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Modeling and mitigation of long-range wakefields for advanced linear colliders

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Abstract. The luminosity requirements of TeV-class linear colliders demand use of intense charged beams at high repetition rates. Such features imply multi-bunch operation with long current trains accelerated over the km length scale. Consequently, particle beams are exposed to the mutual parasitic interaction due to the long-range wakefields excited by the leading bunches in the accelerating structures. Such perturbations to the motion induce transverse oscillations of the bunches, potentially leading to instabilities such as transverse beam break-up. Here we present a dedicated tracking code that studies the effects of long-range transverse wakefield interaction among different bunches in linear accelerators. Being described by means of an efficient matrix formalism, such effects can be included while preserving short computational times. As a reference case, we use our code to investigate the performance of a state-of-the-art linear collider currently under design and, in addition, we discuss possible mitigation techniques based on frequency detuning and damping.

1. Introduction

Advanced experiments in the field of high energy particle physics rely on high luminosity electron/positron linear colliders working at the TeV scale. In the last decades, several conceptual facilities of such type have been investigated exploring techniques ranging from X-band ($\sim 12\,\mathrm{GHz}$) rf-linacs to superconductivity and plasma acceleration [1–3]. A recent design proposed by SLAC [4] exploits advanced rf concepts to achieve high gradient performances relying on the mature and consolidated experience with C-band ($5.712\,\mathrm{GHz}$) technology [5, 6]. The design project, known as C^3 or the "Cool Copper Collider", allows to work with $\sim 120\,\mathrm{MeV/m}$ accelerating gradients by means of state-of-the-art techniques such as distributed coupling [7] and cryogenic cooling of the accelerating structures [8, 9].

In all the above cases, the collective interaction among charged bunches constitutes a major concern since it can cause unstable motion due to *beam break-up* (BBU) effects [10, 11]. In particular, particles traveling off-axis in the accelerating structures excite parasitic dipole fields in the form of higher order modes (HOMs) supported by the structures themselves which deflect the trajectories of the trailing charges. In this paper we introduce a dedicated tracking code studying the long-range wakefield interaction caused by self-induced dipole modes in linacs and we exploit such a tool to investigate the C^3 machine. The code describes the bunches within the rf-pulse as a sequence of rigid macro-particles with no internal structure performing transverse oscillations in presence of an external focusing optics and acceleration. The combination of such fields in an alternating gradient FODO lattice with length L_c, average betatron function β_x and phase advance $\mu_x = L_c/\beta_x$ provides the following transfer map [12]

$$\begin{pmatrix} x \\ x' \end{pmatrix} \mapsto \sqrt{\frac{\gamma}{\gamma + \gamma' L_c}} \begin{pmatrix} \cos\mu_x & \beta_x \sin\mu_x \\ -\frac{1}{\beta_x}\sin\mu_x & \cos\mu_x \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix} \tag{1}$$

which is responsible for the adiabatic damping of the transverse oscillations and characterizes the motion in absence of collective effects.

2. The process of beam break-up

The interaction between two particles with time separation τ induced by a resonant mode with angular frequency ω_0, shunt impedance $R_\perp$ and quality factor Q is described by the dipole wake-function per unit length

$$w_\perp(\tau) = \frac{\omega_0 R_\perp}{Q} e^{-\alpha\tau} \sin(\omega_n \tau) \tag{2}$$

with $\omega_n = \omega_0\sqrt{1-(2Q)^{-2}}$ and $\alpha = \omega_0/2Q$ [13]. Deflected charges moving off-axis cause further excitation of the resonant mode intensifying the parasitic interaction. A quantitative analysis for such a process has been introduced by Mosnier [14] whose approach describes the bunch train as a sequence of point-like macro-particles accelerated inside a machine with uniform betatron function where the mutual interaction is due to a single dipole HOM which exhibits the same parameters in each cell of the linac. Such a model shows that the motion becomes highly unstable if the ratio of the HOM frequency and the bunch repetition rate is close to an integer so that the beam-mode interaction fulfills a resonant condition. Mitigation is possible by breaking the coherent interplay through a spread in frequency of the dipole modes [15, 16]. Indeed, it can be shown that frequency spread reduces the HOM quality factor with a consequent shortening of the corresponding decay time.

