Lightweight Horse Saddletree Through Reverse Engineering and Lattice Structure Design

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Abstract. Additive Manufacturing (AM) is currently making the relevance of lattice structure solutions increasing, allowing the achievement of high performance/mass ratio, where performance stands for energy absorption, stiffness, and/or insulation. This paper undertakes lattice structure for lightweight design of a horse saddletree. Saddletree is the backbone of a horse saddle, and it is composed of different components. In particular, the spring steel reinforcements inside the saddletree make it the heaviest part of the horse saddle, involving also multiple processes of manufacturing and manual assemblies. This paper aims to lightweight an existing saddletree with a Voronoi lattice solution, reducing several manual assemblies. From the methodological point of view, the lightweight design has been based on a multi-scale approach, carried out via nTopology (static FEA on the original bulk design, implicit geometrical lattice generation from FEA result maps and Boolean operation among lattice results and bulk design implicit model). The original bulk design has been digitally acquired and modeled through Reverse Engineering techniques, so that a specific customized solution may be improved.
A final weight reduction of 76.5% is achieved, providing an example of how topological optimization techniques coupled with AM (in particular Powder Bed Fusion technology) may reduce assembly efforts.

Keywords: Reverse Engineering, Topological Optimization, Voronoi Lattice Structure, FEA, Horse Saddletree
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1 INTRODUCTION

Horse saddle is a horse-riding seat able to provide a firm support with comfort, safety, and dynamic balance to the rider. Typically, a horse saddle is composed of a saddletree, a seat made of hard foam and a leather cover with various holding pins. The saddletree (Figure 1) is the backbone of the horse saddle, and it acts as a shield between the horse and the rider. It uniformly distributes the load of the rider over the horse's back [9], helping to reduce the pressure points on the horse's back and enabling the horse to withstand the rider efficiently without mistreatment.

Saddletrees include metallic elements such as the stirrup bars, the gullet plate, spring steels, and cantle iron. These reinforcements are necessary to strengthen the structure from the fatigue point of view and to provide the proper stiffness with respect to the dynamic loads that may persist [17], but on the other hand, these steel reinforcements make the saddletree as the heaviest part [15] of the horse saddle. This is particularly true for the saddletrees made with laminated wood (Figure 2). However, the laminated wood provides a higher strength but, on the other hand, it reduces the durability [8] due to delamination induced by fatigue loads. In addition, the traditional design of the horse saddletree is followed by numerous manual manufacturing (handcrafts) processes and assemblies. Therefore, it entails the inaccuracies related to the standardization and geometrical symmetry which cause the life cycle problem and uneven distribution of the pressure points over the horse back, resulting in possible pain on the horse back during the riding [7,11]. In addition, the involvement of multiple processes pertaining to manufacturing and manual assemblies increases the total development time of the saddletree, according to [23], arriving to several months for developing a single saddletree model. Therefore, researchers are striving to explore an appropriate saddletree material and manufacturing technology to minimize the above-mentioned issues. Some progresses have been made, in the recent years, through molded solutions able to reduce the number of reinforcements that are necessary to strengthen the structure, using materials that, in some cases, may have better performances in terms of fatigue resistance. These materials could sound also useful in weight reduction. In [23], the reverse engineering technique was exploited successfully for the virtual prototyping of an existing traditional horse saddletree, and subsequently the traditional manufacturing and assembly processes were minimized by introducing the injection molding process with Polypropylene copolymer (PPCP grade 3530). Hence this new innovative solution achieved the desired objectives at a large extent, but it was still an expensive solution with regards to the cost and time incurred on the die prototyping, and provision of special tools.

Figure 1: Different types of English Saddletrees available in the Market [12] (highlighted in red some steel reinforcements).

