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Testing and Defining a Complex Design Through Digital and Physical Models

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Abstract

This paper presents the methodology adopted in an application of shape optimisation and digital fabrication conducted in the field of wooden furniture design. Experience has allowed the authors to define the models that support the creative process, identifying their respective peculiarities and their contribution to the design process as a process of experimentation, which takes place between digital and physical, not only defining the form of the idea but also correctly representing the tested model and its use, and allowing a dialogue between the stylistic requirements of the designed shape and the technical needs of the built one. The phases of the methodology are applied to the project of furnishing the lounge bar and restaurant area of the Oasis Skyview hotel in Doha (Qatar). The coordinated dialogue between the different models allows the definition of the design project, creating a workflow that significantly reduces the distance between the project and construction.

Keywords Modelling · Interoperability · Reverse modelling · Digital fabrication · Computational design

Introduction

Contemporary architecture routinely makes use of digital representation tools to prefigure the design of organic forms, so much so that these have become identifiable with the so-called 'digital turn' (Carpo 2017), and with digital fabrication techniques for their realisation.

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The increasing availability of computing resources and the increasing flexibility of the 3D modelling software has pushed architectural research towards forms of increasing complexity. This trend has demanded the introduction into architecture and design of a methodological approach that pursues ‘simplexity’, a theory that proposes a new complementary relationship between complexity and simplicity. Such ‘simple complexity’ is expressed in architecture whenever an apparently complex project is the result of a simple design and building strategy (Kolarevic 2016).

In this sense, the study of form and its optimisation are often called ‘architectural geometry’ or ‘smartgeometry’ and are seen as a direct evolution of descriptive geometry, intended as a tool for the design and control of shape (Pottmann et al. 2007; Burry 2013: 155–165).

In the last ten years, the growing computing power has been complemented by the possibility of storing and processing in real-time an unprecedented amount of data, the Big Data. These have opened the ‘second digital turn’, in which design has moved away from the mathematical determinism of modern science to appropriate heuristics through evolutionary design, optimisation processes and adaptive digital fabrication techniques (Reichert et al. 2014; Carpo 2017).

This contribution aims to methodologically present what has been achieved in a digital fabrication experience that has seen a process of shape optimisation directed at its production through computerised numerical controlled (CNC) machines. The implementation of this process required the *translation* of the design through different *mediums*: physical and digital models (3D), drawings (2D) and data lists (1D), the latter being necessary to control the production (Romero et al. 2010: 365–366).

In a construction process that includes a digital fabrication component, the role of the shape optimisation and engineering expert lies somewhere between the designer and the manufacturer/maker. The translation of the designed shape into a feasible shape, respecting the aesthetic values of the former and the technical specifications of the latter, is the task of this intermediate figure.

Analogue and Digital Prototypes

The definition of complex artefacts requires a multidimensional vision of the project: the idea must be investigated in the abstraction of the digital space as far as evolutionary genesis is concerned, as well as in the concreteness of the real space for the investigation of technical and constructive solutions. In the most advanced experiences, the project matures through the development of the following representations (Migliari 2004) (Fig. 1).

The Conceptual Model

The conceptual model is the initial act of the design process, where the designer begins a communication with himself, explaining the shapes and characteristics

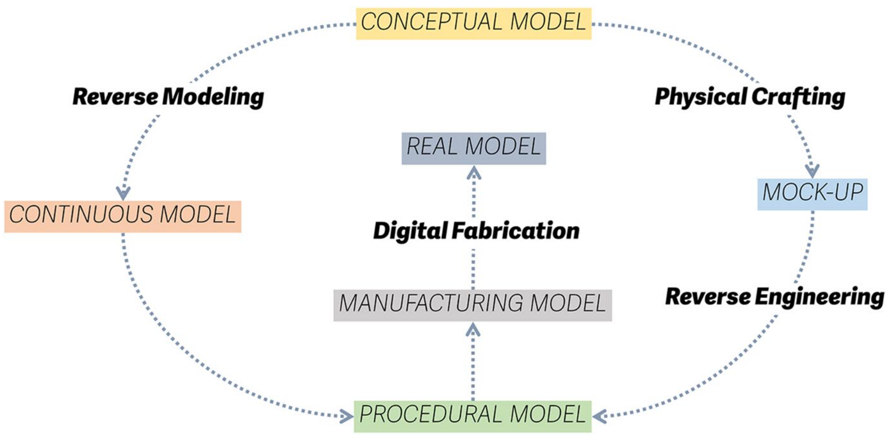

Fig. 1 Interoperability between the models that contribute to the development of the design. From conceptual model to manufacturing model

desired for the project through images. In this phase, the first representation is rapid, free of measurements, cryptic, evocative, multidimensional and method-less; the construction of the model must take place quickly so as not to interrupt the ideational flow. The digital tools for creating these drawings are extremely intuitive, making it possible to create multiple digital prototypes: visualisations with which to assess the chiaroscuro effect and/or the impact of the chosen shapes. The 3D representations that fulfil the requirements of this phase are generally quad-mesh, sub-D or NURBS (Non-Uniform Rational B-Spline); in less controlled cases, designers use hybrid models whose only purpose is to render the model.

The Continuous Model

The continuous model is the result of reverse modelling operations (Russo 2012; Calvano 2013) on the conceptual model and is interposed between the actions that begin with the conceptual model and end with what we will call the manufacturing model. The continuous model is composed of NURBS surfaces connected in the different geometric continuities allowed in the digital environment, thus overcoming the heterogeneity of the conceptual model. The continuous model maintains the aesthetic instance envisaged by the designer but implements its geometric and topological quality in favour of a homogeneous representation, which becomes the correct input to apply the algorithms conceived by the computational designer to generate the manufacturing model.

The Study Mock-up

The study mock-up is a preliminary study maquette used to represent the entire model or part of it in scale (Lim et al. 2008; Das and Das 2019). With the mock-up, the builder investigates the procedures to realise the artefact while maintaining the aesthetic and functional instance of the project. The fabrication of parts or the whole in scale allows the choice and the testing of construction procedures, machinery and materials to return an image that is as faithful as the one conceived. In addition, time and human resources are planned, defining the margins of the work feasibility. The study prototype must not be subject only to aesthetic and functional requirements but must consider the constraints to which the work will be subjected during its production and life cycle: starting from the manufacturing process, through transportation from the factory to the site, to its assembly, without neglecting the dynamics of maintenance to ensure durability over time.