3. The matrix formalism for wakefields

In this section we describe the approach we use in our tracking code for the inclusion of collective effects. The interaction of charged particle beams with a resonant mode can be described in terms of a matrix formalism which allows to account for both the longitudinal and transverse wakefield effects [17, 18]. Deflecting modes admit a lumped element shunt-circuit representation whose voltage accounts for the transverse kick applied to the particles: $c\Delta p_x = qq_0x_0w_\perp(\tau) \doteq qV_\perp(\tau)$, where the subscript zero refers to source particle quantities. Such a voltage is induced by the passing bunches which perturb the resonator and evolves according to the following matrix equation

$$\begin{pmatrix} V(t+\tau) \\ \dot{V}(t+\tau) \end{pmatrix} = e^{-\alpha\tau} \begin{pmatrix} \cos\omega_n\tau + \frac{\alpha}{\omega_n}\sin\omega_n\tau & \frac{1}{\omega_n}\sin\omega_n\tau \\ -\frac{\omega_0^2}{\omega_n}\sin\omega_n\tau & \cos\omega_n\tau - \frac{\alpha}{\omega_n}\sin\omega_n\tau \end{pmatrix} \begin{pmatrix} V(t) \\ \dot{V}(t) \end{pmatrix} \tag{3}$$

As new bunches enter the cavity, the system is continuously perturbed and, therefore, the matrix-transformation alone does not fully describe the evolution. The correct operator has the form $\mathcal{T}(\cdot) = \mathbf{p} + M(\cdot)$ where the argument is multiplied by the matrix M in equation (3) and the additive vector $\mathbf{p}$ accounts for the perturbation. The latter is given by equation (2) and its derivative in the limit $\tau \to 0$: reminding that $V(\tau) \propto w_\perp(\tau)$ one has $\mathbf{p} = q\omega_n \frac{\omega_0 R_\perp}{Q} \Delta x \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ where Δx is the transverse displacement of the bunch from the axis which is responsible for the excitation of dipole modes. In Ref. [19] the details of such a formalism were discussed showing that it reduces the number of operations needed to evaluate the deflecting kick introduced by the wakefields.

3.1. Benchmark test for the matrix formalism

In the remaining of this section we show an example validating the matrix formalism in presence of long-range wakefields. In particular, we consider a fictitious linac described in [14] by Mosnier in order to demonstrate his analytical model for beam break-up effects. In the approximation of moderate BBU in a single betatron wavelength, Mosnier's analysis allows to find the amplitude of the transverse oscillation for each bunch while it provides an asymptotic solution for strong BBU. Table 1 shows the main parameters characterizing the bunch train, the linear accelerator and the resonant mode responsible for the BBU effects.

Table 1. Parameter list for the comparison with Mosnier's theory.

Parameter	**Value**
Number of bunches	100
Bunch population	$2 \cdot 10^9$
Bunch separation	40 ns
Injection energy	10 GeV
Accelerating gradient	100 MeV/m
Avg betatron function	10 m
Linac length	4900 m
f_0 (HOM)	15.7 GHz
Q (HOM)	100
$R_\perp/Q$ (HOM)	$0.45\,\mathrm{M\Omega/m^2}$

In figure 1, a comparison between Mosnier's analytic approach and our tracking code is shown. In this example each bunch is injected with an offset of $50\,\mu$m and the amplitude of the transverse oscillation at the end of the linac is shown together with the maximum envelope provided by Mosnier's theory.

4. The C^3 linac case

The working point of the C^3 linac is described in [4]. A total charge of 75 nC is delivered within a 250 ns long macro-pulse (1428 rf-bucket) which accommodates 75 electron bunches (1 nC each) with a separation of 19 rf periods (~ 3.3 ns). The electron beams are injected at 10 GeV and accelerated up to a final energy of ~ 1 TeV with an average gradient of 117 MeV/m (*i.e.*, overall length $\gtrsim 12$ km) while being focused by a FODO lattice with $\langle \beta_x \rangle = 4$ m.