This paper extends the methodology being used in [23], and it explores the saddletree lightweight design through lattice structure and the adoption of a 3D printable material suitable for Powder Bed Fusion Technology. More in details, this paper investigates how Topological Optimization (TO) techniques based on Voronoi lattice structure solution, may reduce the mass of a saddletree, obtaining a single component. The original bounding shape of the saddletree is preserved as a constraint of the design requirements. This constraint has been considered as a functional constraint and its shape has been acquired from an existing English saddletree via Reverse Engineering (RE) techniques. In the specific workflow of the lightweight design, this step mimics the customized acquisition of the functional surfaces, provided by physical or digital prototypes made according with the horse or the rider (as it happens for example, in the medical or cultural heritage applications [4,6,10,20]).

Figure 2: Investigated Horse Saddle: Saddle upper view (on the left); saddletree dissected cross section (on the right).

Lightweight design involves research in many fields such as development of new materials, innovative design criteria and solutions such as weight reduction by topological optimization [1,2,3,14]. Cellular structures (foams, honeycombs, and lattices) are excellent for producing lightweight parts with structural requirements, having the possibility to tune their parameters and maintain material only where it is required. The resulting cell pattern can be defined by optimizing the stiffness, or the energy absorption or the insulation properties over the mass reduction [13]. Voronoi tessellation is one of the approaches used in computer-aided design and graphic domains to build lattice structure with different geometric characteristics [5,18], and, in particular, Voronoi lattice structure due to the stochastic distribution behavior provides improved structural stiffness enhancing the structural strength against the external loading conditions [16]. From the general perspective of the mathematical theory, topological optimization may be approached in different ways. The most implemented are density-based approaches and level set method. Density-based approaches superimpose on the FEA elements an equivalent material model able to weight their structural relevance, so that they may be switched off, in case of low relevance. Level set method parametrizes the geometry into an implicit description of boundaries that iteratively change to minimize the optimization problem. The adoption of lattice structure introduces a mesoscopic length scale that, from the computational point of view increases the efforts. Nevertheless, its relevance in many engineering fields (bioengineering, additive manufacturing, crashworthiness, etc.) asks for proper approaches as presented in [25], where a review is presented according to a classification into full-scale and multi-scale approaches respectively. Level set methods based on level set function are intrinsically suitable to better explore the interface among different "phases", that from the structural point of view may represent different length of scale (mesoscale versus macroscale geometry) [26]. In the case of lattice structure, the pattern replication of cells is designed according to the final TO results. Density distribution may lead to multigrade materials maximizing performance/mass ratio. CAD smoothing and manufacturing constraints may improve the feasibility of the final shape design.

In this paper, we apply a multi-scale approach based on implicit geometry formulation. To achieve the desired objectives, a CAD based workflow has been followed from reverse engineering up to the design optimization stage, as presented in section 2 together with the presentation of the investigated saddletree. Then in section 3, Reverse Engineering, the digital acquisition, and post-processing of the original design are presented. In section 4, the TO set-up is illustrated and in section 5, Results and Discussion are provided. Finally in section 6, Conclusions are highlighted.

2 WORKFLOW AND CASE STUDY DEFINITION

2.1 Workflow

The applied workflow (Figure 3) integrates the following two stages: Reverse Engineering and lattice based TO, so that the final CAD model may be provided to the Design for AM stage (definition of printing orientation and possible supports, printing parameters, and post processing). In the first stage, the English saddletree is scanned for the CAD model reconstruction. In the second stage, TO is performed with subsequent lattice structure generation to produce the optimized design.

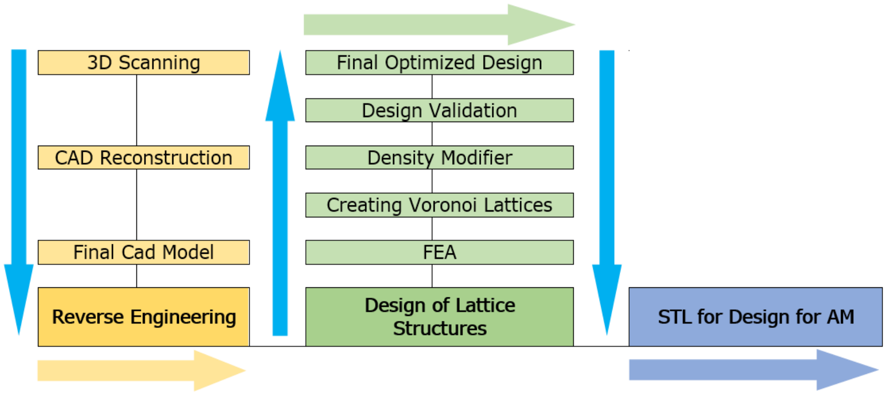


Figure 3: Workflow.

