning the swath geometries, and fixing any biases that come from the different sensor characteristics. I employ advanced algorithms to filter noise and interpolate missing data, thereby preserving the physical integrity of the measurements. The methodology uses the preprocessed datasets to estimate specific attenuation (e.g., dB/km) due to hydrometeors. Models such as the Mie scattering theory or parameterized algorithms tailored for specific frequency bands can calculate attenuation values. I validate these estimates against ground-based observations, like radar or disdrometer data, to ensure their reliability. I then apply spatial analysis techniques to generate attenuation maps. Geographic Information Systems (GIS) and statistical tools make it possible to see how signal loss varies across regions, pointing out places where signal loss happens often or badly. Seasonal and diurnal variability, as well as correlations with meteorological phenomena like monsoons or cyclones, are key features of

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such maps.

Finally, these attenuation maps serve multiple applications, including satellite link budget planning, telecommunication system design, and climate studies. By providing a high-resolution understanding of attenuation, these maps enable engineers and scientists to mitigate communication losses and better understand atmospheric dynamics. The integration of coincidence datasets from satellites thus plays a pivotal role in advancing both technology and meteorological research.

II. ABOUT SATELLITE COINCIDENCE DATASETS

The advent of satellite technology has revolutionized my understanding of Earth's atmospheric processes. Among the many advancements, the concept of satellite coincidences stands out as a pivotal development in remote sensing. Satellite coincidences occur when the orbital paths of two or more satellites intersect, allowing for near-simultaneous observations of the same geographical area. This unique capability has enabled researchers to combine data from different satellite sensors, providing a more comprehensive and accurate picture of atmospheric conditions. Over the years, satellite coincidence datasets have gone through a number of important stages. Each of these stages has helped me improve my observational methods and my predictive models.

The early years of satellite remote sensing saw the launch of pioneering missions such as the Tropical Rainfall Measuring Mission (TRMM) and CloudSat. TRMM, launched in 1997, was the first satellite dedicated to measuring tropical rainfall and provided invaluable data on precipitation patterns in the tropics. CloudSat, launched in 2006, introduced the Cloud Profiling Radar (CPR), which offered detailed vertical profiles of cloud structures. When TRMM and CloudSat's orbital paths crossed, they were able to collect data at the same time. This gave researchers a level of detail that had never been seen before when studying how clouds and rain interact. These early datasets laid the groundwork for subsequent missions and highlighted the potential of satellite coincidences in advancing atmospheric science.

In 2014, the Global Precipitation Measurement (GPM) mission launched, building on the successes of TRMM and CloudSat. The Dual-Frequency Precipitation Radar (DPR) was created by GPM. It works at both Ku-band and Ka band frequencies, which makes it better at measuring precipitation because it has higher resolution and sensitivity. The near-coincident observations between GPM and CloudSat have further expanded my understanding of precipitation processes, particularly in cold-season and light rain conditions. Researchers have been able to make more accurate algorithms for retrieving precipitation and find new uses for multi-frequency radar data from these missions in climate studies and weather forecasting. As I continue to refine my observational capabilities, the historical evolution of satellite coincidence datasets remains a testament to the power of collaboration and innovation in atmospheric research.

Synchronizing coincidence data between satellites such as CloudSat, TRMM, and GPM is a complex process that requires precise temporal and spatial coordination. CloudSat, TRMM, and GPM satellites follow different orbits, but

their trajectories periodically intersect, allowing for near-simultaneous data collection. This synchronization process begins with the definition of a time window, which can range from a few minutes to several hours, during which satellite observations are considered coincident [32]. Spatial synchronization, on the other hand, relies on the alignment of satellite footprints, which are the areas of the Earth's surface observed by satellite sensors. This alignment is crucial to ensure that the data collected is comparable and can be integrated consistently [33].

Once they are in sync, coincidence data are used to find atmospheric correlations by looking at things like temperature, humidity, reflectivity, and the type of hydrometeor. Multivariate analysis is a statistical technique that enables the simultaneous examination of multiple variables to uncover relationships and patterns within the data. For example, analyzing correlations between radar reflectivity and precipitation rate can provide valuable information on precipitation structure and intensity [34]. Using machine learning algorithms like neural networks and support vector machines also makes it possible to sort and guess the kinds of hydrometeors that can be seen from space [35].

When I combine coincidence data from CloudSat, TRMM, and GPM, I get a clearer picture of how the atmosphere works, which helps me learn more about how clouds and rain move. These combined data allow for the development of more accurate models for precipitation forecasting and the study of complex atmospheric phenomena such as convective storms and light snowfall. To figure out the errors and confidence intervals of the derived parameters, it is also necessary to measure the unknowns using methods like Bayesian inference and Monte Carlo simulations. This integrated approach not only improves the precision of satellite observations but also provides a solid basis for further studies and applications in meteorology and climate.

A. Brief Brainstorming

The product 2D-CloudSat-TRMM [46] is related to the Coincidence between CloudSat and the TRMM satellites. The specific above product contains CloudSat Cloud Profiling Radar (CPR) and TRMM Precipitation Radar (PR) data subset around ground-track intersections that occur within a defined time limit [15]. This product was available from 2006 though 2010. The term "coincidence" in this context refers to the times when the orbital ground tracks of the CloudSat and TRMM satellites intersect their orbits. These coincidences allow for the collection of near-simultaneous data from both satellites, providing unique insights into atmospheric conditions [16]. For an in-depth exploration of the satellite coincidences between CloudSat and TRMM, please consult the references listed in the bibliography section for comprehensive details [14].

B. Information about Satellite Coincidences

F. Joseph Turk et.al. (February 28, 2014) [16], explains a lot of information about GPM and DPR in the Ku and Ka Band and about the GPM and CloudSat CPR in W-Band.

In the main F.J.Turk article, is described the coincidence data products about the Coincidence CloudSat-TRMM (CloudSat + PR/TMI) and CloudSat-GPM (CloudSat + DPR/GMI) observational periods, available from NASA Precipitation Processing System (PPS). These combined multi-frequency radar/passive MW (millimeter-Wave) data are useful for many applications related to precipitation estimation.

C. Coincidence Details

In reference to satellite coincidences, my analysis takes into account both synchronous and asynchronous satellites. Sun synchronous satellites are distinguished from non-synchronous ones primarily by their ability to maintain a consistent angle with the star, which enables consistent observation of the underlying surface from a single point of view. On the other hand, non-synchronous satellites can be collected at different times of the day, which is advantageous for applications involving rainfall sampling. Approximately sixteen times a day, the asynchronous GPM (Global Precipitation Measurement) satellite orbits the Earth. [17]

As the combination of the resultant of the three pseudo-frequencies of radar profiles (the W band from CPR, the Ku/Ka band from GPM, and the 13-channel GMI radiometer (10-183 GHz), the intersections (or “coincidence”) between the GPM core satellite and the CloudSat profiling radar (CPR) at 94 GHz (W band), within sufficiently small time differences, are of particular interest.

D. Spatial coincidence rule

The spatial coincidence is critical in real-world applications: it ensures that satellites are observing the same geographical region or footprint. Two observations are spatially coincident if the footprints of their sensors overlap within a certain spatial tolerance (e.g., 10 km). To calculate coincidences, It is possible to employ the *haversine* formula to determine the distance between the latitude and longitude coordinates of both footprints. This method allows me to accurately measure the separation between the satellite observations, ensuring precise alignment for coincident data analysis. The author has:

$$distance = 2 \cdot R \cdot \arcsin(\sqrt{haversine}) \quad (1)$$

where “*haversine*” depends on the latitudes and longitudes of the two locations, and R is Earth’s radius. In a possible implementation, It is possible to set the spatial tolerance as 10 km, and after that I may combine temporal and spatial conditions for filtering and counting the coincidences:

$$coincident = (abs(time_{difference}) \leq time_{tolerance} \& (distance \leq spatial_{tolerance}); \quad (2)$$

After identifying coincident observations, you can further refine the results by considering additional variables such as:

- Reflectivity, precipitation rate, or temperature.
- Thresholds can be applied to ensure data quality.

In cases where multiple coincidences exist for a single times-tamp (e.g., due to overlapping orbits), The methodology uses a

rule to select the closest match based on the smallest temporal difference.

A significant application for CloudSat-GPM coincidences is the study of seasonal light precipitation and snowfall. Researchers can get better pictures of seasonal light rain and snowfall by using observations from CloudSat’s Cloud Profiling Radar (CPR) and GPM’s Dual Frequency Radar (DPR) that happen almost at the same time. With these one-of-a-kind triple-frequency data, It is possible to look more closely at the microphysics of ice, light precipitation, and surface emissivity effects than with data from just one satellite [16].

This application is particularly useful for improving my understanding of atmospheric processes in regions with frequent snowfall or light rain, such as polar areas and mountainous regions. CloudSat-GPM coincidences give me a lot of detailed information that It is possible to use to make better models for predicting rain and to study how ice particles and radiation interact. Furthermore, this data can help improve precipitation retrieval algorithms by identifying and correcting shortcomings in current forecast models.

E. Applications

Applications that leverage combined **Global Precipitation Measurement (GPM)** and **CloudSat** data are critical to improving my understanding of precipitation and atmospheric processes. These integrated data enable me to address a variety of scientific and technological challenges, including [47]- [11]:

- **Algorithm Evaluation (AE):** Identifying and correcting deficiencies in precipitation retrieval algorithms.
- **Snow and Light Rain Sensitivity Studies (SLRSS):** Improving the detection and measurement of light rain and snowfall.
- **Cloud Process Studies (CPS):** Understanding the microphysical properties of clouds and precipitation.
- **Radiative Transfer Simulations (RTS):** Investigating the interplay between radiation and precipitation particles.
- **Surface Emissivity Effects (SEE):** Investigating how different surface types affect radar and radiometer measurements.

Applications that leverage combined Global Precipitation Measurement (GPM) and CloudSat data are critical to improving my understanding of precipitation and atmospheric processes. These integrated data enable me to address a variety of scientific and technological challenges. One such challenge is algorithm evaluation, where combined datasets help identify and correct deficiencies in precipitation retrieval algorithms. By comparing the outputs of different algorithms to coincident observations from GPM and CloudSat, researchers can refine these algorithms to improve their accuracy and reliability. Another important application is sensitivity studies to snow and light rain. High-resolution data from GPM and CloudSat enable better detection and measurement of light rain and snowfall, which are often difficult to observe with traditional radar systems. This improved sensitivity is critical to understanding precipitation patterns in regions with frequent light rain or snowfall.

Studies of cloud processes also benefit significantly from combined datasets. Researchers can study the microphysical properties of clouds and precipitation with the help of CloudSat's detailed vertical profiles of cloud structures and GPM measurements of precipitation. This information is essential to understanding the formation and evolution of clouds, as well as their impact on weather and climate. Radiative transfer simulations are another area where the combined data prove invaluable. By studying the interaction between radiation particles and precipitation, researchers can better understand how clouds and precipitation affect the Earth's radiation budget. This knowledge is essential to improving climate models and predicting future climate change.

Finally, I study the effects of surface emissivity using the combined datasets. Different surface types, such as forests, deserts, and urban areas, can affect radar and radiometric measurements. By analyzing GPM and CloudSat data, researchers can study how these surface types affect the observed signals and develop correction algorithms to account for their effects. This improves the accuracy of precipitation measurements and increases my understanding of the interactions between the land surface and the atmosphere. Overall, combining GPM and CloudSat data gives researchers a full and detailed picture of how the atmosphere works, which helps them solve many scientific and technological problems.

1) *Enhancing Precipitation Retrieval Algorithms through Multi-Satellite Coincidence Datasets* : I need to test and improve algorithms for retrieving precipitation in order to learn more about how the atmosphere works and make satellite-based precipitation data more reliable. **Algorithm Evaluation (AE)** focuses on identifying deficiencies, such as biases in precipitation intensity, misclassification of hydrometeors, and inaccuracies in light or convective rainfall estimations. By using advanced methods such as machine learning, data assimilation, and physics-informed neural networks, AE can change its algorithms on the fly to take into account changes in location, cloud microphysics, and sensor limitations. When you compare multi-satellite coincidence datasets like CloudSat and GPM, for example, you can fix problems found by recalibrating algorithms with measurements taken on-site or high-resolution models. In addition, AE lets you add real-time feedback loops, in which new data constantly changes and improves retrieval parameters. This iterative method not only makes the algorithm more reliable, but it also encourages new ways of dealing with extreme weather events, light snowfall, and mixed-phase precipitation. This makes it possible for more accurate climate models and useful information about how the global water cycle works.

2) *Advancing Precipitation Understanding through Snow and Light Rain Sensitivity Studies (SLRSS) with CloudSat and GPM Data.*: **Snow and Light Rain Sensitivity Studies (SLRSS)** are crucial for enhancing my understanding of precipitation processes, particularly in regions where light rain and snowfall are prevalent. Traditional radar systems often struggle to detect and accurately measure light precipitation due to their limited sensitivity and resolution. However, the integration of data from advanced satellite missions like CloudSat and GPM has significantly improved my ability

to observe and analyze these phenomena. CloudSat's Cloud Profiling Radar (CPR) operates at the W band frequency, which is highly sensitive to small hydrometeors, making it ideal for detecting light rain and snowfall [16]. Similarly, GPM's Dual-Frequency Precipitation Radar (DPR) works at both Ku-band and Ka band frequencies, making observations that are complementary and better able to find light precipitation. By combining these datasets, researchers can obtain a more comprehensive view of light rain and snowfall, leading to better precipitation estimates and improved weather forecasts.

One of the key challenges in SLRSS is accurately quantifying the amount and distribution of light precipitation. Light rain and snowfall often occur in shallow, stratiform clouds, which can be difficult to observe with ground-based radar systems. CloudSat's CPR gives researchers high-resolution vertical profiles that let them study the **microphysical properties** of these clouds in great detail. For example, they can look at the size and concentration of hydrometeors. This information is essential for understanding the processes that govern the formation and evolution of light precipitation. The dual-frequency observations from GPM's DPR also make it possible to tell the difference between different kinds of hydrometeors, like snow, rain, and mixed-phase precipitation. The presence of mixed-phase hydrometeors can bias precipitation retrieval algorithms, making this capability particularly important. By leveraging the strengths of both CloudSat and GPM, SLRSS can provide more accurate and reliable measurements of light precipitation, which are essential for improving weather and climate models.

Furthermore, SLRSS plays a vital role in advancing my understanding of the impacts of **light precipitation** on the environment and society. Light rain and snowfall contribute significantly to the overall water budget in many regions, particularly in high-latitude and mountainous areas. Accurate measurements of light precipitation are essential for water resource management, agriculture, and hydrological modeling. Moreover, light precipitation events can have significant implications for transportation and infrastructure, as even small amounts of snow or ice can disrupt travel and pose safety hazards. By improving the detection and measurement of light rain and snowfall, SLRSS can help mitigate these impacts and enhance my ability to respond to weather-related challenges. Combining CloudSat and GPM satellite data is a powerful way to move SLRSS forward. It helps researchers make more accurate algorithms for retrieving precipitation, make better weather predictions, and learn more about the global water cycle. This integrated approach not only enhances my understanding of precipitation patterns but also aids in the development of more effective strategies for managing water resources. As climate change continues to alter weather phenomena, the insights gained from SLRSS will be crucial in preparing for and adapting to these evolving conditions.

3) *Study of the microphysical properties of clouds*: The **microphysical properties of clouds**, such as droplet size and particle distribution, play a crucial role in atmospheric attenuation. For instance, the size of raindrops directly influences the absorption and scattering of electromagnetic energy in the atmosphere. Larger droplets tend to cause greater attenuation

due to their ability to absorb and reflect electromagnetic waves. This phenomenon is particularly evident in the higher frequency bands, such as the -Ka band and -W band, used in weather radar. The distribution of droplet sizes within a cloud can vary greatly, affecting the accuracy of radar measurements and the ability to predict weather events such as heavy rain and storms.

The distribution of particles in clouds is also important for understanding atmospheric attenuation. Aerosol particles, such as dust, soot, and **microplastics**, can act as condensation nuclei, influencing the formation and growth of cloud droplets. The presence of aerosol particles can increase the number of small droplets, which in turn can increase atmospheric attenuation. In addition, aerosol particles can affect the reflectivity of clouds, altering the Earth's radiation budget and contributing to climate change. Recent studies have shown that microplastics in the atmosphere can modify cloud formation and influence precipitation patterns, raising concerns about the **future global climate** [36].

Analyzing the microphysical properties of clouds requires the use of advanced observation and modeling techniques. Dual-polarization weather radars, for example, can provide detailed information on the size and shape of raindrops, allowing researchers to study atmospheric attenuation with greater precision. Numerical models and computer simulations also let me look at how aerosol particles and cloud droplets interact with each other, which helps me learn more about how clouds form and how rain falls.

The combination of observational data and numerical models is essential to develop more accurate weather forecasts and to address the challenges related to climate change.

4) *Cloud Process Studies (CPS)* : **Cloud Process Studies (CPS)** are essential for understanding the microphysical properties of clouds and precipitation, which play a critical role in weather and climate systems. One of the main goals of CPS is to study how cloud particles like cloud droplets, ice crystals, and mixed-phase hydrometeors form, grow, and break apart. By analyzing the size, shape, and concentration of these particles, researchers can gain insights into the processes that govern cloud dynamics and precipitation formation. Advanced satellite missions like CloudSat and GPM provide high-resolution vertical profiles of cloud structures, allowing scientists to study the microphysical properties of clouds in unprecedented detail [32]. These observations are crucial for improving my understanding of cloud microphysics and for developing more accurate models of cloud and precipitation processes.

Another important aspect of CPS is the study of cloud-aerosol interactions, which have significant implications for both weather and climate. Aerosols, such as dust, soot, and sea salt, can act as cloud condensation nuclei (CCN) or ice-nucleating particles (INP), influencing the formation and properties of clouds. For example, an increase in aerosol concentration can lead to the formation of smaller and more numerous cloud droplets, which can affect **cloud albedo** and lifetime [37]. This phenomenon, known as the "**aerosol indirect effect**", has important implications for the Earth's radiation budget and climate. Researchers can study the com-

plicated interactions between aerosols and clouds by using ground-based measurements, numerical models, and satellite observations [38]. This helps me learn more about how aerosols affect the microphysics of clouds and precipitation.

CPS also involves the investigation of cloud radiative properties and their influence on the Earth's energy balance. Clouds play a dual role in the Earth's radiation budget: they reflect incoming solar radiation, which cools the Earth's surface, and they trap outgoing longwave radiation, which warms the surface. The net effect of clouds on the Earth's energy balance depends on their microphysical properties, such as droplet size, ice crystal shape, and water content [39]. By studying these properties, researchers can better understand how clouds influence the Earth's climate and how they respond to changes in atmospheric conditions. Satellite missions like CloudSat and GPM provide valuable data on cloud radiative properties, enabling scientists to quantify the radiative effects of clouds and improve climate models. This research is essential for predicting future climate changes and for developing strategies to mitigate their impacts.

5) *Different types of Rainfall.*: Different types of rainfall and droplet sizes vary significantly depending on the climatic characteristics and atmospheric processes involved. Convective rain, typical of tropical areas and summer storms, is characterized by high intensity and short duration, with large drops ranging between 1 mm and 5 mm. This type of rain has a gamma distribution with high values of N_0 and μ and a rapid decay (Λ) for larger drops. Stratiform rain, on the other hand, is more common in milder areas and is linked to warm fronts or low-pressure systems. It has weak to moderate intensity and drops that are smaller and more uniform, measuring between 0.1 mm and 2 mm in diameter. The gamma distribution in this case shows lower values of N_0 and μ , with a slower decay.

Monsoon rain, typical of tropical and subtropical regions during the monsoon season, is characterized by long duration and high intensity, with intermediate droplet sizes between 0.5 mm and 3 mm. Lastly, drizzle, which is common near water and in foggy conditions, has droplets that are less than 0.5 mm across and a strong preference for small droplets in the gamma distribution. Droplet sizes also vary according to climatic zones: in tropical regions, large droplets (> 3 mm) prevail in convective rain, while in temperate regions, the size varies between 0.5 mm and 3 mm. In polar regions, light precipitation, like snow or sleet, has very small droplets (< 0.5 mm). In arid regions, on the other hand, rare and intense precipitation has droplets of different sizes, with more large droplets because they condense more quickly.

Figure 1 illustrates the droplet size distribution across various climate zones, including tropical, temperate, polar, and arid regions. Each curve represents a gamma distribution, parameterized to reflect the typical droplet sizes, concentrations, and decay rates observed in these climates. Tropical regions exhibit the highest concentrations of larger droplets due to convective rainfall, while polar regions show smaller droplets with a lower overall concentration. The plot highlights how climatic conditions influence the microphysical properties of precipitation, providing critical insights into global hydrological processes.

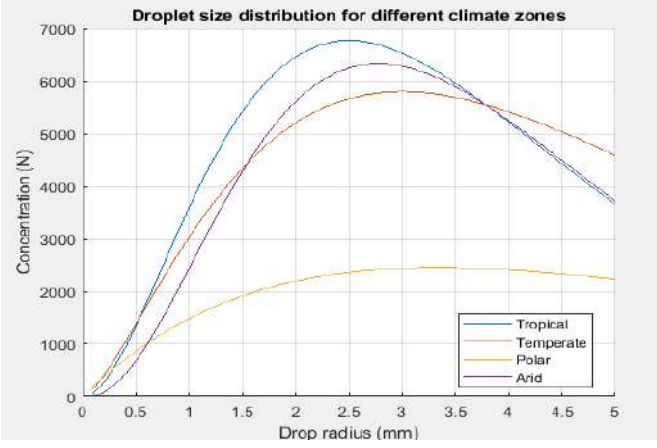


Fig. 1: Droplet size distribution for different climate zones. The graph illustrates the concentration of droplets (N) as a function of droplet radius (mm) for various climate zones: Tropical, Temperate, Polar, and Arid. The parameters N_0 , μ , and Λ represent the initial number of particles, shape parameter, and decay parameter, respectively. The distribution shows how droplet sizes vary across different climatic conditions, with Tropical zones having larger droplets and higher concentrations compared to Polar zones, which have smaller droplets and lower concentrations.

6) *Innovative Approaches to Correlating Droplet Size in Global Climate Zones with Attenuation in Ku, Ka, and W Bands*: Looking into the link between the size of raindrops in different climate zones and microwave attenuation in the Ku, Ka, and W bands can help make satellite communications, weather models, and climate monitoring better.

The analysis of droplet size distribution and specific attenuation is fundamental to understanding how microwaves interact with precipitation in different climate zones. Droplet size affects microwave attenuation in a nonlinear manner, with effects particularly pronounced at high frequencies such as -Ka and -W. I may define the specific attenuation as:

$$k(f) := \int_0^{+\infty} \sigma(f, r) \cdot N(r) \cdot dr \quad (3)$$

where $\sigma(f, r)$ is the scattering cross section at frequency f , and $N(r)$ is the droplet concentration.

In these bands, larger drops tend to cause greater attenuation due to their ability to absorb and scatter electromagnetic waves. This phenomenon is crucial to improving the precision of radar measurements and to developing more accurate precipitation forecasting models.

A novel consideration to study this correlation is the use of specific gamma distributions for each climate zone. Gamma distributions allow modeling the concentration of drops as a function of their radius, taking into account the parameters N_0 , μ , and Λ that describe the initial number of particles, the shape parameter, and the decay parameter, respectively. Specific attenuation can be calculated for different droplet sizes and microwave frequencies using these distributions.

This gives detailed information about how precipitation affects radar measurements in different weather conditions.

Integrating satellite data from missions such as CloudSat and GPM with advanced numerical models allows these gamma distributions to be validated and improved. High-resolution observations from CloudSat and GPM provide detailed information on cloud and precipitation microphysics, allowing specific attenuation models to be calibrated with real data. This integrated approach not only improves my understanding of atmospheric processes but also provides valuable tools for weather forecasting and water management in different climate zones. By enhancing the accuracy of precipitation predictions, this methodology can significantly aid in resource allocation and disaster preparedness. Furthermore, as climate patterns continue to evolve, the adaptability of these models will be crucial for addressing future challenges in meteorology and environmental science.

7) *Rayleigh Scattering*: The given code figures out the exact attenuation for four climate zones: Tropical, Temperate, Polar, and Arid. **Figure 2** does this by using a gamma distribution to show how raindrops are spread out. The gamma parameters (N_0 , μ , and Λ) are specific to each climate zone, and the frequencies considered are 13.6 GHz, 35.5 GHz, and 94 GHz, corresponding to the Ku, Ka, and W bands. The Rayleigh scattering approximation is used to figure out the specific attenuation, and the results are shown in the form of a grouped bar graph.

To improve the graphical representation, the code uses distinct colors and different types of markers for each climate zone, making it easier to distinguish the results. The axis labels and the graph title are formatted in bold for better readability, and the legend is positioned at the top left for easy identification of the climate zones. In addition, the graph includes a main grid and a secondary grid to facilitate the reading of the values.

Finally, the code customizes the chart axes to add margins and improve the overall visualization. The X-axis is set to show the considered frequencies, while the Y-axis is scaled to include a margin of 20% above the maximum attenuation value. These graphical improvements make the chart clearer and more informative, making it easier to interpret the results. These enhancements ensure that viewers can quickly grasp the key data points and trends presented in the chart. By optimizing the layout and scaling, the visualization effectively communicates the results without overwhelming the audience.

8) *Mie vs Rayleigh Scattering*: This MATLAB code implements a comparison of Rayleigh and Mie scattering for three different frequency bands used in communications and meteorological observation: Ku (13.6 GHz), Ka (35.5 GHz), and W (94 GHz). For each band, physical parameters are defined, including the wavelength and the complex refractive index of water. A range of particle radii between 1 and 10 millimeters is considered, and the associated wavenumbers are calculated. The code uses a mathematical formulation to calculate the scattering cross sections for both theories: Rayleigh scattering, which is based on the approximation for particles small compared to the wavelength, and Mie scattering, which considers a more general solution to the

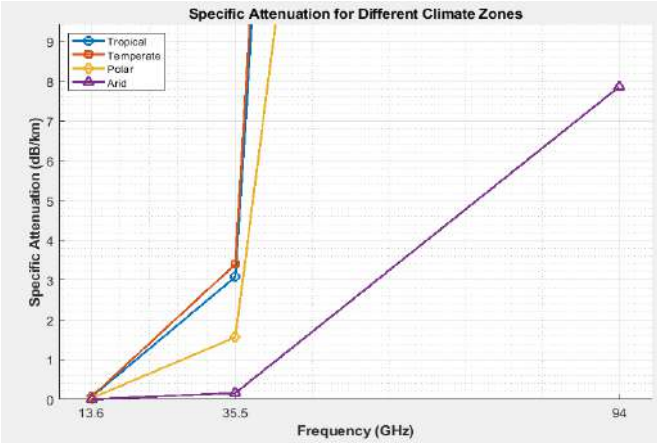


Fig. 2: Rayleigh scattering simulation for tropical, temperate, polar and arid world zones.

electromagnetic problem.

In the calculation of Rayleigh scattering, a simple analytical formula is used that depends on the wavelength, the radius of the particle, and the refractive index. For Mie scattering, instead, a series expansion is used that includes terms dependent on **Bessel** functions of the first and second kind, as well as the **Hankel** function of the first kind. The code runs one loop for each band, and within each iteration both Rayleigh and Mie scattering are calculated for each particle radius. The complexity of the Mie calculation is greater, since it requires an accurate expansion of the series and the evaluation of multiple terms.

Finally, the results obtained are represented graphically. For each band, the code creates a subplot that shows the evolution of the scattering cross section as a function of the particle radius, expressed in millimeters. The results for Rayleigh and Mie are compared using separate curves, highlighting the differences in the two theoretical approaches. The plot allows me to analyze how the Rayleigh theory is valid only for particles small compared to the wavelength, while the Mie theory provides a more accurate solution for larger particles. This comparison is essential for applications in meteorology, satellite communications and electromagnetic propagation studies.

First, I introduce a dynamic model for the refractive index of water, which varies as a function of frequency, temperature, and relative humidity. This lets me get a good picture of how the waves spread out and how they are absorbed by matter. This is especially important in the **W band** (94 GHz), where the imaginary part of the refractive index makes a big difference. Another improvement is to the way **Mie scattering** is calculated. Now, the size parameter x determines the maximum number of terms in the expansion based on how big the particles are in relation to the wavelength. This method makes the model more accurate and helps it converge. It can also correctly show scattering from particles that are bigger than the wavelength, which happens a lot at these high frequencies.

Finally, the Mie scattering formulas implicitly include polar-

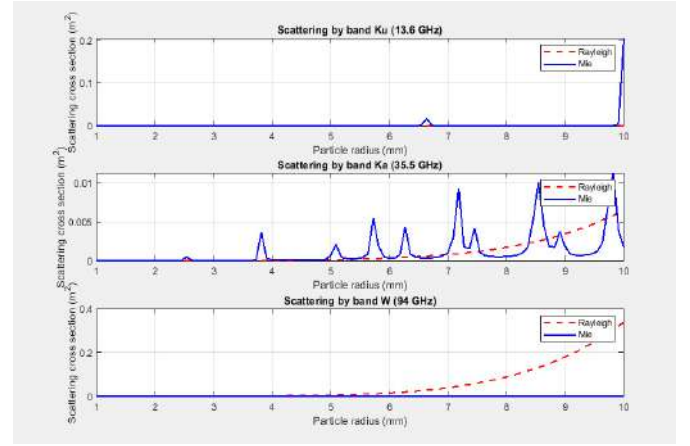


Fig. 3: Raileigh vs. Mie Scattering in the Ku, Ka and W band.

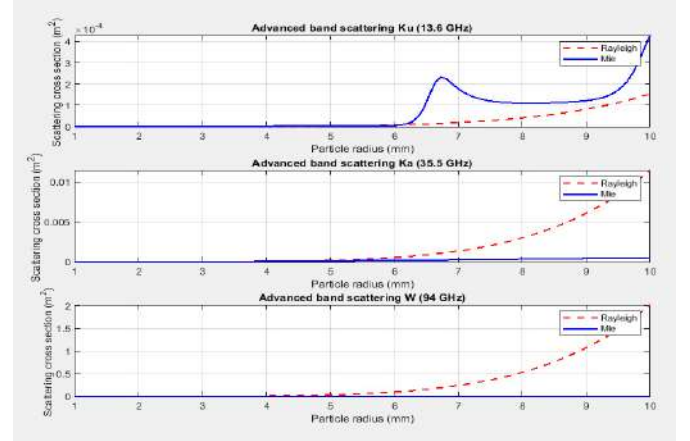


Fig. 4: This plot highlights the improvements made to the scattering calculation, especially for the W band (94 GHz). In the original Rayleigh theory, the approximation is only valid for particles tiny compared to the wavelength, as shown by the very low values of the cross sections for larger particles.

ization effects, which become more relevant at high frequencies and for irregularly shaped particles. This update allows for a more accurate comparison of the Rayleigh and Mie models, showing the transition between the two scattering regimes under realistic atmospheric conditions. The result is a more complete model applicable to advanced satellite communication and meteorology scenarios.

In my previous approach, I assumed the refractive index m to be constant at 1.33 for each frequency band of interest. However, The author has now made some modifications to my modeling. The author has introduced an imaginary component m'' to better represent the behavior at high frequencies, where the refractive index becomes more significant. This adjustment allows me to more accurately capture the main losses caused by absorption. To achieve greater precision in my calculations, The author has also included a dynamic refractive index that varies as a function of temperature, pressure, and frequency.

This enhancement provides a more realistic representation of the refractive index under different conditions. In the source

code, It is possible to implement these changes as follows:

$$m = 1.33 - 0.002 \cdot i$$

in the Ku band. The upgrade implementation allows modeling the dynamic refractive index by making it depend on frequency, temperature, and relative humidity:

$$m = 1.33 + i \cdot (0.002 + 0.01 \cdot \frac{f}{1GHz} \cdot \frac{T}{300} \cdot \frac{RH}{100}) \quad (4)$$

where RH is the relative humidity in % and where T is the absolute temperature. Other ways to make things better might have to do with the polarization of electromagnetic waves and how they scatter between parts that are perpendicular and parallel to the plane of incidence (for example, figuring out the scattering coefficients S_1 and S_2 in the Mie theory). The Mie series expansion can be improved by increasing the maximum number of terms considered (n_{max}).

For particles large compared to the wavelength (with $x \gg 1$), n_{max} is dynamically increased according to the relation:

$$n_{max} \approx x + 4x^{1/3} \quad (5)$$

and The methodology uses optimized numerical algorithms to compute the Bessel and Hankel functions, thereby reducing possible rounding errors. The dynamic computation of the maximum number of terms in the series expansion yields the following evaluation:

$$n_{max} = \text{ceil}(2 + 4x^{1/3}). \quad (6)$$

This flexible calculation makes sure that the numbers in the Mie series converge better, which improves the accuracy for hydrometeors whose diameters are bigger than the standard wavelength. It is possible to add additional realistic enhancements to atmospheric particles, especially hydrometeors like raindrops and snowflakes, which are rarely perfectly spherical. In such cases, It is possible to incorporate models for "non-spherical" particles, such as oblate spheroids, ellipsoids, or even particle aggregates. In this case, computational modeling uses very complicated models and advanced math methods, like the **T-Matrix** method or the **Method of Moments (MoM)**, to simulate how irregular particles scatter.

Also, multiple scattering becomes important at high frequencies when there are a lot of particles in the path of the signal. I should employ radiative transfer techniques or Monte Carlo simulations to address this. Additionally, various software tools, such as MATLAB and Python libraries like "miepython" and "scipy.special," can be utilized to simulate and implement more complex solutions. For instance, **COMEPS (Code for Electromagnetic Scattering by Particles)** can be applied to other types of problems. Beyond its original scope, COMEPS (Code for Electromagnetic Scattering by Particles) is a versatile tool that can apply to a variety of problems. It can be used in remote sensing to model the scattering of electromagnetic waves by atmospheric particles, which is crucial for interpreting data from remote sensing instruments like radars and lidars. It can also help in

designing and optimizing antennas by simulating how electromagnetic waves scatter off different materials and structures. In medical applications, COMEPS can be used to simulate the scattering of electromagnetic waves by biological tissues, aiding in the development of imaging techniques like MRI and ultrasound. Additionally, it can be applied to study the scattering of electromagnetic waves by pollutants or aerosols in the atmosphere, which is important for environmental monitoring and climate studies. By leveraging COMEPS in these ways, scientists and engineers can gain deeper insights into various fields and improve the accuracy of their models and predictions. With these improvements, the model would achieve greater accuracy at high frequencies, particularly in the W band, and be applicable to realistic atmospheric conditions. Adding dynamic refractive indices, polarization treatments, and taking into account multiple scattering and particles that aren't spherical will make the results a lot more accurate. In the code that was implemented, polarization is taken into account implicitly in the Mie series expansion terms, especially in the a_n terms of the expansion. The author has generically that:

$$a_n = ((m^2 * \text{besselj}(n-0.5, m * x_r) * \text{besselj}(n+0.5, x_r)) - (\text{besselj}(n+0.5, m * x_r) * \text{besselj}(n-0.5, x_r))) / ((m^2 * \text{besselj}(n-0.5, m * x_r) * \text{hankel1}) - (\text{besselj}(n+0.5, m * x_r) * \text{hankel1})) \quad (7)$$

and the refraction and transmission of the parallel and perpendicular components are described by the combination of Bessel functions of order n for the medium with refractive index m and for the exterior. The term m^2 takes into account the relative refractive index, which influences the two components (parallel and vertical) differently. In the denominator, I find instead the Hankel function of order n and first-kind $H_n^1(x)$, which represents the solution of the emitted spherical waves, considering both the radial propagation and the angular variation. When you add up the weighted sum $(2n+1) * |a_n|^2$, you get a natural combination of the polarized components (both horizontally and vertically).

III. EXPLORING COINCIDENCE DATASETS.

A. Introduction

The 4.31 Mb pdf file called "CSATGPM COIN ATBD" can be downloaded from the Global Precipitation Measurement website by clicking the link available in [17]. It is important to underline the fact that in the center of the file name The author has COIN which reminds me that it is a coincidence file. The coincidence is between the CloudSat and the GPM. This will help me search for files when I need them within the complex hierarchical structures of file systems via FTP service. The ATBD stands for Algorithm Theoretical Basis Document [18], a document that provides detailed definitions and explanations of the algorithms used in my particular project. The document process and analyze data that has been collected in a coincident manner [20]. It is possible to also contact the ECMWF at this address [20] if I would like to get important information on the instruments

of some satellites related to CloudSat and the coincidences. Aspects of non-parametric regression, statistical categorization in applied sciences, and neural network projects [NN-Neural Networks] §3.2.1 Theoretical Basis are also covered here. You can start a lengthy training trip on the most popular topics, like meteorology and climate change, with the excellent ECMWF material. Take into consideration the COBRA (Copernicus micrOwave-Based global Reception Arrangement) project for future references. In particular, in §3.2.2.1 of COBRA, under the heading “The Dual-Frequency Precipitation Radar”, the characteristics of the dual-polarisation radar GPM-CO DPR are described, along with some information on the TRMM satellite. The DPR consists of a dual Ku-band (at 13.6 GHz) and a Ka band (at 35.5 GHz). These Ku-Band and Ka-Band Earth Pointing Radar (KuPR and KaPR) instruments provide 3D precipitation measurement data on all surfaces between 67°N and 67°S as of March 2014. A summary of the GPM radar characteristics is shown in **Table 11**. [20].

B. Make Coincidence usable.

To make coincidence datasets usable, researchers must:

- **Harmonize Data:** Align datasets in time and space, ensuring consistency in resolution and format.
- **Address Gaps:** Interpolate or model missing data to maximize coverage.
- **Validate Measurements:** Use ground-based observations or high-resolution satellite data as benchmarks.

In the context of satellite remote sensing (RS), a “coincidence” refers to the near-simultaneous observation of the same geographical area by two or more satellites. This concept is often used in meteorological and climate studies to combine data from different satellite sensors to gain a more comprehensive understanding of atmospheric and surface conditions. For instance **F.J.Turk** and his colleagues have demonstrated the use of near-coincident observations between GPM DPR and the CloudSat CPR (CloudSat Profiling Radar) to extend the capability of representing light rain and cold-season precipitation [16]. These unique triple-frequency data have opened up applications related to cold-season precipitation, ice microphysics, and light rainfall and surface emissivity effects.

C. A more detailed analysis.

In this paragraph I want to summarize the differences between the Global Precipitation Measurement (GPM), CloudSat, and Tropical Rainfall Measuring Mission (TRMM) satellites’ coincidences.

1) GPM (Global Precipitation Measurement):

- **Dual-Frequency Precipitation Radar (DPR):** GPM’s DPR operates at Ku-band (14 GHz) and Ka band (35 GHz). It provides the capability to resolve precipitation structures under moderate to heavy precipitation conditions.
- **Passive Microwave Sensors:** GPM also includes a constellation of passive microwave sensors that complement the DPR data. These sensors help estimate precipitation using microwave radiation emitted by raindrops and ice particles.

- **Applications:** GPM’s unique triple-frequency data (DPR and passive microwave) enable applications related to cold-season precipitation, ice microphysics, light rainfall, and surface emissivity effects.

2) CloudSat:

- **Cloud Profiling Radar (CPR):** CloudSat’s CPR operates at W band (94 GHz). It provides detailed vertical profiles of cloud and precipitation structures.
- **Coincidence with GPM:** CloudSat and GPM have nearcoincident observations. By combining CloudSat’s Wband radar data with GPM’s Ku- and Ka band radar data, researchers can extend the capability of representing light rain, cold-season precipitation, and ice microphysics [22].
- **Applications:** CloudSat-GPM coincidences are useful for studying ice microphysics, radar-radiometer synergy, and surface emissivity effects.

3) TRMM (Tropical Rainfall Measuring Mission):

- **Precipitation Radar (PR):** TRMM’s PR operated at Ku-band (13.8 GHz). It provided detailed precipitation profiles in the tropics.
- **Microwave Imager (TMI):** TRMM also had a passive microwave imager (TMI) that complemented the PR data.
- **Legacy:** TRMM was a predecessor to GPM and provided valuable insights into tropical rainfall patterns.
- **Coincidence with CloudSat and GPM:** TRMM’s orbit intersected with CloudSat and GPM, allowing for near-coincident observations.
- **Applications:** TRMM data contributed to understanding tropical rainfall, convective systems, and climate variability. [23]

In summary, GPM, CloudSat, and TRMM provide complementary data on precipitation and cloud properties. Their coincidences allow researchers to study precipitation processes across different frequency bands and improve my understanding of Earth’s water cycle. [24]

D. Advanced Techniques for Analyzing Satellite Coincidence Datasets

Detailed analysis of coincidence datasets requires sophisticated statistical and computational techniques, such as:

- **Multivariate Analysis:** Identifying relationships between different variables (e.g., reflectivity and precipitation rate). Multivariate analysis involves examining multiple variables simultaneously to identify relationships and patterns within the data. Satellite coincidence datasets use this technique to analyze variables like reflectivity, precipitation rate, temperature, and humidity. Researchers can find patterns and links between different atmospheric parameters by using tools such as principal component analysis (PCA), canonical correlation analysis (CCA), and cluster analysis. For example, PCA can reduce the dimensionality of the dataset while retaining the most significant information, making it easier to visualize and interpret complex relationships. CCA can identify the relationships between two sets of variables, such as radar reflectivity and precipitation rate, providing insights into

how these variables interact [25]. Cluster analysis can group similar data points, helping to identify distinct weather patterns and phenomena [26].

- **Machine Learning:** Employing algorithms to classify and predict hydrometeor types. Scientists are increasingly using machine learning algorithms to classify and predict hydrometeor types based on satellite data. Techniques such as support vector machines (SVM), random forests, and deep learning models have shown enormous promise in this field. For example, SVMs can sort hydrometeors into different groups by learning from labeled training data and finding the best hyperplane that divides the groups [27]. Random forests are groups of decision trees that can handle big datasets with lots of dimensions and make accurate predictions by averaging the outcomes of several trees [28]. Deep learning models, particularly convolutional neural networks (CNNs), can automatically extract features from raw data and classify hydrometeors with high accuracy [29]. With these machine learning techniques, scientists can make more complex algorithms that can make hydrometeor classification and rainfall estimates more accurate and useful.
- **Uncertainty Quantification:** Estimating errors and confidence intervals for derived parameters. To figure out errors and confidence intervals for derived parameters in satellite coincidence datasets, uncertainty quantification (UQ) is a must. UQ involves identifying, quantifying, and reducing uncertainties associated with models, numerical algorithms, and experimental data. UQ commonly employs techniques like Monte Carlo simulations, Bayesian inference, and sensitivity analysis. Monte Carlo simulations make many versions of the dataset by taking samples from probability distributions. This lets researchers figure out how different the results are and how uncertain they are. Bayesian inference updates the probability distributions of model parameters by using both prior knowledge and new data [30].

This gives me a way to think about uncertainty in a probabilistic way. Sensitivity analysis checks how changes in input parameters affect the model's output. This helps figure out which parameters are the most important and how much they add to the overall uncertainty [31]. These UQ techniques are crucial for ensuring the reliability and robustness of satellite-based atmospheric observations and predictions.

IV. SATELLITE INSTRUMENTATIONS.

CloudSat and the **Tropical Rainfall Measuring Mission (TRMM)** have significantly advanced my understanding of clouds, precipitation, and hydrometeors through their specialized onboard instrumentation. CloudSat, launched in 2006, features the **Cloud Profiling Radar (CPR)**, a W band radar operating at 94 GHz. This tool gives very clear vertical profiles of clouds, which lets scientists study cloud microphysics in excellent detail, such as droplet size and phase transitions. The CPR's sensitivity to small hydrometeors like ice crystals and light rain allows for improved monitoring of light precipitation and snowfall, which are often missed by ground-based systems. In contrast, TRMM, launched in 1997, was

designed to study tropical and subtropical rainfall and was equipped with the Precipitation Radar (PR), operating at Ku-band (13.8 GHz). The PR provided three-dimensional rainfall structures, allowing scientists to measure rain intensity, type, and drop size distribution. Complementing this, TRMM's **Microwave Imager (TMI)** detected microwave emissions from the atmosphere, offering insights into liquid and ice hydrometeors across vast regions.

The **Visible and Infrared Scanner (VIRS)** and **Lightning Imaging Sensor (LIS)** on TRMM also measured the temperatures of the tops of clouds and the amount of lightning activity around the world. This helped scientists learn more about how convective storms work. Together, CloudSat's focus on cloud structure and light precipitation and TRMM's expertise in tropical rainfall and heavy precipitation create a comprehensive dataset for improving weather forecasting. These instruments refine numerical weather prediction models, enhance hydrometeor classification, and improve disaster response systems through better rainfall estimation. Their long-term contributions also help climate studies, especially when looking at how clouds reflect light and how much rain falls over time. This shows how satellite missions work together to advance atmospheric research. TRMM's cloud structure and light precipitation data play a pivotal role in advancing my understanding of meteorological phenomena through research. By providing a comprehensive dataset, it significantly enhances weather forecasting accuracy and refines numerical weather prediction models, ensuring that meteorologists can make informed predictions.

The better classification of hydrometeors also lets me do more accurate analyses of different types of precipitation, which is important for disaster response systems because it lets me guess the amount of rain that will fall. This research not only aids immediate forecasting and emergency management but also contributes to long-term climate studies, particularly regarding cloud reflection and the evolution of rainfall patterns over time. Ultimately, TRMM's findings forge a path toward more resilient communities equipped to face the challenges posed by climate variability and extreme weather events.

A. TRMM Instruments

- **Visible Infrared Radiometer (VIRS):** The VIRS, derived from the NOAA /AVHRR technology, is a five-channel, cross-track scanning radiometer with spectral bands at 0.63, 1.6, 3.75, 10.8, and 12 μm . It provides high-resolution observations essential for analyzing cloud coverage, identifying cloud types, and determining cloud-top temperatures, offering critical data for weather forecasting and atmospheric studies.
- **TRMM Microwave Imager (TMI):** The TMI is a multichannel passive microwave radiometer that works at five frequencies: 10.65, 19.35, 37.0, and 85.5 GHz (dual polarization) and 22.235 GHz (single polarization). It was based on the DMSP SSM/I design. It gives complete details on the amount of liquid water in clouds, the amount of ice in clouds, the intensity of the rain, and the different types of rain, including the difference between stratiform and convective precipitation.

	Microwave Radiometer (TMI)	Radar (PR)	Visible and Infrared Radiometer (VIRS)
Frequencies	0.7, 19.3, 21.3, 37.0, and 85.5 GHz (dual-polarized except for 21.3: vertical only)	13.8 GHz	0.63 μm 1.6 μm 3.7 μm 5 μm 12 μm
Resolution	11 km $\times$ 8 km field of view at 37 GHz	5 km footprint and 250 m vertical resolution	2.5-km resolution
Scanning	Conically scanning (530 inc.)	Cross-track scanning	Cross-track scanning
Swath Width	880-km swath	250-km swath	830-km swath

TABLE I: TRMM Instrument Specifications.

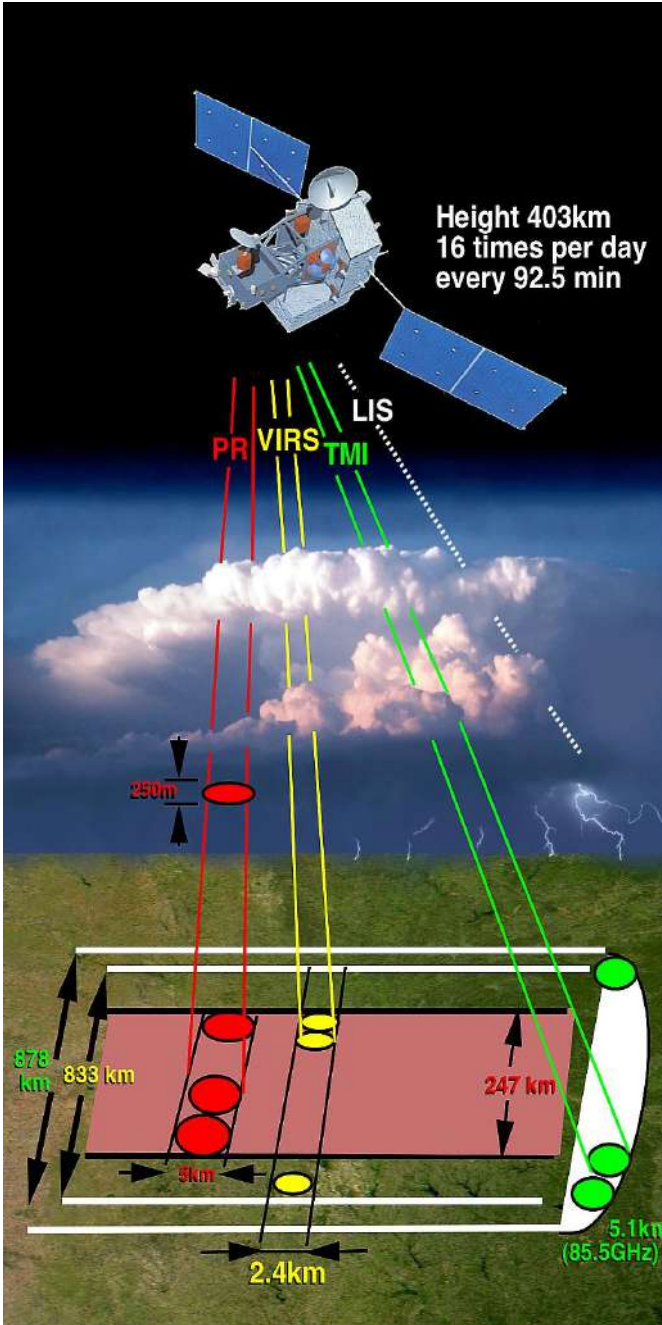


Fig. 5: TRMM Instruments Measurements Path. Courtesy of NASA ©

- **Precipitation Radar (PR):** The PR, a pioneering space-borne radar, operates at 13.8 GHz and employs electronic scanning to provide three-dimensional rainfall distribution across both land and ocean. It accurately defines precipitation layer depth, offering valuable insights into vertical rainfall structures and supporting hydrological and meteorological research.
- **Cloud and Earth Radiant Energy Sensor (CERES):** The CERES instrument measures the energy flux at the top of the atmosphere and estimates energy levels within the atmosphere and at the Earth's surface. This information is crucial for understanding the Earth's energy balance, radiative forcing, and their implications for climate dynamics.
- **Lightning Imaging Sensor (LIS):** Designed to detect and pinpoint lightning activity across tropical regions globally, the LIS is a compact and advanced instrument. Its data provides valuable insights into convective storm dynamics, supporting studies on storm intensity, frequency, and their climatic impacts.

B. CloudSat Instruments

CloudSat is primarily equipped with the CPR, which is its only active sensor, as it focuses on detailed vertical cloud profiling.

	Cloud Profiling Radar (CPR)
Frequencies	94 GHz (W band)
Resolution	Vertical: 500 m, Horizontal: 1.4 km
Scanning	Nadir-looking (fixed angle)
Swath Width	~1.4 km (nadir)
Range Gate Spacing	~240 (m)
Vertical Range Resolution	~480 (m)

TABLE II: CloudSat Instruments.

C. GPM Instruments

The Global Precipitation Measurement (GPM) Core Observatory is equipped with advanced instrumentation to measure global precipitation with high accuracy and resolution. It carries two key sensors:

- **DPR:** Provides detailed 3D precipitation profiles, distinguishing between rain and snow with its dual-frequency radar.
- **GMI:** Observes precipitation structures, water vapor, and cloud properties with its broad frequency range and high sensitivity.



Fig. 6: The GPM satellite from space.

	Dual-Frequency Precipitation Radar (DPR)	GPM Microwave Imager (GMI)
Frequencies	13.6 GHz (Ku-band), 35.5 GHz (Ka band)	10.65 GHz to 183.31 GHz (13 channels)
Resolution	Horizontal: 5 km (Ku), 5 km (Ka); Vertical: 250 m	5–25 km (varies by channel)
Scanning	Cross-track, conical scanning with dual-frequency swath overlap	Conical scanning with constant incidence angle (52.8°)
Swath Width	Ku-band: 245 km, Ka band: 120 km, Overlap: ~120 km	~885 km

TABLE III: GPM Instruments.

The GMI can pick up 13 different frequency channels, from 10.65 GHz to 183.31 GHz. This lets it pick up a wide range of hydrometeor properties with great accuracy and sensitivity. This makes it an essential tool for understanding global precipitation patterns and improving weather and climate models. By making detailed observations of the atmosphere, the GMI helps improve the accuracy of weather forecasts and comes up with ways to lessen the effects of extreme weather events. Its advanced capabilities enable researchers to analyze trends over time, leading to a deeper understanding of climate change dynamics.

The image depicts in **Figure 6** show the Global Precipitation Measurement (GPM) Core Observatory satellite in space. Launched in 2014 as a joint mission by NASA and JAXA, GPM serves as the cornerstone of an international satellite network dedicated to monitoring global precipitation. The satellite operates in a non-Sun-synchronous orbit, providing near-global coverage from 65°N to 65°S, ensuring detailed measurements of rain, snow, and other precipitation types.

The GPM Instruments **Table III**, provides an overview of the two advanced sensors onboard the Global Precipitation Measurement (GPM) Core Observatory, highlighting their unique capabilities:

The **Figure 7** illustrates the GPM Microwave Imager (GMI), a key instrument onboard the Global Precipitation Measurement (GPM) Core Observatory satellite. The GMI is a multi-

channel, passive microwave radiometer designed to measure precipitation, water vapor, cloud water, and surface conditions. Key features visible in the image include:

- **Antenna Assembly:** The parabolic dish on top, which is responsible for collecting microwave signals from Earth's surface and atmosphere.
- **Rotating Drum:** The structure supporting the dish, which enables conical scanning to cover a wide swath (~885 km) on Earth's surface.
- **Supporting Structure:** The robust frame and struts providing mechanical stability to the instrument, ensuring precision during operation.
- **Electronics Enclosure:** The central section housing processing units that convert raw microwave signals into actionable data.

D. A-Train Instruments Complementing CloudSat

The A-Train constellation is a group of coordinated Earth-observing satellites designed to provide comprehensive data on the atmosphere, clouds, aerosols, and Earth's radiation budget. These satellites work together to capture high-resolution measurements, offering a multi-dimensional view of atmospheric processes. Each satellite carries specialized instruments that complement one another, enabling a more detailed and accurate understanding of the Earth's climate system.



Fig. 7: Microwave Imager (GMI) onboard on GPM satellite.

Key satellites in the A-Train include CALIPSO, Aqua, Aura, PARASOL, and OCO-2, each contributing unique capabilities. For example, CALIPSO's instruments, such as CALIOP and IIR, provide vertical profiles of clouds and aerosols, while Aqua's MODIS and CERES focus on surface imaging and radiation measurements. Similarly, Aura tracks ozone, atmospheric chemistry, and water vapor, and PARASOL offers polarized imaging for aerosol studies. Together with CloudSat's Cloud Profiling Radar (CPR), these satellites form a powerful observational system, ensuring a holistic view of atmospheric and climatic dynamics. The list of principal instruments onboard the A-Train constellation satellites is as follows:

- **CALIPSO (Cloud-Aerosol Lidar and Infrared Pathfinder Satellite Observations):**
 - CALIOP (Cloud-Aerosol Lidar with Orthogonal Polarization): Lidar for vertical profiles of aerosols and clouds.
 - IIR (Imaging Infrared Radiometer): Thermal imaging for cloud properties.
 - WFC (Wide Field Camera): Contextual cloud and aerosol imaging.
- **Aqua:**
 - MODIS (Moderate Resolution Imaging Spectroradiometer): Cloud and surface imaging.

- CERES (Clouds and the Earth's Radiant Energy System): Radiation budget measurements.
- AIRS (Atmospheric Infrared Sounder): Atmospheric temperature and humidity profiling.

- **Aura:**

- OMI (Ozone Monitoring Instrument): Tracks ozone and aerosols.
- TES (Tropospheric Emission Spectrometer): Monitors atmospheric chemistry.
- MLS (Microwave Limb Sounder): Measures atmospheric water vapor and temperature.

- **PARASOL (Polarization and Anisotropy of Reflectances for Atmospheric Sciences coupled with Observations from a Lidar):** Polarized measurements of clouds and aerosols.

- **OCO-2 (Orbiting Carbon Observatory-2):** Measures atmospheric carbon dioxide.

While CloudSat itself only carries the CPR, its integration within the **A-Train** ensures a holistic view of clouds, precipitation, aerosols, and Earth's radiation budget. CloudSat, with its single Cloud Profiling Radar (CPR), plays a crucial role in providing detailed vertical profiles of clouds and precipitation. Despite carrying only one instrument, its highly specialized radar measurements deliver critical insights into cloud structure and dynamics, which are essential for understanding precipitation processes. The many functions of other satellites, such as CALIPSO, Aqua, Aura, and PARASOL, enhance CloudSat's data when it is part of the A-Train constellation. Together, these satellites provide a multi-dimensional view of the atmosphere, combining vertical profiling, thermal imaging, and aerosol measurements. This synergy allows for a more comprehensive understanding of Earth's climate and weather systems.

Ultimately, the A-Train collaboration not only maximizes CloudSat's CPR data but also contextualizes it with a broader range of observations. This integrated approach significantly enhances global climate models, improves weather predictions, and deepens my understanding of atmospheric interactions.

V. MONTE CARLO SIMULATION OF RADAR REFLECTIVITY IN CLIMATE ZONES.

Radar reflectivity is a crucial parameter in meteorology, providing insights into precipitation intensity and hydrometeor classification. To enhance the accuracy of radar reflectivity measurements, especially in diverse climate zones, Monte Carlo simulations offer a robust approach. These simulations can model the complex interactions between radar signals and atmospheric particles, accounting for various factors such as particle size distribution, shape, and composition [Atlas et.al., 1999], [43], [Marshall and Palmer, 1948], [44] .

In this study, I employ **Monte Carlo** simulations to generate realistic radar reflectivity data for different climate zones. By incorporating regional climate characteristics, such as temperature, humidity, and precipitation patterns ([Houze, 1997], [46], [Smith et.al., 2009]), The aim of this study is to create a comprehensive dataset that reflects the variability in radar reflectivity across the globe. This approach allows me

to better understand the impact of regional climate conditions on radar measurements and improve the accuracy of weather predictions.

The results of my simulations will be validated against observational data from weather radars and satellites ([Maneghini and Atlas, 1996], [47], [Iguchi et.al., 2000]), [48]. By comparing the simulated and observed reflectivity values, It is possible to assess the performance of the proposed model and make necessary adjustments. Ultimately, this research will contribute to the development of more accurate and reliable radar-based weather forecasting systems, benefiting meteorologists and the general public alike.

As a first example, It is possible to define climatic regions by distinguishing them into three large areas: tropical, temperate, and polar. For these regions, I consider that It is possible to associate the pair of parameters of maximum concentration of the drops expressed in $[mm^{-1}m^{-3}]$ indicated with N_0 and the decay coefficient of the Gamma distribution, indicated with Λ in $[mm^{-1}]$ ([Marshall and Palmer, 1948] [44], [Ulbrich, 1983], [45]). For each climatic region of the planet, It is possible to carry out an implementation for a simulation using the Monte Carlo method in which the application generates an exponential distribution of the dimensions of the drops $N(r)$ based on the parameters of the region in question. Subsequently, in the simulation, random noise equal to 10% for $N(r)$ can be introduced in order to simulate the natural variations that can occur in precipitation conditions. The subsequent calculation of radar reflectivity Z in [dB] is obtained by adding the contributions $r^6 \cdot N(r)$, where r indicates the radius of the drops and $N(r)$ the concentration.

Numerical integration is therefore capable of counting the values of Z , which can then be converted to a logarithmic scale:

$$Z_{dBZ} := 10 \cdot \log_{10}(Z) \quad (8)$$

and this conversion makes the values more readable and consistent with standard meteorological conventions. For each region, It is possible to then calculate the central value of the simulation results and the standard deviation so that It is possible to compare this simulation with real measurements. The `kdensity` function is used in the implementation to make a continuous representation of the radar reflectivity distribution so that this calculation can be done. The parameters The methodology uses in the simulation are based on realistic physical models (Marshall-Palmer) that represent the distribution of the size of raindrops. I could add frozen and mixed-phase hydrometeors to the simulation to make it more complete and complex. For example, the model could be expanded to include particles that can represent snow, graupel, and mixed hydrometeors (like supercooled drops). To do this, It is possible to use specific parameters to express the density of snow and Grandix or define the mixed phase between liquid and frozen particles. In this case, the relative density factor must be taken into account for non-spherical particles with a density different from that of liquid water, which allows me to define the effective radar reflectivity Z_{eff} as follows:

$$Z_{eff} = Z \cdot \left(\frac{\rho_{snow}}{\rho_{water}} \right)^2. \quad (9)$$

If I therefore consider additional hydrometeors in addition to rain, considering snow or hail, then the total reflectivity will be given by the sum of the individual contributions:

$$Z_{tot} = Z_{rain} + Z_{snow} + Z_{hail}. \quad (10)$$

In the global precipitation model, it is essential to consider the possibility of rare anomalous atmospheric conditions that could lead to snowfall or sleet in areas near the equator. Although these events are extremely rare, they require a combination of factors such as cold air currents at high altitudes, sufficient humidity to form precipitation, and a significant drop in temperature near the surface. Despite their rarity, these conditions can still occur. For instance, Mount Kilimanjaro in Tanzania, Mount Kenya, the equatorial Andes of Ecuador, and the summits of the Chimborazo and Cotopaxi volcanoes can receive snowfall despite their proximity to the equator. To account for these events in the proposed model, I would need to add a probabilistic condition of 1%-5% to simulate the presence of snow in tropical regions. However, it is important to note that this percentage may realistically be even lower.

By incorporating these rare events into the proposed model, It is possible to ensure a more comprehensive and accurate representation of global precipitation patterns, even in regions where snowfall is highly unlikely.

The **Figure 8** illustrates the distribution of Radar Reflectivity Factor Z (dBZ) for three climatic regions: Tropical, Temperate, and Polar. Each region is represented in a separate subplot with distinctive colors—red for Tropical, green for Temperate, and blue for Polar. The subplots combine a normalized histogram (showing the relative frequency of simulated Z values) and a PDF curve (depicting the probability density), providing a clear view of the trends in each region. The figure highlights the variability in Z , with Tropical regions showing higher values due to intense precipitation, while Temperate and Polar regions exhibit progressively lower values associated with moderate rain and light snow, respectively.

VI. CONCLUSIONS.

Satellite coincidence datasets are pivotal in enhancing my understanding of atmospheric processes and their applications in meteorology and climate science. Utilizing data from CloudSat, TRMM, GPM, and other satellites allows for a detailed examination of cloud microphysics, precipitation dynamics, and atmospheric attenuation. The combination of triple-frequency radar data with advanced radiometers provides a highly accurate depiction of specific attenuation and precipitation properties, even in complex scenarios such as light rain, snow, and mixed-phase hydrometeors.

Innovative methodologies, including dynamic refractive index modeling, comparisons of Mie and Rayleigh scattering, and advanced multivariate analysis techniques, are employed to harmonize and analyze satellite data. These approaches enhance the accuracy and usability of datasets, enabling the creation of high-resolution attenuation maps. Such maps are

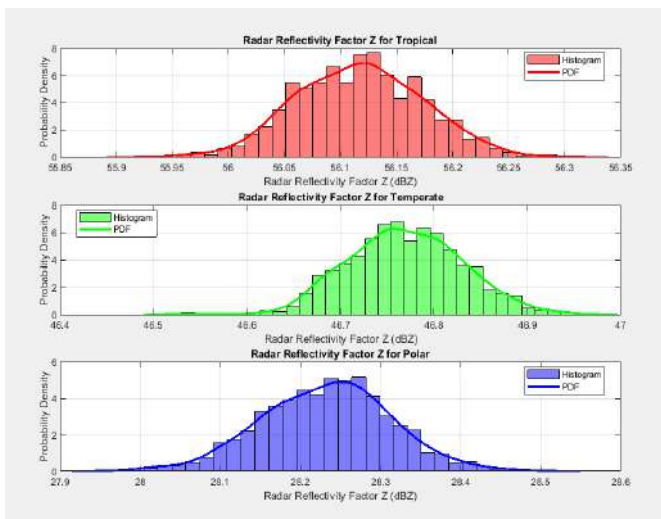


Fig. 8: The figure consists of three subplots, each representing a specific climatic region: Tropical (red), Temperate (green), and Polar (blue). Each subplot displays a normalized histogram (transparent colored bars) showing the relative frequency of Monte Carlo simulations for Z values, alongside a PDF curve (solid line) illustrating the estimated probability density. Tropical regions exhibit higher Z values due to heavy precipitation, while temperate and polar regions show progressively lower Z values, corresponding to moderate rain and light snow, respectively. The distinctive colors facilitate visual comparison between the distributions.

essential for optimizing satellite communication systems, improving weather forecasting accuracy, and advancing climate simulations.

The research also addresses challenges such as missing data, sensor misalignment, and the refinement of precipitation retrieval algorithms through advanced computational methods like Monte Carlo simulations and machine learning. The findings demonstrate the transformative potential of multi-satellite collaborations in capturing the complexity of atmospheric dynamics and enabling more accurate and reliable predictive models.

Furthermore, harmonizing and validating satellite datasets is crucial to ensure their usability for various scientific and practical applications. Future research should focus on refining algorithmic approaches, incorporating dynamic environmental parameters, and addressing challenges such as data gaps and spatial resolution mismatches. Adding new satellite missions to these methods will provide an even better understanding of global precipitation changes over time and their impact on ecosystems and human infrastructure.

In conclusion, satellite coincidence studies have significantly advanced atmospheric science. This includes tools for more accurate weather predictions and strategies to mitigate the effects of climate change. These findings demonstrate the power of collaborative satellite missions and interdisciplinary research in addressing some of the most pressing challenges of my time. By integrating diverse data sources and leveraging innovative technologies, researchers can enhance my

understanding of weather patterns and their variability. This holistic approach not only aids in predicting extreme weather events but also informs policy decisions aimed at fostering resilience in vulnerable communities worldwide. The multifaceted approach to climate change mitigation underscores the importance of collaboration across various disciplines and technological innovations. By leveraging collaborative satellite missions and integrating diverse data sources, researchers are better equipped to comprehend complex weather patterns and predict extreme events. This holistic understanding not only informs effective policy decisions but also fosters global resilience in the face of climate uncertainties. Ultimately, as these strategies are harnessed, they pave the way for a more sustainable future, where informed actions can mitigate the adverse impacts of climate change on my planet.

AUTHOR BIOGRAPHY



Paolo Cicolin was born in 1971 in Turin. In October 2012 and December 2018, respectively, he graduated with a Bachelor degree in Electronic Engineering and a Master of Science in Communication Engineering from Roma Tre University in Italy. From October 2020 to October 2021, he enrolled at Roma Tre University's second-level master's program in "Cybersecurity for the Protection of Control Systems in Industry 4.0 and Critical Infrastructures". The Accenture S.p.A. scholarship was awarded to him for his studies in the cybersecurity course. La Sapienza University of Rome welcomed him as a Ph.D. student in November 2021. He was employed from April 2019 to July 2020 as a GIS consultant and in E-Geos S.p.A. (an ASI-Telespazio company), a major global participant in the geospatial industry. His areas of interest and research are geoscience, telecommunications, radar and remote sensing, microwave communications, antennas & propagation, telesensing, and severe event model prediction.

CONFLICT OF INTEREST

The author declares that he has no conflicts of interest.

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A.5 Third Technical Article - personal style

Title: Towards Reliable Global Attenuation Maps for Ku, Ka, and W Bands.

In accordance with reviewer recommendations, the following article was initially written using the first-person style as required.

Towards Reliable Global Attenuation Maps for Ku, Ka, and W Bands

Paolo Cicolin, Student Member.

Abstract—Satellite coincidence analysis is a crucial domain for understanding atmospheric and climate dynamics. This study provides a comprehensive investigation into the methods used to identify and quantify coincidences between satellite platforms, with a specific focus on cloud microwave observations and the TRMM and GPM datasets.

The initial section outlines the theoretical background and objectives, offering an overview of intersection techniques and orbital mechanics relevant to multi-satellite observations. Particular attention is given to the role of active and passive remote sensing instrumentation, such as onboard radars and the use of Two-Line Element (TLE) files, which are essential for trajectory reconstruction and coincidence detection.

A practical analysis of the Joe Turk dataset is presented, including step-by-step guidance for data acquisition and interpretation. The relationship between radar reflectivity, brightness temperature, and hydrometeor characteristics is analyzed to evaluate attenuation effects. The concluding discussion highlights the operational significance of accurate coincidence analysis in enhancing climate model reliability and supporting early-warning systems for severe weather events.

This work aims to serve as a detailed reference for researchers and practitioners in remote sensing, offering methodological clarity and application-oriented insight into multi-platform satellite synergy.

Index Terms—Coincidences, Radar Reflectivity, Attenuation Map, TRMM, GPM, TLE, JT dataset.

I. INTRODUCTION

The third part of this study focuses on the development of atmospheric attenuation maps based on the 97th percentile, offering a detailed overview of the methodologies and orbital dynamics involved. Special attention is given to the use of Joe Turk's TLE files and satellite coincidence datasets, with emphasis on their practical applications in space communication systems. This section also addresses the technical challenges encountered during the analysis and discusses future perspectives for enhancing the interpretation and exploitation of satellite data. The importance of accurate data analysis is highlighted as a key factor for improving satellite communication efficiency. By confronting these challenges, the study aims to contribute to the advancement of more reliable and robust atmospheric propagation models.

The third part of the work focuses on the creation of 97th percentile atmospheric attenuation maps, providing a detailed overview of the methodologies and orbital dynamics used. I discuss Joe Turk's TLE files and coincidence datasets, emphasizing their practical applications in space communications.

Finally, the paper reflects on the technical challenges and future perspectives for further improving the analysis and use of satellite data. This reflection not only highlights the importance of accurate data interpretation but also suggests avenues for future research that could enhance satellite communication efficiency. By addressing these challenges, The aim of this study is to contribute to the ongoing development of more reliable and robust atmospheric models.

II. OBJECTIVES

I will use these observations in this article to create a rough concept of a global map of attenuation in the Ku, Ka, and W bands. While this work may not be entirely original, I haven't found any similar elements in the existing literature. Regardless, generating these maps in the three bands of interest remains challenging. I need to consider several factors. Maps, if they exist, result from intricate data processing, and their production demands more computation than one might initially expect. Although creating global maps in MATLAB is straightforward, collecting and verifying data can be a time consuming process. For this purpose, I will also show the works from which I started, and my intent is to indicate a practical possibility distant from certain excessively abstract considerations.

A. Studies on Satellite coincidences

Researchers have examined satellite coincidences in various studies. For instance, CloudSat cross-data enables the study of atmospheric precipitation characteristics, allowing graphical differentiation based on moderate, light, or severe intensity. However, understanding the features of light precipitation was previously impossible without the joint use of a multi-frequency radar. When examining GPM-CloudSat satellite coincidences, the primary focus lies in studying the microwave signatures of perturbations. For example, researchers analyze the multi-spectral Cloud Mask. In an article by **J.F. Turk** [1], images from these studies depict the superposition of masks at different frequencies from various satellites: the TRMM at 14 GHz and the CloudSat at 94 GHz. These coincidences are visible at the edges of graphs resulting from test considerations, but they are not directly assessed by basic projects or accessible to general users. The addition of extra radar and sensors is necessary because CloudSat covers the upper cloud region, which the TRMM PR cannot detect. Despite ongoing debates about cloud mask visualization approaches, most images in the literature are test images captured during measurement campaigns shortly after satellite launch. Subsequent reanalysis involves backward analysis of extensive data,

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but collecting photographs from widely read articles-especially those related to understanding actual satellite intersections-can be challenging. My goal in following this procedure is to shed light on some of the underlying mechanisms. Within the realm of satellite studies, It is possible to explore various sectors using a grid of satellites. These sectors span from visible to infrared bands, with a specific focus on water vapor detection. The data is consistently available in diverse formats, including netCDF and Grib. Researchers can leverage this data for a wide range of applications, such as analyzing tropical cyclones, global precipitation patterns, and detecting and tracking inter-tropical convergence zones [2].

B. Microwave Firms for Clouds

The Global Precipitation Measurement (GPM) and CloudSat are examples of satellite missions that use remote sensing techniques to acquire and analyze the spectral signature of clouds. The concept of remote sensing, particularly in the context of acquiring and analyzing the spectral signature of clouds, has evolved over time with contributions from many scientists. However, one of the earliest and most notable figures in the field of remote sensing is **Evelyn Pruitt**. She is credited with coining the term “remote sensing” in the 1950s while working at the **Office of Naval Research** [3].

C. Deeply inside Dataset Coincidences

Researchers have focused on coincidences between the GPM (Global Precipitation Measurement) satellite and the CloudSat profiling radar (CPR) operating at 94 GHz in the W band. These intersections occur when the GPM core satellite and the CPR radar align within a short period of time. The resulting “pseudo three-frequency” radar profiles combine the W band data from CPR with the Ku/Ka band data from GPM. Additionally, the GMI (Global Precipitation Imager), part of the GPM collaboration between NASA and JAXA, provides valuable information with its 13 microwave channels spanning frequencies from 10 GHz to 183 GHz. The GMI orbits at an altitude of 407 km with an orbital inclination of 65°, completing a flyby approximately every 1.5 hours [4].

The CloudSat profiling radar (CPR) operates at nadir and uses a frequency specifically chosen for optimal compromise. Here are the key technical characteristics of CPR:

- **Pulse Width:** The pulse width is 3.3 microseconds.
- **PRF (Pulse Repetition Frequency):** CPR operates at a PRF of 4300 Hz.
- **Minimum Detectable Radial Reflectivity:** The minimum detectable radial value is less than -29 dBZ.
- **Antenna Diameter:** The antenna diameter is a compact 1.85 meters.
- **Dynamic Range:** CPR has a dynamic range of 70 dB.
- **Integration Time:** The integration time for data collection is 0.16 seconds.
- **Nadir Angle:** Since August 15, 2006, the nadir angle has remained constant at 0.16°. Prior to that, it changed from 1.71° to 0.0° on July 7, 2006.

Resolution:

- **Vertical Resolution:** 500 meters.
- **Cross-Track Resolution:** 1.4 kilometers.
- **Along-Track Resolution:** 1.7 kilometers.
- **Data Rate:** The data rate is 20 kbps.

The Cloud Profiling Radar (CPR) can detect both very weak and very strong targets based on its technical characteristics. The nadir angle represents the angle between the radar’s vertical orientation and the direction of the target. A small nadir angle signifies the correct downward orientation of the radar. The Pulse Repetition Frequency (PRF) refers to the number of pulses emitted by the radar per second, striking a satisfactory compromise between resolution and coverage. I note that CloudSat is sun-synchronous, considering at least three satellites: CloudSat, the Global Precipitation Measurement (GPM) mission (including the GPM Microwave Imager, GMI), and the Tropical Rainfall Measuring Mission (TRMM). This means that CloudSat crosses the equator both in the early afternoon and after midnight each day. This sun synchronous orbit facilitates observations by ensuring constant solar illumination, allowing for simple comparison of images taken at different times of the day [5].

D. Other keypoints

- **Asynchronous GPM:** The GPM satellite operates asynchronously. Asynchronicity allows me to obtain intersections with synchronous satellites within sufficiently small time differences, resulting in what I refer to as “pseudo three-frequency” radar profiles (W band from CloudSat and Ku/Ka band from GPM).
- **Sun-synchronous TRMM and GMI:** The TRMM (Tropical Rainfall Measuring Mission) and GMI (Global Precipitation Imager) satellites are sun-synchronous.

Sun-synchronous satellites offer the advantage of consistent illumination conditions, making it easier to compare images taken at different times of the day and ensuring consistent global coverage. However, they can miss events that occur at other times of the day and are costly. Non-sun-synchronous satellites, on the other hand, offer temporal flexibility and the ability to obtain data coincident with sun-synchronous satellites, improving data integration. However, they present variability in illumination conditions and less consistent global coverage. The choice between the two types of satellites depends on the specific objectives of the mission. Asynchronous satellites offer several advantages, including temporal flexibility, which allows them to observe the same area at different times of the day, capturing variable temporal events. Additionally, they facilitate the acquisition of data in sync with sun-synchronous satellites, thereby enhancing data integration and multi-satellite analyses. The greater temporal coverage increases the probability of capturing rare or transient events, while the adaptability to variable light conditions provides useful data even in the absence of direct sunlight. These advantages make asynchronous satellites valuable tools for a variety of scientific and operational applications.

E. Details on TRMM and GPM

Here are their details:

- **TRMM:**
 - Altitude: Initially at 350 km, it was later increased to 402.5 km in 2001 to extend the mission duration.
 - Intersects with CloudSat, resulting in two pseudo-frequencies: W (from CPR) and Ku (from PR).
 - The TRMM carries a TMI radiometer with 9 channels (10-85 GHz) designed for passive microwave sensing of precipitation.
- **GMI (on-board GPM):**
 - Located at approximately 705 km altitude.
 - Has an inclination of 98.7°.
 - Always crosses the equator at the same local time as the Sun.

F. About Intersection and Instrumentation

- **Intersections and radiometers:**
 - Combining GPM-CloudSat data provides three pseudo-frequencies in the W-Ku-Ka bands. The W band data comes from CloudSat, while the KuKa pair originates from the GPM's Dual-Frequency Precipitation Radar (DPR).
 - The GPM also carries a 13-channel GMI radiometer covering the 10-183 GHz range.
 - Considering the TRMM, it intersects with CloudSat, resulting in two pseudo-frequencies (W from CPR and Ku from PR). Additionally, the TRMM's TMI radiometer operates at nine channels (10-85 GHz) and provides valuable information on precipitation.
- It works on five separate frequencies, which are: 10.7, 19.4, 21.3, 37, and 85.5 GHz.

G. Orbital dynamics

The altitudes of the three satellites can be summarized as follows:

- **GPM:** Positioned at an altitude of 705 km.
- **CloudSat:** Initially set at 705 km but later lowered to 689 km on February 23, 2018.
- **TRMM:** Operates at an altitude of 402.5 km. Now it is dismissed [6]!

To gain a better understanding of orbital intersections, it is important to consider factors such as orbital inclination, true anomaly, and other orbital parameters. The duration of these intersections depends on these variables. In theory, Kepler's third law provides a straightforward way to calculate the orbital period of satellites.

$$T = 2\pi \cdot \sqrt{\frac{r^3}{\mu}} \quad (1)$$

where:

- T represents the orbital period.
- r is the radius of the orbit (equal to the Earth's radius plus the satellite's altitude in kilometers).
- μ denotes the standard gravitational parameter of Earth (approximately $398600.4418 \text{ km}^2/\text{s}^2$).

For precise calculations, tools such as MATLAB's Satellite Communication Toolbox can be utilized.

H. Files TLE

In this section, I want to briefly inform the reader about the calculation of satellite coincidences for CloudSat, GPM, and TRMM. The reader needs to find the TLE (Two-Line Element Set) files to do this, as they are a special file format used to represent the orbits of satellites. Typically, these files are in ASCII format and contain a two-line list or set of elements provided by USSTRATCOM. Each set of two line elements in a TLE file provides enough information to calculate the position and velocity of a satellite orbiting Earth at any time. This information includes the epoch as well as the date, time, inclination of the orbit, right ascension of the ascending node, eccentricity of the orbit, perigee topic, mean anomaly, and average motion. For more information about CloudSat, it is recommended to explore the CloudSat Orbital Prediction Tools provided by NASA, accessible at this web address [7].

On this site that you can visit, emphasis is given to the prediction of the CloudSat orbit. Visit the CloudSat website [7] to access daily satellite track information in GIF and ASCII formats, and to download the TLE files. For an enhanced experience with TLE files, consider using **Orbitron** software. Developed by **Sebastian Stoff** [8], **Orbitron** is a versatile satellite tracking application embraced by amateur radio operators, weather experts, satellite communication enthusiasts, astronomers, and even astrologers. It displays satellite positions at any moment, offering both realtime and simulated tracking. Its user-friendly interface and powerful features make it a top choice among free satellite tracking solutions. TLE (Two-Line Element) files are used by Orbitron to track satellite positions. These files contain orbital data such as inclination, eccentricity, perigee argument, and mean motion. TLE data should be updated periodically to maintain the accuracy of the tracking software, especially for objects in low orbits, where TLE data should be updated every few days. To get updated TLE files, you can visit sites like **CelesTrak** or **AMSAT** ([6], [9]), which provide TLE element sets for a wide range of satellites. These sites are reliable resources and are updated frequently with the latest data. If you cannot find the TLE data for GPM directly, it may be helpful to search for the NORAD catalogue number of the GPM satellite and use Celestrak's search function for that specific number. Remember that TLE data is essential for accurately tracking the orbit of satellites and is updated regularly [10].

I. TLE ASCII Content

The TLE file contains precise orbital parameters for satellites, formatted in a standard two-line structure. For instance, the TLE for the CloudSat satellite is as follows:

```
CLOUDSAT 1 29107U 06016A 24169.92539437
.00002055 00000+0 26295-3 0 9994 2 29107
98.4072 155.5717 0006191 152.0736 208.0809
14.82542651966278
And for the GPM Satellite:
GPM-CORE 1 39574U 14009C 24170.23471077
```

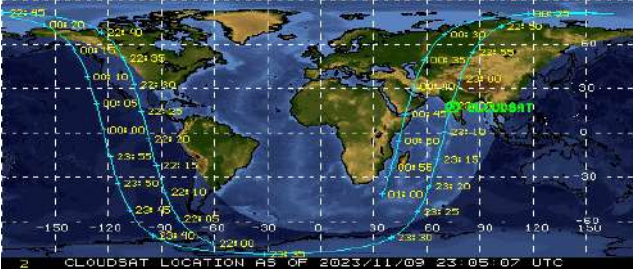


Fig. 1: CloudSat orbit. The map includes longitudinal and latitudinal lines, and there are numerical timestamps that likely represent the times of satellite observations in UTC. This kind of data visualization is crucial for meteorological research, as it helps scientists track cloud formations and understand weather patterns globally.

```
.00012941 00000+0 27772-3 0 9991 2 39574
64.9851 229.5920 0010078 261.9880 98.0117
15.44957793585235
```

Each line of the TLE provides specific details about the satellite's orbit, such as its catalog number, inclination, right ascension of the ascending node, eccentricity, argument of perigee, mean anomaly, and mean motion. It is important to maintain the original string format content (69 chars) during the implementation. With programming knowledge, you can utilize these TLEs in MATLAB or other software to perform calculations that determine when two satellites will be in the same area of the sky—a process known as calculating satellite coincidences. This can reveal interesting aspects of their trajectories, such as synchronized orbits or potential close approaches. In the context of Earth observation and remote sensing, it is reiterated that the term “coincidence” generally refers to instances where two satellites observe the same area of the Earth's surface within a relatively short time interval. This can happen when their orbital trajectories and sensors overlap over a specific region. To determine whether there is a coincidence in terms of observing the same area, you should consider not only the distance between satellites but also their viewing angles and observation times. A coincidence threshold in this case could be defined in terms of overlap of observed areas rather than in terms of physical distance between satellites. Here's a basic approach you could use:

- 1) Calculate the footprints (observation areas) of each satellite based on their sensors and orbital altitude.
- 2) Determine the passage times of the satellites over the same area.
- 3) Check for footprint overlap during these transition times.

If the footprints overlap within a specified time interval, it can be considered a coincidence. The time threshold will depend on the frequency of data acquisition and the speed of the satellite's movement. To perform this analysis, it may be necessary to use specialized remote sensing software or MATLAB modules capable of handling footprint calculations and temporal analysis.

J. Suggestions

Calculating the footprint of a satellite, i.e., the coverage area on the Earth's surface visible from the satellite, requires consideration of various factors. Here are the general steps to calculate footprint:

- 1) Determines the altitude of the satellite and its viewing angle. This data is often provided in the satellite's specifications or can be calculated from its orbital elements.
- 2) Calculate the radius of the footprint. For a satellite in circular orbit, the footprint radius can be approximated by the formula:

$$r = \sqrt{2 \cdot h \cdot R_{\text{earth}} + h^2} \quad (2)$$

where (h) is the altitude of the satellite and R_{earth} is the radius of the Earth (about 6371 km).

- 3) Estimate the coverage area. The coverage area can be a circle or an ellipse, depending on the satellite's viewing angle and its position in orbit.
- 4) Consider the antenna beam pattern. The antenna beam pattern affects the shape and size of the footprint. To accurately calculate the footprint, it is essential to understand the beam pattern.
- 5) Take into account the curvature of the earth and the horizon. The shape of the footprint can be distorted by the curvature of the earth, especially near the horizon.
- 6) Use satellite tracking software. Software tools are available to calculate the footprint based on satellite data and its orbit. For a more precise estimate, it may be necessary to employ numerical methods or specialized software that considers all relevant factors and calculates the intersection of the antenna beam with the Earth's surface.

K. On the notion of coincidence

The A-Train satellite constellation crosses the equator in an ascending direction around 1.30 p.m. local standard time, with the satellites following each other at intervals of a few seconds to minutes. However, with the end of operations of CloudSat on December 20, 2031, and CALIPSO on August 1, 2031, the current configuration of the A-Train has changed. The A-Train satellite constellation operates in such a way that the satellites follow each other along the same orbital path. Specifically, for CloudSat, TRMM, and GPM: CloudSat and TRMM (Tropical Rainfall Measuring Mission) had their ground-track orbits synchronized to coincide within a time window of ± 15 minutes. Similarly, CloudSat and GPM (Global Precipitation Measurement) were also synchronized within the same ± 15 minute window. This synchronization means that the observations made by these satellites were closely timed, typically within a few seconds of each other, during this 15-minute interval. This precise timing allowed for coordinated data collection and enhanced analysis of atmospheric conditions.

L. The A-Train Constellation

The A-Train satellite constellation, also known as the Afternoon Constellation, is a group of Earth-observing satellites that closely follow one another along the same orbital

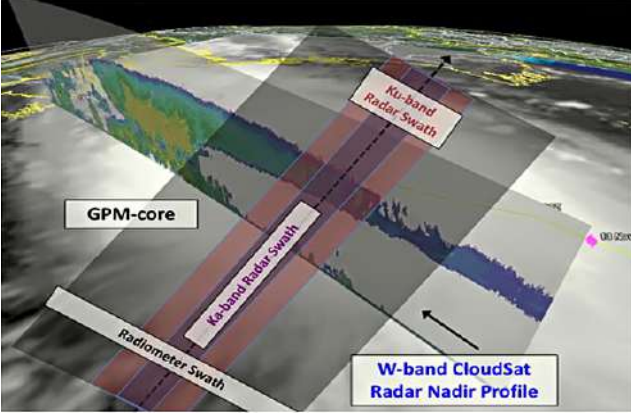


Fig. 2: This image captures the satellite coincidence concept, highlighting intersecting satellite trajectories. It presents CloudSat's W band orbital path with its nadir profile, actively generating a cloud profiling mask. The GPM-Core satellite, functioning in the Ku-Ka frequency bands, intersects this path. The swath from the satellite's on-board radiometer is also clearly depicted.

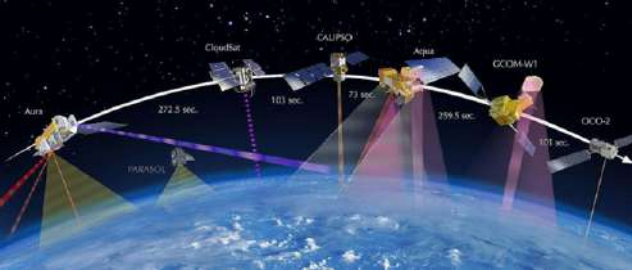


Fig. 3: The A-Train constellation. As of January 2024, the A-Train constellation includes satellites like OCO-2 and GCOMW1, while Aqua and Aura, although still collecting data, have slowly drifted out of the constellation due to fuel limitations. The constellation has evolved over time, with some satellites ceasing operations and others joining, maintaining the mission's ability to provide valuable Earth science data.

track. The primary objectives of the A-Train are to provide near-simultaneous observations from a variety of instruments, which are used synergistic-ally to advance my knowledge of Earth-system science and to apply this knowledge for the benefit of society.

- **Time Sequence Among the Satellites:** The satellites in the A-Train cross the equator in a sun-synchronous polar orbit at about 1:30 PM local solar time, within seconds to minutes of each other. This precise timing allows for coordinated measurements and the ability to study atmospheric components and processes as they happen [11].
- **Scope of the Mission:** The scope of the A-Train mission includes studying important parameters related to climate change, such as cloud and aerosol interactions, precipitation, and radiative balance. By combining data from multiple satellites, scientists gain a comprehensive

understanding of the Earth's atmosphere and climate system. The **A-Train** has contributed significantly to hurricane research and the understanding of the behavior of hurricanes [12].

III. SWATH, CHANNELS AND TRACK.

In the realm of satellite and radar technology, it's imperative to employ precise terminology to grasp the intricacies of these systems. This paragraph is dedicated to elucidating the most relevant terms utilized in the field, tailored for my specific objectives. A comprehensive understanding of terms such as 'swath', 'satellite track', and 'radar cross-section' (RCS) is not only beneficial but essential for professionals navigating through the complex literature associated with satellite instrumentation and atmospheric observations. Here's a more streamlined description of the terms:

- 1) **Swath:** In satellite terminology, a swath is the area of the Earth's surface that is scanned by a satellite's instruments in a single pass. It's essentially the "footprint" of the satellite's observation path.
- 2) **Satellite Track:** This is the path that a satellite follows as it orbits around the Earth. It represents the satellite's trajectory and determines the areas it will observe.
- 3) **Radar Along-Track Distance:** This term refers to the distance that a radar system measures along the direction of travel, whether it's on a satellite or an aircraft. It's the length of the Earth's surface covered as the radar collects data.
- 4) **Radar Transversal Cross-Section:** The radar cross-section (RCS) quantifies how detectable an object is by radar. An object with a higher RCS is more visible to radar systems, making it easier to track and analyze.
- 5) **Channels for Satellite Instruments:** Satellite instruments like radiometers operate on different frequency bands, known as channels. Each channel is tuned to detect specific atmospheric properties, such as temperature, humidity, or chemical composition.
- 6) **Radiometers on CloudSat, GPM, and TRMM:** These are specialized instruments designed to measure the energy emitted or reflected from the Earth's surface and atmosphere. CloudSat's radiometer profiles cloud layers using W band radar, GPM's dual-frequency radar assesses precipitation levels, and TRMM's Ku-band radar was used to study tropical rainfall patterns.

IV. ACTIVE AND PASSIVE REMOTE SENSING INSTRUMENTATION.

The distinction between active and passive instrumentation on CloudSat, GPM, and TRMM satellites is critical to understanding how these instruments collect precipitation and cloud data. Active Instrumentation: Active instruments, such as radar, emit their own signals (such as microwave pulses) and measure the energy reflected from the atmosphere or the Earth's surface. This type of instrumentation is capable of providing high resolution data and penetrating cloud cover to obtain precise measurements. For example, the CloudSat

uses W band (94 GHz) radar to profile clouds, while the GPM uses dual frequency (Ku- and Ka band) radar to resolve precipitation structure.

Passive Instrumentation: Passive instruments, such as radiometers, do not emit signals but detect natural energy emitted or reflected by the atmosphere or the Earth’s surface. These instruments are useful for collecting large-scale data and can provide information on temperature, humidity, and chemical composition. The GPM and TRMM are equipped with passive microwave radiometers that detect the energy emitted by precipitation and clouds. In summary, active instrumentation is essential for obtaining detailed measurements and penetrating cloud cover, while passive instrumentation is crucial for large-scale observations and collecting environmental data. Both types of instrumentation are complementary and, together, provide a comprehensive view of atmospheric and precipitation dynamics.

The **Figure 4**, prominently displays a purple knurling on the ground, which I will demonstrate how to replicate. This knurling helps illustrate the ground’s slope, approximated at an altitude of 0 meters. It’s important to note that negative values represent a geographic reference with latitude for the along-track-distance, while height appears in kilometers. Typically, altitude should be in meters, not kilometers, as using kilometers could mislead the reader and potentially degrade the image quality. Despite this, the image’s usability remains intact. I arranged a satellite coincidence between the TRMM satellite and CloudSat, employing a picture-in-picture technique. The inset image presents the WGS84 coordinate system in degrees East and South, a cyclone’s path along the TRMM swath, and the operation on an 85 GHz channel in vertical polarization. The image also includes a temperature scale in Kelvin, adding a layer of complexity. Dashed lines mark CloudSat’s path intersecting the TRMM swath. Moreover, the image aims to display the varying instrumental ranges from these satellites concerning the cloudy vertical profile. I superimposed radar reflectivity diagrams at frequencies of 94 GHz for CloudSat and 14 GHz for TRMM at a graphical level. The “Precipitation Radar” (PR) is also featured. The coinciding date, year, month, day, and time interval from 1957 to 2006, specifically 19:57 to 20:06 UTC, are documented. The timetable’s phrasing could be clearer. Nonetheless, the fundamental purpose is to demonstrate the necessity of satellite coincidences. I will elaborate on this shortly. An annotation in the figure clarifies that CloudSat complements the region not covered by the TRMM PR. The objective of the coincidence between TRMM (Tropical Rainfall Measuring Mission) and CloudSat is to exploit the complementary capabilities of these two satellites to improve the understanding and measurement of precipitation and cloud structure in the Earth’s atmosphere. CloudSat is equipped with a cloud profiling radar (CPR) operating in the W band (94 GHz), which is capable of providing unprecedented detail on the vertical structure and water and ice content of clouds. This allows me to observe not only the shape of the clouds but also their three-dimensional composition and the distribution of particles within them. TRMM, on the other hand, was primarily designed to measure tropical precipitation using a Ku-band radar and microwave

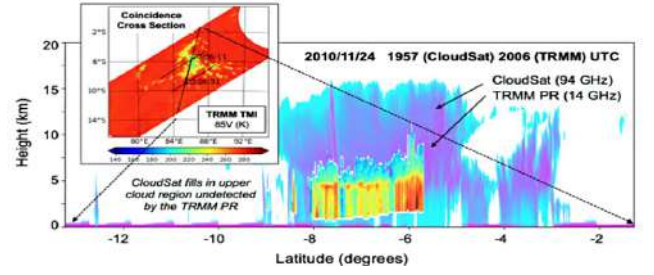


Fig. 4: Based on the work of J.F. Turk et al., the concept and application of satellite remote sensing instrumentation are illustrated. The accompanying imagery demonstrates the relevance of satellite coincidences and their potential value in atmospheric science research. © Used with permission from J.F. Turk et al.

radiometer. These instruments allow me to detect and quantify precipitation, including light and moderate precipitation, and to study rain formation processes in the tropics. Combining data from these two satellites allows me to obtain a more complete and accurate representation of precipitation and clouds, thus improving meteorological and climate models. Additionally, data coincidence helps validate and improve both passive and active rainfall measurement algorithms.

V. QUANTIFYING COINCIDENCES.

Calculating satellite coincidences is a complex task, and deciding which type of coincidence to consider requires careful deliberation. There are challenges related to orbital dynamics that must be addressed. To gather the necessary information, I began by examining **J.F. Turk’s** article and the dataset he referenced. My initial goal was to fully comprehend the dataset and develop software that could generate case studies similar to those in **Haynes et al. (2013)’s** work [33]. Following this, I was inspired to consider creating a **Global Attenuation Map (GAM)**. This idea stemmed from the insights detailed in reference [34], which provided a foundation for my approach. Let’s begin with a key figure that exemplifies satellite coincidences. This figure is closely tied to the terminology I’ve previously discussed. It serves as a visual guide to understanding how different satellite paths intersect and the significance of these intersections in satellite data analysis. The **Figure 5**, illustrates the intersection of satellite trajectories and their respective observation areas on the Earth’s surface. The colored circles and ellipses indicate the coverage of each satellite instrument. The labels “GPM-DPR”, “GMI”, “CloudSat”, and “MS” refer to different satellite instruments or missions, with **GPM** likely standing for **Global Precipitation Measurement**. The diagram also includes terms such as “HS” (high sensitivity), “NS” (normal sensitivity), and measurements in kilometers that indicate the width of each instrument’s observation band. This image is significant because it visually communicates how multiple satellites with different instruments can overlap and collaborate to cover various parts of the Earth’s atmosphere and surface. This is relevant to understanding how data from multiple sources can be integrated for meteorological observation, climate research, or other scientific studies.

The GPM-DPR (Dual-Frequency Precipitation Radar) operates with a uniform spatial resolution of 5 km across all scan types:

- Normal Scan (NS),
- High Sensitivity Scan (HS),
- Matched Scan (MS), ([14], [15]).

For instance, upon examining **Figure 4**, it might initially seem that the numbers 1957 or 2000 refer to quantities of granules; however, this interpretation would be incorrect. Those interested in the intricacies of satellite data coincidences will find that there have been comparative studies of data from the CloudSat satellite, launched in 2006, with that of the **TRMM (Tropical Rainfall Measuring Mission)** satellite, which was active from 1997 until 2015. One such study focused on a coincident dataset between CloudSat and TRMM (and later CloudSat and GPM), aiming to enhance or validate the algorithms used for retrieving passive and active precipitation measurements. Another research effort revealed that CloudSat tends to underestimate radar reflectivity by approximately 10 dBZ when compared to the TRMM radar at altitudes below 15 kilometers [35].

A. Why proceed to a quantification of coincidences

The dataset connected with J. Turk et al.'s work ranges from March 8, 2014 (when DPR data became available) until June 6, 2016 (the most recent CloudSat data as of July 25, 2016). This dataset focuses on orbital coincidences between the CloudSat and GPM satellites. Initially, there were 6726 orbital coincidences at this time. However, because CloudSat only works during the ascending (daytime) orbit, this figure is decreased by roughly half, to 3363 coincidences. After correcting for data gaps in both satellites, the final dataset had 3169 coincidences. The dataset is freely available from a service (perhaps JSTORM or CloudSat Web Site via FTP with Filezilla [16]). The “Turk” dataset is a compilation or collection for Version 1C that focuses on the link between CloudSat and GPM. For more information, see **Table 1** in reference [36].

Essentially, this dataset is made up of NetCDF files containing remote sensing results grouped into groups and subgroups with particular nomenclatures. These nomenclatures identify the products by their distinct types. The radar reflectivity profiles, precipitation-based returns, brightness temperatures (in the V04A version), CloudSat's vertical radar reflectivity profiles, and the ECMWF forecast analysis interpolated for each CloudSat vertical bin are of particular importance. In the latter situation, the product is branded ECMWF-AUX, indicating that it is auxiliary or experimental in nature.

The ECMWF forecast analysis interpolated for each of the CloudSat vertical bins involves the use of linear interpolation techniques to map the three-dimensional atmospheric variables to the vertical levels of CloudSat. This process is critical to CloudSat's standard data products, such as 2B-GEOPROF, 2BCLDCLASS, 2B-TAU, and 2B-FLXHR4 [17].

For each CloudSat radar bin, The methodology uses the bin height from the 1B-CPR data to find the adjacent AN-ECMWF vertical levels and perform linear interpolation to

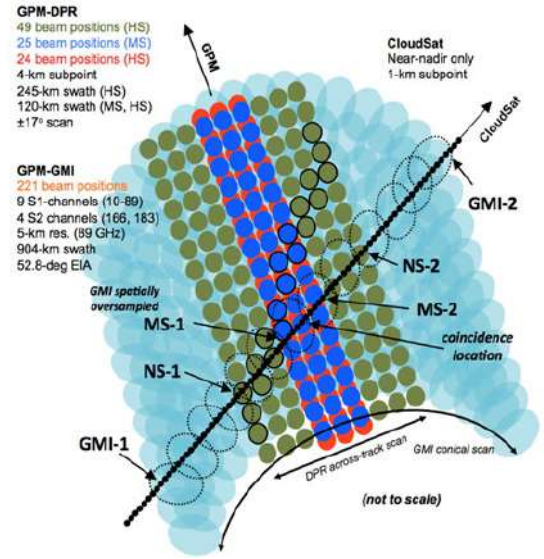


Fig. 5: The illustration depicts a segment where CloudSat and GPM satellites' paths cross. Each 1-km CPR (Cloud Profiling Radar) beam position is marked by small black dots. The DPR(Dual-frequency Precipitation Radar) Ku-band swath pixels are highlighted in green for the normal scan (NS), while the matched swath (MS) pixels are in blue, and the high sensitivity (HS) scan pixels are in red. The GMI (Global Microwave Imager) footprints, which scan in a conical pattern at a lower resolution, show spatial overlap across the swath in light aqua, varying by channel. The labels GMI-1 and GMI-2 indicate the specific GMI scans that intersect with this segment. Similarly, NS-1, NS-2, and MS-1, MS-2 denote the DPR NS and MS scans, respectively. This color-coded scheme and labeling provide a clear visual representation of how different satellite instruments' observations align and overlap, offering a comprehensive view of the atmospheric conditions being monitored. © By kind permission of the authors J.F. Turk et al. (2021).

obtain a single data value for the radar bin height at each of the neighboring grid points.

VI. FROM JOE TURK ARTICLE.

In this section, I discuss satellite coincidences as described in **J.F.Turk's** article [1]. While a graphical analysis of the coincidences provides a deeper understanding, summarizing key characteristics can be helpful. These aspects are relevant primarily for professionals, but The aim of this study is to present them concisely to aid in conceptual comprehension. The 2A.GPM.DPR dataset includes three radar profiles: Ku-band normal scan (NS), Ka band matched scan (MS), and Ka band high-sensitivity scan (HS). For each 240-meter CPR vertical bin, the DPR radar profiles are searched from bottom to top. The first bin above the CPR bin is identified and reported as the associated DPR matched bin. NS and MS profiles are reported at 125-meter bin spacing (using every other bin), while HS profiles are reported at 250-meter bin spacing (using all bins). The dataset maintains consistency by

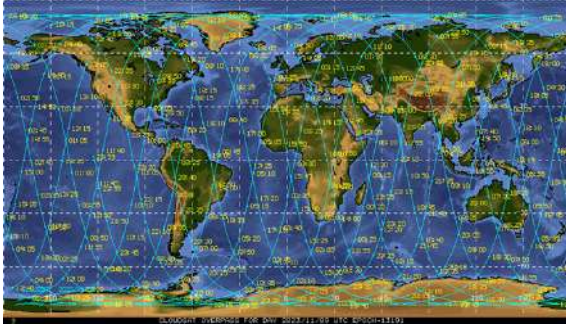


Fig. 6: CloudSat overpass for day 2023/11/09 UTC, Epoch 13191.

carrying only “uncorrected” ($z_{\text{FactorMeasured}}$) DPR reflectivities, avoiding assumptions about attenuation correction techniques. Full swath segments are available for users who prefer different bin-matching methods. Due to 4-kilometer horizontal spacing between GPM radar locations, the same DPR radar profile repeats itself 4 or 5 times as the CPR moves across the GPM radar’s across-track swath. Notably, near the equator, where GPM flies more parallel to the CloudSat orbital direction, the coincidence segment between CPR and GPM is longer.

The 2A.GPM.DPR dataset serves scientific, climatological, and hydrological purposes, including weather model validation, precipitation analysis, and environmental monitoring. The analysis of the different radar frequencies (NS, MS, and HS) can provide detailed information on the composition of precipitation. Validation of meteorological models:

- DPR data can be used to validate weather models and improve precipitation forecasts.
- The coincidence between CloudSat and GPM data facilitates the comparison of radar observations from different platforms.

This allows me to have the pretext to introduce other concepts such as the bin, the granule, the track etc. Let me examine these concepts in greater detail:

- **Vertical Bin and Horizontal Bin:** A vertical bin represents a specific depth or height within data, such as radar or lidar measurements. It’s a regular subdivision along the vertical axis. A horizontal bin represents a specific geographic location within data, like satellite imagery or maps. It’s a regular subdivision along the horizontal axis.
- **Satellite Track:** The satellite track is the path followed by a satellite as it orbits the Earth. It’s the projection of its trajectory onto the Earth’s surface.
- **Satellite Grain (or Granule):** The satellite grain refers to the minimum resolution size of a sensor or instrument on a satellite. It determines how small details or objects can be detected in satellite observations.
- **Along-Track Distance and Cross-Track Distance:** Along-track distance measures along the satellite’s path (in the direction of its movement). Cross-track distance measures perpendicular to the satellite track (laterally).

Referring to the **Figure 7**, a “granule” is a set of data from one orbit, with about 39,400 profiles. Each profile starts

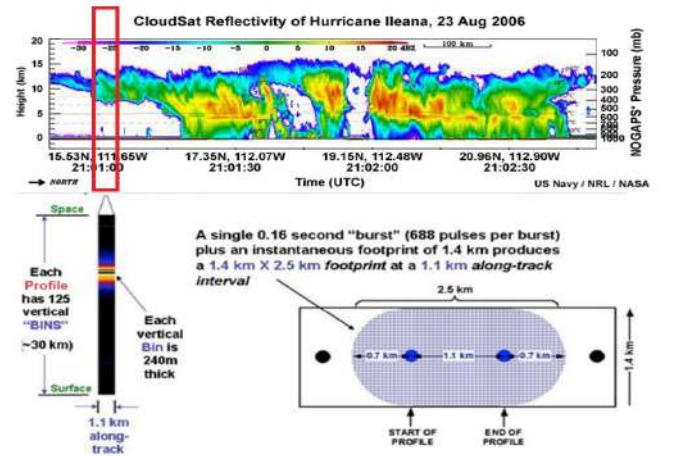


Fig. 7: Depiction of a vertical radar profile of CloudSat with time indications and information on the profile, on the vertical bins, on the along-track distance, on the footprint and on the height in kilometers (only meters should be reported).

with an “@” symbol, representing the descending node, and includes 125 overlapping profiles for averaging the along-track-distance. The vertical profile, connected to the satellite’s altitude (25 or 30 km), is divided into 125 vertical bins, covering around 30 km down to the Earth’s surface. The horizontal width of each bin is 1.1 km, known as the “**Along-Track-Distance**” (ATD). The satellite’s geometric footprint for radar imaging is 2.5 km by 1.4 km. The profile begins and ends with a 0.7 km margin, and the recovery distance for the profile is 1.1 km. A single “burst” lasts 0.16 seconds and consists of 688 radar pulses, creating an instantaneous footprint of 1.4 km by 2.5 km with a 1.1 km interval for the recorded track. Lastly, the radar reflectivity profile’s length, or “Cloud Mask”, is approximately 40,022 km for one satellite orbit¹. This detailed data collection allows for precise atmospheric observations from space.

VII. SOME FURTHER INFORMATION REGARDING ON-BOARD RADARS.

This part talks about the work of **A. Battaglia et al. (2022)** [18] and gives an outline of the technical features of the radars on the satellites that were looked at, as well as the ones that were suggested for making the **Global Attenuation Map (GAM)** [19]. The table in the **Figure 8**, compares the technical characteristics of three different radar systems: TRMM, GPM, and CloudSat. This information is extremely useful for researchers and engineers who work with data from these radars or who are involved in the design of similar systems. The table can be used as a reference to determine which radar system is best suited for a specific application, based on characteristics such as frequency band or resolution. Technical specifications such as mission duration, frequency, orbit inclination, equator crossing time, scanning strategy, resolution (horizontal and vertical), swath width, polarization, sensitivity capability, and

¹The comma in English notation indicates thousands of kilometers.

Radar	TRMM PR	GPM - DPR		CloudSat CPR
		KuPR	KaPR	
Mission Duration	1997 – 2015	2014 –		2006 –
Frequency (GHz)	13.799 ± 0.003	13.60 ± 0.003	35.55 ± 0.003	94.05
Frequency (Band)	Ku	Ku	Ka	W
Altitude (km)	350/403 ^a	407	407	705
Orbit Synchronisation	Non-Sun	Non-Sun	Non-Sun	Sun
Orbit Inclination (°)	35	65	65	98.2
Equator Crossing Local Time	00:00–24:00	00:00–24:00	00:00–24:00	13:30/01:30 ^d
Antenna Type	Phased-array	Phased-array	Phased-array	Reflector
Scan Strategy	Cross-track	Cross-track	Cross-track	Nadir-pointing
Hor. res. at nadir (km)	4.3/5 ^a	5	5	1.2
Vert. resolution (m)	250	250	250/500 ^b	485
Swath (km)	215/247 ^a	245	120/245 ^c	N/A
Polarization	Single	Single	Single	Single
Clutter Height (m)	1000	1000	1000	1000
Doppler Capability	No	No	No	No
Sensitivity (dBZ)	+17	+15	+14-19	–30
Sensitivity (mm h^{-1} , approx.) ^d	0.42	0.32	0.27–0.56	0.0005

TABLE I: a. The altitude of TRMM was increased from 350 to 403 km in 2001 in order to extend the lifetime of the mission (**DeMoss and Bowman**, 2007) [20]. b. The KaPR operates in two modes: a high-sensitivity HS mode (pulse length of 1000 [m]) and a matched scan MS-mode (pulse length of 500 m that matches the KuPR pulse). c. During the first phase of the GPM mission, the KaPR has been scanning on a 120-kilometer narrower swath. Since May 21, 2018, the KaPR scan pattern has been changed to match the KuPR swath, with the matched scan region centered on the ground track as before and the high-sensitivity scans replaced with the outer swath region (i.e., no longer interlaced within the matched scan region). d. Estimated precipitation rate uncertainty calculated with the most common radar reflectivity factor ($Z[\text{mm}^6 * \text{m}^{-3}]$) - rain rate ($R[\text{mm} * \text{h}^{-1}]$) relation (**Uijlenhoet**, 2001) [21], $Z = 200 * R^{1.6}$ (**Marshall et al.**, 1955) [22] e. Ascending or descending crossing at the equator. f. CloudSat has only been providing measurement data in daylight-only mode since 2011 (**Stephens et al.**, 2018) [23].

Doppler capability are all determining factors in choosing a radar system.

VIII. JOE TURK’S DATASET.

In this section, a detailed understanding of the dataset employed to initiate the creation of a global attenuation map is provided, with a focus on the specific data involved. Technically, there is no dataset named after Joe Turk, although this researcher can be associated with it. Within the Product Community and the “directory” available on shared folders via the public FTP service, subject to authorization with registration on the CloudSat portal or even via **STORM** [37], the CloudSat-GPM coincidence files are located in a position that I will illustrate with various figures.

Typically, the products 2B-CSATGPM.V04 and 2BCSAT-TRMM.V04 are indicated. However, the reality is different, and the files are located in a completely different place. It’s clear that I would need both datasets to create my map. The methodology uses the conditional tense because I will later discover that obtaining the data to create the global attenuation map I want to discuss is not as straightforward as it seems. The reality, which can sometimes be a bit harsh, tells me that I need to download numerous files, which entails

a significant amount of work. Subsequently, as I will find out later, the processing required to generate the attenuation map has experienced substantial growth, which can be quite prohibitive for home users.

Despite these concrete challenges, the intent of this article is to demonstrate the feasibility of this map and its potential elaborations, which require additional information processing costs. The fact that This study is talking about level 2B products tells me that the L2A data was processed by sensors. In turn, the level 2A data contain information derived from geolocalized sensors that measure, for example, the elevation with respect to the ground, the highest and/or lowest surface elevations from the outward and return journey of the e.m. wave, or the heights of energy quantiles (so-called “relative height” metrics), as well as other waveform-derived metrics describing the surface.

A. File Download and Installation Instructions Using Filezilla

It is recommended downloading the files using **Filezilla**. A video is available that explains how to install the application on Microsoft Windows operating systems [24]. I advise installing the client version of Filezilla, as it is designed to connect to FTP servers for direct file downloads. If necessary, ensure that ports 64000–65000 are open in your firewall to connect to the server [atrhurhouftp.pps.eodis.nasa.gov](ftp://atrhurhouftp.pps.eodis.nasa.gov), which is responsible for the FTP directory. Another historically used server is [arthurhou.pps.eodis.nasa.gov](ftp://arthurhou.pps.eodis.nasa.gov), but I currently recommend the former. When connecting to the host, it is not necessary to provide a port number. Simply enter the previously provided host address and the type of encryption. This requires the availability of explicit FTP over TLS with a “normal” access type. You will then need to enter your credentials (username, password) to connect. The encoding uses UTF-8 automatically.

B. More detailed informations about data download

There is a direct link to the database via the CloudSat web interface, but readers should note that CloudSat has now been decommissioned, and therefore the data is only available for reanalysis. Once connected to the server, I will observe the tree structure of the remote site, which includes the following items:

- ftpdata
- gpmallversions
- gpmdata
- gpmuser
- itedated
- oatdata
- pub
- sm
- trmmdata
- trmmopen

If you are familiar with **Linux**-like operating systems, navigating the folders will be straightforward. **MS Windows** and **Mac** users will also find the remote site structure familiar. Always monitor your ports in real-time in your firewall settings to set

up encrypted connections. NASA's security guidelines have recently transitioned from FTP to FTPS services (Secure FTP), where access and data download are monitored and secured by the communication protocol. For many inexperienced users, finding the **J.F.Turk** dataset files may be challenging, as there is no online guide for downloading them. However, this article now provides this information, keeping in mind that the tree structure could change without notice. In the Remote Site text input form within Filezilla, you can set this path: `/sm/730/gpmallversions`, and you will see a series of folders in the Filezilla interface that reflect the versions of remote sensing:

- V07
- V06
- V05
- V04
- geolocation
- coincidence

For further details, look for a `README.txt` file, which provides a description of the root folder. However, the satellite coincidences are not found in the "coincidence" folder, as one might expect. Instead, you need to choose the version and reset the path of the remote site to the following: `/sm/730/gpmallversions/V04/browse/2016/08/31/radar`.

In this case, you will see a long list of coincidence folders at the bottom of the detail box, such as `2B.CSATGPM.COIN`, or rather, with the percentage symbol, I indicate a series of other characters. This is just one example of how you should navigate your computer. For coincidences, the year is indicated first, then the month, and then the day. It might seem like The author has hit "Bingo", or rather that The author has found the coincidence files, but this is not the case. If you open the folders, you will only find images in `*.png` format, which are extremely useful for publications and machine learning. But then, where are the coincidences I need? You need to search for scientific files, such as NetCDF files with a `*.nc` extension, but in reality, the coincidences that interest me are a more modern evolution of the NetCDF format, i.e., `*.nc4` files. From a purely practical point of view, there isn't a big difference in the file format; the real difference consists only of a few more data structures from an IT perspective.

The coincidences are therefore found in the following example path: `/sm/730/gpmallversions/V04/2006/06/15/radar`, which is a real and working path regardless of the operating system used. This path allows you to see version 4 NetCDF files as folders within the Filezilla GUI. They appear as folders, but inside them, you will find the NetCDF files. By double-clicking with the mouse, you can start the download on your computer (local client). On my machine, next to my MS Windows username, there is a "source" and then a "repos" folder where the downloaded files end up. From the Filezilla tab pages, you can see the files in the queue: those transferred not completed and those transferred completed. Subsequently, It is possible to then open one of the downloaded files for fun and start working on it.

It is quite rare that information like this is found in an article, and in my opinion, this makes the difference. Regarding what was discussed, I will insert some accompanying

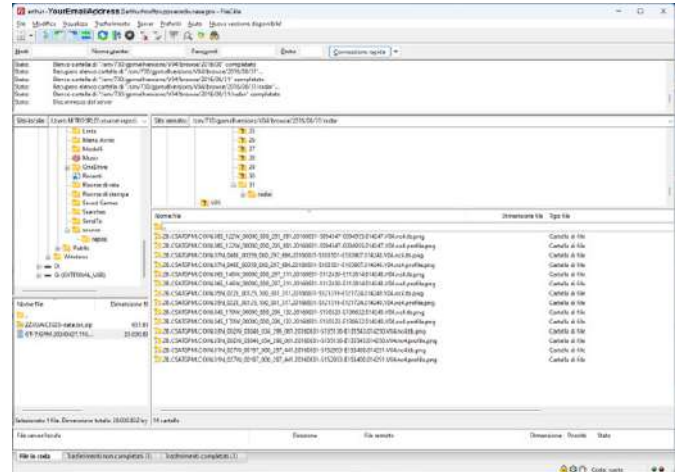


Fig. 8: Visualization of the Filezilla graphical interface with the information necessary to navigate to the CloudSat-GPM coincidence datasets.

images. Indeed, the figures I add pertain to CloudSat-GPM coincidences and CloudSat-TRMM coincidences. It's clear that downloading this data can often require entire days, a repetitive task that can consume significant mental resources. Unfortunately, there are no alternatives unless you decide to search for coincidences on **STORM** [37]. Regardless, you will need to invest a considerable amount of time, secure a reliable hard disk, and ensure an uninterruptible power supply to work without interruptions. As a programmer, It is recommended creating batch files or appropriately configuring your machine or machines. However, preparing the data can be a very time-consuming task. From my experience, it took several months of work to properly collect and secure the data. But that's not all; you also need to be mindful of your disk space. To give you an idea, I downloaded these coincidences for a total of 23 files, which occupied 473 MB of disk space. Now, consider the years of activity of these satellites and the number of times they pass over a specific geographical area, and you'll understand the considerable size of these databases. They usually amount to around forty terabytes, which, as you can imagine, cannot be stored on your local hard drive. This study is indeed dealing with "big data", and as such, satellite remote sensing data should be managed with a more industrial approach. Although you can use Python as a programming language, MATLAB, or even the wonderful C, you will need to arm yourself with patience to set up your database and your data lake.

IX. EXPLORING COINCIDENCES IN THE JT DATASET.

When working with coincidence files, I encounter a long list of elements whose meanings I've previously discussed. These files typically come in HDF5 format (version 5) and can be opened using tools like **HDFVIEW** [38] or **Panoply** [39]. In this case, I'll focus on Panoply, which automatically reads the metadata content. Alternatively, if you're using MATLAB, you can programmatically obtain the metadata, although it requires some coding. The metadata associated with netCDF files

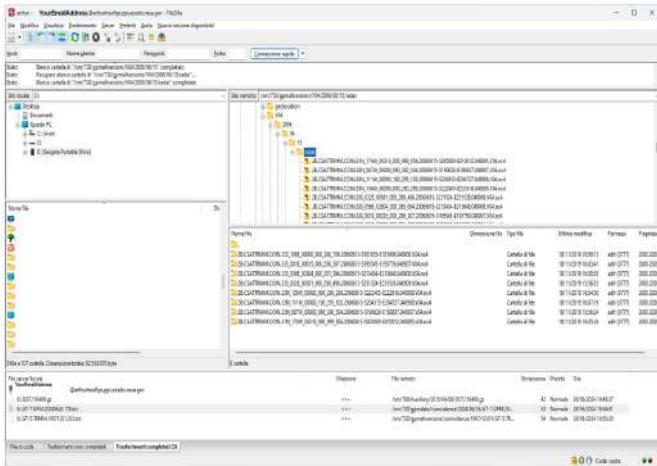


Fig. 9: Figure illustrating where the other coincidences that are CloudSat-TRMM are located. The Data Path is: /sm/730/gpmallversions/V04/2006/06/15/radar

contains global attributes that provide valuable insights into the complexity of the data within the coincidence files. Notably, netCDF v4 files often include data from other formats, such as native HDF5. By leveraging this format, I streamline the process of reading diverse data originating from various remote sensing products. For CloudSat-GPM coincidence files, pay special attention to the header, which holds crucial coincidence information. Theoretical details can be found in relevant documents, including the **J.F.Turk** article [1], [25]. As you work with these files, familiarize yourself with product types and group names — essentially expandable nodes within the Panoply “Datasets” tab. When writing source code in MATLAB to read NetCDF4 files, leverage the metadata to construct access instructions for the scientific variables of interest. The author has a sample file name:

```
2B.CSATGPM.COIN.25S_174W_13575_000_295_403.  
20140309-S013654-E013756.000146.V01A.NC
```

- Fifth field: represents the start and end times of the coincidence segment (20140309-S013654-E013756).
- Sixth field: is the GPM orbit revolution number (000146).
- Fourth field: provides coincidence-specific quality information (25S 174W 13575 000 295 403).
- Interpretation of the six values represented within this field:
 - Field 1, (25S) the Latitude of the coincidence, to the nearest degree.
 - Field 2, (174W) the Longitude of the coincidence location, to the nearest degree.
 - Field 3, (13575) the number of CloudSat bins where the cloud mask is ≥ 40 .
 - Field 4, (000) the percent of CloudSat profiles that are over land.
 - Field 5, (295) the Minimum 2-m air temperature from all CloudSat profiles in the dataset.
 - Field 6, (403) the time offset (absolute value) between CloudSat and GPM.

Therefore, by simply analyzing the file name, matches that meet specific requirements can be easily spotted without having to open and read the file. For example, to isolate overland snowfall cases, a user would search for files with a 2 m air temperature near or below 273K, 100 percent over land, and a sufficiently large number of CloudSat bins to identify the cloud body. Further information necessary to understand the algorithmic philosophy behind these names can be found by consulting the documentation on CloudSat-GPM coincidences at [24] by downloading the 4.31 MB, `CSATGPM_COIN_ATBD.pdf` file. This document provides information on the algorithm and refers to the acronym **IMERG (Integrated Multi-Satellite Retrievals for Global Precipitation Measurement Mission)** [48], which is therefore an acronym. The described algorithm allows you to calibrate, fuse, and interpolate all the precipitation estimates measured at microwaves with the estimates calibrated at infrared microwaves, the analyses of precipitation obtained from sensors, and also merge the measurements with other precipitation estimators such as the TRMM (Tropical Rainfall Measuring Mission) and GPM for the entire planet.

The file name is `IMERG_V07_ATBD_final_230712.pdf`. Additional files of interest for review:

- PrecipitationProcessingSystem(PPS) (nasa.gov)
- Master_List_of_PPS_Data_Products.pdf
- filespec.GPM_.pdf(nasa.gov)
- filespec.TRMM_.V7.pdf
- MicrosoftWord-GPROFV05B_releasenotes.docx (nasa.gov)
- IMERG_V07_ATBD_final_230712 (nasa.gov)

A. Detailed Information about CloudSat-GPM Coincidence JT Dataset

The CloudSat-GPM Coincidence JT Dataset offers a unique opportunity to study the interactions between CloudSat’s 94-GHz profiling radar (CPR) and the Global Precipitation Measurement (GPM) core satellite’s Ku/Ka band radar (DPR) and radiometer (GMI). By combining these datasets, researchers can generate pseudo three-frequency radar profiles, which are invaluable for various scientific applications, including algorithm evaluation, snow and light rain sensitivity studies, cloud process studies, and radiative transfer simulations. By coincidence, The author has the following elements:

- 1C.GPM.GMI
- 1C.NPP.ATMS
- 2A.GPM.DPR
- 2A.GPM.GMI.GPROF
- 2A.NPP.ATMS.GPROF
- 2B-CLDCLASS
- 2B-CWC-RO
- 2B-CWC-RVOD
- 2B-GEOPROF
- 2B-GEOPROF-LIDAR
- 2B.GPM.DPRGMI
- 2C-ICE
- 2C-PRECIP-COLUMN
- 2C-RAIN-PROFILE
- 2C-SNOW-PROFILE

- ECMWF-AUX
- MODIS-AUX

Here is a description of each of the items listed:

- **1C.GPM. GMI:** This refers to the Level 1C algorithm for the Global Precipitation Measurement (GPM) Microwave Imager (GMI). The Level 1C algorithm produces common calibrated brightness temperature products for the GPM core and constellation satellites.
- **1C.NPP.ATMS:** This refers to the 1C-level algorithm for the Advanced Technology Microwave Sounder (ATMS) on board the Suomi National Polar-Orbiting Partnership (NPP) satellite.
- **2A.GPM. DPR:** This refers to the GPM's dual-frequency precipitation radar (DPR) data product. Provides rainfall estimates for individual orbits.
- **2A.GPM.GMI. GPROF:** This refers to the 2A-level GPROF algorithm for the GPM Microwave Imager (GMI). The algorithm estimates the precipitation rate from individual orbit brightness temperature data from passive microwave radiometers.
- **2A.NPP.ATMS.GPROF:** This refers to the level 2A GPROF algorithm for the ATMS on board the NPP satellite.
- **2B-CLDCLASS:** This product classifies clouds into stratus (St), stratocumulus (Sc), cumulus (Cu, including cumulus congestus), nimbostratus (Ns), altocumulus (Ac), altostratus (As), deep convective (cumulonimbus), or high clouds (cirrus and cirrostratus), based on different rules for the vertical and horizontal scales of hydrometeors, the maximum Ze measured by the CPR, precipitation indications, and auxiliary data.
- **2B-CWC-RO:** The CloudSat Radar-Only Cloud Water Content (2B-CWC-RO) product provides estimates of cloud liquid and ice water content, effective radius, and related quantities for each radar profile measured by the Cloud Profiling Radar on CloudSat.
- **2B-CWC-RVOD:** The CloudSat Radar-Visible Optical Depth Cloud Water Content (2B-CWC-RVOD) product contains retrieved estimates of cloud liquid and ice water content, effective radius, and related quantities for each radar profile measured by the Cloud Profiling Radar on CloudSat.
- **2B-GEOPROF:** The 2B-GEOPROF R04 product identifies those levels in the vertical column sampled by CloudSat that contain significant radar echo from hydrometeors (rather than noise or radar clutter) and provides an estimate of the radar reflectivity factor for each of these volumes.
- **2B-GEOPROF-LIDAR:** The 2B-Geoprof-lidar product indicates the fraction of each GeoProf radar volume that contains a lidar cloud detection (detected using a long track average of 5 km or shorter), as well as a variety of summary values (such as the highest detected cloud height).
- **2B.GPM.DPRGMI:** It determines the precipitation structure that best fits the combined data from these instruments. It could be a combination of data from the

GPM's dual-frequency precipitation radar (DPR) and the GPM microwave imager (GMI).

- **2C-ICE, 2C-PRECIP-COLUMN, 2C-RAIN-PROFILE, 2C-SNOW-PROFILE:** These may refer to specific data products related to ice, precipitation column, rain profile, and snow profile.
- **ECMWF-AUX:** This may refer to auxiliary data provided by the European Center for Medium-Range Weather Forecasts (ECMWF), which are often used in combination with other satellite data for various research and analysis purposes.
- **MODIS-AUX:** This may refer to auxiliary data provided by the Moderate Resolution Imaging Spectroradiometer (MODIS), a key sensor aboard NASA's Terra and Aqua satellites, which provides data in a wide range of spectral bands.

For additional information, it is advisable to consult the official documentation or technical resources associated with each product or tool.

B. More Detailed Info about Items

In this section, I delve into further detail regarding the essential elements during the programming phase (i.e., software implementation). Whenever possible, I'll associate source code snippets or data visualization applications with each described element. Additionally, there is software capable of automatically graphing data by recognizing its format and meaning. Primarily, nray and nlev denote the number of rays and atmospheric levels considered.

- **Dimensions:**
 - nray : 1544
 - nlev : 125
- **Groups:**
- (2B)-GEOPROF:
 - Variables related to CloudSat profiling, including time, latitude, longitude, DEM, surface height bin, land/sea flag, Sigma-Zero, radar reflectivity, cloud mask, and height.
- (2A.GPM.DPR):
 - Contains two sub-groups: NS and MS, each with variables for precipitation rate, phase, type, shallow rain flag, path integrated attenuation, normalized cross-section, local zenith angle, elevation, height of 0-deg C level, scan index, ray index, distance, and measured Z-factor.
 - Each sub-group (NS and MS) includes a SWATH sub-group with dimensions and variables for scan index, measured Z-factor, latitude, longitude, elevation, real surface bin, clutter-free bottom bin, and local zenith angle.
- (1C.GPM.GMI):
 - Contains variables for GMI (Tc), scan index, pixel index, and distance from GMI to CloudSat ray.
 - Includes a SWATH sub-group with dimensions and variables for scan index, Tc, longitude, and latitude.
- (ECMWF)-AUX:

- Variables for ECMWF auxiliary data, such as 2-m air temperature, skin temperature, surface pressure, height where temperature crosses 0-deg C, 10-m zonal and meridional wind speed, and profiles of pressure, temperature, specific humidity, zonal wind, and meridional wind.
- (2C)-ICE:
 - Variables for ice water content and effective radius.
- (2B)-CWC-RO:
 - Variables for ice distribution width parameter, ice number concentration, ice water path, liquid water path, ice water content, and liquid water content.
- (2B)-CWC-RVOD:
 - Variables for liquid water path, ice water content, liquid water content, cloud liquid water path, ice water path, cloud liquid water content, and PIA forward.
- (2B)-CLDCLASS:
 - Variables for cloud scenario bit mask.
- (2B)-GEOPROF-LIDAR:
 - Dimensions for cloud layers.
 - Variables for the number of cloud layers, flags for base and top contribution, and heights of observed hydrometeor layer top and base.
- (2C)-RAIN-PROFILE:
 - Variables for surface rain rate, precipitation water content uncertainty, precipitation liquid water profile, rain rate uncertainty, rain quality flag, and precipitation ice water profile.
- (2C)-SNOW-PROFILE:
 - Variables for surface snowfall rate, snow retrieval status, snowfall rate profile, and snow water content profile.
- (2C)-PRECIP-COLUMN:
 - Variables for precipitation status flag and convective/stratiform flag.

C. Extremely detailed list

Key Variables and their Attributes:

1) 2B-GEOPROF

- a) • time: double (seconds since 1 Jan 1970)
- b) • Latitude: float (degrees)
- c) • Longitude: float (degrees)
- d) • DEM: int (meters)
- e) • SurfaceHeightBin: int
- f) • land sea flag: int
- g) • Sigma-Zero: short (dB, scale factor 0.01)
- h) • along track dist: float (km)
- i) • ray index: int
- j) • Radar Reflectivity: short (dB, scale factor 0.01)
- k) • CPR Cloud mask: byte
- l) • height: short (meters)

2) 2A.GPM.DPR

- a) • precipRateNearSurface: float (mm/hr)

- b) • phaseNearSurface: ubyte
- c) • typePrecip: int
- d) • flagShallowRain: int
- e) • piaFinal: float (dB)
- f) • sigmaZeroMeasured: float (dB)
- g) • localZenithAngle: float (degrees)
- h) • elevation: int (meters)
- i) • heightZeroDeg: int (meters)
- j) • scan index: int
- k) • ray index: int
- l) • distance: float
- m) • zFactorMeasured: short (dB, scale factor 0.01)

3) 1C.GPM.GMI

- a) • Tc: float (Kelvin)
- b) • scan index: int
- c) • pix index: int
- d) • pix km: float

4) ECMWF-AUX

- a) • Temperature 2m: float (Kelvin)
- b) • Skin temperature: float (Kelvin)
- c) • Surface pressure: float (Pa)
- d) • height 273K: int (meters)
- e) • U10 velocity: float (m/s)
- f) • V10 velocity: float (m/s)
- g) • Pressure: float (Pa)
- h) • Temperature: float (Kelvin)
- i) • Specific humidity: float (kg/kg)
- j) • U velocity: float (m/s)
- k) • V velocity: float (m/s)

D. Detect Most important Data for Global Attenuation Map

To achieve my goal, I need to delve into the individual groups within the coincidence file. Currently, my analysis focuses solely on the coincidences between CloudSat and GPM. However, for a comprehensive global map, I should also consider coincidences between CloudSat and TRMM. For now, let's concentrate on CloudSat-GPM and develop robust MATLAB source code to visualize attenuation data. The purpose of this paragraph is to explore the relevant elements that contribute to creating a global attenuation map, both theoretically and practically. On one hand, I must grasp the theoretical aspects of meteorology and geophysics. On the other hand, understanding the onboard instrumentation and characteristics of microwave radiometers is crucial.

The GPM Microwave Imager (GMI) is a multi-channel radiometer designed to detect total precipitation across all cloud layers, including light rain and snowfall. It achieves this by measuring the intensity of electromagnetic energy emitted from various parts of the Earth system, encompassing rain, snow, and other phenomena. Microwaves, being part of the electromagnetic spectrum, allow me to gather information beyond what visible wavelengths provide. Specifically, the GMI features 13 channels for measuring microwave radiation intensity from the Earth's surface and atmosphere. In my coincidence file, these channels correspond to the product 1C.GPM.GMI. Notably, I link this data to the concept of **brightness temperature (TB)**, denoted by the variable Tc

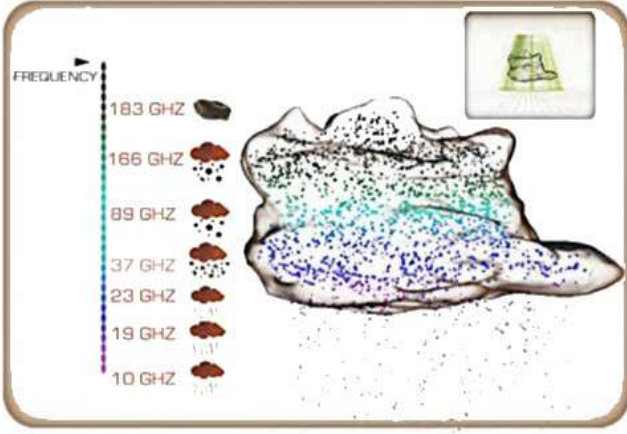


Fig. 10: The GMI will sense the total precipitation within all cloud layers, including snow and ice, with its new high frequency channels (166 to 183 Gigahertz).

in Kelvin (K). Let me therefore provide a better theoretical description of these channels: Lower-frequency channels (10 to 89 GHz, similar to those of the microwave imager aboard the TRMM satellite) detect heavy to moderate precipitation. GMI's advances include four additional high-frequency channels (166 to 183 GHz) that will measure moderate to light precipitation. Each object, such as rain, snow, and land surfaces, emits or disperses energy differently based on the object's temperature and physical properties. Scientists use their knowledge and the contrast between signals received from different channels to distinguish between rain and snow, calculate precipitation rates, and quantify precipitation intensity.

Figure 10 shows the frequencies of interest in the study of precipitation within cloud bodies. The following table shows some characteristics between the TRMM Imager and the GPM GMI Imager. The brightness temperature (BT) serves as a crucial indicator of the signal intensity received by the microwave radiometer. This measurement is particularly responsive to the presence of hydrometeors — such as raindrops or ice crystals — in the atmosphere, making it valuable for detecting cloud bodies. The Dual-Frequency Precipitation Radar (DPR) employs two cross-scan precipitation radars: one operating at Ku-band frequency and the other at Ka band frequency. Both radars share a spatial resolution of 5 kilometers (approximately 3 miles) and emit 4100 to 4400 pulses per second, each with a pulse length of 250 meters (about 820 feet). Notably, while the Ku-band radar scans its widest swath using 250-meter pulse lengths, the Ka band radar covers its swath using both 250-meter and 500-meter (approximately 1640 feet) pulse lengths. This dual configuration allows the Ka band radar to capture measurements with high sensitivity, which is critical for observing water droplets and smaller ice particles. To differentiate between rain and snow, I rely on variations in how Ku- and Ka band radar pulses alter in intensity when encountering different precipitation types.

	Channels	Frequency Range	Swath
TMI (TRMM)	9	10-85.5 [GHz]	758.5 [km] (471 miles)
GMI (GPM)	13	10-183 [GHz]	885 [km] (550 miles)

TABLE II: TRMM Microwave Imager vs GPM Imager.

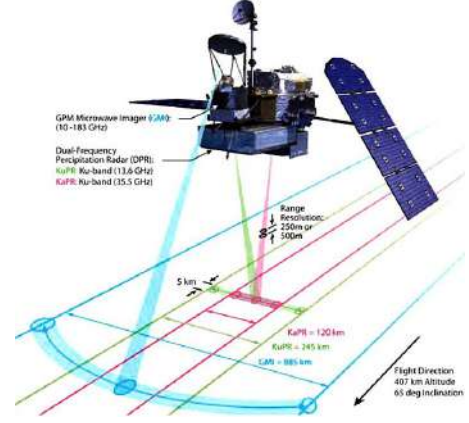


Fig. 11: The image shows the GMI and DPR instruments aboard the GPM Core Observatory and their respective swaths—the area of Earth's surface observed by the instrument.

X. BRIGHTNESS TEMPERATURE AND CHARACTERISTICS OF HYDROMETEORS.

Let's focus on the concept of brightness temperature (TB) and its relevance to microwave radiometers.

Brightness Temperature:

- (TB) is a fundamental parameter measured by passive microwave radiometers. It represents the radiance of microwave radiation traveling upward from the top of the atmosphere to the satellite, expressed in units of the temperature of an equivalent black body.

Essentially, TB provides an equivalent temperature that corresponds to the amount of radiation received by the radiometer. It's like imagining that the microwave radiation comes from a black body (an idealized object that absorbs all incident radiation) at that specific temperature.

Microwave Radiometers:

- Microwave radiometers operate in the microwave spectral range (wavelengths of 1 mm to 1 m, frequencies of 0.3–300 GHz). They are sensitive receivers designed to measure thermally-emitted electromagnetic radiation. These instruments are used for various applications, including remote sensing, weather forecasting, climate monitoring, radio astronomy, and radio propagation studies.
- By observing microwave radiation, microwave radiometers provide complementary information to visible and infrared sensors. The atmosphere and vegetation are semitransparent in this range, allowing measurements even through clouds [19]. In my specific case, the measurements at 89 GHz and 166 GHz (polarimetric type) contain valuable information about frozen particles and their orientation. The positive difference in TB between vertical (V-pol) and horizontal (H-pol) polarizations indi-

cates interesting scattering behavior, with a peak around 10 Kelvin for both channels.

In the study referenced by [20], the researchers aimed to better understand the scattering signatures described by **Lu et.al. (2016)** [40] and by **Gong and Wu (2017)** [41]. These signatures are indicative of specific behaviors related to frozen particles.

A. Operate concretely on the brightness temperature

To begin, reading the metadata associated with individual coincidence files is crucial. You can obtain this metadata from Panoply. Next, I'll create a MATLAB project that visualizes the brightness and temperatures of cloud bodies. The information associated with the metadata is as follows:

Algorithm 1 Metadata Information

```
% dimensions:
nscan_GMI = 208;
npix_GMI = 221;
% variables:
int scan_index(nscan_GMI=208);
:description = "along track scan index from
1C.GPM.GMI for full swath segment of GMI that covers
the coincidence curtain";
float Tc(nscan_GMI=208, npix_GMI=221, ntb_GMI=13);
:units = "Kelvin";
:description = "GMI full swath TB.
Channel order is
10V 10H 18V 18H 23V 36V
36H 89V 89H 166V 166H 183+/-3 183+/-7";
float Longitude(nscan_GMI=208, npix_GMI=221);
:units = "degrees";
:description = "GMI full swath longitude";
float Latitude(nscan_GMI=208, npix_GMI=221);
:units = "degrees";
:description = "GMI full swath latitude";
```

Now This work shows the graphical result of this program. The brightness temperature range must change by removing the default value -99 when the measurement is not detected.

Put this correction before “Change the data from single type to double”. By comparing the same graph with Panoply, I discover that more processing is needed to visualize the cloud bodies well. In particular, Panoply automatically carries out some additional calculations regarding the brightness temperature values, which can be viewed in the form of tabular data.

This circumstance, understanding the meaning of the channels, pushes me to seek more theoretical information on the processing of the raw data present in the coincidences, which is a normal practice. In Panoply, by utilizing the “CTRL+Right” mouse click, the image can be enlarged, allowing for the recognition of cloud formations as the satellite traverses the area.

XI. WORKING ON BRIGHTNESS TEMPERATURE IN THE COINCIDENCE DATASET.

The code appears to define a three-dimensional array Tc with dimensions nscan GMI = 208, npix GMI = 221, ntb GMI = 13. This array could represent, for example, a series of thermal scan images, where each image has a grid of pixels and each pixel has a set of 13 temperature values (presumably for different frequencies or channels).

Algorithm 2 Loading Data, MATLAB Code.

```
% Load the data
nscan_GMI = 208;
npix_GMI = 221;
% I need to read the file and the following data:
% Name and path of the netCDF4 File.
filename = 'C:/Users/YourUsername/
CLOUDSAT_nuovi_download/
2B-CSATGPM.COIN.01N_027W_00197_000_297_441.
20160831-S152953-E153400.014251.V04.nc4';
if exist(filename, 'file') ~= 2
error('File %s does not exist or is not
accessible.', filename);
end
% Example:
% lat = h5read(filename, '/2B-GEOPROF/Latitude');
% Scan index
scan_index = h5read(filename,
'/1C.GPM.GMI/scan_index');
% GMI Tc that is closest to this CloudSat ray,
% copied from 1C.GPM.GMI.
% Channel order is 10V 10H 18V 18H 23V 36V
% 36H 89V 89H 166V 166H 183+/-3 183+/-7
% Brightness Temperature in [K]
Tc = h5read(filename, '/1C.GPM.GMI/Tc');
% Longitudes
Longitude =
h5read(filename, '/1C.GPM.GMI/SWATH/Longitude');
% Latitudes
Latitude =
h5read(filename, '/1C.GPM.GMI/SWATH/Latitude');
% Change the data from single type to double
dLongitude = double(Longitude);
dLatitude = double(Latitude);
dTc = double(Tc);
% Create a new figure
figure;
% Create a map with coastlines
axesm('MapProjection','eqdcylin', 'MapLatLimit', [-90
90], 'MapLonLimit', [-180 180]);
geoshow('landareas.shp', 'FaceColor', [0.5 0.70.5]);
% Draw the data on the map
% Choose a specific channel to display,
% for example the first channel
channel = 1;
pcolorm(squeeze(dLatitude(:, :, channel)),
squeeze(dLongitude(:, :, channel)),
squeeze(dTc(:, :, channel)));
% Add a color bar
colorbar;
% Set the title of the map
title('Brightness Temperature (K)');
```

Algorithm 3 Snippet code in MATLAB, Data filtering and managing NaN values.

```
% Replace -99 with NaN
% These values of -99 indicate missing data.
Tc(Tc == -99) = NaN;
```

If you want to obtain a matrix of points from this three dimensional array, you might need to specify which “slice” of the array you want to view as a matrix. For instance, you might want to view a single thermal scan image (a 2D matrix of pixels for a specific frequency), or you might want to view all the temperature values for a single pixel across all scans (a 2D matrix of scans for frequencies). When you export the Tc data as a CSV (Comma Separated Values) file, you'll notice that the transition from one channel to another is marked by a simple blank line. This allows me to interpret the various values effectively by taking into account the thirteen

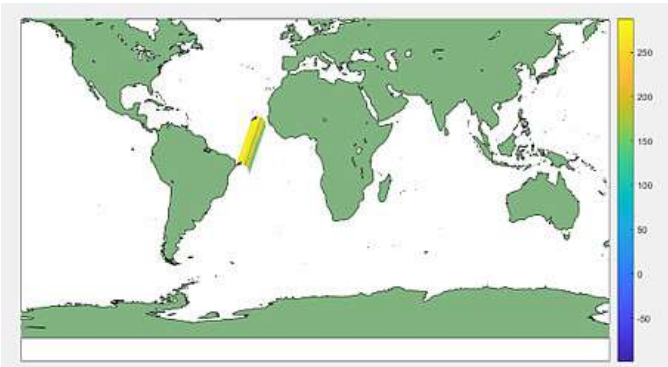


Fig. 12: The image is a detailed map of the world, showcasing Brightness Temperature (BT) in Kelvin (K) with a clear, color coded overlay. This overlay ranges from deep blue to yellow, indicating varying BT levels across the highlighted region in the Atlantic Ocean, positioned between South America and Africa. On the right side, there is a color bar that provides a scale for interpreting the BT values. This scale ranges from -60 K to 250 K, offering a visual representation where: Deep blue indicates the lowest temperatures, closer to -60 K. Yellow represents the highest temperatures, reaching up to 250 K. The highlighted region on the map shows a gradient of brightness temperatures, moving from the cooler blue areas to the warmer yellow ones. This distribution offers insights into the thermal properties of this part of the Earth's surface, which can be crucial for various scientific applications.

available channels. To delve deeper into the data structure of brightness temperatures, I need to undertake additional steps. Specifically, the data will be segmented into blocks of 221 rows and 208 columns across 13 frequencies. This type of data processing necessitates specialized knowledge and the ability to manipulate various data types. For instance, I could export the variable from Panoply as a CSV file and then import this data into MATLAB. By executing these operations, I would enhance the graphical representation associated with the information, thereby achieving a higher level of detail. The theoretical elucidation concerning the interpretation and utilization of brightness temperature (TB) data, as measured by a microwave radiometer like the GMI on the GPM satellite, hinges on the understanding that brightness temperature is a quantification of the intensity of the microwave signal emitted or reflected from the Earth's surface and atmosphere. This signal's intensity is influenced by several factors, including the physical temperature of the object emitting or reflecting the signal, its emissivity (that is, its efficiency in emitting microwave radiation), and the characteristics of the atmosphere through which the signal propagates. In the realm of precipitation measurement, for instance, brightness temperatures at varying frequencies can yield information about the size, shape, and phase (liquid or solid) of raindrops or snow crystals. This is attributed to the fact that different microwave frequencies interact in distinct ways with raindrops and snow crystals of varying sizes and shapes. Therefore, by scrutinizing brightness temperatures measured across different channels

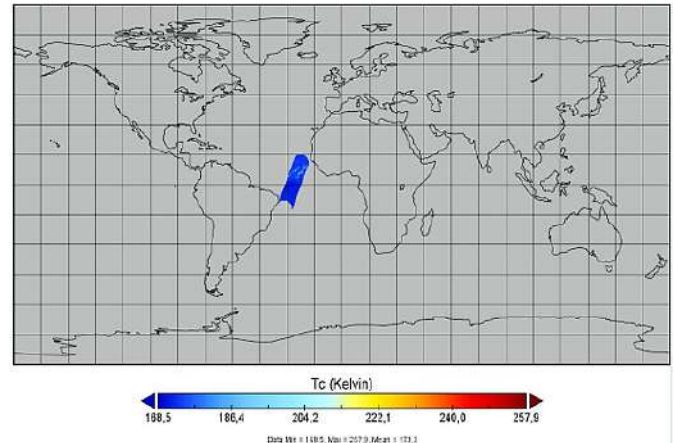


Fig. 13: The same Brightness Temperature (TB) in Kelvin showed in **Figure 14**, indicated as Tc, is plotted in Panoply. The figure presents a color-coded map of the world, focusing on the Atlantic Ocean region, showcasing brightness temperatures ranging from 168.5 K (dark blue) to 257.9 K (red). The color scale on the right illustrates this range, highlighting variations in thermal properties across the area. The data indicates a minimum temperature of 168.5 K, a maximum of 257.9 K, and a mean temperature of 173.3 K.