A study of the higher order modes ringing in the unit accelerating cell was performed with the electromagnetic software CST Studio Suite [20]. A set of 20 modes (excluding degeneracy) was found in the frequency range $5\,\mathrm{GHz} < f < 20\,\mathrm{GHz}$ nine of which possess dipole field components. In the following we investigate the wakefield effects induced by the excitation of such modes and how they affect the beam trajectories.

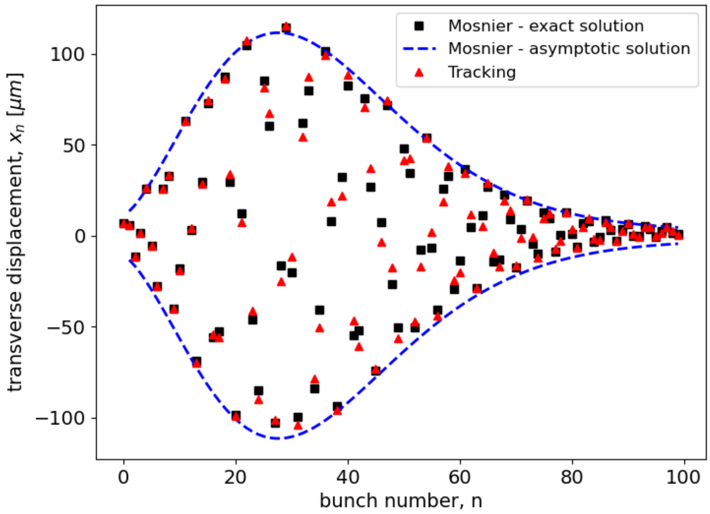

Figure 1. Final amplitude of the transverse oscillations in presence of BBU effects. Comparison with Mosnier's analytical approach.

4.1. Alignment errors

In the example of figure 2 we discuss the effects of injection and alignment errors by comparing the amplitude of the transverse oscillations for each bunch at the end of the linac. The first two curves (blue and red) assume that the accelerating sections are perfectly aligned but all the bunches are injected with a transverse offset $\Delta x = 50$ and $100\,\mu$m respectively. For the last two curves (green and magenta) the injection occurs on-axis but the linac sections are affected by gaussian random offsets with standard deviation $\sigma_{\Delta x} = 50$ and $100\,\mu$m. In order to mitigate statistical fluctuations, the final amplitudes in the latter case are obtained by averaging 50 runs of the tracking code. It can be noticed that in case of random misalignments the dipole modes are continuously excited regardless of the adiabatic damping process and, thus, we observe larger oscillations compared to the case of a simple injection error.

4.2. Detuning techniques

As the previous example has shown, mitigation techniques aimed to keep the amplitude of the transverse oscillations under control are usually necessary. A possible approach is to introduce a frequency spread of the dipole modes. Indeed, slight geometric variations of the unit-cell allow to shift the resonant frequencies of the HOMs without affecting the fundamental mode. Such fluctuations cause each HOM to exhibit a different resonant frequency throughout the subsequent linac sections breaking the coherent interaction with the pulsed beam. In order to apply this technique, a set of seven cavities has been designed from small variations of the original geometry. As an example, figure 3 shows the frequency variations for the TM_{110}-like mode in the unit cell.

Such information can be exploited to introduce a section-by-section frequency spread in our tracking code as we show in figure 4. Here the case of random alignment errors with $\sigma_{\Delta x} = 100\,\mu$m is considered again with the introduction of either uniform or gaussian spread of the HOM frequencies. Breaking the interaction pattern cancels the BBU-effects completely as the displacement of each bunch at the interaction point is close to the amplitude of the first beam.