As described in the next section, the workflow starts from the Reverse Engineering step. It acquires the constrained surfaces that represent the bottom and the top part of the saddletree.

The investigated horse saddle is a traditional English saddle (Figure 2), provided by Uptons saddlery Copenhagen, Denmark (the whole research work was performed in the mutual collaboration with Thürmer Tools, Denmark). The English horse saddle has the standard size of 431.8mm and a weight of 9 kg. The weight of the overall saddletree, made by laminated wood and spring steel where necessary, is estimated to be 2 kg with standard dimensions of length= 411.0 mm, width=260.8 mm, and height=210.0 mm.

The next step concerns with the TO process. It asks for a preliminary FEA static analysis to assess the critical design issues. For the structural static analysis of the saddletree, boundary conditions are defined using a simplified equestrian jumping physics (Figure 4(a)). However, jumping over a fence (Figure 4(b)) and water (Figure 4(c)) are considered as the critical conditions from the analysis point of view, because these two conditions are those where the saddle is subjected to maximum pressure/loads.

Concerning the material, Ultrasint® PA11 black CF (Table 1) is selected, since it is a suitable material for lightweight design and Additive Manufacturing via Selective Laser Melting [24]. Ultrasint® PA11 black CF is a bio-derived powder material, commonly used for advanced applications where high rigidity, impact resistance and strength are required. in fact, this carbon-fiber reinforced material can provide optimal mechanical performances of 3D printed structures.

3 REVERSE ENGINEERING

The saddle is dissected through the middle plane (as shown in Figure 2 on the top) to expose the saddletree. Extreme care is taken to remove the leather flaps, pad, and pins so that the saddletree remains intact. All the sponge fillings are completely removed. Occipital structure scanner (Figure 5) is used for the 3D scanning. It is an IR-light projector scanner, providing very simple construction and operations with fast data acquisition phase.