The Procedural Model

The analogue reasonings made on the mock-up must then be applied to the actual quantities, which must be treated with the same care as was given to the maquette. The computational designer is in charge of the procedural model and their attention is not only directed to the formal outcome and the automation of processes but also to the choice and digitisation of the procedures that generate the resulting manufacturing model (Calvano 2019). The first step in the construction of the model is careful observation of the operations performed for the creation of the mock-up and its translation into geometries, actions and transformations available in the digital environment.

The Manufacturing Model

The manufacturing model is an accurate digital image of what will actually be built; in addition to the aesthetic instance, there is the static instance, i.e., the construction frame that allows the artefact to be built; the manufacturing model is the final goal of the procedural model. It is characterised by several interconnected representations: a 3D and a 2D representation (Kaseman and Graser 2020) enriched with 1D information (texts). The first one is the exact digital image of the real model, with which information related to assembly, machining tolerances and possible interferences between elements can be retrieved. The 2D representation is a reduction of the 3D model, which is more suitable for programming the CNC machines that produce the parts. The generated elements are then arranged on the plane and enriched with text and numbers to provide further information for assembly (1D). The manufacturing model can be described as the ‘digital twin’ of the real model, described next.

The Real Model

The real model is that destined for the site for which the project is designed, the place where the artefact will be subjected to the judgement of time. The real model is the last image of a process that moves between the real and the digital, in a sequence of complementary environments that alternately allow the project development.

Besides being the output of the process, the real model is a further moment of testing. Generally, this model is made up of prefabricated parts whose dimensions are suitable for storage, transport and easy assembly. In more complex cases, the parts are manufactured in the factory to deal with possible discrepancies between the tolerances of the design software and those of the construction machines in a safe environment. Any problems are often resolved by direct handcrafted actions decided upon following human control. Tolerance variation between models is not always a problem—often the lower accuracy inherent in the real model allows operations that are impossible with the mathematical accuracy of digital models (Mathieu et al. 1998).

Principles for the Digital Description of the Continuous Form and its Discretisation

NURBS mathematics is capable of describing continuous, smooth shapes and also makes it possible to control differential properties of geometries, such as the direction of tangents and normals to curves and surfaces, curvatures (bending and torsion) of lines and the Gaussian curvature of surfaces. These are implicit properties of geometry that, by themselves, allow us to describe its shape at any point.

The shape of a one-dimensional parametric space, i.e., a line, can be uniquely described through the concepts of tangent, curvature (bending) and torsion (second curvature). Intuitively, the tangent can be said to indicate the direction of the curve; the bending describes how far a line deviates from being straight while the torsion represents how far a curved line deviates from being flat at the analysed point P (Farin et al. 2002: 39) (Fig. 2).

The graphical representation of the curvature (bending) of a curve is given by the osculating circle, i.e., the circle that best approximates the curve's course at the analysed point P, while the inverse of its radius defines the value of the bending k (Pottmann et al. 2007: 226–227).

The ability to control this property of the curves is fundamental when dealing with the freeform shapes such as those in this experiment, and it is no coincidence

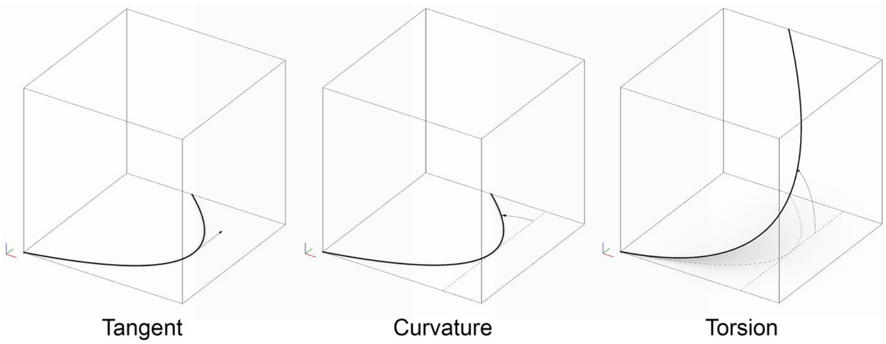

Fig. 2 The differential properties define the shape of a line

that the NURBS software can automatically calculate and display both the osculating circle and the curvature values of lines (Fig. 3).

Furthermore, this concept is particularly useful for extending the concept of curvature from 1D parametric space to 2D space, i.e., surfaces. The shape of a surface is described at each of its points by the Gaussian curvature. This is the product of the principal curvatures k_{min} and k_{max} of the surface at the analysed point P. The principal curvatures of a surface are the curvatures of the principal sections of the surface (Rogers 2001: 197; Pottmann et al. 2007: 495–497).

Some interesting properties of a surface for designing, optimising and producing a shape are derived from its Gaussian curvature; for example, a surface with zero Gaussian curvature is a developable surface and can therefore be manufactured from flat panels without tearing and elastic deformation. This property is also calculated and displayed automatically in the NURBS modelling, usually through false colours maps to make it easy to read (Fig. 4).

The properties mentioned so far refer to the single shape without considering the need to join several geometries and therefore to control their behaviour in the transition between one geometry and another. This aspect is defined as 'continuity' and it is possible to identify at least three levels, depending on the geometrical property to be preserved in the transition between geometries: G0 is the simple position continuity, i.e., the one in which the two geometries share an edge but the respective tangent planes have different positions; G1 is the tangency continuity, i.e., the one in which the two geometries share an edge and the position of the tangent planes along that edge; G2 is the curvature continuity, i.e., the one in which the two geometries share an edge and the value of the Gaussian curvature along that edge (Rogers 2001: 10; Farin et al. 2002: 193).