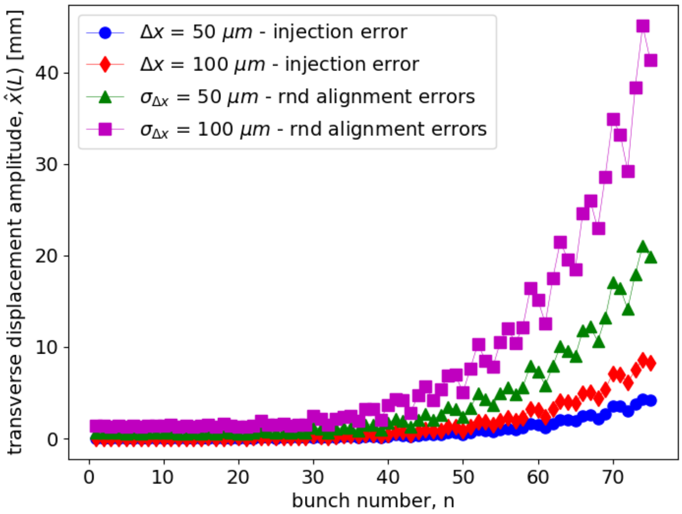

Figure 2. Amplitude of the transverse oscillation for each bunch in the rf-pulse at the interaction point. Both the cases of injection errors and randomly misaligned linac sections are investigated.

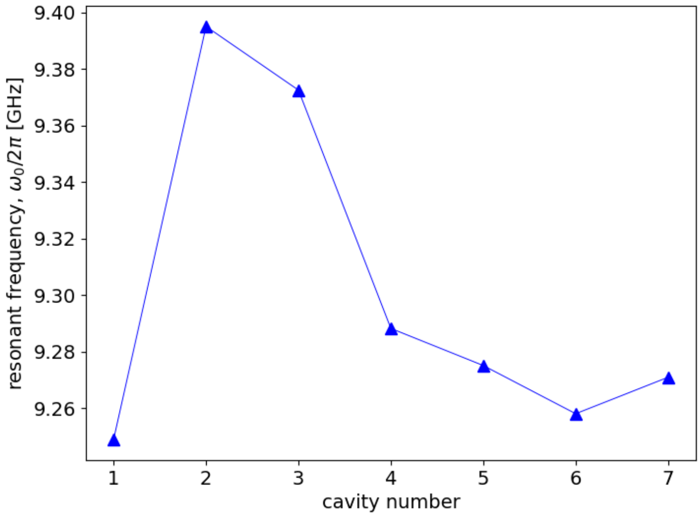

Figure 3. Spread of the resonant frequency of the TM_{110}-like mode for seven geometric variations of the unit-cell cavity.

5. Conclusion

In this paper we introduced a simple and fast tool which allows to investigate BBU effects in linear accelerators operating in multi-bunch mode. In particular, we have studied the performance of a state-of-the-art collider for which the main issues arising from injection and alignment errors were discussed. In addition, mitigation techniques based on a frequency spread of the higher order modes are investigated in order to compensate the beam break-up effects.

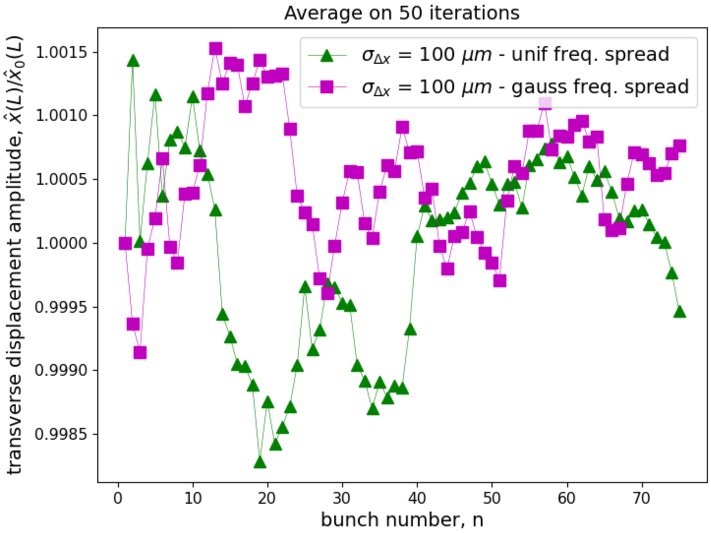

Figure 4. Frequency spread in presence of randomly misaligned sections ($\sigma_{\Delta x} = 100\,\mu$m). The cases of uniform and gaussian spread of the HOMs are investigated and displacements are normalized to the amplitude of the first bunch.

Acknowledgments

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