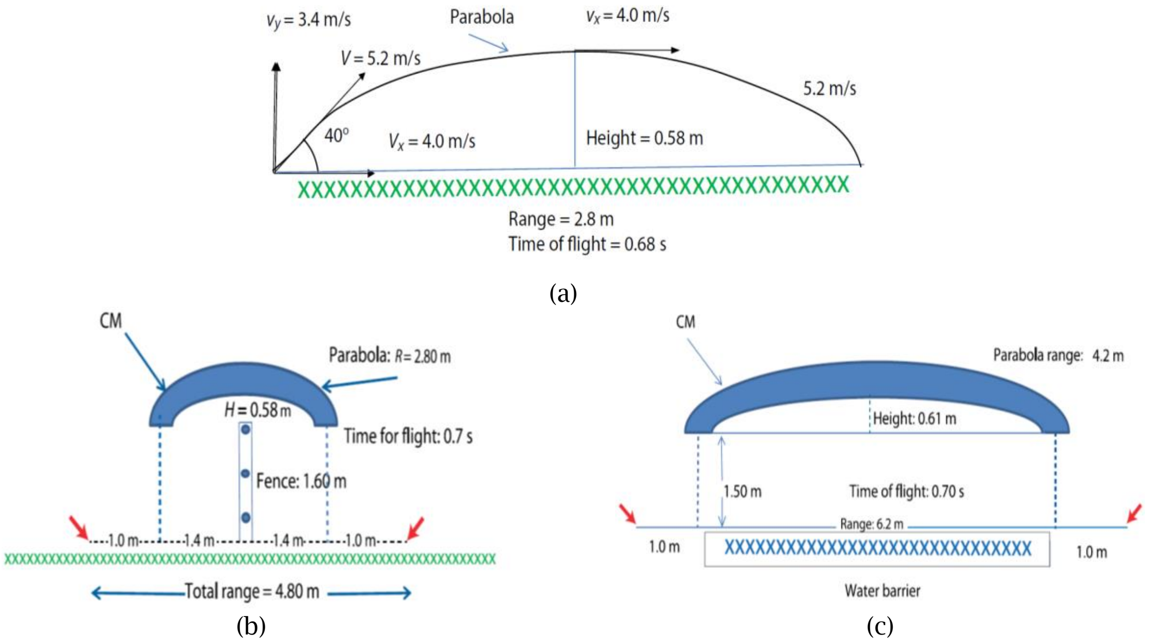


Figure 4: (a) Simplified equestrian Jumping Physics, (b) Jump over Fence, and (c) Jump over Water [21].

Case Study	*Material*	*Young's Modulus (MPa)*	*Poisson's Ratio*	*Bulk Density (kg/m³)*	*Tensile Strength (MPa)*	*Elongation at Break %*
Saddletree	Ultrasint® PA11 black CF	4500	0.43	540	82	7

Table 1: Material Properties of Ultrasint® PA11 black CF [24].

Figure 5: Occipital 3D Scanner.

The steps, reported in Figure 6, were adopted for the 3D scanning. Occipital structure scanner gives 5 preset options to scan (Table 2). The best result for the saddle scan is given by the close-range preset.

During the setting of the scene, the ground plane and the object bounding box on the screen of the scanner are adjusted to fit the entire saddletree model for a proper scan. The scan must work under homogeneous indirect light. Acquisition is made by slowly moving around the object and stopping every ten to fifteen degrees for several seconds to allow a full depth scan.

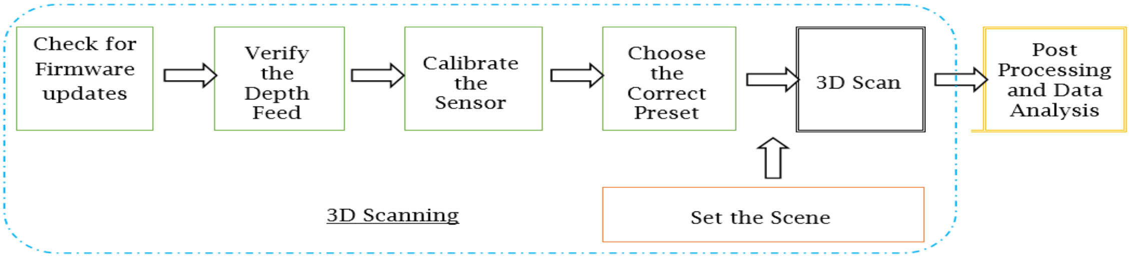


Figure 6: Reverse Engineering of the case study.

Preset type	*Ideal condition to use*	*Range*
Default	This preset is designed to fit most scanning use-cases but is not optimized for any specific use-case	Minimum Range: 570 mm Maximum Range: 10000+ mm
Body Scanning	This preset is designed to fit the body scanning use-case	Minimum Range: 360 mm Maximum Range: 980 mm
Outdoor	This preset is designed to allow the Structure Sensor Mark II to scan outdoors	Minimum Range: 570 mm Maximum Range: 10000+ mm
Room Scanning	This preset is designed to allow the Structure Sensor Mark II to best scan rooms	Minimum Range: 480 mm Maximum Range: 6210 mm
Close-Range	This preset is designed to allow the Structure Sensor Mark II to scan objects close up.	Minimum Range: 350 mm Maximum Range: 900 mm

Table 2: Preset Types for 3D Scanning.

The dissected saddle is fixed on a pole for scanning. Multiple scans are performed to get the best scan with lower average noise. Figure 7(a) shows the rendered saddletree after acquisition.