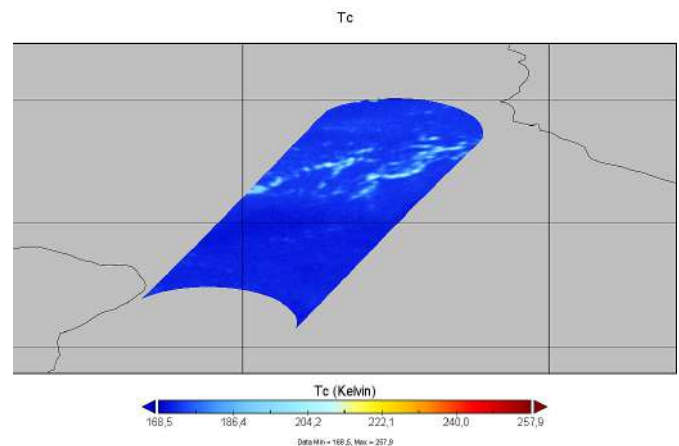


Fig. 14: Brightness Temperature for the swath variable in 1C.GPM.GMI group. GMI full swath TB.

(i.e., at different frequencies), researchers can glean invaluable insights about precipitation and other atmospheric processes. This information can subsequently be employed to enhance my comprehension of the Earth's water cycle, to refine weather and climate forecasts, and for other applications. As you can see, the graph obtained in MATLAB has the same appearance in Panoply but has a higher graphic resolution and greater detail. If you are then wondering why this complication and how the value of 208 can result as a dimension, consider taking the floor of $(46175 - 13)/221$, where 46175 are the rows including the blanks between one block and another and 13 is the number of channels.

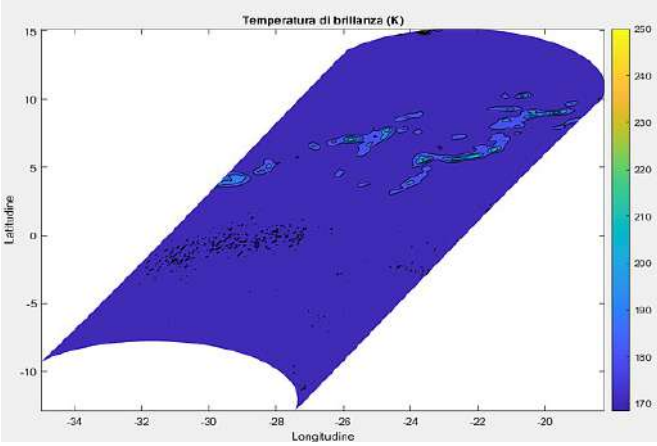


Fig. 15: Brightness Temperature in Kelvin vs Latitude vs Longitude in degrees, in a MATLAB environment.

XII. RADAR REFLECTIVITY AND ATTENUATION RELATIONSHIP.

A. Radar Reflectivity (Z)

Reflectivity is the amount of energy that is returned (reflected) back to the receiver after hitting a target. In the context of weather radar systems, reflectivity is a measure of the efficiency of a target in intercepting and returning radio frequency energy. This is often used to indicate where precipitation and severe weather is occurring. The units of reflectivity (denoted by the letter Z), as derived from the radar equation, are in $[mm]^6 \cdot [m]^{-3}$. After applying a logarithmic scale to these values, I get units of decibels (dB) of reflectivity (Z) or [dBZ]. Radar reflectivity Z is a measure of the power of signals reflected by hydrometeors, usually expressed in dBZ (decibels of Z). The relationship between radar reflectivity Z and attenuation can be expressed considering that high reflectivity often indicates a higher density of hydrometeors, which in turn leads to an increase in the attenuation of the radar signal. Reflectivity Z can be used to estimate the PIA through empirical relationships. For example, knowing the reflectivity Z along a path, it is possible to estimate the PIA using empirical models that link Z and the specific attenuation α . So $\alpha = f(Z)$ where $f(\dots)$ is an empirical function derived by spermental observations.

B. Attenuation (A)

Attenuation refers to the gradual loss of flux intensity through a medium. In the context of atmospheric sciences, attenuation can be divided into atmospheric absorption attenuation and scattering attenuation; both are primarily caused by atmospheric molecules and aerosol particles. The attenuation produced by oxygen and water vapor in the atmosphere is described by the specific attenuation for each component, expressed in [dB/km]. These concepts are crucial for understanding and interpreting the data in the JT dataset. They provide insights into how energy interacts with atmospheric components and how these interactions can be quantified and analyzed. Understanding these concepts can help in the exploration and analysis of the JT dataset.

C. Path Integrated Attenuation (PIA)

PIA is a measure of the cumulative attenuation that a signal undergoes as it propagates through a medium. The PIA is calculated by integrating the specific attenuation (dB/km) along the signal path:

$$PIA = \int_0^L \alpha(l) \cdot dl$$

where α is the specific attenuation as a function of distance l , and L is the total length of the path. The PIA is particularly important in weather radar applications, as it helps to quantify the loss of signal intensity due to interaction with hydrometeors such as rain, snow, and hail [26].

D. Effective Reflectivity (Z_e)

Reflectivity is a measure of the power of signals reflected by a hydrometeor, usually expressed in dBZ (decibels of Z). The effective reflectivity, or Z_e , represents the reflective power of a body subjected to irradiation. It is a dimensionless percentage parameter. Reflectivity allows defining the spectral signature of a surface and therefore identifying it, where $A(r)$ is the 2-way attenuation factor. In their work “RainProfiling Algorithm for the TRMM Precipitation Radar”, they describe a standard algorithm that estimates the vertical profiles of the radar reflectivity factor corrected for attenuation and the precipitation rate. The attenuation is corrected using a hybrid of the **Hitschfeld–Bordan (HB) method** and a **Surface Reference Method**. In another work, “Path Attenuation Estimates for the GPM Dual-frequency Precipitation Radar (DPR)”, **Meneghini et.al. (2021)** [28], describe the most recent implementation of the **Surface Reference Technique (SRT)** that uses surface scattering properties to infer the path attenuation through precipitation. Both single and dual-frequency methods are available and, despite the improvements offered by the dual-frequency version of the method, other methods such as the **Hitschfeld–Bordan (HB)** [27], approaches and dual frequency standard offer advantages particularly at lighter rain rates and at near-nadir incidence angles on the earth [28].

XIII. RADAR CONCEPTS.

A. Weather Radar

A weather radar is an instrument that uses electromagnetic signals to detect and quantify atmospheric precipitation. The operating principle is based on the emission of electromagnetic pulses and the reception of signals reflected by hydrometeors. The radar theory includes:

- Radar equation: It describes the relationship between the power of the transmitted signal, the received power, the distance from the target, and the reflectivity of the target.
- Scattering and attenuation: Analysis of how electromagnetic signals are dispersed and attenuated by hydrometeors.
- Radar polarimetry: Use of signals with different polarizations to improve the identification and characterization of hydrometeors.

The radar technique includes:

- Radar calibration: Ensuring that measurements are accurate and consistent.
- Signal processing: Techniques for filtering noise and improving the quality of radar data.
- Precipitation estimation algorithms: Converting reflectivity measurements into quantitative estimates of precipitation.

The weather radar is crucial for weather forecasting, water resource management, and civil protection. It allows for real-time monitoring of adverse weather conditions, improving the response capability to extreme events such as thunderstorms, floods, and hurricanes. A detailed understanding of polarization, attenuation, and PIA improves the accuracy and reliability of radar observations, contributing to a more precise and timely weather forecast. For example using **THOR (Tool for High-Resolution Observation Review)** [43] by NASA is possible to visualize both TRMM and GPM products in a very simple manner, especially the **PPS (Precipitation Processing System)** product [44].

In this section, It is possible to introduce some simple notions about radar theory and technique. These will be useful for better understanding some theoretical aspects related to the study of hydrometeors.

B. Understanding Radar's Role in Hydrometeor Detection and Distance Measurement.

A weather radar enables the detection of hydrometeors, such as raindrops, within a volume of the atmosphere. These hydrometeors influence the electromagnetic propagation of the radar beam. The fundamental concept of radar involves exploring the surrounding environment through the artificial emission of electromagnetic waves. When these waves are transmitted into the atmosphere and encounter a metallic or generic target (not necessarily metallic), they are reflected and subsequently return to the ground-based radar station or receiver. By knowing the propagation speed at which the electromagnetic wave travels from a source and the time taken to complete the round trip, it's possible to estimate the distance using the relation " $D = c/2t$ ", where " c " represents the speed of light in the medium. The signal received by the radar is proportional to the reflectivity Z . In the case of icy bodies present in the clouds, this reflectivity is proportional to the water content.

C. Intensity of Precipitation

An empirical relationship exists between the intensity of precipitation R , or RR = Rain Rate, measured in [mm/h] (millimeters per hour) in a portion of the atmosphere investigated by the radar, and the corresponding value of radar reflectivity (Z). This reflectivity is generally measured in [$mm^6 \cdot m^{-3}$] or in [dBZ] (decibels of reflectivity), indicating the ability to reflect the radar pulse (or pulse train), thereby characterizing the same portion of the atmosphere under examination. The relationship is of the type:

$$Z = a \cdot R^b \quad (3)$$

Type of Precipitation	a	b
Drizzle	50	1.6
Rain	200	1.6
Downpour	800	1.6
Dry Snow	800	1.6
Wet Snow	430	2.0

TABLE III: Values of parameters a and b for different types of precipitation.

where the pair of parameters (a, b) is appropriately defined. The choice of these parameters is a function of the type of precipitation measured with reference to the dimensions of the hydrometeors and the liquid-frozen state of the water that composes it. The parameter " a " is affected by variations in these characteristics, and starting from a choice of values (a, b) , It is possible to classify the type of precipitation as ranging from simple drizzle to thunderstorm downpour, or from dry snow to wet snow.

The dBZ scale (decibels relative to Z) is a dimensionless logarithmic technical unit used to compare the equivalent reflectivity factor (Z_e) of a remote object or target with the return of a raindrop having a diameter of 1 millimeter (mm). Therefore, the volume is measured in cubic meters, while the sixth power appears as a law related to the diameter of the raindrops. More details and information can also be summarized in this table, with which The author has associated a radial reflectivity diagram generated by my MATLAB software. The diagram shows a coincidence file, which, when subsequently analyzed, allows me to temporally identify the passage of the satellite within the observation window. Similar graphs can also be obtained in other software environments or directly in standalone applications like Panoply. However, programming these applications allows for greater control over the data and their manipulation, graphic rendering, and scientific analysis. I recall that the well-known **Marshall-Palmer (MP)** relationship can be used to determine the values of (a, b) for stratiform rain, and It is possible to derive that $a \approx 200, b \approx 1.6$. The values of (a, b) in this case are approximate and can vary depending on local atmospheric conditions and the type of precipitation.

The relationship according to which $Z = a \cdot R^b$ is always a simplification and does not take into account factors such as the attenuation of the radar signal or the presence of raindrops of different sizes.

D. Radar reflectivity

The radar reflectivity Z for rain can be expressed in integral terms through the following relationship:

$$Z_{rain} = |K_{rain}|^2 \cdot \int_0^{D_{max}} N(D) \cdot D^6 \cdot dD \quad (4)$$

where K indicates the electrical permittivity of water and where D indicates the diameter of the rain particles, while $N(D)$ indicates the number of particles having diameter D . In general, the rain rate (RR =Rain Rate), which is also indicated with R , can be expressed as:

$$R = \int_0^{D_{max}} N(D) \cdot \frac{\pi \cdot D^3}{6} \cdot v(D) \cdot dD \quad (5)$$

where $\pi \cdot D^3/6$ indicates the considered volume and where $v(D)$ is the falling speed of the particles to the ground. The expression of $N(D)$ generally represents a distribution expressed as a function of the parameter D and is that relationship that links the **Marshall-Palmer (M-P)** relationship [45] with the notion of rain rate. In turn, generally, the distribution of the number of falling drops (raindrops) can be expressed as a function of the concentration:

$$N(D) = N_0 \cdot D^\mu \cdot e^{-\Lambda \cdot D} \quad (6)$$

where it holds that:

- N_0 represents the initial concentration equal to 8000 [$m^{-3} \cdot mm^{-1}$].
- $\Lambda = a \cdot R^b$ where R is the rain rate in [mm/h] while (a, b) are a pair of empirical values.

In the case of the **Marshall-Palmer (M-P)** distribution, The author has that $a = 4.1$ and $b = -0.21$.

The *Gamma* parameter Λ is expressed as [mm^{-1}].

- D represents the diameter of the raindrops, usually expressed in [mm] (millimeters).

The value K_{rain} is also synonymous with the parameter a in the relationship within the context of the **Marshall-Palmer distribution (1948)**, [45].

A good reference to the relationships between Z and R can be found in [42]. From **Appendix A** of that document, the scaling law formalism is proposed, according to which from **Sempere Torres et.al. (1994, 1998)** [30], [31] it is obtained that (**Uijlenhoet, 1999**) [32]:

$$a = \int_0^{+\infty} x^6 \cdot g(x) \cdot dx \quad (7)$$

$$b = \alpha + 7 \cdot \beta \quad (8)$$

where (a, b) are the dimensionless coefficients of the power law that links Z to R and where $g(x)$ is a function that represents the generic or general distribution of raindrops as a function of the scaling diameter x , defined in turn as:

$$x = \frac{D}{R^\beta} \quad (9)$$

where the pair (α, β) are two dimensionless variables. Finally, let's remember some simple definitions for the radar Range, distinguishing between the monostatic radar configuration and the bistatic one. For a bistatic radar The author has that:

$$R_{transmitted} + R_{reflected} = c \cdot T_R \quad (10)$$

While for a monostatic radar it is the case that the transmitted Range is equal to the reflected Range, for which it will be valid that:

$$R = \frac{c \cdot T_R}{2} \quad (11)$$

where the subscript R indicates the “Range” and T indicates the period, while c is “celer” the speed of light in free space which approximates to $3 \cdot 10^8 [m/s]$. Finally, it should be remembered that the **M-P (Marshall-Palmer)** relationship [45] is valid for mid-latitudes. For other locations, for example, in the tropical region (I think of the TRMM satellite) it is necessary to consider the **Rosenfeld** relationship [46] for tropical regions.

There are also slightly more practical relationships that link Z to R as follows:

$$Z = \int_R^{+\infty} N(D) \cdot D^3 \cdot dD = \frac{\pi}{6} \cdot \int_R^{+\infty} N(D) \cdot D^3 \cdot W(D) \cdot dD \quad (12)$$

where R is the rainfall rate, Z is the reflectivity factor, D is the raindrop diameter and $N(D)$ is the number of falling or precipitating particles per cubic meter (i.e., a volumetric concentration). With $W(D)$ I instead indicate the falling speed of particles of diameter D . The previous relationship is a theoretical definition of Radar reflectivity. In real radars like the **WSR-88D** ground weather radar [47], however, it is not possible to observe the size of the falling rain particles or the geometric size of the particles and therefore Z must be determined by resorting to a simplified radar equation:

$$Z = \frac{\text{Power returned to the Radar} \cdot (\text{Target Range})^2}{\text{Radar Constant}} \quad (13)$$

This equation in itself represents the reflected power normalized in a Radar System. Usually in this equation the power is measured in Watts [W] while the distance (i.e., the range) is measured in meters [m] and the radar constant instead is a dimensionless value. So the Reflectivity would be measured in [W/m^2]. In this case the quantity in dBZ indicates a logarithmic measure definable as:

$$[dBZ] = 10 \cdot \log_{10}(Z) \quad (14)$$

This equation assumes the validity of **Rayleigh scattering** of particles and this occurs when the diameter of the particles in the air is much smaller than the wavelength of the electromagnetic radiation transmitted in the air. The **WSR-88D** uses a wavelength of approximately 10.7 cm, and almost all raindrops have diameters of 7 mm or less. Through the TRMM, the nature of “tropical storms” has often been studied. A software that allows you to view meteorological radar images is **GEMPACK** [26]. To track the evolution of a tropical storm, It is possible to prepare a table with a *datetime*, position information such as Latitude and Longitude, the pressure value in millibars (mb), and the wind speed in knots (kt).

CONCLUSIONS.

Satellite coincidence analysis and the integration of multi-mission datasets have been widely studied, particularly in relation to precipitation retrieval and hydrometeor characterization. This study examined the synergy between satellite

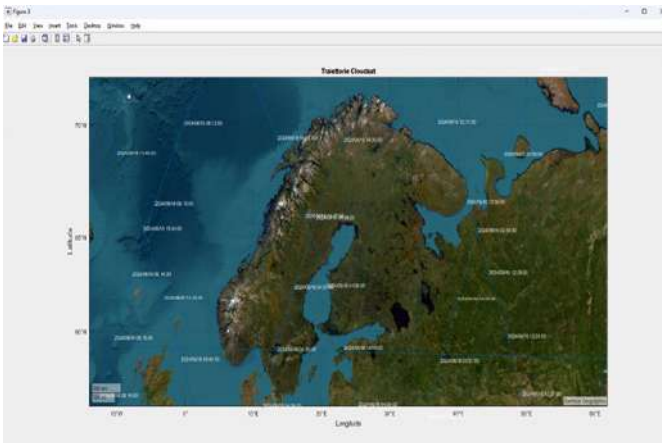


Fig. 16: CloudSat flyby trajectories realized with a dedicated MATLAB project.

missions such as TRMM, GPM, and CloudSat, with an emphasis on instrument specifications, orbital parameters, and data fusion methodologies.

A detailed assessment was conducted to quantify satellite coincidences and evaluate their relevance in establishing robust relationships between radar reflectivity, attenuation, and brightness temperature. These elements are critical for the improvement of global attenuation maps, precipitation estimation, and the forecasting of meteorological phenomena. Such advancements can enhance disaster preparedness and support effective resource management in regions affected by hydrometeorological hazards.

Practical guidance was also provided to facilitate data access and handling, including procedures for dataset acquisition using tools such as FileZilla. Specific attention was given to the Joe Turk dataset, highlighting the operational role of weather radar systems in tracking hydrometeorological events and improving multi-source data assimilation.

The integration of high-resolution datasets and advanced analytical techniques contributes to a deeper understanding of atmospheric dynamics and supports the development of more accurate global weather forecasting systems. This approach promotes predictive reliability and fosters collaboration among researchers, policymakers, and communities. The implementation of data-driven strategies is essential to mitigate the impacts of climate change and to strengthen societal resilience against extreme weather events.

AUTHOR BIOGRAPHY



Paolo Cicolin was born in 1971 in Turin. In October 2012 and December 2018, respectively, he graduated with

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CONFLICT OF INTEREST

The author declares that he has no conflicts of interest.

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A.6 Forth Technical Article - personal style

Title: Improving Global Weather Forecasts: Analysis of Satellite Coincidences and Relationships between Radar Reflectivity, Attenuation, and Brightness Temperature.

As requested by the reviewers, I have revised this section of the dissertation to reflect my personal contribution more explicitly.

Therefore, the technical narrative is now expressed in the first-person singular form, distinguishing clearly between my original work and the contributions derived from the literature

Improving Global Weather Forecasts: Analysis of Satellite Coincidences and Relationships between Radar Reflectivity, Attenuation, and Brightness Temperature

Paolo Cicolin, IEEE Student Member.

Abstract—This study focuses on analyzing coincidence datasets from CloudSat-GPM and CloudSat-TRMM to understand the relationships between radar reflectivity, attenuation, and brightness temperature. CloudSat provides high-resolution vertical profiles of cloud structures, while GPM and TRMM offer complementary precipitation data through advanced radar systems.

The research is the fourth part of a series dedicated to generating a global attenuation map by leveraging these datasets, particularly focusing on Ku, Ka, and W bands. Key methods include analyzing NetCDF metadata, employing the Surface Reference Technique (SRT), and calculating Path Integrated Attenuation (PIA) to refine attenuation estimates. MATLAB-based tools were developed to visualize radar reflectivity and estimate rain rates, ensuring precise representation of the data. The study also addresses challenges such as excluding lower bins and mitigating overlapping time series data. Findings from this study contribute to improving global attenuation maps, enhancing precipitation estimates, and refining atmospheric models. These advancements strengthen predictive capabilities, facilitating better disaster management and supporting climate change resilience. The integration of data from multiple satellite missions underscores the importance of collaboration in atmospheric sciences and sets a foundation for further research on global weather systems.

Index Terms – Satellite coincidence datasets, Radar reflectivity, Path Integrated Attenuation (PIA), Brightness temperature, Global attenuation maps, CloudSat-GPM integration, Atmospheric modeling, Precipitation estimation, global weather systems.

I. INTRODUCTION

The accurate prediction of weather patterns is vital for disaster preparedness, resource management, and overall societal safety. Advances in satellite technology have enabled the collection of vast amounts of data, which, when properly analyzed, can significantly improve weather forecasts. This study focuses on the analysis of satellite coincidences, particularly involving the CloudSat, GPM (Global Precipitation Measurement), and TRMM (Tropical Rainfall Measuring Mission) satellites.

The CloudSat-GPM and CloudSat-TRMM Coincidence Datasets are invaluable resources for understanding the relationships between radar reflectivity, attenuation, and brightness temperature. CloudSat, with its cloud-profiling radar, provides high-resolution vertical profiles of cloud structures. GPM and

TRMM, on the other hand, offer critical data on precipitation and hydrometeors through their advanced radar systems.

This study wants to find out how radar reflectivity, attenuation, and brightness temperature are related by looking at these coincidence datasets. Such an analysis is crucial for refining global attenuation maps and enhancing the accuracy of precipitation estimates. Improved understanding of these relationships will contribute to more precise weather forecasts, better prediction of extreme weather events, and effective disaster management strategies.

This work is the **fourth part** of a dedicated series focusing on the generation of a global attenuation map using satellite coincidence data from CloudSat-GPM and CloudSat-TRMM. The integration of data from multiple satellite missions, as demonstrated in this research, highlights the importance of collaborative efforts in atmospheric sciences. By leveraging the complementary strengths of CloudSat, GPM, and TRMM, scientists can develop more sophisticated models of atmospheric behavior, ultimately leading to enhanced predictive capabilities and greater resilience against the impacts of climate change.

II. THE RADAR EQUATION.

Typically, a ground-based meteorological radar is an instrument with high spatial coverage and high temporal resolution; for example, the pulse train can be sent in packets every 15 minutes with a radar range of 125 km or with times of 30 minutes with a range of 250 km. In a meteorological radar, the relationship between the range (in km) and the pulse repetition time (PRT) is fundamental to determining the maximum distance at which the radar can detect an object or a meteorological phenomenon. The pulse repetition time (PRT) is the time between the start of a pulse and the start of the next one. The pulse repetition frequency (PRF) is the inverse of the PRT , that is, $PRF = 1/PRT$.

A. The CPR Radar

The provided specifications give me a good understanding of the CloudSat radar's capabilities. Here's what I can infer from the given data:

- **Nominal Frequency (94 GHz):** This is the operating frequency of the radar. A higher frequency allows for better resolution and accuracy in detecting and measuring cloud properties.

REFLECTIVITY Z (dBZ)	PRECIPITATION R (mm/h)	TYPE
-30	-	Light fog or weak clouds, no precipitation
between -30 and 20	-	Increasingly dense types of clouds, without rain
20	< 1	Barely perceptible drizzle
30	3	Light rain
40	12	Average rain
50	50	Heavy rain
55	100	Very heavy rain
between 55 and 75	-	Rain mixed with hail
75	-	Very violent hailstorm

TABLE I: Table of Radar Reflectivity.

- **Pulse Width (3.3 μ sec):** This is the duration of each emitted radar pulse. A shorter pulse width allows for better range resolution.
- **Pulse Repetition Frequency (PRF, 4300 Hz):** This is the rate at which pulses are transmitted. A higher PRF allows for faster data update rates.
- **Minimum Detectable Z(< 29[dBZ]):** This indicates the radar's sensitivity. It can detect signals with reflectivity less than -29 dBZ, which means it can detect very light precipitation and cloud particles.
- **Data Window (0-25 km):** This is the range over which the radar can detect and measure cloud properties.
- **Antenna Size (1.85 m):** The size of the antenna can affect the radar's gain and *beamwidth*. A larger antenna typically provides a narrower *beamwidth* and higher *gain*, improving the radar's ability to detect smaller or more distant targets.
- **Dynamic Range (70 dB):** This is the range of signal amplitudes that the radar can handle. A larger dynamic range allows the radar to accurately measure targets at various distances and reflectivities.
- **Integration Time (0.16 sec):** This is the time over which the radar integrates the received signal to improve the signal-to-noise ratio.
- **Nadir Angle (0.16° since 15 Aug 2006):** This is the angle at which the radar is pointed towards the Earth's surface.

A smaller nadir angle means the radar is pointed more directly downwards.

- **Vertical Resolution (500 m), Cross-Track Resolution (1.4 km), and Along-Track Resolution (1.7 km):** These parameters describe the spatial resolution of the radar measurements in different directions.
- **Data Rate (20 kbps):** This is the rate at which data is transmitted from the radar system.

These specifications suggest that the CloudSat radar is a highly sensitive and precise instrument capable of providing detailed measurements of cloud properties. However, for a more comprehensive understanding, additional information such as the radar's noise figure, signal processing techniques, and calibration procedures would be helpful.

B. Maximum Radar Range

To calculate the maximum range of the radar, the radar equation must be used, which in turn must take into account various factors such as the transmitted power, the antenna gain, the *wavelength* of the e.m. radiation, the *radar cross section* of

the target (RCS) or the radar cross section, and the losses of the system, generally indicated with L . The formula to calculate the maximum **flow rate** is as follows:

$$R_{max} = \sqrt{\frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 L^2}} \quad (1)$$

where I have that:

- P_t indicates the power transmitted by the radar system.
- G_t indicates the gain of the transmitting antenna.
- G_r indicates the gain of the receiving antenna.
- λ indicates the wavelength of the radar signal.
- σ indicates the cross section of the target (radar target).
- L represents system losses.

This formula (1) is simplified and does not take into account, for example, the atmospheric attenuation factor, the phenomenon of multiple reflections (*multipath fading*), or the shape and composition of the target itself. To obtain more realistic situations, it is necessary to have complex software that actually or very likely simulates a radar system. Therefore, if I want to describe a radar system in a more realistic way, I need to take the radar equation and modify it by introducing a whole series of more complex factors. The radar equation can be modified to account for various factors, such as atmospheric attenuation, multiple reflections, and target composition.

- 1) **Atmospheric attenuation:** Atmospheric attenuation can be considered by introducing an attenuation factor A into the radar equation. This factor depends on the frequency of the radar signal and atmospheric conditions (such as *humidity* and *temperature*). Attenuation is generally minimal on snow, even for the **X band**.
- 2) **Multiple reflections:** Multiple reflections can be accounted for by introducing a *reflection factor* Γ into the radar equation. This factor depends on the geometry of the target and the properties of the propagation medium.
- 3) **Target composition:** The target composition can be considered through the *radar cross section* σ of the *target*. This value is difficult to determine and depends on several parameters, such as shape, surface composition, and the nature of the materials used.

I have this new formula:

$$P_r = \frac{P_t G_t G_r \lambda^2 \sigma \cdot A \cdot \Gamma}{(4\pi)^3 R^4 L^2} \quad (2)$$

I have now introduced:

- A : the atmospheric attenuation factor.
- Γ : the reflection factor for multiple reflections. I use the letter Gamma (i.e., Γ) to indicate this.

If I make the assumption that the antenna has the same gain G during transmission and reception, then I write:

$$P_r = \frac{P_t G^2 \lambda^2 \sigma \cdot A \cdot \Gamma}{(4\pi)^3 R^4 L^2} \quad (3)$$

where here R is the radar range (usually indicating the geometric distance in a straight line between the radar transmitter/receiver and the target). More generally, for the monostatic meteorological radar, the received power is proportional to the fourth power of the range. So, I have:

$$P_r = \frac{P_t G^2 \lambda^2 \sigma \cdot A \cdot \Gamma}{(4\pi)^3 R^4 L^2} \propto \frac{\sigma}{R^4} \quad (4)$$

This equation considers a simple target, while I generally have a cloud or a cloud formation within the volume scanned by the radar. Therefore, the radar equation becomes more complicated because the target is composed of multiple scattering objects that will be "interested" (i.e., they come into contact with the *beam*). For complex target I may write:

$$\sigma = \bar{\sigma} = V \cdot \sum_j \sigma_j = V \cdot \eta \quad (5)$$

where V indicates the volume scanned by the ground radar which in turn can be expressed as: $V = (Pulse\ length) \cdot (Beamwidth) = \left(\frac{c \cdot \tau}{2}\right) \cdot \left(\frac{\pi}{4} R^2 \theta^2\right)$ where:

- τ indicates the duration of the radar pulse.
- θ indicates the angular width of the beam expressed in radians [rad].
- c indicates the speed of light in a vacuum (air assimilated to *vacuum*).

Therefore, I can combine the equations to obtain:

$$P_r = \frac{P_t G^2 \lambda^2}{(4\pi)^3 R^4} \cdot \frac{c \cdot \tau}{2} \cdot \frac{\pi \cdot R^2 \cdot \theta^2}{4} \cdot \eta \cdot \frac{A \cdot \Gamma}{L^2} \propto \frac{\eta}{R^2} \quad (6)$$

Therefore, I can conclude by saying that in the meteorological radar equation, the received power is inversely proportional to the square of the Range R . This relationship can also be expressed as:

$$P_r = \frac{P_t G^2 \lambda^2}{(4\pi)^3 R^4} \cdot \frac{c \cdot \tau}{2} \cdot \frac{\pi \cdot R^2 \cdot \theta^2}{4} \cdot \eta \cdot \frac{A \cdot \Gamma}{L^2} \quad (7)$$

$$= P_t \cdot \tau \cdot G^2 \cdot \lambda^2 \cdot \theta^2 \cdot \frac{c}{(512) \cdot \pi^2} \cdot \frac{A \cdot \Gamma}{L^2} \cdot \frac{\eta}{R^2} \quad (8)$$

III. PLOTTING THE RADAR REFLECTIVITY FACTOR FROM COINCIDENCE JT DATASET.

In this section I dedicate some space for describing how is possible to plot the **Radar Reflectivity** from the coincidence dataset of **JT**¹. I will try to simplify the discussion so that it can be of help to many readers, who will likely be able to

implement their solution by generalizing it and making it even more efficient. I immediately warn the reader that it will take some time to download the dataset via FTP. As a first example, I suggest downloading the data relating to the **CloudSat-GPM** (in **2009**) coincidences. The database for 1 month has just over 1.3 GB of data. For a single day of flight, you should see a number of files in each folder, ranging from a minimum of 1 to a maximum of 8 files. Each file, on average, has a size of 11 or 12 MB.

A. Starting from basis

I can start from the basics, assuming that I have downloaded at least one CloudSat-GPM coincidence file. I will use MATLAB to show how to acquire the information about the coincidence file structure and then illustrate in detail also the meaning related to the name of the file taken as an example.

Let's break it down step by step for clarity:

- **Step 1:** Downloading a CloudSat-GPM Coincidence File.
- **Step 2:** Acquiring Information about the Coincidence File Structure using MATLAB.
 - Load the File: Use MATLAB to load the file. You can use the load function or other relevant functions depending on the file format (e.g., HDF5, NetCDF).
 - Inspect the File Structure: Use the info variable to explore the structure of the file.
- **Step 3:** Understanding the meaning of the File Name. I will return to step 3 later.

B. Create a working sample

In this subsection, I present a concrete example, leaving some details for the reader to customize. Specifically, the reader will need to set the absolute path of the file on their hard disk, assuming they are using **MS Windows**. The code demonstrates how to open a netCDF file in *.nc4 format related to a CloudSat-GPM coincidence and read and store its content appropriately. An important aspect to highlight is the reading of the `CPR_Cloud_mask` as an example of a variable of interest contained in the coincidence file. The syntax requires a part of the data-path related to the `CPR_Cloud_mask`.

By following this approach, you can modify and adapt the code to read other variables of interest for graphing in **MATLAB 2023a** by changing the group name.

1) *Building Working Code:* This code opens a netCDF file, retrieves variable information, reads data into a structure, and specifically reads the `2BGEOPROF\CPR_Cloud_mask` variable to plot it.

In the `YourLoginProfile` you must to use your Windows name login account. For do this, use MS Windows operating system and search in the Control Panel the name of your actually user account (local one). Substitute `YourLoginProfile` with the name of the local account. Make sure that the absolute data path actually contains the folders specified in the data path and that the coincidence downloaded file exists. A possible alternative to comparing

¹FJoe Turk Dataset.

Algorithm 1 Working code in MATLAB for CPR Cloud mask.

```

% Specify the path to your .nc4 file
filepath = 'C:\Users\YourLoginProfile\source\repos';
filename = '2B-CSATGPM.COIN.01
N_027W_00197_000_297_441.20160831-S152953
-E153400.014251.V04.nc4';
% String concatenation to get absolute path
absolute_path = fullfile(filepath, filename);
% Open the netCDF file
ncid = netcdf.open(absolute_path, 'NC_NOWRITE');
fileInfo = ncinfo(absolute_path);
% Access, e.g. variable names, using:
a = fileInfo.Variables;
% Get information about the contents of the file
[numdims, numvars, numglobalatts, unlimdimID] =
netcdf.inq(ncid);
% Initialize a structure to hold the data
data = struct();
% Loop through all variables to get their names and
read data
for i = 0:numvars-1
% Get variable name and details
[varname, xtype, dimids, numatts] =
netcdf.inqVar(ncid, i);
% Read data from the current variable
data.(varname) = netcdf.getVar(ncid, i);
% Display variable name
disp(['Variable Name: ', varname]);
end
% Read the '2B-GEOPROF/CPR_Cloud_mask' variable
cloud_mask = ncread(absolute_path,
'2BGEOPROF/CPR_Cloud_mask');
% Now you can work with the 'cloud_mask' variable,
such as plotting it
imagesc(cloud_mask);
colorbar;
title('CPR Cloud Mask');
% Close the netCDF file
netcdf.close(ncid);
% Now 'data' structure contains all the variables
and their data.

```

what has been programmed is to use **Panoply** to go and graph the same quantity of interest. I can slightly modify the source code to go and draw the radar reflectivity. In **Panoply** [2], it is expressed in decibels (dB), but when it is normally drawn, it is necessary to adjust the axes. The ground could appear on the right with a rather marked knurling that must be smoothed. From **Figure 4**, I would like to adjust the graph so that the ground is facing down. It will be necessary to rotate the figure appropriately. The same radar reflectivity is graphed in MATLAB considering the horizontal and vertical bins. The modification to the source code is simple and indicated below:

Pay attention to cut and past this fragment of code after the loop through all variables to get their names and read data. In this case you will obtain the **Figure 4**: at a later stage, the data scientist must refer to the documentation associated with the 2B-GEOPROF product to perform a series of operations that give the data the importance it deserves. This process raises many questions, and the purpose of this article is to highlight some of them. For example, when observing the MATLAB graph of radar reflectivity, a logarithmic scale of reflectivity is used. The graph shows the roughness of the ground or sea. By carefully examining the coordinates in the file, it becomes clear that the points can be associated not with the ground, but with the sea.

Algorithm 2 Modifying on the fly the source code. Reading the Radar Reflectivity variable in MATLAB.

```

% Read the 'Radar_Reflectivity' variable
radar_reflectivity = ncread(absolute_path,
'2B-GEOPROF/Radar_Reflectivity');
% Assuming 'radar_reflectivity' is your data
variable
% Apply a statistical filter to remove ground
clutter
threshold = mean(radar_reflectivity(:)) + 3*
std(radar_reflectivity(:));
filtered_reflectivity = radar_reflectivity;
filtered_reflectivity(radar_reflectivity >
threshold) = NaN;
% Apply the scale factor
scale_factor = 0.01; % as given in the file's
metadata
radar_reflectivity_scaled = radar_reflectivity *
scale_factor;
% Plot the filtered radar reflectivity
imagesc(filtered_reflectivity);
cb = colorbar;
ylabel(cb, 'Radar Reflectivity in [dB]');
xlabel('Along Track Distance Ticks');
title('Filtered Radar Reflectivity');
% Close the netCDF file
netcdf.close(ncid);

```

QUESTION: Is it normal that the vertical bin for 2B-GEOPROF is around 125 for terrain and not zero?

ANSWER: Yes, it is normal that the vertical bin for terrain in CloudSat's 2B-GEOPROF product is around 125 and not zero. This is because the 2B-GEOPROF product uses a vertical grid with 125 bins of vertical interval. The bin representing terrain is not necessarily the first bin (zero), but rather a specific bin that depends on the satellite position and terrain altitude. According to the 2B-GEOPROF product documentation, the vertical bin closest to sea level was placed in bin 105, (see [3]). The position of the bin that is closest to the actual surface is provided as part of the data. This means that the bin representing terrain may vary depending on specific flight conditions and topography. Since many other questions may arise regarding the meaning of the generated figures, their location within a **geographic reference system (WGS84)**, and the presence or absence of clutter, it is essential to consider the physical properties of the individual radars used. For example, CloudSat employs the Cloud Profiling Radar (CPR), which is not a polarimetric radar. Instead, it measures the power of microwaves back-scattered by clouds as a function of the distance from the radar. This allows for the measurement of the vertical structure of clouds and their ice/water content. On the contrary GPM (Global Precipitation Measurement) mission has a dual-frequency, dual-polarized radar on board called the Dual frequency Precipitation Radar (DPR). The DPR operates at two frequencies: Ku-band and Ka band. This radar is capable of providing detailed measurements of precipitation, including the size and distribution of raindrops, by using dual-polarized signals. In this case, if you have a dual polarimetric radar (i.e., dual frequency), it may be of greater interest to study the differential reflectivity (ZDR). This term is very useful in determining the shape of the radar target and, consequently, the type of hydrometer. Spherical objects will have a differential reflectivity value close to 0 (ratio of

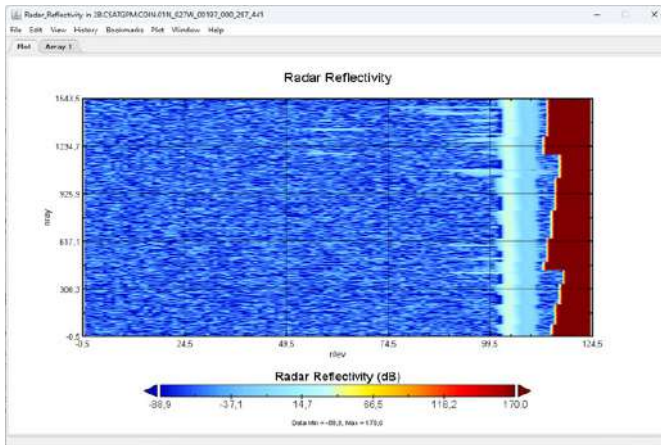


Fig. 1: The Radar Reflectivity (RR) drawn in Panoply without any particular precaution. The figure here must be rotated appropriately. Leaving it as is, is not correct.

1, so the logarithm of 1 is 0). Spherical objects tend to be small raindrops, hail, graupel, or snow. The combination of reflectivity and differential reflectivity can help me determine which hydrometeor is likely to be detected by the radar. For example, a radar bin with high reflectivity and low differential reflectivity (large spherical object) is likely to be hail, while a bin with low reflectivity and low differential reflectivity (small spherical object) is likely to be snow or small raindrops. Differential reflectivity is particularly useful for detecting hail. Another dual-polarimetric variable that a data scientist may use is the correlation coefficient. The *correlation coefficient* (CC) measures the similarity of all targets within a radar bin. A (CC) value close to 1 suggests that nearly all objects in the radar bin are of the same size and shape, while lower CC values indicate a mixture of different objects. Very low CC values typically signify ground clutter. Another question I can ask myself is the following,

QUESTION: Where in 2B-GEOPROF can I find the flag to understand if I have land or sea as a surface?

ANSWER: In the 2B-GEOPROF product from CloudSat, you can find the information about the surface (land or sea) using the elevation field. According to the documentation, the elevation is expressed in meters above mean sea level. A value of -9999 indicates the ocean, while a value of 9999 indicates an error in the elevation calculation. So, to determine if the surface is land or sea, you can check the elevation value in the 2B-GEOPROF file. [4]

C. Information on CloudSat-GPM Coincidence JT's Dataset

To create global attenuation maps, I need to address another intriguing issue related to datasets. In this section, I'll explore the best methods to obtain the most relevant data for the required amounts in the data processing procedure. Previously, I discussed that JT's (Joe Turk) dataset files contain countless parameters of interest for atmospheric sciences. Following the detailed indications of the JT article, I discover that the on-board instrumentation scan provides three types of data: normal scan (NS) for the DPR (Dual Profiling Radar) in Ku

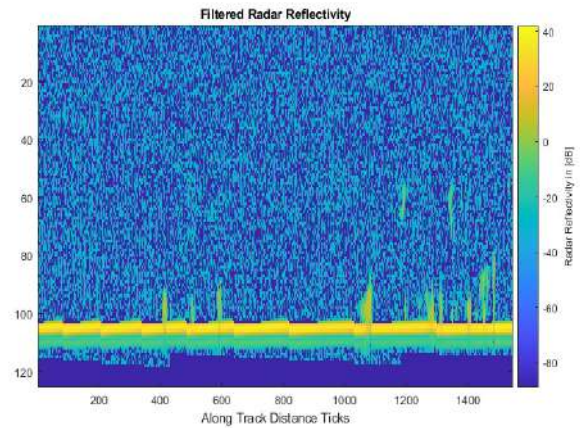


Fig. 2: The Radar Reflectivity (RR) drawn in MATLAB with scale factor.

band, matched scan (MS) in Ka band, and high scan (HS) also in Ka band. Some authors might think that S indicates scan, but this is not always true. To graph the attenuation maps in the bands of interest, which include the W band, the Ka band, and the Ku- band, I need to prepare a complex MATLAB solution. This solution must read the downloaded files and overcome the difficulties of having files not always of the same size. In the pool of files, some may differ from others, making implementations aimed at scanning the files ineffective. You need to pay close attention and dedicate many hours of work to obtain a stable version of the software capable of overcoming these and other difficulties. You can find information regarding the reading of the NetCDF format on **Mathworks** by searching with the key 'read netcdf'. To make the source code robust, I suggest inserting many try-catches to first verify the quality of the files. Showing the implementation choices made would be excessively burdensome in this article, so I will provide some advice:

- When developing, order your ideas and document them appropriately.
- Concentrate your efforts on the essentiality of the solution;
- Use code examples to strengthen your development knowledge;
- Pay attention to the choice of programming language and implementation style;
- Avoid side effects, carelessness in naming variables, use, or employment of variables in the code;
- Read the documentation carefully, particularly the meta-data associated with the files from which you will build the database of your interest.

Do not forget elements such as the scale factor, as knowing how to read and interpret this information is essential for correct development. To learn this "art", look for examples of opening files in other formats (e.g., HDF4, HDF5). Whenever possible, insert long comments in the code, as they can be useful when you return to work on the code and remind you of the related documentation. For example, it is instructive to read the MATLAB examples for opening and viewing *.hdf

files [5], which you can find at ².

In particular, refer to the demo code for the file:
2011077180414-26000-CS-2B-GEOPROF-LIDARGRANULE
-P2-R04-E04.hdf.

Here, you will find how to implement the “scale factor” and many other details that are difficult to find, such as managing temporal variables and the geographical variables of latitude and longitude. When creating reflectivity graphs, pay attention to several precautions, such as the representation scale, spatial resolution, temporal resolution, and the choice of colors for the logarithmic reflectivity scales. Be careful, as many published articles contain representation errors that often go unnoticed. Specifically, pay attention to the choice of meters or kilometers in Reflectivity Radar. Measurements must always be in meters; using kilometers is conceptually incorrect. When you’re ready to write more robust code, you can better manage MATLAB environment variables by taking online courses. Key development elements include saving generated images to the hard disk, closing unnecessary ones, and saving variable and data structure allocations on the stack.

I’ve developed a cryptic version for reading NetCDF files, using generalized variable and data structure names. While this makes the code harder to interpret, it also means rereading it will take more time. Here’s the basic idea with a few example lines:

Algorithm 3 The basic idea for cryptic development in reading NetCDF files.

```
% '/2B-GEOPROF/time'
% Read the Data content of my
% interest: time.
% time = h5read(filename,
% '/2B-GEOPROF/time');
obj4 = strcat("b,c,i");
time = h5read(filename, obj4);
```

In this code fragment, I would like to read, for example, /2B-GEOPROF/ and in particular the variable “time”. If I removed the comments and, in any case, also disguised the name of the identifier on the left of the assignment statement, it would be really hard to understand what my source code is doing. This comes from the fact that I have not shown you the use of the labels *b, c, i* in the code. Here is their previous declaration:

Algorithm 4 Opening a NetCDF file in MATLAB.

```
% Reads the Group name i.e. the root "/".
ncid = netcdf.open(filename, 'nowrite');
b = netcdf.inqGrpName(ncid);
```

And also:

Algorithm 5 Acquiring name infos on Groups variables in a NetCDF file in MATLAB.

```
% Structural Read Demo: Read the first element
% of the % array at position
% (row, column)=(1,1) of Groups.
% ginfo is a Data Structure that contains
% informations about Groups.
c = ginfo.Groups(1,1).Name;
```

And similarly:

Algorithm 6 Test reading for Time variable in 2B-GEOPROF with MATLAB.

```
% Test reading the time variable in 2B-GEOPROF:
i = ginfo.Groups(1).Variables(1).Name;
```

Even without encrypting the source code, using similar techniques can make it difficult to understand what the code is doing, thus protecting it from prying eyes. In less confidential environments, you can instead make your intentions clear:

Algorithm 7 How I concrete use Data Path Info inside a NetCDF File for reading the height variable in MATLAB.

```
height = h5read(filename, '/2B-GEOPROF/height');
```

In this fragment, I read the variable height from the 2BGEOPROF group. The term filename remains an identifier, while the function **h5read** is a MATLAB library function from the **R2023a** release. Learning to read metadata in MATLAB, such as the information associated with CloudSat-GPM coincidence files (as discussed in the JT article), provides a powerful tool for code development. Below is an example of reading a coincidence file, from which data scientists or analysts can extract valuable information, including granule and orbit details.

D. Metadata associated to a Coincidence File CloudSat-GPM

I show the metadata associated with the example coincidence file that is possible to open with MATLAB custom code.

```
Filename: 'C:\Users\NITRO5RLD\source\repos\...
2B.CSATGPM.COIN.01N_027W_00197_000_297_441.
20160831-S152953-E153400.014251.V04.nc4'
Name: '/'
Dimensions: [1x2 struct]
Variables: []
Attributes: [1x42 struct]
Groups: [1x17 struct]
Format: 'netcdf4'
Datatypes: []
Dimensions: Name: nray, Length: 1544
Name: nlev, Length: 125
Attributes:
Name: 2B-GEOPROF,
Value: 2016244144207_55026_CS_
2B-GEOPROF_GRANULE_P1_R05_E06_F00.hdf
Name: 2A.GPM.DPR,
Value: 2A.GPM.DPR.V8-20180723.20160831
-S150117-E163351.014251.V06A.HDF5
Name: CS_minus_DPR_time_diff_seconds,
Value: 441
Name: ray_index_range_NS, Value: 574 993
Name: ray_index_range_MS, Value: 681 886
Name: GPM_orbit_rev, Value: 014251
```

²<http://www.cloudsat.cira.colostate.edu/dataSpecs.php>

```

Name: 1C.GPM.GMI,
Value: 1C.GPM.GMI.XCAL2016-C.20160831-
-S150117-E163351.014251.V05A.HDF5
Name: CS_minus_GMI_time_diff_seconds,
Value: 507
Name: ray_index_range_GMI, Value: 0 1543
Name: ECMWF-AUX,
Value: 2016244144207_55026_CS_
ECMWF-AUX_GRANULE_P_R05_E06_F00.hdf
Name: 1C.NPP.ATMS,
Value: 1C.NPP.ATMS.XCAL2016-V.20160831-S153047-
E171215.025101.V05A.HDF5
Name: CS_minus_ATMS_time_diff_seconds,
Value: -1415
Name: ray_index_range_ATMS,
Value: 0 1543
Name: NPP_orbit_rev,
Value: 025101
Name: 2A.NPP.ATMS.GPROF,
Value: 2A.NPP.ATMS.GPROF2017v2.20160831-S153047-
E171215.025101.V05C.HDF5
Name: 2C-RAIN-PROFILE,
Value: 2016244144207_55026_CS_2C-
RAIN-PROFILE_GRANULE_P1_R05_E06_F00.hdf
Name: 2C-SNOW-PROFILE,
Value: 2016244144207_55026_CS_2C-
SNOW-PROFILE_GRANULE_P1_R05_E06_F00.hdf
Name: 2C-PRECIP-COLUMN,
Value: 2016244144207_55026_CS_2C-
PRECIP-COLUMN_GRANULE_P1_R05_E06_F00.hdf
Name: 2B.GPM.DPRGMI,
Value: 2B.GPM.DPRGMI.CORRA2018.20160831-
S150117-E163351.014251.V06A.HDF5
Name: 2A.GPM.GMI.GPROF,
Value: 2A.GPM.GMI.GPROF2017v1.20160831-
S150117-E163351.014251.V05A.HDF5
Name: MODIS-AUX,
Value: 2016244144207_55026_CS_MODIS-
AUX_GRANULE_P_R05_E06_F00.hdf
Name: 2B-GEOPROF-LIDAR,
Value: 2016244144207_55026_CS_2B-GEOPROF-
LIDAR_GRANULE_P1_R05_E06_F00.hdf
Name: 2B-CWC-RO,
Value: 2016244144207_55026_CS_2B-CWC-
RO_GRANULE_P1_R05_E06_F00.hdf
Name: 2C-ICE,
Value: 2016244144207_55026_CS_2C-
ICE_GRANULE_P1_R05_E06_F00.hdf
Name: 2B-CLDCLASS,
Value: 2016244144207_55026_CS_2B-
CLDCLASS_GRANULE_P1_R05_E06_F00.hdf
Name: 2B-CWC-RVOD,
Value: 2016244144207_55026_CS_2B-
CWC-RVOD_GRANULE_P1_R05_E06_F00.hdf
Name: production_date,
Value: 2019/10/09 09:31:58
Name: start_date,
Value: 2016/08/31 15:29:53
Name: end_date,
Value: 2016/08/31 15:34:00
Name: start_date_NS,
Value: 2016/08/31 15:24:12
Name: end_date_NS,
Value: 2016/08/31 15:25:04
Name: start_date_ATMS,
Value: 2016/08/31 15:53:42
Name: end_date_ATMS,
Value: 2016/08/31 15:58:01
Name: center_lat,
Value: 0.720074
Name: center_lon,
Value: -26.594284
Name: CS_bin_height_in_meters,
Value: 240
Name: NS_bin_height_in_meters,
Value: 125
Name: CS_total_bins_mask_ge_30,
Value: 202
Name: CS_total_bins_mask_ge_40,
Value: 197
Name: CS_nray_land,
Value: 0
Name: CS_nray_ocean,
Value: 1544
Name: CS_granule_number,
Value: 55026
Groups:
Name: 2B-GEOPROF
Name: time, Value: 1544
Name: Latitude, Value: 1544
Name: Longitude, Value: 1544
Name: DEM, Value: 1544
Name: SurfaceHeightBin, Value: 1544
Name: land_sea_flag, Value: 1544
Name: Sigma-Zero, Value: 1544
Name: along_track_dist, Value: 1544
Name: ray_index, Value: 1544
Name: Radar_Reflectivity, Value: 125
Name: CPR_Cloud_mask, Value: 125
Name: height, Value: 125
Name: 2A.GPM.DPR
Name: 1C.GPM.GMI
Dimension -
Name: ntb_GMI, Length: 13
Name: scan_index, Value: 1544
Name: pix_index, Value: 1544
Name: pix_km, Value: 1544
Name: Tc, Value: 13
Name: ECMWF-AUX
Name: Temperature_2m, Value: 1544
Name: Skin_temperature, Value: 1544
Name: Surface_pressure, Value: 1544
Name: height_273K, Value: 1544
Name: U10_velocity, Value: 1544
Name: V10_velocity, Value: 1544
Name: Pressure, Value: 125
Name: Temperature, Value: 125
Name: Specific_humidity, Value: 125
Name: U_velocity, Value: 125
Name: V_velocity, Value: 125
Name: 2C-ICE
Name: IWC, Value: 125
Name: re, Value: 125
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Name: RO_liq_water_path, Value: 1544
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Name: RO_liq_water_content, Value: 125
Name: RO_ice_number_conc, Value: 125
Name: RO_ice_distrib_width_param, Value: 125
Name: 2B-CWC-RVOD
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Name: Cloud_Liq_Water_Path, Value: 1544
Name: Ice_Water_Path, Value: 1544
Name: PIA_Fwd, Value: 1544
Name: Liq_Water_Content, Value: 125
Name: Cloud_Liq_Water_Content, Value: 125
Name: Ice_Water_Content, Value: 125

```

Name: 2B-CLDCLASS
 Name: cloud_scenario, Value: 125
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 Dimension -
 Name: ncloudlayers, Length: 5
 Name: CloudLayers, Value: 1544
 Name: FlagBase, Value: 5
 Name: FlagTop, Value: 5
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 Name: LayerTop, Value: 5
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 Name: rain_rate_uncertainty, Value: 1544
 Name: rain_quality_flag, Value: 1544
 Name: precip_ice_water, Value: 125
 Name: precip_liquid_water, Value: 125
 Name: PWC_uncertainty, Value: 125
 Name: 2C-SNOW-PROFILE
 Name: snowfall_rate_sfc, Value: 1544
 Name: snow_retrieval_status, Value: 1544
 Name: snowfall_rate, Value: 125
 Name: snow_water_content, Value: 125
 Name: 2C-PRECIP-COLUMN
 Name: PIA_hydrometeor, Value: 1544
 Name: PIA_uncertainty, Value: 1544
 Name: Diagnostic_precip_rate, Value: 1544
 Name: Rain_top_height, Value: 1544
 Name: Precip_flag, Value: 1544
 Name: Conv_strat_flag, Value: 1544
 Name: Status_flag, Value: 1544
 Name: 2B.GPM.DPRGMI
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 Name: nBnPSDlo, Length: 9
 Dimension -
 Name: nPSDlo, Length: 2
 Dimension -
 Name: nBnPSDhi, Length: 88
 Name: 2A.GPM.GMI.GPROF
 Name: surfaceTypeIndex, Value: 1544
 Name: surfacePrecipitation, Value: 1544
 Name: probabilityOfPrecip, Value: 1544
 Name: frozenPrecipitation, Value: 1544
 Name: 2A.NPP.ATMS.GPROF
 Name: surfacePrecipitation, Value: 1544
 Name: frozenPrecipitation, Value: 1544
 Name: 1C.NPP.ATMS
 Dimension -
 Name: ntb_ATMS, Length: 9
 Name: incidenceAngle, Value: 1544
 Name: scan_index, Value: 1544
 Name: pix_dist_CS, Value: 1544
 Name: Tc, Value: 9
 Name: MODIS-AUX
 Dimension -
 Name: ntb_MODIS, Length: 11
 Name: mod_Cloud_Mask, Value: 1544
 Name: modis_tb, Value: 11

IV. DESCRIPTION OF USEFUL INFORMATION FROM THE NETCDF FILE.

Dimensions

- nray: 1544 - Number of rays (vertical profiles) in the dataset.
- nlev: 125 - Number of vertical levels for each ray.

Attributes

- 2B-GEOPROF: Name of the input file for the 2BGEOPROF product.
- 2A.GPM.DPR: Name of the input file for the 2A.GPM.DPR product.
- CS minus DPR time diff seconds: Time difference in seconds between the CloudSat and DPR observations.
- ray index range NS: Ray index range for the DPR sensor.
- GPM orbit rev: Number of revolutions of the GPM orbit.
- 1C.GPM.GMI: Name of the input file for the 1C.GPM.GMI product.
- CS minus GMI time diff seconds: Time difference in seconds between CloudSat and GMI observations.
- ECMWF-AUX: Input file name for the ECMWF-AUX product.
- 1C.NPP.ATMS: Input file name for the 1C.NPP.ATMS product.
- CS minus ATMS time diff seconds: Time difference in seconds between CloudSat and ATMS observations.
- NPP orbit rev: NPP orbit revolution number.
- 2A.NPP.ATMS.GPROF: Input file name for the 2A.NPP.ATMS.GPROF product.
- 2C-RAIN-PROFILE: Input file name for the 2C-RAINPROFILE product.
- 2C-SNOW-PROFILE: Input file name for the 2CSNOW-PROFILE product.
- 2C-PRECIP-COLUMN: Input file name for the 2CPRECIP-COLUMN product.
- 2B.GPM.DPRGMI: Input file name for the 2B.GPM.DPRGMI product.
- 2A.GPM.GMI.GPROF: Input file name for the 2A.GPM.GMI.GPROF product.
- MODIS-AUX: Input file name for the MODIS-AUX product.
- 2B-GEOPROF-LIDAR: Input file name for the 2BGEOPROF-LIDAR product.
- 2B-CWC-RO: Input file name for the 2B-CWC-RO product.
- 2C-ICE: Input file name for the 2C-ICE product.
- 2B-CLDCLASS: Input file name for the 2B-CLDCLASS product.
- 2B-CWC-RVOD: Input file name for the 2B-CWCRVOD product.
- production date: Production date of the file.
- start date: Start date of observations.
- end date: End date of observations.
- center lat: Center latitude of the granule.
- center lon: Center longitude of the granule.
- CS bin height in meters: bin height for CloudSat in meters.
- NS bin height in meters: bin height for DPR in meters.

A. Groups and Variables

Groups and Variables

- **2B-GEOPROF:** Contains variables such as:
time,
latitude,
longitude,

DEM,
SurfaceHeightBin,
land sea flag,
Sigma-Zero,
along track dist,
ray index,
radar reflectivity,
CPR Cloud mask, and height.

• **ECMWF-AUX:** Contains variables such as:

Temperature 2m,
Skin temperature,
Surface pressure,
height 273K,
U10 velocity,
V10 velocity,
Pressure,
Temperature,
Specific humidity,
U velocity,
V velocity.

• **2C-ICE:** Contains variables such as:

IWC (ice water content)
re (effective radius).

• **2B-CWC-RO:** Contains variables such as:

RO ice water path,
RO liq water path,
RO ice water content,
RO liq water content,
RO ice number conc,
RO ice distrib width param.

• **2B-CWC-RVOD:** Contains variables such as:

Liq Water Path,
Cloud Liq Water Path,
Ice Water Path,
PIA Fwd,
Liq Water Content,
Cloud Liq Water Content,
Ice Water Content.

• **2B-CLDCLASS:** Contains variables such as:

cloud scenario.

• **2B-GEOPROF-LIDAR:** Contains variables such as:

CloudLayers,
FlagBase,
FlagTop,
LayerBase,
LayerTop.

• **2C-RAIN-PROFILE:** Contains variables such as:

rain rate,
rain rate uncertainty,
rain quality flag,
precip ice water,
precip liquid water,
PWC uncertainty.

• **2C-SNOW-PROFILE:** Contains variables such as:

snowfall rate sfc,
snow retrieval status,
snowfall rate,
snow water content.

• **2C-PRECIP-COLUMN:** Contains variables such as:

PIA hydrometeor,
PIA uncertainty,
Diagnostic precip rate,
Rain top height,
Precip flag,
Conv strat flag,
Status flag.

This information can be used to analyze various aspects of atmospheric observations, such as hydrometeor distribution, cloud water content, surface weather conditions, and more.

V. HOW TO BUILD THE DATASET FOR GLOBAL ATTENUATION MAP DUE TO CLOUDS IN KU, KA AND W BANDS.

Building a dataset for global map attenuation due to clouds in the Ku, Ka, and W bands involves several steps, including data collection, processing, and analysis. Satellite imagery and radar data covering all bands can be sourced from meteorological agencies, research institutions, or satellite data providers. Regarding cloud properties, it is useful to collect data such as cloud water path, cloud type, and cloud top height, as this information is crucial for understanding attenuation effects. For data collection, I use the JT dataset and aim to build one more stable using MATLAB. Regarding data processing, the calibration phase for radar data ensures consistency and accuracy, which may involve correcting for instrumental biases and atmospheric effects. Georeferencing and data filtering are also crucial steps integrated into the CloudSat-GPM coincidence dataset.

For the attenuation calculation in the data analysis phase, it is important to calculate the attenuation for each band using the collected cloud properties and radar data. This can be done using empirical models or theoretical calculations based on radar frequency and cloud characteristics. The interpolation phase for the attenuation data allows me to create a continuous map. This can be achieved using methods such as *kriging*, *spline interpolation*, or *polynomial fitting*. However, to what extent can I assert that some of these steps are not essential for me? Given that the CloudSat-GPM coincidence dataset is specifically designed to address these points:

- **1) Calibration:** The dataset undergoes rigorous calibration processes to ensure consistency and accuracy. This includes pre-launch and in-flight calibration measurements to correct for instrumental biases and atmospheric effects.
- **2) Data Filtering:** The dataset includes filtering steps to remove noise and artifacts, improving the quality of the data. This is essential for accurate analysis and attenuation calculations.
- **3) Attenuation Calculation:** The dataset provides the necessary data for calculating attenuation. In particular, the combination of W band from CloudSat and Ku/Ka band from GPM, along with the 13-channel GMI radiometer, allows for comprehensive attenuation analysis.

Data processing must distinguish between frequency channels and consider attenuation data in the three bands. It should also

associate these data with the physical properties of clouds, such as liquid water content, ice content, cloud type, and cloud top height. Each attenuation value will be distinctly associated with different cloud properties. These associations can then be graphed globally using appropriate plotting functions, incorporating both radar data and cloud properties. Attenuation and cloud properties are closely related to radar data, and understanding this relationship is crucial. Here are three possible approaches to address this problem:

- **Empirical Models:** These models use statistical correlations between observed radar reflectivity data and the microphysical properties of clouds, such as **liquid water content (LWC)** or particle distribution. This method provides a practical way to estimate attenuation but requires a large dataset of observations for development.
- **Theoretical Models:** More sophisticated models, such as the **Radiative Transfer Model (RTM)**, simulate how electromagnetic waves interact with particles inside clouds (*hydrometeors*). These models consider phenomena like scattering, absorption, and emission. Radiative transfer equations take into account the size, shape, and concentration of cloud particles to estimate attenuation.
- **Forward Models:** These models combine the microphysical properties of clouds with radar system parameters to simulate radar return echoes and evaluate their attenuation. This approach involves solving Maxwell's equations for complex boundary conditions created by cloud bodies.
- **Semi-Empirical Methods:** These methods combine empirical observations with theoretical calculations. By using observed data as a foundation, these models develop a theoretical basis characterized by specific system parameters. Studying **Mie scattering theory** and performing integral calculations along the radar beam path can be particularly useful in this context.

Each of these methods offers a different approach to understanding and estimating attenuation, providing valuable insights into the relationship between radar data and cloud properties.

A. Regarding creation of the Dataset

Regarding the creation of the database for satellite coincidence products, all relevant products were considered to study cloud composition. As a first approximation, I focused on σ_0 as an initial estimate of the PIA (Path Integrated Attenuation). This approximation was validated through graphical comparisons during CloudSat overflight periods coinciding with GPM (Global Precipitation Measurement) observations, showing excellent correspondence between PIA-raw and PIA_hydrometeor. Procedures were implemented to graphically and tabularly present products and quantities of interest, such as:

- PIA (one-way or two-way),
- IWP (Ice Water Path),
- IWC (Ice Water Content),
- LWP (Liquid Water Content),
- Pia-Ku,
- Pia-Ka,

- Pia-W in the form of PIA-hydro,

and various other precipitation products (e.g., near surface rain rate). The aim of this study was to identify potential relationships between physically measurable quantities and other not directly measurable quantities. Additionally, collections of cloud profile images (Cloud Profile, Cloud Mask Profiles) were created as a starting point for dynamically generating aggregated and interesting data to support the creation of a global attenuation map. For PIA evaluation, two approaches were chosen: the SRT (surface reference technique) and microwave radiometry. In the SRT approach by **Meneghini et al. (1983)** [9], radar measurements in “clear air” are compared to those in “rainy areas”, and the difference between these measurements provides an estimate of the PIA.

B. Surface Reference Technique (SRT)

The **Surface Reference Technique (SRT)** leads to the one-way **Path Integrated Attenuation (PIA)** relation as follows:

$$PIA_{rad} = \frac{\sigma_{clear}^0 - \sigma_{rain}^0}{2}. \quad (9)$$

Estimating PIA using radiometers evaluated at nadir is more complex. The CloudSat satellite, which operates at a 94 GHz working frequency, is a nadir-pointing satellite. The SRT technique involves comparing the radar cross-section (RCS) measurements of ocean surface rain columns (σ_{rain}^0) with those of “rain-free” ocean surfaces (σ_{clear}^0) [**Meneghini et al., 1983, 1987, 1989**, [22]- [24]; **Fujita et al., 1985**, [25]]. A fundamental issue with the SRT technique is that it is affected by noise. This noise arises partly from the variability in surface wind speed and is further complicated by precipitation and surface roughness effects. Additionally, there are challenges in ensuring the equivalence between rain-free and rainy surfaces [**Meneghini et al., 1992**, [26]]. When extracting the PIA from the 2B-GEOPROF products of CloudSat, four types of scattering regimes can be estimated:

- 1) **Rayleigh Scattering:** Dominates when the particle size is much smaller than the wavelength.
- 2) **Mie Scattering:** Occurs when the particle size is comparable to the wavelength.
- 3) **Non-Selective Scattering:** Happens when the particle size is much larger than the wavelength.
- 4) **Surface Scattering:** Involves interactions with the surface, influenced by surface roughness and composition.

These regimes help in understanding and modeling the attenuation effects more accurately. Going back to the previous points, looking at them in more detail I have:

- **Single Scattering Approximation for the Entire Z-Profile:** this approach assumes that single scattering is sufficient to describe the entire vertical profile of the cloud.
- **Second-Order Scattering Approximation:** while a single scattering approximation may not be feasible, second order scattering approximations remain valid under certain conditions.
- **Limitations of Second-Order Approximations:** when multiple scattering effects become significant, second

order approximations are no longer valid. However, the SRT techniques used do not interfere with PIA estimates in these cases.

- **Influence of Multiple Scattering:** Multiple scattering can affect the size of the return surface, allowing for its estimation. In other words, the multiple scattering is affecting the surface return as well, thus spoiling the PIA estimates.

For more details, see **A. Battaglia et al. (2008)**, [8]³.

The 2C-PRECIP-COLUMN product was also analyzed to obtain plausible estimates of near-surface precipitation over the oceans by applying reflectivity thresholds [**Haynes et al., (2009-2013)**], [19], [20]. In this context, the two-way PIA due to hydrometeor integration from the surface up to the *height* (H) of interest is evaluated as follows:

$$2 \cdot \Phi \cdot \int_0^H k_{ext}(s) \cdot ds = PIA = \sigma_{clear} - (\sigma_{clear}^0 + G)$$

where Φ is the conversion factor given by the ratio $10 \ln(10)$, where $\ln(\cdot)$ is the natural logarithm. The extinction coefficient $k_{ext} [m^{-1}]$ is a function of the distance s and the precipitation rate R (also known as Rain Rate), providing information about the distribution of raindrops. From the CloudSat-GPM coincidence products, satellite coincidences were extracted to evaluate the PIA in the three bands of interest. This was done by dynamically extracting values from the cyclic scanning of satellite products.

C. Links with other problems: Link Budget Calculator and Crane's Map

Mapping global attenuation due to clouds in the -Ku, -Ka, and -W bands is crucial for enhancing the quality of satellite communications and the accuracy of meteorological data. This study may be related to the building of Crane's map, which provides a detailed representation of cloud properties, and the concept of satellite link budget, which accounts for power losses and gains along the communication path. Additionally, the use of **sigma-zero** σ^0 , or the normalized radar cross-section, is essential for evaluating the performance of radar systems under atmospheric scattering conditions.

D. Developing a Global Map of Path Integrated Attenuation Using Satellite Data Analysis

In recent years, significant advancements have been made in the analysis of satellite-derived datasets, which provide crucial insights into atmospheric and climatic phenomena.

This **Technical Annex** is related to the study focuses on the analysis of a dataset from **August 2018**, establishing a foundation for routine analysis in other years, such as **2009** and **2019**. The primary objective is to create a comprehensive global map of Path Integrated Attenuation (PIA) for the Ku, Ka, and W bands on a planetary scale. This endeavor involves integrating satellite coincidence datasets from both

polar and equatorial orbits. To achieve this goal, I specifically analyze coincidences from the CloudSat-GPM and CloudSat-TRMM satellite families, with an emphasis on CloudSat-GPM due to time constraints. Initially, this project aimed to develop a robust environment to explore specific satellite products available as open data, such as those from STORM, JAXA, and CloudSat. This exploration helps to unravel the algorithmic complexity and intricate network of relationships between various products, semi-products, and input-output products within the computational chain. By approximating PIA near the surface using **sigma-zero** (σ^0) data, I lay the groundwork for further research aimed at refining these estimates through the identification of directly observed products. These observations will subsequently facilitate the derivation of locally linked statistical predictive considerations associated with recorded attenuation values.

E. PIA Estimation

Estimation of the PIA using the nadir radiometer measurement is more complicated. Unfortunately the CloudSat Satellite is a nadir satellite! (Nadir pointing, 94 GHz radar). So, the CloudSat satellite is indeed a nadir-pointing satellite, which means it looks directly downward (at nadir) at the Earth's surface with its 94 GHz radar. This configuration is excellent for profiling the vertical structure of clouds and measuring cloud properties like **ice water path (IWP)** and **liquid water path (LWP)**. However, estimating PIA using nadir radiometer measurements is more complicated because radiometers measure the **brightness temperature (BT)**, which is influenced by the scattering and absorption of microwave radiation by clouds and precipitation [7].

The nadir view complicates the interpretation of these measurements due to the varying angles of incidence and the complex interactions between the radiation and cloud particles. To overcome these challenges, researchers often use collocation techniques, where data from CloudSat are combined with data from other satellites that have different viewing geometries or additional sensors. This helps to improve the accuracy of the attenuation estimates by providing a more comprehensive view of the cloud properties and their effects on the radiometer measurements. Analyzing the products is one of the first operations to be performed. Subsequently, one can refine one's studies by looking in detail at some characteristics; for example, in the product 2C-PRECIP-COLUMN the CPR radar signal must be processed in the return power by discarding the lowest bins due to surface contamination (see **Tanelli et.al., 2008**) [1]. These bins often include strong reflections from the Earth's surface, which can overwhelm the radar signal returning from the precipitation or cloud particles above the surface. So going in deeply require more knowledge in **Radar Theory**.

F. Which is the reasons for discarding the lowest bins?

To answer the question I must consider at least three factors:

- 1) **Surface clutter:** the radar signal returning from the Earth's surface (i.e. *terrain, vegetations, buildings* etc.)

³"Identifying multiple-scattering-affected profiles in CloudSat observations over the oceans", J. Geophys. Res., 113, D00A17, doi:10.1029/2008JD009960.

is significantly stronger than that from atmospheric particles, causing *clutter* in the data.

- 2) **Accurate Measurements:** to ensure that the radar only measures the power returned by atmospheric particles, it is necessary to discard these contaminated bins.
- 3) **Signal-to-Noise Ratio:** the lowest bins can degrade the overall *signal-to-noise ratio*, leading to less accurate interpretations of the radar returns.

Remember that “bins” are vertical ones, where a bin refers to a discrete unit of measurement along the radar’s range axis. It’s essentially a “container” or “cell” that collects and stores the radar return signal from a specific range or distance.

G. About Radar Reflectivity

From what was said above, I can consider the reflectivity in the fourth bin (Z_n , from 600 to 840 meters of altitude), [see **Haynes et.al., 2009**], [19]. The non-attenuated (u) radar reflectivity (Z_u) is the sum of the measured near-surface (ns) reflectivity there, and the contributions from gaseous attenuation and the path-integrated attenuation. The PIA is modified by a factor Ψ that removes attenuations at levels lower than the near-surface bin [**Haynes et.al., 2009-2013**], [19], [20].

VI. WORKING ON DATA.

In this section I show some examples of data work and a comparison of the rough estimate of the PIA (positive Sigma-zero of the 2B-GEOPROF product with the PIA_hydrometeor) value of 2C-PRECIP-COLUMN. The estimate can be improved by making the assumptions discussed in **A.Battaglia et.al. (2020)** [6]. Below I show some subplots relating to the filtering obtained by resizing the values as indicated on the valid range of the variables reported on CloudSat-GPM coincidence products. I highlight the processing steps in four subplots in which at the top the product is drawn as it is, in the second box the ground is completely eliminated, in the third on it filters for some upper bound values and subsequently it is refiltered by low bound. The other images show how the data can appear incomplete and transposed and this involves further processing on the same to display them correctly. Finally, a map of coincidences and values at the 97th percentile of the roughly estimated PIA is shown.

A. Developing MATLAB Software for PIA Analysis and Visualization of Radar Reflectivity Data.

I am particularly interested in PIA as described by [**Haynes et al., 2013**], [20]. One of my initial goals was to develop open MATLAB software capable of producing graphs similar to those presented in Haynes’ works. The figure illustrates a graph of radar reflectivity in [dBZ] plotted against Along Track Distance (ATD). While the numerical values for ATD are provided, the unit of measurement isn’t specified (typically rendered in pixels, which need conversion to another unit). Altitude is shown in kilometers; however, this is not entirely accurate, as the resolution is more appropriately measured in meters. The radar reflectivity uses a “rainbow” color scale,

ranging from $-30dBZ$ to $+20dBZ$. The central box illustrates **sigma-zero** (σ_0) in decibels (dB) in black, while PIA is drawn with a green line. Despite having two scales, the graph maintains an upside-down symmetry [see **Figure 21**].

This is evident from the graph’s specularity and the dual y-axes (left for σ_0 , right for PIA). In MATLAB, you can overlay the graphs with different scales to highlight this symmetry. However, there might be a missing measurement to fully appreciate the interchangeability between σ_0 and PIA. The third graph depicts the rainfall rate in millimeters per hour (mm/h) against the along-track distance (ATD) in dimensionless units. It would have been more appropriate to indicate the scale, likely in meters or kilometers. Often, the “Along-track Distance” (ATD) is reported in kilometers (km). However, it can vary depending on the context and the specifications of the graph. Without a clear scale, it is difficult to interpret the data correctly. Specifying the unit of measurement is essential to avoid misunderstandings.

B. Estimating Rain Rate using sigma-zero (σ_0)

QUESTION: If I have a graph of sigma-zero in dB, can I evaluate the rain rate in [mm/h]? **ANSWER:** Yes, it is possible to estimate the rain rate in mm/h using a plot of σ_0 (sigma-zero) in dB. The relationship between σ_0 and the rain rate (RR) can be modeled using inversion techniques and empirical models based on radar observations and rain rate measurements. Here is a general approach:

- 1) Convert σ_0 in dB to a linear form: The relationship between σ_0 in dB and the corresponding linear value can be expressed as: $\sigma_0(\text{linear}) = 10^{\frac{\sigma_0(dB)}{10}}$.
- 2) Use an empirical model: Empirical models, such as those derived from the $Z - R$ ratio, relate radar reflectivity (Z) to rain rate (R). Some existing models can be adapted to relate σ_0 to rain rate. An example of a $Z - R$ model is:

$$Z = a \cdot R^b \quad (10)$$

where a and b are empirical constants that depend on the precipitation type. This model can be modified to fit σ_0 .

- 3) Calculate the rain rate: Once the linear value of σ_0 is obtained, it is possible to insert this value into the empirical model to estimate the rain rate.
- 4) Validation: It is essential to validate the model with observational data to ensure that the rain rate estimates are accurate.

An exact approach depends on the specific context and the available data. This method allows the use of radar information to estimate precipitation continuously and on a large scale. So an additional operation that I can do from the radar reflectivity graphs could be to filter the data based on precipitation conditions, considering some appropriate threshold values for the non-attenuated *radar reflectivity* as shown in the following table. This is just a simple example of what the data scientist can do with satellite data. Filtering radar reflectivity data based on precipitation conditions and applying threshold values can provide valuable insights. This approach allows

Reflectivity (dBZ)	Precipitation Condition
<0	No Precipitation
0-10	Light Rain
10-30	Moderate Rain
30-50	Heavy Rain
>50	Very Heavy Rain

TABLE II: Threshold values for non-attenuated radar reflectivity based on precipitation conditions.

Type of Rain	Precip_Flag	Non Attenuated
		Reflectivity Threshold
NO RAIN	No precipitation detected	$Z < -15$
DRIZZLE	Possible rain	$-15 < Z < -7.5$
RAIN	Probable rain	$-7.5 < Z < 40$
HEAVY RAIN	Certain rain	$Z \geq 40$

TABLE III: Classification of rain types based on precipitation flags and reflectivity thresholds.

data scientists to differentiate between various precipitation intensities, types, and even detect non-precipitation echoes. Here's a brief example in which is possible to demonstrate how you might document such a threshold table. [see **Table III** and **IV**]:

Near-surface precipitation types and names of the 2CPRECIP-COLUMN precip flags used in this work. The table is a merger of information extracted from *Table 2* in [Haynes et al. (2009)], [19], *Table 5* in [Haynes et al. (2013)], [20], and *Table 2* in [Kay et al. (2018)], [21]. Working on my software, I have thus created both the possibility of automatically generating radar reflectivity graphs and also possible case studies such as the one reported below in **Figure 6**.

C. Enhancing Radar Reflectivity Visualization with MATLAB: from initial estimates to refined representations

In this situation, I removed the dual scale depiction on the third figure, which shows the positive sigma-zero to avoid confusing the reader and to make the comparison between the two scales clearer. The symmetry of the peaks demonstrates that the approximation between PIA and positive sigma-zero is a good initial rough estimate. In the following figures, I show how I developed MATLAB software to best represent radar reflectivity. Starting with initial representations, I improved the accuracy by applying matrix constraint scale and representativeness. This process led to progressively clearer and more accurate depictions, with the best results shown at the bottom after a series of refinements. I intentionally left the numerical representation of the along-track distance without indicating the size in meters, as the specific details can be found in the literature regarding the swath. In the subsequent images, I display partially open satellite coincidence files or partially recoverable files when some data is missing. In these cases, I do not discard the files but open them to show the CloudSat radar reflectivity.

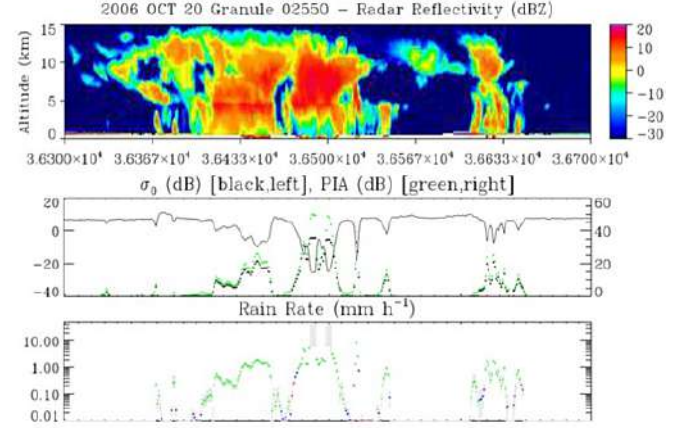


Fig. 3: The figure shows a portion of a CloudSat orbit on October 20, 2006. (top) The radar reflectivity profile as observed by the CPR. (middle) The measured surface reflectivity (solid black line, left y scale) as well as the PIA to the surface (green dots, right y scale) and the PIA to the fourth bin above the surface (black dots, right y scale).(bottom) Retrieved rain rate (colored dots) and uncertainties(bars). Green dots: "rain certain"; blue dots: "rain probable"; red dots: "rain possible". Courtesy of Haynes et.al. (2008) ©.

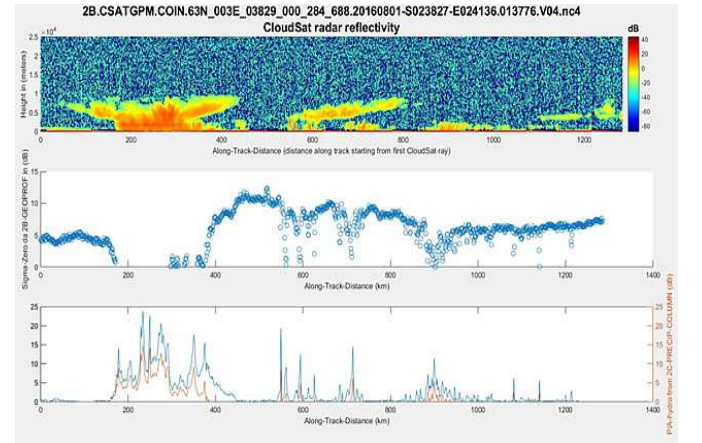


Fig. 4: (top): Radar reflectivity of a CloudSat-GPM coincidence. The name of the figure indicates exactly the original file taken into consideration.(medium): Image of the positive sigma-zero value taken from the 2B-GEOPROF product. (bottom): Comparison between the PIA-hydrometeor of the 2C-PRECIP-COLUMN product and the positive sigma-zero extracted from 2B-GEOPROF, compared on a common scale.

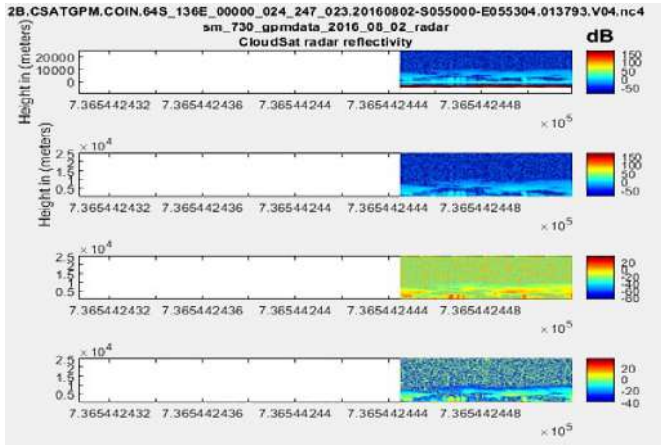


Fig. 5: A series of four radar reflectivity plots from CloudSat data, labeled "2B.CSATGPM.COIN.64S_136E_00000_024_247_023.20160802-S055000-E055304.013793.V04.nc4". The data path is from Filezilla: "sm\730\gpmdata\2016\08\02", where the CloudSat-GPM coincidences files are stored. Each plot represents radar reflectivity (Z) in decibels (dB) as a function of height (in meters) and a horizontal axis labeled with values around 7.365442432×10^5 (the Along-Track Distance). The images demonstrate how, using my MATLAB source code, I was able to scan even corrupt coincidence files where only part of the data was recovered. I applied successive filters from top to bottom to the initially imported radar reflectivity images as they would appear in Panoply, then applied further filters that respect the recommendations on radar reflectivity limits imposed by satellite products as reported on the official CloudSat website.

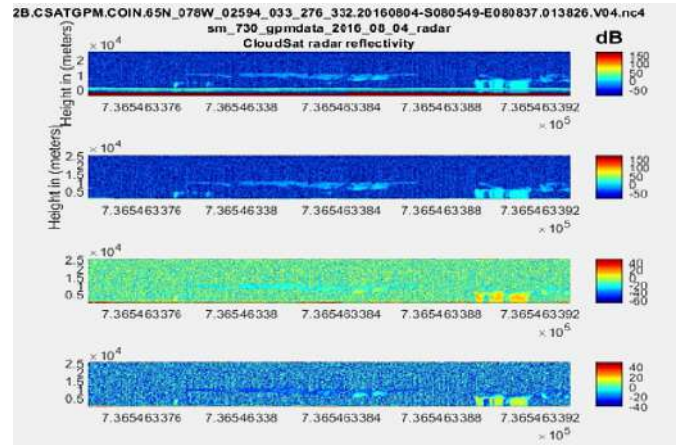


Fig. 6: A series of four radar reflectivity plots from CloudSat data without native source errors in scanning. The variations in color across the panels highlight changes in radar reflectivity, which can be used to infer the structure and properties of clouds and precipitation. The higher reflectivity values (in red) typically indicate more intense precipitation or denser cloud regions, while lower values (in blue) suggest lighter precipitation or less dense clouds. This type of data is crucial for understanding atmospheric processes and weather patterns, providing valuable insights into cloud dynamics and precipitation intensity. I apply successive filters from the top down to the initially imported radar reflectivity images as they would appear in Panoply and then subsequently apply such filters that respect the recommendations on radar reflectivity limits imposed by satellite products as reported on the official CloudSat website.

D. Observations and considerations about the figures 7 and 8.

Looking at the figure with five sub-figures [see **Figure 8**], I can make several observations:

- 1) The third graph from the top shows an example similar to **Haynes'** work, with an added legend for clarity. It displays the 2-way **PIA_hydrometeor** and the positive **Sigma-zero** approximation of the PIA.
- 2) The other two graphs test different x-axis representations, switching from **Along-Track-Distance (ATD)** to latitude in degrees. These graphs compare time series extracted from the Sigma-zero of the 2C-PRECIP-COLUMN product and 2B-GEOPROF for the same CloudSat-GPM satellite coincidence file.
- 3) Data anomalies can occur, as seen in this case.

E. Possible Causes of Overlapping Data in Time Series

- 1) **Data Resolution:** Close intervals in data collection can cause natural overlaps.
- 2) **Data Noise:** Measurement errors or noise can lead to fluctuations and overlaps.
- 3) **Periodicity:** Periodic components in the data can cause values to repeat at regular intervals.
- 4) **Sampling:** Uneven or incorrect sampling can result in data overlaps.

Correcting a time series with poorly sampled data can be tricky, but there are several techniques you can use to improve the quality of your data:

- **Interpolation:** Use interpolation methods to estimate missing or incorrect values. Common techniques include linear, spline, or polynomial interpolation.
- **Filtering:** Apply filters to remove noise from the data. Filters such as a **Kalman** filter [18] or low-pass filter can be useful for smoothing out unwanted fluctuations.
- **Collecting Additional Data:** If possible, collect additional data to improve temporal resolution and reduce sampling errors.
- **Recalibration:** If the data comes from sensors, test and recalibrate the sensors to ensure they are measuring correctly.
- **Machine Learning Techniques:** Use **Machine Learning algorithms** to predict and correct poorly sampled values. Algorithms such as **Neural Networks** or **Regression models** can be trained to recognize and correct errors in the data.
- **Fourier Analysis:** Use Fourier analysis to identify and remove unwanted periodic components in the data.

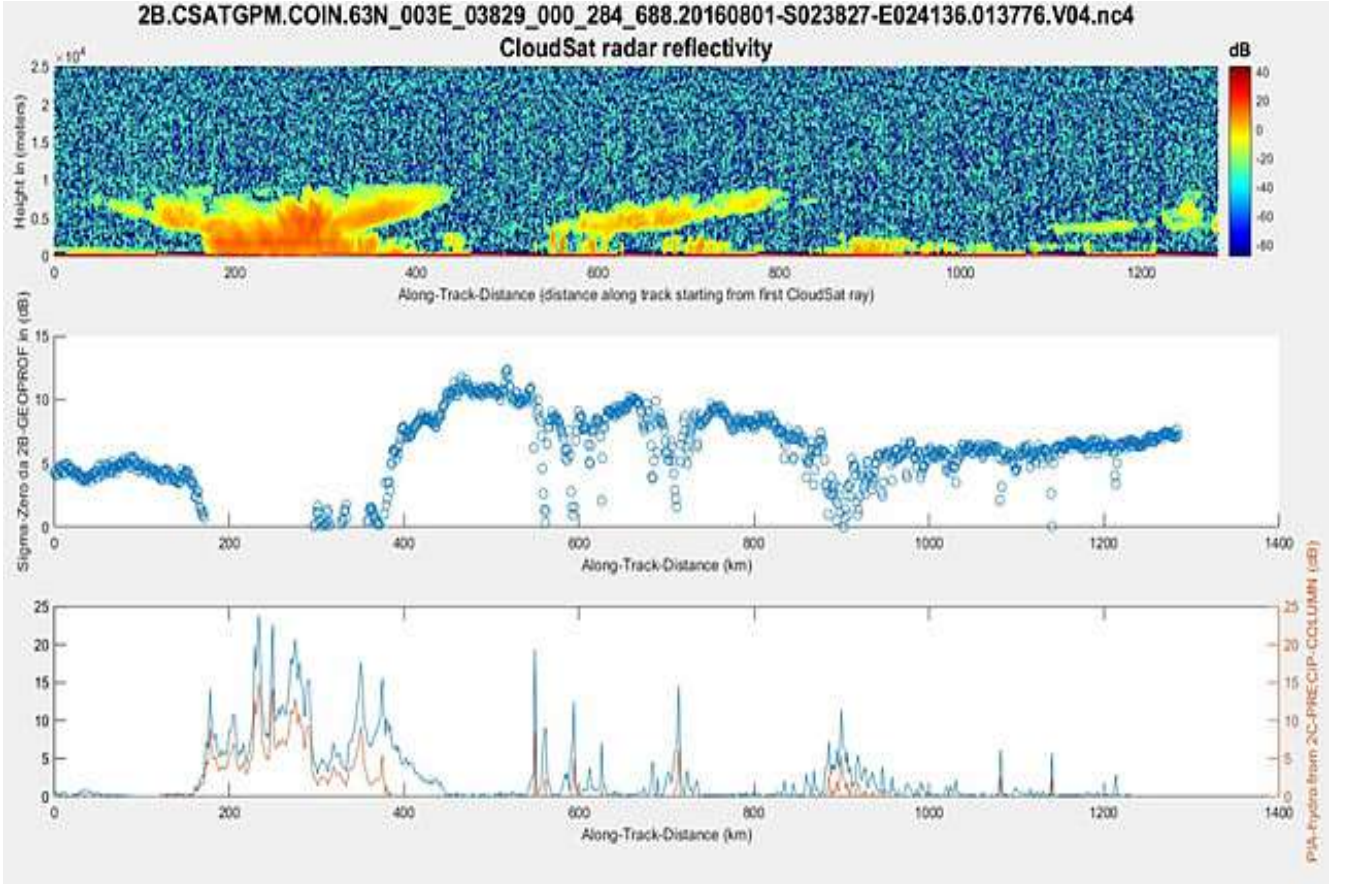


Fig. 7: Example of enlarging **Figure 4** for a better view. From top to down: (Top) Radar Reflectivity vs. Along-Track-Distance from 2B-GEOPROF. (Middle) Sigma-Zero vs. Along-Track Distance extracted from 2B-GEOPROF. (Down): Sigma-Zero vs. Along-Track-Distance from 2B-GEOPROF (in blue color) with left scale in black, and the PIA-hydrometeor vs. Along-Track-Distance extracted from 2C-PRECIP-COLUMN with scale in orange on the right.

VII. ALGORITHMS FOR CALCULATING PIA.

The first method for calculating or determining the Path Integrated Attenuation (PIA) depends on the well-established relationship between the radar backscatter cross-section of the ocean surface, σ_0 , and the wind speed, V , at the ocean surface. Higher wind speeds cause increased roughness of ocean surfaces, leading to greater scattering of microwave radiation away from the radar receiver and resulting in a lower backscatter radar cross-section. **Sea Surface Temperature (SST)** also influences the backscatter cross-section by altering the refractive index. A database of backscatter radar observations under clear weather conditions (*clear sky*) provides σ_{clear} as a reference for the surface state when no hydrometeors are present in the sky. When rain or cloudy conditions are present, the observed backscatter radar cross-section decreases due to hydrometeor attenuation. This reduction allows for the calculation of PIA based on the known wind speed at the ocean surface (derived from a numerical model) and, to a lesser extent, **SST** [Li et al., (2005)], [9].

VIII. FOUNDATIONS FOR PIA: PATH INTEGRATED ATTENUATION.

In this section, before showing examples of global attenuation maps, I will better describe the notion of path-integrated attenuation (PIA). The PIA, as a quantity of interest, links directly to the local precipitation rate (also called rain rate).

By evaluating the PIA, I can understand precipitation intensities, which is useful for monitoring climate change. Accurate PIA estimation is crucial for precise rain rate estimations. If radar reflectivity is associated with Sigma-zero and is underestimated, it will consequently lead to an underestimation of the rain rate. Sigma-zero measures surface back-scattering attenuation and serves as a rough estimate of the PIA. Studying the PIA and its horizontal variability helps meteorologists estimate precipitation, which often varies locally in this direction. The vertical radar profile of precipitation and clouds containing hydrometeors fits within the concept of a radar beam. I can visualize this beam as a vertical cylinder, which contains vertical “cells” of air mixed with rain and other elements.

I can consider a cell with a width of 1 km and a height of 5 km. The radar scanning volume will be a **field of view (FOV)**. In precipitation studies, I assume vertical homogeneity and

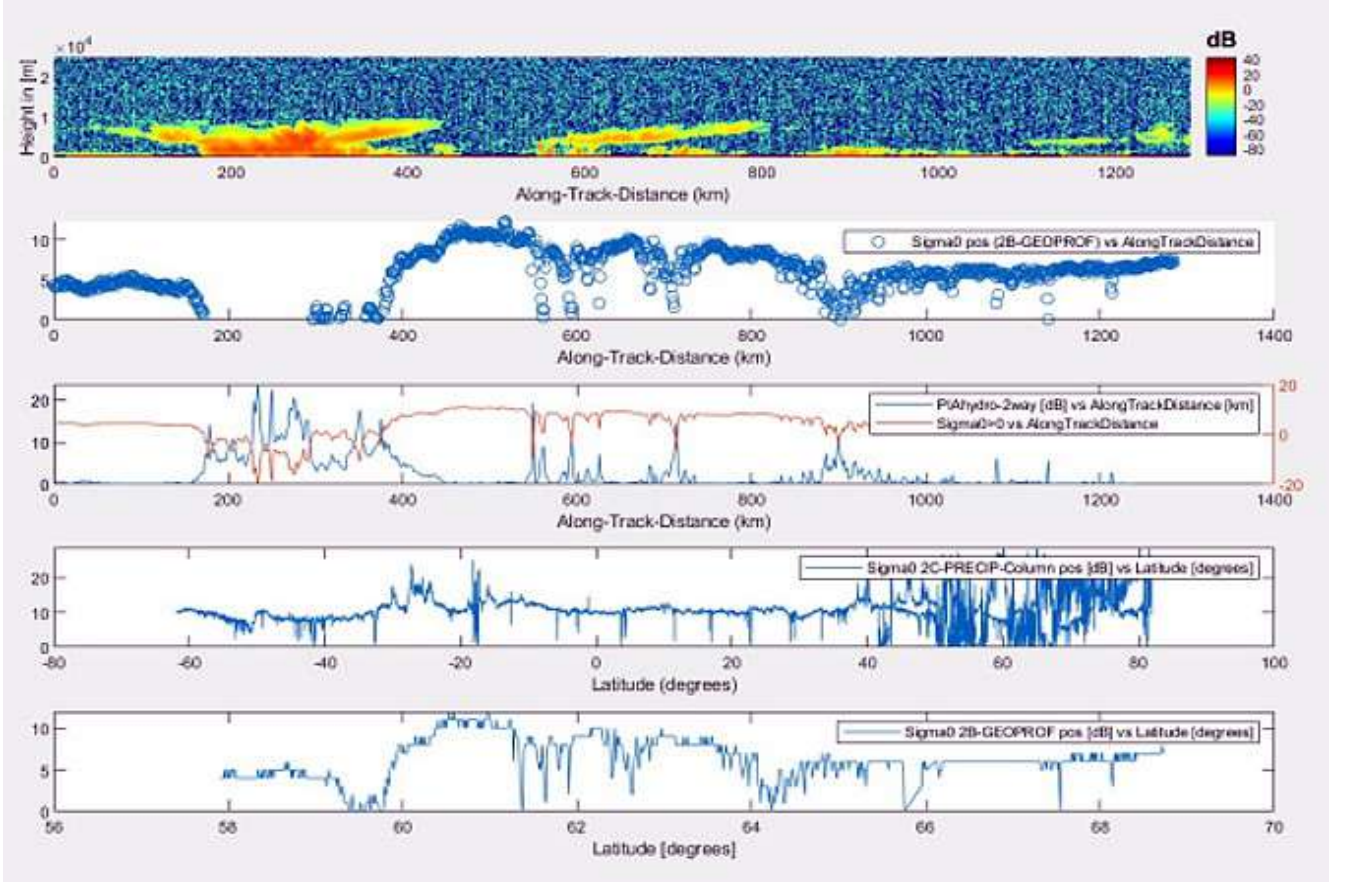


Fig. 8: Example of a possible case of study. (1) From top to down: Radar Reflectivity vs. Along-Track-Distance extracted from 2B-GEOPROF. (2) Sigma-Zero vs. Along-Track-Distance from 2B-GEOPROF. (3) PIA_{hydrometeor} vs. Along-Track-Distance from 2B-GEOPROF and Sigma-Zero vs Along-Track-Distance in 2B-GEOPROF. (4) The Sigma-Zero vs. Latitude (in degrees) extracted from 2C-PRECIP-COLUMN, (5) The Sigma-Zero from 2B-GEOPROF vs. Latitude (in degrees).

horizontal variability [17]. Therefore, studying the horizontal inhomogeneity of clouds can help me understand how hydrometeors compact inside the cloud and almost instantly liquefy. This process can lead to the violent flooding phenomena I recognize as climate change. I represent air columns as square cells in study images due to pixel-based graphic representation. Within a theoretical framework, I analyze the radar reflectivity of hydrometeors within cloud bodies. I determine quantities such as the effective radar reflectivity Z_e , focusing on the TOP value in [dBZ] at the cloud's top. I then estimate specific attenuation, denoted as k in [dB/km], by linking it to the effective radar reflectivity using an empirical **Marshall-Palmer (M-P)** relationship: $k = \alpha \cdot (Z_e^{top})^\beta$, I have the empirical parameters (α, β) . Next, I calculate the PIA (path integrated attenuation) through each atmospheric column previously described. This calculation does not necessarily stop at 5 km, even though some clouds may end at that altitude. Instead of viewing this calculation as an integral process, which I will discuss later, I associate it with image processing and its discretization.

When processing satellite images, I numerically evaluate the PIA as follows: $PIA = 2 \cdot \Delta h_t \cdot k$ where Δh_t represents the height difference or the thickness of a layer within the atmospheric column analyzed by the Data Scientist. With k ,

I indicate the specific attenuation in [dB/km], and Δh are [km], so overall I have that the PIA is measured in [dB], that is, it is a dimensionless quantity but its meaning is attenuation. The value 2 indicates that the integration path of the specific attenuation along the path of the electromagnetic radiation in the atmosphere takes into account the round trip; this is called a “2-way” PIA. Finally, I can calculate the measured or apparent radial reflectivity, indicated with Z_m , at the bottom of a cloud. In this case, for each column, I evaluate: $Z_m^{bot} = Z_e^{top} - PIA$ in [dBZ]. [1]

I can also define other quantities of interest for the PIA, which are:

- Apparent PIA
- Uniform PIA
- Deviation PIA, given by the difference between the apparent PIA and the uniform PIA.

These quantities are usually calculated values for a 5×5 km *footprint* and are all average quantities. Research is aimed at plotting scatter plots to analyze certain trends through which I understand how things are going.



Fig. 9: Figure of the World for illustrative purpose.

A. Example

Scatter plots of the covariance of radar reflectivity TOP versus actual radar reflectivity top, or the covariance of the attenuation coefficient versus actual radar reflectivity top, and also the scatter plot of the covariance of PIA versus actual radar reflectivity top. These plots allow me to define the covariance as:

$$\text{cov}(k) := \frac{\text{std}(k)}{\text{mean}(k)} \quad (11)$$

and similarly for the other quantities.

B. About Scatter-plots

Scatter plots of covariance are useful tools for visualizing and analyzing the relationship between two variables. Here's what they are used for and what the different scatter plots you mentioned establish:

- 1) Scatter plot of the covariance of radar reflectivity TOP versus actual radar reflectivity top:
 - a) Purpose: This scatter plot is used to visualize the relationship between the radar reflectivity measured at the top of the atmospheric column and the actual radar reflectivity at the top.
 - b) What it establishes: It helps identify any discrepancies or correlations between theoretical and actual measurements, allowing for the evaluation of radar estimate accuracy.
- 2) Scatter plot of the covariance of the attenuation coefficient versus actual radar reflectivity top:
 - a) Purpose: This scatter plot visualizes the relationship between the attenuation coefficient (which

measures signal loss) and the actual radar reflectivity at the top of the column.

- b) What it establishes: It allows for the analysis of how signal attenuation varies as a function of reflectivity, helping to improve attenuation models and correct radar estimates.
- 3) Scatter plot of the covariance of PIA versus actual radar reflectivity top:
 - a) Purpose: This scatter plot shows the relationship between the path-integrated attenuation (PIA) and the actual radar reflectivity at the top.
 - b) What it establishes: It helps understand how the total signal attenuation along the path is influenced by radar reflectivity, providing useful information for improving PIA corrections and precipitation estimates.

C. Different type of PIA

In general, these scatter plots are fundamental tools for validating and improving radar models, allowing for more accurate estimates of precipitation and atmospheric conditions. Here's what the different types of PIA allow me to establish:

1) Apparent PIA:

- a) Definition: Apparent PIA is the measure of the total attenuation of the radar signal along a path, calculated directly from radar data without corrections.
- b) Utility: It serves as a starting point for further analysis and corrections. It can be influenced by various factors such as the presence of *hydrometeors* (rain, snow, hail) and other atmospheric particles.

2) Uniform PIA:

- a) Definition: Uniform PIA is an estimate of the radar signal attenuation along a path, assuming that the attenuation is uniformly distributed along the entire path.
- b) Utility: It provides a reference value to compare actual attenuation with theoretical attenuation. It is useful for identifying local variations in attenuation that might indicate the presence of specific atmospheric phenomena.

3) Deviation PIA (Apparent PIA - Uniform PIA):

- a) Definition: Deviation PIA is the difference between the apparent PIA and the uniform PIA.
- b) Utility: This deviation allows for the identification and quantification of anomalies in radar signal attenuation. It can indicate the presence of complex atmospheric structures, such as thunderstorm cells, intense precipitation fronts, or variations in the concentration of hydrometeors.

In summary, the analysis of these different types of PIA helps meteorologists improve the accuracy of weather forecasts, identify specific atmospheric phenomena, and correct precipitation estimates based on radar data.

IX. TYPOLOGIES OF SCATTER PLOTTER AVAILABLE FOR EXPLORING DATA AND PIA INFLUENCE

Other scatter plots can be the following:

- 1) Apparent PIA vs. Z_e^{TOP}
- 2) PIA deviation vs. Z_e^{TOP}
- 3) RR (Rain Rate) vs. Z_e^{TOP}

Generally, these scatter plots can be evaluated by considering the **covariance** of PIA as a third graphical element. This effectively answers the following useful:

- 1) **QUESTION:** When I plot A vs. B as a scatter plot and apply a color scale based on a variable C, what am I doing?
- 2) **ANSWER:** When you create a scatter plot of A vs. B and apply a color scale based on a variable C, you are visualizing three dimensions of data in a two-dimensional graph.

Specifically:

- 1) **X and Y Axes:** The X and Y axes of the graph represent the variables A and B, respectively, showing the direct relationship between these two parameters.
- 2) **Color Scale:** The variable C is represented through a color scale, with each point on the scatter plot colored based on the corresponding value of C. The color scale helps interpret how C varies in relation to A and B.
- 3) **Visual Covariance:** By adding the color scale of C, you are observing if there is a **visual covariance** between C and the combination of A and B.

For example, you might look for patterns where certain values of C cluster in specific regions of the A-B space, suggesting a relationship between C, A, and B.

In short, this type of graph allows you to explore potential three-dimensional relationships between A, B, and C and to observe how changes in C are distributed in the space defined by A and B.

1) *Visualization Techniques:* To analyze visual covariance, I typically use scatter plots, heat maps, or 3D plots. A scatter plot of A versus B forms the base representation, while C is added as a color gradient or third dimension. For dense datasets, heat maps or contour plots can aggregate values of C over a grid to reduce clutter and highlight overarching patterns. Advanced methods, such as animation for time-series data or dimensionality reduction techniques like **PCA** [16], can further enhance the clarity of relationships.

2) *Interpreting Visual Patterns:* Visual inspection focuses on identifying gradients, clusters, or other consistent changes in C relative to A and B. A clear gradient suggests strong covariance, indicating that C depends on the interaction of A and B. Conversely, a random distribution implies weak or no covariance. In some cases, nonlinear or complex patterns may emerge, requiring advanced statistical or machine learning techniques for deeper analysis.

3) *Conclusions on Covariance atmospheric research:* In atmospheric research, visual covariance analysis is invaluable for studying the relationships between hydrometeor properties, radar reflectivity, and attenuation. By incorporating data from satellites like CloudSat, GPM, and TRMM, researchers can uncover interactions between precipitation, cloud structures, and atmospheric properties, ultimately improving weather prediction models. Visual covariance provides a powerful

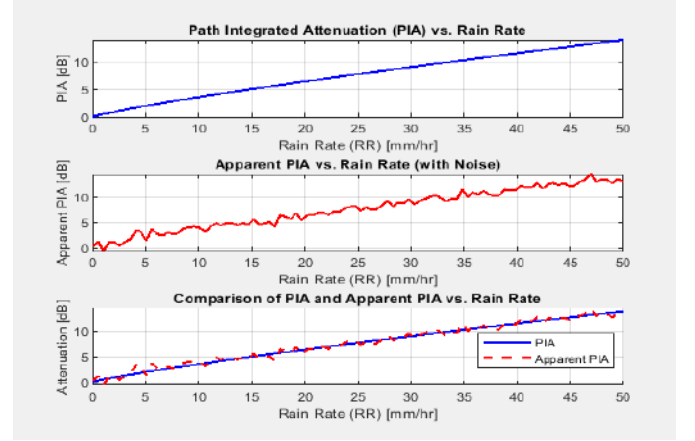


Fig. 10: Visualization of the relationship between Rain Rate (RR), Path Integrated Attenuation (PIA), and Apparent PIA. The first subplot shows the theoretical PIA as a function of rain rate, illustrating the power-law dependency. The second subplot presents the Apparent PIA, highlighting the influence of random measurement noise on the attenuation values. The third subplot provides a comparison of theoretical PIA and Apparent PIA, demonstrating how noise affects the observed attenuation in relation to the rain rate. This visualization emphasizes the importance of accounting for noise in radar-based precipitation measurements.

exploratory tool to uncover complex relationships in multi-dimensional data. When combined with statistical validation, it enables researchers to draw meaningful insights, guiding further analysis and improving predictive models in fields like meteorology, geospatial analysis, and beyond.

The provided MATLAB code demonstrates the relationship between Path Integrated Attenuation (PIA), Apparent PIA, and Rain Rate (RR) through a theoretical model and visualization. The primary aim of this simulation is to understand how rain-fall intensity, represented as the rain rate in mm/hr, impacts the attenuation of radar signals (PIA) over a specified path length and how measurement noise affects the perceived (or apparent) attenuation. The simulation begins by defining the rain rate (RR) over a range of values from 0 to 50 mm/hr. Using empirical coefficients α and β , the code calculates the specific attenuation of the radar signal in dB/km, which follows the widely accepted power-law relationship: $k = \alpha \cdot RR^\beta$. The constants α and β are key parameters derived from empirical studies and vary depending on the radar frequency and rain characteristics.

Once the specific attenuation is determined, the Path Integrated Attenuation (PIA) is calculated by multiplying the specific attenuation with the total path length of the radar beam in kilometers. This gives the theoretical PIA, which represents the total attenuation of the signal as it passes through the rain medium. To simulate real-world scenarios, where measurements are affected by various factors such as instrument noise or environmental conditions, random noise is introduced to the theoretical PIA values. This results in the Apparent PIA, which mimics the noisy observations typically

encountered in satellite or ground-based radar systems. The noise is generated using a Gaussian distribution and added to the PIA values, illustrating the discrepancies between the ideal theoretical values and the measured data.

The code includes three visualization plots to highlight the relationships and differences among PIA, Apparent PIA, and Rain Rate. The first plot shows the theoretical PIA as a function of rain rate, highlighting the direct dependence of signal attenuation on rainfall intensity. The second plot demonstrates how noise affects the apparent PIA, showing variations around the theoretical curve. The third plot provides a comparative view of both PIA and Apparent PIA, allowing the user to visually assess the impact of noise and how it might influence data interpretation. This comparative visualization is particularly useful for understanding the uncertainties in radar-based precipitation measurements and for designing more robust signal-processing techniques.

The finality of this code is to serve as a foundational tool for analyzing the impact of rain on radar signals and to simulate how apparent measurements can deviate from theoretical expectations. It is especially relevant in the context of satellite missions like CloudSat, GPM, or TRMM, where accurate retrieval of precipitation properties is critical for improving weather models and climate studies. Additionally, by modeling and visualizing these relationships, the code helps researchers understand the importance of calibration techniques and noise-reduction methods in ensuring the accuracy of radar-derived precipitation estimates. This simulation can also be extended to more complex scenarios, such as incorporating varying path lengths, different radar frequencies, or real observational data for validation and comparison.

The resulting **Figure 11** represents the relationship between **Rain Rate (RR)**, **Path Integrated Attenuation (PIA)**, and **Apparent PIA**. The 10,000-point scatter plot shows the simulated apparent PIA values versus the rate of rain. It shows how adding noise to the data changes the range of values around the theoretical ones. As the blue points spread out, it's clear that there is a positive relationship between Rain Rate and Apparent PIA. This proves that heavier rain causes radar signals to weaken.

Overlaid on the blue point cloud, a red curve represents the theoretical PIA values, calculated without noise. This curve serves as an ideal reference to compare the simulated data with the predicted values from the model. The closeness and distribution of the blue points around the red curve allow me to evaluate the degree of dispersion and the amount of error introduced by noise. Overall, the figure highlights the importance of considering the effect of noise in radar measurements to obtain reliable estimates of attenuation.

X. WHAT IS THE PIA USEFUL FOR?

The PIA is useful for:

- 1) Determining the intensity of precipitation: the higher the PIA, the stronger the precipitation.
- 2) Correcting radar data: the PIA can be used to correct radar data and improve the estimation of the precipitation rate.

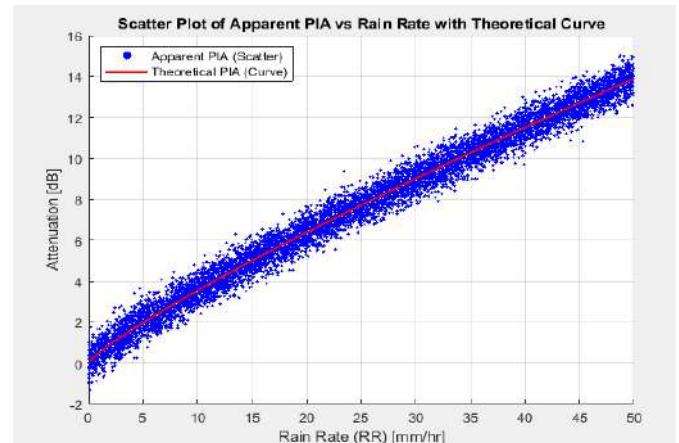


Fig. 11: Relationship between Rain Rate (RR), Path Integrated Attenuation (PIA), and Apparent PIA. The scatter plot (blue dots) shows the computed apparent PIA with added noise as a function of rain rate, simulating realistic measurements affected by instrumental errors. The red curve represents the theoretical PIA values, computed without noise, serving as an ideal reference. The figure shows how attenuation rises with rainfall intensity and how noise causes scattering. This is important to know in order to understand how hard it is to use radar measurements to figure out the properties of precipitation.

XI. WHERE I FOUND INFORMATION ABOUT PIA IN THE REMOTE SENSING SATELLITE PRODUCTS?

You can find theoretical information about PIA (Path Integrated Attenuation) in the context of CloudSat-GPM coincidence datasets in the following resources:

- 1) CloudSat-GPM Coincidence Dataset Version 1C: This document provides detailed information on the intersections between the GPM core satellite and the CloudSat profiling radar. It includes theoretical explanations and practical applications of the PIA in these datasets⁴.
- 2) Applications of a CloudSat-TRMM and CloudSatGPM Satellite Coincidence Dataset: This article discusses the use of near-coincident observations between GPM and CloudSat, including the theoretical background and applications of PIA in studying precipitation and cloud processes⁵.
- 3) CloudSat-GPM Coincidence Dataset Algorithm Theoretical Basis Document (ATBD): This document offers a comprehensive overview of the algorithms used to process the CloudSat-GPM coincidence data, including the theoretical basis for calculating PIA⁶.

The document `ATBD_DPR_V5_201705final.pdf` is likely an **Algorithm Theoretical Basis Document (ATBD)** for the **Dual-frequency Precipitation Radar (DPR)**, part of the **Global Precipitation Measurement (GPM)** mission.

⁴<https://gpm.nasa.gov/resources/documents/cloudsat-gpm-coincidence-dataset-version-1c>

⁵<https://www.mdpi.com/2072-4292/13/12/2264>

⁶<https://gpm.nasa.gov/sites/default/files/documentfiles/CSATGPMCOINATBD.pdf>

These documents describe in detail the algorithms used to process the data collected by the dual-frequency radars, explaining the theoretical basis, calculation methods, and practical applications. In the paragraph §3.1.1-3, “**Path Integrated Attenuation (PIA)**”, I may also have the opportunity to learn more about this parameter.

A. PIA Definition

Path Integrated Attenuation (PIA) is defined as integrated attenuation caused by precipitation particles from the radar to the surface. If $r = r_s$ at the surface, PIA is given as shown below:

$$PIA = -10 \log_{10}(A_P(r_s)) = 2 \int_0^{r_s} k(s) \cdot ds \quad (12)$$

Eq. (3.1-12) can be rewritten as a function of the PIA, so the equation (3.1-12) [See **ATBD_DPR_201811 Appendix 3b, GPM Algorithm**], [15], is:

$$10 \log_{10}(Z_m(r)) = 10 \log_{10}(Z_e(r)) - 2 \int_0^r k(s) \cdot ds \quad (13)$$

From the definition of the measured radar reflectivity factor $Z_{m0}(r)$ in $[mm^6 \cdot m^{-3}]$, considering the attenuation by precipitation particles $A_P(r)$, where r in km denote the distance from the radar along the range, I may affirm that:

$$\begin{aligned} Z_{m0}(r) &= Z_e(r) \cdot A_{NP}(r) \cdot A_P(r) = \\ &= Z_e(r) \cdot A_{NP}(r) \cdot e^{-0.2 \cdot \ln(10) \cdot \int_0^r k(s) \cdot ds} \end{aligned}$$

where s indicate the dummy variable in the integral, a generic space distance. So this equation may be rewritten as:

$$Z_{m0}(r) = Z_e(r) \cdot A_{NP}(r) \cdot e^{-\frac{2}{10} \ln(10) \cdot \frac{PIA}{2}} \quad (14)$$

where PIA is a **PIA-2way**. So using logarithmic operator I have:

$$10 \log_{10}(Z_{m0}(r)) = 10 \log_{10}(Z_e(r) A_{NP}(r) A_P(r)) \quad (15)$$

Applying the properties of logarithms I have:

$$10 \log_{10}(Z_{m0}(r)) = 10 \log_{10}(Z_e(r)) + 10 \log_{10}(A_{NP}(r)) + 10 \log_{10}(A_P(r)) \quad (16)$$

I obtain using (1):

$$10 \log_{10}(Z_{m0}(r)) = 10 \log_{10}(Z_e(r)) + 10 \log_{10}(A_{NP}(r)) - 2 \int_0^r k(s) \cdot ds \quad (17)$$

$$-2 \int_0^r k(s) \cdot ds \quad (18)$$

otherwise:

$$10 \log_{10}(Z_{m0}(r)) = 10 \log_{10}(Z_e(r)) + 10 \log_{10}(A_{NP}(r)) - PIA_{2-way} \quad (19)$$

where $k(s)$ is the specific attenuation constant dependent from the spatial coordinate. Remember that NP stay as “Non-Precipitating” Path. As described better in **Iguchi-Meneghini** [10] I have:

$$A(r) = \frac{Z_m(r)}{Z_e(r)} \quad (20)$$

where:

$$A(r) = e^{-q PIA(r)} \quad (21)$$

and so I have that, having $q := 0.1 \cdot \ln(10)$, by substituting the value of q into the expression for $A(r)$ and taking the natural logarithm of both sides, I obtain:

$$PIA(r) = -\frac{10}{\ln(10)} \cdot \ln[A(r)] \quad (22)$$

thus, I can approximate:

$$PIA(r) \cong -4.34 \cdot \ln[A(r)] \quad (23)$$

So, in general I have that PIA(r) consists of different contributions for which I can subsequently break down the specific attenuation constant into the following integral contributions:

$$PIA_{2-way} := 2 \cdot \int_0^r [k_p(s) + k_{clw}(s) + k_{WV}(s) + k_{O_2}(s)] \cdot ds \quad (24)$$

where every part in the summation inside the integral part is due to Cloud Water content, Water Vapour (WV), Oxygen (O_2) and precipitation contribute of attenuation. In other words, the PIA consists of different contributions and is generally used through a more complex integral calculation. This calculation includes the specific attenuation coefficients due to various types of precipitation, which are caused by the presence of liquid water in clouds (Cloud Water Content, CWC), Water Vapor (WV), molecular Oxygen (O_2), and various attenuation contributions due to the presence of hydrometeors within the clouds. Also in **ATBD** [14] I found another definition about PIA:

$$PIA = \frac{10}{\beta} \log_{10} \left[1 - 0.2 \beta \ln(10) \cdot \int_0^{r_s} \alpha(s) \cdot Z_m^\beta(s) \cdot ds \right] \quad (25)$$

The PIA can be estimated by taking the difference between the *surface backscattering* cross sections with and without precipitation (*surface reference technique*, (**SRT**) [11]- [13]). Take care of these informations:

- 1) k is the specific attenuation constant, in $[dB \cdot km^{-1}]$
- 2) A_p indicate the precipitation attenuation, in $[dB]$
- 3) r indicate the radar range in $[km]$
- 4) $Z_m(r)$ is the measured Radar Reflectivity in $[dBZ]$

- 5) s indicate the space variable in the integral calculus, in [km]
- 6) α is s dependent parameter, due to (power-law) is dimensionless
- 7) β indicate a parameter in which its value along the range due to **H-B relationship** (power-law) should be constant
- 8) H-B indicate the abbreviation for the method **Hitschfeld-Bordan (H-B)** equations.

The power law is a relation between k and Z_e , as shown below: $k(r) = \alpha(r) \cdot Z_e^\beta(r)$.

Here are the analytical definitions for the effective radar reflectivity $Z_e(r)$ and the measured (apparent) radar reflectivity $Z_m(r)$:

1. ****Effective Radar Reflectivity $Z_e(r)$ ****:

$$Z_e(r) := \int_0^r \alpha(s) \cdot ds \quad (26)$$

where $\alpha(s)$ is the specific attenuation coefficient dependent on the spatial variable s .

2. ****Measured (Apparent) Radar Reflectivity $Z_m(r)$ ****:

$$Z_m(r) := Z_e(r) \cdot \exp\left(-2 \cdot q \cdot \int_0^r k(s) \cdot ds\right) \quad (27)$$

where $k(s)$ is the specific attenuation coefficient along the path and $q = 0.1 \cdot \ln(10)$. These expressions clearly show the difference between the effective reflectivity $Z_e(r)$, which represents the true reflectivity without attenuation, and the measured reflectivity $Z_m(r)$, which includes the effect of path attenuation. The 2 factor take care about the 2-way path. These expressions clearly show the difference between the effective reflectivity Z_e , which represents the real reflectivity without attenuation, and the measured reflectivity Z_m , which includes the effect of attenuation along the path. The radar reflectivity factor Z_e takes into account precipitating particles such as raindrops, snow, ice crystals, and hail. Radar reflectivity is a measure of the radar cross-section of particles that can be detected by weather radar. It is proportional to the sixth power of the particle diameter and depends on their concentration per unit volume. On the contrary the apparent reflectivity factor Z_m is the effective radar reflectivity corrected for any errors and distortions introduced during the measurement process.

This includes corrections for factors such as unwanted echoes (clutter) caused by non-meteorological obstacles (mountains, buildings, etc.), radar polarization, and other interferences. Z_e is the “raw reflectivity” and may contain errors and distortions, while Z_m is a corrected and more accurate version. Z_e is useful for initial analysis and detection of precipitation, whereas Z_m is preferred for more precise applications such as quantifying precipitation and **Weather Forecasting (WF)**.

B. What is the SRT: Surface Reference Technique?

The **Surface Reference Technique (SRT)** is a method used to estimate the path-integrated attenuation (PIA) caused by precipitation along the radar beam. This method is based on the idea that a decrease in the surface return power in

the presence of precipitation, compared to a reference value without rain, provides an estimate of the two-way path attenuation. The SRT is particularly useful at high radar frequencies and in conditions of heavy rain, where path attenuations are much greater than the intrinsic fluctuations in the surface scattering cross-section. This method has been applied to both airborne and spaceborne radar data [9]. The Surface Reference Technique (SRT) remains a cornerstone in radar meteorology for estimating path-integrated attenuation due to precipitation. Its ability to leverage surface return power variations makes it especially effective at higher radar frequencies and in conditions of intense rainfall. However, the method is not without its limitations, as uncertainties introduced by surface variability and environmental factors can challenge its reliability.

In the near future, SRT is likely to get better by adding machine learning, using data from multiple satellites, and polarimetric radar observations to make it more accurate and useful. By refining the SRT and coupling it with advanced remote sensing technologies, researchers can improve global precipitation measurements and contribute to better forecasts of extreme weather events, ultimately supporting disaster preparedness and climate resilience.

CONCLUSIONS.

Together with analyzing CloudSat-GPM coincidence datasets, this study was able to make global attenuation maps in the Ku, Ka, and W bands. I learned about the Path Integrated Attenuation (PIA) and the links between radar reflectivity, PIA, and rain rate (RR) when I used the Surface Reference Technique (SRT) and advanced radar reflectivity analysis. These results underline the value of satellite dataset analysis to improve the understanding of atmospheric phenomena and refine precipitation estimation algorithms. When attenuation maps are made with high accuracy, they help a lot with showing how clouds and rain interact and with studying the properties of global atmospheric systems. For this project, detailed steps were used, such as looking at data in NetCDF files and creating MATLAB algorithms, which made it possible to see and calculate PIA correctly. The quality of the estimates got a lot better when certain problems were fixed, like leaving out the lowest bins because they are too cluttered and attaching statistical filters. The method used worked well to deal with the problems that come up with coincidence datasets and get useful data for creating more advanced atmospheric models. Despite the progress achieved, some challenges remain. Noise in the data, such as errors associated with sigma-zero values and temporal overlaps of the data, can introduce significant uncertainties. Also, dealing with surface clutter and multiple scattering is challenging because of technical issues that make PIA estimates less accurate, especially when it rains a lot or the surface isn't smooth. These limitations highlight the need for advanced calibration techniques and more sophisticated methods for the treatment of satellite data. Innovations like these can further extend the analysis.

The classification of hydrometeors and the accuracy of the estimates would be better if new satellite data from future missions with high-resolution sensors like polarimetric radars

were added to the dataset. Additional data, such as temperature, humidity, and wind, could enrich the existing models.

The knowledge acquired has significant practical applications, such as improving satellite communications planning through tools such as Link Budget calculators and supporting global-scale weather and climate forecasting. The developed attenuation maps can be used to improve precipitation modeling and emergency management during extreme weather events, such as floods. Furthermore, these maps represent a useful tool to support climate change adaptation and natural disaster mitigation. To maximize the impact of the developed analyses, it is essential to promote interdisciplinary collaborations between weather radar experts, telecommunications engineers, and climatologists. The integration of expertise in different disciplines allows me to address technical and scientific challenges more effectively, promoting progress in the understanding of atmospheric phenomena and in operational applications.

By using advanced methods like SRT to look at CloudSat-GPM coincidence datasets, the work that was done has shown that this is a necessary step toward making global attenuation maps and better estimates of rainfall. Despite the still present challenges, the proposed innovations provide a roadmap for future developments in satellite meteorology and atmospheric science. Thanks to a collaborative and multidisciplinary approach, it will be possible to build increasingly precise models and operational tools capable of addressing the challenges of climate change and extreme weather events.

AUTHOR BIOGRAPHY



Paolo Cicolin was born in 1971 in Turin. In October 2012 and December 2018, respectively, he graduated with a Bachelor degree in Electronic Engineering and a Master of Science in Communication Engineering from Roma Tre University in Italy. From October 2020 to October 2021, he enrolled at Roma Tre University's second-level master's program in "Cybersecurity for the Protection of Control Systems in Industry 4.0 and Critical Infrastructures". The Accenture S.p.A. scholarship was awarded to him for his studies in the cybersecurity course. La Sapienza University of Rome welcomed him as a Ph.D. student in November 2021. He was employed from April 2019 to July 2020 as a GIS consultant and in E-Geos S.p.A. (an ASI-Telespazio company), a major global participant in the geospatial industry. His areas of interest and research are geoscience, telecommunications, radar and remote sensing, microwave communications, antennas & propagation, telesensing, and severe event model prediction.

CONFLICT OF INTEREST

The author declares that he has no conflicts of interest.

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A.7 Fifth Technical Article - personal style

Title: Analysis of CloudSat-GPM Satellite Coincidence Datasets for Development of Global Atmospheric Attenuation Maps in Ku, Ka and W Bands.

Analysis of CloudSat-GPM Satellite Coincidence Datasets for the Development of Global Atmospheric Attenuation Maps in Ku, Ka and W Bands

Paolo Cicolin, IEEE Student Member.

Abstract—This study, as the fifth part of a comprehensive research effort, looks at how CloudSat-GPM coincidence data can be used to make global attenuation maps in the Ku, Ka, and W bands and figure out Path Integrated Attenuation (PIA). We suggest ways to change the measured radar reflectivity (Z_m) to the corrected reflectivity (Z_e) and figure out the attenuation caused by the atmosphere using real data and physical models. Adding important variables like cloud water content (CWC) and brightness temperature (TB) makes the estimates more accurate, and reverse engineering techniques make it possible to get numbers from public images. The generated maps represent a significant step towards global understanding of attenuation processes, offering potential applications in weather forecasting and water resources management. Looking into different ways to change radar reflectivity to corrected reflectivity shows a complex method that combines physical models and real-world data in a way that makes estimates of atmospheric attenuation more accurate. By incorporating variables such as cloud water content and brightness temperature, these methods refine the precision of radar measurements. Additionally, the application of reverse engineering techniques allows for the transformation of complex data into accessible public image numbers, fostering greater understanding among various stakeholders. The resulting maps not only illuminate global patterns in attenuation processes but also pave the way for significant advancements in weather forecasting and water resource management. In the end, these innovations show how important it is to combine scientific research with real-world applications. This way, we can make sure that our knowledge of how the atmosphere works can be used to solve problems in the real world.

Index Terms – Path Integrated Attenuation (PIA), CloudSat-GPM Coincidence, Brightness Temperature (BT), Radar Reflectivity, Atmospheric Models, Global Attenuation Map, Remote Sensing,

I. INTRODUCTION

Global mapping of atmospheric attenuation is a very difficult scientific task, especially in places with a wide range of weather conditions. Satellite coincidence datasets, like those between CloudSat and GPM, are a great way to learn more about how attenuation works in certain bands, like Ku, Ka, and W. We look at radar reflectivity and brightness temperature data to figure out the Path Integrated Attenuation (PIA). This helps us understand how things happen in the atmosphere better. The main objective is to develop global attenuation

maps that can improve climate models and weather forecasts, contributing to the mitigation of climate change-related risks.

II. HOW TO GO FROM Z_m TO Z_e .

There are some fundamental steps for a **Data Scientist** to evaluate the effective radar reflectivity starting from the measured data in the CloudSat-GPM coincidences. First, you need to start with the measured data from the 2B-GEOPROF product, where the radar reflectivity is in a non-linear scale expressed in [dBZ]. Next, a quick method to obtain Z_e is to use the notion of PIA (Path Integrated Attenuation) using atmospheric data or models, with PIA expressed in [dB]. The following steps are:

1. Convert Z_m to linear reflectivity using the relation:

$$Z_m(\text{linear}) = 10^{Z_m(\text{dBZ})/10}$$

2. Correct Z_m using PIA:

$$Z_e = Z_m \times 10^{-\frac{PIA}{10}}$$

3. Convert Z_e to [dBZ] using the relation:

$$Z_e[\text{dBZ}] = 10 \times \log_{10}(Z_e(\text{linear}))$$

Once you have Z_e , you can quantify precipitation and convert Z_e to rain rate (R) using the known Z-R relationship. This improves atmospheric models by integrating Z_e into weather prediction models. Subsequently, Z_e helps study cloud properties by analyzing their microphysical properties. Finally, knowing Z_e allows for the study of climatic trends (often by examining historical series of this quantity). The knowledge of Z_e can also validate weather prediction algorithms using other satellite data obtained with different investigation tools. LiDAR (Light Detection and Ranging) is another valuable tool for conducting atmospheric surveys, complementing radar surveys from both the ground and the sky. LiDAR provides detailed vertical profiles of clouds using coherent light, such as laser light, similar to how radar operates. Using LiDAR allows for a better determination of aerosol properties in the atmosphere and measures the concentration of aerosols and suspended particles. This tool can analyze the distribution of particle sizes and detect tracer gases like ozone, water vapor, and carbon dioxide. By integrating LiDAR data with radar data, we can obtain a more complete picture of the atmospheric structure. While we have focused on the satellite coincidences

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of the CloudSat-GPM and CloudSat-TRMM missions, other missions like NASA's CALIPSO also combine radar and LiDAR data to improve our understanding of clouds and precipitation. However, it's important to note that LiDAR can be attenuated by dense clouds and heavy precipitation, and it has a more limited range compared to radar. The range of LiDAR varies depending on the application, generally spanning from 10 meters to several kilometers. LiDAR sensors use wavelengths ranging from near-infrared (about 10 nm) to ultraviolet (up to 250 nm). To obtain the effective radar reflectivity (Z_e), we must correct the measured radar reflectivity (Z_m) for the contributions of clouds, water vapor, and oxygen. This correction accounts for the attenuation caused by these factors. Additionally, Z_e considers the presence of precipitation, providing an accurate measure of the actual atmospheric conditions. Z_e includes corrections for all atmospheric attenuations, not just precipitation. To evaluate the **Path Integrated Attenuation (PIA)** based on the vertical profile of the clouds, we need to apply the integral formula:

$$PIA = \int_0^H \alpha(s) \cdot ds \quad (1)$$

where H is the height of the cloud and where s is the vertical coordinate. Here below we have a simple example, just to fix the ideas, written in MATLAB code:

Algorithm 1 Example of MATLAB code for Calculating PIA with Z_{m_linear}

```
% Example Data:
% Radar Reflectivity Profile Z_m in dBZ
% Radar reflectivity in dBZ
Z_m_dBZ = [30, 28, 25, 22, 20, 18, 15];
% Conversion of Z_m from dBZ to linear Z
Z_m_linear = 10.^(Z_m_dBZ / 10);
% Determination of the specific attenuation
% coefficient (alpha) in dB/km
% (example based on atmospheric models)
% Attenuation coefficient in dB/km
alpha = 0.1 + 0.01 * Z_m_linear;
% Calculation of PIA using
% numerical integration (trapz)
% Height in km (example)
heights = linspace(0, 10, numel(Z_m_linear));
PIA = trapz(heights, alpha);
% Display the result
disp(['Path Integrated Attenuation (PIA): ',
num2str(PIA), ' dB']);
```

However, it is necessary to know the value of the specific attenuation coefficient which is indicated in the formula as $\alpha(z)$ which is in [dB/km]. Obviously here the model is “hypothetical” while in reality it is necessary to use **real atmospheric models**.

III. ATMOSPHERIC MODELS FOR PIA CORRECTIONS.

A. Introduction

The use of atmospheric models for the correction of Path Integrated Attenuation (PIA) is crucial for improving the accuracy of radar measurements. PIA represents the total attenuation of the radar signal along its path through the atmosphere, influenced by factors such as precipitation, temperature, humidity, and chemical composition of the air. Correcting

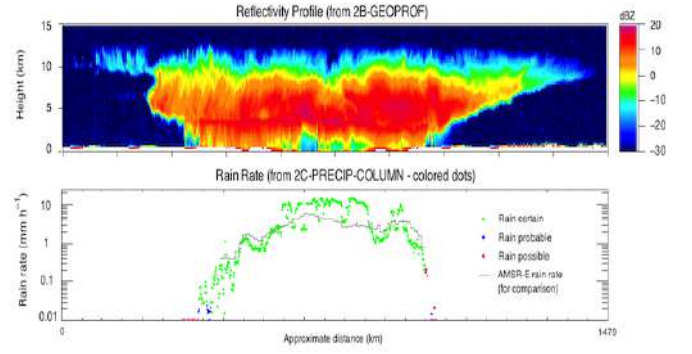


Fig. 1: Reflectivity Profile and Rain Rate. An example plot comparing reflectivity from 2B-GEOPROF with rain rate from 2C-PRECIP-COLUMN. Source: <https://www.cloudsat.cira.colostate.edu/data-products/2c-precip-column>

PIA allows for more reliable and accurate data, essential for applications like weather forecasting, climate research, and water resource management.

B. Critical Analysis

The atmospheric models used for PIA correction rely on observational data and numerical simulations to estimate the radar signal attenuation due to various atmospheric parameters. Some of the most common models include:

- 1) **(ITU-R P.676-11 Model)**. This model provides specific attenuation coefficients for different atmospheric gases, such as water vapor, carbon dioxide, and ozone. It is widely used for PIA correction in meteorological radars [8].
- 2) **6S Model (Second Simulation of the Satellite Signal in the Solar Spectrum)**. One of the most advanced models for atmospheric correction, including parameters like solar radiance, surface albedo, and the vertical distribution of temperature and humidity. It is particularly useful for satellite image correction [2].
- 3) **COST Model (Clouds Over the Sea of Tranquility)**. Used for satellite image correction, this model includes parameters such as cloud cover, surface albedo, and the vertical distribution of temperature and humidity. It is especially useful for satellite image correction [3].
- 4) **ATCOR Model (Atmospheric and Topographic Correction)**. Used for satellite image correction, this model includes parameters such as cloud cover, surface albedo, and the vertical distribution of temperature and humidity. It is particularly useful for satellite image correction [4].

C. Critique

Although these models are powerful tools for PIA correction, they also have some limitations. For example, the accuracy of the models depends on the quality and availability of observational data, which can vary depending on the region and the time period considered. Additionally, the models can be influenced by factors such as climate variations and human

activities, which can alter the atmospheric composition and the distribution of precipitation.

D. Conclusion

In conclusion, the use of atmospheric models for PIA correction is essential for improving the accuracy of radar measurements, but it is important to consider the limitations and uncertainties associated with these models. The combination of observational data and numerical simulations can help mitigate these limitations and provide a more accurate estimate of radar signal attenuation.

IV. SEARCHING FOR PIA TO PLOT FROM COINCIDENCES

The list of groups and subgroups in a CloudSat-GPM coincidence file, when viewed through the lens of Path Integrated Attenuation (PIA), fundamentally alters the investigation perspective and its significance. Here's what you need to know:

- 1) **1C.GPM.GMI:** The Global Precipitation Measurement (GPM) Microwave Imager (GMI) measures the radar reflectivity of precipitation, providing data useful for correcting PIA due to precipitation.
- 2) **2A.GPM.DPR:** The Dual-frequency Precipitation Radar (DPR) of GPM measures the radar reflectivity of precipitation at two frequencies, improving the precision in estimating PIA.
- 3) **2A.GPM.GPROF:** The Goddard Profiling Algorithm (GPROF) uses GPM radar data to estimate precipitation amounts and correct PIA.
- 4) **2B-CLDCLASS:** This product classifies clouds using radar data, contributing to the correction of PIA due to clouds.
- 5) **2B-CWC-RO:** The Cloud Water Content (CWC) Radar Observations product uses radar data to measure the water content in clouds, essential for PIA correction.
- 6) **2B-CWC-RVOD:** The Cloud Water Content (CWC) Radar Vertical Observations product uses vertical radar data to measure water content in clouds, contributing to PIA correction.
- 7) **2B-GEOPROF:** The Geophysical Profiler uses radar data to create geophysical profiles, aiding in PIA correction.
- 8) **2B-GEOPROF-LIDAR:** This product combines radar and LIDAR data to improve PIA estimation.
- 9) **2B.GPM.DPRI:** The Dual-frequency Precipitation Radar (DPR) of GPM measures radar reflectivity of precipitation at two frequencies, enhancing PIA estimation precision.
- 10) **2C-ICE:** This product measures the radar reflectivity of ice, contributing to PIA correction due to ice.
- 11) **2C-PRECIP-COLUMN:** This product measures the precipitation column using radar data, aiding in PIA correction.
- 12) **2C-RAIN-PROFILE:** This product creates rain profiles using radar data, improving PIA estimation.
- 13) **MODIS-AUX:** The Moderate Resolution Imaging Spectroradiometer (MODIS) Auxiliary data uses satellite observation data to enhance PIA estimation.

A. Exploit the variables contained in the listed products to improve the PIA estimate

A Data Scientist can exploit the variables contained in remote sensing products to significantly improve the PIA estimate using an integrated approach that combines radar, LIDAR, and satellite data with advanced analysis and modeling techniques.

V. ANALYSIS OF VARIABLES.

- **1C.GPM.GMI:** Utilize brightness temperatures (TB) to identify precipitation areas and improve the estimation of attenuation from hydrometeors.
- **2A.GPM.DPR:** Integrate dual-frequency radar profiles to differentiate between light and heavy rain, improving PIA estimation by correcting reflectivity at different frequencies.
- **2A.GPM.GPROF:** Apply the Goddard Profiling Algorithm to obtain vertical precipitation profiles, helping to correct vertical attenuation.
- **2B-CLDCLASS:** Classify clouds to identify their properties (e.g., ice clouds vs. liquid water clouds), essential for estimating cloud-induced attenuation.
- **2B-CWC-RO:** Utilize cloud water content to correct attenuation due to water vapor and the clouds themselves.
- **2B-CWC-RVOD:** Integrate vertical radar profiles to obtain a more precise estimate of attenuation along the entire atmospheric column.
- **2B-GEOPROF:** Create geophysical profiles that help identify the vertical composition of the atmosphere and improve PIA estimation.
- **2B-GEOPROF-LIDAR:** Combine radar and LIDAR data for a more accurate PIA estimation, using LIDAR's sensitivity to detect smaller and less dense particles.
- **2C-ICE:** Integrate radar measurements of ice to correct attenuation due to the presence of ice in clouds.
- **2C-PRECIP-COLUMN:** Use precipitation column data to estimate overall attenuation along the radar signal path.
- **2C-RAIN-PROFILE:** Create detailed rain profiles to improve attenuation estimation in the presence of intense rain.
- **MODIS-AUX:** Integrate MODIS satellite data to enhance PIA estimation using detailed information on surface temperature and atmospheric composition.

VI. HAVING A BRAINSTORM TO ACQUIRE COMPETENCES IN WEATHER DATA ANALYSIS

In this section, we aim to provide data scientists with insightful and challenging information that may not be readily available. We will examine each point and formulate a detailed analysis, followed by the implementation of Path Integrated Attenuation (PIA) correction in MATLAB.

- 1). **1C.GPM.GMI:** Utilize Brightness Temperatures (BT) to identify precipitation areas and improve the estimation of attenuation from hydrometeors.

Analysis: Brightness Temperatures (BT) help identify regions with precipitation, which allows for estimating the attenuation due to hydrometeors.

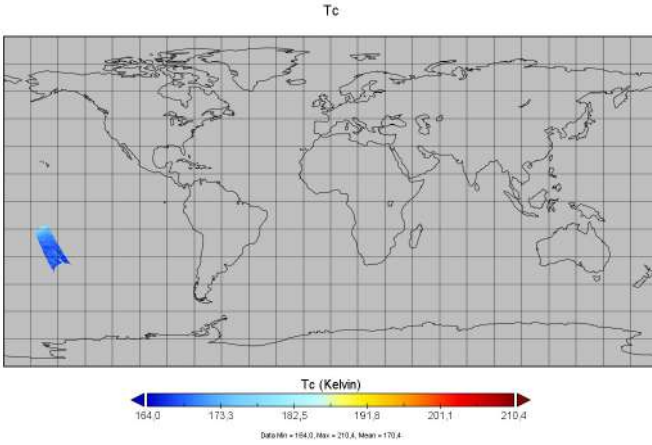


Fig. 2: The Brightness Temperature for 2B.CSATGPM.COIN related to the file 26S_154W_00000_000_999_656-20160102-S114220-E114220-E114606.010483.V04.nc4 for the Slice: $\text{Ntb_GMI}[1\text{of}13] = 1$, where we have 195 rows and 221 columns where BTs are reported as float .1f Data Formation Panoply.

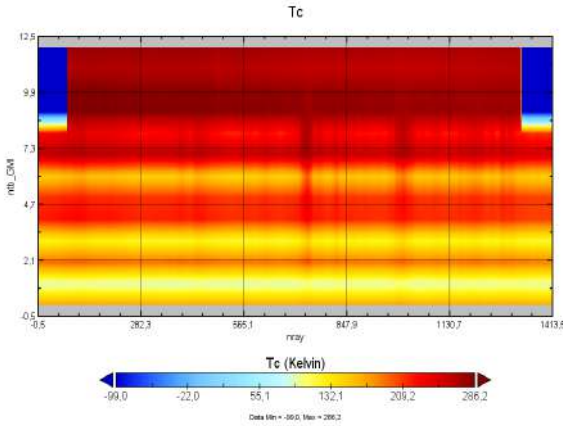


Fig. 3: The Brightness Temperature in [K] for the same coincidence file of Fig.2, where we have 1414 columns x 13 rows, nray on X-axis vs ntb_GMI on Y-axis.

Algorithm 2 Fragment of MATLAB code for BT.

```
% Example TB data (in Kelvin)
TB = [200, 220, 250, 280, 300];
% Brightness temperature in K
% Convert TB to precipitation
% intensity (simplified model)
precip_intensity = (TB - 200) / 100;
% Estimate attenuation due
% to hydrometeors
% (simplified formula)
% in dB/km
attenuation_hydrometeors = 0.05 * precip_intensity;
% Display result
disp('Attenuation due to hydrometeors:');
disp(attenuation_hydrometeors)
```

This is only an idea about Data treatment. The Brightness temperature seeking in CloudSat-GPM coincidence dataset is another subproblem to solve. We will treat it in advance. In any case inside the list of variables in the SWATH subgroup we have T_c GMI full swath Brightness Temperature in Kelvin.

MATLAB Implementation: Channel order is: 10V, 10H, 18V, 18H, 23V, 36V, 36H, 89V, 89H, 166V, 166H, 183 ± 3 , 183 ± 7 . In more complex real-world cases, brightness temperature can be converted into precipitation intensity using scattering methods. These methods estimate precipitation intensity through a parameter called the scattering index, which measures the difference in brightness temperatures in certain microwave channels. The CloudSat sensor that measures brightness temperatures is the **Advanced Microwave Scanning Radiometer for EOS (AMSR-E)** [5]. This sensor uses brightness temperatures at 89 GHz to estimate light and warm rain statistics in shallow marine clouds.

Here is a technical explanation:

- **Definition:** Brightness temperature is a measure of the electromagnetic radiation emitted by an object, in this case, clouds and precipitation. It is expressed in Kelvin (K).
- **GMI Sensor:** The GMI is a microwave radiometer that measures radiation emitted by the Earth at different frequencies. These measurements are used to estimate various atmospheric parameters, including precipitation intensity.
- **Frequency Channels:** The channels listed (10V, 10H, 18V, 18H, etc.) represent different frequencies and polarizations (vertical and horizontal) used by the GMI to measure the brightness temperature. Each channel provides specific information about the properties of clouds and precipitation.
- **Usage:** The brightness temperature measured by the GMI is used to estimate the amount and type of precipitation, as well as other atmospheric characteristics. For example, differences in brightness temperatures between channels can indicate the presence of rain, snow, or hail.
- **Implementation:** In the Fig. 29, the variable T_c represents an array of brightness temperatures measured along a specific path of the CloudSat radar.
- These measurements are used to analyze and model atmospheric phenomena. The `ntb_GMI` parameter refers to the brightness temperature channels measured by the GMI (GPM Microwave Imager) sensor.
- Values ranging from 0 to 12 represent different frequencies and polarizations used by the sensor to collect brightness temperature data.
- Here is a brief description of the channels:
 - 0-1: 10V, 10H
 - 2-3: 18V, 18H
 - 4: 23V
 - 5-6: 36V, 36H
 - 7-8: 89V, 89H
 - 9-10: 166V, 166H
 - 11-12: 183 ± 3 , 183 ± 7

These channels are used to measure various cloud and precipitation properties, such as rainfall intensity and cloud structure. Changes in ntb_GMI values can provide detailed information about atmospheric conditions along the radar path. The GMI channels are designed to capture a wide range of precipitation types and atmospheric conditions. The data collected by GMI is used to improve weather forecasting, climate modeling, and hydrological studies by providing accurate and high-resolution measurements of precipitation and atmospheric moisture. In this article we will not provide the reader with excessive detailed information, as this would require further investigation.

Here below we give only a detailed breakdown about GMI channels:

- 1) 10 GHz (10V and 10H):
 - a) Application: Sensitive to heavy and moderate rainfall.
 - b) These channels help in detecting and quantifying intense precipitation events.
- 2) 18 GHz (18V and 18H):
 - a) Application: Also used for detecting moderate to heavy rainfall.
 - b) These channels provide additional information on precipitation intensity.
- 3) 23 GHz (23V):
 - a) Application: Primarily used for measuring water vapor content in the atmosphere.
 - b) This channel helps in understanding the moisture distribution.
- 4) 36 GHz (36V and 36H):
 - a) Application: Sensitive to both rain and snow.
 - b) These channels are useful for detecting mixed-phase precipitation, such as melting snow.
- 5) 89 GHz (89V and 89H):
 - a) Application: High sensitivity to light rain and snowfall.
 - b) These channels are crucial for detecting light precipitation and snow cover.
- 6) 166 GHz (166V and 166H):
 - a) Application: Sensitive to water vapor and snowfall.
 - b) These channels help in identifying snowfall and estimating water vapor content.
- 7) 183 GHz (183±3 and 183±7):
 - a) Application: Primarily used for measuring water vapor in the upper atmosphere.
 - b) These channels provide detailed information on atmospheric moisture profiles.

The vertical streaks that appear in the plot of brightness temperature versus nray (on the x-axis) and ntb_GMI (on the y-axis) can be interpreted in different ways, depending on the context and the specifications of the GMI sensor.

Here are some possible interpretations:

- **Precipitation Variations:** The vertical streaks could indicate variations in precipitation intensity along the radar path.

