Ph.D. Dissertation

Essays on Macrodynamic Theory, Historical Time and Climate Change

by

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"Omnia, Lucili, aliena sunt, tempus tantum nostrum est; in huius rei unius fugacis ac lubricae possessionem natura nos misit, ex qua expellit quicumque vult."

Seneca, Epistulae morales ad Lucilium I

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This dissertation is dedicated to all of you.

Avanti Picerno!

Abstract

The dissertation is organized in 3 chapters dealing with the timescale of macrodynamic growth models in the short and in the long run, as well as with the analysis of the climate-economy interplay over the business cycle.

The first essay provides an analytical solution to the differential equation that regulates the motion of the neo-Kaleckian model in the short run. After presenting a simple open economy neo-Kaleckian model with government activity, the essay analytically derives an expression for the time of adjustment, defined as the time required for the system to make a k percent adjustment from one steady-state to another.

The second chapter seeks to answer the question of *when is the long run* in long-run growth models driven by demand. By making use of numerical integration, the essay analyses the time of adjustment from one steady-state to the other in two well-known demand-led growth models: the Sraffian Supermultiplier and the fully-adjusted version of the neo-Kaleckian model.

The third chapter of the dissertation presents a business cycle model encompassing the short-run effect of mobilizing green investment to achieve longer-term climate goals. In doing so, the chapter focuses on the dynamics of green and brown investment, assessing whether the interplay between green and capital formation, on one hand, and CO2 emissions, on the other, may allow for conditions of coupling or decoupling – speeding up or slowing down the path towards net-zero emissions.

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Introduction

It is interesting to note that the term *macrodynamics* was coined before the term *macroeconomics*, as documented by Velupillai (2008) and Zambelli (2011). In particular, *macrodynamics* appears for the first time in Ragnar Frisch's seminal contribution *'Propagation problems and impulse problems in dynamic economics'*: "the macro-dynamic analysis [...] tries to give an account of the fluctuations of the whole economic system taken in its entirety" (Frisch, 1933, p. 156). Frisch subsequently adds that "obviously in this case it is impossible to carry through the analysis in great detail", thus already sketching a point made more clearly by Robinson and Eatwell (1973, p. 54) – and referenced in chapter 3 – according to which "models must be simplified. A map at the scale of 1:1 is of no use to a traveller". From this observations, two points follow. The first, Frisch argues, is that macrodynamic analysis could be carried out in great details only if one confines "to a purely *formal* theory" (*ibid.*, italics in the original). The second, and probably most important aspect introduced by Frisch (1933) regards the treatment of time in macrodynamic analysis: models must remain fairly simplified as otherwise "it would hardly be possible to study such fundamental problems as the *exact time shape* of the solutions" (*ibid*, italics in the original). Unlike the timeless dimension of (most of) modern macroeconomics, macrodynamic theory is hence, by definition, rooted in time. More specifically, macrodynamic theory focuses of aggregate fluctuations and economic growth in a simplified and yet relevant manner, in such a way to explicitly account for the time dimension of models' solutions and adjustment processes. All chapters of this dissertation are grounded in this approach.

In particular, the dissertation is organized in 3 chapters dealing with the timescale of macrodynamic growth models in the short and in the long run, as well as with the analysis of the climate-economy interplay over the business cycle.

Chapter 1 (“How Short is the Short Run in the Neo-Kaleckian Growth Model?”) provides an analytical solution to the differential equation that regulates the motion of the neo-Kaleckian model in the short run, thus discussing the *exact time shape* of the model’s solution. After presenting a simple open economy neo-Kaleckian model with government activity, the essay analytically derives an expression for the time of adjustment, defined as the time required for the system to make a k percent adjustment from one steady-state to another. The essay is forthcoming in revised form in the *Review of Political Economy*.

In the second chapter (“When is the Long Run? – Historical Time and Adjustment Periods in Demand-led Growth Models”) I seek to answer the question of *when is the long run* in demand-led growth models. By making use of numerical integration, the essay analyses the time of adjustment from one steady-state to the other in two well-known demand-led growth models: the Sraffian Supermultiplier and the fully-adjusted version of the neo-Kaleckian model. A revised version of the essay has been published online in *Metroeconomica* in May 2022; the printed version is forthcoming in the November 2022 issue of the journal.

The third chapter of the dissertation (“Reduction of CO_2 Emissions, Climate Damage and the Persistence of Business Cycles: A Model of (De)coupling”) presents a business cycle model encompassing the short-run effect of mobilizing green investment to achieve longer-term climate goals. In doing so, the chapter focuses on the dynamics of green and brown investment, assessing whether the interplay between green and capital formation, on one hand, and CO2 emissions, on the other, may allow for conditions of coupling or decoupling – speeding up or slowing down the path towards net-zero emissions.

While a common trait is particularly clear for the first two chapters of the dissertation, which have in common the interest in the analysis of the traverse between steady-state positions in the short and long run, there is – I believe – a *fil rouge* between all chapters. As recalled in the epigraph, Seneca in his

Epistulae morales ad Lucilium warned that *nothing is ours except time*. In a little humble way, this dissertation is an attempt to put this maxim into practice in macrodynamic analysis, from the methodological focus on traverse between steady states to the more pressing need of exploring ways to move faster to a low-carbon future.

Bari, Italy
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1 How Short is the Short Run in the Neo-Kaleckian Growth Model?

Because of the domination of the equilibrium mode of thought, most economists unkowingly evacuate time from their analysis, exactly like Mr. Jourdain spoke prose: equilibrium economics is really timeless economics.

Henry (1987, p. 472)

Abstract

The paper provides an analytical solution to the differential equation that regulates the motion of the neo-Kaleckian model in the short run. After presenting a simple open economy neo-Kaleckian model with government activity, the paper analytically derives an expression for the time of adjustment, defined as the time required for the system to make a k percent adjustment from one steady-state to another. The solution shows that there is an inverse relationship between the time of adjustment and (i) the strength of the Keynesian stability condition; (ii) the behavior of entrepreneurs underlying their decisions to more rapidly/slowly respond to changes in goods market conditions. Last, the model is calibrated for the US, showing that vicinity of the new equilibrium is reached after a period of about 5 quarters under a baseline calibration. By formally analyzing the out-of-equilibrium trajectory of the neo-Kaleckian model, this contribution moves away from the method of comparative dynamics and provides a historical-time representation of the model's traverse.

Keywords: neo-Kaleckian Model; Time; Adjustment Period; Traverse; Effective Demand; Growth; Distribution

JEL codes: E11; E12; E17; O41

1.1 Introduction

The neo-Kaleckian growth model has been mainly criticized because of its failure to provide a long-run convergence of the rate of capacity utilization to the normal one (Dávila-Fernández et al., 2019; Girardi and Pariboni, 2019; Skott, 2012). A partial admission of the difficulties of neo-Kaleckian models in explaining long-run phenomena has also been recently recognized by Lavoie (2018, p.9): "Maybe the mistake was to speak of long-run equilibria; perhaps there would have been no controversy if from the beginning we had called them medium-run equilibria."

While the Kaleckian literature and its critiques have focused on issues related to the stability of the Neo- and Post-Kaleckian models of growth and distribution (Del Monte, 1975; Franke, 2017; Lavoie, 2010; Skott, 2010), little to no attention has been paid to the formal analysis of the traverse from one steady-state position to another. As a consequence, even if we admit that the neo-Kaleckian model ought to be restricted to short or medium-run analysis, it is still left to know what the short and medium runs actually are. More specifically, what needs to be proven is that the neo-Kaleckian model moves between steady-state positions in a time span that the existing literature identifies as either short or medium run.

Accordingly, the first research goal of this paper is to seek an analytical solution to the differential equation that describes the short-run adjustment mechanism of a simple open economy neo-Kaleckian model with government activity. Second, the paper aims to explicitly find a solution of the system in terms of the time of adjustment, thus exploring *how short is the short run* in the neo-Kaleckian model by means of model calibration. Methodologically, the paper follows the line of research pioneered by Sato (1963, 1964, 1980) in analyzing the adjustment period in Neoclassical growth models.

The remainder of the paper is organized as follows. Section 1.2 presents a simple open economy neo-Kaleckian model with government activity, characterized by the endogeneity of the rate of capacity utilization in the short run. Section 1.3 discusses the ordinary differential equation that explains the motion of the neo-Kaleckian system in the short run, providing a general solution to it. Subsequently, the resulting equation is then expressed in terms of the adjustment period t_k required for a k percent adjustment from one steady-state position to another second one. Section 1.4 calibrates the model for the US in line with existing studies and BEA data, showing that the neo-Kaleckian model provides for a very fast pace of adjustment of saving to investment. Last, Section 1.5 concludes, summarizing the findings of the paper.

1.2 A Simple Open Economy neo-Kaleckian Model with Government Activity

This Section presents a simple version of an open economy neo-Kaleckian model with government activity for the analysis of short-run dynamics.

In order to derive the growth model, let us first start with the output equation of an open economy with government activity:

$$Y_t = C_t + I_t + G_t + (X_t - M_t) \tag{1.1}$$

where the current level of aggregate output (Y_t) is defined as the sum of aggregate consumption (C_t), private investment (I_t), public expenditures (G_t) and net exports $(X_t - M_t)$. Consumption, investment, government spending, exports and imports can be modelled as follows:

$$C_t = \overline{C_{0t}} + c(1-t)Y_t \tag{1.2}$$

$$I_t = [\alpha_t + \beta u_t]K_t \tag{1.3}$$

$$G_t = \overline{G_t} \tag{1.4}$$

$$X_t = \overline{X_t} \tag{1.5}$$

$$M_t = mY_t \tag{1.6}$$

Equation (1.2) assumes that aggregate consumption is partly induced - via the tax-adjusted propensity to consume $c(1-t)$ - and partly autonomous from the current level of income ($\overline{C_{0t}}$). Investment (Equation 1.3) is modeled in line with the neo-Kaleckian treatment of capital formation as (linearly) dependent on the rate of capacity utilization ($u_t = Y_t/Y^p$), as postulated by Steindl (1952) and formalized in the 80s by Dutt (1984); Rowthorn (1981); Taylor (1983) and

Amadeo (1986). More specifically, the parameter α reflects "the animal spirits of firms, for instance expectations about the future trend rate of sales growth" (Lavoie, 2014, p. 361), while the parameter β represents the sensitivity of the investment rate to changes in the actual rate of capacity utilization (u_t). Both α and β are assumed to be positive.[1] Government spending (Equation 1.4) and exports (Equation 1.5) are both treated as autonomous expenditures, the first because public consumption and investment depend on the arbitrary decisions of the general government, the second because exports depend on foreign demand, which depends in turn on foreign income. For the sake of simplicity, imports of goods and services are assumed to be linearly dependent on the level of income, via the propensity to import m (Equation 1.6).

Given Equations (1.2) and (1.4), and considering that $s = 1 - c(1-t)$ is the tax-adjusted propensity to save, we can write the domestic saving equation as follows:

$$S_t = Y_t - C_t - G_t = Y_t - \overline{C_{0t}} - c(1-t)Y_t - \overline{G_t} = sY_t - \overline{C_{0t}} - \overline{G_t} \tag{1.7}$$

Dividing Equation (1.7) by the capital stock (K_t), we can obtain the saving rate (σ_t), with v denoting the capital-capacity ratio. :

$$\sigma_t = \frac{S_t}{K_t} = s\frac{Y_t}{K_t} - \frac{\overline{C_{0t}}}{K_t} - \frac{\overline{G_t}}{K_t} = s\frac{Y_t}{Y^p}\frac{Y^p}{K_t} - \frac{\overline{C_{0t}}}{K_t} - \frac{\overline{G_t}}{K_t} = \frac{su}{v} - \frac{\overline{C_{0t}}}{K_t} - \frac{\overline{G_t}}{K_t} \tag{1.8}$$

The accumulation rate (g_t) is obtained by dividing Equation (1.3) by the capital stock (K_t):

$$g_t = \frac{I_t}{K_t} = \alpha + \beta u_t \tag{1.9}$$

Lastly, given Equations (1.5) and (1.6), we can obtain the net export rate (b_t):

$$b_t = \frac{X_t - M_t}{K_t} = \frac{\overline{X_t}}{K_t} - m\frac{Y_t}{Y^p}\frac{Y^p}{K_t} = \frac{\overline{X_t}}{K_t} - \frac{mu_t}{v} \tag{1.10}$$

[1]Since the analysis is restricted to short-run dynamics, the paper abstains from the consideration of a normal degree of utilization, in line with the original vision of Steindl (1952) and Kalecki (1954). Therefore, the model does not provide for a return to a normal degree of capacity utilization, under the assumption - widely acknowledged by Kaleckian authors - that the rate of capacity utilization is an endogenous variable, at least in the short run. For a more-in-depth discussion, see Hein (2014), Lavoie (2014) and Blecker and Setterfield (2019).

As discussed by Blecker and Setterfield (2019, p. 192), the goods market equilibrium condition requires that the saving rate has to be equal to the sum of the accumulation and net export rates:

$$\sigma_t = g_t + b_t \tag{1.11}$$

Therefore, after equating and rearranging Equations (1.9), (1.10) and (1.11), we can obtain the short-run goods market equilibrium as follows:

$$\frac{(s+m)u^*}{v} - z = \alpha + \beta u^* \tag{1.12}$$

where z denotes the ratio of autonomous expenditures to the capital stock. Similarly to Lavoie (2016), the ratio is assumed to be constant in the short run:

$$z = \frac{\overline{Z_t}}{K_t} = \frac{\overline{C_{0t}}}{K_t} + \frac{\overline{G_t}}{K_t} + \frac{\overline{X_t}}{K_t} \tag{1.13}$$

Last, let us solve the model for the equilibrium rate of capacity utilization (u^*):

$$u^* = \frac{\alpha + z}{(s+m)/v - \beta} = \frac{(\alpha + z)v}{s + m - \beta v} \tag{1.14}$$

The model leads to a stable equilibrium if and only if the denominator in equation (1.14) is positive. This implies that the short-run stability condition is met if saving adjusts faster than investment and the trade balance to changes in the rate of utilization, as discussed by Hein (2014, p.290).

The simple open economy version of the neo-Kaleckian model presented here maintains all the fundamental properties of Kaleckian analysis:[2]

[2] It is worth noting that the paper relies on the consideration of a unique economy-wide tax-adjusted propensity to save. The main reason is to move beyond the traditional Cambridge assumption that wage earners do not save, thus making the analysis in Sections 1.3 and 1.4 more consistent with economic reality (Barbieri Góes, 2020). This way, however, issues related to shifts in the functional distribution of income take a back seat. In order to bring them back, the analysis should be extended by modeling the economy-wide propensity to save as equal to the average of the propensities to save out of wages and out of profits weighted by the respective factor shares and assuming the former to be greater than the latter, in line with the Kaleckian and Post-Keynesian literature. For the sake of analytical tractability, the paper abstains from this further step, that would however permit to recover two further postulates of Kaleckian analysis, i.e. the ideas that demand and growth are wage-led and that the paradox of cost holds in the short run. For a more extensive discussion, see Hein (2014, Sec. 7.2).

1. Growth is demand-led through the investment channel;
2. The rate of capacity utilization is endogenous in the short-run, bearing the brunt of the adjustment of saving to investment and the trade balance;
3. A positive change in the animal spirits parameter (α) boosts accumulation (Equation 1.3);
4. The paradox of thrift holds in the short run: an increase in the economy-wide tax-adjusted propensity to save (s) lowers the equilibrium utilization and accumulation rates;

Having sketched the basics of the model and its steady-state, let us now move to the consideration of out-of-equilibrium dynamics, formally analyzing the characteristics of the short-run traverse.

1.3 Analysis of the Adjustment Period

Firms are assumed to react to any supply-demand mismatch in the goods market through quantity adjustments. More specifically, with the principle of effective demand at work, firms will increase "output and hence the rate of capacity utilization whenever aggregate demand [D_t below, *note of the author*] exceeds aggregate supply" (Lavoie, 2014, p. 363).[3] Framing the adjustment in terms of changes in the utilization rate, it follows that:

$$\begin{aligned}\frac{du}{dt} &= u_t \frac{dY/dt}{Y_t} = \frac{\mu(D_t - Y_t)}{Y_t}\frac{Y_t}{Y^p} = \\ &= \frac{\mu(I_t + X_t - M_t - S_t)}{K}\frac{K}{Y^p} = \mu v(g_t + b_t - \sigma_t)\end{aligned} \tag{1.15}$$

where μ is a parameter measuring the intensity and speed with which supply adjusts to demand. The parameter needs to be positive for the adjustment to be possible in the assumed direction, but not greater than 1 (instantaneous adjustment): $0 < \mu \leq 1$.

[3]Given that the process takes place in the short run, potential output will not increase with output changes. Therefore, the percentage change of the rate of capacity utilization will be equal to that of output.

Rewriting and rearranging equation (1.15) in light of equations (1.9, 1.10 and 1.11), it follows that:

$$\frac{du}{dt} = \mu\left[(\alpha + z)v - (s + m - \beta v)u_t\right] \tag{1.16}$$

Equation (1.16) is of key importance, as it constitutes the first-order linear differential equation that explains the motion of the neo-Kaleckian model in the short run. It postulates that entrepreneurs adjust the utilization of productive capacity on the basis of goods market conditions. More specifically, whenever investment demand and the trade balance fall short of (exceeds) the supply of savings, the rate of capacity utilization will decrease (increase) to match the new equilibrium in the goods market, making possible the *ex-post* adjustment of saving to investment and net exports. Moreover, the equation captures all the fundamental properties of the neo-Kaleckian model moving towards its new steady-state, postulating that changes in the rate of capacity utilization are positively related to changes in the animal spirits parameter (α) and the autonomous demand-capital ratio (z), and negatively related with changes in the tax-adjusted propensity to save (s), in line with the paradoxes of thrift. The general solution[4] of equation (1.16) is given by:

$$u_t = \frac{(\alpha + z)v - C\exp[-t\mu(s + m - \beta v)]}{s + m - \beta v} \tag{1.17}$$

where C is the constant of integration.[5]

Let us consider the case of an increase in the parameter capturing animal spirits (α).[6] Accordingly, from equation (1.14), it follows that the old and new

[4]The ordinary differential equation in equation (1.16) can be easily solved with most statistical softwares. For a formal proof, see Appendix A.1.

[5]For further discussion, see Appendix A.1.

[6]It is worth stressing that the mathematical derivation would yield the same result for the time of adjustment t_k even if the initial change would be in s, m, β or v. The analysis starts with a change in the parameter α merely because the mathematical derivation becomes more straightforward. In other terms, a shock in the parameters determining the Keynesian stability condition would affect the speed of the dynamic adjustment, but not its time structure, which is regulated by equation (1.26) below.

steady-state values of the capacity utilization rate are, respectively:

$$u_0^* = \frac{(\alpha_0 + z)v}{s + m - \beta v} \quad \text{and} \quad u_1^* = \frac{(\alpha_1 + z)v}{s + m - \beta v} \tag{1.18}$$

Since $\alpha_1 > \alpha_0$, the new equilibrium rate of capacity utilization (u_1^*) will be greater than the initial one (u_0^*), i.e. $u_1^* > u_0^*$.

When the increase in animal spirits – from α_0 to α_1 – occurs at time $t = 0$, the system is still in its initial steady state corresponding to u_0^*, beginning the process of convergence to the position corresponding to the new equilibrium u_1^*. Accordingly, as the adjustment mechanism is now triggered, the general solution of the differential equation (1.16) will reflect the new value of the animal spirits parameter (α_1). In other terms, equation (1.17) at time $t = 0$ becomes:

$$u_0 = \frac{(\alpha_1 + z)v - \mathbf{C}\exp[-0\mu(s + m - \beta v)]}{s + m - \beta v} = \frac{(\alpha_1 + z)v - \mathbf{C}}{s + m - \beta v} \tag{1.19}$$

However, as discussed before, at $t = 0$ the system is in its short-run initial equilibrium, implying that u_0 in equation (1.19) must be equal to u_0^* in equation (1.18):

$$\frac{(\alpha_1 + z)v - \mathbf{C}}{s + m - \beta v} = \frac{(\alpha_0 + z)v}{s + m - \beta v} \tag{1.20}$$

Simplifying and rearranging, we have that the constant of integration C is equal to:

$$C = (\alpha_1 - \alpha_0)v \tag{1.21}$$

Therefore, Equation (1.17) can be rewritten as follows:

$$u_t = \frac{(\alpha_1 + z)v - (\alpha_1 - \alpha_0)v\exp[-t\mu(s + m - \beta v)]}{s + m - \beta v} \tag{1.22}$$

At this stage, we ought to consider the difference between the two steady-states in equation (1.18):

$$\Delta u^* = u_1^* - u_0^* = \frac{(\alpha_1 - \alpha_0)v}{s + m - \beta v} \tag{1.23}$$

Let us now denote with t_k the time period corresponding to a k (percent) adjustment to the new steady-state value u_1^*. Accordingly, the amount of the adjustment in capacity utilization at time t_k is given by $k\Delta u^* = u_k - u_0^*$, implying that:

$$u_k = u_0^* + k\Delta u^* = \frac{(\alpha_0 + z)v + kv(\alpha_1 - \alpha_0)}{s + m - \beta v} \tag{1.24}$$

where u_k is the value of u_t at time t_k. Therefore, u_k must be equal to u_t in equation (1.22) with $t = t_k$. Equating the former with equation (1.24), it follows that:

$$\frac{(\alpha_1 + z)v - (\alpha_1 - \alpha_0)v\exp[-t_k\mu(s + m - \beta v)]}{s + m - \beta v} = \frac{(\alpha_0 + z)v + kv(\alpha_1 - \alpha_0)}{s + m - \beta v} \tag{1.25}$$

Simplifying and rearranging, we can explicitly solve equation (1.25) in terms of the adjustment period t_k, as follows:

$$t_k = \frac{-\ln(1 - k)}{\mu(s + m - \beta v)} \tag{1.26}$$

Equation (1.26) provides an analytical relation between the adjustment period (more specifically, a k percent of the adjustment) and the other relevant parameters of the neo-Kaleckian model presented in Section 1.2. At first glance, it can be easily noted that there is an inverse relationship between the strength of the Keynesian stability condition and the the time of adjustment, i.e. the greater $(s + m - \beta v)$, the smaller the k percent adjustment period t_k. Moreover, the time of adjustment t_k is inversely related with the parameter μ, that captures the speed and intensity with which entrepreneurs decide to adjust production to demand in the goods market.

Taken together, the two conditions mentioned in the previous paragraph imply that the time required for the utilization rate to adjust to a new steady-state position is fundamentally influenced by (i) the structure of production and demand embedded in the parameters determining the Keynesian stability condition (s, m, β and v) and (ii) the behavior of entrepreneurs underpinning their decisions to more rapidly/slowly respond to an aggregate demand shock by adjusting production (μ). In other terms, the more responsive is production to aggregate demand changes, and the more dynamic the behavior of entrepreneurs to such changes, the shorter will be the adjustment period.

