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Original research article

Detailed analysis for temperature-dependent and temperature-independent Goos–Hänchen shift

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ABSTRACT

In this work, we have presented the deep insight of Goos–Hänchen shift depending upon the temperature when a plane incident electromagnetic beam of light enters from a rare to a denser medium, keeping conditions of total internal reflection intact. A simple well known Drude's model has been used for the purpose of thermal analysis of this shift. Through this model, the temperature-dependent permittivity of absorbing or complex medium has been found through the temperature-dependent plasma and damping wavelengths. Both transverse electric and magnetic incident electromagnetic fields have been considered in this regard. On an air-metal interface, firstly, Goos–Hänchen shift has been found when a temperature-independent dielectric function of a complex medium has been considered, and secondly, by analyzing the temperature-dependent dielectric function in the simplest way by using the stationary-phase method. This study is useful to understand the behavior of Goos–Hänchen shift around the temperature-dependency of the medium and how the angle of incidence impacts in this scenario.

1. Introduction

Generally, when a light beam incident on an interface which possesses different refractive indices, the beam bounces back at the same point as by the prediction of geometrical optics, but in 1947, it was observed that when the light beams that travel from media with different refractive indices deviate from the path of geometrical optics and travel a short distance along the interface. This optical phenomenon is referred to as lateral displacement or Goos–Hänchen shift (GH), and can be seen in [Fig. 1\(a,b\)](#) [1–4].

Mostly, this shift is mentioned under the situation of total internal reflection, when beams of light travel from optically sparse to a denser medium [5]. Artmann's work on the GH shift by using the stationary-phase method serves as a backbone for the theoretical interpretation [6]. Now it is more evident that deviating behavior of reflected rays exists and has been proved by considering a simple air-glass interface, shone by the Gaussian beam [7]. As Goos–Hänchen shift is a non-specular phenomenon and shows very interesting behavior for different media. The amplitude of this shift can be tuned as well, and many research groups are working on the electrically tunable GH-shift [8]. Temperature-dependent Goos–Hänchen shift is one step forward in this regard and has opened a new door in advanced optoelectronics, temperature-monitoring sensors and surface-plasmon resonance sensors [9,10]. Recently, plenty of literature has been developed, which leads to remarkable results ending with applications in optical switching and in sensing technology, including plasmon resonance sensors, oscillating wave-sensor and temperature-dependent sensors [9–11], lasers [12], and acoustic [13]. GH-shift also shows positive as well as negative behavior lean upon the media or refractive index

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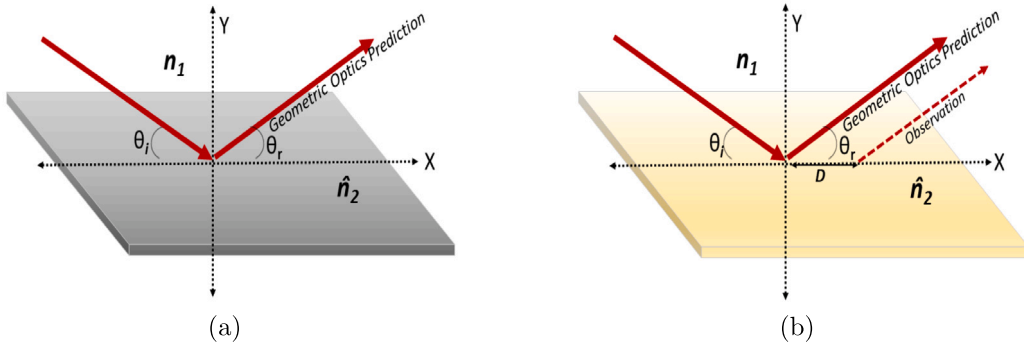


Fig. 1. Structural diagram for a planar interface. (a) Geometrical optics prediction for an incident beam at an interface; (b) observation of GH shift for an incident beam at a planar interface.