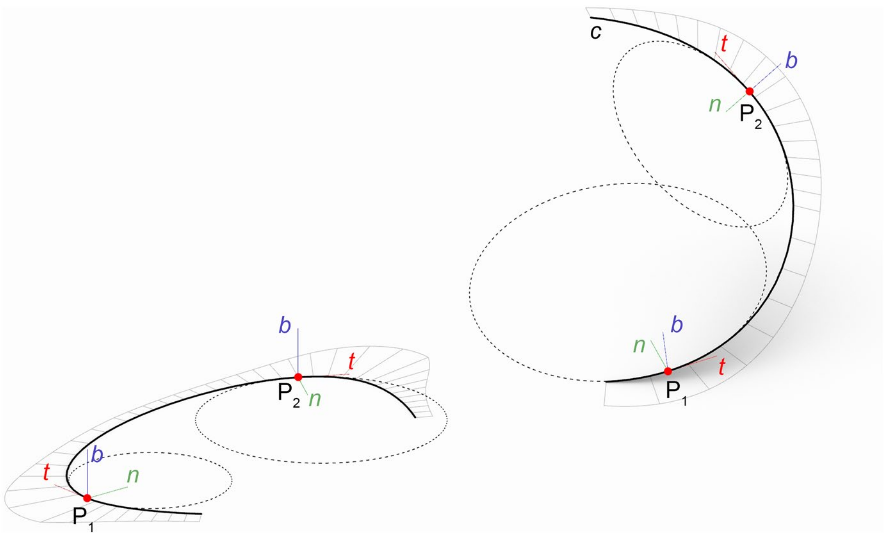

Fig. 3 The curvature of the lines represented through their osculating circles at different points P, the curvature graph and the Frenet trihedron

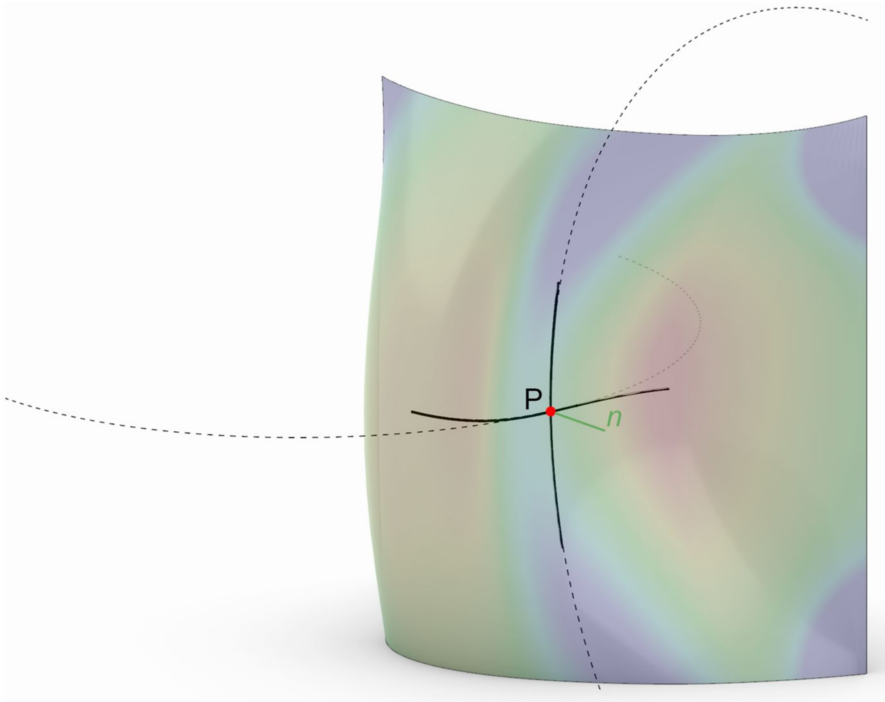

Fig. 4 The Gaussian curvature of a surface represented in false colours with the main sections at point P and their respective curvatures

The importance of the choices made along the joining edges between surfaces is evident, especially if one considers the important implications for the design of surfaces with a glossy finish. Reflections clearly show the type of continuity adopted and, exploiting this property, reflection lines are used in NURBS modelling to graphically display the type of continuity (Pottmann et al. 2007: 503–508) (Fig. 5).

The complexity of shapes enabled by NURBS modelling has given rise to the need for developing methods of shape discretisation to start the process from the

Fig. 5 Study of the reflection behaviour of pairs of surfaces connected in geometric continuity. From left to right: G0, G1, G2

ideal shape to its realisation. This issue has been the focus of research in architectural geometry for about 15 years and has seen the definition of various shape tessellation procedures.

The tessellation of surfaces can be the result of a process of aggregation or subdivision, outlining regular, semi-regular or irregular patterns. Observation of the contemporary architectural panorama shows how often the logic of shape parcelling is strongly linked to some of the previously mentioned mathematical properties, such as the Gaussian curvature of the surface (Chang 2018).

Generally, the realisation of double-curved surfaces is done by subdividing the 2D domain into variable slats avoiding surface tears (Wang et al. 2014). New digital procedures also allow the extrapolation of clusters of equal modules covering the parts of surfaces with similar curvature (Calvano and Sacco 2016); the described process is instructed by the designer by setting constraints and geometric variables within robust digital processes to tessellate complex shapes. The downside lies in the gap generated between these minimal elements, which is generally absorbed by small deformations of the tessellations during assembly or by the tolerances of the engineering elements with which real models are assembled. The most common standard algorithms for the discretisation of a continuous shape, both 1D and 2D, can be traced back to two operating principles:

- *scale-independent algorithms*, which subdivide the parametric domain of the geometry and produce a tendentially homogeneous pattern;
- *scale-sensitive algorithms*, which work according to measures specified at the conversion stage such as the minimum or maximum edges length of the desired target mesh, or the maximum acceptable distance between the initial NURBS geometry and the target mesh; the last parameter returns a variable tessellation related to the Gaussian curvature of the initial surface.

The second approach was used in the application presented in the next section for the discretisation of continuous lines. In particular, two geometric characteristics were considered: the arrow of each arc, which describes the maximum acceptable distance between the initial continuous curve and the target polyline, and the chord of each arc, which represents the length of the segments that make up the target polyline. The combination of these two values also can describe the boundary rectangle of each arc and therefore its overall dimensions (Fig. 6).