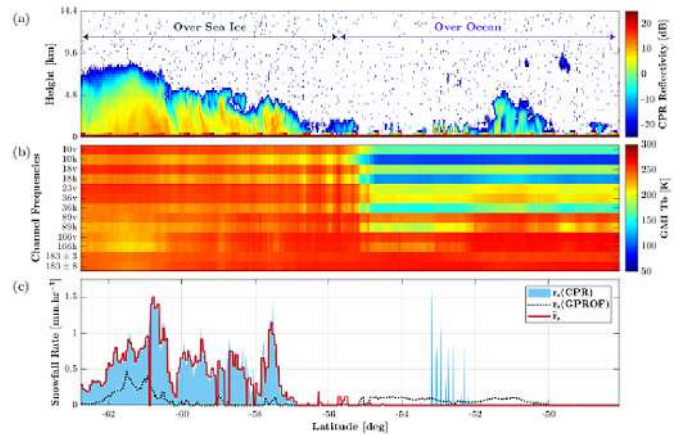


Fig. 4: Example of a Study Case from J.Turk article (see: <https://www.mdpi.com/2072-4292/13/12/2264>).

- For example, a sudden increase in brightness temperature could indicate an area of heavy rain or snow.
- **Cloud Structures:** The streaks could represent different cloud structures, such as cloud layers with different reflectivity properties or water content.
 - These variations can be detected by different GMI channels.
- **Instrumental Effects:** In some cases, vertical streaks could be due to instrumental effects or artifacts in the data collected by the sensor.
 - This can happen due to calibration errors or interference.
- **Weather Phenomena:** Streaks may also be caused by specific weather phenomena, such as weather fronts or changes in cloud composition.
 - For example, a cold front may cause a sudden change in the detected **brightness temperature (BT)**.

For a deeper understanding, it may be helpful to consult the technical documentation of the GMI sensor or dataset used. An example of this type of study that would take us far from the considerations of attenuation and global mapping that we set out to describe is present in an article by J.F.Turk [6]. This graph shows (see Fig.4) the snowfall rate in millimeters per hour versus latitude. The lines represent snowfall retrieval data from CPR (Cloud Profiling Radar) and the official GPM product. Looking at the graph, one can interpret how the variations in snowfall rate along different latitudes indicate the spatial distribution of snowfall. The differences between the lines can highlight the accuracy and reliability of different methods of snowfall retrieval.

- 2). **2A.GPM.DPR:** Integrate dual-frequency radar profiles to differentiate between light and heavy rain, improving PIA estimation by correcting reflectivity at different frequencies.

Analysis: Dual-frequency radar helps in distinguishing rain intensity, improving PIA correction.

MATLAB Implementation:

Algorithm 3 Fragment of MATLAB code for Attenuation Correction.

```
% Example dual-frequency
% reflectivity data (in dBZ)
% Ku-band reflectivity
Z_Ku = [20, 25, 30, 35, 40];
% Ka-band reflectivity
Z_Ka = [18, 23, 28, 33, 38];
% Differential reflectivity
dBZ_diff = Z_Ku - Z_Ka;
% Estimate rain intensity
% and attenuation correction
rain_intensity = dBZ_diff / 10;
attenuation_dual_freq =
0.03 * rain_intensity; % in dB/km
% Display result
disp('Attenuation correction
for dual-frequency radar:');
disp(attenuation_dual_freq);
```

This source code is just a simple starting point, just to fix the ideas. There are then more advanced, detailed implementations to estimate the rain rate and the attenuation correction using the notion of a specific differential phase (Kdp) between the Ku and the Ka band.

- 3). **2A.GPM.GPROF**: Apply the Goddard Profiling Algorithm to obtain vertical precipitation profiles, helping to correct vertical attenuation.

Analysis: The GPROF algorithm uses vertical profiles to refine attenuation estimates.

MATLAB Implementation:

Algorithm 4 Fragment of MATLAB code for Vertical Precipitation Profile.

```
% Example vertical precipitation
% profile (in mm/h)
% Rain rate at different heights
R_profile = [1, 2, 3, 4, 5];
% Convert rain rate to attenuation
(simplified model)
% Attenuation in [dB/km]
attenuation_profile = 0.1 * R_profile;
% Integrate to get
% vertical attenuation
PIA_vertical = trapz(attenuation_profile);
% Display result
disp('Vertical Path Integrated Attenuation (PIA):');
disp(PIA_vertical);
```

To create more complex and detailed examples, it's essential to calculate different attenuation coefficients for various atmospheric components. This approach enables a comprehensive PIA calculation by distinguishing attenuation from rain, clouds, and water vapor, focusing on these three fundamental aspects.

- 4). **2B-CLDCLASS**: Classify clouds to identify their properties (e.g., ice clouds vs. liquid water clouds), essential for estimating cloud-induced attenuation.

Analysis: Cloud classification helps in determining the type of attenuation (ice vs. liquid water).

MATLAB Implementation:

Algorithm 5 Fragment of MATLAB code for Data Cloud Classification.

```
% Example cloud classification data
cloud_type = {'Ice', 'Liquid'};
% Attenuation factors for
% different cloud types (simplified)
attenuation_ice = 0.02;
attenuation_liquid = 0.05;
% Calculate attenuation
% for each cloud type
attenuation_clouds = zeros(size(cloud_type));
for i = 1:length(cloud_type)
if strcmp(cloud_type{i}, 'Ice')
attenuation_clouds(i) = attenuation_ice;
else
attenuation_clouds(i) =
attenuation_liquid;
end
end
% Display result
disp('Attenuation due to cloud type:');
disp(attenuation_clouds);
```

The classification of hydrometeors is much more complex than this example and we provide a clue in the next section of the study.

- 5). **2B-CWC-RO**: Utilize cloud water content to correct attenuation due to water vapor and the clouds themselves.
 - **Analysis:** Cloud water content data allows for correcting attenuation due to water vapor and clouds.

MATLAB Implementation:

Algorithm 6 Fragment of MATLAB code for Data Cloud Water Content.

```
% Example cloud water
% content data (in g/m^3)
CWC = [0.1, 0.2, 0.3, 0.4, 0.5];
% Convert CWC to
% attenuation (simplified model)
attenuation_CWC = 0.01 * CWC; % in dB/km
% Display result
disp('Attenuation due to cloud water content:');
disp(attenuation_CWC);
```

To convert the Liquid Water Content (LWC) into attenuation here we used a simplified model, just to fix the ideas. For more concrete real examples it is necessary to perform some evaluations regarding how the CWC varies with the altimetric height. In the different bands considered (Ku, Ka and W) each band responds differently to the presence of water in the clouds and using the numerical integration we could calculate the PIA for each band. Then the study of the attenuation in the Ku, Ka and W bands and the search for PIA(Ku), PIA(Ka) and PIA(W) will become more concrete in the analysis. The 2B-CWC-RVOD product from CloudSat is designed to provide estimates of cloud liquid water content. So, in this case it's opportunistic going to see this group in detail in advance. A detailed Official document about 2B-CWC-RVOD is available on line at this address:¹.

Is important to sottolineate that for the CloudSat-GPM coincidence there may be some data problems. The reason

¹<https://www.cloudsat.cira.colostate.edu/cloudsat-static/info/dl/2b-cwc-rvod/2B-CWC-RVODPDICD.PR04.20081023.pdf>

why Cloud Water Content (CWC) from CloudSat data is not always available can be due to several factors:

- 1) **Geolocation Errors:** There were periods when CloudSat experienced geolocation errors, which affected the quality of the radar data.
- 2) **Mode of Operation:** CloudSat operated in “Daylight-Only Operations” (DO-Op) mode due to anomalies in the gyroscopes, reducing the amount of data collected.
- 3) **Battery Anomalies:** Battery anomalies caused interruptions in data collection.
- 4) **A-Train Formation:** CloudSat left the A-Train formation, reducing the availability of data combined with other satellites such as CALIPSO.
- 5) **Variable Pulse Rate:** The frequency of the radar’s pulsars was periodically adjusted to manage power consumption, affecting the radar’s sensitivity.

CloudSat was decommissioned in March 2024 after almost 18 years of operation. It has not been directly replaced by another satellite with the same radar capabilities, but there are other satellites that continue to monitor clouds and the atmosphere, such as CALIPSO (CloudAerosol Lidar and Infrared Pathfinder Satellite Observation). However, in terms of studying cloud properties and precipitation, **PACE (Plankton, Aerosol, Cloud, ocean Ecosystem)** is a new satellite that studies in detail the biology of the oceans, aerosols and clouds. Although it may seem useless, due to the existence of satellite data reanalysis, we believe this article can still be useful for future investigations in Remote Sensing.

- 6) **2B-CWC-RVOD:** Integrate vertical radar profiles to obtain a more precise estimate of attenuation along the entire atmospheric column.

Analysis: Vertical radar profiles provide a detailed view of attenuation throughout the atmospheric column.

MATLAB Implementation:

Algorithm 7 Fragment of MATLAB code for Data Radar Vertical Profile.

```
% Example vertical radar
% profile data (in dB/km)
attenuation_profile = [0.1, 0.2, 0.3, 0.4, 0.5];
% Integrate to get total
% attenuation along the
% atmospheric column
total_attenuation = trapz(attenuation_profile);
% Display result
disp('Total attenuation along atmospheric column:');
disp(total_attenuation);
```

Since it may often happen that the CWC data from the coincidences are not available, then some considerations must be made:

- 1) a) We can derive the CWC from other more complex calculation procedures, such as those that exploit the knowledge of the PIA. In this case, it is good practice to separate the attenuation due to clouds from that of rain and water vapor. Specific theoretical models can be used that connect the attenuation of clouds to the CWC.
- 2) b) Other remote sensing satellite missions can be used, in particular NASA’s Aqua mission, which uses **MODIS (Moderate Resolution Imaging Spectroradiometer)** to

measure the CWC globally. MODIS collects data on a large scale, allowing the creation of monthly CWC maps that show the global distribution of cloud water content.

- 7) **2B-GEOPROF:** Create geophysical profiles that help identify the vertical composition of the atmosphere and improve PIA estimation.

Analysis: Geophysical profiles provide information about the atmospheric composition, aiding in PIA correction.

MATLAB Implementation:

Algorithm 8 Fragment of MATLAB code for Geophysical Data Profile.

```
% Example geophysical
% profile data
% (e.g., humidity, temperature)
% Humidity profile is in %
humidity_profile = [50, 55, 60, 65, 70];
% Temperature profile is in [Celsius]
temperature_profile = [20, 18, 16, 14, 12];
% Calculate attenuation correction
% based on humidity
% and temperature (simplified)
attenuation_geo = 0.01 * (humidity_profile - 50)
+ 0.01 * (20 - temperature_profile);
% in dB/km
% Display result
disp('Attenuation correction based on geophysical
profiles:');
disp(attenuation_geo);
```

For a more realistic calculation of attenuation in the Ku, Ka and W bands, we can use specific models for propagation loss in the atmosphere, taking into account factors such as frequency, humidity, temperature, atmospheric pressure and the concentration of gases such as oxygen and water vapor. A common example is the **ITU-R P.676** attenuation model [9]. Please note that for best accuracy, you should implement the full model taking into account all relevant atmospheric factors.

- 8) **2B-GEOPROF-LIDAR:** Combine radar and LIDAR data for a more accurate PIA estimation, using LIDAR’s sensitivity to detect smaller and less dense particles.

Analysis: Combining radar and LIDAR data improves PIA estimation by leveraging LIDAR’s sensitivity.

MATLAB Implementation:

Algorithm 9 Fragment of MATLAB code for Radar and Lidar Data.

```
% Example radar and
% LIDAR data (in dB/km)
attenuation_radar = [0.3, 0.4, 0.5, 0.6, 0.7];
attenuation_lidar = [0.2, 0.3, 0.4, 0.5, 0.6];
% Combine data to
% improve PIA estimation
combined_attenuation = 0.5 * (attenuation_radar +
attenuation_lidar);
% Display result
disp('Combined radar and LIDAR attenuation:');
disp(combined_attenuation);
```

In order to enhance the basic example, it is essential to consider the specific bands of interest (Ku, Ka, and W). To achieve a more accurate representation, we must begin with the geophysical profiles of humidity, temperature, and atmospheric

pressure. Utilizing the ITU-R P.676 recommendations, we can calculate the specific attenuation for each frequency. Subsequently, the data should be combined to refine the estimation of the Path Integrated Attenuation (PIA).

- 9). **2C-ICE:** Integrate radar measurements of ice to correct attenuation due to the presence of ice in clouds.
 - **Analysis:** Radar measurements of ice are used to estimate attenuation due to ice in clouds.

MATLAB Implementation:

Algorithm 10 Fragment of MATLAB code for Ice Data Content.

```
% Example ice content data
% (in g/m^3)
ice_content = [0.01, 0.02, 0.03, 0.04, 0.05];
% Convert ice content to
% attenuation (simplified model)
attenuation_ice = 0.02 * ice_content; % in dB/km
% Display result
disp('Attenuation due to ice content:');
disp(attenuation_ice);
```

For a more complex calculation of the attenuation due to ice content in the Ku, Ka, and W bands, we can take into account more factors and use a more accurate model as suggested by the **ITU-R P.676** recommendations. The best method to derive the **IWC (Ice Water Content)** of clouds using the radar reflectivity profile in dBZ is to combine radar data with lidar data and other in situ measurements. This approach, known as the lidar-radar method, is particularly effective as it allows for improved accuracy in IWC estimates, especially in the presence of ice particles of various sizes and shapes. Here is an overview of the lidar-radar method:

- 1) **Radar:** The radar provides reflectivity measurements (Z) of the clouds, which are related to the density and size of ice particles.
 - 2) **Lidar:** The lidar measures the optical scattering of ice particles, providing information on the size distribution of the particles.
 - 3) **Data Integration:** By combining radar and lidar data, it is possible to obtain a more accurate estimate of the IWC, as the lidar helps resolve uncertainties related to the shape and size of ice particles that the radar might not detect accurately.
- 10). **2C-PRECIP-COLUMN:** Use precipitation column data to estimate overall attenuation along the radar signal path.

Analysis: Precipitation column data provides an overall view of attenuation along the radar signal path. Here's a basic MATLAB implementation to estimate overall attenuation along the radar signal path using precipitation column data.

MATLAB Implementation:

Algorithm 11 Fragment of MATLAB code for Estimate overall Attenuation using Precipitation Data Column.

```
% Sample MATLAB code to estimate
% overall attenuation using
% precipitation column data
% Define constants
frequency = 10;
% Frequency in GHz
k = 0.1;
% Specific attenuation
% coefficient (dB/km per mm/hr)
rain_rate = 5; % Rain rate in mm/hr
% Load precipitation
% column data (example data)
precip_column = [0, 1, 2, 3, 4, 5];
% Example precipitation
% column data in mm/hr
% Calculate specific
% attenuation for each column
specific_attenuation = k * precip_column;
% Integrate specific attenuation
% along the radar signal path
overall_attenuation = trapz(specific_attenuation);
% Display the result
fprintf('Overall attenuation along the radar signal
path:
%.2f dB\n', overall_attenuation);
```

VII. CLASSIFICATION OF HYDROMETEORS

A. Cloud Droplets

- Description: Small water droplets suspended in clouds.
 - Size: 2-50 micrometers.
 - Effect on Radar Reflectivity: Generally have low radar reflectivity

B. Rain Drops

- Description: Larger water droplets falling from clouds.
 - Size: 0.1-6 mm.
 - Effect on Radar Reflectivity: High radar reflectivity, especially in the Ku and Ka bands.

C. Ice Crystals

- Description: Ice particles that can have various shapes such as needles, columns, or plates.
 - Size: 20-2000 micrometers.
 - Effect on Radar Reflectivity: Variable, depending on shape and concentration.

D. Snowflakes

- Description: Aggregates of ice crystals forming snowflakes.
 - Size: 1-5 mm.
 - Effect on Radar Reflectivity: Moderate radar reflectivity, depending on density and size.

E. Hail

- Description: Balls or pieces of ice falling from clouds during thunderstorms.
 - Size: 5-200 mm.
 - Effect on Radar Reflectivity: Very high radar reflectivity, especially in the S and X bands.

F. Graupel

- Description: Soft, opaque ice particles, similar to hail but smaller and softer.
 - Size: 2-5 mm.
 - Effect on Radar Reflectivity: High radar reflectivity, but less than hail.

VIII. SOME DETAILS ABOUT FILE NAMING CONVENTION AND MEANING FOR COINCIDENCE DATASET.

Here is a more detailed explanation of the various components of the file name:

2B.CSATGPM.COIN.26S_154W_00000_000_999_656_.20160102-S114220-E114606.010483-V04.nc4.

For CloudSat-GPM coincidences:

- 2B: This indicates the level of data processing. In this case, it is level 2B data, which refers to derived and geophysical data.
- CSATGPM: This signifies that the data is a coincidence between CloudSat observations and GPM (Global Precipitation Measurement).
- COIN: This is an abbreviation for “coincidence”, indicating that the file contains coincidence data between CloudSat and GPM.
- 26S_154W: These are the geographic coordinates (latitude and longitude) of the center point of the observation area. Specifically, it is located at 26 degrees south and 154 degrees west.
- 00000_000_999_656: These are identifiers specific to the dataset. These numbers can represent various parameters, such as the satellite ID, sensor ID, and other dataset specific information.
- 20160102: This is the date the data was acquired, in the format YYYY:MM:DD. In this case, it represents January 2, 2016.
- S114220-E114606: These are the start and end times of the data acquisition, in the format HH:MM:SS. The acquisition started at 11 : 42 : 20 and ended at 11 : 46 : 06.
- 010483: This is a sequence number or unique identifier for the file.
- V04: This indicates the version of the dataset. In this case, it is version 4.
- nc4: This is the file extension, indicating that it is a NetCDF (Network Common Data Form) version 4 file.

These components of the file name provide detailed information about the content and characteristics of the dataset, making it easier to identify and use the data for scientific analysis. However, it may happen that due to this coincidence the file does not contain data of interest because they have not been detected. In this case, it is then necessary to trace the reason for the anomaly.

IX. ANALYSIS OF PIA’S TYPOLOGY.

Analyzing the CloudSat-GPM coincidence satellite products relative to the PIA we have:

1) 2B-CWC-RVOD:

- a) – PIA Fwd
- 2) **2C-PRECIP-COLUMN:**
 - a) – PIA hydrometeor
 - b) – PIA uncertainty
- 3) **2A.GPM.DPR/MS/:**
 - a) – piaFinal
- 4) **2A.GPM.DPR/NS/:**
 - a) – piaFinal
- 5) **2B.GPM.DPRGMI/MS/:**
 - a) – pia
- 6) **2B.GPM.DPRGMI/NS/:**
 - a) – pia ku

Here are the variables starting with PIA and their references:

- **PIA_Fwd:** Detailed information on this term in the context of meteorological radars and precipitation measurements can be found in the paper by **Frederic Tridon et al.** (2020) [10].
- **PIA_hydrometeor:** This term describes the properties of atmospheric precipitation, such as water droplets or ice crystals. Further details can be found in the document by **Terry Schuur (2015)** [9].
- **PIA_uncertainty:** This term refers to the uncertainty associated with the measurements and estimates of atmospheric parameters. More information can be found in the context of (PIA) in the paper by **Guy Delrieu et.al.** (2020) [7].
- **PIAFinal:** This term refers to the Ku-band PIA from DPR MS swath copied from piaFinal in 2A.GPM.DPR.
- **pia:** This term refers to the combined PIA ku and PIA ka from 2B.GPM.DPRGMI.CORRA product.
- **pia_ku:** in the \MS\ this term describes PIA ku from DPRGMI combined NS swath, copied from 2B.GPM.DPRGMI.CORRA product. In the \NS\ this term describes PIA ku from DPR NS swath, copied from piaFinal in 2A.GPM.DPR product.

X. CLOUDSAT-GPM TYPE 2A AND 2B PRODUCTS.

CloudSat-GPM Type 2A and 2B products differ mainly in the type of data they provide and their use:

A. Product 2A

- Description: CloudSat-GPM Product 2A offers raw radar reflectivity data collected by satellites. This data is crucial for analyzing the structure of clouds and precipitation.
- Use: Researchers use it primarily for detailed studies of cloud microphysics and to enhance precipitation detection algorithms.

B. Product 2B

- Description: CloudSat-GPM Product 2B processes the raw data from Product 2A to deliver more detailed and derived information, such as the liquid water and ice content in clouds.
- Use: It serves more advanced applications, including climate modeling and weather forecasting, by providing a more comprehensive view of cloud and precipitation properties.

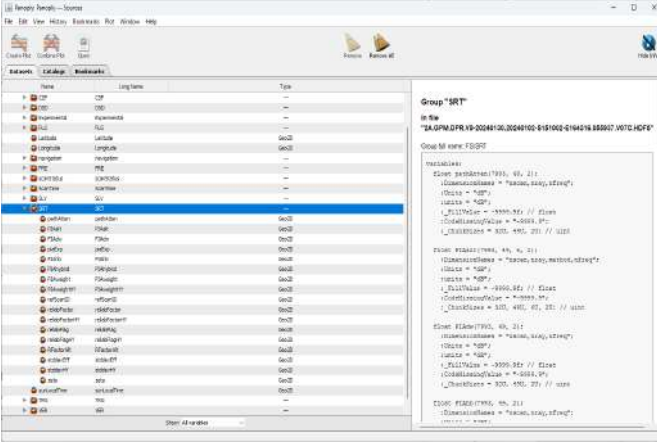


Fig. 5: Example in searching data of 2A.GPM.DPR.V9 for SRT (Surface Reference Technique) and PIAhb (Hitschfeld-Bordan).

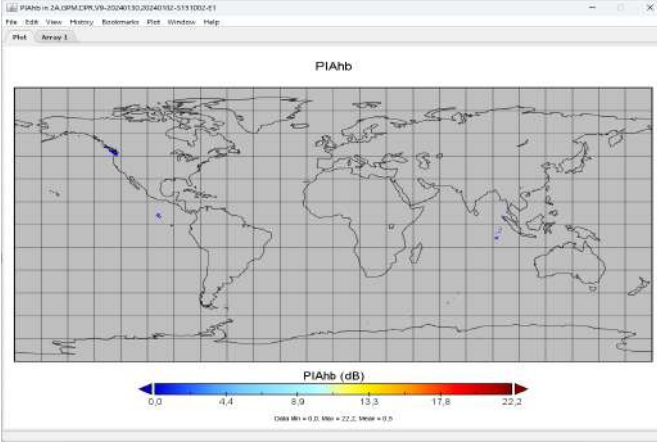


Fig. 6: Graphing PIAhb (due to Hitschfeld-Bordan) of the 2A.GPM.DPR.V9 drawn in Panoply.

XI. WHICH PIA I NEED?

In this section we start from a good knowledge forward the Global Map creation, without corrections (we are referring about absence of the attenuation contribute). To create a global attenuation map, use PIA hydrometeor. This parameter shows the total attenuation caused by hydrometeors along the radar signal path, providing an overall view of the attenuation due to rain, snow, ice, and other hydrometeors.

The other parameters serve specific purposes:

- **PIA_Fwd:** Calculates forward attenuation, useful for specific directional analysis.
- **PIA_uncertainty:** Indicates the uncertainty in the attenuation estimate, useful for evaluating measurement precision.
- **PIA_W = PIA_ku + PIA_ka:** sums the -Ku and -Ka band attenuations, useful for combined analysis but not specific for a global map.
- **PIA_ku:** Shows attenuation specific to the Ku band.
- **PIA_ka:** Shows attenuation specific to the Ka band.

Frequency	Polarization	Band	Sensitive
10.65 GHz	Horizontal	X	to heavy rain
10.65 GHz	Vertical	X	to heavy rain
18.7 GHz	Horizontal	Ku	to moderate rain
18.7 GHz	Vertical	K	to moderate rain
23.8 GHz	Vertical	Ka	to water vapor
36.5 GHz	Horizontal	Ka	to light rain and snow
36.5 GHz	Vertical	W	to light rain and snow
89.0 GHz	Horizontal	W	to snow and ice
89.0 GHz	Vertical	W	to snow and ice
166 GHz	Horizontal	W	to snow and ice
166 GHz	Vertical	W	to snow and ice
183.31 ± 3 GHz	Vertical	W	to water vapor
183.31 ± 7 GHz	Vertical	W	to water vapor

TABLE I: GMI Channels and Their Sensitivities.

Path-Integrated Attenuation (PIA) is a measure of the total attenuation (reduction in signal strength) that a radar signal experiences as it travels through the atmosphere. This attenuation is caused by hydrometeors, which are particles of water or ice suspended in the atmosphere, such as rain, snow, ice, and water vapor. When a radar signal is transmitted from a satellite or ground-based radar, it travels through the atmosphere and interacts with hydrometeors. These interactions cause the signal to scatter and absorb energy, leading to a reduction in the signal's strength. The PIA quantifies this reduction by measuring the difference in signal strength between the transmitted and received signals. The GPM Microwave Imager (GMI) on the GPM satellite has multiple channels, each sensitive to different frequencies and types of hydrometeors. By analyzing the data from these channels, scientists can calculate the PIA for various types of hydrometeors. Here's how it works:

- **Rain:** Channels sensitive to frequencies around 10.65 GHz and 18.7 GHz are used to calculate the PIA for rain. These frequencies are particularly effective at detecting liquid water droplets.
- **Snow and Ice:** Higher frequency channels, such as those around 36.5 GHz and 89.0 GHz, are used to calculate the PIA for snow and ice. These frequencies are more sensitive to solid precipitation particles.
- **Water Vapor:** Channels around 23.8 GHz and 183.31 GHz are used to calculate the PIA for water vapor. These frequencies are sensitive to the presence of water vapor in the atmosphere.

The bands are classified as follows:

- X-Band: 8-12 GHz
- Ku-Band: 12-18 GHz
- K-Band: 18-27 GHz
- Ka-Band: 27-40 GHz
- W-Band: 75-110 GHz

In the data that are stored within satellite coincidences, there are numerous quantities of interest that can be of interest to us:

- a) PIA (Path Integrated Attenuation)
- b) IWP (Ice Water Path)
- c) LWC (Liquid Water Content)
- d) Pia-Ka
- e) Pia-Ku

- f) Pia-W
- g) IWC (Ice Water Content)
- h) LWP (Liquid Water Path)

XII. CORRELATIONS BETWEEN PARAMETERS.

- PIA (Path Integrated Attenuation):
 - Pia-Ka, Pia-Ku, Pia-W: These are specific types of PIA measured at different radar frequencies (Ka-band, Ku-band, W-band). The correlation between PIA and these specific measurements is direct, as they all represent the integrated attenuation along the path at different frequencies.
 - IWP (Ice Water Path) and IWC (Ice Water Content): PIA can be used to estimate the amount of ice present in clouds, as attenuation is influenced by the presence of ice particles.
 - LWC (Liquid Water Content) and LWP (Liquid Water Path): Similarly, PIA can be correlated with the liquid water content in clouds, as liquid water contributes to the attenuation of the radar signal.
- IWP (Ice Water Path):
 - IWC (Ice Water Content): IWP is an integrated measure of the total amount of ice along a vertical column, while IWC represents the concentration of ice at a specific point. These two parameters are closely correlated.
- LWC (Liquid Water Content):
 - LWP (Liquid Water Path): LWC represents the concentration of liquid water at a specific point, while LWP is the integrated measure along a vertical column. These two parameters are also closely correlated.
- Pia-Ka, Pia-Ku, Pia-W:
 - These parameters are correlated with each other as they represent the integrated attenuation along the path at different radar frequencies. The differences between them can provide useful information about the distribution and properties of particles in clouds. Now is natural do a question: Isn't talking about LWC and LWP a repetition? While Liquid Water Content (LWC) and Liquid Water Path (LWP) are related, they are not the same and serve different purposes in meteorology and atmospheric sciences:
 - LWC (Liquid Water Content): This measures the amount of liquid water present in a unit volume of air. It is typically expressed in grams per cubic meter (g/m^3). LWC is used to understand the microphysical properties of clouds and fog.
 - LWP (Liquid Water Path): This measures the total amount of liquid water present in a column of air extending from the surface to the top of the atmosphere. It is typically expressed in kilograms per square meter (kg/m^2).
 - LWP is used to understand the overall water content in the atmosphere and is important for studying cloud formation and precipitation.

XIII. REFERRING TO THE MAIN ARTICLE CONTENT: “GLOBAL-SCALE MAPPING OF ATMOSPHERIC ATTENUATION AT KU, KA AND W BANDS USING MULTI-SATELLITE COINCIDENCE DATASETS”.

For a detailed and in-depth understanding of the following topics:

- The primary and secondary objectives of the project
- General cost considerations for the development of the global atmospheric attenuation map (GMAP)
- A detailed description of the algorithm used to generate the global map across the W-, Ku-, and Ka-bands
- Application of the Path Integrated Attenuation (PIA) forward model
- A step-by-step outline of the algorithm's essential processes
- Guidelines for interpreting metadata

XIV. INITIAL STEPS IN THE ALGORITHM FOR RADAR REFLECTIVITY MAPPING

The initial steps of the algorithm involve plotting the radial reflectivity in the W-band. Negative values are filtered out to eliminate terrain slopes, as the analysis assumes a flat terrain. The code provides the flexibility to select how radar reflectivity is plotted by adjusting the ordinate axis. The altimetric height can be displayed in either kilometers or meters. It is important to note that representing the resolution in kilometers results in a lower resolution. Consequently, misrepresenting kilometers as meters in scientific articles constitutes a significant error.

Key variables of interest include latitude, longitude, height, time, and radar reflectivity. Additionally, the scale factor plays a crucial role in data visualization. Careful attention must be given to both the scale factor and the offset of the data before graphing.

A. Fundamental Aspects of Data Visualization

- **Scale Factor:** Determines how large or small the values appear in the graph.
For instance:
 - If the data ranges from 0 to 100, a corresponding scale should be used to maintain accuracy.
 - If data ranges from 0 to 1, a smaller scale is required to ensure proper representation.
- **Offset:** A value added to or subtracted from the data before plotting to shift it vertically.
For example:
 - If the data starts at 10, but the graph needs to start at 0, subtracting 10 (negative offset) ensures proper alignment.

These two factors are essential for ensuring clear and accurate data representation, thereby improving graph readability and interpretability.

1) **Tip: Ensuring Accurate Radar Reflectivity Visualization:**

When working with radar reflectivity data, always verify the scale factor and offset before plotting. Incorrect scaling can lead to misinterpretations, especially when comparing different altitude resolutions. For example,

presenting kilometers as meters introduces significant errors in resolution, which should be explicitly avoided in scientific articles.

Algorithm 12 Scaling factor management.

The scaling equation is:

$$\text{DATA FILE VALUE} = (\text{SCIENCE VALUE} * \text{FACTOR}) + \text{OFFSET}$$
 - or -

$$\text{SCIENCE VALUE} = (\text{DATA FILE VALUE} - \text{OFFSET}) / \text{FACTOR}$$

2) Remark:

For further useful applications, refer to [1]. In general, radar reflectivity, which is stored as a matrix, is denoted as “T” (short for “Table”) in MATLAB code. I want to demonstrate an example of how one can obscure the understanding of the source code for those reviewing it by introducing a deliberately cryptic syntax. This matrix is obtained using the “dark statement” method, explained in the next tip:

Algorithm 13 Cryptic mode for acquire NetCDF property in MATLAB with some attributes and data obfuscated by some name variables.

```
% From 2B-GEOPROF Radar Reflectivity
obj5 = strcat(",b,c,b,l,");
T = h5read(filename, obj5);
```

3) Tip: A crypting-way to Design the code when working on reserved data:

This value must be converted from single to double precision in MATLABB for accurate plotting and graphing.

Algorithm 14 Converting data acquired in double numerical format in MATLAB.

```
Tdata = double(T);
```

It is recommended to convert all variables to double precision before graphing in MATLAB for improved accuracy.

- **In the second step** of my research, I have developed and implemented a loop that processes the appropriately downloaded and ordered satellite CloudSat-GPM coincidence files. This loop generates real-time radar reflectivity images in the W band. By processing one file at a time, the loop opens each file and generates the corresponding graph.
- In this research I have developed a complex code to recognize even partially corrupted files, as previously demonstrated with concrete examples that are not possible to achieve in Panoply. This capability can be a strong point in developing autonomous source code. The code is part of SNEM-2 framework, entirely developed in MATLAB.

Finally,

- **In the third (critical) step**, I have built, as a product of the research, a Source Code that adheres to the radar reflectivity value constraints published by the CloudSat web portal. This code filters the data to improve graphic rendering. The radar reflectivity is graphed with respect

to both the along-track-distance (ATD) and the latitude. Additionally, there are complex controls to ensure the data is not reversed in the reading sequences, among other checks.

B. Plotting and Storing PIA values Using MATLAB

In the **fourth fundamental step** of this research, I have managed the PIA entity. For this I have realized another recommendation described in the **fragment code** comment, contained in the **Algorithm 15**:

Algorithm 15 Efficient Management of Path Integrated Attenuation (PIA) in MATLAB: suggested URL resource.

```
% PIA (Path Integrated Attenuation) reading
expressed in [dB]
% CloudSat Radar-Visible Optical
% Depth Cloud Water Content Product.
% More INFO @:
https://www.cloudsat.cira.colostate.edu/
data-products/2b-cwc-rvovd
```

C. Selecting the Appropriate PIA for Global Map Generation

The initial approach considered in this work is to take into account the Forward PIA (indicated as **PIA_Fwd**) of the 2B-CWC-RVOD product.

Algorithm 16 Testing Statements for reading Attributes inside a NetCDF File for Coincidence JT (mantained by Joe Turk) Dataset in MATLAB.

```
piaFinMS = h5read(filename,
'/2A.GPM.DPR/MS/piaFinal');
piaFinNS = h5read(filename,
'/2A.GPM.DPR/NS/piaFinal');
piaD_MS = h5read(filename, '/2B.GPM.DPRGMI/MS/pia');
piaKuco_NS = h5read(filename, '/2B.GPM.DPRGMI/NS/pia
ku');
```

In Algorithm 16, the variable `piaKuco` represents the PIA(Ku) combined with **Normal Scan (NS)**. Since multiple PIA values are available in the satellite JT Dataset Coincidence, it is recommended to conduct test by plotting and comparing them using **Thor** and **Panoply** software application.

D. Analysing PIA_hydrometeor in the JT Coincidence CloudSat-GPM Dataset

The next steps for the graphic rendering of the **PIA_hydrometeor** are outlined as follows:

- 1) **Data Collection:** collect the data by scanning the coincidence files you want to analyze. Then, superimpose the individual tracks. To verify this process, use both Thor and Panoply to check the operations in MATLAB.
- 2) **Track Superposition:** Overlay the individual tracks to construct a comprehensive visualization.
- 3) **Verification:** Validate the process using Thor and Panoply, ensuring consistency with MATLAB operations.

These steps were fully implemented in the present research to ensure accurate processing and visualization of the **PIA_hydrometeor** dataset.

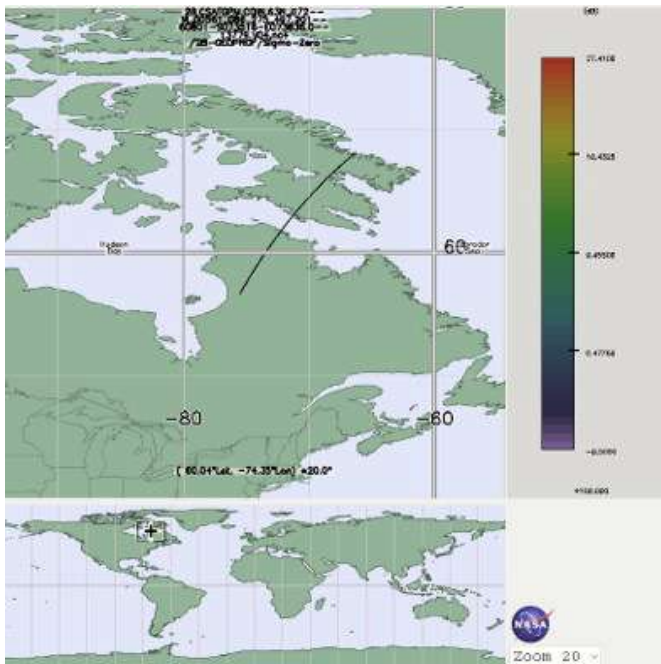


Fig. 9: Graphical Visualization of Sigma-Zero (σ_0) for Coincidence file 2B.CSATGPM.COIN63N-072W-00561-086-275-49710150801-S073516.013779.V04.nc4 in Thor.

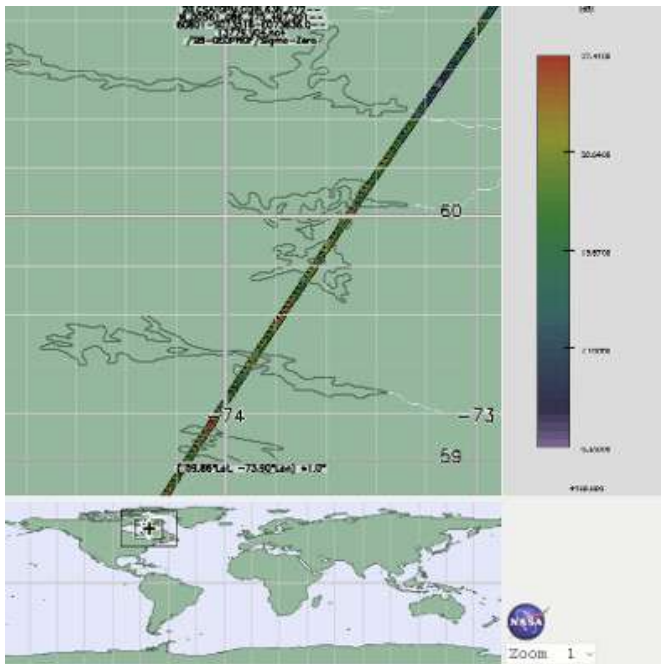


Fig. 10: Adjusting the Zoom factor in Thor for Viewing the Sigma-Zero Swath.

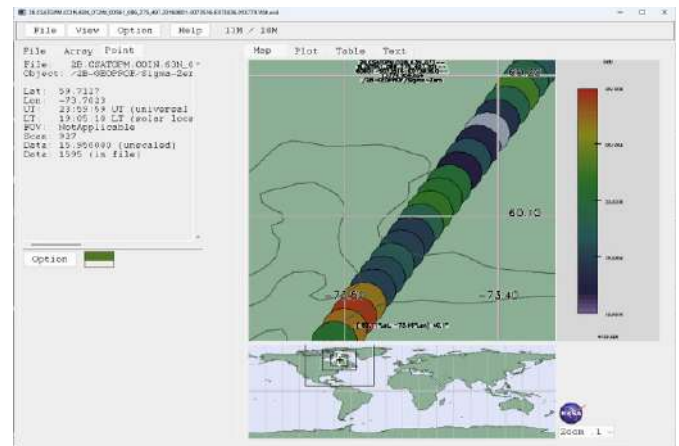


Fig. 11: Zooming on the Sigma-Zero of the product 2BGEOPROF in Thor.

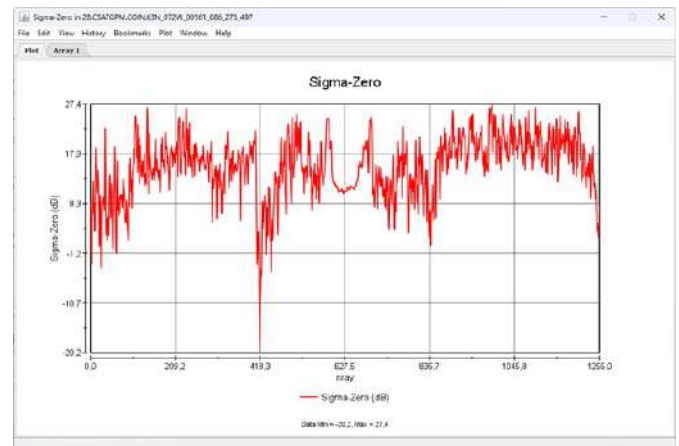


Fig. 12: Same trend of Sigma-Zero drawn in Thor now viewed in Panoply.

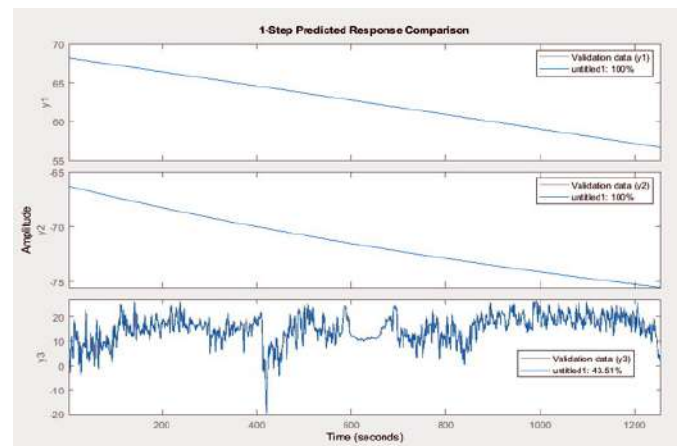


Fig. 13: Sigma-Zero Trend plotted in MATLAB.

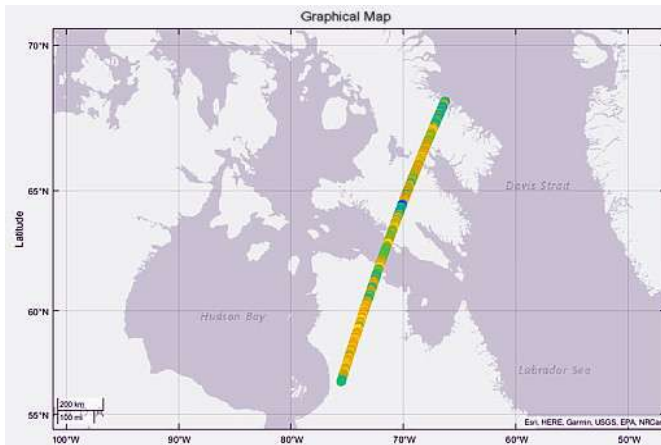


Fig. 14: Sigma-Zero 1D values plotted as 2D with Latitude and Longitude in MATLAB. We use the same data of the Fig. 9. It's shown a passage over Canada.

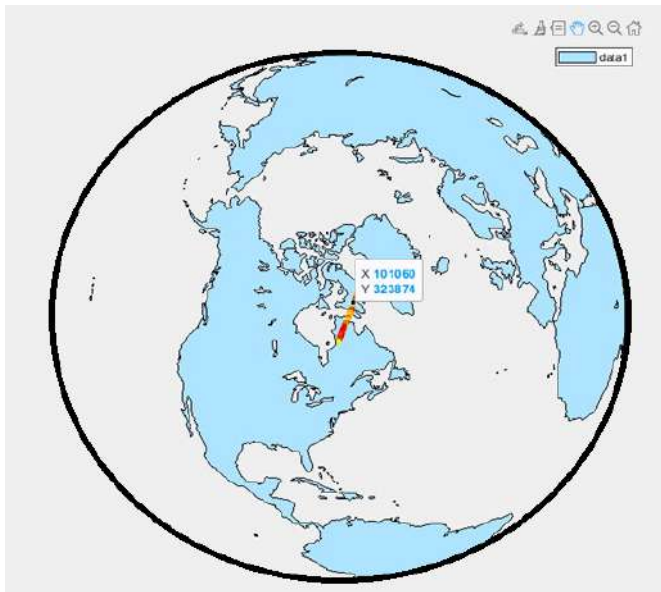


Fig. 15: Sigma-Zero 1D values plotted as 3D using Globe 3D Geometry of the Earth in MATLAB.

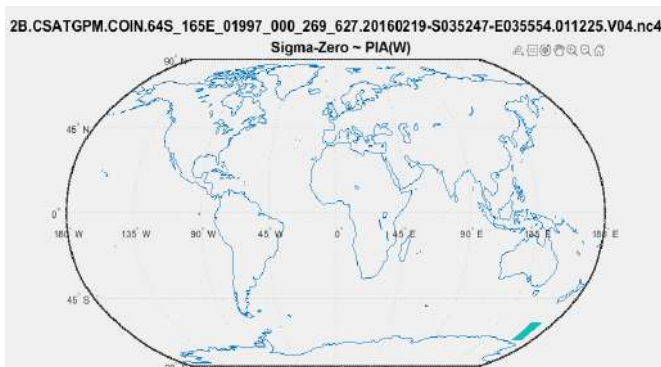


Fig. 16: Activating pixels using the 97th percentile technique: South Pole passage example.

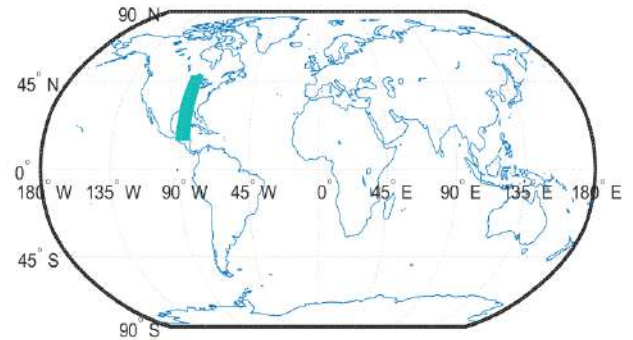


Fig. 17: Activating pixels using 97th percentile technique for (Sigma-Zero): North Pole passage example.

accurately estimating signal loss in satellite communication systems operating at frequencies near 35 GHz. Battaglia's attenuation maps are derived by combining multiple GPM (Global Precipitation Measurement) products to account for the different contributing factors.

Each map employs a distinct color scale tailored to its specific data range, ensuring accurate visual representation. For full access to the maps, readers are referred to the original article by Battaglia et al. [16]. Inspired by this approach, similar climatological products can be developed; however, downloading and processing four years of satellite data via FTP remains a significant logistical challenge.

Additional technical details regarding the use of PIA (Path-Integrated Attenuation) are available on page 1813 of the article "GPM-Derived Climatology of Attenuation due to Clouds and Precipitation at Ka-Band" (Battaglia et al., 2020) [15]. It is important to note that reproducing maps of this nature is far from trivial. When applying the 97th percentile technique—although alternative statistical methods may also be considered—and analyzing CloudSat-GPM coincidence data for the entire month of January, several spatial inconsistencies emerge. These non-punctual features follow the satellite swath pattern on the Earth's surface, complicating the construction of a coherent and uniformly gridded image, such as the one targeted in this work. Recognizing and understanding this intrinsic limitation is, in my view, essential for any robust interpretation or extension of such climatologies.

XVI. REVERSE ENGINEERING A PUBLISHED ARTICLE TO EXTRACT A FUNCTIONAL DATASET.

In cases where data is unavailable or difficult to obtain due to time or cost constraints, reverse engineering the data published by A. Battaglia et al. [16] can be an effective approach. To extract the PIA gaseous and PIA cloud attenuation functions from the images in MATLAB, the following general steps can be followed:

- 1) **Load Images:** Use the `imread` function to load the map images into MATLAB.
- 2) **Extract Data:** If the images contain color-coded numeric data, map the colors to their corresponding attenuation values and convert them into numeric values.

- 3) **Create Functions:** After converting the data into numeric values, construct functions that represent the attenuation as a function of geographic location.

XVII. REVERSE ENGINEERING A PIA GLOBAL MAP FROM BATTAGLIA ET AL. ARTICLE (2020).

In this section, I demonstrate the process of reverse engineering on a Global PIA Map published in Battaglia et al. (2020) [16]. The first step is to capture the image extracted from the article in a readable format and then transfer it to MATLAB for the numerical acquisition. I suppose that the image called PIA_gas 97percentileBATTAGLIA.png was the identical image that is present in the original article about PIA_GAS.

Example 1. Importing the Figure.

Algorithm 17 Working basic code for importing a PNG image extracted from A. Battaglia's et al. (2020).

```
img = imread('PIA_gas97percentileBATTAGLIA.png');
[lat grid, lon grid] = meshgrid(linspace(-90, 90,
size(img, 1)),
linspace(-180, 180, size(img, 2)));
PIA function = @(lat, lon) interp2(lon grid, lat
grid, double(img(:,:,1)), lon, lat);
figure;
imagesc(lon grid(1, :),
lat grid(:, 1), img);
title('PIA Attenuation with Original Colors');
colorbar;
xlabel('Longitude');
ylabel('Latitude');
```

Because the original image has a color bar, we suggest to recreate the color scale with this fragment of code:

Algorithm 18 Recreating the Color Scale form the original Image using the following Code Fragment.

```
cmap = [ 0.0, 0.0, 1.0; 0.0, 0.0, 1.0; 0.0, 1.0,
1.0; 0.0,
1.0, 0.0; 1.0, 0.5, 0.0; 1.0, 0.0, 0.0; 0.5, 0.0,
0.0; 1;
```

A. Working Code for Numerical Generation of the Global Map in MATLAB.

The following MATLAB code demonstrates how to process an image, extracted form the article by Battaglia et al. (2020) [16], and convert it into numerical representation. This allows for further manipulation and analysis of the data, such as plotting, mapping, or applying statistical functions. The code reads the image, processes the color values, and converts them into numerical data based on the color scale, enabling more detailed analysis of the PIA values.

Example 2. MATLAB code example for extracting data from the image

The code below demonstrates how to import the image, extract color values, and convert them into a corresponding numerical matrix. This numerical data can then be used for analysis, such as calculating attenuation values or generating maps.

Algorithm 19 Reverse Engineering Code in MATLAB to convert the imported image into a Numerical Representation.

```
load('attenuation grayscale matrix.mat', 'attenuation
grayscale');
[m, n] = size(attenuation grayscale);
[x, y] = meshgrid(1:n, 1:m);
[xq, yq] = meshgrid(linspace(1, n, 10*n),
linspace(1, m, 10*m));
attenuation approx = interp2(x, y, attenuation
grayscale, xq, yq, 'cubic');
figure;
subplot(1,2,1);
imagesc(attenuation grayscale);
colormap(gray);
colorbar;
title('Original Attenuation Matrix');
xlabel('Longitude');
ylabel('Latitude');
subplot(1,2,2);
imagesc(attenuation approx);
colormap(gray);
colorbar;
title('Approximate Attenuation Matrix');
xlabel('Longitude');
ylabel('Latitude');
```

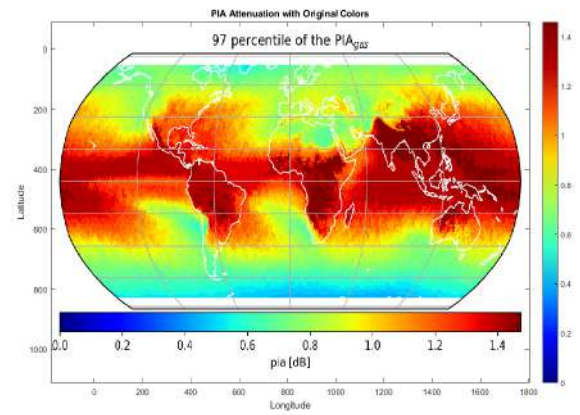


Fig. 18: Original image from Battaglia's article (2020): 97 percentile PIA_gas Global Map Selection.

In this step, I have chosen the 97th percentile PIA_Gas global map from Battaglia et al. (2020) as the reference image for further processing. This map provides a detailed representation of PIA values, which are essential for understanding the global distribution of attenuation caused by gaseous absorption in the atmosphere.

The data from this image will be reverse-engineered and converted into a numerical format for further analysis in MATLAB.

B. Reverse Engineering Color Map: Working Code in Algorithm 20

In the following working code from Algorithm 20, I demonstrate how to correctly implement the reverse engineering of the color map. This process is optimized by performing an RGB scouting on the original color map, which is depicted along the horizontal dimension. The vertical dimension represents the color map acquired through the reverse engineering

process, allowing for accurate transformation of the image into a numerical format.

Example 3. MATLAB Code for Reverse Engineering and Interpolating the PIA_{Gas} Image

The following MATLAB code demonstrates the reverse engineering process of the PIA_{Gas} image from Battaglia et al. (2020) [16]. The code extracts the color data from the image and interpolates it to recreate the color map, which can then be used for further numerical analysis.

Algorithm 20 Working code for Reverse Engineering Battaglia’s PIA_{Gas} Image with interpolated color map recreated in MATLAB.

```
% Load the image
img = imread('BattagliaImage
withColorBarOriginal.png');
% Creates a latitude and longitude grid
[lat_grid, lon_grid] = meshgrid(linspace(-90, 90,
size(img, 1)), linspace(-180, 180, size(img, 2)));
% Define the PIA function
PIA_function = @(lat, lon) interp2(lon_grid',
lat_grid', double(img(:,:,1)), lon, lat);
% Displays the original image
figure;
imagesc(lon_grid(1, :), lat_grid(:, 1), img);
title('PIA Attenuation with Original Colors');
colorbar;
xlabel('Longitude');
ylabel('Latitude');
% Define the interpolated color map with the
specified RGB values (0-255)
cmap = [
0, 1, 134;
1, 11, 254;
0, 154, 254;
49, 255, 199;
160, 255, 84;
253, 218, 0;
255, 87, 0;
182, 0, 0;
133, 0, 0;
];
cmap = cmap / 255;
% Convert RGB values from 0-255 to 0-1
interpolated_cmap = interp1(linspace(0, 1,
size(cmap, 1)), cmap, linspace(0, 1, 256));
% Apply interpolated color map
colormap(interpolated_cmap);
% Adjust color bar to have step 0.2 and range from 0
to 1.46
clim([0 1.46]);
colorbar('Ticks', 0:0.2:1.46); % Example of using
PIA function
lat = 0; % Sample latitude
lon = 0; % Sample longitude
PIA_value = PIA_function(lat, lon);
disp(['PIA value at (', num2str(lat), ', ', num2str(
lon), '): ', num2str(PIA_value)]);
```

Figure 19 illustrates a key step in our research, which involves the reconstruction and reuse of scientific data published in Battaglia et al. (2020). By selecting the 97th percentile PIA_{Gas} global attenuation map, we demonstrate the feasibility of extracting and re-analyzing high-value climatological information using reverse-engineering techniques.

The map shown has been imported from the original publication and processed in MATLAB using custom scripts developed for this study. These scripts enable the translation of

color-coded image data into quantitative attenuation values on a global geographic grid. This approach allowed us to recover spatially distributed PIA_{Gas} values with high fidelity, enabling further analysis and comparison with new satellite datasets (e.g., CloudSat-GPM coincidences).

The reconstruction of such data—often difficult to access in raw numerical format—represents a core methodological contribution of this work, making legacy climatological maps accessible for extended scientific usage, model validation, and training of surrogate AI models. illustrates a key step in our research, which involves the reconstruction and reuse of scientific data published in Battaglia et al. (2020). By selecting the 97th percentile PIA_{Gas} global attenuation map, we demonstrate the feasibility of extracting and re-analyzing high-value climatological information using reverse-engineering techniques.

The map shown has been imported from the original publication and processed in MATLAB using custom scripts developed for this study. These scripts enable the translation of color-coded image data into quantitative attenuation values on a global geographic grid. This approach allowed us to recover spatially distributed PIA_{Gas} values with high fidelity, enabling further analysis and comparison with new satellite datasets (e.g., CloudSat-GPM coincidences).

The reconstruction of such data—often difficult to access in raw numerical format—represents a core methodological contribution of this work, making legacy climatological maps accessible for extended scientific usage, model validation, and training of surrogate AI models.

Example 4. Importing and Interpolating Colors from the PIA_Gas Image.

Algorithm 21 Concrete MATLAB code for Importing and Interpolating Colors from the PIA_Gas Image.

```
% Load image
imagePath = 'BattagliaPIAgas.png'; % Replace with
the path to your image
img = imread(imagePath);
% Extract unique colors from image
[rows, cols, ~] = size(img);
reshapedImg = reshape(img, [], 3);
[uniqueColors, ~, idx] = unique(reshapedImg,
'rows');
% Normalize color values to [0, 1]
normalizedColors = double(uniqueColors) / 255;
% Reduce colors to 256 primary shades
numColors = 256;
colorMap = interp1(linspace(0, 1,
size(normalizedColors, 1)),
normalizedColors, linspace(0, 1, numColors));
% Reconstruct the image using the reduced color map
[~, colorIdx] = min(pdist2(colorMap,
normalizedColors), [], 1);
coloredImg = reshape(colorMap(colorIdx(idx), :),
rows, cols, 3);
% Display the original image and the one with the
reduced color map
figure;
subplot(1, 2, 1);
imshow(img);
title('Original Image');
subplot(1, 2, 2);
imshow(coloredImg);
title('Image with 256 Shades Color Map');
% Define the color map (from dark blue to dark red)
customColorMap = [
0, 0, 0.5; % Dark Blue
0, 0, 1; % Blue
0, 1, 1; % Cyan
0, 1, 0; % Green
1, 1, 0; % Yellow
1, 0.5, 0; % Orange
1, 0, 0; % Red
0.5, 0, 0 % Dark red
];
% Add color bar with custom color map
colormap(customColorMap);
colorbar('Ticks', linspace(0, 1, 8), 'TickLabels',
0:0.2:1.4);
```

All the algorithms presented in this work are original contributions, entirely developed by the author throughout the research activity. Each script or computational procedure has been conceived, implemented, tested, and validated with the aim of addressing specific scientific and technical challenges related to the reconstruction, processing, or modeling of atmospheric attenuation data. When relevant, explanatory comments or technical annotations—such as the one provided for Algorithm 21—have been included to clarify their structure, purpose, and scientific contribution. These annotations are intended to support reproducibility and facilitate the understanding of methodological innovations introduced in this research.

XVIII. CONSIDERATIONS FOR PIA IN GLOBAL MAP GENERATION.

Path Integrated Attenuation (PIA) varies depending on the frequency band and atmospheric conditions. The following is

an overview of typical PIA values for the Ku, Ka, and W bands:

- **Ku Band (12-18 GHz):** PIA can vary significantly, but on average, it is around 1-3 dB for moderate rain conditions. However, in heavy rain, PIA can exceed 10 dB.
- **Ka Band (26.5-40 GHz):** Due to the greater sensitivity to precipitation, PIA values in the Ka band are generally higher than in the Ku band. Typical values range from 3 to 10 dB, with peaks exceeding 20 dB during heavy rain.
- **W Band (75-110 GHz):** The W band is highly sensitive to attenuation caused by clouds and precipitation. PIA values can easily exceed 10 dB and reach up to 30 dB or more in very heavy rain.

For a more comprehensive understanding of this topic, I recommend reading [Liao, Meneghini, 2019], [17].

XIX. TECHNIQUES FOR GENERATING GLOBAL ATTENUATION MAPS IN KU, KA AND W BANDS.

This paragraph aims to demonstrate how, after analyzing the uncorrected data, we can generate global attenuation maps in the three bands of interest. We will begin by simulating the data due to the previously mentioned time and computational limitations. Using reverse engineering techniques on the images, we obtained numerical values for a four-year period (see A. Battaglia et al., 2020, [15]).

We will start by discussing the processing techniques used to create the maps, beginning with the random generation of attenuation in the three bands. This assumes that it is possible to obtain species for the highest frequencies of the representations. These representations require extensive research and the predictive ability to formulate hypotheses on a planetary scale, which carries both risks and benefits. In the final part of the article, we will address the theoretical aspects, considering them as actions that the **Data Scientist** aims to achieve.

A. Creating Synthetized Datasets with Reasonable dB Values

We begin by generating a global attenuation map for a specific band using range information available in the literature. For this process, MATLAB is preferred. The globe image is initially populated with random values across all latitudes and longitudes, ensuring that these values fall within a plausible range. This technique allows for experimentation and the determination of the most effective method for presenting the data.

Example 5. Algorithm 22: MATLAB Code for Generating a Global Map with Random Attenuation Values

Algorithm 22 Synthetized Global Map with reasonable PIA values.

```
figure();
data=rand(72,144);
rv = [0.4 90 0];
worldmap world
C = load("coastlines.mat");
for k=1:length(C)
plotm(C(k).coastlat, C(k).coastlon, 'k')
end geoshow(data, rv, 'displaytype', 'texturemap');
plotm(C.coastlat, C.coastlon, 'k');
title('Global Map with fake attenuation values');
```

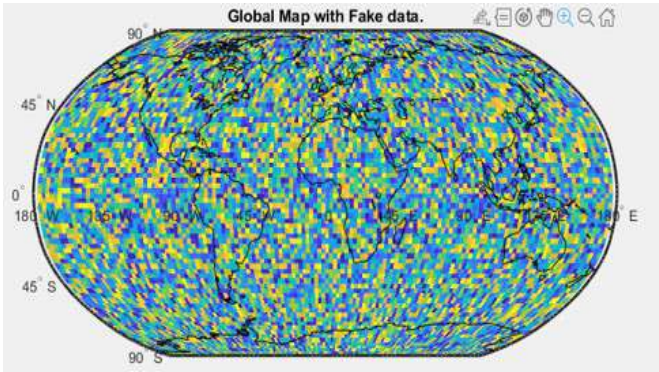


Fig. 21: Global Map of Synthetized or simulated Random attenuation data from **Algorithm 22**. Values are in the range 0–35 [dB]

To represent a PIA(W) between 5 dB and 40 dB on a global map, we can consider different geographical zones where such attenuation values might be plausible. The influence of soil moisture on Path Integrated Attenuation (PIA) varies significantly across different climate and environmental zones, with updated estimates providing a more realistic picture.

- **Coastal and Arid Regions:** Soil moisture ranges between 2% and 8% (VWC), resulting in a relatively low PIA of 5 to 10 dB.
- **Temperate Regions (Plains and Hills):** Soil moisture ranges between 8% and 15%, leading to a PIA of 10 to 15 dB.
- **Dense Forests and Mountainous Areas:** Soil moisture content increases to 15–25%, resulting in a PIA of 15 to 20 dB.
- **Tropical Forests and Equatorial Areas:** Extremely high soil moisture levels, ranging from 25% to 35%, lead to the highest PIA values of 20 to 25 dB.
- **Polar Regions:** Typically see frozen soil with a surface moisture content of 2-8%, resulting in a minimal PIA of 0 to 5 dB.
- **Urban Areas:** Sealed soil restricts moisture to 5-10%, resulting in a PIA of 5-15 dB. Structural obstacles dominate attenuation more than soil moisture.
- **Industrial Zones:** High human activity and pollution can cause greater attenuation, with PIA values ranging from 10 to 20 dB.

These estimates provide a comprehensive and refined understanding of the dynamic relationship between **Soil Moisture (SMO)** and **Path Integrated Attenuation (PIA)**, two key variables that characterize the distribution and movement of water within the Earth–atmosphere system. In meteorological and hydrological contexts, PIA quantifies the cumulative signal loss experienced by radar systems as they traverse precipitation fields, while SMO reflects the water retained at the land surface. Precipitation events contribute directly to both variables: they enhance PIA through increased atmospheric attenuation and simultaneously elevate SMO via infiltration into the soil.

Moreover, soil moisture itself influences the dielectric properties of the ground, modifying radar wave reflection and absorption, and thus exerting a measurable feedback effect on

PIA. This bidirectional interaction enables the combined use of satellite-based or radar-derived observations to simultaneously retrieve precipitation characteristics and surface soil moisture content. Such synergistic retrievals are crucial for improving applications in agriculture, water resource management, flood prediction, and numerical weather modeling.

By analyzing the coupled behavior of SMO and PIA, researchers can gain deeper insights into infiltration processes, water retention, and runoff generation, ultimately leading to more accurate representations of the local and regional hydrological cycle in both models and observational frameworks.

attenuation	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	14.7365	18.6465	12.3714	15.4427	11.6945	5.7900	22.0870	7.7027	15.4733	24.6599	16.0242	7.5242	38.9957	23.0360	7.9180
2	22.5585	10.4541	4.3374	3.1771	8.7376	3.4207	16.2111	1.7319	30.9929	22.0254	16.0396	27.9476	28.9523	16.1952	19.8950
3	11.7782	14.2197	15.1612	17.0952	20.0388	33.6401	17.0744	17.0454	28.8051	19.4340	26.0774	23.0005	8.9770	20.6437	9.6225
4	30.3870	21.2306	16.9857	18.1235	6.8700	5.8038	20.5629	6.3705	12.8623	0.0664	26.0709	30.7514	17.1421	12.0060	11.8944
5	35.4742	4.1020	1.7500	15.0007	32.2016	17.5360	32.9740	31.5886	4.8647	33.5170	32.5886	8.8544	8.3242	15.6760	8.9352
6	12.1832	20.0357	5.5514	15.0745	24.0654	2.0710	25.4087	26.6327	16.4504	31.0263	10.1072	5.2125	7.1162	26.6736	26.3385
7	34.2381	7.8625	5.2368	3.2113	20.0161	31.0612	12.4818	6.7854	8.8955	1.0761	15.2759	14.1635	7.7365	13.2342	12.8421
8	14.0004	38.4805	21.1133	18.9439	9.4812	12.8440	21.7131	6.7512	6.7020	11.8804	25.4051	13.8807	17.7021	16.2730	27.4344
9	21.5214	15.3678	11.3807	18.8805	7.2654	23.2779	18.9186	11.1370	26.8386	38.7652	21.7084	8.6002	4.0388	9.8489	1.8338
10	12.0107	7.7577	6.8005	17.3833	23.4816	16.5332	15.5000	16.4656	4.4688	33.4446	15.2743	17.2758	18.3462	15.6239	4.8357
11	2.4501	1.5325	23.3676	16.0564	18.1057	18.0306	16.0907	36.9736	5.1635	25.3084	1.6758	6.7632	23.1564	26.7100	26.4102
12	14.4817	20.5300	12.2116	9.1165	34.7354	26.8778	17.2504	26.7772	28.1418	11.2282	33.6235	19.5163	32.8768	42.827	12.7254
13	6.3576	24.4071	30.7901	41.896	6.1321	38.5819	21.2440	1.6532	9.8805	32.4044	26.057	19.8436	8.3468	3.7722	32.6020
14	21.4088	2.0620	13.3681	0.7643	24.2336	28.8754	10.0342	31.4805	25.5409	16.2154	33.5848	24.8906	11.3367	10.0531	20.0001
15	34.3030	6.8066	54.5532	25.1341	31.6931	6.8064	14.7195	2.0056	34.0771	7.2444	14.3845	8.7579	15.1621	7.5600	14.2164
16	2.6104	24.0451	5.3048	26.6186	9.6287	16.2060	26.1628	1.4016	7.7075	1.5218	6.3032	0.6604	17.1521	15.3245	18.6369
17	2.0612	22.8816	3.8116	25.7059	24.2056	12.2864	11.0517	14.6757	26.0004	11.7249	22.2639	1.6847	27.8117	25.6871	14.5814
18	11.5403	14.4935	31.7604	21.3104	1.2587	5.8892	10.0104	3.4487	27.7025	10.3004	20.0206	16.6175	32.1586	15.0342	28.6225
19	36.5581	5.5864	26.8137	22.8813	26.1002	18.3064	17.1179	27.1336	0.6238	27.1465	7.1775	24.6804	21.6865	25.2661	12.7240
20	16.6832	23.9369	7.3665	11.1132	25.7911	5.5916	12.9451	25.6759	15.2141	25.4031	7.4564	25.4409	23.4455	15.6677	1.2530
21	3.1589	19.3707	17.4504	2.2678	6.7391	10.4619	31.2828	25.6723	15.4805	12.0741	1.8270	14.7620	1.4504	10.1077	16.7361
22	9.9889	20.0522	32.1214	33.5531	16.8945	7.2887	3.7520	20.8205	25.8907	2.7026	33.4640	21.7207	25.5462	14.6880	1.1076

Fig. 22: Synthetric Attenuation DataTable with Simulated values.

XX. CONTRIBUTION OF SOIL MOISTURE TO THE PIA ESTIMATION.

To include the contribution of soil moisture in different territorial zones, we can consider how soil moisture varies according to climatic and geographical conditions. Soil moisture contributes indirectly to atmospheric Path Integrated Attenuation (PIA), mainly by influencing the local microclimate and the water vapor content in the air near the ground. Saturated soil can promote evaporation, increasing water vapor, which in turn increases atmospheric attenuation at higher frequencies, such as microwaves and millimeter waves. In addition, soil moisture can alter the reflecting and absorbing properties of the Earth's surface, influencing wave propagation through reflection, multipath, and interference.

However, its contribution to PIA is marginal compared to the main atmospheric factors, such as precipitation, clouds, and aerosols. This effect becomes relevant only under specific conditions, such as in tropical regions or during thawing in polar areas. In different parts of the Earth's microclimates, soil moisture has a very different effect on Path Integrated Attenuation (PIA). In general, though, it is not as important as factors like clouds, precipitation, and aerosols. In desert and arid areas, the contribution is negligible, with an impact on PIA of less than 1%, due to the almost total absence of soil moisture. In coastal and subtropical areas, soil moisture contributes minimally, with an impact on PIA ranging between 1% and 5%. In temperate plains and hills, this contribution can increase up to 10% due to increased seasonal evaporation, which in turn increases local water vapor. The role of soil

moisture is more important in dense forests and mountain areas, where it has an effect of 10% to 15%. In tropical and equatorial areas, where it rains all the time and the soil is always wet, the effect can be up to 20%. Conversely, polar regions and permafrost or frozen soils limit the contribution of soil moisture. Lastly, in cities and industrial areas, soil moisture contribution remains limited (5% to 10%), with infrastructure and made-man barriers having the most significant effect on atmospheric attenuation.

A. Impact of Soil Moisture on PIA(W) Global Attenuation and Evaluation of Moisture Estimation Contribution

In this section, I present an experimental simulation designed to estimate the potential contribution of **Soil Moisture (SM)** to global PIA(W) attenuation by working with synthetic or “proxy” data. This approach enables a preliminary evaluation of the SM impact in the absence of spatially resolved real measurements. As part of this research, I developed a MATLAB-based methodology that integrates soil moisture effects into PIA modeling by assigning representative **volumetric water content (VWC)** values—expressed as percentages—to different territorial zones.

The proposed simulation reflects a conceptual extension of traditional attenuation models by incorporating SM-dependent modulation of water vapor content and surface dielectric properties. This allows for the differentiation of soil moisture effects across diverse climatic and geographical areas (e.g., arid, tropical, temperate, and polar zones). The framework provides a first-order estimation of soil-driven attenuation variability and lays the groundwork for future integration of real satellite-derived SM products in PIA(W) modeling workflows. I have considered in this research:

- 1) **Coastal and Arid Regions:** Soil moisture levels between 2% and 8% volumetric water content (VWC) typically result in relatively low Path Integrated Attenuation (PIA), with values ranging from 5 to 10 dB. Such conditions are characteristic of semi-arid regions, dry grasslands, and frozen soils during transitional thaw periods.
- 2) **Temperate Regions (Plains and Hills):** Soil moisture levels ranging between 8% and 15% volumetric water content are typically associated with moderate Path Integrated Attenuation (PIA) values, estimated between 10 and 15 dB. These conditions are representative of temperate plains, agricultural zones, and transitional climates where evaporation and infiltration are balanced.
- 3) **Dense Forests and Mountainous Areas:** When soil moisture content increases to intermediate-to-high levels—typically between 15% and 25% volumetric water content—Path Integrated Attenuation (PIA) correspondingly rises, with estimated values ranging from 15 to 20 dB. Such conditions are commonly found in temperate forested regions, mountainous terrains, or post-rainfall environments with high retention capacity.
- 4) **Tropical Forests and Equatorial Areas:** Extremely high soil moisture levels, typically ranging from 25% to 35% volumetric water content, are associated with the

highest observed values of Path Integrated Attenuation (PIA), which can reach between 20 and 25 dB. These conditions are characteristic of saturated soils in tropical, equatorial, or flooded environments, where both surface water retention and increased atmospheric humidity contribute significantly to signal attenuation.

- 5) **Polar Regions:** Frozen soils typically exhibit low surface moisture content, generally ranging between 2% and 8% volumetric water content (VWC). Under these conditions, the resulting contribution to Path Integrated Attenuation (PIA) is minimal, typically within the range of 0 to 5 dB.
- 6) **Urban and Industrial Areas:** Sealed or impervious soils typically retain limited surface moisture, generally within the range of 5% to 10% VWC. In such conditions, the corresponding Path Integrated Attenuation (PIA) is estimated between 5–15 dB and may reach 10–20 dB in more saturated scenarios. However, in urban and industrial environments, structural obstacles—such as buildings and infrastructure—play a more dominant role in signal attenuation than soil moisture itself.

These estimates offer a more accurate depiction of the relationship between soil moisture and PIA. In meteorological and hydrological contexts, PIA and soil moisture content (SMO) reflect the presence and distribution of water within the earth-atmosphere system. Precipitation along a given path significantly affects PIA, as it represents the cumulative attenuation of radar signals. A large portion of the precipitation infiltrates into the soil, increasing surface moisture.

Additionally, soil moisture alters the dielectric properties of the soil, which in turn influences radar signal reflection and absorption, leading to measurable effects on PIA. This relationship enables the use of satellite or radar data to simultaneously estimate soil water content and precipitation characteristics, providing valuable insights for agriculture, hydrology, and weather forecasting. Studying these combined variables enhances the understanding of infiltration and drainage processes, improving models that describe the local hydrological cycle.

To improve the realism, noise or ripple signals can be added to the edges of regions, and the gaps between humidity values can be made bigger.

This example serves as a starting point for research projects. However, it remains a simplified model and may require improvements and adaptations to meet specific requirements. To refine the model, I recommend considering the following adjustment: when including the influence of atmospheric physics, particularly gases like oxygen and water vapor, it is essential to incorporate parameters such as the absorption coefficient. For more accurate global results, integrating real meteorological data for humidity and temperature is suggested. Furthermore, when using advanced propagation models it is crucial to incorporate the ITU-R propagation models to account for various atmospheric and geographical factors.

Lastly, if more effect like those that weaken the signal are taken into account, they may include the effects of particulate matter in the air, signal reflection and diffraction, and terrain and vegetation.

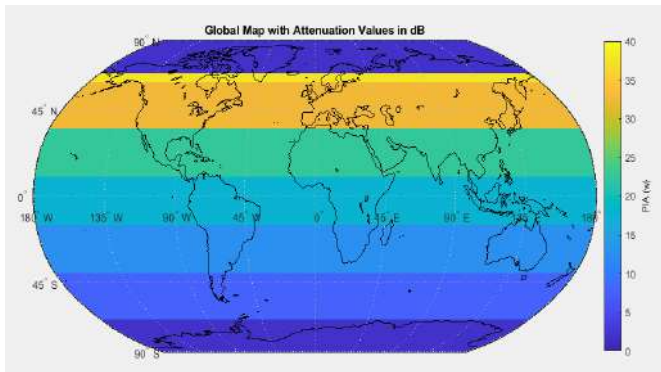


Fig. 23: PIA(W) Attenuation for Region of Interest with Plausible Values.

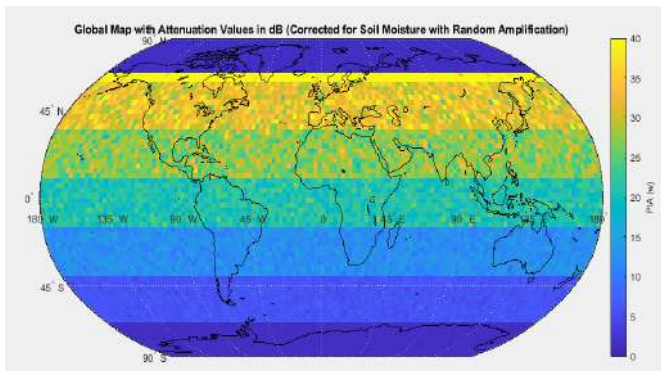


Fig. 24: With Random Soil Moisture Content, PIA(W) values become non-uniform in Geographically Neighboring Areas.

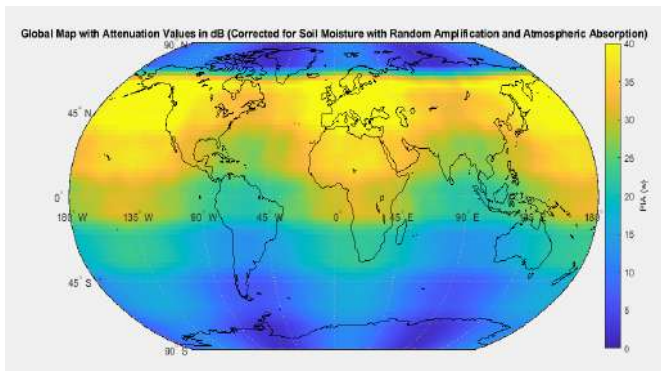


Fig. 25: Synthetic PIA(W) values with Random Noise in Regional Boundaries. Ripple Effects over regions, Random Moisture Influence, and Water Vapor with Blurring Effect.

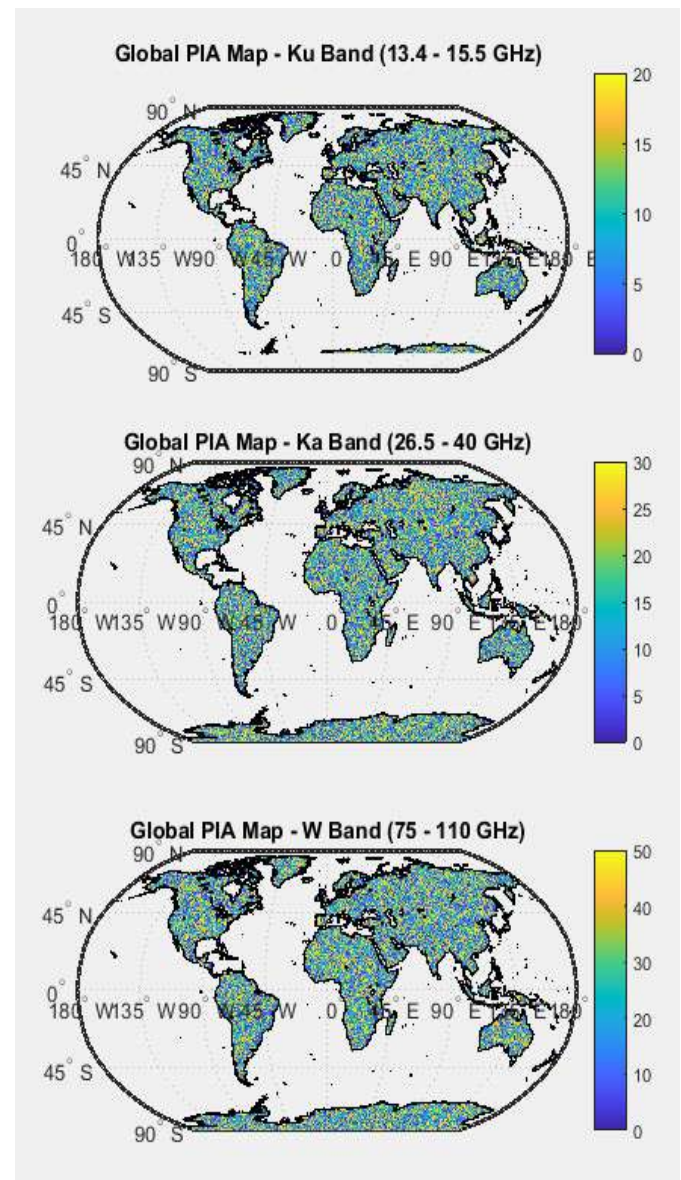


Fig. 26: Imaginary Plotting of PIA-W, PIA-Ku and PIA-Ka from Synthetic Data Over Land: for presentation purposes with Scale in dB.

B. Global PIA Map Across Ku, Ka, and W Bands with Synthetic Random Generation

In this paragraph I in breaf present images related to the PIA World Map generated randomly within predefined ranges without any specific precaution. This is done solely for demonstration purposes.

C. Generating Synthetic Global PIA Maps over Lands and Oceans.

In this paragraph, I add the Source Code in MATLAB for demonstration purposes. This code serves as a demonstration tool for visualizing synthetic PIA maps across different frequency bands using MATLAB. It provides a foundation that can be adapted for real-world data, with further improvements

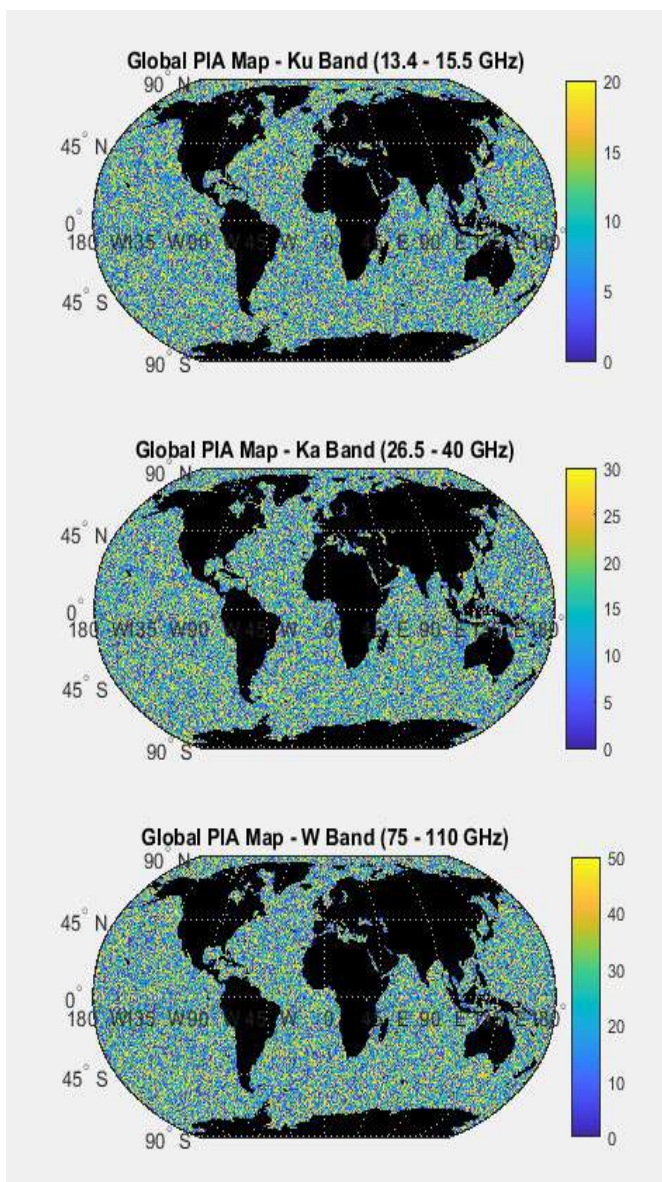


Fig. 27: Imaginary Plotting of PIA-W, PIA-Ku and PIA-Ka from Synthetic Data over Oceans: for Presentation Purposes with Scale in dB.

such as incorporating actual meteorological or environmental data.

Key Points:

- **Synthetic Data:** The PIA values are randomly generated for the purpose of demonstration, simulating attenuation levels across different frequency bands.
- **Land Area Boundaries:** Only land areas are used for generating the PIA values, and the generated maps reflect random attenuation values for these regions.
- **Visualization:** The use of different color maps (*parula* for Ku Band and *warbler* for Ka and W Bands) helps distinguish different PIA maps.

Algorithm 23: Description and Explanation

The process involves the following steps:

- 1) **Latitude and Longitude Mesh Grid:** The code creates a mesh grid of latitude and longitude with the `meshgrid` function. The latitude range is from -90 to 90 degrees, and the longitude range is from -180 to 180 degrees.
- 2) **Initialization of PIA Arrays:** Three arrays (`PIA_Ku`, `PIA_Ka`, and `PIA_W`) are initialized to `NaN` (Not a Number) values, which represent areas where no data is available.
- 3) **Loading Land Area Data:** The land area shapefile is read using `shaperead`, with geo-coordinates enabled, to obtain the shapes of continents.
- 4) **Loop Over Land Areas:** The code iterates over each land area in the shapefile. For each land mass, the longitude and latitude of the land are extracted. The function `inpolygon` is used to identify the pixels within the boundaries of the continent.
- 5) **Random Data Assignment:** For the pixels that lie within the land areas, random values are assigned to the PIA arrays:
 - a) `PIA_Ku`: Random values between 0 and 20 (dB) are assigned for Ku Band.
 - b) `PIA_Ka`: Random values between 0 and 30 (dB) are assigned for Ka Band.
 - c) `PIA_W`: Random values between 0 and 50 (dB) are assigned for W Band.
- 6) **Plotting the PIA Maps: The code then generates three subplots:**
 - a) The first subplot displays the PIA map for the Ku Band (13.4 - 15.5 GHz), using the `pcolorm` function for the map projection and colorbar to show the color scale. The colormap *parula* is used, and the color scale is set from 0 to 20 dB.
 - b) The second subplot displays the PIA map for the Ka Band (with values between 0 and 30 dB), using the colormap *warbler*.
 - c) The third subplot shows the PIA map for the W Band (with values between 0 and 50 dB), using the colormap *warbler* again.
- 7) **Displaying Minimum and Maximum PIA Values:** Finally, the minimum and maximum values of `PIA_Ku`, `PIA_Ka`, and `PIA_W` are displayed in the command window using the `disp` function in MATLAB. This allows to view the range of PIA values generated.

Example 7. Global PIA Map over Oceans

Key differences from Algorithm 23

- 1) **PIA Value Range:**
 - **Previous Code:** Random values for `PIA_Ku`, `PIA_Ka` and `PIA_W` were generated within a specific range for each continent, using `rand(sum(in_continent),1)` function to assign values for each land area. The values were specified per region (land).
 - **New Code:** Random PIA values are generated globally for all latitudes and longitudes. The `PIA_Ku`,

Algorithm 23 World PIA Map over Lands.

```

lat = -90:1:90;
lon = -180:1:180; [Lon, Lat] = meshgrid(lon, lat);
PIA_Ku = NaN(size(Lat));
PIA_Ka = NaN(size(Lat));
PIA_W = NaN(size(Lat));
land = shaperead('landareas', 'UseGeoCoords', true);
for k = 1:length(land)
    lon_continent = land(k).Lon;
    lat_continent = land(k).Lat;
    in_continent =
    inpolygon(Lon, Lat, lon_continent, lat_continent);
    PIA_Ku(in_continent) =
    20 * rand(sum(in_continent(:)), 1);
    PIA_Ka(in_continent) =
    30 * rand(sum(in_continent(:)), 1);
    PIA_W(in_continent) =
    50 * rand(sum(in_continent(:)), 1);
end
figure;
subplot(3, 1, 1);
worldmap([-90 90], [-180 180]);
pcolorm(Lat, Lon, PIA_Ku);
colorbar;
clim([0 20]);
title('Global PIA Map - Ku Band (13.4 - 15.5 GHz)');
colormap parula;
hold on;
end
hold off;
worldmap([-90 90], [-180 180]);
pcolorm(Lat, Lon, PIA_Ka);
colorbar;
clim([0 30]); colormap warbler;
hold on;
end
hold off;
worldmap([-90 90], [-180 180]);
pcolorm(Lat, Lon, PIA_W);
colorbar;
clim([0 50]);
colormap warbler;
hold on;
end hold off;
disp([min(PIA_Ku( isnan(PIA_Ku))),
max(PIA_Ku( isnan(PIA_Ku)))]);
disp('Min and Max PIA Ka:');
disp([min(PIA_Ka( isnan(PIA_Ka))),
max(PIA_Ka( isnan(PIA_Ka)))]);
disp('Min and Max PIA W:');
disp([min(PIA_W( isnan(PIA_W))),
max(PIA_W( isnan(PIA_W)))]);

```

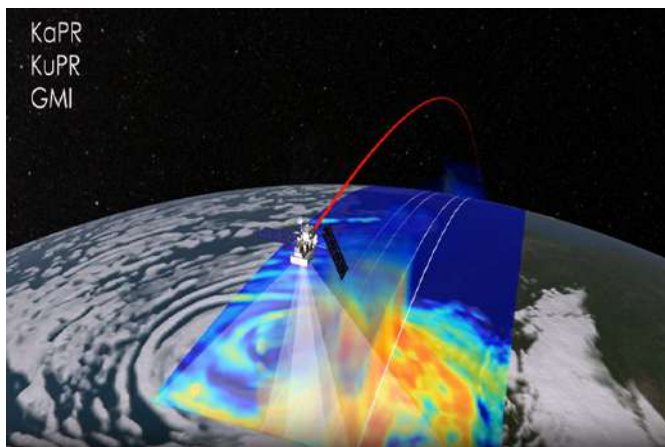


Fig. 28: Image of Satellite while is scanning Earth in the Ku, Ka Band with GMI sensor.

Algorithm 24 World PIA Map over Oceans

```

lat = -90:1:90;
lon = -180:1:180; [Lon, Lat] = meshgrid(lon, lat);
PIA_Ku = 20 * rand(size(Lat));
PIA_Ka = 30 * rand(size(Lat));
PIA_W = 50 * rand(size(Lat));
figure;
subplot(3, 1, 1);
worldmap([-90 90], [-180 180]);
pcolorm(Lat, Lon, PIA_Ku);
colorbar;
caxis([0 20]);
title('Global PIA Map - Ku Band (13.4 - 15.5 GHz)');
colormap(parula);
hold on;
geoshow('landareas.shp', 'FaceColor', 'k');
hold off;
subplot(3, 1, 2);
worldmap([-90 90], [-180 180]);
pcolorm(Lat, Lon, PIA_Ka);
colorbar;
caxis([0 30]);
title('Global PIA Map - Ka Band (26.5 - 40 GHz)');
colormap(parula);
hold on;
geoshow('landareas.shp', 'FaceColor', 'k');
hold off;
subplot(3, 1, 3);
worldmap([-90 90], [-180 180]);
pcolorm(Lat, Lon, PIA_W);
colorbar;
caxis([0 50]);
colormap(parula);
hold on;
geoshow('landareas.shp', 'FaceColor', 'k');

```

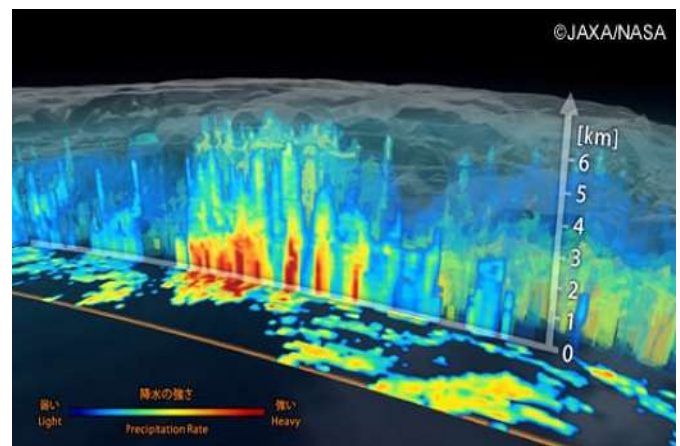


Fig. 29: A JASA/NASA Image for Precipitation Rate from Earth to Space. ©

PIA_Ka, and PIA_W values are assigned uniformly across the globe (without any geographical segmentation). This simplifies the random data generation but lacks regional specificity.

2) Plotting Method:

- **Previous Code:** The code used a loop over the land area data, and PIA values were only assigned to land regions, making it more region-specific.
- **New Code:** The PIA values are applied uniformly to the entire map (without distinguishing land from water), and the geoshow function overlays the land

areas on top of the random global data. This provides a global view without regional boundaries for PIA values.

3) Color Scaling:

- **Previous Code:** The PIA values in each band were displayed using different color scale, but with more control over the specific ranges, adjusting them per land area.
- **New Code:** The color scale (caxis) for each subplot is set specifically for each PIA band (Ku:0–20 dB, Ka: 0–30 dB, W:0–50 dB), but now this applies to the whole global map, not just land areas.

4) Data Storage:

- **Previous Code:** The PIA values for each landmass were calculated dynamically during the iteration over the land shapefile and stored in separate variables.
- **New Code:** All PIA values for the entire globe are generated at once and stored in matrices corresponding to PIA_Ku, PIA_Ka and PIA_W.

The new code is more simplified and provides a global visualization of PIA values without distinguishing between land and water. It generates synthetic data and displays it in a clean, global format. In contrast, the previous code worked with regional data and generated PIA values specifically for land areas, providing more detailed and localized information.

Description for the Algorithm 24

The following steps describe the code's functionality:

- 1) **Latitude and Longitude Grid Generation:** The latitude (lat) and longitude (lon) values are defined, and a mesh grid is created with the meshgrid function. This grid represents the entire world (from -90° to 90° latitude and -180° to 180° longitude).
- 2) **Random Data Generation for PIA:**
 - a) PIA_Ku: Random values are generated for the Ku band (13.4 - 15.5 GHz) using rand, scaled to the range 0–20 dB.
 - b) PIA_Ka: Random values for the Ka band (26.5 - 40 GHz) are generated in the range 0–30 dB.
 - c) PIA_W: Random values for the W band (75 - 110 GHz) are generated in the range 0–50 dB.
- 3) **Plotting the Global PIA Maps:**
 - a) Subplots: Three subplots are created to display the PIA maps for the Ku, Ka, and W bands.
 - i) Ku Band: The first subplot displays the PIA map for the Ku band. It uses pcolor to plot the data, with caxis([0 20]) to set the color axis scale from 0 to 20 dB. The map uses the parula colormap and overlays land areas using geoshow('landareas.shp', 'FaceColor', 'k').
 - ii) Ka Band: The second subplot displays the PIA map for the Ka band with a color axis scale from 0 to 30 dB and uses the parula colormap. The land areas are similarly shown with geoshow.
 - iii) W Band: The third subplot displays the PIA map for the W band with a color axis scale from 0 to 50 dB and the parula colormap. Land areas are again shown.

- 4) **Colorbars:** A colorbar is included in each subplot to indicate PIA values with respect to the selected colormap.
- 5) **Geographical Overlays:** The geoshow function overlays land and water areas on the global map to distinguish land from water regions.

XXI. USING THE KRIGING TECHNIQUE TO GENERATE GLOBAL PIA MAPS.

The Kriging technique is a geostatistical interpolation method that is particularly useful when working with discrete, pixelated maps. Regular grids typically represent spatial data in such cases, with each pixel containing a measured or estimated value. However, these measurements can be sparse or irregular, complicating the process of obtaining a continuous and accurate representation of the studied phenomenon. Kriging uses the spatial correlations between known points to estimate values at unmeasured points. This method not only provides more accurate estimates than other interpolation techniques but also offers a measure of the uncertainty associated with each estimate.

In practice, Kriging can significantly enhance the quality of discrete pixelated maps, making them more reliable for analyzing and visualizing spatial data.

I have applied the Kriging technique to several examples to demonstrate this, and the following images present the results. In this section, I aim to revisit the project in light of the Kriging technique. A global attenuation map in the W band or nearby frequency bands could have numerous valuable applications for the community.

In the frequency range between 75–110 GHz, two key atmospheric attenuation phenomena are observed: a maximum attenuation around 75 GHz and a relative minimum at approximately 96 GHz. Attenuation is the loss of signal power as it travels through a transmission medium. Geometric divergence, atmospheric absorption, terrain effects, and objects in the slant path are some of the factors that can affect attenuation. Additional factors such as propagation through vegetation, industrial sites, and urban areas also play a role in affecting attenuation.

A. The Importance of Creating a Global Attenuation Map

Creating a global attenuation map would provide valuable insights into how various environmental variables affect signal propagation across different regions worldwide. Such a map should be regularly updated and enhanced with additional factors, including **Crane's** analysis and other relevant models, to identify optimal transmission frequencies for diverse global regions and atmospheric conditions. Technologies like weather radars could benefit significantly from this map, improving the efficiency and accuracy of signal transmission in different environmental contexts.

B. Crane models

Global meteorological mapping widely uses crane models, essential tools for predicting rain-induced signal attenuation. These models divide the globe into different rain climate

regions, each characterized by specific distributions of precipitation intensity and frequency. The 1985 Crane model, for instance, uses updated atmospheric distribution tables and a map of rain regions to calculate signal attenuation over terrestrial and satellite paths. The scientific value of Crane models lies in their ability to give precise estimates of signal attenuation, which are necessary for designing and improving communication systems on land and in space.

These models consider variables such as transmitter frequency, ground station elevation angle, station location and altitude, and Earth's curvature. Crane models have been proven correct by satellite beacon observations and measurements of paths on land, showing a standard deviation that is within acceptable limits between predicted and observed values [18]–[20].

Regarding the creation of **Global Attenuation Maps (GAMs)**, our study emphasized the operational feasibility and developed the necessary MATLAB source code for data acquisition. However, the man-hours required for this operation are quite prohibitive due to the extensive volume of data necessary for processing. This data needs to be checked for accuracy and analyzed for **CloudSat-TRMM coincidences**, such as when the satellites cross the equator. First, we looked at orbital passages over the polar regions (Antarctic and Antarctic) and created attenuation maps by using the **97th percentile method** to find the average values in pixels (see **Figs. 16–17**). This approach enabled the extraction of individual latitude-longitude boxes with precision.

However, attempting to process all satellite passes simultaneously overwhelmed the MATLAB working memory, leading to a crash. This issue arose because the hardware resources available were insufficient to handle the large volume of data required to generate the global attenuation map in a single operation. Adopting a more incremental approach can mitigate this problem. Instead of processing all the data at once, the proposed method built the map by sequentially processing smaller subsets of satellite overpass data in the form of orbital bundles or limited time event lines, preventing system overload. By using dynamic memory use, this method distributes memory consumption over time, preventing system failure and allowing the map to be generated without the immediate need for hardware upgrades.

XXII. TECHNICAL SUGGESTIONS FOR REAL-WORLD ANALYSIS OF COINCIDENCE DATASETS.

To assist the reader in understanding how to extract sensitive data for our research from satellite coincidences, a concrete example related to radar reflectivity will be provided. In this example, we assume that the goal is to accumulate the content of a variable of interest into an appropriate data structure in MATLAB. This variable will subsequently be analyzed, studied, and potentially visualized in graphical form.

Example 8. Algorithm 25: Listing and Storing Radar Reflectivity Data from NetCDF Files

Algorithm 25: Description and Explanation

This MATLAB script is designed to list the groups and variables within NetCDF files located in a specified directory.

Algorithm 25 Extracting Information from Coincidence Dataset.

```

directory = 'C:\Users\MyUserName\Desktop\Data';
fileList = dir(fullfile(directory, '*.nc4'));
for i = 1:length(fileList)
    fileName = fullfile(directory, fileList(i).name);
    ncid = netcdf.open(fileName, 'NC_NOWRITE');
    fprintf('File: listGroupsAndVars(ncid, ");
    netcdf.close(ncid);
end
function listGroupsAndVars(ncid, indent)
[numgrps, numvars, ~, ~] = netcdf.inq(ncid);
for i = 0:numvars-1
    varname = netcdf.inqVar(ncid, i);
    fprintf('end
    grpIDs = netcdf.inqGrps(ncid);
    for i = 1:length(grpIDs)
        grpname = netcdf.inqGrpName(grpIDs(i));
        fprintf('%sGroup: %s n', indent, grpname);
        listGroupsAndVars(grpIDs(i), [indent ' ']);
    end
end
end

```

Here's how it works:

- 1) **Directory Setup:** The directory path is set where the NetCDF files are located (*.nc4 files).
- 2) **Loop Through Files:** The script loops through each NetCDF file in the directory and opens the file in read-only mode (NC_NOWRITE).
- 3) **List Groups and Variables:** For each file, the `listGroupsAndVars` function is called. This function is responsible for listing both the groups and variables contained in the NetCDF file.
- 4) **Close File:** After processing each file, it is closed using `netcdf.close`.
- 5) **Listing Groups and Variables:**
 - The `listGroupsAndVars` function recursively lists groups and their associated variables.
 - It first retrieves the number of groups and variables in the file and iterates through the variables, printing their names.
 - It then lists any subgroups (if applicable), recursively calling itself to handle nested groups.

Key Points:

- This code is useful for inspecting the structure of NetCDF files, especially for large files with nested groups and variables.
- The recursive function `listGroupsAndVars` helps navigate hierarchical data structures within the NetCDF file.

In the first listing, which describes **Algorithm 25**, we demonstrate how to acquire information about the contents of files stored in the Data folder, such as NetCDF files. The second listing illustrates how to store arrays of radar reflectivity structures.

Example 9. Algorithm 26: MATLAB Code for Extracting Radar Reflectivity and Geospatial Data from NetCDF Files

This MATLAB code processes NetCDF files stored in a specific directory and extracts radar reflectivity data (referred

to as `zFactorMeasured`) for two types of measurements: NS (Normal Scan) and MS (Medium Scan). Here's a breakdown of what the code does:

1) **Directory Setup and File Listing:**

- The directory where the NetCDF files are stored is specified (C:\Users\MyUserName\Desktop\Data).
- A list of all NetCDF files (*.nc4) in the specified directory is obtained using `dir`.

2) **Pre-allocation of Data Storage:**

- Arrays (using cell arrays) are pre-allocated to store the data for each file:
 - `zFactorMeasuredData_NS`: Stores radar reflectivity data for the Normal Scan (NS).
 - `zFactorMeasuredData_MS`: Stores radar reflectivity data for the Medium Scan (MS).
 - `latitudeData_NS` and `longitudeData_NS`: Store the latitude and longitude for the Normal Scan.
 - `latitudeData_MS` and `longitudeData_MS`: Store the latitude and longitude for the Medium Scan.

3) **Loop Through Files:**

- The code loops through all NetCDF files in the directory (`fileList`), and for each file, it opens the file in read-only mode using `netcdf.open`.

4) **Accessing Groups and Variables:**

- The code accesses specific groups and variables inside the NetCDF file:
 - 2A.GPM.DPR group (the main data group).
 - NS and MS subgroups, corresponding to Normal Scan and Medium Scan, respectively.
 - SWATH data within each scan type.

5) **Reading Variables:**

- The radar reflectivity data (`zFactorMeasured`) is extracted for both Normal and Medium Scans.
- The latitude (`Latitude`) and longitude (`Longitude`) values are also extracted for both scans.

6) **Storing Data:**

- The extracted `zFactorMeasured`, `Latitude`, and `Longitude` values for each scan type (NS and MS) are stored in the corresponding pre-allocated arrays.

7) **Closing the NetCDF File:**

- After all the required data is extracted for the current file, the NetCDF file is closed using `netcdf.close(ncid)`.

8) **Displaying the Data:**

- Once all files are processed, the radar reflectivity data for both NS and MS scans is displayed using the `disp` function.

- NetCDF Handling:** The code handles reading and extracting variables from nested groups inside NetCDF files using `netcdf` functions.

Algorithm 26 Storing Radar Reflectivity Arrays in MATLAB from a Coincidence Dataset.

```

directory =
'C:\Users\MyUserName\Desktop\Data';
fileList = dir(fullfile(directory, '*.nc4'));
zFactorMeasuredData_NS = cell(length(fileList), 1);
zFactorMeasuredData_MS = cell(length(fileList), 1);
latitudeData_NS = cell(length(fileList), 1);
longitudeData_NS = cell(length(fileList), 1);
latitudeData_MS = cell(length(fileList), 1);
longitudeData_MS = cell(length(fileList), 1);
for i = 1:length(fileList)
    fileName = fullfile(directory, fileList(i).name);
    ncid = netcdf.open(fileName, 'NC_NOWRITE');
    grpID = netcdf.inqNcid(ncid, '2A.GPM.DPR');
    nsID = netcdf.inqNcid(grpID, 'NS');
    swathID_NS = netcdf.inqNcid(nsID, 'SWATH');
    varid_NS =
netcdf.inqVarID(swathID_NS, 'zFactorMeasured');
zFactorMeasuredData_NS{i} =
netcdf.getVar(swathID_NS, varid_NS);
latid_NS = netcdf.inqVarID(swathID_NS, 'Latitude');
latitudeData_NS{i} =
netcdf.getVar(swathID_NS, latid_NS);
lonid_NS =
netcdf.inqVarID(swathID_NS, 'Longitude');
longitudeData_NS{i} =
netcdf.getVar(swathID_NS, lonid_NS);
msID = netcdf.inqNcid(grpID, 'MS');
swathID_MS = netcdf.inqNcid(msID, 'SWATH');
varid_MS =
netcdf.inqVarID(swathID_MS, 'zFactorMeasured');
zFactorMeasuredData_MS{i} =
netcdf.getVar(swathID_MS, varid_MS);
latid_MS =
netcdf.inqVarID(swathID_MS, 'Latitude');
latitudeData_MS{i} =
netcdf.getVar(swathID_MS, latid_MS);
lonid_MS =
netcdf.inqVarID(swathID_MS, 'Longitude');
longitudeData_MS{i} =
netcdf.getVar(swathID_MS, lonid_MS);
netcdf.close(ncid); end
disp('Data zFactorMeasured
related to NS:');
disp(zFactorMeasuredData_NS);
disp('Data zFactorMeasured per MS:');
disp(zFactorMeasuredData_MS);

```

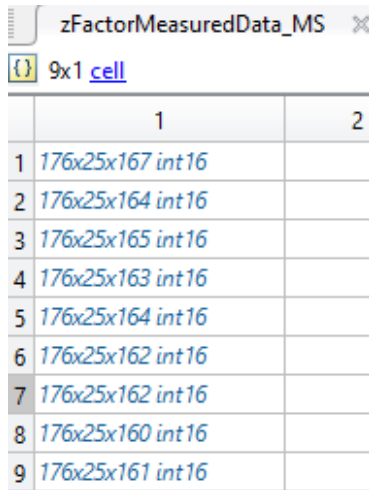
A. Practical Suggestions for Dataset Treatment and Visualization

The CloudSat-GPM coincidence radar reflectivity dataset is organized as a matrix with dimensions of 125 rows by 1272 columns, containing floating-point values. This results in a total of 159,000 elements. The primary variable of interest in this dataset is the "Radar Reflectivity".

- Matrix Dimensions:** the dataset is structured as a 125 (rows) by 1272 (columns) matrix, where each element represents a radar reflectivity value.
- Data Type:** the values in the matrix are of float type, allowing for precise representation of radar reflectivity measurements.

Key Points:

- Data Extraction:** The code extracts radar reflectivity, latitude, and longitude for both Normal Scan and Medium Scan from multiple NetCDF files.
- Storing and Displaying:** The data is stored in cell arrays and displayed after processing all the files.



	1	2
1	176x25x167 int16	
2	176x25x164 int16	
3	176x25x165 int16	
4	176x25x163 int16	
5	176x25x164 int16	
6	176x25x162 int16	
7	176x25x162 int16	
8	176x25x160 int16	
9	176x25x161 int16	

Fig. 30: Visualization of the Radar Reflectivity Data Structure collected in MATLAB after completing the Reading Cycle.

- 3) **Total Elements:** the total number of elements in the matrix is 159,000, calculated by multiplying the number of rows (125) by the number of columns (1272).
- 4) **Variable of Interest:** the key variable in the dataset is radar reflectivity, which quantifies the reflectivity of radar signals. This variable is essential for analyzing precipitation and cloud properties.

This dataset is particularly valuable for the study of atmospheric phenomena, combining data from the CloudSat and GPM (Global Precipitation Measurement) missions. The CloudSat radar operates at higher frequencies (W-band, 94 GHz), providing detailed vertical profiles of clouds, while the GPM radar operates at lower frequencies (KuKa-band, 14–35 GHz), capturing precipitation structures [21].

In this simple example, instead of collecting the 2D quantities of Radar Reflectivities, we demonstrate how to collect the `zFactorMeasured` variable from the DPR-MS-`zFactorMeasured` product contained within the 2A.GPM.DPR group. The process is broken down as follows:

- 1) **Identify the Product and Group:** the focus is on extracting the `zFactorMeasured` variable from the DPR-MS-`zFactorMeasured` product, which is located within the 2A.GPM.DPR group.
- 2) **Access the Data:** appropriate MATLAB functions are used to access and read the data from the specified product and group. This typically involves opening the NetCDF file and navigating to the desired group and variable.
- 3) **Extract the Variable:** the `zFactorMeasured` variable is extracted from the dataset. This variable represents the radar reflectivity factor measured by the Dual-Frequency Precipitation Radar (DPR).
- 4) **Store the Data:** the extracted data is stored in a suitable MATLAB data structure, such as an array or matrix, for further analysis and visualization.

In this case the magnitude is always a matrix of 125 columns and 1272 rows, which we can then graph by associating both

Latitude and *Longitude* as we showed previously in Fig. 17. To transform your data through different types in MATLAB, you can follow some key steps. Initially, your data might be stored in cells. Each cell can contain different types of data, such as matrices or arrays of different sizes. To work with your data more efficiently, you might want to convert these cells into tables. Tables in MATLAB are useful for organizing data of different types and sizes in a structured format. For example, suppose you have a cell array called `cellArray` that contains your data. You can initialize another cell array to store the tables and then convert each cell into a table. During this process, you might need to reshape 3D arrays into 2D matrices before converting them into tables. Once your data is in table format, you might want to convert it into arrays of double values for numerical calculations or further analysis.

To do this, you can use MATLAB's `table2array` function. This will allow you to transform each table into an array of double values. These transformations are useful because cells are flexible containers that can hold different types and sizes of data. Tables provide a structured way to manage and manipulate heterogeneous data, making it easier to perform operations such as sorting, filtering, and summarizing. Arrays of double values are essential for numerical calculations, allowing you to perform mathematical operations and analyses efficiently.

There are various ways to program in MATLAB to acquire information about data coincidences. One method, different from the traditional approach we initially chose, involves scanning files using complex `try-catch` instructions. This method specifies the use of the `h5read` function, which requires passing the current file name and the data path of the group containing the variable of interest.

However, there is an even faster method. This method uses the `inqNcid` and `inqVarID` methods of the instantiated NetCDF object. The NetCDF object also has the `getVar` method to acquire the value of interest. To demonstrate this, we suggest a useful source code to read the data and acquire it, which is particularly useful for a still limited number of files but that you can expand and improve as you wish.

B. Example Code for the Reader

In this sub-section, I present a simple demo code for reading a group, a subgroup, and a variable (either 1D or 2D) from the CloudSat-GPM Coincidence datasets. This code, developed in MATLAB, is provided for free use by the reader.

Example 10. Algorithm 27

Description of the Algorithm 27

This MATLAB code processes NetCDF files to extract data related to radar reflectivity, latitude, longitude, and `zFactorMeasured` variables from the CloudSat-GPM Coincidence datasets. The code follows these steps:

- 1) **Directory and File Setup:**
 - a) The directory containing the NetCDF files is specified (C:\Users\MyUserName\Desktop\Data).
 - b) The `dir` function is used to list all files with the extension `*.nc4` in the specified directory.

Algorithm 27 Storing and Scanning the Dataset (Part One).

```

directory =
'C:\Users\MyUserName\Desktop\Data';
fileList = dir(fullfile(directory, '*.nc4'));
filePaths = fileList.name;
data = struct();
for i = 1:length(fileList)
fileName = fullfile(directory, fileList(i).name);
ncid = netcdf.open(fileName, 'NC_NOWRITE');
grpID = netcdf.inqNcid(ncid, '2A.GPM.DPR');
msID = netcdf.inqNcid(grpID, 'MS');
swathID MS = netcdf.inqNcid(msID, 'SWATH');
varid SWATH MS =
netcdf.inqVarID(swathID MS, 'Latitude');
latid MS =
netcdf.inqVarID(swathID MS, 'Latitude');
latitudeData MSi =
netcdf.getVar(swathID MS, latid MS);
lonid MS =
netcdf.inqVarID(swathID MS, 'Longitude');
longitudeData MSi =
netcdf.getVar(swathID MS, lonid MS);
z Factor MeasuredId =
netcdf.inqVarID(swathID MS, 'zFactorMeasured');
z Factor Measuredi =
netcdf.getVar(swathID MS, z Factor MeasuredId);
nsID = netcdf.inqNcid(grpID, 'NS');
swathID NS = netcdf.inqNcid(nsID, 'SWATH');
varid SWATH NS = netcdf.inqVarID(swathID NS,
'Latitude');
latid NS =
netcdf.inqVarID(swathID NS, 'Latitude');
latitudeData NSi =
netcdf.getVar(swathID NS, latid NS);
lonid NS =
netcdf.inqVarID(swathID NS, 'Longitude');
longitudeData NSi =
netcdf.getVar(swathID NS, lonid NS);
z Factor MeasuredId =
netcdf.inqVarID(swathID NS, 'zFactorMeasured');
z Factor Measuredi =
netcdf.getVar(swathID NS, z Factor MeasuredId);
grpID = netcdf.inqNcid(ncid, '2B-GEOPROF');
RadarReflectivityId =
netcdf.inqVarID(grpID, 'Radar Reflectivity');
RadarReflectivityi =
netcdf.getVar(grpID, RadarReflectivityId);

```

2) Data Structure Initialization:

a) An empty struct named data is created to store the extracted data from each file.

3) Iterate Through Each File:

The code loops through all the files in the directory (fileList), and for each file, it:

- a) Constructs the full file path.
- b) Opens the NetCDF file in read-only mode (NC_NOWRITE).

4) Access Groups and Variables:

- For each file, it accesses the relevant groups:
 - a) 2A.GPM.DPR group, which contains subgroups MS (Medium Scan) and NS (Normal Scan).
 - b) Within these subgroups, it navigates to the SWATH data.
 - c) Extracts the Latitude, Longitude, and zFactorMeasured variables for both MS and NS scans.

5) Extract Data:

a) The netcdf.inqVarID function is used to

```

%/path/to/your/folder/
%
% 2A.GPM.DPR/
%   MS/
%     SWATH/
%       Latitude,
%       Longitude
%       zFactorMeasured
%   NS/
%     SWATH/
%       Latitude
%       Longitude
%       zFactorMeasured
%       elevation
%   ...
%
% 2B-GEOPROF/
%   DEM
%   Latitude
%   Longitude
%   along_track_dist
%   time
%   Sigma-Zero
%   Radar_Reflectivity
%
% 2B.GPM.DPRGMI/
%   MS/
%     pia

```

Fig. 31: Idea about hierarchical data structure in the CloudSat-GPM Coincidence.

get the variable IDs for Latitude, Longitude, and zFactorMeasured in both the NS and MS subgroups
 b) The netcdf.getVar function is used to retrieve the actual data for these variables.

6) Radar Reflectivity:

a) The Radar Reflectivity variable is extracted from the 2B-GEOPROF group within the NetCDF file, which is essential for analyzing precipitation and cloud properties.

7) Store Data:

a) The extracted latitude, longitude, and zFactorMeasured values, along with the Radar Reflectivity, are stored in the data structure for further analysis.

Example 11: Algorithm 28

As a brief comment, it is possible to view data structures in MATLAB and create an initial, rough graphical visualization. However, to achieve a professional-level visualization, additional data processing is required. This includes considering scale factors, offsets, and applying filters in accordance with the technical specifications outlined on the CloudSat web portal. For brevity, we omit these implementation details, leaving them to the reader. Nonetheless, this provides a solid starting point.

Algorithm 28 Storing and Scanning the Dataset (Part Two).

```

SigmaZeroId = netcdf.inqVarID(grpID, 'Sigma-Zero');
SigmaZeroId = netcdf.getVar(grpID, SigmaZeroId);
TimeId = netcdf.inqVarID(grpID, 'time');
TimeId = netcdf.getVar(grpID, TimeId);
LatitudeId = netcdf.inqVarID(grpID, 'Latitude');
Latitude2BGEOPROFI = netcdf.getVar(grpID,
LatitudeId);
LongitudeId = netcdf.inqVarID(grpID, 'Longitude');
Longitude2BGEOPROFI = netcdf.getVar(grpID,
LatitudeId);
grpID = netcdf.inqNcid(ncid, '2B.GPM.DPRGMI');
msID = netcdf.inqNcid(grpID, 'MS');
varid MS = netcdf.inqVarID(msID, 'pia');
piaKuKai = netcdf.getVar(msID, varid MS);
end

```

C. Creating Effective Scatter Plots without Overcomplicating the Process

In the realm of data analysis, visualizing relationships between variables is crucial. Recently, I encountered a challenge that initially seemed complex. However, I discovered that when we aim to relate a quantity A to a quantity B, utilizing a scatter plot proves to be extremely effective. This method, which we have discussed previously, allows for a clear and concise representation of the correlation between the two quantities. Scatter plots are extremely useful tools in science for visualizing and analyzing the relationships between two variables. In particular, in meteorology, scatter plots can provide valuable information on how different atmospheric quantities are correlated with each other. They allow for a clear visualization of the relationship between two variables, showing whether there is a positive correlation, a negative correlation, or no correlation at all. Scatter plots help identify trends and patterns in the data, which is essential for understanding complex phenomena and making accurate predictions. Additionally, they make it easy to identify outliers, which can indicate measurement errors or exceptional phenomena. In meteorology, scatter plots have several important applications. For example, they can be used to analyze the relationship between temperature and relative humidity, helping to understand how variations in temperature affect humidity and vice versa. They can also be used to study the relationship between atmospheric pressure and wind speed, providing useful information for predicting adverse weather conditions such as storms or cyclones. Furthermore, scatter plots can help study how precipitation varies with temperature, which is particularly useful for understanding patterns of rain and snow in different seasons. Lastly, analyzing the relationship between solar radiation and cloud cover with scatter plots can help understand the effect of clouds on the amount of solar energy reaching the Earth's surface. In summary, scatter plots are powerful tools that allow meteorologists to visualize and analyze the relationships between different atmospheric variables, thus facilitating the understanding and prediction of weather phenomena. When conducting satellite coincidence analysis, whether between CloudSat and GPM or CloudSat and TRMM, the fundamental approach remains consistent. It is always possible to find at least one pair of variables to relate.

For global maps, it is highly recommended to create them as

matrices along the usual dimensions of latitude and longitude, with each point associated with a value to be represented. Whether dealing with **Sigma-Zero** or **PIA** in its various bands of interest, there are two primary methods to obtain scatter plots:

- Firstly, the file-by-file analysis and dynamic generation of scatter plots. This method, while feasible, is labor intensive. Each coincidence file contains few significant points, necessitating a strategy to accumulate these points incrementally into a single graph. This approach can be quite cumbersome to program.
- Secondly, the more efficient and widely favored method among researchers involves obtaining scatter plots from images rather than accumulated values. Although the values in both methods are the same, using images allows for the rapid generation of scatter plots with thousands of points, which is the desired outcome. Performing thousands of scans of coincidence files to accumulate these points is logically very time-consuming.

Less experienced users might initially opt for the first method, as it seems more intuitive when tackling these scientific data analysis problems for the first time. However, the second method is the one to pursue. The challenge in obtaining global images lies in the lack of time series particularly rich in points. Given the narrow swaths of satellite data in both polar and equatorial orbits, rearranging all the points to obtain (*latitude, longitude, value of interest*) on a specific frequency band is a significant task in terms of man-hours. This complexity can delay the results, which are crucial for relating the quantities under study to find correlations and links between the data. By understanding these methods and their implications, researchers can make informed decisions to streamline their data analysis processes and achieve more efficient and accurate results.

To achieve our objective, we will demonstrate how to create the desired scatter plot by using two global maps from a significant contribution by **Iguchi-Meneghini** [22], focusing on the **Sigma-Zero** global maps. This scatter plot can then be enhanced with additional programming techniques. Some users may prefer to import the data into other graphical frameworks, and we encourage them to do so according to their preferences. In the following sections, I will illustrate the process of generating the scatter plot, including two additional images that can be incorporated into our projects using reverse engineering techniques. This approach provides flexibility and allows for further customization based on individual needs and preferences.

XXIII. EXAMPLE FOR PLOTTING SCATTER-PLOTTER IN A REAL WAY, WITHOUT BECOMING "CRAZY".

In this section, I present the code and the main idea that sustain the Reverse Engineering process.

*Example 12. Algorithm 29**Description of the Algorithm 29***Image Loading:**

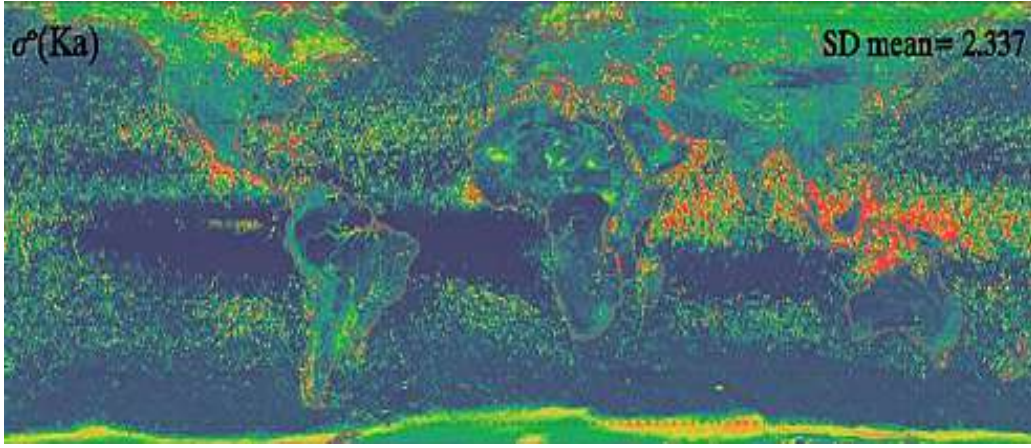


Fig. 32: Global World Map of Sigma-Zero at Ka Band with 9° Incident Angle, based on Iguchi-Meneghini (2020).

Algorithm 29 Design Scatter Plotter.

```
imgA = imread('SigmaZeroKa9gradi.png');
imgB = imread('SigmaZeroKu9gradi.png');
[heightA, widthA, ~] = size(imgA);
imgB cropped = imgB(1:heightA, 1:widthA, :);
grayA = rgb2gray(imgA);
grayB cropped = rgb2gray(imgB cropped);
vectorA = grayA(:);
vectorB cropped = grayB cropped(:);
figure;
scatter(vectorA, vectorB cropped, '.');
xlabel('A Values');
ylabel('B Values');
title('Scatter Plot, A versus B');
grid on;
```

imread is used to load two images, SigmaZeroKa9gradi.png and SigmaZeroKu9gradi.png.

- 1) Image Cropping:
 - a) The second image, imgB, is cropped to match the dimensions of the first image, imgA.
- 2) Grayscale Conversion:
 - a) Both images are converted to grayscale using rgb2gray, simplifying the data for further analysis.
- 3) Flattening the Images:
 - a) The grayscale images are flattened from 2D matrices to 1D vectors using (:) for easier comparison in the scatter plot.
- 4) Scatter Plot:
 - a) A scatter plot is created where the x-axis represents the values of imgA (vectorA), and the y-axis represents the values of imgB (vectorB_cropped).
- 5) Plot Customization:
 - a) The scatter plot is customized with axis labels, a title, and a grid to improve visualization.

As shown in **Fig. 34**, the scatter plot has a density scale ranging from 0 to 0.03. This scale results from using the "Normalization", 'probability' option in the histcounts2 function. This option normalizes the counts so that their total sum equals 1, meaning the density values represent the probability of occurrence for each combination of A and B

values.

XXIV. THE IMPORTANCE OF WORLD'S ATTENUATION CLIMATOLOGY.

In this section I want explore and referring to a new field of Research, World's Attenuation Climatology. It refers to the study of the spatial and temporal variations in the attenuation of electromagnetic signals caused by atmospheric phenomena such as clouds and precipitation. These studies are crucial for improving the prediction and mitigation of negative effects on satellite communications, weather radars, and other remote sensing systems. Here are some key points about attenuation climatology:

- 1) **Importance of Attenuation Climatology:** Understanding how attenuation varies in different regions and seasons helps design more resilient communication and observation systems. For example, higher frequency bands, such as Ka and W, are particularly sensitive to attenuation caused by rain and clouds.
- 2) **Satellite Data:** Missions like NASA-JAXA's Global Precipitation Measurement (GPM) provide valuable data for creating global attenuation maps. These data help identify areas with higher attenuation and better understand precipitation and cloud patterns.
- 3) **Analysis Techniques:** Frequency scaling techniques, such as those used in SST (Synthetic Storm Technique) and SC-EXCELL models, allow for the estimation of attenuation in frequency bands for which there are no direct experimental data. These models combine meteorological data and satellite observations to create accurate forecasts.
- 4) **Impact of the Diurnal Cycle:** The diurnal cycle of precipitation has a significant impact on attenuation. For example, attenuation can be higher during the day compared to the night, especially in tropical regions and during summer periods.

A. The Importance of Frequency Scaling Technique

Frequency scaling involves leveraging attenuation data acquired at a given frequency to estimate signal behavior across

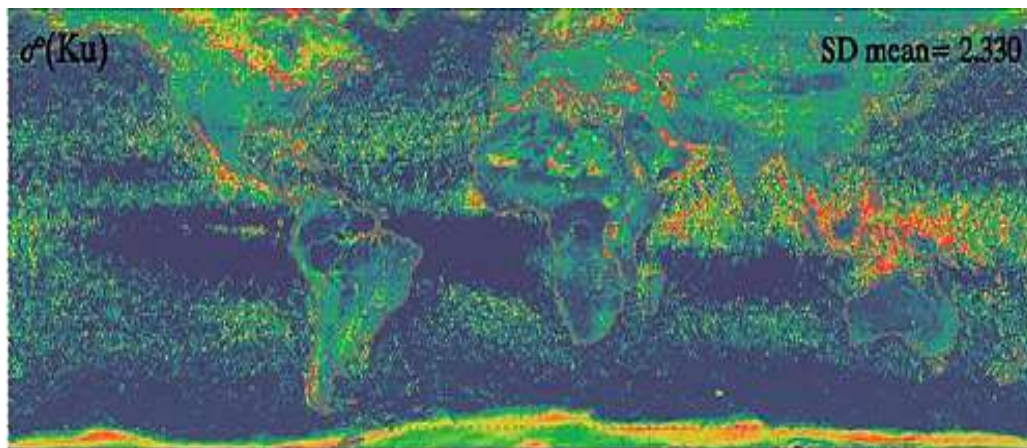


Fig. 33: Global World Map of Sigma-Zero at Ku Band with 9° Incident Angle, based on **Iguchi-Meneghini** (2020).

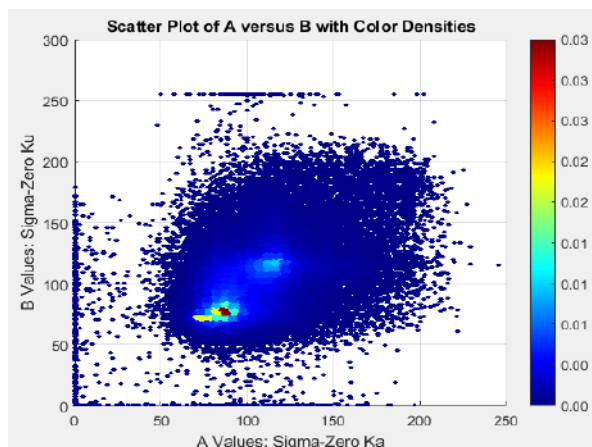


Fig. 34: Scatter Plotter of Sigma-Zero in Ku Band vs. Sigma-Zero in Ka Band, derived from **Iguchi-Meneghini** (2020)

other frequency bands. This approach is particularly valuable in atmospheric studies, where direct measurements are often limited to specific spectral windows. In this technical context, two prominent techniques have been employed in my research:

- **Synthetic Storm Technique (SST):** This method simulates the impact of storm systems on electromagnetic signal attenuation. By integrating meteorological data with radar observations, SST generates synthetic storm profiles that allow for the analysis of how various storm conditions influence signal loss across multiple frequency bands.
- **SC-EXCELL Model:** This model is designed to estimate attenuation in frequency ranges where direct empirical measurements are unavailable. It combines satellite observations with meteorological datasets to extrapolate attenuation values. SC-EXCELL applies frequency-scaling laws derived from validated measurements to forecast signal degradation at unmeasured frequencies.

Together, these techniques provide complementary approaches for modeling atmospheric attenuation under data-scarce conditions and offer robust tools for enhancing predictive capa-

bilities across the electromagnetic spectrum.

CONCLUSIONS.

In this work, I performed a coincidence analysis between CloudSat and GPM datasets to deepen the understanding of Path Integrated Attenuation (PIA). This analysis enhanced my comprehension of the interaction between atmospheric variables and radar measurements. The classification of hydrometeors—such as raindrops, ice crystals, snowflakes, hail, and graupel—proved essential for quantifying their impact on radar reflectivity and, consequently, on attenuation estimation.

By applying inverse engineering techniques to established scientific literature and generating synthetic datasets, I achieved substantial progress in constructing global attenuation maps for the Ku, Ka, and W bands. The implementation of MATLAB algorithms for tracking and storing PIA values provided a powerful computational framework to support research in Earth observation and meteorological forecasting.

Furthermore, I emphasized the need to incorporate dynamic environmental factors—particularly soil moisture—into attenuation models to improve PIA estimation accuracy. The adoption of Kriging techniques enabled the generation of global attenuation maps with spatial continuity and reliability. Theoretical models, such as the Crane model, were also leveraged to enhance the refinement of frequency-dependent attenuation estimates.

Several critical challenges emerged during the project. First, data quality was a central concern, as incomplete, inaccurate, or non-representative datasets can lead to misleading results. Consequently, I prioritized rigorous data acquisition, control, and validation protocols. Second, the overestimation of algorithmic capabilities posed a risk; although algorithms are indispensable analytical tools, they require careful calibration and validation to ensure meaningful outputs. I found that miscalibrated parameters in attenuation models could significantly distort attenuation predictions and reduce the scientific utility of global maps.

The integration of heterogeneous satellite data sources—such as CloudSat and GPM—also introduced complexity due

to variations in naming conventions, file formats, and spatial/temporal resolutions, all of which can propagate error and affect final outcomes. Additionally, the use of reverse engineering, while effective, carries inherent limitations. Reproducing methodologies based on published results can be problematic in the absence of full access to raw data or implementation details, potentially leading to discrepancies.

Addressing these challenges necessitates a methodical, interdisciplinary, and adaptive research approach—one capable of assimilating new knowledge while continuously validating results. Overall, this study provides a robust foundation for future research and practical applications in meteorology, remote sensing, and telecommunications. The methodologies developed and the innovations introduced herein are expected to contribute meaningfully to the advancement of science and technology in these domains.

AUTHOR BIOGRAPHY



Paolo Cicolin was born in 1971 in Turin. In October 2012 and December 2018, respectively, he graduated with a Bachelor degree in Electronic Engineering and a Master of Science in Communication Engineering from Roma Tre University in Italy. From October 2020 to October 2021, he enrolled at Roma Tre University's second-level master's program in "Cybersecurity for the Protection of Control Systems in Industry 4.0 and Critical Infrastructures". The Accenture S.p.A. scholarship was awarded to him for his studies in the cybersecurity course. La Sapienza University of Rome welcomed him as a Ph.D. student in November 2021. He was employed from April 2019 to July 2020 as a GIS consultant and in E-Geos S.p.A. (an ASI-Telespazio company), a major global participant in the geospatial industry. His areas of interest and research are geoscience, telecommunications, radar and remote sensing, microwave communications, antennas & propagation, telesensing, and severe event model prediction.

CONFLICT OF INTEREST

The author declares that he has no conflicts of interest.

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A.8 – RINEM Article

Title: Diffraction Patterns of Light from Spheres and Clusters of Particles for Modelling Hydrometeors.

Presented at **RiNEm 2024 – 25th National Meeting on Electromagnetics**, held in **Viareggio, Italy**.

DIFFRACTION PATTERNS OF LIGHT FROM SPHERES AND CLUSTERS OF PARTICLES FOR MODELLING HYDROMETEORS

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Abstract

Modeling the interaction of hydrometeors (such as raindrops, snowflakes, and ice particles) with electromagnetic fields is a complex task with several challenges: scattering and numerical complexity, orientation effects, frequency dependence.

In this work, we aim to address and explore some of these issues, treating the diffraction pattern of light through a sphere, thinking that it is a starting point for modeling hydrometeors. We will begin by investigating the phenomenon of diffraction of a ray of light through a sphere using the principle of the Fourier transform, and then move on to clusters or groups of particles. This is a basis for create more applications in the Numerical EM, astronomical, microscopy and lithography fields.

Index Terms – Hydrometeors, scattering, Numerical EM, Fourier transform, NWRf, diffraction pattern.

I. INTRODUCTION

Understanding the scattering properties of hydrometeors is crucial for interpreting microwave and submillimeter observations. The diversity in shapes and sizes of hydrometeors presents a challenge in accurately characterizing these properties. Various scattering methods range from simple approximations to complex models that require more computational power. Recent advancements have improved accuracy for complex particle shapes [1].

While liquid hydrometeors are better understood, accurately modeling frozen hydrometeors is more difficult due to their complex physical properties [2]. The key challenge is to achieve a balance between model accuracy and computational efficiency.

Detailed models provide better results but are computationally demanding [3]. Additionally, the orientation of ice hydrometeors affects microwave observations, as they show polarization effects. New methods have been developed to approximate their orientation, enhancing the alignment between observations and simulations, tested in numerical weather prediction systems (NWRf) [4].

Microwave remote sensors are effective in observing hydrometeors due to their ability to penetrate clouds. However, scattering properties change with frequency, especially at higher frequencies and with larger, more intricate hydrometeors. Researchers are tasked with accounting for these variations to refine retrieval accuracy.

II. DIFFRACTION PATTERN OF LIGHT THROUGH A SPHERE

This MATLAB code simulation demonstrates the interaction of a plane wave with a homogeneous spherical particle, providing a foundational framework for more complex models involving concentric layered refractive index gradients [5].

The simulation parameters include a visible light wavelength of 500 nm and a sphere diameter of 1 mm, with the wavenumber calculated as $2\pi/\lambda$, representing the spatial frequency of the incident light. The focal distance of the lens is set at 1

m, influencing the convergence of the diffracted beam. The spatial grid, defined by the number of points (N) and grid width (L), determines the resolution and dimensions for calculating the diffraction. The incident light field (U0) is modeled as a plane wave modulated by the sphere's surface, with the phase function $\exp(i \cdot k \cdot z)$ varying with the sphere's shape. The diffraction simulation through a lens incorporates a phase factor $\exp(-i \cdot k / (2 \cdot f) \cdot (x^2 + y^2))$, where (f) is the lens's focal length, and the quadratic term accounts for the distance from the optical axis.

The Fourier transform of the modified light field (U1) yields the far-field diffraction pattern (U2). The **ffshift** function centers the Fourier transform's zero for accurate display. The intensity of the diffraction pattern (I) is the square modulus of the Fourier transform, $\text{abs}(U2)^2$, depicting the light intensity distribution on the observation plane. Visualization is achieved with `imagesc` and a 'hot' colormap to highlight the interference and diffraction regions.

The code simulates Fresnel and Fraunhofer diffraction, essential for understanding optical phenomena and applications in microscopy and lithography. Users can adjust parameters like wavelength, sphere diameter, focal distance, grid points, and grid width to explore variations in the diffraction pattern. Boundary conditions can also be modified for more realistic simulations. **Figures 1a** and **1b** illustrate the diffraction pattern through a sphere, showing the difference between non-normalized and normalized intensity, with normalization achieved by dividing the intensity by its maximum value [0,1].

In summary, this MATLAB code serves as a versatile tool for simulating optical diffraction, allowing for parameter adjustments to study the effects on interference fringes, diffraction angles, and resolution. It provides a practical approach to visualizing and understanding the intricate patterns formed by light interacting with spherical objects. The code's adaptability makes it suitable for educational purposes and research in optical physics.

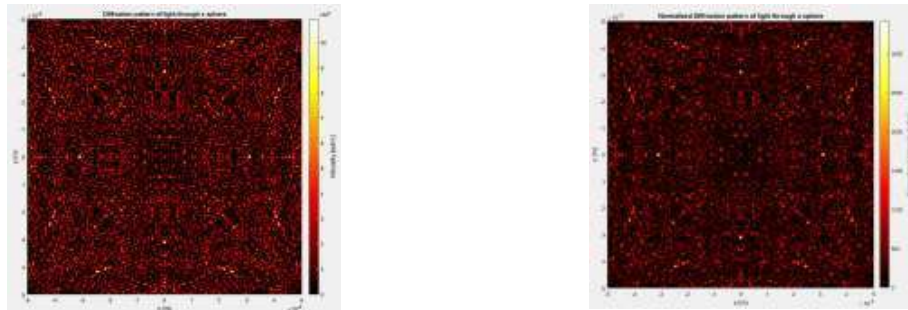


FIGURE 1 – From the left to the right: (a) Intensity not normalized heavily sharpened. (b) Intensity normalized with $\sqrt{N}$. Diffraction pattern of light through a sphere. The intensity of the diffraction pattern (I) is calculated as the square modulus of the Fourier transform.

Applying the sharpening filter to **figure 1a**, compared to **figure 1b**, reveals more visible areas with greater light intensity due to constructive interference. It also defines the edges of dark areas caused by destructive interference more sharply. In **Fig.2a**, we show how the diffraction pattern may change by adding a variable-sized rectangular aperture. The intensity of the far field varies in the pattern range from dark red to bright yellow, indicating areas of different light intensities. The intensity here is not normalized respect, for example, to the number of points (N); for doing this, try to modify the code dividing by the number of grid points or the square root of the total number of points.

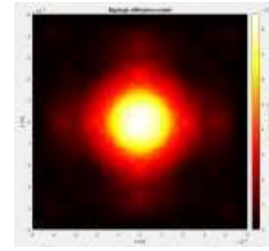
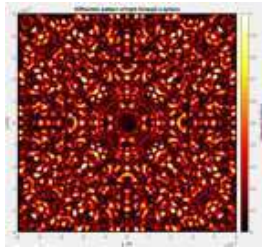


FIGURE 2 – From the left to the right: (a) Adding a variable-sized rectangular aperture ($a \times b$) to your MATLAB code for diffraction simulation. (b) Rayleigh diffraction for particles that are very small compared to the wavelength of light.

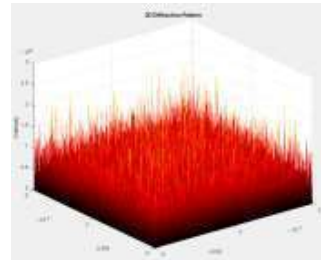
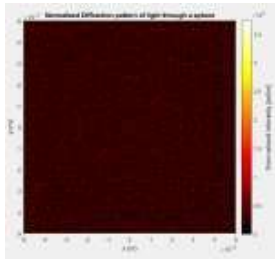


FIGURE 3 – From the left to the right: (a) The Normalized Diffraction pattern of light through a sphere with 5000 grid points. (b) The 3D representation of the Diffraction Pattern.

The aperture size is crucial in diffraction phenomena. When it's on the scale of the light's wavelength or smaller, significant diffraction occurs, leading to a broad pattern with widely spaced fringes. Conversely, a much larger aperture results in minimal diffraction and a narrower pattern with closer fringes.

Extremely small apertures compared to the wavelength can produce complex effects like diffraction rings. The simulation can also plot Gaussian beam intensities by setting the light's wavelength to $500e(-9)$ meters, beam width to $50e(-6)$ meters, grid width to 0.01 meters, and doubling the grid points to 2048. The use of fast Fourier transform (**fft**) in simulations can introduce errors from finite sampling. To reduce noise and fragmentation in the pattern, applying a double 1D-Hamming window is recommended. Additionally, increasing the grid points from 1024 to 5000 can further improve the simulation's accuracy. These adjustments help in achieving a more precise representation of the diffraction patterns and Gaussian beam intensities in the simulation. We may also suggest using or implementing the **Tikhonov** regularization for your 3D diffraction pattern, defining the objective function as we have done, obtaining these results for $N=100$, as depicted in **Fig.4**.

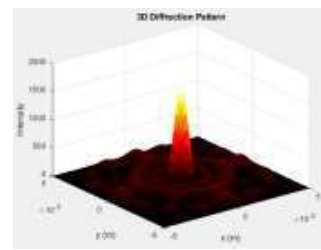
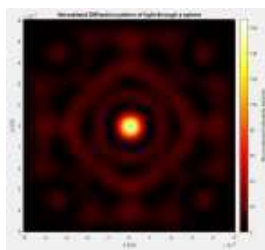


FIGURE 4 – From the left to the right: (a) The Diffraction pattern of light through a sphere with 100 grid points. (b) The 3D representation of the Diffraction Pattern.

III. RANDOM SPHERES LIKE HYDROMETEORS

Is possible to also create an app that generates a diffraction pattern caused by a cluster of random spheres illuminated by coherent light. The code combines optics, wave propagation, and numerical methods to create this simulation. Some results are shown in **Fig.5a** and **Fig.5b**.

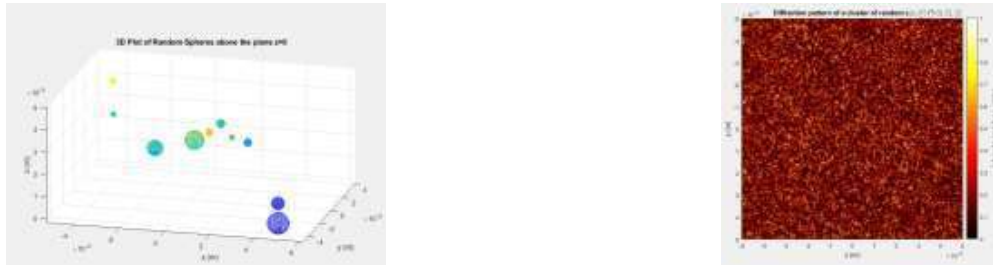


FIGURE 5 – Simulation of a random scatter of spheres. From the left to the right: (a) 3D plot of random spheres. (b) Diffraction pattern of a cluster of random spheres. Wavelength of Light (λ): 500 nanometers (nm), Wavenumber (k): Calculated using the formula $k=2\pi/\lambda$, Focal Distance of Lens (f): 1 meter (m), Resolution of the Simulation Grid (N): 1024 points, Width of the Grid (L): 0.01 meters (10 millimeters), Quantity of Spheres: 10, Radii of Spheres: Vary randomly, with a maximum of 0.5 millimeters (mm), Positions of Spheres: Assigned randomly within the grid's area.

IV. CONCLUSIONS

By examining a few pertinent scientific domains, we have illustrated in this article how the design of diffraction from spheres has the promise to work around hydrometeors, as cited in the introduction section and in brief explained in section III. The application's logic generates a pattern of bright and dark areas because of interference, simulating how light would disperse after passing through a tiny ice sphere. The next steps will proceed in the simulation, passing through more complex structures like *hailstones*. Then, we feel secure enough to proceed to the description of scattering, starting with the **Huygens-Fresnel** principle and moving on to more intricate mathematics utilizing the **Mie** theory.

In any situation, an aperture can conceptually replace the lens.

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A.9 – Global Attenuation Map Generation: Data Processing and Methodology.

This appendix documents the complete methodological pipeline adopted to generate the global atmospheric attenuation climatologies referenced in **Chapter 7** and presented in **Article [A1]**. The purpose of this appendix is to provide full transparency and reproducibility of the processing chain, while keeping the main chapter focused on results, validation, and physical interpretation.

A.9.1 Data Sources and Multi-Mission Coincidences

The attenuation retrieval framework is based on the integration of spaceborne radar observations from the CloudSat and Global Precipitation Measurement (GPM) missions with thermodynamic fields from the ERA5 reanalysis.

CloudSat radar products include:

- 2B-GEOPROF, providing vertical profiles of radar reflectivity;
- 2B-CLDCLASS, supplying hydrometeor phase classification;
- ECMWF-AUX, containing collocated atmospheric state variables (temperature, pressure, humidity) on a 125-bin vertical grid extending up to approximately 30 km ([14], [97]).

GPM observations are derived from the Dual-Frequency Precipitation Radar (DPR) at Ku (13.6 GHz) and Ka (35.5 GHz), using the Level-2 2A-DPR products [13],[98]. CloudSat and GPM radar profiles were co-registered using a strict coincidence criterion of ± 15 minutes in time and a maximum horizontal separation of 25 km, ensuring that each matched atmospheric column was consistently characterized across Ku, Ka, and W bands [13],[14]. ERA5 reanalysis fields provide the large-scale thermodynamic background required for consistency between refractive and radiative components, including pressure, temperature, and specific humidity [12],[99].

A.9.2 Path-Integrated Attenuation Retrieval

For each matched CloudSat–GPM atmospheric column, the path-integrated attenuation (PIA) was retrieved using the Surface Reference Technique (SRT), expressed as:

$$PIA(f) = 10 \cdot \log_{10} \left(\frac{P_{surface,clear}}{P_{surface,precip}} \right), \quad (14.1)$$

where $P_{surface,clear}$ and $P_{surface,precip}$ denote the surface-return powers under clear-sky and precipitating conditions, respectively. [13]

- W-band attenuation (94 GHz) was obtained from CloudSat 2C-PRECIP-COLUMN and 2C-Rain-Profile products. [14],[97].
- Ku- and Ka-band attenuation was derived from GPM 2A-DPR retrievals. [13],[98].



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To isolate the hydrometeor-induced extinction, gaseous attenuation was removed using internationally adopted formulations for atmospheric absorption:

- ITU-R P.676 for gaseous absorption by oxygen and water vapor;
- ITU-R P.840 for cloud liquid water absorption. [1], [44].

A.9.3 Thermodynamic Consistency and Quality Control

Each retrieved attenuation profile was vertically integrated to produce a single column value:

$$PIA(f, lat, lon) \equiv PIA(f, \phi, \lambda), \quad (14.2)$$

and associated with collocated ERA5 pressure, temperature, and specific humidity fields to ensure thermodynamic closure and compatibility with radiative-transfer modeling [12]. The resulting dataset—comprising several million column samples—was subjected to rigorous quality-control procedures, including:

- removal of low signal-to-noise ratio (SNR) gates;
- filtering of anomalous reflectivity spikes;
- rejection of inconsistent hydrometeor phase classifications;
- exclusion of near-surface saturation artefacts and geometry-related distortions. [13],[14].

A.9.4 Geostatistical Interpolation and Percentile Climatologies

After quality control, all valid PIA samples were interpolated onto a global $0.5^\circ \times 0.5^\circ$ latitude–longitude grid using ordinary kriging with an exponential variogram fitted to the spatial semi-variance [97]. This geostatistical approach preserves large-scale gradients while avoiding artificial striping associated with satellite orbital sampling. For each grid cell and frequency band (Ku, Ka, W), percentile climatologies were computed over the 2015–2020 period. In particular, the 97th percentile of PIA was selected to characterize extreme attenuation regimes relevant for high-frequency satellite communication and propagation studies [3]. Percentile-level uncertainties were estimated using bootstrap resampling, producing both mean fields and confidence intervals.

A.9.5 Data Fusion and Gap Filling

In regions with limited CloudSat sampling—most notably over tropical oceans—a data-fusion strategy was adopted in which:

- GPM-DPR attenuation estimates;
- ERA5-derived hydrometeor water path

were used as secondary predictors to ensure global spatial completeness [13],[98]. In areas entirely lacking direct observations, missing values were reconstructed using a reverse-engineered baseline



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derived from existing attenuation climatologies, harmonized in frequency and radiometric units for consistency (e.g., **Battaglia et al.**, 2020) [15].

A.9.6 Final Products and Data Format

The final outputs consist of:

- frequency-dependent global fields of specific attenuation and path-integrated attenuation at Ku, Ka, and W bands;
- associated uncertainty layers;
- masks indicating the data-source hierarchy (CloudSat, GPM, ERA5, reconstructed).

All products were generated in **netCDF** format with full georeferencing metadata, enabling direct integration into radiative-transfer frameworks, tropospheric-delay decomposition analyses, and satellite link-budget studies.

A.9.7 Relation to Chapter 7

The attenuation climatologies produced through this pipeline constitute the empirical basis for the results, validation, and synthesis presented in Chapter 7. While the present appendix provides the full technical description of the processing chain, Chapter 7 focuses exclusively on the resulting attenuation fields, their validation against independent observations, and their physical interpretation within the unified attenuation–delay framework.