The data post processing is made using Autodesk Meshmixer (Figure 7(b)), including defeaturing of unnecessary edges, hanging supports and noise filtering. The 3D scanned model is scaled at its reference length and an optimized stl tessellation is made and mirrored to achieve the entire part. The mirrored model is checked for holes and non-manifold elements and further smoothing operation is carried out to smoothen the mesh. The final saddletree mesh has 90619 vertices and 181234 triangles. Hence, the model needs to be a solid body to proceed for the following step of Topology Optimization (Figure 7(c)).

4 TOPOLOGICAL OPTIMIZATION

Topology Optimization based on Voronoi lattice structures is performed using nTopology. Its modeling is based on implicit geometries and a multi-scale approach that is also able to support basic FEA and design for AM. The pre-processing step of the analysis includes:

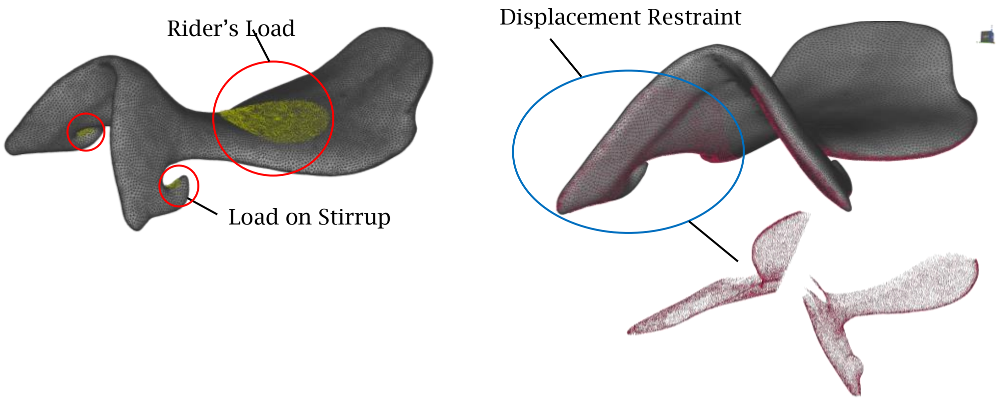
- Conversion of the CAD body into FE body. However, before conversion it is necessary to convert the model into an implicit geometry to be meshed again. From the STL mesh provided by the Reverse Engineering step, the solid mesh is obtained (adopted element: tetrahedral).

Figure 7: Postprocessing: (a) acquisition with rendering, (b) Fully Edited Saddletree, and (c) Converted Solid B-Rep Saddletree.

- Definition of material, boundary, and load conditions. Boundary and load conditions are derived from the computations reported in Section 2. There are three areas where loads are acting on the saddletree: top of the saddletree, where load is acting downward due to the rider's weight, and the left and right iron stirrups where loads are acting downward when the rider is in the standing position (Figure 8(a)). The computed load on the saddletree is 1200 N (weight of the rider) in the downward (negative Z-axis) direction. During the simulation, the full load (1200 N) is applied in the sitting position while half of this is load (600 N) is applied on each stirrup bar in the standing position. Constraints have been defined assuming that the bottom part of the pommel and the backside remain fixed.
- FEA static analysis to excerpt the input point map for the optimization (Von Mises stress results or displacement results). The map creates a field data to be converted into a lattice density and truss thickness. The field is used to edit densities, thicknesses, and other values that affect the TO.

