Crystal slow extraction of positrons from the Frascati DAΦNE collider

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The slow high-efficiency extraction from a ring positron accelerator (SHERPA) project's aim is to develop an efficient technique to extract a positron beam from one of the accelerator rings composing the DAΦNE complex at the Frascati National Laboratory of INFN, setting up a new beam line able to deliver positron spills of O(ms) length, excellent beam energy spread and emittance. The most common approach to slowly extract from a ring is to increase betatron oscillations approaching the third order tune resonance to gradually eject particles from the circulating beam. SHERPA proposes a paradigm change for lepton machines using coherent processes in bent crystals to kick out positrons from the ring, a cheaper and less complex alternative. A description of this innovative nonresonant extraction technique is reported in this manuscript, including its performance preliminary estimation.

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I. INTRODUCTION

The positron annihilation into dark matter experiment (PADME) [1,2] has been designed to search for a new kind of dark sector light particle, like a "dark photon" or an axionlike mediator, seen as a peak in the missing mass spectrum of monophoton events in $e^+e^- \to \gamma X$ annihilations of positrons on target. Very high luminosity is achieved in a fixed-target collision scheme, albeit at the price of a reduced center-of-mass energy below 20 MeV, exploiting the positron beam coming from the LINAC [3] of the DAΦNE complex.

The DAΦNE [4,5] Φ-Factory is an electron positron collider at the ϕ meson resonance center-of-mass energy (1.02 GeV), where the beams are stored in two main rings (MRe, MRp) a hundred meters long. A LINAC with electron and positron source and a small damping ring, which reduces the emittance of both beams, is used for the injection.

The LINAC pulses can also be diverted to a separate transfer line where secondary beams can be produced on a dedicated target, serving a beam test facility (BTF) with two experimental areas for high and medium-low intensity applications [6,7]. A schematic layout from the end of the LINAC is shown in Fig. 1.

From 2018 to 2020, PADME has taken data using secondary and primary positrons in the BTF-1 beam line; the main limitation to the sensitivity of the experiment comes from the maximum tolerable rate in the veto detectors, thus requiring to increase the positron beam bunch length as much as possible.

The DAΦNE LINAC was configured to produce pulses as long as 300 ns [8], more than a factor 10 longer than the design 10 ns required for injection into the collider, at the expense of a lower maximum accelerating field and higher energy spread; thus, beam pulses in the 0.43 to 0.49 GeV energy range were produced.

The maximum tolerable rate for the PADME detectors is of the order of 10^2 positrons/ ns, so that with 300 ns long pulses the maximum positron population cannot exceed 3×10^4 positrons/pulse.

With such a positron beam density, a maximum of 1.5×10^{13} positrons on target (POT) can be reached in one year of operation, given the LINAC maximum repetition rate of 50 Hz. In order to get a significant increase in the reach of the experiment, it would be then necessary to further extend the duration of the LINAC pulses.

Increasing the particle density will produce a random veto level so high to cause a signal acceptance reduction greater than the corresponding luminosity gain [1].

As an alternative to LINAC modifications [9], it has been proposed to use one of the rings of the DAΦNE complex as a pulse stretcher [10].

Two options were put forward: one third of integer resonant extraction [11] and *ultraslow extraction* using coherent effects in bent crystals. The latter is the object of this paper, in which we report on the studies aiming at