Summing up, the inspection of the equation 1.25 allows to state the following fundamental results:

1. The adjustment period does not depend neither on the initial nor on the new value of animal spirits (α);
2. The adjustment period does not depend neither on the initial nor on the new value of the autonomous demand-capital ratio (z);
3. The greater the propensity to save (s), the shorter the adjustment period;
4. The greater the propensity to import (m), the shorter the adjustment period;
5. The greater the capital-capacity ratio (v), the longer the adjustment period;
6. The greater the sensitivity of accumulation to changes in the rate of capacity utilization (β), the longer the adjustment period;
7. The greater is the speed and intensity of the adjustment of production to demand (μ), the shorter the adjustment period;
8. The greater the percentage of adjustment (k), the longer the adjustment period.

1.4 Parameter Values and Adjustment Time

This section provides a parameter calibration of the neo-Kaleckian model, in order to find an approximate time length for a given percentage of the adjustment to a new steady-state. By relying on existing studies and BEA data, the calibration is carried out in light of the empirical evidence for the US economy in the period between 2002 and 2019, i.e. the years encompassing the Great Moderation and the Global Financial Crisis, before the COVID-19 Recession.

In order to be able to coherently interpret the results in calendar time, it is important to point out that we need to assume *a priori* that the adjustment of saving to investment does not occur faster than the unit period inherent in the data (Gandolfo, 2012). In other terms, if we were to use an annual calibration (as most of the existing literature does), we would need to assume that the adjustment does not take place within a year. In the opposite case, it would be difficult to derive a plausible discrete-time representation of the adjustment process, as showed by Gandolfo (2012). For this reason, using an annual calibration is somewhat problematic in the case of fast processes. Accordingly, the model is calibrated at a quarterly frequency, under the more realistic assumption that the adjustment does not occur at higher frequencies (daily, weekly or monthly). Calibrating the accumulation rate and all other relevant parameters to account for quarter-on-quarter growth ensures that the unit period can be interpreted as a single quarter. Therefore, assuming that a quarter is a sufficiently small time step, we can then coherently provide a continuous-time representation of a discrete process.

In order to calibrate the quarterly capital-capacity ratio (v), let us decompose it as follows:

$$v = \frac{K}{Y^p} = \frac{K}{I}\frac{I}{Y}\frac{Y}{Y^p} = \frac{h_t u_t}{g_t} \tag{1.27}$$

Therefore, the capital-capacity ratio depends positively on the investment share (h_t) and on the rate of capacity utilization (u_t) and negatively on the accumulation rate (g_t). The benchmark value of the ratio is obtained from the analysis of capital dynamics in the US, in line with Fazzari et al. (2020, Supplementary Appendix). The authors abstain from the complicated matter of measuring capital and the problem of aggregating heterogeneous capital goods, thus not relying on BEA fixed assets data. Instead, they make use of national accounts and investment data to calibrate the capital-actual output ratio. In particular, they do so by starting from the empirical observation of the average investment share from 2002 to 2016 (equal to 12.5%) and of the annual gross capital accumulation rate (10.9%) - obtained as the sum of a yearly growth rate of 2.5% and a 8.4% depreciation rate. In quarterly frequency, the latter observation implies an accumulation rate of 2.62%.[7] With a private non-residential investment share of 12.65% and a rate of capacity utilization of 77% - equal to the average

[7]The quarterly growth rate is obtained using the formula $g_{qtr} = (1 + g_{yr})^{1/4} - 1$.

measure of utilization from 2002 to 2019 - Equation (1.27) yields a quarterly capital-capacity ratio of 3.72.[8]

The value of the economy-wide propensity to save (s) is set to 0.5, in line with the empirical estimation of Blecker et al. (2022) and the recent evidences and calibration exercise by Fazzari et al. (2020, Supplementary Appendix). The value of the propensity to import (m) is obtained by calculating imports of goods and services in percent of GDP from 2002Q1 to 2019Q4 and averaging the time series; the result yields $m = 17\%$.

The expected growth rate of sales (α) is calibrated using quarterly real GDP growth as a proxy of expected revenues, yielding an average growth rate of 0.51% at quarterly rates from 2002 to 2019. Furthermore, the parameter that captures the impact of the rate of capacity utilization on accumulation (β) and the autonomous demand-capital ratio (z) are both set to match the above mentioned steady-state values of the degree of capacity utilization and the quarterly accumulation rate.[9] Let us now move to the discussion of the value of the parameter capturing the speed and intensity of the adjustment of production to demand (μ). Given the difficulty associated with inferring it from empirical evidences, we assume it to be 0.75 in the baseline scenario, then allowing it to vary between a lower value of 0.5 and a higher value of 0.9. This implies that every quarter entrepreneurs respond to goods market conditions by adjusting production in a order of magnitude between 50% and 90% of the change in demand.

The parameter values are summarized in Table 1.1.

[8]The adopted value of the investment share is just slightly above the one used by Fazzari et al. (2020), as the data is extended until the last quarter of 2019. In order to measure capacity utilization, the paper makes use of the average value of the Federal Reserve Board (FRB) measure of utilization from 2002 to 2019 (for data sources, see Appendix A.2). It should be noted that there is no definite consensus on whether the FRB index is the most appropriate measure of the degree of capacity utilization. For a critical discussion, the reader should refer to Nikiforos (2016) and Gahn and González (2020). However, the empirical controversies on the use of FRB data are centered on the discussion of the stationarity of the series and thus on the opportunity of using it to properly measure long-run variations of utilization. The purpose of the current exercise is rather different, as the average value of the rate of capacity utilization is used as a mere benchmark; the adoption of a different measure of utilization to calibrate the model would have no effect on the overall results.

[9]It is worth noting that since neither α nor z have an effect on the length of the adjustment period, their calibration is merely carried out for expositional purposes.

Table 1.1 Parameter values

Par.	Description	Value	Source
v	Capital-capacity ratio (quarterly)	3.7204	Author's calculation, based on Fazzari et al. (2020)
s	Propensity to save	0.5	Blecker et al. (2022); Fazzari et al. (2020)
m	Propensity to import	0.17	Author's calculation, based on BEA data (See Appendix A.2)
α	Animal spirits	0.0051	Author's calculation, based on BEA data (See Appendix A.2)
β	Impact of u_t on the accumulation rate	0.0274	Author's calculation
z	Autonomous demand-capital ratio	0.1126	Author's calculation
μ	Speed of adjustment (quarterly)	0.75; 0.5; 0.9	Author's assumption
k	Percentage of the adjustment	0.90; 0.99	–

Source: author's calculation, various sources (see Appendix A.2)

Under the baseline parameter constellation (with $\mu = 0.75$), we can now explicitly compute the adjustment period.[10] Defining vicinity to the new steady-state position as 90% of the total adjustment, it follows that:

$$t_{0.90} = \frac{-\ln(1-0.90)}{0.75(0.5+0.17-0.0274\times 3.72)} \approx 5 \text{ quarters} \approx 1 \text{ year} \tag{1.28}$$

Therefore, the model approaches the new steady-state in about 1 year, reaching it almost entirely (99% of the total adjustment) in about 2 years:

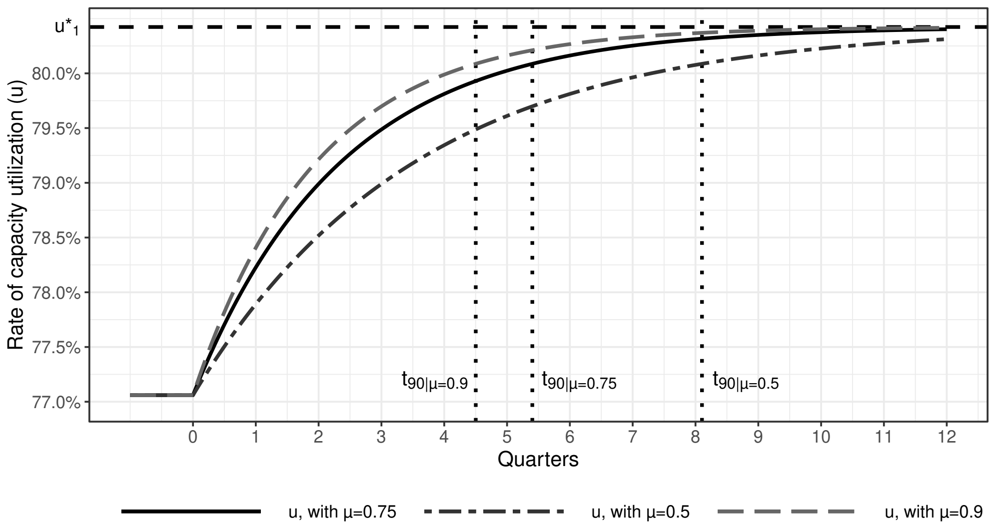
$$t_{0.99} = \frac{-\ln(1-0.99)}{0.75(0.5+0.17-0.0274\times 3.72)} \approx 10 \text{ quarters} \approx 2 \text{ year} \tag{1.29}$$

The results slightly change when we assume a different speed of adjustment of production to demand (μ). In particular, reducing μ to 0.5 lengthen the time required for a 90% and 99% adjustment to about 8 and 16 quarters, respectively. Conversely, a faster adjustment in the goods market ($\mu = 0.9$) produces a slight reduction of the time required to approach the new steady-state position u_1^* (with $t_{0.90} \approx 4$ quarters and $t_{0.99} \approx 9$ quarters).

Figure 1.1 provides a graphical illustration of the adjustment process under the parameter calibration described above, following an initial increase in the expected growth rate of sales (α) and allowing for three different values of μ. The dotted lines match the time needed for a 90% adjustment to the new equilibrium u_1^* under the three different parameter sets.

[10]Since the main scope of the paper is analytical rather than empirical, it does not include a sensitivity analysis, thus deriving the qualitative results from the benchmark values reported above. However, the interested reader may easily perform a re-parameterization of the neo-Kaleckian model using the resource reported in the Online Appendix A.3.

Figure 1.1 The adjustment of the rate of capacity utilization to an increase in α at $t = 0$



Source: authors' representation

Therefore, under the baseline parameter calibration, vicinity (90%) of the new equilibrium in the model is reached after a period of about 5 quarters (9 quarters for 99% of the adjustment). This consideration implies that, in historical time, the neo-Kaleckian model presented here is characterized by a relatively fast pace of adjustment, compatible with the time span of short-run processes as defined by Angeletos et al. (2020).

Before drawing conclusions from the analysis conducted above, two important remarks are in order. First, the analysis of the short-run traverse in the neo-Kaleckian model rests on a framework that, although simple, embeds an open economy with government activity and autonomous consumption spending. Conducting the same calibration exercise on a simpler model that does not account for foreign trade, government activity and/or autonomous consumption may lead to misleading conclusions regarding the time of adjustment needed for the transition between steady states.[11] Second, even though the calibration exercise is conducted in light of empirical evidences for the US economy, this does not imply that economic reality follows the same adjustment path postulated by the model. In other terms, the analysis does not provide any empirical

[11] I wish to thank Robert Blecker for pointing this out to me.

support whatsoever to the Kaleckian claim that the rate of capacity utilization is endogenous in the short run, nor to the implication of a stable convergence of saving to investment. Rigorous econometric analysis aimed at supporting or disproving Kaleckian investment and output theory is therefore still needed, leaving space to further research on the matter.

1.5 Concluding Remarks

The paper presents a simple open economy neo-Kaleckian model with autonomous components of aggregate demand. Most importantly, it finds an analytical solution to the differential equation that regulates the motion of the neo-Kaleckian model in the short run. In line with the methodology introduced by Sato (1963, 1964, 1980), the analysis provides and discusses a general solution to the ordinary differential equation that explains out-of-equilibrium dynamics in the model. Subsequently, the effect of an increase in animal spirits is considered, rewriting the general solution of the neo-Kaleckian model in terms of the time of adjustment t_k, i.e. the time required for the system to make a k percent adjustment to the new steady-state.

The explicit analysis of the short-run traverse in the neo-Kaleckian model yields few fundamental results. First, the time of adjustment is not affected by changes neither in the animal spirits parameter (α) nor in the autonomous demand-capital ratio (z). Second, the adjustment period depends negatively on the propensity to save (s) and on the propensity to import (m). Third, t_k is in a direct relation with the capital-capacity ratio (v) and with the sensitivity of accumulation to changes in the rate of capacity utilization (β). Fourth, there is an indirect relation between the speed and intensity of the adjustment of production to demand (μ) and the time of adjustment. Taken together, these conditions imply that the time it takes for the utilization rate to adjust is largely determined by (i) the structural determinants of production and demand embedded in the Keynesian stability condition and (ii) the behavior of entrepreneurs underlying their decisions to adjust production more quickly or slowly in response to a change in goods market conditions. In other words, the more responsive is production to aggregate demand changes, and the more dynamic the behavior of entrepreneurs to such changes, the shorter will be the adjustment period.

Last, the paper performs a parameterization of the neo-Kaleckian model in line with empirical evidences and recent Post-Keynesian literature. The calibration exercise shows that, under a reasonable parameter constellation, vicinity of the new equilibrium - defined as 90% of the total adjustment - is reached after a period of about 4 to 9 quarters (depending on the value of μ), and the model almost settles in the new steady state (99% of the adjustment) after about 9 to 16 quarters. This result, implying a relatively fast pace of adjustment compatible with short-run processes, provides more solid foundation to Lavoie's (2018, p. 9) claim - reported in the introduction - that the neo-Kaleckian model is better suited for short and medium-run analysis rather than for giving a proper representation of long-run macrodynamics. While the investment theory upon which the neo-Kaleckian model rests needs to be further assessed empirically, the analysis of the short-run traverse conducted in the present contribution calls for a closer connection between the neo-Kaleckian model of growth and distribution and Kalecki's original business cycle theory. As the neo-Kaleckian model appears to be moving between steady-state positions at business cycle frequencies (Angeletos et al., 2020), the former is consistent with Kalecki's idea that the short run is characterized by damped oscillations perturbed continuously by stochastic shocks that generate semi-regular cyclical movements (Kalecki, 1971, pp. 134-135).

On a more general level, the analysis conducted in the paper points to the importance of explicitly taking into account the time scale of steady-state growth models when describing their comparative dynamic effects and policy implications, thus coherently combine logical-time analysis and real-world historical time, as advocated by Joan Robinson (1980). In this respect, the paper has analytically showed the validity of the line of argument put forward by Henry (1987), Park (1995) and Lavoie (2016, p.183-184) on the importance of paying more attention to the values that the relevant variables of a system take *during the traverse* rather than to their *potential* steady-state values. Whilst the ultimate assessment of the validity of the neo-Kaleckian model for policy analysis ought to rest on rigorous empirical investigation, this contribution wishes to set the ground for a new agenda for Kaleckian authors and demand-led growth theorists, suggesting to move away from the comfortable but limited realm of comparative dynamics and think more carefully about the properties exhibited by economic models *during the traverse*. The comparison between steady-state positions is undoubtedly useful to grasp the logic of a model as it moves from one equilibrium to another, but it needs to be coupled with a precise description of the model's out-of-equilibrium trajectory if we want to provide a valid representation of a real-world economy operating in historical time on human time scales.

2 When is the Long Run? – Historical Time and Adjustment Periods in Demand-led Growth Models

If we throw away information about the time dimension, we are reducing still further our limited understanding of the relationship between these models and the real world.

Atkinson (1969, p.137)

Abstract

In recent years, Post-Keynesian models of growth and distribution have substantially shifted their focus from short to long-run analysis. While many authors have focused on the convergence of demand-led growth models to a fully-adjusted equilibrium, relatively little attention has been given to the time required to reach this long-run position. In order to fill the gap, this paper seeks to answer the question of *when is the long run* in demand-led growth models. By making use of numerical integration, it analyses the time of adjustment from one steady-state to the other in two well-known demand-led growth models: the Sraffian Supermultiplier and the fully-adjusted version of the neo-Kaleckian model. The results show that the adjustment period is generally beyond an economically meaningful time span, suggesting that researchers and policy makers ought to pay more attention to the models' predictions during the traverse rather than focusing on steady-state positions.

Keywords: Neo-Kaleckian model; Sraffian Supermultiplier; time; adjustment period; traverse; effective demand; growth

JEL codes: B51; E11; E12; B41

2.1 Introduction

This paper takes Joan Robinson seriously.[1] In her famous 1980 article, Robinson claimed that "to construct models that cannot be applied is merely an idle amusement" (p. 223-224). Yet, the construction of any supposedly realistic model cannot abstain from the consideration that historical time – rather than logical time – rules reality. Accordingly, it is "a common error to confuse a comparison of static positions with a movement between them" (*ibid.*, p. 228). This contribution is chiefly interested in the duration of the movement between steady-state positions in demand-led growth models.

In recent years, Post-Keynesian models of growth and distribution have substantially shifted their focus from the short run – or from "chain*(s)* of short-period situations" (Kalecki, 1971, p. 165) – to long-run modeling. While Post-Keynesian growth theory benefited from this shift, gaining more rigor and coherency, more fundamental questions were often overlooked; in particular, few or no academic discussions can be found as regards the essential question of *when is the long run* (Robinson, 1980, p. 226) and how we can evaluate growth models in historical time. In other terms, inquiries about the nature and duration of the traverse effectively fell by the wayside. Bringing these issues to the front of the debate is thus of key importance to avoid committing Post-Keynesian growth theory to what we might call the 'Marshallian leap', making "the step from a model to reality by an act of faith" (*ibid.*).

Along these lines, the present contribution seeks to shed light on a dormant debate on traverse analysis and the persistence of out-of-equilibrium dynamics, thus recovering and deepening Joan Robinson's insights on the differences between logical and historical time in economic analysis. Accordingly, the main research goal is to analyze the time of adjustment in two prominent demand-led models focused on the role of autonomous demand in driving long-run growth, namely the Sraffian Supermultiplier model and the long-run version of the neo-Kaleckian model presented by Allain (2015) and Lavoie

[1] The *incipit* of the paper draws upon the opening line of one of most influential articles in the field of neoclassical growth models, i.e. Mankiw et al. (1992).

(2016)[2]. More specifically, in accordance with the line of research pioneered by Sato (1963, 1980), Sato (1966) and Atkinson (1969), the paper adopts the method of numerical integration to solve the systems of differential equations regulating the out-of-equilibrium dynamics of the two models. In order to do that, we calibrate both models in line with the existing theoretical and empirical literature.

The paper is organized as follows. Section 2.2 presents the two models under scrutiny, i.e. the Sraffian Supermultiplier model and the long-run version of the neo-Kaleckian model presented by Allain (2015) and Lavoie (2016). Section 2.3 discusses the adopted parameter calibration, then presenting the numerical solution of the two models and our main findings. Section 2.4 discusses the sensitivity of the models' time of adjustment following a change in the parameter space. Last, Section 2.5 concludes, discussing the interpretation and implications of the results.

2.2 Sraffian and Kaleckian Long-run Growth Models

This section provides a synthetic review of the models under scrutiny. A more in-depth discussion of the Sraffian Supermultiplier model (Subsection 2.2.1) can be found in Girardi and Pariboni (2016); Serrano (1995b); Serrano and Freitas (2017) and Gallo (2019). As regards the long-run version of the neo-Kaleckian model (Subsection 2.2.2) with autonomous demand and Harrodian dynamics, see Allain (2015, 2018, 2021) and Lavoie (2016).

In order to make the Supermultiplier and neo-Kaleckian frameworks fully comparable, the two models are presented for an open economy with gov-

[2]Some words on the rationale behind the choice of the two models are in order. First, both models rely on the role of autonomous components of demand in driving economic growth. Given that "the literature on autonomous growth has itself been cast in terms that are intrinsically long run" (Skott, 2019, p. 238), the comparison of the two models allow to coherently answer the question that inspires the paper, i.e. *when is the long run*. Second, both models are demand-led, allowing to summarize the compatibilities and divergences of Kaleckian and Sraffian insights on growth in a relatively simple way. Third, both models reach a fully adjusted position – equaling the actual and normal rate of capacity utilization in the long run – thus preventing the emergence of the second Harrod problem.

ernment activity. Moreover, we include a linear depreciation rate of physical capital.[3]

2.2.1 The Sraffian Supermultiplier Model

Following Serrano and Freitas (2017), this Subsection presents the Sraffian Supermultiplier model assuming an open economy with government activity. The model can be represented as a 3-equation in 3 variables – autonomous demand growth (g_t^Z), the investment share (h_t) and the rate of capacity utilization (u_t:)

$$g_t^Y = g_t^Z + \frac{h_t \gamma (u_t - u_n)}{s + m - h_t} \tag{2.1}$$

$$g_t^K = \frac{h_t u_t}{v} - \delta \tag{2.2}$$

$$g_t^Z = \overline{g^Z} \tag{2.3}$$

Equation (2.1) describes the evolution of economic activity as depending on autonomous demand growth (g_t^Z) plus an additional proportional rate of growth of output resulting from the supermultiplier when capacity utilization is not at its normal degree (u_n), i.e. the second term of the equation. Moreover, s indicates the "tax-adjusted marginal propensity to save" (Girardi and Pariboni, 2015, p. 526) and γ is "a parameter that measures the reaction of the growth rate of the marginal propensity to invest to the deviation of the actual degree of capacity utilization" (Serrano and Freitas, 2017, p. 74). Assuming a constant capital-capacity ratio (v), the evolution of capital accumulation is given by the rate of growth of capacity output minus the depreciation rate (δ), as in equation (2.2).[4] Lastly, equation (2.3) constitutes the closure of the model for an exogenously given rate of growth of autonomous demand ($\overline{g^Z}$).

The model settles in its long-run steady state when the fully-adjusted position (Vianello, 1985) is reached, i.e. $u_t = u_n$ and actual output and capital grow at

[3]For the derivation of variables from levels to growth rates, see Appendix B.1. The list of variables used in the paper is reported in Appendix B.2, while a list and description of parameters can be found in Table 2.1 below.

[4]Under the assumption of fully-induced investment, it ought to be noted that this is a mere accounting identity, as showed in Appendix B.1.

the same pace, i.e. $g_t^Y = g_t^K$. Therefore, the long-run equilibrium position of the model is characterized by:

$$h^* = \frac{v}{u_n}(\overline{g^Z} + \delta) \tag{2.4}$$

$$u^* = u_n \tag{2.5}$$

$$g^{Z*} = \overline{g^Z} \tag{2.6}$$

Accordingly, in the long run all growth rates ought to equal the exogenous expansion of autonomous components of demand, i.e $g^* = g^{K*} = g^{Y*} = \overline{g^Z}$.

Let us now analyze more-in-depth the process of economic growth and out-of-equilibrium dynamics. The adjustment to the long-run equilibrium is carried out by the two endogenous variables of the system, i.e. the rate of capacity utilization u_t and the investment share h_t. In line with Serrano and Freitas (2017), the two adjustment mechanisms[5] are modeled as follows:

$$\dot{u} = u_t(g_t^Y - g_t^K) \tag{2.7}$$

$$\dot{h} = h_t\gamma(u_t - u_n) \tag{2.8}$$

Substituting equation (2.1 and 2.2) into equation (2.7), we obtain the system of two first-order non-linear differential equations that will be solved numerically in Section (2.3):

$$\begin{cases} \dot{u} = & u_t\left[g_t^Z + \dfrac{h_t\gamma(u_t-u_n)}{s+m-h_t} - \dfrac{h_t}{v}u_t + \delta\right] \\ \dot{h} = & h_t\gamma(u_t-u_n) \end{cases} \tag{2.9}$$

Summarizing, discrepancies between actual and normal degrees of capacity utilization can only be of transient nature, producing growth effects in the short but not in the long run, in which the fully-adjusted position is reached.[6] More specifically, during the adjustment process when $u_t \gtreqless u_n$, it follows that

[5]Henceforth, changes of a variable over time will be denoted with the dot symbol, e.g. $\dot{u} = du/dt$.