Application

The specific experience on which the described methodology is based was the realisation of the permanent solid wood furniture of the hall on the 28th floor of the Oasis Skyview hotel in Doha (Fig. 7), which houses the lounge and the food zone of the structure. The architectural design is by the French firm Studio Jacques Garcia (http://studiojacquesgarcia.com/) while the definition of the construction technique, the production of the components and their assembly on site was carried out by Devoto Design (https://www.devotodesign.it/en/).

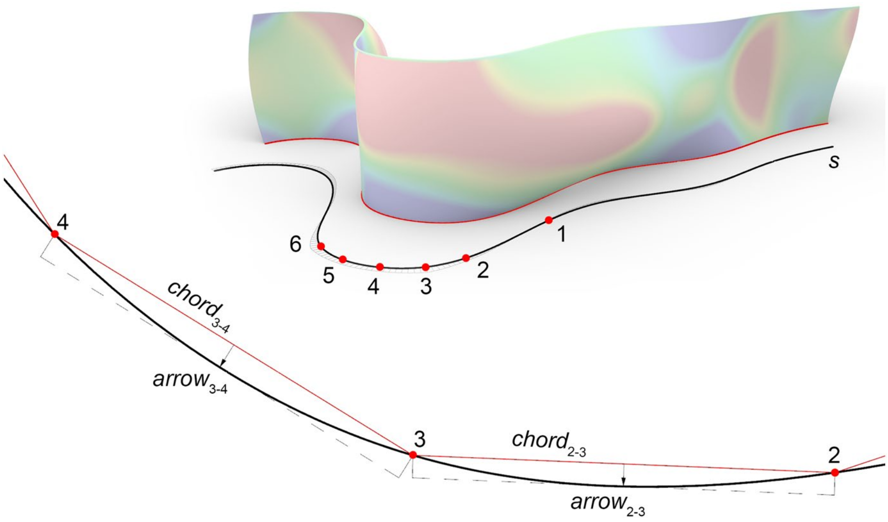

Fig. 6 The approach adopted for the discretization of the *s* sections of a double curvature surface, considering their curvature variations

Fig. 7 Interior of the 28th floor of the Oasis Skyview hotel in Doha. The picture shows the room before the installation

The final realisation is composed of black walnut walls in double curvature that wrap around the different areas facing the hall. The wooden structures reach a maximum height of 10 m and curve in different directions; the construction technology adopted consists of thousands of wooden strips which, stacked one on top of the other, form a sort of 3D puzzle that reconstructs the shape of the walls designed by the architect. The methodology applied is divided into three phases that relate physical and digital models which, each with their own particularities, are necessary for testing and defining the process that goes from conception to realisation of the project.

From Conceptual Model to Continuous Model

The purpose of this phase is to create a homogeneous digital model that guarantees compliance with the geometric and topological qualities desired by the designer and the technical specifications indicated by the manufacturer, depending on the chosen construction technique.

The entire setup is made up of eight walls of considerable morphological variety: walls 5-6-7-8 are double-curved, with a height equal to one storey and built with a single structural skin; wall 2 is a cylindrical tower while wall 4 is a tower with a variable section, both characterised by a height of about 10 m and built with a single structural skin; walls 1 and 3 are double-curved, characterised by variable height, with openings or crossings, and are built with a double structural skin solution (Fig. 8).

Geometrically, all walls, except the cylindrical tower 2, are characterised by the composition of double-curved surfaces in continuity of curvature (G2). The translation of the conceptual model into the continuous model required the analysis of several properties of the designed shape. If the conceptual model did not meet the minimum requirements and could not be adequately corrected, it was remodelled by reverse modelling of the shape. In the latter cases, each wall was decomposed into main surfaces and connecting surfaces in order to construct a topologically correct model consisting of as few patches as possible. In addition, each model underwent further optimisation operations to obtain a parametric structure that efficiently

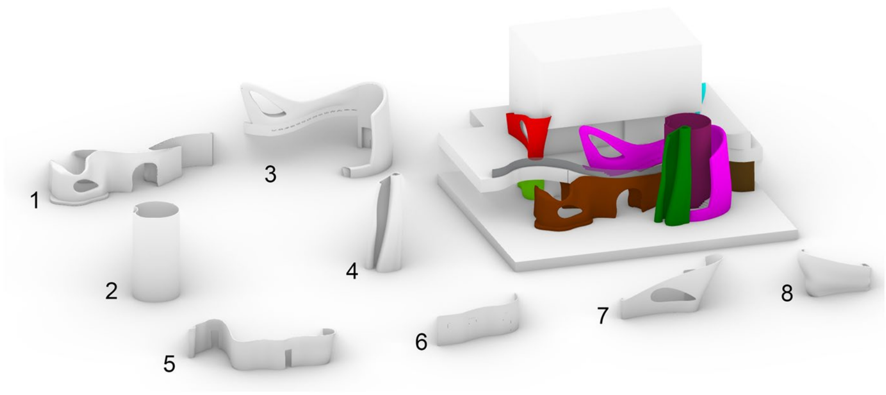

Fig. 8 The set-up is divided into its eight constituent walls. The walls are juxtaposed according to their geometric and constructional characteristics (1-3, 2-4, 5-6-7-8)

described the geometric properties of the surfaces, through an optimised density and orientation of the parameters (u, v). Each wall was analysed to fix any topological errors such as open and non-manifold edges to ensure the presence of curvature continuity between surfaces and the respect of technical specifications indicated by the manufacturer, such as wall thickness. The aim of the model optimisation phase was to preserve or implement the aesthetic values of the designed shapes and to ensure the macroscopic geometric properties (dimensions) of the buildable shapes.

The next phase was the engineering of the shapes during which the architectural details such as doors, window frames, parapet housings, flashings and rails necessary for assembly on site were determined.

Finally, the continuous model, optimised and re-engineered, made it possible to start organising production by successive layers, which, on this occasion, were subdivided into overlapping sectors that considered the height of the strips of strips and their quantity (Fig. 9). The creation of these models was fundamental to guarantee a smooth and error-free translation between 3D models, 2D drawings and 1D data lists, which characterises the subsequent algorithmic discretisation phase of the shape.