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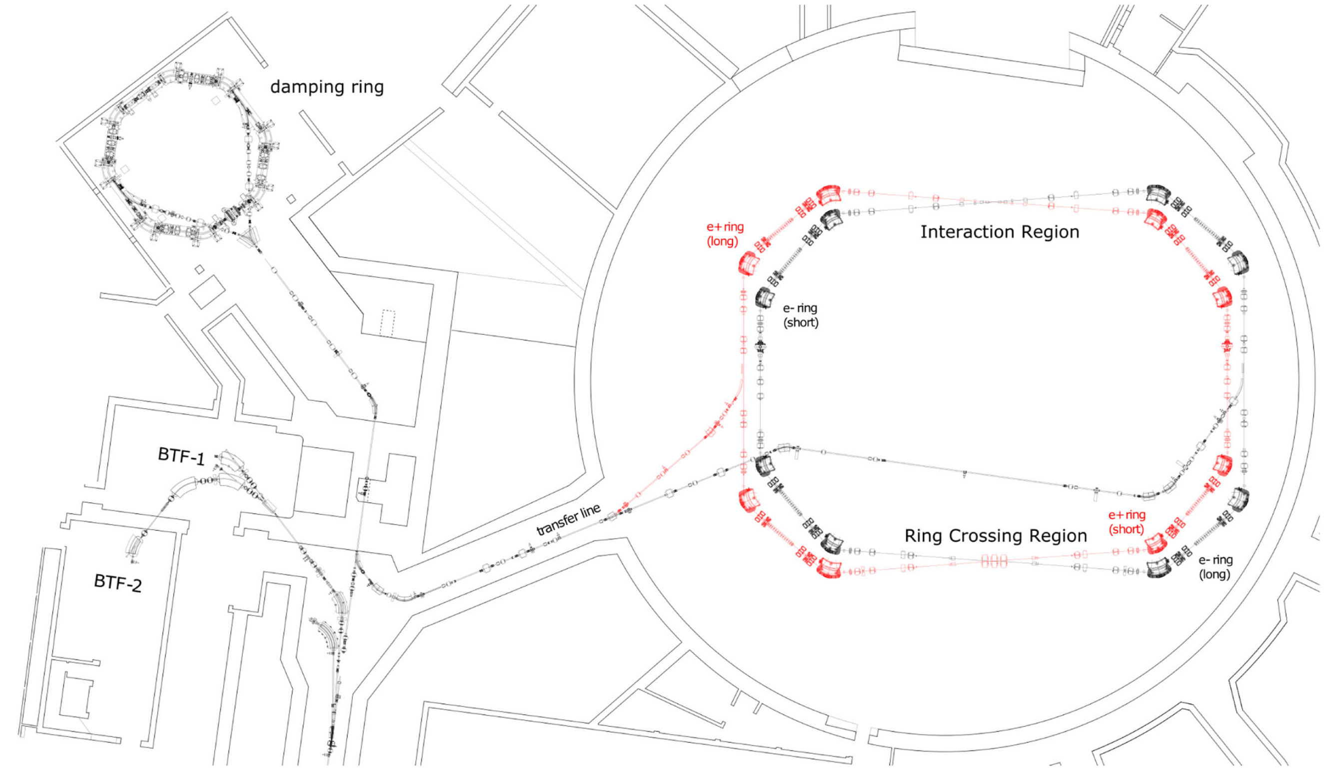

FIG. 1. DAΦNE complex schematic layout: the beam from the LINAC (bottom left) can be driven either to the BTF-1 and BTF-2 lines (middle left) or to a damping ring (top left) and from there injected counterclockwise into the electron main ring or clockwise into the positron one (in red).

increasing the statistics of a PADME-like experiment by at least 3 orders of magnitude, i.e., extending the positron beam duration in the hundreds of μs range. The sensitivity of PADME to dark sector candidates is currently limited in the range of couplings $\epsilon \sim 10^{-3}$ with order 10^{13} POT. Being the sensitivity background dominated, it will scale like $\sqrt{N_{\mathrm{POT}}}$, i.e., a factor of ~30, moving the sensitivity down in the range of $\epsilon \sim 10^{-4}$.

Different options have been studied, also considering technical and practical aspects aiming at the design of a realistic and efficient implementation.

II. THE SHERPA EXTRACTION PROJECT

The *ultraslow extraction* of a particle beam is performed by the turn-by-turn extraction of a small portion of the bunch population that is stored in an accelerator ring.

Slow high-efficiency extraction from a ring positron accelerator (SHERPA) is aimed at studying the possibility of ultraslow positron extraction, aided by a bent crystal, from one of the rings composing the DAΦNE complex: the damping ring (DR) or the positron main ring (MRp).

The crystal extraction is based on the possibility to deflect charged particles using the channeling effect in bent single crystals. High-energy charged particles impinging on the crystal with small angles relative to the lattice planes move oscillating between two neighboring planes, and consequently can be deflected by the crystal bend angle [12].

This process and its application for slow extraction from the ring accelerator have been studied and experimentally proved during the past decades. In particular, at the U-70 Institute of High Energy Physics (IHEP) Russian synchrotron [13–15] and at the European Organization for Nuclear Research (CERN) Super Proton Synchrotron (SPS) [16–19] crystal nonresonant slow extraction has been successfully obtained for high-energy hadrons.

Applying the same technique in DAΦNE is challenging, but anyway possible. For sub-GeV leptons it is necessary to use ultrathin crystals (~20 μm), instead of some mm, to reduce the interactions of crystal electrons on channeled particles pushing them out of the lattice potential well ("electron dechanneling").

We have obtained very promising preliminary results in simulations which have foreseen few modifications at the current DAΦNE complex configuration: first, the inclusion of a crystal, and the structure to hold it and move it, into one of the vacuum chambers where the beam is circulating; second, the modifications of the injection and extraction lines and of the lattice to optimize the extraction efficiency and the parameters of the extracted beam. In case the main ring is used the installation of an extraction septum is also needed.