[6]For a discussion of the stability of the system, see Appendix B.1.

$\dot{h} \gtreqless 0$, and whenever $g_t^Y \gtreqless g_t^K$, then $\dot{u} \gtreqless 0$. The rest of the paper will focus on evaluating in historical time the transiency of these effects.

2.2.2 The Long-run neo-Kaleckian Model with Autonomous Expenditures and a Harrodian Mechanism

The Allain-Lavoie long-run version of the neo-Kaleckian model can be presented as the following 3-equations system in 3 variables – autonomous demand growth (g_t^Z), animal spirits (α_t) and the autonomous demand-capital ratio (z_t):

$$g_t^I = \alpha_t + \beta(u_t - u_n) \tag{2.10}$$

$$g_t^S = \frac{(s+m)u_t}{v} - z_t \tag{2.11}$$

$$g_t^Z = \overline{g^Z} \tag{2.12}$$

Equation (2.10) constitutes the conventional version of the neo-Kaleckian investment function with a normal rate of capacity utilization. The term α_t captures animal spirits, which along with z_t, vary in the long run to prevent the emergence of the second Harrod problem, as we will see later.[7] Equation (2.11) represents the saving function proposed by Lavoie (2016) in line with Serrano (1995a,b); it incorporates in the neo-Kaleckian model a "non-proportional saving function with a constant term that in the long run grows at an exogenously given rate" (Lavoie, 2016, p. 173).

In the short run, animal spirits and the autonomous demand-capital ratio are assumed to be constant. Accordingly the *ex-post* equality of the growth rates of investment and saving yields the following short-run equilibrium rate of growth of investment and saving:

$$g_{sr}^{I*} = g_{sr}^{S*} = \alpha + \beta(u_{sr}^* - u_n) \tag{2.13}$$

[7] Amadeo (1986) was the first author to associate the constant term in the investment function with animal spirits, and the paper maintains his terminology. However, as acknowledged by Lavoie (2016), α_t could be interpreted as capturing all determinants of investment unexplained by the model, "such as technological change, the profit rate or the profit share, credit or monetary conditions, the leverage ratio of firms, radical uncertainty and so on" (*ibid.*, p. 177).

Solving for the short-run goods market equilibrium of $g_t^S = g_t^I$, it follows that the short-run rate of capacity utilization is equal to:

$$u_{sr}^* = \frac{(\alpha + z - \beta u_n)v}{s + m - \beta v} \tag{2.14}$$

If not by a fluke, the short-run rate of capacity utilization u_{sr}^* – that brings about the goods market equilibrium – will diverge from its long-run value u_n. More specifically, short-run discrepancies between the actual and normal rates of capacity utilization are given by:

$$u_{sr}^* - u_n = \frac{(\alpha + z)v - (s + m)u_n}{s + m - \beta v} \tag{2.15}$$

Consequently, the equilibrium accumulation rate in the short run is given by:

$$g_{sr}^{K*} = g_{sr}^{I*} - \delta = \alpha + \beta(u_{sr}^* - u_n) - \delta \tag{2.16}$$

where g_{sr}^{K*} is the growth rate of the capital stock corresponding to the goods market equilibrium.[8]

However, during the traverse towards the long-run steady state, animal spirits α and the z ratio will vary, ensuring the long-run convergence of economic growth to autonomous demand growth ($g^* = g^{K*} = g^{Y*} = \overline{g^Z}$) and of the actual rate of capacity utilization towards its normal degree ($u = u_n$). Therefore, the long-run equilibrium position of the model is characterized by:

$$z^* = \frac{(s + m)u_n}{v} - \overline{g^Z} - \delta \tag{2.17}$$

$$\alpha^* = \overline{g^Z} + \delta \tag{2.18}$$

$$u^* = u_n \tag{2.19}$$

$$g^{Z*} = \overline{g^Z} \tag{2.20}$$

As mentioned above, the long-run adjustment process is carried out through changes in animal spirits and in the autonomous demand-capital ratio. More

[8] In order to make the Allain-Lavoie model fully comparable with the Supermultiplier, a small amendment is introduced, including a linear depreciation rate of the capital stock. The novelty does not alter significantly the long-run equilibrium results, as showed in Appendix B.1.

specifically, animal spirits react to discrepancies between the short-run equilibrium of the capacity utilization rate – i.e. the one that ensures the *ex-post* adjustment of saving to investment – and the normal degree. Furthermore, the z ratio adjusts to discrepancies between the exogenous growth rate of autonomous demand and the short-run equilibrium rate of economic growth[9].

$$\dot{\alpha} = \mu(g_{sr}^{I*} - \alpha_t) = \beta\mu\,(u_{sr}^* - u_n) \tag{2.21}$$

$$\dot{z} = z_t\left(\overline{g^Z} - g_{sr}^{K*}\right) \tag{2.22}$$

Substituting equations (2.15) and (2.16) into the above equations, we obtain the system of two first-order non-linear differential equations describing out-of-equilibrium dynamics in the long-run neo-Kaleckian model:

$$\begin{cases} \dot{\alpha} = & \beta\mu\left[\dfrac{(\alpha_t + z)v - (s+m)u_n}{s+m-\beta v}\right] \\ \dot{z} = & z_t\left[\overline{g^Z} - \alpha_t - \beta\left(\dfrac{(\alpha_t + z)v - (s+m)u_n}{s+m-\beta v}\right) + \delta\right] \end{cases} \tag{2.23}$$

As discussed by Lavoie (2016, p. 185-186), the system is dynamically stable "when there is short-run Keynesian stability as long as the effect of Harrodian instability is not overly strong". As showed in Appendix B.1, this implies that the system converges towards its long-run equilibrium when $s+m-\beta v > 0$ and $\mu < \dfrac{(s+m)u_n}{v} - \overline{g^Z} - \delta$. Moreover, it is worth stressing that – similarly to the Supermultiplier model - "the growth rate of autonomous expenditures cannot be too large, for otherwise the share of autonomous consumption expenditures would need to be negative" (Lavoie, 2016, p. 193). In our framework:

$$z^* > 0 \quad \Longrightarrow \quad \overline{g^Z} < \frac{(s+m)u_n}{v} - \delta \tag{2.24}$$

[9] For the discussion of the derivation and the economic rationale of the two adjustments, see Allain (2015, 2018), Lavoie (2016) and Skott (2017). Regarding the animal spirits adjustment, the paper adopts the specification suggested by Allain (2015, 2018) rather than the one put forward by Lavoie (2016), who expresses the adjustment in terms of the growth rate of α. However, as noted by Skott, (2017, p. 188) "There is no reason [...] to assume that the rate of change should be proportional to the level of γ [*α in the notation of this paper*] for any given discrepancy", as would result from Lavoie's specification ($\hat{\alpha} = \mu(g_{sr}^{I*} - \alpha_t) \Rightarrow \dot{\alpha} = \alpha_t\mu(g_{sr}^{I*} - \alpha_t)$).

2.3 Numerical Solution

Since an analytical solution to the two systems of differential equations cannot be found, the method of numerical integration is adopted. Accordingly, the first challenge is to provide a sound calibration of the models' structural parameters.

2.3.1 Parameter Calibration and Initial Values

Parameter values are set in accordance with the empirical evidences for the US economy in the post-war period, as well as in line with previous model calibrations.

The values assigned to the parameters are summarized in Table 2.1.

Table 2.1 Parameter values

Par.	Description	Value	Source
δ	Depreciation rate (annual)	0.084	Fazzari et al. (2020)
u_n	Normal rate of capacity utilization	0.8242	Setterfield and Budd (2011)
v	Capital-capacity ratio (annual)	0.9890	Author's calculation, based on Fazzari et al. (2020)
s	Propensity to save	0.5	Fazzari et al. (2020)
m	Propensity to import	0.17	Gallo (2022a), Girardi and Pariboni (2016)
γ	Sensitivity of the investment share to $u_t - u_n$	0.15	Nomaler et al. (2021)
β	Sensitivity of the investment rate to $u_t - u_n$	0.25	Allain (2021)
μ	Sensitivity of animal spirits to $u_t - u_n$	0.18	Author's calculation, based on Allain (2021)

Source: author's calculation, various sources

The value of the annual depreciation rate (δ) is taken from Fazzari et al. (2020). As discussed by the authors in their Supplementary Appendix, the value is consistent with the empirical evidences for the US economy. The normal rate of capacity utilization ($u_n = 82.42\%$) is set in accordance to Setterfield and Budd (2011). As regards u_n, it is worth mentioning that the value matches the empirical evidences for other advanced capitalist economies, e.g. it is relatively close to the value (0.8104) calculated by Gallo (2019).[10] The capital-capacity ratio (v) is also obtained from Fazzari et al. (2020). By using investment

[10] Consistent with the models presented in Section 2.2, treating the normal degree of capacity as parametric implies that it is not affected by temporary changes in demand. For a more detailed critical discussion of the notion of normal capacity, the interested reader may refer to Ciccone (1986); Kurz (1986). For an empirical support of the idea that normal utilization is exogenous to the level of demand, see Haluska et al. (2021b) and Haluska et al. (2021a).

data, the authors estimate a long-run capital-output ratio in the US equal to 1.2. Therefore, given $u_n = 82.42\%$, the capital-capacity ratio will be equal to $v = \frac{K}{Y_p} = \frac{K}{Y_n}\frac{Y_n}{Y_p} = 1.2 \times 0.8242 = 0.9890$. The benchmark values of the propensities to save and to import are set in accordance to the empirical evidence in the US economy as discussed by Girardi and Pariboni (2016), Fazzari et al. (2020) and Gallo (2022a).

Last, the supermultiplier-specific parameter γ – which measures the reaction of the investment share to changes in the utilization rate – is taken from Nomaler et al. (2021). The remaining parameters β and μ (specific to the neo-Kaleckian model) are both taken from Allain (2021).[11] Since these sensitivities greatly influence the numerical solution of the two systems of differential equations under scrutiny, more attention should be given to them. Therefore, the next section will assess how changing the value of these parameters affects the time of adjustment in the two models.

A discussion of the choice of the initial conditions is now in order (Table 2.2). First, we ought to recall that the main goal of the exercise conducted in this Section is to show the persistence of out-of-equilibrium dynamics following an increase in autonomous-demand growth. Accordingly, let us suppose that prior to the shock the economy was in its fully-adjusted position $u_0 = u_n = 82.42\%$, growing at an exogenously given annual growth rate of autonomous demand of 2.5%.[12] Accordingly, from equation (2.4) and (2.5) and on the basis of the parameter calibration discussed above, it follows that the initial value of the investment share (h_0) in the Supermultiplier model is equal to 13.08% – in line with the empirical evidences for the US economy (Fazzari et al., 2020; Gallo, 2022a; Girardi and Pariboni, 2016). Similarly, equations (2.17) and (2.18) imply that $z_0 = 43.93\%$ and $\alpha_0 = 10.9\%$ in the amended neo-Kaleckian model .

[11] It ought to be noted that the value of μ in the present calibration exercise is slightly above the one in Allain (2021), who sets its value equal to $\mu = 0.4z^*$. Accordingly, since in this paper the derived equilibrium autonomous demand-capital ratio (z^*) is higher, μ will be higher as well.

[12] The value of the year-on-year growth rate of autonomous demand is taken from Fazzari et al. (2020). It is worth noting that this is consistent with the empirical evidences for the US economy; according to the definition of autonomous demand used by Girardi and Pariboni (2016), the average annual growth rate of the variable in the US for the period 1979-2013 is just slightly higher (2.54%).

Table 2.2 Initial conditions

Variable	Description	Value
g_0^Z	Autonomous demand (annual) growth rate	0.035
u_0	Capacity utilization rate	0.8242
α_0	Animal spirits	0.109
z_0	Autonomous demand-capital ratio	0.4493
h_0	Investment share	0.1308

Source: author's calculation

At time $t = 0$, the annual growth rate of autonomous demand permanently increases from 2.5% to 3.5% (e.g. as a consequence of an increase in government spending), thus affecting the long-run growth path of both models and giving rise to the long-run traverse discussed in the next subsection.

2.3.2 How Long is the Long Run?

This subsection shows by means of numerical integration the behavior of the two models following a a permanent increase in the growth rate of autonomous demand, relying on the calibration summarized in Tables 2.1 and 2.2.

Before moving to the discussion of the simulation results, an important consideration is in order. Throughout the paper, calendar time is considered at a yearly frequency. At this stage, the reader may well wonder about the legitimacy of interpreting one time step as one year. The reason comes from the calibration itself, which is carried out so as to ensure that all relevant variables and growth rates are compatible with yearly processes (e.g. autonomous demand growth is around 2.5% per year). Accordingly, under the assumption that the long-run adjustment of capacity to demand does not occur faster than the unit period considered (Gandolfo, 2012), the yearly calibration allows to coherently interpret the out-of equilibrium trajectories in calendar time with $dt = 1$ year, as more extensively discussed by Gallo (2022a).

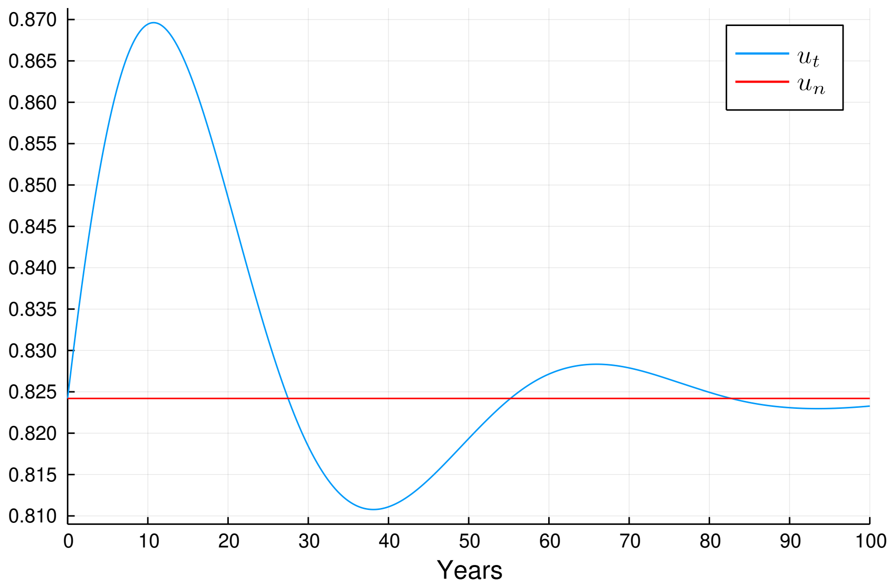
The Long-Run Convergence in the Supermultiplier Model

As discussed in Subsection (2.2.1), the two adjusting variables of the Sraffian Supermultiplier model are the rate of capacity utilization and the investment share. In the long-run steady-state, they should come back, respectively, to the normal rate of capacity utilization (u_n) and to the equilibrium investment share (h^*) given by equation (2.4).

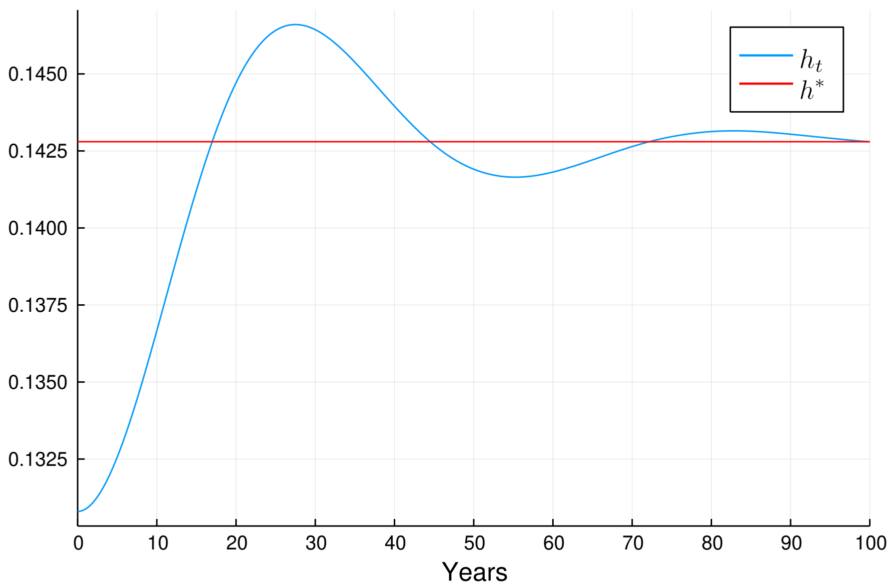
Figure 2.1 shows the behavior of the two adjusting variables in the long run, following an increase in the growth rate of autonomous demand at time 0, from 2.5% to 3.5%.[13].

[13]It is worth noting that Freitas and Serrano (2013, p. 41) report a graph that is very similar to the ones below. However, they express time as logical indexes ($t_0, t_1, ...$) instead of historical time (months, quarters, years, etc.).

Figure 2.1 The long run in the supermultiplier model



(a) The long-run dynamic of the rate of capacity utilization



(b) The long-run dynamic of the investment share

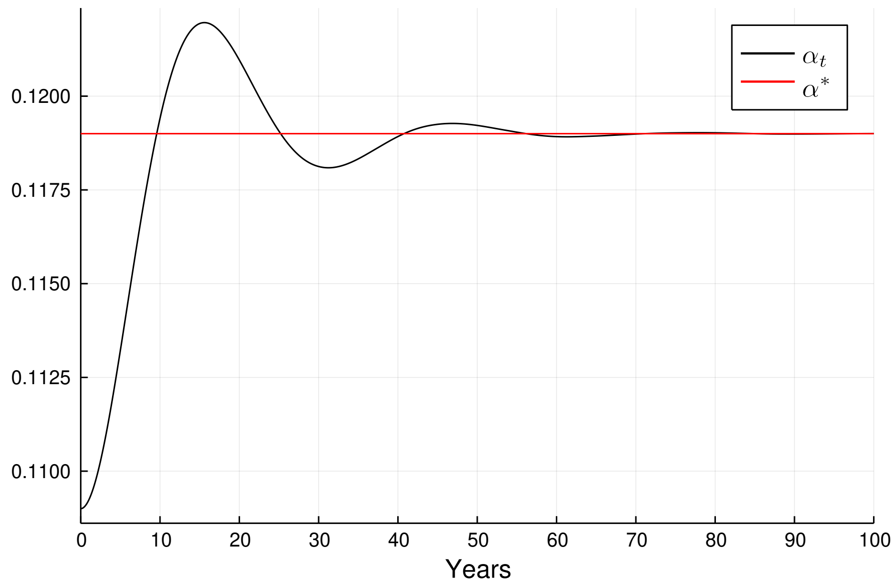
Following the permanent autonomous demand shock at time 0, output will increase as well, and hence entrepreneurs will push more on the utilization of productive capacity. More specifically, in the first phase of the long-run traverse, the output growth rate will be greater than the accumulation rate, i.e. $g_t^Y > g_t^K$. Following the demand shock, entrepreneurs will thus increase their utilization of productive capacity ($\dot{u} > 0$) through equation (2.7). In this first phase, the economy will be characterized by a situation of above-normal utilization ($u_t > u_n$), triggering the investment share adjustment ($\dot{h} > 0$), as per equation (2.8). The gap between the accumulation rate and output growth is closed only after a period of about 10 years, after which the actual rate of capacity utilization starts to decrease again towards the normal rate ($\dot{u} < 0$). However, as long as the gap between u_t and u_n remains positive, the investment share will keep rising. The investment share peak is reached only after more than 25 years, corresponding to a temporary situation of normal utilization. However, as long as the actual rate of utilization keeps decreasing ($\dot{u} < 0$), the economy will enter a period of under-utilization of productive capacity, which leads in turn to an investment share adjustment of reverse sign.

The economy proceeds through damped oscillations following the pattern described above, converging towards its long-run equilibrium position. Only after about 50 years do the dynamics of the rate of capacity utilization and the investment share begin to stabilize around their steady-state values. Generally speaking, the simulation postulates that it takes a very long period of time for the model to settle down in the fully-adjusted equilibrium.

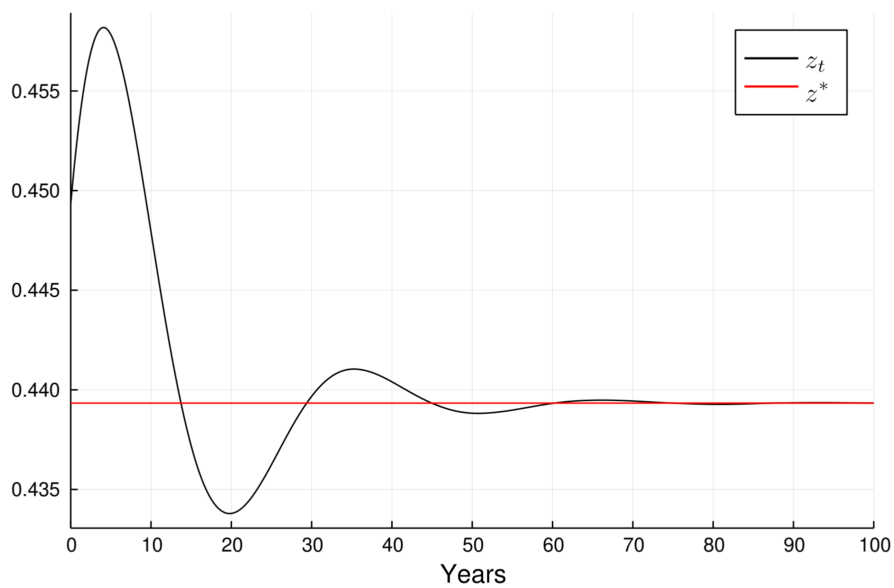
The Long-Run Convergence in the Amended neo-Kaleckian Model

In the model presented in Subsection (2.2.2), the two adjusting variables are the autonomous demand-capital ratio (z_t) and the animal spirits proxy variable (α_t). In the steady state, their values are given by equations (2.17 and 2.18). Figure 2.2 shows the behavior of the two adjusting variables in the long run, following the same increase in the growth rate of autonomous demand described above.

Figure 2.2 The long run in the amended neo-Kaleckian model



(a) The long-run dynamic of animal spirits



(b) The long-run dynamic of the autonomous demand-capital ratio

An increase in the growth rate of autonomous expenditures above the the accumulation rate will generate an increase in the value of the autonomous demand-capital ratio, i.e. $\dot{z} > 0$ via equation (2.22). Entrepreneurs will hence absorb the demand boom by pressing additional capital resources into productive use, resulting in an increase in the short-run rate of capacity utilization (u^*_{sr}). As u^*_{sr} rises above the normal rate of capacity utilization, the Harrodian mechanism (equation 2.21) will be activated, resulting in an increase in animal spirits $\dot{\alpha} > 0$. At the same time, the increase in u^*_{sr} will compensate the effect of the higher autonomous demand growth rate, gradually closing the gap between $\overline{g^Z}$ and g^K_{sr}. When the latter exceeds the former after about 5 years, the z ratio will begin its descent towards its long-run position. In this time span, α will keep rising until the discrepancy between the short-run utilization rate and the normal rate remains positive; however, as $\dot{z}$ is now negative, the gap between u^*_{sr} and u_n is shrinking. Under the parameter constellation discussed above, it takes about 20 years for this gap to be closed after an initial 1% increase in g^Z_t. After the actual rate of capacity utilization has fallen short of the normal rate, the Harrodian mechanism will work in the opposite direction, i.e. $\dot{\alpha} < 0$. The process will go on until both α_t and z_t stabilize around their long-run steady-state values at which point the traverse will end.