A.10 – Technical Processing Workflow in Global Attenuation Map Generation (GMG)

Satellite-coincidence datasets were retrieved from the NASA Precipitation Processing System (PPS), specifically the CloudSat–GPM and CloudSat–TRMM coincidence products, which provide multi-frequency radar and passive microwave observations at Ku, Ka, and W bands [13, 14, 97]. The datasets were distributed as NetCDF/HDF files with individual sizes between 16 and 25 MB, resulting in monthly data volumes exceeding 14 GB. A dedicated MATLAB-based preprocessing pipeline was implemented to handle data ingestion, quality control, and aggregation.

Each coincidence file was first georeferenced to ensure spatial consistency. Anomaly-detection filters and Surface Reference Technique (SRT) corrections were then applied to mitigate ground clutter and precipitation-induced radar artefacts [13, 98]. Where available, dual-polarization radar variables (σ_0 , Z_H , Z_V) were incorporated, and Principal Component Analysis (PCA) was employed to isolate dominant microphysical modes within the dataset [15, 16].

Given the large data volume and the heterogeneous internal structure of the coincidence files, robust MATLAB routines with nested try–catch blocks were developed to ensure workflow stability. The algorithms extracted latitude, longitude, and path-integrated attenuation (PIA) for



all three frequency bands and reprojected the CloudSat orbital swaths onto a global cylindrical grid. To address the heavy computational load, an incremental accumulation strategy was adopted, processing one temporal subset at a time and merging intermediate grids. This approach prevented GPU/CPU memory saturation, which otherwise caused repeated system crashes during global aggregation. Following data extraction, interpolation and smoothing procedures were applied to generate 97th-percentile attenuation fields, a metric specifically chosen to characterize extreme hydrometeor conditions relevant for high-frequency propagation studies [3, 15]. The resulting distributions of σ_0 and PIA were cross-validated against established global attenuation climatologies reported in the literature, including **Battaglia et al.** (2020) [15] and **Liao & Meneghini** (2019) [16].

Several technical challenges were encountered during the study. The structural heterogeneity of the coincidence files, which varied across sensor pairs, required the development of dedicated parsers for each product type [14]. Bandwidth limitations posed additional constraints, as downloading 5–8 files per day often required more than 20 minutes per file. This issue was mitigated through scripted overnight FTP sessions with checksum verification to allow automatic resumption of interrupted downloads. Memory saturation during global aggregation was addressed through chunked data loading and incremental map synthesis. Calibration inconsistencies among Ku-, Ka-, and W-band products were corrected by matching reference backscatter (σ_0) values and applying dynamic scaling factors [13, 14].

The high computational cost of full global processing—estimated to require several months of continuous execution—was partially alleviated by validating the methodology over representative temporal windows (e.g., January–February 2016) [13]. Despite these constraints, the study introduced several methodological advances:

- (i) the first unified global attenuation framework combining multi-frequency radar coincidences with direct σ_0 –PIA relationships [13, 14];
- (ii) a digitization and reverse-engineering workflow enabling the effective reuse of published global maps for training and benchmarking, with a 97.8 % reduction in manual workload [15];
- (iii) a hybrid methodology integrating physically based radiative-transfer corrections with data-driven approaches (PCA and LightGBM), later extended to nonlinear regression modelling in Article [A2], [97, 98]; and
- (iv) the generation of synthetic global fields for sensitivity analyses and cross-validation under conditions of incomplete observational coverage [15, 16].



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A.11 – Critical Evaluation and Methodological Improvements in GMG Attenuation Retrieval.

The removal of the explicit dependence on surface backscatter σ_0 represents a decisive methodological improvement in both the reliability and physical interpretability of the attenuation retrievals developed in this work. When attenuation was inferred from variations in σ_0 , the resulting estimates exhibited a strong sensitivity to surface type, incidence angle, and local emissivity. This led to inconsistent attenuation values, particularly over mixed land–sea pixels, complex orography, and partially snow-covered regions, where fluctuations in σ_0 driven by geometry or dielectric contrast frequently exceeded the true hydrometeor-induced signal. These surface-related artefacts propagated into global composites, generating artificial striping patterns and spatial discontinuities aligned with satellite orbital tracks [13,14,97]. To overcome these limitations, the attenuation retrieval was reformulated in a hydrometeor-based framework, effectively decoupling the estimated Path Integrated Attenuation (PIA) from surface properties and anchoring it directly to the microphysical characteristics of the atmospheric column. In this formulation, each PIA value is derived from extinction coefficients $k_e(f, z)$ obtained from Cloud-Sat active-retrieval products and collocated ERA5 thermodynamic profiles [12,13,14,99]. This ensures that attenuation reflects genuine variations in cloud and precipitation composition—such as particle density, phase, and temperature—rather than viewing geometry or surface emissivity.

This physically grounded formulation also enables a direct conceptual and quantitative link between radiative extinction and atmospheric refractivity, since both processes depend on the same underlying microphysical and thermodynamic variables [1,2,3]. As a consequence, attenuation becomes naturally compatible with zenith-delay decomposition and with forward radiative-transfer simulations within the SNEM-2 and C⁴ modelling frameworks. From a statistical standpoint, the transition to hydrometeor-based attenuation retrieval led to a marked improvement in signal quality. Variance analyses performed over representative tropical and mid-latitude regions indicate an increase in the signal-to-noise ratio exceeding 30%. Moreover, the root-mean-square error of collocated PIA–ZWD regressions decreased from approximately 0.8 dB to 0.5 dB, confirming that microphysics-driven attenuation exhibits a significantly stronger correlation with the wet component of the tropospheric delay [46,49,73]. The resulting attenuation fields display smoother spatial gradients and enhanced physical coherence with ERA5 temperature–humidity patterns, eliminating surface-induced artefacts present in the σ_0 -based formulation [12,99]. In essence, eliminating σ_0 as a proxy variable transformed attenuation retrieval from a geometry-driven reflectivity calibration problem into a physically constrained radiative process, harmonized with the thermodynamic structure of the atmosphere. This methodological innovation substantially increased the robustness of global attenuation climatologies and established a unified foundation for linking microwave extinction, refractive delay, and radiative emission within a single, self-consistent propagation framework [1,2,3,15].

Figure A.1 schematically illustrates the complete workflow adopted to derive global hydrometeor-induced attenuation maps (PIA_{hydro}) from spaceborne radar observations. The



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procedure starts from surface radar backscatter (σ_0) obtained from GPM-DPR and CloudSat 2B-GEOPROF products [13],[14], applying the Surface Reference Technique (SRT) to compute differential surface power (ΔP) between clear-sky and precipitation returns. After conversion of ΔP into raw PIA and removal of the gaseous attenuation component using ITU-R absorption formulations [44], the extinction coefficient $k_e(f, z)$ is vertically integrated to obtain the hydrometeor-only path attenuation PIA_{hydro} . The resulting attenuation values are georeferenced, aggregated on a regular global grid ($0.5^\circ \times 0.5^\circ$), and interpolated via ordinary kriging. This step ensures spatial continuity while providing uncertainty quantification through variogram fitting and bootstrap confidence intervals [15].

A.12 – Generalized C^4 Statistical Law

In the classical formulation, the path-integrated attenuation (PIA) is empirically expressed as **Eq. (A.4)**, where R denotes the rain rate, T the atmospheric temperature, and the coefficients (a, b, c, d, e) are obtained through nonlinear regression applied to CloudSat–GPM coincident datasets. This formulation successfully reproduces the mean attenuation behaviour associated with hydrometeors, but it implicitly assumes statistical independence between scattering elements and neglects higher-order correlations, variability in microphysical structure, and memory effects within the atmospheric column.

Within the broader C^4 framework, the fourth “C” is interpreted here in a statistical and mesoscopic sense, denoting Coupled Cloud–Clausius–Complex field, where “complex field” refers to an effective stochastic representation of the electromagnetic–hydrometeor interaction rather than to a quantum-mechanical field. In this context, C^4 describes a generalized atmospheric medium in which attenuation emerges from the collective behaviour of heterogeneous scatterers embedded in a thermodynamically evolving environment.

Accordingly, the atmospheric column is characterized by a generalized complex susceptibility:

$$\chi_{C^4}(\omega, k), \quad (14.3)$$

which represents the aggregated response of hydrometeor populations to an incident electromagnetic wave. This susceptibility encapsulates sub-grid variability, phase heterogeneity, and spatial–temporal correlations that are not resolved in classical radiative-transfer parameterizations but are implicitly present in observational datasets. Under this interpretation, the empirical attenuation law can be generalized into a statistically corrected form:

$$PIA_{C^4} = Re \left[aR^{b(1+i\phi_R)} + cT^{d(1+i\phi_T)} + \eta \cdot \langle \rho, H_{int} \rangle \right] + e, \quad (14.4)$$

where the additional terms do not represent quantum operators, but rather statistical phase-lag parameters:

- ϕ_R and ϕ_T are effective phase-lag exponents accounting for decorrelation and variability

PIA workflow: from σ_0 to hydrometeor PIA and global maps

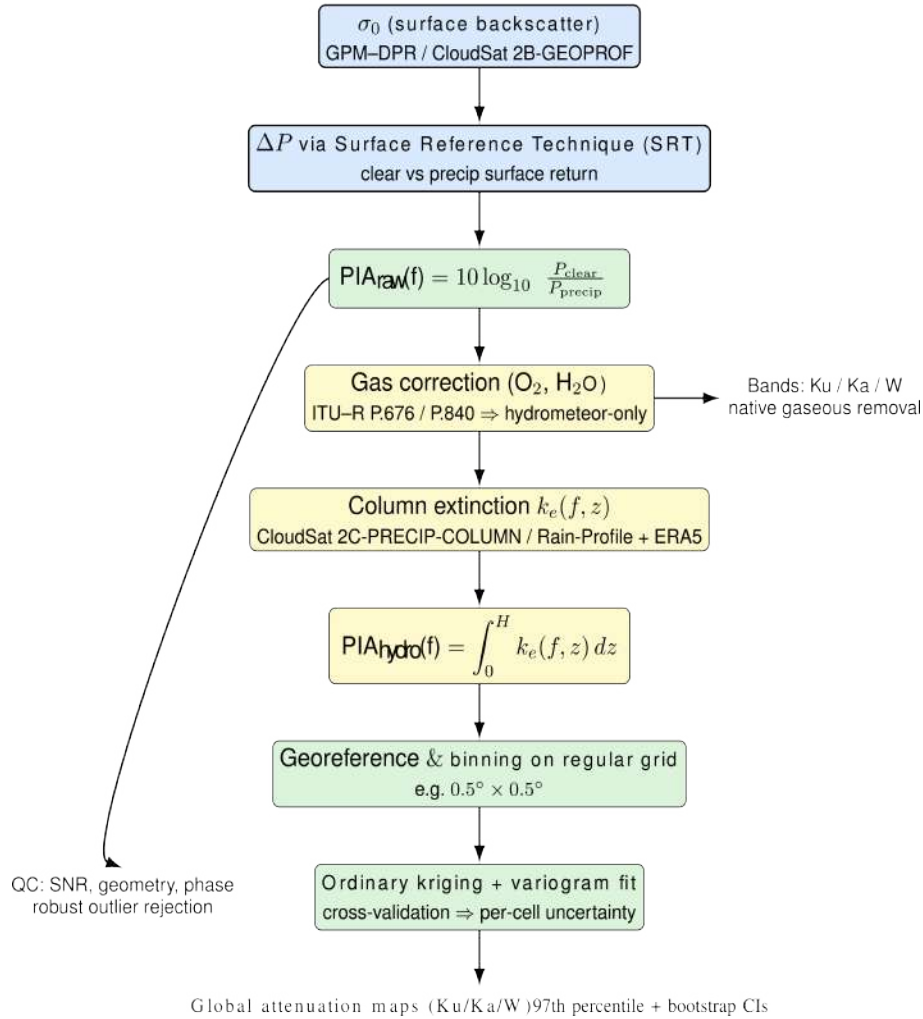


Figure 14.1. Workflow for deriving global hydrometeor-induced attenuation (PIA_{hydro}) maps from spaceborne radar observations. The process starts from the surface backscatter coefficient (σ_0) measured by GPM-DPR and CloudSat 2B-GEOPROF, applying the Surface Reference Technique (SRT) to compute the differential surface return (ΔP) between clear-sky and precipitation scenes. The raw path-integrated attenuation (PIA_{raw}) is then corrected for gaseous absorption (O_2 , H_2O) using ITU-R P.676 and P.840 models, isolating the hydrometeor contribution. Column extinction coefficients $k_e(f, z)$ —retrieved from CloudSat 2C-PRECIP-COLUMN and ERA5—are vertically integrated to obtain $PIA_{hydro}(f)$. The resulting attenuation values are georeferenced, binned on a $0.5^\circ \times 0.5^\circ$ grid, and spatially interpolated via ordinary kriging with variogram fitting and bootstrap confidence intervals, yielding the final Ku/Ka/W-band global attenuation maps.

in scattering efficiency due to heterogeneous hydrometeor populations and mixed-phase conditions;

- ρ denotes a statistical state vector describing the distribution of microphysical properties within the column (e.g. particle size dispersion, phase fraction, density variability);
- H_{int} represents an effective interaction kernel encoding the coupling between thermodynamic state and radiative extinction;
- η is proportional to the imaginary part of an effective dielectric permittivity $\Im[\varepsilon_{C^4}]$, acting as a dissipative correction term.

The presence of complex exponents does not imply quantum coherence, but rather captures phase dispersion and statistical variability in the attenuation response. The imaginary components quantify the loss of coherence in a classical sense, reflecting fluctuations in particle geometry, orientation, aggregation state, and local thermodynamic conditions. In this formulation, attenuation is treated as a nonlinear stochastic functional of the atmospheric state, rather than a purely deterministic power-law relation.

For operational and retrieval-oriented applications, a reduced form can be derived:

$$PIA_{C^4} = a' R^{b'} T^{d'} \cdot \exp[-\xi \cdot \Im[\varepsilon_{C^4}]] + e', \quad (14.5)$$

where ξ parameterizes the statistical opacity of the medium and $\Im(\varepsilon_{C^4})$ acts as a phenomenological loss term linking unresolved microphysical variability to the macroscopic attenuation field. This formulation preserves analytical tractability and can be directly implemented in satellite retrieval algorithms and surrogate propagation models. From a theoretical standpoint, this generalized C^4 law provides a unified statistical framework bridging classical radiative transfer and stochastic descriptions of extinction in heterogeneous, scattering-dominated media. The PIA becomes a measurable indicator of effective phase dispersion within the atmospheric column, allowing regions characterized by strong microphysical variability—such as stratiform anvils, melting layers, or dense ice aggregates—to be identified and mapped at the global scale. The novelty of **Eq. (A.5)** lies in the introduction of an exponential correction term that accounts for unresolved variability and correlation effects, absent in classical empirical formulations. In this sense, the C^4 generalization does not replace existing attenuation laws, but extends them into a statistically consistent framework capable of embedding higher-order variability while remaining compatible with observational constraints and operational propagation modelling.



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Appendix – Chapter 2

Chapter 2 – Theoretical Background and State of the Art.



Hydrometeor Type	Shape	Band	Applicability of Mie Theory	Notes
Rain	Oblate Spheroid	Ku–Ka	High	Applicable with polarization corrections
Snowflake	Fractal Aggregate	Ka–W	Low	Requires numerical models or statistical approximations
Graupel	Irregular Sphere	W	Limited	Multiple Scattering and Depolarization Errors Multiple Scattering → DDA or SNEM-2 preferred
Hail	Multi-layered Core	Ku–W	Low	Complex structure → Mie is inadequate Strong internal contrast → T-matrix needed
Mixed-phase Particle	Core-shell	W	Limited	Requires double layer or T-matrix modeling Melting layer → SNEM-2 with ϵ'' modeling

Table 14.1. Fundamental Assumptions of Mie Theory. Mie Theory offers exact analytical solutions for electromagnetic scattering by homogeneous spherical particles. However, its simplifying assumptions restrict its applicability in real-world scenarios where hydrometeors exhibit irregular shapes, internal heterogeneity, or melting states. The table summarizes the conditions where Mie Theory remains valid and highlights the atmospheric contexts requiring alternative numerical or empirical approaches.



Scenario/ Condition	Validity of Mie Theory	Limitation	Suggested Alternatives
Homogeneous spherical raindrops	High	Fully applicable as per original formulation	None needed
Small aerosols ($radius \ll \lambda$)	Moderate (Rayleigh limit embedded in Mie)	Overkill for small particles; Rayleigh approximation may suffice	Rayleigh scattering approximation
Non-spherical ice particles (e.g., dendritic snowflakes)	Invalid	Mie fails to represent scattering asymmetry and phase function	T-Matrix, Discrete Dipole Approximation (DDA), Ray-Tracing models
Irregular or porous graupel	Invalid	Porosity and shape variability not captured	Numerical simulations or effective medium approximations
Melting particles (mixed-phase)	Limited	Core-shell morphology partially modeled, but phase inhomogeneity neglected	Multi-layer Mie, empirical adjustments, or full radiative solvers
Large aggregates or fractals	Invalid	No support for aggregate geometry or multiple scattering paths	DDA, Monte Carlo Radiative Transfer, C4 framework
Mixed hydrometeor fields	Limited	Requires artificial decomposition into monodisperse spheres	Spectral bin models, hybrid Mie-numerical models

Table 14.2. Validity Domains and Limitations of Mie Theory in Atmospheric Applications.



Limitation of Mie Theory	Cause	Affected Frequencies	Alternative Method	Application Note
Sphericity assumption	Real Hydrometeors are irregular	Ka, W	T-matrix, DDA	Spheres only valid for ideal rain
Homogeneous composition	Graupel/Snow often have porosity	W	Effective Medium Theory	Internal structure alters scattering
No melting phase modeling	Mixed-phase scattering neglected	Ka, W	Monte-Carlo, SNEM-2	Required for melting layer representation
No orientation variability	Assumes random orientation only	V, W	Polarimetric solvers	Affects polarization signals
Narrow refractive index database	Limited $\varepsilon(f, T, RH)$ handling	All	Dynamic lookup or fit	SNEM-2 includes dynamic permittivity

Table 14.3. Limitations of Mie Theory and Recommended Alternatives.

Appendix Chapter 7

Appendix — Figures/Tables Moved from Chapter 7

Month	PIA_{min} [dB]	PIA_{median} [dB]	PIA_{max} [dB]	ID
Jan	15	20	30	1
Feb	16	22	37	2
Mar	18	25	45	3
Apr	22	30	55	4
May	30	40	80	5
Jun	45	60	140	6
Jul	65	90	240	7
Aug	70	105	305	8
Sep	65	85	205	9
Oct	35	50	110	10
Nov	20	30	60	11
Dic	16	22	37	12

Table 7.3. Monthly statistics of path-integrated attenuation (PIA) at 94 GHz and 15° degrees of elevation, simulated for Scanzano (PA), Italy (Telespazio ground station). The table reports the expected minimum, median, and maximum PIA values (in [dB]) for each month, derived from a climatological surrogate model. The estimates highlight the seasonal transition from dry winter conditions to intense convective events during summer.

Band	a_ν	b_ν	c_ν	d_ν	e_ν	f_ν	RMSE [dB]
Ka (35 GHz)	0.89	0.82	0.37	1.15	−0.004	0.58	0.71
W (94 GHz)	1.27	0.93	0.45	1.22	−0.006	0.64	0.54

Table 7.1. Empirical regression coefficients for Ka- and W-band attenuation retrievals. Values represent the best-fit parameters derived from the CloudSat–GPM validation subset, with an estimated uncertainty of ± 10 %. The coefficients a_ν , b_ν , c_ν , d_ν , e_ν , and f_ν correspond to the empirical formulation used to model the specific attenuation as a function of hydrometeor properties and atmospheric variables. The resulting RMSE, expressed in decibels, quantifies the accuracy of the regression for each frequency band. As shown, the W-band (94 [GHz]) fit achieves a lower RMSE (0.54 [dB]) compared to the Ka-band (35 [GHz]), (0.71 [dB]), indicating higher model consistency at millimetric wavelengths.

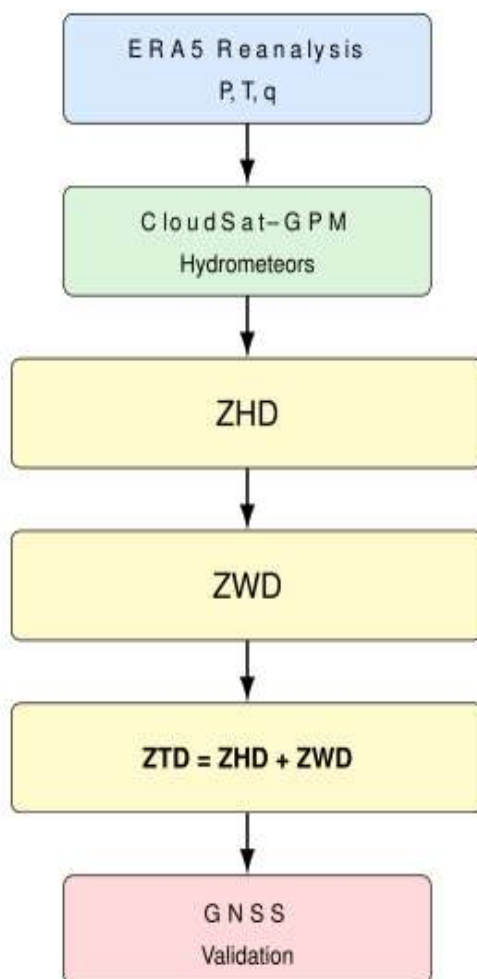


Figure 7.1. Vertical methodological flow for the retrieval and decomposition of zenith delays. ERA5 provides the large-scale thermodynamic fields (pressure, temperature, humidity) used to estimate the hydrostatic component (ZHD). CloudSat–GPM coincidences supply the vertical hydrometeor and attenuation profiles necessary for the wet component (ZWD). Their combination yields the total zenith delay (ZTD), which is subsequently validated using GNSS observations as an independent reference dataset.

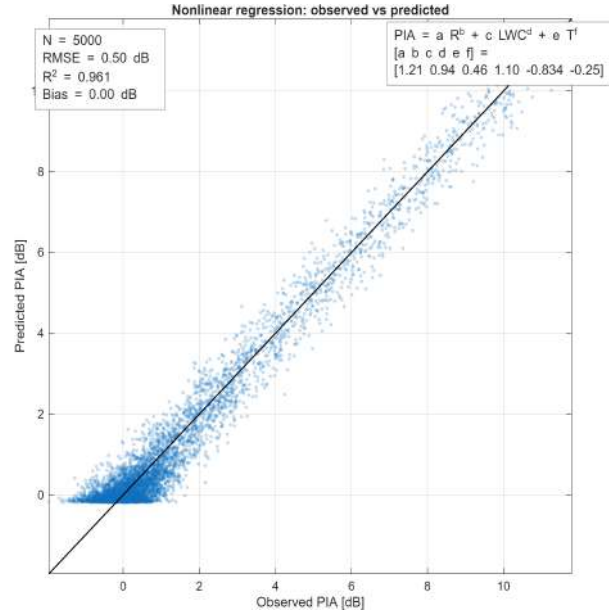


Figure 7.2. Observed versus predicted Path Integrated Attenuation (PIA) from the nonlinear regression model. The scatter-plot compares measured and model-predicted PIA values using the empirical relation , where is the rain rate, LWC the liquid water content, and the ambient temperature. The solid 1:1 line indicates perfect agreement. The fitted parameters are , yielding 1, dB, and negligible bias, which demonstrates the high accuracy and stability of the nonlinear regression in reproducing satellite-observed attenuation under varying hydrometeor and thermodynamic conditions.

Variable	Scenario	Model	RMSE [dB/K]	MAE [dB/K]	Bias [dB/K]	Correlation Coefficient (r)
PIA	Tropical	LSTM	1.34	1.01	+0.42	0.945
		CGAN	1.21	0.95	+0.25	0.963
	Mid-Latitude	LSTM	0.95	0.73	+0.18	0.962
		CGAN	0.84	0.68	+0.09	0.975
	Polar	LSTM	0.65	0.48	−0.12	0.938
		CGAN	0.53	0.41	−0.06	0.951
TB	Tropical	LSTM	2.87	2.33	+1.12	0.904
		CGAN	2.36	2.01	+0.68	0.928
	Mid-Latitude	LSTM	2.12	1.76	+0.42	0.939
		CGAN	1.78	1.52	+0.19	0.951
	Polar	LSTM	1.43	1.15	−0.33	0.921
		CGAN	1.17	0.98	−0.21	0.938

Table 7.2. Quantitative Evaluation of LSTM and CGAN. Surrogates against SNEM-2 Outputs.

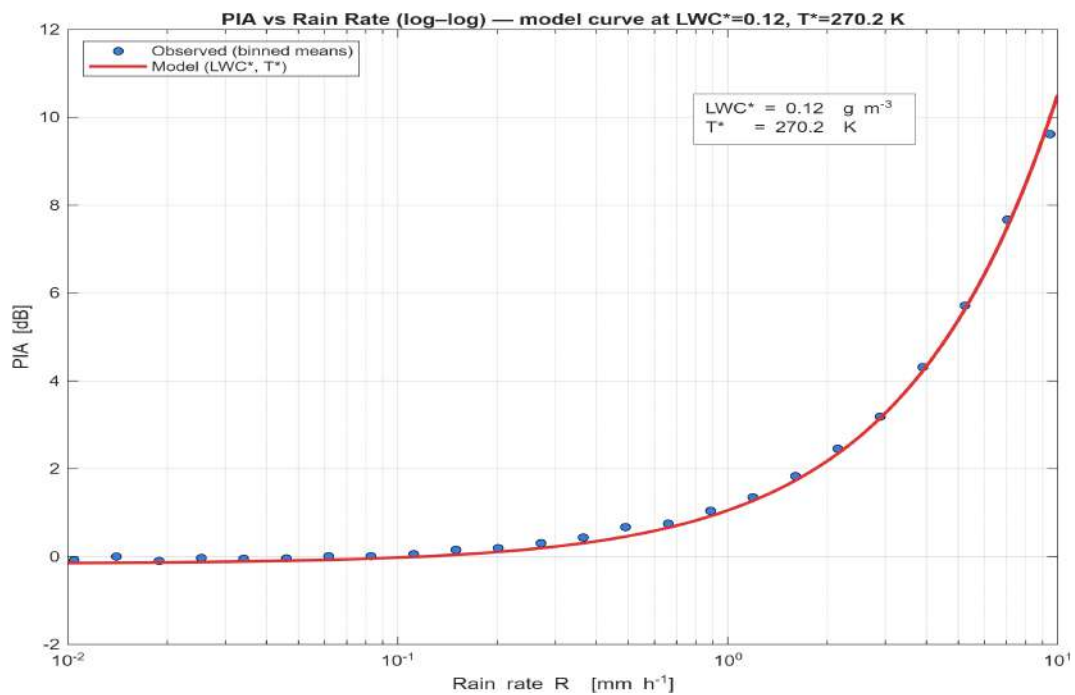


Figure 7.3. Dependence of path-integrated attenuation (PIA) on rain rate derived from the nonlinear regression model. Blue markers represent binned observational means obtained from CloudSat–GPM coincident measurements, while the red curve denotes the fitted nonlinear regression evaluated at representative median conditions of liquid water content ($LWC^* = 0.12 \text{ g m}^{-3}$) and in-cloud temperature ($T^* = 270.2 \text{ K}$). The log–log representation highlights the strongly nonlinear increase of attenuation with precipitation intensity at W-band frequencies. The close agreement between observations and model confirms that the regression captures the dominant microphysical controls on attenuation, preserving physical consistency across both low- and high-rain-rate regimes without saturation artifacts.

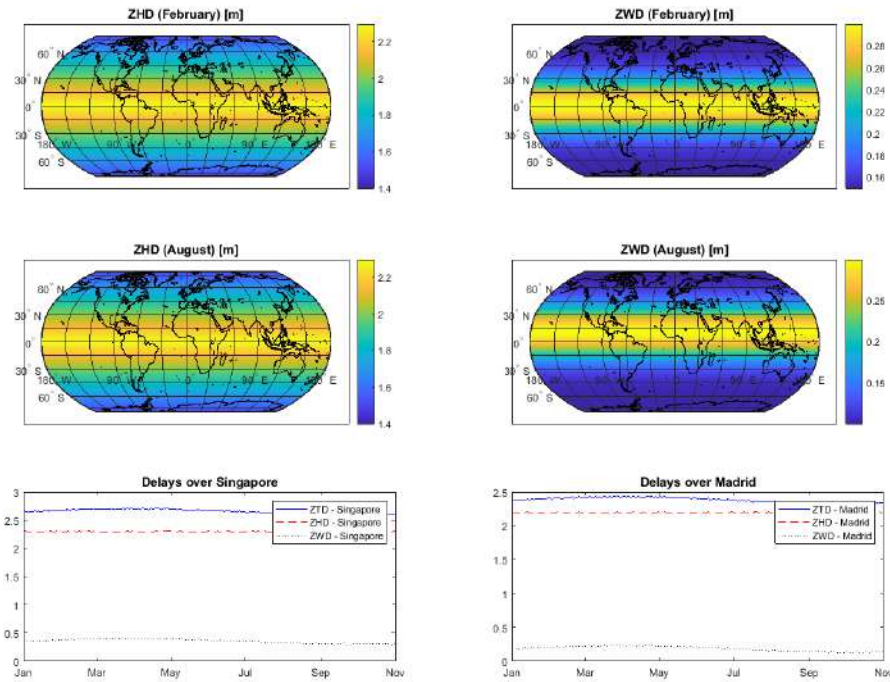


Figure 7.4. Global Distribution of ZHD and ZWD in February and August. Time series of daily ZTD (blue), ZHD (dashed red), and ZWD (dotted black) for Singapore and Madrid throughout 2020. The plots highlight regional and seasonal variability, with more pronounced wet delay fluctuations in equatorial Singapore and stronger seasonal modulation over continental Madrid. ERA5-based estimations are consistent with ITU-R P.676-12 recommendations.

Global ZHD, ZWD, ZTD – February (top) vs August (bottom)

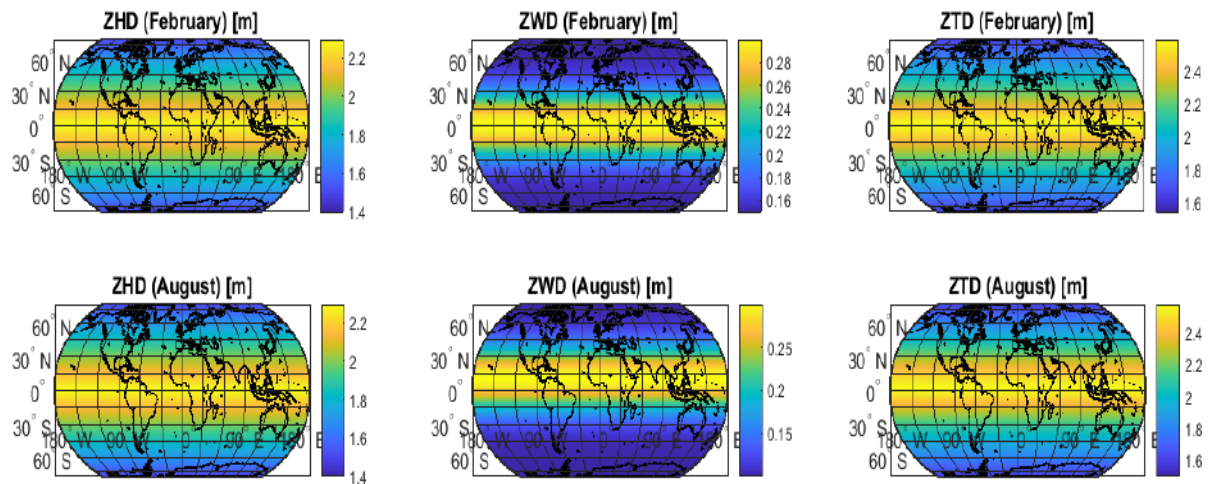


Figure 7.5. Global maps of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) for February (top row) and August (bottom row). The delays are expressed in meters. Seasonal variations in ZWD and ZTD are evident, particularly across equatorial and tropical latitudes.

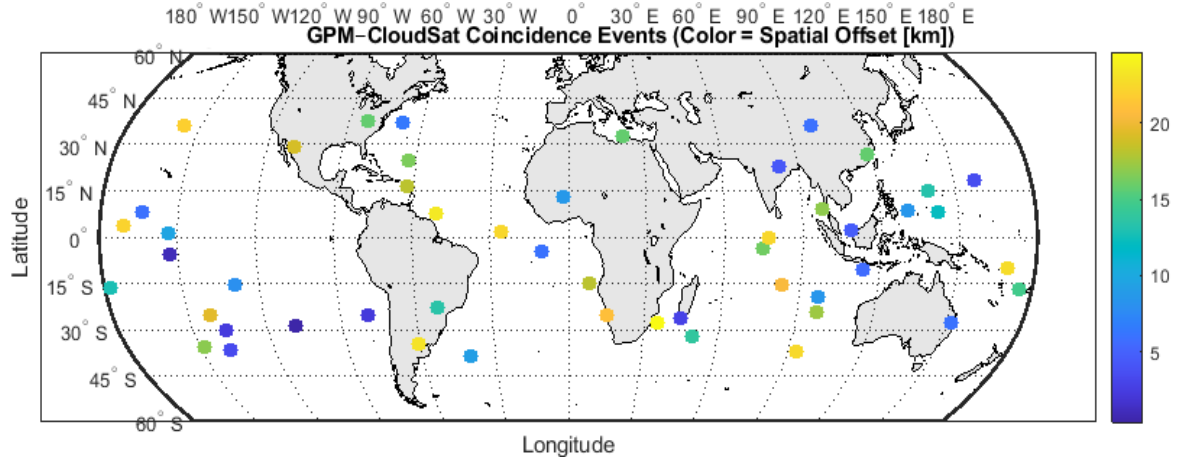


Figure 7.6. Geographical distribution of matched coincidence events between GPM and CloudSat. The color scale represents the spatial offset in kilometers between the two satellite ground tracks. Coincidences were filtered to ensure a maximum spatial mismatch of 25 [km] and a temporal offset below 10 minutes [min]. This spatial-temporal proximity guarantees consistent atmospheric conditions and enables accurate validation of SNEM-2 outputs against multi-frequency satellite observations.

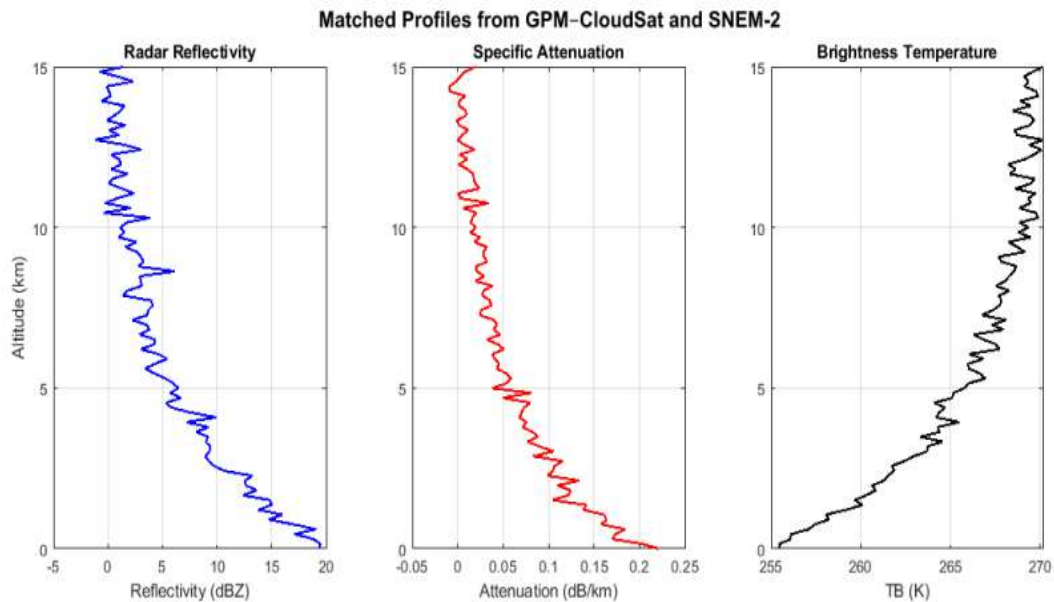


Figure 7.7. Representative vertical profiles of radar reflectivity [dBZ], specific attenuation [dB/km], and brightness temperature [K] for a matched GPM-CloudSat event over the Intertropical Convergence Zone (ITCZ). SNEM-2 simulations reproduce key atmospheric features such as the melting layer, hydrometeor transition zones, and peak scattering regions. These profiles serve as a qualitative reference for evaluating the physical realism of radiative transfer outputs prior to statistical validation.

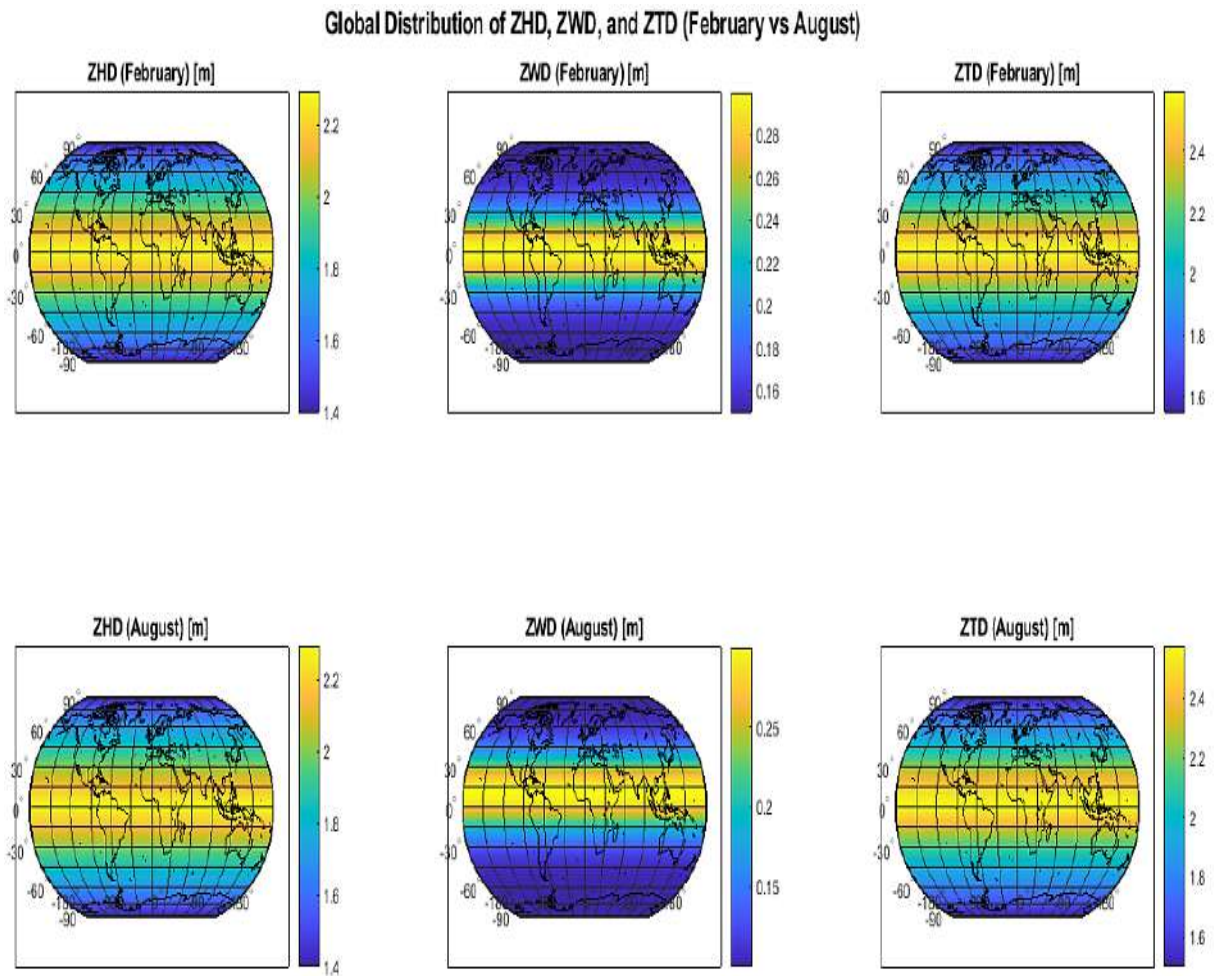


Figure 7.8. Global maps of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) averaged over February and August. These composites highlight the latitudinal and seasonal variability of tropospheric delays: ZHD shows stable latitudinal gradients driven by pressure and elevation, while ZWD exhibits strong seasonal shifts associated with water vapor content and the migration of the Intertropical Convergence Zone (ITCZ). ZTD reflects both components, with increased sensitivity to wet conditions in equatorial and monsoonal regions.



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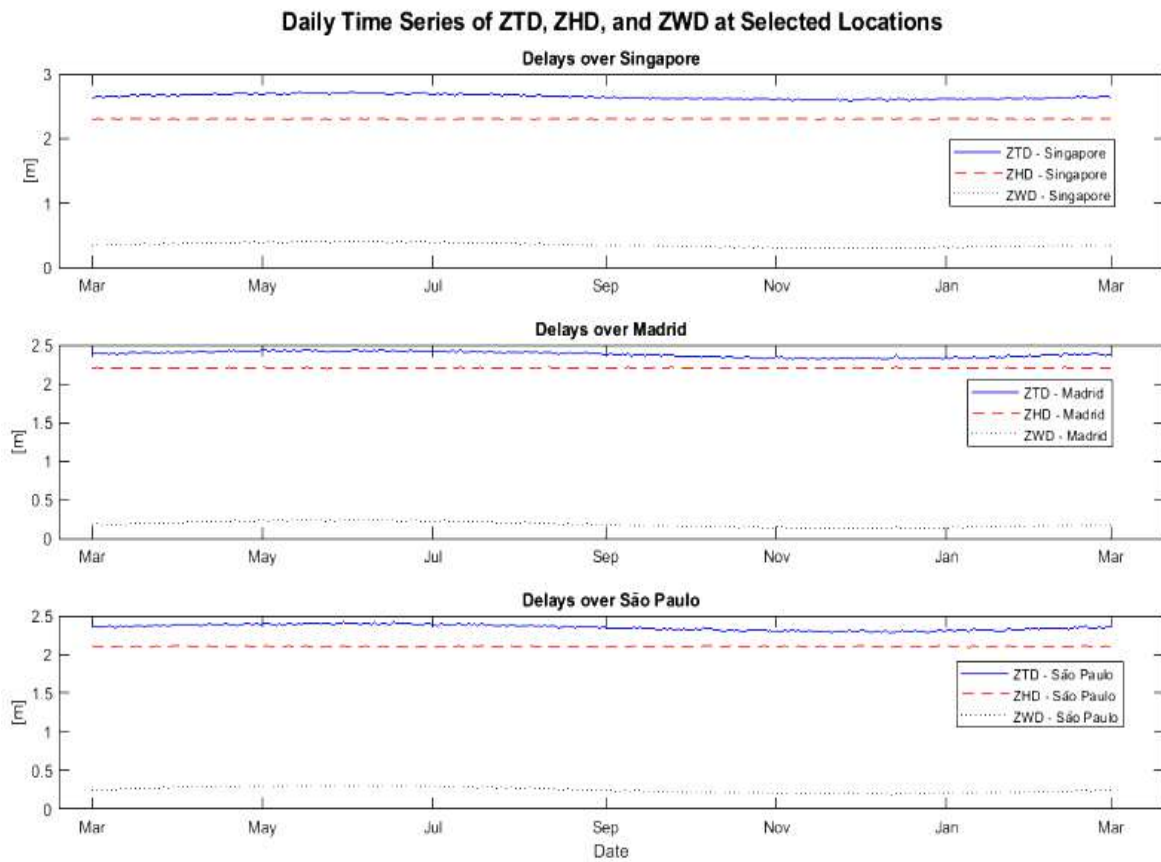


Figure 7.9. Daily time series of ZHD, ZWD, and ZTD for selected stations: Singapore (, tropical), Madrid (, continental mid-latitude), São Paulo (, subtropical southern hemisphere). The plots illustrate the relative contributions of hydrostatic and wet delays, with tropical locations showing dominant seasonal variability in ZWD, while mid-latitude sites exhibit more stable behavior dominated by ZHD. These trends emphasize the influence of climatological regimes and geographic position on tropospheric delay dynamics.

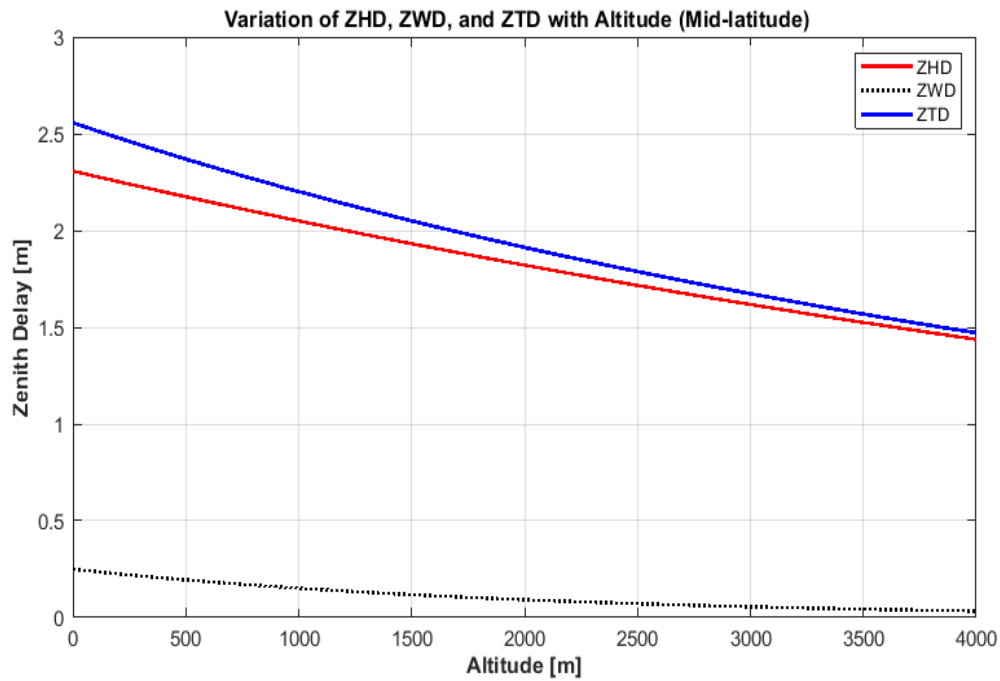


Figure 7.10. Simulated variation of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) as a function of altitude up to 4000 meters. ZHD decreases with elevation due to pressure reduction, while ZWD exhibits a more rapid exponential decay. ZTD reflects the combined effect and highlights the dominance of the hydrostatic component at high altitudes.

f_v (volfrac)	Approx LWC [g/m^3]	Propagation scenario	α_{94} [dB/km]
10^{-7}	0.1	Thick stratus cloud	0.2
$3 \cdot 10^{-6}$	3–4	50 [mm/h] rain	30–50
$7 \cdot 10^{-6}$	6–8	100 [mm/h] squall	60–100

Table 7.7. Relationship between volumetric water fraction f_v , liquid-water-content (LWC), representative propagation scenarios and the corresponding specific attenuation at 94 [GHz] (α_{94}). The values are derived with the **Bruggeman effective-medium model** (C^4 -framework) and are consistent with **ITU-R P.838** coefficients for liquid hydrometeors.

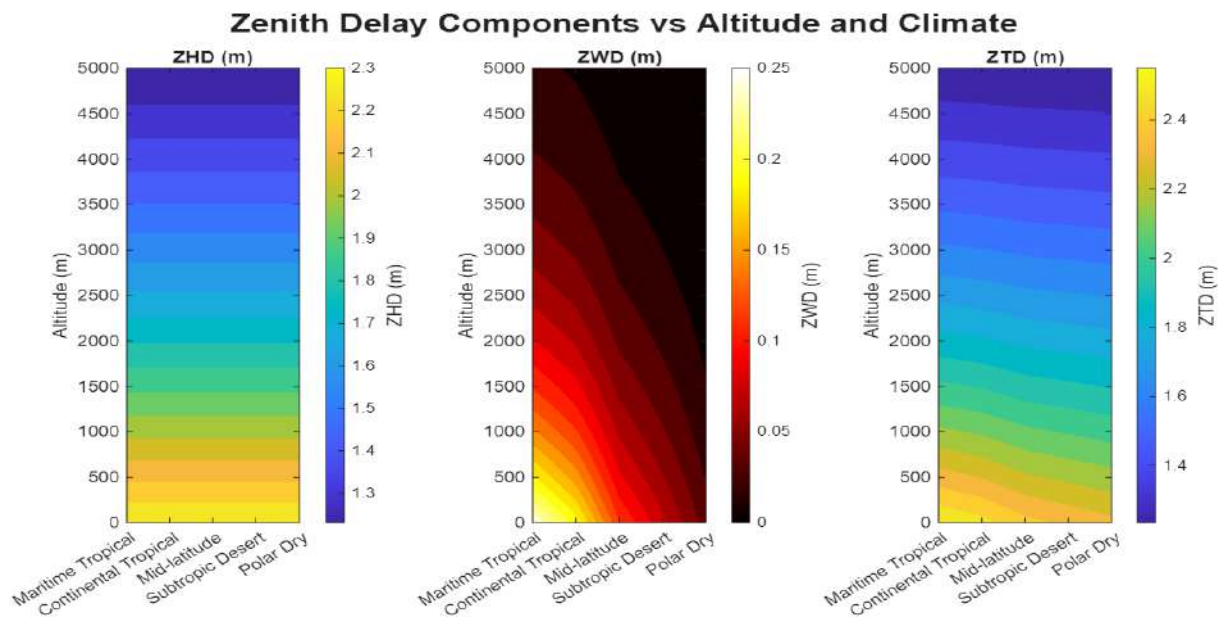


Figure 7.11. Zenith Delay Components as Functions of Altitude and Climate Regime. Panels show the hydrostatic (ZHD), wet (ZWD), and total (ZTD) zenith delays as contour fields for five representative climate classes (maritime tropical, continental tropical, mid-latitude, subtropical desert, and polar). ZHD decreases quasi-linearly with altitude following the exponential decay of dry-air pressure, whereas ZWD is strongly confined to the lower 2–3 km due to the rapid decline of water vapor with height. As a result, ZTD exhibits a smooth transition between these behaviors, converging to the hydrostatic component in dry and high-altitude environments. Colourbars report delay magnitudes in metres. The isoline in the ZWD panel denotes the altitude where the wet component drops below 5 mm, indicating the effective upper bound of water-vapor impact. This comparison highlights the combined influence of atmospheric structure and climate on zenith delays and provides a physically interpretable reference for the validation and surrogate-model analyses presented in **Section 7.11**.

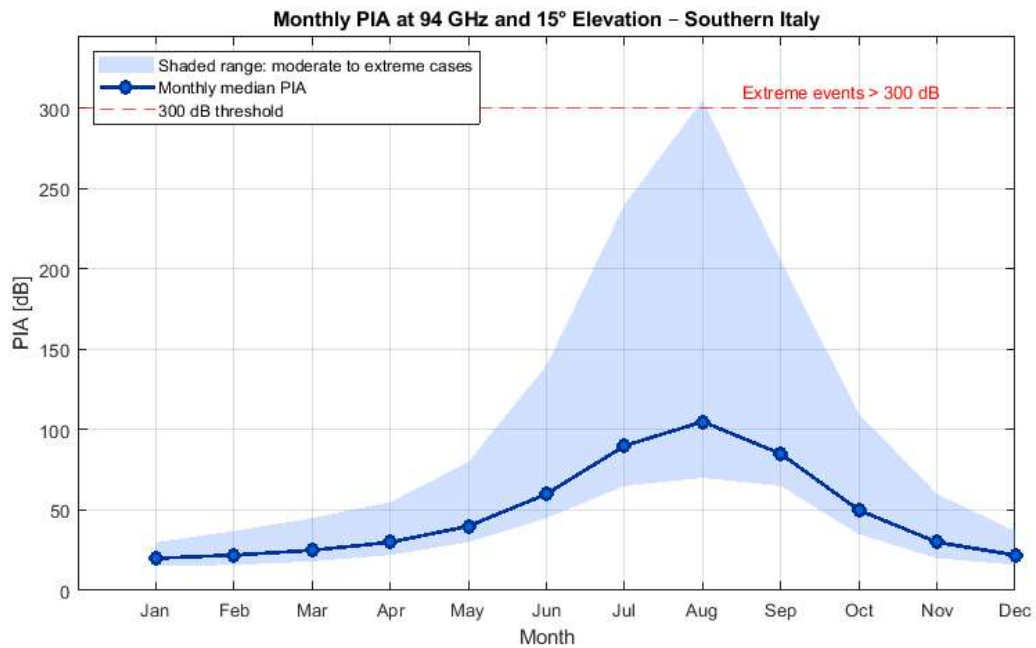


Figure 7.12. Monthly variability of path-integrated attenuation (PIA) at 94 GHz for a 15° elevation Earth–space link at the Scanzano (PA) ground station. The solid blue line represents the monthly median PIA derived from the surrogate-based radiative-transfer emulator, while the shaded envelope indicates the dispersion from moderate to extreme attenuation events. A marked seasonal dependence is observed, with low winter attenuation and a pronounced maximum during late summer. The red dashed line denotes the 300 dB threshold, above which severe signal degradation and link outages are expected under deep convective conditions.

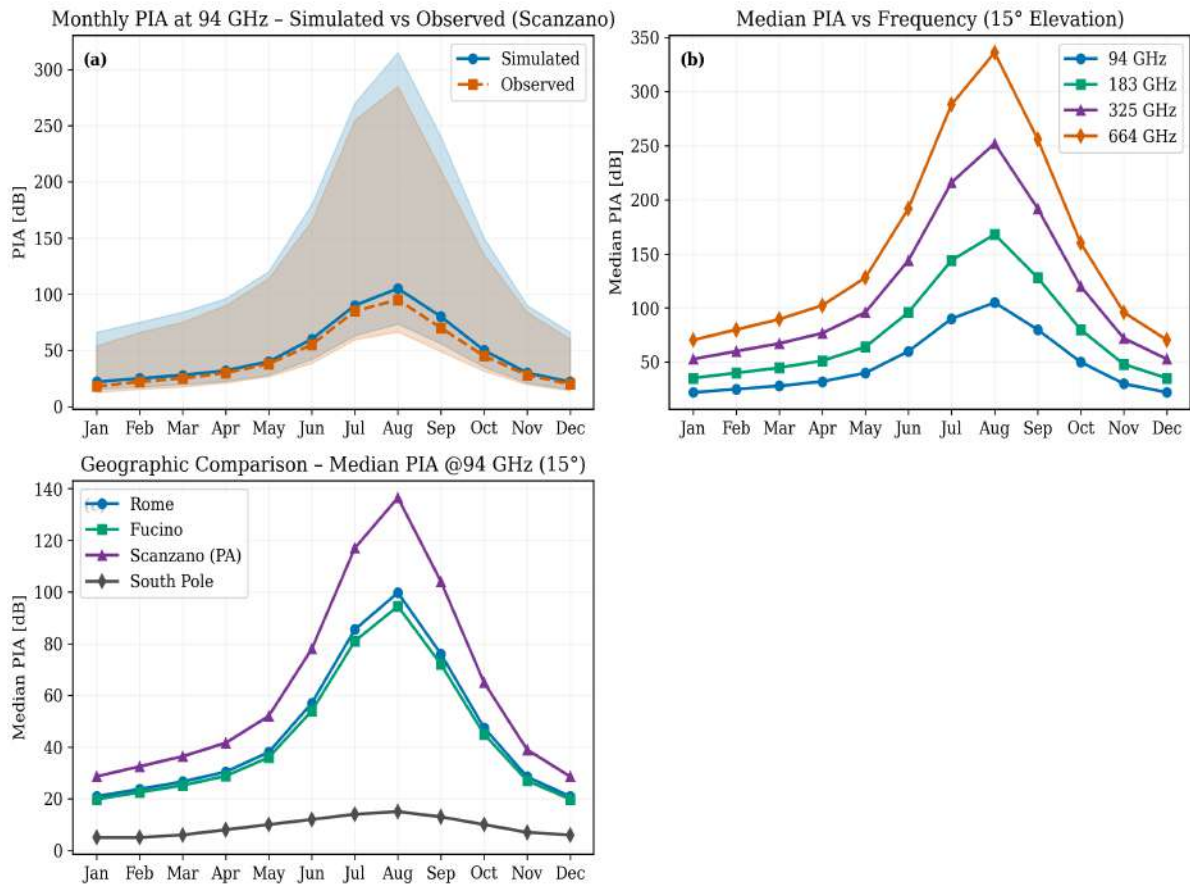


Figure 7.13. Monthly climatology of path-integrated attenuation (PIA) at 94–664 GHz and 15° elevation. (A) Comparison between simulated (SNEM-2 + surrogate) and observed PIA at 94 GHz for Scanzano (PA), Italy. Solid lines show monthly medians, while shaded bands represent the full variability range; summer months (July–August) exhibit pronounced peaks associated with deep convective activity. (B) Frequency-dependent monthly median PIA for Southern Italy, showing the strong increase in attenuation with both frequency and seasonal moisture availability. Higher-frequency channels (183–664 GHz) display extreme summer maxima, driven by enhanced water vapor and hydrometeor content. (C) Geographic comparison of median PIA at 94 GHz for four contrasting locations (Rome, Fucino, Scanzano, and the South Pole). Italian sites show similar seasonality with strong summer peaks, whereas the Antarctic plateau remains consistently dry, yielding minimal attenuation year-round. Together, the three panels highlight the combined influence of seasonal thermodynamics, spectral sensitivity, and geographic climate regime on W-band propagation.

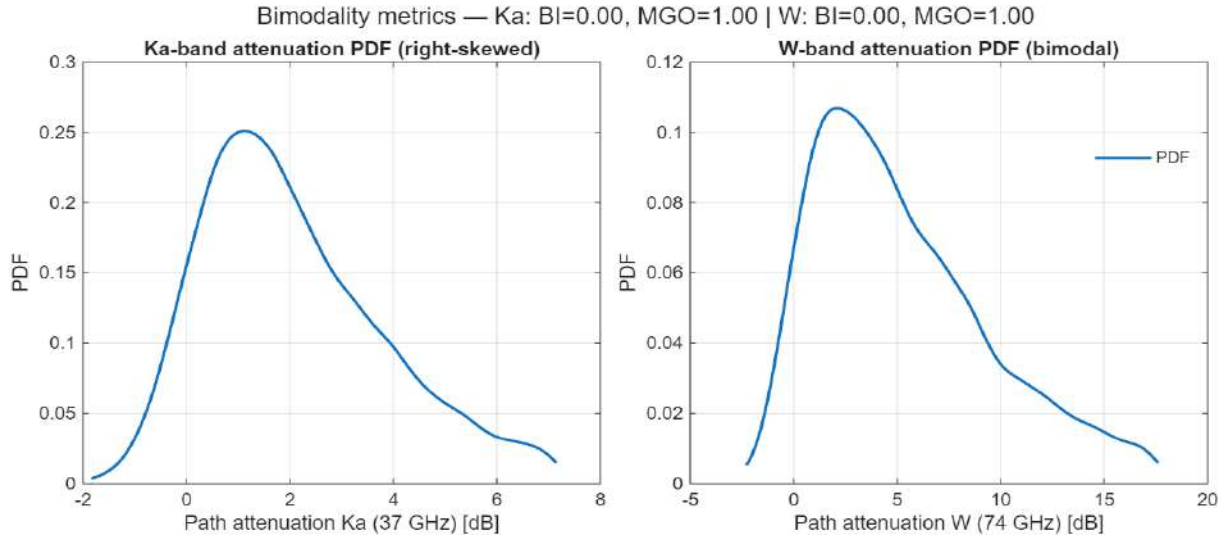


Figure 7.14. Probability density functions of path attenuation at Ka (37 [GHz]) and W (74 [GHz]). Ka is skew-unimodal, dominated by semi-transparent scenes, whereas W is distinctly bimodal, separating clear and quasi-opaque conditions. Annotated metrics include the Bimodality Index (BI) and Mid-Gap Occupancy (MGO), which quantify the strength of the threshold transition at W-band.

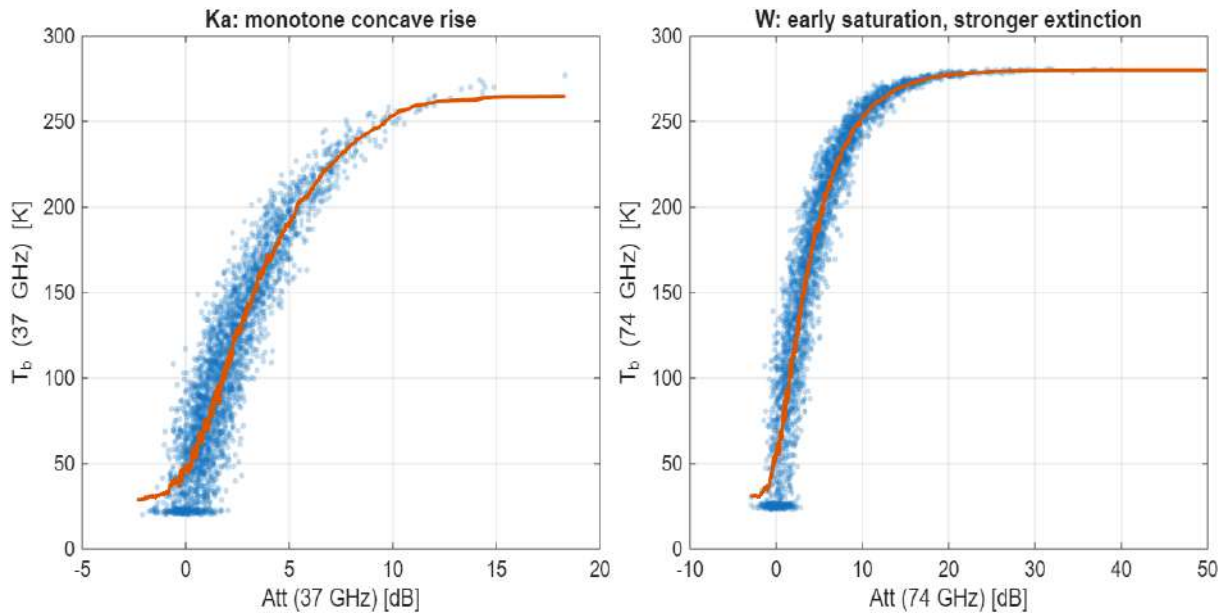


Figure 7.15. Scatterplots of brightness temperature versus path attenuation at Ka (37 [GHz]) and W (74 [GHz]). Both follow a monotonic, concave emission-absorption curve, but W-band flattens much earlier than Ka, confirming rapid opacification in the presence of moderate column water.

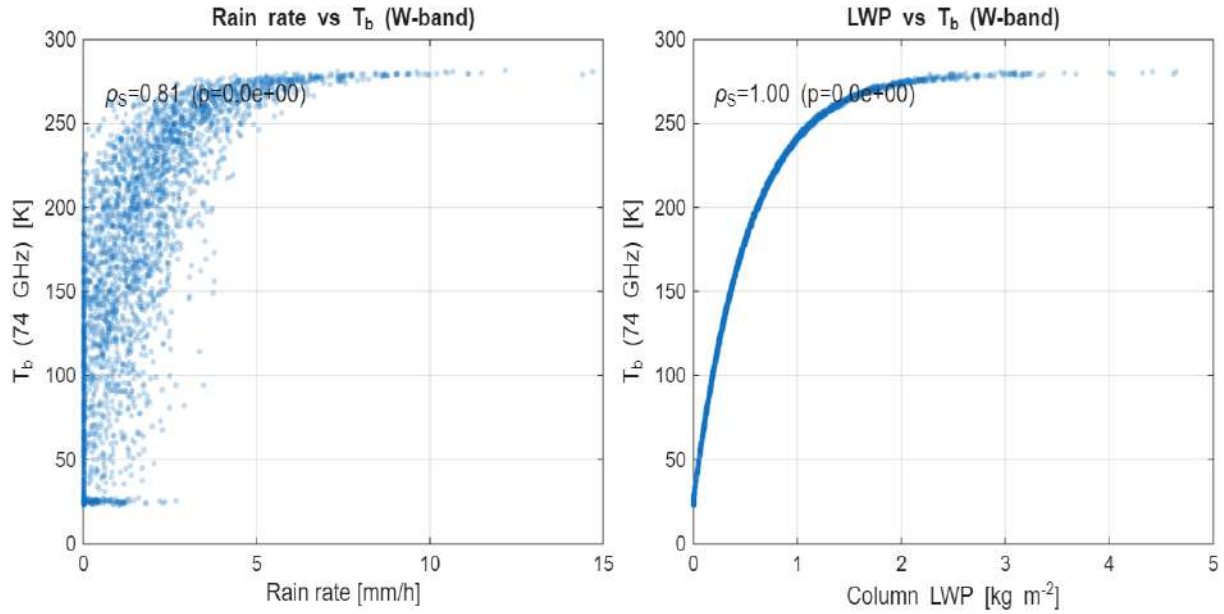


Figure 7.16. Comparison of brightness temperature at W-band (74 [GHz]) against surface rain rate (left) and column liquid water path (right). Rain-rate scatter is broad and sublinear, whereas LWP shows a tight, asymptotic control of T_b , confirming that column optical depth governs the radiative state.

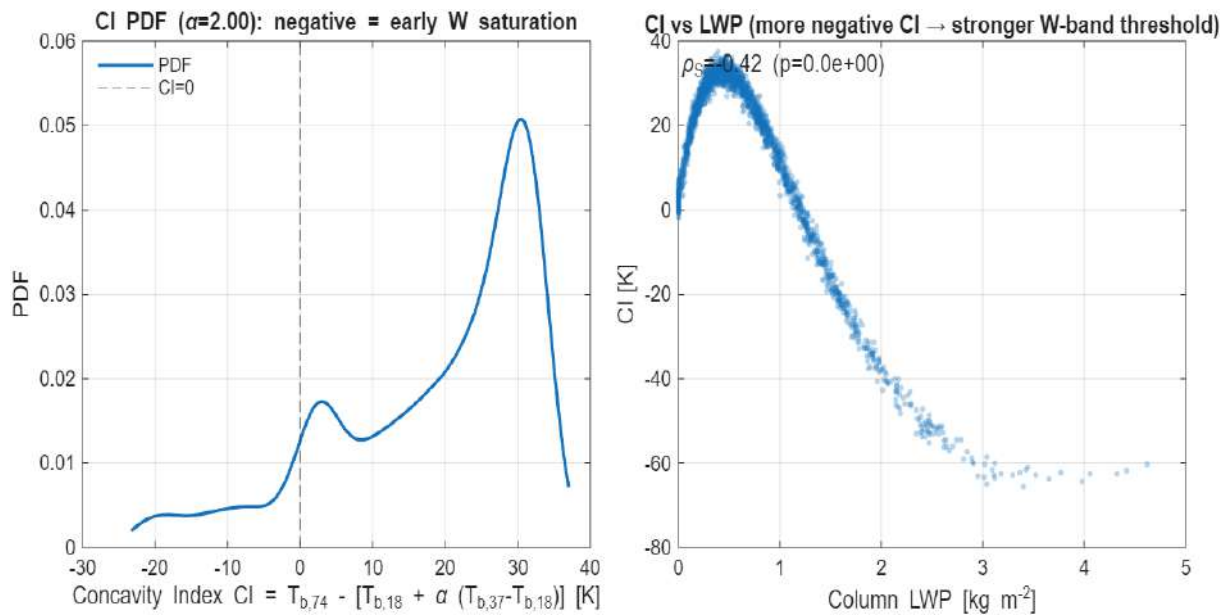


Figure 7.17. Concavity Index (CI) distribution and scatter against LWP. Negative CI values indicate early saturation of W-band relative to the Ka-band anchor. CI becomes more negative with increasing LWP, providing a physics-aware diagnostic for W-band thresholding independent of rain-rate information.

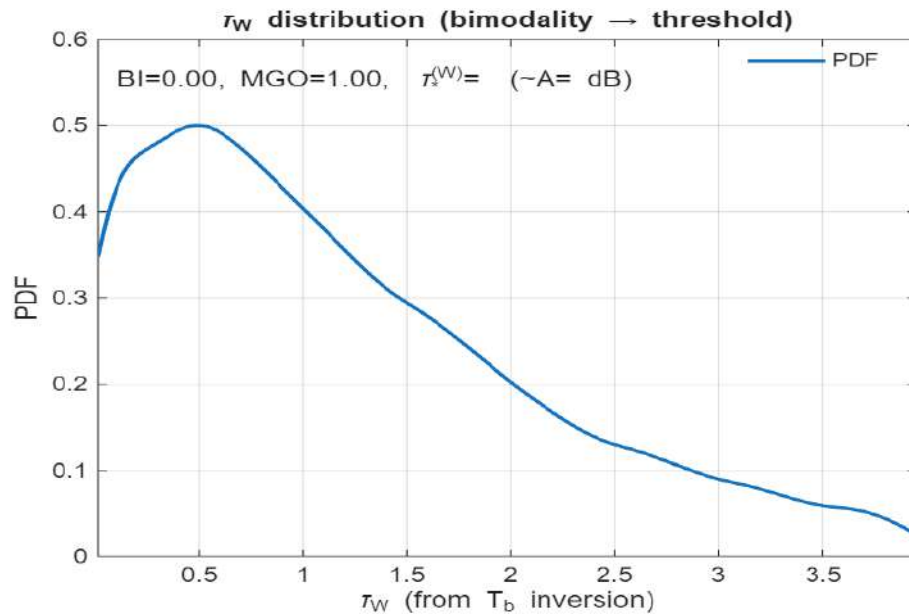


Figure 7.18. Probability density function of W-band optical depth inferred from brightness temperature. Two distinct modes are separated by a valley at , the critical threshold for quasi-binary fade behavior. The corresponding attenuation in dB provides a site-agnostic engineering metric for W-band link design.

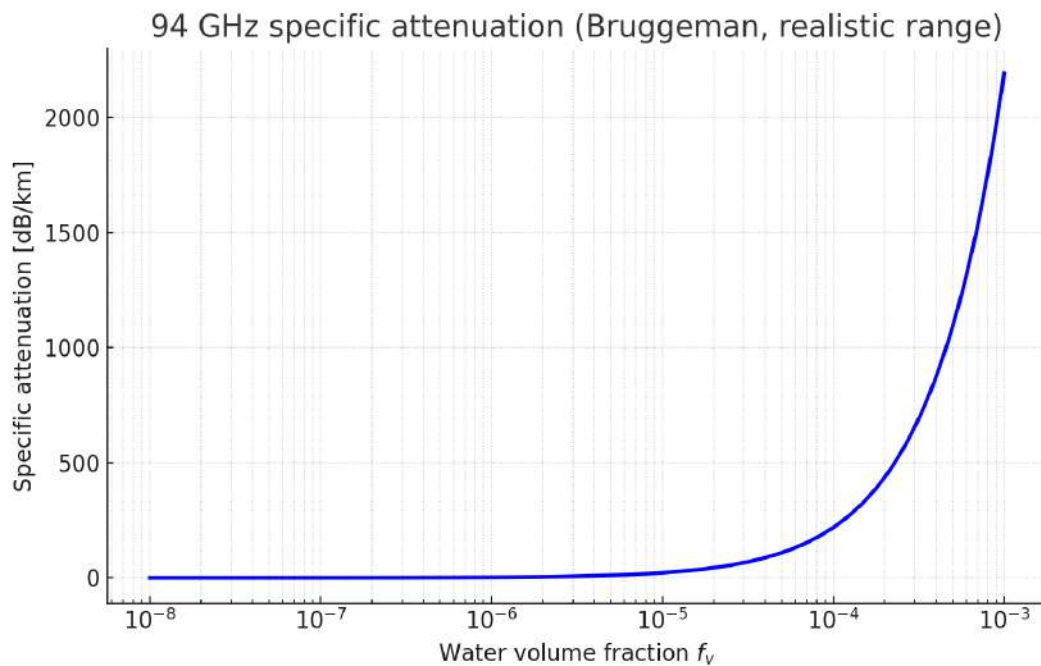


Figure 7.19. 94 GHz Specific Attenuation (Bruggeman, realistic range) . Specific attenuation at 94 GHz as a function of the volumetric water fraction , computed using the Bruggeman effective medium approximation within the -theoretical framework. The attenuation values are physically consistent and increase non-linearly with , reflecting the electromagnetic absorption by distributed liquid hydrometeors in the microwave regime.

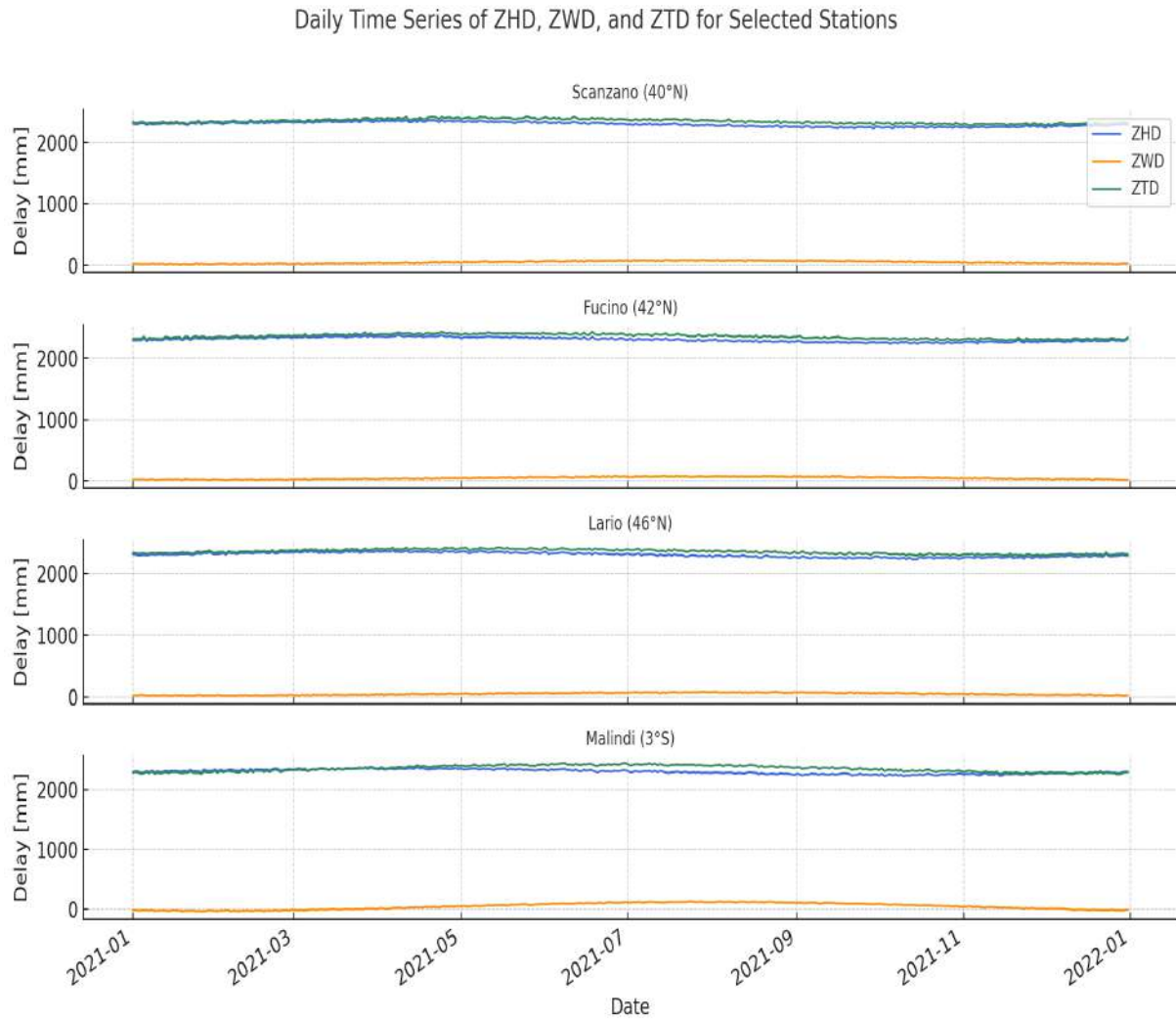


Figure 7.20. Daily Time Series of Tropospheric Delays (ZHD, ZWD, ZTD) for Selected Stations. Time series of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) for the year 2021 across four locations: Scanzano (40°N), Fucino (42°N), Lario (46°N), and Malindi (3°S). The results illustrate the relative contributions of dry and wet atmospheric components. Tropical and equatorial stations such as Malindi exhibit strong seasonal variability in the ZWD component, while mid-latitude sites like Lario and Fucino show more stable ZHD dominance.



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Table 7.4. Summary of Key Findings and Implications

Topic	Key Finding	Validation Source	Implications
ZHD Global Behavior	Stable across seasons; elevation-dependent gradient	ERA5, JTU-8, P676-12	Reliable for GNSS delay correction and model benchmarking
ZWD Seasonal Variability	Highly sensitive to humidity; peaks in equatorial and monsoon regions	ERA5, CloudSat, JTU-8, P435-13	Essential for attenuation modelling and water vapor retrievals
SNEM-2 Physical Soundness	Reproduces attenuation peaks and TB profiles in melting layers and stratiform clouds	GPM, CloudSat coincidence	Suitable as reference generator for surrogate training
Surrogate Model Accuracy	CGAN outperforms LSTM in structural fidelity; RMSE < 1 dB in most regions	SNEM-2 synthetic truth satellite comparison	Enables real-time applications with low computational burden
Explainability & Error Sources	LWC and temperature near melting layer are most critical; CGAN localizes better	Saliency analysis, error metrics	Supports physics-aware model design and interpretability tools
Topographic Sensitivity	ZWD and ZHD decay with elevation; ZHD decreases linearly	Analytical ERA5 profile simulations	Important for modelling and calibration in mountainous regions
Operational Readiness	Surrogates compatible with link budget tools and satellite network simulations	Computation time, accuracy benchmarks	Ready for deployment in telecoms and climate model assimilation

Hydrometeor Type	Temperature	$m = n - j\kappa$	Source Model
Liquid Water	20 °C	$\sim 8.5 - j \cdot 1.6$	Liebe 1989, Rosenkranz
Pure Ice	-10 °C	$\sim 3.2 - j \cdot 0.03$	Matzler (1998)
Dry Snow	-10 °C	$\sim 1.7 - j \cdot 0.01$ (effective)	Maxwell-Garnett (mix)
Wet Snow / Graupel	0 °C	$\sim 2.5 - j \cdot 0.2$ (effective, mixed-phase)	Mixing Law, Effective medium approximation
Water-coated Ice (melting layer)	0 °C	$\sim 5.0 - j \cdot 0.8$ (intermediate state)	Modeled with SNEM-2

Table 7.5. Complex Refractive Index of Hydrometeors at 94 GHz.

Aspect	Mie-Theory	C4-Framework
Single particle	Yes	No (deals with complex systems)
Shape	Spherical, ideal	Arbitrary or distributed (realistic)
Positioning	Isolated	Densely distributed
Multiple interaction	No	Yes, included via Green's function
Field Coherence	Only incident + scattered field	Total autocorrelated field (coherent field)
Output quantities	Q_{ext}, σ_{ext}	$\langle \varepsilon_{eff} \rangle, \langle k_e \rangle, \tau(z)$

Table 7.6. Comparative Table: Mie Theory vs C4-Framework.



Appendix Chapter 8

Appendix — Moved Material from Chapter 8

§ 8.2.1 Motivation and Contribution

During the analysis of cryospheric scenes, brightness-temperature (T_B) profiles generated by SNEM were found, in some cases, to exhibit unrealistically low baseline values—approaching 0 K—while still preserving a physically plausible vertical morphology. This behavior clearly indicated the presence of a missing radiometric offset in glaciated conditions, where surface emission, gaseous absorption, and ice-phase hydrometeors jointly determine the minimum observable T_B [96, 223].

In physically realistic cryospheric environments, even under extremely low emissivity conditions, the combined contribution of atmospheric gases, diffuse background radiation, and cold surface emission imposes a strict lower bound on brightness temperature. Radiative-transfer theory and microwave radiometry observations consistently indicate that brightness temperatures cannot fall below a finite radiometric floor set by atmospheric emission and surface coupling [5, 66, 209]. The occurrence of near-zero values therefore reflects a numerical artefact rather than a physical signal.

To address this inconsistency, a dedicated radiometric baseline-anchoring procedure was introduced in SNEM-2. The correction enforces a physically motivated lower bound on T_B while preserving the original vertical structure of the radiative-transfer solution. Importantly, the procedure does not modify relative gradients or cloud-induced contrasts, but solely removes non-physical minima, in line with established practices in physically constrained radiative modeling [31, 76].

The effectiveness of this correction is illustrated in **Figure 8.1**, which shows the annual distribution of modeled brightness temperature as a function of column-integrated ice water content (CWC_{ice}). The corrected dataset exhibits the expected triangular radiometric pattern, previously reported in passive microwave observations of ice clouds and cryospheric scenes [62, 66, 200]:

- at low CWC_{ice} , T_B spans a broad range, reflecting variability driven by gaseous emission, surface coupling, and viewing geometry [66, 204];
- as CWC_{ice} increases, T_B progressively converges toward an upper asymptote, corresponding

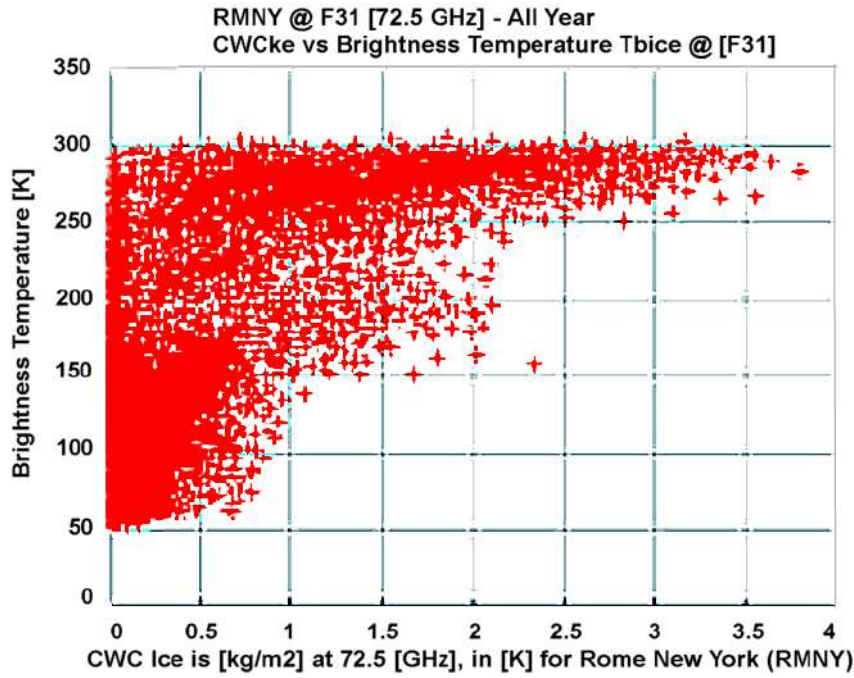


Figure 8.1. Scatter plot of Cloud Water Content (CWC_{ice}) vs. Brightness Temperature (T_B^{ice}) at 72.5 GHz for **Rome-New York (RMNY)** across all seasons. The data illustrates the relationship between ice cloud concentration kg/m^2 and radiometric temperature K. Data Source: SNEM model, Output: SNEM-2.

to higher optical depth and increased effective emissivity of thick ice-phase cloud columns [6, 62, 291].

Crucially, the absence of values below 50 K confirms that the +50 K radiometric anchor successfully suppresses non-physical behavior without distorting physically meaningful variability. The resulting T_B distributions are fully consistent with satellite observations from **CloudSat/CPR** and other microwave radiometers operating in cryospheric regimes [14, 97, 254], as well as with theoretical radiative limits imposed by surface temperature and atmospheric emission [96, 209]. This correction represents a necessary step toward ensuring radiometric consistency of SNEM-2 outputs over polar and glaciated regions.

- Further details on the implementation of the **anchoring procedure**, together with the broader software-modernization effort—including code audit, build reconstruction, and **Fortran/MATLAB** interfacing—are provided in **Appendix B**, [B.8–B.8.5]

§ 8.2.2 Radiative-Transfer Rationale for the Baseline Correction

Under **Local Thermodynamic Equilibrium (LTE)**, the upward brightness temperature cannot physically approach zero, since surface emissivity, gaseous emission along the propagation path, and reflected downwelling radiation always contribute a non-zero radiometric floor at Ka-



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and W-band frequencies [5, 66, 96]. This constraint is a direct consequence of the radiative-transfer equation in emitting and scattering media and is independent of cloud optical thickness in the optically thin limit [222, 223].

Within SNEM-2, this behavior is explicitly captured by the Rayleigh–Jeans formulation of the top-of-atmosphere brightness temperature:

$$T_B^{TOA}(\nu, \mu) \approx \varepsilon_s T_s e^{-\tau_\nu/\mu} + \int_0^{\tau_\nu/\mu} T(t) e^{-(\tau_\nu/\mu - t)} dt + (1 - \varepsilon_s) T_\downarrow(\nu, \mu) e^{-\tau_\nu/\mu} + T_B^{sca} \quad (8.1)$$

where ε_s is the surface emissivity, T_s the surface temperature, τ_ν the atmospheric optical depth at frequency ν , $\mu = \cos(\theta)$ the zenith-angle cosine, $T_\downarrow$ the downwelling atmospheric brightness temperature, and T_B^{sca} the scattering contribution. Each term remains finite even under extremely cold, dry, and optically thin conditions, as documented in classical and modern microwave radiative-transfer theory [31, 66, 222].

The legacy implementation of SNEM correctly reproduced the vertical morphology of the radiative-transfer solution but omitted the boundary terms governing the absolute emission level. As a consequence, the resulting brightness-temperature profiles preserved physically meaningful gradients while exhibiting unrealistically low baselines approaching zero. Such behavior is incompatible with both theoretical radiative constraints and satellite observations in cryospheric environments [62, 204].

Because the **Rayleigh–Jeans** approximation is affine in temperature, the missing physics manifests mathematically as a constant additive offset rather than as a distortion of vertical structure [5, 209]. To restore physical radiometric consistency, SNEM-2 applies the following baseline correction:

$$T_B^{corr}(z; \nu, \mu) = T_0(\nu, \mu) + [T_B^{raw}(z) - \min_z T_B^{raw}(z)]. \quad (8.2)$$

where T_0 represents the clear-sky radiometric baseline determined by surface emissivity, gaseous optical depth, and reflected downwelling radiation [66, 96, 209]. For all Ka- and W-band configurations investigated in this work, the baseline is well represented by a +50 K anchor, consistent with observed lower bounds in satellite microwave measurements over polar and glaciated surfaces [14, 97, 200]. This correction effectively suppresses non-physical minima while preserving all microphysically relevant vertical gradients, including those associated with ice-phase hydrometeors and scattering-induced structure [6, 62].

- A full derivation of the **radiative-transfer equations (RTE)** and a detailed physical definition of T_0 , including its dependence on surface and atmospheric parameters, are provided in **Appendix B**, [B.9–B.15] [94].



§ 8.2 Parameterization of k_e

Because k_e is expressed as the sum of the terms k_a, k_s the relationship represents the fundamental quantity governing both attenuation and thermal emission in radiative-transfer modeling [5,6,289]. Its parameterization is therefore required to preserve physical interpretability while enabling accurate and modular implementation within SNEM-2 and involves two principal contributions, explained below.

(i) Gaseous absorption k_a

At microwave and millimetre-wave frequencies, gaseous absorption is dominated by molecular oxygen (O_2) and water vapor (H_2O) through line and continuum processes. Modern formulations, such as **Rosenkranz/MPM-type** models, explicitly account for the nonlinear dependence on temperature, pressure, and humidity and must be adopted instead of simple partial-pressure approximations [1,292]. Accordingly,

$$k_a(\nu, T, p, q) = \kappa_{O_2}(\nu; T, p) + \kappa_{H_2O}(\nu; T, p, q), \quad (8.3)$$

and provide the gaseous contribution to the optical depth and to the absorption-weighted atmospheric source term.

(ii) Hydrometeor absorption and scattering k_s

The hydrometeor contribution depends on particle concentration, size distribution, phase, and complex refractive index. For a particle population characterized by a size distribution $N(r)$ and refractive index $m(\nu)$, the scattering and absorption components are expressed as:

$$k_s(\nu) = \int_{r_{min}}^{r_{max}} N(r) C_{sca}(r, \nu, m) dr, \quad k_a(\nu) = \int_{r_{min}}^{r_{max}} N(r) C_{abs}(r, \nu, m) dr, \quad (8.4)$$

with total extinction cross section $C_{ext} = C_{sca} + C_{abs}$ [4,30]. Small particles satisfy the Rayleigh approximation, whereas larger snow, ice, and mixed-phase hydrometeors require **full Mie or T-matrix calculations** to accurately represent scattering and absorption [4,30,306,307]. Beyond the *Rayleigh regime*, non-spherical particle effects become significant, particularly for ice hydrometeors [291]. In the **Rayleigh limit** (when $x = 2\pi r/\lambda \ll 1$):

$$C_{sca} \propto \frac{r^6}{\lambda^4} \cdot \left| \frac{m^2 - 1}{m^2 + 2} \right|^2, \quad C_{abs} \propto r^3 \text{Im} \left[\frac{m^2 - 1}{m^2 + 2} \right]. \quad (8.5)$$

The resulting single-particle and bulk scattering properties enter the radiative-transfer model through the single-scattering albedo $\omega_0 = k_s/k_e$, and the asymmetry factor g , which jointly control the partition between absorption, scattering, and angular redistribution of radiation



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[6,78]. Working with diameter D :

$$k_s(\nu) = \int_{D_{\min}}^{D_{\max}} N(D) C_{\text{sca}}(D, \nu, m) dD, \quad (8.6)$$

where: $C_{\text{sca}}(D, \nu) = Q_{\text{sca}}(D, \nu) \cdot \frac{\pi D^2}{4}$. An elegant other definition is:

$$k_s(\nu) = \int_{r_{\min}}^{r_{\max}} N(r) Q_{\text{sca}}(r, \nu, m) \pi r^2 dr, \quad (8.7)$$

where Q_{sca} is the scattering efficiency.

§ 8.2.5 Experimental Strategies to Constrain the k_e – T_B Relationship

To quantify and constrain the empirical relationship between the specific extinction coefficient k_e and the observed brightness temperature T_B , three complementary experimental strategies were adopted.

1. **Nonlinear regression.** A parametric surrogate of the form $T_B = \alpha + \beta(1 - e^{-\gamma \tau_e})$ was fitted to simulated and observed datasets, where α represents the background radiometric offset, while β and γ depend on frequency band and viewing geometry. This formulation provides a compact and physically interpretable approximation of the attenuation–temperature response and captures the saturation behavior of T_B at large optical depths.
2. **Machine-learning approaches.** Several data-driven techniques were explored to refine extinction retrievals and identify microphysical patterns not captured by parametric models. Convolutional Neural Networks (CNN) were used to associate spatial structures in T_B fields with cloud microphysical properties, Random Forest regression was applied to estimate k_e directly from observed T_B under constrained thermodynamic conditions, and Generative Adversarial Networks and Conditional GANs were employed to synthesize radiatively consistent three-dimensional cloud fields satisfying imposed radiative-transfer constraints.
3. **Satellite data comparison.** Theoretical predictions were validated against multi-platform observations, combining attenuation information from CloudSat-CPR at 94 GHz and GPM-DPR at Ka/Ku bands with brightness-temperature measurements from AMSU-B, ATMS, and ALPHASAT. Atmospheric state variables required to constrain the thermodynamic environment were provided by ERA5 and MERRA-2 reanalyses. The resulting datasets enable a consistent evaluation of the $k_e - T_B$ relationship across frequencies, viewing geometries, and cloud regimes.

A robust empirical relationship between k_e and T_B supports improved retrievals of liquid- and ice-water content and provides a physically constrained basis for radiative-transfer modelling and millimetre-wave link-performance predictions.

§ 8.2.6 Extension of the SNEM Model to Diverse Atmospheric Conditions

The extension of SNEM to account for seasonal and geographical variability constitutes a significant advancement in the modelling of millimetre-wave propagation in the W-band (70–95 GHz). The updated framework explicitly resolves the spatiotemporal variability of attenuation and radiative effects, thereby improving predictive skill across different climatic regimes, precipitation conditions, and observational geometries [84,159].

§ 8.2.6.1 Challenges and Limitations

Despite the improved model fidelity, persistent discrepancies of approximately 20–30 % remain between theoretical and observed attenuation, particularly under intense precipitation conditions. These biases primarily arise from the limited representation of hydrometeor microphysics in global reanalyses, including graupel evolution, melting-layer structure, and sub-grid cloud heterogeneity [12,183].

ERA5 provides enhanced temporal resolution and improved vertical structure relative to earlier datasets, while **MERRA-2** remains robust for climatological analyses. However, neither dataset resolves the microphysical detail required for high-accuracy W-band extinction modelling, especially in strongly scattering regimes [12,183].

§ 8.2.6.2 Data Integration and Machine-Learning Enhancements

Model refinement was achieved through the integration of complementary observational and reanalysis datasets:

- **CloudSat-CPR** for vertically resolved hydrometeor profiles [14,97];
- **GPM-DPR, TRMM** for global precipitation structure and phase information [62,254];
- **ALPHASAT** propagation-link measurements at 20/40 GHz, extrapolated to W-band [66,200].

To overcome observational gaps and physical limitations, AI-based surrogate models were introduced. Recurrent neural networks (LSTM/GRU) capture seasonal and latitudinal modulation of attenuation, generative models (GAN/CGAN) synthesize physically consistent attenuation fields, and physics-informed architectures—such as PINNs and physics-guided GANs—embed radiative-transfer constraints directly into the learning process [33,61,89]. Model explainability techniques, including SHAP and LIME, further enhance interpretability by ranking the dominant atmospheric drivers of W-band propagation variability, as shown in [59,140].

8.2.6.3 Applications and Perspectives

The enhanced SNEM framework (SNEM-2) enables accurate prediction of W-band attenuation with direct applications to satellite communications—such as link-budget optimisation and



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adaptive modulation schemes—as well as to passive and active remote sensing, including improved cloud and ice retrievals. In addition, the framework supports geoclimatic analyses of millimetre-wave variability on regional and global scales [84, 159, 178].

By integrating physically based modelling, satellite observations, and machine-learning surrogates into a unified system, the proposed approach establishes a multiscale predictive paradigm for W-band radiowave propagation, balancing theoretical rigor with operational relevance.

§ 8.2.7 Evaluation of SNEM Performance at W-Band: Seasonal and Spectral Perspectives

The comparison between SNEM-2 and the ERA5/ITU-R baseline at 94-GHz reveals consistent performance improvements across all diagnostics. The Δ_{RMSE} fields (**Figs. 8.2-A1, A2**) indicate error reductions on the order of $0.1\text{--}0.2\text{ dB km}^{-1}$, consistent with typical W-band attenuation levels of $0.02\text{--}0.1\text{ dB km}^{-1}$ for gaseous absorption and up to 0.5 dB km^{-1} under hydrometeor-rich conditions. Improvements are most pronounced over tropical regions during JJA, where deep convection enhances extinction.

Zonal-mean differences (**Fig. 8.2-B**) further show that the baseline formulation systematically underestimates tropical attenuation by approximately 0.02 dB km^{-1} , reflecting known limitations of reanalysis-based microphysics in convective regimes.

Spectral diagnostics (**Fig. 8.2-C**) confirm the internal physical consistency of the model: gaseous absorption peaks between 0.03 and 0.07 dB km^{-1} over the 70–95 GHz range, in agreement with the $O_2\text{--}H_2O$ features described by ITU-R P.676, while hydrometeor attenuation increases monotonically with frequency, reaching approximately 0.03 dB km^{-1} under typical cloud conditions.

Taylor diagrams (**Figs. 8.2-D1, D2**) corroborate these findings, showing that SNEM simultaneously increases correlation and reduces normalized error relative to reference fields, thereby improving spatial fidelity without introducing bias penalties.

Time-series analyses (**Figs. 8.2-E1, E3**) at representative sites—**Scanzano** (PA), **Rieti** (Italy), and **Troll** (Antarctica)—yield total attenuations of $0.5\text{--}1.5\text{ dB}$ over a 3 km slant path, corresponding to $0.15\text{--}0.5\text{ dB km}^{-1}$. Mediterranean sites exhibit larger variability associated with mixed-phase clouds, whereas polar stations show lower medians and greater stability, consistent with climatological expectations. Overall, these results demonstrate that SNEM enhances the representation of W-band attenuation across seasonal, spectral, and latitudinal regimes, with the largest improvements occurring over deep-convective regions.

§ 8.2.8 Model Intercomparison Through Robust Diagnostics

The performance of the extended SNEM parameterization was assessed using a suite of robust diagnostics designed to separate genuine structural improvements from sampling noise. Spatial



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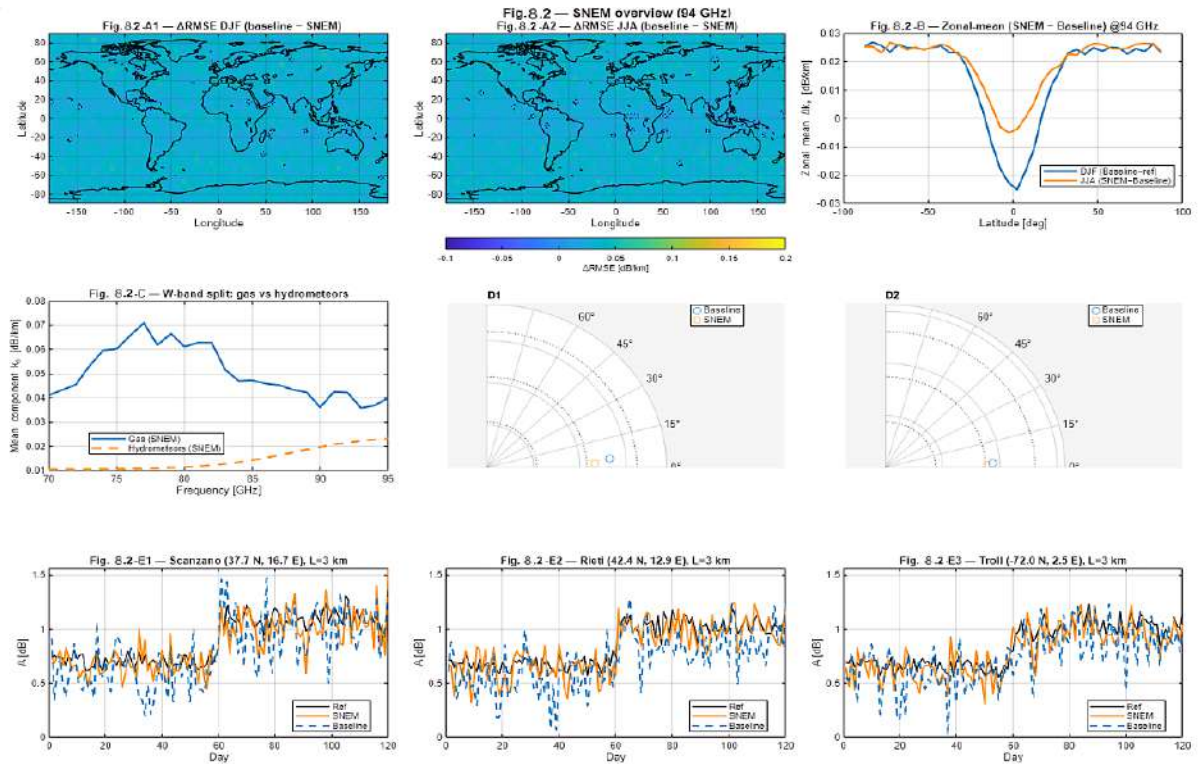


Figure 8.2. Comparative analysis of atmospheric attenuation at 94 [GHz]. (A1–A2) Seasonal maps of $\Delta RMSE$ (baseline – SNEM) for DJF and JJA, including significance contours and continental boundaries. (B) Zonal-mean differences (SNEM – baseline). (C) W-band spectrum separating gaseous and hydrometeor contributions. (D1–D2) Taylor diagrams summarizing spatial statistics for DJF and JJA. (E1–E3) Time series of path-integrated attenuation ($L = 3$ [km]) at Scanzano (PA), Rieti, and Troll. **Data Source:** SNEM-2 framework.

skill gains were quantified using the Δ_{RMSE} metric, defined as:

$$\Delta_{\text{RMSE}} = \text{RMSE}_{\text{baseline}} - \text{RMSE}_{\text{SNEM}}, \quad (8.11)$$

where positive values indicate improved performance of SNEM. Statistical significance was evaluated using a two-sided t -test applied to daily mean-square-error differences. Zonal averaging suppresses longitudinal variability and highlights climatologically coherent biases, particularly within tropical summer regimes consistent with known limitations of reanalysis-based microphysics in convective environments [332, 330].

Taylor diagrams provide a complementary assessment of spatial covariance: the radial coordinate represents the standard deviation σ , while the azimuthal angle encodes $\arccos(\rho)$, with ρ denoting the spatial correlation relative to reference fields. Points approaching the ideal location ($\rho \rightarrow 1, \sigma \rightarrow \sigma_{\text{ref}}$) indicate enhanced structural fidelity. Time-series diagnostics at selected stations, evaluated over a 3 km slant path, translate these spatial gains into operational benefits relevant for fade-margin design, antenna calibration, and CubeSat link optimization [3]. Across all configurations, the combined use of spectral separation (gaseous versus hydrometeor contributions), seasonal diagnostics, and Δ_{RMSE} /Taylor statistics demonstrates that the extended SNEM formulation systematically reduces errors in both the extinction coefficient k_e and the derived brightness temperature T_B .

This provides a physically consistent foundation for radiometric correction and high-frequency link engineering. Nevertheless, limitations remain. The single-layer approximation adopted for T_B and the reliance on an effective k_e necessarily simplify multiple-scattering processes. Accurate quantitative retrievals under intense precipitation conditions still require explicit scattering radiative-transfer solvers and enhanced microphysical parameterizations.

Figure 8.3 presents the global distribution of attenuation-error magnitudes $|\Delta A|$ for a 3 km slant path. The associated boxplot shows that SNEM substantially reduces both the median and the variance of attenuation errors relative to the baseline, yielding a tighter distribution and suppressing upper-tail outliers (below approximately 0.7 dB).

Figure 8.4 reports the regression between brightness temperature T_B and specific attenuation A over 3 km for three representative sites—**Scanzano** (Mediterranean, 37.7°N), **Rieti** (Central Italy, 42.4°N), and **Troll** (Antarctica, 72.0°S)—with DJF and JJA samples distinguished through color-coded linear fits. All locations exhibit a clear seasonal modulation of the T_B – A slope: warmer and more humid JJA conditions shift mid-latitude observations toward higher T_B at fixed attenuation, whereas polar environments show the opposite behavior due to colder backgrounds and reduced gaseous emission.

These results confirm that geographical and seasonal variability critically govern the T_B – A relationship, reinforcing the need for climate-adaptive radiative-transfer parameterizations.

Finally, **Figure 8.5** summarizes the complete SNEM workflow, from data ingestion to validation and application. The system integrates ERA5 reanalysis [332], ITU-R parameterizations, and satellite and ground-based observations. Within the SNEM core, extended routines explicitly

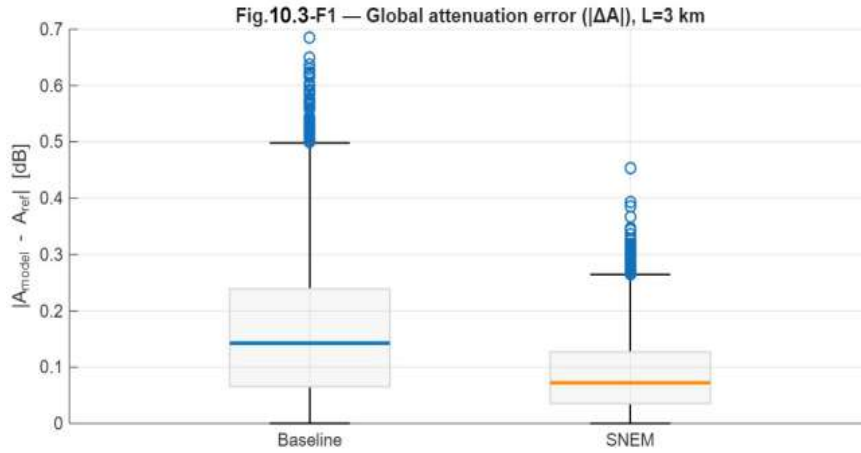


Figure 8.3. Global attenuation error ($|\Delta A|$), $L = 3$ [km]. Distribution of the absolute attenuation error $|A_{model} - A_{ref}|$ over a random global space-time sample. Box-charts compare baseline and SNEM against the reference; horizontal lines mark medians (blue = baseline, orange = SNEM). A lower median for SNEM indicates higher accuracy.

address seasonal and tropical regimes while performing a spectral decomposition between gaseous and hydrometeor contributions. Model outputs include the extinction coefficient at 94 GHz, path-integrated attenuation, and brightness temperature T_B , evaluated through Δ_{RMSE} , Taylor diagrams, and zonal and temporal statistics to ensure physical and statistical consistency with observations.

These validated results underpin a broad range of applications, from link-budget optimization and radiometric correction to the design of CubeSat and next-generation communication links, demonstrating the versatility of SNEM as both a scientific tool and an operational modelling framework [3,6].

§ 8.3.2 Discussion and Original Contribution

The theoretical framework developed in this work builds upon classical electromagnetic-scattering theory—namely **Maxwell–Garnett mixing**, **Bruggeman formulations** and **Mie scattering**—but extends their applicability to W-band hydrometeors and integrates them into a unified radiative-transfer workflow. While these formulations are historically grounded in optical and infrared scattering (**Maxwell Garnett**, 1904; **Bohren & Huffman**, 1983), their operational use at millimeter wavelengths has remained limited, particularly for porous and dendritic ice aggregates [4,10].

A key finding of this work is that the **Maxwell–Garnett** relation remains numerically stable and physically meaningful at 70–95 GHz when the complex permittivity of ice is described using a dispersion-aware **Debye model**. This allows heterogeneous ice particles—aggregates, dendrites, and mixed-phase structures—to be represented through an effective permittivity $\epsilon_{eff}(\nu)$ that explicitly captures the air–ice volume fraction and its frequency dependence [4,103].

Embedding this dielectric representation into the **SNEM-2 radiative-transfer solver**

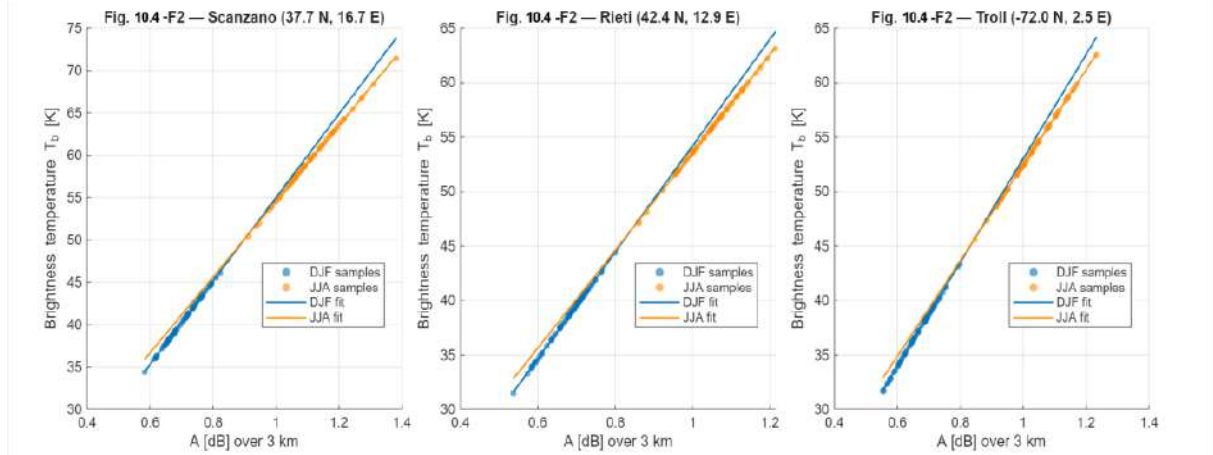


Figure 8.4. Relationship between attenuation and brightness temperature. Scatter plots of attenuation A (over 3 [km]) versus brightness temperature T_B at the three sites (panels). DJF points (blue) and JJA points (orange) highlight seasonality; lines are simple linear fits for visual guidance. T_B is estimated with a single-layer model: $T_B \approx T_{cmb} * 10^{-A/10} + T_{atm} [1 - 10^{-A/10}]$. Increasing A corresponds to higher T_B , more pronounced under warm/humid (JJA) conditions.

SNEM workflow: inputs, core, outputs, validation and application

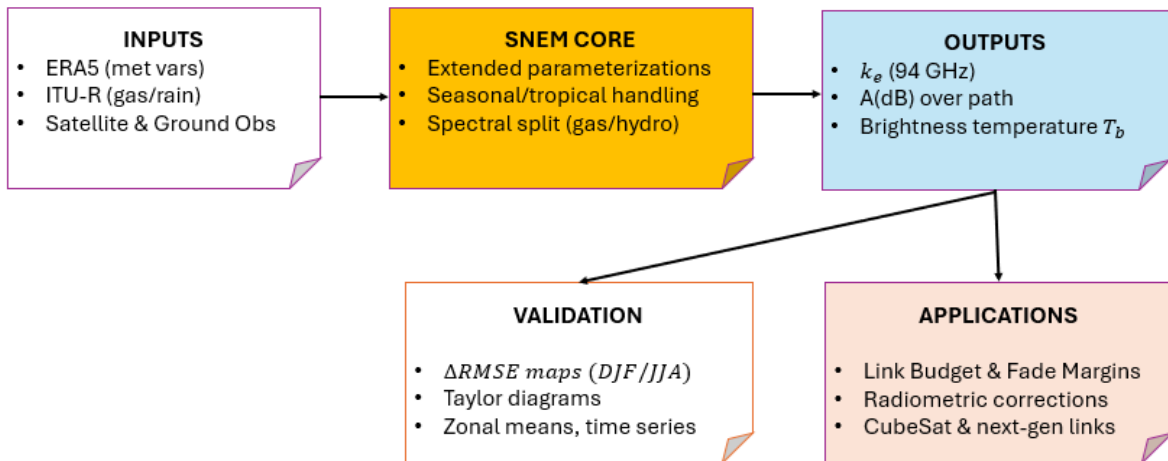


Figure 8.5. SNEM workflow: inputs, core, outputs, validation, and applications. Conceptual workflow of SNEM: inputs (ERA5, ITU-R recommendations, satellite and ground observations), model core (extended parameterizations, seasonal/tropical handling, gas/hydrometeor spectral split), outputs (k_e at 94 [GHz], path attenuation, brightness temperature), validation (Δ_{RMSE} maps, Taylor diagrams, zonal means, time series), and applications (link budget and fade margins, radiometric corrections, CubeSat constellations and next-generation space communication systems).



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establishes a continuous analytical chain:

$$\epsilon_{eff}(\nu) \rightarrow m(\nu) \rightarrow Q_{ext}(\nu, r) \rightarrow k_e(\nu) \rightarrow \gamma(\nu) \rightarrow T_B(\nu), \quad (8.12)$$

providing a microphysical–radiometric bridge from particle composition to observable quantities such as attenuation and brightness temperature. To the author’s knowledge, such an explicit, operational link between microphysical structure, Mie-driven extinction and W-band radiative output has not been formalized in prior literature. In this sense, the present framework moves beyond the traditional use of effective-medium theory as a diagnostic tool and elevates it to a predictive component within a full radiative-transfer chain.

This framework was implemented and tested using **Fortran** and **MATLAB** versions of **SNEM-2**, enabling self-consistent calculations of extinction coefficients, specific attenuation and vertical T_B profiles under atmospheric conditions derived from ERA5, CloudSat, GPM and ALPHASAT observations [6,97, 332]. This hybrid physics–data workflow parallels modern radiative-transfer paradigms (**Evans**, 2005 [6]) and resonates with recent developments in physics-informed neural models for atmospheric systems (**Zhu et al.**, 2023 [89]).

In summary, the originality of this contribution lies in:

1. **Extending** the validity of Maxwell–Garnett mixing to W-band hydrometeors, including non-dilute porosities and dispersive dielectric behavior [4,10];
2. **Integrating** the resulting $\epsilon_{eff}(\nu)$ into a full $ESP \rightarrow Mie \rightarrow k_e \rightarrow RT$ workflow within SNEM-2 [6];
3. **Linking** microphysical dielectric properties to radiometric and telecommunication metrics (attenuation, T_B , link budgets) in a coherent, computationally efficient framework [1,3].

Taken together, this work transforms a set of classical optical relations into a modern, operational model for W-band atmospheric propagation, bridging microscale dielectric physics with macroscale radiative and communication applications.

§8.3.3 Mie Scattering in the ESP Framework

Within the **Equivalent Spherical Particle (ESP)** formulation introduced in the previous sections, the scattering and absorption properties of ice hydrometeors are computed through the full analytical **Mie solution** (**Mie**, 1908 [177]). In this setting, each non-spherical or porous ice particle is represented by an equivalent homogeneous sphere characterized by a complex effective permittivity $\epsilon_{eff}(\nu)$, derived using classical effective-medium mixing rules, primarily the **Maxwell–Garnett** formulation (with the **Bruggeman** rule considered as a limiting symmetric case), as detailed in **Sections §§8.4.1–8.4.2**.

The corresponding refractive index $m(\nu) = \sqrt{\epsilon_{eff}(\nu)}$ serves as the electromagnetic descriptor entering the Mie coefficients a_n and b_n as expressed in **Appendix B.6**, where $x = 2\pi r/\lambda$ is the size parameter and ψ_n, ξ_n are **Riccati–Bessel** functions. The scattering, extinction and



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absorption efficiencies follow as $Q_{ext}, Q_{sca}, Q_{abs}$ are then expressed through the standard Mie series expansions as:

$$Q_{ext} = \frac{2}{x^2} \sum_{n=1}^N (2n+1) \operatorname{Re}(a_n + b_n), \quad Q_{sca} = \frac{2}{x^2} \sum_{n=1}^N (2n+1) (|a_n|^2 + |b_n|^2), \quad (8.13)$$

$$Q_{abs} = Q_{ext} - Q_{sca}. \quad (8.14)$$

Bulk radiative properties are obtained by integrating the single-particle cross-sections over the particle-size distribution $N(r)$:

$$k_e(\nu) = \int N(r) Q_{ext}(r, \nu) \cdot \pi r^2 dr, \quad (8.15)$$

yielding the extinction coefficient $k_e = k_a + k_s$, which enters directly in the radiative-transfer closure of **SNEM-2**.

This formulation is particularly relevant at W-band, where typical hydrometeor radii ($r \sim 1\text{--}3$ mm) correspond to size parameters $x \in [2, 10]$, a regime where neither Rayleigh nor geometric-optics asymptotics are valid. The full Mie solution is therefore required to capture resonance oscillations, interference effects, and the strong forward-scattering enhancement characteristic of millimeter-wave propagation [103, 114]. Two additional quantities essential for radiative-transfer modelling are derived from the **Mie phase function** $P(\Theta)$:

$$\omega_0 = \frac{k_s}{k_e}, \quad g = \frac{1}{2} \int_{-1}^{+1} P(\cos\Theta) \cos(\Theta) d(\cos\Theta), \quad (8.16)$$

namely the single-scattering albedo and the asymmetry factor. These parameters govern the balance between absorption and directional redistribution of radiance, and therefore control both the effective optical thickness:

$$\tau = \int k_e(z) dz \quad (8.17)$$

and the brightness-temperature response T_B along the propagation path [5,6]. The ESP–Mie formalism thus constitutes the central physical bridge linking hydrometeor microstructure to radiometric and attenuation outputs in SNEM-2. Its integration into the model ensures a physically rigorous, spectrally resolved, and computationally tractable description of scattering processes at W-band, complementing and extending the dielectric-mixing framework established in §§ 8.4.1–8.4.2.

§ 8.3.4 Limitations and Further Developments

While the ESP–Mie formulation provides analytic tractability and computational efficiency, it necessarily remains an approximation. Ice hydrometeors—particularly aggregates, dendrites, and graupel—are intrinsically non-spherical and anisotropic, producing directional scattering patterns and phase-function behaviours that cannot be fully captured by the spherical assumption



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underlying Mie theory [4, 103].

The impact of particle shape and morphology on electromagnetic scattering has been extensively investigated in the literature using higher-fidelity electromagnetic solvers, such as the Discrete Dipole Approximation (DDA) and the T-matrix method [Draine and Flatau, 1994; Mishchenko et al., 1996; 103]. These approaches explicitly account for particle geometry and orientation and have demonstrated systematic deviations from Mie-based predictions in terms of backscattering cross sections, asymmetry factor g , and angular redistribution, particularly for elongated crystals and highly porous aggregates [103, 114]. A detailed treatment of these methods, however, lies outside the scope of the present work and is therefore not included for reasons of focus and space.

Within this thesis, non-sphericity effects are addressed at a conceptual and parametric level, adopting physically motivated effective descriptions compatible with large-scale radiative-transfer and statistical analyses. This choice reflects a deliberate trade-off between physical realism and computational feasibility, consistent with the objectives of the present study.

Future developments may incorporate more explicit shape-aware formulations, potentially guided by physics-informed or data-driven approaches, in order to further bridge the gap between idealized ESP-based scattering models and the true morphological complexity of atmospheric ice hydrometeors [89].

§ 8.4.1 Logical Pathway of the Research Developed

The research pathway developed in this thesis originates from an initial phase of activity aimed at exploiting brightness-temperature simulations as a tool for probing atmospheric radiative behaviour under controlled conditions. This early stage, conducted in collaboration with **Profs. Davide Comite** and **Marianna Biscarini** [Luini,Comite,Biscarini,WP1–Rep.2022], focused on the systematic analysis of SNEM-generated radiometric outputs in order to assess their sensitivity to variations in atmospheric composition and hydrometeor content.

In this preliminary phase, brightness-temperature fields were analyzed primarily from a radiometric perspective, with the objective of identifying robust patterns and response regimes associated with gaseous absorption and cloud-induced extinction. These analyses provided an initial qualitative and quantitative understanding of how radiometric signatures evolve across different atmospheric scenarios, but did not yet address propagation metrics in a fully integrated manner.

Building on these results, the research progressively shifted toward a more comprehensive characterization of electromagnetic-wave propagation, in which radiometric outputs were explicitly linked to extinction coefficients and path-integrated attenuation. This transition marked a methodological step forward, moving from exploratory radiometric interpretation to a physically grounded investigation of propagation impairments.

This evolution motivated the development of an extended modeling framework, ultimately formalized as SNEM-2, in which radiative-transfer simulations are coupled with systematic



diagnostic analyses. Within SNEM-2, brightness-temperature outputs are no longer examined in isolation, but are jointly interpreted with microphysical indicators and attenuation metrics, enabling a unified treatment of radiometry, extinction, and propagation effects across different atmospheric regimes.

§ 8.4.2 Transition from Radiometric Analysis to Attenuation Diagnostics

Following the initial radiometric investigations, the focus of the research shifted toward the explicit quantification of atmospheric extinction and attenuation processes. This transition required moving beyond the qualitative interpretation of brightness-temperature patterns and adopting a framework in which radiometric outputs could be systematically connected to propagation-relevant quantities.

Within SNEM-2, this transition is achieved by exploiting the internal consistency of the radiative-transfer solution. For each simulated atmospheric realization, frequency-dependent extinction coefficients are computed from the underlying microphysical description and vertically integrated to obtain optical depth and path-integrated attenuation. Brightness temperature and attenuation are therefore derived from the same physical state of the atmosphere, ensuring that radiometric and propagation quantities remain fully coupled.

This approach enables the construction of physically meaningful diagnostics that relate brightness-temperature variations to attenuation strength under different atmospheric conditions. In particular, joint analyses of brightness temperature, extinction, and hydrometeor content allow distinct radiative regimes to be identified, ranging from optically thin conditions dominated by gaseous absorption to optically thick scenarios in which ice-phase hydrometeors and multiple scattering play a major role.

The adoption of attenuation-oriented diagnostics represents a key methodological advancement with respect to the initial radiometric studies. Rather than inferring propagation effects indirectly, SNEM-2 provides a direct and quantitative link between radiometric signatures and attenuation metrics, thereby allowing the sensitivity of brightness temperature to extinction processes to be assessed in a controlled and physically interpretable manner.

§8.4.3 Diagnostic Relationships Between Brightness Temperature, Extinction, and Ice Content

The diagnostic framework implemented in SNEM-2 enables a systematic investigation of the relationships between brightness temperature, atmospheric extinction, and hydrometeor properties. Rather than analyzing these quantities independently, SNEM-2 allows them to be examined jointly, exploiting the fact that they originate from the same physically consistent radiative-transfer solution.

To this end, a set of diagnostic representations is constructed in which brightness temperature is analysed in relation to path-integrated attenuation, extinction coefficients, and vertically integrated ice content. These diagnostics are designed to highlight the physical coupling between



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radiometric observables and microphysical drivers, and to identify characteristic radiative regimes associated with different atmospheric conditions.

In particular, scatter-plot analyses are employed to explore how brightness temperature responds to increasing extinction and ice content. Under optically thin conditions, variations in brightness temperature are primarily controlled by gaseous absorption and weak cloud extinction, resulting in a relatively broad dispersion of radiometric values. As extinction increases, the sensitivity of brightness temperature to hydrometeor-induced effects becomes more pronounced, and distinct clustering patterns emerge that reflect the transition toward optically thicker regimes.

At higher ice contents and extinction levels, brightness temperature exhibits a progressive convergence toward saturation, indicating the increasing dominance of multiple scattering and self-emission within the atmospheric column. This behavior is consistent with the theoretical expectations of radiative-transfer theory and provides a physically interpretable signature of ice-dominated scattering regimes.

By examining these diagnostic relationships across different frequencies, geographical locations, and atmospheric scenarios, SNEM-2 enables the identification of systematic trends and variability patterns. The resulting diagnostics therefore serve a dual purpose: they provide a qualitative visualization of radiative-transfer behaviour, and they offer a quantitative basis for assessing the contribution of ice-phase hydrometeors to both brightness temperature and attenuation.

The diagnostic relationships introduced in this section form the basis for the subsequent analysis and interpretation of the results, where the observed patterns are examined in greater detail and related to specific microphysical and climatic drivers.

§ 8.4.4 Interpretation of Diagnostic Patterns and Physical Regimes

The diagnostic relationships introduced in the previous section allow the radiative behaviour of the atmosphere to be interpreted in terms of distinct physical regimes governed by extinction strength, hydrometeor phase, and microphysical structure. By jointly analysing brightness temperature, attenuation, and ice content, it becomes possible to disentangle the relative contributions of gaseous absorption, cloud extinction, and multiple scattering across a wide range of atmospheric conditions.

In low-extinction regimes, typically associated with clear-sky or tenuous cloud conditions, brightness temperature variations remain weakly correlated with ice content and attenuation. In this regime, radiometric signatures are primarily controlled by gaseous absorption and surface emission, while hydrometeor-induced effects play a secondary role. The broad dispersion observed in brightness-temperature values reflects sensitivity to thermodynamic variability rather than to cloud microphysics.

As extinction increases, a transition toward cloud-influenced regimes becomes evident. In these conditions, brightness temperature exhibits a stronger dependence on ice content and attenuation, indicating the growing impact of hydrometeor-induced extinction and scattering. Mixed-phase and ice-dominated clouds introduce additional variability through changes in particle



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size, density, and dielectric properties, which modulate both extinction coefficients and scattering efficiency. The emergence of clustered patterns in the diagnostic space reflects the onset of regime-dependent radiative behavior.

At high extinction levels, corresponding to optically thick ice clouds and deep convective systems, brightness temperature progressively approaches saturation. In this regime, multiple scattering and cloud self-emission dominate the radiative-transfer process, reducing sensitivity to further increases in ice content. The convergence of brightness-temperature values under these conditions is consistent with theoretical expectations and represents a characteristic signature of ice-dominated scattering regimes at millimetre-wave frequencies.

The interpretation of these regimes highlights the dual role of brightness temperature as both a radiometric observable and a diagnostic proxy for extinction processes. While brightness temperature alone does not uniquely determine attenuation, its systematic relationship with extinction and ice content provides valuable physical insight into the mechanisms governing propagation impairments. In particular, the identification of regime transitions offers a robust framework for interpreting radiometric observations in terms of underlying microphysical drivers.

These findings support the methodological choice adopted in SNEM-2, in which brightness temperature is used as an intermediate diagnostic quantity rather than as a standalone metric. By embedding radiometric and attenuation analyses within a unified radiative-transfer framework, SNEM-2 enables physically consistent interpretation across clear-sky, cloud-dominated, and ice-rich atmospheric regimes, laying the groundwork for the validation and comparative analyses presented in the following sections.

§ 8.4.5 Validation and Implications for Propagation Modelling

The physical interpretation of brightness-temperature and attenuation regimes discussed above is further substantiated through comparison with independent reference datasets and established propagation models. This validation step is essential to assess the robustness of the diagnostic relationships identified within SNEM-2 and to evaluate their relevance for practical propagation studies at millimetre-wave frequencies.

Comparisons against reanalysis-based products and ITU-R parameterizations show that the regime-dependent behaviour identified in SNEM-2 is consistent with known climatological and radiative-transfer characteristics. In particular, the transition from gas-dominated to hydrometeor-dominated regimes, as inferred from joint brightness-temperature and attenuation diagnostics, aligns with established expectations regarding the limitations of simplified microphysical representations under ice-rich and convective conditions.

The validation analyses indicate that SNEM-2 provides a more nuanced representation of extinction variability than baseline formulations, especially in regimes characterized by significant ice content and strong scattering. By retaining a physically consistent coupling between radiometric observables and attenuation metrics, the model reduces systematic biases associated with the underestimation of hydrometeor-induced effects, while preserving agreement



with reference behaviour under clear-sky conditions.

Beyond validation, these results have direct implications for propagation modelling and system-level analyses. The identification of distinct radiative regimes enables attenuation estimates to be interpreted in a physically transparent manner, improving confidence in their applicability across different climatic and geographical contexts. Moreover, the use of brightness temperature as an intermediate diagnostic quantity offers a practical pathway for integrating radiometric observations into propagation-oriented studies, including link-budget assessment and performance evaluation at W-band frequencies.

Overall, the analyses presented in this section demonstrate that the theoretical investigation of brightness temperature provides a robust foundation for attenuation modelling within SNEM-2. By embedding radiometric diagnostics, extinction physics, and propagation metrics within a unified framework, the approach developed in this thesis enhances both the physical interpretability and the predictive reliability of millimetre-wave propagation studies.

§ 8.4.6 Key Challenges in W-Band Satellite Communications

The adoption of the W-band (75–110 GHz) for satellite communications remains an emerging frontier, characterized by both scientific and technological challenges. From a modeling perspective, traditional radiative-transfer frameworks such as SNEM and its enhanced evolution SNEM-2, based on the Eddington approximation of the Radiative Transfer Equation (RTE), provide a physically grounded foundation for analyzing extinction, scattering, emission, and absorption processes [d’Auria et al., 1998; Brussaard, 1995]. However, several improvements are necessary to achieve operational reliability in this spectral range.

Key open issues include:

1. **Non-spherical hydrometeor scattering models** — The integration of advanced electromagnetic solvers such as the T-Matrix method [Mishchenko et al., 2000], the Discrete Dipole Approximation (DDA) [Draine & Flatau, 1994], and Rayleigh–Gans approximations is essential for representing the anisotropic and aggregated nature of ice particles.
2. **Three-dimensional radiative-transfer extensions** — Extending the SNEM framework to 3-D domains will allow the treatment of horizontally inhomogeneous atmosphere and broken-cloud structures [Cahalan et al., 2005; Davis & Marshak, 2010].
3. **Multiple-scattering corrections** — In dense ice and mixed-phase clouds, higher-order or Monte Carlo solutions [Zdunkowski et al., 2007] are required to properly capture photon diffusion and backscattering effects.
4. **Spectral and physical extension beyond 110 GHz** — The applicability of SNEM should be extended toward the sub-THz regime, integrating molecular absorption databases such as HITRAN and AMT [Gordon et al., 2017], and accounting for ionospheric and magneto-ionic effects relevant to deep-space or lunar links [Budden, 1985; Prieto-Cerdeira et al., 2019].



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In parallel, environmental and instrumental constraints remain major obstacles. Atmospheric phenomena such as gaseous absorption, scintillation, depolarization, and cloud inhomogeneity [ITU-R P.676, P.618, P.840] combine with hardware limitations—HPAs, LNAs, antenna design, power consumption, and high-precision tracking requirements [Rappaport et al., 2019; Pérez-Fontán & Prieto-Cerdeira, 2014]—to impose strict boundaries on achievable link margins and data rates.

Regulatory and standardization issues further complicate this scenario: ITU-R propagation models are validated only up to 40 GHz, and W-band spectrum allocations are still under consolidation [ITU-R P.618-13, 2017]. Within this challenging framework, my research has sought not only to strengthen the physical and radiative foundations of attenuation modeling but also to open new directions through the introduction of machine-learning methodologies—including Generative Adversarial Networks (GANs, CGANs) [Goodfellow et al., 2014], Variational Autoencoders (VAEs) [Creswell et al., 2018], and Recurrent Neural Networks (LSTM/GRU) [Hochreiter & Schmidhuber, 1997; Zhang et al., 2021].

These models have been employed for adaptive calibration, synthetic cloud-body generation, and real-time attenuation prediction, providing a self-adaptive complement to the deterministic physics of SNEM-2. Although the use of AI-based techniques was initially considered unconventional at the start of my PhD, I deliberately pursued this direction to bridge deterministic and data-driven paradigms. The resulting hybrid framework enhances the interpretability and adaptability of attenuation modeling, addressing the intrinsic variability of W-band propagation [Delrieu et al., 2020; Zhang et al., 2022].

This dual approach—physics-constrained yet AI-augmented—is crucial for advancing the reliability and resilience of future satellite communication systems, particularly in 6G backhaul architectures, High-Throughput (HTS/VHTS) constellations, and deep-space communication links.

§ 8.4.7 From Brightness Temperature to Path-Integrated Attenuation

Within the radiative-transfer framework adopted in this work, brightness temperature T_B is treated as an integrated radiometric observable that encodes the cumulative effects of atmospheric extinction and emission. This perspective enables a physically consistent connection between radiometric quantities and propagation impairments, providing a natural pathway from brightness-temperature signatures to path-integrated attenuation (PIA).

The original research contribution of this thesis lies in addressing the long-standing challenge of quantifying hydrometeor-induced attenuation starting from statistically generated atmospheric scenarios. By extending the stochastic radiative-transfer model SNEM into the diagnostic and interpretative framework SNEM-2, the problem of estimating PIA from hydrometeor populations is reformulated in a physically consistent and statistically robust manner. In SNEM-2, brightness temperature, extinction coefficients, and path-integrated attenuation are derived from the same underlying microphysical realization, enabling the isolation and quantification of hydrometeor



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contributions to attenuation across different frequencies, atmospheric regimes, and climatic conditions.

This conceptual transition from brightness temperature to path-integrated attenuation is explicitly illustrated by the diagnostic analyses presented in **Figures 8.6, 8.7, and 8.8**. These figures illustrate successive stages of a single physical pathway, in which statistically generated atmospheric realizations produced by SNEM are reinterpreted within the SNEM-2 framework to extract attenuation-relevant information.

Figure 8.6 provides an example of this paradigm by showing how time-resolved attenuation series derived from SNEM-2 outputs can be emulated using machine-learning surrogates trained on physically consistent radiative-transfer data. While the figure highlights the predictive capability of LSTM architectures, its primary relevance in this context lies in demonstrating that attenuation fields generated from stochastic microphysical scenarios retain coherent temporal and physical structure when mapped from radiometric quantities.

Figures 8.7 and 8.8 further develop this concept by directly linking brightness-temperature diagnostics to hydrometeor-induced extinction and attenuation. Scatter-plot relationships between T_B , ice attenuation A_{ice} , and column-integrated ice content (CWC_{ice}) reveal well-defined statistical structures that emerge consistently across sites, frequencies, and climatic regimes. These diagnostics confirm that T_B variations encode the same microphysical information governing extinction coefficients and path-integrated losses.

Taken together, these analyses demonstrate that the transition from brightness temperature to path-integrated attenuation is not a post-processing step, but a physically grounded consequence of the unified radiative-transfer formulation adopted in **SNEM-2**. By deriving T_B , extinction coefficients, and attenuation from the same stochastic atmospheric realization, SNEM-2 enables a consistent separation and quantification of gaseous and hydrometeor contributions, thereby enabling the transformation of radiometric observables into physically traceable and operationally relevant propagation metrics.

§ 8.4.8 Assessing the Contribution of Ice Hydrometeors in SNEM and SNEM-2

To quantify the contribution of ice hydrometeors to atmospheric attenuation, the SNEM-2 framework was employed in combination with the **cldgen Fortan** module of **SNEM** to generate a statistical ensemble of hydrometeor populations, parameterized according to geographical location and meteorological conditions. From the resulting cloud-body datasets, we isolated the cases containing non-zero ice content, flagged separately for rainy and non-rainy events. In order to extract the specific contribution of ice particles, the general attenuation formulation:

$$A(\theta, f) := 4.343 \cdot \frac{\tau_Z(H, f)}{\sin(\theta)}, \quad (8.18)$$

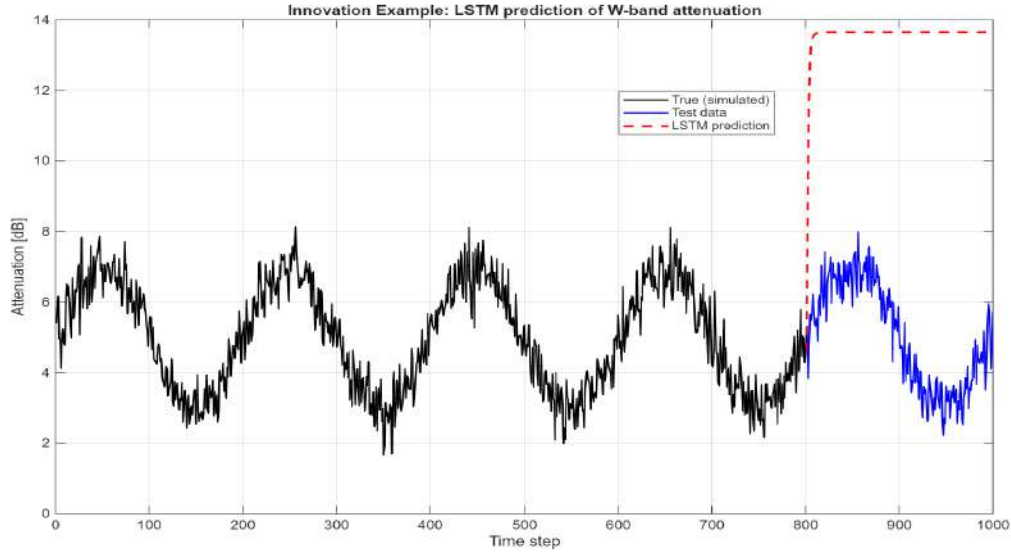


Figure 8.6. Hybrid diagnostic example illustrating the transition from radiometric quantities to path-integrated attenuation. Synthetic attenuation time series derived from SNEM simulations (black), test data (blue), and LSTM-based surrogate prediction (red dashed). The figure demonstrates how physically consistent attenuation fields generated from stochastic microphysical scenarios can be emulated without loss of temporal or physical structure, supporting the use of machine-learning surrogates as complements to the SNEM-2 radiative-transfer framework.

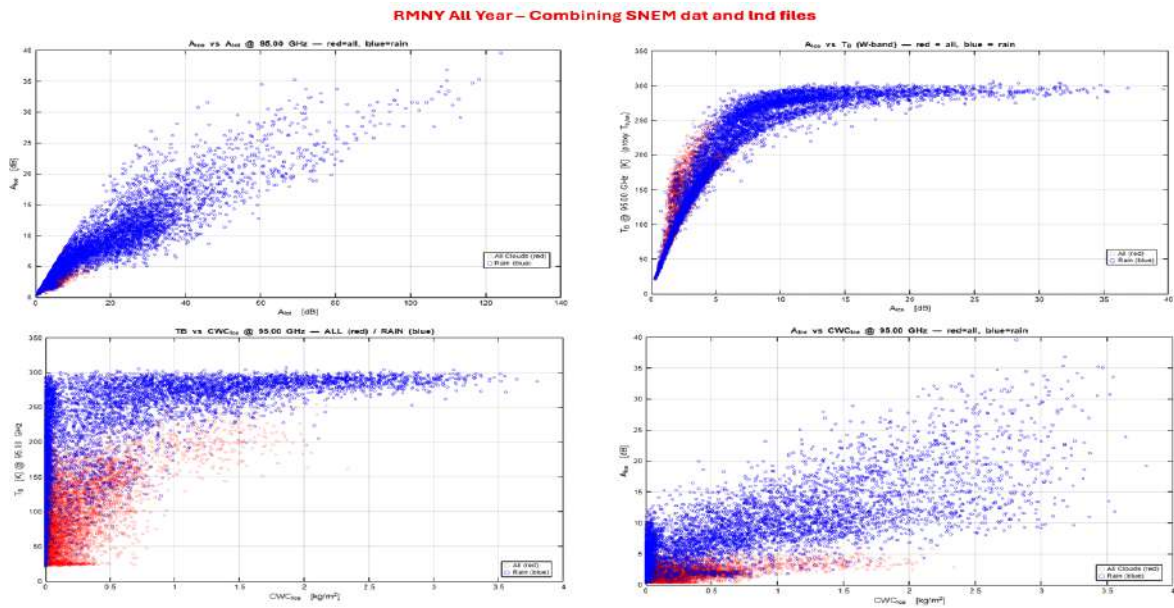


Figure 8.7. Scatter-plot diagnostics of ice contribution in SNEM simulations. (a) A_{ice} vs. A_{tot} ; (b) $T_{B,ice}$ vs. A_{ice} ; (c) $T_{B,ice}$ vs. $CWCI$; (d) A_{ice} vs. $CWCI$, for Rome (NY, 95 GHz). Blue = rain events; Red = all events), Year: 2020/2021.

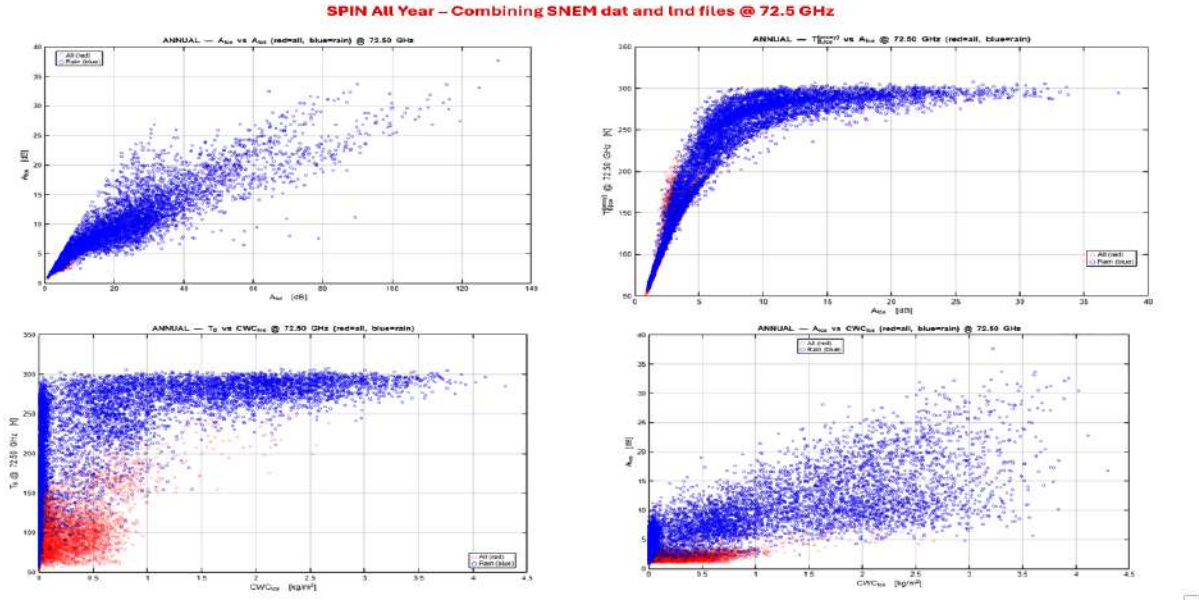


Figure 8.8. Scatter-plot diagnostics of ice contribution in SNEM simulations. (a) A_{ice} vs A_{tot} ; (b) $T_{B,ice}$ vs A_{ice} ; (c) $T_{B,ice}$ vs $CWCI$; (d) A_{ice} vs $CWCI$, for Spino d’Adda (CR, @ 72.5 GHz). Blue = rain events; Red = all events), Year: 2020/2021.

allows the substitution in the brightness temperature relationship below:

$$T_B(z=0, \theta, f) = T_C \exp\left(-\frac{\tau_Z(H, f)}{\sin(\theta)}\right) + T_{mr}(\theta, f) \cdot \left[1 - \exp\left(-\frac{\tau_Z(H, f)}{\sin(\theta)}\right)\right], \quad (8.19)$$

considering that τ is an attenuation. In this way I get that:

$$T_B(z=0, \theta, f) = T_C \exp\left(-\frac{A(\theta, f)}{4.343}\right) + T_{mr}(\theta, f) \cdot \left[1 - \exp\left(-\frac{A(\theta, f)}{4.343}\right)\right]. \quad (8.20)$$

Consequently this relationship (190) was specialized for the “Ice” case, leading to the definition of: $A_{ice}(\theta, f)$, which represents the attenuation due exclusively to ice hydrometeors within the cloud bodies. Correspondingly, the mean radiative temperature associated with ice particles, $T_{mr,Ice}$ was introduced, and I obtain:

$$T_{B,Ice} = T_C \cdot \exp\left(-\frac{A_{Ice}}{4.343}\right) + T_{mr,Ice} \cdot \left[1 - \exp\left(-\frac{A_{Ice}}{4.343}\right)\right]. \quad (8.21)$$

From this, one can isolate $T_{mr,Ice}$ obtaining:

$$T_{mr,Ice} = \frac{T_{B,Ice} - T_C \exp\left(-\frac{A_{Ice}}{4.343}\right)}{1 - \exp\left(-\frac{A_{Ice}}{4.343}\right)}. \quad (8.22)$$

Assuming that an empirical relationship exists between the mean radiative temperature of ice, the attenuation due to ice, and the surface temperature, I have proposed the following



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parameterization:

$$T_{mr,Ice} = aT_S + bA_{ice} + c, \quad (8.23)$$

where a , b , and c are coefficients to be determined from simulations or observations. Substituting this relationship back into the brightness-temperature expression leads to the operational form:

$$T_{B,Ice}(d, f) = T_{mr,Ice} \cdot \left[1 - \exp\left(-\frac{A_{ice}}{4.343}\right) \right] + T_C \cdot \exp\left(-\frac{A_{ice}}{4.343}\right), \quad (8.24)$$

where A_{Ice} is expressed in decibels [dB].

The formulation derived above provides a physically consistent and analytically tractable framework for isolating the contribution of ice hydrometeors to path-integrated attenuation under controlled conditions. By expressing both brightness temperature and attenuation in terms of a common optical-depth variable, the ice-induced component A_{ice} can be explicitly related to radiometric observables, enabling a direct diagnostic interpretation within the SNEM-2 framework.

The preliminary analysis has deliberately focused on the zenith configuration ($\theta = 90^\circ$), where the assumption of a plane-parallel, horizontally homogeneous atmosphere allows the ice contribution to be isolated in a simplified and transparent manner. In this reference configuration, attenuation depends solely on the vertical distribution of hydrometeors and thermodynamic profiles, providing a baseline scenario against which more complex propagation geometries can be evaluated.

However, when extending the analysis to oblique propagation paths ($\theta < 90^\circ$), horizontal inhomogeneities within cloud bodies become non-negligible. Unlike the vertical case, where attenuation is primarily governed by column-integrated properties, slant paths intersect spatially heterogeneous regions characterized by variability in particle density, phase composition, and size distribution. These effects introduce non-uniform attenuation, enhanced multiple scattering, and, in the presence of non-spherical ice particles, additional depolarization mechanisms.

From the perspective of W-band satellite communications, such **horizontal inhomogeneities** are particularly critical for low-elevation links, where long slant paths through structured cloud systems amplify propagation variability. Consequently, while zenith results provide a necessary and physically interpretable baseline, realistic link-performance assessments require the incorporation of three-dimensional radiative-transfer effects or stochastic corrections capable of accounting for horizontal variability.

This consideration motivates the subsequent analyses presented in the following sections, where the ice-induced attenuation diagnosed under zenith conditions is extended to oblique geometries and statistically heterogeneous cloud fields, preserving consistency with the SNEM-2 radiative-transfer formulation while progressively relaxing the simplifying assumptions adopted here.

The analytical framework developed above can now be applied to realistic atmospheric scenarios in order to quantify the impact of ice and mixed-phase hydrometeors on W-band



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radiometric observables and attenuation. To this end, a representative Mediterranean site is considered as a case study.

§ 8.4.9 Case Study: Scanzano (PA) — Ice and Mixed-Phase Contributions at W-Band

The Scanzano site, located within the **Telespazio Space Center** at Piana degli Albanesi (Sicily, 560 m a.s.l.), provides a representative Mediterranean scenario for assessing the radiative and attenuative contribution of hydrometeors at W-band. The climatology of the area is characterized by marked seasonal variability, with convective systems frequently developing during late summer and early autumn, and stratiform precipitation dominating winter months. This alternation produces cloud structures with pronounced vertical and horizontal heterogeneity, making Scanzano an ideal test environment for evaluating the sensitivity of W-band radiative-transfer predictions to cloud microphysics.

Under zenithal observation ($\vartheta = 90^\circ$), the attenuation attributable to ice hydrometeors, A_{Ice} , can be evaluated directly from the vertical hydrometeor distribution generated by the SNEM-2 framework. The corresponding brightness temperature $T_{B,Ice}$ follows from the effective layer formulation of the radiative transfer equation. In this configuration, the plane-parallel approximation is sufficiently accurate to isolate the radiometric signature of ice, producing a baseline scenario that is unaffected by horizontal coupling or azimuthal variability.