Based on previous experimental results of particle channeling through bent crystals with electrons at about 1 GeV, reported in [20,21], the best performance of a silicon bent crystal of 30 μm thickness along the beam direction is about 1 mrad of deflection. The *channeling efficiency*, defined as the percentage of particles deflected by channeling with respected to the total population, was measured to be at the level of ~20% for electrons, and according to channeling theory efficiency for positrons is expected to be even higher. All the other crystal parameters are reported in [20,21].

The optical parameters of the rings were modified and tuned in simulations, using MAD-X [22], to allow the circulating beam to interact with the crystal and deflect particles for the slow extraction, while still allowing its injection and storage.

A standard solution is the "local extraction," in which the crystal and the extraction point are separated by a drift and the particles are directly kicked in the septum by the crystal without crossing any magnetic elements. It was discarded because 1 mrad of deflection would need a long free space, not available in the DAΦNE rings, to separate the extracted beam from the circulating by some millimeters and allowing it to enter in the septum (e.g., 10 mm = 10 m × 1 mrad).

Instead, the most promising scheme was found in the so-called "nonlocal" crystal extraction: a deflection is imparted by a crystal at one point of the ring starting an oscillation and allowing particles to reach a septum, with the adequate transverse displacement, in a later point of the ring. Positrons could also encounter the crystal multiple times, getting kicks or be lost. In fact, if not channeled, their energy loss through the crystal is negligible (~KeV) with respect to the machine energy acceptance.

In the nonlocal extraction, the transverse displacement Δx_2 at point 2 (the septum location) due to a kick $\Delta x'_1$ given at point 1 (the crystal location) can be calculated from the equation

$$\Delta x_2 = \sqrt{\beta_1 \beta_2} \sin(2\pi\Delta\mu)\Delta x'_1, \qquad (1)$$

where β_1 and β_2 are the optics Twiss β functions at the locations 1 and 2, $\Delta\mu$ is the phase advance between points 1 and 2 (in 2π units), and $\Delta x'_1$ is the deflection produced by the crystal. Equation (1) can be seen as the displacement produced by the propagation of a kick given the linear optics transport element R_{12} as explained by Wiedemann in [23]. Here we are considering only the effect of the crystal, a change of angle without change of transverse position. In order to have maximum displacement Δx_2, the Twiss beta functions should be as large as possible and $\Delta\mu = \frac{1}{4}$.

In the following we describe the simulation and results obtained using the DAΦNE rings model for the two main options: using the smaller damping ring (Sec. II A) and extracting from one of the two main rings (MRp in particular, Sec. II B).

A. Beam extraction from the damping ring

The DR could provide an adequate extracted beam quality and could be considered as a good option because its structure is already used for beam extraction therefore requiring simpler modifications. In particular, the present extraction septum and the connected beam line can be used.

Particle bunches with a maximum population of 10^{10} electrons or 10^9 positrons are accelerated by the LINAC to 510 MeV and injected at 49 Hz into the 30 m long DR where the beam emittance is reduced by synchrotron radiation emission with a damping time of about 20 ms. Then bunches are extracted at 2 Hz and transported to the DAΦNE MRs for injection. For DAΦNE the bunch duration is about 10 ns: for the slow extraction it can be increased up to about 100 ns filling the DR circumference.

The proposed crystal location is shown in Fig. 2, together with the extraction septum, which is the same used to extract the beam towards the DAΦNE main rings.

We use the MAD-X model of the DR and the PTC tracking [24] libraries in MAD-X to perform multiturn particle tracking for a small sample of the particle population, varying in the order of 10^2 to 10^4 particles.

For the initial simplified model we take into account the particle energy loss per turn and the beam pipe aperture to determine the time and location of particle losses, while changes in the beam aperture are used to model the movement of the crystal inside the vacuum chamber.

Particle tracking and theoretical estimations show that positrons on axis, with an energy offset of −1.0%, with respect to the nominal one, will arrive at the crystal location with a horizontal displacement of −7.5 mm.