Even though the long-run traverse is somewhat shorter than that of the supermultiplier model, vicinity of the new equilibrium is reached after a period of more than 30 years. In other terms, when evaluated in historical time, both the supermultiplier and the amended neo-Kaleckian model share a very slow pace of adjustment. The asymptotic convergence to the fully-adjusted equilibrium is a sluggish one.

2.4 Sensitivity Analysis

As noted earlier in the paper, the existing literature has already extensively discussed issues related to the stability of the long-run equilibria of the two models.[14] Therefore, this section will confine itself to assess the sensitivity of the models' speed of adjustment when the parameter space is modified.

In order to assess the speed of convergence of a system of differential equation, there exist known analytical methods based on eigenvalue computation. For instance, Gabaix et al. (2016) use the dominant eigenvalue, i.e. the largest in absolute value, to provide a convenient description of the speed of the dynamic

[14]See Appendix B.1 for the derivation of the stability conditions of both models.

adjustment. However, this method is not available for the models under scrutiny. As showed in Appendix B.1, the eigenvalues for both models are complex with nonzero imaginary parts and hence cannot be ordered.

Therefore, the sensitivity analysis would need to rely on numerical methods only. In order to do that, a convenient visualization tool is provided in the Online Appendix B.3 of this paper. With the aid of a web app, the interested reader could easily perform a re-parametrization of the two models, within the broad ranges reported in Table 2.3.

Table 2.3 Parameters and exogenous variables - minimum and maximum values

Par.	**Description**	**Min. Value**	**Max. Value**
g^Z	Autonomous demand growth	0.01	0.12
δ	Depreciation rate (annual)	0.01	0.2
u_n	Normal rate of capacity utilization	0.5	1
v	Capital-capacity ratio (annual)	0.7	3
s	Propensity to save	0.2	0.6
m	Propensity to import	0	0.3
γ	Sensitivity of the investment share to $u_t - u_n$	0	1
β	Sensitivity of capital formation to $u_t - u_n$	0	1

Source: author's calculation

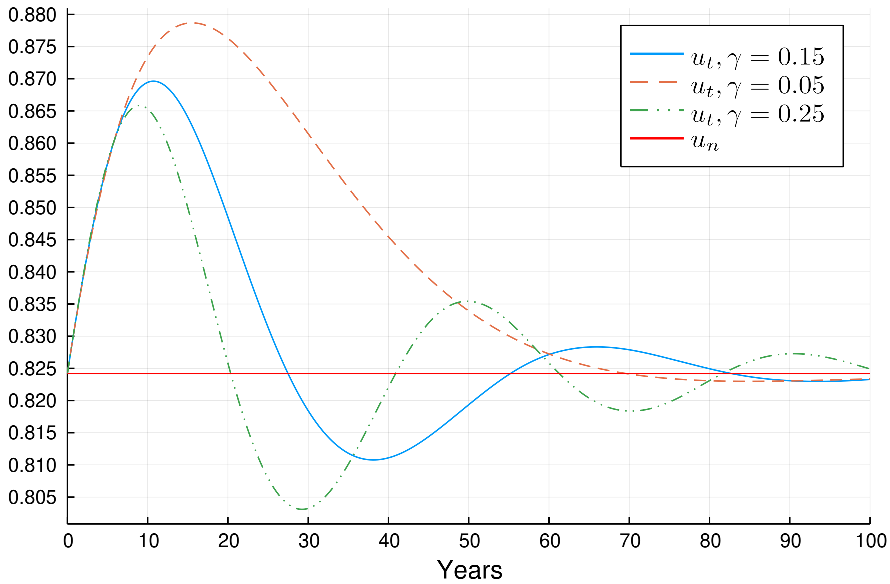
While all parameters influence – to different degrees – the *magnitude* of the dynamic adjustments and the stability of the long-run equilibria, one could easily verify that the two reaction coefficients γ and β are the only ones that sensibly influence the *speed* of adjustment of the Supermultiplier and of the amended neo-Kaleckian model, respectively. Unfortunately, these two parameters are exactly the ones for which we do not have sufficient empirical support. Whilst the existing literature provides a sufficiently solid ground to justify the baseline values for most parameters, these foundations become more shaky when it comes to the reaction coefficients, as also noted by Nomaler et al. (2021) for the Supermultiplier model.

Solving numerically the system for bigger and smaller values of γ and β would allow to assess how the two parameters affect the speed of the dynamic adjustment in the Supermultiplier and the amended neo-Kaleckian model, respectively.

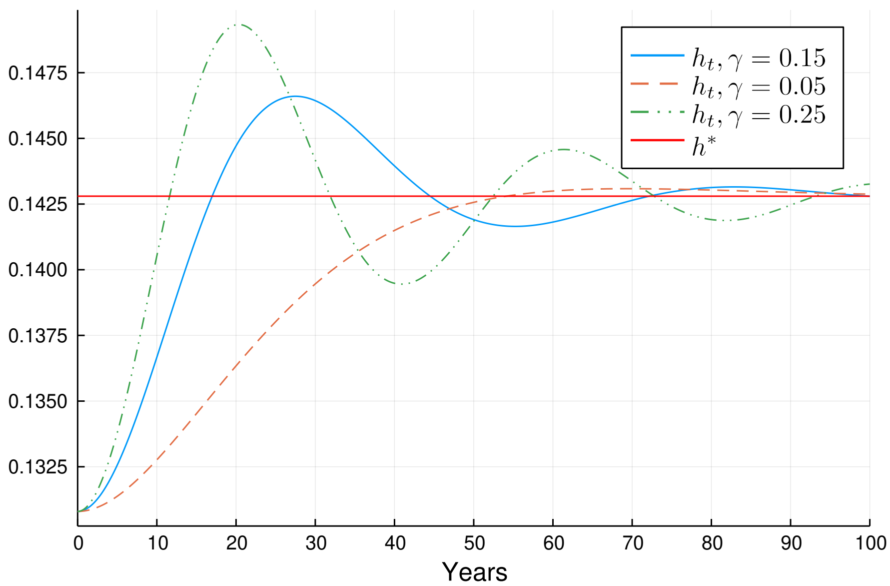
Let us start with the Supermultiplier (Figure 2.3). A reduction of γ from a baseline value of 0.15 to 0.05 stabilizes the system, making the adjustment slower but less persistent.[15] In the first phase of the long-run traverse, the increase of the rate of capacity utilization is bigger with $\gamma = 0.05$; after a 1% increase in the growth rate of autonomous demand, u_t peaks only after a period of about 15 years. Following that, the model slowly converges towards the fully-adjusted position. Moreover, this case provides a good occasion to stress another important point in traverse analysis of demand-led growth models (and hence valid both for the Supermultiplier and for the amended neo-Kaleckian model). As it can be seen, a reduction of γ (or similarly of s and m or an increase of u_n, u_0, $\overline{g^Z}$, v or δ) has the effect of increasing the maximum value reached by the rate of capacity utilization during the adjustment. At and near the peak, u_t may become bigger than one, which would be logically inconsistent (an economy cannot *at any time* achieve an income level higher than the maximum one determined by its potential). In this sense, as noted by Lavoie and Ramírez-Gastón (1997, p. 162): "to look at the requirements of the steady state is insufficient to assess whether or not the new steady state is possible; the rate of capacity utilization must also remain below unity at all times during the traverse". Let us now assess what happens in the opposite case, looking at the traverse trajectory corresponding to an increase in γ to a value of 0.25. In this case, the time required to approach the long-run equilibrium position after the initial shock decreases, while making the model less stable, with the dumped fluctuations being not completely absorbed after a period of more than 70 years.

[15]For sufficiently small values of γ, the eigenvalues becomes real and distinct, and the system could converge monotonically towards the long-run equilibrium. See Appendix B.1 for further discussion.

Figure 2.3 The long-run traverse in the supermultiplier model with changing γ



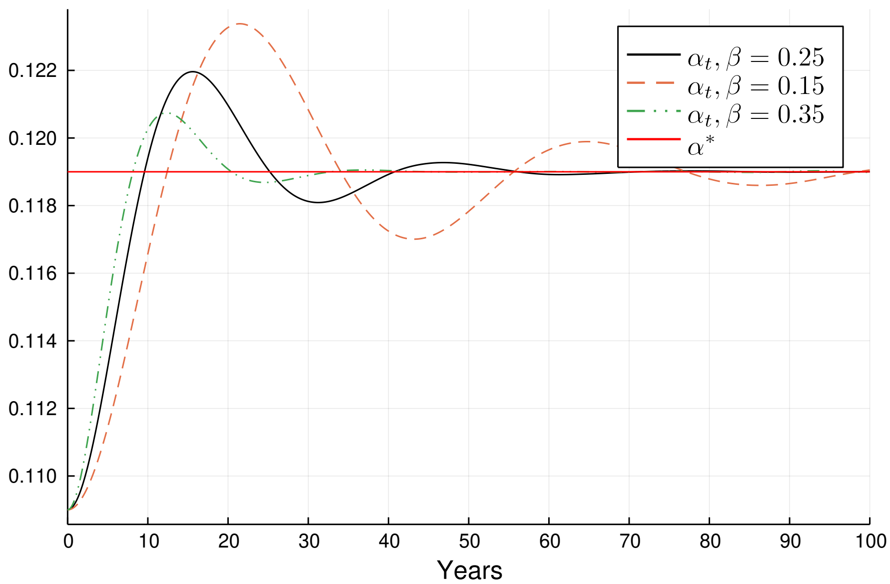
(a) The long-run dynamic of the rate of capacity utilization



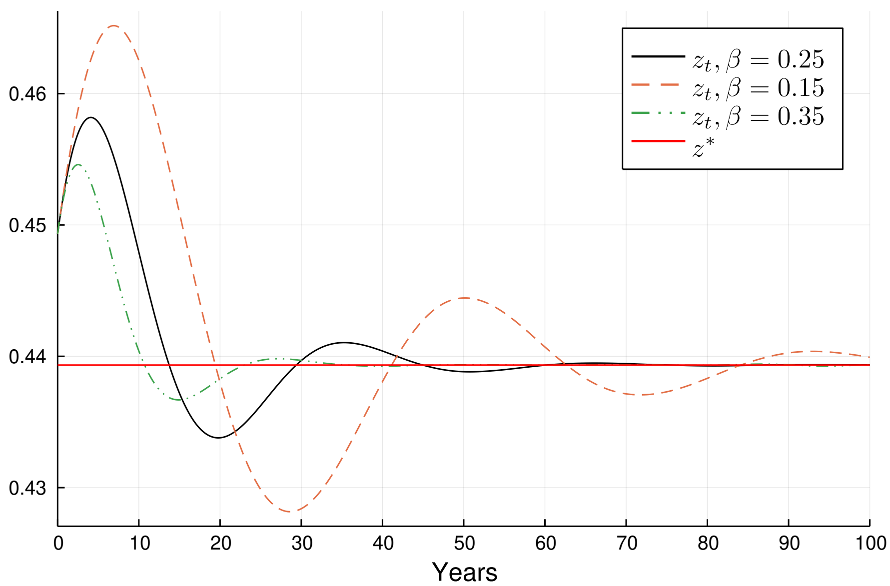
(b) The long-run dynamic of the investment share

Let us now discuss what happens in the Allain-Lavoie model if the sensitivity of accumulation to the discrepancy between the actual and the normal utilization rates (β) changes (Figure 2.4). In particular, if β increases from the baseline value of 0.25 to 0.35, the time of adjustment is reduced and the model stabilizes faster, approaching the long-run equilibrium in less than 30 years. Conversely, with a lower β of 0.15, the model becomes less stable and takes more time to converge. Similar to what has been discussed in the previous paragraph, with a lower β one must be careful about whether the traverse path ensures that the rate of capacity utilization remains below unity during the entire process. As all time steps over the long-run traverse correspond to a situation of short-run equilibrium in the goods market, this condition could be easily verified by computing the value of the rate of capacity utilization via equation (2.14). Besides β, the other parameters that determine the value of u^*_{sr} also ensure whether or not the rate stays below unity at all times. In particular, u^*_{sr} may rise above 1 with a lower s, m and u_n or with a higher α_0, z_0, v and δ.

Figure 2.4 The long-run traverse in the amended neo-Kaleckian model with changing β



(a) The long-run dynamic of animal spirits



(b) The long-run dynamic of the autonomous demand-capital ratio

In both cases, a change of ± 0.1 in both γ and β does not alter the general conclusion in Section 2.3 regarding the relative speeds of adjustment of the two models. Regardless, further econometric analysis would be needed to assess the size of these reaction parameters (provided the empirical soundness of the assumed adjustments), allowing to determine both whether the two models predict stability of the long-run equilibrium and to have reliable point estimates of the predicted adjustment to the fully-adjusted position.

2.5 Concluding remarks

The paper has attempted to answer Joan Robinson's (1980) question *'when is the long run?'*, evaluating in historical time the long-run traverse in two prominent demand-led growth models, namely the Supermultiplier model and the long-run neo-Kaleckian model. In doing so, it provided a description of the temporal sequence needed to achieve a new long-run position after an initial increase in the growth rate of autonomous demand. After presenting the two models, the paper discussed a reasonable calibration in line with the existing theoretical and empirical literature. The calibration allowed to provide a numerical solution of the systems of differential equations that regulate out-of-equilibrium dynamics in both models. The simulation exercise showed that the convergence to the fully-adjusted equilibrium is sluggish, with adjustment periods of about 50 and 30 years for the Supermultiplier and the neo-Kaleckian model, respectively. Furthermore, the analysis assessed how changes in the parameter space affect the adjustment periods in both models. More specifically, it showed that the reaction coefficients of the investment share (for the Supermultiplier) and of the investment rate (for the neo-Kaleckian model) affect sensibly the duration of the long-run traverse, leaving scope for further empirical research to derive point estimates of these coefficients.

A conclusion as tempting as naive that could be drew from the exercise is that – if interpreted as a length of time – *the long run may be longer than expected*. As the simulations presented in the previous sections have shown, the two models under scrutiny share a very slow pace of adjustment. In other terms, in historical time the adjustment period to a new steady-state position may be long enough to be economically meaningless.

The long run, however, is not a length of time, but a process. Accordingly, as Robinson (1965, p. 17) notes, "it is absurd, though unfortunately common, to talk as though 'in the long run' we shall reach a date at which the equilibrium corresponding to today's conditions will have been realized". In the length of time spanning from a change in today's conditions to the realization of a new equilibrium, further changes are likely to affect the growth process: history has a pervasive influence on the determination of economic outcomes and growth processes (Setterfield, 1995, 1997). Rather than focusing, as the existing literature sometimes does, on the mere comparison between two equilibrium positions, researchers should pay more attention to the properties that characterize the models' trajectories during the traverse, e.g. by discussing out-of-equilibrium growth effects, path dependency and so on (Morlin et al., 2021). The examination of the models' timescale and adjustment period is a fundamental piece of information and a key factor for understanding the relation between the theoretical framework and the real world. Very rarely this information is exploited for economic analysis and policy recommendations, with researchers and policy makers finding themselves more at ease with thinking in logical rather than historical time.

Lastly, it is worth stressing that the goal of this exercise has not been to quantify the actual duration of the traverse, but first and foremost to shed light on issues and methods that have not received the deserved attention by growth theorists. On the methodological side, the results presented in the paper have been derived by making use of numerical methods of analysis to solve two systems of differential equations that cannot be solved analytically. The mathematical tool is well known by economists and growth theorists, but neglected for the analysis of the traverse and out-of-equilibrium dynamics. Using more thoroughly these methods may result in a significant gain of explanatory power of the models used for the analysis of real-world economies.

3 Reduction of CO_2 Emissions, Climate Damage and the Persistence of Business Cycles: A Model of (De)coupling

La cause unique de la dépression
c'est la prospérité.

Schumpeter (1927, p. 7)

Abstract

Following the pioneering work of Goodwin (1951, The nonlinear accelerator and the persistence of business cycles, *Econometrica*), the paper presents a business cycle model encompassing the short-run effect of mobilizing green investment to achieve longer-term climate goals. The model assumes that cumulative CO2 emissions follow a logistic pathway; the carrying capacity of the system, however, may be above some tipping points after which major climate degradation occurs. After this threshold has been hit, climate damage and environmental degradation will have a negative impact on output and investment dynamics over the business cycle, implying that even a fast and strong green transition may not be sufficient to substantially reduce emissions. Furthermore, the paper focuses on calibration and simulation of the model in line with recent empirical evidences for the US, thus discussing the necessary conditions for which the investment channel contributes to greenhouse emissions reductions in line with the established targets.

Keywords: green investment, ecology-economy feedback, climate change, climate damage, business cycle

JEL codes: C53; C63; E17; E32; Q57

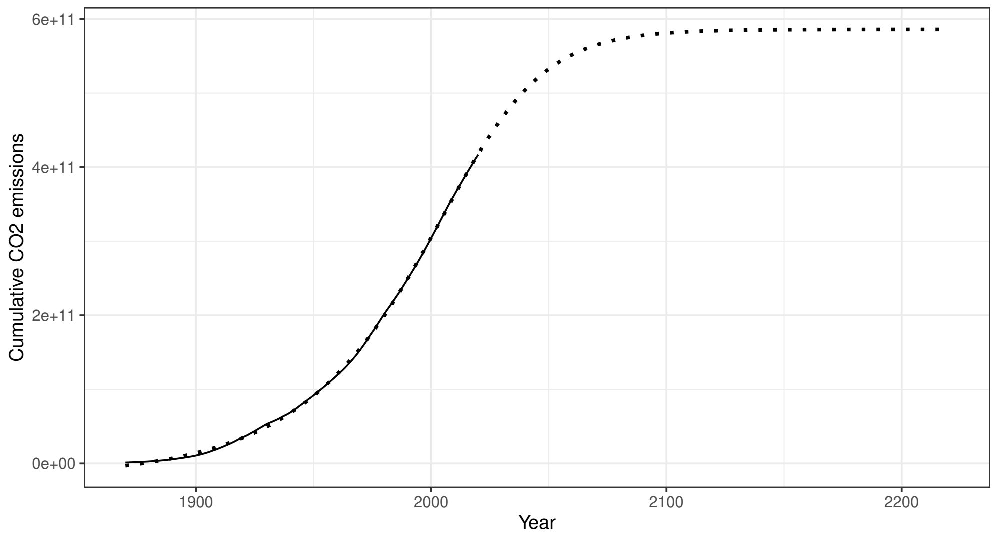
3.1 Introduction

The latest report of the IPCC (Intergovernmental Panel on Climate Change, 2022, ch. 1, p. 12) argues that “limiting human-induced global warming to a specific level requires limiting cumulative CO_2 emissions, reaching at least net zero CO_2 emissions, along with strong reductions in other greenhouse gas emissions”. In the path towards net zero, it seems likely that single countries and the world as a whole will follow a pathway similar to some sort of sigmoid function. After growing exponentially from the start of the First Industrial Revolution, human-induced emissions are likely to stabilize and eventually cease as low-carbon emitting technologies and circular products become fully integrated in the economy. In this respect, the case of advanced economies already provides some preliminary evidence of such dynamics. For instance, looking at the case of the U.S. (figure 3.1), it appears that there is strong evidence that the time series of cumulative CO_2 emissions may conform to a logistic trend.

However, the carrying capacity of the system (corresponding to net zero emissions), is likely to be beyond some tipping points after which major climate degradation occurs. Moreover, *rebus sic stantibus*, the time span needed to achieve the established targets might take longer than expected. Therefore, a key policy priority should be to ‘flatten the curve’ of cumulative emissions, achieving the net zero targets established by the Paris Agreement by speeding up the transition towards green investment and technologies.

While the long-term effects of the green transition have been extensively studied, little attention has been given to the interactions between green investment, climate change and environmental degradation over the business cycle. Accordingly, the goal of this paper is to build a stylized business cycle model encompassing green investment and climate damage, inspired by the pioneering work of Goodwin (1951). In particular, the analysis seeks to assess how the interactions between the dynamic multiplier and the non-linear accelerator principle may explain short-run economic fluctuations, as well as to assess the effect of the green transition by modeling the shift towards green investment. Moreover, the paper aims at incorporating the negative effect of climate damage and environmental degradation on business cycle fluctuations. Last, the paper

Figure 3.1 Cumulative CO_2 emissions (solid) and logistic fit (dotted) in the United States



Source: author's representation, see Appendix C.3

seeks to calibrate and simulate the model in light of recent empirical evidences for the US, showing how the developed framework allows to draw policy implications regarding the strength and speed required for green investment to substantially reduce greenhouse emissions in line with the established targets.

The article proceeds as follows. After the literature review in section 3.2, the model is presented in section 3.3, discussing the assumed dynamics and mutual feedback mechanisms between macroeconomic dynamics, green and brown investment, and CO_2 emissions. Section 3.4 and 3.5 discuss parameter calibration and model simulation, respectively. The effect of climate damage is embedded in the baseline model in section 3.6, further discussing and simulating its effects. Last, section 3.7 concludes, summarizing the key arguments and discussing scope for further research.

3.2 Literature Review

3.2.1 Studies on Absolute and Relative Decoupling

The existing literature distinguishes between absolute and relative decoupling of economic and environmental variables. In particular, absolute decoupling (also called de-linking) occurs when economic growth does not lead to increased environmental impact, i.e. "resource use declines, irrespective of the growth rate of the economic driver" (UN Environment Programme, 2011, p. 5). Conversely, relative decoupling occurs when resource use increases at a slower rate compared to a given economic variable of interest; the association between the environmentally relevant variables and the economically relevant ones is still positive, but smaller than 1. In general, studies on the extent of decoupling focus on the relation between greenhouse gas emissions (in particular CO_2 emissions) and GDP growth.

Recent contributions by Wiedenhofer et al. (2020) and Haberl et al. (2020) provide a comprehensive survey of the existing literature on decoupling of GDP and greenhouse gas emissions. More specifically, Wiedenhofer et al. (2020) conduct a full-text analysis of 835 empirical studies on the relationship between economic growth (GDP), resource use (materials and energy) and greenhouse gas emissions, finding that almost half of the articles surveyed are based on single-country analysis (mostly on China and the US). They also note that while the literature is dominated by a variety of rather sophisticated modeling and econometric methods, the "statistical complexity of the method of analysis does not automatically translate into more robust insights" (Wiedenhofer et al., 2020, p. 12). Building upon the bibliometric mapping of Wiedenhofer et al. (2020), Haberl et al. (2020) show that the empirical literature frequently finds evidence of relative decoupling between economic growth and greenhouse gas (often CO_2) emissions. Conversely, absolute long-term decoupling is empirically rarely found and "generally only occurs during periods of low GDP growth" (Haberl et al., 2020, p. 32). Recent studies published after the bibliometric mapping of Wiedenhofer et al. (2020) also find similar results. In particular, Cohen et al. (2022) analyze the trends and cyclical behaviors of CO_2 emissions and GDP in the twenty major emitting countries. They show that there is empirical evidence of statistically significant relative decoupling in all but two cases – i.e., Thailand and Vietnam – while absolute long-term decoupling can be observed only in few advanced economies (France, Germany and the UK).