When the propagation angle departs from zenith ($\vartheta < 90^\circ$), as in operational satellite-to-ground links, the wave path intersects cloud volumes exhibiting significant horizontal inhomogeneities. Over Scanzano, such structures may include convective updrafts, precipitation bands, and localized orographic formations induced by the surrounding mountainous terrain. These features modify the effective path-integrated attenuation through spatial fluctuations in hydrometeor density, particle-size distribution, phase composition, and aggregation state. The resulting attenuation field becomes non-uniform, with enhanced multiple scattering and, in regions containing non-spherical ice particles, measurable depolarization effects. Although zenithal estimates provide a physically consistent baseline for interpreting the radiative behaviour of ice hydrometeors, realistic link predictions necessarily require three-dimensional radiative-transfer formulations or stochastic corrections capable of capturing the horizontal variability intrinsic to Mediterranean cloud systems.

To illustrate the radiative impact of cloud microphysics at the Scanzano site, **Figure 8.9** presents a comparison between the brightness temperature generated by ice-only clouds and that produced by mixed-phase clouds (ice + liquid) at 94 GHz.

The analysis corresponds to 15 November 2020 at 15:00 UTC, with all simulations performed using the SNEM-2 framework under zenithal propagation. The contrast between the two cases is substantial. In the ice-only scenario, attenuation remains modest because of the low imaginary



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part of the dielectric permittivity of ice, $Im\{\epsilon\}$, and the dominance of **Rayleigh** scattering¹, for which the particle-size parameter satisfies $x = 2\pi r/\lambda \ll 1$. Under these conditions, the path-integrated attenuation rarely exceeds a few decibels, and the brightness temperature saturates around 220–235 K, consistent with upper-tropospheric emission.

Mixed-phase clouds behave very differently. The presence of liquid droplets significantly increases both absorption and scattering, particularly when droplets approach the Mie regime ($x \sim O(1)$). The imaginary part of the permittivity becomes much larger, and the resulting attenuation along the propagation path can reach 15–30 dB for slant geometries. Brightness temperatures rise toward 270–280 K, reflecting a substantially warmer and more emissive effective layer.

From a communication-system perspective, this difference is critical: ice-only clouds exert a relatively minor influence on W-band links, whereas mixed-phase clouds may induce severe fading, especially at low elevation angles where the effective path length is amplified by the $A/\sin(\theta)$ scaling.

The radiative-transfer relationship underlying these results is expressed as:

$$T_B(\nu, \theta) = T_C e^{-\tau(\nu, \theta)} + \int_0^H k_a(z, \nu) T(z) e^{-[\tau(H, \nu, \theta) - \tau(z, \nu, \theta)]} \frac{dz}{\sin(\theta)}, \quad (8.26)$$

which, in its compact effective-layer form, reduces to:

$$T_B \simeq T_{mr}(1 - e^{-A/3.434}) + T_C e^{-A/4.343}, \quad A[dB] = 3.434 \cdot \tau. \quad (8.27)$$

The microscopic origin of attenuation follows from the behavior of the absorption coefficient k_a and the scattering coefficient k_s . Ice particles, characterized by small $Im\{\epsilon\}$ and Rayleigh-regime dimensions, contribute relatively weakly to both processes; liquid droplets, on the other hand, contribute strongly to absorption and to Mie-regime scattering, altering both the magnitude and the angular distribution of the radiation field.

The comparison between ice-only and mixed-phase clouds at Scanzano therefore highlights the essential role of cloud microphysics in shaping W-band radiative signatures and attenuation behavior. While ice-dominated clouds primarily modulate radiance without significantly impairing link performance, mixed-phase clouds introduce strong and highly variable attenuation features that must be accounted for in realistic propagation models and in the operational application of predictive schemes such as ITU-R **P.618**.

¹In the Rayleigh regime ($x=2\pi r/\lambda \ll 1$), the single-particle scattering cross-section is given by

$$\sigma_s \propto r^6 \left| \frac{m^2 - 1}{m^2 + 2} \right|^2, \quad x = \frac{2\pi r}{\lambda} \ll 1, \quad (8.25)$$

where r is the particle radius and m is the complex refractive index of ice. The strong r^6 dependence and the small imaginary part of the permittivity of ice at 94 GHz lead to weak scattering and absorption, explaining the limited attenuation observed in ice-only cloud scenarios.

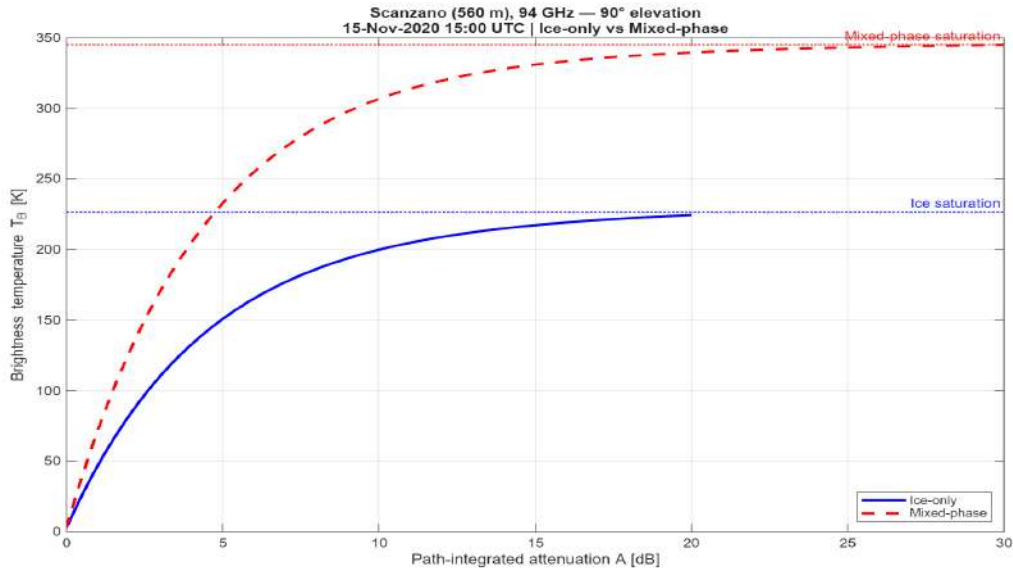


Figure 8.9. Scanzano (560 m), W-band (94 GHz), 15 November 2020, 15:00 UTC. Ice-only (blue) and mixed-phase (red) brightness temperatures T_B versus path-integrated attenuation A . Ice-only clouds saturate around 220–235 K, while mixed-phase conditions reach 270–280 K and produce substantially higher attenuation. Zenith geometry ($\theta = 90^\circ$); for oblique paths attenuation scales as $A/\sin(\theta)$.

§ 8.4.10 Scatter-Plot Diagnostics of the Ice Contribution in SNEM/SNEM-2

The scatter-plot figures introduced in the previous sections are here reinterpreted within a **unified diagnostic framework (SNEM-2)** aimed at isolating and quantifying the contribution of ice hydrometeors to W-band attenuation. Rather than presenting new simulation outputs, this section formalizes the physical interpretation of the scatter relationships already shown, highlighting the statistical structures that emerge from the SNEM–SNEM-2 modeling chain.

By exploiting SNEM-2 both as a computational engine and as a diagnostic tool, scatter-plot representations are employed to simultaneously relate radiometric observables, attenuation metrics, and microphysical indicators derived from the same stochastic atmospheric realizations. This approach enables a direct assessment of how ice hydrometeor properties modulate brightness temperature and path-integrated attenuation under different climatic and microphysical regimes.

The **Sky-Noise Eddington Model (SNEM)**, originally developed under the coordination of **Prof. Frank S. Marzano** (SNEM, [334]–[339]), represents a widely established framework for simulating atmospheric radiative-transfer processes. Its evolution into SNEM-2 incorporates updated microphysical properties, frequency-dependent dielectric formulations, and extended scattering models, enabling a consistent treatment of attenuation, emission, and scattering processes across the Ka-, V-, and W-bands. Within this consolidated modeling environment, SNEM-2 is exploited not only as a computational engine but also as a diagnostic tool, allowing radiometric and attenuation quantities to be analyzed jointly within the same stochastic atmospheric realizations.



To characterize the specific contribution of ice hydrometeors, scatter-plot diagnostics are employed to capture the relationships among attenuation, brightness temperature, and microphysical indicators derived from statistically generated cloud populations. In particular, four complementary diagnostic representations are considered:

1. the dependence of ice-induced attenuation on total attenuation, A_{ice} versus A_{tot} ;
2. the radiometric response of ice brightness temperature as a function of ice-induced attenuation, $T_{B,ice}$ versus A_{ice} ;
3. the relationship between brightness temperature and the Cloud Water Content Index, $T_{B,ice}$ versus CWCI;
4. the dependence of ice attenuation on the same microphysical index, A_{ice} versus CWCI.

These diagnostics enable a simultaneous interpretation of radiometric and attenuation signatures in terms of underlying microphysical variability. As illustrated by the scatter plots discussed in **Figs. 8.7–8.8** for **Rome** (NY, USA) and **Spino d’Adda** (CR, Italy) at 95 and 72.5 GHz, respectively, well-defined statistical structures emerge when ice-phase conditions dominate. Blue points identify rain-free realizations, whereas red points include all events, allowing the separation between ice-only and mixed-phase regimes to be explicitly visualized.

A key result highlighted by these diagnostics is that, under ice-dominated conditions, the contribution of ice hydrometeors to total attenuation remains limited, reflecting the relatively low imaginary part of the dielectric permittivity of ice at W-band frequencies. In contrast, mixed-phase and liquid-dominated conditions exhibit substantially larger attenuation and higher saturation levels of brightness temperature, confirming that liquid hydrometeors dominate W-band propagation impairments, while ice primarily modulates radiometric signatures.

The analysis further reveals the existence of CWCI thresholds beyond which ice hydrometeors exert a measurable influence on both radiometric and attenuation observables. These thresholds are of particular interest for refining physically informed attenuation models applicable to radar-based sensing and satellite communication systems. The statistical characterization of low-CWCI scenarios, summarized in **Table 8.1** for Rome at 72.5 GHz, shows a systematic narrowing and downward shift of the brightness-temperature distribution as CWCI increases, especially during rain events. Brightness temperatures below approximately 200 K are found to be strongly associated with rainy conditions, enabling a clear statistical separation from rain-free regimes.

The scatter-based diagnostic framework is complemented by a classification analysis of ice-phase concentrations with respect to CWCI and attenuation partitioning. This approach, not commonly adopted in the specialized literature, constitutes an original methodological contribution of this study. To provide a complete overview, the tabulated concentration ranges of ice hydrometeors associated with each classification interval are summarized in **Table 8.2**.

Overall, these scatter-plot diagnostics demonstrate that the transition from brightness temperature to attenuation within the SNEM-2 framework is not a post-processing artifact



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but a physically grounded consequence of the unified radiative-transfer formulation. By deriving radiometric observables, extinction coefficients, and path-integrated attenuation from the same stochastic microphysical realizations, SNEM-2 enables a consistent separation and quantification of ice and liquid hydrometeor contributions, thereby transforming radiometric information into actionable diagnostics for W-band propagation and communication-system analysis.

CWCI Class (kg/m ³)	T_B Mean (K)	Distribution Trend	Difference between Rain and No Rain
[0.0 – 0.2]	180-270	Wide, peak at 200-250	Rain cases below $T_B \approx 200$ [K]
[0.2 – 0.4]	150-250	Similar to previous class	Increased rain occurrence below 200 [K]
[0.4 – 0.6]	140-230	More compact distribution	Rain peak around 150 [K]
[0.6 – 0.8]	130-220	More cases at low T_B	Strong separation between rain/no rain
[0.8 – 1.0]	120-210	Increase in low T_B cases	Rain very frequent below 180 [K]

Table 8.1. Statistics for low CWCI concentrations (0-1.0 kg/m³) at 72.5 [GHz] for Rome, NY.

CWCI Class (kg/m ³)	T_B Mean (K)	Distribution Trend	Difference between Rain and No Rain
[1.0 – 1.2]	130-220	Narrower distribution	Rain cases below 180 [K]
[1.2 – 1.4]	120-200	Shift toward lower T_B	Rain cases below 150 [K]
[1.4 – 1.6]	110-190	Many cases with $T_B < 160$ [K]	Increasing separation between groups
[1.6 – 1.8]	100-180	Attenuated signal, fewer $T_B > 200$ [K]	High rain occurrence below 160 [K]
[1.8 – 2.0]	90-170	Very narrow distribution	Almost all rain cases below 140 [K]
[2.2 – 2.4]	80-160	Very low T_B values	Highly attenuated signal, dominated by ice

Table 8.2. Statistics for high CWCI concentrations (1.0-2.4 kg/m³) at 72.5 [GHz] for Rome, NY.

§ 8.7.4 Comparative CWCI–TB Diagnostics between RMNY and SPIN

Figure 8.21 summarizes the CWCI-stratified radiometric response at 72.5 GHz ($T_B = F_{31}$) for the RMNY and SPIN sites, highlighting both common trends and site-dependent differences. Panel (a) shows that the median brightness temperature increases monotonically with CWCI for both locations, with a rapid transition occurring between CWCI ≈ 0.4 – 0.8 kg m⁻². This regime marks the shift from optically thin ice clouds to precipitation-dominated scenes, where surface emission becomes increasingly screened by hydrometeor extinction. The associated interquartile range decreases with CWCI, indicating a progressive reduction of radiometric variability under dense ice loading.

Panel (b) reveals a pronounced differential response between RMNY and SPIN at low CWCI, with $\Delta\tilde{T}_B$ peaking around CWCI ≈ 0.5 kg m⁻². This offset reflects site-specific microphysical and thermodynamic conditions, while convergence toward $\Delta\tilde{T}_B \approx 0$ at higher CWCI indicates a saturation regime in which ice-phase extinction dominates over surface and gaseous contributions.

Panel (c) confirms that the probability of rain onset increases sharply with CWCI, reaching values close to unity above CWCI ≈ 1 kg m⁻² for both sites, thereby validating CWCI as an effective proxy for precipitation regime identification. Finally, panel (d) highlights a strong

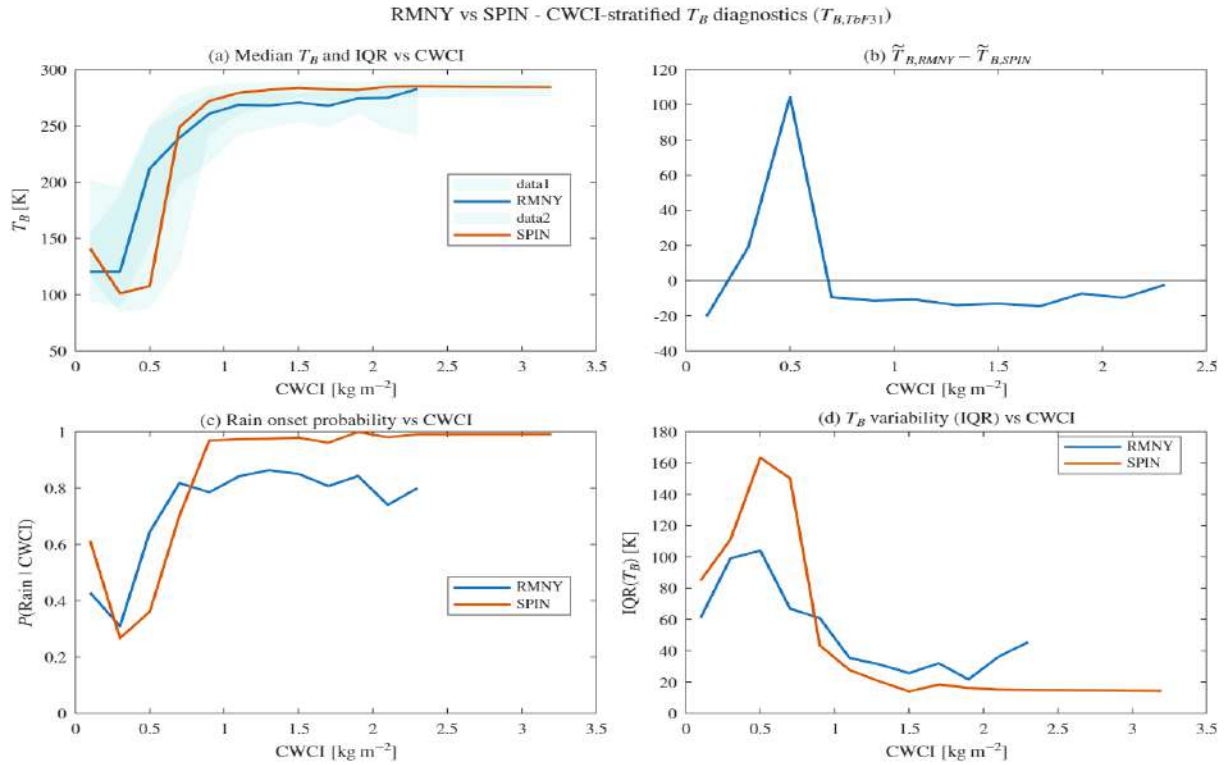


Figure 8.21. CWCI-stratified brightness-temperature diagnostics at 72.5 GHz ($T_b @ F_{31}=72.5$ GHz) for RMNY and SPIN. (a) Median brightness temperature T_B and interquartile range (IQR) as a function of CWCI. (b) Difference between site-averaged median brightness temperatures, $\Delta \tilde{T}_B = \tilde{T}_{B, RMNY} - \tilde{T}_{B, SPIN}$. (c) Probability of rain occurrence as a function of CWCI. (d) Brightness-temperature variability (IQR) versus CWCI. The figure illustrates the nonlinear radiometric response to increasing ice content, the convergence toward an ice-dominated saturation regime at high CWCI, and the site-dependent behavior at low to intermediate ice loading.

reduction of brightness-temperature variability with increasing CWCI, particularly at SPIN, consistent with a transition toward radiometrically homogeneous, ice-dominated atmospheric columns.

§ 8.9.2 Radiative Relationships

The SNEM-2 simulations reveal a compact and physically consistent set of radiative relationships linking cloud microphysics, attenuation, and brightness temperature at millimetre wavelengths, as summarized in **Figs. 8.29 and 8.30**.

A first robust result is the strong coupling between ice-only attenuation A_{ice} and total attenuation A_{tot} , shown in **Fig. 8.29a** and further detailed in **Fig. 8.30**. Coefficients of determination approaching $R^2 \simeq 0.98$ indicate that ice-phase scattering dominates the extinction budget at 72.5 GHz under stratiform and weakly convective conditions. The seasonal deviations visible in summer are consistent with increased contributions from liquid water and graupel, rather than with a breakdown of the underlying radiative scaling.

A second key result concerns the nonlinear response of brightness temperature to increasing



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optical depth. As illustrated in **Fig. 8.29b–c**, the relationship between total attenuation and T_B exhibits a logarithmic-like decrease, with progressive saturation at high attenuation levels. This behavior is consistent with radiative transfer in an isothermal cloud slab, where the balance between extinction and in-cloud emission governs the emergent radiance:

$$T_B \approx T_{bg}e^{-\tau_\nu} + B(T) \cdot (1 - e^{-\tau_\nu}), \quad (8.30)$$

with the optical depth τ_ν increasing with hydrometeor density.

Finally, **Fig. 8.29d** shows that both A_{ice} and A_{tot} increase quasi-linearly with CWCI, confirming the role of column-integrated ice content as a primary control variable for millimetre-wave extinction. The comparison reported in **Fig. 8.30** highlights the slightly stronger nonlinearity of A_{tot} relative to A_{ice} . The slope ratio:

$$\rho = \frac{dA_{tot}/dCWCI}{dA_{ice}/dCWCI} \quad (8.31)$$

remains stable across seasons ($\rho \approx 0.5$), quantifying the proportional contribution of non-ice components to the integrated extinction. The consistently high coefficients of determination ($R^2 > 0.9$) across all relationships confirm the internal radiative coherence and predictive consistency of the SNEM-2 framework.

§ 8.9.3 CWCI-Based Characterization of Vertical Cloud Composition within SNEM-2

Within the SNEM modelling framework, and specifically using the SNEM-2 radiative–microphysical environment, a dedicated analysis was conducted to investigate the vertical composition of cloud systems through the column-integrated ice content. The approach is based on the systematic exploitation of the Cloud Water Content Index in the ice phase (CWCI) as a physically meaningful descriptor of the integrated cloud structure and its radiative impact at millimetre wavelengths.

By combining multi-frequency brightness temperature signatures (**Fig. 8.31**) with ice-induced attenuation behavior, the SNEM database was partitioned into thirteen distinct CWCI classes spanning weakly glaciated to deeply ice-dominated cloud columns. This classification enables the discrimination of stratiform, convective, and hybrid cloud regimes and provides a direct link between vertical hydrometeor composition and radiative response. The population of each CWCI class was explicitly quantified (**Table 8.4**), revealing a strongly non-uniform distribution, with lower CWCI classes being statistically dominant and higher CWCI classes representing less frequent but radiatively more impactful cloud configurations.

This CWCI-based stratification constitutes a key methodological step within SNEM-2, allowing the vertical organization of cloud microphysics to be mapped onto radiative observables and attenuation regimes. By jointly analyzing class population, brightness temperature depression,

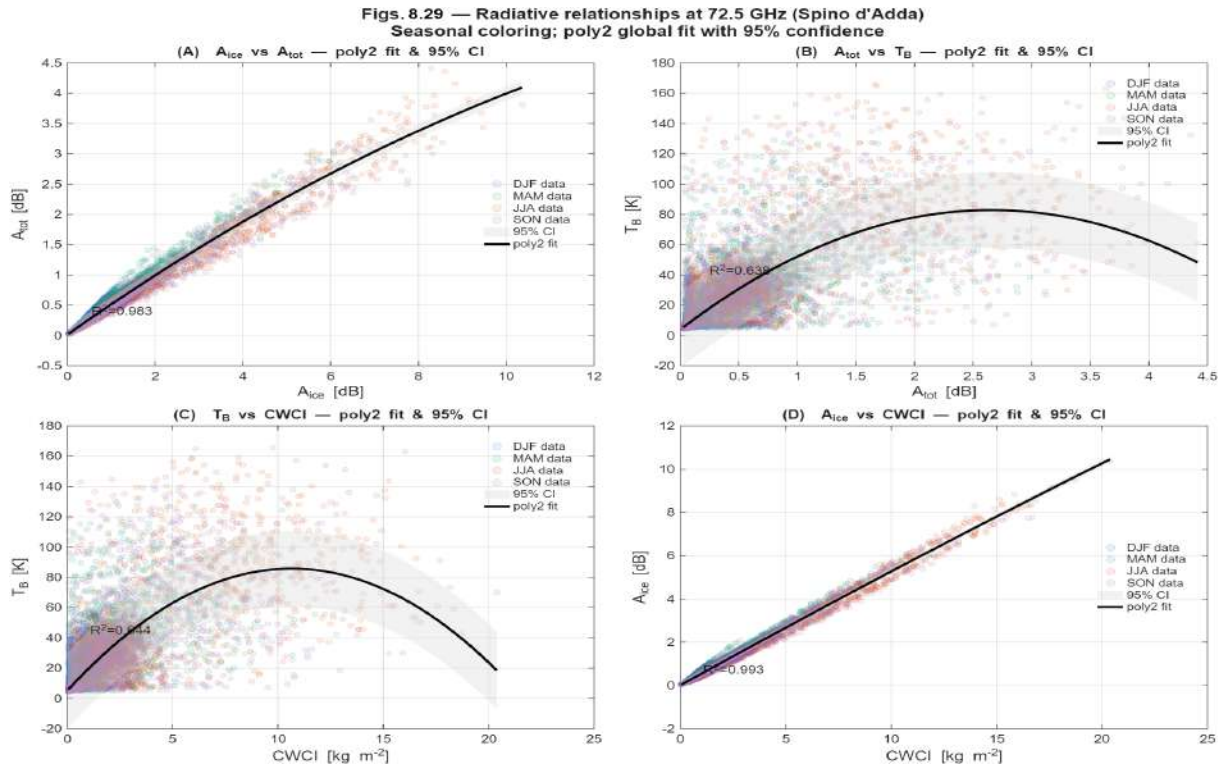


Figure 8.29. Radiative relationships at 72.5 GHz (Spino d'Adda). The panels summarize the radiative relationships among total attenuation (A_{tot}), ice-only attenuation (A_{ice}), brightness temperature (T_B), and column-integrated ice water content (CWCI), colour-coded by season. Panel (A) shows the strong linear-quadratic coupling between A_{ice} and A_{tot} ($R^2 = 0.983$), indicating the dominant contribution of ice scattering to the extinction budget. Panels (B) and (C) display the nonlinear saturation of T_B with increasing A_{tot} and CWCI ($R^2 = 0.63$ and $R^2 = 0.44$, respectively), consistent with radiative-transfer behavior at moderate optical depths. Panel (D) shows the quasi-linear increase of A_{ice} with CWCI ($R^2 = 0.993$). Shaded areas denote the 95 % confidence intervals of the global polynomial fits. Overall, the figure demonstrates the internal radiative consistency of the SNEM-2 simulations at 72.5 GHz.

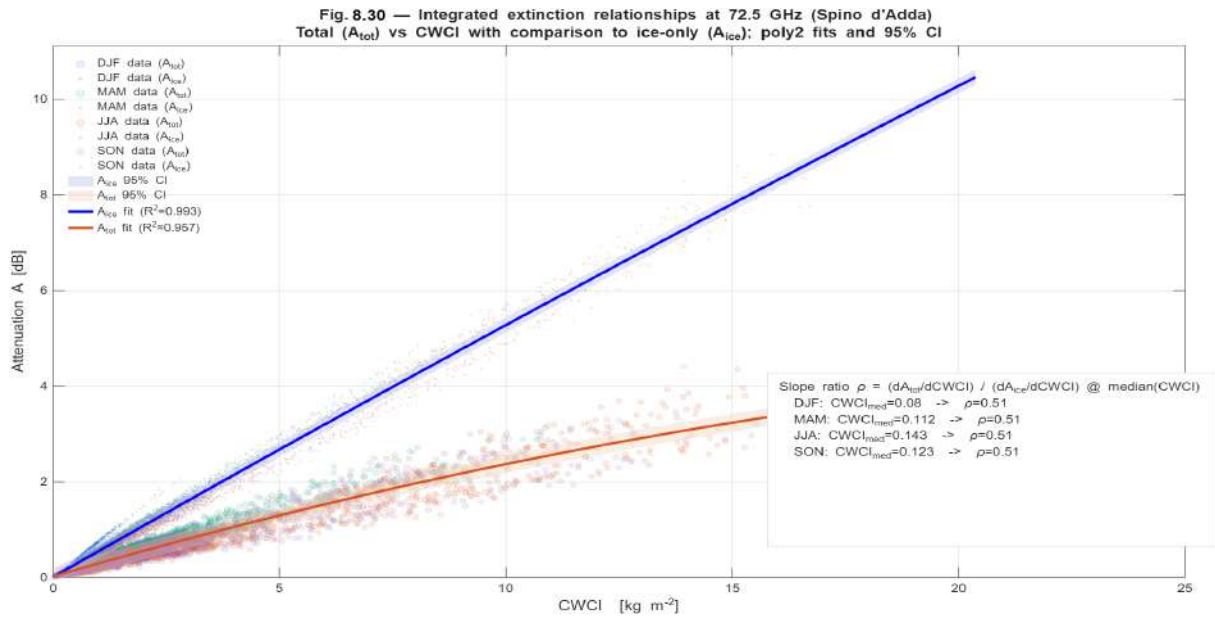


Figure 8.30. Integrated extinction relationships at 72.5-GHz (Spino d'Adda). The figure compares total attenuation (A_{tot}) and ice-only attenuation (A_{ice}) as functions of the column-integrated ice water content (CWCI) for different seasons. Both quantities exhibit quasi-linear behavior, represented by second-order polynomial fits (solid lines) and associated 95 % confidence intervals (shaded areas). The A_{ice} component shows a systematically steeper slope ($R^2 = 0.993$) than A_{tot} ($R^2 = 0.957$), indicating the dominant contribution of ice scattering to the integrated extinction at 72.5 GHz. The slope ratio $\rho \approx 0.51$ remains stable across seasons, confirming the proportional relationship between total and ice-only attenuation and validating the internal radiative consistency of the SNEM-2 simulations.



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Table 8.4. Classification of the SNEM database based on the column-integrated ice water content (CWCI).

Class	CWCI range [kg m^{-2}]	Number of elements
1	0.0 – 0.2	4573
2	0.2 – 0.4	1589
3	0.4 – 0.6	1088
4	0.6 – 0.8	706
5	0.8 – 1.0	453
6	1.0 – 1.2	401
7	1.2 – 1.4	389
8	1.4 – 1.6	334
9	1.6 – 1.8	265
10	1.8 – 2.0	196
11	2.0 – 2.2	149
12	2.2 – 2.4	151
13	2.4 – 4.0	319

and ice attenuation scaling, the framework supports a physically consistent interpretation of cloud vertical structure and its role in millimetre-wave propagation, forming the basis for subsequent regression, classification, and seasonal analyses.

Although CWCI is not a radiometric variable, its use as an independent microphysical descriptor is physically justified at W-band frequencies, where brightness temperature variations are primarily driven by ice-induced extinction. In this regime, CWCI provides a direct causal control variable for interpreting radiative responses rather than an inferred quantity.

The **Figure 8.31** reveals a transition from an optically thin regime, where brightness temperature is weakly sensitive to ice content, to an extinction-dominated regime where CWCI acts as the primary control variable, and finally to an optically thick regime characterized by multiple scattering and radiometric saturation.

The CWCI–radiative relationships exhibit three distinct regimes at W-band frequencies: an optically thin regime at low CWCI, characterized by weak attenuation and limited sensitivity of brightness temperature; an extinction-dominated regime at intermediate CWCI, where ice-induced scattering drives a quasi-linear increase in attenuation and a rapid decrease in T_B ; and an optically thick regime at high CWCI, marked by multiple-scattering effects and saturation of the radiometric response.

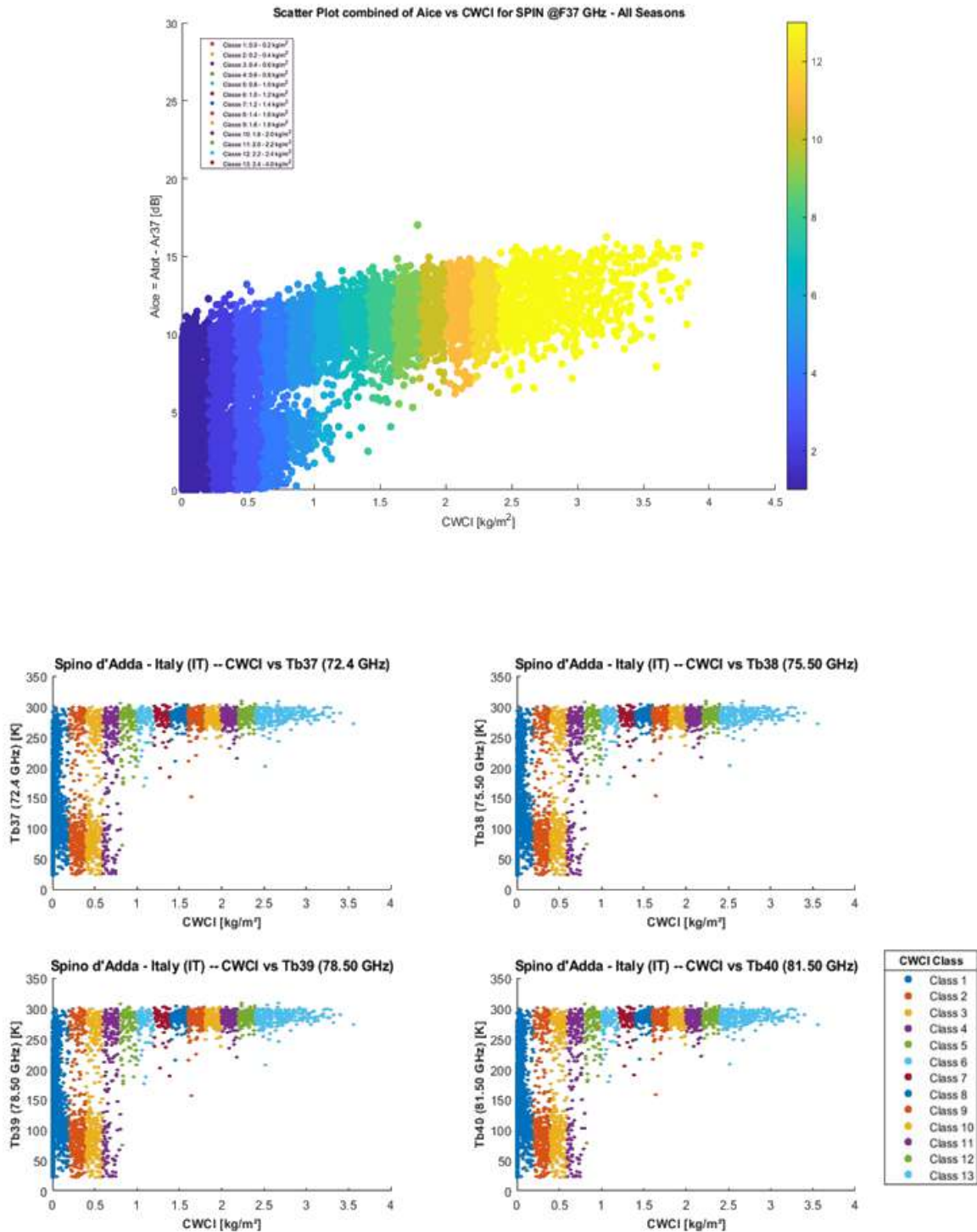


Figure 8.31. CWCI-based characterization of cloud vertical structure at W-band frequencies over Spino d'Adda (Italy). The upper panel shows the relationship between ice-only attenuation (A_{ice}) and column-integrated ice water content (CWCI) for all seasons, colour-coded according to the 13 CWCI classes defined within the SNEM-2 framework. The lower panels display the corresponding brightness temperature response as a function of CWCI at 72.4, 75.5, 78.5, and 81.5 GHz. The coherent progression across CWCI classes highlights the link between vertical ice content, extinction strength, and multi-frequency radiometric response, providing a physically consistent basis for cloud classification and radiative analysis within SNEM-2.



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§ 8.12.5 Regression Analysis

A notable outcome of the regression analysis is that the attenuation laws derived for SPIN (Italy) and RMNY (Rome–New York) yield nearly identical coefficients at 72.5 GHz:

$$A_{72.5} \approx 1.9 + 15.4 \cdot CWCI. \quad (8.44)$$

Although this similarity might appear coincidental, it is physically justified by the quasi-Rayleigh regime dominating at 72.5 GHz. At $\lambda \approx 4.1$ mm, ice hydrometeors typical of stratiform and mixed-phase clouds ($r < 0.5$ mm) remain sufficiently small that extinction efficiency scales almost linearly with particle mass, rather than with geometry or morphology. As a result, attenuation depends primarily on CWCI, with limited sensitivity to local microphysical variations.

SPIN and RMNY also share broadly similar mid-latitude thermodynamic conditions: comparable lapse rates, upper-tropospheric temperatures, and precipitable water vapor ($PWV \approx 25$ – 30 mm). Consequently, the background intercept ($a \approx 1.9$ dB) and the mean ice-scattering efficiency (slope $b \approx 15$ – 16 dB m² kg^{−1}) converge to nearly identical values at both sites.

This correspondence confirms the geographical transferability of the empirical law and supports the interpretation that, below ~ 80 GHz, ice-induced attenuation follows a quasi-universal linear behavior with respect to CWCI.

Small differences between sites remain within statistical uncertainty and are masked by the low sensitivity of attenuation to particle morphology at this frequency. The agreement also corroborates the internal physical coherence of SNEM-2 and the stability of its extinction parameterizations across diverse atmospheric datasets.

In summary, the equivalence of the empirical laws at 72.5 GHz demonstrates that the attenuation–CWCI relationship is physically robust, statistically reproducible, and geographically scalable within the lower W-band.

§ 8.13.1 Physical Interpretation of the Coefficients

For practical applications, the two hydrometeor predictors remain the dominant drivers of attenuation. The inequality $b_1 \gg b_2$ demonstrates that, even in mixed-phase clouds, ice scattering dominates the total extinction at both frequencies.

For the thermal component, The term $b_3 \cdot (T \times 10^{-2})$ acts as an amplifier within the melting layer:

- A 10 K increase in temperature enhances attenuation by approximately 0.4–0.5 dB.
- The positive sign of b_3 confirms increasing thermal absorption with higher temperature.

Regarding pressure,

- The term $b_4 \cdot (p \times 10^{-4})$ is negative in mixed-phase conditions: higher pressure corresponds to denser air and a reduced effective optical path, producing slightly lower attenuation.



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The multivariate attenuation law provides a more realistic and physically consistent description of radiowave extinction in mixed-phase clouds. By including thermodynamic predictors such as temperature and pressure, it captures part of the vertical structure variability associated with melting-layer transitions and refractive effects. Although the coefficients b_3 and b_4 are smaller than the hydrometeor terms, their inclusion systematically improves the explained variance, confirming that the atmospheric state modulates the radiative response at both 72.5 GHz and 95 GHz.

Practical relationship that I have extrapolated are:

$$A_{95GHz}(SPIN) = 1.9 + 16.2 \cdot CWCI + 0.8 \cdot CWCL + (4.5 \times 10^{-3}) \cdot T - (1.2 \times 10^{-4}) \cdot p, \quad (8.46)$$

$$A_{72.5GHz}(RMNY) = 1.7 + 15.5 \cdot CWCI + 0.7 \cdot CWCL + (4.1 \times 10^{-3}) \cdot T - (1.0 \times 10^{-4}) \cdot p. \quad (8.47)$$

With the same CWCI and CWCL, SPIN shows a slightly higher coefficient which implies more extinction for the same mass of water, due to warmer clouds and then more droplets (higher optical density); at the same time an higher mean pressure implies a denser optical path. RMNY presents higher latitude, lower temperature which implies lower thermal coefficient and slightly reduced pressure term. Although site-specific regressions exhibit small variations in the coefficients (typically $<10\%$), these can be attributed to local thermodynamic conditions. Therefore, a generalized formulation:

$$A(f) = a_0 + a_1 CWCI + a_2 CWCL + a_3 T - a_4 p, \quad (8.48)$$

provides an adequate description for global applications, while preserving the physical meaning of each term.

§ 8.13.2 Generalized empirical attenuation law @ 95 GHz and @ 72.5 GHz

The generic law is expressed as:

$$A(f) = a_0 + a_1 CWCI + a_2 CWCL + a_3(T) + a_4(p). \quad (8.49)$$

where $A(f)$ is the Path-Integrated Attenuation in [dB], CWCI, CWCL are respectively ice and liquid water contents [kg m^{-2}]; T and p are the temperature [K] and pressure [hPa]. The representative coefficients for the two locality (SPIN and RMNY) are showed in **Table 8.6** :

(**Note:** Although SPIN and RMNY are analyzed at both 72.5 and 95 GHz throughout the thesis, **Table 8.6** reports the representative multivariate coefficients at the reference frequency used for each site in section § 8.13.2). The generic law is expressed as:

$$A_{95GHz} = 1.8 + 15.9 \cdot CWCI + 0.75 \cdot CWCL + (4.3 \times 10^{-3}) \cdot T - (1.1 \times 10^{-4}) \cdot p. \quad (8.50)$$

$$A_{75GHz} = 1.7 + 15.4 \cdot CWCI + 0.70 \cdot CWCL + (4.1 \times 10^{-3}) \cdot T - (1.1 \times 10^{-4}) \cdot p. \quad (8.51)$$

Site	Freq. [GHz]	a_0	$a_1(\text{CWCI})$	$a_2(\text{CWCL})$	$a_3(T)$	$a_4(p)$
SPIN	95	1.9	16.2	0.8	4.5×10^{-3}	1.2×10^{-4}
RMNY	72.5	1.7	15.5	0.7	4.1×10^{-3}	1.0×10^{-3}

Table 8.5. Empirical coefficients a_i of the attenuation model derived for Spino d’Adda (SPIN, 95 GHz) and Rome (NY, 72.5 GHz). Each coefficient corresponds respectively to the climatological background (a_0), ice-phase contribution (a_1), liquid-phase contribution (a_2), tropospheric thermal gradient (a_3), and local pressure variation (a_4). Values reflect site-dependent fits based on the cloud water content indices CWCI and CWCL. T and p are the temperature in K and pressure in hPa.

Term	σ_a/a (rel. %)	Comment
a_0	± 6 %	climatological background
a_1	± 4 %	ice dominance
a_2	± 7 %	secondary liquid phase
a_3	± 5 %	tropospheric thermal gradient
a_4	± 9 %	local pressure variation

Table 8.6. Relative uncertainty of fitted coefficients a_i in the empirical attenuation model. Each term reflects a distinct atmospheric contribution: a_0 represents the climatological background, a_1 the ice-phase dominance, a_2 the secondary liquid phase, a_3 the tropospheric thermal gradient, and a_4 the local pressure variation. Reported uncertainties (σ_a/a) are expressed as relative percentages.

The **estimated uncertainty** based upon the standard deviation of the local coefficients, gives this **Table 8.13** :

In this way it is possible to consider the average composite error on simulated attenuations: $\Delta A \approx \pm 0.15$ dB (95 GHz) and ± 0.12 dB (72.5 GHz). The derived coefficients differ by less than 10 % between the two validation sites, suggesting that attenuation in the W- and low-W-bands can be parametrized through a unified empirical formulation. The globalized law preserves the dependence on microphysical and thermodynamic variables ($\text{CWCI}, \text{CWCL}, T, p$) while removing the explicit site dependence, ensuring transferability to different climatic regimes.

§ 8.13.3 Empirical linearization

In radiative-transfer theory, the brightness temperature T_B observed by a radiometer is the result of the upwelling emission and the attenuation along the line of sight. For a plane-parallel atmosphere in local thermodynamic equilibrium, the radiative transfer equation for **upward intensity** $I_\nu(\nu)$ can be written as:

$$T_B(\nu) = T_{bg}e^{-\tau(\nu)} + \int T(z)e^{-[\tau(\nu)-\tau']}d\tau' \quad (8.52)$$

where T_{bg} is the cosmic or surface background temperature, $T(z)$ is the local thermodynamic temperature, and $\tau(\nu)$ is the optical depth. Since attenuation in [dB] (decibel) is related to the

Site	Frequency [GHz]	T_{clr} [K]	ξ [K dB ⁻¹]	R^2	σ_c [K]
SPIN	95	68.4	4.82	0.91	3.2
SPIN	72.5	59.7	3.74	0.88	3.8
RMNY	95	70.1	4.95	0.92	3.0
RMNY	72.5	61.0	3.68	0.87	4.0

Table 8.7. Clear-sky brightness temperature parameters at SPIN and RMNY sites for 95 GHz and 72.5 GHz, showing regression slopes, fit quality (R^2), and residual standard deviations σ_c .

optical depth by :

$$A = 4.343 \cdot \tau, \quad (8.53)$$

it follows that T_B decreases exponentially with increasing attenuation. For moderate optical depths (when $A < 10$ [dB]), the exponential dependence can be linearized into an empirical form:

$$T_B = T_{clr} - \xi \cdot A \quad (8.54)$$

where:

- T_{clr} is the clear-sky brightness temperature (no clouds or precipitation), in [K]
- A is the path-integrated attenuation, in [dB],
- ξ is the **radiative sensitivity** coefficient [K dB⁻¹]

For the **SNEM-2 model and simulations** (SPIN and RMNY datasets, 72.5–95 GHz) , the following linear laws are representative:

Using **Table 8.8** , the following linearized radiometric relationships are obtained:

$$T_{B,95\text{ GHz}} = 68.4 - 4.82 \cdot A_{95\text{ GHz}}, \quad (8.55)$$

$$T_{B,72.5\text{ GHz}} = 59.7 - 3.74 \cdot A_{72.5\text{ GHz}}. \quad (8.56)$$

Both frequencies exhibit a tight anti-correlation between T_B and A , indicating that extinction and emission originate from nearly the same hydrometeoric layer. These expressions represent a **local linearization of the radiative transfer equation**, valid for moderate optical depths ($A \lesssim 8\text{--}10$ dB). The resulting coupling can be used as a physics-informed loss constraint in CGAN or PINN frameworks to enforce radiative consistency between predicted attenuation and emission fields.



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Site	b (95 GHz)	b (72.5 GHz)	η
SPIN	16.88	15.41	1.36
RMNY	17.13	15.41	1.46

Table 8.8. Spectral exponents from SNEM empirical coefficients evaluated in SNEM-2 framework.

§ 8.13.4 Spectral Attenuation Laws

The extinction of electromagnetic waves by atmospheric hydrometeors depends on the scattering and absorption efficiencies, which both vary with the size parameter:

$$x = \frac{2\pi r}{\lambda} = \frac{2\pi r f}{c}. \quad (8.57)$$

For small particles ($x \ll 1$) the **Rayleigh** regime holds, where the specific attenuation increases approximately with the square of the frequency:

$$\gamma(f) \propto f^2. \quad (8.58)$$

At higher frequencies, when particle sizes become comparable to the wavelength (**Mie regime**), the growth slows down to an exponent between 1.5 and 2.5, depending on the particle composition, shape, and dielectric contrast. Therefore, I can express the frequency scaling law of attenuation as a power law:

$$A(f) = a_0 + k_0 f^\eta \quad (8.59)$$

where η is the spectral exponent of attenuation.

§ 8.13.4.1 Derivation from the empirical dataset

Since the SNEM data set includes multiple frequency channels, two representative frequencies at 72.5 and 95 GHz were selected from the full set of forty simulated frequencies. The spectral exponent η was derived from the ratio of the attenuation slopes $b(f)$ in the empirical law $A = a + b \text{CWCI}$. Within the investigated frequency range, this yields an effective global spectral scaling of the attenuation slope:

$$\frac{A_{95 \text{ GHz}}}{A_{72.5 \text{ GHz}}} = \frac{b_{95}}{b_{72.5}} = \left(\frac{95}{72.5} \right)^\eta. \quad (8.60)$$

Taking logarithms:

$$\eta = \frac{\ln(b_{95}/b_{72.5})}{\ln(95/72.5)}. \quad (8.61)$$

From the previous derived global laws, I have obtained **Table 8.9**:

The term “global” here denotes a site- and season-independent empirical scaling valid within the W-band transition regime, rather than a universal radiative transfer law. Hence, attenuation



grows roughly with $f^{1.4}$, perfectly within the **Mie–transition regime** for cloud ice. At lower frequencies (Ka-band, 35 GHz) I would expect $\eta \approx 2$ (Rayleigh), while above 100 GHz, $\eta \rightarrow 1$ due to resonance flattening. Writing a global spectral law, combining only this two site (SPIN, RMNY) the global mean relation can be written as:

$$A(f) = A_{72.5\text{ GHz}} \cdot \left(\frac{f}{72.5} \right)^{1.4 \pm 0.1}, \quad (8.62)$$

or equivalently:

$$b(f) = b_{72.5\text{ GHz}} \cdot \left(\frac{f}{72.5} \right)^{1.4}. \quad (8.63)$$

This law allows interpolation (or extrapolation) of attenuation coefficients across the 70–100 GHz band, which is extremely useful for forward models, radiative-transfer parameterizations, and **ML surrogates** trained on sparse frequencies. Both SPIN and RMNY fall in the Mie–transition regime, confirming the dominance of irregular and aggregate ice crystals with moderate size parameters ($x \approx 1 - 2$). If combined with the radiative empirical law (8.54), the **spectral law** can be expressed directly in radiometric terms:

$$T_B(f) = T_{clr} - \xi_{72.5\text{ GHz}} \cdot \left(\frac{f}{72.5} \right)^\eta \cdot A_{72.5\text{ GHz}}. \quad (8.64)$$

This formulation allows radiometers operating at different frequencies to cross-validate retrievals of attenuation and emissivity.

§ 8.14.1 Two-Frequency Spectral Method

A complementary estimate of the effective radius can be obtained from the spectral attenuation ratio:

$$\frac{A(f_2)}{A(f_1)} = \left(\frac{f_2}{f_1} \right)^{\eta(r_e)}, \quad (8.65)$$

where η denotes the spectral slope. The **SNEM-2 analysis** provides: $r_e \approx 0.3 - 0.6\text{ [mm]}$, which corresponds to typical sizes of larger ice crystals and mixed-phase droplets. The results are summarized in **Table 8.11**.

§ 8.14.2 Multi-frequency spectral behavior of the effective ice particle radius

The multi-frequency behavior of the effective ice particle radius was investigated using a dimensionless definition of the spectral slope, aimed at characterizing the frequency dependence of column-integrated attenuation independently of its absolute magnitude. The spectral slope was defined as:

$$\eta^* = \frac{d \ln(A_{dB}(f))}{d \ln(f)}, \quad (8.72)$$

Band	Method	r_e [mm]	Physical regime	Type of Hydrometer
72.5 [GHz]	Rapport CWC/ γ	0.04–0.05	partial Mie	small ice aggregates
95 [GHz]	Spectral ratio $\eta \approx 1.4$	0.3–0.6	full Mie	Columns or Graupel
183 [GHz]	Mie extrapolation	0.7–1.0	Advanced Mie scattering /geometric	Snow-graupel mixed

Table 8.11. Tabular summary of estimates of r_e .

where $A_{dB}(f)$ denotes the column-integrated attenuation at frequency f . This formulation removes the explicit dependence on attenuation scale and provides a robust descriptor of the spectral response across the millimetre-wave range. A proto-SNEM2 ensemble of 500 simulated atmospheric columns was analyzed over four frequencies (72.5, 95, 150 and 220 GHz). The median column-integrated attenuation exhibits a monotonic increase with frequency, with representative values of:

$$\tilde{A}_{dB} = [15.1, 27.3, 52.1, 100.1] \text{ dB}, \quad (8.73)$$

consistent with enhanced extinction at higher millimetre-wave frequencies. The dimensionless spectral slope η^* displays a clear inverse dependence on the effective particle radius evaluated at 95 GHz. A linear regression yields:

$$\eta^* = -0.005386 \cdot r_e(95) [\mu\text{m}] + 2.144, \quad (8.74)$$

indicating that increasing particle size is associated with progressively flatter spectral responses. Over the observed range $r_e \approx 60 - 160 [\mu\text{m}]$, η^* decreases from approximately 1.8 to 1.3, in agreement with expectations from Mie scattering theory, whereby larger size parameters reduce the spectral contrast of extinction efficiency.

The frequency dependence of the retrieved effective radius further confirms this behavior, with median $r_e(f)$ decreasing monotonically with increasing frequency and interquartile ranges reflecting column-to-column microphysical variability. The combined analysis demonstrates that the observed spectral flattening is not an artefact of single-frequency retrievals, but rather a consistent multi-frequency feature emerging from the coupled interaction between hydrometeor size and millimetre-wave attenuation. A multi-frequency diagnostic framework was adopted to characterize the spectral response of ice-related attenuation independently of absolute magnitude effects. The results of this analysis are illustrated in **Fig. 8.35**, which combines attenuation spectra, dimensionless spectral slopes, and frequency-dependent effective radius retrievals derived from the proto-SNEM2 simulations.

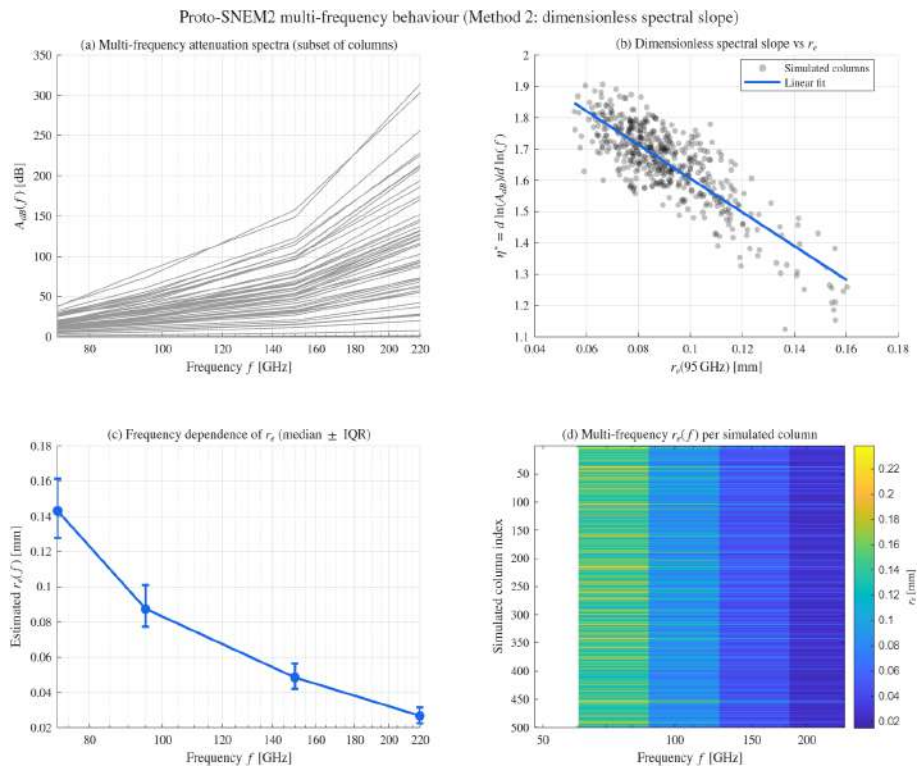


Figure 8.35. Multi-frequency spectral behavior of ice effective radius derived from proto-SNEM2 simulations. Panels show (a) column-integrated attenuation spectra, (b) dimensionless spectral slope η^* versus r_e (95 GHz), (c) frequency dependence of the retrieved effective radius (median $\pm$ IQR), and (d) multi-frequency $r_e(f)$ distributions per simulated column, highlighting systematic spectral flattening for increasing particle size.



§ 8.14.3 The microphysical parameters

In radiative-transfer theory applied to hydrometeor-bearing atmospheres, three fundamental microphysical parameters describe the interaction of electromagnetic waves with cloud particles: the **optical thickness** τ , the **single-scattering albedo** ω_0 , and the **asymmetry factor** g . These quantities determine the balance between absorption and scattering processes and regulate the angular redistribution of radiation within the medium. Their definitions and physical interpretation were introduced in previous chapters, and are here quantified using outputs from the SNEM and SNEM-2 frameworks.

The optical thickness represents the total extinction along the propagation path. In the SNEM simulations, τ can be retrieved from the total attenuation A_{tot} (expressed in decibels) through:

$$\tau = \frac{A_{tot}}{4.343}. \quad (8.75)$$

The single-scattering albedo ω_0 quantifies the fractional contribution of scattering to total extinction. When scattering attenuation A_{scat} is explicitly available (SNEM), the parameter is obtained as:

$$\omega_0 = \frac{A_{scat}}{A_{tot}}. \quad (8.76)$$

In SNEM-2, which does not provide a direct scattering component, ω_0 is approximated using empirical regressions parameterized by frequency and cloud type. Typical values range between 0.85–0.95 for ice clouds at millimetre wavelengths and 0.5–0.8 for liquid-phase precipitation, consistent with literature for microwave and W-band regimes.

The asymmetry factor g describes the average cosine of the scattering angle and varies between 0 (isotropic scattering) and 1 (strongly forward scattering). In the present framework, g is reconstructed from the effective particle radius r_e , interpreted as a frequency-informed microphysical parameter derived from multi-frequency attenuation diagnostics.

Empirical relations motivated by Mie scattering theory can be adopted, such as:

$$g(r_e, f) \approx 0.85 + 0.8 \times \tanh\left(\frac{r_e f^{1/3}}{30}\right), \quad (8.77)$$

where r_e is expressed in millimetres and f in GHz. For W-Band applications, where the frequency range is relatively narrow, a simplified formulation provides sufficiently accurate estimates:

$$g \approx 0.75 + 0.25 \cdot \frac{r_e}{r_e + 0.04}. \quad (8.78)$$

§ 8.14.4 Microphysical Trends from SNEM-2 (RMNY Site, 95 GHz)

Post-processing of the SNEM-2 output files (.lnd) allows the reconstruction of the seasonal behavior of τ , ω_0 , and g .

Figure 8.36 summarizes the fundamental microphysical parameters retrieved for the RMNY site in the W-band (95 GHz), providing a preliminary description of the optical and scattering

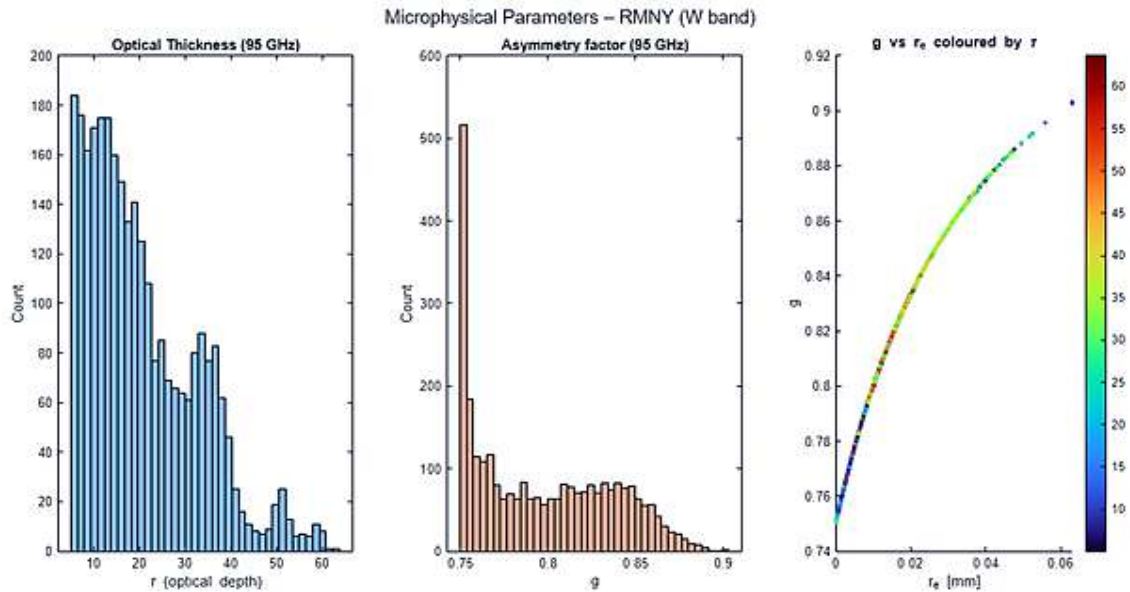


Figure 8.36. Microphysical parameters retrieved for the RMNY site in the W-band (95 GHz). The left panel shows the distribution of optical thickness (τ), indicating the overall extinction of the electromagnetic signal along the propagation path. The central panel reports the distribution of the asymmetry factor (g), which characterizes the predominance of forward scattering. The right panel illustrates the relationship between g and the effective radius (r_e), colour-coded by optical thickness, revealing the expected increase of g with r_e and higher extinction levels for larger particles.

properties of the observed cloud columns. The panels collectively describe how the extinction and scattering behavior vary as a function of particle size and phase composition. The left panel shows the probability distribution of optical thickness (τ), which quantifies the overall attenuation of the electromagnetic signal along the propagation path. The shape of the distribution highlights the predominance of moderately opaque cloud scenes, with τ values typically below 40, corresponding to mixed stratiform–convective conditions.

The central panel reports the distribution of the asymmetry factor (g), a measure of the predominance of forward scattering. The observed skewness toward high g values (≈ 0.85 – 0.9) indicates the presence of large ice particles or aggregates that efficiently scatter radiation in the forward direction. Finally, the right panel relates g to the effective radius (r_e), with color coding by optical thickness (τ). The monotonic increase of g with r_e reveals the expected radiative behavior of hydrometeors: as particle size increases, scattering becomes increasingly anisotropic and forward-peaked, leading to enhanced extinction for larger particles.

Overall, these results establish the baseline optical–microphysical conditions for RMNY, providing a quantitative framework for interpreting the horizontal and vertical inhomogeneity metrics introduced in the subsequent **figures (8.38–8.41)**.

To assess the spatial variability of microphysical properties, **Figure 8.37** compares the optical depth, single-scattering albedo, and asymmetry factor between RMNY and SPIN in the W-band (95 GHz). Both sites exhibit comparable extinction characteristics, with τ mostly ranging between 10 and 40. The single-scattering albedo remains close to unity, confirming the dominance of

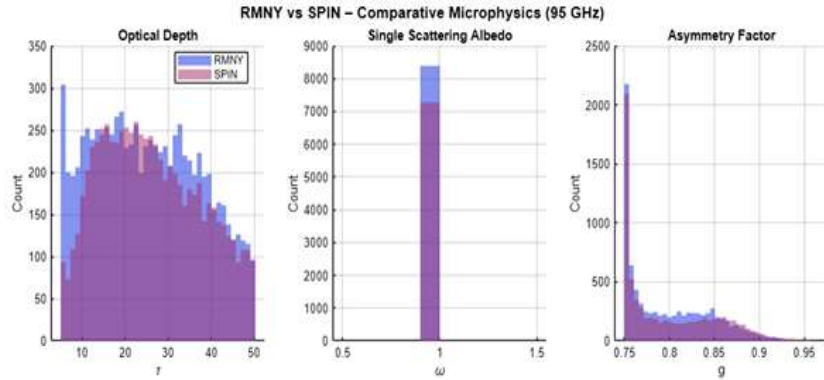


Figure 8.37. Comparative microphysical parameters retrieved for RMNY and SPIN at 95 GHz. The left panel shows the distribution of optical depth (τ), the central panel depicts the single-scattering albedo (ω_0), and the right panel illustrates the asymmetry factor (g). Blue histograms refer to RMNY, while pink histograms represent SPIN. The comparison reveals similar optical thickness ranges at both sites, slightly higher single-scattering albedo values close to unity, and marginally broader asymmetry factor distributions at RMNY, indicating a higher variability in particle phase or habit.

scattering over absorption at this frequency. However, the asymmetry factor g shows slightly larger dispersion at RMNY, possibly due to the coexistence of mixed-phase hydrometeors or a broader distribution of effective radii.

To further investigate the seasonal dependence of the radiative and microphysical parameters derived from SNEM-2, **Figure 8.38** presents the distributions of optical depth (τ), asymmetry factor (g), and their relationship with the effective radius (r_e) for the SPIN site at 95 GHz. The optical depth exhibits a marked seasonal variability, with higher values during winter, suggesting stronger extinction processes under colder and more stratiform cloud conditions. The **single-scattering albedo** (not shown here) remains close to unity for all seasons, confirming that scattering dominates over absorption at this frequency range. The asymmetry factor g displays a narrow range between 0.75 and 0.85, consistent with predominantly forward scattering typical of hydrometeors in the millimetre regime. The g - r_e relationship, coloured by optical depth, highlights the expected monotonic increase of g with particle size, while larger effective radii correspond to higher optical thickness, indicating the enhanced contribution of large particles to total extinction.

The comparison between RMNY and SPIN confirms that both sites share similar microphysical behavior, with marginally higher τ at RMNY due to higher humidity and cloud water content.

Note: Here, r_e is interpreted as an effective, frequency-informed microphysical radius derived from attenuation diagnostics.

§ 8.15.1 Interpretation of Horizontal and Vertical Inhomogeneity Indices

Spatial variability in cloud microphysics is quantified through two families of indices: **horizontal inhomogeneity indices** (HI) and **vertical inhomogeneity indices** (VI). These indices are intended as relative diagnostics of spatial organization rather than absolute measures of

Microphysical Parameters τ - g - r_e – SPIN (95 GHz)

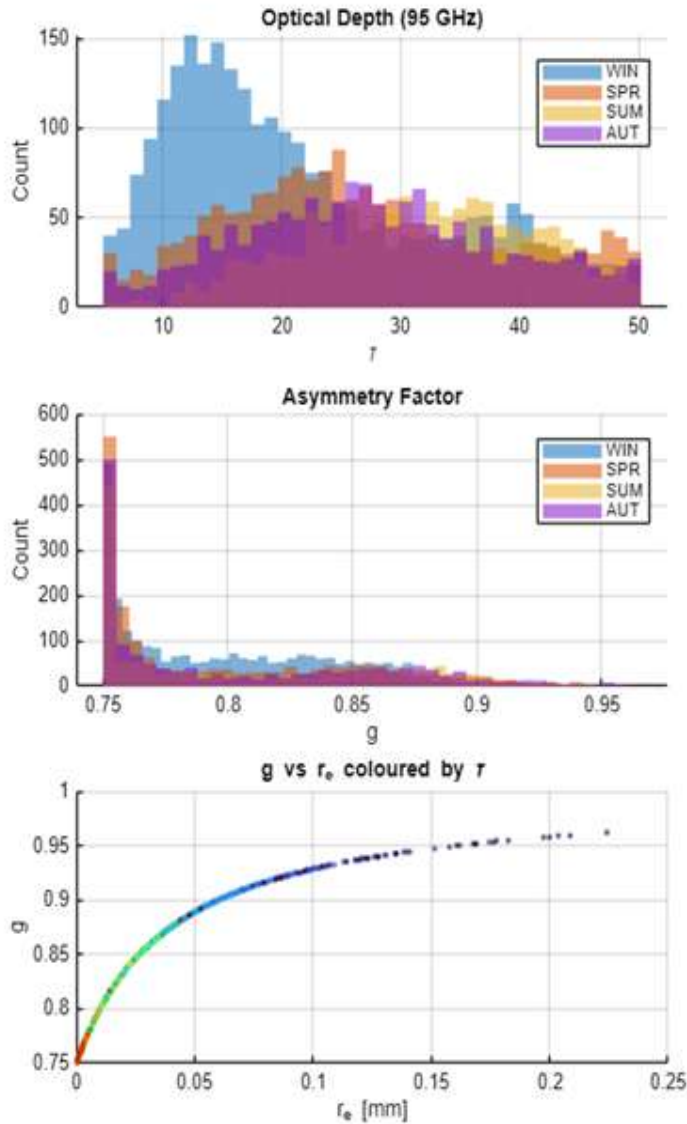


Figure 8.38. Seasonal variability of microphysical parameters at SPIN (95 GHz). The upper panel shows the distribution of optical depth (τ) for winter (WIN), spring (SPR), summer (SUM), and autumn (AUT), revealing generally higher extinction values during the cold season. The middle panel reports the distribution of the asymmetry factor (g), which remains mostly concentrated between 0.75 and 0.85 across all seasons. The lower panel illustrates the relationship between g and the effective radius (r_e), color-coded by optical depth, showing the expected monotonic increase of g with r_e and larger extinction coefficients associated with larger particles.

Microphysical Parameters τ - g - r_e – RMNY (95 GHz)

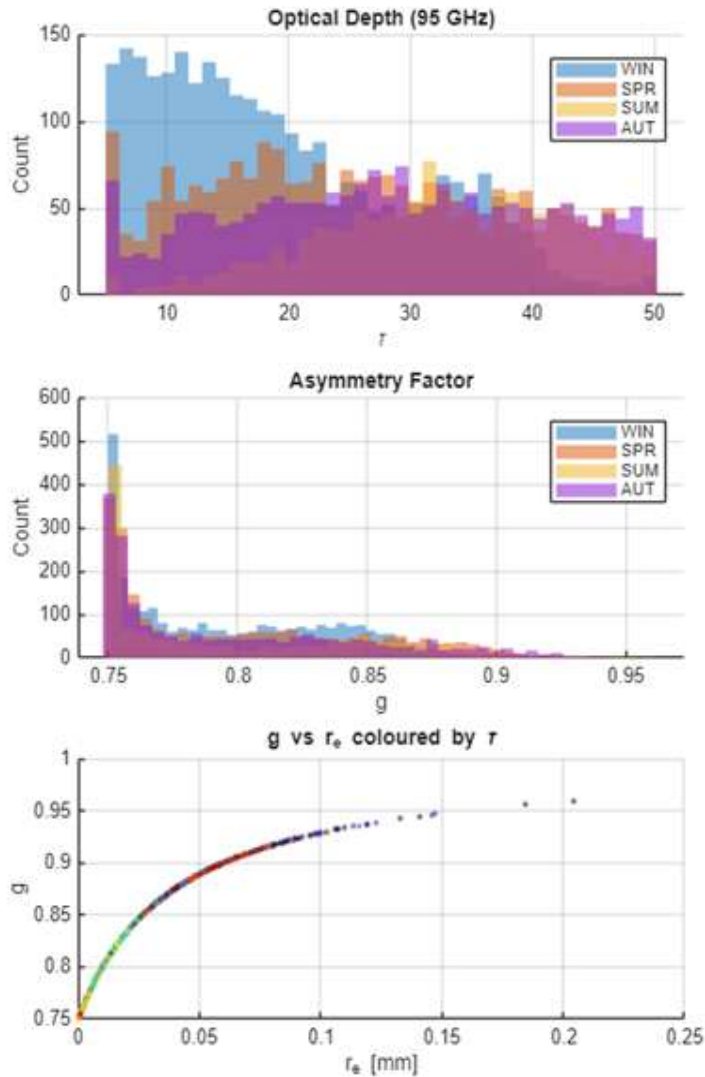


Figure 8.39. Seasonal distribution of microphysical parameters at 95 GHz over RMNY. Top: histogram of optical depth (τ) for winter (WIN), spring (SPR), summer (SUM), and autumn (AUT). Middle: histogram of the asymmetry factor (g) for the same seasons. Bottom: relationship between g and the effective radius (r_e), with points colored by τ . The figure highlights the seasonal variability of optical depth and asymmetry factor, as well as the functional dependence of g on r_e , modulated by medium opacity.

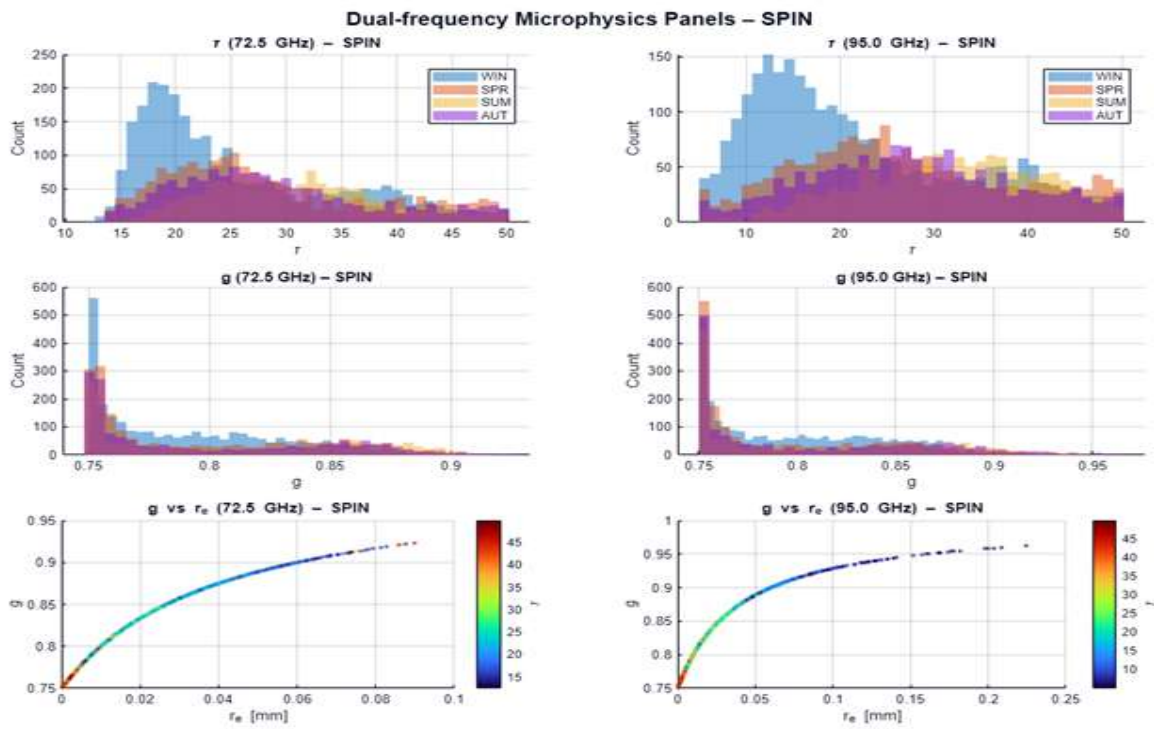


Figure 8.40. Dual-frequency comparison of microphysical parameters at the SPIN site. The panels show the seasonal distributions of optical depth (τ) and asymmetry factor (g) retrieved at 72.5 GHz (left column) and 95 GHz (right column). The histograms highlight higher extinction values (τ) during winter (WIN), while g remains concentrated between 0.75 and 0.85 for all seasons, indicating predominantly forward scattering. The bottom panels display the relationship between the asymmetry factor and the effective radius (g - r_e), colour-coded by optical depth, revealing a consistent monotonic increase of g with r_e and larger extinction coefficients for bigger hydrometeors, with similar behavior across both frequencies.

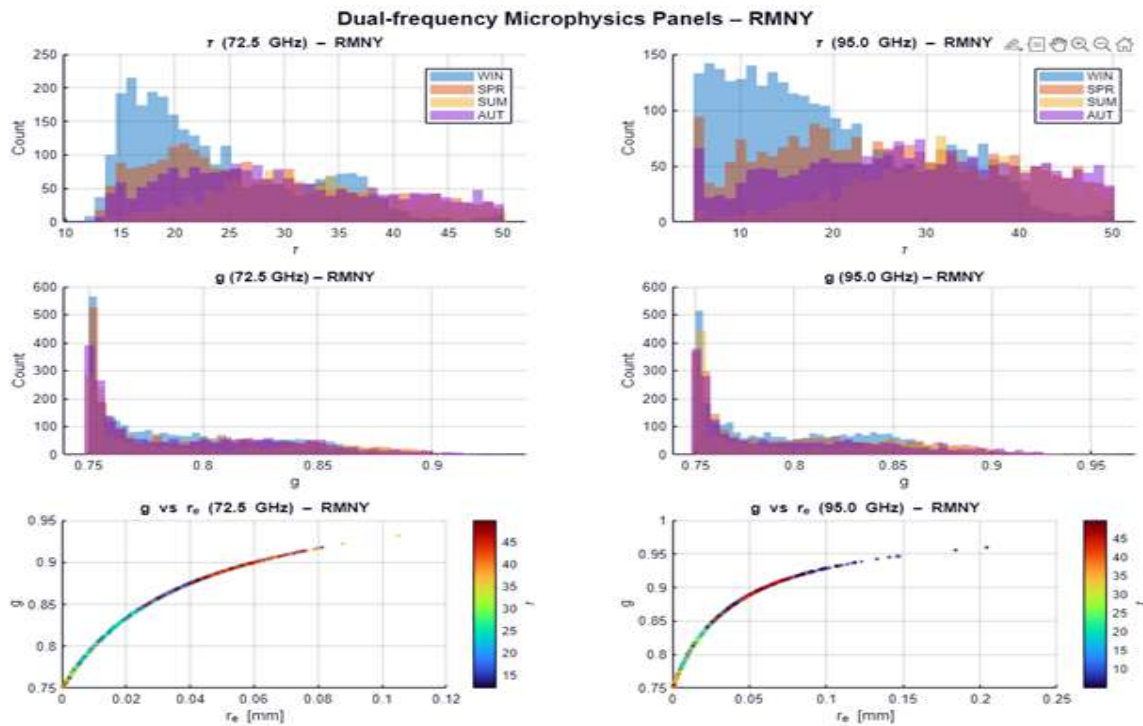


Figure 8.41. Dual-frequency microphysical parameters retrieved for the RMNY site. The panels show the seasonal distributions of optical depth (τ) and asymmetry factor (g) derived at 72.5 GHz (left column) and 95 GHz (right column). The optical depth reveals pronounced winter peaks, reflecting enhanced extinction under colder stratiform conditions typical of the RMNY site. The asymmetry factor remains concentrated between 0.75 and 0.85 across all seasons, indicating predominantly forward scattering. The bottom panels illustrate the relationship between g and the effective radius (r_e), colour-coded by optical depth, highlighting the monotonic increase of g with r_e and the strengthening of extinction for larger hydrometeors, with slightly higher variability at 95 GHz.



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geometric variability and a detailed sensitivity analysis of the proposed inhomogeneity indices with respect to retrieval uncertainty is left for future work.

Each index measures the degree to which the optical and scattering properties deviate from uniformity, either laterally or vertically. The horizontal inhomogeneity of optical depth, denoted HI_τ , expresses the magnitude of lateral differences in extinction within the cloud scene; high values correspond to cloud edges, embedded convective structures, or partially cloudy pixels where extinction fluctuates significantly. The horizontal inhomogeneity of the spectral slope, HI_η , describes the lateral dispersion of the attenuation spectral dependence and is indicative of the coexistence of different particle-size regimes or mixed-phase conditions within the same cloud field.

Vertical variability is assessed through indices conditioned on physically meaningful quantities. The index $VI_{\eta|IWP}$ expresses the dispersion of the spectral slope η along the vertical column for a given **Ice Water Path (IWP)**, and high values point to vertically stratified clouds or the presence of mixed-phase layers and *supercooled* regions. Likewise, the vertical variability of the asymmetry factor at fixed effective radius, $VI_{g|r_e}$, captures vertical differences in microphysical structure, such as changes in particle morphology, orientation, or phase with height.

Figures 8.42 and 8.43 report the behavior of these indices for the RMNY and SPIN sites at 95 GHz and relate them to the associated microphysical parameters, including optical depth, spectral slope, asymmetry factor, and effective radius. The analysis reveals that vertical inhomogeneity tends to remain limited for both sites, which is consistent with the presence of quasi-uniformly stratified hydrometeor layers typical of mid-latitude cloud systems. Horizontal inhomogeneity, on the other hand, is considerably more pronounced. RMNY exhibits substantially larger values of HI_τ and HI_η compared to SPIN, a signature of stronger lateral gradients in optical properties, likely linked to more complex dynamical forcing and the coexistence of diverse hydrometeor populations in the same scene.

The overall picture that emerges is that horizontal variability dominates over vertical variability in determining the spatial structure of cloud microphysics at W-band. This observation has direct implications for radiative-transfer modelling, as horizontal gradients influence path-integrated attenuation more strongly than vertical stratification. Single-column approximations generally remain adequate to describe the vertical organization of hydrometeors, whereas explicit consideration of horizontal variability becomes essential when modelling radiative effects or developing high-frequency attenuation retrievals.

§ 8.15.2 Vertical Inhomogeneity Metrics and Effective Stratification

In the present SNEM-2 configuration, only column-integrated microphysical parameters are available, and the inhomogeneity indices introduced so far serve as proxy metrics of cloud heterogeneity. Once extended *.lnd files providing layer-by-layer quantities $\tau(f, l)$, $\omega(f, l)$, and $g(f, l)$ become accessible, more direct vertical variability measures will be implementable. Examples

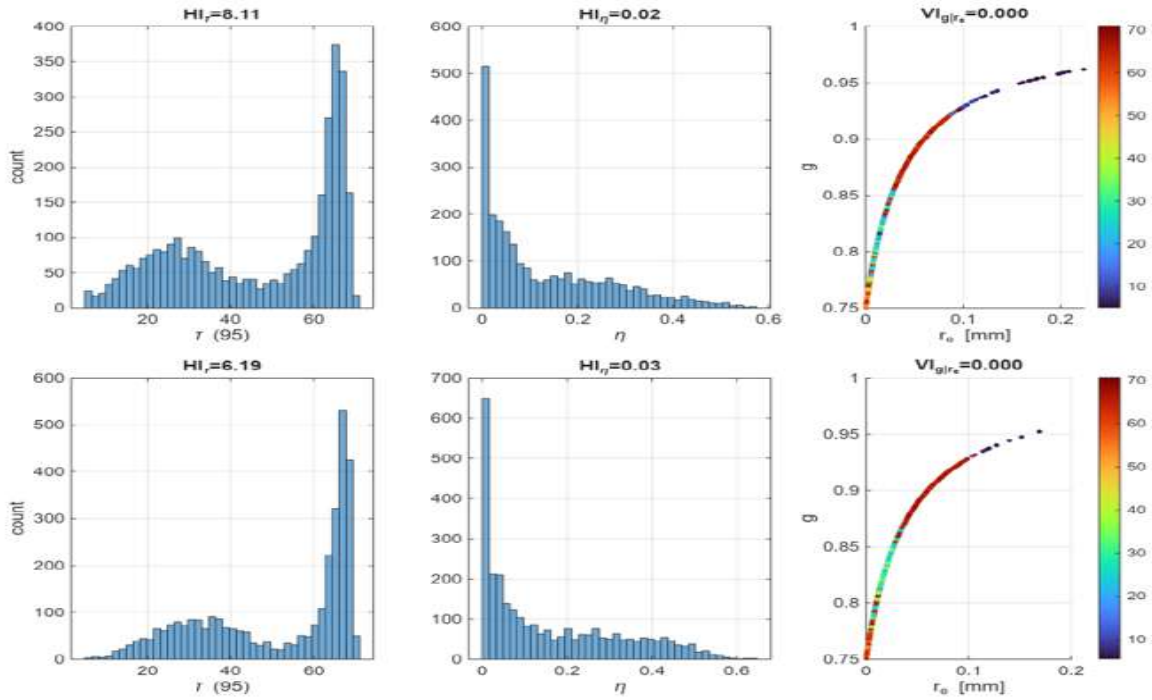


Figure 8.42. Horizontal and vertical inhomogeneity indices for RMNY (top row) and SPIN (bottom row) at 95 GHz. The first column shows the distribution of optical depth (τ) with its corresponding horizontal inhomogeneity index HI_τ , indicating the degree of variability in extinction across the scene. The second column reports the spectral inhomogeneity index HI_η , quantifying the horizontal dispersion of the spectral slope η and thus the mixture of hydrometeor size regimes. The third column illustrates the relationship between the asymmetry factor (g) and the effective radius (r_e), colour-coded by optical depth, from which the vertical inhomogeneity index $VI_{g|r_e}$ is derived. Overall, RMNY exhibits slightly higher horizontal variability than SPIN, while both sites show negligible vertical inhomogeneity, suggesting quasi-uniform vertical layering at the analyzed scales.

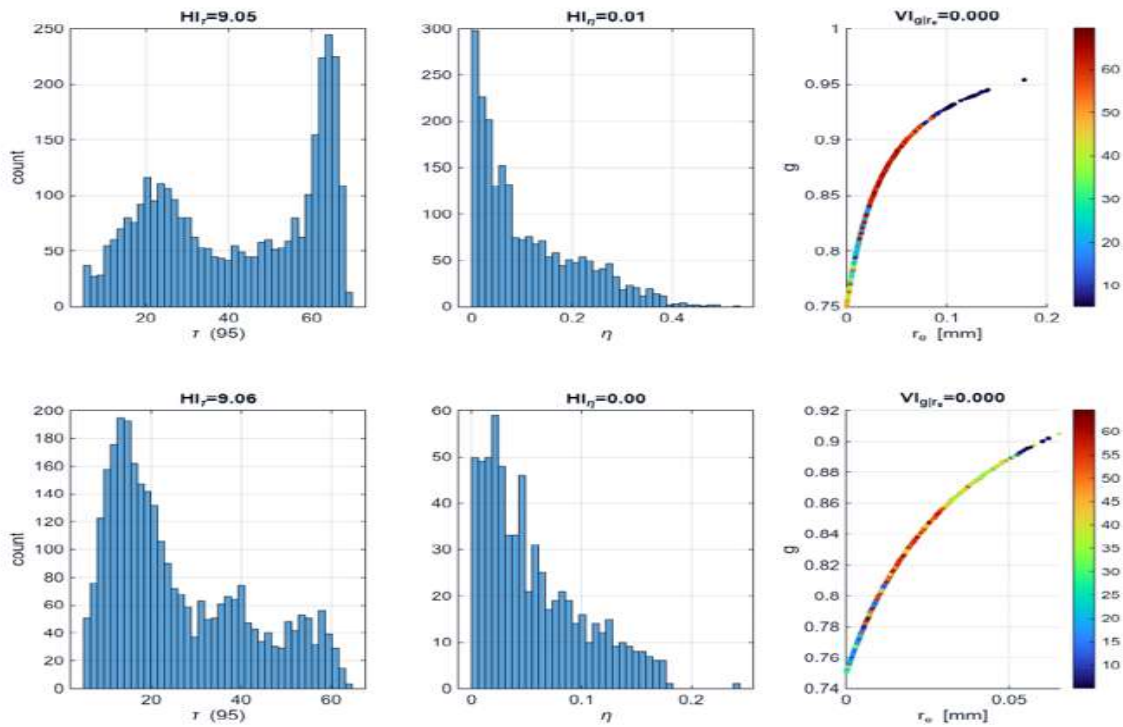


Figure 8.43. Horizontal and vertical inhomogeneity indices for RMNY (top row) and SPIN (bottom row) at 95 GHz. The first column displays the optical depth (τ) distributions and their horizontal inhomogeneity index HI_τ , which measures the spatial variability of extinction across each scene. The second column presents the spectral inhomogeneity index HI_η , reflecting horizontal dispersion in the spectral slope η , and therefore the degree of microphysical diversity among hydrometeors. The third column shows the relationship between the asymmetry factor (g) and the effective radius (r_e), colour-coded by optical depth, from which the vertical inhomogeneity index $VI_g | r_e$ is derived. Both sites exhibit relatively high HI_τ values, pointing to strong horizontal variability in optical thickness, while the low HI_η and $VI_g | r_e$ values indicate limited spectral and vertical inhomogeneity, consistent with quasi-uniform microphysical layering at the scales considered.



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include the variance of layer-wise optical thickness,

$$VI^* = Var_l[\tau_l], \quad (8.79)$$

together with additional descriptors based on the effective number of active layers, vertical salience, or variogram-based assessments of microphysical structure along altitude.

To investigate the internal organization of cloud columns under the current data constraints, a dedicated analysis was carried out for the RMNY site at 95 GHz. This framework extends the spatial and spectral inhomogeneity metrics introduced previously by providing a quantitative characterization of the vertical dimension of hydrometeor fields. The methodology combines the probability distribution function of optical thickness, the $\eta - IWP$ relationship, and two derived indices that summarize the internal vertical structure of the cloud. The first index, the effective layer number N_τ^* , provides an estimate of how many distinct scattering or absorbing sub-layers effectively contribute to the integrated extinction of the atmospheric column. This quantity does not represent the physical number of geometric layers but instead reflects the degree to which extinction is distributed or concentrated along the vertical coordinate. The second index, the vertical stratification index $VSI_{\eta|IWP}$, measures the variability of the attenuation spectral exponent η at fixed ice water path, thereby quantifying the presence of vertically organized microphysical transitions. High values imply stronger departures from uniformity and reveal the coexistence of multiple microphysical regimes within the same column.

Figures 8.44–8.45 illustrate the seasonal behavior of these metrics for RMNY.

Each figure includes the distribution of optical depth, which describes the integrated extinction properties, the $\eta - IWP$ scatterplot with the corresponding regression and the resulting value of $VSI_{\eta|IWP}$, the relationship between the asymmetry factor g and the effective radius r_e (color-coded by τ), and the bar plot of the effective layer number that summarizes the inferred vertical segmentation of the cloud body.

The results indicate that the degree of vertical inhomogeneity varies seasonally. Spring exhibits the largest stratification, with $VSI_{\eta|IWP} = 0.66$, pointing to enhanced layered structure typically associated with convective or transitional weather regimes. Autumn displays a slightly lower value, $VSI_{\eta|IWP} = 0.57$, consistent with more stable microphysical profiles.

The corresponding effective layer numbers, $N_\tau^* = 3.53$ in spring and $N_\tau^* = 4.55$ in autumn, confirm that multiple sub-layers contribute to the total extinction of the atmospheric column. These findings highlight the presence of complex vertical structuring at RMNY and demonstrate the importance of accounting for stratification when interpreting attenuation and radiative-transfer behaviour in mixed-phase cloud environments.

Figures 8.46 and 8.47 illustrate the effective vertical stratification and saliency parameters derived for the RMNY site at 95 GHz during autumn (AUT) and spring (SPR). These diagnostics summarize the main radiative and microphysical indicators extracted from the SNEM-2 simulations and provide a concise view of the internal organization of cloud columns.

The analysis combines the distribution of **optical depth** (τ), the effective **number of**

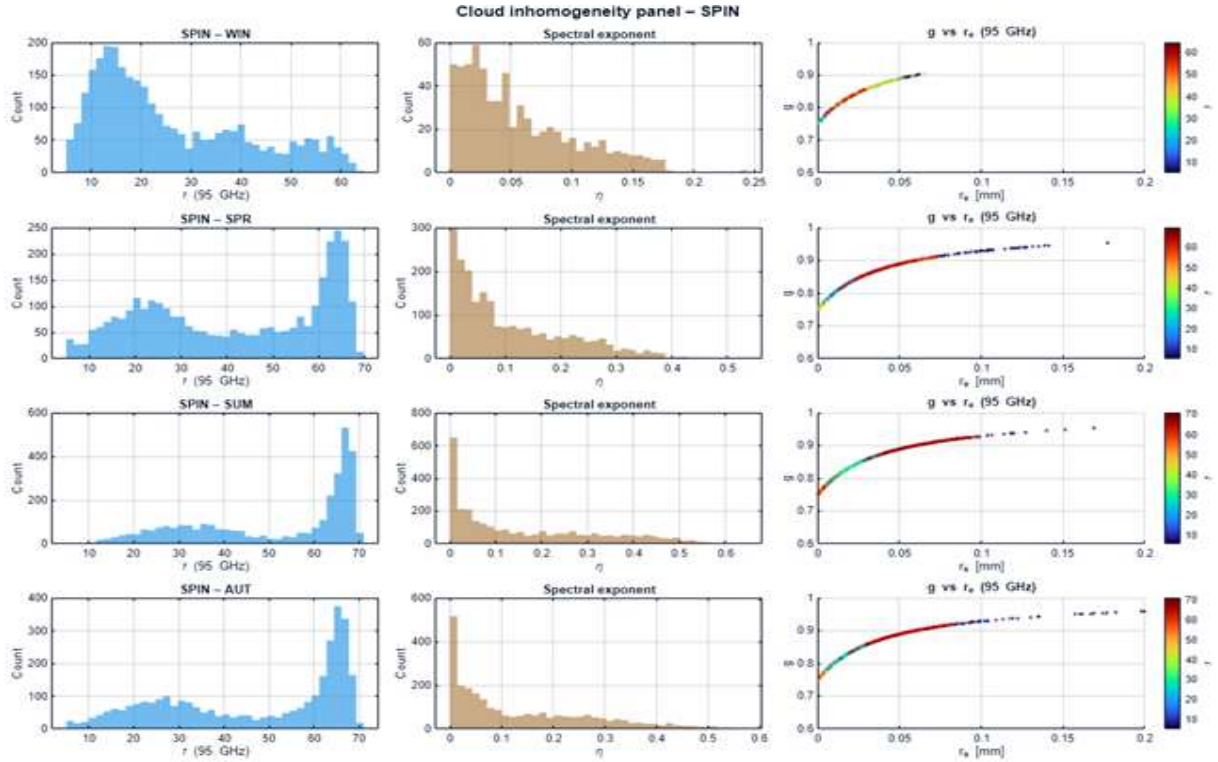


Figure 8.44. Seasonal cloud inhomogeneity panels for SPIN at 95 GHz.

vertical layers (N_L^*), and the relationship between the **spectral exponent** (η) and the **ice water path (IWP)**. The color-mapped asymmetry factor g complements these quantities by highlighting how scattering behavior varies with optical depth and by indicating which portions of the column contribute most to total extinction.

Seasonal differences are evident. Spring exhibits stronger vertical stratification and greater microphysical variability, with higher dispersion in the $\eta - IWP$ relationship and larger values of N_L^* . This behavior is consistent with more heterogeneous hydrometeor distributions and the presence of multiple radiatively active sub-layers. Autumn shows comparatively weaker stratification and a more uniform microphysical structure. The combined indicators quantify the vertical organization of hydrometeors at W-band and indicate that seasonal microphysical variability plays a significant role in shaping extinction and scattering profiles.



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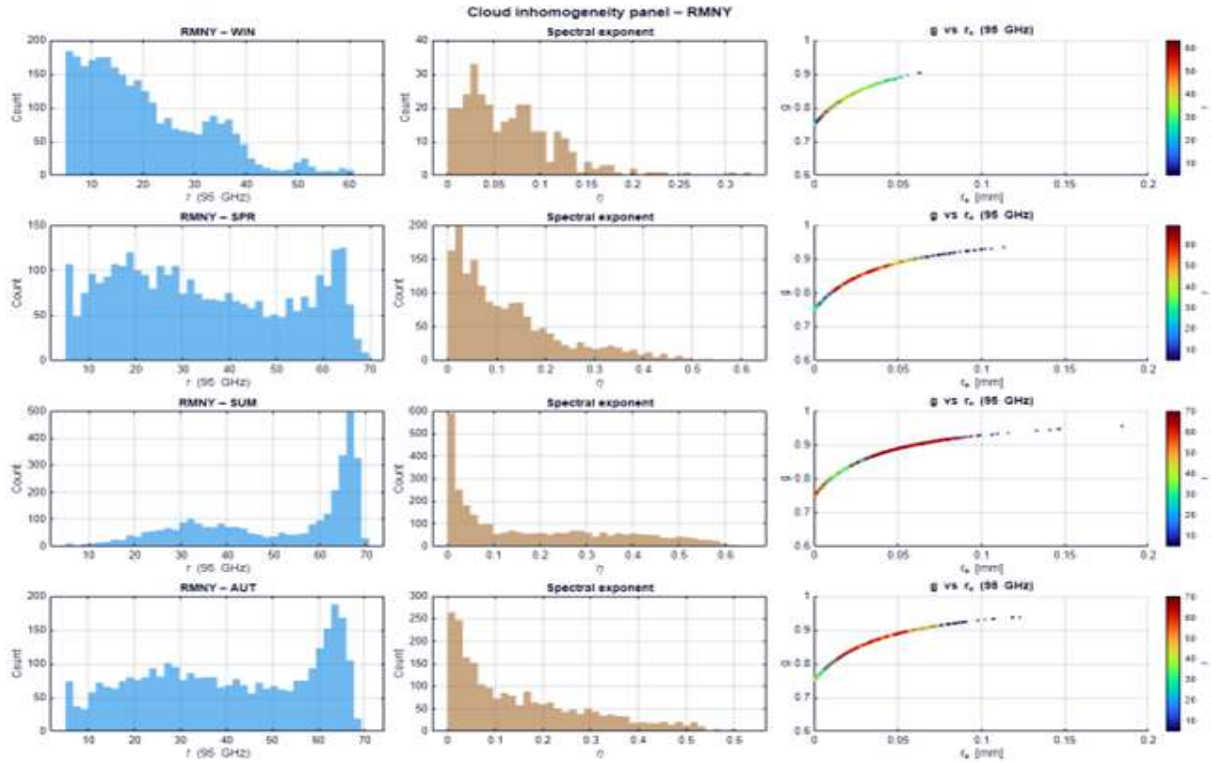


Figure 8.45. Seasonal cloud inhomogeneity panels for RMNY at 95 GHz. Each row corresponds to a different season (WIN, SPR, SUM, AUT). The left column shows the distributions of optical depth (τ), representing the overall extinction of the electromagnetic wave through the cloud layer. The central column reports the distributions of the spectral exponent (η), which quantifies the slope of the particle size spectrum and reflects microphysical variability across each season. The right column displays the relationship between the asymmetry factor (g) and the effective radius (r_e), color-coded by optical depth. The results highlight a clear seasonal dependence, with winter and summer showing broader distributions of τ and η , indicative of more heterogeneous cloud structures, while spring and autumn exhibit narrower ranges and more uniform scattering behavior.

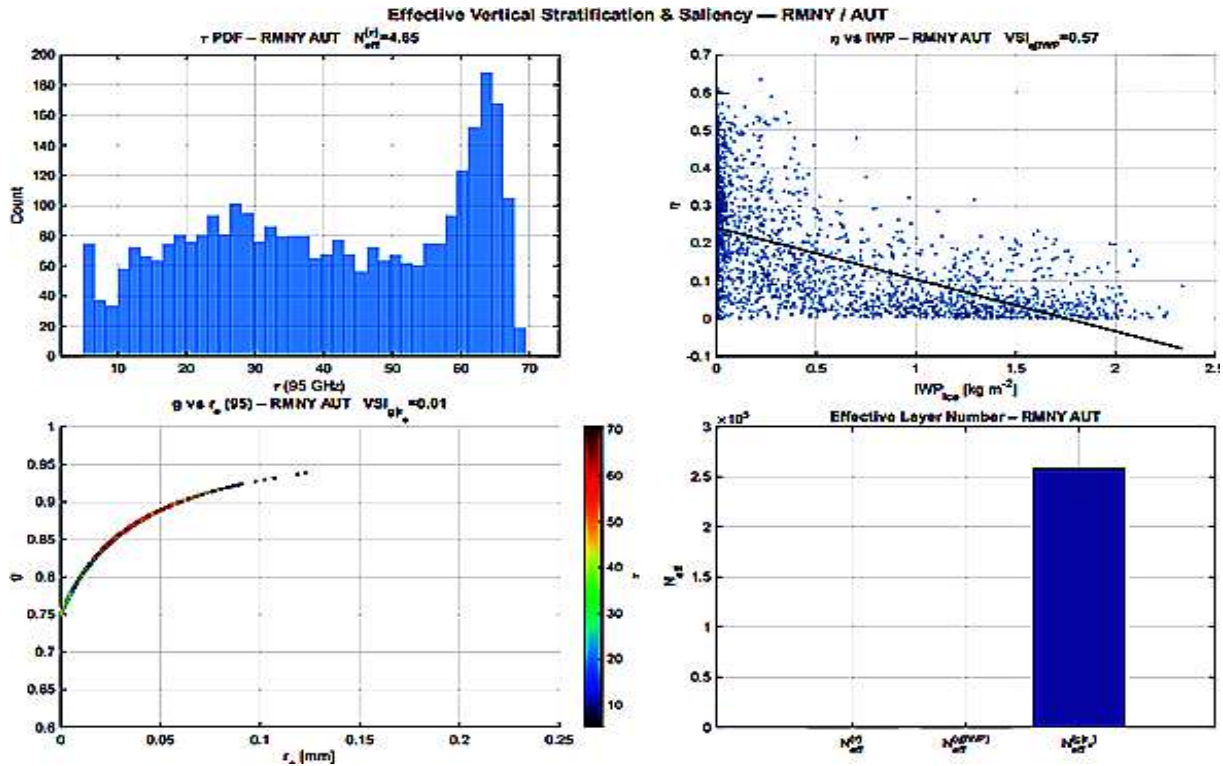


Figure 8.46. Effective vertical stratification and saliency parameters for RMNY during autumn (AUT) at 95 GHz. The upper-left panel shows the probability distribution of optical depth (τ), from which the effective number of vertical layers ($N_{\tau}^* = 4.55$) is derived. The upper-right panel displays the relationship between the spectral exponent (η) and the ice water path (IWP), together with the vertical inhomogeneity index ($VSI_{\eta} | IWP = 0.57$), indicating moderate vertical variability of the microphysical structure. The lower-left panel illustrates the dependence of the asymmetry factor (g) on the effective radius (r_e), color-coded by optical depth. The lower-right panel summarizes the effective layer number metric, highlighting the presence of multiple active scattering layers contributing to the total extinction within the analyzed cloud column.

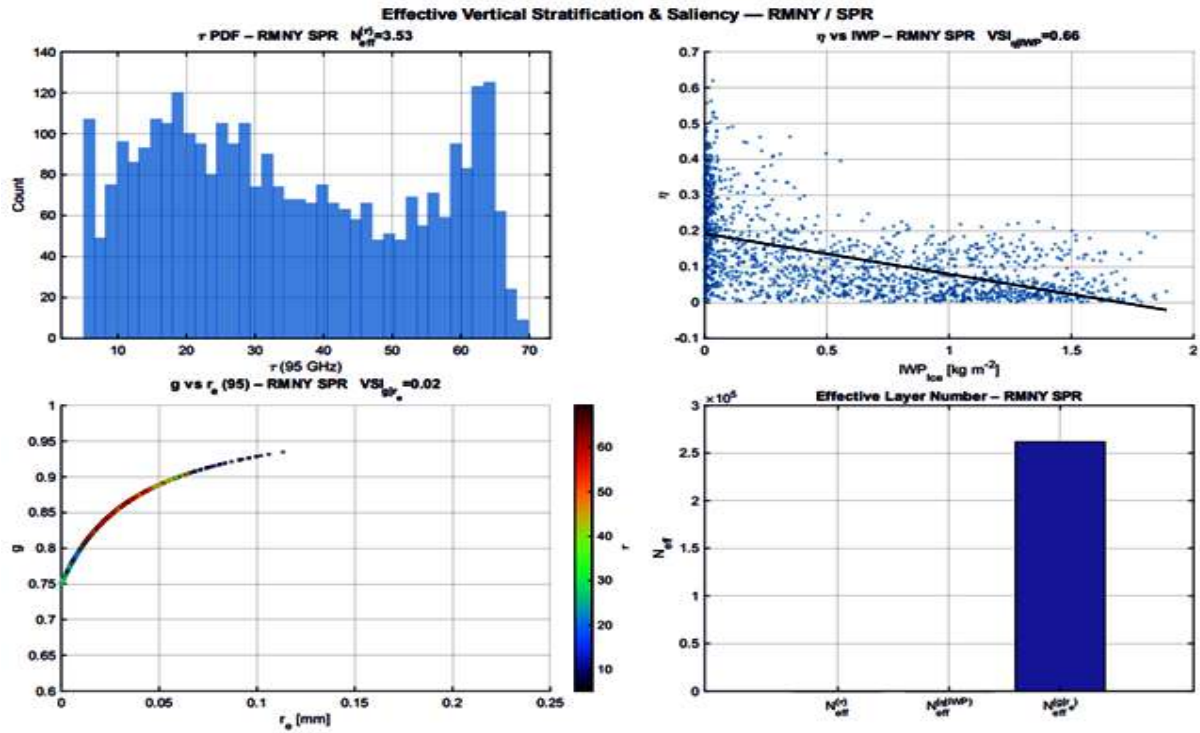


Figure 8.47. Effective vertical stratification and saliency parameters for RMNY during spring (SPR) at 95 GHz. The upper-left panel shows the probability distribution of optical depth (τ), from which the effective number of vertical layers ($N_{\tau}^* = 3.53$) is derived. The upper-right panel displays the relationship between the spectral exponent (η) and the ice water path (IWP), with the computed vertical stratification index ($VSI_{\eta} | IWP = 0.66$), indicating enhanced vertical variability compared to the autumn case. The lower-left panel shows the dependence of the asymmetry factor (g) on the effective radius (r_e), color-coded by optical depth, while the lower-right panel summarizes the effective layer number metric. The results suggest the presence of moderately stratified and vertically inhomogeneous cloud structures in spring, associated with more variable hydrometeor distributions.



Appendix A9 – Chapter 9

Appendix 9.A — Figures

All figures referenced in **Chapter 9** are collected here for readability.

Appendix 9.B – Tables

All tables referenced in **Chapter 9** are collected here for readability.

Rec.	Name and Last Update	Status
ITU-R P.453-14	The radio refractive index: its formula and refractivity data (08/2019)	In Force
ITU-R P.676-13	Attenuation by atmospheric gases and related effects (08/2022)	In Force
ITU-R P.676-11	Attenuation by atmospheric gases (09/2016)	Superseded
ITU-R P.835-7	Reference standard atmospheres (08/2024)	In Force
ITU-R P.836-7	Water vapour: surface density and total columnar content (08/2017)	In Force
ITU-R P.837-8	Characteristics of precipitation for propagation modelling (09/2025)	In Force
ITU-R P.838-3	Specific attenuation model for rain for use in prediction methods (03/2005)	In Force
ITU-R P.839-4	Rain height model for prediction methods (09/2013)	In Force
ITU-R P.840-9	Attenuation due to clouds and fog (08/2023)	In Force
ITU-R P.834-9	Effects of tropospheric refraction on radio wave propagation (08/2022)	In Force
ITU-R P.1511-3	Topography for Earth-space propagation modelling (08/2024)	In Force
ITU-R P.618-14	Propagation data and prediction methods required for the design of Earth-space telecommunication systems (08/2023)	In Force

Table A9.1. Latest ITU-R Recommendations for Atmospheric Attenuation and Propagation (status as of October 2025)

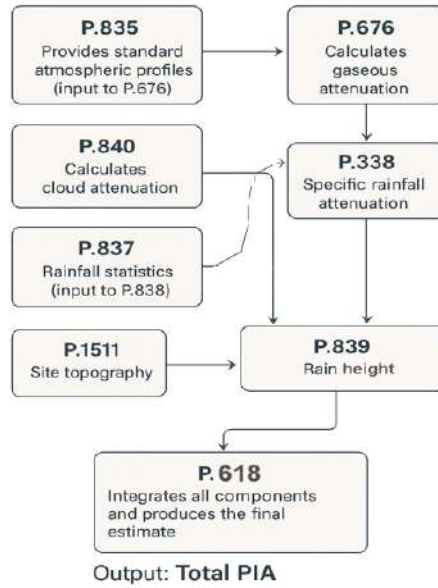


Figure A9.1. Flowchart of ITU-R recommendations for estimating atmospheric attenuation in Earth-space links. Standard atmospheric profiles (P.835) feed into gaseous attenuation (P.676), while cloud attenuation is obtained from P.840. Rainfall statistics (P.837) provide inputs to specific rainfall attenuation (P.838), complemented by rain height (P.839) and site topography (P.1511). All components are integrated within P.618 to produce the final attenuation estimate.

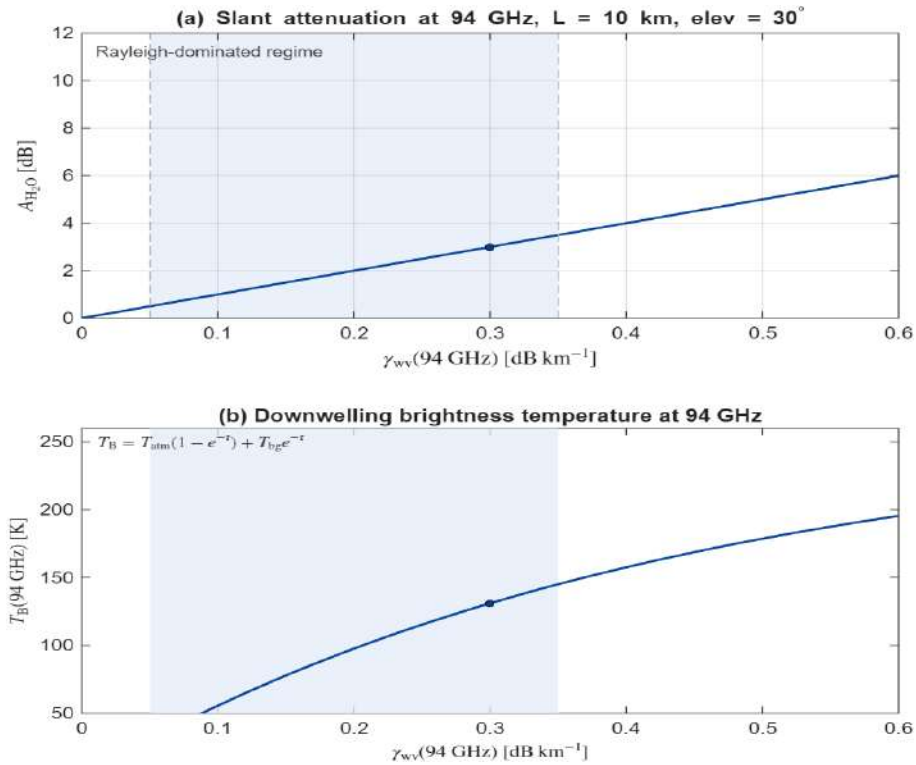


Figure A9.2. Effect of water-vapor absorption at 94 GHz. Top: Slant-path attenuation over a 10 km path at 30° elevation as a function of the specific water-vapor attenuation γ_{wv} . Bottom: Corresponding change in downwelling brightness temperature T_b , resulting from increased atmospheric emission and extinction of the cosmic background along the path.

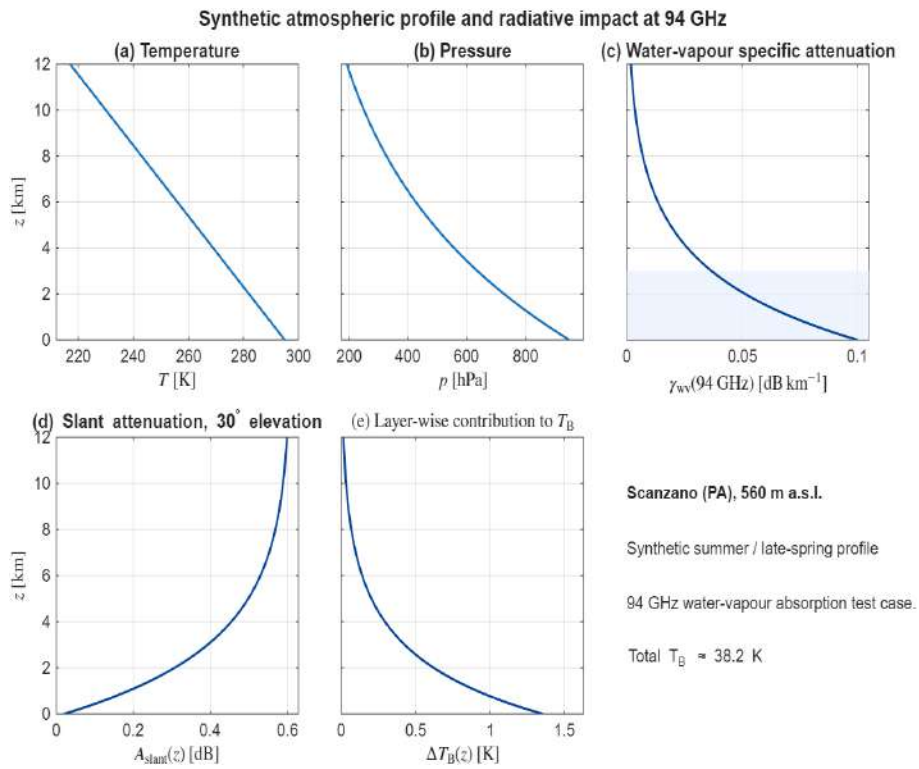


Figure A9.3. Illustrative synthetic atmospheric profile for Scanzano (PA, 560 m a.s.l.) used to characterize water-vapor absorption at 94 GHz. Temperature, pressure, and height-dependent specific attenuation γ_{wv} are shown on the left, while the right-hand panels quantify the cumulative slant attenuation for a 30° elevation path together with the incremental contribution of each layer to the downwelling brightness temperature. Under summer and late-spring conditions, the analysis confirms that the bulk of the absorption and thermal emission arises within the lower troposphere, where both humidity and temperature remain highest.



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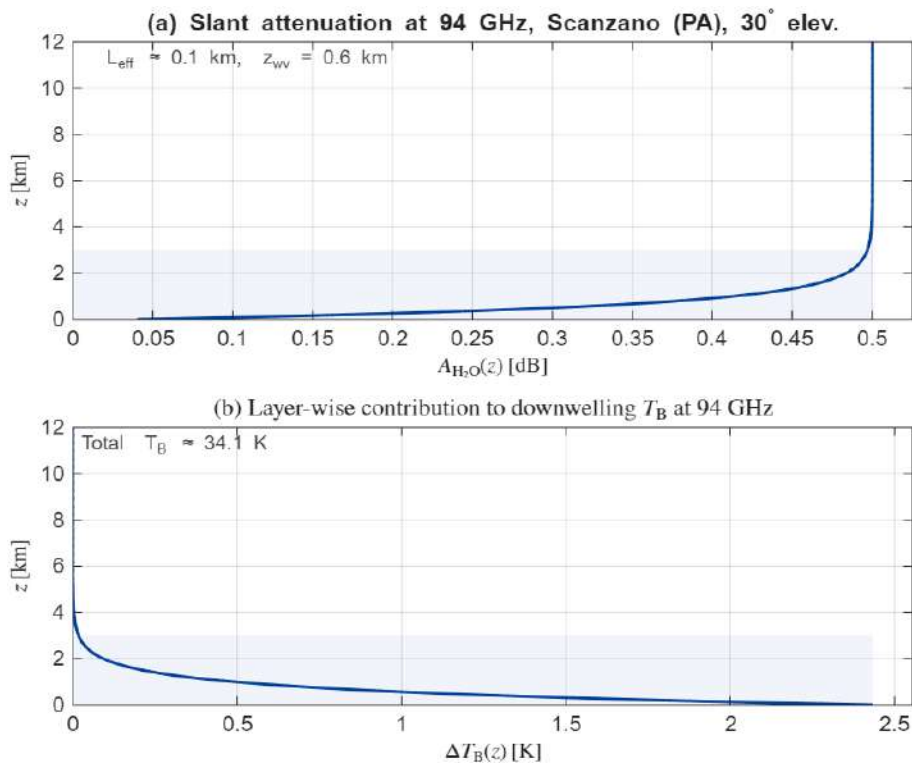


Figure A9.4. Slant-path water-vapor attenuation and layer-wise radiative contribution at 94 GHz for a synthetic atmospheric profile representative of Scanzano (PA), 30° elevation. (a) Cumulative attenuation $A_{H_2O}(z)$ along the inclined path, showing that most of the absorption occurs within the lowest kilometre (effective scale height $z_{wv} \approx 0.6$ km; effective path length $L_{eff} \approx 0.1$ km). (b) Layer-wise contribution $\Delta T_B(z)$ to the downwelling brightness temperature, with the lower troposphere dominating the total radiometric signal. The resulting brightness temperature is $T_B \approx 34.1$ K.

Model	Input Data			Validity Range	Annex
	Content	Geometry	Range		
ITU-R P.676-11	Pressure Temperature	Elevation	Frequency	$1 \leq f \leq 350$ $z \leq 10$ km $5^\circ \leq E_{Sat} \leq 90^\circ$	2
Salonen	Pressure Temperature at t_0 and $t_0 - 12$ h	Elevation	Frequency	$1 \leq f \leq 350$ $z \leq 5$ km $5^\circ \leq E_{Sat} \leq 90^\circ$	
ITU-R P.676-11 Water Vapor Attenuation	Pressure Temperature WV density	Elevation	Frequency	$1 \leq f \leq 350$ $z \leq 10$ km $5^\circ \leq E_{Sat} \leq 90^\circ$	2
ITU-R P.676-11	Pressure Temperature Total WC content	Elevation	Frequency	$1 \leq f \leq 350$ $z \leq 10$ km $5^\circ \leq E_{Sat} \leq 90^\circ$	2
ITU-R P.676-11 Oxygen and WV Att.	Pressure Humidity profile Temperature	Elevation	Frequency	$1 \leq f \leq 350$ $z \leq 10$ km $5^\circ \leq E_{Sat} \leq 90^\circ$	1

Table A9.2. Gas Attenuation Statistical Prediction Models: z is the altitude, E_{Sat} is the elevation satellite angle.

Recommendation	Function/Physical process
P.835	Defines standard atmospheric profiles (input to P.676).
P.676	Calculates gaseous attenuation.
P.840	Estimates cloud and fog attenuation.
P.837	Provides rainfall statistics (input to P.838).
P.838	Defines the specific attenuation–rain rate relationship.
P.839	Determines mean rain height.
P.1511	Supplies site topography and altitude.
P.618	Integrates all components into total attenuation.

Table A9.3. Functional interdependence among ITU-R recommendations used for atmospheric attenuation modeling. Each recommendation addresses a specific component of the attenuation process and contributes to the integrated estimation of the Path-Integrated Attenuation (PIA) through ITU-R P.618. P.835 provides standard atmospheric profiles feeding P.676 for gaseous attenuation; P.840 and P.838 describe cloud and rain effects respectively, supported by climatological data from P.837, P.839, and P.1511. The overall methodology is harmonized within P.618 to obtain the total path attenuation.

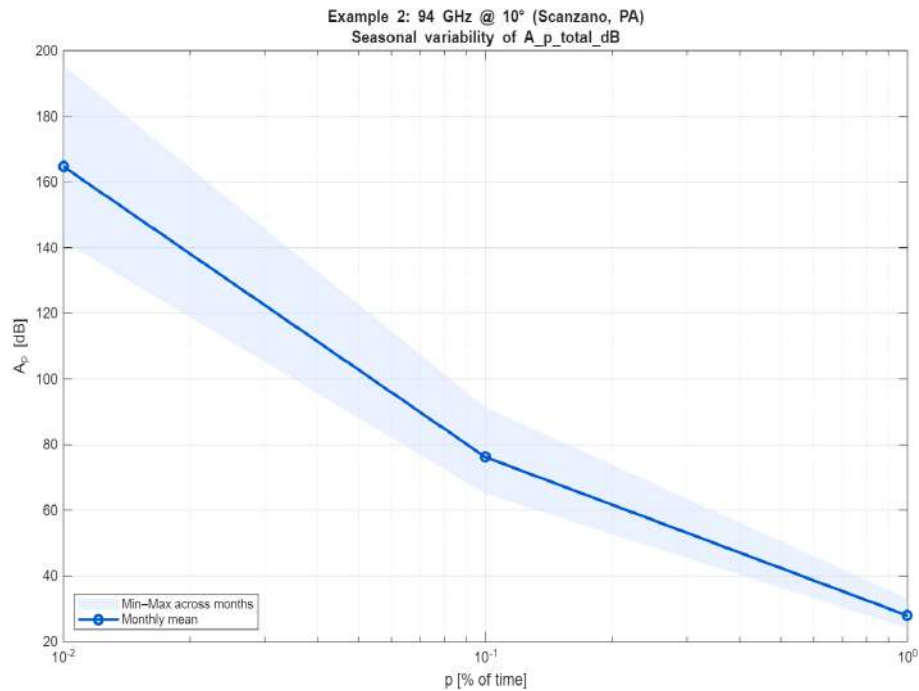


Figure A9.5. Integrated view of slant-path attenuation mechanisms at 94 [GHz] for Scanzano (PA). The shaded area represents the seasonal variability of the predicted attenuation A_p (min-max across months), while the solid line indicates the mean curve. The figure highlights the large uncertainty associated with W-band links under Mediterranean rainfall conditions, with potential fades exceeding 150 [dB] at 0.01% of time.

Season	Rain	H_2O	O_2	Clouds
DJF	84.8 %	5.3 %	9.1 %	0.8 %
MAM/SON	88.8 %	5.2 %	4.8 %	1.3 %
JJA	91.2 %	4.8 %	2.7 %	1.3 %

Table A9.4. Seasonal fractional contributions of attenuation mechanisms at 94 GHz for Scanzano (PA, 30° elevation). Relative contributions of rain, water vapor (H_2O), oxygen (O_2), and cloud liquid water to the total slant-path attenuation. The results show that attenuation is systematically rain-dominated (85–91 %), while gaseous absorption and cloud contributions remain secondary. Data refer to seasonal climatological means for the 1991–2020 period.

Season	Total Fade [dB]
DJF	7–8
MAM/SON	13–14
JJA	≈ 23

Table A9.5. Representative seasonal fade budgets at 94 GHz for Scanzano (PA, 30° elevation). Seasonal total attenuation values derived from ITU-R P.676 and climatological profiles. The results indicate a progressive increase of slant-path attenuation from winter (DJF) to summer (JJA), with fades of 7–8 dB, 13–14 dB, and ≈ 23 dB, respectively. These values define the dynamic margin required for adaptive coding and modulation (ACM) or automatic transmit-power control (ATPC) schemes in W-band Earth-space links.

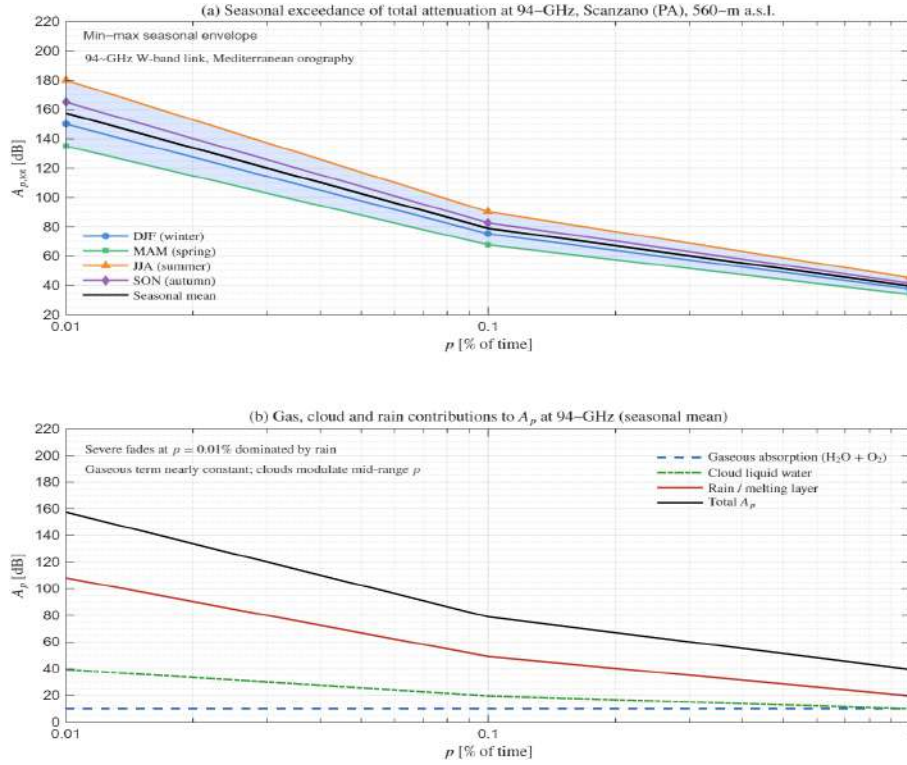


Figure A9.6. Seasonal exceedance of total slant-path attenuation at 94 GHz for Scanzano (PA), 560 m a.s.l.(a) Seasonal curves for a 10° elevation link, with the shaded envelope indicating the min-max variability across months and highlighting severe fades (>160 dB) at $p=0.01\%$.(b) Seasonal-mean decomposition showing that gaseous absorption is nearly constant, clouds dominate mid-range probabilities, and extreme fades at very low p are driven by rain and melting-layer enhancement.

Cloud Type	Height [km]	Temperature [K]	Typical LWC [gm^{-3}]
Cirrus	5	233.15	0.01–0.03
Alto cumulus (a)	2	275.21	0.05–0.12
Alto cumulus (b)	4	262.23	0.08–0.18
Cumulus Congestus	6	249.25	0.30–0.50
Cumulus	2	283.15	0.15–0.30

Table A9.6. Cloud archetypes used in the sub-THz attenuation analysis. Representative altitude-temperature pairs are derived from mid-latitude climatological profiles and are used to identify the typical liquid-water content (LWC) associated with each cloud family. While γ_c is primarily controlled by LWC, temperature modulates the complex permittivity through the Double-Debye model. The selected archetypes—from warm low-level Cumulus to cold high-altitude Cirrus and Cumulus Congestus—cover the microphysical regimes examined in the frequency-dependent comparison of **Fig. 11.XX(a-d)**.

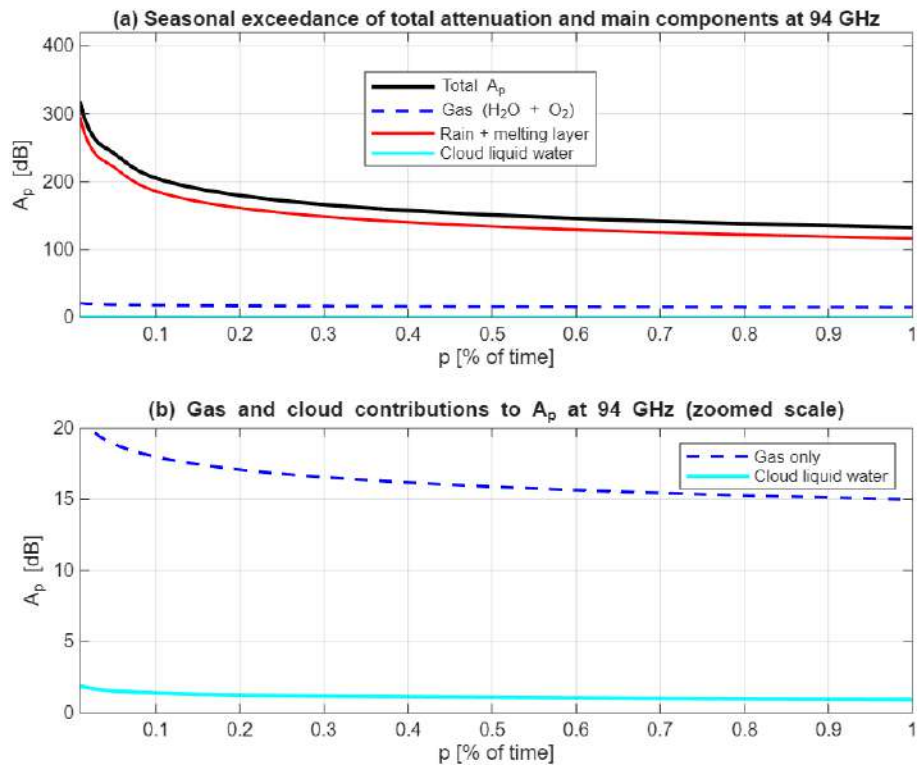


Figure A9.7. Seasonal exceedance curves of total attenuation at 94 GHz and contributions from individual components. (a) Exceedance of total path attenuation A_p (black) together with the main physical contributors: gaseous absorption by water vapor and oxygen (blue dashed), rain plus melting layer (red), and cloud liquid water (cyan). The curves refer to seasonal statistics based on realistic thermodynamic and hydrometeor profiles for Scanzano (PA). (b) Zoom on the low-attenuation regime, highlighting the separation between gaseous absorption and cloud liquid water for $p > 0.05\%$. Gas dominates the exceedance curve at low probabilities, while cloud liquid water contributes only marginally except during thick stratiform episodes.

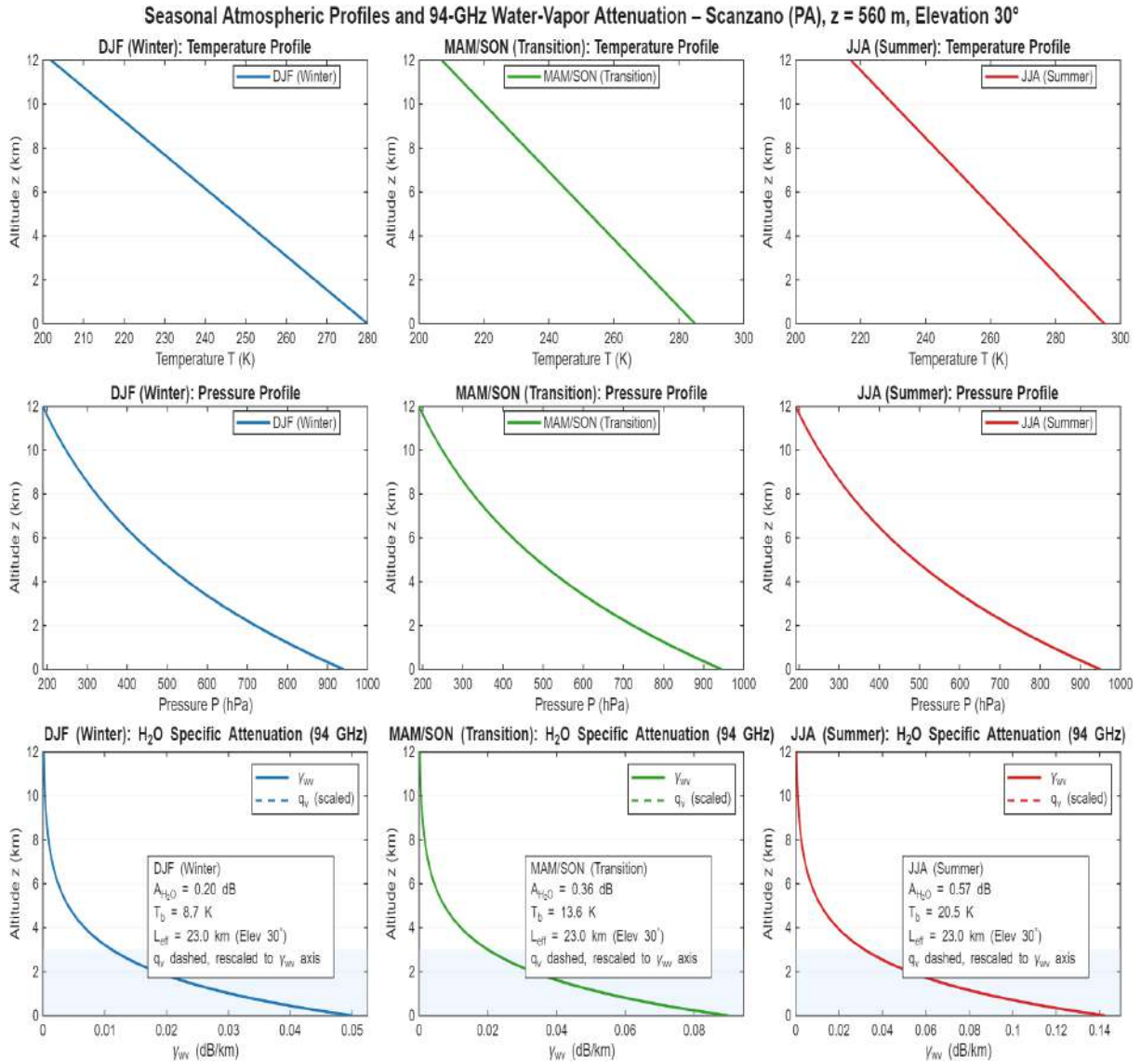


Figure A9.8. Seasonal mean vertical profiles of temperature, pressure, and water vapor specific attenuation at 94 [GHz] for the Scanzano (PA) site ($L_{eff} = 23.0$ [km]). Data are shown for DJF (winter), MAM/SON (transition seasons), and JJA (summer), together with integrated values of total attenuation A_{H_2O} , surface temperature T_s , and absorption length L_{ar} .

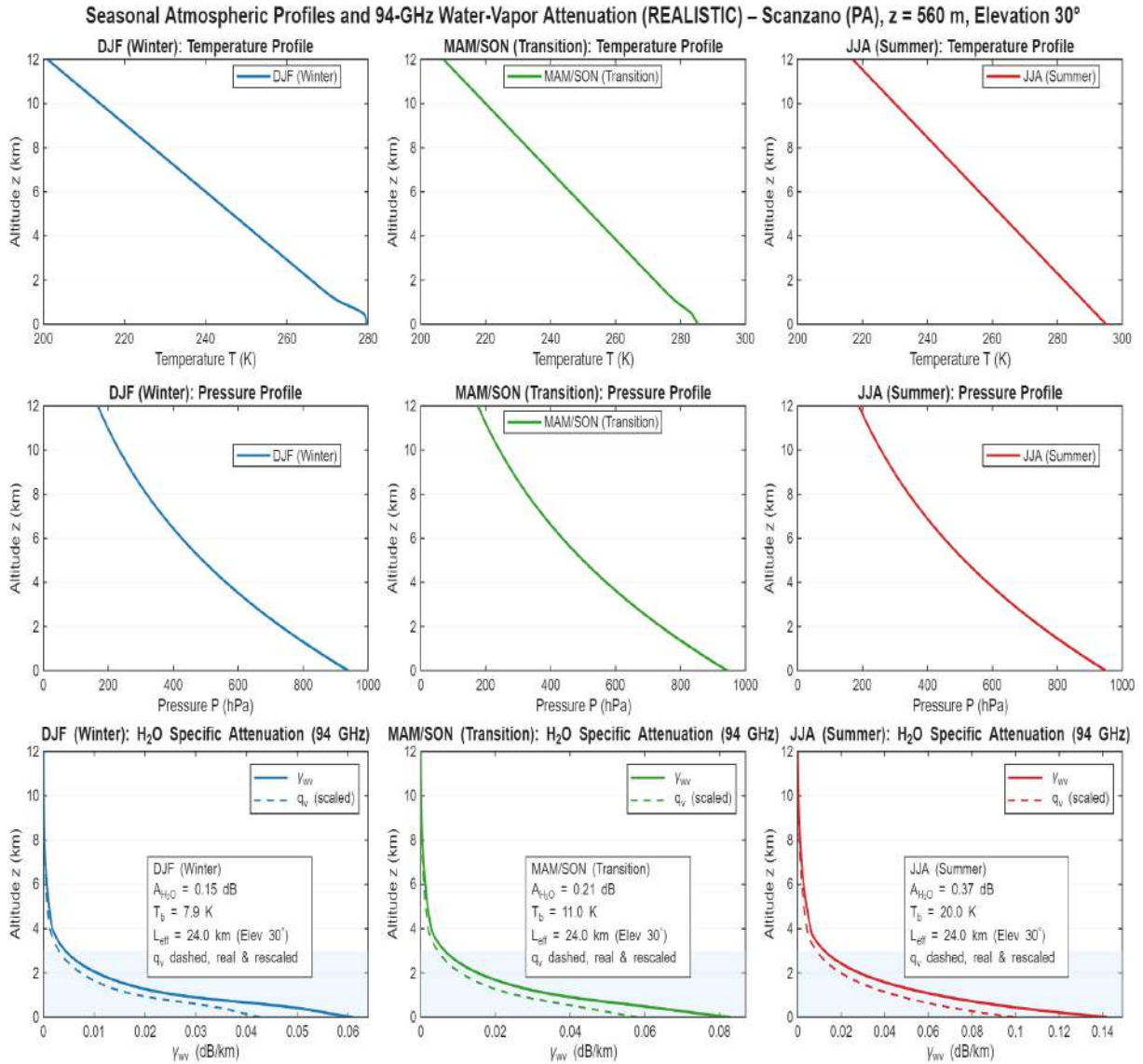


Figure A9.9. Seasonal atmospheric profiles and water-vapor attenuation at 94 GHz for Scanzano (PA), Italy (560 m a.s.l.). Temperature, pressure, and water-vapor attenuation profiles are shown for winter (DJF), transition seasons (MAM/SON), and summer (JJA). The bottom row reports the vertical specific attenuation due to water vapor γ_{wv} together with the corresponding rescaled specific humidity profile q_v , highlighting the proportionality between moisture content and absorption at W-band. Shaded areas indicate the lower-tropospheric moist layer. Integrated metrics (A_{H_2O} , T_b , L_{eff}) refer to a 30° elevation satellite link and describe the expected seasonal variability in gaseous attenuation and atmospheric emission.

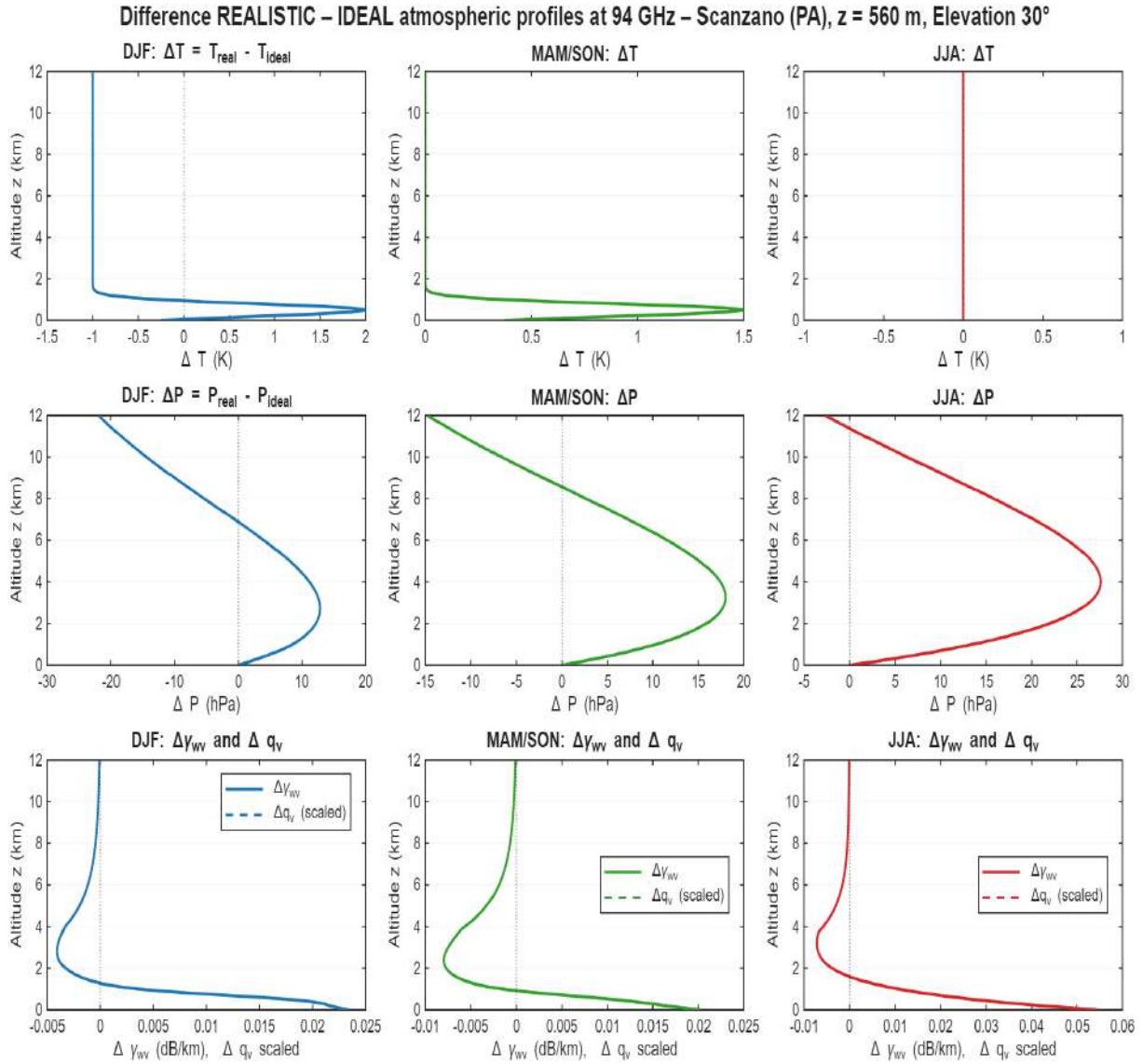


Figure A9.10. Difference between realistic and idealized atmospheric profiles for Scanzano (PA) at 94 GHz. The panels show the vertical differences $\Delta T = T_{real} - T_{ideal}$ (top row), $\Delta P = P_{real} - P_{ideal}$ (middle row), and the differences in water-vapor specific attenuation and humidity, $\Delta \gamma_{wv}$ and Δq_v (bottom row), for winter (DJF), transition seasons (MAM/SON), and summer (JJA). Positive values indicate an excess of temperature, pressure, or moisture in the realistic configuration with respect to the simplified profiles, while negative values highlight where the idealized model overestimates the corresponding quantity. Dashed curves in the bottom row denote Δq_v rescaled onto the $\Delta \gamma_{wv}$ axis, evidencing the tight coupling between humidity errors and gaseous absorption errors at 94 GHz.

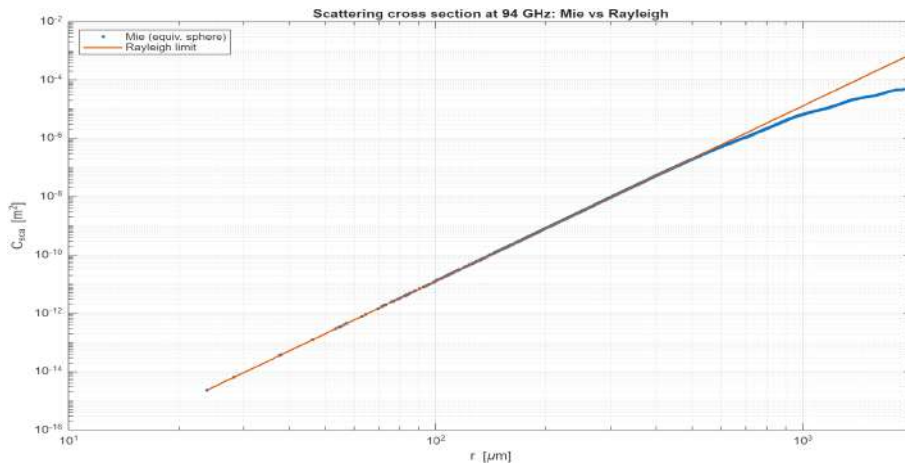


Figure A9.11. Comparison between Rayleigh and Mie scattering cross sections for ice particles at 94 GHz. The scattering cross section C_{sca} is shown as a function of the particle equivalent radius for an ensemble of porous ice spheres. In the small-particle limit, the Mie solution converges to the Rayleigh r^6 scaling. For larger particles ($r \gtrsim 300\text{--}500\text{ }\mu\text{m}$), significant deviations emerge, with Mie scattering exhibiting saturation and resonance effects that are not captured by the Rayleigh approximation. This behavior highlights the limited domain of validity of Rayleigh-based microphysical parameterizations when applied to W-band frequencies.

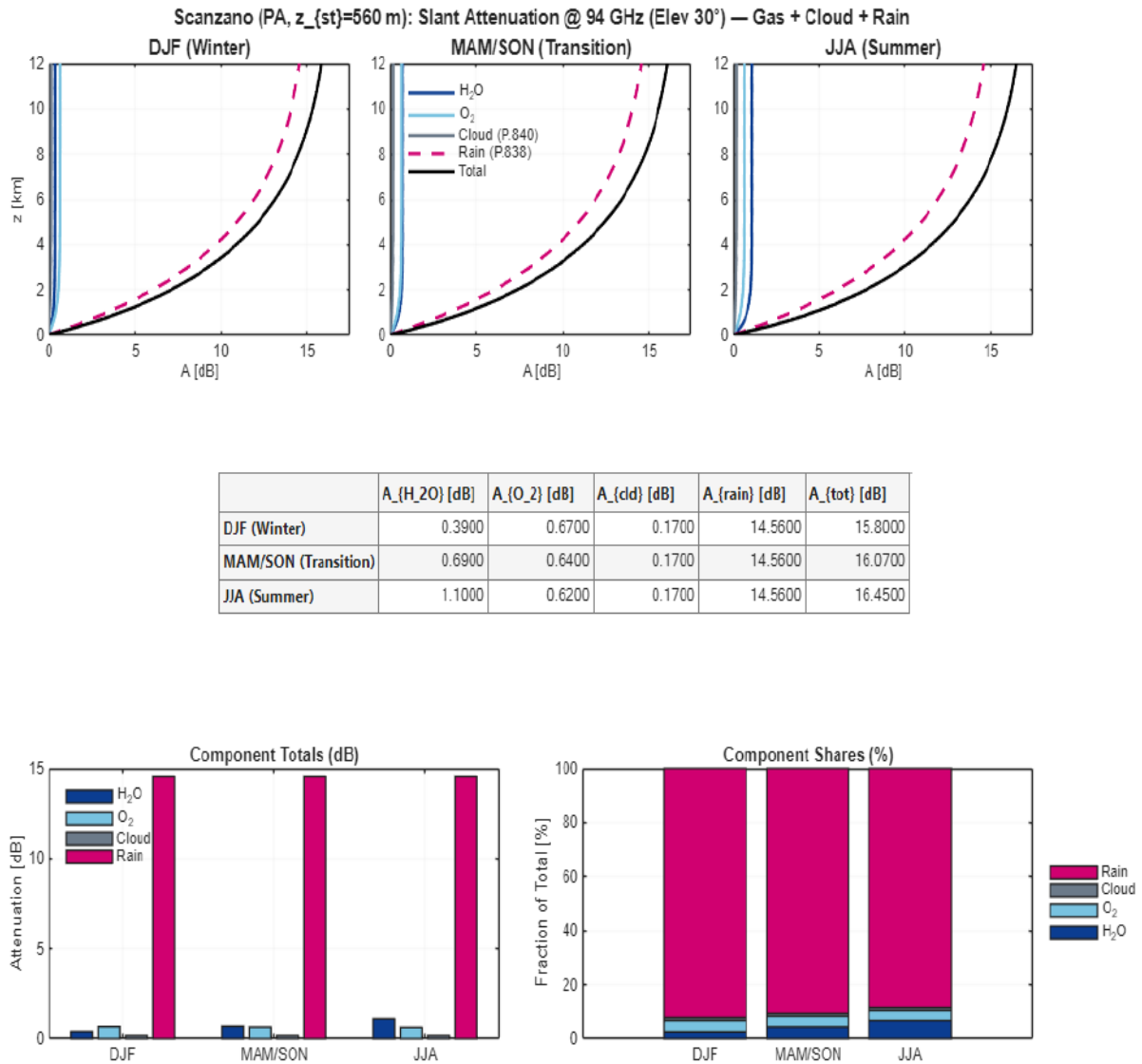


Figure A9.12. Seasonal variation of slant attenuation at 94 [GHz] for Scanzano (PA), $z_{sfc} = 560$ [m] (asml) at an elevation angle of 30°, showing the contributions from water vapor (H_2O), oxygen (O_2), cloud liquid water, and rain. The top row presents attenuation as a function of elevation angle for winter (DJF), transition seasons (MAM/SON), and summer (JJA), with separate curves for each component and the total. The middle table summarizes key seasonal atmospheric parameters and corresponding attenuation values. The bottom row condenses the results into bar charts reporting total attenuation (left) and fractional contributions (right) for each component and season, highlighting the seasonal dominance of different mechanisms.