of media. Negative GH-shift has received considerable attention and has many applications in electromagnetic regime [14–16]. In recent studies, composite GH-shift has been explored, which is a combination of two shifts, when a Gaussian beam impinges on the interface and explains how the GH-shift changes its behavior for both s and p-polarization. Large positive and negative GH-shift has been observed when an incident beam hits on metal grating when the condition of surface plasmon resonance (SPR) has been satisfied. In these studies, it has been proved theoretically and experimentally as well that the incident beam width and grating depth influence the GH-shift [7,17–19]. In the recent past, a number of research groups have investigated the Goos–Hänchen shift in the context of temperature dependency while considering different media, including complex media [20,21], absorbing media [22,23], and metamaterials [24]. Goos–Hänchen shift has been explored by using different kinds of incident beams on different typologies of media. In the literature have been derived the new mathematical expression giving a valuable interpretation of the behavior of the shift in this regard. For example, Airy beams and vortexes when incident on the Dirac metamaterial and graphene-based substrate surfaces are reported [25–28].

Here we have presented the explicit studies for Goos–Hänchen shift while considering both temperature-dependent and temperature-independent absorbing media. Moreover, Drude’s model is taken into account in order to dig deep into the temperature-dependent wavelengths, i.e., plasma and damping wavelengths [29]. Moreover, for a better understanding of temperature-dependent reflectivity, contour plots have been reported, Fig. 4.

2. Mathematical model

For the sake of better understanding, we have divided our analysis into two cases, temperature-dependent and temperature-independent GH-shift, and considered both s-polarization and p-polarization in this regard. For the first case, experimental values of the refractive index of metals like silver ($n_{Ag} = 0.14 + i5.50$), gold ($n_{Au} = 0.18 + i5.39$), and copper ($n_{Cu} = 0.260 + i5.26$) have been considered [30]. And for the second case, temperature-dependent refractive index or permittivity has been taken into account for investigating the temperature-dependent Goos–Hänchen shift (GH_T). For this purpose, a famous Drude–Lorentz formulation is used, and we only take gold metal as a second medium due to its chemical stability and extensive usage in modern electronic technology [11].

In this study, a plane incident electromagnetic beam of light (wavelength $\lambda = 826$ nm and its intensity is one volt per meter (1 V/m)) travel from the medium one with refractive index n_1 , air in this scenario, to the second medium with complex refractive index \hat{n}_2 . Both temperature-dependent and temperature-independent cases have been investigated on the same geometry: difference depends upon the second medium’s dependency on temperature. Starting from the Fresnel equations, it is possible to find GH-shift through the formulation given by Artmann’s stationary phase method [6]. A basic equation for finding the GH-shift is [5,31],

$$D = -\frac{1}{\kappa} \frac{d\delta}{d\theta} \tag{1}$$

$\kappa = \frac{2\pi}{\lambda}$, represents the wave number of the medium, δ and θ show the phase of medium and the angle of incidence, respectively. Reflection coefficients for both s-polarization and p-polarizations are given by [32],

$$R_p = \frac{\hat{n}_2 \cos \theta_i - n_1 \cos \theta_t}{\hat{n}_2 \cos \theta_i + n_1 \cos \theta_t} \tag{2}$$

$$R_s = \frac{n_1 \cos \theta_i - \hat{n}_2 \cos \theta_t}{n_1 \cos \theta_i + \hat{n}_2 \cos \theta_t} \tag{3}$$

respectively. θ_i is the angle of incidence, while θ_t represents the transmission angle. Studying the optical properties of metals in deep details is not easy: therefore, we assumed Drude’s model as a good approximation in this regard [33]. Further, the metal is a complex medium or absorbing medium: therefore, it exhibits the complex permittivity, and Drude’s model gives the simple