3.2.2 Business Cycles and the Cyclical Properties of Emissions

Over the past years, macroeconomic research has been characterized by a renewed interest in endogenous business and financial cycles (Beaudry et al., 2020; Borio, 2014; Stockhammer et al., 2019). In particular, a recent contribution by Beaudry et al. (2020, p. 42) has provided support to the idea business cycle phenomena are strongly endogenous, rather than being caused by persistent exogenous disturbances:[1]

> [...] it could be that the economy is locally unstable, or close to unstable, in that there are not strong forces that tend to push it towards a stable resting position. Instead, the economy's internal forces may endogenously favor cyclical outcomes, where booms tend to cause busts, and vice versa. (Beaudry et al., 2020, p. 42)

This revived interest in business cycle analysis has mostly focused on the interplay between financial and real cycles, while relatively little efforts have been made to model the relation between economic variables and greenhouse emissions over the cycle. Against this background, the empirical evidences suggest that the interaction between emissions and GDP over the business cycle is macro-critical, deserving further attention. More specifically, Doda (2014) builds a database of CO_2 emissions, GDP and GDP per capita for 122 countries,

[1]The idea that business cycles are the result of endogenous rather than exogenous forces is far from new. Without pretending to be exhaustive, it is worth referencing nonlinear deterministic theories of cycles in the product market. Following the early applications of Le Corbeiller (1933) and Frisch (1933), this strand of the literature has been developing with the seminal contributions of Kalecki 1954), Samuelson (1939) and Kaldor (1940), who tried to couple the dynamic properties of the Keynesian multiplier with different forms of investment acceleration. The literature has been particularly flourishing in the 50s, with the notable contributions of Hicks (1950) and by Goodwin (1948, 1950, 1951), which constitute the backbone of the model presented in section 3.3. For an extensive review of the literature, see Semmler (1986) and Semmler (1994).

showing four salient facts: (i) emissions are typically procyclical; (ii) the correlation between emissions procyclicality and GDP per capita is positive; (iii) emissions tend to be more volatile than GDP over the cycle; (iv) the correlation between the cyclical volatility of emissions and GDP per capita is negative. Other country-case studies have focused on the asymmetric properties of the cyclical relation between output and emissions. In particular, Klarl (2020) studies the response of CO_2 emissions to the business cycle in the U.S. using monthly data from 1973 to 2015. By using four different filtering techniques, the author shows that emissions elasticity are above one in times of recession and below one in normal times. A similar asymmetry for the case of the U.S. is also documented by Sheldon (2017) and Gozgor et al. (2019). Along these lines, the recent contributions of Cohen et al. (2018, 2019, 2022) have attempted to distinguish between trends and cycles in emissions and GDP, using national data for the world's 20 largest emitters (Cohen et al., 2018), aggregate and provincial Chinese data (Cohen et al., 2019), and cross-country data for 178 countries (Cohen et al., 2022). In particular, they distinguish between the Environmental Kuznets Curve (trend component) and the Environmental Okun's Curve (cyclical component), separating them by means of different filtering techniques. In their most comprehensive study (Cohen et al., 2022), the authors find that, on one hand, advanced developed economies show evidence of long-term decoupling, while on the other the strong cyclical relation between emissions and real GDP growth appears not to have weakened neither in advanced nor in developing economies.

Against this background, the model presented in section 3.3 aims at capturing the role of green investment in explaining the strong and asymmetric relation between CO_2 emissions and the business cycle, further discussing the conditions for longer-term decoupling.

3.2.3 Climate Damage and Tipping Points

As noted by Lenton (2011, p. 201), "a climate 'tipping point' occurs when a small change in forcing triggers a strongly nonlinear response in the internal dynamics of part of the climate system, qualitatively changing its future state". In particular, human-induced climate change builds in small changes in the ecosystem that accumulate over time, which after a critical threshold produce large-scale singular events of irreversible nature. Examples of climate tipping points are the melting of the Greenland ice sheet, as well as coral reefs bleaching and the deterioration of the Amazon rainforest.

While there is a broad consensus on the human-induced nature of climate damage, the likelihood of these large-scale tipping events is fundamentally uncertain.[2] This is reflected by the variety of different approaches to modeling environmental degradation, damage and tipping points in climate economics. More specifically, the relevant question for the sake of our discussion relates to the deterministic vis-à-vis stochastic nature of climate tipping points. As documented by Lontzek et al. (2015) while focusing on Integrated Assessment Models (IAMs), most approaches model tipping points as fundamentally deterministic, whereas only few contributions include stochastic climate damages. Furthermore, most studies assume that the impact of a tipping event is instantaneous, "whereas in reality impacts will accumulate over time at a rate determined by the dynamics of the system that has been tipped" (*ibid.*, p. 441).

Regarding deterministic treatments of climate disasters, Crépin and Nævdal (2019) provide a modeling framework for endogenous catastrophic risk, engendering path dependent outcomes as a result of the delays between physical variables and the hazard rate. Mittnik et al. (2020) present a more comprehensive macroeconomic model of climate change encompassing climate disasters risk; more specifically, the authors build a multi-phase dynamic model showing that disaster events deterministically give rise to capital losses and increased risk premia, thus calling for adaptation policies to reduce the economic vulnerability to these events. Conversely, few examples of papers adopting a stochastic approach to climate damage and tipping points are Dumas and Ha-Duong (2005), Cai et al. (2013) and Lemoine and Traeger (2014).

[2]For an empirical estimation of the economic costs associated with extreme whether events see Frame et al. (2020).

3.2.4 Environmental Research and the Limits of Economic Modeling

Part of the existing literature on climate change and the climate-economy literature is also (rightly) concerned with the inherent limits of economic modeling in the context of climate change. Given the theoretical purpose of this contribution, it is worth to briefly discuss these challenges, understanding the limits of the analysis and the scope for further research.

As noted by Bonen et al. (2014, p. 45), "the most understandable and unavoidable concern of any climate model is the (im)precision of its formal representation of natural processes" and, in particular, the lack of expertise by economists in modeling and estimating climatic phenomena. These difficulties are also acknowledged by the most eminent voices of what we might call mainstream climate economics.

In particular, Nordhaus (2013) takes a significant step-back compared to past efforts in the development of his DICE model, by acknowledging the limits of programming the ecosystem in Integrated Assessment Models. The inclusion in the model a single damage function forces to rely on other's estimates; in other terms, the approach taken by Nordhaus (2013) in designing DICE 2013R implies that economists should confine themselves with the mere reparametrization of their models as climate science advances, rather than making efforts to incorporate more complicated damage functions as the environment changes.[3]

As argued by Bonen et al. (2014), this stance puts economic models in a subordinate position, but at the same time the approach they advocate allows for a more efficient division of labor between disciplines. While "[m]odel inaccuracies are, of course, not eliminated by reducing complexity" (Bonen et al., 2014, p. 49), the simplification of climate change models allows to capture in a more straightforward way the key element of interest in the complex economy-environment relation. In this sense, it is probably worth recalling

[3]At the same time, this approach may undermine the empirical estimation of the model, as parametrizations usually need to rely on outdated country-/sector-/region-specific studies. As (Tol, 2009, p. 38) notes in his review of the literature, "[e]stimates are often based on extrapolation from a few detailed case studies, and extrapolation is to climate and levels of development that are very different from the original case study. Little effort has been put into validating the underlying models against independent data".

Robinson and Eatwell (1973, p. 54) remark: "models must be simplified. A map at the scale of 1:1 is of no use to a traveller. The art of setting up models is to cut out all complications inessential to the point at issue, without eliminating the features necessary for safe guidance."

In some respect, the model presented in the next section moves along these lines; while not pretending to be a realistic model of the environment – especially because it does not model energy use – it aims at capturing in a simple framework the complex interaction emerging from the interaction between business cycles, green investment, and CO_2 emissions.

3.3 The Model

This section presents a stylized business cycle model aimed at capturing the interactions over the cycle between output, green and brown sources of investment, and CO_2 emissions. The subsections below will deal in a point-by-point manner with the building blocks of the model. The baseline model merely focuses on business cycle fluctuations, thus abstaining from the consideration of long-run economic growth.[4]

3.3.1 Output determination and the dynamic multiplier

In his 1951 *Econometrica* article, Richard Goodwin presents a business cycle model combining the dynamic multiplier with a non-linear accelerator mechanism.[5] The author presents different versions of the model, starting from a simple (linear) version and subsequently relaxing the assumptions to allow for nonlinearities. While it is not useful here to present the entire derivation of the Goodwin (1951) model, it is worth recalling its final equation for output

[4] In this respect, the model is similar to the contribution of Dejuán et al. (2022). Of course, abstaining from the consideration of long-run growth does not imply denying its importance. In particular, it is worth noting that economic growth may hinder the goal of flattening the curve of CO_2 emissions, given that empirical studies usually find that growth is positively correlated with emissions (Cohen et al., 2018, 2022). Appendix C.2 proposes two possible ways to extend the model so as to reconcile trend and cycle of real output. See Greiner et al. (2010) for a growth model that allows for global warming and abatement spending.

[5] The first continuous-time formulation of the dynamic multiplier can be found in Goodwin (1948).

determination over the business cycle.[6] With the principle of the dynamic multiplier at work,[7] output is assumed to evolve in line with the following forced second-order differential equation:

$$\ddot{Y} = \frac{1}{\epsilon\theta}\left[\overline{Z} + \dot{\kappa} - (s+m)Y - (\epsilon + (s+m)\theta)\dot{Y}\right] \tag{3.1}$$

where:

- $\overline{Z}$ is given by the sum of all autonomous components of demand, i.e. autonomous consumption, autonomous investment, government spending, and exports. Since we are interested in business cycle fluctuations, we abstain from economic growth by assuming a constant level of autonomous demand, i.e. $Z = \overline{Z}$;
- $\dot{\kappa}$ is private induced investment. As we will see in the next section, it depends on the accelerator principle;
- s is the economy-wide tax-adjusted propensity to save;
- m is the propensity to import;
- ϵ is the time-lag of the dynamic multiplier;
- θ is the investment lag. More specifically, it captures the time elapsing "between decisions to invest and the corresponding outlays" (Goodwin, 1951, p.11).[8]

The underlying idea of equation 3.1 is that the multiplier process takes time. As noted by Goodwin (1948, p. 112), it is difficult to see how the assumption of an instantaneous multiplier – as in the standard Keynesian output-expenditure model – could be tenable as a realistic modeling device. Instead, he argues that a certain time lag is always necessary:

> Money spent has no mystical virtue by which it stretches itself in the spending. As it circulates through the economy, it gradually

[6]See Appendix C.1 for the full derivation of the output equation.

[7]Equation 3.1 is obtained by rearranging equation (5e) in Goodwin (1951) for an open-economy with government activity. It is alike the first equation presented by Sordi (2006), with only minor differences concerning the notation. For derivation and further discussion, see Appendix C.1.

[8]The notation for all variables and parameters used in the text is summarized in Appendix C.6.

produces further incomes, but only over a period of time (Goodwin, 1948, p. 112).

The amount of time necessary for income propagation throughout the economy via the multiplier depends on the value of the lag ϵ: the bigger is ϵ, the more gradual will be the adjustment of supply to demand. Moreover, as discussed more extensively in Appendix C.1, another lag (θ) is introduced by Goodwin (1951) to capture the time that elapses between the decision to invest and the actual purchase of new capital goods. The value assigned to θ will be most likely affected by institutional as well as behavioral factors: the bigger is θ, the more time will be needed for investment decisions to be realized.

A more thorough discussion of investment dynamics is in order in the next subsection.

3.3.2 Investment and capital dynamics

As argued by Sordi (2006), the nonlinear accelator principle discussed by Goodwin (1951) can be well approximated by a piecewise linear investment function of the type:

$$\dot{\kappa} = \begin{cases} \dot{\kappa}^* \ \text{ if } v\dot{Y} > \dot{\kappa}^* & \text{CEILING} \\ v\dot{Y} \ \text{ if } \dot{\kappa}^{**} \leq v\dot{Y} \leq \dot{\kappa}^* & \text{MID-RANGE} \\ \dot{\kappa}^{**} \text{ if } v\dot{Y} \leq \dot{\kappa}^{**} & \text{FLOOR} \end{cases} \tag{3.2}$$

where v is the acceleration coefficient.

In line with Goodwin (1951), net private induced investment depends on the acceleration principle ($\dot{\kappa} = v\dot{Y}$) over some middle range, but passes to complete inflexibility when a floor ($\dot{\kappa}^{**}$) or a ceiling ($\dot{\kappa}^*$) are hit.[9] For the sake of simplicity, capital is assumed to be non-depreciating, so that capital dynamics correspond to investment dynamics.

Assuming that the floor differs – in absolute value – from the ceiling implies the possibility of distinguishing in a rather simple way between booms and slumps. As noted by Goodwin (1951, p.4), "with nonlinear theory we may

[9] For a detailed review of the literature on the accelerator principle, see Zambelli (2011).

make the depression as different from the boom as we wish", thus allowing to account for the asymmetric properties of the business cycle discussed by the empirical literature. In particular, it is usually found that downturns are sharper than expansions, i.e. the cycle is characterized by "relatively small amplitude during mature expansions and substantial variation during and immediately following recessions" (Morley and Piger, 2012, p. 208).

Unlike the previous literature, the paper distinguish between two sources of induced investment, i.e. investment in green and brown technologies. Therefore, total investment ($\dot{\kappa}$) is given by the sum of green ($\dot{\kappa}_{green} = \alpha\dot{\kappa}$) and brown investment ($\dot{\kappa}_{brown} = (1-\alpha)\dot{\kappa}$), both considered as shares of $\dot{\kappa}$:

$$\dot{\kappa} = \alpha\dot{\kappa} + (1-\alpha)\dot{\kappa} \tag{3.3}$$

The share of green investment in total investment (α) is treated as time-dependent, as we shall see in the next section.

These considerations give rise to a slightly more complex investment function:[10]

$$\dot{\kappa} = \begin{cases} \dot{\kappa}^* & \text{if } [\alpha v_g + (1-\alpha)v_b]\dot{Y} > \dot{\kappa}^* \\ [\alpha v_g + (1-\alpha)v_b]\dot{Y} & \text{if } \dot{\kappa}^{**} \leq [\alpha v_g + (1-\alpha)v_b]\dot{Y} \leq \dot{\kappa}^* \\ \dot{\kappa}^{**} & \text{if } [\alpha v_g + (1-\alpha)v_b]\dot{Y} < \dot{\kappa}^{**} \end{cases} \tag{3.4}$$

where v_g is the acceleration coefficient of green investment and v_b the acceleration coefficient of brown investment. At this stage, it is important to note that these coefficients will usually differ, i.e. $v_g \neq v_b$. In other terms, the two types of investment need not to be equally responsive to output changes.

3.3.3 The dynamics of green and brown investment

The model assumes that, as a consequence of technology diffusion and adoption, green investment will gradually replace over time investment in brown capital.

[10] At the current stage, a piecewise linear function may serve as a good approximation of Goodwin's investment theory. However, it ought to be noted that more complex investment functions could be used to better model the non-linearity of investment. See e.g., Zambelli (2015).

This modeling choice is based on the Schumpeterian notion of swarming of innovations (Nakicenovic and Grübler, 2013; Perez, 2003). In particular, innovation and the structural change associated with it will lead to a gradual reduction of investment in (brown) carbon-intensive capital in favor of greener technologies in two fundamental ways: (i) carbon-intensive production processes become outdated, making no longer convenient to invest in them; (ii) the availability of convenient low-carbon emitting technologies will enhance investment in them, greening previously carbon-intensive production processes.[11]

The significance of the 'swarm' of innovation and the modeling tool associated with it is well captured by Goodwin (1990, p. 86):

> There is considerable agreement that the archetypical innovation begins very weakly; then gradually proves its worth, becomes better known, along with improved design and adaptation to diverse uses; finally it decelerates gradually as it is completely integrated into the economy. Thus it tends to have a quadratic trajectory, happily represented by the logistic.

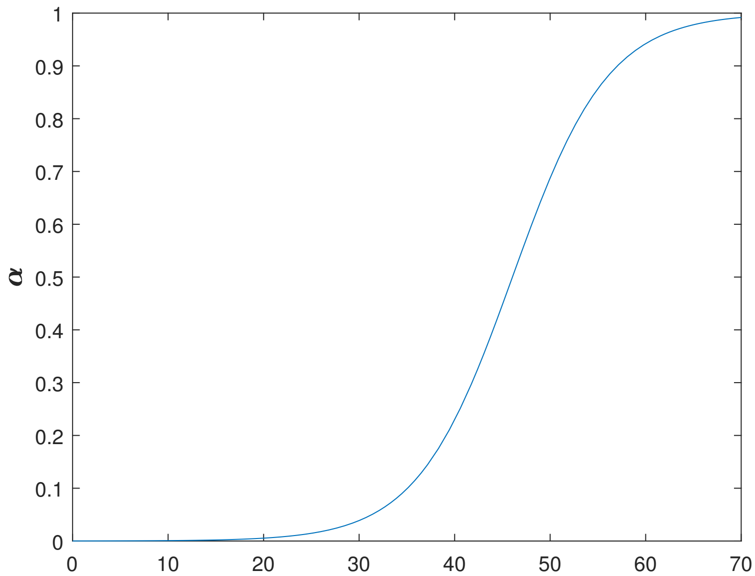
Accordingly, the share of green investment in total investment is assumed to evolve according to the following logistic differential equation:

$$\dot{\alpha} = r\alpha(1-\alpha) \text{ with } 0 \leq \alpha \leq 1 \tag{3.5}$$

where r is the maximum growth rate of α, i.e. the highest rate at which green replaces brown investment.

[11] The practical significance of this shift is far from straightforward and deserves attention. Since the purpose of this contribution is mainly theoretical, the study of how to better characterize, distinguish, and measure different types of green investment is left to further research. Broadly speaking, green investment might be conceived as any expenditure in abatement technologies, including (but not limited to) investment in renewable energy and resource efficiency (manufacturing, waste, buildings, transport, and cities) and in natural capital (agriculture, fisheries, water resources, forests), in line with UN Environment Programme (2011).

Figure 3.2 The dynamics of the share of green investment in total investment



Source: author's representation

Given that α is a share, its value is bounded between 0 (all investment is in brown technologies) and 1 (all green investment) – as in figure 3.2.[12]

3.3.4 Modeling CO_2 Emissions

As it has been touched upon in the introduction and estimated in Appendix C.3, the stock of cumulative CO_2 emissions in the atmosphere is also likely to follow a logistic pathway: its human-induced growth – started with the First Industrial Revolution – is likely to cease as low-carbon emitting technologies and circular products become fully integrated in the economy. Accordingly, its change over time is assumed to evolve in line with the following logistic

[12] It is important to note here that the analysis conducted in the paper assumes away the possibility of rebound effects. Given their empirical relevance in the context of the transition towards resource-conserving technologies, we leave to further research the incorporation of these effects in the model. For a survey on the rebound effect, see Greening et al. (2000). A modeling attempt to account for this effect is presented in Carnevali et al. (2021).

differential equation:[13]

$$\dot{CO_2} = \eta CO_2 \left(1 - \frac{CO_2}{CO_2^{max}}\right) \tag{3.6}$$

where η is the maximum growth rate of emissions and CO_2^{max} is the carrying capacity.[14]

At this stage, it ought to be noted that technological progress and climate mitigation strategies may have a positive impact in reducing the flow of emissions, but not the previously accumulated stock. Accordingly, we differentiate equation 3.6, subsequently incorporating the effect of the investment channel in increasing or reducing CO_2^{max}. In order to do so, we incorporate an interaction term between green investment ($\alpha\dot{\kappa}$), brown investment ($(1-\alpha)\dot{\kappa}$), and change in cumulative emissions ($\dot{CO_2}$), weighted by a reaction coefficient β_1, which measures the effect of the investment-emissions interaction:[15]

$$\ddot{CO_2} = \eta \dot{CO_2} \left[1 - \frac{2CO_2}{CO_2^{max} - \beta_1 \left(\alpha(1-\alpha)\dot{\kappa}^2 \dot{CO_2}\right)}\right] \tag{3.7}$$

Given that the term in square brackets in equation 3.7 is strictly positive for meaningful values of the variables, the sign of β_1 is of key interest to assess the effect of investment on emissions.[16] If $\beta_1 < 0$, then an increase in total in-

[13] The theoretical analysis assumes a 2-parameter logistic function to make the notation more compact and the differentiation simpler. However, empirical analysis of cumulative carbon emissions ought to rely on 4- or 5-parameters logistic curves in order to allow for asymmetries (see Appendix C.3).

[14] It is important to stress that the carrying capacity of the system should not be interpreted as an ecological limit, as many tipping points may lie below it. More specifically, weather extremes and climate disasters could produce major effects before the limit is approached, as it will be discussed in Section 3.6.

[15] It is worth discussing further what determines the denominator of equation 3.7. Given the interaction between the flow of CO_2 emissions, on one hand, and green and brown investment, on the other, the denominator of the equation (which we might call the investment-influenced carrying capacity) would be given by $CO_2^{max} - \beta_1 \dot{\kappa}_g \dot{\kappa}_b \dot{CO_2} = CO_2^{max} - \beta\alpha\dot{\kappa}(1-\alpha)\dot{\kappa}\dot{CO_2}$. Rewriting the expression in a more compact format yields the formulation in equation 3.7, characterized by a quadratic investment term ($\dot{\kappa}^2$).

[16] As the scope of the paper is mainly theoretical, for the sake of simplicity we only include the influence of the investment channel on emissions reduction. While this allows to make the model more tractable and the exposition clearer, it is by no means realistic to assume that other components of aggregate demand (e.g. consumption, government spending, etc.) do not play any role in affecting the flow of emissions. This leaves scope to further empirical research,

vestment will reduce CO_2 emissions, thus contributing to the goal of 'flattening the curve'. Vice versa, if β_1 is positive (because of prevalence of brown technologies and/or of a slow reaction of emissions to the green-brown investment interaction), it follows that increased capital formation will negatively affect emissions. Therefore, in the context of the paper, the notions of coupling and decoupling will be associated with the effect that the investment channel has on CO_2 emissions, negative for $\beta_1 < 0$ (decoupling) and positive for $\beta_1 > 0$ (coupling). The remainder of the paper will show how the system behaves if the condition is/is not met vis-à-vis a baseline scenario in which the carrying capacity is not affected by the investment channel.

Furthermore, it should be noted that equation 3.7 is broadly consistent with the Kaya Identity: the portion of the identity attributable to energy intensity (energy-GDP ratio) and fuel mix (CO2-energy ratio) is treated as deterministic through $\eta \dot{CO_2}$, while the impact of economic activity is endogenized via the investment channel, as discussed above.