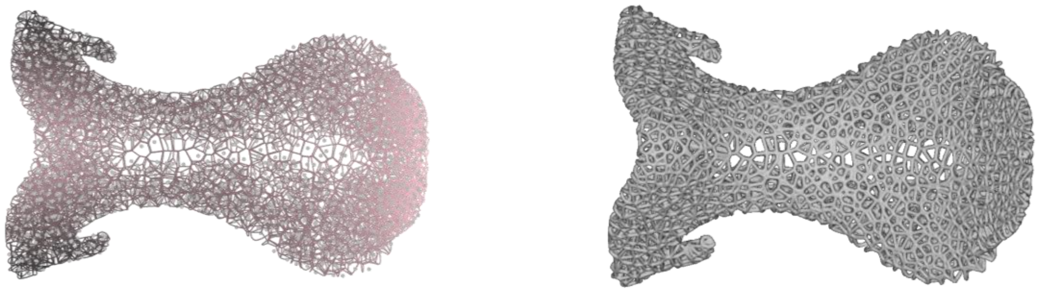
The "Voronoi Volume Lattice" generation is defined creating seed points in the volume of the original saddle, assuming a randomness value of 15. The seed spacing is filled through the density modifier modeled by a ramp operator. The density modifier defines the Voronoi lattice density according to the range of the Von Mises stress taken from the static FEA. The minimum cell distance (5 mm) is given according to the maximum Von Mises stress value, the maximum cell distance (10 mm) according to the minimum value of Von Mises stress. The field values of the map are then scaled to this range obtaining the graded geometry. A "clamp function" is used to scale the min-max and also the distances outside this range (Figure 9(a)). The next step is to assign a thickness to truss of the lattice. Figure 9(b) shows the final results after applying the ramp operator within the range [0.25÷5] mm.

By doing this, the Voronoi volume lattice has been imposed over the whole FE body, however, it is necessary to define the non-design spaces that are associated to the functional surfaces. Therefore, the original FEA volume is divided into design and non-design spaces (Figure 10(a)). Non design space includes the boundary of the saddletree where connection with the saddle is defined; the center of the seat supporting the padding foam. After this step, the Voronoi volume lattices are generated into the design space so that the final optimized design is achieved (Figure 10(b)). From the structural point of view the Non-Design Space also includes the most stress/deformed areas.



(a) (b)

Figure 8: Defining Boundary Conditions: (a) Loads, and (b) Applied Constraints.



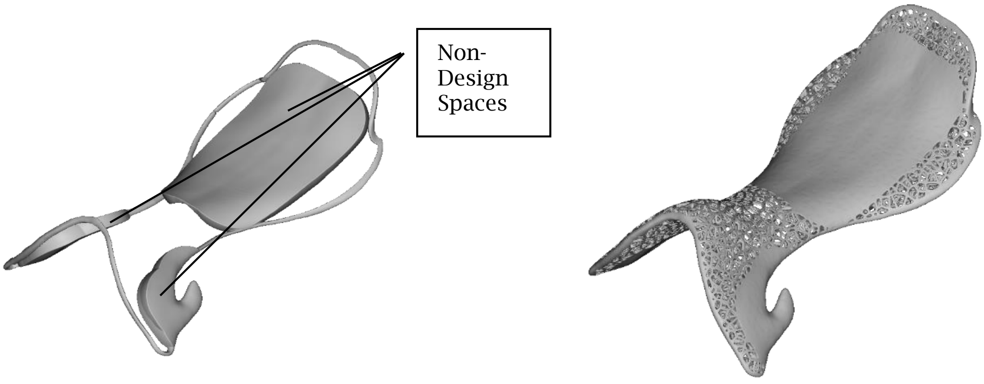
(a) (b)

Figure 9: (a) Voronoi Volume Lattices Without Thicken Beam, and (b) Thickened Voronoi Lattices.

Due to this consideration according to the fact that stochastic lattice solutions are usually able to increase the stiffness of the parts, and that the cell pattern accomplishes the stress map derived from the applied loads, the final stiffness and resistance are expected to guarantee the required performances, as also demonstrated in literature [19,22].