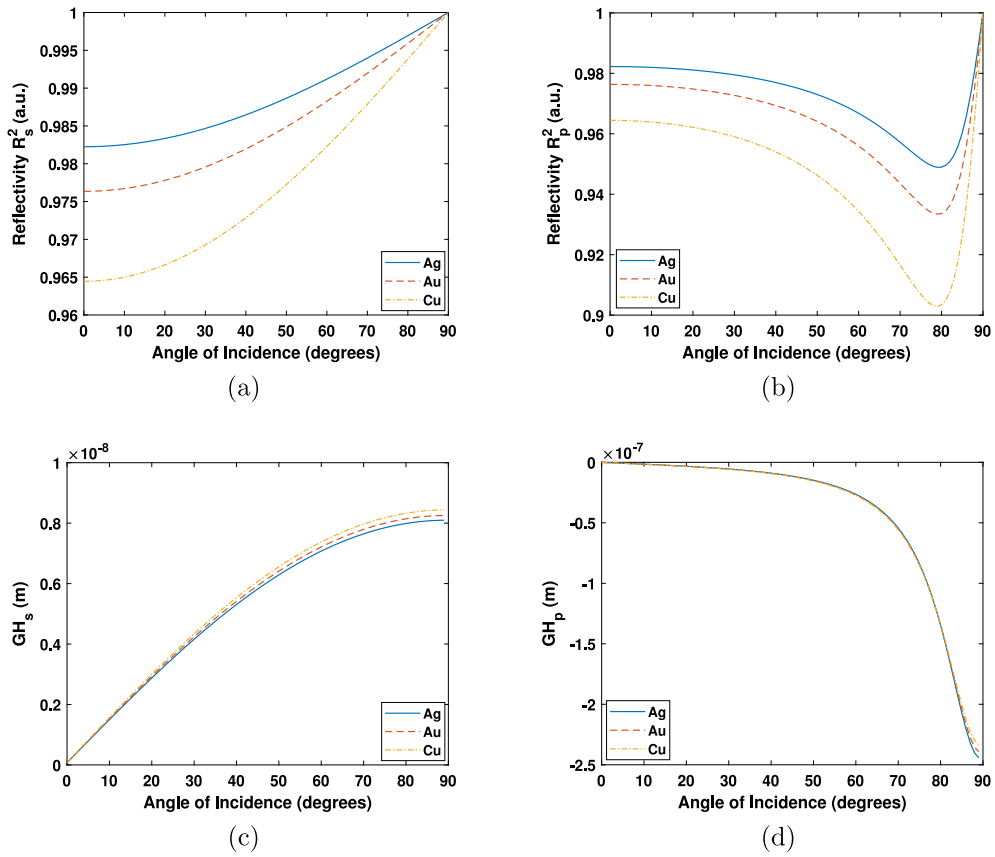


Fig. 2. Behavior of reflectivity $R_{s,p}^2$ and Goos-Hänchen shift $GH_{s,p}$ for the angle of incidence. (a) Behavior of Reflectivity for s -polarized incident beam, (b) Behavior of Reflectivity for a p -polarized incident beam; (c) GH-shift for a s -polarized incident beam; (d) GH-shift for a p -polarized incident beam.

relation for handling the complex permittivity, especially when it depends upon wavelength and temperature [9]. Therefore, Temperature-dependent permittivity as a function of temperature can be given by Drude’s model [9],

$$\hat{\epsilon}_2(T) = 1 - \frac{\lambda^2 \lambda_c(T)}{\lambda_p^2(T)(\lambda_c(T) + i\lambda)} \tag{4}$$

In the above relation, λ_p is the plasma frequency, λ_c represents collision, and incident wavelength is denoted by λ . Temperature-dependent plasma wavelength λ_p directly related to the density and effective mass of the electrons, which also depends upon temperature, given by [34],

$$\frac{1}{\lambda_p(T)} = \frac{1}{\lambda_p(T_o)} [1 + 3\gamma_m(T - T_o)]^{(-1/2)} \tag{5}$$