While the ultimate validity of the modeling assumptions ought to be empirically verified, it is worth noting that equation 3.7 embeds a framework that is flexible enough to take into account both the case of decoupling between emissions and economic activity – which may be observable in some advanced developed economies – and the one of coupling, more likely observable in emerging market economies.

3.3.5 The Full Model

For the sake of the argument, it may be worth summarizing the four equations in four unknowns (output, investment, green investment share in total investment, and CO_2 emissions) that constitute the basis of the model:

$$\ddot{Y} = \frac{1}{\epsilon\theta}\left[\overline{Z} + \dot{\kappa} - sY - (\epsilon + s\theta)\dot{Y}\right] \tag{3.8}$$

aimed at estimating the value associated with the reaction coefficient β_1, as well as of other β_i associated with other demand components.

$$\dot{\kappa} = \begin{cases} \dot{\kappa}^{*} \\ [\alpha v_g + (1-\alpha) v_b]\dot{Y} \\ \dot{\kappa}^{**} \end{cases} \tag{3.9}$$

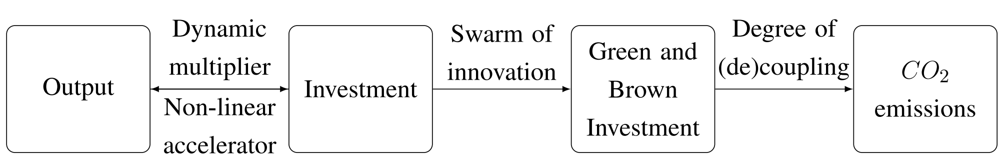
$$\dot{\alpha} = r\alpha(1-\alpha) \qquad \text{GREEN SHARE of } \dot{\kappa} \tag{3.10}$$

$$\ddot{CO_2} = \eta \dot{CO_2} \left[1 - \frac{2CO_2}{CO_2^{max} - \beta_1 \left(\alpha(1-\alpha)\dot{\kappa}^2 \dot{CO_2}\right)}\right] \tag{3.11}$$

The ‘economic module’ of the model (i.e, output and investment) is determined by the interaction between the dynamic multiplier and the non-linear accelerator, allowing the system to exert persistent and endogenous cycles. The ‘environmental module’ is instead determined by the logistic curve describing the dynamics over time of cumulative CO_2 emissions and it is affected by the economic sphere via the investment channel. More specifically, the interaction, on one hand, between green and brown investment – in turn depending on the storm of innovation underlying the dynamics of their shares in total investment – and the flow of emissions, on the other, may be such that to allow for coupling or decoupling of emissions and economic activity.

The basic structure of the model is summarized in figure 3.3.

Figure 3.3 Structure of the model



Source: author’s representation

3.4 Parameter calibration and Initial Conditions

Parameter values are set in accordance with the empirical evidences for the US in the last ten years (2012-2022). The model is calibrated at a quarterly frequency. It is important to stress that the calibration exercise is purely illustrative in nature, as proper evaluation of the model would require further empirical and econometric investigation.

Parameters underlying the trajectory of cumulative CO_2 emissions are estimated. More specifically, the values assigned to the maximum growth rate of cumulative CO_2 emissions (η) and to its carrying capacity (CO_2^{max}) are both taken from the estimation procedure described in Appendix C.3. The series of cumulative CO_2 emissions is indexed (2012=100) for the sake of tractability, which implies a value of $CO_2^{max} = 156.63$. The parameter η is then obtained by converting the estimated annual rate of 0.0506 to quarterly frequency ($1.0506^{1/4} - 1 = 0.0124$). The remaining parameters are in line with the macroeconomic performances of the US economy following the 2007-2008 crisis. In particular, the values for the tax-adjusted propensity to save and for the propensity to import are taken from Fazzari et al. (2020) and Girardi and Pariboni (2016), respectively.[17] The value for the time-lag of the dynamic multiplier is taken from Sordi (2006), while the investment lag is set to 3.5 in order to match the periodicity of the investment cycle. Values for the ceiling and floors of total investment are set in line with the maximum and minimum values, respectively, of the cyclical component of investment data in the period 2012-2022, filtered using the Baxter-King filter which is usually adopted for trend-cycle decomposition of macroeconomic data.[18] In absence of sufficient empirical evidences, the acceleration coefficients of green and brown investment are assumed to be equal ($v_b = v_g = 4$), both set in order to match a persistent business cycle of about 10 quarters peak-to-bottom. For the sake of simplicity, we assume that autonomous demand is entirely determined by its long-run trend, implying that over the business cycle $\overline{Z} = 0$.[19]

[17]Further justification for the values used for these parameters can be found in Gallo (2022b).

[18]For the trend-cycle decomposition of the investment and GDP series, see Appendix C.4.

[19]See Appendix C.2 for an attempt based on long-run autonomous demand growth to reconcile trend and cyclical movements in the model.

Table 3.1 Parameter values

Par.	**Description**	**Value**
β_1	Reaction of cumulative CO_2 emissions to green investment	$\pm$0.075
ϵ	Time-lag of the dynamic multiplier	0.4
η	Maximum growth rate of cumulative CO_2 emissions	0.0124
θ	Lag between decisions to invest and the corresponding outlays	3.5
CO_2^{max}	Carrying capacity of cumulative CO_2 emissions	156.63
$\dot{\kappa}^*$	Ceiling of (induced) investment	150
$\dot{\kappa}^{**}$	Floor of (induced) investment	-85
m	Propensity to import	0.17
r	Maximum growth rate of the green investment share	0.045
s	Tax-adjusted propensity to save	0.5
v_b	Acceleration coefficient of brown investment	4
v_g	Acceleration coefficient of green investment	4

Source: author's calculation

Parameter values are summarized in Table 3.1.

The value of the maximum growth rate of the green investment share (r) is set to match a diffusion process of green technologies of about 200 quarters (50 years), in line with the Schumpeterian literature (see e.g. Goodwin 1990).

For exposition purposes, β_1 takes two custom values to allow for the two cases under scrutiny, i.e. coupling (β_1>0) and decoupling ($\beta_1 < 0$). It is important to note that the arbitrary values adopted here need to be updated through further empirical analysis.

Initial conditions (Table 3.2) are set to match empirical evidences as of the first quarter of 2012, in order to assess recent trends and evaluate future pathways. The initial values of output and investment are set equal to the value of the cyclical components of the two variables in the first quarter of 2012, as implied by the Baxter-King filtering procedure described above. The initial value of cumulative CO_2 emissions corresponds to the index number 100, while $\dot{CO}_{2,0}$ is set equal to the estimated change of the variable divided by 4, in order to match the quarterly frequency of the calibration.

Table 3.2 Initial conditions

Variable	Description	Value
Y_0	Output	15.38
$\dot{Y}_0$	Output change	29.5
$\dot{\kappa}_0$	Total investment	60.39
α_0	Share of green investment in total investment	0.064
$CO_{2,0}$	Cumulative CO_2 emissions	100
$\dot{CO}_{2,0}$	Quarterly CO_2 emissions	0.38

Source: author's calculation

As noted by Inderst et al. (2012, p. 6), in absence of sufficiently disaggregated national accounts data, it is almost impossible to obtain a proper measurement of green investment; its very definition, in fact, would need further discussion, as green investment could "be stand-alone, a sub-set of a broader investment theme or closely related to other investment approaches such as SRI (socially responsible investing), ESG (environmental, social and governance investing), sustainable, long-term investing or similar concepts". For the sake of simplicity, we will use for calibration a notion more strictly related to investment in renewable energy, using the OECD data on renewable energy supply in percentage of total energy supply as a proxy of the green investment share (see Appendix C.5). More specifically, the initial value α_0 reflects the data point of the series in 2012.

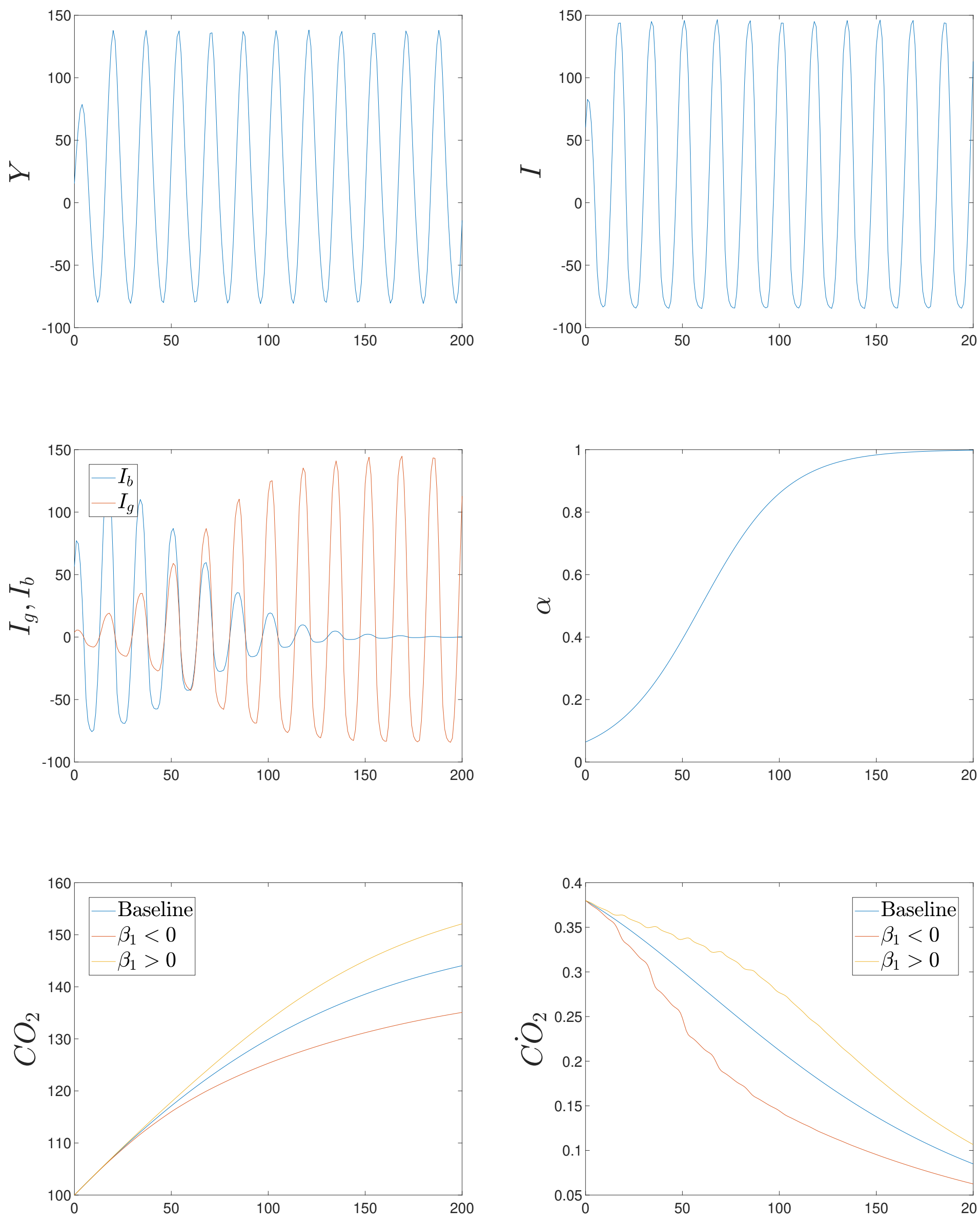
3.5 Simulation Results

As mentioned earlier, the results of the simulation – and thus the overall implications of the model – will heavily depend on the parameter constellation. In particular, the investment channel might be either emissions enhancing or mitigating, depending on the condition $\beta_1 \gtrless 0$ discussed in Subsection 3.3.4.

Figure 3.4 show three possible trajectories depending on the parameter configurations discussed in the previous section: a counterfactual scenario in which there is no interplay between investment and emissions ($\beta_1 = 0$, blue line), a scenario in which investment is climate-mitigating ($\beta_1 < 0$, orange line) and, vice versa, a third scenario in which investment is emissions-enhancing (β_1>0, yellow line).

As the simulation results show, the model produces a persistent and self-sustaining business cycle, with endogenous fluctuations of output and total investment. Given the logistic dynamics of the green investment share (α), green capital formation diffuses through amplifying waves up to $\alpha = 1$. Conversely, brown investment waves dampens up to the point in which brown technologies are fully replaced by resource-conserving ones. The two plots at the bottom summarize the behavior of cumulative emissions (stock) and emission flows to the investment-emissions feedback. Against the baseline scenario of no interplay ($\beta_1 = 0$), the feedback mechanism works in the direction of reducing or enhancing emissions depending on the sign of β_1. Given this reaction coefficient, the stronger is the transition towards green technologies, the stronger will also be the effect on emissions – and vice versa. In both cases, the model simulation shows that the change in CO_2 emissions is procyclical both in the case of coupling and decoupling, in line with empirical findings (Doda, 2014).

Figure 3.4 Simulation results



Source: author's representation

3.6 Incorporating Climate Damage in the Model

The model presented in the previous sections maintains a rather optimistic stance, drawing upon the potentially positive effect of green investment on CO_2 emissions reduction. However, transitioning towards green capital and technologies is far from straightforward, as it requires the radical reconversion of most production processes. Even if the goal is achieved, it is not granted that the transition will be sufficiently quick and strong as required by net-zero strategies. As the ecological literature shows, the world may have already hit some tipping points (see subsection 3.2.3). Beyond these thresholds, human-induced climate change may have large impacts not only on long-run growth, but also on the business cycle (e.g. because of the changing patterns of rainfall, floods, famine, etc). Extreme weather events past some tipping points will become all the more likely as the level of greenhouse gas emissions builds in the atmosphere. In turn, the resulting disruptions of supply chains following these extreme events might result in increased instability and uncertainty, likely affecting investors' behavior not only in the longer run, but also in the shorter term. This could be captured by amending the investment function of the model presented in section 3.3. More specifically, the analysis assumes that climate degradation reduces at the same time both the investment floor and ceiling, thus making recessions over the business cycle more severe and expansions milder. As emissions accumulate in the atmosphere, the investment ceiling and floor will thus be reduced by two amounts corresponding to $\gamma_1 CO_2$ and $\gamma_2 CO_2$, where γ_1 and γ_2 are two reaction coefficients measuring the magnitude of the respective effects.[20]

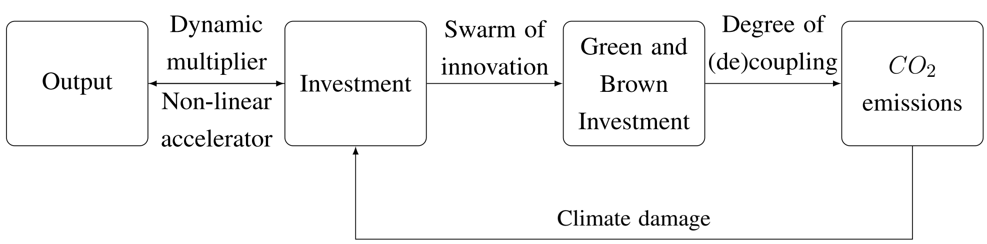
[20]The amended model assumes a purely deterministic mechanism through which climate change impacts the 'economic module'. However, it ought to be noted that weather extremes and climate disasters are largely unpredictable both in time and space, thus affecting business cycle dynamics in a stochastic way as well. The partly deterministic and partly stochastic nature of climate damage will be incorporated in future versions of the paper.

Without amending – for the sake of simplicity – the piecewise linear structure of the model, it follows that:

$$\dot{\kappa} = \begin{cases} \dot{\kappa}^* - \gamma_1 CO_2 & \text{if } [\alpha v_g + (1-\alpha)v_b]\dot{Y} > \dot{\kappa}^* - \gamma_1 CO_2 \\ [\alpha v_g + (1-\alpha)v_b]\dot{Y} & \text{if } \dot{\kappa}^{**} - \gamma_2 CO_2 \leq [\alpha v_g + (1-\alpha)v_b]\dot{Y} \leq \dot{\kappa}^* - \gamma_1 CO_2 \\ \dot{\kappa}^{**} - \gamma_2 CO_2 & \text{if } [\alpha v_g + (1-\alpha)v_b]\dot{Y} < \dot{\kappa}^{**} - \gamma_2 CO_2 \end{cases} \tag{3.12}$$

The structure of the amended model is summarized in figure 3.5. Compared to what has been presented in section 3.3, the causation now runs not only from the economic module to the environmental one, but also in the opposite direction, as environmental degradation negatively affects investment.

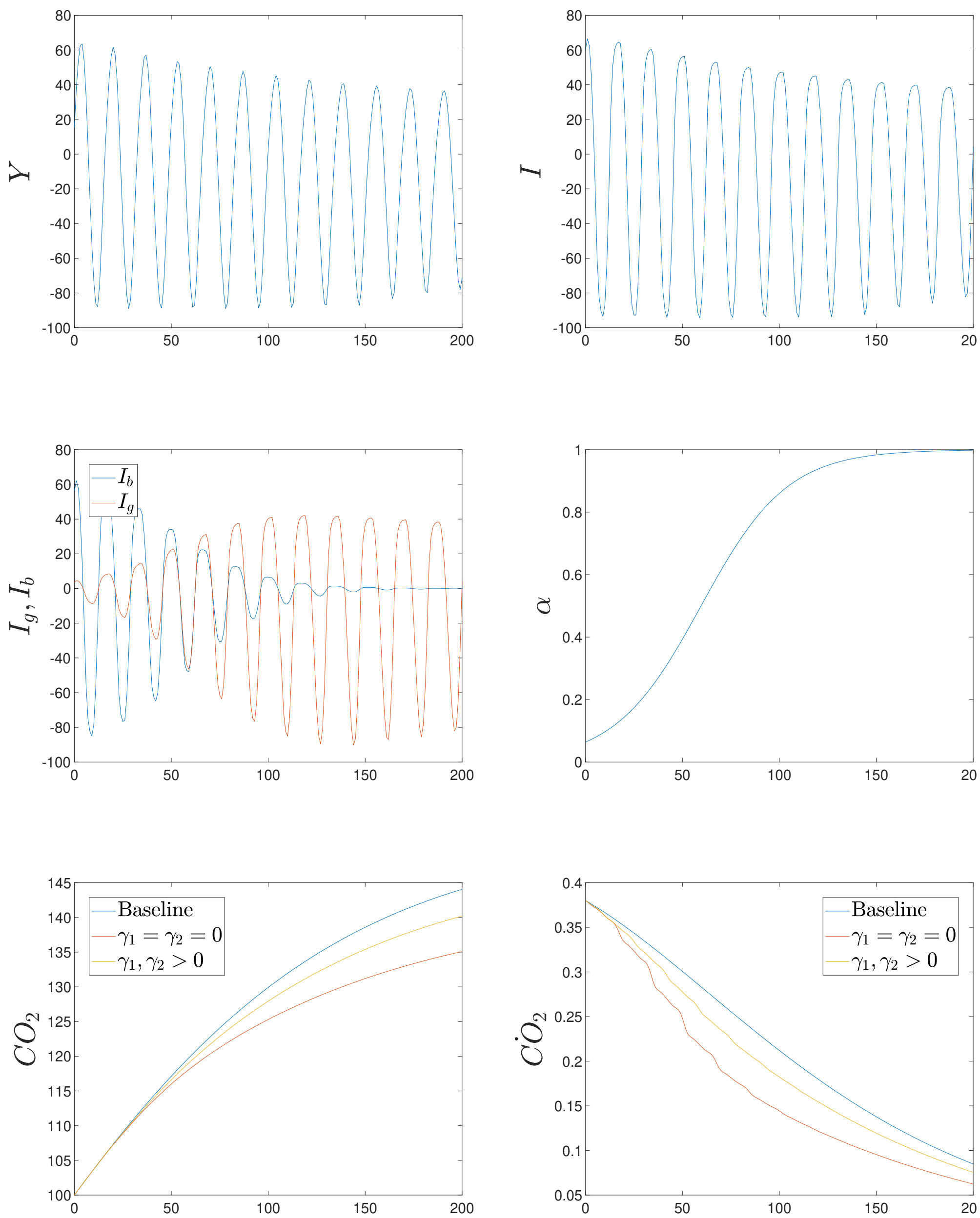
Figure 3.5 Structure of the model



Source: author's representation

Let us now simulate the effect of climate damage in the amended model. For the sake of clarity, the analysis merely focuses on the case of decoupling, i.e. $\beta_1 < 0$. Assuming that weather extremes and disasters have a stronger effect over the cycle on the investment ceiling than on the floor ($\gamma_1 = 0.8, \gamma_2 = 0.1$), climate damage reduces the scope of the transition towards green technologies as a mitigation tool. As showed in figure 3.6, climate degradation has the effect of dampening the cycle, making expansion phases milder and turmoil sharper; thus, by diminishing the interplay between private investment and emissions, it reduces *ceteris paribus* the possibility of the investment channel to be climate-mitigating. In other terms, under this scenario, even a sufficiently fast and strong green transition may not be sufficient to flatten the curve of cumulative emissions in order to approach net-zero targets.

Figure 3.6 Simulation results – the case of decoupling with climate damage



Source: author's representation

3.7 Conclusion

The paper develops a theoretical framework to assess the effect of transitioning towards green investment on business cycle fluctuations and CO_2 emissions reduction. More specifically, it assesses how mobilizing capacity in green technologies might contribute to sustain the cycle, on one hand, while having the effect of reducing emissions, thus flattening the curve of cumulative CO_2 emissions in line with the established climate targets. In order to do so, it relies on the endogenous business cycle theory of Goodwin (1951), which allows to produce a self-sustaining business cycle based on the interaction between the dynamic multiplier and the non-linear acceleration principle. By modeling the transition from brown to green investment as a logistic diffusion process, the model shows that the investment channel might have a positive or negative impact on emissions depending on the timing and magnitude of the transition, as well as on the sign of the sensitivity of emissions to the green-brown investment interplay (β_1). Furthermore, the model contributes to the analytical understanding of the relation between emissions, output and investment by capturing the short-run procyclicality of emissions found in the empirical literature (e.g. Doda 2014).

Moreover, an amended version of the model is presented in order to account for climate damage. Provided that locked-in emissions are likely to permanently alter the environment, human-induced climate change may have large impacts on output and investment dynamics over the business cycle. More specifically, extreme weather events past some tipping points will likely result in disruptions of supply chains, engendering increased instability and uncertainty, and likely affecting investors' behavior. This has been captured by amending the model in order to allow for climate damage in the model's investment function. The results show that climate degradation has the effect of making expansion phases milder and turmoil sharper, hindering the potentially positive impact of the green transition. Under this scenario, even a sufficiently fast and strong green transition may not be enough to flatten the curve of cumulative emissions in order to approach net-zero targets.

Summarizing, the paper has developed a theoretical framework which is flexible enough to account both for the case of coupling and decoupling of output and emissions over the business cycle. Therefore, the adopted modeling strategy allows to account at the same time for the case of advanced economies – in which decoupling is more likely – and developing ones – more likely characterized by a situation of coupling. By focusing on the investment channel over the business cycle, the paper contributes to the study of the shorter run economy-ecology feedback, which has received relatively little attention in recent years. While the calibration and simulation of the model has been informative in clarifying the basic assumed levers of this feedback, the model presented in this paper – both in its baseline and amended versions – ought to be confronted with the data both in advanced and developing countries. In particular, by assessing the validity of the model, further empirical analysis would allow to draw relevant policy implications on the strength and speed required for green investment – both public and private – to substantially reduce CO_2 emissions in line with the established targets.