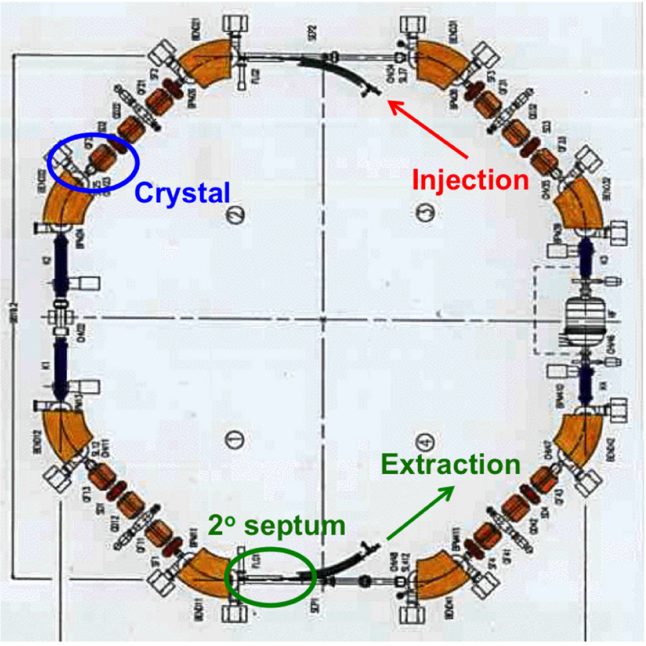

FIG. 2. Accumulator ring, location of the bent crystal and extraction.

Those particles could interact with the crystal and thus be kicked by 1 mrad, producing a larger horizontal displacement of about −11 mm at the septum position. In order to effectively achieve the slow extraction of those positrons it is necessary to adjust the initial position of the beam, the crystal position and the ring parameters.

The crystal will be placed on a remote controlled handling system able to move it along the horizontal axis and optimize its angular orientation with respect to the circulating beam. Once the machine and beam parameters have been set, the crystal will be aligned and left in place during extraction, new injections or refilling of the accelerator. In fact, the crystal position does not significantly reduce the ring acceptance.

Since positrons in the DR lose 1.0% of energy by synchrotron radiation in 1000 turns, if the energy spread of the injected beam is ±1%, particles with −1.0% are extracted soon after injection and particles with +1.0% are extracted after 2000 turns, therefore the spill duration is about 200 μs, given that the rf radio frequency (rf) is kept off, thus allowing positrons to lose naturally energy and "diffuse" until they reach the crystal.

In order to improve over the initial result, the optics Twiss functions have been modified to obtain a larger β_x at the crystal and septum locations, while at the same time achieving a phase advance difference of 0.875 (in 2π units) between the two. Figure 3 shows the Twiss functions, including the approximate location of the crystal and the extraction point.

We start the simulations considering a very small emittance, of 0.1 mm mrad, in order to remove the effect of the beam size on the particle distribution. The energy spread of the beam is uniformly distributed between ±1%, which is the energy acceptance of the ring.

Considering the septum located at −20 mm away from the beam pipe center, we bring the beam close to the septum by adjusting the kicker for off-axis injection. The injection oscillation brings the beam close to the septum, which is 2.5 mm thick, and the crystal provides the additional jump to pass the septum thickness. Figure 4 shows the distribution in the horizontal (H) phase space of particles at the extraction septum when they are injected 10 mm away from the beam pipe center and tracked for 18 turns.

In the following we proceed to evaluate the effect of the bent crystal on the beam position at the extraction septum, using a beam deflection of 1 mrad, in agreement with the experimental results of a silicon crystal ~30 μm thick [20,21].

In the simulations this is equivalent to select from every turn the particles that have reached the transverse offset allowed by the crystal position, then add them a 1 mrad kick and track the particle to the extraction septum. Once they reach the septum they are no longer considered in the simulation.

As shown in Fig. 5 the effect of the crystal at the first turn is to displace the particles by about 4.0 mm at the extraction septum. The position of the crystal is chosen in order to interact with the particles which are very close to the extraction septum in order to extract them with the 1 mrad kick. The crystal is placed at −30 mm from the beam center in the horizontal plane.

In Fig. 6 the phase space of the same particles seen at the crystal location is shown at the first turn and at turn 500. The particles are tracked until they reach the position of

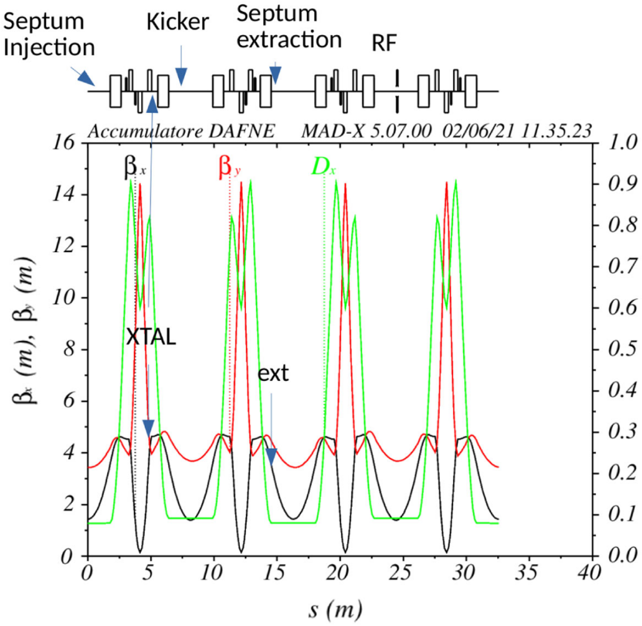

FIG. 3. One possible configuration for the optics Twiss functions of the DAΦNE DR to allow crystal slow extraction.