Here $\lambda_p(T_o) = 168.26$ nm represents the plasma wavelength of gold at the room temperature of $T_o = 300$ K. Thermal expansion coefficient of bulk metal is given by $\gamma_m = 1.42 \times 10^{-5}$ K⁻¹. For finding collision wavelength $\lambda_c(T)$, the scattering from both phonon and electron parts must be considered [9].

$$\frac{1}{\lambda_c(T)} = \left[\frac{2\pi c \epsilon_o}{\lambda^2 \sigma(0)} \right] \frac{1}{10} + \left(\frac{T}{T_D} \right)^5 \int_0^{T_D/T} \frac{z^4 dz}{(e^z - 1)} + \frac{\pi^2 \Gamma \Delta}{24 \hbar E_F} \left[(K_B T)^2 + \left(\frac{\hbar c}{\lambda} \right)^2 \right] \tag{6}$$

In Eq. (6), the first term indicates the contribution from phonon–electron scattering and the second term indicates the electron–electron scattering. Further, $\sigma(0) = 7.576 \times 10^7$ Ω⁻¹ m⁻¹ represents the DC conductivity of gold at Debye temperature of $T_D = 170$ K, and $E_F = 5.53$ eV is the Fermi energy, $\Gamma = 0.55$ represents the average amount of the Fermi surface of the scattering, and $\Delta = 0.77$ represents the umklapp scattering, $\hbar = 1.05457 \times 10^{-34}$ J s is the Plank’s constant and $k_B = 1.38062 \times 10^{-23}$ J K⁻¹ is the Boltzmann’s constant [9,34].