From Mock-up and Continuous Model to Procedural Model

The methodological approach to the case study presented saw the use of algorithms for shape tessellation with which to control various aspects. The role of the computational designer is between input and output, building a path that puts together variables and geometric constraints with the aim of designing the procedural

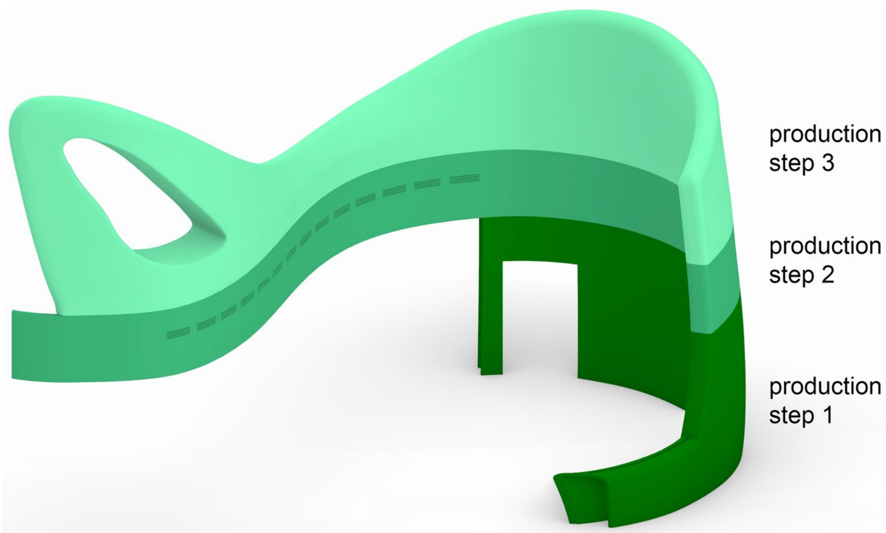

Fig. 9 The wall production steps were programmed using the continuous model. The picture shows the production steps for wall 3

model. In the case study presented, the first constraints were those imposed by the manufacturer, who assembled portions of solid wood with a height of 43 mm and compounded them together to create flat panels with a size of 1200 × 1350 mm (Fig. 10), elements from which to derive the different slats for manufacturing.

Nesting of the parts gave further indication of the length of the individual elements; the aim was to fit the maximum number of slats within a single panel. The best optimisation is achieved by producing straight slats, but this was not possible due to the complexity of the designed shapes. For this reason, it was decided to cut the areas of greater curvature with shorter elements and, conversely, to cut the areas of lesser curvature with longer elements. The direct consequence of this choice was

Fig. 10 Construction and assembly stages of the panels used for milling the slats that make up the walls of the installation: from the raw boards to the finished panels used for the processing

to obtain elements with a reduced concavity, allowing maximum matching of the parts during nesting. The length of the parts was further constrained by the need to facilitate storage in warehouses, transport by container and handle within the site.

Static and aesthetic considerations made on the mock-up further enriched the construction algorithm; the tessellation of vertical architectural elements was done by clamping the slats but without creating continuous vertical joints to make the wall more solid.

The positioning of the elements to be cut in the panel was guided by further aesthetic considerations: the proximity of the slats in the part to be built had to be repeated in the panel so as to create a continuity of grain between the parts once they were placed side by side on the final artefact; this was to avoid sudden variations in the grain.

The procedural model generated by the computer writing (the code) of the steps described above led to the generation of the manufacturing model.

From Procedural Model to Manufacturing Model

The solving algorithm of the procedural model is conceived backwards respecting the conditions imposed by the invariants. Below, we have listed the main points of the procedural model for generating the 3D (the digital twin), 2D (the curves useful for guiding the CNC machine) and 1D (the textual information to be associated with the 3D and 2D representations) representations that define the manufacturing model:

– make multiple cuts with horizontal planes spaced by a dimension equal to the height of the panel to be milled;
– check the length of the parts within a pre-set range;
– check the length of the parts in relation to the curvature of the surface to be panelled;
– create the misalignment of vertical joints between consecutive panels;
– ensure homogeneity of the wood grain.

The digitisation of the process can be done through any programming language. In this application, Grasshopper was chosen for its widespread use in architecture and design. The role of visual programming language (VPL) in digital fabrication processes is generated in the CAD environment both in the management of complex shapes as well as in the compiling of CAM files to drive the numerical control machines engaged in the milling of parts.

To explain the steps of the procedure, let us take as an example one of the walls that constituted the Skyview set-up, wall number 7. The input to the procedure was represented by the closed polysurface from which the external polysurface was extracted.

The first point of the process listed above was solved by ‘slicing’ the model of wall 7 with a series of horizontal planes at a distance of 43 mm, equal to the thickness of the panels to be milled with the CNC milling machine.

Figure 11 shows the part of the procedural model used to create the boundary box around the chosen wall; a vertical edge was extracted from it and used as the normal axis for the planes slicing the wall. The planes have their origin on the axis at a distance of 43 mm from each other. The outputs, in addition to the section planes, are the closed curves resulting from the intersection between the planes and the polysurface of the wall.

The generated curves were subsequently squared within boundary rectangles; these served to break the polysurface into strips. The cuts are successful if we slightly enlarge the rectangles to avoid any geometric ambiguity due to tangency

