Second-Harmonic Dual-Frequency Microcomb Generation in a Fundamental Frequency Temporal Cavity Soliton Fabry-Perot

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Abstract: We show a novel example of space-time analogy, where a dissipative soliton microcomb generates a traveling-wave temporal Fabry-Perot for trapping a dual-frequency dispersive comb, generated at the second-harmonic of the pump laser.

There is much research interest in the study of optical pulse propagation in spatiotemporal dispersive media. Interesting effects arise in the presence of a refractive-index modulation, both in space and time, which is induced by a traveling-wave perturbation. In this context, intriguing space-time analogies such as temporal reflection and refraction, total internal reflection, and temporal guiding from a moving boundary have been revealed [1]. An optical soliton permits to introduce, via cross-phase-modulation, a traveling-wave refractive index perturbation which is seen by a weak dispersive wave propagating at a different wavelength [2, 3]. In addition, the propagation of a soliton pair provides the temporal analog of a waveguide or a Fabry–Perot resonator [4, 5].

Here we introduce a new example of a wave-propagation effect that can be explained in terms of space-time analogies. Specifically, we show that dissipative Kerr solitons (DKS) associated with a coherent frequency comb in the anomalous dispersion regime of a microresonator may be used to create a Fabry-Perot resonator temporal cavity for dispersive waves at different carrier frequencies. In the relevant case where the microresonator has both quadratic and cubic nonlinearities, a time-stationary dispersive wave is spontaneously generated by the fundamental-frequency (FF) soliton comb via phase-matched intra-cavity second-harmonic (SH) generation [6]. Based on numerical experiments, we show that the usual sweeping of the frequency detuning of the FF coherently injected CW laser across the cavity resonance leads to the simultaneous generation of a stable pair of coupled combs, comprising a FF soliton comb and bichromatic SH comb.

Optical frequency comb generation in the presence of both quadratic and cubic nonlinearities has been experimentally demonstrated in different experimental platforms, such as aluminum nitride with intermodal phase-matching for SH generation [7], and Si_3N_4 with both intrinsic $\chi^{(3)}$ and quasi-phase-matched photo-induced $\chi^{(2)}$ [8]. For a doubly-resonant cavity, the generation of coupled combs associated with DKS and modulation instabilities have been recently theoretically predicted in microresonators with both $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearities [6, 9–11]. Now we consider a different case, where the properties of the SH comb generated in the normal dispersion regime are determined via cross-phase modulation by a temporal Fabry-Perot formed by FF DKSs propagating in the anomalous dispersion regime.

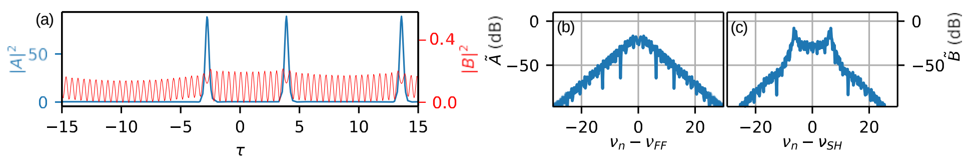

Fig. 1. (a) a SH oscillatory wave trapped by a temporal Fabry-Perot cavity soliton; (b,c) The associated spectra of the FF and SH field.

First, we numerically solved the coherently driven and damped mean-field equations for a passive cavity with quadratic and cubic nonlinearities [9, 10, 12–14], coupling the FF field A to its SH field B. When the pump laser frequency is scanned across a cavity resonance, eventually stable DKSs emerge at the FF forming an effective temporal Fabry-Perot which confines a weaker oscillatory wave at the SH (Fig. 1 (a)). In panel (b-c) we report the associated spectra, and we observe a double peaked SH profile.

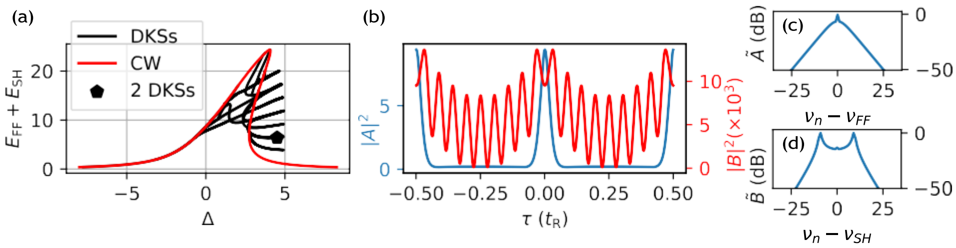

Fig. 2. (a) Bifurcation diagram; (b) DKS solitons at the FF (blue curve) and dual-frequency comb at the SH (red curve); (c) DKS comb at the FF and (d) dual-frequency comb at the SH.

The bifurcation analysis (Fig. 2 (a)) shows steady bifurcations which connect the homogeneous continuous wave (CW) solution to the DKSs branches. We locate a particular solution (2 DKSs) and we report the temporal profile in Fig. 2 (b). We can see a FF two solitons solution trapping a much weaker synchronous broad pulse with fast oscillating intensity and exponentially decaying wings at the SH. We can observe the corresponding spectra in panels (c-d). Because of the much-reduced intensity of the SH comb, it is expected that its properties can be described in terms of a linearized model. When neglecting the self-phase-modulation term for the weak SH field and the walk-off, one obtains a linear equation for the SH comb. In the presence of a DKS comb at the FF, we find that the SH field has a dominant Fourier mode $B \simeq B_{k_*} cos(k_* \tau)$, where

$$k_* = \sqrt{\frac{2\Delta - 2\gamma_{21}\frac{S^2}{\Delta^2}}{\eta_2}}.$$

Here Δ is the cavity detuning, γ_{21} is the cross-phase modulation coefficient, η_2 is the (normal) chromatic dispersion coefficient at the SH, and S is the CW pump amplitude at the FF. This equation, which can be generalized to non-zero walk-off, is in excellent agreement with direct numerical simulations. When multiple DKS are present in the FF comb, multiple reflections of the counterpropagating SH waves occur inside the temporal soliton Fabry-Perot, which enhances the SH comb, whose amplitude is inversely proportional to the temporal separation among the two adjacent DKSs. Remarkably, even in the presence of a significant linear group-velocity mismatch, the dual-comb generation remains stable.

We anticipate that our findings will have a significant impact in the development of octave spanning coherent microcombs based on the cooperative action of quadratic and cubic nonlinearities. Funding agencies: ANR-22-CE92-0065, DFG-505515860 (Quadcomb); EU - NRRP, CUP B53C22004050001, PE00000001 - program "RESTART"; Marie Sklodowska-Curie Actions (101149506), Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – Project-ID 258734477 – SFB 1173.

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