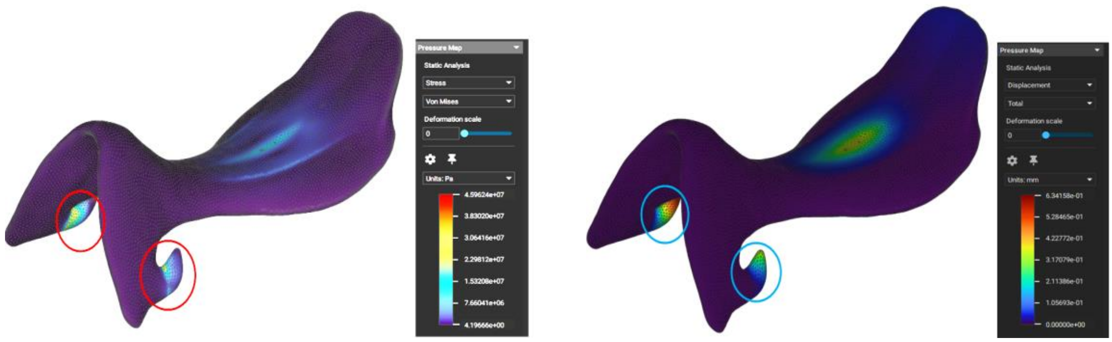
5 RESULTS AND DISCUSSION

From the numerical simulations, the maximum Von Mises stress is of 45.96 MPa and it is observed at the joints of stirrup bars (Figure 11(a)). These are the areas which are subjected to the force exerted by the rider's feet when the rider stands on the stirrup iron. The stirrup iron is connected to the stirrup bars, which have a small cross-sectional area. However, the obtained stress value is below the tensile strength (82 MPa) of the selected material, and therefore provides a safety factor of approximately 1.78. In addition, the maximum total displacement of 0.634 mm is observed at the tip of the stirrup bars on both sides of the saddletree (Figure 11(b)). Similar results are obtained by adopting the Displacement Point Map as field data and also through different order of magnitude for seed randomness.



(a) (b)

Figure 10: (a) Design and Non-Design Spaces, and (b) Final Optimized Design.



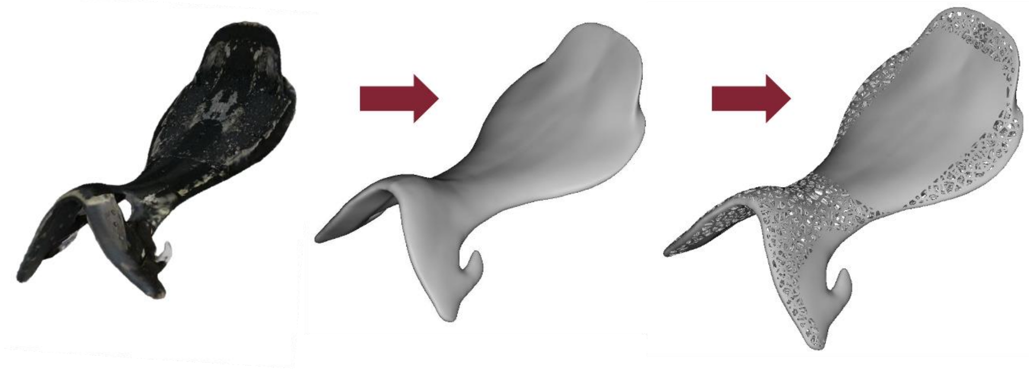
(a) (b)

Figure 11: Initial Design: (a) Highlight of Maximum Von Mises Stress, and (b) Highlight of Maximum Total Displacement.

From the lightweight design point of view, the final optimized design results to have a mass reduction passing from 2 kg (original saddletree) to 0.47 kg (final optimized design). Thanks to lattice solution, this represents a mas reduction of more than 75% in comparison to the saddletree made by injection molding. Power Bed Fusion should be applied with the selected material.

From the methodological point of view, the adoption of 3D acquisition of the starting CAD model confirms the reverse engineering versatility to support shape customization in part design. Then, the adoption of a TO through nTopology allows to tune the lattice generation based on Von Mises distribution maps used to constraint the seed spacing and the cell strut thickness. Doing so, the problem is divided into steps with different significant length scales, from the part length scale of the FEA to the meso-scale of the lattice generation. Functional surfaces are used here as non-design space where bulk material is maintained, though Boolean operations between the lattice structure and the original implicit models.