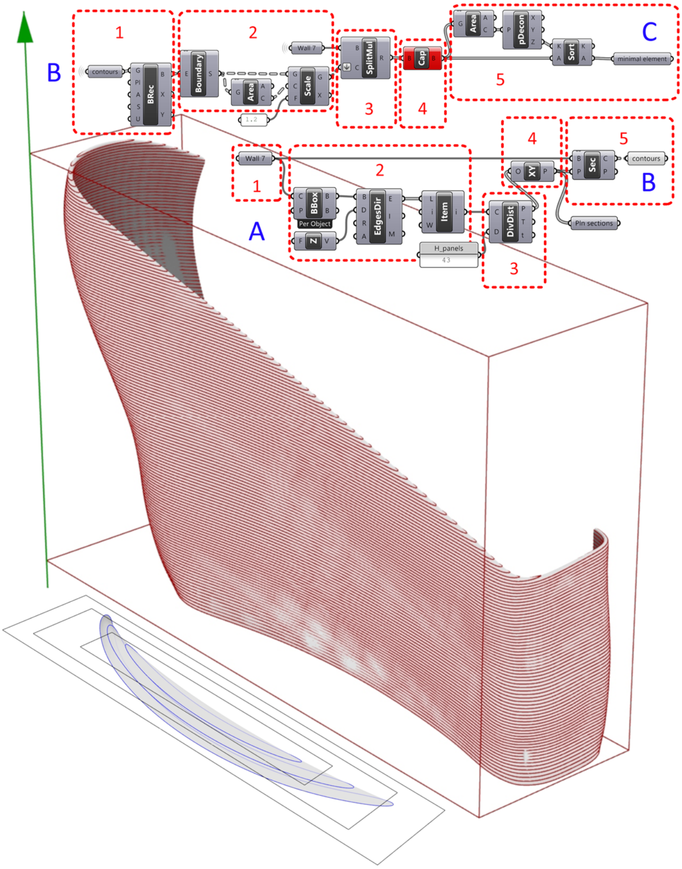

Fig. 11 Procedural model part that cuts the continuous model into solid strips for subsequent construction of the manufacturing model. A–B, below: (1) acquisition of the continuous model; (2) construction of the boundary box and extraction of a vertical edge; (3) location of points every 43 mm on the isolated vertical segment; (4) setting of horizontal planes with origin on the extracted points; (5) multiple sections of the wall by intersecting the horizontal planes with the continuous model. B–C, above: (1) Construction of the boundary rectangle around each section curve; (2) Construction of a surface inside the rectangle and slight scaling with respect to the centroid; (3) Cutting the wall with the rectangular surfaces; (4) Closing open polysurfaces into closed solids; (5) Extraction of the Z-coordinate of the solid strips' centroids to order the elements in the list from lowest to highest

between entities. Multiple cutting of the solid returns open polysurfaces above and below and and these must be closed with flat trimmed surfaces; the result was closed polysurfaces that had to be ordered from bottom to top.

The main part of the process was the tessellation of the curves guided by the manufacturing rules of the factory. The closed polysurfaces just generated had to be broken in relation to their curvature, which is variable for the length of the wall.

To simplify the process, we take the external polysurface of wall 7 as input, which sliced with the planes generated in the first part of the procedure, returns a sequence of open curves able to synthesise the curvature of the wall (Fig. 12).

To subdivide the solid strips, we then worked on a sequence of open curves of degree 3, which reduces the complex problem of the curvature of the surface to a problem in the plane repeated for the number of sections generated (for wall 7 there were 130). This operation was done thanks to a component that allows the transformation of the degree 3 curves into polylines starting from the tessellation criteria mentioned in Section "Principles for the Digital Description of the Continuous Form and itsDiscretisation". The Curve to Polyline (ToPoly) component mainly adopts 2 criteria for drawing the new polyline:

- minimum and maximum length of the segments constituting the entity;
- distance between the midpoint of each segment and the curve to be tessellated.

The first condition made it possible to obtain parts that conformed to the dimensions of the panel to be milled (maximum length 1350 mm) while also introducing a minimum size to obtain panels that could be handled and machined by the CNC milling machine (minimum 800 mm).

The second condition, using a maximum deflection of 60 mm, made it possible to reduce the size of the pieces in the areas with greater curvature, leaving longer segments in the areas with less curvature. The operations defined up to now have a geometric result: a series of polylines with the vertices belonging to the section curves. To gain a feedback on the model for fabrication, each vertex becomes the

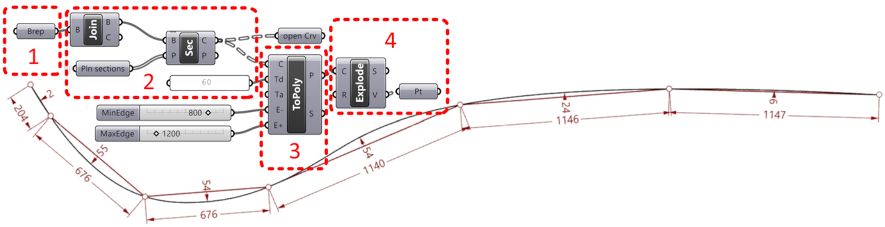

Fig. 12 Part of the procedural model that defines the chords of the slats in relation to the curvature of the solid strips and the dimensional constraints established by the manufacturer. (1) Acquisition of the external surface of the continuous model; (2) Section of the surface with horizontal planes at a set distance; (3) Discretization of the degree 3 curve into a polyline composed of segments with a maximum distance from the original curve of 60 mm and a range of lengths defined by parametric E− and E+; (4) Use of the vertices of the polyline to break the main curve

origin of a vertical plane perpendicular to the curve; the generated planes will be used to break the created 3D strips to prefigure the different slats.

To avoid alignment problems between the vertical joints of adjacent overlapping strips, each vertex identified in the previous phase has been displaced (Fig. 13) with a positive or negative value along the direction tangent to the curve.

The amplitude of the displacement was determined by a random value belonging to a range of 300 mm defined by its limits; the displaced points no longer belong to the curves and are therefore projected onto them. In some cases, this will result in points approaching closer than the set minimum distance; therefore, in this part of the algorithm, a component (Cull Duplicates) has been introduced to detect points that are too close and eliminate one of them. The remaining points are the origin of the vertical and perpendicular planes to the curve that are used to break up the 3D strips, prefiguring the different tiles to be manufactured. A final part of the code flips the slats in the plane, extrapolates the upper and lower curves for the slats consisting of six faces, sets the anchor holes