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A Appendix for Chapter 1

A.1 Proof of the General Solution in Equation 1.17

1. In order to prove that equation (1.17) is the general solution of the ordinary differential equation (1.16), let us first conveniently simplify the notation. In particular, let us denote with the term K the Keynesian stability condition, i.e. $K = s + m - \beta v > 0$. Therefore, equation (1.16) becomes:
$$\frac{du}{dt} = (\alpha + z)\mu v - \mu K u_t \tag{A.1}$$
2. Rewrite equation (A.1) in the form $dy/dt + p_t y_t = q$, as follows:
$$\frac{du}{dt} + \mu K u_t = (\alpha + z)\mu v \tag{A.2}$$
which implies that $p_t = \mu K$ and $q = (\alpha + z)\mu v$.
3. Let us find the integrating factor (η_t), i.e. the continuous function that satisfies the condition $\eta_t p_t = \eta_t'$, as follows:
$$\eta_t = \mathbf{e}^{\int \mu K dt} = \mathbf{e}^{\mu K t} \tag{A.3}$$
4. Let us now multiply all the terms in the differential equation (A.2) by the integrating factor:
$$\mathbf{e}^{\mu K t}\frac{du}{dt} + \mu K \mathbf{e}^{\mu K t} u_t = (\alpha + z)\mu v \mathbf{e}^{\mu K t} \tag{A.4}$$

$$\left(\mathbf{e}^{\mu K t} u_t\right)' = (\alpha + z)\mu v \mathbf{e}^{\mu K t} \tag{A.5}$$

5. Integrating both sides of equation (A.5), it follows that:

$$\int \left(\mathbf{e}^{\mu K t} u_t\right)' dt = \int (\alpha + z)\mu v \mathbf{e}^{\mu K t} dt \tag{A.6}$$

$$\mathbf{e}^{\mu K t} u_t + k = \frac{(\alpha + z)\mu v}{\mu K} \mathbf{e}^{\mu K t} + c \tag{A.7}$$

6. Subtracting k from both sides, we get:

$$\mathbf{e}^{\mu K t} u_t = \frac{(\alpha + z) v}{K} \mathbf{e}^{\mu K t} + c - k \tag{A.8}$$

7. Both c and k are unknown constants and so the difference is also an unknown constant. Therefore, we can write the difference as $c_1 = c - k$:

$$\mathbf{e}^{\mu K t} u_t = \frac{(\alpha + z) v}{K} \mathbf{e}^{\mu K t} + c_1 \tag{A.9}$$

8. We now have only one constant of integration c_1. It should be noted that the constant c_1 is negative for economically meaningful initial values of the rate of capacity utilization (if $u_0 > 0$, then $c_1 < 0$). For convenience, let us then define another constant C as $C = -c_1/K$. Therefore, equation (A.9) becomes:

$$\mathbf{e}^{\mu K t} u_t = \frac{(\alpha + z) v \; \mathbf{e}^{\mu K t} - C}{K} \tag{A.10}$$

9. Multiplying both sides by $\mathbf{e}^{-\mu K t}$, we can obtain the general solution to the ODE that regulates out-of-equilibrium dynamics in the model, as follows:

$$u_t = \frac{(\alpha + z) v - C \, \mathbf{e}^{-\mu K t}}{K} \tag{A.11}$$

10. Last, substituting $K = s + m - \beta v$, we can write u_t as follows:

$$u_t = \frac{(\alpha + z) v - C \exp[-t\mu(s + m - \beta v)]}{s + m - \beta v} \tag{A.12}$$

A.2 Data sources

- Capacity Utilization, Rate, All industry, SA, Federal Reserve Board (FRB), https://fred.stlouisfed.org/series/TCU
- Gross Domestic Product, Overall, Total, Constant Prices, SA, USD, 2012 Chained Prices, BEA - Bureau of Economic Analysis, U.S. Department of Commerce, https://www.bea.gov/data/gdp/gross-domestic-product
- Imports, Goods and Services, Total, Constant Prices, SA, USD, 2012 Chained Prices, BEA - Bureau of Economic Analysis, U.S. Department of Commerce, https://www.bea.gov/data/gdp/gross-domestic-product
- Private Fixed Investment, Nonresidential, Total, Constant Prices, SA, USD, 2012 Chained Prices, BEA - Bureau of Economic Analysis, U.S. Department of Commerce, https://www.bea.gov/data/gdp/gross-domestic-product

All weblinks last accessed on September 26, 2021.

A.3 Online: Sensitivity analysis – Chapter 1

The interested reader could easily perform a re-parameterization of the neo-Kaleckian model under scrutiny through the following interactive Web App - created with Shiny R: https://ettoregallo.shinyapps.io/Short_run_NKM/

B Appendix for Chapter 2

B.1 Derivation of the models, Stability and Equilibrium

Open Economy with Government Activity

Let us start from the output equation of an open economy with government activity:

$$Y_t = C_t + I_t + G_t + (X_t - M_t) \tag{B.1}$$

where the current level of aggregate output (Y_t) is defined as the sum of aggregate consumption (C_t), private investment (I_t), public expenditures (G_t) and net exports $(X_t - M_t)$. Consumption, government spending, exports and imports can be modeled as follows:

$$C_t = C_{Yt} + \overline{C_{0t}} = c(1-t)Y_t + \overline{C_{0t}} \tag{B.2}$$

$$G_t = \overline{G_t} \tag{B.3}$$

$$X_t = \overline{X_t} \tag{B.4}$$

$$M_t = mY_t \tag{B.5}$$

Equation (B.2) assumes that aggregate consumption is partly induced - via the tax-adjusted propensity to consume $c(1-t)$ - and partly autonomous from the current level of income ($\overline{C_{0t}}$). Autonomous consumption can be understood as 'that part of aggregate consumption financed by credit and, therefore, unrelated to the current level of output resulting from firms' production decisions' (Freitas

and Serrano, 2015, p.4). Government spending (equation B.3) and exports (equation B.4) are both treated as autonomous, the first because public consumption and investment depend on the arbitrary decisions of the general government, the second because exports does not depend on the level of national income, but on that of the rest on the world. For the sake of simplicity, imports of goods and services are assumed to be linearly dependent of the level of income, via the propensity to import m (equation B.5).
The modeling choice regarding aggregate investment is what effectively constitutes the main difference between the Sraffian Supermultiplier and the Neo-Kaleckian model, as showed below.

Sraffian Supermultiplier Model
According to the baseline Supermultiplier model, private investment is treated as fully induced (equation B.6), reflecting the simple idea that at the aggregate level firms will invest only as long as there is demand for their products. Therefore, I_t can be model *sic et simpliciter* as the product of the investment share (h_t) times national income.

$$I_t = h_t Y_t \tag{B.6}$$

Since $\dot{K}_t = I_t - \delta K_t$, the accumulation rate can be derived as follows:

$$g_t^K = \frac{\dot{K}_t}{K_t} - \delta = \frac{I_t}{K_t} - \delta = \frac{h_t Y_t}{K_t} - \delta = h_t \frac{Y_t}{Y^p}\frac{Y^p}{K_t} - \delta = \frac{h_t u_t}{v} - \delta \tag{B.7}$$

where Y^p is full-capacity output. Let us now solve for the level of output, substituting equations (B.2, B.3, B.4, B.5 and B.6) in equation (B.1):

$$Y_t = \left(\frac{1}{s+m-h_t}\right)\left(\overline{C_{0t}} + \overline{G_t} + \overline{X_t}\right) = \left(\frac{1}{s+m-h_t}\right) Z_t = SM_t Z_t \tag{B.8}$$

where s denotes the tax-adjusted propensity to save, i.e. $s = 1 - c(1-t)$.
Differentiating equation (B.8), we obtain the growth rate of output as the sum of the growth rate of autonomous demand and of the supermultiplier, under the assumption that the investment share behaves in line with equation (2.8):

$$g_t^Y = g_t^Z + \frac{h_t \gamma (u_t - u_n)}{s+m-h_t} \tag{B.9}$$

Lastly, the model closure is given by the assumption of an exogenously given growth rate of autonomous demand:

$$g_t^Z = \overline{g^Z} \tag{B.10}$$

Let us now analyze the stability of the fully-adjusted equilibrium, whose necessary and sufficient condition is that the determinant of the Jacobian's matrix evaluated at the equilibrium point with $u^* = u_n$ and $h^* = \frac{v}{u_n}(\overline{g^Z} + \delta)$ is positive and its trace is negative:

$$\boldsymbol{J}^* = \begin{bmatrix} \left[\frac{\partial \dot{h}}{\partial h}\right]_{h^*,u^*} & \left[\frac{\partial \dot{h}}{\partial u}\right]_{h^*,u^*} \\ \left[\frac{\partial \dot{u}}{\partial h}\right]_{h^*,u^*} & \left[\frac{\partial \dot{u}}{\partial u}\right]_{h^*,u^*} \end{bmatrix} \tag{B.11}$$

$$= \begin{bmatrix} 0 & \frac{\gamma v(\overline{g^Z}+\delta)}{u_n} \\ -\frac{u_n^2}{v} & (\overline{g^Z}+\delta)\left(\frac{\gamma v}{s+m-\frac{v}{u_n}(\overline{g^Z}+\delta)} - 1\right) \end{bmatrix} \tag{B.12}$$

$$det\ \boldsymbol{J}^* = \gamma u_n(\overline{g^Z} + \delta) \tag{B.13}$$

$$tr\ \boldsymbol{J}^* = (\overline{g^Z} + \delta)\left(\frac{\gamma v}{s + m - \frac{v}{u_n}(\overline{g^Z} + \delta)} - 1\right) \tag{B.14}$$

Since γ, u_n and $\overline{g^Z}$ are assumed to be positive, the determinant is necessarily positive. Similarly to Freitas and Serrano (2015), the stability condition boils down to the sign of the $Tr\ \boldsymbol{J}^*$, which is ensured by the following condition:

$$1 - s + m + \gamma v + \frac{v}{u_n}(\overline{g^Z} + \delta) < 1 \tag{B.15}$$

where $1 - s + m$ may also be interpreted as the tax and imports-adjusted propensity to spend. Equation (B.15) implies three conditions:

1. The value of the reaction parameter γ should be sufficiently low, implying that induced investment ought not to adjust capacity to demand too fast outside the fully-adjusted position (Freitas and Serrano, 2015). In other terms, the effect of Harrodian instability needs not to be overly strong;
2. The growth rate of autonomous demand $\overline{g^Z}$ cannot be too large;

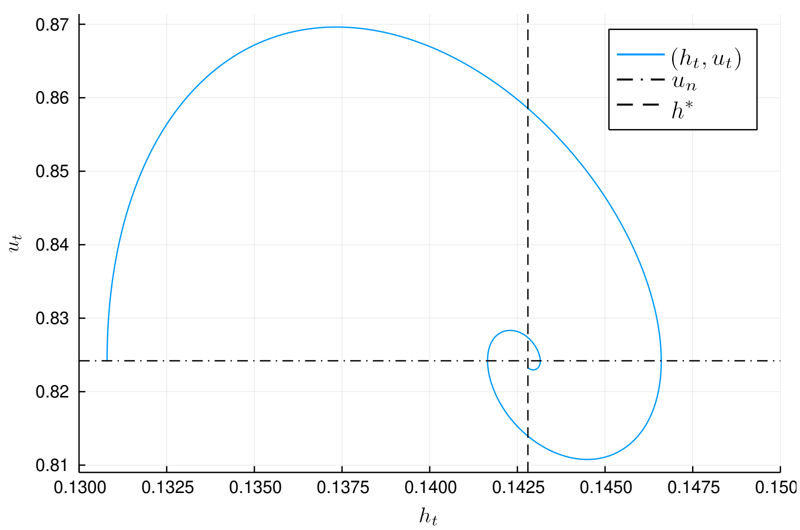
3. The tax and imports-adjusted propensity to spend $(1-s+m)$ needs not to be too large and it must be smaller than unity in the entire adjustment process.

If the condition in equation (B.15) is fulfilled, then the system converges to its long-run equilibrium. The trajectory of the system depends on the discriminant of its eigenvalues:

$$\lambda_{1,2} = \frac{tr\,J^* - \sqrt{\Delta}}{2} \quad \text{with} \quad \Delta = (tr\,J^*)^2 - 4det\,J^* \tag{B.16}$$

Therefore, if $tr\,J^*, \Delta < 0$ and $det\,J^* > 0$, the eigenvalues will be complex with nonzero imaginary part; u_t and h_t will converge via damped oscillation (spiral sink). Conversely, if $tr\,J^* < 0$ and $det\,J^*, \Delta > 0$, both eigenvalues will be real and distinct; the system will converge monotonically towards the fully-adjusted position (real sink). Relying on the parameter calibration discussed in Section 2.3, the first case applies. Figure B.1 shows the resulting phase plane.

Figure B.1 Phase plane of the amended Supermultiplier model



Source: authors' representation

Long-run Neo-Kaleckian Model

Neo-Kaleckian models treat capital formation as dependent on the rate of capacity utilization. More specifically, adopting the formulation proposed for the first time by Amadeo (1986), the investment rate will depend on the secular growth rate of sales (α_t) plus discrepancies between the actual and the normal or 'planned' (Steindl, 1952) utilization rates, via the parameter β:

$$I_t = [\alpha_t + \beta(u_t - u_n)]K_t \tag{B.17}$$

which - under the assumption of a linear depreciation coefficient - implies that the accumulation rate will be equal to:

$$g_t^K = g_t^I - \delta = \frac{I_t}{K_t} - \delta = \alpha_t + \beta(u_t - u_n) - \delta \tag{B.18}$$

The saving equation in levels is then given by:

$$\begin{aligned} S_t &= Y_t - C_t - G_t - (X_t - M_t) = \\ &= [1 - c(1-t) + m]Y_t - \left(\overline{C_{0t}} + \overline{G_t} + \overline{X_t}\right) = (s+m)Y_t - Z_t \end{aligned} \tag{B.19}$$

Dividing everything by the capital stock and multiplying/dividing the first term on the right-hand side by full-capacity output, it follows that:

$$g_t^S = \frac{S_t}{K_t} = (s+m)\frac{Y_t}{Y^p}\frac{Y^p}{K_t} - \frac{Z_t}{K_t} = \frac{(s+m)u_t}{v} - z_t \tag{B.20}$$

Same as for the Supermultiplier model, the model is closed by the assumption of an exogenously given growth rate of autonomous demand - equation (B.10) above.

Let us now evaluate the Jacobian matrix in the long-run fully-adjusted equilibrium $\alpha^* = \overline{g_t^Z} + \delta$ and $z^* = \frac{(s+m)u_n}{v} - \overline{g^Z} - \delta$:

$$\begin{aligned} \boldsymbol{J}^* &= \begin{bmatrix} \left[\frac{\partial \dot{\alpha}}{\partial \alpha}\right]_{\alpha^*, z^*} & \left[\frac{\partial \dot{\alpha}}{\partial z}\right]_{\alpha^*, z^*} \\ \left[\frac{\partial \dot{z}}{\partial \alpha}\right]_{\alpha^*, z^*} & \left[\frac{\partial \dot{z}}{\partial z}\right]_{\alpha^*, z^*} \end{bmatrix} = \\ &= \begin{bmatrix} \frac{\beta v \mu}{s+m-\beta v} & \frac{\beta v \mu}{s+m-\beta v} \\ -\left[\frac{(s+m)u_n}{v} - \overline{g^Z} - \delta\right]\left(1 + \frac{\beta v}{s+m-\beta v}\right) & \frac{-\beta v}{s+m-\beta v}\left[\frac{(s+m)u_n}{v} - \overline{g^Z} - \delta\right] \end{bmatrix} \end{aligned} \tag{B.21}$$

$$det\ \boldsymbol{J}^* = \frac{\beta v \mu}{s+m-\beta v}\left(\frac{(s+m)u_n}{v} - \overline{g^Z} - \delta\right) \tag{B.22}$$

$$tr\ \boldsymbol{J}^* = \frac{\beta v}{s+m-\beta v}\left[\mu - \frac{(s+m)u_n}{v} + \overline{g^Z} + \delta\right] \tag{B.23}$$

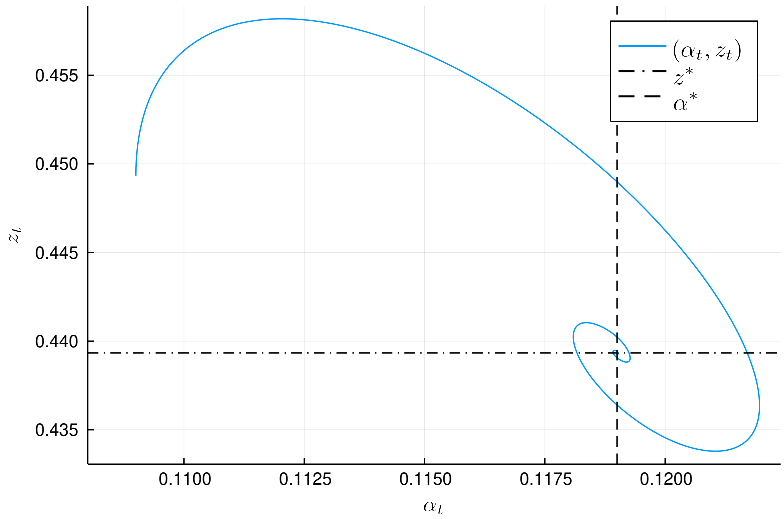
Given that β and v are assumed to be positive, the determinant is positive whenever the Keynesian stability condition holds ($\beta < (s+m)/v$) and the equilibrium autonomous demand-capital ratio z^* is positive, i.e. whenever $\overline{g^Z} < (s+m)u_n/v - \delta$. If the Keynesian stability condition holds, then it can be shown that the trace is negative whenever:

$$\mu < \frac{(s+m)u_n}{v} - \overline{g^Z} - \delta \Longrightarrow \mu < z^* \tag{B.24}$$

Taken together, the stability conditions of the long-run Neo-Kaleckian model imply that:

1. The value of the reaction parameter β should be sufficiently low, implying that the reaction of capital formation to discrepancies in utilization rates is not too strong;
2. The growth rate of autonomous demand $\overline{g^Z}$ cannot be too large;
3. Similarly to the Supermultiplier model, capacity ought to adjust fairly slowly to demand, i.e. the Harrodian mechanism need not to be overly strong (equation B.24);

Figure B.2 Phase plane of the amended neo-Kaleckian model



Source: authors' representation

As discussed above, the discriminant of the system's eigenvalues will determine its trajectory. Similar to the Supermultiplier model, the calibration of the model suggests that – at least in the baseline parametrization – the eigenvalues are complex ($\Delta < 0$). Therefore, the systems converges through damped oscillations towards the fully-adjusted position (spiral sink).
Figure B.2 shows the 2D phase space plot of the Allain-Lavoie system.

B.2 List of Variables – Chapter 2

α_t	Animal spirits (also, expected growth rate of sales)
h_t	Investment share (also, marginal propensity to invest)
g_t^I	Investment rate
g_t^K	Growth rate of the capital stock
g_t^S	Saving rate
g_t^Y	Growth rate of output
g_t^Z	Growth rate of autonomous demand
u_t	Capacity utilization rate
z_t	Autonomous demand-capital ratio

B.3 Online: Sensitivity Analysis – Chapter 2

The interested reader could easily perform a re-parametrization of the two models through the following interactive Web App – created with Shiny R: http://ettoregallo.shinyapps.io/When_is_the_long_run

C Appendix for Chapter 3

C.1 Output Determination – Derivation of Equation 3.1

This appendix summarizes the derivation of the dynamic multiplier in Goodwin (1951), expanding the original model to an open economy with government activity. For further discussion, see Sordi (2006). Time subscripts are omitted to simplify the notation; here and elsewhere, time indexes will be included in brackets only when needed.

First, let us start with the output equation of an open economy with government activity:

$$Y = C + I + G + X - M \tag{C.1}$$

where the current level of aggregate output (Y) is defined as the sum of aggregate consumption (C), private investment (I), public expenditures (G), exports (X) less imports (M).

Consumption, investment, government spending, exports and imports can be modeled as follows:

$$C = \overline{C_0} + c(1-t)Y - \epsilon_1 \dot{Y} \tag{C.2}$$

$$I = \bar{I} + \dot{\kappa} - \epsilon_2 \dot{Y} \tag{C.3}$$

$$G = \overline{G} \tag{C.4}$$

$$X = \overline{X} \tag{C.5}$$

$$M = mY \tag{C.6}$$

Equation (C.2) assumes that aggregate consumption is partly induced - via the tax-adjusted propensity to consume $c(1-t)$ - and partly autonomous from the current level of income ($\overline{C_0}$). Moreover, a time-lag is introduced to account for the dynamic adjustment in the multiplier (ϵ_1). Likewise, private investment (equation C.3) is given by the sum of autonomous ($\bar{I}$) and induced ($\dot{\kappa}$) investment minus the time-lag of the dynamic multiplier (ϵ_2); further assumptions about investment dynamics are made in Subsection 3.3.2. Both lags (ϵ_1 and ϵ_2) capture the time necessary to find and purchase/sell consumption or investment goods when income increases/decreases: accordingly, they can be seen as "a kind of saving or disinvestment [...] resulting from a changing level of income" (Goodwin, 1951, p. 9). Through this assumption, the instantaneous multiplier is replaced with a dynamic multiplier, allowing to account for the fact that "the process of multiplication takes time, and in any dynamical situation it is important to take this into account" (*ibid.*): Government spending (equation C.4) and exports (equation C.5) are both treated as autonomous, the first because public consumption and investment depend on the arbitrary decisions of the general government, the second because exports does not depend on the level of national income, but on that of the rest on the world. For the sake of simplicity, imports of goods and services are assumed to be linearly dependent on the level of income, via the propensity to import m (equation C.6).

Substituting equations (C.2, C.3 C.4, C.5 and C.6) in equation (C.1), we then obtain:

$$\epsilon\dot{Y} + (s+m)Y = \overline{Z} + \dot{\kappa} \tag{C.7}$$

where s is the tax-adjusted propensity to save ($s = 1 - c(1-t)$), $\overline{Z}$ is the sum of all autonomous components of aggregate demand ($\overline{Z} = \bar{I} + \overline{G} + \overline{X}$), and ϵ represents the sum of the two time-lags of the dynamic multiplier in consumption and investment ($\epsilon = \epsilon_1 + \epsilon_2$).