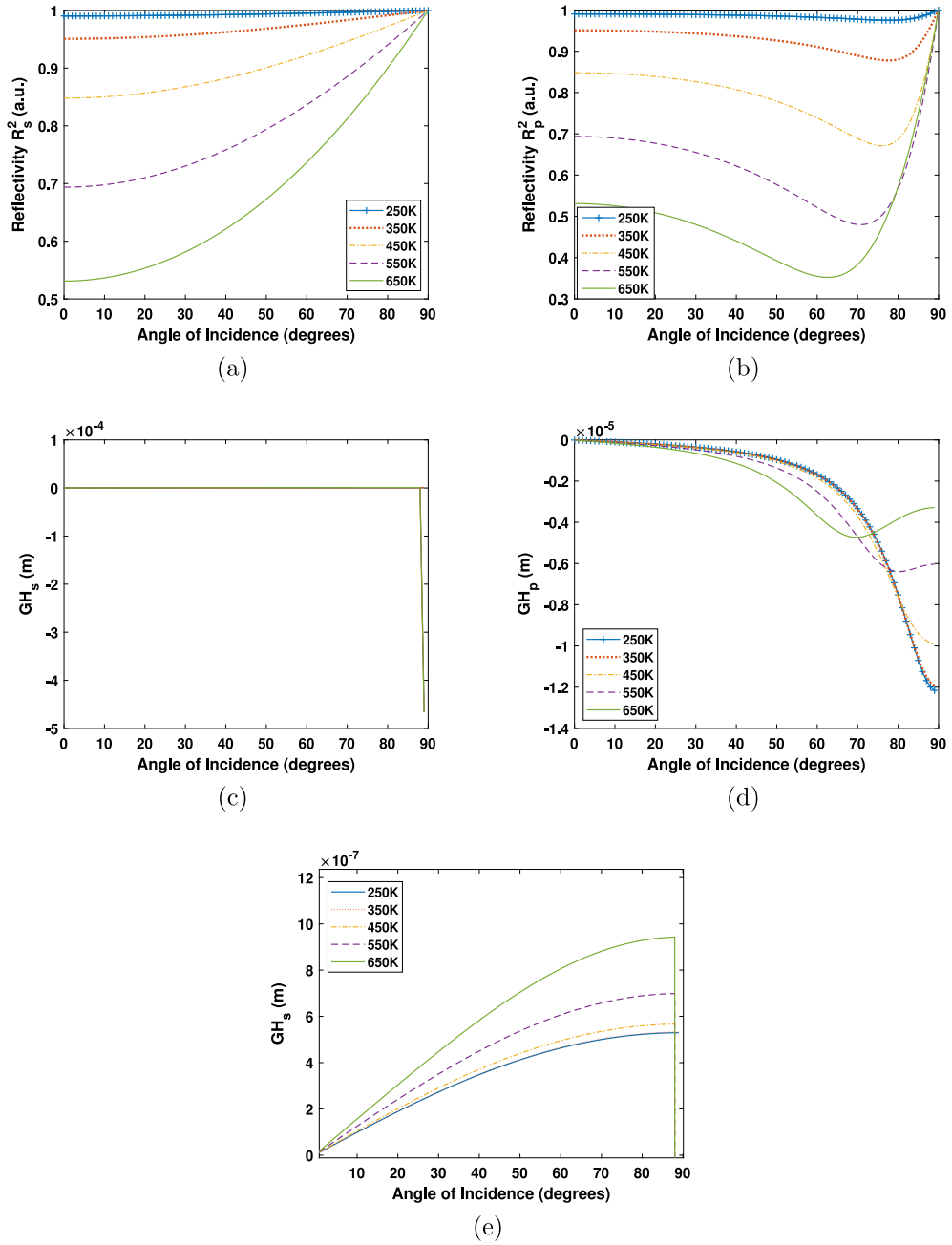


Fig. 3. Behavior of reflectivity $R_{s,p}^2$ and Goos-Hänchen shift $GH_{s,p}$. (a) Behavior of Reflectivity for a s -polarized light, (b) Behavior of Reflectivity for a p -polarized incident beam, (c) GH-shift for a s -polarized incident beam, (d) GH-shift for a p -polarized incident beam, (e) Sub-figure (zoom view of Fig. 3(c)) of GH-shift for a s -polarized incident beam.

The Phase of reflection coefficient for both s -polarization and p -polarization is given by [35]

$$\delta_p = \text{Im} \left[\ln \left[\frac{\hat{n}_2^2 \cos \theta - n_1 (\hat{n}_2^2 - n_1^2 \sin^2 \theta)^{1/2}}{\hat{n}_2^2 \cos \theta + n_1 (\hat{n}_2^2 - n_1^2 \sin^2 \theta)^{1/2}} \right] \right] \quad (7)$$

$$\delta_s = \text{Im} \left[\ln \left[\frac{n_1 \cos \theta - (\hat{n}_2^2 - n_1^2 \sin^2 \theta)^{1/2}}{n_1 \cos \theta + (\hat{n}_2^2 - n_1^2 \sin^2 \theta)^{1/2}} \right] \right] \quad (8)$$

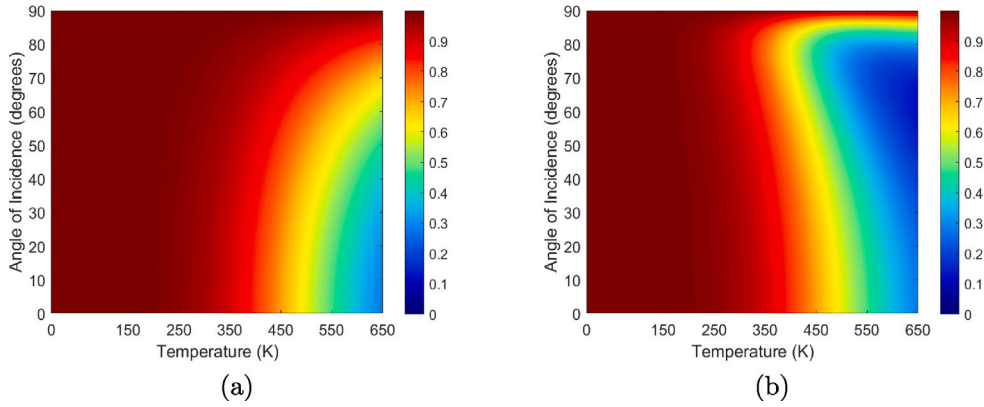


Fig. 4. Contour plots of reflectivity as a function of temperature, incident angle (vertical axes), temperature (horizontal axes), and color bar indicates the reflectivity from 0.2 (blue) to 0.9 (red), (a) Reflectivity when the incident field is s -polarized, (b) Reflectivity when the incident field is p -polarized. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