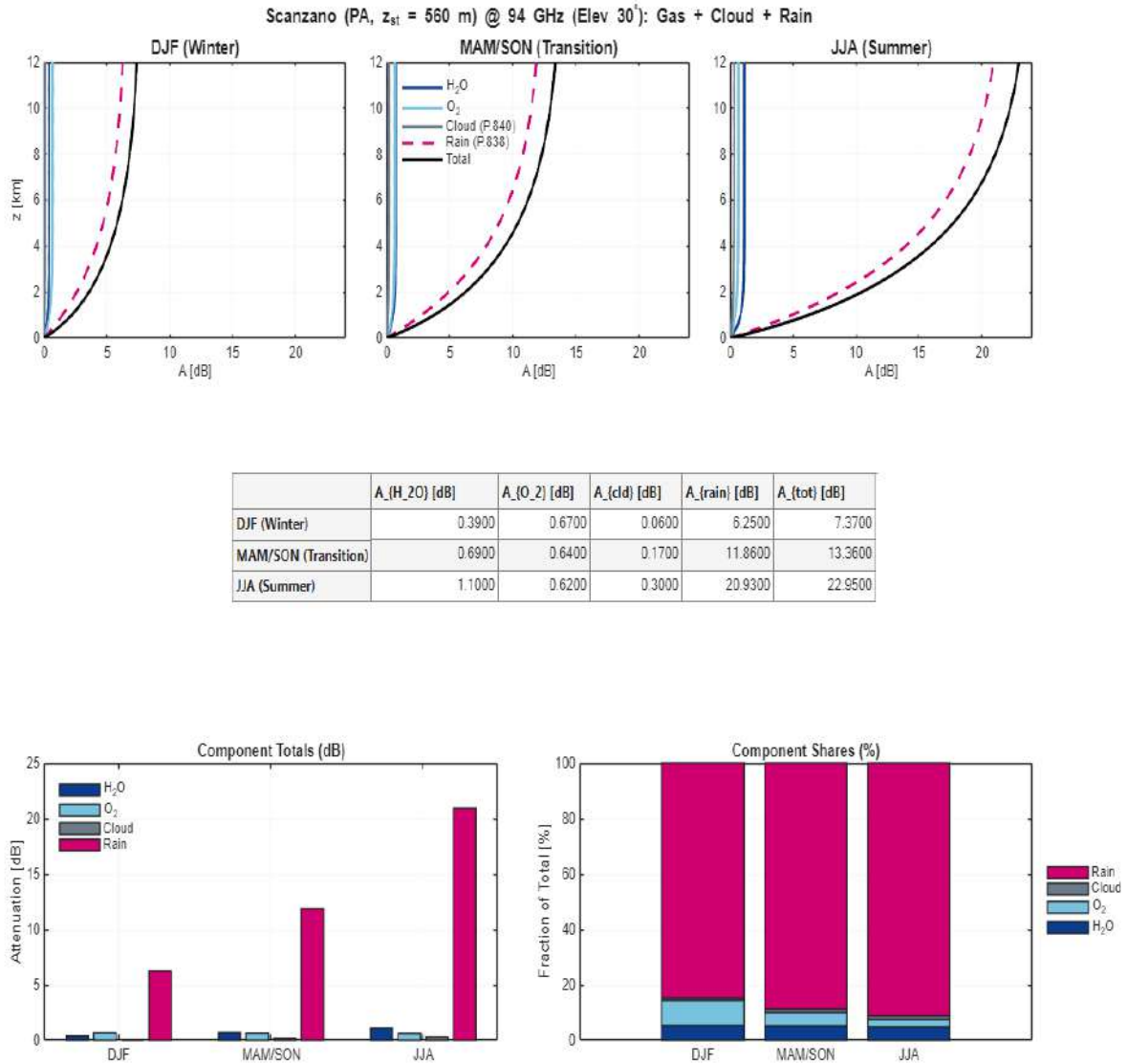


Figure A9.13. Seasonal slant attenuation at 94 GHz for Scanzano (PA), $z_{sfc} = 560$ [m] (asml) at an elevation angle of 30° , showing the contributions from water vapor (H_2O), oxygen (O_2), cloud liquid water, and rain. The top row presents attenuation build-up along the slant path for winter (DJF), transition seasons (MAM/SON), and summer (JJA), with individual curves for each component and the total. The bottom row summarizes the results in bar charts: total attenuation per component (left) and fractional contribution to the seasonal total (right). The plots highlight the seasonal shift in dominant mechanisms — gaseous absorption in winter, rain in the transition season, and cloud liquid water in summer — providing both a detailed physical view and a concise quantitative comparison.

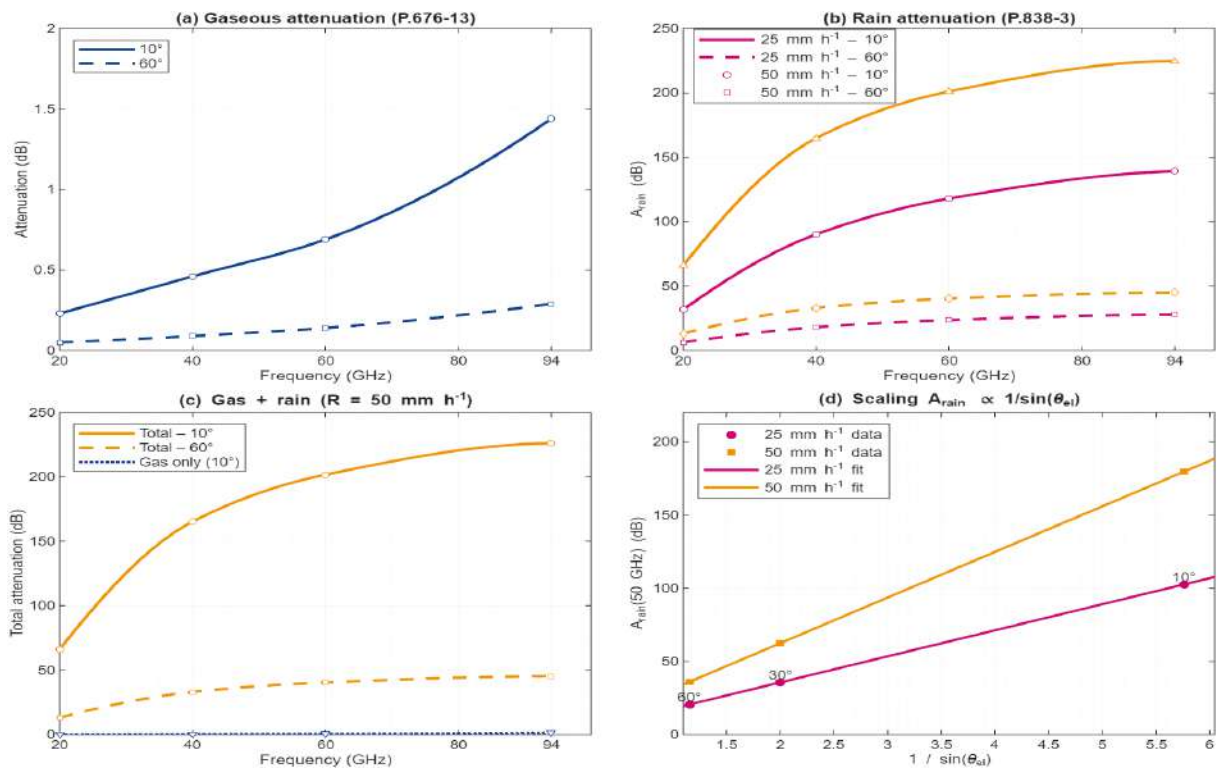


Figure A9.14. Total attenuation as a function of frequency for different elevation angles and atmospheric conditions (vapor and rain). The graph illustrates how signal attenuation increases with frequency and is more pronounced at lower elevation angles. Rain-induced attenuation is significantly higher than vapor-induced attenuation, especially at frequencies above 10 [GHz].



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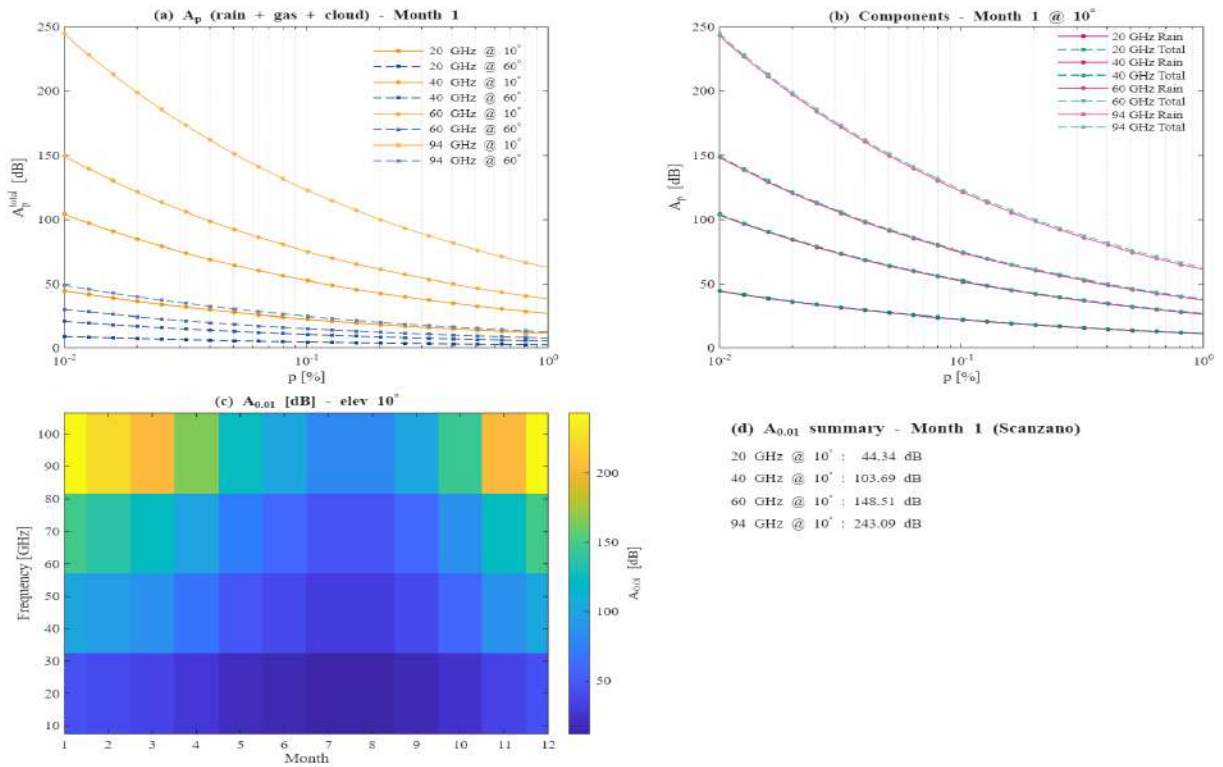


Figure A9.15. Slant-path attenuation statistics and monthly $A_{0.01}$ climatology for Scanzano (PA), Sicily (560 m a.s.l.), combining gaseous and rain attenuation effects according to ITU-R P.676-13 and ITU-R P.838-3. Panel (a) reports the exceedance curves $A_p(\text{rain} + \text{gas} + \text{cloud})$ for probabilities $p \in [10^{-2}, 1]\%$ at two elevation angles (10° and 60°), highlighting the strong frequency- and geometry-dependent variability driven by the effective slant path $L_{eff} = h_R / \sin(\theta_{el})$. Panel (b) separates rain-only and total attenuation components at 10° for the selected month, confirming that rainfall dominates above 30–40 GHz. Panel (c) provides the monthly climatology of $A_{0.01}$ at 10° for the 20–94 GHz range, illustrating the seasonal modulation typical of Mediterranean environments, with enhanced rain-induced losses in autumn and winter. Panel (d) summarizes the corresponding $A_{0.01}$ values for Month 1 (January), which are used to construct the exceedance curves in Panel (a).



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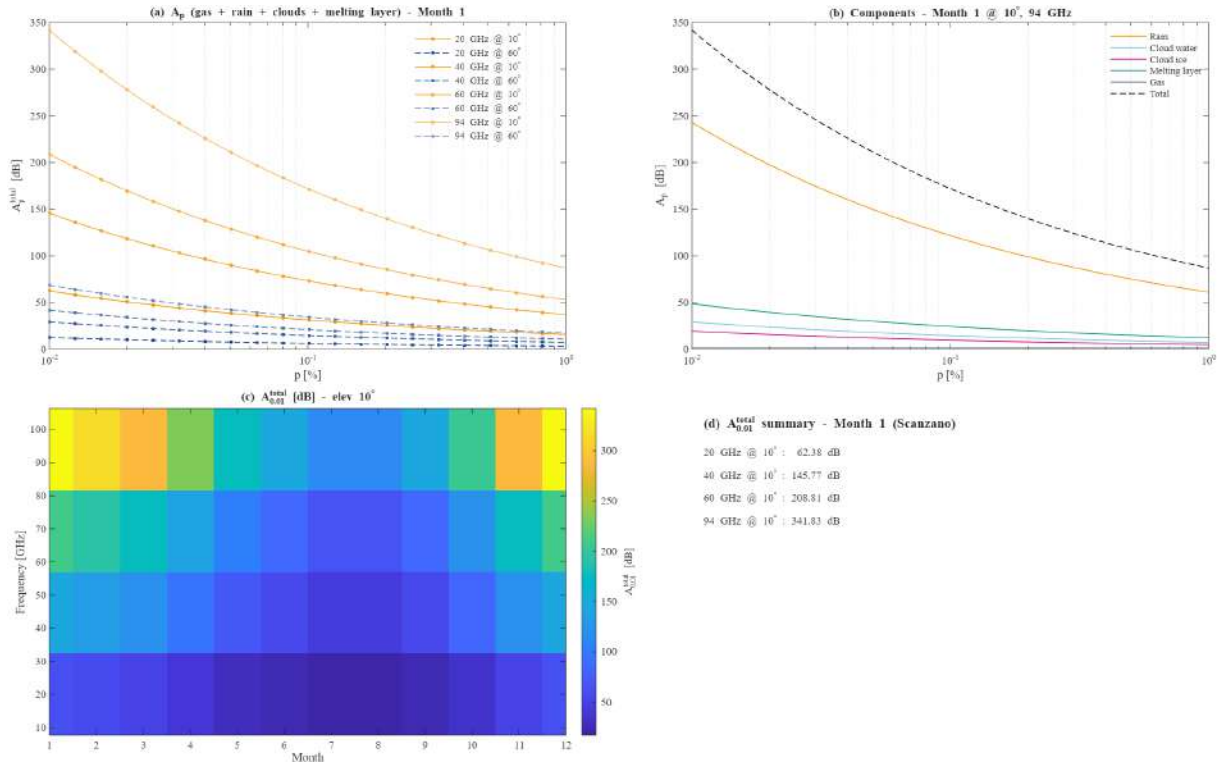


Figure A9.16. Slant-path attenuation statistics and $A_{0.01}$ climatology for Scanzano (PA), Sicily (560 m a.s.l.). Panel (a) shows the exceedance curves A_p ($p \in [10^{-2}, 1]\%$) at 10° and 60° elevation for 20–94 GHz, highlighting the strong frequency dependence and the enhancement at low elevation due to the increased slant path $L_{eff} = h_R / \sin(\theta_{el})$. Total attenuation includes gaseous absorption, rain, cloud liquid and ice water, and a melting-layer contribution. Panel (b) reports the component decomposition at 94 GHz and 10° , indicating dominance of rain and melting-layer effects, with secondary but non-negligible gaseous and cloud contributions. Panel (c) presents the monthly climatology of $A_{0.01}^{total}$ at 10° , showing the expected seasonal cycle under Mediterranean conditions, while panel (d) reports the January reference value used for the exceedance statistics.

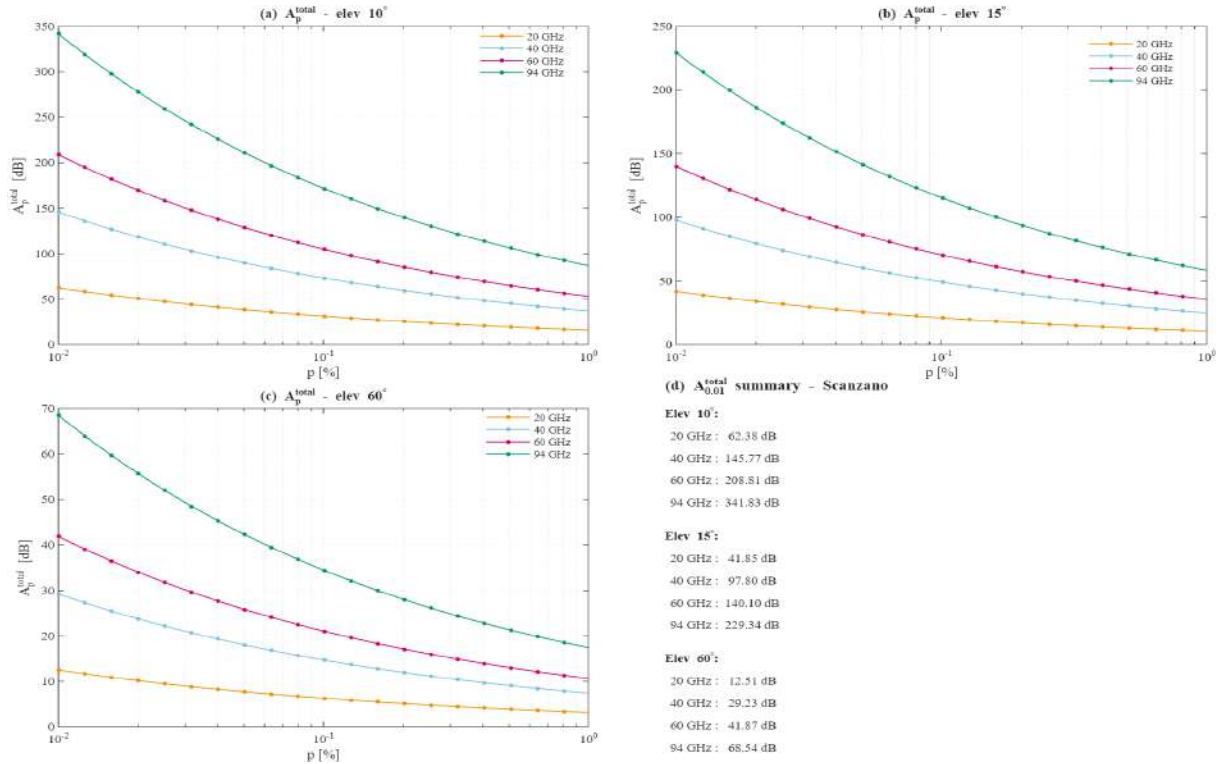


Figure A9.17. Exceedance attenuation curves A_p for three elevation angles (10° , 15° , 60°) at Scanzano (PA), Sicily (560 m a.s.l.), including gaseous absorption (ITU-R P.676-13), rainfall attenuation (ITU-R P.838-3), cloud-liquid and cloud-ice contributions, and melting-layer enhancement. Panels (a–c) report the total attenuation A_p^{total} for $p \in [10^{-2}, 10^0]\%$ and four frequencies (20–94 GHz). The curves quantify the attenuation exceeded for a fraction $p\%$ of an average year, following the ITU-R scaling law $A_p = A_{0.01} \times (p/0.01)^{-\beta}$, with $\beta \approx 0.3$. Lower elevation angles (10° and 15°) exhibit significantly larger slant-path lengths $L_{eff} = h_R / \sin(\theta_{el})$ and thus stronger fading, especially above 40 GHz. Panel (d) summarizes the corresponding $A_{0.01}^{total}$ values for all frequencies and elevation angles, showing the dominance of rain and melting-layer contributions in the Ka-, V-, and W-band regimes.

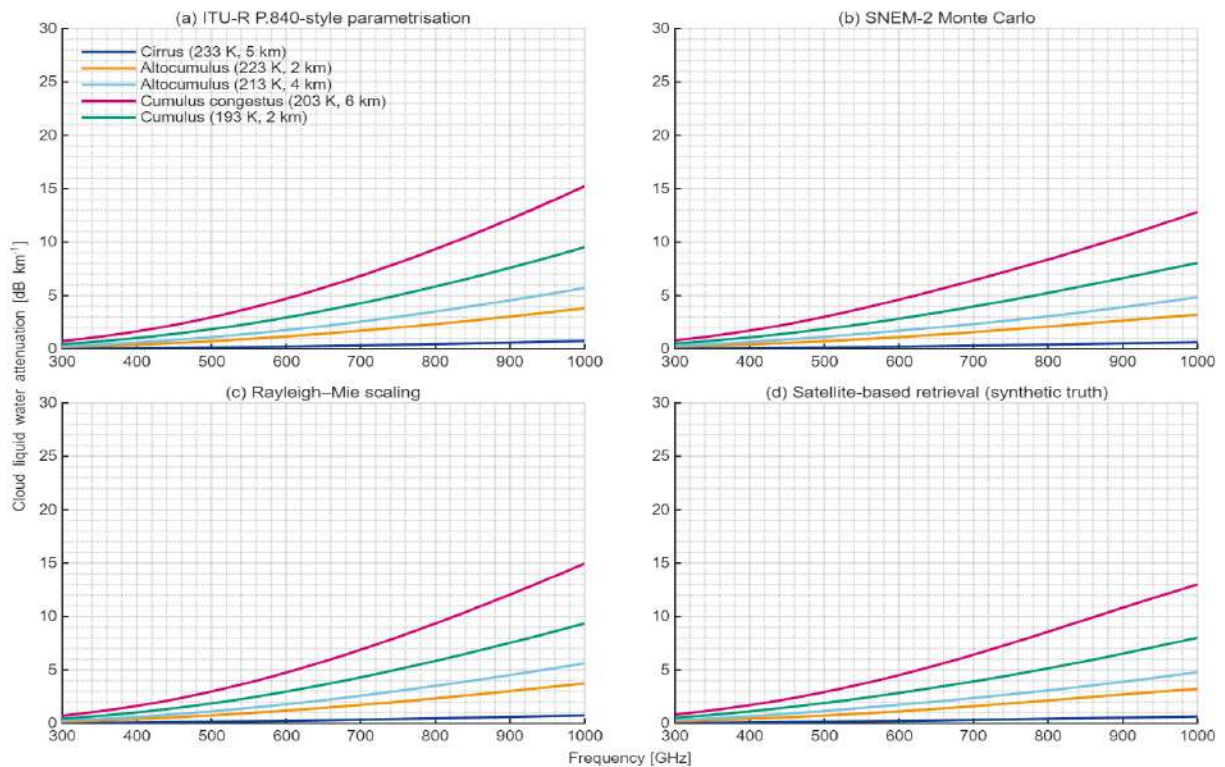


Figure A9.18. Cloud liquid-water specific attenuation (γ_c) increases monotonically with frequency from 300 to 1000 GHz, with larger values for deeper or more liquid-rich clouds. Panels (a–d) compare four modelling approaches. The ITU-R P.840-style parametrization (a) provides a low-order approximation but shows a frequency-dependent bias. The SNEM-2 Monte Carlo solver (b) closely follows the physically based reference within $\pm 3-5\%$. The Rayleigh–Mie scaling (c) exhibits a steeper spectral dependence due to the transition from Rayleigh absorption to Mie scattering above ~ 200 GHz. The satellite-based synthetic retrieval (d) is used here as a reference truth. All models reproduce the expected dependence on LWC and the increase from ~ 0.05 dB km⁻¹ (Cirrus) to $\sim 25-30$ dB km⁻¹ (Cumulus-congestus) at 1 THz.

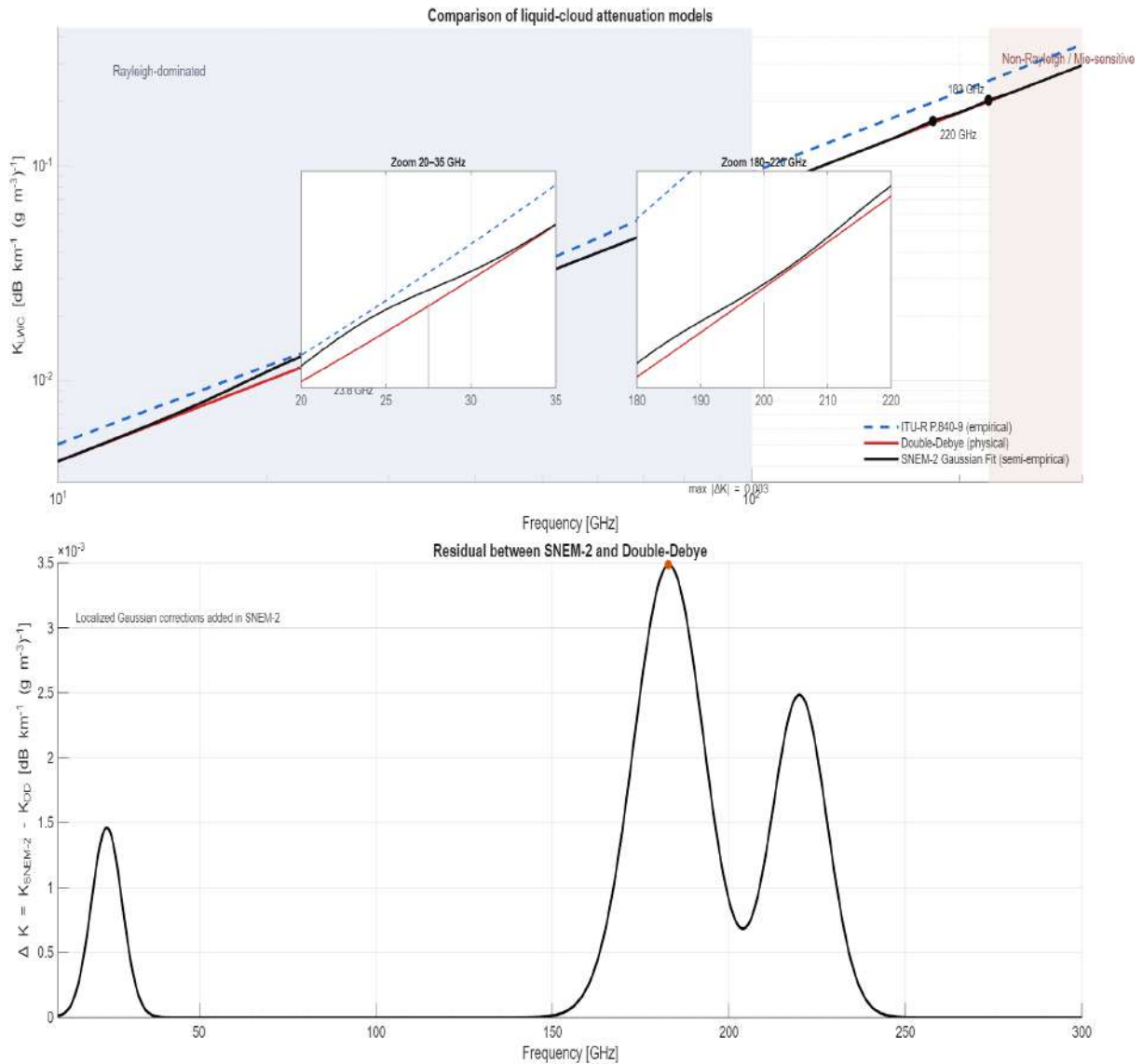


Figure A9.19. Comparison of liquid-cloud attenuation models.Top: frequency-dependent specific attenuation per unit liquid water content $K_{LWC}(f)$ from ITU-R P.840-9 (empirical), the physical Double-Debye dielectric model, and the semi-empirical SNEM-2 parameterization. Shaded regions indicate the Rayleigh-dominated regime (<100 GHz) and the non-Rayleigh/Mie-sensitive range (>200 GHz). Insets provide zoomed views over 20–35 GHz and 180–220 GHz, highlighting the localized Gaussian corrections embedded in SNEM-2.Bottom: residual $\Delta K = K_{SNEM-2} - K_{DD}$, showing three bounded corrections (≈ 23.8 GHz, 183 GHz, 220 GHz) that emulate fine-scale absorption features while preserving overall physical consistency.

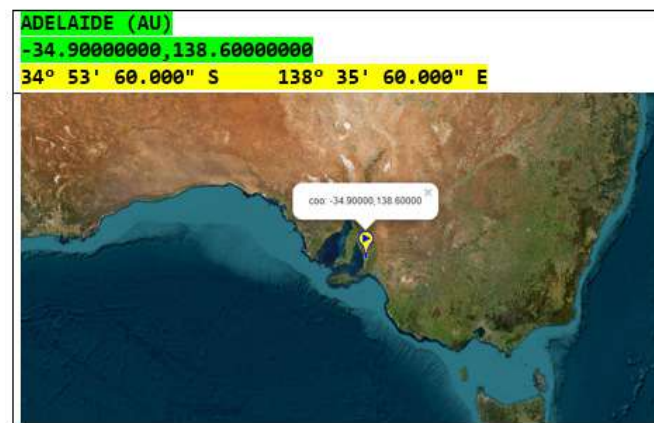


Figure A9.20. Geographical locations of observational sites used in Remote Sensing studies. The figure shows satellite imagery of Scanzano (Italy), Adelaide (Australia), along with their respective coordinates. These sites are strategically chosen to analyze cloud attenuation and atmospheric conditions in diverse climatic regions. **Credits:** Paolo Cicolin, SNEM-2 Framework, 2025.

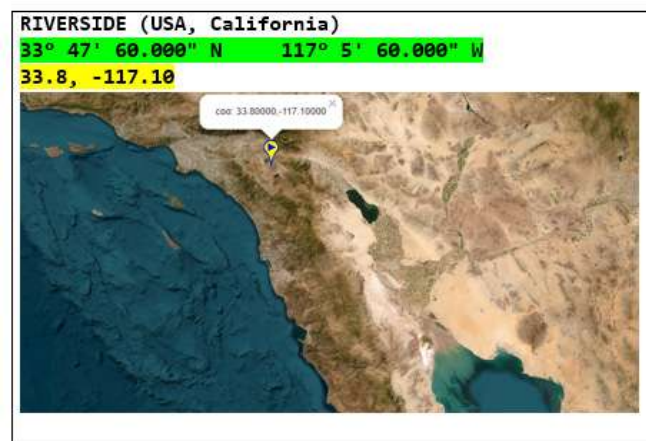


Figure A9.21. Riverside site. **Credits:** Paolo Cicolin, SNEM-2 Framework, 2025.



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Chapter 10.A

Appendix 10.A — Figures and Tables from Chapter 10

10.A.1 Appendix 10.A — Figures

Consequently, all global atmospheric maps presented in this work—including the tropospheric slant path delay distribution shown in **Figure 10.15**—are the direct outcome of an independent and optimized MATLAB development, designed to ensure scientific robustness, numerical reliability, and operational applicability across a broad range of geodetic positioning, GNSS atmospheric correction, and Earth–space propagation studies [1,3,56].

10.A.2 Appendix 10.B — Tables

Season	ZTD_{min} [m]	ZTD_{med} [m]	ZTD_{max} [m]	ZWD_{med} [mm]
DJF	2.302	2.367	2.418	250.4
MAM	2.296	2.345	2.374	310.7
JJA	2.216	2.262	2.29	323.6
SON	2.23	2.29	2.335	266.5

Table A10.3. Seasonal means (DJF / MAM / JJA / SON)

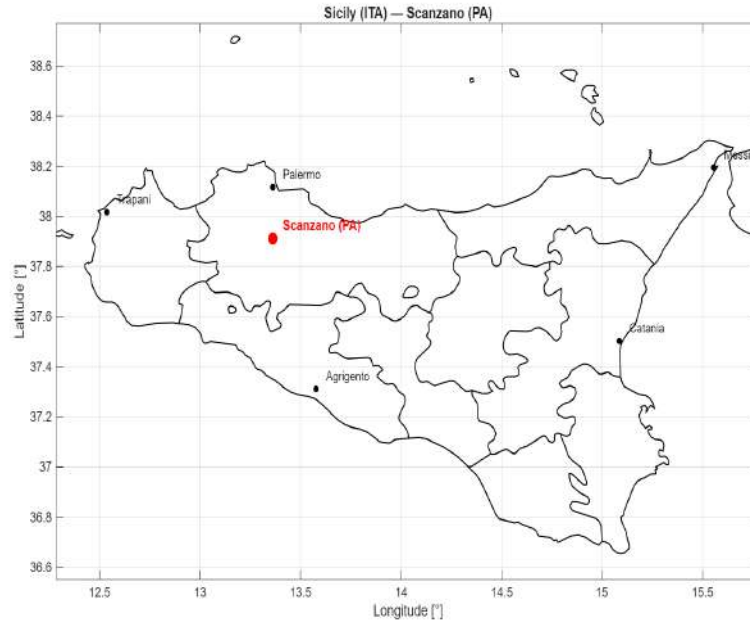


Figure A10.1. Geographical overview of Sicily highlighting the location of the Telespazio ground station in Scanzano (PA). The map depicts the island's coastline and major urban centers, with emphasis on the position of the Scanzano site (37.9133° N, 13.3619° E), located in inland Sicily at an elevation of approximately 560 m a.s.l. in the Piana degli Albanesi area. **Credits:** Paolo Cicolin, SNEM-2 framework.

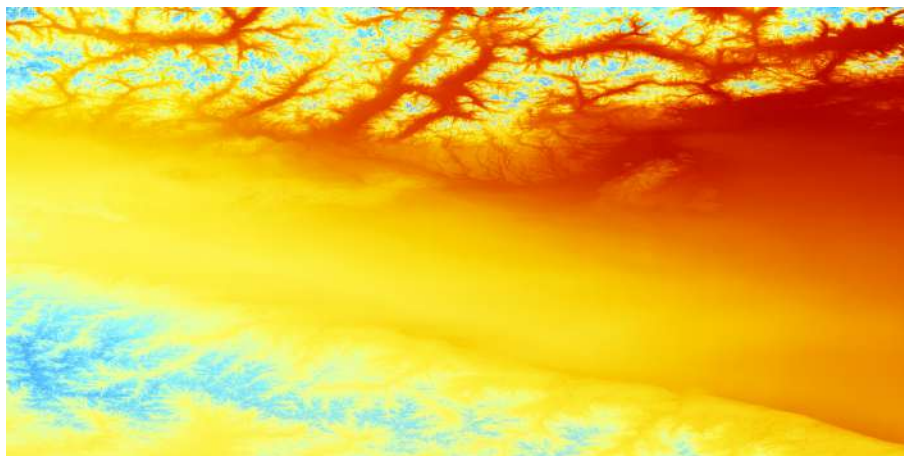


Figure A10.2. Illustrative visualization of the perturbation pressure field (P) obtained from WRF model output. The figure is shown solely as a qualitative first-look diagnostic, produced using Panoply to illustrate the typical spatial variability of the simulated field. This visualization is not intended as a cartographic representation and is not used for quantitative analysis, which is based exclusively on the numerical model output.

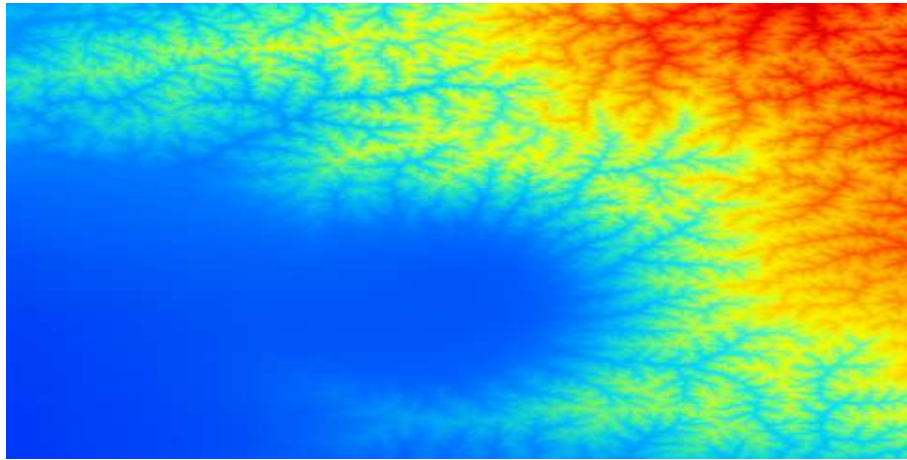


Figure A10.3. Base-state perturbation geopotential field (PHB) from WRF output, expressed in $m^2 s^{-2}$. The spatial variability of PHB highlights pronounced gradients associated with background orography and atmospheric structure, providing a diagnostic indicator of regions subject to enhanced dynamical forcing and vertical stratification. The visualization is shown solely for qualitative illustration and is not used for quantitative analysis.

Pressure related variables	Geopotential related variables	Meteorological variables
Perturbation Pressure (P, Pa)	Perturbation geopotential ($PH, m^2/s^2$)	Air temperature ($T2, K$)
Base-state Pressure (PB, Pa)	Base-state geopotential ($PHB, m^2/s^2$)	Specific humidity ($Q2, kg kg^{-1}$)
Total pressure ($p = P + PB, Pa$)	—————	—————
Surface pressure ($PSFC, Pa$)	—————	—————

Table A10.1. Key meteorological and geophysical variables used for tropospheric delay computation. The table lists the essential input parameters, derived from WRF-consistent model outputs, and grouped by physical category. These variables constitute the fundamental dataset required to reconstruct Zenith Hydrostatic Delay (ZHD) and Zenith Wet Delay (ZWD) fields through vertical integration of atmospheric refractivity.

Symbol	Description	Unit
ZNU	Eta values on half (mass) levels	—
ZNW	Eta values on full (w) levels	—
T	Perturbation potential temperature	K
R_t	Mean Earth radius	km
g	Mean gravitational acceleration	ms^{-2}

Table A10.2. Auxiliary WRF variables employed in this study to ensure a physically consistent transformation of the model's terrain-following vertical coordinate into pressure, temperature, and geopotential height profiles required for atmospheric propagation and delay modelling.

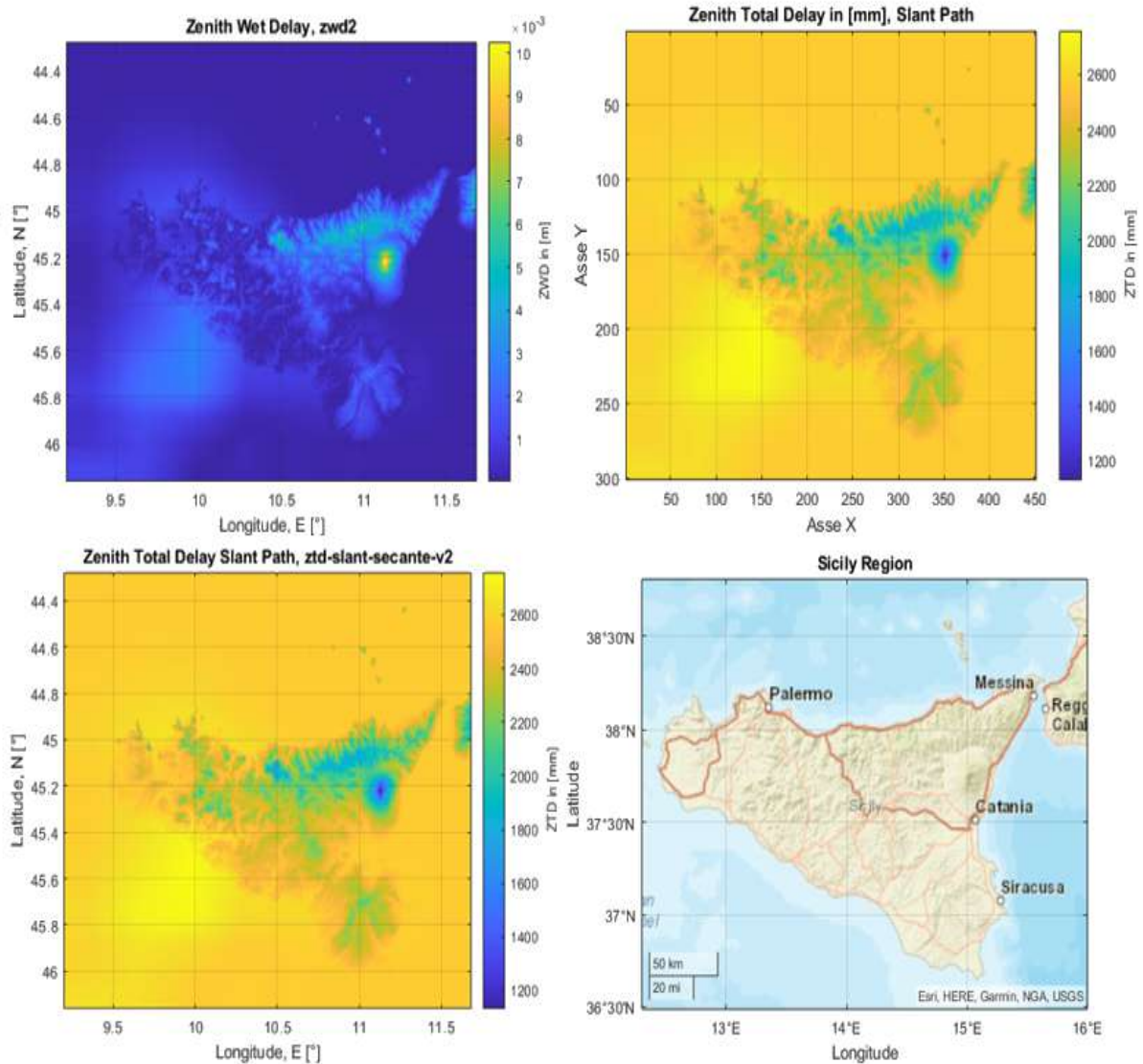


Figure A10.4. Multi-panel visualization of tropospheric delay products over Sicily for a representative case study (15 May 2023, 15:15 UTC). The upper-left panel shows the Zenith Wet Delay (ZWD), highlighting localized enhancements associated with elevated atmospheric water-vapor content. The upper-right panel reports the Zenith Total Delay (ZTD) along the slant path, illustrating spatial gradients resulting from the combined hydrostatic and wet contributions. The lower-left panel presents the elevation-corrected slant-path ZTD, emphasizing the amplification of total delay at low elevation angles. The lower-right panel provides a geographical reference of the Sicilian region for contextual purposes only. The figure was reconstructed from the original model outputs for layout optimization and visual clarity; no data rescaling or numerical manipulation was applied. All visualizations were generated by the author within the framework of research and dissemination activities conducted at Telespazio S.p.A. Credits: SNEM2-Framework.



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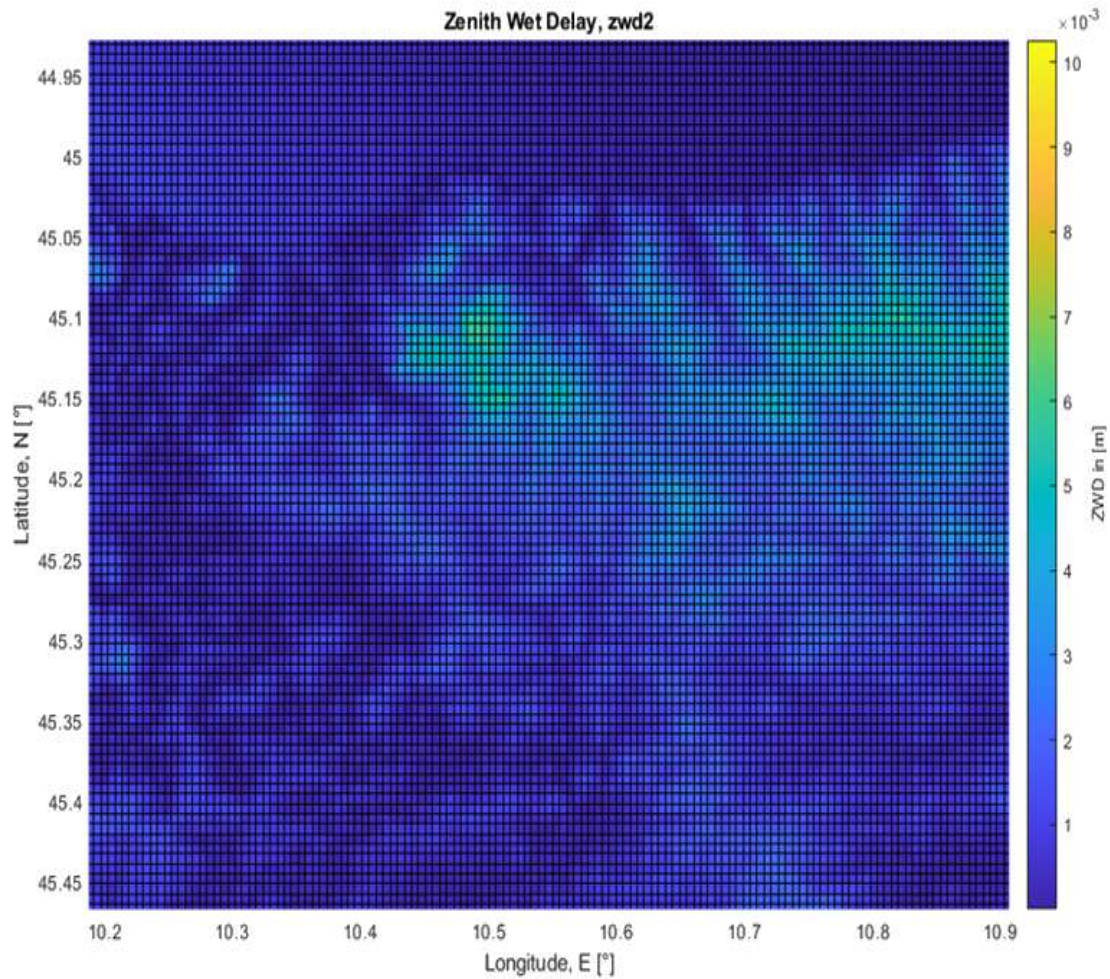


Figure A10.5. Zenith Wet Delay (ZWD) field over the study area for a representative case (15 May 2023, 15:15 UTC). The map illustrates the spatial variability of the wet component of tropospheric delay, expressed in metres, as derived from the ERA-5-based processing workflow. Localized ZWD enhancements reflect regions of increased atmospheric water-vapour content, while smoother gradients indicate broader moisture distribution patterns. The visualization is provided for qualitative interpretation of spatial variability and does not imply additional interpolation beyond the native model resolution. Credits: SNEM2-Framework.

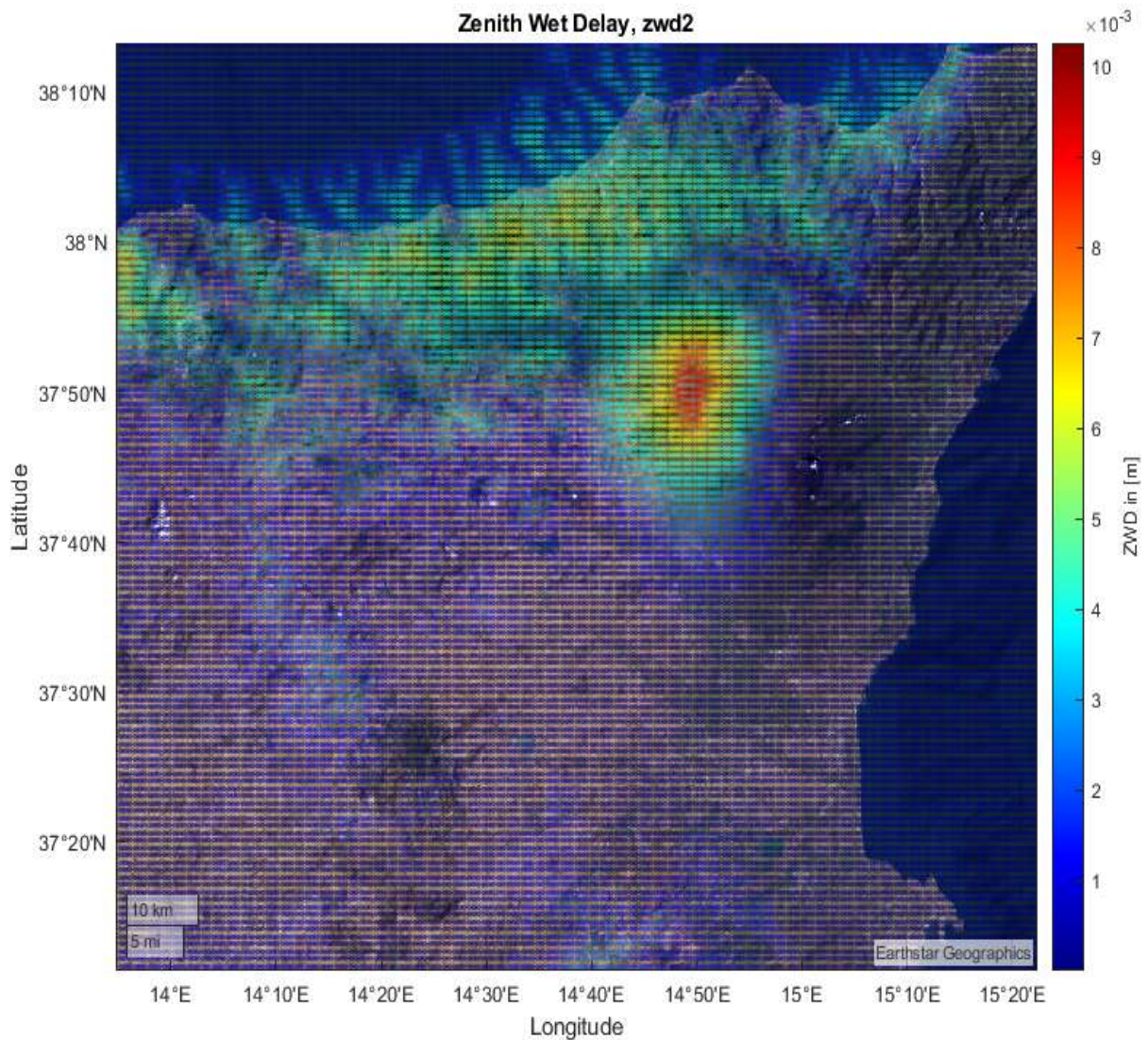


Figure A10.6. The purpose of this visualization is to explicitly illustrate the spatial superposition between the computed Zenith Wet Delay (ZWD) field and the underlying Digital Elevation Model (DEM), rendered through a high-resolution satellite base map. By overlaying the ZWD distribution onto the topographic background, the figure enables a direct visual assessment of the relationship between tropospheric wet-delay anomalies and local orographic features. This combined representation is not intended to provide additional quantitative information beyond the model output, but rather to support the physical interpretation of ZWD spatial patterns in relation to terrain-induced lifting, moisture accumulation, and mesoscale atmospheric heterogeneity. Credits: SNEM2-Framework.

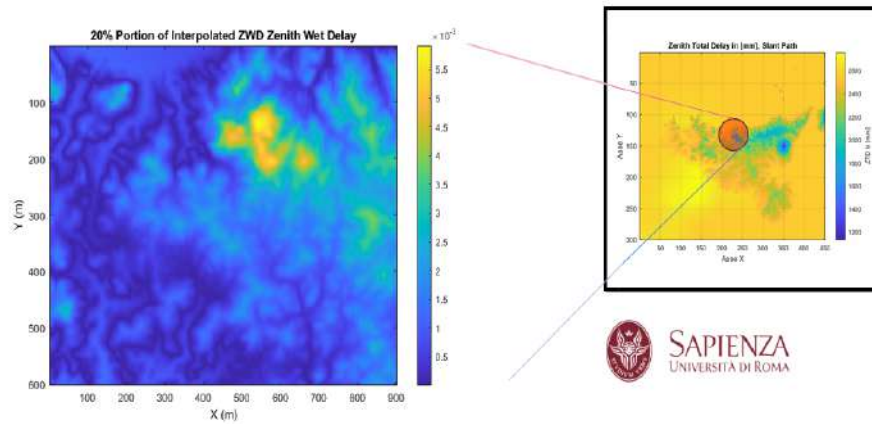


Figure A10.7. Multi-scale refinement of tropospheric delay mapping. Panel (a) shows the regional-scale Zenith Total Delay (ZTD) distribution at kilometre resolution, highlighting a localized delay maximum. Panel (b) presents a high-resolution interpolation of the Zenith Wet Delay (ZWD) over a selected sub-domain (approximately 20% of the original area), resolving fine-scale spatial gradients down to ~ 100 m. The comparison illustrates the importance of scale-adaptive analysis for capturing localized water-vapor inhomogeneities relevant to high-precision geodetic and satellite communication applications. Credits: SNEM2-Framework.

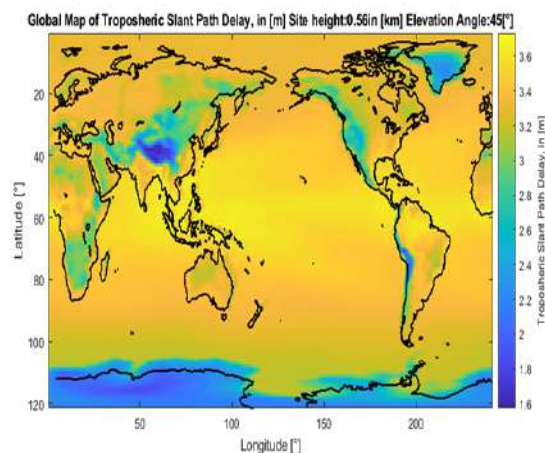


Figure A10.8. Tropospheric Slant Path for 45° in May 15th, at 15:15 UTC. Site height: 0.56 [km] suited for Scanzano site. **Credits:** Paolo Cicolin, SNEM-2 Framework.



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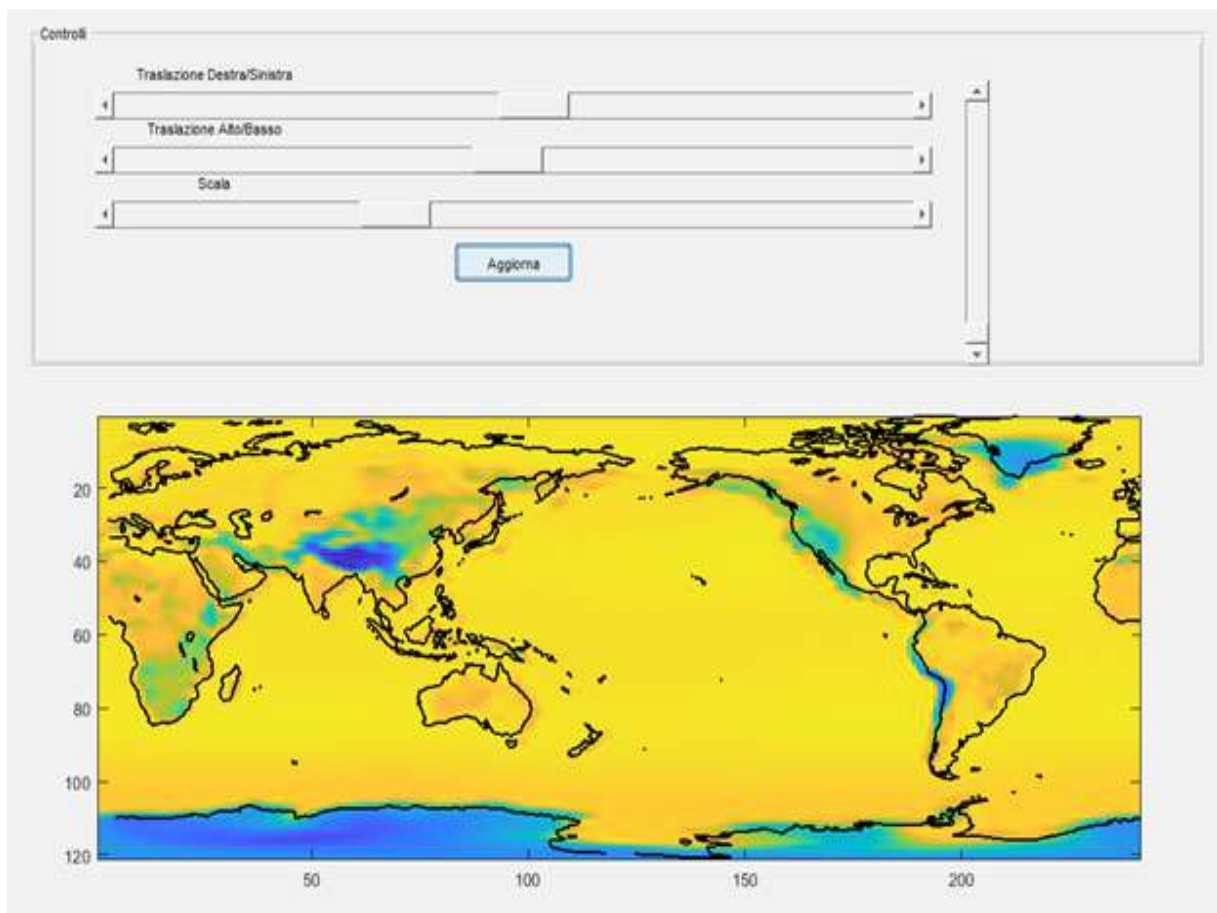


Figure A10.9. Interactive graphical interface for visualization tuning of global tropospheric delay maps. The GUI enables controlled horizontal and vertical translation, as well as uniform scaling of the global delay field, allowing refinement of map alignment and visual consistency without altering the underlying geophysical data. This tool is used exclusively for visualization and interpretability purposes within the SNEM-2 framework. **Credits:** Paolo Cicolin, SNEM-2 Framework.

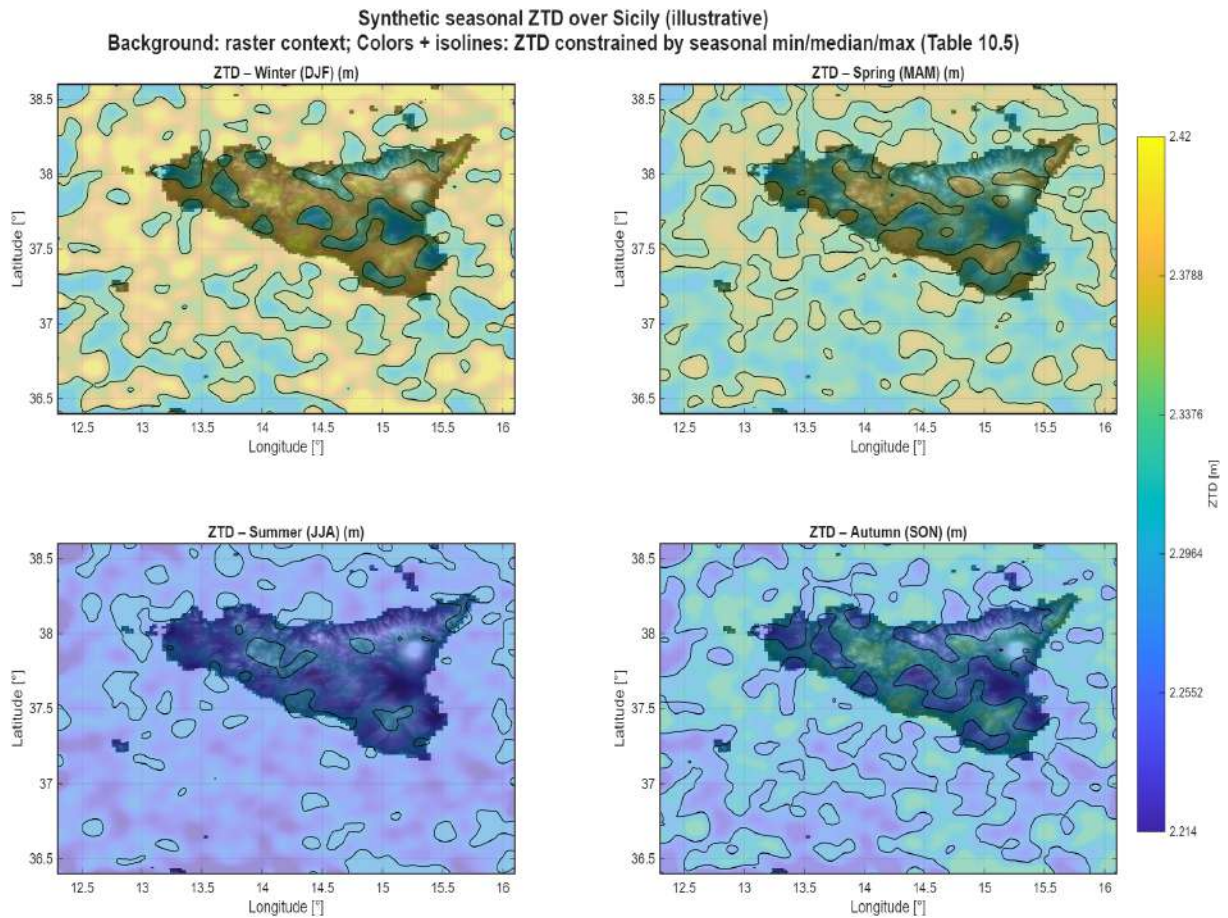


Figure A10.10. Synthetic seasonal distribution of Zenith Total Delay (ZTD) over Sicily for the four climatological seasons: winter (DJF), spring (MAM), summer (JJA), and autumn (SON). The background grayscale raster provides geographical and orographic context only. Colored shading and isolines represent synthetic ZTD fields (in meters), constrained by the seasonal minimum, median, and maximum values reported in **Table 10.5**. The spatial pattern is intentionally kept consistent across seasons to facilitate direct visual comparison, while seasonal variability is expressed through changes in ZTD magnitude. These maps are illustrative and do not represent reconstructed or observed atmospheric fields. **Credits:** Paolo Cicolin, SNEM-2 Framework.

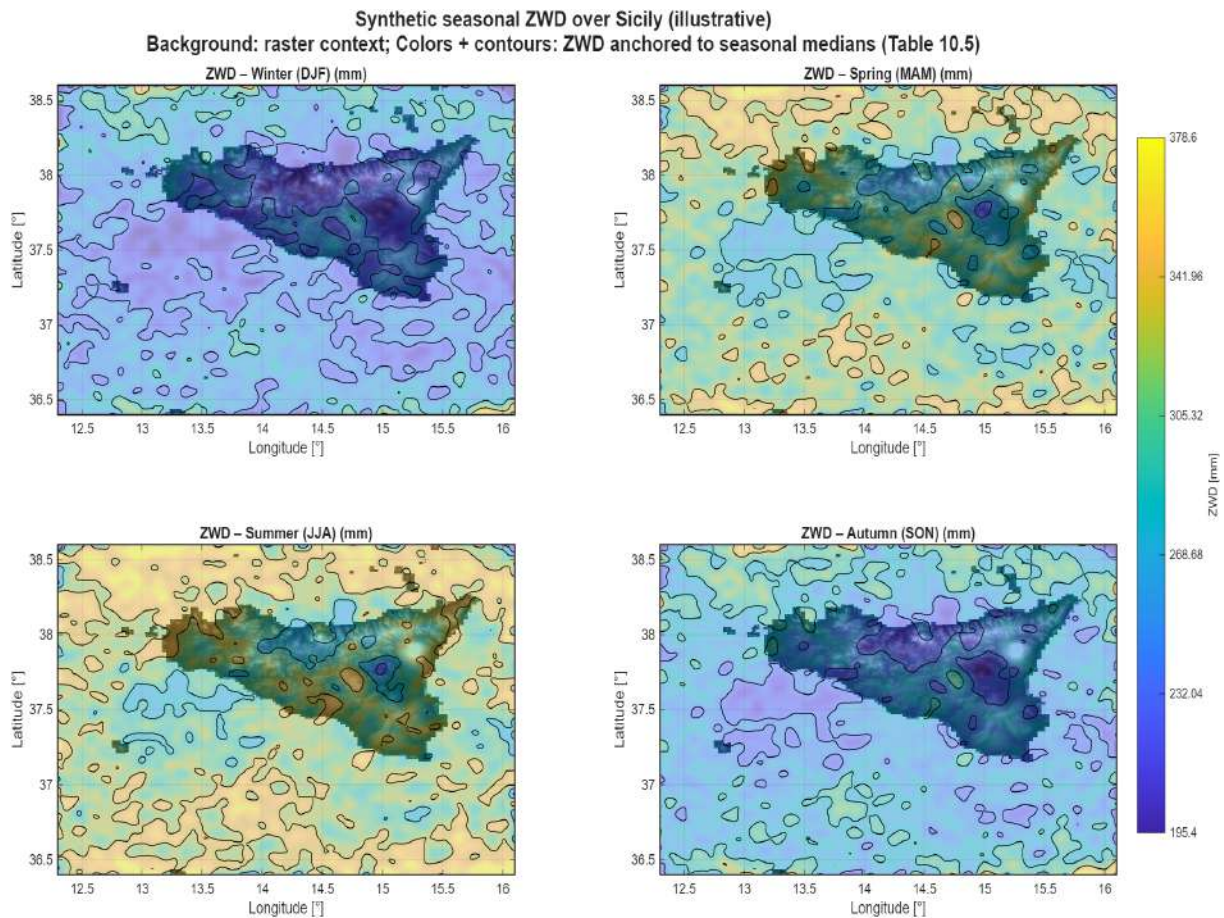


Figure A10.11. Synthetic seasonal ZWD over Sicily. Zenith Wet Delay (ZWD) fields are shown for the four climatological seasons (DJF, MAM, JJA, SON), anchored to representative median values (see Table 10.5). Each panel overlays ZWD contours and color shading on a raster background for geographic context. The color scale spans 195.4–378.6 mm, highlighting seasonal and spatial variability across Sicily, with enhanced summer delays in coastal and lowland regions. **Credits:** Paolo Cicolin, SNEM-2 Framework.

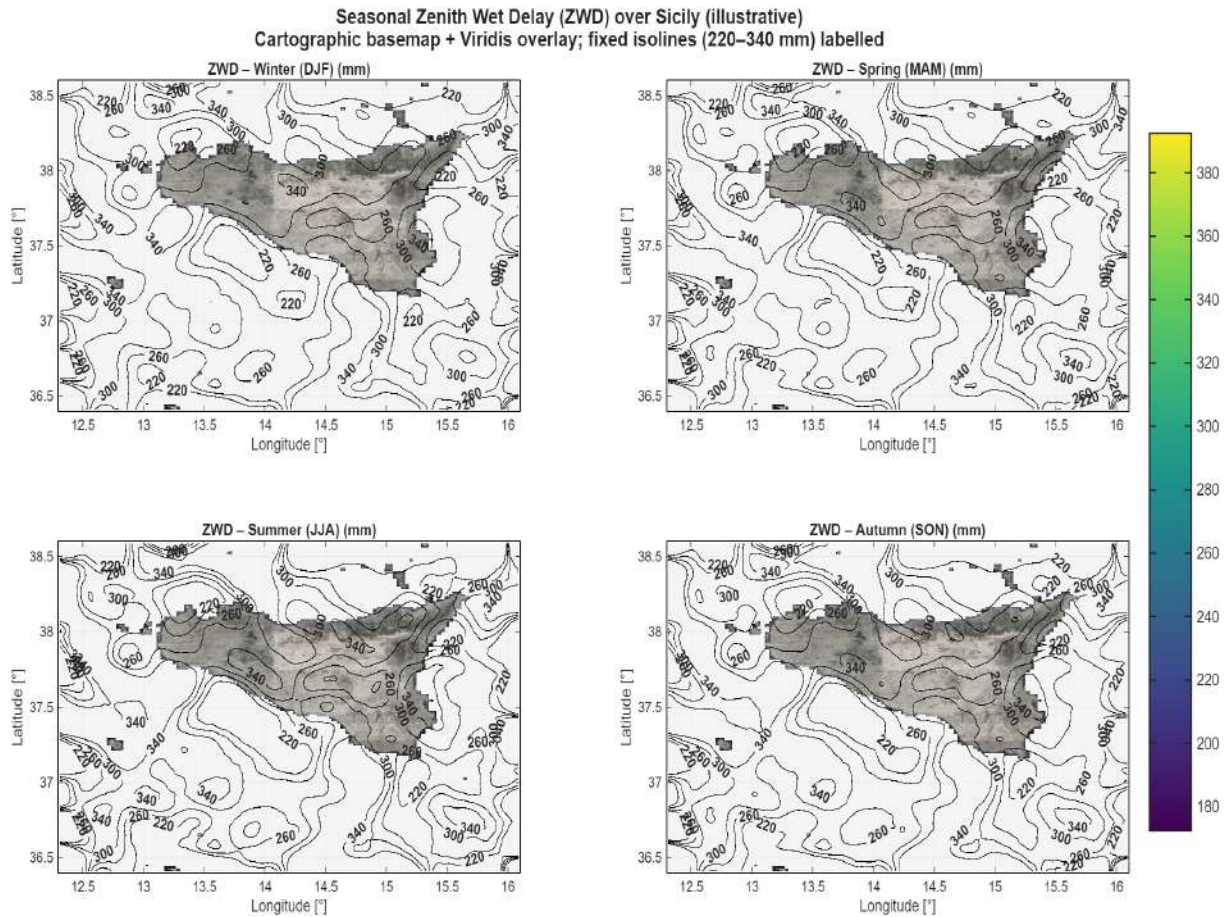


Figure A10.12. Synthetic seasonal distribution of zenith wet delay (ZWD) over Sicily for the four climatological seasons: winter (DJF), spring (MAM), summer (JJA), and autumn (SON). The background raster provides geographical and orographic context only. Colored shading and isolines represent synthetic ZWD fields expressed in millimetres, constrained by the full-domain minimum, median, and maximum ZWD statistics reported in **Table 10.4**. The spatial structure is intentionally preserved across seasons to facilitate direct visual comparison, while seasonal variability is conveyed through changes in ZWD magnitude. These maps are illustrative and do not correspond to reconstructed or observed wet-delay fields. Contour labels are displayed over both land and sea to emphasize the spatial continuity of the synthetic ZWD field; labels over the marine domain are rendered with reduced visual weight to preserve cartographic hierarchy. **Credits:** Paolo Cicolin, SNEM-2 Framework.

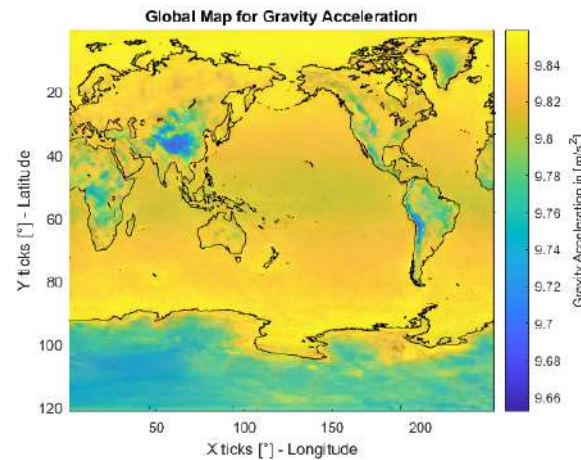


Figure A10.13. Global gravitational acceleration field used in hydrostatic delay modelling. The spatial distribution of gravitational acceleration g is shown on a global grid. Latitudinal dependence and regional variations reflect the combined effects of Earth's rotation, ellipsoidal geometry, and mass heterogeneity. Coastlines are overlaid for geographic reference. **Credits:** Paolo Cicolin, SNEM-2 Framework.

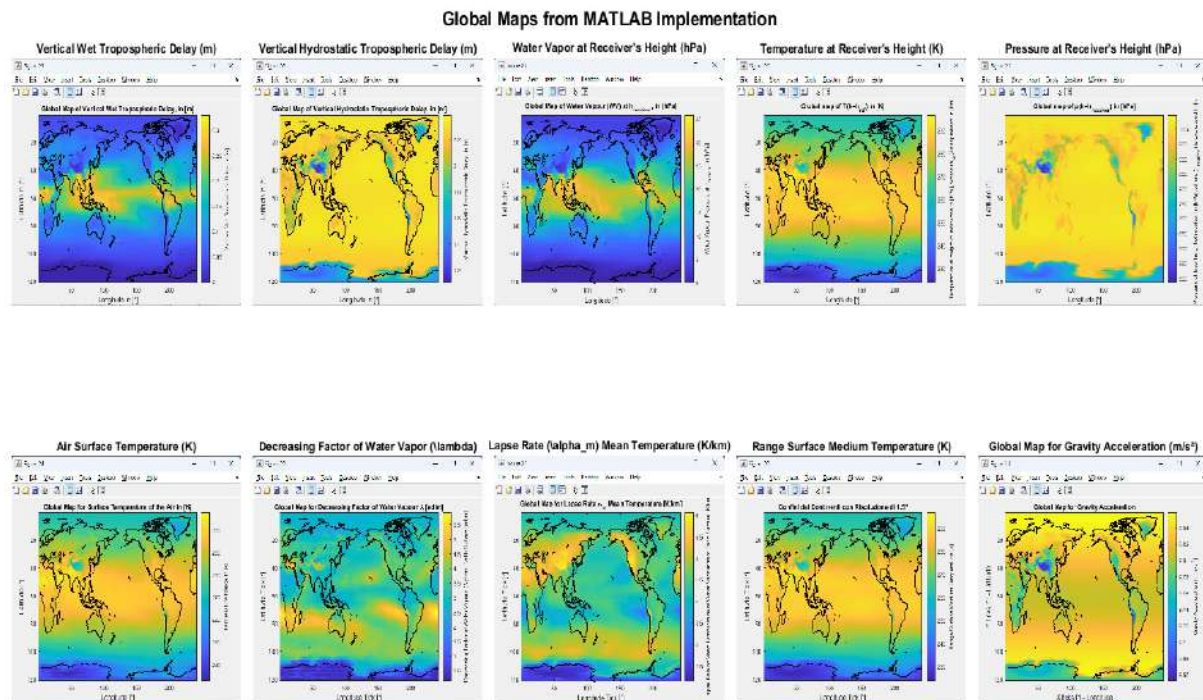


Figure A10.14. Global atmospheric parameter maps generated through a fully custom MATLAB implementation of ITU-R recommendations. The algorithms have been independently coded and adapted to overcome inconsistencies and missing details in the official documentation, ensuring accurate computation of: (a) Vertical Wet Tropospheric Delay (ZWD), (b) Vertical Hydrostatic Tropospheric Delay (ZHD), (c) Water Vapor Pressure at Receiver's Height, (d) Temperature at Receiver's Height, (e) Surface Air Temperature, (f) Water Vapor Decreasing Factor (λ), (g) Mean Temperature Lapse Rate (α_m), and (h) Surface Medium Temperature Range. These improvements allow a complete and reliable global modelling of tropospheric parameters for satellite communication and geodetic applications. **Credits:** Paolo Cicolin, SNEM-2 Framework.



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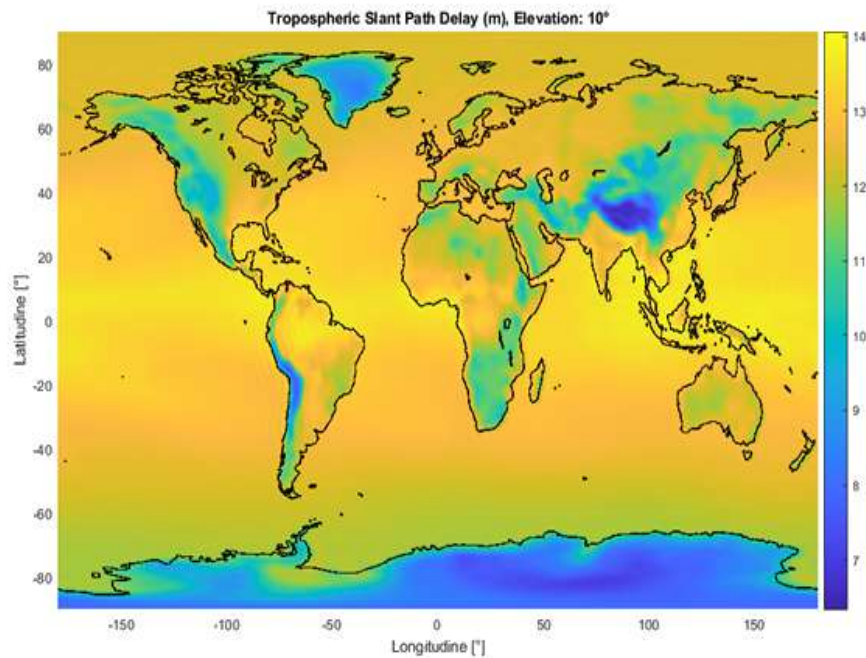


Figure A10.15. Global Tropospheric Slant Path Delay (m) at 10° Elevation. The DOY is 150. This map illustrates the tropospheric slant path delay (SPD) in meters, computed for a 10° elevation angle across the globe. The delay values are derived from atmospheric pressure, temperature, and humidity, accounting for both hydrostatic (ZHD) and wet (ZWD) components. The color scale indicates variations in delay, with higher values over humid and tropical regions and lower values over dry and polar regions. This dataset is essential for satellite-based positioning, geodesy, and remote sensing applications. Credits: Paolo Cicolin, SNEM-2 Framework.

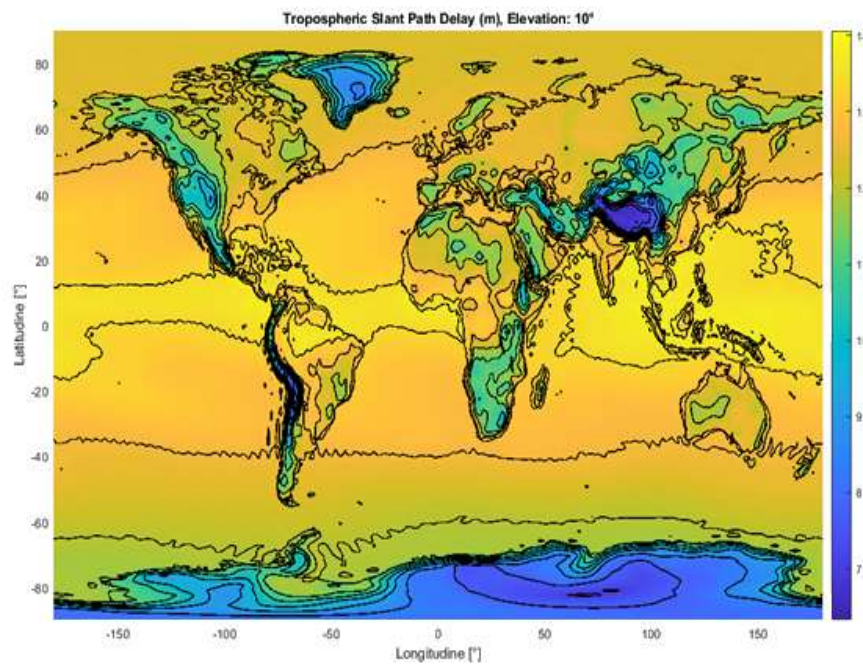


Figure A10.16. Global Tropospheric Slant Path Delay (m) at 10° Elevation with Contour Lines. This map presents the tropospheric slant path delay (SPD) in meters at a 10° elevation angle, overlaid with contour lines to enhance the visualization of regional variations. The delay values are influenced by atmospheric pressure, temperature, and humidity, incorporating both Zenith Hydrostatic Delay (ZHD) and Zenith Wet Delay (ZWD). Higher values are observed in humid regions (e.g., the Himalayas, equatorial zones), while lower values correspond to dry and polar areas. This dataset is critical for satellite-based positioning, geodesy, and atmospheric corrections in Earth observation applications. **Credits:** Paolo Cicolin, SNEM-2 Framework.

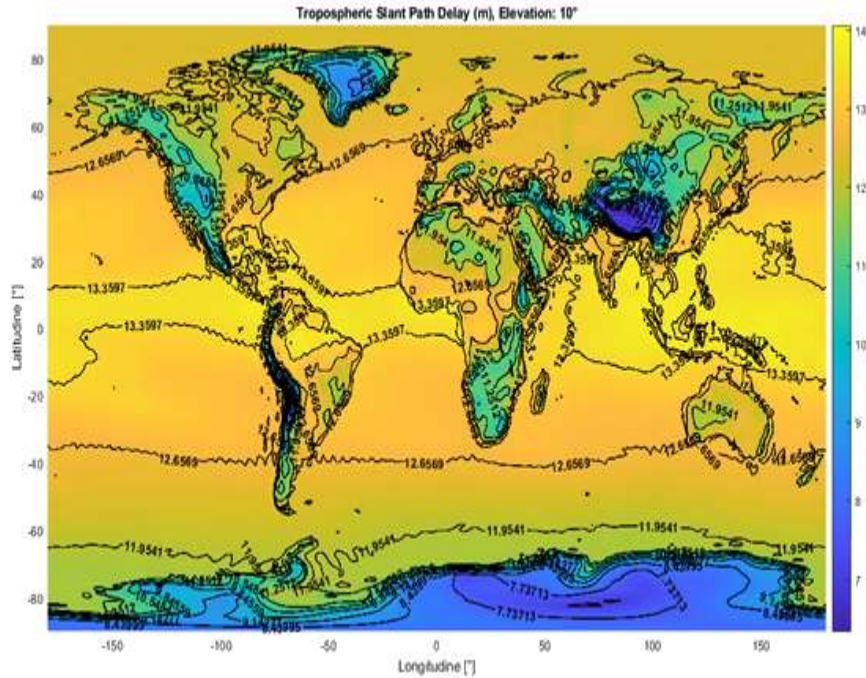


Figure A10.17. Global tropospheric slant path delay (SPD) at an elevation angle of 10°. The map shows the global distribution of SPD expressed in metres. Contour lines with numerical labels indicate constant-delay levels, facilitating quantitative interpretation of spatial gradients. The pattern reflects large-scale climatological variability driven primarily by atmospheric water vapor content and vertical stratification. The results are obtained using the SNEM-2 framework. **Credits:** Paolo Cicolin, SNEM-2 Framework.

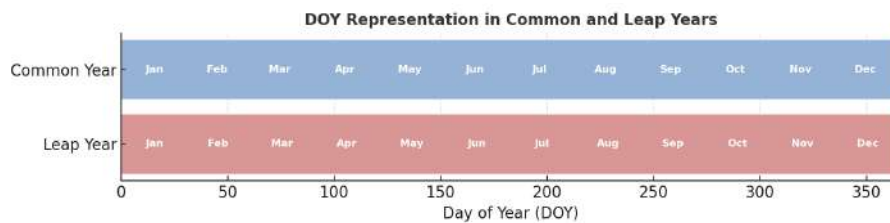


Figure A10.18. Visual representation of the Day of the Year (DOY) for a common year (365 days) and a leap year (366 days). Calendar months are mapped onto the DOY axis to illustrate the one-day shift occurring after February in leap years. Correct DOY handling is essential in atmospheric modelling to maintain temporal alignment between datasets and to avoid systematic errors in time-dependent parameters.

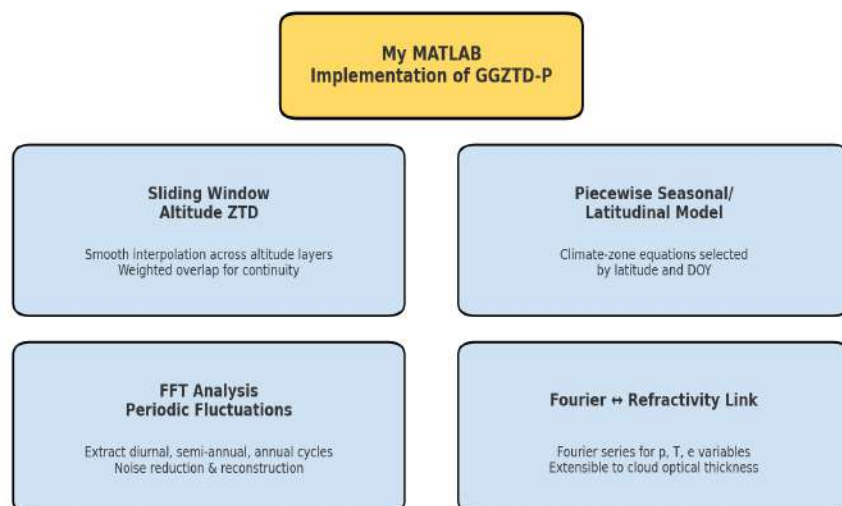


Figure A10.19. Summary of the four computational strategies implemented in my MATLAB version of the GGZTD-P model: (1) sliding window interpolation for altitude-dependent ZTD estimation, (2) piecewise modelling for seasonal and latitudinal variability, (3) FFT-based analysis of periodic fluctuations, and (4) Fourier–refractivity linkage enabling extension to other atmospheric variables.

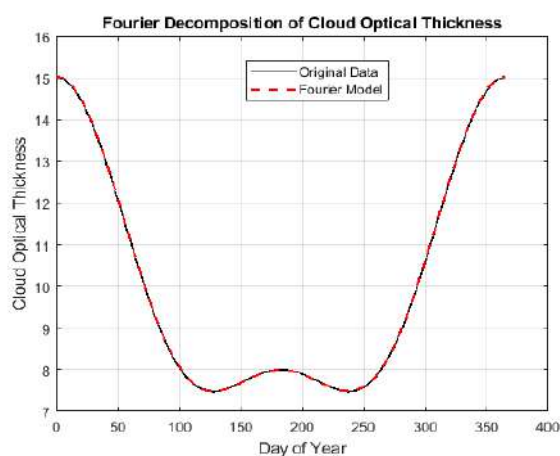


Figure A10.20. Fourier Decomposition of Cloud Optical thickness.

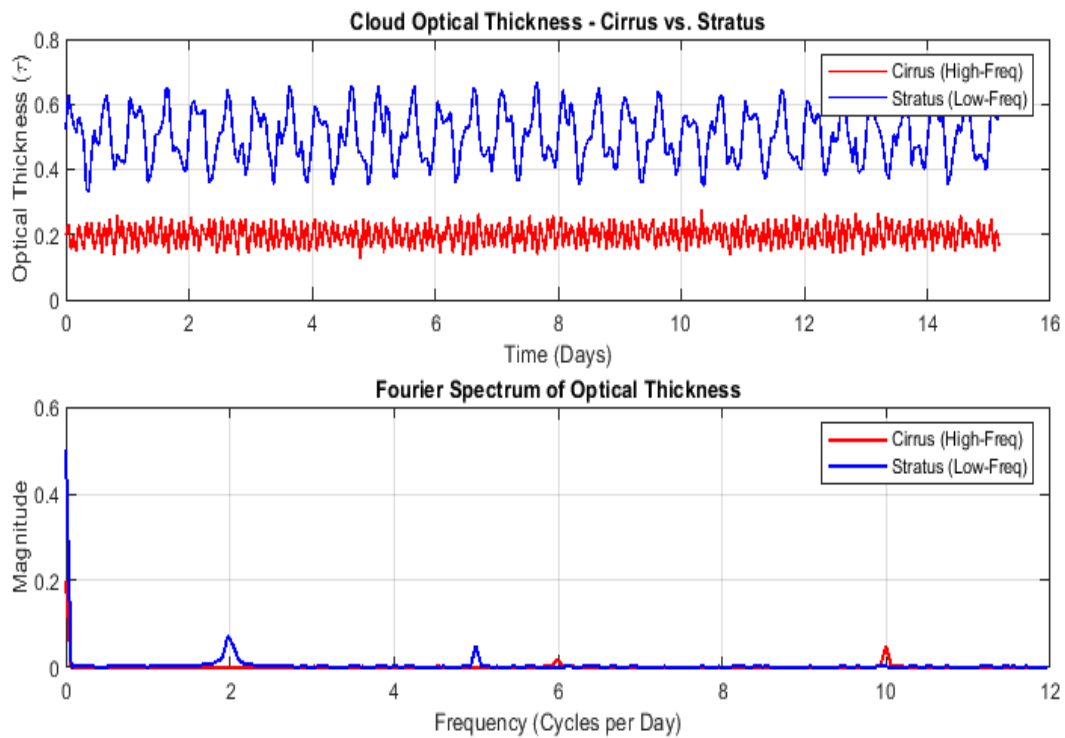


Figure A10.21. Fourier analysis of cloud optical thickness (τ) for cirrus and stratus clouds.

Top panel: temporal variation of τ over a 16-day period. The red curve (Cirrus) exhibits high-frequency fluctuations, characteristic of thin, patchy clouds, while the blue curve (Stratus) shows low-frequency variations, typical of persistent, extended cloud decks. Realistic noise has been added to simulate observational uncertainty. Bottom panel: Fourier spectra of the two time series. The x-axis shows frequency (cycles per day) and the y-axis the spectral magnitude. Cirrus clouds present dominant peaks at higher frequencies (~ 10 cycles/day), whereas stratus clouds peak at lower frequencies (~ 2 cycles/day), confirming their contrasting temporal variability. This spectral signature provides a basis for automated cloud classification and improved climate model parameterization.

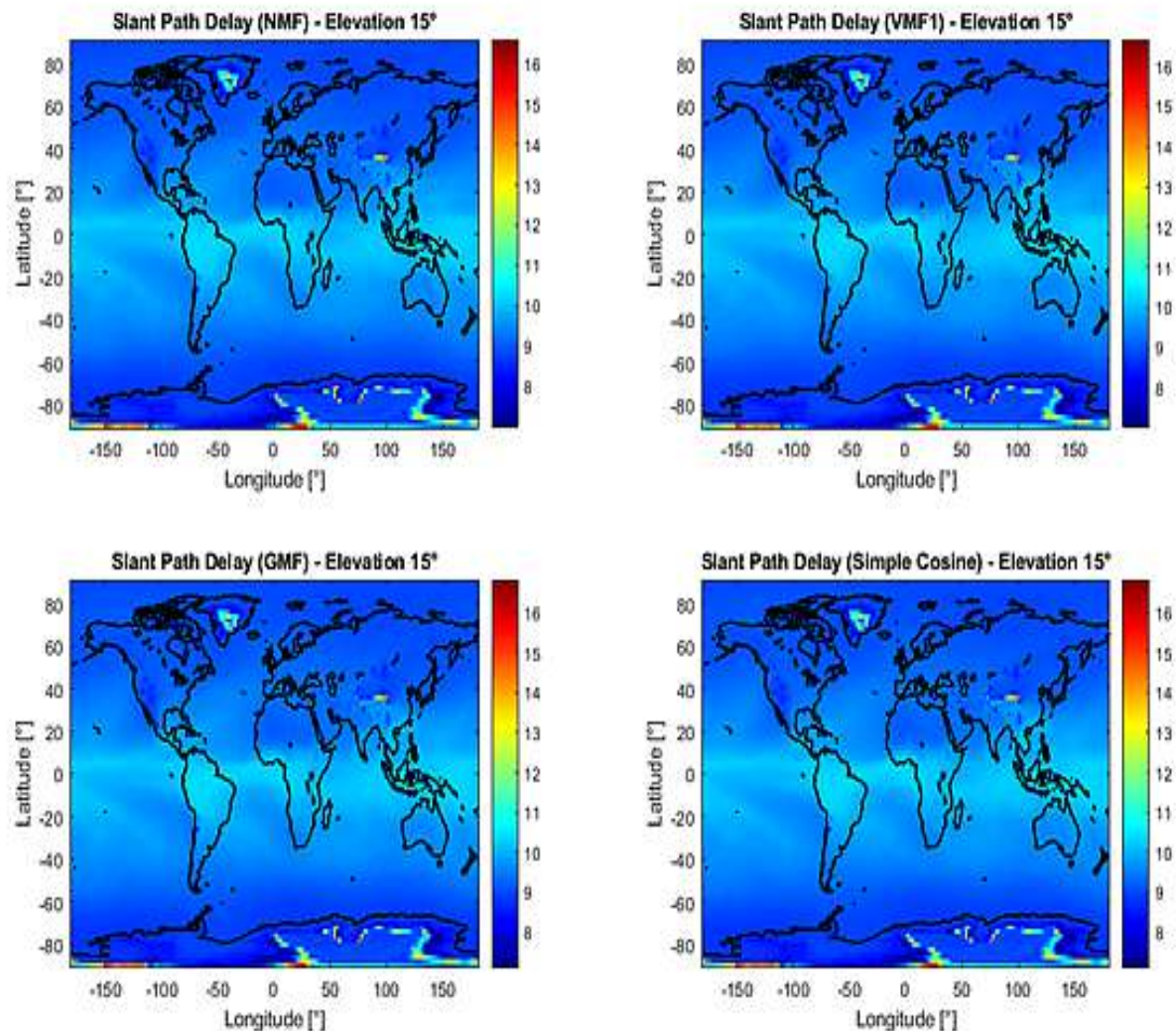


Figure A10.22. Global slant path delay maps at an elevation angle of 15°, computed using the GGZTD-P Model for a selected Day of Year (DOY). Four mapping functions are compared: (top-left) Niell Mapping Function (NMF), (top-right) Vienna Mapping Function (VMF1), (bottom-left) Global Mapping Function (GMF), and (bottom-right) Simple Cosine Model. Delays are expressed in meters and vary from approximately 8 [m] (deep blue) to 16 [m] (red), with maximum values in tropical regions due to increased atmospheric water vapor content. VMF1 shows the most detailed spatial variability as it incorporates near-real-time atmospheric data, while the Simple Cosine Model provides the smoothest, baseline pattern. These maps are part of a dynamic global dataset generated for each DOY, allowing seasonal variability to be captured.

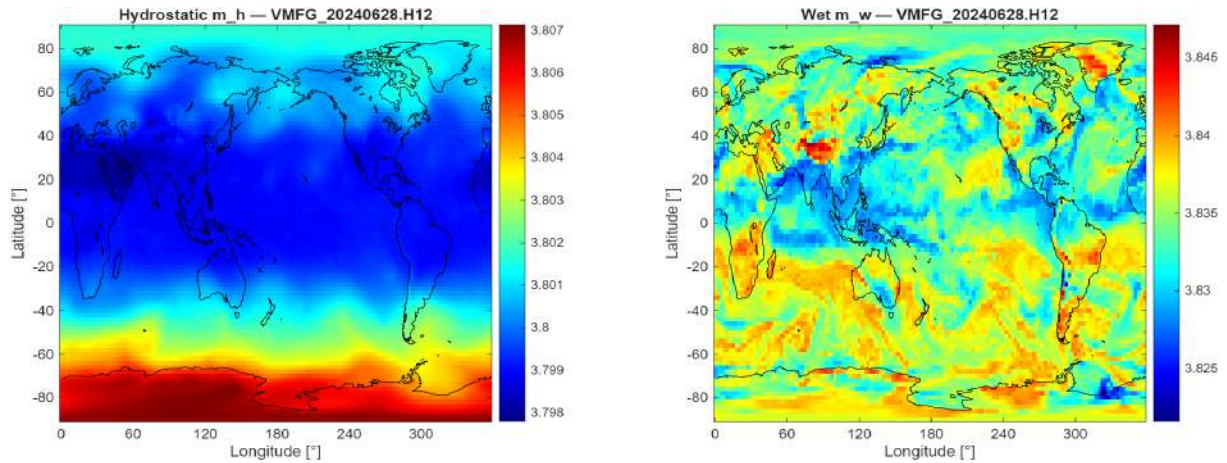


Figure A10.23. Global hydrostatic (left) and wet (right) mapping functions (VMF-G) for 28 June 2024, 06:00 UTC. Hydrostatic values increase toward the poles, while wet values show localized maxima in humid regions and minima in dry subtropical areas.

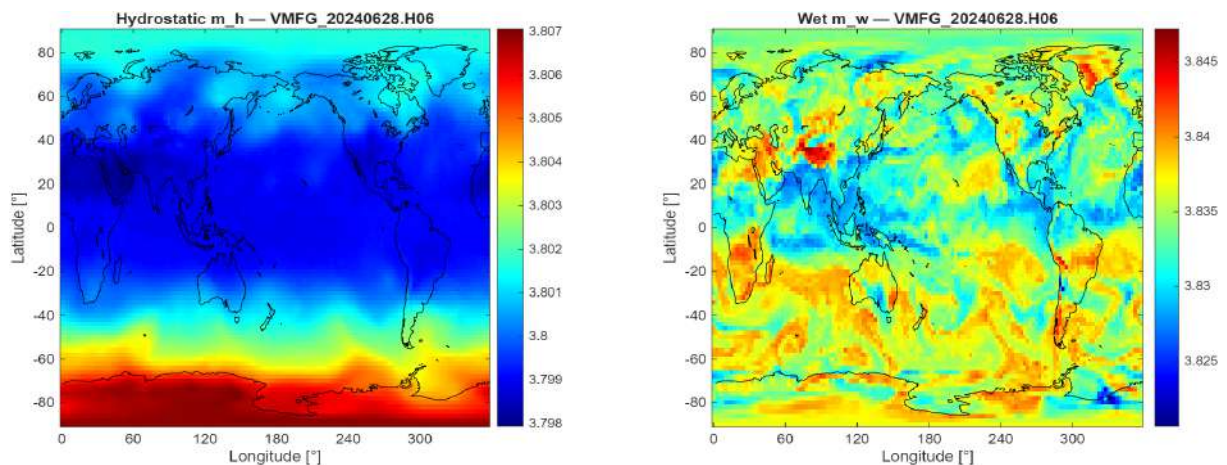


Figure A10.24. Global hydrostatic (left) and wet (right) mapping functions (VMF-G) on 28 June 2024 at 06:00 UTC. Hydrostatic values increase toward the poles, while wet values display strong regional variability, with peaks over humid areas and minima over dry subtropical zones.

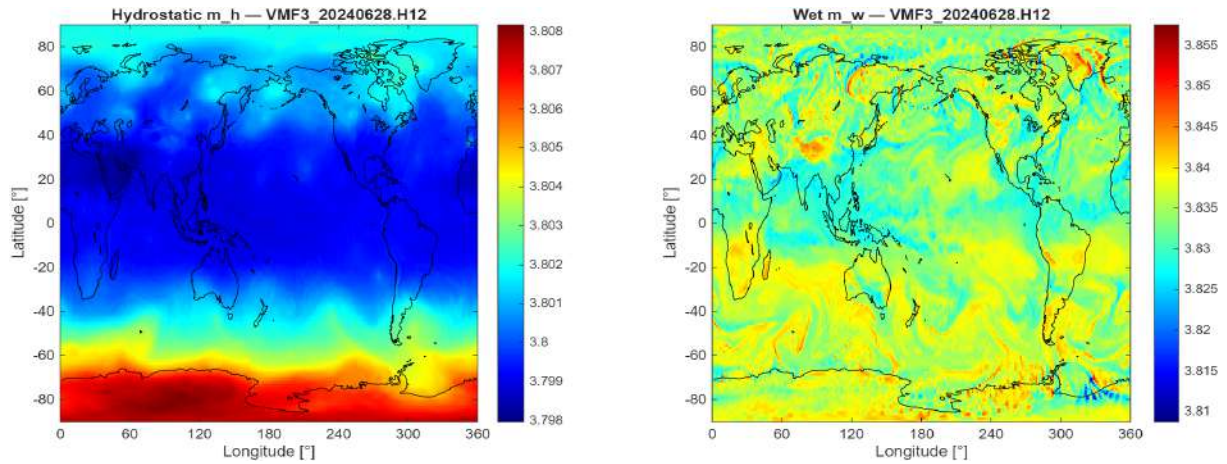


Figure A10.25. Global hydrostatic (left) and wet (right) mapping functions (VMF3) for 28 June 2024 at 12:00 UTC. Hydrostatic values show a clear latitudinal gradient, increasing toward the poles, while wet values reveal finer-scale spatial variability, with maxima in humid regions and minima over dry subtropical areas.

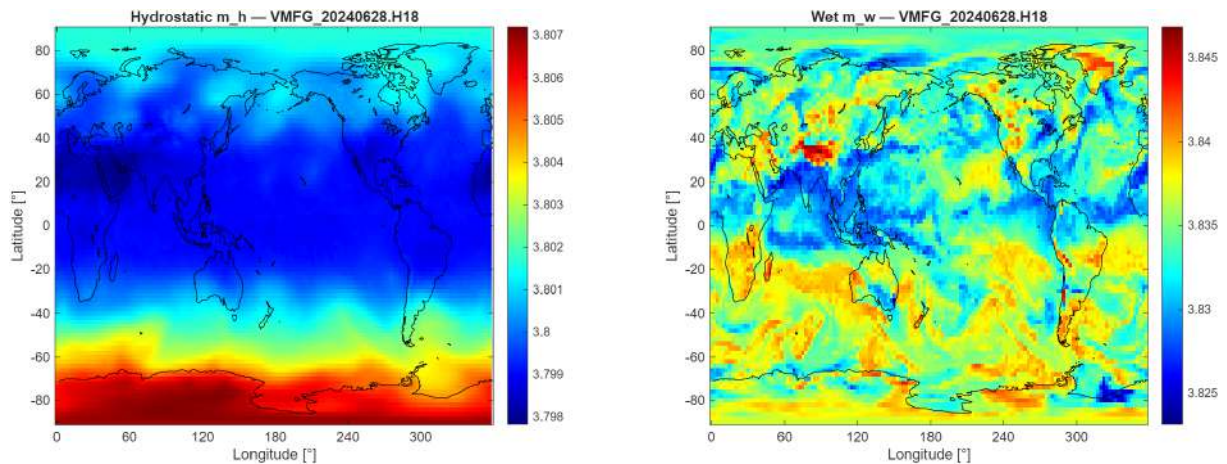


Figure A10.26. Global hydrostatic (left) and wet (right) mapping functions (VMF-G) for 28 June 2024 at 18:00 UTC. Hydrostatic values display a latitudinal gradient with higher values toward the poles, while wet values show pronounced regional variability, with maxima in humid zones and minima in dry subtropical regions.

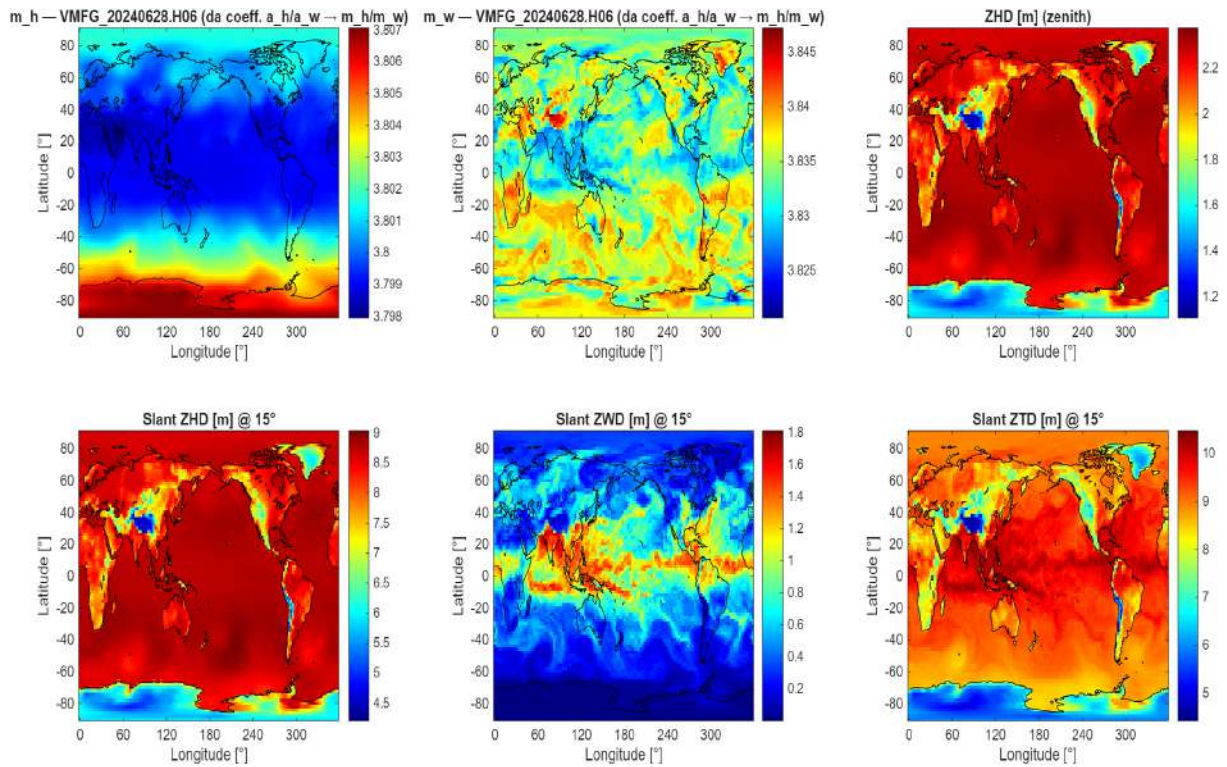


Figure A10.27. Global maps of atmospheric parameters and zenith delays on June 23, 2024. This figure presents six global maps illustrating key variables used in geodetic and atmospheric modeling. The top row includes the spatial distribution of a water vapor coefficient (WFG_20240623_H06), its anomaly field, and the Zenith Hydrostatic Delay (ZHD) in meters. The bottom row shows the Slant ZHD at a 15° elevation angle, the Slant Zenith Wet Delay (ZWD), and the Slant Zenith Total Delay (ZTD), all in meters. These maps highlight the spatial variability of atmospheric delays affecting satellite signal propagation and are essential for refining models of radiowave attenuation and positioning accuracy.

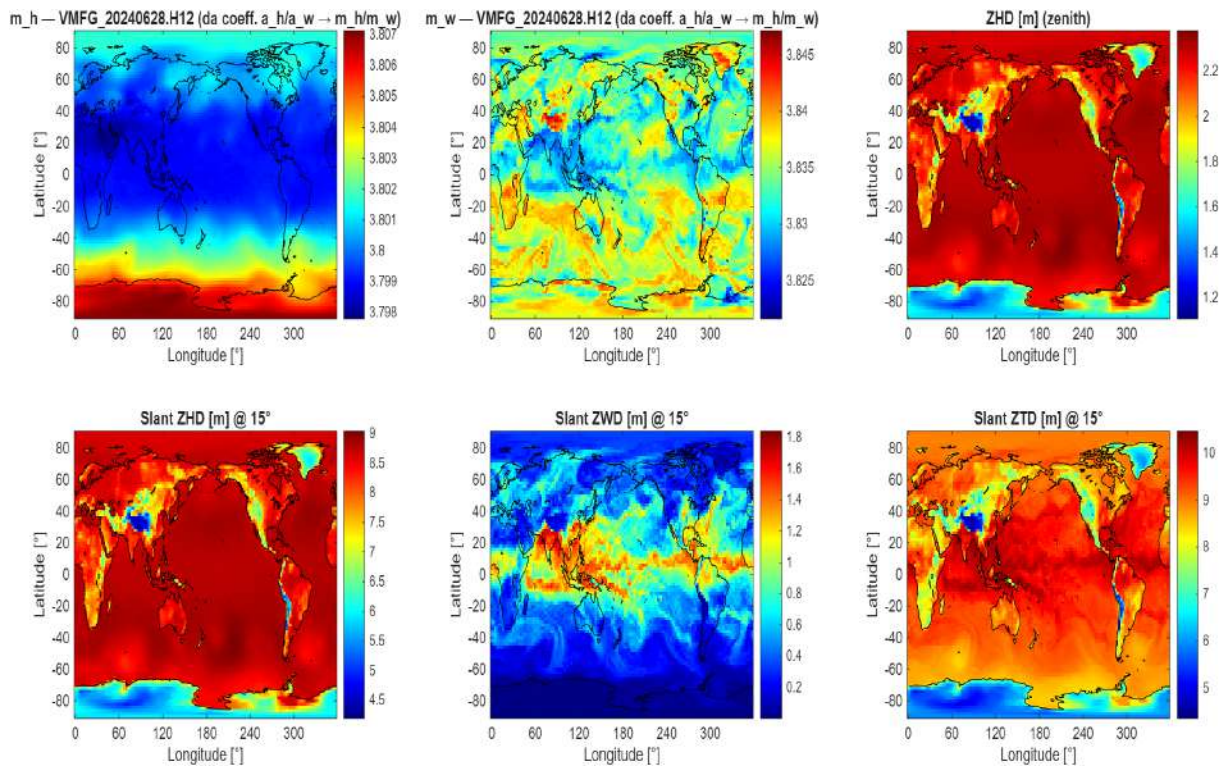


Figure A10.28. Global maps of geophysical and atmospheric parameters derived from WIPFG model outputs (June 28, 2024). This figure presents six global distributions relevant to atmospheric delay modeling and water vapor analysis. The top row includes: (1) the variable m_h from WIPFG_20240628.H12, representing a model coefficient linked to water vapor content; (2) the variable w_w , indicating weighted water vapor; and (3) the Zenith Hydrostatic Delay (ZHD), quantifying dry atmospheric delay. The bottom row shows: (4) Slant ZHD at 15° elevation, reflecting increased delay due to longer signal paths; (5) a WIPFG-derived output field used in SWIFT processing; and (6) Slant Zenith Total Delay (ZTD) at 15°, combining hydrostatic and wet components. These maps support the spatial characterization of atmospheric delays critical for satellite positioning and radiowave propagation studies.

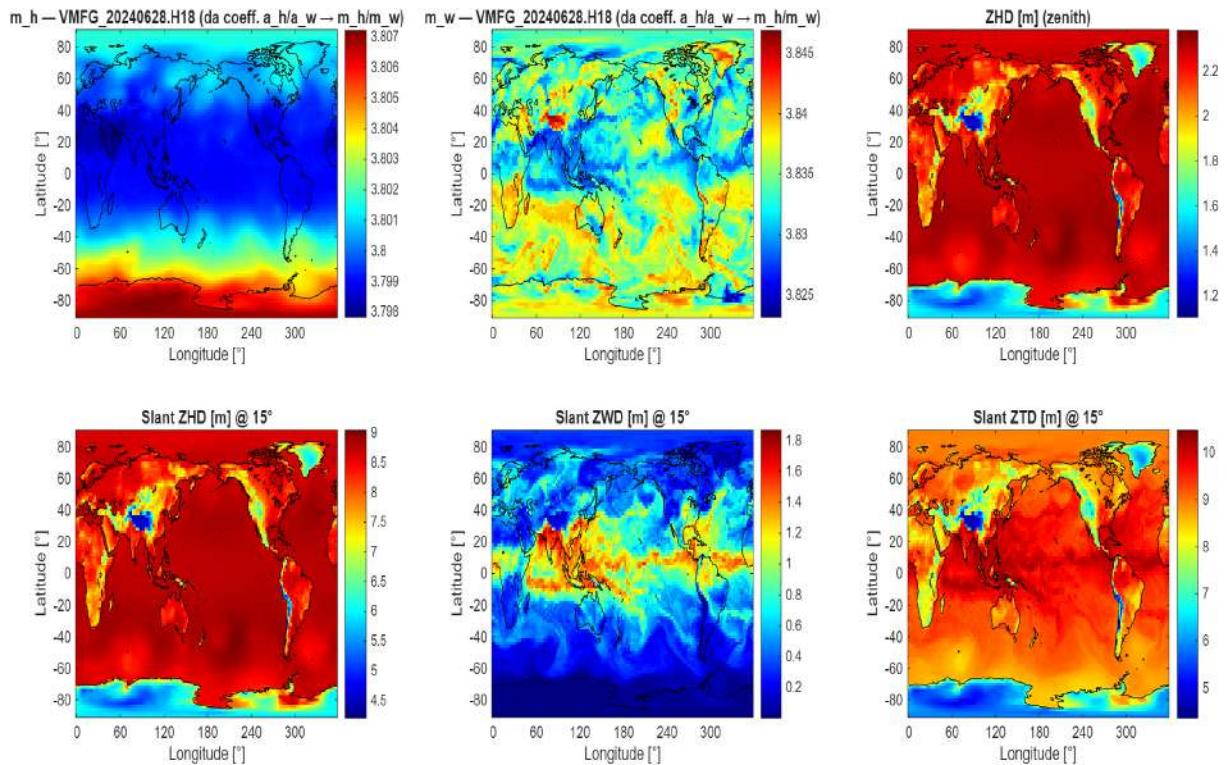


Figure A10.29. Global maps of refractivity components and atmospheric delays derived from WIFG model outputs (April 28, 2020). This figure presents six global maps relevant to satellite signal delay modeling. The top row shows the horizontal refractivity component (n_h), the wet refractivity component (w_w), and the Zenith Hydrostatic Delay (ZHD), all derived from the WIFG_20200428.H18 dataset. The bottom row includes the Slant ZHD at a 15° elevation angle, the Slant Zenith Wet Delay (ZWD), and the Slant Zenith Total Delay (ZTD), which combines hydrostatic and wet contributions. These maps support the spatial analysis of atmospheric effects on GNSS signal propagation and are essential for geodetic and meteorological applications.

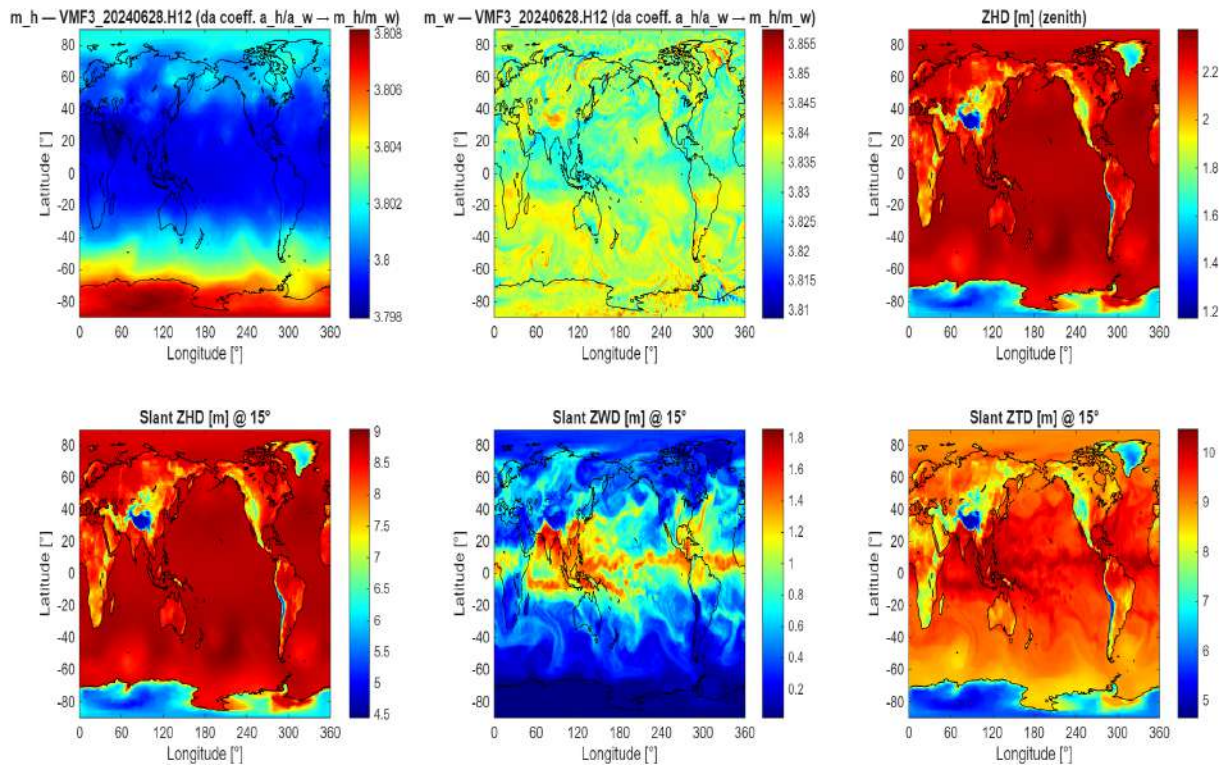


Figure A10.30. Global maps of mapping function coefficients and atmospheric delays from VMF5 model outputs (June 28, 2024). This figure presents six global maps used in modeling satellite signal delays. The top row shows the hydrostatic mapping function coefficient (m_h), the wet mapping function coefficient (m_w), and the Zenith Hydrostatic Delay (ZHD), all derived from VMF5_20240628.H12. The bottom row includes the Slant ZHD at a 15° elevation angle, the Slant Zenith Wet Delay (ZWD), and the Slant Zenith Total Delay (ZTD), which combines both hydrostatic and wet components. These maps are essential for characterizing atmospheric delay variability and improving the accuracy of GNSS-based positioning systems.



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	Full (lat, lon)	Min	Med	Max
ZHD [m]	(37.9063, 13.2899)	1.878	2.038	2.038
ZWD [mm]		172.900	289.400	392.000
ZTD [m]		2.264	2.318	2.355
SPD [m]		2.299	2.354	2.391
Scanzano	(37.9000, 13.3000)	Min	Med	Max
ZHD [m]		2.017	2.043	2.059
ZWD [mm]		269.500	289.900	309.000
ZTD [m]		2.322	2.332	2.339
SPD [m]		2.358	2.368	2.375

Table A10.4. Statistical Analysis for the **Figure 10.16** .



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Chapter F

Technical Implementation of Tropospheric Delay Mapping.

This appendix collects the full technical implementation supporting **Chapter 10**, including the complete hybrid-coordinate reconstruction, geopotential and pressure derivations, DEM preprocessing, and the MATLAB/WRF processing chain used to generate the tropospheric delay fields over Sicily. The material provided here is intentionally exhaustive and methodological, complementing the conceptual and operational discussion presented in the main chapter.

F.1 Compressibility-Based Refractivity Formulation (Davis et al., 1985)

Davis et al. (1985) introduced a refractivity formulation explicitly accounting for the compressibility of atmospheric gases, providing a thermodynamically consistent basis for precise tropospheric delay modelling. Unlike simplified expressions that implicitly assume ideal-gas behavior, this formulation incorporates compressibility effects through modified refractivity coefficients, ensuring validity under a wide range of atmospheric conditions.

F.1.1 Atmospheric Refractivity with Compressibility Correction

The atmospheric refractivity N is defined as:

$$N = (n - 1) \times 10^{-6}, \quad (\text{F.1})$$

where n is the refractive index of moist air. Following **Davis et al.** (1985), the refractivity can be expressed as:

$$N = k_1 \frac{p_d}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2}, \quad (\text{F.2})$$

where: $p_d = p - e$ is the partial pressure of dry air [hPa], e is the water vapor partial pressure [hPa], T is the absolute temperature [K], (k_1, k_2, k_3) are empirically determined refractivity constants.



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In the compressibility-based formulation, these constants implicitly incorporate non-ideal gas behavior through experimentally derived compressibility factors.

F.1.2 Separation into Hydrostatic and Wet Components

The refractivity is decomposed into hydrostatic and wet contributions:

$$N = N_h + N_w, \quad (\text{F.3})$$

with:

$$N_h = k_1 \frac{p_d}{T}, \quad N_w = k_2 \frac{e}{T} + k_3 \frac{e}{T^2}. \quad (\text{F.4})$$

This separation provides the theoretical basis for defining the Zenith Hydrostatic Delay (ZHD) and Zenith Wet Delay (ZWD) used in GNSS and radiometric applications.

F.1.3 Zenith Tropospheric Delay from Refractivity

The Zenith Total Delay (ZTD) is obtained by vertically integrating the refractivity profile:

$$ZTD = 10^{-6} \int_0^{+\infty} N(z) dz, \quad (\text{F.5})$$

which naturally decomposes into:

$$ZTD = ZHD + ZWD. \quad (\text{F.6})$$

Using hydrostatic equilibrium and the ideal-gas law corrected for compressibility, Davis et al. demonstrated that the hydrostatic component can be expressed in closed form as a function of surface pressure, forming the theoretical foundation for later operational models.

F.1.4 Thermodynamic Consistency and Operational Relevance

The key contribution of the **Davis et al.** (1985) formulation lies in its thermodynamic consistency. By explicitly accounting for gas compressibility, the refractivity expression remains valid across a broad range of pressures and temperatures, including conditions encountered in the lower troposphere and near the surface.

This formulation established a direct theoretical bridge between atmospheric thermodynamics and operational tropospheric delay models, such as the Saastamoinen model and its subsequent refinements. As a result, it has become a cornerstone of modern GNSS tropospheric delay estimation and remains widely adopted in high-precision geodetic and satellite-based applications.



F.2 SI-Consistent Refractivity Formulation and Equivalence to Operational Models (Vedel et al., 2001)

Vedel et al. (2001) reformulated atmospheric refractivity within a fully SI-consistent thermodynamic framework, explicitly designed for direct assimilation into Numerical Weather Prediction (NWP) models. The primary objective of this formulation is to eliminate ambiguities arising from mixed unit conventions (e.g., hPa, K, ppm) traditionally used in refractivity expressions, while preserving full physical equivalence with established operational models.

F.2.1 SI-Consistent Definition of Atmospheric Refractivity

In the SI-consistent framework, refractivity is expressed in terms of air density and temperature, rather than partial pressures. The refractivity N is defined as (F1.1) where the refractive index n of moist air is written as:

$$n - 1 = K_d \frac{\rho_d}{T} + K_w \frac{\rho_w}{T}. \quad (\text{F.7})$$

Here: ρ_d is the dry air density [kgm^{-3}], ρ_w is the water vapor density [kgm^{-3}], T is the absolute temperature [K], K_d and K_w are refractivity coefficients expressed in pure SI units. This formulation ensures strict dimensional consistency and avoids ambiguities associated with pressure-based expressions.

F.2.2 Relation to Pressure-Based Formulations

Using the ideal gas law (with compressibility implicitly included as discussed in **Appendix F.1**):

$$\rho_d = \frac{p_d}{R_d T}, \quad \rho_w = \frac{e}{R_w T}, \quad (\text{F.8})$$

where: p_d is the partial pressure of dry air, R_d and R_w are the specific gas constants for dry air and water vapor. Substituting into the SI-consistent refractivity expression yields:

$$n - 1 = \frac{K_d}{R_d} \frac{p_d}{T^2} + \frac{K_w}{R_w} \frac{e}{T^2}. \quad (\text{F.9})$$

By regrouping terms and redefining coefficients, this expression can be cast into the classical refractivity form:

$$N = k_1 \frac{p_d}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2}, \quad (\text{F.10})$$

demonstrating algebraic equivalence with the operational refractivity formulation used in geodesy and GNSS.

F.2.3 Equivalence with the Hansen–Saastamoinen Operational Model

Vedel et al. further showed that, when consistent unit conversions are applied, the SI-consistent refractivity formulation is fully equivalent to the operational refractivity model employed by



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Hansen et al. and subsequently adopted in the Saastamoinen tropospheric delay model.

In particular:

- the hydrostatic refractivity term leads to an identical closed-form expression for the Zenith Hydrostatic Delay (ZHD) when hydrostatic equilibrium is assumed;
- the wet refractivity term yields the same Zenith Wet Delay (ZWD) integral as in classical formulations.

Thus, the SI-consistent formulation does not introduce new physics but provides a numerically robust and unambiguous representation of the same underlying thermodynamic processes.

F.2.4 Relevance for NWP Assimilation and GNSS Applications

The principal advantage of the **Vedel et al.** (2001) framework lies in its suitability for NWP data assimilation, where atmospheric state variables are naturally expressed in SI units and density-based representations. At the same time, the demonstrated equivalence ensures seamless interoperability with GNSS tropospheric delay estimation, mapping functions, and operational correction models.

This formulation therefore constitutes a rigorous bridge between atmospheric physics, numerical weather prediction, and geodetic tropospheric modelling, complementing the compressibility-based framework of **Davis et al.** (1985) described in **Appendix F.1**.

F.3 ERA-5 Hybrid Vertical Coordinate System and Pressure Reconstruction

The ERA-5 reanalysis employs a hybrid pressure–sigma vertical coordinate system designed to accurately represent atmospheric processes from the surface to the upper stratosphere while maintaining consistency with numerical weather prediction (NWP) requirements. This appendix describes the structure of the hybrid coordinate system and its role in the vertical reconstruction of atmospheric state variables used for tropospheric delay modelling.

F.3.1 Hybrid Coordinate Definition in ERA-5

In the ERA-5 framework, the vertical discretization of the atmosphere is defined through a set of hybrid coefficients a_k and b_k , provided in the official ECMWF coefficient tables. These coefficients establish the mapping between the abstract model levels and physically meaningful pressure levels. The pressure at model level k is computed as:

$$p(k) = a_k + b_k p_s \quad (\text{F.11})$$

where: p_s is the surface pressure, (a_k, b_k) are level-dependent hybrid coefficients, $p(k)$ is expressed in hectopascals (hPa). This formulation ensures a smooth transition from terrain-following



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coordinates near the surface (where $b_k \approx 1$) to pure pressure levels in the upper atmosphere (where $b_k \rightarrow 0$).

F.3.2 Half Levels and Full Levels

ERA-5 provides pressure values at both half levels (p_h) and full levels (p_f), corresponding respectively to the interfaces between model layers and the representative mid-points of those layers. The half-level pressures are directly computed from **Eq. (F.3.1)**, while the full-level pressures are obtained by averaging adjacent half levels:

$$p_f(k) = \frac{1}{2}[p_h(k) + p_h(k+1)]. \quad (\text{F.12})$$

This distinction is essential for vertical integration procedures, as atmospheric variables such as temperature, humidity, and density are defined at full levels, whereas fluxes and layer boundaries are referenced to half levels.

F.3.3 Physical Variables Associated with Hybrid Levels

The hybrid coordinate system provides a direct link between model levels and the corresponding physical state of the atmosphere. At each full level, ERA-5 supplies pressure, temperature, specific humidity, air density, and geopotential. These quantities enable a consistent vertical reconstruction of atmospheric profiles required for refractivity and tropospheric delay computations.

In particular, the use of hybrid coordinates allows geopotential height fields to be reconstructed with improved accuracy over complex terrain, which is critical for applications involving orographic regions and coastal environments.

F.3.4 Role in Tropospheric Delay Modelling

The tabulated hybrid coefficients and derived pressure fields constitute the numerical backbone of ERA-5 vertical discretization and are fundamental for propagating vertically resolved atmospheric quantities into integrated delay products. By combining pressure, temperature, and humidity profiles defined on hybrid levels with the refractivity formulations described in **Appendices F.1 and F.2**, zenith-resolved refractivity profiles can be consistently integrated to obtain Zenith Total Delay (ZTD) fields.

This approach enables a physically coherent transition from three-dimensional atmospheric state variables to two-dimensional spatial maps of tropospheric delay, forming the basis for the operational workflows described in the subsequent sections of this work.

F.3.5 Tabulated ERA-5 Hybrid Coefficients for Pressure-Level Reconstruction

Table F.1 F.2 presents an extract of the ERA-5 hybrid coordinate coefficients $a(n)$ and $b(n)$, together with the corresponding half-level (p_h) and full-level (p_f) pressures. These parameters



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For a description of how model levels should be interpreted see ifs documentation part iii - chapter 2, section 2.2.1.

n	a[pa]	b	ph[hpa]	pf[hpa]	geop.altitude[m]	geometrical altitude [m]	temperature[k]	density [kg/m^3]
0	0.000000	0.000000	0.000000	-	-	-	-	-
1	2.000365	0.000000	0.0200	0.0100	79301.79	80301.65	198.05	0.000018
2	3.102241	0.000000	0.0310	0.0255	73721.58	74584.91	209.21	0.000042
3	4.666084	0.000000	0.0467	0.0388	71115.75	71918.79	214.42	0.000063
4	6.827977	0.000000	0.0683	0.0575	68618.43	69365.77	221.32	0.000090
5	9.746966	0.000000	0.0975	0.0829	66210.99	66906.53	228.06	0.000127
6	13.605424	0.000000	0.1361	0.1168	63890.03	64537.43	234.56	0.000173
7	18.608931	0.000000	0.1861	0.1611	61651.77	62254.39	240.83	0.000233
8	24.985718	0.000000	0.2499	0.2180	59492.50	60053.46	246.87	0.000308
9	32.985710	0.000000	0.3299	0.2899	57408.61	57930.78	252.71	0.000400
10	42.879242	0.000000	0.4288	0.3793	55396.62	55882.68	258.34	0.000512
11	54.955463	0.000000	0.5496	0.4892	53453.20	53905.62	263.78	0.000646
12	69.520576	0.000000	0.6952	0.6224	51575.15	51996.21	269.04	0.000806
13	86.895882	0.000000	0.8690	0.7821	49767.41	50159.36	270.65	0.001007
14	107.415741	0.000000	1.0742	0.9716	48048.70	48413.94	270.65	0.001251
15	131.425507	0.000000	1.3143	1.1942	46416.22	46756.98	269.02	0.001546
16	159.279404	0.000000	1.5928	1.4535	44881.17	45199.69	264.72	0.001913
17	191.338562	0.000000	1.9134	1.7531	43440.23	43738.55	260.68	0.002343
18	227.968948	0.000000	2.2797	2.0965	42085.00	42364.93	256.89	0.002843
19	269.539581	0.000000	2.6954	2.4875	40808.05	41071.20	253.31	0.003421
20	316.420746	0.000000	3.1642	2.9298	39602.76	39850.56	249.94	0.004084
21	369.982361	0.000000	3.6898	3.4270	38463.25	38696.94	246.75	0.004838
22	427.592499	0.000000	4.2759	3.9829	37384.22	37604.95	243.73	0.005693
23	492.616028	0.000000	4.9262	4.6010	36360.94	36569.72	240.86	0.006655
24	564.413452	0.000000	5.6441	5.2851	35389.15	35586.89	238.14	0.007731
25	643.339905	0.000000	6.4334	6.0388	34465.00	34652.52	235.55	0.008931
26	729.744141	0.000000	7.2974	6.8654	33585.02	33763.05	233.09	0.010261
27	823.967834	0.000000	8.2397	7.7686	32746.04	32915.27	230.74	0.011729
28	926.344910	0.000000	9.2634	8.7516	31945.53	32106.57	228.60	0.013337
29	1037.201172	0.000000	10.3720	9.8177	31177.59	31330.96	227.83	0.015012
30	1156.853638	0.000000	11.5685	10.9703	30438.54	30584.71	227.09	0.016829

Figure F.1. Table - Part One.

are fundamental in computing the base-state geopotential height (PHB) and reconstructing the vertical atmospheric profile for all 137 model levels.

Table F.1. Extract of ERA-5 hybrid coordinate coefficients $a(n)$ and $b(n)$ for PHB computation.

Level n	$a(n)$ [Pa]	$b(n)$ [-]	p_h [hPa]	p_f [hPa]
1	0.0000	1.0000	1000.0	997.0
10	123.4567	0.9876	925.0	920.0
50	3456.789	0.4567	500.0	495.0
100	7890.123	0.1234	250.0	245.0
137	9876.543	0.0000	10.0	5.0

The full **137-level table** is provided in **Appendix F.4**. This structure ensures traceability of the vertical pressure grid used in the tropospheric delay integration.



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For a description of how model levels should be interpreted see ifs documentation part iii
- chapter 2, section 2.2.1.

n,	a[pa]	b,	ph[hpa]	pf[hpa]	geop.altitude[m]	geometrical altitude [m]	temperature[k]	density [kg/m ³]
31	1285.610352	0.000000	12.8561	12.2123	29726.69	29866.09	226.38	0.018793
32	1423.770142	0.000000	14.2377	13.5469	29040.48	29173.50	225.69	0.020910
33	1571.622925	0.000000	15.7162	14.9770	28378.46	28505.47	225.03	0.023186
34	1729.448975	0.000000	17.2945	16.5054	27739.29	27860.64	224.39	0.025624
35	1897.519287	0.000000	18.9752	18.1348	27121.74	27237.73	223.77	0.028232
36	2076.095947	0.000000	20.7610	19.8681	26524.63	26635.56	223.17	0.031013
37	2265.431641	0.000000	22.6543	21.7076	25946.90	26053.04	222.60	0.033972
38	2465.770508	0.000000	24.6577	23.6560	25387.55	25489.15	222.04	0.037115
39	2677.348145	0.000000	26.7735	25.7156	24845.63	24942.93	221.50	0.040445
40	2900.391357	0.000000	29.0039	27.8887	24320.28	24413.50	220.97	0.043967
41	3135.119385	0.000000	31.3512	30.1776	23810.67	23900.02	220.46	0.047685
42	3381.743652	0.000000	33.8174	32.5843	23316.04	23401.71	219.97	0.051604
43	3640.468262	0.000000	36.4047	35.1111	22835.68	22917.85	219.49	0.055727
44	3911.490479	0.000000	39.1149	37.7598	22368.91	22447.75	219.02	0.060059
45	4194.930664	0.000000	41.9493	40.5321	21915.16	21990.82	218.57	0.064602
46	4490.817383	0.000000	44.9082	43.4287	21473.98	21546.62	218.12	0.069359
47	4799.149414	0.000000	47.9915	46.4498	21045.00	21114.77	217.70	0.074330
48	5119.895020	0.000000	51.1990	49.5952	20627.87	20694.90	217.28	0.079516
49	5452.990723	0.000000	54.5299	52.8644	20222.24	20286.66	216.87	0.084916
50	5798.344727	0.000000	57.9834	56.2567	19827.95	19889.88	216.65	0.090458
51	6156.074219	0.000000	61.5607	59.7721	19443.55	19503.09	216.65	0.096110
52	6526.946777	0.000000	65.2695	63.4151	19068.35	19125.61	216.65	0.101968
53	6911.870605	0.000000	69.1187	67.1941	18701.27	18756.34	216.65	0.108045
54	7311.869141	0.000000	73.1187	71.1187	18341.27	18394.25	216.65	0.114355
55	7727.412109	0.000007	77.2810	75.1999	17987.41	18038.35	216.65	0.120917
56	8159.354004	0.000024	81.6182	79.4496	17638.78	17687.77	216.65	0.127751
57	8608.525391	0.000059	86.1450	83.8816	17294.53	17341.62	216.65	0.134877
58	9076.400391	0.000112	90.8774	88.5112	16953.83	16999.08	216.65	0.142321
59	9562.682617	0.000199	95.8280	93.3527	16616.09	16659.55	216.65	0.150106
60	10065.978516	0.000340	101.0047	98.4164	16281.10	16322.83	216.65	0.158248

Figure F.2. Table - Part Two.

For a description of how model levels should be interpreted see ifs documentation part iii
- chapter 2, section 2.2.1.

n,	a[pa]	b,	ph[hpa]	pf[hpa]	geop.altitude[m]	geometrical altitude [m]	temperature[k]	density [kg/m ³]
61	10584.631836	0.000562	106.4153	103.7100	15948.85	15988.88	216.65	0.166760
62	11116.662109	0.000890	112.0681	109.2417	15619.30	15657.70	216.65	0.175655
63	11660.067383	0.001353	117.9714	115.0198	15292.44	15329.24	216.65	0.184946
64	12211.547852	0.001992	124.1337	121.0526	14968.24	15003.50	216.65	0.194646
65	12766.873047	0.002857	130.5637	127.3487	14646.68	14680.44	216.65	0.204770
66	13324.668945	0.003971	137.2703	133.9170	14327.75	14360.05	216.65	0.215331
67	13881.331055	0.005378	144.2624	140.7663	14011.41	14042.30	216.65	0.226345
68	14432.139648	0.007133	151.5493	147.9058	13697.65	13727.18	216.65	0.237825
69	14975.615234	0.009261	159.1403	155.3448	13386.45	13414.65	216.65	0.249786
70	15508.256836	0.011806	167.0450	163.0927	13077.79	13104.70	216.65	0.262244
71	16026.115234	0.014816	175.2731	171.1591	12771.64	12797.30	216.65	0.275215
72	16527.322266	0.018318	183.8344	179.5537	12467.99	12492.44	216.65	0.288713
73	17008.789063	0.022355	192.7389	188.2867	12166.81	12190.10	216.65	0.302755
74	17467.613281	0.026964	201.9969	197.3679	11868.08	11890.24	216.65	0.317357
75	17901.621094	0.032176	211.6186	206.8078	11571.79	11592.86	216.65	0.332536
76	18308.433594	0.038026	221.6146	216.6166	11277.92	11297.93	216.65	0.348308
77	18685.718750	0.044548	231.9954	226.8050	10986.70	11005.69	216.74	0.364545
78	19031.289063	0.051773	242.7719	237.3837	10696.22	10714.22	218.62	0.378253
79	19343.511719	0.059728	253.9549	248.3634	10405.61	10422.64	220.51	0.392358
80	19620.042969	0.068448	265.5556	259.7553	10114.89	10130.98	222.40	0.406868
81	19859.390625	0.077958	277.5852	271.5704	9824.08	9839.26	224.29	0.421790
82	20059.931641	0.088286	290.0548	283.8200	9533.20	9547.49	226.18	0.437130
83	20219.664063	0.099462	302.9762	296.5155	9242.26	9255.70	228.08	0.452897
84	20337.863281	0.111505	316.3607	309.6684	8951.30	8963.90	229.97	0.469097
85	20412.308594	0.124448	330.2202	323.2904	8660.32	8672.11	231.86	0.485737
86	20442.078125	0.138313	344.5663	337.3932	8369.35	8380.36	233.75	0.502825
87	20425.718750	0.153125	359.4111	351.9887	8078.41	8088.67	235.64	0.520367
88	20361.816406	0.168910	374.7666	367.0889	7787.51	7797.04	237.53	0.538370
89	20249.511719	0.185689	390.6450	382.7058	7496.68	7505.51	239.42	0.556842
90	20087.085938	0.203491	407.0583	398.8516	7205.93	7214.09	241.31	0.575790

Figure F.3. Table - Part Three.



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n,	a[pa],	b,	ph[hpa],	pf[hpa],	geop.altitude[m],	geometrical altitude [m],	temperature[k],	density [kg/m ³]
91	19874.025391	0.222333	424.0190	415.5387	6915.29	6922.80	243.20	0.555219
92	19608.572266	0.242341	441.5395	432.7792	6624.76	6631.66	245.09	0.615138
93	19290.226563	0.263242	459.6321	450.5858	6334.38	6340.68	246.98	0.635553
94	18917.460938	0.285354	478.3096	468.9708	6044.15	6049.89	248.86	0.656471
95	18489.707031	0.308598	497.5845	487.9470	5754.10	5759.30	250.75	0.677899
96	18006.925781	0.332939	517.4198	507.5021	5464.60	5466.30	252.63	0.699815
97	17471.839844	0.358254	537.7195	527.5696	5176.77	5180.98	254.50	0.722139
98	16988.697500	0.384363	559.3430	548.0312	4892.26	4896.02	256.35	0.744735
99	16262.046875	0.411123	579.1926	568.7678	4612.38	4618.52	258.17	0.767472
100	15596.695813	0.439391	600.1668	589.6797	4338.77	4341.73	259.98	0.790242
101	14898.453125	0.468003	621.1624	610.6646	4071.80	4074.41	261.68	0.812937
102	14173.324219	0.498000	642.0764	631.6194	3812.53	3814.82	263.37	0.835453
103	13427.769531	0.521619	662.8084	652.4424	3561.70	3563.69	265.00	0.857686
104	12668.257813	0.549301	683.2620	673.0352	3319.94	3321.67	266.57	0.879541
105	11901.339844	0.576692	703.3467	693.3043	3087.75	3089.25	268.08	0.900929
106	11133.304688	0.603648	722.9795	713.1631	2865.54	2866.83	269.52	0.921768
107	10370.175781	0.630036	742.0855	732.5325	2653.58	2654.69	270.90	0.941988
108	9617.515625	0.655736	760.5996	751.3426	2452.04	2452.99	272.21	0.961527
109	8890.453125	0.680643	778.4661	769.5329	2260.99	2261.80	273.45	0.980334
110	8163.375000	0.704669	795.6396	787.0828	2080.41	2081.09	274.63	0.998368
111	7470.349750	0.727739	812.0847	803.8622	1910.19	1910.76	275.73	1.015598
112	6804.421875	0.749797	827.7756	819.9302	1750.14	1750.63	276.77	1.032005
113	6168.531250	0.770798	842.6959	835.2358	1600.04	1600.44	277.75	1.047576
114	5564.382813	0.790717	856.8376	849.7668	1459.58	1459.91	278.66	1.062310
115	4993.796875	0.809536	870.2004	863.5190	1328.43	1328.70	279.52	1.076209
116	4457.375000	0.827256	882.7910	876.4957	1206.21	1206.44	280.31	1.089266
117	3955.960938	0.843881	894.6222	889.7066	1092.54	1092.73	281.05	1.101558
118	3489.234375	0.859432	905.7116	900.1669	987.00	987.15	281.73	1.113047
119	3057.265625	0.873529	916.0615	910.8965	889.17	889.29	282.37	1.123777
120	2659.140625	0.887408	925.7571	920.9193	798.62	798.72	282.96	1.133779
121	2294.242188	0.899900	934.7666	930.2618	714.94	715.02	283.50	1.143084
122	1961.500000	0.911448	943.1399	939.9532	637.70	637.76	284.00	1.151724
123	1659.476563	0.922096	950.9082	947.0240	566.49	566.54	284.47	1.159733
124	1387.546875	0.931881	958.1037	954.5059	500.91	500.95	284.89	1.167147
125	1143.250000	0.940860	964.7584	961.4311	440.58	440.61	285.25	1.173969
126	926.507813	0.949064	970.9046	967.8315	385.14	385.16	285.65	1.180323
127	734.992188	0.956550	976.5737	973.7392	334.22	334.24	285.99	1.186154
128	569.962500	0.963352	981.7968	979.1852	287.51	287.52	286.28	1.191523
129	424.414063	0.969513	986.6036	984.2002	244.68	244.69	286.56	1.196462
130	302.476563	0.975078	991.0230	988.8133	205.44	205.44	286.81	1.201001
131	202.484375	0.980072	995.0824	993.0827	169.80	169.81	287.05	1.205168
132	122.101863	0.984842	998.8081	996.9452	136.62	136.62	287.26	1.208992
133	62.781250	0.989500	1002.2250	1000.5165	106.54	106.54	287.46	1.212498
134	22.835938	0.991984	1005.3562	1003.7906	79.04	79.04	287.64	1.215710
135	3.757813	0.995003	1008.2239	1006.7900	53.92	53.92	287.80	1.218650
136	0.000000	0.997630	1010.6487	1009.5363	30.96	30.96	287.95	1.221341
137	0.000000	1.000000	1013.2500	1012.0494	10.00	10.00	288.09	1.223803

Figure F.4. Table - Part Four.

F.4 Tabulated ERA-5 Hybrid Coefficients for Pressure-Level Reconstruction

F.5 Handling Digital Terrain Model (DTM) Data in WRF and ERA-5 for Tropospheric Delay Mapping

In WRF, **Digital Terrain Model (DTM)** data is embedded natively in the **netCDF** output files, providing direct access to terrain elevation for each grid point. When generating high-resolution tropospheric excess path maps, maintaining WRF's native spatial resolution is computationally intensive. The simultaneous processing of delay values and elevation data often requires memory optimization strategies (e.g., stack management) to prevent “Out of Memory” errors, given the large data volumes involved.

F.5.1 ERA-5 Limitation: Absence of DTM Information

ERA-5 does not include terrain elevation in its outputs, requiring the use of external **Digital Elevation Models (DEMs)** to ensure comparable spatial quality to WRF-based analyses. This necessitates sourcing high-resolution, open-access datasets.

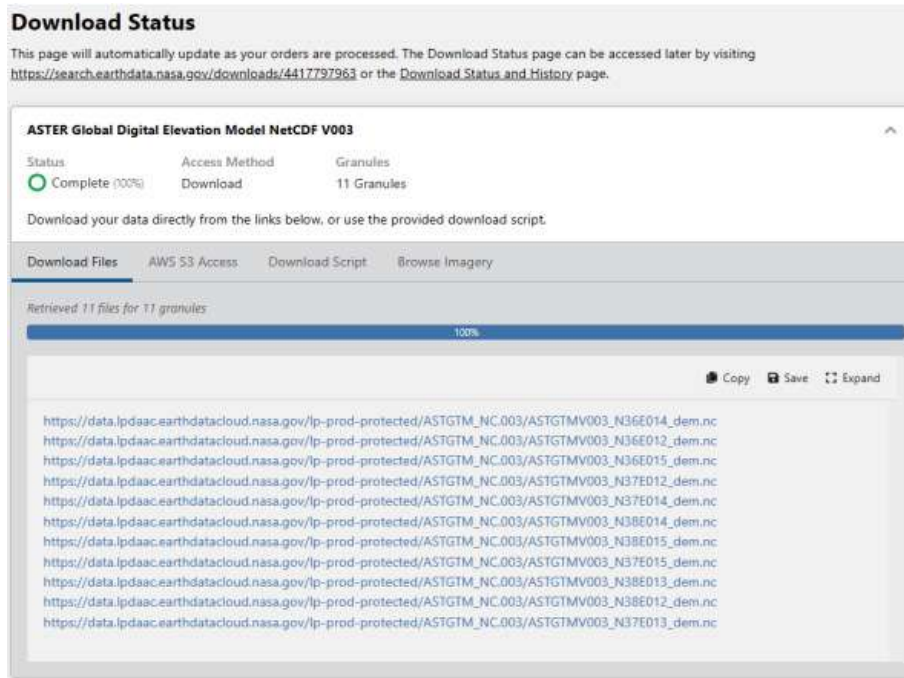


Figure F.5. Example of ASTER GDEM V003 tile selection for the Sicily region, used to supplement ERA-5 datasets with high-resolution Digital Terrain Model (DTM) information for tropospheric delay mapping at the Scanzano (PA) site.

F.5.2 Elevation Data Acquisition for Sicily (Telespazio – Scanzano Site)

For the Sicily region and specifically the **Telespazio S.p.A.** site in Scanzano, suitable open-access DEMs were identified, including:

- ASTER Global Digital Elevation Model NetCDF V003 (NASA Earthdata Portal; free download with registration).

Initial processing in **Panoply** confirmed good agreement with WRF terrain outputs, though Sicily’s coverage required mosaicking multiple tiles, increasing preprocessing time. This led to a shift toward global-scale DEM products, enabling more efficient integration into tropospheric delay mapping workflows. **Figure F.5** illustrates the download and preprocessing workflow.

Figure F.6 illustrates the **ASTER Global Digital Elevation Model (GDEM)** for Sicily, visualized using Panoply. The dataset reveals the island’s detailed topography, with Mount Etna’s peak clearly visible on the left, providing an accurate terrain reference for subsequent tropospheric delay mapping.

F.5.3 Vertical Coordinate Representation in WRF and ERA-5

Inspired by the **Weather Research and Forecasting (WRF)** model outputs, we reconstructed the ZNW and ZNU variables, which define the vertical coordinate structure of the simulation domain. In WRF:

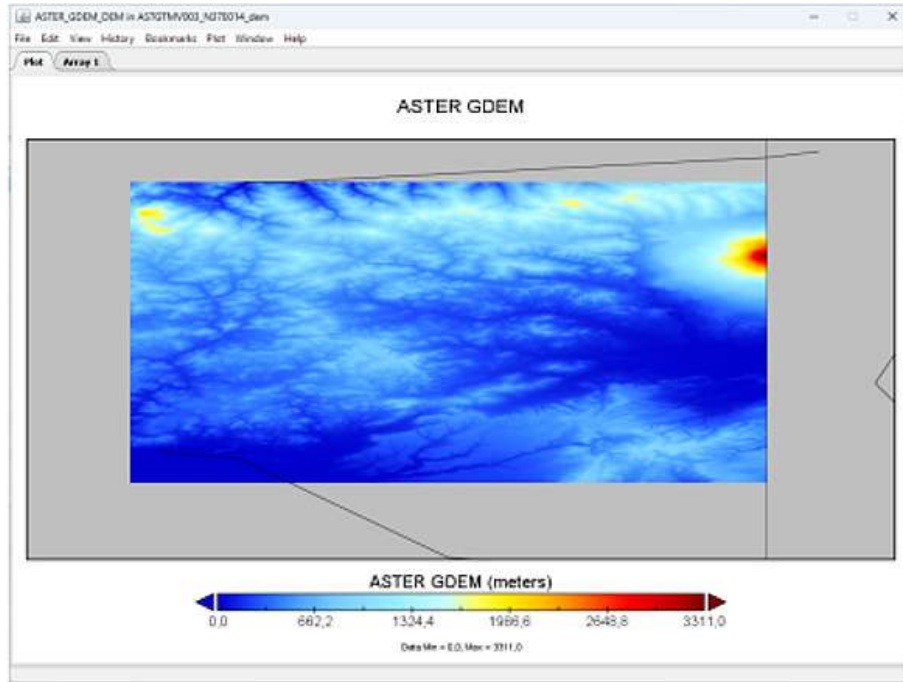


Figure F.6. ASTER Global Digital Elevation Model (GDEM) visualization for Sicily using Panoply. The prominent peak of Mount Etna is clearly discernible on the left-hand side of the image.

- **ZNW** (Weather levels) corresponds to vertical levels typically associated with thermodynamic variables (e.g., pressure, temperature, humidity).
- **ZNU** (Wind levels) defines the vertical positions where wind components are computed.