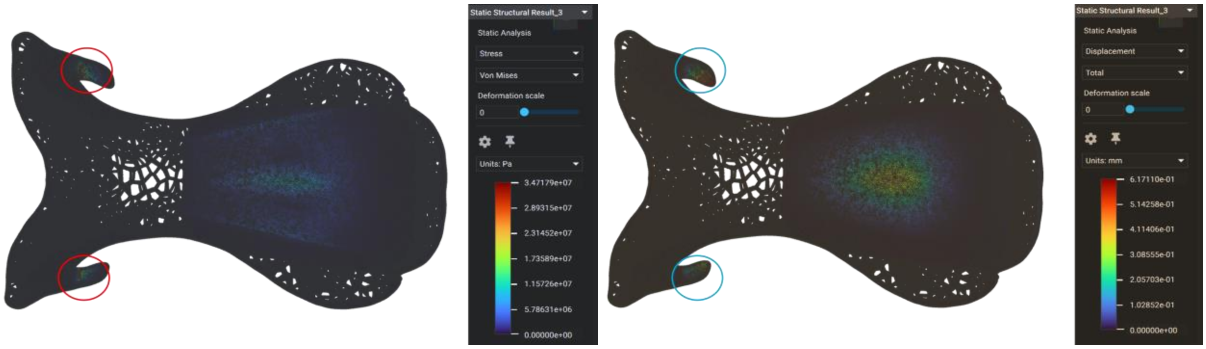
Moreover, specific simulation of the achieved stiffness (Max Force/Max Deflection) can be carried out and compared with reference data, in different riding conditions. Finally, a proper design for AM will be investigated in the next, in association to a sensitivity analysis to different cell typologies.



(a) (b) (c)

Figure 12: (a) Scanned Saddletree, (b) Fully Edited Saddletree, and (c) Final Optimized Saddletree.

FEA of the optimized design is carried out for the design validation purpose. The FEA results shows that the maximum Von Mises stress is 34.7 MPa, in the same areas where it was observed in the FEA of the initial/original design, i.e., at joints of stirrup bars (Figure 12(a)). Similarly, the maximum total displacement of 0.617 mm is observed in the same areas (Figure 12(b)) as found in case of the initial design.



(a) (b)

Figure 13: Optimized Design: (a) Highlight of Maximum Von Mises Stress, and (b) Highlight of Maximum Total Displacement.

However, it is observed that both the maximum Von Mises stress and maximum total displacement are reduced with the optimized design which improves stiffness of the saddletree, as well as it increases the factor of safety (FoS) and provides a fail-safe design. Summary of the FEA and optimization results is shown below (Table 3 & 4).

Design Status	*Maximum Von Mises Stress (MPa)*	*Maximum Total Displacement (mm)*	*FoS*	*Increase in FoS (%)*
Initial Design	45.96	0.634	1.78	32.58
Optimized Design	34.72 (-24.5%)	0.617 (-2.7%)	2.36	

Table 3: Summary of the FEA Results

Saddletree Status	*Material*	*Mass (kg)*	*Final Mass Reduction (%)*
Original Saddletree	Laminated Wood and Steel Reinforcements	2	76.5
Optimized Saddletree	Ultrasint® PA11 black CF	0.47	

Table 4: Summary of the Optimization Results.

CONCLUSIONS

The case study demonstrated the feasibility of the lattice structure lightweight design of the saddletree based on Voronoi volume lattice. The new optimized design preserves the original shape of the saddletree, as required. Adopting a bio-derived powder material (suitable for both dynamic performance and AM) for the optimized design of the saddletree, around 76.5% mass reduction along with a FoS of 2.36 are obtained successfully, thanks to the Voronoi lattice structure. The obtained structure also accomplished the stiffness requirements under operative load conditions, thanks to the stochastic pattern with graded density, that was defined trough the Von Mises stress map. Hence, the final optimized design is a monolithic structure, eliminating several manual assembly processes. In addition, the manufacturing process will be based on Power Bed Fusion (PBF) additive technologies, which will replace the conventional manufacturing processes (previously used) with a single process without requiring any special tools. This may provide a significant reduction in terms of manufacturing time and costs maintaining or improving the performances in terms of stiffness.

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