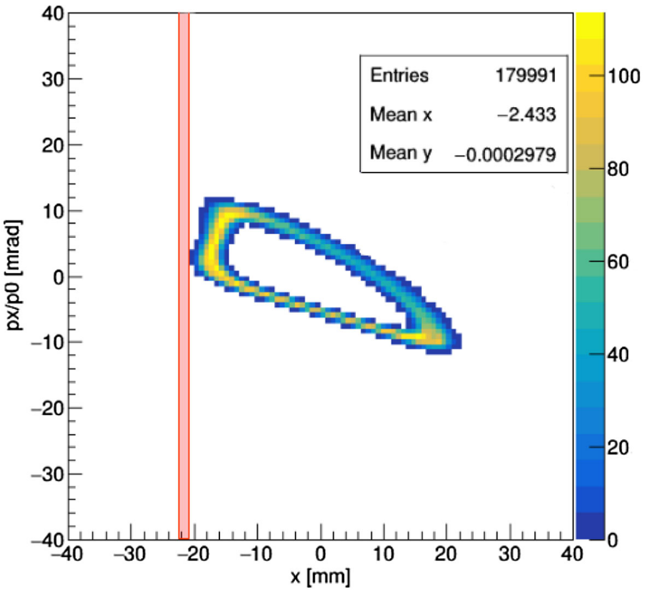

FIG. 4. H phase space plot at the extraction septum of particles injected with a +10 mm offset from beam pipe center and tracked for 18 turns. The extraction septum is represented by a red region.

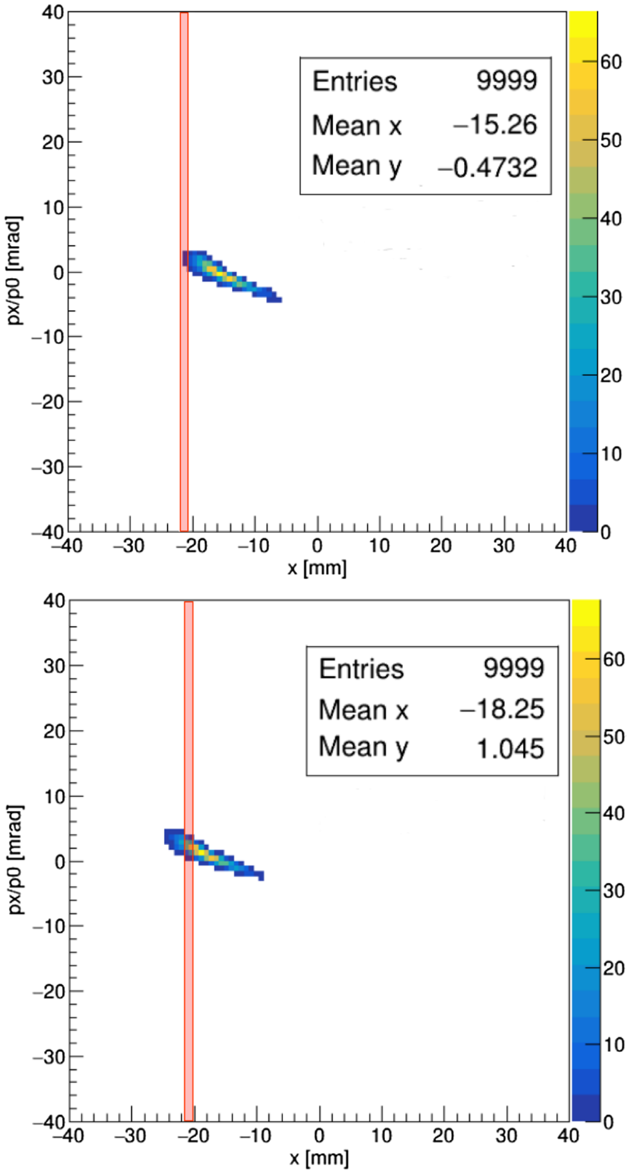

FIG. 5. H phase space plot at the extraction septum of particles injected with a +10 mm offset from beam pipe center and tracked for 1 turn. Top: particles do not receive a kick by the crystal. Bottom: particles receive a 1 mrad kick by the crystal. Particles close to the septum are moved out of it, in the extraction channel.