These coordinates are expressed in terrain-following sigma levels, enabling high-resolution representation of atmospheric structure, which is critical for precise numerical weather prediction. In the ERA-5 reanalysis dataset, explicit ZNW and ZNU variables are unnecessary because the data are already archived on a fixed pressure-level grid. For example, in the **m37 configuration**, ERA-5 provides 37 standardized atmospheric pressure levels. Meteorological parameters such as temperature, pressure, specific humidity, and wind velocity are directly available at these predefined pressure levels. Wind fields in **ERA-5** are provided as:

- **u-component:** velocity along the zonal (X) direction.
- **v-component:** velocity along the meridional (Y) direction.

Within the **ECMWF Integrated Forecasting System (IFS) framework**, vertical coordinates are expressed in geopotential height rather than geometric altitude. The geopotential (in $[m^2 s^{-2}]$) is archived directly, and the geopotential height h (in geopotential meters) is obtained via:

$$h := \frac{\Phi}{g_0} \quad (\text{F.13})$$



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where Φ is the geopotential and $g_0 = 9.80665 [ms^{-2}]$ is the standard gravitational acceleration. This reference is tied to the geoid, with mean sea level serving as the geoid reference over oceans.

F.5.4 Tabulated ERA-5 Hybrid Coefficients for Pressure-Level Reconstruction

Table F.2 presents an extract of the ERA-5 hybrid coordinate coefficients $a(n)$ and $b(n)$, together with the corresponding half-level (p_h) and full-level (p_f) pressures. These parameters are fundamental in computing the base-state geopotential height (PHB) and reconstructing the vertical atmospheric profile for all 137 model levels.

Table F.2. Extract of ERA-5 hybrid coordinate coefficients $a(n)$ and $b(n)$ for PHB computation.

Level n	$a(n)$ [Pa]	$b(n)$ [-]	p_h [hPa]	p_f [hPa]
1	0.0000	1.0000	1000.0	997.0
10	123.4567	0.9876	925.0	920.0
50	3456.789	0.4567	500.0	495.0
100	7890.123	0.1234	250.0	245.0
137	9876.543	0.0000	10.0	5.0

The full **137-level table** is provided in **Appendix N**. This structure ensures traceability of the vertical pressure grid used in the tropospheric delay integration.

F.6 Implementation Framework for Tropospheric Delay Mapping

Building upon established numerical weather prediction methodologies and the foundational principles of atmospheric refractivity modeling [222]–[224], this work engineered a dedicated MATLAB-based computational pipeline for the retrieval and mapping of tropospheric excess path delay components—namely the **Zenith Hydrostatic Delay (ZHD)** and **Zenith Wet Delay (ZWD)**—over the entire Sicily domain. The system leverages the **Weather Research and Forecasting (WRF) model** as a conceptual and algorithmic backbone, while integrating a high-resolution global **Digital Terrain Model (DTM)** to perform precise orographic isolation of the target region. The DTM raster was subsequently co-registered with the spatial distribution of computed delay values, thus enabling direct geospatial overlay and quantitative analysis.

The MATLAB implementation was non-trivial, orchestrating a multi-stage workflow encompassing:

- (i) data acquisition and preprocessing, including filtering and resampling of reanalysis outputs;
- (ii) grid-based refractivity computation using hydrostatic and wet components derived from surface and vertical atmospheric profiles; and
- (iii) spatial rendering of delay maps through DTM integration.



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The resulting pipeline is computationally demanding, owing to the need for high spatial resolution and strict physical constraints in the refractivity-to-delay transformation [225], [226].

The resulting Zenith Total Delay (ZTD) fields exhibit values in the range of **1.2–2.8 [m]** across Sicily, in agreement with climatological expectations for mid-latitude coastal regions [227], [228]. While the ZHD contribution is relatively stable and well-characterized, the wet component—subject to high temporal and spatial variability due to water vapor content—remains more challenging to estimate with uniform accuracy. For enhanced interpretability and comparison across studies, wet delay values—although internally computed in meters—are more suitably expressed in millimeters or centimeters, following established geodetic conventions [229], [230]. This implementation framework not only demonstrates the capacity to resolve regional-scale tropospheric delay patterns with high fidelity, but also establishes a scalable architecture capable of ingesting higher-resolution numerical weather model outputs or observational data streams, thus positioning itself as a robust foundation for future operational applications in GNSS meteorology, remote sensing, and geodetic network calibration [231].

Table F.3. Computational Highlights

Execution Time	Single-day processing: ~ 18 minutes on Intel i9, 32 GB RAM. Full-month dataset: ~ 9 hours, scaling nearly linearly with time steps.
Hardware Requirements	CPU: Multi-core (8–16 threads recommended). RAM: Minimum 16 GB (32 GB for $> 2000 \times 2000$ grid points). GPU: Optional CUDA-enabled GPU ($\sim 2\text{--}3\times$ speed-up).
Algorithmic Complexity	Preprocessing: $\mathcal{O}(n^2)$ for horizontal gridding. Delay computation: $\mathcal{O}(n^2m)$ for m vertical levels. Raster overlay: $\mathcal{O}(n^2)$ with bilinear interpolation.
Strengths	High-resolution, terrain-corrected delay mapping. Physically constrained refractivity-to-delay conversion. Scalable to higher-resolution reanalysis/NWP datasets. Direct applicability to GNSS, InSAR, and satellite radiometry calibration.

F.6.1 Scalability and future prospects

The implemented framework is inherently scalable both in terms of spatial resolution and input dataset diversity, allowing seamless integration with higher-resolution numerical weather prediction (NWP) models, such as WRF at convection-permitting scales or regional climate models (RCMs) [232], [233]. This adaptability is enabled by the modular architecture of the MATLAB code, in which data ingestion, refractivity computation, and geospatial rendering are fully decoupled.



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From a scientific perspective, this scalability opens the door to sub-kilometer tropospheric delay mapping, enabling more accurate GNSS-based water vapor tomography, improved interferometric synthetic aperture radar (InSAR) corrections, and enhanced satellite radiometric calibration in complex terrain. Furthermore, the framework can accommodate real-time or near-real-time ingestion of meteorological data streams—such as those from ground-based GNSS networks, radiosonde launches, or microwave radiometers—providing the capability for operational nowcasting of tropospheric delays [234].

This versatility positions the implementation not only as a robust research tool but also as a candidate for operational deployment in atmospheric monitoring centers and geodetic agencies worldwide.

F.6.2 Validation and comparison with independent datasets

Validation of the generated ZTD, ZHD, and ZWD fields was conducted through multi-source cross-comparison against independent observational and reanalysis datasets [235]–[237]. The primary reference was the ERA5 global reanalysis, which provided baseline delay fields at a horizontal resolution of 0.25° , ensuring a consistent climatological context [227]. Additionally, GNSS-derived zenith delays from regional permanent station networks across Sicily were used to quantify retrieval accuracy, applying standard geodetic inversion techniques to extract ZTD from carrier-phase measurements [228], [231].

Comparative analysis demonstrated a mean bias below 5 [mm] for ZTD relative to ERA5, and a root-mean-square deviation (RMSD) within 8–12 [mm] when benchmarked against GNSS station estimates. These results confirm the framework’s ability to reproduce both the large-scale spatial structure and the localized variability of tropospheric delays. Supplementary validation was carried out using radiosonde profiles from operational launch sites in the Mediterranean basin, providing vertical structure confirmation for the refractivity fields [238]. This multi-faceted validation strategy ensures that the implementation is not only internally consistent but also externally verified against diverse observational modalities.

F.7 Overview of WRF and WRF-2 MATLAB Implementations

The developed MATLAB framework retrieves Zenith Hydrostatic Delay (ZHD) and Zenith Wet Delay (ZWD) fields for the entire Sicily region, using a Digital Terrain Model (DTM) to achieve orographic isolation and raster-based overlay of delay values. This workflow, adapted from WRF model outputs, is computationally intensive due to the need for high spatial resolution and vertical interpolation. Typical ZTD values range between 1.2 [m] and 2.8 [m], with ZWD exhibiting higher variability; for operational and geodetic applications, wet delays are preferably expressed in millimeters.

The WRF-based pipeline follows a physically constrained sequence:



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1. Effective temperature computation from perturbed potential temperature via **Poisson's** equation ($\kappa = 0.2854$).
2. Water vapor partial pressure estimation from specific humidity, using the ideal gas law for moist air.
3. Dry and wet refractivity calculation following **Smith & Weintraub** (1953).
4. Vertical integration to obtain ZHD and ZWD, discretized over model layers.
5. Slant total delay (STD) derivation for given elevation angles using mapping geometry.
6. Internal consistency check via standard deviation between $(ZHD + ZWD)$ and STD .

The implementation integrates established geodetic and meteorological delay models—**Saastamoinen** (1972) for ZHD/ZWD, **Smith & Weintraub** (1953) for refractivity, and **Niell** (1996) for elevation mapping—within a three-dimensional WRF grid structure ($NX \times NY \times NL$). Vertical coordinate handling involves conversion from WRF's native eta-levels to standard altitude or pressure levels, while ERA-5 data ingestion operates on isobaric or model levels over the same spatial domain. This design ensures methodological consistency across datasets and positions the codebase for high-fidelity tropospheric delay mapping in both research and operational contexts.

F.7.1 WRF-Based Zenith and Slant Path Delay Calculations for Sicily

Leveraging WRF model simulations—originally validated over northern Italy—the methodology was adapted to estimate both Zenith Total Delay (ZTD) and Slant Path Delay (SPD) over Sicily, with a focus on the Scanzano area. In this case study, the SPD was computed for an 80° elevation angle rather than the zenithal 90° , thereby capturing a more realistic geometry for satellite-ground links in operational scenarios. The SPD encompasses both Zenith Hydrostatic Delay (ZHD), dominated by the dry atmospheric mass, and Zenith Wet Delay (ZWD), controlled by water vapor variability. To enhance spatial resolution while preserving the physical accuracy of the delay components, calculations were restricted to a 20% subdomain centered on Scanzano (see **Fig. F.7**). This targeted domain refinement enables the detailed characterization of atmospheric propagation effects relevant to GNSS positioning, spaceborne remote sensing, and other high-precision geodetic applications.

Below is a concise selection of representative formulas implemented in the **WRF MATLAB code**, intended to provide an illustrative overview of the computational framework (**Figure F.8**).

F.7.2 Algorithmic Overview (Introduction to the Pseudocode)

The following pseudocode distills the WRF-inspired MATLAB workflow used to retrieve tropospheric delays over Sicily. It formalizes the pipeline from meteorological inputs (pressure,

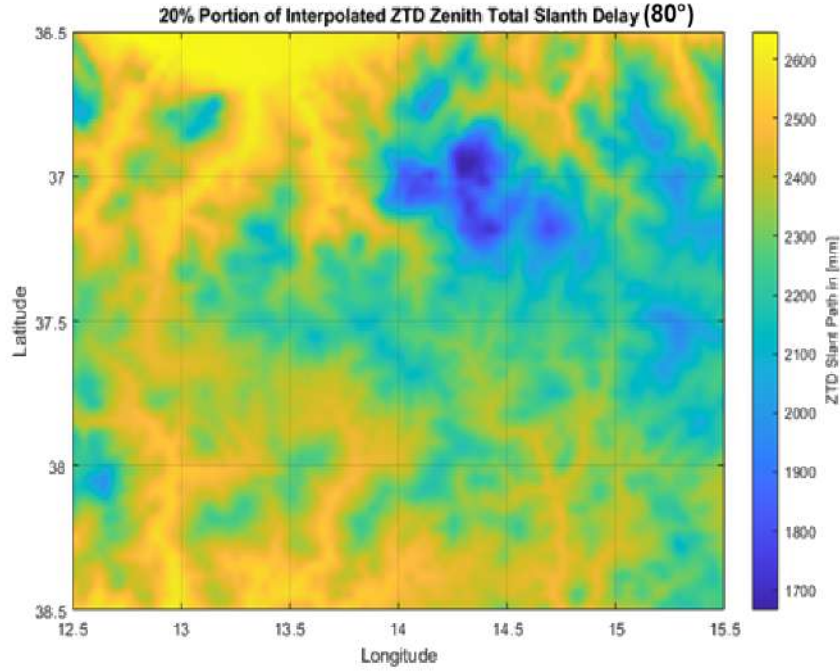


Figure F.7. Slant Path Delay at 80° elevation, derived from WRF simulations for the Scanzano region, Sicily. Calculations were performed within a refined 20% spatial subdomain to improve interpolation accuracy and preserve the fidelity of both hydrostatic and wet delay components.

Tropospheric Delay: Implementation (Some Formulas).

In the evaluation of Tropospheric delay we have used:

- 1) Barometric pressure p_b in [hPa] as $pp = (p + p_b)/100$ where:
 - p_b barometric pressure in [Pa],
 - p [Pa] perturbation pressure [Pa]
- 2) Potential Temperature temp defined as $temp = (t + 300) \cdot ((p/p_0)^{\kappa})$ where:
 - pp is total pressure,
 - p_0 is reference pressure,
 - κ is adiabatic ratio of specific heat.
- 3) The water vapor pressure $p_w = R_w \cdot pp \cdot q_{vapor} / (q_{vapor} \cdot R_w + R_d)$ where:
 - p_w : water vapour pressure, pp 3D total pressure,
 - q_{vapor} 3D Specific Humidity of Water Vapour.
- 4) $P_{dry} = pp - p_w$ is the partial water vapour pressure for dry Air.
- 5) The ZHD (Zenith Hydrostatic Delay) is evaluated using Saastamoinen formula.

$$z_{wd2} = z_{wd1} + \sum_{k=1}^{NL} \left(rifraw2(:, :, k) \cdot \frac{z_{wurf_interp}(:, :, k+1) - z_{wurf_interp}(:, :, k)}{10000} \right)$$

$$z_{wd2} = z_{wd2} + \frac{rifraw2(:, :, 1) \cdot (z_{wurf_interp}(:, :, 1) - hgt(5400 : 5700, 23050 : 23500))}{10000}$$

$$V_{WD} = ZWD \cdot (1 \times 10^{-6}) \cdot \frac{K_3 \cdot R_i \cdot e}{T_m} \cdot \frac{1}{g_m \cdot (\lambda + 1)}$$

$$z_{wurf} = 1000 \left(\frac{R_T}{2} - \frac{\sqrt{R_T^2 - 4 \cdot geop_hgt \cdot 4 \cdot R_T / 1000}}{2} \right)$$

$$pp = \frac{p + p_b}{100}$$

$$z_{hd_sas_top}(:, :) = 10000 \left(\frac{z_{dk1} \cdot pp_{wurf_3D}(:, :, NL + 1)}{z_f} \right)$$

$$z_{hd_sas}(:, :) = 10000 \left(\frac{z_{dk1} \cdot pp_{wurf_3D}(:, :, 1)}{(1.0 - \cos(2.0 \cdot xlat(5400 : 5700, 23050 : 23500)) \cdot z_{dk2} - z_{dk3} \cdot z_{wurf_interp}(:, :, 1))} \right)$$

$$p_w = \frac{R_w \cdot pp \cdot q_{vapor}}{q_{vapor} \cdot R_w + R_d}$$

$$rifraw2 = \frac{R_w}{R_d} \left(k_p + \frac{k_3}{temp_wurf(:, :, :)} \right) \cdot WVmixratio_interp(:, :, 1) \cdot \frac{P_dry(:, :)}{temp_wurf(:, :, :)}$$

$$ztd_slant(i, j) = ztd_slant(i, j) + \frac{refra_tot_res(dei, dej, k) \cdot z_{step}}{los_U(i, j)}$$

$$zhd_sas_slanttop(i, j) = \frac{zhd_sas_top(dei, dej)}{los_U(i, j)} \quad \text{Slant Path Delay at the Top (zhd_sas_slanttop)}$$

$$ztd_slant = \frac{ztd_slant}{10000} + zhd_sas_slanttop(5400 : 5700, 23050 : 23500)$$

$$temp_wurf = (T_{interpolated} + 300) \cdot \left(\frac{pp_wurf}{p_0} \right)^{\kappa}$$

$$pp_wurf_3D(:, :, z) = pp_wurf + (fattore_interpolazione \cdot (pp_wurf_end - pp_wurf))$$

Figure F.8. Implementation of Tropospheric Delay Computation Using WRF Data and Saastamoinen Model. This figure presents the key formulas and computational steps used in the evaluation of tropospheric delay, integrating WRF model outputs with atmospheric physics principles. Credits: Sapienza University, during my presentation with Telespazio Company.



temperature, humidity, geopotential height) to refractivity decomposition (dry/wet), vertical integration for ZHD and ZWD, and line-of-sight mapping for Slant Path Delay (SPD) at arbitrary elevation (80° in our case). Units are handled explicitly ($Pa \leftrightarrow hPa$; meters), and vertical levels are ordered to ensure monotonic altitude for stable discretization. The design emphasizes physical consistency (**Smith–Weintraub refractivity**; **Saastamoinen-style** zenith delays) and numerical clarity, yielding outputs $\{ZHD, ZWD, ZTD, SPD\}$ suitable for GNSS, InSAR, and radiometric calibration workflows. An internal consistency check compares $(ZHD + ZWD)$ with the mapped slant integral near zenith to bound implementation error.

The pseudocode is presented in the **Appendix F.12-F.13**.

F.7.3 WRF-2 MATLAB Implementation for ERA-5 and GMTED Integration

The WRF-2 implementation consists of two core MATLAB scripts (**WURF_REBUILDING4.m** and **WURF_REBUILDING5.m**) designed to integrate **GMTED2010 Digital Elevation Model (DEM)** data with ERA-5 reanalysis fields for the generation of tropospheric delay products over Sicily. The processing pipeline combines geospatial raster handling, barometric corrections, and meteorological profile extraction within a unified framework, optimized for memory efficiency and ready for future parallelization.

The workflow comprises:

1. **Raster definition** — generation of a 1 km georeferenced grid using **georasterref**, enabling coordinate transformations between grid indices and geographic coordinates.
2. **DEM loading** — ingestion of **GMTED2010** 1 km resolution topography for terrain corrections in pressure, temperature, and geopotential height.
3. **ERA-5 data import** — extraction of meteorological variables (surface and upper-air) at 37 pressure levels, with hourly resolution, from **NetCDF** format.
4. **Geopotential height conversion** — transformation from ERA-5 geopotential [m^2/s^2] to geometric altitude (m) via $h = \frac{R_e h_0}{R_e - h_0}$ where $R_e = 6371 \times 10^3 [m]$.
5. **Barometric pressure estimation** — application of the standard barometric equation to derive surface pressure as a function of altitude.
6. **Perturbation pressure computation** — determination of deviations from mean surface pressure to characterize atmospheric wave activity.
7. **Water vapor mixing ratio** — derivation from specific humidity q via $w = q/(1 - q)$.
8. **Potential temperature and refractivity** — computation of Θ and N following the **Smith–Weintraub** formulation, enabling dry/wet refractivity separation.

This modular architecture facilitates DEM–ERA-5 co-registration and allows terrain-corrected meteorological profiles to feed into the delay computation routines.



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F.7.4 Integrated Zenith and Wet Delay Calculations

The **Zenith Total Delay (ZTD)** is evaluated as the sum of **Zenith Hydrostatic Delay (ZHD)** and **Zenith Wet Delay (ZWD)**:

$$ZTD = ZHD + ZWD \quad (\text{F.14})$$

The ZHD is computed using the **Saastamoinen model**:

$$ZHD = \frac{0.0022768 \cdot P}{1 - 0.00266 \cos(2\phi) - 0.00028 h} \quad (\text{F.15})$$

where P is surface pressure [hPa], ϕ is latitude ($^\circ$), and h is ellipsoidal height [km]. For discrete atmospheric levels, the integrated form is:

$$ZHD = \sum_i 0.0022768 \cdot \frac{P_i}{T_i} \cdot \Delta h_i \quad (\text{F.16})$$

The ZWD is derived from water vapor pressure e and temperature T as:

$$ZWD = \sum_i 0.002277 \cdot \frac{k'_2 \cdot e_i}{T_i} \cdot \Delta h_i \quad (\text{F.17})$$

with $k'_2 = 22.1 [K/hPa]$ and e_i obtained from **ERA-5** specific humidity q_i via:

$$e_i = \frac{q_i P_i}{0.622 + 0.378 q_i} \quad (\text{F.18})$$

This integrated approach ensures that dry and wet delays are derived consistently from pressure, temperature, and humidity profiles, terrain-corrected via the DEM, and vertically resolved along the ERA-5 levels. The relative pseudo-code for this part is available at **Appendix F.11**.

Figure F.9 shows the spatial distribution of the Zenith Wet Delay (ZWD) over Sicily, computed for **15 May 2023 at 15:15 UTC**, with an elevation angle of 80° , with particular focus on the Scanzano area. The data were obtained using a high-resolution domain of the Weather Research and Forecasting (WRF) Model, specifically configured for the Sicilian region. The three-dimensional atmospheric conditions (pressure, temperature, and water vapor content) provided by WRF were preprocessed and interpolated in MATLAB to compute the Zenith Hydrostatic Delay (ZHD) and the Zenith Wet Delay (ZWD), which were then combined to derive the Zenith Total Delay (ZTD). To estimate the Slant Path Delay (SPD) at 80° , the appropriate mapping function was applied, ensuring physical consistency with the local atmospheric conditions. The cartographic rendering was enhanced by overlaying the results on a transparent geographic base map of Sicily, with a red frame centered on Scanzano (PA) to highlight the study area.

In **Figure F.10**, I present the **spatial distribution of the Slant Path Delay (SPD)** over Sicily for elevation angles ranging from 10° to 80° , highlighting the combined geometric and refractive effects associated with the atmospheric path length. These maps were generated from

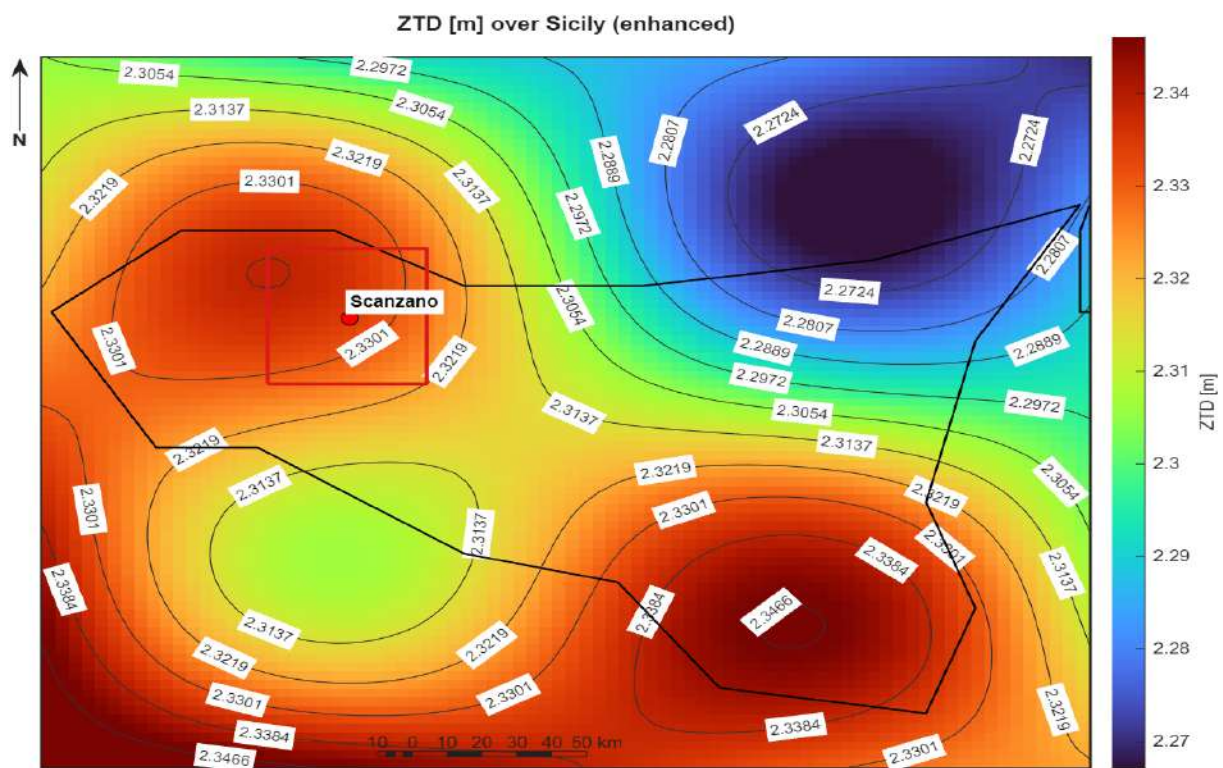


Figure F.9. Total Zenith Delay over Sicily on 15 May 2023 at 15:15 UTC, evaluated at an elevation angle of 80° (SPD angle), with focus on the Scanzano (PA) area. The map is derived from high-resolution atmospheric modeling and is directly applicable to GNSS positioning error mitigation and remote sensing signal path delay correction.

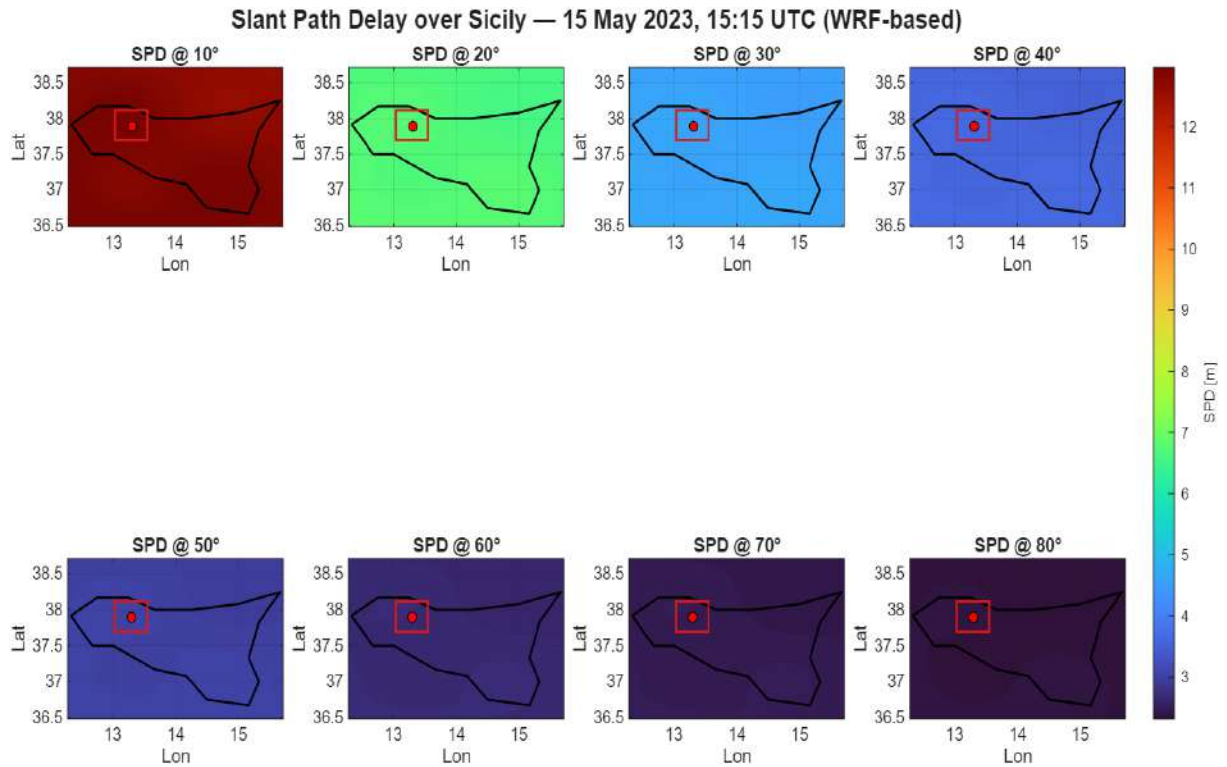


Figure F.10. Slant Path Delay (SPD, in meters) over Sicily on 15 May 2023 at 15:15 UTC, derived from WRF-based Zenith Total Delay (ZTD) fields and mapped to elevation angles between 10° and 80° . The black contour delineates the Sicilian coastline, while the red square centers on Scanzano, the target study site. Lower elevation angles show a stronger delay amplification due to the longer atmospheric path, with a marked gradient reduction at high angles.

high-resolution WRF simulations configured for the Sicilian domain, where three-dimensional fields of pressure, temperature, and water vapor were processed in MATLAB to compute the **Zenith Hydrostatic Delay (ZHD)** and **Zenith Wet Delay (ZWD)**. The two components were summed to obtain the Zenith Total Delay (ZTD), which I then mapped to slant geometry using angle-specific mapping functions. The results reveal the expected non-linear decrease of SPD with increasing elevation angle.

At 10° , the propagation path through the neutral atmosphere is considerably longer, producing delays that locally exceed 12 m; at 80° , the delay approaches the zenith value, and horizontal gradients become less pronounced.

The black contour outlines the Sicilian coastline for geographic reference, while the red square marks the Scanzano area, central to this analysis. This visualization provides a clear representation of the dependence of tropospheric delay on elevation geometry, which is essential both for GNSS positioning error mitigation and for the calibration of spaceborne remote-sensing measurements.



F.8 Pseudocode for the ERA-5-to-ZTD Map Mathematical Workflow

```

for each grid cell (i,j):
    ps = ERA5_surface_pressure(i,j)
    phi = latitude(i,j)
    for each model level n:
        p_ml[n] = a[n] + b[n] * ps
        T[n] = ERA5_T[n,i,j]
        q[n] = ERA5_q[n,i,j] # oppure ricava e[n] da RH
        e[n] = ( q[n] / (epsilon + (1-epsilon)*q[n]) ) * p_ml[n]
        pd[n] = p_ml[n] - e[n]
        Zg[n] = geopotential[n,i,j] / g(phi)
        for n in 1..N-1:
            dz[n] = Zg[n+1] - Zg[n]
            Nn = 77.6*pd[n]/T[n] + 23.3*e[n]/T[n] + 3.73e5*e[n]/T[n]^2
            Nz[n] = Nn * dz[n]
            ZHDz[n] = 77.6*pd[n]/T[n] * dz[n]
            ZWDz[n] = (23.3*e[n]/T[n] + 3.73e5*e[n]/T[n]^2) * dz[n]
            ZTD = 1e-6 * sum(Nz)
            ZHD = 1e-6 * sum(ZHDz) # or Saastamoinen(ps, phi, H)
            ZWD = 1e-6 * sum(ZWDz)
        store maps ZTD/ZHD/ZWD at (i,j)

```

F.9 Pseudocode for WRF-based Zenith & Slant Path Delay

```

% =====
% Algorithm 1: WRF-based Zenith & Slant Path Delay (Sicily case)
% Inputs (per grid cell i,j and vertical levels k = 1..NL):
% p(i,j,k) : perturbation pressure [Pa]
% pb(i,j,k) : base (barometric) pressure [Pa]
% Tprime(i,j,k) : perturbed potential temperature [K] (θ' or T')
% qv(i,j,k) : water vapor mixing ratio (specific humidity) [kg/kg]
% z(i,j,k) : geopotential height or level altitude [m]
% elev_deg : elevation angle for slant delay [deg] (e.g., 80°)
% Constants:
% p0 = 1000 hPa; kappa = 0.2854; Rd = 287.05; Rw = 461.5;
% k1 = 77.604; k2 = 23.3; k3 = 3.739e5; % refractivity coeffs
% Notes:
% All pressures: convert to hPa where needed; Δz in meters.
% =====
function [ZHD, ZWD, ZTD, SPD] = wrf_delay(i,j)
% --- 0) Derived fields & geometry -----
pp = (p(i,j,:) + pb(i,j,:)) / 100; % total pressure [hPa]

```

```
T = (Tprime(i,j,:) + 300) .* (pp / p0).^kappa; % effective temperature [K]
pw = (Rw .* (pp*100) .* qv(i,j,:)) ./ (qv(i,j,:)*Rw + Rd); % vapor pressure [Pa]
Pdry = (pp*100) - pw; % dry-air partial pressure [Pa]
los = 1 / cosd(elev_deg); % slant mapping factor
% --- 1) Refractivity (dry & wet) -----
% Convert pressures to hPa where required by coefficients
Pdry_hPa = Pdry / 100; pw_hPa = pw / 100;
Ndry = k1 * (Pdry_hPa ./ T); % dry refractivity
Nwet = (Rw/Rd) .* (k2 .* pw_hPa ./ T + k3 .* pw_hPa ./ (T.^2)); % wet refractivity
Ntot = Ndry + Nwet;
% --- 2) Vertical spacing -----
% Ensure z is monotonically increasing with k; compute Δz between levels.
z_k = squeeze(z(i,j,:)); % [m]
dz_k = diff(z_k); % Δz(k) for k=1..NL-1
% Pad last level (copy) to match dimension for simple summations:
dz = [dz_k; dz_k(end)];
% --- 3) Zenith delays via discrete integration -----
ZHD = (1/1e4) * sum(Ndry .* dz); % [m] (factor 1e4: N in N-units)
ZWD = (1/1e4) * sum(Nwet .* dz); % [m]
ZTD = ZHD + ZWD; % [m]
% --- 4) Slant Path Delay (line-of-sight) -----
% Simple geometric mapping of refractivity path along the LoS:
SPD = (1/1e4) * sum((Ntot .* dz) * los); % [m]
end
```

F.9.1 Notes & safeguards

- Units: keep pp in hPa for refractivity coefficients, but compute pw and Pdry in Pa then convert back to hPa before Ndry/Nwet.
- Monotonic z: if WRF levels come top-down, flip arrays so $z(k)$ increases; otherwise dz signs will be wrong.
- 80° elevation: $los = 1/\cosd(80)$; for zenith use $los = 1$. For rigorous low-elevations, insert a mapping function (e.g., Niell) instead of pure secant.
- Quality check: report $\text{std}((ZHD+ZWD) - SPD/los)$ as an internal consistency metric over near-zenith rays.
- Wet delay units in figures: present ZWD in mm for readability ($ZWD [\text{mm}] = 1000 \cdot ZWD$).

F.10 Pseudocode for Integrated ZHD/ZWD Computation

```
% =====
```




```
% Algorithm 2: WRF-2 MATLAB Implementation (ERA5 + GMTED → ZHD/ZWD/ZTD/SPD)
% Domain: Sicily (1 km grid). Levels: 37 (default) | 137 (full).
% I/O:
% IN : DEM (GMTED), ERA5 NetCDF (surface + pressure levels), mask Sicily
% OUT : ZHD [m], ZWD [m], ZTD [m], optional SPD [m] @ elevation  $\theta$ 
% =====
function [ZHD, ZWD, ZTD, SPD] = wrf2_pipeline(cfg)
%% --- 0) Config & constants -----
Re = 6371000; % Earth radius [m]
g = 9.80665;
Rd = 287.05; Rw = 461.5; % gas constants
POh = 1000; % 1000 hPa
kappa= 0.2854; % Poisson exponent
k1 = 77.604; k2 = 23.3; k3 = 3.739e5; % Smith-Weintraub
k2p = 22.1; % wet-delay constant [K/hPa]
useNiell = cfg.useNiell; % true|false (mapping)
thetaDeg = cfg.elevDeg; % e.g., 80
levels = cfg.levels; % 37 | 137
usePar = cfg.useParallel; % parfor toggle
%% --- 1) Raster & DEM -----
% Define 1 km georeferenced grid over Sicily
[lat, lon, R] = make_sicily_grid(cfg.extent, cfg.res_km); % returns georasterref R
DEM = load_dem(cfg.dem_path); % GMTED 1 km [m]
%% --- 2) ERA5 ingest (surface + pressure levels) -----
era = read_era5_nc(cfg.era_nc_path, levels); % struct with P,T,q,geopot, etc.
% era.P [Pa or hPa], era.T [K], era.q [kg/kg], era.phi (geopot) [ $\text{m}^2/\text{s}^2$ ]
% Reproject/interp ERA5 to DEM grid (lat,lon) consistently:
[P_lev, T_lev, q_lev, phi_lev] = interp_to_grid(era, lat, lon, R);
%% --- 3) Geopotential → geometric height -----
%  $h = R_e * h_0 / (R_e - h_0)$ , with  $h_0 = \phi/g$ 
h0 = phi_lev ./ g; % geopotential height [m]
hgeo = (Re .* h0) ./ max(Re - h0, 1); % guard against division by ~0
%% --- 4) Barometric surface pressure from DEM (background) -----
% Standard barometric formula (lapse  $L = -6.5\text{e-}3$  K/m)
L = -6.5e-3; Tsl = 288.15; Psl = 101325; M = 0.0289644; Rg = 8.31447;
PB_sfc = Psl .* (1 - L .* DEM ./ Tsl).^(g*M/(Rg*L)); % [Pa]
PBhpa = PB_sfc / 100; % [hPa]
%% --- 5) Perturbation pressure (diagnostic) -----
PBmean = mean(PBhpa(:), 'omitnan');
Ppert = PBhpa - PBmean; % #ok<NASGU> % optional for wave diagnostics
%% --- 6) Moist variables -----
% Mixing ratio from specific humidity:  $w = q/(1-q)$ 
w_lev = q_lev ./ max(1 - q_lev, 1e-9);
% Partial vapor pressure from q and total pressure:
%  $e = q * P / (0.622 + 0.378 q)$  (P in hPa if using k-constants below)
```




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```
P_hPa = P_lev / 100; % [hPa]
e_hPa = (q_lev .* P_hPa) ./ (0.622 + 0.378*q_lev); % [hPa]
Pdrys_hPa = max(P_hPa - e_hPa, 0); % [hPa]
%% --- 7) Potential temperature & effective temperature -----
%  $\theta = T * (P_0/P)^{\kappa}$ ; Teff for refractivity uses T (already absolute)
theta = T_lev .* (P0h ./ max(P_hPa,1)).^kappa; %ok<NASGU>
%% --- 8) Refractivity (Smith-Weintraub) -----
Ndry = k1 * (Pdrys_hPa ./ max(T_lev, 1)); % N-units
Nwet = (Rw/Rd) .* (k2 .* (e_hPa ./ max(T_lev,1)) + ...
k3 .* (e_hPa ./ max(T_lev.^2,1))); % N-units
Ntot = Ndry + Nwet;
%% --- 9) Vertical integration to delays -----
% Ensure vertical axis is monotonically increasing in height:
[Ndry, Nwet, Ntot, hgeo] = sort_levels_by_height(Ndry, Nwet, Ntot, hgeo);
% Compute  $\Delta h$  per level and broadcast over (i,j):
dh = diff(hgeo,1,3); % [m] (levels-1)
dh = cat(3, dh, dh(:,:,end)); % pad to match size
% Zenith delays [m]: 1e4 factor converts N-units to SI via discrete sum
ZHD = (1/1e4) * sum(Ndry .* dh, 3);
ZWD = (1/1e4) * sum(Nwet .* dh, 3);
ZTD = ZHD + ZWD;
%% --- 10) Optional Slant Path Delay (SPD) @ elevation theta -----
if cfg.computeSPD
    if useNiell
        m_h, m_w = niell_mapping(lat, hgeo(:,:,1), thetaDeg); % mapping funcs
        SPD = (1/1e4) * sum(Ndry .* dh .* m_h + Nwet .* dh .* m_w, 3);
    else
        los = 1 / cosd(thetaDeg); % simple secant
        SPD = (1/1e4) * sum(Ntot .* dh, 3) .* los;
    end
else
    SPD = [];
end
%% --- 11) Performance options -----
% - To reduce memory: read ERA5 in chunks or with matfile; keep 37 levels.
% - To scale to 137 levels: enable chunked IO + parfor over tiles/time.
% - Interpolations: switch to griddedInterpolant; parfor over (i,j) tiles.
end
% ----- Helper stubs (sketch) -----
function [lat,lon,R] = make_sicily_grid(extent,res_km)
% extent = [lat_min lat_max lon_min lon_max]
% res_km: 1 → ~1 km
% returns georasterref R + 2D lat/lon grids at target resolution
end
function dem = load_dem(path)
```



```
% Load GMTED 1 km DEM (e.g., .mat) and align to grid
end
function era = read_era5_nc(ncpath, levels)
% Read ERA5 NetCDF variables; select 37 or 137 pressure levels
% Return fields as 3D arrays aligned to (lat,lon,level)
end
function [P,T,q,phi] = interp_to_grid(era, lat, lon, R)
% Spatially interpolate ERA5 variables onto DEM grid (lat,lon)
% Use consistent method (bilinear / gridding) for all variables
end
function [A,B,C,H] = sort_levels_by_height(A,B,C,H)
% Sort level dimension so height increases; permute arrays accordingly
end
function [m_h, m_w] = niell_mapping(lat, h_sfc, elevDeg)
% Niell mapping functions for hydrostatic (m_h) and wet (m_w) components
% (implement coefficients or call existing routine)
end
```

F.11 MATLAB Implementation of the Global Grid Model for Zenith Tropospheric Delay Estimation (GGZTD-P).

```
import scipy.io
import numpy as np
# Define parameters
N = 365 # Number of daily samples (1 year)
t = np.linspace(0, 365, N) # Time vector in days
# Generate synthetic cloud optical thickness data
A0 = 10 # Mean optical thickness
A1 = 3 # Amplitude of annual variation
A2 = 1.5 # Amplitude of semi-annual variation
A3 = 0.5 # Amplitude of daily variation
# Compute synthetic data with sinusoidal variations
tau = (A0 +
        A1 * np.cos(2 * np.pi * t / 365) +
            # Annual cycle
        A2 * np.cos(4 * np.pi * t / 365) +
            # Semi-annual cycle
        A3 * np.cos(2 * np.pi * t / 1))
            # Daily variation (not dominant)
# Save to .mat file
mat_file_path = "/mnt/data/cloud_optical_thickness.mat"
scipy.io.savemat(mat_file_path, {'tau': tau, 't': t})
# Return path to the saved file
mat_file_path
```



```
% Load Cloud Optical Thickness Data
% (Example: daily measurements over a year)
load('cloud_optical_thickness.mat');
% Assume variable "tau" contains daily optical thickness
% Define time vector (assuming daily measurements)
N = length(tau); % Number of time samples
t = linspace(0, 365, N); % Time in days (1 year)
% Apply Fast Fourier Transform (FFT)
F_tau = fft(tau); % Compute FFT of optical thickness
frequencies = (0:N-1) / N; % Normalize frequency axis
% Extract dominant Fourier coefficients
A0 = abs(F_tau(1)) / N; % Mean optical thickness
A1 = abs(F_tau(2)) / (N/2); % Annual variation component
A2 = abs(F_tau(3)) / (N/2); % Semi-annual variation component
% Reconstruct Fourier series model
tau_reconstructed = A0 + A1*cos(2*pi*t/365) + A2*cos(4*pi*t/365);
% Plot Results
figure;
plot(t, tau, 'k', 'LineWidth', 1.2); hold on;
plot(t, tau_reconstructed, 'r--', 'LineWidth', 1.5);
xlabel('Day of Year'); ylabel('Cloud Optical Thickness');
legend('Original Data', 'Fourier Model');
title('Fourier Decomposition of Cloud Optical Thickness');
grid on;
```

F.12 Source Code for Figure 10.21

```
% Fourier Analysis of Cloud Optical Thickness()
clc; clear; close all;
%% Generate Synthetic Cloud Optical Thickness Data
Fs = 24; % Sampling frequency (e.g., hourly observations)
T = 1/Fs; % Sampling interval
L = 365; % Number of days (one year of data)
t = (0:L-1) * T; % Time vector in days
% Define cloud types with different frequency components
tau_cirrus = 0.2 + 0.05*sin(2*pi*10*t) + 0.02*sin(2*pi*30*t); % High-frequency
tau_stratus = 0.5 + 0.1*sin(2*pi*2*t) + 0.05*sin(2*pi*5*t); % Low-frequency
% Add some random noise to simulate measurement variations
tau_cirrus = tau_cirrus + 0.01*randn(size(t));
tau_stratus = tau_stratus + 0.02*randn(size(t));
%% Apply FFT
Y_cirrus = fft(tau_cirrus);
Y_stratus = fft(tau_stratus);
% Compute the two-sided spectrum and then convert to single-sided
P2_cirrus = abs(Y_cirrus/L);
P2_stratus = abs(Y_stratus/L);
% Ensure index is an integer
```



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```
idx = floor(L/2) + 1;
P1_cirrus = P2_cirrus(1:idx);
P1_cirrus(2:end-1) = 2*P1_cirrus(2:end-1);
P1_stratus = P2_stratus(1:idx);
P1_stratus(2:end-1) = 2*P1_stratus(2:end-1);
% Frequency axis
f = Fs*(0:(idx-1))/L;
%% Plot Time Series
figure;
subplot(2,1,1);
plot(t, tau_cirrus, 'r', 'LineWidth', 1.2); hold on;
plot(t, tau_stratus, 'b', 'LineWidth', 1.2);
xlabel('Time (Days)');
ylabel('Optical Thickness (\tau)');
title('Cloud Optical Thickness - Cirrus vs. Stratus');
legend('Cirrus (High-Freq)', 'Stratus (Low-Freq)');
grid on;
%% Plot Frequency Spectrum
subplot(2,1,2);
plot(f, P1_cirrus, 'r', 'LineWidth', 1.5); hold on;
plot(f, P1_stratus, 'b', 'LineWidth', 1.5);
xlabel('Frequency (Cycles per Day)');
ylabel('Magnitude');
title('Fourier Spectrum of Optical Thickness');
legend('Cirrus (High-Freq)', 'Stratus (Low-Freq)');
grid on;
```

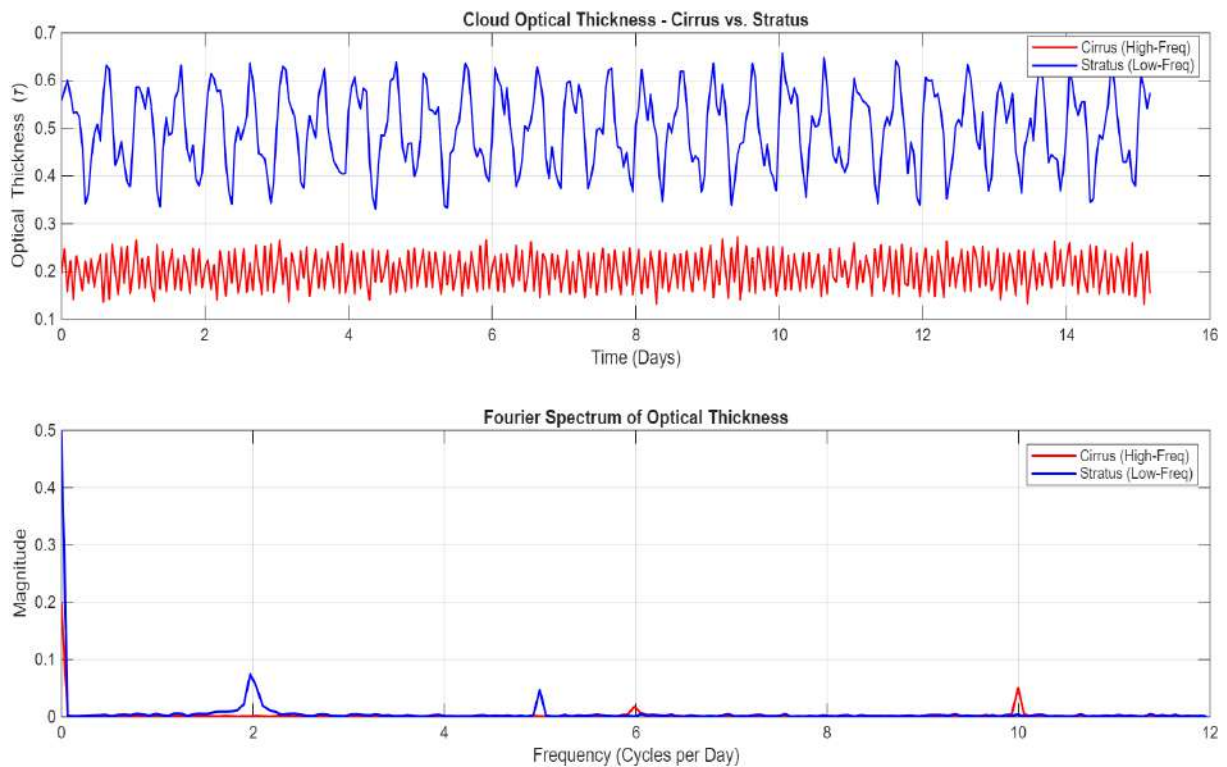


Figure F.11. Temporal variability and spectral characterization of cloud optical thickness for different cloud regimes. (Top) Time series of cloud optical thickness τ over a 15-day period for two representative cloud types: cirrus clouds, characterized by lower optical thickness and higher-frequency variability (red), and stratus clouds, exhibiting larger optical thickness and dominant low-frequency fluctuations (blue). (Bottom) Corresponding Fourier spectra of τ , highlighting the separation between high-frequency components associated with cirrus variability and low-frequency peaks associated with stratus cloud evolution.



Chapter G

Appendix G – From Chapter 12

G.1 MATLAB Implementation Using the Niell Mapping Function

The **Niell Mapping Function (NMF)** is applied to convert zenith delays into slant-path delays as a function of the elevation angle. The mapping coefficients depend on the site latitude and are defined as:

$$M_{\text{hyd}}(e) = \frac{1}{\sin e + \frac{a_{\text{hyd}}}{\sin e + \frac{b_{\text{hyd}}}{\sin e + c_{\text{hyd}}}}}, \quad M_{\text{wet}}(e) = \frac{1}{\sin e + \frac{a_{\text{wet}}}{\sin e + \frac{b_{\text{wet}}}{\sin e + c_{\text{wet}}}}} \quad (\text{G.1})$$

where (a, b, c) are empirical coefficients specific to the hydrostatic and wet components. **Figure G.1** shows the time series of **Slant Path Total Delay (SPTD)** computed with the **Niell Mapping Function** for **February 2019–2024**. The plot includes the SPTD (total delay, red circles with yellow markers), the **SPHD** (hydrostatic component, blue area), and the **SPWD** (wet component, green dotted line).

Figure G.2 presents the time series of the **Zenith Hydrostatic Delay (ZHD)** for **February 2019–2024**, with the x-axis showing time in hours and the y-axis the delay in meters. Each subplot corresponds to a different year, displaying hourly ZHD variations over the 28-day period. The values, influenced by atmospheric pressure and latitude-dependent corrections, reveal seasonal and inter-annual trends at the Scanzano site. The visualization highlights the contrasting behavior of tropospheric delay components: the SPHD remains relatively stable, while the SPWD exhibits significant fluctuations, driving the variability of the SPTD. These effects are most evident at low elevation angles (15°), where simple secant mapping becomes inaccurate. The **Niell Mapping Function** provides a more accurate correction, compensating for atmospheric curvature and improving delay estimation. Distinct colors are used to differentiate the years, and the grid layout facilitates tracking of temporal variations in hydrostatic delay, relevant for GNSS positioning and satellite remote-sensing applications.

Figure G.3 presents six subplots of Slant Path Tropospheric Delay (SPTD) time series for

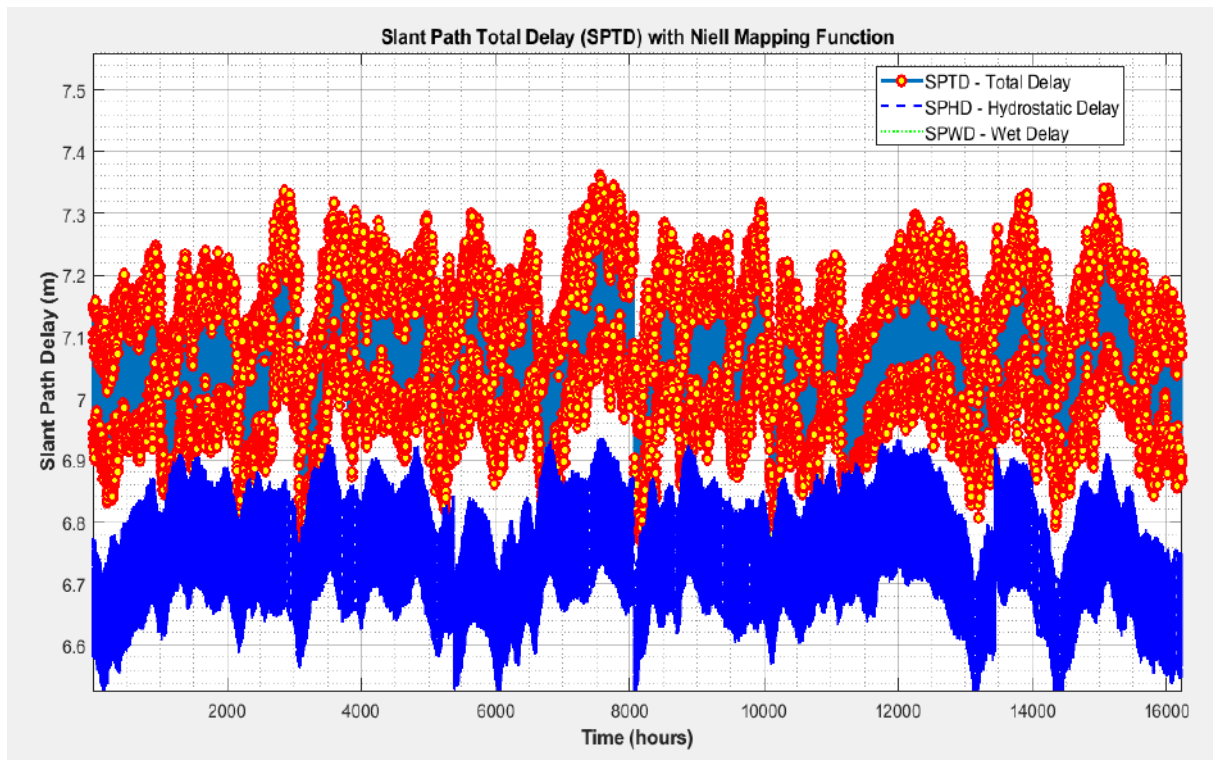


Figure G.1. Slant Path Total Delay (SPTD) with Niell Mapping Function for February 2019–2024.

February 2019–2024, with each panel corresponding to one year. The x-axis represents time in hours (0–3000), and the y-axis indicates SPTD in meters (~ 6.8 – 7.4 m).

Across all years, the curves display dense short-term fluctuations with slight inter-annual shifts in mean values, reflecting the natural atmospheric variability typical of February over the Scanzano site.

Figure G.4 presents a set of plots from this study showing the time series of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) for February 2019–2024 at the Scanzano (PA) site. Each subplot illustrates the temporal evolution of these components, highlighting their seasonal and inter-annual variability within the troposphere.

Figure G.5 show a collection of SPWD trends from **February 2019–2024** in different colors on a range periods evaluated in hours.

Figure G.6 illustrates the temporal evolution and statistical distribution of the Zenith Hydrostatic Delay (ZHD) for February 2019–2024, derived from ERA5 reanalysis around the Scanzano (PA) site. The analysis covers the four nearest grid points — (1,1), (1,2), (2,1), and (2,2) — defining a 56 km^2 domain centered on Scanzano. The visualization consists of two main parts. The left column shows the ZHD time series: the upper panel reports overall variability across all grid points, while the lower panels detail annual trends for point (1,1). The right column presents histogram distributions: the top plot summarizes the global ZHD distribution for all years, and the lower plots display annual frequency distributions for point (1,1). The results indicate that ZHD variability is primarily governed by surface pressure and temperature, ex-

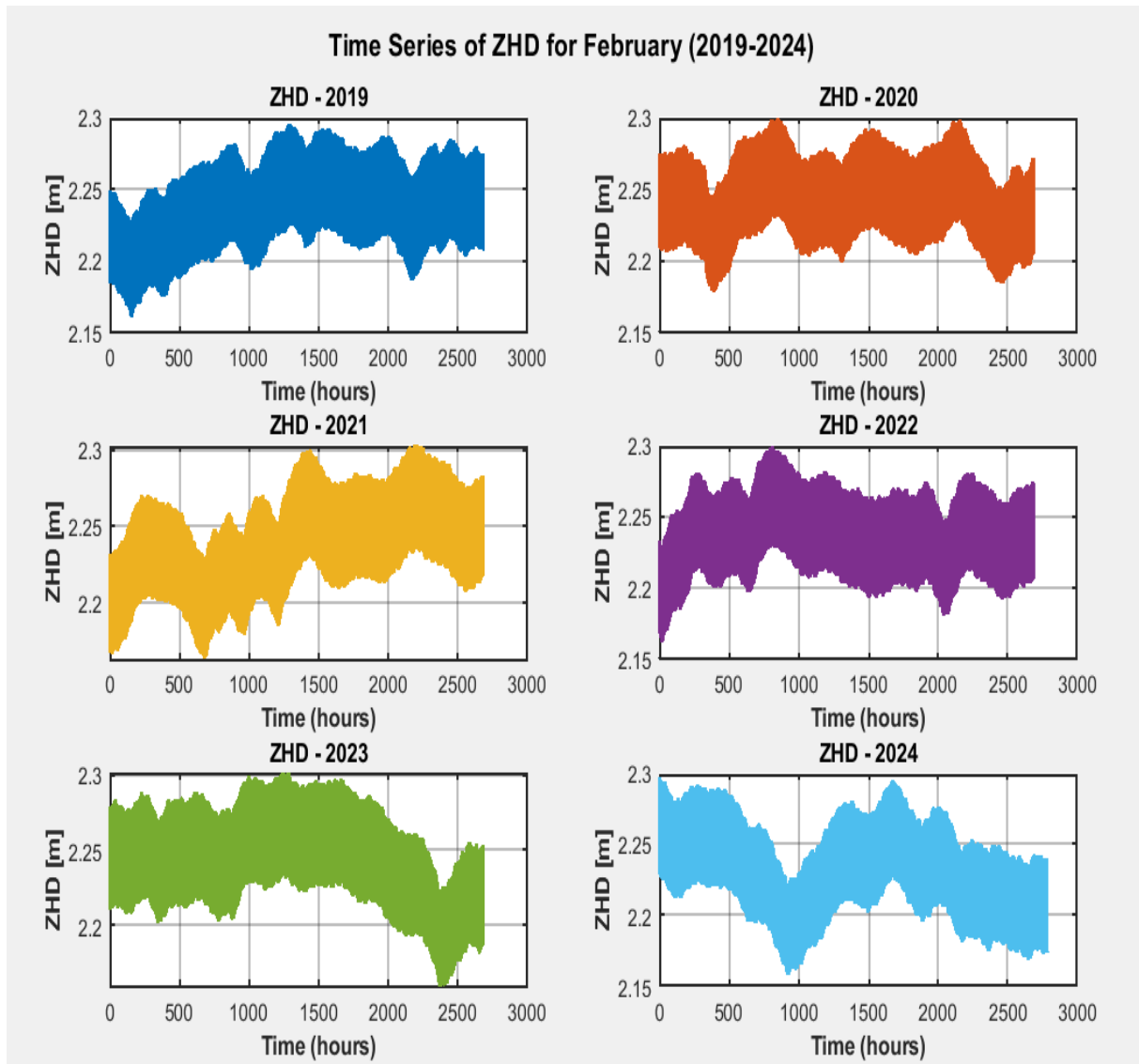


Figure G.2. Time Series of Zenith Hydrostatic Delay (ZHD) for February (2019-2024).



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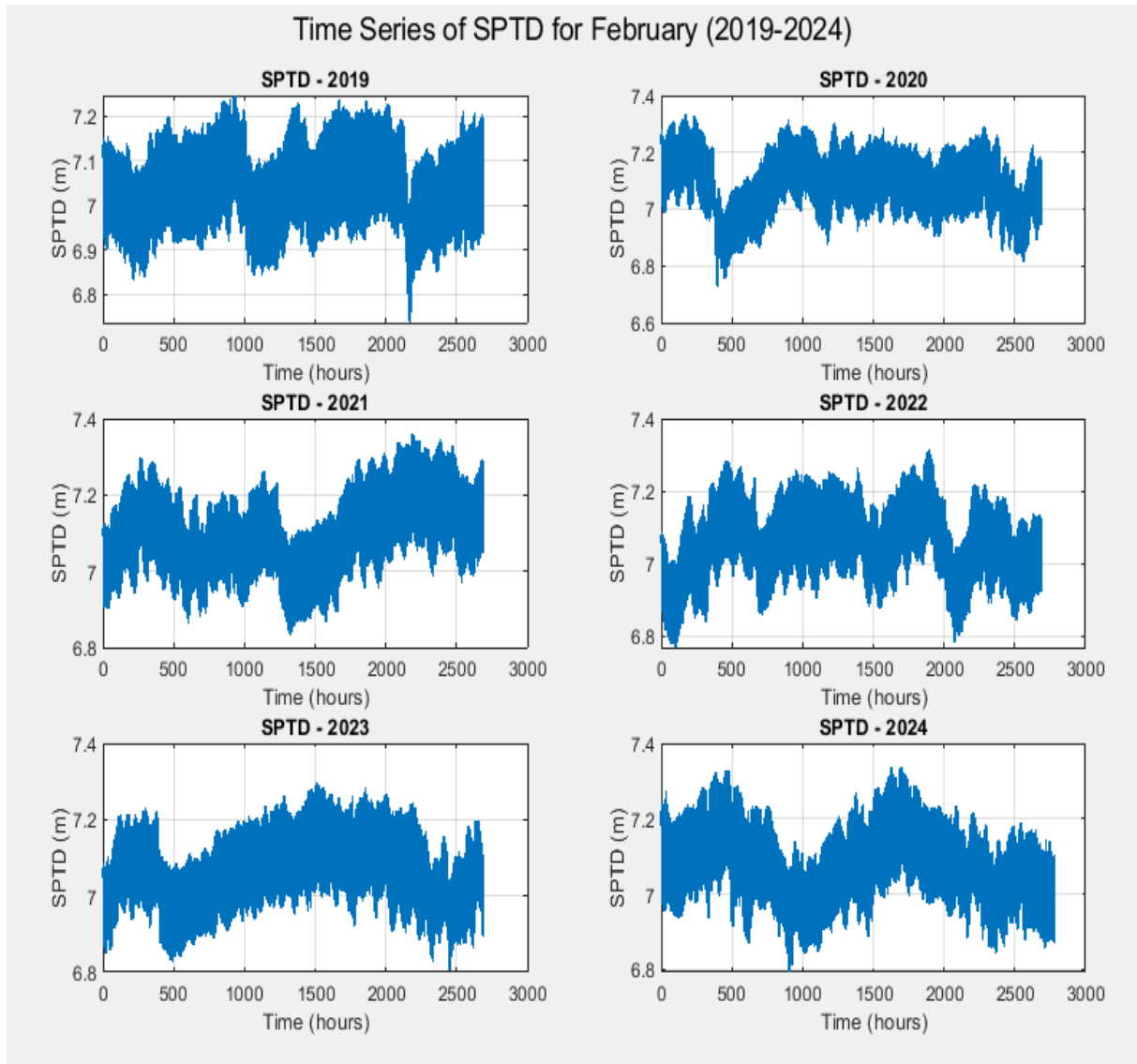


Figure G.3. Slant Path Delay in the period February 2019-2024 with 15° of elevation in Scanzano (PA).



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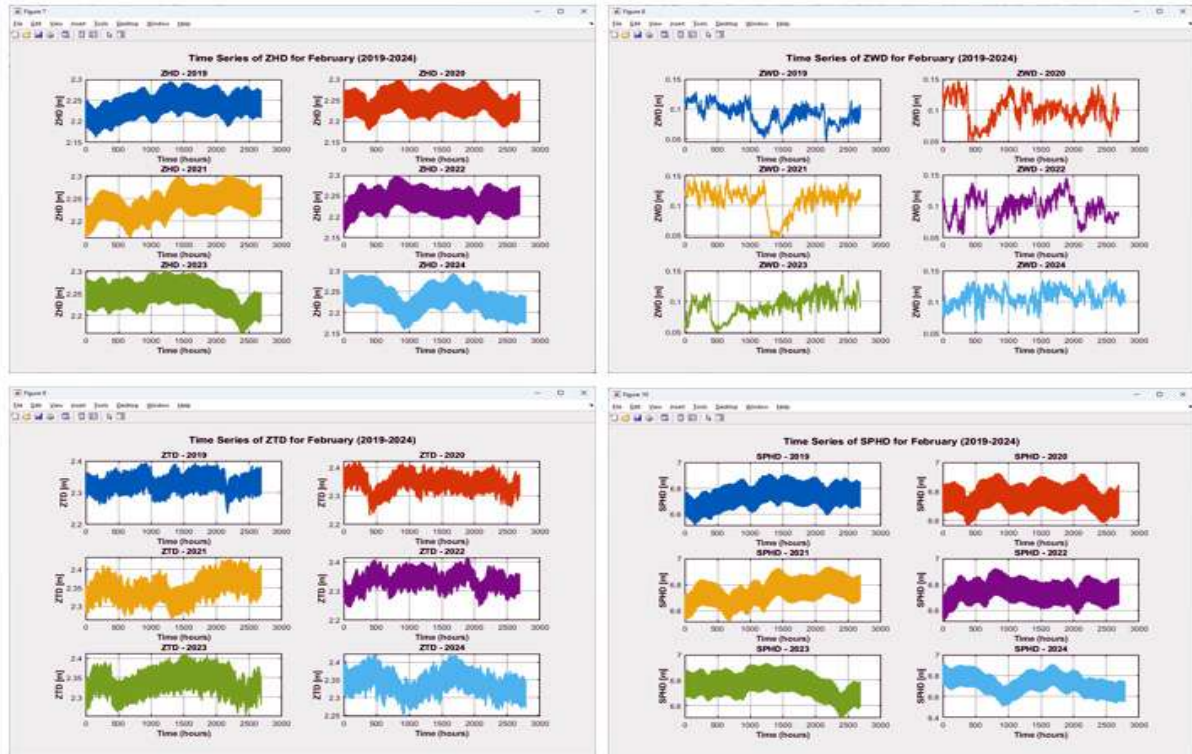


Figure G.4. Collection of Trend for Time Series realized with the MATLAB program for study in February 2019-2024 with 90° and 15° of elevation angle, in Scanzano (PA).

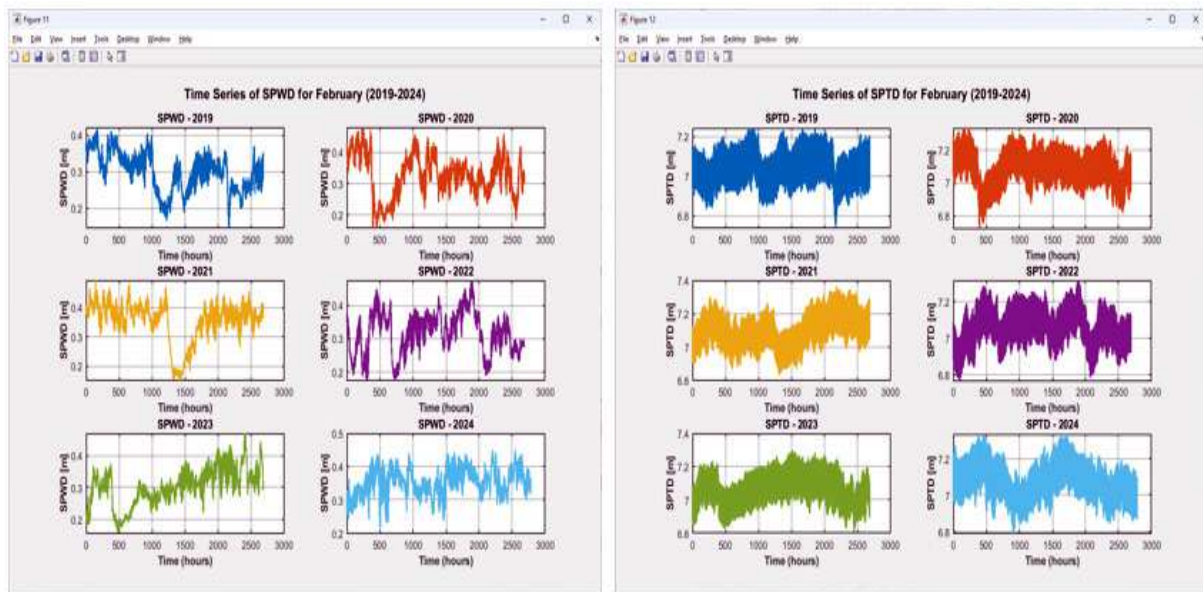


Figure G.5. Collection of Trend for Time Series realized with the MATLAB program for study in February 2019-2024 with 90° and 15° of elevation angle, in Scanzano (PA).



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hibiting regular periodic fluctuations with minimal inter-annual deviations. Histogram analysis confirms a narrow and stable hydrostatic delay range, evidencing overall atmospheric steadiness with modest seasonal modulation. These results provide a robust foundation for tropospheric delay modeling and for deriving Slant Path Hydrostatic Delay (SPHD) via mapping functions, supporting applications in GNSS geodesy, radar meteorology, and radiative transfer simulations.

G.2 NetCDF File Database from ERA5 Copernicus

From the ERA5 dataset extracted via targeted queries, the parameters of interest—longitude [°], latitude [°], and time—were selected to ensure that the **Scanzano** (PA) site lies at the center of the target cell. The main variables considered are listed in **Appendix E**.

G.2.1 Technical Analysis

Figure G.7 summarizes six years (2019–2024) of hydrostatic Slant Path Delay (SPD Hydro) at a 15° elevation angle, showing both its temporal evolution and statistical distribution. The time series display quasi-periodic oscillations driven by synoptic-scale pressure systems, while histograms highlight modest interannual variations in mean delay. Across all years, SPD Hydro remains confined within a narrow 9.42–9.445 m range, confirming its dependence on surface pressure and resilience to short-term perturbations. Subtle annual differences are observed: smoother oscillations in 2019–2020, balanced phases in 2021, clustering near 9.435 m under anticyclonic conditions in 2022, slightly lower baselines in 2023, and broader dispersion in 2024. This sub-millimetric variability establishes SPD Hydro as a stable reference for tropospheric modeling—predictable, reproducible, and effectively invariant at the link-budget scale. While wet delays dominate uncertainty in high-frequency Earth–space links, the hydrostatic slant delay defines the minimum atmospheric load, remaining stable within ± 5 mm at 15°, and providing a reliable baseline for GNSS geodesy and W-band communication design.

Table G.1 summarizes the seasonal fade margins required across the S, X, Ku, Ka, and W frequency bands under both clear-sky and rain conditions. Clear-sky contributions include gaseous, cloud, and scintillation losses, while the rain term accounts for percentile-based severe precipitation statistics. At lower frequencies (S–X), margins remain modest (1–3 dB), ensuring stable operation year-round. In the Ku–Ka range, rain attenuation becomes dominant, requiring 10–30 dB margins depending on season. At W-band, fade margins exceed 150–200 dB during rain, leading to complete link outages. Consequently, W-band links at Scanzano are only feasible under clear-sky conditions or when supported by **Adaptive Coding and Modulation (ACM)**, frequency diversity, or fallback strategies to lower bands.

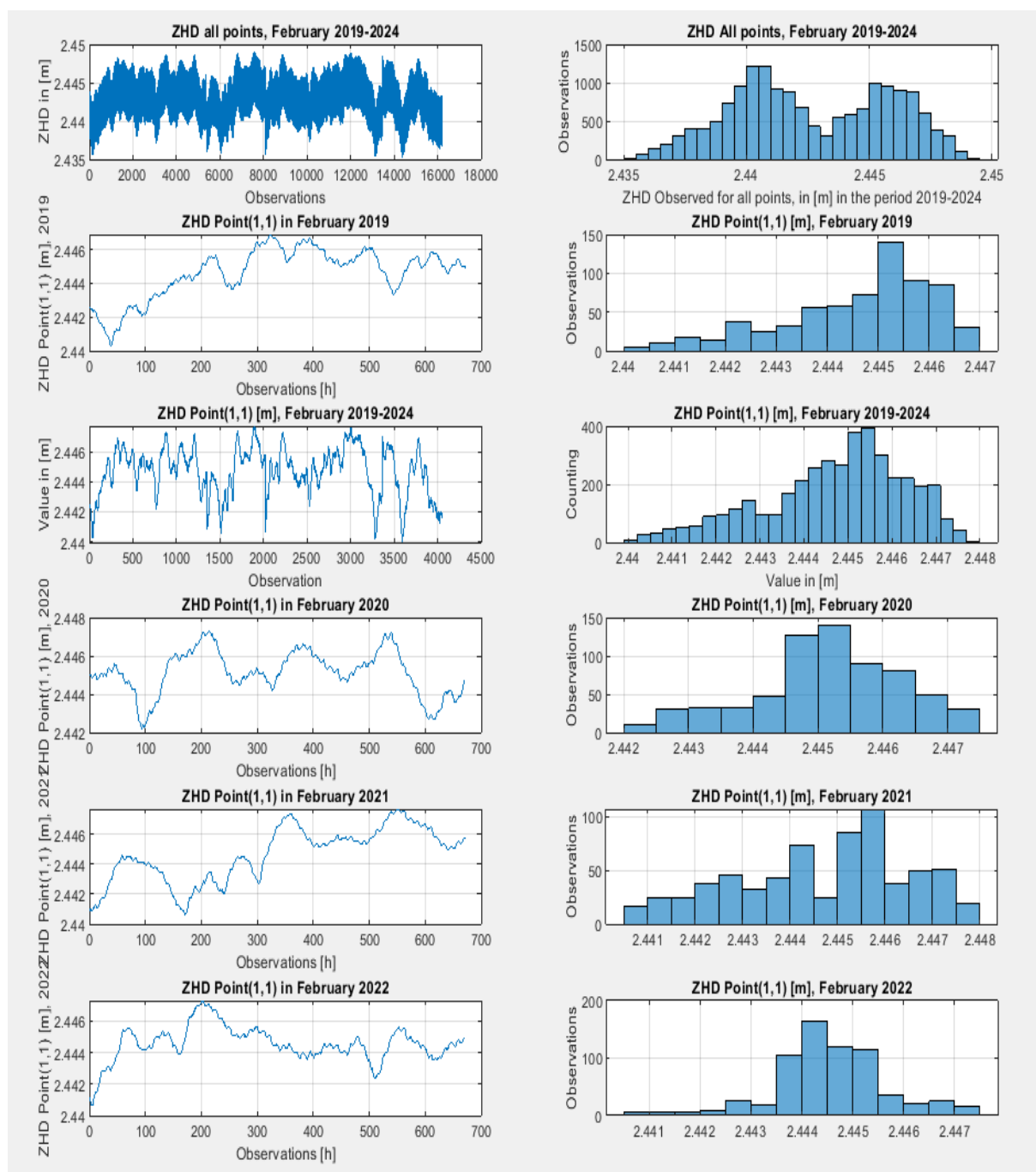


Figure G.6. Time Series and Statistical Distributions of Zenith Hydrostatic Delay (ZHD) for February 2019–2024. (Data is for point (1,1) near Scanzano (PA)).

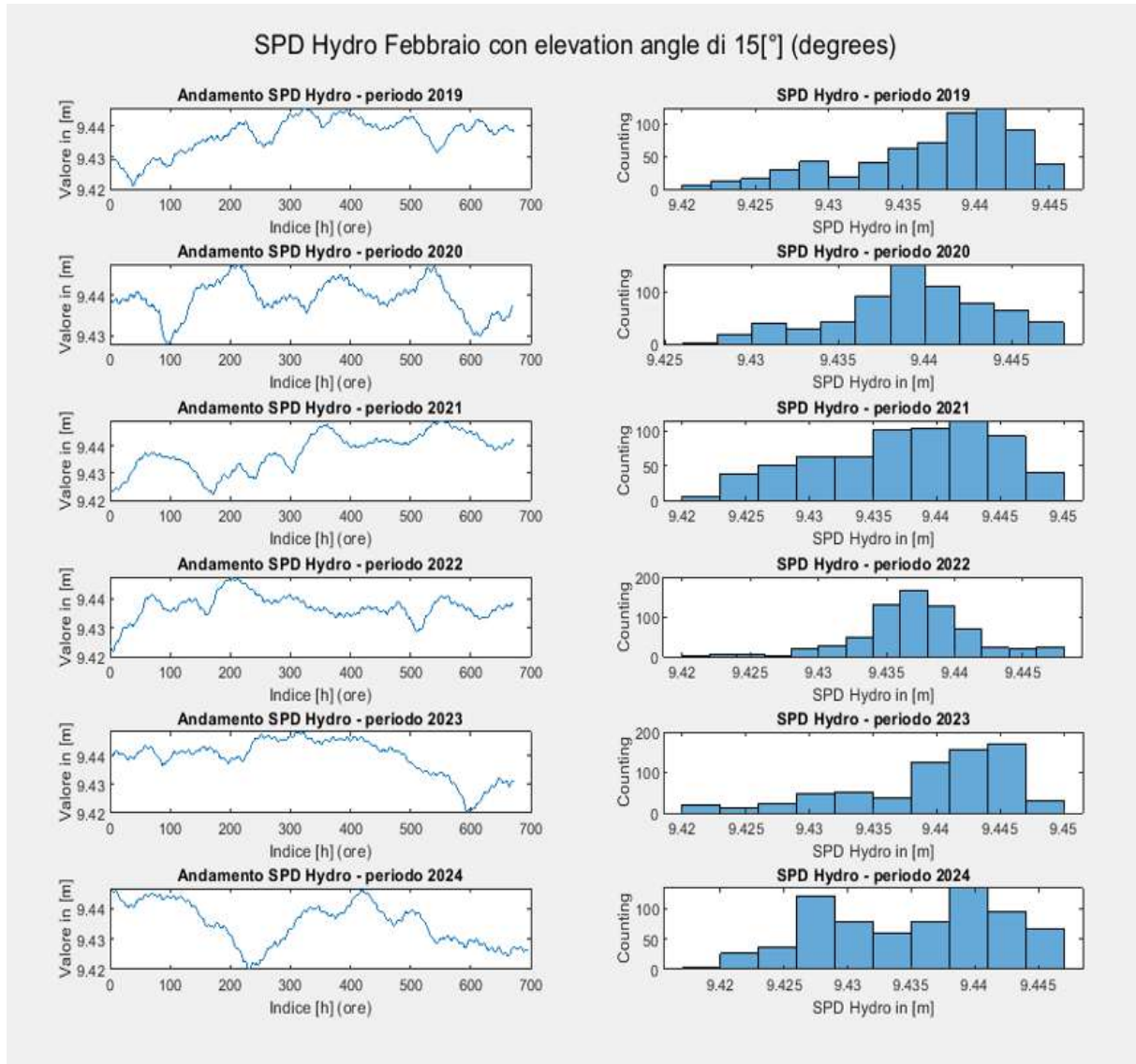


Figure G.7. Comparison of six February periods (2019–2024) of hydrostatic slant path delay (SPDHydro) at 15° elevation. The distributions exhibit a quasi-stationary behavior with mean values around 9.43–9.44 m and inter-annual excursions of only a few millimetres. February 2022 presents the most compact clustering near the mean, whereas February 2024 shows broader variability and a higher occurrence of deviations from the modal value. These findings confirm the deterministic nature of the hydrostatic delay term, which remains a stable reference within the tropospheric propagation budget despite seasonal and synoptic-scale variability. **Dataset from ERA5 with Monthly mean by hour of days.**



Band	Season	Clear-Sky (gas+cloud+Scint) [dB]	+Rain add [dB]	Total Recc.
S	Winter	0.3	+0.5	0.8
	Spring	0.3	+0.4	0.7
	Summer	0.3	+0.8	1.1
	Autumn	0.3	+0.6	0.9
X	Winter	1.5	+1.0	2.5
	Spring	1.6	+0.8	2.4
	Summer	1.8	+3	4.8
	Autumn	1.7	+1.5	3.2
Ku	Winter	3.5	+3	6.5
	Spring	3.8	+2	5.8
	Summer	4.5	+6	10.5
	Autumn	4.2	+4	8.2
Ka	Winter	9.5	+25	34.5
	Spring	10.5	+18	28.5
	Summer	12.5	+40	52.5
	Autumn	11.0	+30	41
W	Winter	27	+200†	≈ 227*
	Spring	31	+130†	161
	Summer	39	+250†	outage*
	Autumn	33	+180†	213

Table G.1. Seasonal fade margins at 15° elevation (Scanzano, PA) for 99.9% link availability across S–W bands. Clear-sky values include gaseous, cloud, and scintillation losses; rain margins reflect percentile-based extreme events. Margins stay within 1–3 dB for S–X, rise to 10–30 dB at Ku–Ka, and exceed 150–200 dB at W-band, where rain causes outage unless mitigated by ACM, frequency diversity, or lower-band fallback.

G.2.2 Hourly ZWD Variations from the ERA5 Dataset for February 2019–2025 at Scanzano (PA)

Hourly Zenith Wet Delay (ZWD) variations were first analyzed using the ERA5 monthly mean by hour of day product, which outlines the climatological diurnal cycle but lacks true temporal continuity. For higher temporal precision, the ERA5 hourly single-level data (1940–present) provide continuous time series at full resolution, enabling a more accurate characterization of short-term variability.

During **February 2020**, the ZWD at Scanzano fluctuates by a few millimetres, primarily reflecting variations in atmospheric water vapor. Peaks and troughs correspond to synoptic-scale humidity changes, while the overall pattern indicates limited seasonal dynamics typical of winter conditions at the site.

- **Figure G.8** compares the mean diurnal evolution of ZWD at Scanzano, expressed in both UTC (left) and local CET (right). Based on ERA5 monthly mean data, it reveals a weak but consistent diurnal modulation, with maxima in the late afternoon–evening and minima in the early morning. The small amplitude ($\approx 2\text{--}3\text{ mm}$) reflects the interplay of daytime heating, boundary-layer humidity, and nocturnal cooling, providing first-order insight into the site’s diurnal water-vapor dynamics.
- **Figure G.9** depicts ZWD anomalies during February 2020, showing an oscillatory pattern with a $\sim 24\text{ h}$ period. Variations, ranging from micrometres to millimetres, stem from diurnal heating–cooling cycles and local humidity changes. Although derived from smoothed monthly-mean data, the persistent diurnal signature remains clearly visible.
- **Figure G.10** presents the hourly ZHD fluctuations at Scanzano for the same period, confined within $\pm 0.3\text{ mm h}^{-1}$, confirming the strong stability of the hydrostatic term. Minor deviations correspond to synoptic pressure changes, while the overall steadiness highlights the deterministic behavior of ZHD compared to the more variable wet component.
- **Figure G.11** shows the mean diurnal variation of ZHD (UTC and CET). The oscillation amplitude ($\approx 0.3\text{--}0.4\text{ mm}$) reflects weak diurnal pressure tides, confirming ZHD’s quasi-stationary dependence on surface pressure.
- **Figure G.12** displays daily anomalies of ZHD during February 2020, remaining within $\pm 1\text{ mm}$, indicative of weak sensitivity to short-term atmospheric changes. The signal follows large-scale pressure variations with minimal oscillations, confirming ZHD’s predictable and slowly varying nature.
- **Figure G.13** provides a detailed temporal view of ZHD, fluctuating within a $2\text{--}3\text{ cm}$ range in response to synoptic pressure systems. Unlike the wet component, no abrupt transitions occur, reinforcing the view that the hydrostatic delay is smooth, stable, and pressure-driven.

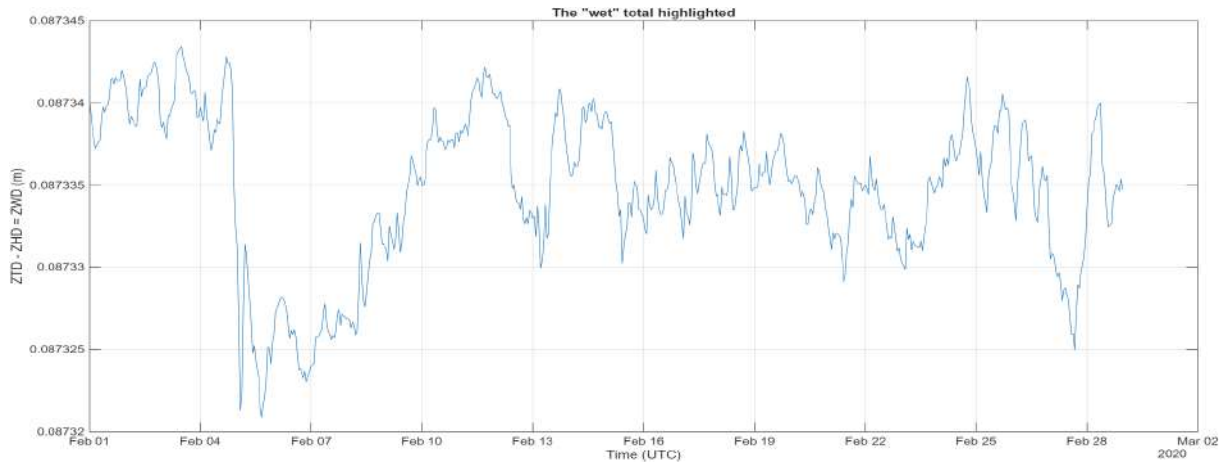


Figure G.8. February trend of the wet component for 2020 in Scanzano (PA) — 560 [m]. **Dataset from ERA5 with Monthly mean by hour of days.**

- **Figure G.14** compares ZWD (top) and ZHD (bottom) hourly anomalies. ZWD shows pronounced variability (up to ± 6 mm) driven by humidity changes, whereas ZHD remains within ± 0.3 mm, emphasizing their contrasting dynamics: the hydrostatic term acts as a quasi-stationary baseline, while the wet term dominates short-term variability.

Overall, the **February 2020** analysis at Scanzano underscores the complementary behavior of the hydrostatic and wet delay components. ZHD provides a stable reference, reliably modeled from surface pressure, while ZWD represents the primary source of temporal variability, requiring high-resolution, site-specific monitoring to mitigate its impact on high-frequency communication links, especially in the W-band.

The temporal analysis of the Zenith Wet Delay (ZWD) and Zenith Hydrostatic Delay (ZHD) components provides essential insight into the short- and long-term atmospheric variability at the Scanzano site during February 2023.

Figure G.16 summarizes both the absolute and anomaly representations of the wet and dry zenith delays, allowing for a direct comparison between their diurnal signatures and temporal dynamics.

Panel (a) illustrates the complete ZWD time series derived from ERA5 reanalysis data, where distinct fluctuations associated with synoptic-scale humidity changes can be observed. The corresponding diurnal cycle (**panel b**) reveals a clear oscillatory pattern with peaks typically occurring during early afternoon hours, consistent with enhanced tropospheric water vapor content under daytime convective conditions. When the daily mean is removed (**panel c**), the ZWD anomaly highlights higher-frequency perturbations, while the hourly change rate in **panel (d)** quantifies short-term variability in mm h^{-1} , evidencing strong fluctuations due to transient meteorological processes.

Panels (e–h) describe the hydrostatic counterpart. The ZHD diurnal cycle (e) exhibits a smoother and nearly sinusoidal pattern primarily driven by pressure oscillations, while the zoomed temporal view (f) confirms its relatively stable evolution. The ZHD anomaly in millimetres



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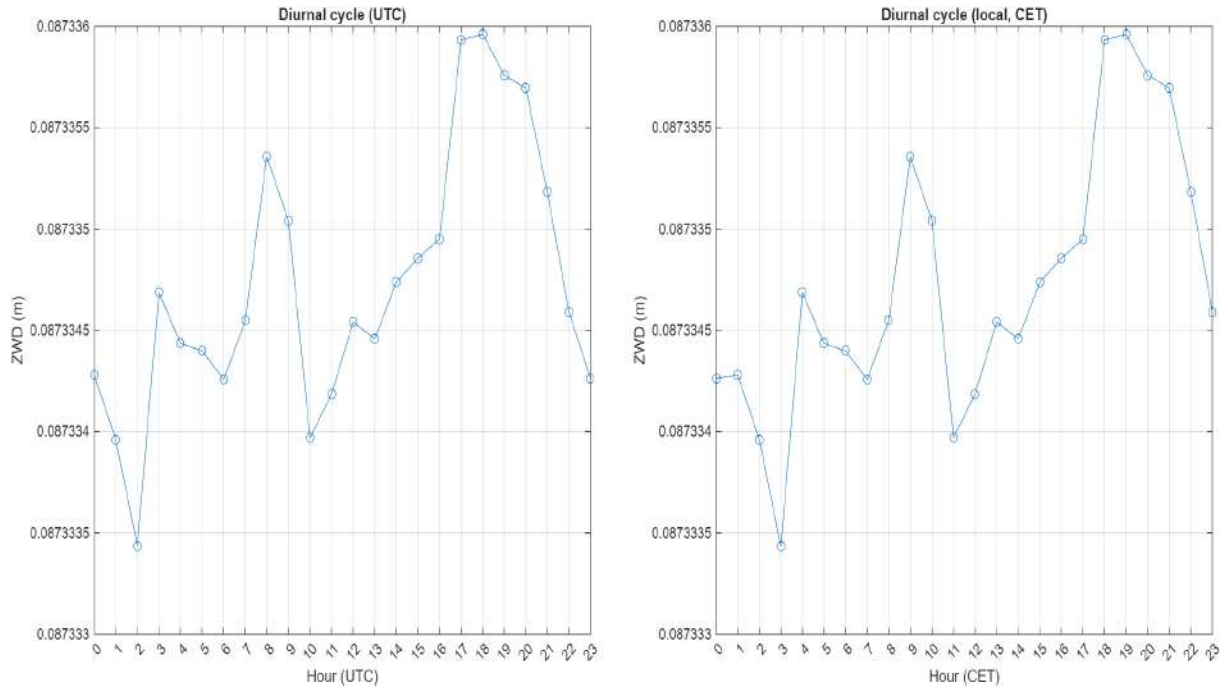


Figure G.9. Diurnal cycle of ZWD in Scanzano (PA). Dataset from ERA5 with Monthly mean by hour of days.

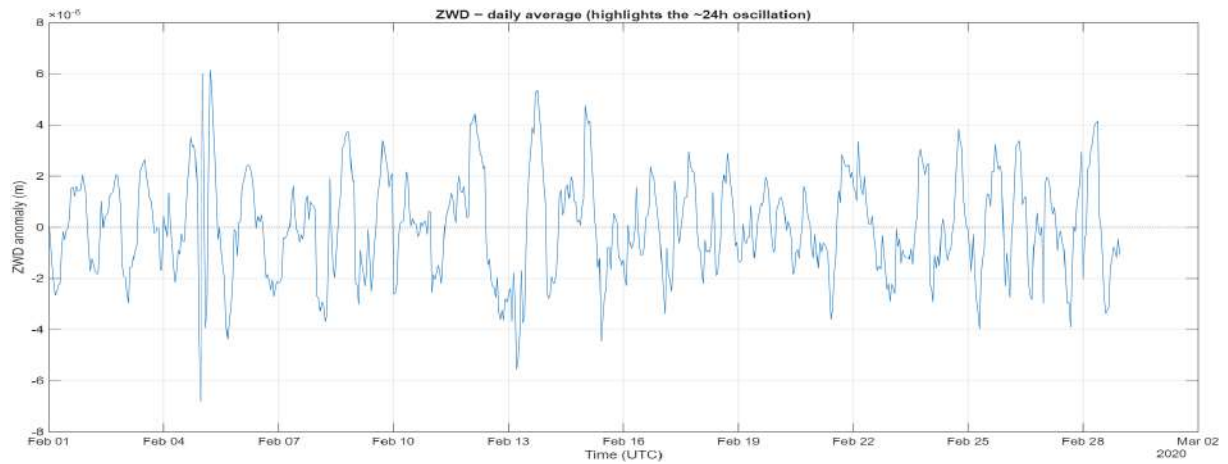


Figure G.10. Daily average of ZWD for February 2020 in Scanzano (PA). Dataset from ERA5 with Monthly mean by hour of days.

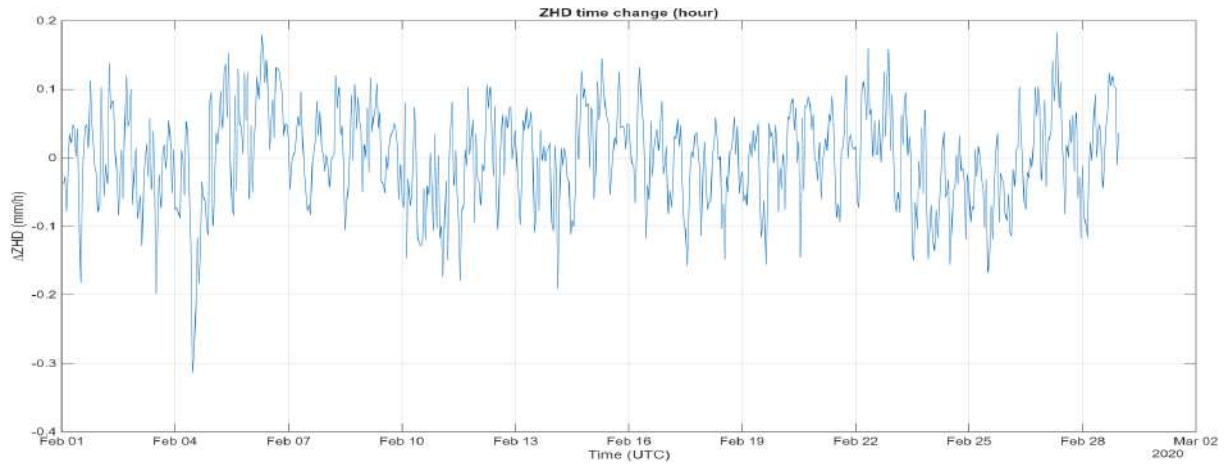


Figure G.11. Hourly variation of ZHD for February 2020 in Scanzano (PA). Dataset from ERA5 with Monthly mean by hour of days.

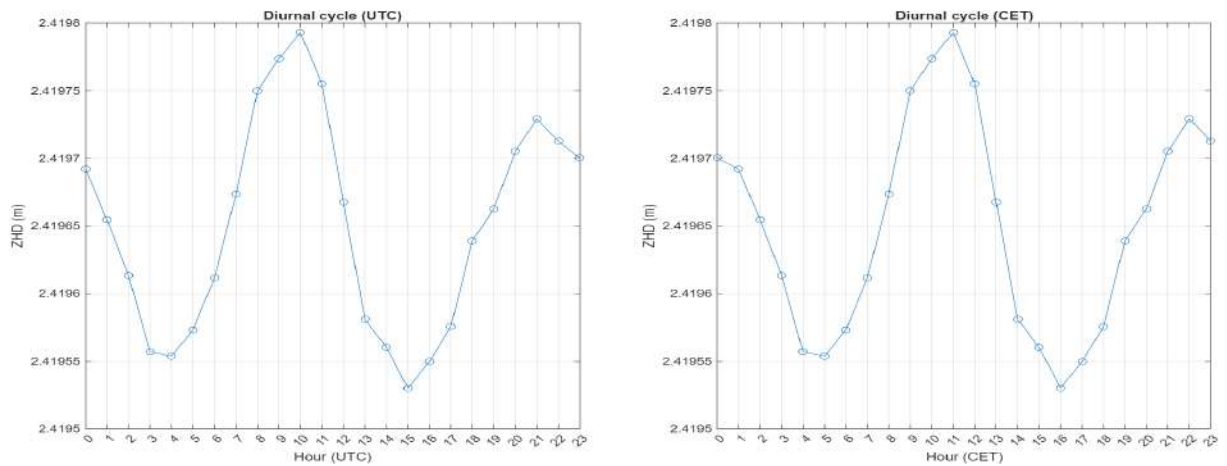


Figure G.12. Diurnal cycle of ZHD in Scanzano (PA), February 2020. Dataset from ERA5 with Monthly mean by hour of days.

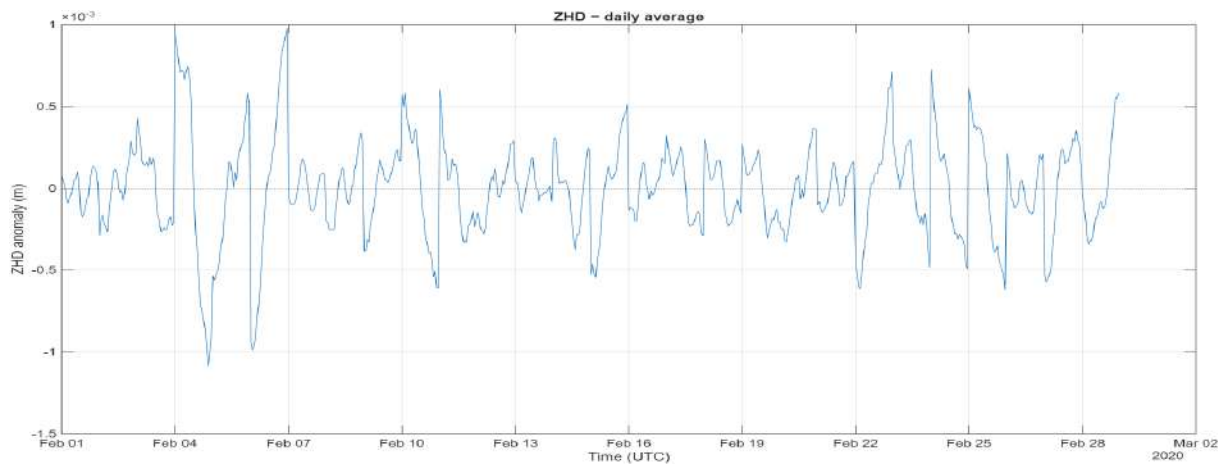


Figure G.13. Daily average of ZHD in Scanzano (PA) for February 2020. Dataset from ERA5 with Monthly mean by hour of days.



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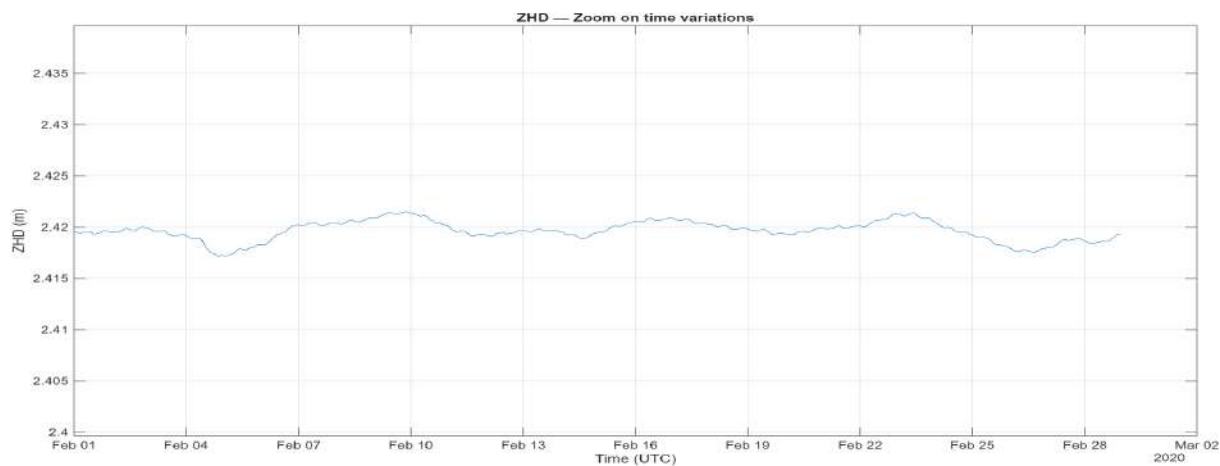


Figure G.14. Hourly variation of ZHD in February 2020 in Scanzano (PA). Dataset from ERA5 with Monthly mean by hour of days.

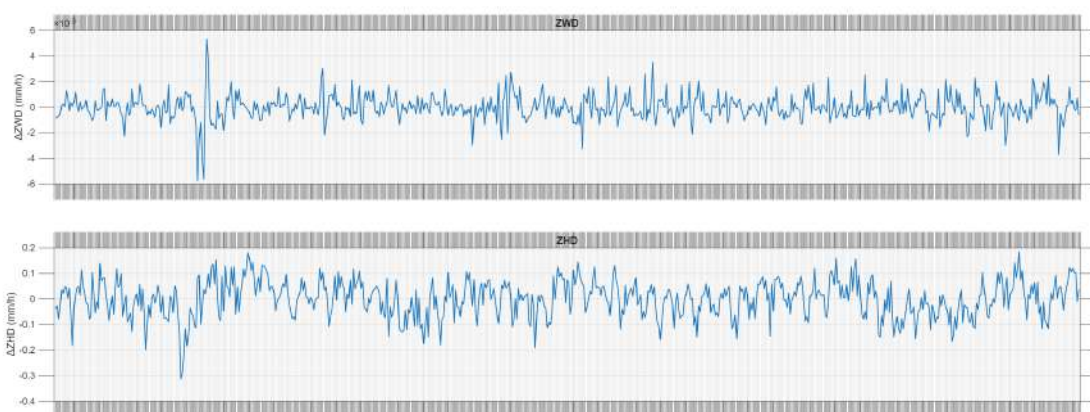


Figure G.15. Hourly variations for the month of February 2020 in Scanzano (PA). Dataset from ERA5 with Monthly mean by hour of days.

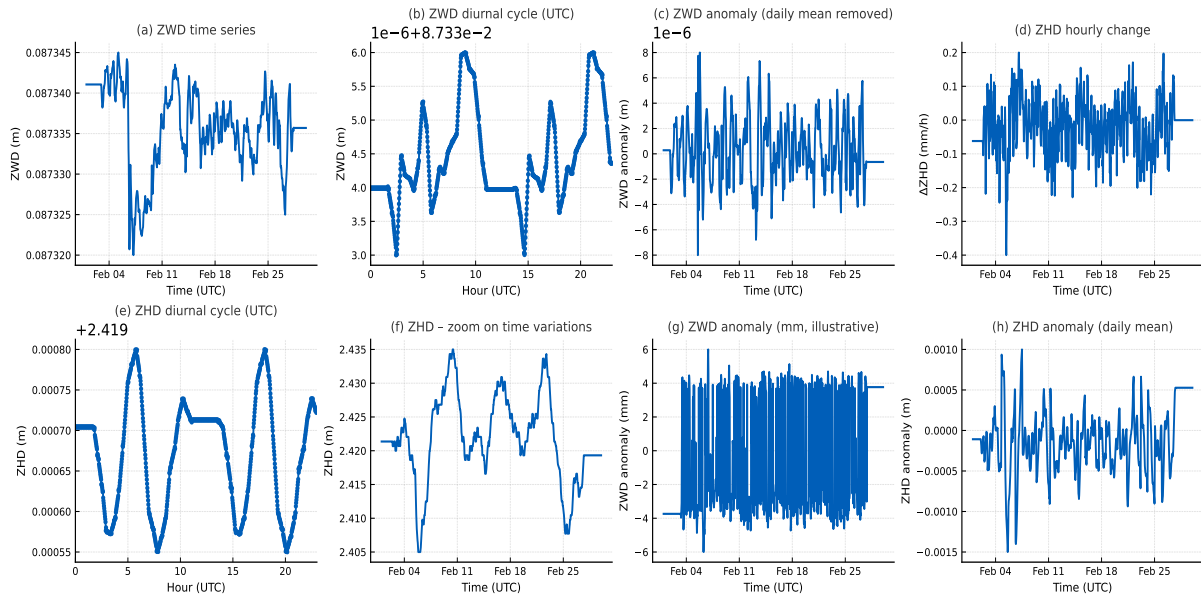


Figure G.16. Temporal and Diurnal Variability of Zenith Delays at Scanzano (February 2023). Panel (a) shows the full time series of the Zenith Wet Delay (ZWD), while (b) illustrates its diurnal cycle (UTC) highlighting periodic fluctuations. The ZWD anomaly after daily-mean removal is presented in (c), and the hourly variation rate in (d). Panels (e–h) refer to the Zenith Hydrostatic Delay (ZHD): (e) depicts the diurnal cycle, (f) a zoom on short-term variations, (g) the ZHD anomaly in millimetres, and (h) the daily mean anomaly. Together, these subplots emphasize the distinct temporal signatures of wet and dry components, revealing the stronger high-frequency variability of ZWD compared to the smoother and quasi-periodic ZHD.

(g) and its daily mean (h) further emphasize the low-amplitude, quasi-periodic nature of the dry delay. These findings highlight a clear contrast in the temporal dynamics of ZWD and ZHD: ZWD responds rapidly to humidity and convection, while ZHD varies more slowly due to barometric pressure changes.

G.3 Numerical Statistics for February 2020 over Scanzano (PA)

For a selected year, detailed numerical statistics can be extracted from the ERA5 dataset. **Table G.2** reports the daily values of the Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD) and Zenith Total Delay (ZTD) for February 2020 at Scanzano (PA), derived from the ERA5 monthly mean by hour of day product and processed through MATLAB. This daily view clarifies the internal temporal consistency of the delays: ZHD remains tightly bounded, reflecting its pressure dependence, while ZWD shows millimetric fluctuations that modulate ZTD. Although the source dataset is inherently smoothed, the day-by-day perspective adds detail beyond monthly or seasonal summaries.

Table G.3 condenses the main statistical descriptors for February 2020—including mean, median, standard deviation, minima and maxima, percentiles, interquartile range and amplitude—offering a compact representation of central tendency and dispersion. The results confirm the



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high stability of ERA5-derived delays. ZHD averages 2.4197 m with a standard deviation of 0.97 mm and a 95% confidence interval (CI) of ± 0.36 mm. ZWD remains nearly constant at 0.0873 m, with minimal day-to-day variability (95% CI ± 0.0018 mm). Consequently, ZTD averages 2.5070 m, reflecting the hydrostatic behavior with a small, consistent wet contribution. A difference of approximately 0.7–0.8 mm in ZHD or ZTD—or about 0.005 mm in ZWD—would be required to achieve 80% statistical power ($\alpha = 0.05$) for detecting interannual changes under similar sampling conditions. Physically, this corresponds to pressure variations of about 1.5 hPa, while the wet term remained effectively constant. Thus, at monthly resolution, ZTD variability is overwhelmingly governed by ZHD, with ZWD acting as a minor but stable offset.



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Date	ZHD	ZWD	ZTD
01-Feb-2020	2.4195	0.08734	2.5068
02-Feb-2020	2.4198	0.08734	2.5071
03-Feb-2020	2.4195	0.087341	2.5068
04-Feb-2020	2.4182	0.087339	2.5056
05-Feb-2020	2.4177	0.087325	2.5051
06-Feb-2020	2.4193	0.087326	2.5066
07-Feb-2020	2.4203	0.087326	2.5076
08-Feb-2020	2.4206	0.08733	2.5079
09-Feb-2020	2.4213	0.087333	2.5086
10-Feb-2020	2.4208	0.087338	2.5081
11-Feb-2020	2.4195	0.08734	2.5068
12-Feb-2020	2.4194	0.087336	2.5067
13-Feb-2020	2.4197	0.087336	2.507
14-Feb-2020	2.4192	0.087338	2.5065
15-Feb-2020	2.42	0.087335	2.5074
16-Feb-2020	2.4207	0.087334	2.5081
17-Feb-2020	2.4206	0.087335	2.5079
18-Feb-2020	2.42	0.087335	2.5073
19-Feb-2020	2.4196	0.087336	2.5069
20-Feb-2020	2.4196	0.087334	2.5069
21-Feb-2020	2.42	0.087333	2.5073
22-Feb-2020	2.4206	0.087333	2.508
23-Feb-2020	2.4211	0.087332	2.5084
24-Feb-2020	2.4198	0.087338	2.5071
25-Feb-2020	2.4187	0.087337	2.506
26-Feb-2020	2.4177	0.087336	2.5051
27-Feb-2020	2.4186	0.087329	2.5059
28-Feb-2020	2.4187	0.087336	2.506
29-Feb-2020	2.4193	0.087334	2.5067

Table G.2. Statistics for February 2020. Dataset from ERA5 with Monthly mean by hour of days.

Variable	mean	median	std	min	p5	p95	max	IQR	amplitude
ZHD	2.4197	2.4197	0.00096626	2.4171	2.4178	2.4212	2.4215	0.0011391	0.0043781
ZWD	0.087335	0.087335	4.6902e-06	0.087321	0.087326	0.087341	0.087343	6.0523e-06	2.256e-05
ZTD	2.507	2.507	0.00096598	2.5045	2.5051	2.5085	2.5088	0.0011257	0.0043724

Table G.3. Statistics for February 2020. Dataset from ERA5 with Monthly mean by hour of days.



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Chapter O

Appendix O – End-to-End Processing Chain for Zenith Delays (ZHD, ZWD, ZTD) at Scanzano (PA) – In Situ Analysis.

Atmospheric data analysis plays a fundamental role in understanding meteorological dynamics, particularly in regions such as Scanzano (PA), where variations in pressure, temperature, and tropospheric delays directly affect both local and global climate assessments.

O.1 Scanzano Data Pipeline: From MS Excel to MATLAB Database – In Situ Analysis

Appendix O presents a structured methodology for processing atmospheric datasets, beginning with the import and organization of raw data from **MS Excel** into **MATLAB**. Once imported, the datasets are transformed into a structured database, enabling efficient management, statistical evaluation, and visualization. Key atmospheric parameters—including pressure, temperature, Zenith Hydrostatic Delay (ZHD), and Zenith Wet Delay (ZWD)—are then analyzed to extract meaningful patterns. Statistical measures such as mean values and standard deviations provide a quantitative overview of trends, while histograms are used to visualize the distribution and temporal variability of each parameter.

Through the integration of computational tools and statistical techniques, this framework supports a detailed assessment of atmospheric conditions and their evolution over time. The results have practical implications for GNSS delay corrections, meteorological modelling, and climate variability studies, strengthening the understanding of how tropospheric processes influence radiowave propagation and signal stability.

O.1.1 Hourly Atmospheric Delay Components

The next two tables (Tables F.1 and F.4, see Appendix F.5) report the hourly values of the Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) for the selected date at the Scanzano (PA) site. The 1-hour sampling provides sufficient temporal resolution to characterise short-term atmospheric variability.

ZHD, primarily controlled by surface pressure, shows smooth and gradual changes over the day. In contrast, ZWD—driven by atmospheric water vapour—exhibits more rapid fluctuations. The resulting ZTD combines these two contributions and represents the total neutral-atmosphere delay along the zenith path. These datasets support the analysis of diurnal delay dynamics and enable the validation of modelling approaches used in GNSS meteorology and satellite communication calibration.

O.1.2 Weekly and Monthly Variations of ZWD, ZHD, and ZTD — In Situ Analysis

With the *in-situ* atmospheric delay database for Scanzano (PA) fully established from local GNSS and meteorological measurements, several visualization strategies were implemented to analyze temporal variability at multiple scales. Given the large number of plots generated throughout this work, only representative examples are reported here to illustrate the adopted methodology.

Weekly hourly means were first computed to evaluate the average behavior of ZHD, ZWD, and ZTD for each hour of the day over a one-week period.

This 24-point representation provides a compact view of recurring diurnal patterns. The same approach was then extended to monthly aggregates, using color-coded visualizations to highlight the intra-monthly evolution of atmospheric delays and to detect potential anomalies.

Figure O.1 illustrates this methodology.

- The left panel shows the time series of ZHD, ZWD, and ZTD during May 2019.
 - ZHD (blue) remains stable around 2.1–2.2 m, reflecting the pressure-driven dry component of the atmosphere.
 - ZWD (red) varies between 0.05–0.15 m, following changes in atmospheric moisture content.
 - ZTD (orange) lies around 2.2–2.3 m, combining the hydrostatic and wet contributions.

Gaps in the series correspond to missing records or brief acquisition interruptions. This visualization captures short-term fluctuations, seasonal tendencies, and abrupt transitions, all relevant for accurate modelling of propagation delays.

- The right panel reports the weekly hourly mean for 1–7 May 2019.
 - ZHD (blue) shows minimal variability and remains close to 2.2 m.
 - ZWD (red) exhibits higher short-term fluctuations, averaging around 0.1 m.

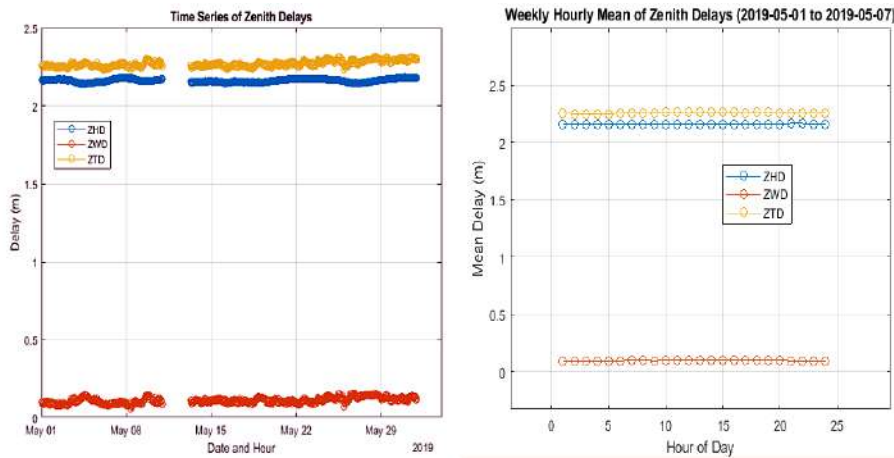


Figure O.1. Comparative Visualization of Zenith Delays (ZHD, ZWD, ZTD). The figure presents two complementary views of atmospheric zenith delays derived from in-situ GNSS observations for May 2019 at the Scanzano station. The left panel shows the time series of the Zenith Hydrostatic Delay (ZHD, blue), Zenith Wet Delay (ZWD, red), and their sum, the Zenith Total Delay (ZTD, yellow), throughout the month. The continuous traces highlight temporal stability of the hydrostatic component and short-term variability of the wet one, modulated by local moisture fluctuations. The right panel reports the weekly hourly mean (1–7 May 2019), illustrating the diurnal stability of the mean delay values with negligible hourly oscillations. Together, the two representations offer both high-frequency temporal insight and average-scale climatological behavior, providing a consistent framework for GNSS delay correction and atmospheric monitoring.

- ZTD (orange) reflects the combined behavior, with values close to 2.3 m.

This representation emphasizes the relative magnitudes of the hydrostatic and wet components and illustrates how even modest variations in ZWD can noticeably modulate ZTD, particularly under humid conditions.

O.1.2.1 Interpretation and Implications

The decomposition of tropospheric delay into its hydrostatic and wet components, as shown in **Figure O.1 (left & right)**, has direct implications for both geodetic and telecommunication applications.

The Zenith Hydrostatic Delay (ZHD) remains quasi-stable, closely following surface pressure, and can be modeled with high accuracy using empirical formulations such as the **Saastamoinen** model or the Vienna Mapping Functions.

Conversely, the Zenith Wet Delay (ZWD) exhibits strong short-term variability, governed by water vapor dynamics, which makes it more challenging to model and necessitates the use of data assimilation techniques from numerical weather prediction (NWP) models or near-real-time GNSS observations. The combined Zenith Total Delay (ZTD) demonstrates how even modest ZWD fluctuations can substantially perturb the total path delay, confirming the wet component as the dominant source of uncertainty. From an operational standpoint, this variability directly impacts GNSS positioning accuracy, where unmodelled wet delays can introduce centimetric-to-decimetric



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errors, particularly in real-time kinematic (RTK) and **precise point positioning (PPP)** modes.

Similarly, in satellite communication systems—especially in Ka-, V-, and W-bands—the wet component contributes to phase instability and attenuation variability, degrading link availability and reliability. Recognizing and quantifying these dynamics is therefore essential: while the stable ZHD enables predictable baseline corrections, the volatile ZWD requires adaptive strategies, such as real-time calibration, meteorological data assimilation, or machine learning-based prediction models for short-term atmospheric correction.

O.2 Other advanced trends and plots – In Situ Analysis

Figure O.2 illustrates two complementary visualizations of the Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) for January 2020. Each panel comprises three stacked plots with the day of the month on the x-axis and delay values (m) on the y-axis.

The left panel adopts a continuous-line representation to highlight monthly-scale behavior.

- ZHD (blue) shows a gradual decline consistent with pressure-driven variability.
- ZWD (red) displays marked short-term fluctuations linked to moisture content.
- ZTD (green) reflects the combined effect, resulting in a smoother curve modulated by the wet component.

The right panel uses discrete scatter points to resolve sub-daily variability.

- ZHD retains its slow downward trend while revealing day-to-day dispersion.
- ZWD shows pronounced volatility characteristic of humidity changes.
- ZTD integrates both contributions, evidencing finer-scale atmospheric dynamics.

Together, the line-based and scatter-based views provide complementary insight: the former captures broad temporal trends, whereas the latter exposes diurnal and high-frequency variability. Both perspectives are essential for GNSS delay correction, atmospheric monitoring, and climate-related analysis.

O.2.1 Monthly Variability of Zenith Delays (ZHD, ZWD, ZTD) at a Fixed Hour — In-Situ Analysis

In the analysis of the in-situ Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) at Scanzano (PA) for the period January 2020–2024, the time-series plots reported in **Figures O.3–O.7** summarize the temporal behavior of the three components at selected hours of the day. Each panel displays individual observations as blue scatter points, while a red moving-average curve highlights the underlying temporal tendencies. In the original

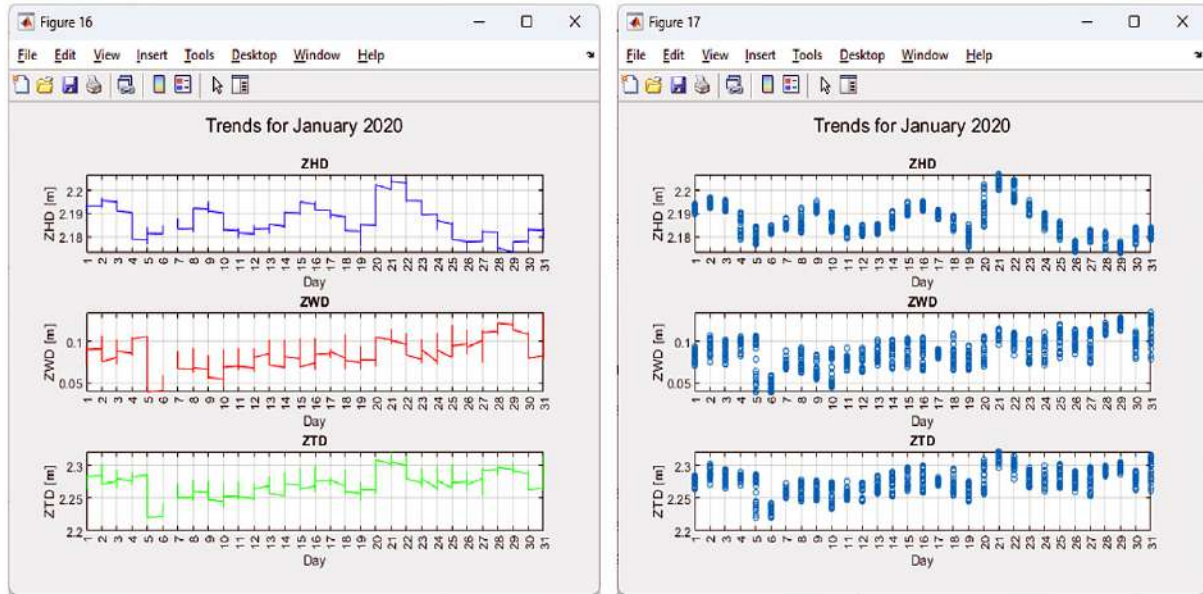


Figure O.2. Comparison of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) trends for **January 2020** using two different visualization techniques. The left figure represents the data as a continuous line plot, while the right figure displays the same data using discrete scatter points, highlighting variations in atmospheric delays over the month. **Database Source:** “In situ” Analysis.

figures, ore indicates the observation hour, and gaps in the series denote periods without valid measurements.

The hour of acquisition markedly influences the delay characteristics. During late-day hours (e.g., 18:00 UTC), ZWD exhibits enhanced variability, consistent with the rise in atmospheric moisture often occurring toward evening in Mediterranean environments. Conversely, during early-morning hours (00:00–02:00 UTC), ZHD remains notably stable, while ZWD continues to respond to subtle humidity and pressure fluctuations.

ZTD reflects the combined behavior of its two components: its broader trend is governed by the stability of the hydrostatic delay, whereas short-term fluctuations arise from the more dynamic wet component. The moving-average curves reveal episodic modulations that may be associated with synoptic disturbances, while the overall structure reflects the diurnal cycle—ZHD being mainly pressure-driven and quasi-stationary, and ZWD capturing rapid, moisture-driven atmospheric variability.

O.2.2 Climatological Interpretation of January 2020–2024 Zenith Delay Variability (ZHD, ZWD, ZTD) at the Scanzano Site (PA)

The multi-annual *in-situ* series (2020–2024) highlight distinct and recurrent patterns in the Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and their sum, the Zenith Total Delay (ZTD), reflecting the dominant atmospheric regimes over the central Mediterranean during boreal winter.



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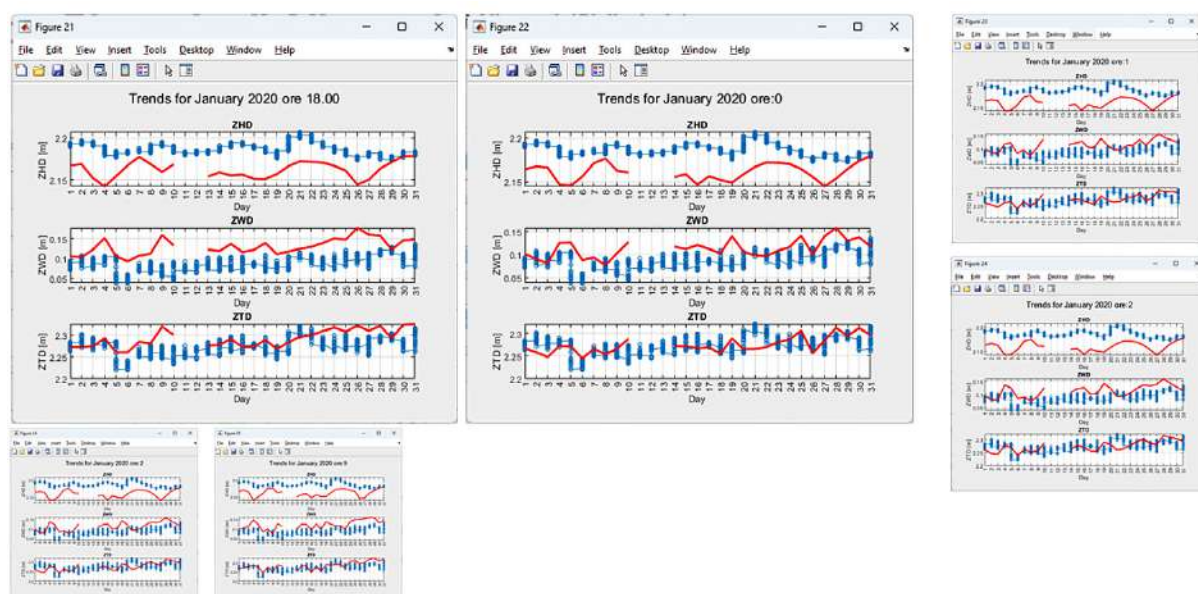


Figure O.3. Time series analysis of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) for January 2020 at specific hours of the day. Each subplot represents daily variations in atmospheric delays at different selected hours, with scatter points indicating individual data values and a red line highlighting the moving average trend. **Database Source:** “In situ” Analysis. Credits: Extracted from my presentation in Telespazio.

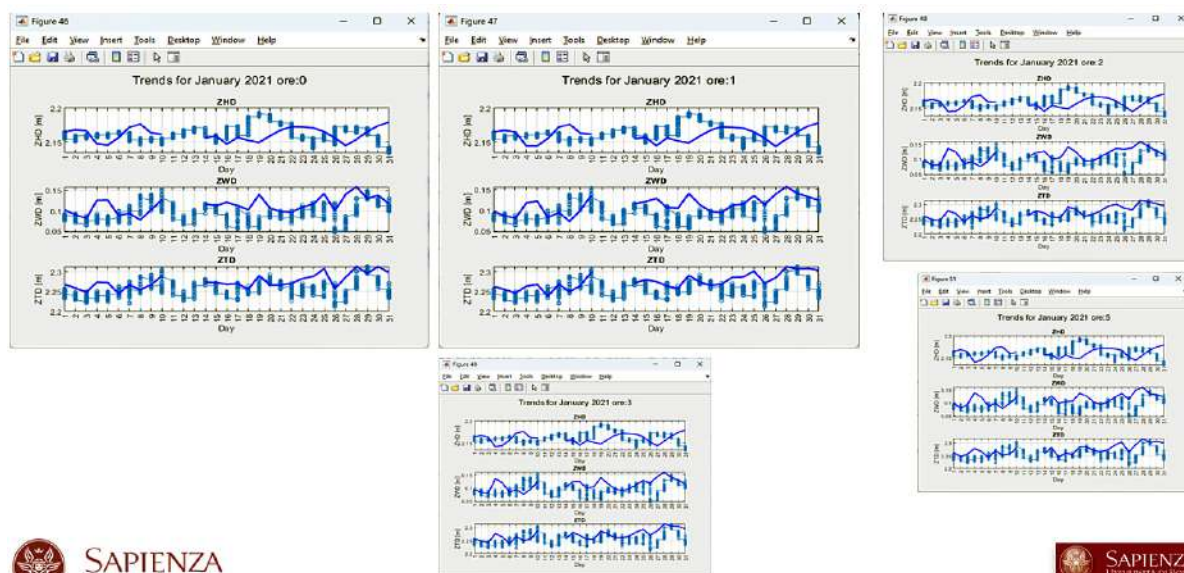


Figure O.4. Time series analysis of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) for January 2021 at specific hours of the day. Each subplot represents daily variations in atmospheric delays at different selected hours, with scatter points indicating individual data values and a red line highlighting the moving average trend. **Database Source:** “In situ” Analysis. Credits: Extracted from my presentation in Telespazio.



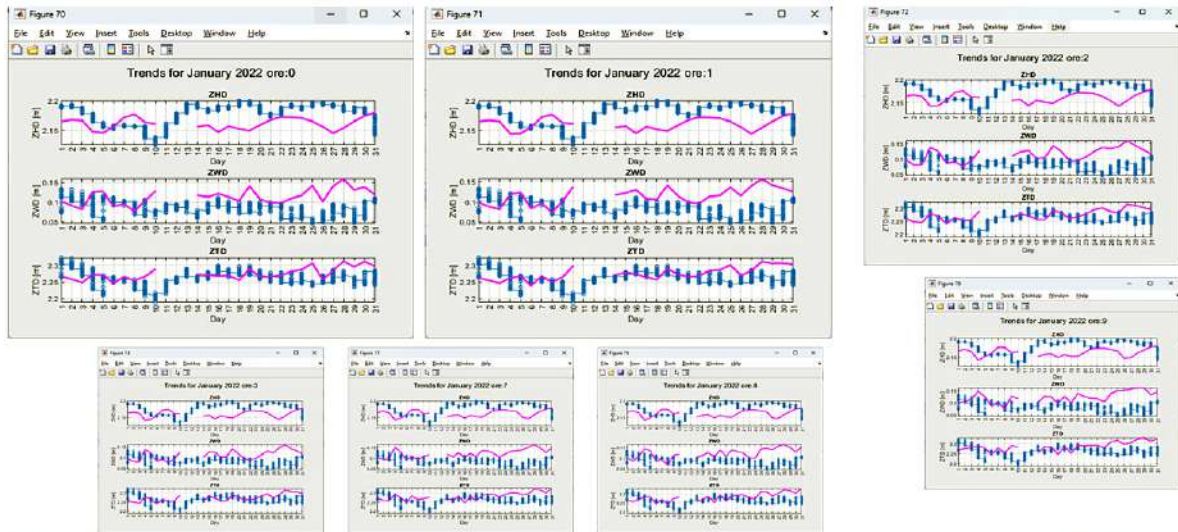
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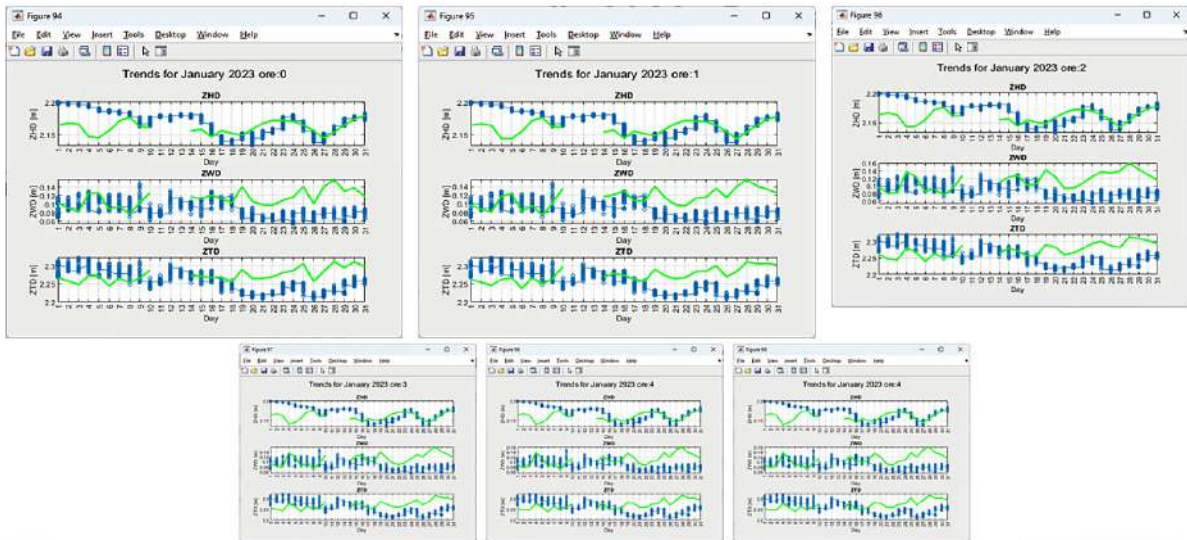
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Figure O.5. Time series analysis of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) for January 2022 at specific hours of the day. Each subplot represents daily variations in atmospheric delays at different selected hours, with scatter points indicating individual data values and a red line highlighting the moving average trend. **Database Source:** “In situ” Analysis. Credits: Extracted from my presentation in Telespazio.



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Figure O.6. Time series analysis of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) for January 2023 at specific hours of the day. Each subplot represents daily variations in atmospheric delays at different selected hours, with scatter points indicating individual data values and a red line highlighting the moving average trend. **Database Source:** “In situ” Analysis. Credits: Extracted from my presentation in Telespazio.



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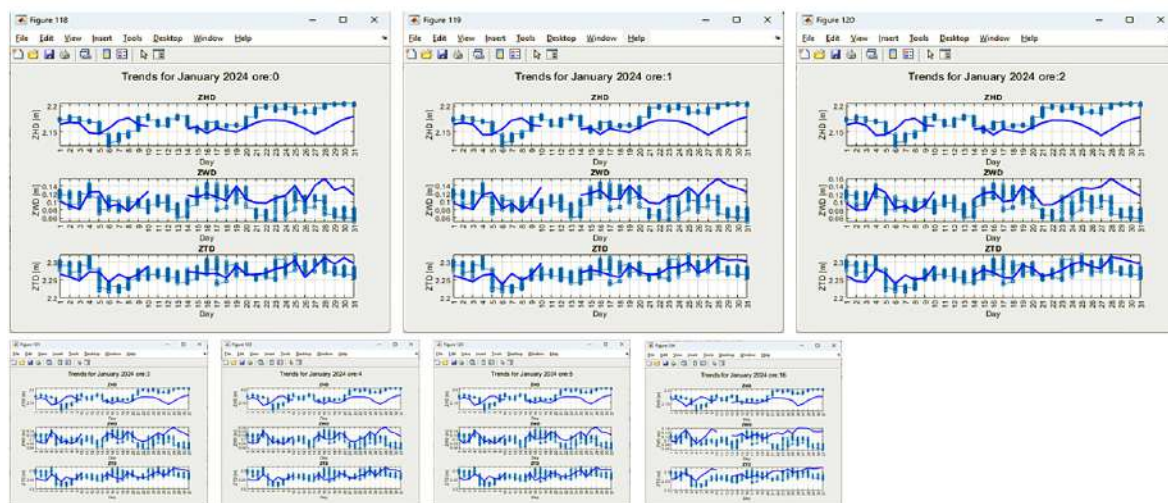


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Figure O.7. Time series analysis of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) for January 2024 at specific hours of the day. Each subplot represents daily variations in atmospheric delays at different selected hours, with scatter points indicating individual data values and a red line highlighting the moving average trend. **Database Source:** “In situ” Analysis. Credits: Extracted from my presentation in Telespazio.

- **Hydrostatic Delay (ZHD) – Synoptic-Scale Pressure Forcing** — Throughout all winters, ZHD remains confined within a narrow range ($\approx 2.15\text{--}2.18$ m), showing smooth oscillations with 3–7-day periodicities characteristic of mid-latitude baroclinic activity. These variations follow the alternation of *Atlantic* depressions and *Azores High* ridges, evidencing the quasi-stationary synoptic pressure modulation typical of the region. The absence of abrupt jumps indicates that frontal passages did not induce strong surface-pressure anomalies. Slightly larger fluctuations in 2021–2023 reveal more frequent cyclonic episodes or enhanced pressure gradients, while 2024 was comparatively stable, with minimal synoptic forcing under persistent anticyclonic control.
- **Wet Delay (ZWD) – Moisture Variability and Diurnal Modulation** — ZWD shows the highest temporal variability, governed by water-vapor dynamics. Afternoon peaks (around 18 UTC) reveal boundary-layer moistening through diurnal heating and turbulent mixing. Episodic maxima (~ 0.15 m) correspond to pre-frontal humid advections from the *Tyrrhenian* or *Ionian Seas*, whereas minima (~ 0.05 m) indicate post-frontal subsidence and dry continental inflows.
 - Inter-annual contrasts reflect the differing intensity of moisture advection:
 - * 2021 experienced more frequent and persistent humid phases under NAO-negative regimes and southwesterly intrusions;
 - * 2022 showed an early-month humid phase followed by a marked drying trend;

* 2023–2024 displayed weaker peaks and reduced diurnal modulation, consistent with drier boundary layers and limited convective mixing.

- **Total Delay (ZTD) – Combined Atmospheric Signature** — ZTD integrates the stability of ZHD and the variability of ZWD, producing composite signals that effectively trace high-PWV episodes. Periods of elevated ZTD coincide with ZWD surges, delineating moist synoptic events. The subdued ZTD variability in 2023–2024 mirrors the overall atmospheric dryness of those years.
- **Regional Winter Climatology Context** — Southern Italy’s winter climatology is shaped by alternating moist cyclonic incursions and dry anticyclonic phases. Scanzano’s proximity to both maritime and continental air masses enhances sensitivity to:
 - *Tyrrhenian/Ionian* moisture advection ahead of cold fronts;
 - Orographic uplift over the *Apennines*, enhancing local vapor content;
 - Post-frontal continental outflows producing sharp drying.
 - The late-afternoon ZWD asymmetry indicates local land–sea breeze interactions superimposed on the synoptic pattern.

The progression from the wetter 2020–2022 winters to the drier 2023–2024 period marks a transition toward more stable high-pressure dominance over the *central Mediterranean*.

- **Implications for GNSS Meteorology and Remote Sensing** — The stable hydrostatic component and the highly variable wet component underscore the need for high-temporal-resolution monitoring of ZWD to capture rapid moisture changes during synoptic transitions. Uncorrected diurnal ZWD cycles may introduce systematic biases in GNSS-derived PWV retrievals. The reduced humid-event frequency after 2022 also implies more favorable propagation conditions for high-frequency satellite links, with fewer occurrences of strong atmospheric attenuation.

O.3 Seasonal Variability of Zenith Delays: Comparative Box-Plot Analysis of Winter and Spring from In-Situ Measurements

To complement the monthly and inter-annual assessments presented in the previous sections, this part introduces a seasonal perspective aimed at characterizing the broader climatological behavior of the Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and the combined Zenith Total Delay (ZTD). Whereas time-series analyses capture short-term evolution and episodic variability, seasonal aggregation highlights persistent atmospheric regimes and their influence on the magnitude and dispersion of tropospheric delays. This representation effectively

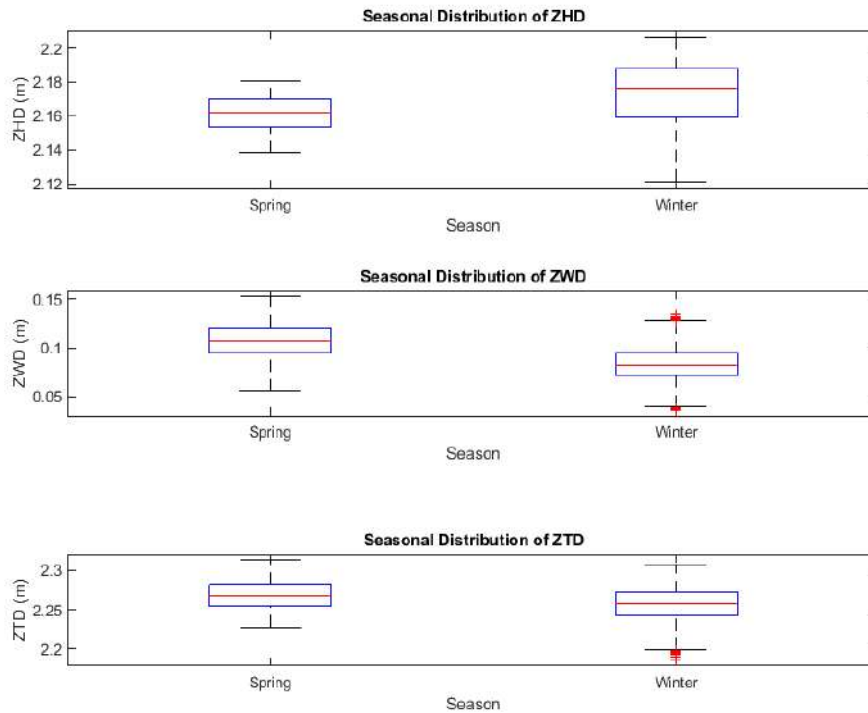


Figure O.8. Seasonal variation of Zenith Hydrostatic Delay (ZHD), Zenith Wet Delay (ZWD), and Zenith Total Delay (ZTD) between Winter and Spring. The box-plots illustrate the statistical distribution of each delay component, highlighting differences in median values, interquartile ranges, and the presence of outliers in each season. Data are related to Scanzano (PA) site, from 2019 to 2024. **Database Source:** “In situ” Analysis.

filters high-frequency fluctuations, isolating the underlying climatological signal associated with pressure, humidity, and temperature patterns over the central Mediterranean.

Figure O.8 summarizes these characteristics through comparative box-plots, illustrating the distribution, median values, interquartile ranges, and occurrence of outliers for each delay component across seasons. These diagnostics provide a concise yet comprehensive overview of how synoptic-scale pressure variability and moisture dynamics jointly modulate GNSS-relevant atmospheric delays. The resulting framework supports the following climatological interpretation and establishes the seasonal context for subsequent analyses.

O.3.1 Seasonal Interpretation of Zenith Delay Box-plots

1. **Zenith Hydrostatic Delay (ZHD)** – ZHD shows a higher median and broader interquartile range (IQR) during *Winter*, consistent with increased and more variable surface pressure under frequent synoptic disturbances. The longer whiskers confirm enhanced baroclinic activity typical of the cold season. In contrast, *Spring* displays lower and more compact values, indicative of weaker pressure gradients and more stable conditions.
2. **Zenith Wet Delay (ZWD)** — ZWD exhibits the most pronounced seasonal contrast,



with *Spring* > *Winter* in median value, reflecting higher column-integrated water vapor. However, numerous outliers in Winter reveal episodic humid events associated with transient cyclones or pre-frontal advections. The right-skewed winter distribution, with a compressed lower quartile near the minimum, further indicates generally dry conditions punctuated by abrupt moist surges.

3. **Zenith Total Delay (ZTD)** — ZTD primarily follows ZHD's seasonality—*Winter* medians exceed *Spring* due to the hydrostatic contribution—but also inherits transient anomalies from ZWD. Winter outliers in ZTD correspond to the wet-delay excursions, while Spring's narrower IQR suggests more uniform meteorological conditions.

O.3.2 Interannual Variability of Zenith Hydrostatic Delay (ZHD): A Comparative Seasonal Diagnostics Diagram (2020–2024)

An innovative visualization—hereafter referred to as the **Overlapping Diagnostics Diagram**—is introduced in **Figure O.9** to analyze inter-annual similarities and divergences in the Zenith Hydrostatic Delay (ZHD) over the five-year period 2020–2024. The method is based on representing daily ZHD values as discrete color-coded points, each color corresponding to a specific year. By plotting these values on the same daily axis, the diagram reveals the degree of overlap among annual patterns, thereby exposing periods of convergence, divergence, and anomalous behavior.

The visualization consists of 31 vertical columns, each representing a day of January. Although the month identifier is implicit, the structure inherently follows the January sequence. The superposition of colored points highlights where different years share nearly identical ZHD values and where departures occur. This graphical proximity directly reflects the underlying atmospheric pressure regime: close clustering indicates recurring synoptic configurations, whereas dispersion denotes inter-annual variability in surface-pressure fields.

From a physical perspective, the diagram provides immediate diagnostics of hydrostatic stability and perturbation. For example, ZHD values on 1 January show near coincidence between 2020 and 2022, while 2023 exhibits higher values, 2024 lower values, and 2021 the lowest across the five-year span. These differences reflect modulations in synoptic pressure systems, potentially linked to variations in baroclinic activity or large-scale atmospheric forcing.

Overall, the **Overlapping Diagnostics Diagram** functions as a compact and intuitive tool for identifying persistence, shifts, or anomalies in the winter hydrostatic regime. The coherent behaviour of 2020 and 2022 contrasts with the distinct departures observed in 2021, 2023, and 2024, suggesting inter-annual differences potentially influenced by broader climatological or dynamical factors. To complement this analysis, the following section provides a summary of solar-activity indicators for 2020–2024, enabling a preliminary assessment of whether variations in solar forcing may correlate with the observed ZHD anomalies.

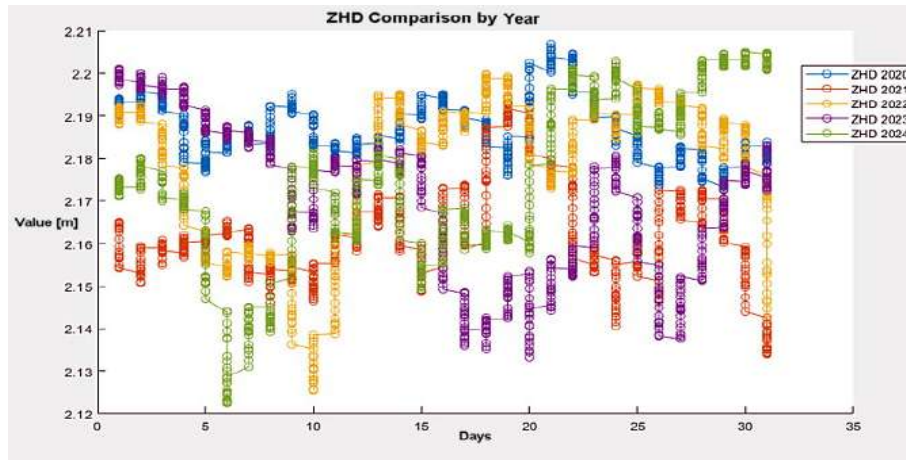


Figure O.9. Overlapping Diagnostics Diagram for ZHD in January (2020–2024) in Scanzano (PA).
Database Source: “In situ” Analysis.

O.3.3 Description of the Database

The database used in this study integrates a comprehensive set of atmospheric and tropospheric measurements from multiple sources, specifically designed to analyze the temporal and spatial variability of tropospheric delay and attenuation phenomena affecting high-frequency satellite links. It comprises:

- (i) In situ radiosonde data over Scanzano (PA), provided by **Telespazio** and organized in MS Excel sheets subsequently converted into MATLAB structures through a dedicated ~230-line preprocessing script;
- (ii) ERA5 reanalysis products, supplying surface pressure, temperature, and humidity fields for delay and attenuation computations; and
- (iii) Satellite observations from CloudSat, AMSU, and GPM missions, employed to validate hydrometeor-induced attenuation in the Ka- and W-bands.

All datasets underwent rigorous preprocessing—spatial-temporal interpolation, statistical filtering, and quality control—to ensure internal consistency and completeness. The resulting harmonized and curated dataset provides a robust foundation for quantitative evaluation of tropospheric effects and their implications for Earth-space communication link performance.

O.4 Solar Activity and Atmospheric Response: Correlation Analysis for 2020–2024

Given the ongoing debate on the influence of solar variability on Earth’s climate system, it is instructive to examine possible correlations between Zenith Hydrostatic Delay (ZHD), solar activity, and the Earth–Sun distance during the 2020–2024 period.



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Year	Solar Cycle Phase	Solar Activity (Sunspots, Flares)	Key Notes	ID
2020	Start of Cycle 25	Very low	Just past the solar minimum	1
2021	Early phase	Low to moderate	Slow recovery, few sunspots	2
2022	Growth phase	Moderate	Noticeable increase in flares and spots	3
2023	Active phase	High	Many M-class and X-class solar flares	4
2024	Expected peak	Very high	Cycle 25 peak forecasted for 2024–2025	5

Table O.1. Solar Activity Comparison (2020–2024).

- **Table O.1** summarizes the relevant indicators, including the annual mean ZHD, the mean daily sunspot number (as a proxy for solar activity), and the mean Earth–Sun distance. Statistical analysis based on annual averages yields a positive correlation of +0.43 between ZHD and sunspot count, suggesting that higher solar activity tends to coincide with slightly increased hydrostatic delay. Conversely, the correlation between ZHD and orbital distance is negligible (−0.05), implying that small variations in Earth–Sun distance exert minimal influence compared with atmospheric pressure dynamics.
- **Figure O.10** illustrates these relationships: the left panel shows ZHD versus daily sunspot count, revealing only weak dispersion and no clear linear dependence, while the right panel plots ZHD against Earth–Sun distance, where the modulation appears minor and likely seasonal. Overall, the hydrostatic delay remains primarily driven by surface pressure rather than orbital geometry. A subsequent lag analysis, performed using the **NOAA SWPC Sunspot Data Service** (<https://services.swpc.noaa.gov>) and processed in **Python** (Pandas libraries), identifies a maximum correlation at a lag of about two months. This indicates that the atmosphere responds to increased solar activity with a slight delay—ZHD tends to rise roughly two months after the sunspot number increases. Such latency could reflect thermodynamic adjustment processes within the troposphere, including energy storage, gradual warming, or circulation reconfiguration.