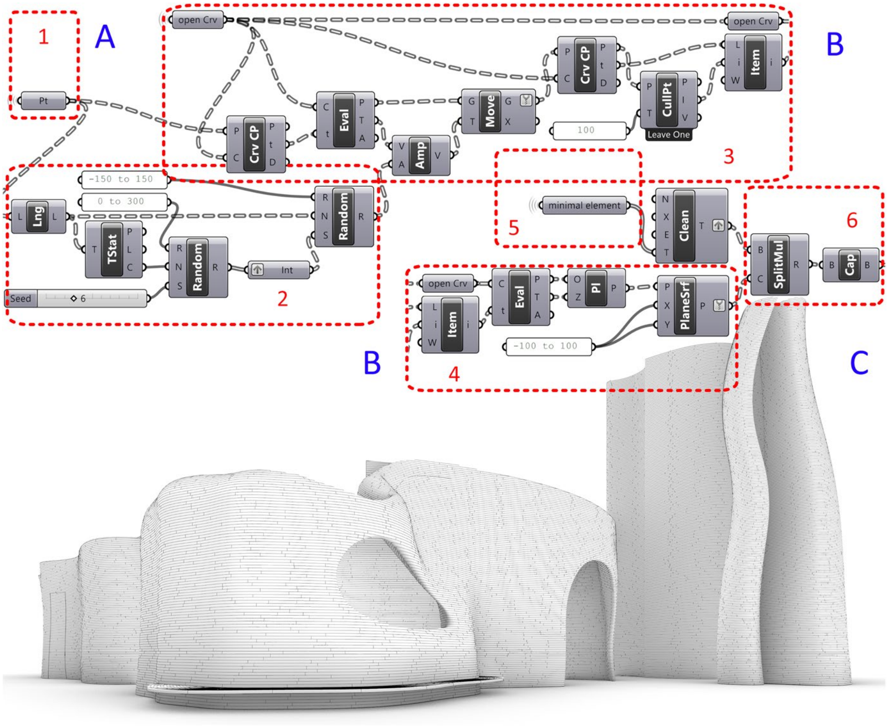

Fig. 13 Above: Procedural model for offsetting vertical joints between consecutive solid strips. Below: 3D representation of the manufacturing model: (1) Acquisition of the points constructed in the previous code; (2) Construction of a list of positive and negative numbers associated with the displacement of the points along the original curve; (3) Displacement of the points along the tangents to the points themselves and re-projection onto the curve; (4) Construction of rectangular surfaces perpendicular to the original curve passing through the projected points; (5) Acquisition of the polysurfaces created in the previous code; (6) Trimming the polysurfaces into smaller solid blocks

and prints the code texts for assembling the elements. The procedure led to the definition of the manufacturing model.

With the 3D representations, the final configuration of the real model was illustrated in digital space. Each tile was represented by a single closed BREP appropriately enriched with textual information that identifies, by means of an alphanumeric code, the position of the panel in relation to the wall of which it is a part.

A comparison between the curvature pattern and the distribution of the slats reveal the relationship between the wall's Gaussian curvature and the criterion for the discretization of the horizontal sections that has been adopted in this application. In the case of wall 5, two different digital representations can be compared: a false-colour map considering the absolute values of the Gaussian curvature and the panelled wall coloured according to the length of the slats. Only the absolute value of the curvature of the walls has been considered since, for their discretization into horizontal slats, it is relevant to know only the entity of the curvature, regardless of its sign. The false-colour map represents in cold tones the parts of the surface with a low curvature value while it adopts warm colours for the parts with higher curvature values. The same colour map was applied to the tessellated wall, this time taking into consideration the length of the tessellations obtained through the proposed discretization algorithm. It is possible to observe that the proposed algorithm guarantees the close relationship with the Gaussian curvature of the surface while reducing the geometrical problem to the discretization of its horizontal plane sections: the slats coloured with cold colours are the longest and correspond to the parts of the surface with lower curvature values, while those showing warm colours are shorter and are applied to the parts of the surface with higher curvature values (Fig. 14).

The manufacturing model, being the 3D photograph in a digital environment of the real model, consists of standard slats and others defined by functional details (doors, frames, air intakes, guides, etc.).

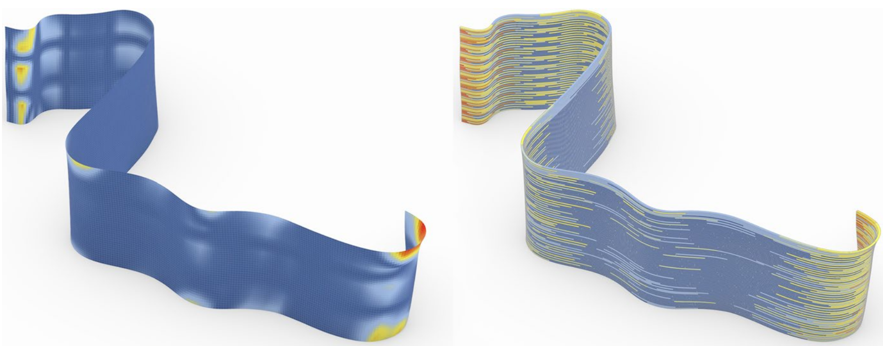

Fig. 14 Wall 5: a false-colour map represents the distribution of the absolute values of the Gaussian curvature of the surface (left); the wall slats coloured in relation to their length (right)

The 3D representation of the complex slat was also used to guide the CNC machine, in the case where the shape consisted of more than six faces, which could not be described by 2D processing.

Most of the information for driving the CNC machines was provided by the means of 2D representations consisting of 1D paths. The outlines of the upper (blue) and lower (green) surfaces are extracted from each slat; the two closed curves, separated in space by the height of the slat, constitute the generating curves of the ruled surface that connects the blue closed curve and the green closed curve. The generating lines of the surface are the directions that the axis of the milling machine's tip must interpolate in order to cut the wooden panel correctly and obtain each individual slat (Fig. 15).