−20 mm or more at the extraction septum. After ∼ 500 turns the first particles pass through the bent crystal receiving the 1 mrad kick.

From the turn 500, the rate of particles passing through the crystal at −30 mm and achieving a displacement of −20 mm or more at the extraction septum is plotted in Fig. 7. There is a peak in the rate at the beginning, which can be reduced by further optimization of the initial conditions. After the first peak, the rate of particles is rather stable up to about 1500 turns after the interaction with the crystal.

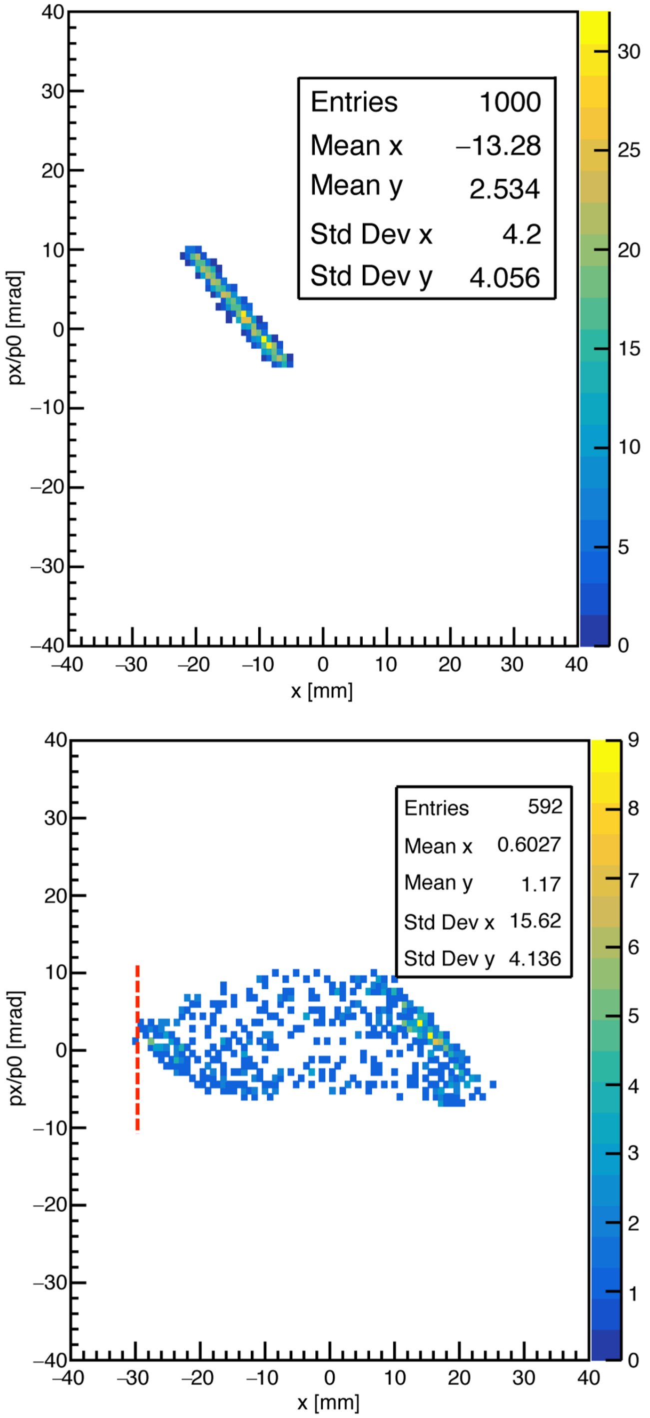

FIG. 6. H phase space of particles achieving −20 mm of displacement, or more, at the extraction point plotted at the crystal. Top: turn 1. Bottom: turn 500. Particles migrate to negative horizontal offsets due to dispersion and energy loss up to go through the crystal (represented by the dashed red line in the plot).