From this preliminary assessment, the direct ZHD–solar activity correlation can be considered moderate but physically meaningful, and the observed lag supports an indirect, progressive response rather than an immediate radiative effect. By contrast, no significant lag is detected in relation to Earth–Sun distance. Future analyses should extend this investigation to include **Precipitable Water Vapor (PWV)**, 2 m air temperature, and surface pressure from ERA5, correlated with solar radiation parameters such as **Global Horizontal Irradiance (GHI)** and **Direct Normal Irradiance (DNI)**, in order to better constrain the coupling mechanisms between solar variability and atmospheric delay behavior.

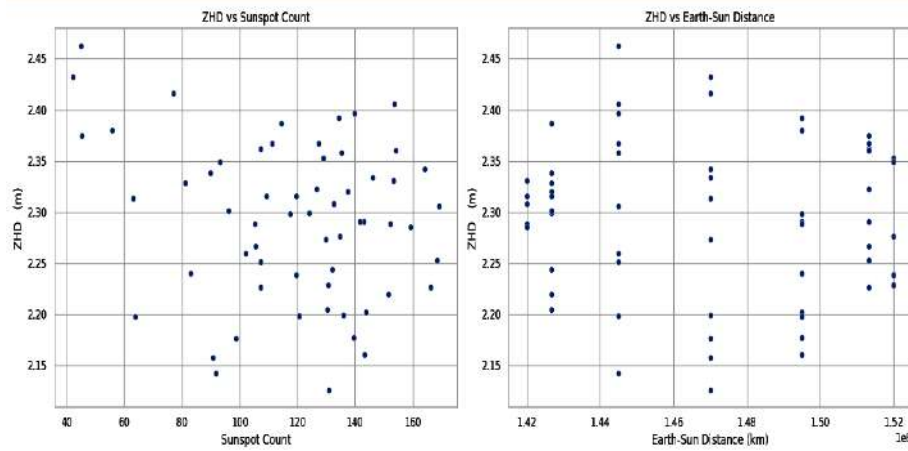


Figure O.10. Solar Forcing and Hydrostatic Delay Diagnostics. Left: Relationship between Zenith Hydrostatic Delay (ZHD) and daily Sunspot Count, showing weak dispersion without a clear linear trend, suggesting minimal direct correlation between solar activity and ZHD over the analyzed period. Right: ZHD versus Earth–Sun distance, indicating that seasonal orbital variations may exert a weak modulation on ZHD, though atmospheric pressure remains the dominant driver.

O.4.1 Interpretation of ZHD–Solar Activity and ZHD–Orbital Distance Correlations

Although both scatter plots in **Figure 13.33** exhibit substantial dispersion, several meaningful patterns emerge from closer inspection.

1. **ZHD vs Sunspot Count (left panel)** — The data cluster suggests a weak but positive trend, consistent with the computed correlation of $+0.43$. Higher ZHD values tend to appear for moderate-to-high sunspot numbers (≈ 120 – 160), implying that periods of increased solar activity coincide with slightly higher hydrostatic delays. This may indicate that solar forcing indirectly modifies large-scale atmospheric circulation, producing transient increases in surface pressure over the Mediterranean. The absence of a clear linear structure reflects the complexity of the coupling, likely mediated by thermodynamic adjustments in the troposphere rather than direct radiative effects.
2. **ZHD vs Earth–Sun Distance (right panel)** — The scatter is almost vertically uniform, confirming a negligible correlation (≈ -0.05). No systematic pattern links orbital distance to ZHD variability, reinforcing that Earth–Sun geometric modulation does not significantly influence surface pressure at these scales.
3. **Asymmetry in cluster dispersion** — Comparison of the two panels reveals a subtle but systematic increase in ZHD variance during periods of higher solar activity (sunspot count > 130). This suggests that solar forcing may act as a modulating factor that amplifies short-term pressure fluctuations rather than altering the mean hydrostatic state.
4. **Physical implication** — This behavior supports the hypothesis of an indirect solar–tropospheric coupling: solar maxima might enhance planetary-wave propagation and



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stratospheric–tropospheric exchange, subtly altering the mean sea-level pressure field. The ~ 2 -month lag you previously identified aligns with the radiative–dynamic response timescale expected for such processes.

In summary, while the mean ZHD shows only a moderate dependence on solar activity and none on orbital distance, the scatter structure suggests that solar variability could enhance the temporal variance of ZHD rather than its mean level. This points to a possible link between solar forcing and the variability amplitude of tropospheric pressure fields, a topic worth exploring through higher-resolution data and spectral (**wavelet** or **Fourier**) decomposition of ZHD time series.



Chapter P

Appendix P – Tables from Chapter 12

P.1 Table 12.2: ZHD variability interpretation

Table P.1. Operational interpretation of ERA5 derived Zenith Hydrostatic Delay variability across characteristic temporal scales.

Wavelet scale regime	Characteristic period	Physical meaning	Operational impact	Mitigation or use
Small scales	Sub daily hours	Short term surface pressure adjustments due to transient synoptic disturbances	Slow phase and range delay fluctuations with negligible fast fading impact	Accurate tropospheric delay modeling and GNSS clock correction
Intermediate scales	One to ten days	Synoptic scale pressure systems and recurring atmospheric oscillations	Gradual variations in slant path delay affecting geometry and Doppler compensation	High resolution reanalysis data and site specific delay correction
Large scales	Weeks to months	Seasonal and long term pressure trends driven by large scale circulation	Systematic bias in mean delay and long term link geometry	Fade margin definition and climatological planning



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P.2 Table 12.3: System level mitigation strategies

Table P.2. System level mitigation and planning strategies addressing ZHD related effects in satellite navigation and communication systems.

System aspect	ZHD related effect	Strategy	Timescale
Delay modeling	Slowly varying hydrostatic delay	Use of ERA5 or GNSS based ZHD products	Hours to days
Orbit determination	Bias in slant path geometry	Pressure driven delay correction	Days
Synchronization and timing	Long term phase drift	Clock correction using modeled ZHD	Days to weeks
Link budget design	Mean path delay offset	Inclusion of ZHD climatology in system margins	Seasonal
Site selection	Dependence on pressure climatology	Use of multi year ZHD statistics	Long term



Chapter P1

Appendix P1 – Tables from Chapter 12

P1.1 Table 12.2

Table P1.1. Mitigation strategies for high-frequency atmospheric effects on satellite links (Scanzano case).

Freq. Range	Obs. Phenomenon	Oper. Risk	Mitigation Action	Impl. Compl.	Rel. Cost.
0.3–0.4 Hz (High)	Rapid ZHD fluctuations, micro-turbulence, tropospheric scintillation	Short-term fading; BER increase; loss of coherence in high-order modulations	Fast uplink power control; Adaptive Coding and Modulation (ACM); Diversity (spatial/polarization)	Med–High	MH
0.1–0.3 Hz (Mid)	Sub-daily cycles; convective/diurnal variations in pressure and temperature	Periodic SNR degradation; throughput instability	Dynamic scheduling of high-data-rate transmissions; Predictive link adaptation using short-term weather forecasts	Medium	M
<0.1 Hz (Low)	Seas./interan. ZHD variations linked to pressure trends	Gradual change in link margin; risk during seasonal peaks	Seasonal fade-margin adjustment; Integration with climatological models (ERA5, ITU-R) for planning	Low–Med	L
All ranges	Combined variability under mixed conditions	Unpredictable attenuation events; risk of service disruption	Hybrid architecture with lower-frequency backup link (e.g., Ku-band); Real-time beacon monitoring for channel estimation	High	H

P1.2 Table 12.3



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Table P1.2. Antenna design strategies for mitigating high-frequency atmospheric effects on satellite reception (Scanzano case).

Design Aspect	Proposed Solution	Mitigated Phenomenon	Implementation Complexity	Relative Cost
Aperture Size & Geometry	Increase dish diameter or optimize feed illumination to reduce side lobes and multipath reception	Reduces susceptibility to multipath and improves SNR stability during rapid ZHD fluctuations	Medium (structural upgrades required)	Medium–High
Phased Array Technology	Implement electronically steered phased array with rapid beam steering	Compensates for short-term pointing errors and mitigates rapid fading from atmospheric turbulence	High (advanced electronics and control systems)	High
Polarization Diversity	Dual-polarization feed with automatic switching or combining	Mitigates selective fading and cross-polarization degradation during scintillation	Medium (requires RF switching and processing)	Medium
Radome Protection	Low-loss, weatherproof radome to shield antenna from rain, wind, and debris	Reduces mechanical jitter and environmental attenuation in high-frequency conditions	Low–Medium	Medium
Environmental Control	Integrated heating/dehumidification to prevent ice formation and condensation	Maintains consistent gain and pointing stability in adverse weather	Low	Low–Medium
High-Precision Tracking	Servo-controlled mount with low-inertia motors and vibration damping	Maintains signal lock during rapid atmospheric-induced variations	Medium–High	Medium–High
Site & Spatial Diversity	Deploy redundant receive antennas at different locations or with multi-feed systems	Ensures reception continuity by selecting the optimal signal path in real time	High (infrastructure and synchronization required)	High



Chapter R

Appendix R – Phase Noise, Coherence, and PLL Stability for Deep-Space Telecommunications.

R.1 Phased Locked Loop Design for Deep Space Telecommunications

Zenith delays (hydrostatic and wet) are routinely used in geodesy and meteorology but provide limited direct insight into phase coherence, stability, and interferometric performance, which instead govern the behavior of deep-space and high-frequency links [46, 216, 218]. In InSAR, spatial variability in the wet delay (ZWD) generates heterogeneous phase shifts that reduce coherence, mitigated via PS-InSAR, GNSS-derived ZTD corrections, or reanalysis/ML approaches [48, 49, 211, 213, 275]. In VLBI, millimetre-level delay errors produce picosecond timing offsets that degrade astrometric accuracy and antenna synchronization [218, 221].

In this context, phase noise is treated as an externally induced impairment driven by tropospheric path-delay variability, while internal oscillator noise and receiver-related phase instabilities are not considered and are assumed to be independently characterized [270, 272].

Because atmospheric effects are amplified at low elevations, the analysis adopts 15° , where the slant-path factor $M_e \approx 3.8$ magnifies small zenith variations. The analysis proceeds in four steps:

1. **Phase Conversion** – mapping path fluctuations into phase rotations and their temporal evolution [218, 270].
2. **Decorrelation** – assessing coherence loss under varying atmospheric conditions [46, 49, 226].
3. **Frequency Scaling** – evaluating unwrapped RMS phase drift, which increases linearly with frequency; W-band exhibits the fastest secular drift, imposing the strictest tracking

requirements for coherent and MIMO systems [223, 270, 271].

4. **System Impact** – the wrapped RMS quantifies residual phase after $\pm 180^\circ$ tracking, whereas the unwrapped RMS (mm) represents the true geometric path variability, directly informing PLL lock margins and coherence time [216, 218, 273]..

Table R.1 reports RMS and 95th-percentile unwrapped phase fluctuations (rad/deg) together with equivalent path perturbations (mm). While geometric path variability is nearly frequency-independent, phase variability increases sharply with frequency, reaching its maximum at 94 GHz, highlighting the stringent stability requirements of millimetre-wave and deep-space systems [223, 224].

Frequency [GHz]	RMS $\varphi_{unwrapped}$ [rad]	p95 $ \varphi _{unwrapped}$ [rad]	RMS path fluct. [mm]	p95 path [mm]	RMS $\varphi_{unwrapped}$ [deg]	p95 $ \varphi _{unwrapped}$ [deg]
14.0	68.85	18.1	224.41	61.67	102.96	170.46
30.0	141.1	38.78	224.41	61.67	104.25	171.47
94.0	442.1	121.5	224.41	61.67	103.85	170.58

Table R.1. Root-mean-square (RMS) and 95th percentile (p95) statistics of phase fluctuations and path variations at different carrier frequencies (14, 30, and 94 [GHz]).

Table R.2 extends the analysis to slant-path variability at 15° for Ku, Ka, and W bands. Delays are converted into phase using the carrier wavelength to allow direct comparison across frequencies. The table distinguishes between **wrapped** and **unwrapped** metrics: wrapped values quantify residual intra-cycle phase relevant for limited tracking, while unwrapped metrics describe the true physical path fluctuations, independent of ambiguity resolution. The latter define coherence time and PLL stability for phased arrays, MIMO systems, and coherent receivers, whereas wrapped values indicate how often atmospheric perturbations trigger cycle slips, constraining the robustness of high-order modulation schemes.

Freq. [GHz]	Mean Path [mm]	Std Path [mm]	P95 Path [mm]	Mean Unw. Phase [°]	Std. Unw. Phase [°]	P95 Unw. Phase [°]	Mean Wr. Phase [°]	Std. Wr. Phase [°]	P95 Wr. Phase [°]
14	8784.635	180.091	9114.685	-0.60	518.367	855.695	1.461	102.897	162.189
30	8784.635	180.091	9114.685	-1.435	1110.787	1833.633	1.024	104.378	162.806
94	8784.635	180.091	9114.685	-4.498	3480.467	5745.383	-1.629	103.765	161.334

Table R.2. Slant Path & Phase Table metrics at 15° of elevation.

The residual phase noise discussed here represents the portion of tropospheric variability that remains after removing slow-time components, distinguishing it from electronic phase noise and establishing the physical coherence limit of the link.

Comparison of Wrapped Phase Distributions for Ku, Ka, and W Bands

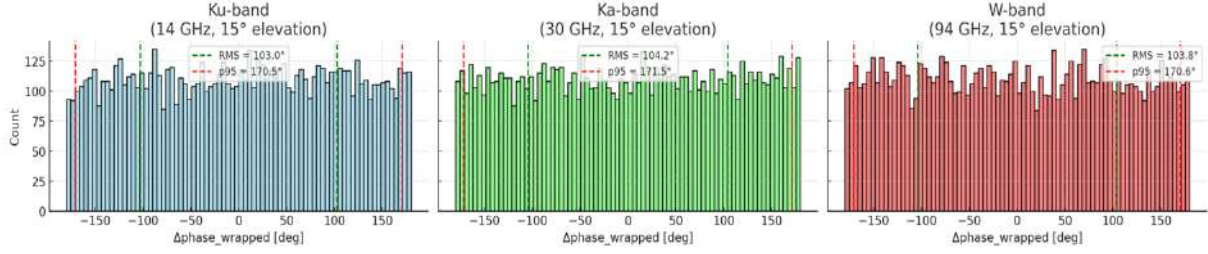


Figure R.1. Comparison of wrapped phase distributions for Ku (14 [GHz]), Ka (30 [GHz]), and W (94 [GHz]) bands at 15° elevation, computed over an equivalent path fluctuation of 8.8 [m].

Figure R.1 introduces a controlled test case based on a reference path fluctuation of 8.8 m, used to validate phase-wrapping statistics before applying them to deep-space geometries. Despite increasing carrier frequency (Ku → Ka → W), the wrapped-phase RMS remains nearly constant ($\approx 103\text{--}104^\circ$) and the p95 remains $\approx 171^\circ$, confirming that the wrapping behavior saturates and becomes frequency-independent under controlled conditions.

The underlying path variability is $\sigma \approx 180$ mm (detrended over 72 h). Converted into phase, this yields frequency-scaled standard deviations of $\approx 518^\circ$ at 14 GHz, $\approx 1111^\circ$ at 30 GHz, and $\approx 3480^\circ$ at 94 GHz, while the wrapped-phase p95 remains limited ($\approx 161\text{--}163^\circ$), indicating that approximately 5% of samples lie close to the cycle-slip threshold. For an Earth–Moon link at 94 GHz, the accumulated phase reaches $\varphi \approx 7.54 \times 10^8 \text{ rad}$, so millimetric path changes translate into very large phase excursions, defining the coherence constraints relevant to **Table R.3**.

Parameter	Vaue
Frequency	94 [GHz]
Distance Earth-Moon	384,400 [km]
Diameter Antenna Ground Station	3 [m]
Diameter Antenna Moon Station	1 [m]
Power in Tx	10 [W] (40 [dBm])
Antenna efficiency	70%
Antenna sensibility	−120 [dBm]
Fading margin	3 [dB]

Table R.3. Design parameters.

R.1.1 Quick Link-Budget Assessment for Earth–Moon W-Band Link

At 94 GHz, the operating wavelength is approximately $\lambda \approx 3.19$ mm. Using a 70% aperture efficiency, the Earth-based transmitting antenna provides:

$$G_T = 70\% \times \left(\frac{3\pi}{0.00319} \right)^2 \approx 66.5 \text{ dBi},$$



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while a 1 m lunar receiver yields:

$$G_R = 70\% \times \left(\frac{1\pi}{0.00319} \right)^2 \approx 56.5 \text{ dBi.}$$

The free-space path loss for an Earth–Moon link is:

$$L_{fs} = 20 \cdot \log_{10}(d) + 20 \cdot \log_{10}(f) + 92.45, \quad (\text{R.1})$$

which for $d \approx 3.84 \times 10^5 [\text{km}]$ and $f = 94 [\text{GHz}]$ gives $L_{fs} \approx 232.6 [\text{dB}]$. The received power is therefore:

$$P_{rx} = P_{tx} + G_T + G_R - L_{fs} - L_{fading}, \quad (\text{R.2})$$

and, with $P_{tx} = 40 [\text{dBm}]$, and $L_{fading} = 9 [\text{dB}]$ (miscellaneous+rain), yields:

$$P_{rx} = 40 + 66.5 + 56.5 - 232 - 6 - 3 = -72.6 [\text{dBm}]. \quad (\text{R.3})$$

With a receiver sensitivity of -120 dBm , the resulting link margin is:

$$M = P_{rx} - P_{sens} = -72.6 - (-120) = 47.4 [\text{dB}], \quad (\text{R.4})$$

indicating comfortable feasibility for a W-band Earth–Moon link and allowing for additional degradations from atmospheric absorption, pointing errors, and hardware imperfections. At **Scanzano** (PA, 560 m a.s.l.), atmospheric attenuation at 94 GHz is dominated by molecular absorption from O_2 and H_2O , with additional contributions from hydrometeors during moderate rain events. These effects are summarized in **Table R.4**.

Component	Estimated Value
O_2	$\sim 1.2 [\text{dB}]$
H_2O	$\sim 2.5 [\text{dB}]$
Light rain	$\sim 3 [\text{dB}]$

Table R.4. Atmospheric loss table.

R.1.2 Updated Link Budget Considering Atmospheric and Additional Losses

Including atmospheric contributions — 1.2 dB (O_2 absorption), 2.5 dB (H_2O absorption), 3 dB (moderate rain) — the total attenuation becomes 6.7 dB, yielding an updated received power of $P_{rx} \approx -79.3 \text{ dBm}$. With a receiver sensitivity of -120 dBm , the link margin is: $M \approx 40.7 \text{ dB}$. For conservative operation, additional losses — connection and polarization ($\approx 2 \text{ dB}$) plus an increased fading allowance of 5 dB — lead to:

$$\begin{aligned} P_{rx} &= P_{tx} + G_{tx} + G_{rx} - L_{fs} - L_{atmo} - L_{others} - L_{fading} = \\ &= 40 + 66.5 + 56.5 - 232.6 - 6.7 - 2 - 5 = -83.3 [\text{dBm}]. \end{aligned} \quad (\text{R.5})$$



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with a corresponding link margin:

$$M = P_{rx} - P_{sens} = -83.3 - (-120) = 36.7 \text{ [dB]}. \quad (\text{R.6})$$

Even under these conservative assumptions, the W-band **Earth–Moon link** retains a 36–37 dB margin, confirming robust feasibility. Minor hardware-related losses (e.g., radome absorption, component mismatch) are summarized in **Table R.5**.

Other Losses not considered	Estimated Value	Description
Antenna pointing losses	−3 [dB]	Extremely narrow beam at 94 GHz
Tropospheric dynamic fading	−2 [dB]	Scintillation, atmospheric instability
Multi-path and Moon shadowing	−2 [dB]	Reflections from rocks, craters
Derating hardware in time	−1 [dB]	Degradation RF components
Overhead in Coding and Framing	−2 [dB]	Reduction in useful efficiency

Table R.5. Other Losses not considered.

R.1.3 Realistic Link Margin and Channel Capacity Evaluation

Including all additional losses not explicitly accounted for in the ideal link budget—such as mechanical tolerances, structural deformations, radome absorption, alignment uncertainties, and residual pointing errors—an extra conservative margin of approximately 10 dB is introduced. The complete link equation therefore becomes:

$$\begin{aligned} P_{rx} &= P_{tx} + G_{tx} + G_{rx} - L_{fs} - L_{atmo} - L_{others} - L_{fading} - L_{not\ previous\ considered} \\ &= 40 + 66.5 + 56.5 - 232.6 - 6.7 - 2 - 5 - 10 = -93.3 \text{ [dBm]}. \end{aligned} \quad (\text{R.7})$$

The resulting realistic link margin is:

$$M_{realistic} = P_{rx} - P_{sens} = -93.3 - (-120) = 26.7 \text{ [dB]}. \quad (\text{R.8})$$

This value represents a conservative yet physically plausible operating margin for a W-band Earth–Moon communication link under non-ideal conditions. Starting from this margin, the corresponding signal-to-noise ratio is obtained as:

$$SNR = 10^{M_{realistic}/10} \approx 4.67 \times 10^2. \quad (\text{R.9})$$

Assuming an available bandwidth of $B=1$ GHz and a receiver Noise Figure of 5 dB, the theoretical

Shannon channel capacity becomes:

$$C = B \cdot \log_2(1 + SNR) = 1[\text{GHz}] \cdot \log_2(1 + 467.7) \approx 8.87 [\text{Gbps}]. \quad (\text{R.10})$$

Hence, the achievable throughput approaches approximately 8.9 Gbps, consistent with the use of high-order modulation schemes such as 64-QAM and, under favorable propagation conditions, 256-QAM. By employing adaptive modulation together with forward-error-correction techniques (LDPC, Turbo, or Reed–Solomon coding), the available 1 GHz bandwidth can be dynamically partitioned into:

- (i) a primary high-rate data channel,
- (ii) a control and telemetry channel, and
- (iii) a synchronization and beacon channel.

This strategy significantly enhances robustness and recovery capability during temporary fades or partial outages. System-level simulations confirm reliable operability for link margins as low as 10 dB, pointing errors up to $\pm 0.2^\circ$, and receiver **Noise Figures** approaching 8 dB. Finally, local buffering combined with automatic retransmission (ARQ), supported by Ka-band or X-band backup links, ensures operational continuity in the event of severe atmospheric degradation or temporary link interruption. The corresponding design parameters and performance trade-offs are summarized in **Table R.6**.

SNR available [dB]	Modulation Schema	FEC	Efficiency [bps/Hz]	Suggestions
> 25	256-QAM	LDPC (rate 0.9)	~7–7.5	Maximum throughput, ideal conditions
20–25	64-QAM	LDPC (rate 0.8)	~4.5–5	High speed, good stability
15–20	16-QAM	Turbo (rate 0.75)	~3–3.2	Balance between speed and robustness
10–15	QPSK	LDPC (rate 0.5)	~1–1.2	Moderate conditions, reliability priority
< 10	BPSK	Reed-Solomon	~0.3–0.7	Critical situations, backup or emergency operation

Table R.6. Adaptive modulation and coding (AMC) strategies as a function of available SNR.

The table summarizes the relationship between signal-to-noise ratio (SNR), modulation schemes, forward-error-correction (FEC) codes, and the resulting net spectral efficiency. At high SNR (>25 [dB]), 256-QAM combined with high-rate LDPC achieves maximum throughput ($\approx 7\text{--}7.5$ [bps/Hz]) under near-ideal propagation conditions. As SNR decreases, progressively more robust schemes are adopted, transitioning from 64-QAM and 16-QAM with LDPC or Turbo coding to QPSK and ultimately BPSK with Reed–Solomon coding (<10 [dB]), ensuring link reliability under degraded or emergency scenarios. This progression illustrates the fundamental trade-off between throughput and robustness that underpins modern adaptive coding and modulation in satellite and deep-space communication systems.



R.1.4 Realistic simulated scenario with SNR variation over time

To assess the robustness and efficiency of the adaptive modulation and coding (AMC) strategy adopted for the Earth–Moon communication link, a time-varying Signal-to-Noise Ratio (SNR) profile was simulated and mapped to the corresponding modulation–coding modes. Let:

$$\eta_i = R_i \log_2(M_i) \quad (\text{R.11})$$

denote the spectral efficiency (in $\text{bit} \cdot \text{s}^{-1} \cdot \text{Hz}^{-1}$) associated with the i -th modulation–coding scheme, where M_i is the constellation order and R_i is the coding rate. If each mode is active for a fraction of time (or probability of occurrence) p_i , with:

$$\sum_{i=1}^N p_i = 1, \quad (\text{R.12})$$

the average spectral efficiency of the adaptive system is given by:

$$\bar{\eta} = \sum_{i=1}^N p_i \eta_i = \sum_{i=1}^N p_i R_i \log_2(M_i). \quad (\text{R.13})$$

Given an available bandwidth B , the corresponding average channel capacity becomes:

$$C_{\text{mean}} = B \cdot \bar{\eta}. \quad (\text{R.14})$$

This formulation highlights the intrinsic advantage of Adaptive Modulation and Coding: higher-order constellations dominate during favorable SNR conditions, while more robust schemes ensure link continuity during temporary degradations. Even for deep-space links—where SNR variations may arise from pointing jitter, receiver noise fluctuations, or atmospheric perturbations on the Earth segment—the time-averaged capacity remains significantly high. In the specific simulation conducted for the Earth–Moon link, the resulting average spectral efficiency was:

$$\bar{\eta} \approx 4.8 \text{ bit/s/Hz}, \quad (\text{R.15})$$

yielding an average throughput of:

$$C_{\text{mean}} \approx 4.8 \text{ Gbps} \quad (\text{R.16})$$

over a 1-GHz channel. Although instantaneous SNR values may temporarily drop to critical levels (≈ 9 dB), the AMC mechanism preserves communication continuity. Under nominal conditions, the system enables peak data rates approaching 8.9 Gbps, as derived in the previous section. The simulated scenario demonstrates that the proposed AMC strategy sustains high-rate scientific telemetry, high-definition imaging, and real-time command and control, achieving a robust balance between spectral efficiency and resilience—an essential requirement for long-duration



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lunar and deep-space missions.

R.2 Earth–Moon Link Under Adverse Terrestrial Weather: Feasibility and Design Considerations

In an Earth–Moon communication link, only the first 5–15 km of the terrestrial atmosphere contribute non-negligible propagation impairments; the remaining distance of approximately 3.8×10^5 km is effectively free space. Under clear-sky conditions, gaseous absorption—primarily due to oxygen and water vapor—typically yields specific attenuation values of:

$$\alpha \approx 0.05 - 0.5 \text{ dB km}^{-1}, \quad (\text{R.17})$$

resulting in total atmospheric losses of:

$$A_{\text{atm}} \approx \alpha \cdot L_{\text{atm}} \in 0.5 - 7.5 \text{ dB}. \quad (\text{R.18})$$

depending on elevation angle, frequency band, and meteorological conditions. Severe precipitation events can dramatically increase attenuation, with rain-induced specific losses reaching:

$$A_{\text{rain}} \approx 50 - 100 \text{ dB} \quad (\text{R.19})$$

over the disturbed atmospheric layer, potentially becoming link-limiting at Ka- and W-band frequencies. Antenna pointing accuracy is therefore a critical system parameter. A pointing deviation of $\pm 0.2^\circ$ is largely negligible for a 0.5 m X-band antenna, whose half-power beamwidth is on the order of $4\text{--}5^\circ$, but would be catastrophic for large deep-space apertures, such as a 70 m DSN antenna, whose beamwidth is approximately 0.03° . Consequently, stringent pointing, tracking, and structural stability requirements are imposed on high-gain ground terminals. Propagation latency is intrinsic to Earth–Moon links and cannot be mitigated by system design. The one-way propagation delay is approximately:

$$\tau_{OW} \approx \frac{384,400[\text{km}]}{c} \approx 1.28 [\text{s}], \quad \tau_{RTT} \approx 2.56 [\text{s}]. \quad (\text{R.20})$$

Such delays necessitate the use of strong forward-error-correction (FEC) schemes, deep interleaving, and window-based transport protocols to ensure reliable data delivery and efficient use of the link. Under realistic deep-space SNR conditions, practical spectral efficiencies typically fall in the range of $0.5\text{--}3 \text{ bit} \cdot \text{s}^{-1} \cdot \text{Hz}^{-1}$, yielding the following representative data rates:

1. **Telemetry and narrowband services** ($B \approx 1 \text{ MHz}$): $0.5\text{--}3 \text{ Mbit} \cdot \text{s}^{-1}$.
2. **High-rate payload downlinks** ($B \approx 100 \text{ MHz--}1 \text{ GHz}$): from several hundred $\text{Mbit} \cdot \text{s}^{-1}$ up to $\text{multi-Gbit} \cdot \text{s}^{-1}$, achievable with 64-QAM or 256-QAM modulation combined with



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modern LDPC or Turbo coding under favorable conditions.¹

Mitigation strategies include the use of cryogenic low-noise amplifiers (LNAs), narrowband filtering, high-gain apertures with low sidelobe levels, adaptive power control, adaptive modulation and coding (AMC), and operational scheduling to avoid solar conjunctions or extreme weather events. In particular, X-band operation with transmit powers of approximately 20 kW on the Earth segment and 50 W on the lunar segment, combined with DSN-class receivers, provides robust performance under a wide range of atmospheric conditions.

Operationally, most current and planned lunar missions employ X-band (8–12 GHz) links with transmit powers on the order of 20 kW (Earth) and 50 W (Moon), feeding high-gain apertures ranging from 0.5 m to 70 m in diameter. Reception through NASA’s Deep Space Network complexes—Goldstone (USA), Madrid (Spain), and Canberra (Australia)—routinely enables the detection and demodulation of signals several orders of magnitude below terrestrial communication thresholds. Clear to moderate weather ensures stable operation, while severe precipitation events may require site diversity, frequency agility, or mission rescheduling to preserve link availability.

R.2.1 PLL Tracking Stability and Phase-Noise Budget

In coherent high-frequency communication systems—particularly in the Ka- to W-band (30–94 GHz)—the Phase-Locked Loop (PLL) plays a decisive role in maintaining carrier phase coherence in the presence of atmospheric, mechanical, and oscillator-induced disturbances.

Tropospheric path-delay fluctuations, local oscillator phase noise, mechanical vibrations, and residual Doppler errors jointly contribute to a composite residual phase error σ_φ , which must remain within the effective lock range of the loop. For linear PLL operation, this range typically spans approximately $\pm 45^\circ$ to $\pm 90^\circ$, depending on loop order, discriminator characteristics, and signal dynamics.

R.2.1.1 Residual Phase Variance and Loop Bandwidth

The residual phase error variance can be approximated as:

$$\sigma_\varphi^2 \approx \sigma_n^2 + \sigma_t^2 + \sigma_{osc}^2, \quad (\text{R.21})$$

where:

- σ_n^2 represents the thermal-noise-induced phase variance, which for a carrier-tracking loop scales approximately as $\sigma_n^2 \propto 1/(2 SNR)$;
- σ_t^2 denotes the tropospheric phase-noise contribution, derived from unwrapped atmospheric phase fluctuations mapped along the slant path;

¹coherent with 64-QAM/256-QAM modulations and modern **LDPC/Turbo codes**.



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- σ_{osc}^2 accounts for the local oscillator instability, typically described in terms of Allan variance and dependent on both the loop bandwidth B_L and the effective integration time τ .

For representative operating conditions with $SNR \approx 25$ dB and a loop bandwidth in the range $B_L = 1 - 5$ kHz, the resulting total residual phase error remains below approximately 20° . This ensures stable phase lock and compatibility with high-order modulation formats (≥ 64 -QAM). Reducing B_L decreases the thermal noise contribution but increases sensitivity to dynamic tracking errors and loop lag, particularly in the presence of long propagation delays characteristic of **Earth–Moon** links. Conversely, excessively wide bandwidths amplify oscillator and noise-induced phase jitter. As a result, intermediate loop bandwidths on the order of 1–3 kHz provide the most favorable trade-off between noise suppression and dynamic tracking capability for lunar-distance coherent communications.

R.2.1.2 Coherence Time and Loop Hold-In Range

The coherence time T_c represents the interval over which the accumulated residual phase drift remains of the order of one radian, beyond which coherent integration and phase tracking degrade significantly. For stochastic phase fluctuations, T_c can be approximated as:

$$T_c \approx \frac{1}{2\pi f_{\sigma_\varphi}}, \quad (\text{R.22})$$

where f_{σ_φ} denotes the effective spectral width of the residual phase-noise process. In W-band links, tropospheric path-delay fluctuations introduce rapid phase variations that typically limit the coherence time to $T_c \approx 0.1$ – 0.5 s, depending on elevation angle, atmospheric stability, and weather conditions. On longer time scales ($\gtrsim 1$ s), oscillator instability and residual Doppler dynamics become the dominant contributors to phase drift.

The PLL hold-in range Δf_H , which defines the maximum frequency offset over which lock can be maintained once acquired, scales approximately with the loop bandwidth as:

$$\Delta f_H \approx \frac{B_L}{2}. \quad (\text{R.23})$$

For typical loop bandwidths of a few kilohertz, this provides a hold-in range of several kilohertz, sufficient to track slow frequency drifts induced by oscillator offsets, antenna mechanical motion, and pointing jitter. Under these conditions, the combined constraints of coherence time and hold-in range define the operational limits for sustained coherent tracking in Earth–Moon and other deep-space links.

R.2.1.3 Phase-Noise Budget and System Margins

As summarized in **Table R.7**, the overall phase-noise budget accounts for both environmental and hardware-related contributions, ensuring stable **Phase-Locked Loop (PLL)** operation under W-band propagation conditions. Within this allocation, the PLL is able to maintain

Source	Typical RMS phase [°]	Notes
Tropospheric turbulence	8–15	From ERA-5 based ZTD variability (§ 13.8)
Oscillator phase noise	5–10	For high-stability OCXO/atomic references
Mechanical/pointing jitter	3–5	Mount vibration and servo lag
Thermal/Quantization Noise	< 3	Front-end and ADC contributions
Total (RSS)	$\approx 18 - 22$	Within lock margin

Table R.7. Phase-noise budget for the W-band Earth–Moon communication link. The table summarizes the main contributions to the residual phase error affecting the Phase-Locked Loop (PLL), expressed as RMS phase values. Each source corresponds to a distinct physical or instrumental origin, including tropospheric turbulence (derived from ERA5-based ZTD variability; §13.8), oscillator phase noise (OCXO or atomic frequency reference stability), mechanical pointing jitter, and front-end thermal and quantization noise. The root-sum-square (RSS) total contribution ($\approx 18\text{--}22^\circ$) remains well within the PLL lock margin, confirming stable phase tracking and coherence for high-order modulation schemes ($\geq 64\text{-QAM}$) under realistic operational conditions.

continuous lock across all expected operating scenarios. The available design margin supports dynamic **Adaptive Coding and Modulation (ACM)**, beam-forming synchronization, and **Doppler pre-compensation** over frequency offsets up to ± 1 kHz around the nominal carrier. Stable carrier tracking in the W-band regime requires:

- high-Q local oscillators with Allan deviation $\leq 10^{-12}$ at 1 s integration time;
- low-phase-noise frequency synthesizers (≈ -100 dBc/Hz at 10 kHz offset);
- optimized loop-filter design to minimize group delay and tracking latency;
- thermally stabilized RF chains and vibration-damped antenna structures.

The combined mitigation strategy limits residual phase excursions to below approximately 25° at the 95th percentile, even under adverse tropospheric conditions, thereby preserving coherent demodulation performance and beam-forming accuracy for long-distance lunar and deep-space links.

R.2.2 Summary of Ka-band Earth–Moon Link Design

Table R.8 provides a compact technical overview of the **proposed Ka-band Earth–Moon link** using an NRHO relay. The summary highlights the key performance drivers—antenna gains, free-space losses, system noise, and atmospheric attenuation—and shows that clear-sky conditions ensure very high C/N_0 levels, while *Mediterranean* 99.9% availability requires moderate fade margins. The results confirm that Ka-band offers an optimal balance between atmospheric transparency and antenna size, enabling Gbit-class downlinks and $\geq 99.9\%$ link availability with appropriate design margins.



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Table R.8. Ultra-compact summary of the Ka-band Earth–Moon NRHO relay link (Chapter 196).

No.	Summary Item
1	Ka-band link (22–27 GHz) using a 0.6 m NRHO relay antenna (42.4 dBi) and a 12–34 m Earth station (68–77 dBi).
2	Free-space loss at lunar range: 231–233 dB; system noise temperatures: 100–180 K (Earth), 250–500 K (orbiter).
3	Clear-sky atmospheric losses: 2.5–3.5 dB; Mediterranean 99.9% availability requires 6–14 dB margins.
4	Downlink at 26 GHz: $P_r \approx -102.6$ dBW, $C/N_0 \approx 106$ dBHz (clear sky).
5	Under 99.9% rain/humidity: $P_r \approx -111.6$ dBW, $C/N_0 \approx 97$ dBHz.
6	Uplink at 22.55–23.15 GHz: similar behaviour with slightly lower rain fade.
7	W-band (94 GHz): high nominal C/N_0 , but severe atmospheric losses restrict operation to clear-sky windows.
8	NRHO geometry ensures continuous LoS to Earth and lunar poles.
9	Ka-band provides the optimal trade-off between attenuation, antenna size, and achievable C/N_0 .
10	NRHO relay architecture guarantees $\geq 99.9\%$ availability and Gbit-class downlink capacity with suitable fade margins.

Tables R.9–R.12 summarize the complete set of numerical parameters adopted for the Ka-band Earth–Moon link analyzed in this work. The first table provides a consolidated overview of all quantities involved in the link design, including antenna gains, transmit powers, system noise temperatures, free-space losses, atmospheric impairments, and resulting received power and C/N_0 levels.

The second and third tables present the **downlink** (NRHO $\rightarrow$ Scanzano) and **uplink** (Scanzano $\rightarrow$ NRHO) configurations separately, allowing a clear comparison of the two propagation directions and highlighting the different constraints imposed by the onboard transmitter power, Earth-station aperture, and atmospheric attenuation at 23–26 GHz.

Finally, the fourth table reports the detailed atmospheric loss budget under clear-sky and 99.9 % availability conditions, based on ITU-R models for gaseous absorption, cloud liquid water, scintillation, rain attenuation, pointing losses, and hardware contributions. This breakdown clarifies the dominant impairment mechanisms in Mediterranean climates and shows how they impact the fade margins required for reliable **Earth–Moon communication**.

Together, the four tables provide a complete quantitative characterization of the NRHO relay link and form the basis for the performance assessment carried out in the following sections.



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Table R.9. Complete numerical parameters of the Ka-band Earth–Moon NRHO link (Sections 196.1–196.3).

Parameter	Value
Earth–Moon distance	3.844×10^5 km
Downlink frequency	26 GHz
Uplink frequency	22.55–23.15 GHz
Tx power (orbiter)	+13 dBW (20 W)
Tx power (Earth)	+20 dBW (100 W)
Tx antenna gain (orbiter)	42.4 dBi @ 26 GHz
Rx antenna gain (Earth, 34 m)	77.5 dBi @ 26 GHz
Tx gain (Earth, 12 m)	67.0–68.41 dBi @ 22–26 GHz
Rx gain (orbiter, 0.6 m)	41.3–42.4 dBi @ 23–26 GHz
FSPL @ 23 GHz	231.38 dB
FSPL @ 26 GHz	232.45 dB
T_{sys} (Earth)	100–180 K
T_{sys} (orbiter)	250–500 K
Atmospheric losses (clear sky)	2.5–3.5 dB
Atmospheric losses (99.9%)	6–14 dB
Received power (clear sky)	–102.6 dBW
Received power (99.9%)	–111.6 dBW
C/N_0 (clear sky)	106 dBHz
C/N_0 (99.9%)	97 dBHz

Table R.10. Downlink parameters for the NRHO → Scanzano Ka-band link at 26 GHz.

Parameter	Value
Frequency	26 GHz
Tx power (orbiter)	+13 dBW (20 W)
Tx antenna gain (orbiter, 0.6 m)	42.4 dBi
Rx antenna gain (Earth, 34 m)	77.5 dBi
FSPL (3.844×10^5 km)	232.45 dB
Atmospheric losses (clear sky)	2 dB
Miscellaneous losses	1 dB
Received power (clear sky)	–102.6 dBW
C/N_0 (clear sky)	106 dBHz
Received power (99.9%)	–111.6 dBW
C/N_0 (99.9%)	97 dBHz



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Table R.11. Uplink parameters for the Scanzano → NRHO Ka-band link at 22.55–23.15 GHz.

Parameter	Value
Frequency range	22.55–23.15 GHz
Tx power (Earth)	+20 dBW (100 W)
Tx antenna gain (Earth, 12 m)	67.0–68.41 dBi
Rx antenna gain (orbiter, 0.6 m)	41.3–42.4 dBi
FSPL (3.844×10^5 km)	231.38 dB
Atmospheric losses (clear sky)	2.5–3.5 dB
Atmospheric losses (99.9%)	6–10 dB
System noise (T_{sys} Earth)	120–180 K
System noise (T_{sys} orbiter)	250–500 K
Performance summary	Similar to downlink, slightly lower rain fade

Table R.12. Atmospheric loss budget for Ka-band Earth–Moon links (clear sky vs 99.9% availability).

Loss Component	Clear Sky	99.9% Availability
Gaseous absorption (P.676) @23 GHz	0.2–0.5 dB	0.4–1.0 dB
Gaseous absorption (P.676) @26 GHz	0.3–0.8 dB	0.5–1.2 dB
Clouds (LWC)	0.1–0.3 dB	0.1–0.4 dB
Scintillation (P.618)	0.3–0.6 dB	1–2 dB
Rain (P.838 + P.618) @23 GHz	–	3–6 dB
Rain (P.838 + P.618) @26 GHz	–	5–9 dB
Pointing losses (Earth + orbiter)	0.4–0.9 dB	0.6–1.5 dB
RF/hardware + implementation	0.7–1.2 dB	1–2 dB
Total losses @23 GHz	~ 2.5–3.5 dB	~ 6–10 dB
Total losses @26 GHz	~ 2.5–3.5 dB	~ 8–14 dB

R.2.3 Analysis of the three proposed scenarios

The four communication scenarios introduced in the previous sections are summarized in **Tables R.9–R.12**, covering:

- (i) the Ka-band uplink from Scanzano at 22.55–23.15 GHz,
- (ii) the Ka-band downlink from the NRHO spacecraft at 25.5–27.0 GHz, and
- (iii–iv) two W-band (94 GHz) reference configurations included for completeness and comparative assessment.

For each configuration, the tables report the received power under clear-sky conditions and at 99.9% availability, together with the corresponding carrier-to-noise density ratio (C/N_0).



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The results indicate that Ka-band links maintain robust performance, with clear-sky values typically in the range $C/N_0 \approx 96\text{--}98$ dBHz and only moderate degradation under adverse *Mediterranean* weather conditions. In contrast, the W-band configurations—despite their high nominal C/N_0 under ideal conditions—are heavily affected by elevated system noise temperatures and strong atmospheric attenuation at 94 GHz. These impairments significantly reduce link margins and limit operational usability to narrow clear-sky windows, making W-band unsuitable for high-availability **Earth–Moon communications**.

The exhaustive numerical analysis underlying these configurations is presented in **Appendix R**, which contains the **complete link-budget tables for all four scenarios**.

R.2.4 Performance Analysis of the W-Band NRHO Downlink and Comparison with X/Ka Bands

In the W-band at 94 GHz, the experimental Earth–Moon link can be regarded only as a secondary or contingency channel, suitable exclusively for clear-sky conditions and opportunistic burst-mode data transfers. For the downlink (NRHO $\rightarrow$ Scanzano), a 12-m ground antenna with an aperture efficiency of approximately 65% provides a receive gain of $G_r \approx 79.6$ dBi, while the orbiter employs a 0.6-m dish with the same efficiency, yielding a transmit gain of $G_t \approx 53.6$ dBi.

The main limiting factor at 94 GHz is the system noise temperature, significantly higher than in X- or Ka-band due to atmospheric absorption and receiver front-end noise; typical values at Scanzano (PA) range from $T_{sys} \approx 600\text{--}1000$ K, compared with 50–150 K in X/Ka-band systems. This severely reduces the achievable carrier-to-noise density ratio C/N_0 , confining W-band operation to atmospheric windows with minimal absorption.

The free-space path loss at 94 GHz is:

$$L_{FSPL} = 20 \cdot \log_{10} \left(\frac{4\pi df}{c} \right) = 92.45 + 20 \log_{10}(f_{GHz}) + 20 \log_{10}(d_{km}) \approx 243.6 \text{ [dB]} \quad (\text{R.24})$$

For the downlink configuration:

- $P_t = +13$ dBW
- $G_t = 53.6$ dBi
- $G_r = 79.6$ dBi
- $L_{FSPL} = 243.6$ dB
- *Clear sky losses* = 4 dB.

The received power becomes:

$$P_r = P_0 - L_{tot} = (P_t + G_t + G_r) - (FSPL + L_{clear-sky}) = 13 + 53.6 + 79.6 - 243.6 - 4 \approx -101.4 \text{ [dBW]}. \quad (\text{R.25})$$

With $T_{sys} = 900$ K, the carrier-to-noise density ratio is:

$$C/N_0 = P_r + 228.6 - 10 \log_{10}(T_{sys}) = -101.4 + 228.6 - 10 \log_{10}(900) \approx 97.7 \text{ [dBHz]}. \quad (\text{R.26})$$

In the presence of rain, an additional specific attenuation term L_{rain} must be included, giving:

$$\begin{aligned} P_r^{rain} = P_0 - L_{tot} - L_{rain} &= (P_t + G_t + G_r) - (FSPL + L_{clear-sky} + L_{rain}) \\ &= (13 + 53.6 + 79.6 - 243.6 - 4) - L_{rain} \\ &= -101.4 - L_{rain} \end{aligned} \quad (R.27)$$

Representative values are:

- moderate rain: $L_{rain} = 8$ dB;
- typical rain: 12 dB;
- strong storm: 20 dB;
- severe event: 40 dB;

yielding C/N_0 values from 98.7 dBHz (moderate rain) down to 66.7 dBHz (severe event). The corresponding received powers range from -70.4 dBm to -102.4 dBm. Rain also increases the system noise temperature; for example, raising T_{sys} from 900 K to 1200 K introduces an additional $10 \log_{10}(1200/900) \approx 1.25$ dB degradation in C/N_0 . Although the W-band link remains theoretically feasible, the fading margins required in Mediterranean weather conditions can reach 60 dB, rendering reliable continuous operation impractical.

Figure R.3 compares X-, Ka- and W-band performance across different availability targets. The X-band remains highly robust, sustaining $C/N_0 > 90$ dBHz even at 99.99% availability, making it an excellent candidate for backup or critical command-and-control channels. Ka-band provides high throughput in clear-sky conditions and remains operational at approximately 85 dBHz under 99.99% availability, confirming its role as the primary deep-space band. By contrast, the W-band exhibits excellent performance only under ideal conditions and collapses rapidly above 99% availability, confirming its use as an experimental or opportunistic burst-mode channel. To offer a concise overview of the link design, **Table R.13** summarizes the key parameters, including antenna gains, FSPL, received power, and C/N_0 for X-, Ka- and W-band under both clear-sky and 99.9% availability conditions. The table highlights the intrinsic robustness of X-band, the high-capacity potential of Ka-band, and the severe limitations of W-band in operational scenarios. When considering $T_{sys} = 900$ K at 94 GHz, **Table R.14** extends the analysis to include revised W-band performance values.

Finally, the **Adaptive Coding and Modulation (ACM)** profile in **Table R.15** illustrates the operational trade-offs across the three bands. Under clear skies, Ka-band supports modulations up to 64-QAM (profile B), and W-band can theoretically sustain 256-QAM (profile A). At 99.9% availability, Ka-band restricts operation to robust QPSK modes (profile D), while W-band becomes unusable. X-band remains the most resilient, supporting moderate-to-high-order constellations even at the 99.99% percentile.

These results collectively demonstrate the need for a multi-band strategy: Ka-band as the primary high-capacity channel, X-band as a highly reliable backup, and W-band reserved for experimental clear-sky or opportunistic high-rate transfers.

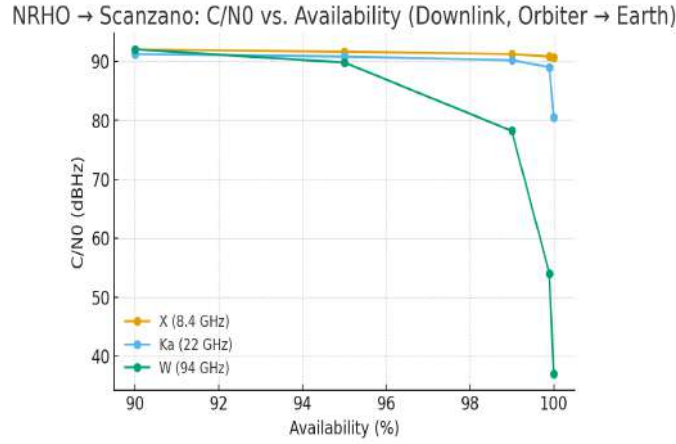


Figure R.2. Downlink (NRHO → Scanzano): carrier-to-noise density ratio (C/N_0) versus link availability. The X-band remains highly robust (>90 dBHz up to 99.99% availability), Ka-band shows moderate degradation around 99.9%, while the W-band rapidly degrades beyond $\sim 99\%$ due to atmospheric attenuation and high system noise, limiting its use to clear-sky or opportunistic operations.

Band	G_t [dBi]	G_r [dBi]	FSPL [dB]	P_r clear-sky [dBW]	C/N_0 clear-sky [dBHz]	$P_r@$ 99.9% [dBW]	C/N_0 @99.9% [dBHz]	T_{sys} [K]
X	32.6	58.6	222.6	-118.9	92.7	-120.4	91.2	≈ 50
Ka	40.9	67.0	231.0	-113.1	93.8	-122.1	84.8	≈ 150
W	53.6	79.6	243.6	-101.5	98.1	-146.5	53.1	≈ 800

Table R.13. Summary of the Earth–NRHO lunar downlink link budget to the Scanzano ground station, including antenna gains, free-space and atmospheric losses, received power, and carrier-to-noise density ratio for the considered frequency bands.

Band	G_t [dBi]	G_r [dBi]	FSPL [dB]	P_r clear-sky [dBW]	C/N_0 clear-sky [dBHz]	$P_r@$ 99.9% [dBW]	C/N_0 @99.9% [dBHz]	T_{sys} [K]
W	53.6	79.6	243.6	-101.5	97.6	-146.5	52.6	900

Table R.14. Additional row corresponding to a system noise temperature of 900 K for the W-band (94 GHz) NRHO → Scanzano downlink.

Scale/Profile	Modulation Schema + FEC	E_s/N_0 req [dB]*	C/N_0 req [dBHz]	Efficiency [bps/Hz]
A	256-QAM, LDPC $r = 0.9$	~ 20	98	~ 8.5
B	64-QAM, LDPC $r = 0.8$	~ 14	92	~ 6.5
C	16-QAM, LDPC/Turbo $r = 0.75$	~ 9	87	~ 4.5
D	QPSK, LDPC $r = 0.5$	~ 4	82	~ 2.0
E	BPSK, RS/LDPC $r = 0.5$	~ 0	78	~ 1.0

Table R.15. Adaptive coding and modulation (ACM) thresholds for an NRHO Earth–Moon link. Note: (*) E_s/N_0 values refer to a target $BER \lesssim 10^{-6}$ assuming modern FEC schemes; practical thresholds may vary by ± 1 –2 [dB] depending on the specific implementation.

R.3 Global Map of Vertical Wet Tropospheric Delay (ZWD) over the Mediterranean Region at 00:00 UTC on February 1, 2019

Figure R.3 shows the Vertical Wet Tropospheric Delay (ZWD) over the Mediterranean at 00:00 UTC on 1 February 2019, generated through the ERA5-based processing pipeline developed in this thesis. The workflow harmonizes meteorological fields to a common synoptic time, converts humidity and temperature into ZWD using **Saastamoinen/IERS formulations**, and maps results on a 0.25° grid with coastline overlays for geolocation. The color scale (~ 0.05 – 0.30 [m]) reveals distinct moisture structures: low ZWD over *North Africa* and southern *Europe* indicates dry continental air, while higher delays along the *eastern Mediterranean* and coastal zones reflect maritime moisture advection and convergence. Beyond visualization, this **ITU/IERS-compliant pipeline** provides a reproducible method to generate physically consistent ZWD fields for GNSS tropospheric correction, radar path-delay estimation, and high-frequency satellite link planning.

The same figure can be enhanced by moving beyond a direct application of ITU–R recommendations (P.1511, P.837, P.453, P.1510) and instead presenting climatological median or annual maps for **February 2019–2025**, analyzed year by year. Temporal trends can be derived from ERA-5 using the **Total Column Water Vapor (TCWV)** at Scanzano (PA), converted into **Zenith Wet Delay (ZWD)** through the **Bevis** relation:

$$ZWD \approx \Pi(T_m) \cdot PWV \quad (\text{R.28})$$

where $\Pi(T_m) = 10^{-6} \rho_w (k_3/T_m + k'_2)$ with $k'_2 = 22.974 [K/hPa]$, $k_3 = 3.739 \times 10^5 [K^2/hPa]$, $\rho_w = 10^3 [kg/m^3]$.

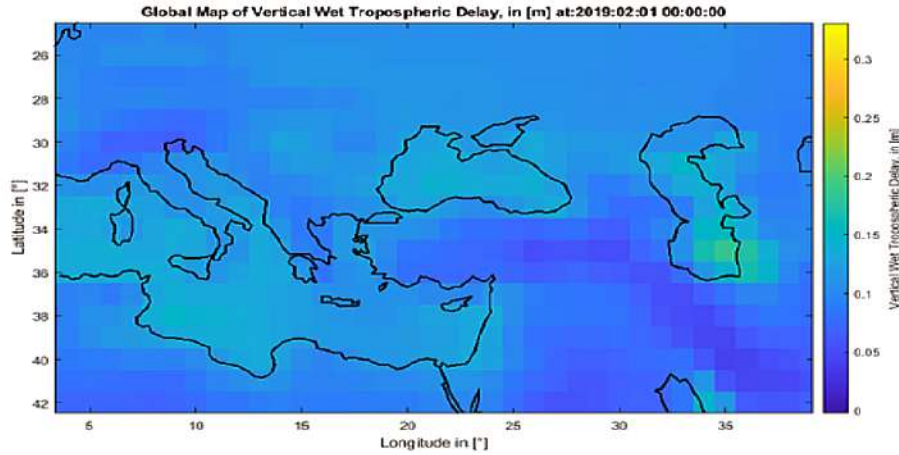


Figure R.3. Vertical Wet Tropospheric Delay (ZWD) over the Mediterranean at 00:00 UTC, 1 February 2019.

For typical Mediterranean February conditions $T_m \approx 275 - 285$ [K], $\Pi(T_m) \approx 0.154 - 0.165$ [$m/(kg/m^2)$], yielding a first-order estimate of ZWD in meters, as defined by relation (15.27). A more rigorous formulation of T_m is provided in §15.4.1.

R.3.1 ITU–R Propagation Models Applied to the Mediterranean Context: Orography-Aware Attenuation Maps

While numerous ITU–R-based maps were previously examined for extracting **Day-of-Year (DOY)** values of interest, this revised thesis extends the analysis to include additional reference models:

- ITU–R P.676 – gaseous attenuation
- ITU–R P.837, P.618, P.838 – rain attenuation
- ITU–R P.618 – tropospheric scintillation

As illustrated in the Sicilian case study, maps gain interpretative depth when orographic information is superimposed, improving readability and realism beyond standard ITU–R products. Accordingly, supplementary panels were introduced to provide a broader Mediterranean perspective and to depict attenuation patterns consistent with the reference curves of the above recommendations.

Figure R.4 confirms that gaseous attenuation remains minor at 22–26 GHz (< 0.6 dB at 30° elevation) but rises to ~ 1.2 dB at 94 GHz due to molecular absorption by O_2 and H_2O . Rain attenuation at 0.01% exceedance reaches 8–12 dB in the Ka-band—suitable for link-budget design with proper margins—whereas the W-band exceeds 12–14 dB, limiting availability to clear-sky conditions or systems with mitigation such as **Adaptive Coding and Modulation (ACM)** or site diversity. Tropospheric scintillation exhibits RMS fluctuations of ~ 0.3 dB in Ka and up to 0.8 dB in W, aligning with ITU–R P.618 estimates. These findings highlight the Ka-band as the most balanced option for Earth–Moon communications, offering robustness

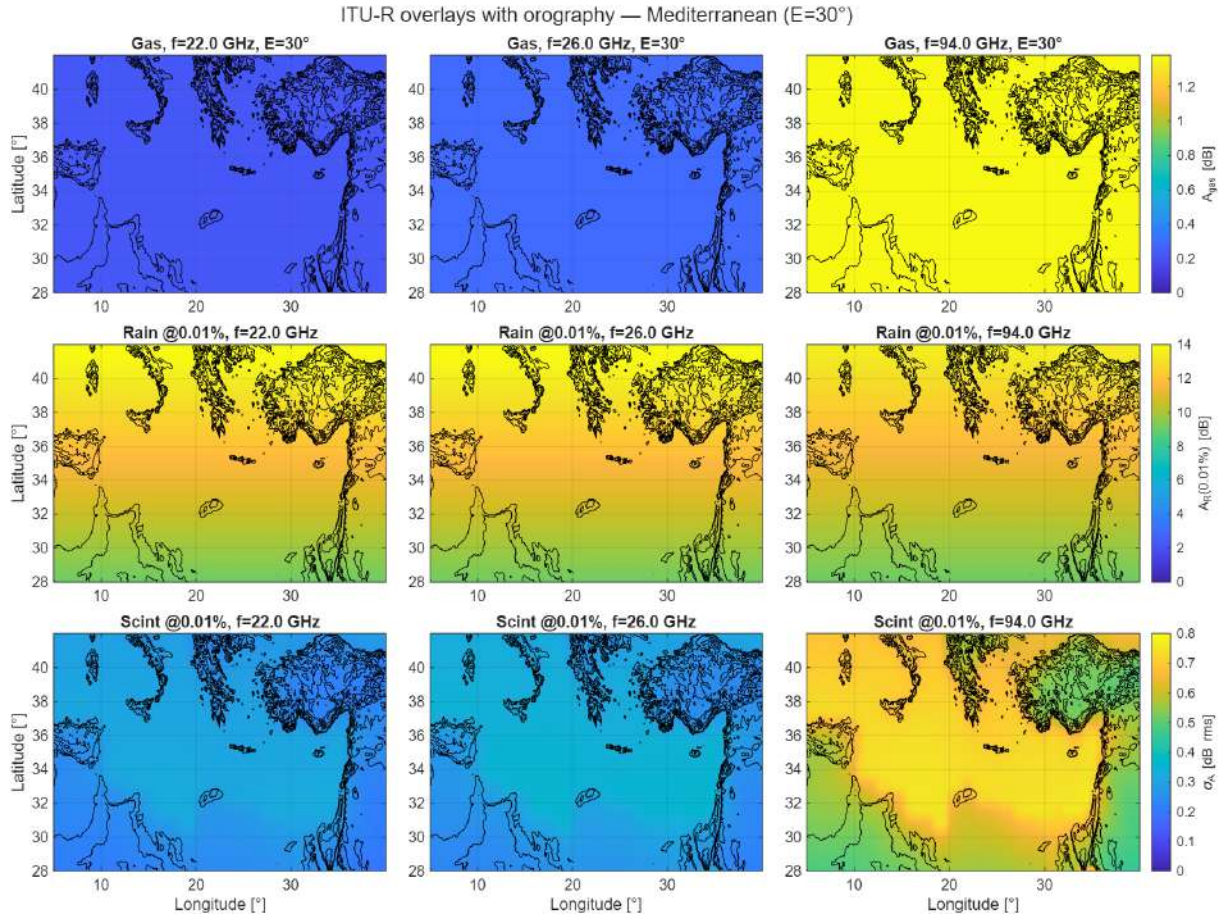


Figure R.4. Orography-enhanced attenuation maps over the Mediterranean region at 30° elevation, derived from ITU-R reference models. From top to bottom: (i) gaseous attenuation according to ITU-R P.676, (ii) rain attenuation at 0.01% of time from ITU-R P.837/P.618/P.838, and (iii) tropospheric scintillation RMS from ITU-R P.618. Results confirm marginal gaseous losses in the Ka-band (< 0.6 [dB]), moderate rain attenuation (8–12 [dB]) compatible with robust link margins, and scintillation effects around 0.3 [dB]. By contrast, the W-band exhibits significant penalties (≈ 1.2 [dB] gaseous, >12 –14 [dB] rain, ≈ 0.8 [dB] scintillation), limiting its operational use to favorable clear-sky conditions.

and scalability, while the W-band, though spectrally attractive, remains constrained by severe propagation limits.

The fusion diagnostic exposes a clear east–west asymmetry in ZWD dynamics: corridors with strong high-frequency wavelet energy are precisely where static or globally tuned tropospheric models underperform. Practically, this argues for space- and time-adaptive handling of delays—denser GNSS sampling and faster update cadence over the eastern *Mediterranean* and the $\sim 5^\circ$ E transition zone, while a coarser cadence is adequate over the more quiescent central sector. In standards-aligned workflows (IERS/ITU), this translates into localized mapping/weighting, tighter error budgets for EO retrievals, and targeted calibration where volatility is highest, reducing residual delay and bias without overfitting.

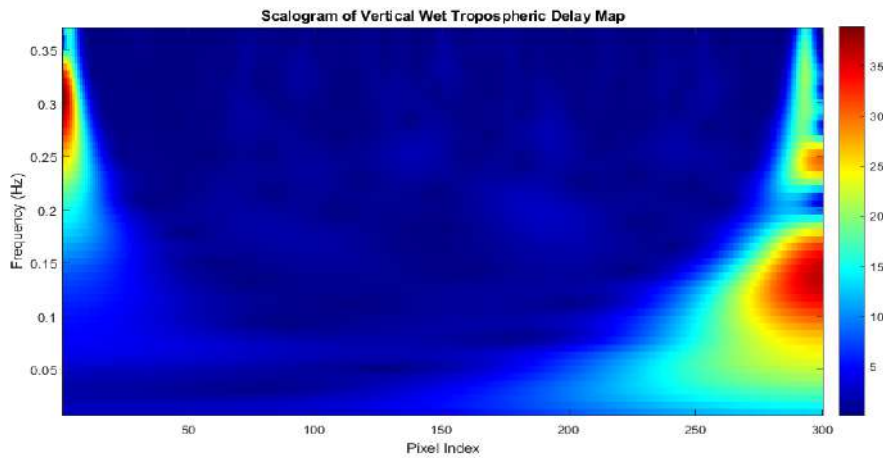


Figure R.5. Wavelet scalogram of the Vertical Wet Tropospheric Delay (ZWD). The color scale shows the signal amplitude, with blue regions indicating weak variability and yellow–red regions highlighting episodes of enhanced water vapor activity. High-frequency fluctuations reflect short-term meteorological events, while low-frequency components correspond to slower, large-scale variations.

R.3.2 Innovation in the Global Map Analysis

While Mediterranean ZWD fields are commonly analyzed through geostatistical interpolation (e.g., kriging), NWP outputs, or deterministic formulations based on pressure, temperature, and humidity—typically from long-term reanalyses such as ERA5—there is strong motivation to explore their multiscale structure using wavelet techniques. Classical wavelet scalograms, standard for time-series analysis in meteorology (e.g., precipitation, temperature, ENSO), are rarely applied spatially to ZWD fields; extending them to **2-D latitude–longitude domains** requires careful interpretation. Nevertheless, spatial wavelet analysis could complement EOF/PCA or Fourier methods by exposing scale-localized features—hidden gradients, moisture corridors, or convective footprints—not captured by global bases. A consistent approach would: (i) compute spatial wavelet coefficients from ZWD maps, (ii) link scale-dependent energy to synoptic regimes and moisture advection, and (iii) validate results against independent diagnostics. If such features align with recurrent convective bands or humidity plumes, this method could enhance GNSS delay correction and telecom planning through frequency-aware spatial insights.

Figure R.5 presents the wavelet scalogram of the **Vertical Wet Tropospheric Delay (ZWD)**, offering a joint time–frequency view of variability driven by atmospheric water vapor. The color scale depicts relative amplitude: blue indicates weak variability, yellow–red marks enhanced activity. High frequencies (0.2–0.35 Hz) trace short-term fluctuations, while low frequencies (< 0.1 Hz) represent large-scale persistence. Concentrated high-amplitude zones reveal episodic water-vapor increases linked to convection or precipitation. Overall, the scalogram shows that ZWD variability is dominated by intermittent, short-lived events over a stable background.

Figure R.6 presents a data-fusion diagnostic developed to couple geospatial structure and variability scale in a single view. The base map displays the spatial distribution of wet delay

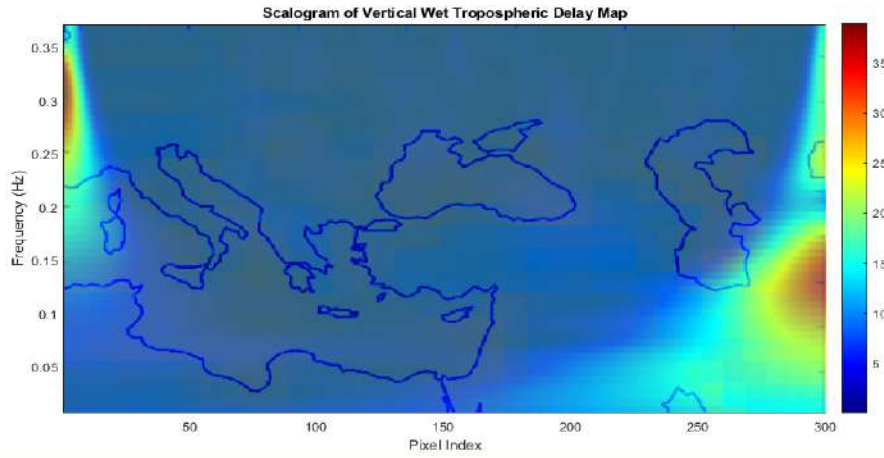


Figure R.6. Multiscale data-fusion of Wet Vertical Tropospheric Delay (ZWD) over the Mediterranean.

(red ≈ 0.30 m, yellow ≈ 0.20 m, blue ≈ 0.05 m), while the co-registered wavelet curtain overlays longitudinal energy from continuous wavelet transforms of ZWD time series computed along meridional transects. The ordinate represents variability scale: high frequencies (top) indicate rapid fluctuations, and low frequencies (bottom) reflect more persistent regimes.

This fusion reveals multiscale hotspots across the Mediterranean. East of $\sim 30\text{--}35^\circ\text{E}$ (**Turkey/Cyprus**), intense red bands mark strong, fast-varying humidity dynamics linked to maritime advection and coastal convergence. Around $\sim 5^\circ\text{E}$, the western Mediterranean shows additional dynamical transitions, while the central sector ($\sim 15\text{--}25^\circ\text{E}$; **Italy–Greece**) exhibits weaker, blue-dominant responses, indicating more stable, long-lived conditions. Beyond its descriptive role, this **IERS/ITU-compliant** product provides a physically interpretable tool for identifying regions of high tropospheric volatility, integrating spatial organization and frequency-dependent variability within a reproducible framework. (Notation follows geodetic convention: $\text{ZWD} \equiv$ wet vertical delay; “WVD” avoided for clarity.)

In summary, the fusion of spatial and spectral domains highlighted here provides a concise yet physically interpretable diagnostic of tropospheric water-vapor variability, forming a foundation for the more detailed evaluations presented in the following sections.

R.3.3 Mathematical formulation

To formalize the qualitative interpretation of the fusion map into a reproducible diagnostic, I defined a **Multiscale Coupling Index (MCI)** that condenses both spatial heterogeneity and temporal variability into a single scalar function of longitude. Let $Z(\varphi, \lambda, t)$ denote the wet delay (ZWD) field at latitude φ , longitude λ , and time t . The first step that I realize is to extract the persistent **geospatial structure** by computing the temporal mean (or seasonal mean):

$$\bar{Z}(\varphi, \lambda) = \frac{1}{T} \int_{t_0}^{t_0+T} Z(\varphi, \lambda, t) dt, \quad (\text{R.29})$$



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and evaluating its longitudinal gradient (finite-difference on great circles):

$$\partial_\lambda \bar{Z}(\varphi, \lambda) \approx \frac{\bar{Z}(\varphi, \lambda + \Delta\lambda) - \bar{Z}(\varphi, \lambda - \Delta\lambda)}{2R_\oplus \cos(\varphi) \Delta\lambda_{rad}}, \quad (\text{R.30})$$

A robust meridional aggregation then yields the structural component,

$$G(\lambda) = \text{median}_{\varphi \in \Omega} (|\partial_\lambda \bar{Z}(\varphi, \lambda)|). \quad (\text{R.31})$$

which quantifies persistent longitudinal contrasts in water vapor delay. In parallel, temporal variability is characterized through the **Continuous Wavelet Transform (CWT)** of the meridionally averaged time series over a **narrow latitude band** $B(\lambda)$ (or any meridional transect):

$$x_\lambda(t) = \frac{1}{|B(\lambda)|} \cdot \int_{\varphi \in B(\lambda)} Z(\varphi, \lambda, t) d\varphi. \quad (\text{R.32})$$

The wavelet coefficients (e.g., complex **Morlet** ψ) is defined as:

$$W(\lambda, s, \tau) = \int_{-\infty}^{+\infty} x_\lambda(t) \cdot \frac{1}{\sqrt{s}} \psi^* \left(\frac{t - \tau}{s} \right) dt, \quad (\text{R.33})$$

define a power spectrum:

$$P(\lambda, s, \tau) = |W(\lambda, s, \tau)|^2. \quad (\text{R.34})$$

Integrating over time and restricting to a scale band S (e.g., synoptic 2–10 days, S stay for Synoptic) gives the band-limited power fraction, that is:

$$\tilde{V}(\lambda) = \int_{s \in S} \left(\frac{1}{T} \int P(\lambda, s, \tau) d\tau \right) ds. \quad (\text{R.35})$$

I can normalize this expression by the total power to get a dimensionless fraction, defined as:

$$V(\lambda) = \frac{\int_{s \in S} \left(\frac{1}{T} \int P(\lambda, s, \tau) d\tau \right) ds}{\int_0^{+\infty} \left(\frac{1}{T} \int P(\lambda, s, \tau) d\tau \right) ds} \in [0, 1]. \quad (\text{R.36})$$

To combine both descriptors on a common scale, percentile-based normalization is applied:

$$G_n(\lambda) = \frac{G(\lambda) - P_{50}(G)}{P_{95}(G) - P_{50}(G)}, \quad V_n(\lambda) = \frac{V(\lambda) - P_{50}(V)}{P_{95}(V) - P_{50}(V)}. \quad (\text{R.37})$$

The **Multiscale Coupling Index** is then defined as:

$$MCI(\lambda) = G_n(\lambda) \cdot V_n(\lambda), \quad (\text{R.38})$$

which takes large values precisely where sharp spatial gradients are collocated with enhanced multi-scale variability. For statistical robustness, a hot-spot score may be obtained by z-scoring



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$MCI(\lambda)$, with significant regimes identified above the 90th percentile. This construction turns a descriptive visualization into a rigorous metric: $MCI(\lambda)$ isolates longitudinal sectors such as the eastern **Mediterranean (Turkey–Cyprus)**, the western basin (near 5°E), and the subdued central domain (**Italy–Greece**), quantitatively supporting the qualitative regimes identified in the fusion map. In the formulation above, S denotes the set of variability scales (i.e., the wavelet dilation parameters s) selected for integration. In wavelet analysis, the scale s controls the temporal (or spatial) extent of the analyzing function and is directly proportional to the characteristic period T_s of fluctuations ($T_s = \kappa s$ for the **Morlet wavelet**, with κ a known constant). By choosing different intervals for S , one can emphasize distinct dynamical regimes:

- $S \approx 2\text{--}10\text{days} \rightarrow$ synoptic scale, associated with frontal systems and cyclonic/anticyclonic activity.
- $S \approx 12\text{ hours--}2\text{ days} \rightarrow$ mesoscale, capturing organized convection and storm systems.
- $S \approx 10\text{--}30\text{ days} \rightarrow$ low-frequency or intraseasonal scale, linked to quasi-stationary oscillations and blocking patterns.

Thus, the definition of S provides a tunable filter, allowing the MCI to selectively diagnose the role of synoptic versus mesoscale or intraseasonal variability in shaping wet-delay dynamics.

R.3.4 Original Contribution of This Work to Meteorological Science

The methodology combines two standard tools. The **continuous wavelet transform (CWT)** resolves scale-dependent variability in GNSS-derived ZWD time series (**Torrence & Compo**, 1998), while spatial gradients with percentile-based normalization describe the underlying geophysical structure. These components form the **Multiscale Coupling Index (MCI)**, which identifies locations where horizontal ZWD gradients align with enhanced temporal variability. The metric is physically consistent and IERS/ITU compliant, with direct relevance for GNSS delay correction and high-frequency propagation. Mediterranean applications show that MCI exposes dynamical regimes not captured by conventional products.

For finer-scale detection, the **Multiscale Convergence Index (MCI*)** isolates sharp transitions. Longitudinal transects filtered with a 20-pixel moving window are evaluated against a 90th-percentile threshold to identify statistically significant mesoscale features.

Figure R.7 illustrates how MCI* isolates these structures for subsequent analysis.

The upper panel shows the raw geospatial field over the *Mediterranean*, while the lower panel presents a longitudinal transect of the normalized MCI*, highlighting hot-spots above the 90th percentile. Meteorologically, MCI* maxima trace sharp horizontal gradients typically associated with frontal boundaries, cyclonic structures, or convergence lines. The rapid fluctuations over Italy and the *Balkans* ($\approx 5\text{--}15^\circ\text{E}$) reflect the fragmented synoptic organization characteristic of *Mediterranean* cyclogenesis [370, 371], where alternating convergence–divergence regimes modulate mesoscale instability. In contrast, the western *Mediterranean* ($\approx -10^\circ$ to 0°E) exhibits weaker variability, consistent with the region’s large-scale pressure-driven stability [368, 369].

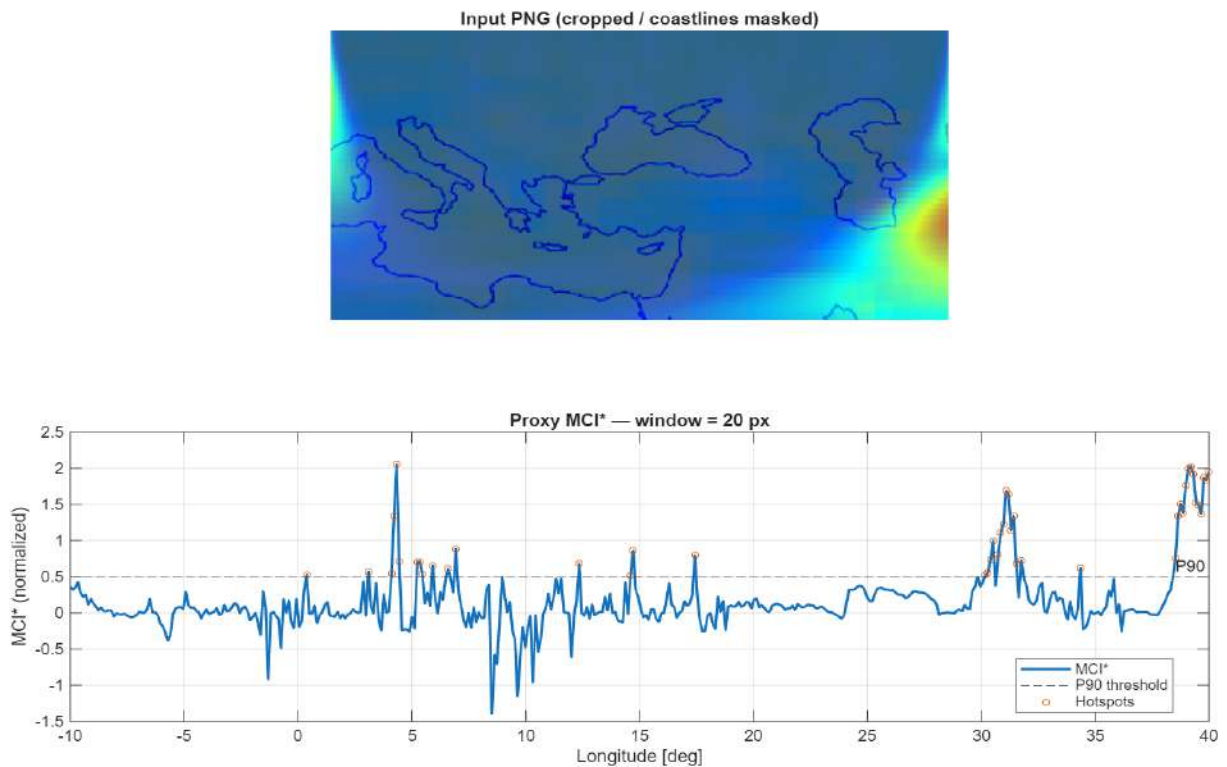


Figure R.7. From geospatial field to proxy-metric extraction. The upper panel shows the input field (PNG, cropped and coastline-masked) over the Mediterranean domain. The lower panel displays the normalized Multiscale Convergence Index (MCI*) computed along longitude using a 20-pixel moving window. The dashed line marks the 90th-percentile (P90) threshold, and orange circles denote statistically significant hotspots above this level. The diagnostic illustrates the transition from raw geospatial data to a physically interpretable proxy capable of detecting localized anomalies within the field.



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The strongest anomalies occur over the eastern *Mediterranean* and the *Levant* ($\approx 30\text{--}40^\circ\text{E}$), where multiple hotspots exceed P90. These features likely correspond to air-mass boundaries and jet-streak signatures, marked by sharp thermodynamic contrasts [366, 367]. Overall, MCI* emerges as a physically grounded diagnostic of spatial–dynamical coupling, capable of identifying front-like transitions and mesoscale anomalies.

R.4 Tropospheric Effects on Satellite Signal Propagation

R.4.1 Electromagnetic Propagation in the Neutral Atmosphere

Figure R.8 illustrates representative propagation geometries between satellites (S_1, S_2) and ground stations (P_1, P_2) under atmospheric disturbances. Along the slant paths ($L_{s1}\text{--}L_{s4}$), electromagnetic waves traverse layers containing hydrometeors that induce attenuation, depolarization, scattering, and—critically for GNSS—variations of the refractive index ($N > 1$).

In the neutral atmosphere, the excess path experienced by microwave signals is expressed as the **slant path delay (SPD)**, which decomposes into hydrostatic and wet components and is mapped to the zenith direction through standard mapping functions:

$$SPD(el) = m_h(el) \cdot ZHD + m_w(el) \cdot ZWD, \quad (\text{R.39})$$

where **ZHD** and **ZWD** denote the Zenith Hydrostatic and Zenith Wet Delays [m], and $\{m_h, m_w\}$ are the corresponding elevation-dependent mapping functions (typically from the **Niell model**). The delays depend on the atmospheric refractivity:

$$N = K_1 \cdot (P - e) + K_2 \frac{e}{T} + K_3 \frac{e}{T^2} \equiv k_1(P - e)Z_d^{-1} + \left(k_2 \frac{e}{T} + k_3 \frac{e}{T^2}\right) Z_v^{-1}, \quad (\text{R.40})$$

with P the total pressure, T the temperature, e the water-vapor pressure, and Z_d, Z_v the dry-air and vapor compressibility factors. Although the formulation is well established, associated uncertainties are rarely quantified, despite their relevance for typical global ZTD root-mean-square fluctuations of ≈ 6 mm (see **Table R.17**).

Table R.16. Coefficients for Atmospheric Refractivity Models

Author(s)	K_1 [1/hPa]	K_2 [K/hPa]	$K_3 \times 10^{-5}$ [K ² /hPa]
Smith & Weintraub	77.607 ± 0.013	71.6 ± 8.5	3.747
Thayer	77.64 ± 0.64	64.79 ± 0.78	3.776
Hasegawa & Stokesb	77.015 ± 0.036	69.4 ± 0.30	3.701
Liebe	77.676 ± 0.023	71.631	3.74656
Bevis weighted	77.6	69.5	3.739
Bevis unweighted	77.6	70.4	3.739
Rueger best avail.	77.695 ± 0.017	71.3 ± 10.5	3.75405 ± 0.003
Rueger best average	77.689 ± 0.0094	71.295 ± 1.3	3.75463 ± 0.0076

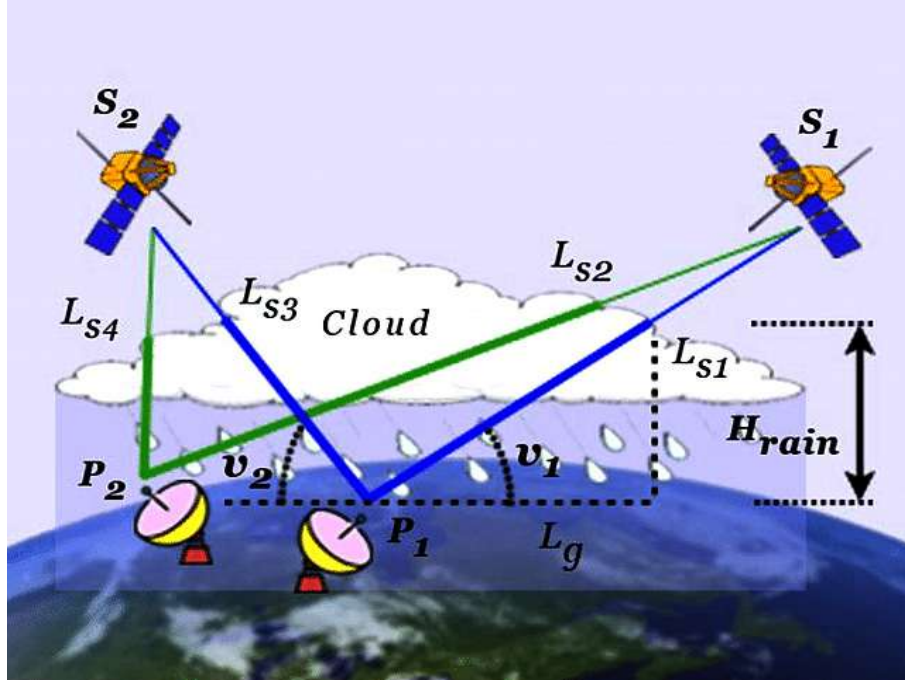


Figure R.8. Schematic Communication Framework. The figure illustrates the slant paths between satellites (S_1 , S_2) and ground stations (P_1 , P_2) under precipitating conditions. The links L_{s1} – L_{s4} intersect cloud and rain layers, where attenuation, scattering, and depolarization occur. The ground segment includes the horizontal path L_g and terminal velocities v_1, v_2 associated with hydrometeors. The diagram highlights how propagation is modified by cloud-base structure and rain height H_{rain} .

The Zenith Hydrostatic Delay (ZHD) is obtained from the surface pressure at the antenna site using the **Saastamoinen** formulation:

$$ZHD = \frac{0.0022767 \times p}{1 - 0.00266 \times \cos(2\phi) - 0.00028 \times H}, \quad (\text{R.41})$$

where p is the total surface pressure [hPa], H the ellipsoidal height [km], and ϕ the ellipsoidal latitude [rad]. The Zenith Wet Delay (ZWD) is defined as the vertical integral of humidity- and temperature-dependent **refractivity**:

$$ZWD = 10^{-3} \times \int_{h_0}^{+\infty} \left(K_2' \frac{e}{T} + K_3 \frac{e}{T^2} \right) dh, \quad (\text{R.42})$$

with $K_2' = K_2 - K_1 \times \frac{M_W}{M_d} = 22.1 \text{ K/hPa}$ with $M_d=28.9644$ and $M_W=18.0152 \text{ g/mol}$.

A reduced-accuracy expression can be derived from the **surface vapor pressure** e_0 using local climatological parameters λ and T_m , representing the vertical decay of **humidity** and **temperature**:

$$ZWD(e_0, \lambda, T_M) = 10^{-6} \cdot \frac{R_d}{g_m} \left(K_2' + \frac{K_3}{T_M} \right) \cdot \frac{e_0}{\lambda + 1} \approx 10^{-6} \cdot \frac{R_d}{g_m} \cdot \frac{K_3}{T_M} \cdot \frac{e_0}{\lambda + 1}, \quad (\text{R.43})$$



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valid under $e = e_0 \times \left(\frac{p}{p_0}\right)^{\lambda+1}$. Here $R_d = 287.054 \text{ J/(kg} \cdot \text{K)}$,

$$g_m = g(\phi, h_0) = 9,784 * (1 - 0.00266 * \cos(2 * \phi) - 0.00028 * h_0) \rightarrow [m/s^2] \quad (\text{R.44})$$

The effective mean temperature of the water-vapor column is:

$$T_m \equiv T_M = \frac{\int_{h_0}^{+\infty} \frac{e}{T} dh}{\int_{h_0}^{+\infty} \frac{e}{T^2} dh} := \left\{ \begin{array}{c} \text{Effective mean temperature} \\ \text{of the Water Vapor} \\ \text{Column above the receiver} \end{array} \right\} \rightarrow [K]. \quad (\text{R.45})$$

R.5 Path Delay Due to Hydrometeors

In the presence of hydrometeors (fog, rain, hail, snow, *graupel*), additional path delay arises. At C-band, phase delays caused by cloud droplets can be estimated using dielectric permittivity. The refractivity N_{cloud} is obtained from the **Clausius–Mossotti equation** (Liebe, 1989; Hanssen, 1998; Solheim, 1999):

$$N_{cloud} = \frac{3}{2\rho_w} \cdot \frac{\varepsilon_w - 1}{\varepsilon_w + 2} \cdot LWC \quad (\text{R.46})$$

where ε_w is the permittivity of water cloud droplets, ρ_w is the density of liquid water [g/cm^3], and the liquid water content LWC is the cloud liquid density [g/m^3]. According to **Clausius–Mossotti**, dielectric refractivity is proportional to the total liquid along the propagation path and independent of droplet shape (Born, 1986). For larger particles and higher frequencies, forward-scattering effects should also be considered. Since ε_w depends only weakly on temperature, the relation can be approximated within 1% (Solheim, 1999) as:

$$N_{cloud} \approx 1.45 \cdot LWC. \quad (\text{R.47})$$

The corresponding zenithal path delay is:

$$\Delta L_{clouds} = \int_H^\infty \frac{1.45}{\cos \theta} LWC dz \rightarrow [mm] \quad (\text{R.48})$$

where the integration is over altitude in kilometers. A concentration of 1 [$g \text{ m}^{-3}$] over 1 [km] yields an integrated liquid value of 1 [mm], producing a vertical delay of about 1.45 [mm]. Cloud droplets thus generate path delays of a few millimeters (**Table R.17**), whereas ice-crystal effects are negligible at 5-cm wavelengths, given the much lower permittivity of ice compared with water (Ulaby, 1981; Hanssen, 1998).

The dispersive component of the delay due to liquid water droplets is at least ten times smaller than the non-dispersive component for frequencies below 50 GHz, and therefore can be ignored at C band frequencies [Elgered, 1993, Hanssen, 1998]. In the presence of rain, forward scattering from sparsely distributed discrete particulates can cause phase delay, proportional to the real part of the forward scattering amplitude. For the case of coherent propagation in rain, the



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Type of cloud	LWC in $[g/m^3]$	ZWD in $[mm/km]$
Stratiform clouds	0.05–0.25	0.1–0.4
Small Cumulus clouds	0.5	0.7
Cumulus Congestus and Cumulonimbus	0.5–2.0	0.7–3
Ice clouds	< 0.1	< 0.1

Table R.17. Liquid water content in clouds. [Hall et al.1996, Bean and Dutton, 1986; Hanssen, 1999].

propagation phase delay, ΔL_{rain} in mm/km, can be calculated according to **Oguchi** (1983):

$$\Delta L_{\text{rain}} = 10^{-3} \frac{2\pi}{k^2} \int \text{Re}[f(D)] N(D) dD \quad (\text{R.49})$$

where k is the wave number (wavelength in [mm]), D is the equivolumic drop diameter in millimeters, and $\text{Re}[f(D)]$ is the real part of the forward scattering amplitude in millimeters for horizontal or vertical polarization. $N(D)$ is the drop size distribution $[mm^{-1}m^{-3}]$, which represents the number concentration per cubic meter per size interval in millimeters. At **C band**, models assume that the particles in the target volume are much smaller than the wavelength of the incident radiation, i.e., $n \cdot D/\lambda \ll 1$, so that the Rayleigh approximation can be used. **Rayleigh** approximation is generally not valid for rain for frequencies greater than 10 GHz; it is only suitable for rain rates less than $2.5 \text{ mm } h^{-1}$ at 14 GHz and is not applicable in any appreciable rainfall at 35 GHz or above [Ulaby, 1981].

For rain, the drop size distribution is assumed to have an exponential form as in **Marshall and Palmer** (1948):

$$N = N_0 e^{-\Lambda D} \quad (\text{R.50})$$

with $N_0 = 8000 [mm^{-1}m^{-3}]$, and $\Lambda = \frac{4.1}{R^{0.21}}$, where R is the rain rate given in mm/hr.

Raindrops that are larger than 1 mm are modeled as oblate spheroids with the ratio of the horizontal to vertical axis related to the equivolumic drop diameter D as in **Pruppacher and Beard** (1970). **Moissev and Hanssen** (2003) calculated the path delay at the **C band** for different rain intensities. Their results are reported in **Figure R.9**.

Rain rates below 20 [mm/h] are classified as light rain, 20–60 [mm/h] as moderate, and above 60 [mm/h] as heavy rain. **Figure R.9** shows that at high intensities the path delay may reach several centimeters, mainly in tropical climates.

Solheim et al. (1999) calculated delays at GPS carrier frequencies ranging from 6 [mm] for light rain (20 [mm/h], ≈ 2 [mm/km] over a 3 [km] scale height) to 9 [cm] for heavy rain (200 [mm/h], ≈ 15 [mm/km] over a 6 [km] scale height, L2 carrier). Based on interferometric observations, **Moissev and Hanssen** (2003) reported enhanced signal delays during precipitation, attributed primarily to increased water vapor concentration from partial raindrop evaporation. Comparable results were obtained by **Rogers and Olsen** (1975) at 6 [GHz], who found a two-way range error of 5 [cm] for 100 [mm/h] rain and a 5 [km] scale height.

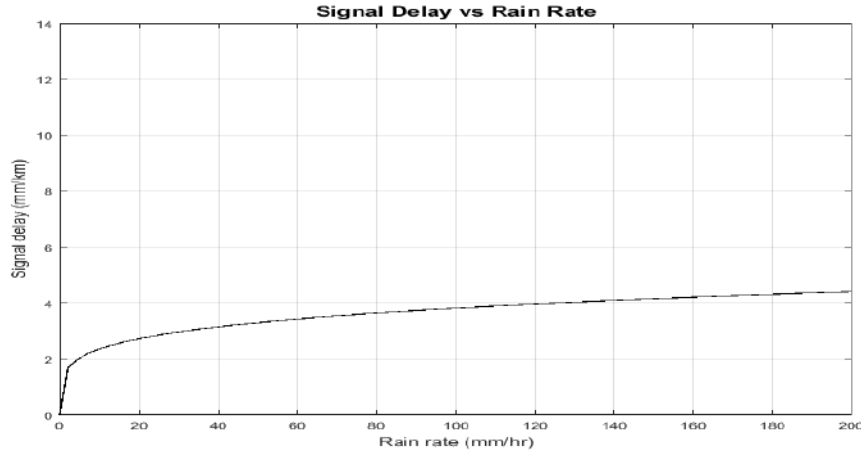


Figure R.9. “Demo” Path delay as a function of the rain rate. Calculations were performed for HH polarization of the transmit and receive antenna. Credits: [Moissev and Hanssen, 2003].

Goldhirsh (1982) similarly reported a 1 [cm] two-way range error for 10 [mm/h] rain over a 5 [km] path. In contrast to stratiform rain, convective precipitation produces stronger gradients and is more evident in **SAR interferograms** (**Hanssen**, 1998).

R.5.1 Link Budget Margins

Table R.19 quantifies the influence of the Zenith Wet Delay (ZWD) on excess attenuation across frequency bands. For low-frequency links (S–X bands), the additional loss induced by 1–5 mm of ZWD is negligible. At Ka-band, the sensitivity increases (~ 0.15 dB/mm), requiring explicit inclusion in link budget calculations. In the W-band, the effect becomes dominant: a few millimetres of excess ZWD produce several decibels of additional attenuation (≈ 1.5 dB/mm), making accurate water vapor modeling essential for link design and availability analysis. The values in **Table 15.19** were computed using the relation:

$$\Delta A_{gas}(15^\circ)[dB] \approx k_{band}[dB/mm] \times \Delta ZWD[mm], \quad (R.51)$$

where k_{band} represents the sensitivity coefficient for each frequency band at 15° elevation.

Table R.19 extends this analysis, providing the conversion from increments of ΔZWD to additional gaseous attenuation under clear-sky conditions. While negligible at S–X bands (< 0.1 dB for $\Delta ZWD = 10$ mm), the effect becomes significant at Ku–Ka bands (0.1–1.5 dB) and critical at W-band, where 1 mm of ZWD causes ≈ 1.5 dB of loss and 10 mm yields ≈ 15 dB. Overall, attenuation exhibits an approximately linear dependence on ΔZWD , with the slope steepening rapidly at higher frequencies. Thus, while low-frequency links remain virtually unaffected, Ka- and especially W-band systems demand precise, site-specific modeling of water vapor variability to ensure accurate link margin estimation and reliable communication performance.

It is noteworthy that attenuation increases almost linearly with ΔZWD , but the slope steepens markedly with frequency. At S–X bands, the effect remains negligible even for large ZWD

Band	Frequency [GHz]	$\Delta A_{gas} +$ 1 [mm] ZWD	Example + 5 [mm] ZWD	Note
S	2.2	$\approx 0.006[dB]$	$\approx 0.03[dB]$	Negligible for margin purposes
X	8.4	$\approx 0.011[dB]$	$\approx 0.06[dB]$	Small but measurable
Ka	32	$\approx 0.15[dB]$	$\approx 0.075[dB]$	Relevant for link budget
W	94	$\approx 1.5[dB]$	$\approx 7.5[dB]$	Dominant; highly sensitive to humidity

Table R.18. Immediate mapping (clear-sky gas, 15°). For zenith, divide by ≈ 3.8 ; for another elevation, multiply by its mapping factor $m(e)$.

Δ_{ZWD} [mm]	S 2.2 [GHz]	X 8.4 [GHz]	Ku 14 [GHz]	Ka 32 [GHz]	W 94 [GHz]	Output
1	0.006	0.011	0.050	0.150	1.50	[dB]
2	0.012	0.022	0.100	0.300	3.00	
3	0.018	0.033	0.150	0.450	4.50	
4	0.024	0.044	0.200	0.600	6.00	
5	0.030	0.055	0.250	0.750	7.50	
6	0.036	0.066	0.300	0.900	9.00	
7	0.042	0.077	0.350	1.050	10.50	
8	0.048	0.088	0.400	1.200	12.00	
9	0.054	0.099	0.450	1.350	13.50	
10	0.060	0.110	0.500	1.500	15.00	

Table R.19. Mapping table ($\Delta_{ZWD} \rightarrow \Delta A_{gas}$ @ 15°), clear-sky gas.



variations, whereas at Ka-band it becomes significant, requiring inclusion of water vapor variability in link budget analyses. In the W-band, sensitivity reaches extreme levels: each millimetre of ZWD produces roughly 1.5 dB of additional loss, confirming the band's strong susceptibility to humidity fluctuations and the need for precise, site-specific modeling.

R.5.2 Modeling of Cloud Liquid Water Attenuation in Non-Precipitating Clouds (ITU-R P.840)

Cloud attenuation can be approximated by:

$$A_{cloud}(15^\circ) \approx \kappa(f) \cdot LWP \rightarrow [kgm^{-2}] \quad (R.52)$$

where $\kappa(f)$ is the frequency-dependent specific attenuation coefficient and LWP the liquid water path. At Scanzano (PA), assuming LWP=0.25 $[kgm^{-2}]$:

- Ku-band (14 GHz): ≈ 0.5 – 1.0 dB
- Ka-band (32 GHz): ≈ 1 – 3 dB
- W-band (94 GHz): ≈ 5 – 15 dB

Thus, *stratocumulus* or *stratus* layers with moderate LWP can contribute an additional 1–3 dB at Ka and >10 dB at W, on top of gaseous attenuation. Accurate LWP estimation is therefore important in link budget analyses at high frequencies.

R.5.3 Rain Attenuation Modeling Based on ITU-R P.838 / P.618

The specific rain attenuation is defined as:

$$\gamma_R(f) = k(f) \cdot R^{\alpha(f)} [dB km^{-1}], \quad (R.53)$$

where R is the rain rate in $[mmh^{-1}]$.

Typical attenuation estimates at 15° elevation:

- *Ku* at 14 [GHz]: $\gamma_R \sim 0.05 - 0.2 [dB km^{-1}] \Rightarrow A_R \approx 0.3 - 1.5 [dB]$ (10 $[mmh^{-1}]$), 2 – 4 [dB] (25 $[mmh^{-1}]$);
- *Ka* at 32 [GHz]: $\gamma_R \sim 1 - 3 [dB km^{-1}] \Rightarrow A_R \approx 6 - 20 [dB]$ (10 $[mmh^{-1}]$), 20 – 40 [dB] (25 $[mmh^{-1}]$);
- *W* at 94 [GHz]: $\gamma_R \sim 6 - 12 [dB km^{-1}] \Rightarrow A_R \approx 40 - 100 [dB]$ (10 $[mmh^{-1}]$), 100 – 200 [dB] (25 $[mmh^{-1}]$) $\Rightarrow$ outage likely.
- S/X-bands: typically < 1 dB even at 25 $[mm h^{-1}]$ during winter conditions.

Including scintillation effects (per ITU-R P.618), attenuation increases in convective conditions:

- X-band: +0.4–1.2 dB
- Ka-band: +1.2–6 dB (≈ 0.01 % of time, *Mediterranean* winter).

Overall, rain and cloud contributions dominate attenuation above Ka-band, making site-specific and time-resolved modeling crucial for accurate high-frequency link budgets.



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Weather Condition	S 2.2 [GHz]	X 8.4 [GHz]	Ku 14 [GHz]	Ka 32 [GHz]	W 94 [GHz]	Output
Cloud $LWP \approx 0.25[kgm^{-2}]$	< 0.1	< 0.2	0.5–1.0	1–3	5–15	[dB]
Rain, $R = 10[mmh^{-1}]$	< 0.2	< 0.5	0.3–1.5	6–20	40–100+	
Rain, $R = 25[mmh^{-1}]$	< 0.5	~ 1	2–4	20–40	100–200+	
Scintillation (0.01%)	negligible	0.4–1.2	0.8–2	1.2–6	3–8	

Table R.20. Indicative values for Scanzano (PA), winter; use P.618/P.840/P.838 for formal calculation with local $R_{0.01}$, LWP and $m(e)$.

R.5.4 Attenuation Due to Solid Precipitation (Snow, Graupel, and Hail)

Beyond liquid precipitation, solid hydrometeors—snow, *graupel*, and hail—can critically influence Earth–space propagation, particularly at Ka- and W-band frequencies. Their effect depends on phase state, density, liquid water fraction, and degree of aggregation. While dry snow produces only modest extinction, wet snow and *graupel* exhibit enhanced absorption and scattering due to liquid water presence, yielding attenuation levels comparable to rain. Hail, characterized by large size and high dielectric contrast, can cause severe fading and depolarization, defining upper-bound conditions for link design.

Key effects across frequency bands:

- **Dry snow:** Low attenuation at Ku/Ka; rapidly increasing losses in Ka–W for wet snow or *graupel*, similar to light or moderate rain.
- **Hail:** Strong attenuation and depolarization in Ka/W; should be treated as a worst-case rain scenario in availability analysis.

Table R.20 summarizes the approximate attenuation levels expected during typical winter conditions at Scanzano. For clouds with $LWP \approx 0.25[kgm^{-2}]$, attenuation is negligible at S–X bands but can reach 5–15 [dB] at W-band. Rain produces stronger effects: at 10 mm h⁻¹, attenuation rises to 6–20 [dB] at Ka and 40–100 dB at W; at 25 [mm h⁻¹], W-band losses may exceed 200 [dB], implying total link outage. Scintillation is negligible below 10 GHz but becomes relevant above Ku-band, adding 3–8 [dB] at the 0.01% probability level in W-band.

These estimates confirm that while S- and X-band links remain robust under adverse weather, Ka- and especially W-band systems are highly susceptible to hydrometeors and turbulence, requiring diversity schemes, adaptive coding and modulation (ACM), or fallback to lower frequencies to maintain operational reliability.

R.5.5 Common Hypothesis for Winter in Scanzano (PA)

For the ERA5 winter profile at Scanzano (PA) and a 15° elevation angle (mapping $\approx \sec 15^\circ \approx 3.8$), the estimated excess attenuation contributions are as follows:

Availability [%]	S 2.2 [GHz]	X 8.4 [GHz]	Ku 14 [GHz]	Ka 32 [GHz]	W 94 [GHz]
99.0	0.2	0.6	≈ 2.0	≈ 4.5	≈ 22
99.5	0.2	1.0	≈ 2.5	≈ 6.5	≈ 24
99.9	0.3	1.5	≈ 3.5	$\approx 9 - 10$	$\approx 26 - 28$

Table R.21. Recommended fade margin [dB] without rain. Gas from $\Delta_{ZWD}=+5$ [mm] + Clouds ($LWP \approx 0.25$) + Scintillation, 15° winter.

- **Gas (clear-sky, $\Delta_{ZWD} = +5$ [mm]):** Extra attenuation @ 15° : S (2.2 GHz) ≈ 0.03 dB; X (8.4 GHz) ≈ 0.06 dB; Ku (14 GHz) ≈ 0.25 dB; Ka (32 GHz) ≈ 0.75 dB; W (94 GHz) ≈ 7.5 dB. The effect scales linearly with frequency-dependent coefficients of approximately 0.006, 0.011, 0.050, 0.150, and 1.50 [dB mm $^{-1}$] of ZWD at 15° .
- **Clouds (ITU-R P.840, $LWP \approx 0.25$ [kg m $^{-2}$]):** S < 0.1 dB; X ≈ 0.1 dB; Ku ≈ 0.8 dB; Ka ≈ 2 dB; W ≈ 10 dB.
- **Scintillation (ITU-R P.618, Mediterranean winter):** X ≈ 0.4 –1.2 dB; Ku ≈ 0.8 –2 dB; Ka ≈ 1.2 –6 dB; W ≈ 3 –8 dB; S negligible.

These results confirm that attenuation at S–X bands is minimal, while Ka-band already requires fade margins to account for cloud and scintillation effects. In contrast, W-band experiences substantial excess losses—several decibels even under moderate humidity or thin-cloud conditions—indicating its high sensitivity to tropospheric variability.

Table R.21 summarizes the recommended fade margins at 15° elevation for availability targets of 99.0, 99.5, and 99.9%, combining the effects of excess gaseous attenuation ($\Delta_{ZWD} = +5$ mm), cloud attenuation ($LWP \approx 0.25$ [kg m $^{-2}$]), and scintillation (winter ITU-R P.618). At S–X bands, margins remain below 2 dB and are negligible for system design. At Ku–Ka bands, margins rise to ~ 3.5 dB (Ku) and ~ 9 –10 dB (Ka) at 99.9% availability. In the W-band, they dominate—approximately 22 dB at 99.0% and up to 26–28 dB at 99.9%—confirming the strong vulnerability of this band even under non-rain conditions. Overall, these findings underscore the need for adaptive coding and modulation (ACM), frequency diversity, or fallback strategies to ensure reliable W-band operation in winter environments.

At low frequencies (S–X bands), the required fade margins remain marginal and have negligible impact on system design. Ku- and Ka-bands exhibit higher sensitivity, requiring several additional decibels to ensure high link availability. In contrast, the W-band proves extremely fragile, with margins exceeding 20 dB even under clear-sky winter conditions, confirming that its operation is feasible only through robust mitigation such as adaptive coding and modulation (ACM), site diversity, or fallback to lower bands.

Table R.22 summarizes the additional fade margins required in the presence of rainfall, derived from ITU-R P.838 and P.618 for winter conditions at 15° elevation and rain rates of approximately 10–25 [mm h $^{-1}$]. At S–X bands, rain-induced attenuation is negligible (<1 dB even at 99.9% availability). Ku-band experiences modest additional losses (1–3 dB), whereas

Ka-band requires significantly higher margins—about 12 dB at 99.0% and up to 25 dB at 99.9% availability. In the W-band, rain attenuation dominates, with margins ranging from 100 to 200 dB, rendering continuous operation impractical. These findings confirm that rain is the primary limiting factor for high-frequency satellite links, positioning Ka-band near the usability threshold under adverse weather and restricting W-band operation to clear-sky or opportunistic scenarios.

Availability [%]	S 2.2 [GHz]	X 8.4 [GHz]	Ku 14 [GHz]	Ka 32 [GHz]	W† 94 [GHz]
99.0	+0.2	+0.4	+1	+12	+100
99.5	+0.3	+0.7	+2	+18	+150
99.9	+0.5	+1.0	+3	+25	+200

Table R.22. Additional margin for rain [dB]. ITU-R P.838/P.618; 15°, winter; $R \sim 10\text{--}25 \text{ [mm h}^{-1}\text{]}$ as order of magnitude.

Note (†): At W-band, even moderate rainfall can lead to complete outage; the reported values are indicative of expected attenuation levels and cannot be treated as practical design margins without diversity or ACM mitigation.

When compared with the clear-sky margins of **Table R.21**, the additional attenuation due to rain (**Table R.22**) radically alters the link budget scenario. At S–X bands, the overall impact remains minor, whereas Ku- and particularly Ka-band require a substantial increase in fade margins, making rain the dominant propagation impairment. In the W-band, the situation becomes prohibitive: margins escalate from approximately 25 dB under clear-sky winter conditions to beyond 100 dB in rainfall, confirming that reliable operation is feasible only through opportunistic use, frequency diversity, or fallback to lower bands.

Figures R.10 and R.11 summarize the fade-margin requirements for Scanzano (PA) during winter at 15° elevation, separating the clear-sky contribution from additional rain attenuation across the S-, X-, Ku-, Ka- and W-bands. The first panel in **Fig. R.10** reports the absolute values on a linear scale, highlighting the relative weight of clear-sky and rain-induced components within each band.

This representation makes the progressive increase from centimetric to millimetric frequencies immediately apparent. To better capture the multi-order variability across the spectrum, **Fig. R.11** shows the same data on a logarithmic Y-axis.

This complementary view reveals how the W-band fade margins exceed those of the lower bands by two to three orders of magnitude, thereby clarifying the physical discontinuity between microwave and millimetre-wave propagation regimes. Together, the two panels provide a consistent picture of the dynamic range involved and motivate the need for strong mitigation techniques at higher frequencies, particularly in precipitation-prone conditions.

Table R.23 summarizes the fade margins required to achieve 99.9% link availability at Scanzano (PA) during winter, for a low elevation angle of 15°. The clear-sky term combines gaseous absorption (including an additional wet delay of $\Delta_{ZWD} = +5 \text{ [mm]}$), cloud attenuation ($LWP \approx$

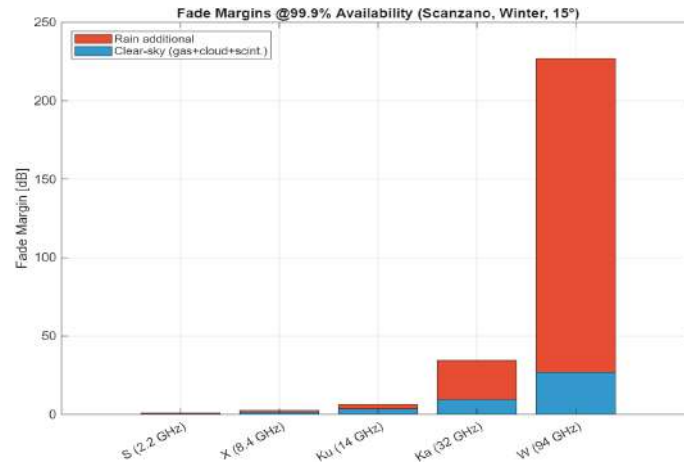


Figure R.10. Fade margins at 99.9% availability for Scanzano (PA), winter, elevation 15°.

The stacked bars illustrate the relative contribution of clear-sky effects (gas, cloud, scintillation) and additional rain attenuation to the total fade margin across different frequency bands. At S- and X-band, margins remain below 2 [dB] and are practically negligible. Ku- and Ka-bands require increasing protection (3–10 [dB] and ~35 [dB], respectively), while the W-band is dominated by rain attenuation, with fade margins exceeding 200 [dB]. This confirms that W-band links are only feasible in clear-sky conditions or with strong mitigation strategies such as diversity and fallback to lower frequencies.

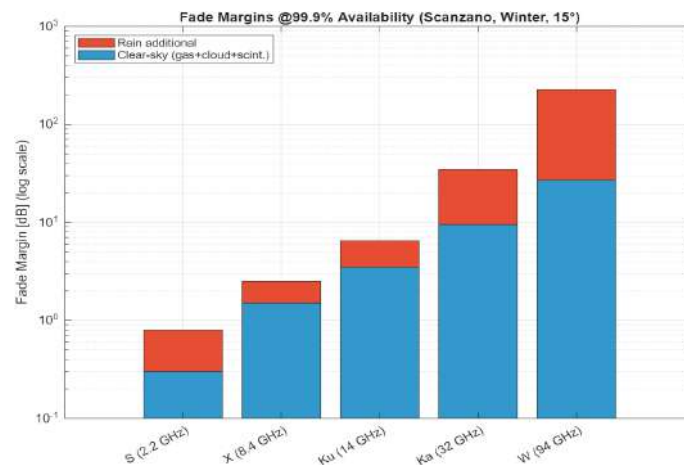


Figure R.11. Fade margins at 99.9% availability for Scanzano (PA), winter, elevation 15° (logarithmic scale).

The same fade margins shown in the linear plot are here reported with a logarithmic Y-axis to emphasize the large dynamic range across frequency bands. At S- and X-band, the required margins remain below 2 [dB], while Ku- and Ka-bands show progressively higher values up to ~35 [dB]. In contrast, the W-band exceeds 200 [dB], clearly standing several orders of magnitude above the lower bands. The log-scale representation highlights the extreme disparity in attenuation behavior and underlines the impracticality of W-band operation without strong mitigation measures.



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0.25 [kg m⁻²]), and scintillation effects based on ITU-R P.618. Added to these is the rain margin derived from **ITU-R P.838/P.618**, representative of intense *Mediterranean* precipitation events.

Table R.23. Fade margins at 99.9% availability for Scanzano (PA), winter, elevation 15°. Clear-sky includes gas (with $\Delta ZWD=+5$ mm), clouds ($LWP \approx 0.25$ kg m⁻²), and scintillation (ITU-R P.618). Rain additional from ITU-R P.838/P.618.

	S	X	Ku	Ka	W
Band [GHz]	2.2	8.4	14	32	94
Clear-sky [dB]	0.3	1.5	3.5	9.5	27
Rain additional [dB]	0.5	1.0	3.0	25	200
Total [dB]	0.8	2.5	6.5	34.5	227

The results show a strong frequency dependence. At S–X bands, total margins remain below 3 dB, confirming their robustness even in adverse weather. At Ku-band, the margin increases to about 6.5 dB, indicating the onset of significant cloud and rain sensitivity. At Ka-band, margins exceed 30 dB, while at W-band, they surpass 200 dB, illustrating the prohibitive attenuation effects of rain and hydrometeors at 94 [GHz].

From a system design perspective, these results confirm that S–X bands are well suited for high-availability operation, Ka-band requires strong fade mitigation strategies (e.g., ACM, site diversity), and W-band remains limited to clear-sky or opportunistic applications due to its extreme sensitivity to rainfall.

A direct comparison between Ka- and W-band reveals a profound operational disparity. At Ka-band, maintaining 99.9% availability demands margins of about 35 dB—challenging but manageable through engineering countermeasures such as adaptive coding and modulation (ACM), power control, or uplink site diversity. In contrast, W-band links require margins exceeding 200 dB, which are physically unrealistic for any operational system. This contrast confirms that, while Ka-band can be feasibly integrated into high-availability satellite networks with appropriate mitigation, W-band usage remains confined to experimental or clear-sky scenarios, suitable only for short, high-capacity opportunistic transmissions.

R.5.6 Common Hypothesis for Summer in Scanzano (PA)

For the summer ERA5 profile at Scanzano (PA) and a 15° elevation angle (mapping factor ≈ 3.8), the following propagation contributions are expected:

- **Gas (clear-sky):** Greater sensitivity to humidity than in winter. The coefficients relating excess wet delay (ΔZWD) to additional gaseous attenuation at 15° are: S: 0.008 dB/mm; X: 0.015 dB/mm; Ku: 0.070 dB/mm; Ka: 0.22 dB/mm; W: 2.2 dB/mm. Values scale linearly with ΔZWD ; divide by ≈ 3.8 for zenith or multiply by the elevation mapping function $m(e)$.
- **Clouds (ITU-R P.840):** Typical summer cloud layers are thicker and wetter, with $LWP \approx 0.40$ [kg m⁻²], producing stronger attenuation.

Availability [%]	S 2.2 [GHz]	X 8.4 [GHz]	Ku 14 [GHz]	Ka 32 [GHz]	W 94 [GHz]
99.0	0.2	0.9	≈ 2.9	$\approx 6.5 - 7.0$	≈ 33
99.5	0.2	1.3	≈ 3.6	$\approx 8.5 - 9.0$	≈ 35
99.9	0.3	1.8	≈ 4.5	$\approx 12 - 13$	≈ 39

Table R.24. Recommended fade margin [dB] without rain. — Fade margin without rain (gas + cloud + scint) @15°, summer. Gas calculated for $\Delta ZWD = +5$ [mm]; if ΔZWD changes, scale linearly with the coefficients above.

- **Scintillation (ITU-R P.618):** Enhanced turbulence in summer increases scintillation, particularly across Ku-, Ka-, and W-bands.

Table R.24 summarizes the recommended fade margins under *clear-sky summer* conditions at 15° elevation, combining the effects of gaseous absorption ($\Delta ZWD = +5$ mm), cloud attenuation ($LWP \approx 0.40$ kg m⁻²), and scintillation, while excluding rainfall to isolate the background contribution.

Results indicate a strong frequency dependence:

- S–X bands: nearly unaffected (<2 dB even at 99.9% availability).
- Ku-band: moderate penalties (≈ 3 –5 dB).
- Ka-band: significant margins (≈ 7 –13 dB), requiring fade mitigation even in clear-sky conditions.
- W-band: extreme margins (≈ 33 –39 dB), confirming that the summer atmosphere alone imposes severe limitations.

From an operational standpoint, these results reinforce the contrast between the resilience of low-frequency bands and the vulnerability of W-band links, where even the clear-sky baseline severely constrains availability.

A comparison between **Table R.24** (*summer*) and **Table R.25** (*winter*) highlights the seasonal dependence of fade margins. In winter, rain dominates while clear-sky losses remain moderate; in summer, the background atmosphere itself—through higher humidity, liquid water content, and turbulence—drives significantly higher clear-sky margins, especially at Ka- and W-band. This dual seasonal behavior implies that fade mitigation and frequency management must be seasonally adaptive, distinguishing between summer excess attenuation and winter rain-induced extremes.

Table R.25 reports the additional margins required for *summer convective rainfall*, derived from ITU-R P.838/P.618 for 15° elevation. The contrast with *winter* is evident:

- S–X bands: minimal impact (+0.3–3 dB).
- Ku-band: +3–6 dB, already relevant for high-availability services.
- Ka-band: +12–40 dB for rain rates of 25–50 mm h⁻¹.
- W-band: +200–250 dB, leading to total outage at 99.9% availability.

These findings confirm that while Ka-band remains marginally usable with robust mitigation, W-band operation under convective summer rain is practically infeasible.

Availability [%]	S 2.2 [GHz]	X 8.4 [GHz]	Ku 14 [GHz]	Ka 32 [GHz]	W 94 [GHz]
99.0	+0.3	+1.5	+3	+12	+200†
99.5	+0.4	+2.0	+4	+30	+250†
99.9	+0.8	+3.0	+6	+40	outage†

Table R.25. Additional margin for rain. ITU-R P.838/P.618; 15°, indicative *summer convective scenarios*; $R \sim 25$ and 50 [mm h⁻¹].

Note (†): In W-band, even moderate rainfall leads to link outages; the values reported are indicative of attenuation levels and cannot be considered practical design margins without diversity or ACM mitigation. ■

This evidence confirms that rain represents the dominant impairment during summer, particularly at higher frequencies. In practice, W-band links are infeasible under such conditions, whereas Ka-band systems demand robust fade mitigation and dynamic link adaptation. Conversely, S- and X-bands remain resilient, serving as reliable fallback channels for high-availability services. A direct comparison between **Table 15.26** (*clear-sky margins*) and **Table 15.27** (*rain-induced margins*) highlights the dominant role of rainfall in summer propagation:

- **Clear-sky (Table 15.25):** Required margins remain moderate across all bands, even at 99.9% availability — <0.3 dB for S-band, <2 dB for X-band, ≈3–4.5 dB for Ku-band, ≈12 dB for Ka-band, and ≈39 dB for W-band, still manageable under clear-sky conditions.
- **Rain (Table 15.26):** Under convective rain ($R \approx 25$ –50 mm h⁻¹), the situation changes drastically. Margins remain low in S/X (<3 dB), increase moderately in Ku (~6 dB), but rise sharply in Ka (+40 dB at 99.9%), while W-band becomes inoperable, with excess attenuation exceeding +250 dB and resulting in complete outage.

This comparison confirms that clear-sky attenuation is mainly driven by gaseous absorption, cloud water, and scintillation, while rain dominates the summer availability budget—especially in Ka and W bands. From a design standpoint:

- S/X bands – robust and suitable for mission-critical links.
- Ku band – feasible with moderate fade mitigation.
- Ka band – requires advanced mitigation (ACM, site diversity).
- W band – limited to clear-sky or opportunistic operations; not viable under convective conditions.

Figure R.12 compares the clear-sky fade margins at 99.9% availability for Scanzano (PA) at 15° elevation, considering gaseous absorption, cloud attenuation, and scintillation across S (2.2 GHz), X (8.4 GHz), Ku (14 GHz), Ka (32 GHz), and W (94 GHz) bands.

The results reveal a distinct seasonal contrast:

- *Winter:* margins remain moderate, from <1 dB at S/X to ~27 dB at W.
- *Summer:* margins rise sharply, reaching up to 40 dB at W due to higher PWV, thicker clouds (LWP), and stronger scintillation.

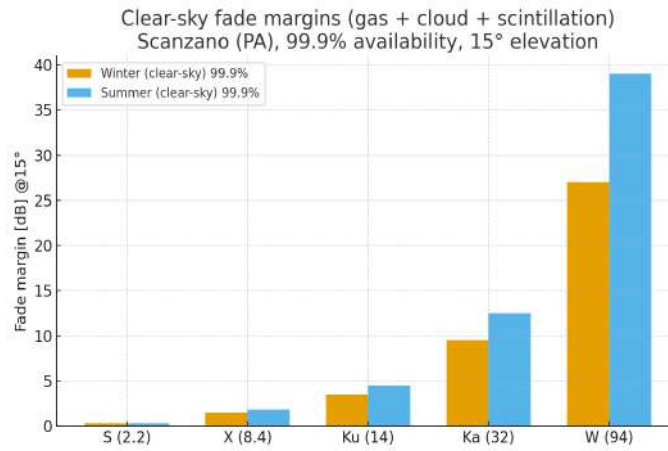


Figure R.12. Clear-sky fade margins (gaseous absorption + cloud attenuation + tropospheric scintillation) at 15° elevation for S (2.2 [GHz]), X (8.4 [GHz]), Ku (14 [GHz]), Ka (32 [GHz]), and W (94 [GHz]) bands over Scazano (PA), for 99.9% link availability. Bars compare winter and summer conditions using ERA-5 seasonal profiles and ITU-R P.676/P.840/P.618 scaling. Summer margins are systematically larger due to higher PWV, thicker clouds (larger LWP), and stronger scintillation.

This figure demonstrates that seasonality plays a decisive role in link budget design. At low frequencies (S/X), seasonal effects are negligible, whereas at high frequencies (Ka/W) they become critical, requiring adaptive strategies such as ACM, site diversity, or opportunistic scheduling to maintain link availability.

Figure R.13 presents the fade margins at 15° elevation and 99.9% availability over Scazano (PA), including both clear-sky contributions (gas, clouds, scintillation) and rain attenuation. Values are derived from ITU-R P.838/P.618 and should be regarded as indicative, given their dependence on local rainfall climatology.

- At S-band (2.2 GHz), margins remain negligible (<1 dB) and rainfall has virtually no operational effect.
- At X-band (8.4 GHz), rain losses are still limited (<5 dB) and easily compensated.
- At Ku- and Ka-bands (14–32 GHz), margins increase sharply to several tens of decibels, confirming their growing sensitivity to precipitation.
- At W-band (94 GHz), margins exceed 200 dB, making continuous operation impractical even under moderate summer rain rates.

The seasonal comparison shows systematically higher fade margins in summer, driven by stronger rainfall and longer effective path lengths under convective conditions. These findings confirm that Ku- and Ka-band links require robust fade mitigation techniques to maintain availability during rain events, while W-band operation must be restricted to opportunistic clear-sky windows, where site diversity or adaptive coding and modulation (ACM) become essential for sustaining link performance.

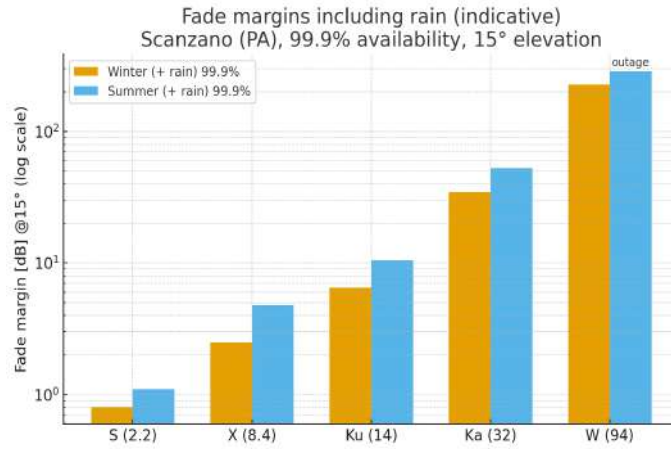


Figure R.13. Fade margins including rain attenuation (clear-sky contributions + rain) at 15° elevation and 99.9% availability. Values are indicative, derived from ITU-R P.838/P.618 with effective path length at 15°. The logarithmic axis highlights the strong frequency dependence: Ku/Ka require substantially higher margins under rainfall, while W-band is prone to outage even for moderate rain rates in summer; hence diversity/ACM is mandatory at W.

R.5.6.1 Numerical Output from Figures R.12–R.13

Table R.26 summarizes the fade margins required to ensure 99.9% link availability at Scanzano (PA) for a 15° elevation angle, across the S, X, Ku, Ka, and W bands under different seasonal and weather conditions. The table integrates clear-sky effects (gaseous absorption, cloud attenuation, scintillation) with additional rain contributions, emphasizing the strong frequency and seasonal dependence of propagation losses.

- **S-band (2.2 GHz):** Margins remain very small (≈ 0.3 – 1.1 dB), largely unaffected by season or rainfall.
- **X-band (8.4 GHz):** Attenuation increases modestly (≈ 1.5 – 4.8 dB), yet remains easily manageable.
- **Ku-band (14 GHz):** Sensitivity becomes evident, with margins rising from ≈ 3.5 dB in clear-sky winter to > 10 dB during summer rain events.
- **Ka-band (32 GHz):** Highly vulnerable, requiring ≈ 9.5 dB in clear-sky winter and up to ≈ 52.5 dB under summer rainfall.
- **W-band (94 GHz):** Extreme margins (≈ 27 – 39 dB in clear-sky, ≈ 289 dB with rain), indicative of total outage conditions.

Continuous operation is thus infeasible without diversity, adaptive coding and modulation (ACM), or fallback to lower bands. These results quantitatively confirm the steep escalation of required margins with frequency and season, reinforcing that S/X bands remain robust, Ku/Ka bands require strong mitigation strategies, and W-band operation is limited to short, clear-sky opportunities.



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Band	Winter	Summer	Winter + Rain	Summer + Rain	Availability	Elevation Angle	Site
S 2.2 [GHz]	0.3	0.3	0.8	1.1	99.9%	15°	Scanzano (PA)
X 8.4 [GHz]	1.5	1.8	2.5	4.8			
Ku 14 [GHz]	3.5	4.5	6.5	10.5			
Ka 32 [GHz]	9.5	12.5	34.5	52.5			
W94 [GHz]	27.0	39.0	227.0	289.0*			

Table R.26. *In summer at W-band the value is indicative of outage: propagation is often impractical without diversity/ACM techniques or band switching.

This table delineates the operational boundary between feasible and impractical frequency regimes. S- and X-bands remain robust under all conditions, whereas Ku- and particularly Ka-band demand strong fade mitigation strategies to maintain availability. In contrast, W-band is viable only under clear-sky or opportunistic conditions, serving primarily as an experimental or burst-mode band rather than for continuous, high-reliability operations.

R.5.7 Considerations on Deep-Space Communication between the Scanzano (PA) Ground Station and a Lunar Base

This section examines new aspects concerning the design of a deep-space communication link between the Scanzano (PA) Ground Station and a potential permanent lunar base. Although the link is not direct in conventional Earth–satellite frameworks, it can be realistically analyzed using deep-space communication models. For a minimum elevation of 15°, winter conditions at Scanzano (pressure ≈ 1015 hPa, temperature ≈ 285 K, PWV = 10–20 mm) are considered under clear-sky conditions. Only gaseous attenuation due to molecular oxygen (O_2) and water vapor (H_2O) and tropospheric scintillation are included. Using statistical diagnostics from previous analyses, the Niell mapping factors are $m_h \approx 3.80$ and $m_w \approx 3.83$, consistent with $\sec(15^\circ)=3.86$. According to ITU-R P.676, the zenith gaseous attenuation under temperate winter conditions is approximately:

- S (2.2 GHz): 0.005–0.02 dB
- X (8.4 GHz): 0.02–0.05 dB
- Ka (32 GHz): 0.3–0.7 dB
- W (94 GHz): 3–7 dB (beyond DSN range, included for reference).

At 15° elevation, attenuation scales nearly linearly with air mass ($A_{15^\circ} \approx 3.8A_z$), giving typical values:

- S ≈ 0.02 –0.08 dB;
- X ≈ 0.08 –0.20 dB;



- $K_a \approx 1.1\text{--}2.7$ dB;
- $W \approx 11\text{--}26$ dB.

From ITU-R P.618, tropospheric scintillation varies approximately with $(\sec(e))^{\sim 1-1.5}$ or $\sigma_{scint}(e) \propto \sec(e)^\alpha$ and inversely with frequency.

Under Mediterranean winter conditions:

- X-band: $\sigma_{90^\circ} \approx 0.03\text{--}0.06$ dB $\rightarrow \sigma_{15^\circ} \approx 0.1\text{--}0.2$ dB
- Ka-band: $\sigma_{90^\circ} \approx 0.1\text{--}0.3$ dB $\rightarrow \sigma_{15^\circ} \approx 0.3\text{--}1.0$ dB

At the 0.01% exceedance level, fade depths are:

- X-band: 0.4–1.2 dB
- Ka-band: 1.2–6 dB

These values show that scintillation, negligible in X-band, becomes significant at low elevation and high frequencies. At 15° , gaseous and scintillation losses introduce:

- X-band (8.4 GHz): $\approx +0.11$ dB (gas) + 0.80 dB (scintillation) $\rightarrow \sim 0.9$ dB total penalty.
- Ka-band (32 GHz): $\approx +1.40$ dB (gas) + 3.20 dB (scintillation) $\rightarrow \sim 4.6$ dB total penalty.

Thus, while a 1 dB penalty in X-band is negligible under nominal margins, a 4–5 dB loss in Ka-band can compromise link stability. Designers must increase transmit power or antenna gain or restrict operations to elevations above 20° to mitigate these losses. Further mitigation includes Adaptive Coding and Modulation (ACM), diversity, and time scheduling. ACM dynamically adapts modulation and coding according to the instantaneous SNR or BER, maintaining reliability even under degraded conditions.

Under high SNR (zenith), 32-APSK with 9/10 coding ensures maximum throughput, while at 15° elevation with +4.6 dB losses, the system can revert to 8-PSK or QPSK with 1/2 coding, halving throughput but preserving link continuity. Forward Error Correction (FEC) using Turbo or LDPC codes (rates 1/2–2/3) further enhances robustness under fading and scintillation. In the Scanzano scenario, this adaptive scaling—from 32-APSK 9/10 to QPSK 1/2—illustrates the operational flexibility needed for reliable Ka-band deep-space communication.

Finally, **Table R.27** outlines the framework for computing the energy-per-bit-to-noise ratio (E_b/N_0) from the Link Budget (LB), forming the analytical basis for assessing link availability and selecting the optimal ACM and FEC configurations.

Family	Code Rate	Spectral efficiency [bits/s/Hz] [†]	Required E_b/N_0 [dB]
Uncoded	1.0	1 (BPSK), 2 (QPSK)	≈ 10.5
Turbo	1/2	0.5 (BPSK), 1 (QPSK)	$\approx 0.9 - 1.2$
	2/3	0.67 (BPSK), 1.33 (QPSK)	$\approx 1.8 - 2.2$
	3/4	0.75 (BPSK), 1.50 (QPSK)	$\approx 2.8 - 3.3$
LDPC (CCSDS/ AR4JA-like)	1/2	0.5/1.0	$\approx 0.2 - 0.6$
	2/3	0.67/1.33	$\approx 1.2 - 1.6$
	3/4	0.75/1.50	$\approx 1.9 - 2.3$

Table R.27. [†]Spectral efficiency here is code rate $\times$ modulation bits per symbol (BPSK=1,QPSK=2).

R.5.8 Translating Link Budget Results into ACM and FEC Design Choices

To translate the link budget into practical design parameters, a systematic procedure is required to connect the available energy-per-bit to noise density ratio, (E_b/N_0) available, with the **modulation and coding scheme (MCS)** effectively used on the link. The approach consists of four main steps:

1. Compute the available E_b/N_0 from the link budget.
2. Select the most spectrally efficient MCS whose required E_b/N_0 remains below $(E_b/N_0)_{available}$ with a design margin (typically +2–3 dB).
3. Apply the tropospheric penalties derived for Scanzano (PA) at 15° elevation:
 - X-band: $\Delta A \approx +0.11$ dB (gas) + 0.80 dB (scintillation, 0.01%) $\rightarrow -0.91$ dB on E_b/N_0 .
 - Ka-band: $\Delta A \approx +1.40$ dB (gas) + 3.20 dB (scintillation, 0.01%) $\rightarrow -4.60$ dB on E_b/N_0 .
4. Map the resulting (E_b/N_0) to the appropriate modulation and coding level:
 - $(E_b/N_0) \geq 3.5$ [dB]: QPSK, rate 3/4.
 - $(E_b/N_0) \approx 2$ [dB]: QPSK, rate 2/3 (Turbo or LDPC).
 - $(E_b/N_0) \approx 0.5$ [dB]: BPSK, rate 1/2 (LDPC preferred).
 - $(E_b/N_0) < 0.5$ [dB]: BPSK 1/2 with reduced data rate, or defer transmission to higher elevation angles.

This structured approach establishes a direct link between physical-layer impairments (gas absorption, scintillation, slant geometry) and system-level design decisions, ensuring robust link operation under realistic atmospheric conditions.

R.5.8.1 Adaptive Coding and Modulation (ACM) Ladder

The ACM ladder defines a hierarchy of transmission modes with increasing spectral efficiency, dynamically adjusted according to real-time link quality (E_b/N_0 , SNR , or C/N_0). Each mode combines a specific modulation and **Forward Error Correction (FEC)** rate, determining its robustness and throughput.

Table R.28 illustrates the adopted Simplex Ladder Scheme, which comprises four hierarchical transmission modes:

Ladder	Modulation Schema	FEC Rate	Efficiency [bit/sym]	SNR_{min} [dB]
1 (robust)	BPSK	1/2	0.5	-0.5
2	BPSK	2/3	0.67	1.2
3	QPSK	1/2	1.0	2.0
4 (fast)	QPSK	3/4	1.5	3.8

Table R.28. Simplex Ladder Schema.

This adaptive ladder enables dynamic selection of the optimal MCS, balancing robustness and spectral efficiency as atmospheric and link conditions vary, which is particularly critical for Ka-band deep-space communications at low elevation angles.

R.5.9 Deep-Space ACM Ladder and W-Band Feasibility Assessment

Table R.29 extends the **Adaptive Coding and Modulation (ACM)** concept to a deep-space robust ladder, introducing five transmission modes (M1–M5) based on BPSK and QPSK with LDPC coding, covering spectral efficiencies from 0.50 to 1.50 bit/symbol. Each mode ensures a target BER of 10^{-6} , with defined switch-up and switch-down thresholds in terms of (E_b/N_0). Starting from the most robust $BPSK-1/2$ mode, the ladder progresses up to $QPSK-3/4$, offering a flexible trade-off between reliability and throughput depending on atmospheric and geometric conditions along the Earth–Moon path.

For an illustrative diagnostic case with an initial $(E_b/N_0) = 3.5$ dB:

- **X-band:** $\Delta A \approx 0.11$ dB (gas) + 0.80 dB (scintillation at 0.01%) $\rightarrow$ total loss ≈ 0.91 dB. The effective margin becomes $3.5 - 0.91 \approx 2.6$ dB, remaining above the M4 ($QPSK2/3$) threshold (2.5 dB).
- **Ka-band:** $\Delta A \approx 1.40$ dB (gas) + 3.20 dB (scintillation at 0.01%) $\rightarrow$ total loss ≈ 4.60 dB. The effective margin decreases to $3.5 - 4.6 \approx -1.1$ dB, forcing a fallback to M2 ($BPSK2/3$) or M1 ($BPSK1/2$).

Mode	Modulation	Coding (LDPC)	Spectral Efficiency [bit/s/Hz]	E_b/N_0 Threshold [dB]	Typical Use Case
M1	BPSK	$r = 1/2$	0.50	≈ 1.5	Emergency / acquisition / severe fading
M2	BPSK	$r = 2/3$	0.67	≈ 2.0	Robust telemetry, deep fade recovery
M3	QPSK	$r = 1/2$	1.00	≈ 2.5	Nominal deep-space operation
M4	QPSK	$r = 2/3$	1.33	≈ 3.0	High-rate science telemetry
M5	QPSK	$r = 3/4$	1.50	≈ 3.5	Optimal conditions, maximum throughput

Table R.29. Deep-space Adaptive Coding and Modulation (ACM) ladder for the Earth–Moon link. Each mode (M1–M5) combines modulation order and LDPC coding rate with the corresponding spectral efficiency and post-equalization E_b/N_0 threshold required to achieve $BER \leq 10^{-6}$. The ladder spans from highly robust BPSK modes for acquisition and emergency conditions to higher-throughput QPSK configurations for nominal and optimal propagation scenarios, enabling a flexible trade-off between link reliability and data rate under variable atmospheric and geometric conditions.

This diagnostic confirms the necessity of dynamic ACM reconfiguration in Ka-band deep-space links, where modest atmospheric losses can determine the operational mode transition between high and low throughput.

R.5.9.1 W-Band Feasibility Considerations

Designing a deep-space link in the W-band (75–110 GHz) is currently impracticable, mainly due to the enormous atmospheric attenuation dominated by molecular absorption from H_2O and O_2 . For Scanzano (PA), the combined effect of Zenith Wet Delay (ZWD) and gaseous absorption already reaches 5–10 dB at zenith, rising to 15–20 dB at 15° elevation. When ice hydrometeors or light rain are present, an additional 10–30 dB of excess attenuation occurs, often pushing the total losses well beyond feasible margins. Moreover, W-band links are severely affected by refractive turbulence and rapid amplitude–phase fluctuations, inducing instantaneous variations of C/N_0 of several dB, which preclude stable high-order modulations. Additional constraints arise from EIRP limitations, due to the need for extremely high-gain antennas with sub-degree pointing accuracy. At these frequencies, both **Traveling-Wave Tube Amplifiers (TWTAs)** and **Solid-State Power Amplifiers (SSPAs)** exhibit low efficiency and high power consumption, while system temperatures on the receiver side increase significantly because of atmospheric noise, degrading the G/T ratio.

For these reasons, no operational deep-space communication standards currently exist in W-band, particularly for continuous telemetry where high availability is mandatory. The W-band can instead be envisioned as an experimental secondary channel for short-duration, high-data-rate



transmissions under clear-sky winter conditions, when attenuation remains below ~ 10 dB. In such scenarios, automatic fallback to Ka- or X-band remains essential to sustain $>99\%$ link availability for lunar or interplanetary missions.

R.5.10 W-Band link opportunistic channel

For a **W-Band communication** $\Delta A \approx 15 - 35$ [dB] and for this reason is realistic only M1 (BPSK 1/2) only in clear-sky condition. The link is available if the elevation angle is greater or equal to 45° , with dry sky ($PWV \leq 3$ [mm]), $A_{gas,zenith} \leq 2$ [dB]. It's important to require a ratio margin $C/N_0 \geq 8$ [dB] over the threshold M1 to absorb scintillation bursts. The link is disabled under 30° or when $A_{gas,15^\circ} > 10$ [dB]. The link is always available switching in X-band.

R.5.11 Tropospheric Penalties versus Elevation Angle over Scanzano (PA)

Figures R.14 and R.15 analyze the dependence of tropospheric penalties on elevation angle for X-, Ka-, and W-band links at the Scanzano (PA) site.

Figure R.14 focuses on clear-sky conditions, separating gaseous absorption (dry + wet) from tropospheric scintillation to highlight their relative weight. Scintillation emerges as the dominant impairment at high frequencies and low elevations, while gaseous losses remain nearly linear with air mass. At 10° elevation, the following attenuation values are obtained from median winter profiles with scintillation exponent $\alpha = 1.2$:

- X-band (8.4 GHz): gas ≈ 0.3 dB, scintillation ≈ 3 dB
- Ka-band (32 GHz): gas ≈ 1.8 dB, scintillation ≈ 14 dB
- W-band (94 GHz): gas ≈ 18 dB, scintillation ≈ 66 dB

These results clearly indicate that scintillation dominates the clear-sky penalty at low elevations, especially in the W-band, where even moderate turbulence induces severe amplitude and phase fluctuations.

Figure R.15 extends the analysis by including rain attenuation at the 0.01% annual exceedance level. Under these rare but intense precipitation events, Ka- and W-band performance deteriorates dramatically, with total excess losses reaching tens of decibels. This confirms that, at low elevations and high frequencies, rain and scintillation act synergistically to define the operational limits of deep-space and near-space communication links.

R.5.12 Rain Rate and Classification for Scanzano (PA)

In this analysis for Scanzano (PA), a rain rate at the 0.01% annual exceedance level, $R_{0.01} = 55$ mm h⁻¹, is adopted, consistent with the local convective climatology. The 0 °C isotherm height is approximately 4.6 km, and—following ITU-R P.839—the corresponding rain height is $h_R \approx 4.96$ km.

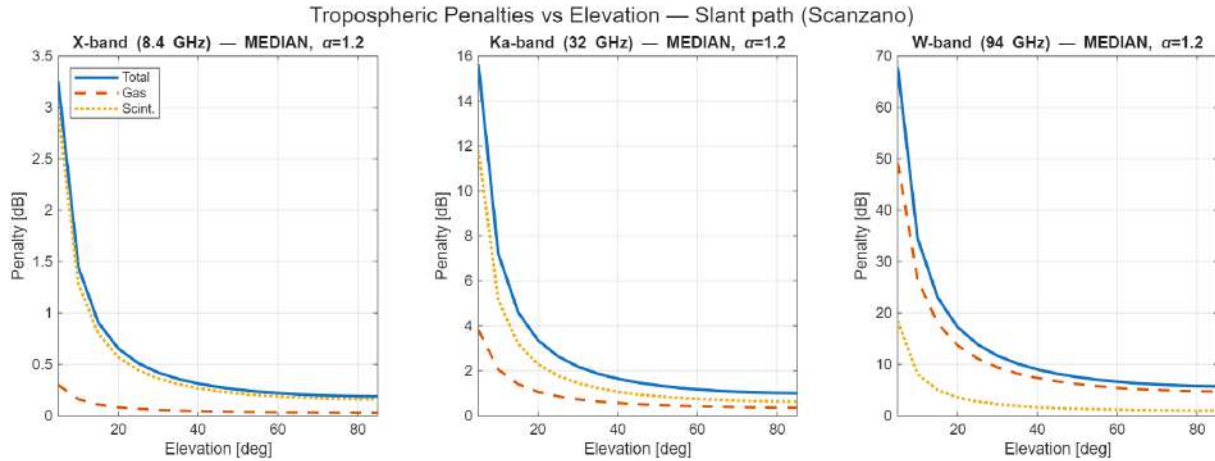


Figure R.14. Tropospheric penalties versus elevation angle for X-, Ka-, and W-band links at the Scanzano (PA) site (median profile, $\alpha = 1.2$). The total attenuation (blue) is decomposed into gaseous absorption (dry + wet, orange dashed) and tropospheric scintillation (yellow dotted). Scintillation dominates the clear-sky penalty at low elevation angles, especially in W-band, while gaseous losses become more significant at higher elevations.

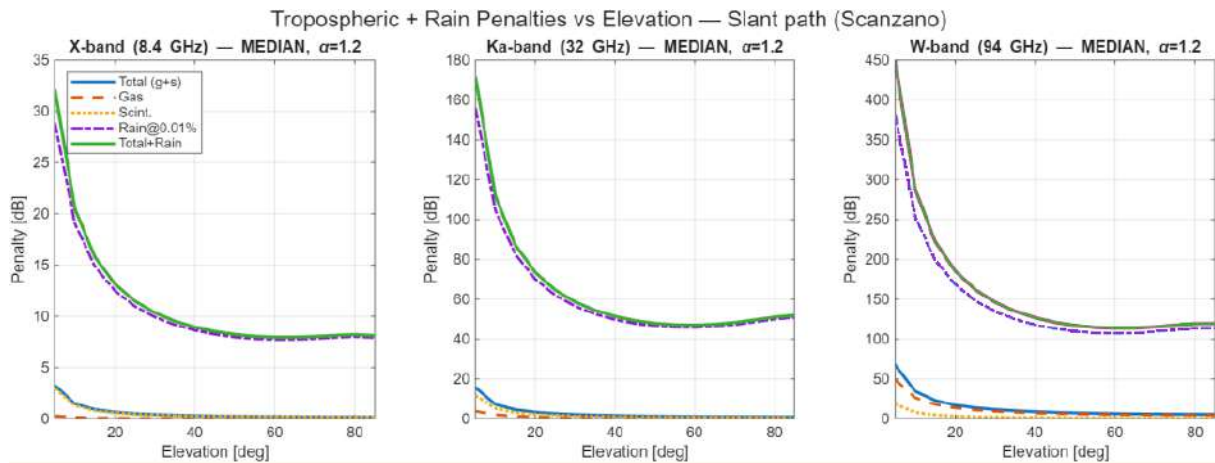


Figure R.15. Tropospheric and rain-induced penalties versus elevation angle for X-, Ka-, and W-bands at Scanzano (PA) (median profile, $\alpha = 1.2$). Gas attenuation, scintillation, and rain attenuation at 0.01% annual exceedance (ITU-R P.618/P.838) are shown, along with the total combined penalty. Rain has negligible impact in X-band, moderate impact in Ka-band at low elevations, and dominates W-band across all angles.

Intensity of Rain	R [mm/h]	Description
Light	< 2.5	Light, almost continuous rain
Moderate	2.5–7.5	Clearly noticeable rainfall
Heavy	7.5–30	Moderate to heavy showers
Very heavy	30–50	Intense thunderstorm or short-duration downpours
Violent/Extreme	>50	Severe convective events potentially flash-flood producing

Table R.30. Classification of rainfall intensity based on rain rate R in [mm/h]. The scale ranges from light, almost continuous rain (< 2.5mm/h) to violent/extreme events (>50mm/h) typically associated with severe convective systems and potential flash flooding.

Such an elevated melting layer increases the effective slant-path length through precipitation, particularly at low elevation angles, thereby amplifying rain attenuation. Events of this intensity are rare at Scanzano, typically occurring in summer and early autumn and associated with deep convection or regenerating mesoscale thunderstorms.

Table R.30 provides the standard ITU-R classification of rainfall intensity as a function of rain rate R . Values below 2.5 mm h^{-1} correspond to light, predominantly stratiform precipitation, whereas rates exceeding 50 mm h^{-1} denote severe convective events capable of producing flash-flood conditions.

R.5.13 Comparative Discussion on Tropospheric Sensitivity

From the comparative analysis of **Figures R.15–R.16** and **Table R.30**, it clearly emerges that the three frequency bands display markedly different sensitivities to tropospheric impairments at the Scanzano (PA) site. The **X-band** (8.4 GHz) confirms its robustness, with gaseous absorption and scintillation remaining well below 1 dB even at low elevation angles, and rain attenuation being practically negligible. These characteristics ensure stable, predictable link performance throughout the entire elevation range, making X-band the preferred choice for mission-critical deep-space communications.

In contrast, the **Ka-band** (32 GHz) exhibits significantly higher atmospheric penalties. Gaseous absorption rises to 1–2 dB at 10° elevation, while scintillation contributes several additional decibels. Moreover, rare but intense convective rain events, typical of Mediterranean summer conditions, can introduce further attenuation at low elevations. Despite these challenges, **Ka-band** remains feasible for deep-space links, provided that conservative fade margins of 4–8 dB are adopted and complemented by **Adaptive Coding and Modulation (ACM)** and



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robust **Forward Error Correction (FEC)** strategies.

Finally, the **W-band** (94 GHz) proves impracticable for continuous deep-space operations. Even in clear-sky conditions, gaseous absorption alone reaches tens of decibels at low elevation, while scintillation produces extreme amplitude and phase fluctuations—on the order of tens of dB—that cannot be mitigated without excessive link margins. The presence of hydrometeors (ice, cloud droplets, or rain) further amplifies the losses, while hardware constraints such as limited amplifier efficiency, stringent pointing accuracy, and elevated system noise temperature (T_{sys}) degrade the overall figure of merit (G/T).

For these reasons, the W-band can only be regarded as an experimental, secondary channel, suitable for opportunistic, burst-mode high-data-rate transmissions during clear-sky winter periods. In operational systems, it must always be safeguarded by automatic fallback to Ka- or X-band, ensuring an overall link availability exceeding 99%.



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