As the dielectric constant or refractive indices are complex in our case for the second medium and can be inter-convertible by using the following relation [36],

$$\begin{aligned}\hat{n} &= n + i\kappa \\ \hat{\epsilon} &= \epsilon_r + \epsilon_i = \hat{n}^2 \\ \hat{n}(T) &= \sqrt{\hat{\epsilon}(T)}\end{aligned}$$

3. Results and discussion

The basic idea behind this study is to explore the Goos–Hänchen shift when it depends upon temperature-dependent complex dielectric constant and to explore the factors which contribute to making temperature-dependent Goos–Hänchen shift different from simple temperature-independent shift. For the temperature-independent GH-shift values of dielectric constant are fixed and do not depend upon temperature, results for reflectivity and Goos–Hänchen shift are shown in Fig. 2. For the temperature-dependent GH-shift we have used temperature-dependent permittivity or complex dielectric constant as a function of temperature for a gold only, results for reflectivity and GH-shift can be seen in Fig. 3.

In Figs. 2(a) and 3(a), it can be seen that the reflectivity R_s^2 increases with the increase in the angle of incidence for s -polarization in both cases of temperature-independent and temperature-dependent permittivity, respectively. And maximum reflection has been occurred at 90° for all the values of temperature. For p -polarization, in Figs. 2(b) and 3(b), decreasing behavior of reflectivity R_p^2 can be seen with respect to the angle of incidence; however, this trend starts to increase after a specific value of the angle of incidence has been reached. In Fig. 3(b), due to temperature dependency, reflectivity decreases with the temperature (250 K to 650 K) while the angle of incidence shifts towards bigger values due to temperature-dependent parameter.

Figs. 2(c) and 3(c, e) represent the positive GH-shift for s -polarization, in both scenarios (temperature-independent case and temperature-dependent case) of Goos–Hänchen. Fig. 3(e) shows a more detailed view of Fig. 3(c). For p -polarization, negative Goos–Hänchen shifts are observed in Figs. 3(d) and 4(d). In Fig. 3(d), GH-shift shows the same trend for all three metals and its value is decreasing and has a minimum value at 90° , while in Fig. 4(d), it can be seen that for temperature-dependent Goos–Hänchen shift at lower temperatures (250 K and 350 K) it shows almost similar behavior just like in temperature-independent case, but as the temperature rises after 350 K, shift displays its deviating behavior at the specific values of angle of incidence, i.e., drops at certain angles and then starts to rise again. In addition, the maximum rise of this shift can be seen for the maximum temperature (650 K). Finally, in Fig. 4(a,b), contour plots of reflectivity (for both s and p -polarization) as a function of temperature have been shown for the better understanding of the concept of the temperature dependency on reflectivity and the angle of incidence. In this Figure, vertical axes display incident angle and horizontal axes show the rise in the temperature and color-bar indicates the reflectivity from blue to red area.

4. Conclusion

In this work, we have seen how temperature-dependent refractive index or permittivity and angle of incidence impact the reflectivity and Goos–Hänchen shift. For both polarization (s and p), reflectivity is inversely related to the temperature, i.e., an increase in reflectivity gives a decrease in the values of temperature. Moreover, GH-shifts are positive and negative for s and p polarization. This study may be helpful to develop a temperature-sensitive devices based on this shift. This shift parameter can be effectively used to control the temperature according to the requirements.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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