The future program is to repeat the simulations using the full beam size of the injected beam and optimizing parameters as the ring tune, injection kickers, crystal

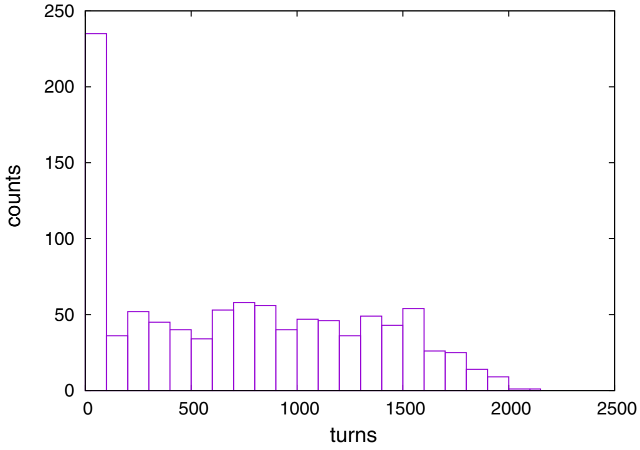

FIG. 7. Rate of particles kicked by the crystal achieving −20 mm of displacement, or more, at the extraction point. The number of particles is shown on the vertical axis, while the number of turns after the first interaction with the crystal is shown on the horizontal axis. The initial tracked population is 10^3 particles and each turn corresponds to 0.1 μs. Each bin corresponds to 100 turns.

location and distance from the axis. Then a simplified simulation of the crystal interaction has to be added in order to evaluate how many particles receive a kick by the crystal. These particles are tracked until they are extracted or lost. Particles that do not receive a kick are tracked in the following passages at the crystal until they are extracted or lost.

B. Simulation studies of the DAΦNE collider positron beam extraction with a bent crystal

The DAΦNE collider is composed by two rings and the structure of both main rings is similar. Any of the two has great flexibility due to the numerous devices (quadrupoles, kickers, beam monitors, sextupoles, etc.) to manage and control the beam parameters for an optimized beam quality in terms of intensity, emittance and rate of extraction. However, this option requires greater efforts in terms of human resources, design, installation, commissioning and operation. Differently from the accumulator ring options, the minimum set of modifications indeed includes the installation of the extraction septum and a suitable transport line.

Starting from the DAΦNE collider configuration used for the SIDDHARTA2 (2019) [5] experiment, we have modified the optics model, finding different promising options for the crystal nonlocal extraction.

The best configuration is the one with the crystal positioned before the rings crossing point (IP2), shown in Fig. 8, and the extraction septum placed downstream the IP2. The extraction would be thus performed only a few meters downstream the crystal, along the same straight section of the MRp ring.

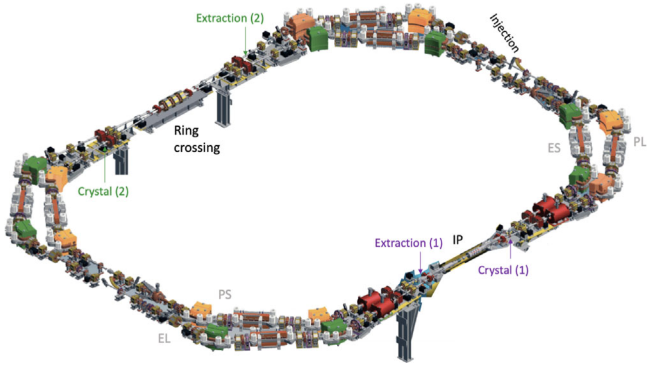

FIG. 8. Nonlocal extraction scheme locations using the DAΦNE positron main ring; particles are injected in the long external arc (PL) and circulate clockwise: in option (1) the crystal and the septum are placed upstream and downstream the main experiment interaction point, in case (2) they are placed around the ring crossing region on the opposite side.

In this configuration, a positron with an energy offset of the order of −0.7% will encounter the crystal, positioned at 8 mm from the circulating beam axis, at the sixth turn in the machine, and will be extracted in the same turn, as shown in Fig. 9.