Each element has been associated with textual information with which to refer the slat to its wall. The textual code was mainly designed to support the sequential assembly of the slats in the construction phases of the real model. The variables that allow the positioning of the minimum wooden elements in the wall are as follows:

- n_p – the number of the wall to which it belongs that goes from 1 to 8;
- n_y – the height position of the slat considering its horizontal reference strip that goes from 00 to n;
- n_x – the position of the minimum element along the strip to which it belongs, from 00 to n;

The result is the reduction in textual form of multidimensional information to be associated to each single wall in the following way: n_p n_y n_x providing the gap between the pieces of information. For dimensions n_y and $n_{x,}$ the range has no fixed final limit since it depends on the height and length of the wall, while the number of walls is known after the operations explained in Section "From Conceptual Model

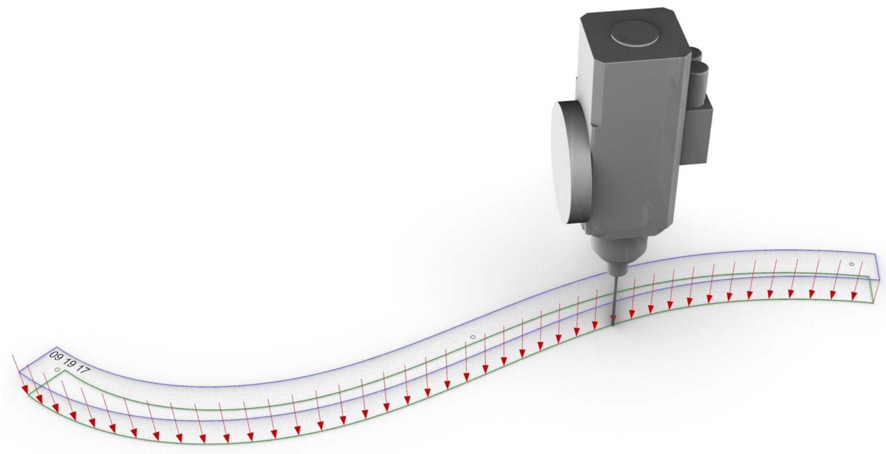

Fig. 15 The upper and lower contours of the slat are the information used to control the CNC milling machine. The image also shows the holes for anchoring and the text with the coordinates for positioning on site

to Continuous Model". The code formatted in this way plays the role of metadata in the manufacturing model, while it is engraved on the upper surface of the tiles in the real model. The manufacturing model had the role of showing the quality but also the quantity of the work done, whose final numbers can be read in Table 1.

Conclusions

The optimisation and digital fabrication experience presented allowed the *simplexity* criterion to be tested on a real process in which the fabrication of complex organic surfaces with double curvature was solved through a process of geometric simplification of the problem, which was taken from 2D parametric space back to 1D space, creating digital algorithms that took account of the construction technologies (Fig. 16).

Among the outcomes achieved by the process is the logical systematisation of the model types that generally gravitate around the design project: a series of representations characterised by different constructive rules that through explicit representation procedures (VPL) can be related to generate different responsive images of the same design model. The creation of relationships between the models allows the dialogue between the players in the design process (designer, computational designer, structural engineer, manufacturer) during the design process, from conception to construction. The knowledge of the differential and topological properties of shapes and their effects on digital models is an important requirement for the simplification of a complex problem and for its resolution through efficient implementation techniques. Furthermore, experimentation has shown that NURBS modelling is an effective way of representing complex shapes and that the introduction of algorithmic procedures has added a high level of efficiency to this representation. The ability of the algorithmic modelling to prefigure, through VPL, not only the shape but also the construction technique, the production phases of the single elements and their construction site is, in fact, an essential added value for the efficient realisation of complex projects, composed of tens of thousands of customised pieces but responding to a single logic.

Table 1 Number of panels per wall

Walls	No. of slats
Wall 1	7641
Wall 2	2606
Wall 3	6744
Wall 4	3284
Wall 5	1824
Wall 6	1241
Wall 7	1159
Wall 8	1395
Total	25,894

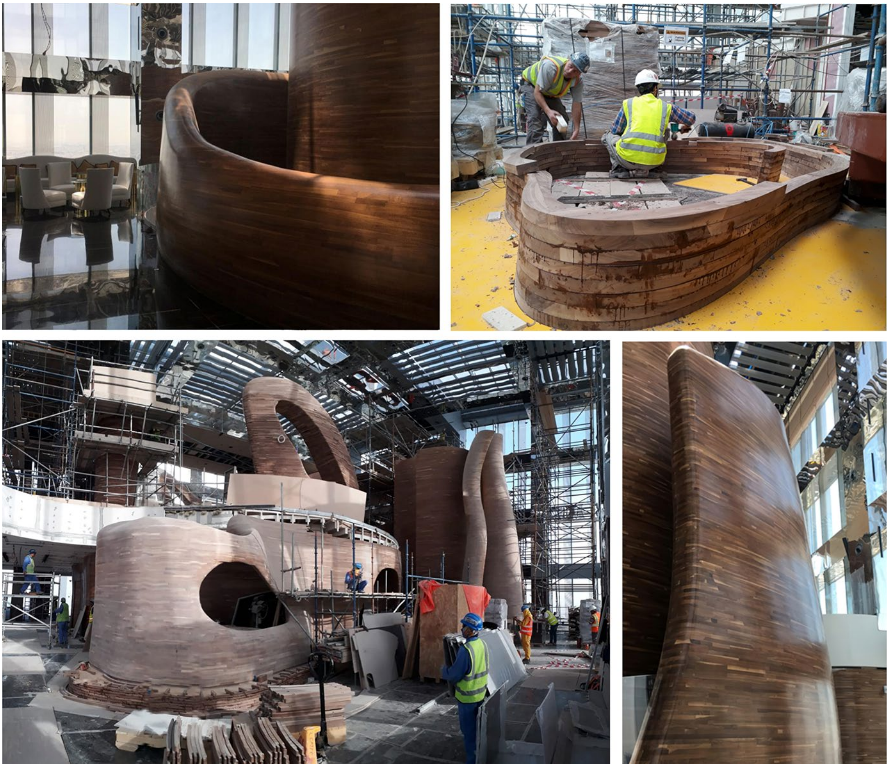

Fig. 16 Detail photos of the finished artefact and some phases during the construction on site of the real model

It should also be emphasised that the introduction of algorithms implements the possibility of automating modifications and updates of the designed shapes and the production of the artefacts themselves, which would not otherwise be possible through direct modelling. This feature also gives the modelling a strong value of repeatability that can be exploited internally within a single project, as in the case of the eight walls tested, but also between different projects, narrowing the gap between the concept of method and tool.

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