Solving for the level of output, we obtain a rather simple expression of the dynamic multiplier:

$$Y = \frac{1}{s+m}\left[\overline{Z} + \dot{\kappa} - \epsilon\dot{Y}\right] \tag{C.8}$$

Besides the time lag of the dynamic multiplier, Goodwin (1951) introduces a second lag to come closer to a realistic investment theory, i.e. the lag θ capturing the time elapsing "between decisions to invest and the corresponding outlays" (*ibid, p. 11*). Accordingly, investment decisions at time t – based on actual induced investment $\dot{\kappa(t)}$ – would correspond to investment outlays only at time $t+\theta$. The bigger is the lag θ, the bigger will be the discrepancy between the decision to invest and the actual expenditure. Therefore, equation C.7 ought to be rewritten as:

$$\epsilon\dot{Y}(t+\theta)+(s+m)Y(t+\theta)=\overline{Z}+\dot{\kappa}(t) \tag{C.9}$$

Let us now expand the two leading terms $\dot{Y}(t+\theta)$ and $Y(t+\theta)$ on the left-hand side in a Taylor series, then dropping all but the first two terms in each, as follows:

$$\epsilon\theta\ddot{Y}+(\epsilon+(s+m)\theta)\dot{Y}+(s+m)Y=\overline{Z}+\dot{\kappa} \tag{C.10}$$

Rearranging and dividing by $\epsilon\theta$, equation 3.1 is thus readily obtained.

C.2 Reconciling Trend and Cycles: Two Possible Modeling Alternatives

C.2.1 Autonomous Demand Growth

The first straightforward way to reconcile the short-run cyclical movements of GDP with its long-run dynamics, thus generating a growth cycle, is to relax the assumption of constant autonomous demand. This is what we might call the demand-side solution, as it assumes that long-run growth is driven by autonomous demand dynamics. In particular, this solution is consistent with post-Keynesian contributions dealing with the role of autonomous non-capacity creating components of demand in driving long-run growth.[1]

For the sake of simplicity, let us assume – in line with the existing literature – that the growth rate of autonomous demand is exogenously given, such that:

$$\dot{Z} = \log(\tau V \exp(\overline{g_z} t)) \tag{C.11}$$

where $\tau = 0.05$, $V = 20$ (Braga et al., 2022), and the quarterly growth rate of autonomous demand (g_z) is set equal to 0.0074, corresponding to an annual growth rate of 3% (Girardi and Pariboni, 2016).

Autonomous demand growth has the effect of providing a trend to output, thus reconciling short-run fluctuations and longer run economic growth. Moreover, an interaction term ($Z\dot{CO}_2$) is introduced in the denominator of equation C.15 in order to account for the interplay between emissions and economic growth, weighted by a reaction coefficient β_2.

While it is left to further research the incorporation in the model of the interaction between climate damage and economic growth, it might that, as tipping points are hit, long-run growth is likely to be negatively affected by climate degradation. This could be captured by introducing a drift in the growth rate of autonomous demand (g_z), making it time- and/or state-dependent as suggested by Greiner et al. (2005).

Therefore, the complete model now becomes:

[1] See, among others, Allain (2015); Gallo (2022b); Lavoie (2016); Serrano (1995a).

$$\ddot{Y} = \frac{1}{\epsilon\theta}\left[Z + \dot{\kappa} - sY - (\epsilon + s\theta)\dot{Y}\right] \tag{C.12}$$

$$\dot{\kappa} = \begin{cases} \dot{\kappa}^* \\ [\alpha v_g + (1-\alpha)v_b]\dot{Y} \\ \dot{\kappa}^{**} \end{cases} \tag{C.13}$$

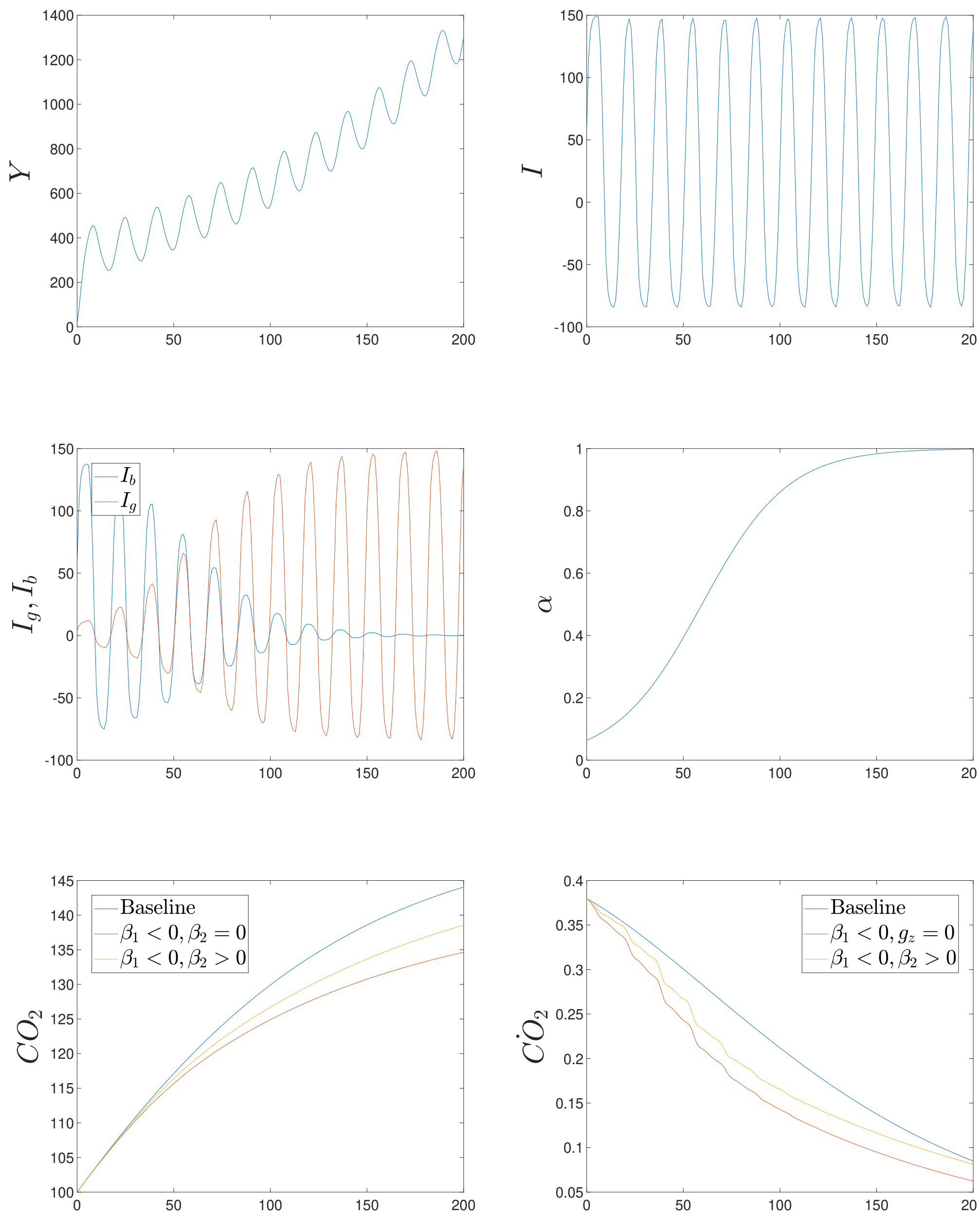
$$\dot{\alpha} = r\alpha(1-\alpha) \tag{C.14}$$

$$\ddot{CO_2} = \eta \dot{CO_2}\left[1 - \frac{2CO_2}{CO_2^{max} - \beta_1\left(\alpha(1-\alpha)\dot{\kappa}^2\dot{CO_2}\right) + \beta_2 Z\dot{CO_2}}\right] \tag{C.15}$$

$$\dot{Z} = \log(\tau V \exp(\overline{g_z}t)) \tag{C.16}$$

The empirical literature shows that the relation between trend emissions and trend GDP is positive – even though the correlation is getting weaker in advanced economies (Cohen et al., 2018, 2022). This is tantamount to assuming $\beta_2 > 0$ in the amended model with autonomous demand growth. Accordingly, the simulation results in figure C.1 assume $\beta_2 = 0.2$. By considering the case of decoupling ($\beta_1 < 0$), the introduction of positive autonomous demand growth reduces the scope for climate mitigation by transitioning to green investment. It is important to note that the cyclical properties of emissions are preserved after the introduction of economic growth in the model.

Figure C.1 The model with a autonomous demand growth



Source: author's representation

C.2.2 Deterministic growth trend

A second possible solution is to assume that output will follow the same time trend of potential output. This solution – that we might call the supply-side solution – is consistent with equation (9) in Braga et al. (2022). More specifically, the output trend can be modeled as follows:

$$\dot{Y}_{trend} = \log(\tau V \exp(g_n t)) \tag{C.17}$$

where g_n is the growth rate of potential output (exogenously given). As before, $\tau = 0.05$ and $V = 20$.

Accordingly, the change of actual output will be given by:

$$\dot{Y} = \dot{Y}_{cycle} + \dot{Y}_{trend} \tag{C.18}$$

The solution and its implications are similar to the ones advanced in the previous subsection, with the only difference that the interaction between growth and emissions is now given by the term $\beta_2 Y CO_2$ in the denominator of equation C.22:

$$\ddot{Y}_{cycle} = \frac{1}{\epsilon\theta}\left[\overline{Z} + \dot{\kappa} - sY_{cycle} - (\epsilon + s\theta)\dot{Y}_{cycle}\right] \tag{C.19}$$

$$\dot{\kappa} = \begin{cases} \dot{\kappa}^{*} \\ [\alpha v_g + (1-\alpha)v_b]\dot{Y}_{cycle} \\ \dot{\kappa}^{**} \end{cases} \tag{C.20}$$

$$\dot{\alpha} = r\alpha(1-\alpha) \tag{C.21}$$

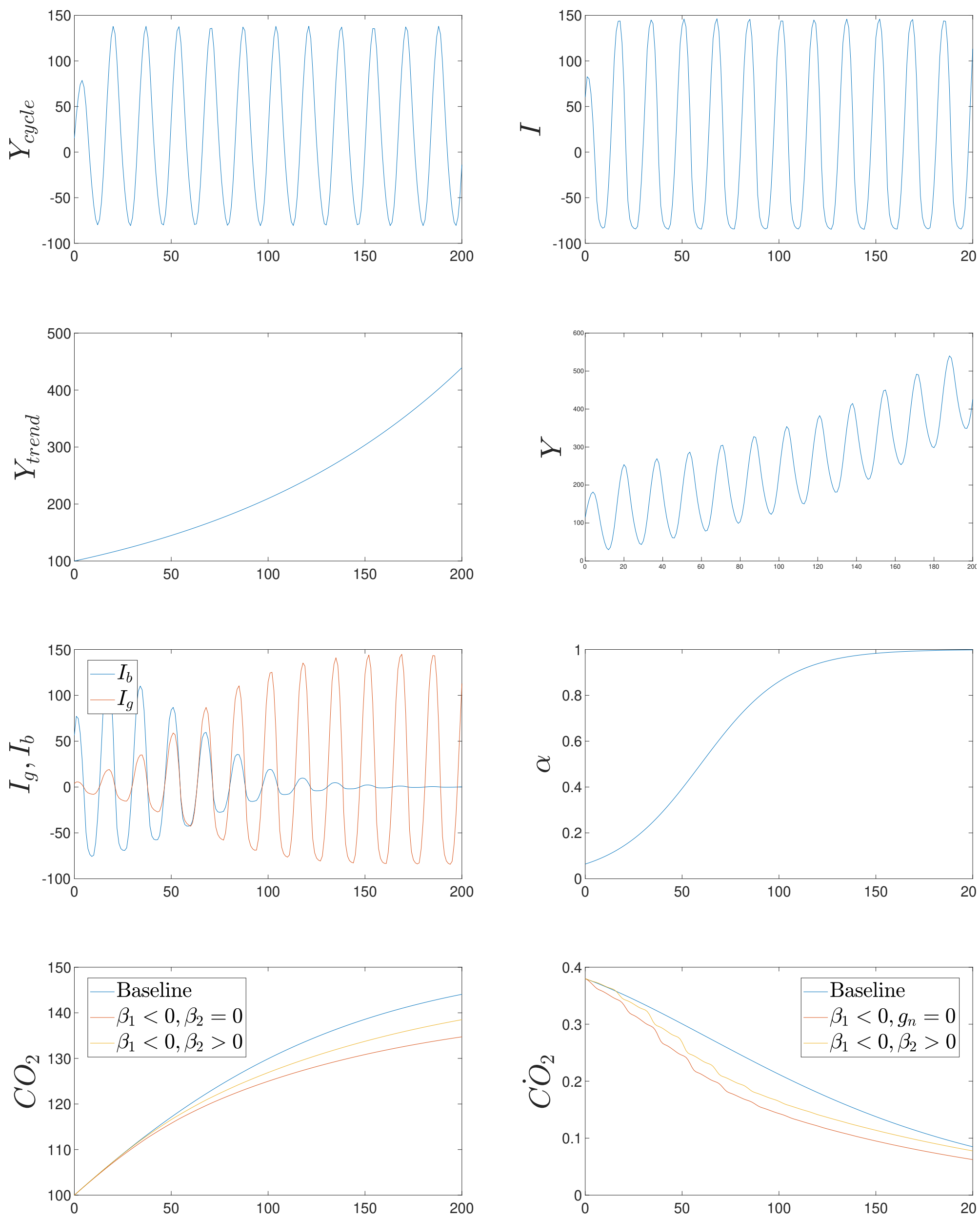
$$C\ddot{O}_2 = \eta C\dot{O}_2\left[1 - \frac{2CO_2}{CO_2^{max} - \beta_1\left(\alpha(1-\alpha)\dot{\kappa}^2 C\dot{O}_2\right) + \beta_2 Y C\dot{O}_2}\right] \tag{C.22}$$

$$\dot{Y}_{trend} = \log(\tau V \exp(g_n t)) \tag{C.23}$$

$$\dot{Y} = \dot{Y}_{cycle} + \dot{Y}_{trend} \tag{C.24}$$

Figure C.2 shows the simulation results of the amended model with a deterministic trend given by the growth rate of potential output.

Figure C.2 The model with a deterministic growth trend



Source: author's representation

C.3 A Five-Parameter Logistic Regression Model for Cumulative Carbon Emissions in the US

The model presented in Section 3.3 relies on the assumption of a 2-parameters logistic functions (η and CO_2^{max}) to describe the trajectory of cumulative emissions in the absence of any influence on their flow via the investment channel. The assumption of such a function, however, rules out the possibility of asymmetries and asymptotic properties of the curve. Therefore,

$$CO_2(t) = CO_2^{min} + \frac{CO_2^{max} - CO_2^{min}}{\left(1 + \exp\left(\eta(C - t)\right)\right)^S} \tag{C.25}$$

where

- CO_2^{min} constitutes the horizontal asymptote when $t \to -\infty$
- CO_2^{max} constitutes the horizontal asymptote when $t \to +\infty$, i.e. it measures the carrying capacity;
- η is the slope factor, i.e. the maximum growth rate of emissions that can be achieved;
- S is the asymmetry factor ($S = 1$ implies symmetry);
- C is the location factor (when $S = 1$, C is the midpoint between the two asymptotes)

$$t_{\text{mid}} = C - \frac{\log\left(2^{\frac{1}{S}} - 1\right)}{\eta} \tag{C.26}$$

Given our interest in estimating the midpoint between the two asymptotes (t_{mid}), let us rewrite equation C.26 in terms of C, i.e. $C = t_{\text{mid}} + \log\left(2^{\frac{1}{S}} - 1\right)/\eta$. Plugging it into equation C.25, we obtain the following equation to be estimated:

$$CO_2(t) = CO_2^{min} + \frac{CO_2^{max} - CO_2^{min}}{\left(1 + \exp\left(\log\left(2^{\frac{1}{S}} - 1\right) + \eta(t_{\text{mid}} - t)\right)\right)^S} \tag{C.27}$$

Fitting the curve in equation C.27 thus directly returns an estimate of t_{mid} and its standard error.

The estimation procedure is conducted using the R function `SSfpl`, which returns 5 starting values for the 5 parameter of the logistic function in equation C.27, given the time series of cumulative CO_2 emissions for the US from 1870 to 2020 (see Appendix C.5).

Table C.1 Estimation results

	Estimate	Std. Error	t value	Pr(>|t|)	
CO_2^{min}	-1.90E+10	1.20E+09	-15.777	<2e-16	***
η	5.06E-02	4.34E-03	11.663	<2e-16	***
t_{mid}	1.27E+02	2.18E+00	58.303	<2e-16	***
CO_2^{max}	5.86E+11	2.33E+10	25.112	<2e-16	***
S	4.81E-01	5.17E-02	9.304	<2e-16	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.884e+09 on 146 degrees of freedom

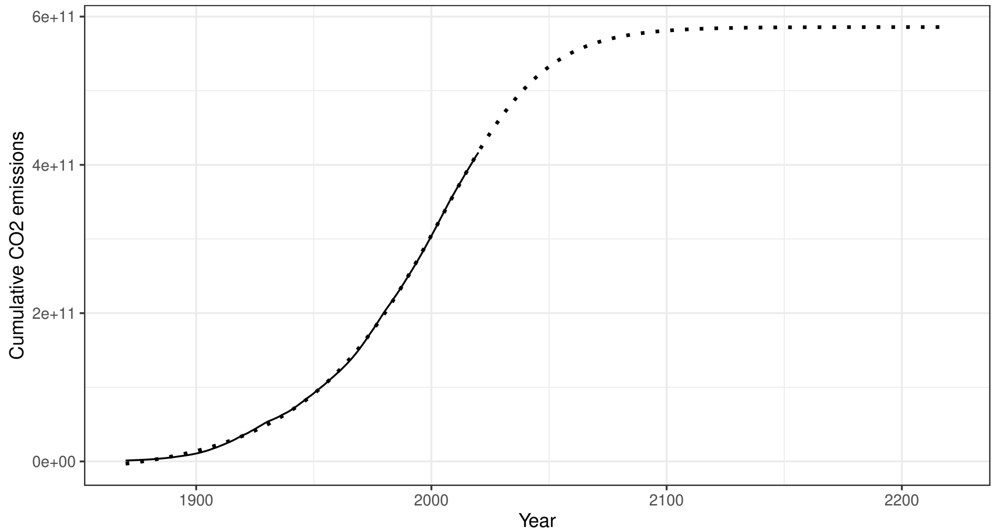
Number of iterations to convergence: 291

Achieved convergence tolerance: 1.49e-08

Source: author's calculation

The results of the estimation are reported in table C.1. Figure C.3 – which also serves to motivate the scope of the paper in the Introduction – shows how the fitted values resulting from the estimation compare with the actual data.

Figure C.3 Fitted values (dotted) vs. actual data (solid)

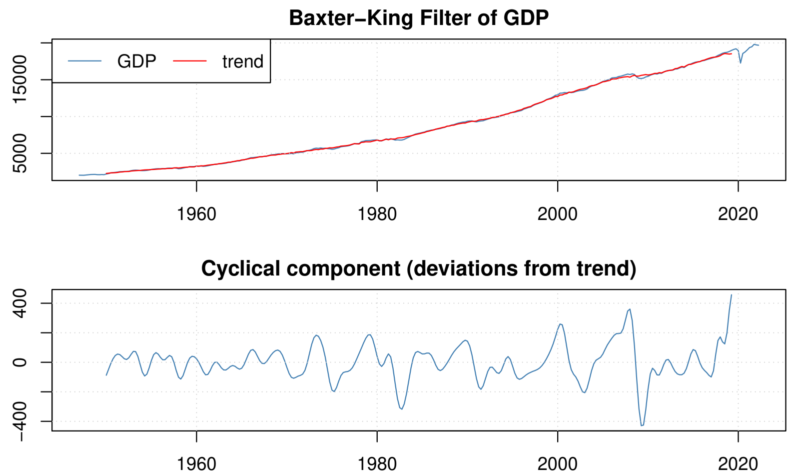


Source: author's representation

The simulated values generated from the estimation procedure just described are indexed (2012=100) and subsequently used for the parametrization of the model, as it has been discussed in section 3.4.

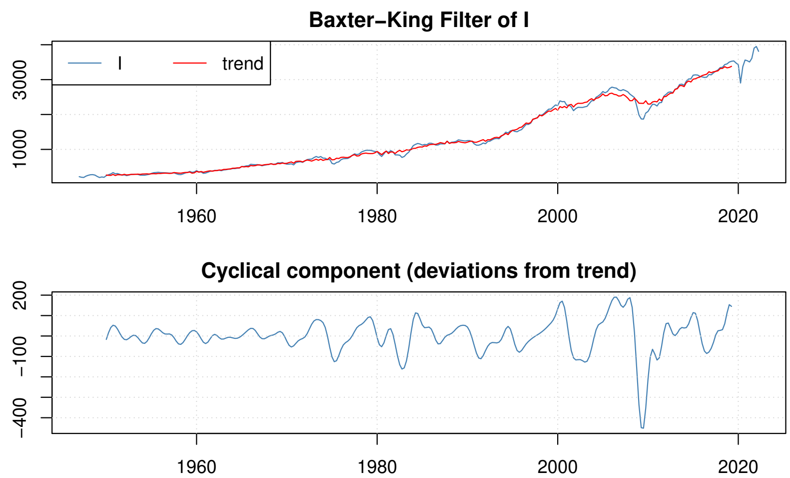
C.4 Trend-cycle Decomposition of Investment and GDP

Figure C.4 Baxter-King filter, Gross domestic product, 1947Q1-2022Q2, United States



Source: author's representation

Figure C.5 Baxter-King filter, Gross private domestic investment, 1947Q1-2022Q2, United States



Source: author's representation

C.5 Data Sources and Accessibility

- Cumulative CO_2 emissions, available at https://ourworldindata.org/co2-dataset-sources
- Gross Domestic Product, Overall, Total, Constant Prices, SA, USD, 2012 Chained Prices, BEA - Bureau of Economic Analysis, U.S. Department of Commerce, https://www.bea.gov/data/gdp/gross-domestic-product
- Gross private domestic investment, Total, Constant Prices, SA, USD, 2012 Chained Prices, BEA - Bureau of Economic Analysis, U.S. Department of Commerce, https://www.bea.gov/data/gdp/gross-domestic-product
- Renewable energy supply in percentage of total energy supply, available at https://stats.oecd.org/Index.aspx?DataSetCode=GREEN_GROWTH

Data are publicly available. Codes to reproduce the results of this paper are available upon request.

C.6 List of Variables and Parameters – Chapter 3

α	Share of green investment in total investment
β_1	Reaction of cumulative CO_2 emissions to green investment
ϵ	Time-lag of the dynamic multiplier
η	Growth constant of cumulative CO_2 emissions
θ	Lag between decisions to invest and the corresponding outlays
ϕ	Adjustment speed of brown investment
CO_2	Cumulative CO_2 emissions
CO_2^{max}	Carrying capacity of cumulative CO_2 emissions
$\dot{\kappa}$	Induced investment
$\dot{\kappa_g}$	Green (induced) investment
$\dot{\kappa_b}$	Brown (induced) investment
$\dot{\kappa}^*$	Ceiling of (induced) investment
$\dot{\kappa}^{**}$	Floor of (induced) investment
m	Propensity to import
r	Growth constant of the green investment share
s	Tax-adjusted propensity to save
v_b	Acceleration coefficient of brown investment
v_g	Acceleration coefficient of green investment
Y	Output
$\overline{Z}$	Autonomous components of aggregate demand (constant)

Please note that parameters/variables used in the previous appendices but not in the main text are not reported here.