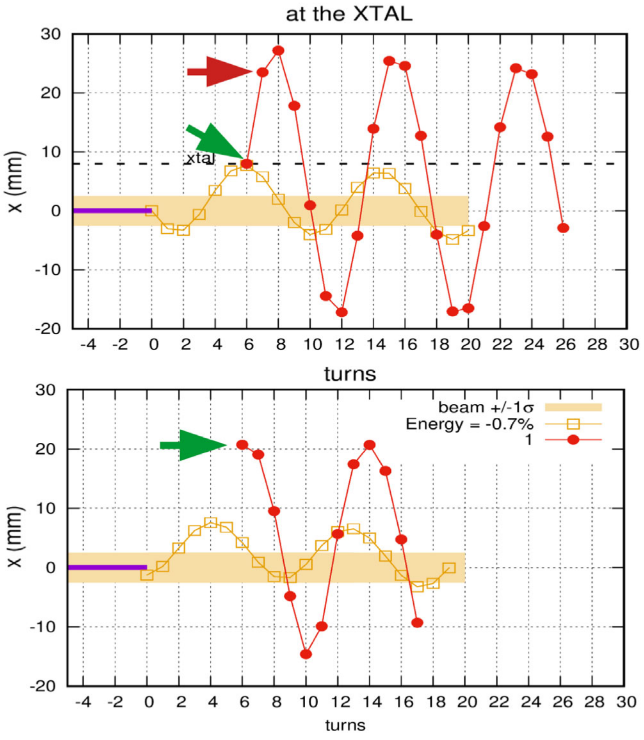

FIG. 9. Turn by turn single particle tracking results seen at the crystal location (top) and the extraction septum (bottom). The particle trajectory is initially set to zero position and angle and at turn zero it loses 0.7% of its energy. At turn 6 it interacts with the crystal and it reaches 20 mm of displacement at the septum. If not extracted, the particle continues to oscillate with a large amplitude.

Due to the large horizontal beta β_x of about 17 m both at the crystal and septum locations, the transverse displacement obtained at the extraction point is about 20 mm, enough to reach the extraction side of the septum magnet. This example shows that the 1 mrad kick given by crystal is sufficient for particle extraction. In order to slowly displace the particles on the crystal position different methods can be used; the simplest one is to exploit the energy loss due to synchrotron radiation, turning off the radio frequency cavity, and placing the crystal in a dispersive region. Another possibility is to excite the beam with a band limited white noise applied to one of the ring horizontal kickers, increasing slowly the horizontal beam size. Further studies are needed to choose the best configuration, to optimize all the optical parameters and to evaluate the expected extraction rate.

In any case, this preliminary study already shows that it is possible to slow extract positrons from the actual DAΦNE ring using a bending crystal with a realistic deflection, i.e., already achieved with a lepton beam of comparable momentum.

III. FUTURE WORK AND CONCLUSIONS

The crystal extraction solutions proposed here have to be optimized in terms of injection and optics parameter, rf tuning, crystal position and septum features to obtain the best result in terms of extracted beam quality.

In particular the extracted beam main parameters to be optimized are spill length, intensity, emittance and energy spread.

For the main ring configuration, a resonant solution has been also investigated [11] and it could be implemented using the crystal in place of the electrostatic thin septum. This would make the extraction more efficient since the crystal thickness is much smaller than a feasible electrostatic septum and its installation is certainly simpler and less expensive. This option is still under study.

Also for the DR, a resonant solution and a hybrid “resonant-nonresonant” crystal extraction is under investigation. Since the DR is already equipped with an extraction septum, the insertion of the crystal holder in the vacuum chamber is the only hardware modification needed to extract the beam, allowing to perform a complete experimental study of the slow extraction varying all the beam and ring parameters.

It is important to emphasize that all the results reported above have been obtained considering a 100% channeling efficiency of the crystal, with respect to ~80% expected by theory and very preliminary simulations for positrons. As a consequence, the extraction efficiency and the expected rate are a little bit overestimated.

Similar results are also expected for electron beams, albeit with an expected lower crystal deflection efficiency. In fact, experimental results and simulations demonstrated a crystal channeling efficiency of ~20% [20,21].

Also in the worst scenario of extraction with a shorter beam spill lengths of 100 μs and very low positron extraction efficiency of 5%, this solution would improve, for example, the PADME sensitivity by a factor between 50 and 100 with respect to the plain LINAC beam [25].

Monte Carlo and analytical simulation studies, especially on the particles behavior in the crystal, are ongoing with the aim to adapt the existing codes optimized for high-energy hadrons also to sub-GeV leptons. The data that will be collected characterizing the SHERPA crystals will be also used for simulation benchmark.

Particle loss and machine protection studies are foreseen in the near future with the aim to optimize the final extraction efficiency and taking under control machine component activation or damages.

Concerning crystal radiation damages, several previous studies performed with high energy and high intensity hadron beams [26–29,29] guarantee the crystal reliability in terms of performance and operational lifetime.

If SHERPA will succeed in its final objective of achieving the crystal-assisted extraction, the very first sub-GeV primary positron slowly extracted beam will be delivered. This would open the possibility of managing low-energy positron and electron beams in a small storage ring. The possibility to extract high-quality spills of O(ms) will guarantee new scenarios for several fixed-target positron experiments: reducing the pileup, the background, the energy spread, their sensitivity will greatly increase the physics reach of PADME-like experiments. Moreover, the study of positron beam steering using bent crystals will provide a know-how that can be applied, in the near future, for several accelerating machine aspects, like beam collimation, extraction and splitting, contributing to a general improvement in the particle accelerator field. For the DAΦNE complex, this project could be an important chance to upgrade its performance, widening its use-cases in a different research line, also for fundamental physics.

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