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Large constellations for fast acquisition and delivery of information: Design and analysis by stereographic projection onto the equatorial plane

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Summary

Recent space projects are designed by satellite constellations with a large number of spacecraft, a global character (i.e., from equatorial to high inclination orbits), and the possibility to transfer information to different satellites of the constellation (inter satellite link) in order to deliver the information to the ground as soon as possible. To cope with a large number of parameters, a fast tool for constellation design, performance evaluation and networking strategies is needed. The aim of this paper is to obtain the performance of any constellation in less than 1 s, even when the number of satellites in the constellation and the duration of the analysis is large (e.g., more than 200 satellites in a period of some days). The proposed algorithm is based on analytical formulae obtained by using the stereographic projection on the equatorial plane of the satellite orbits and the projection of the target and ground station orbits. It is believed that the two-dimensional projection proposed here can offer some advantages with respect to the spatial analysis of satellite orbits and their ground tracks, such as the reduction in time required to calculate station/satellite and satellite/satellite encounter conditions, and the clear and simple representation of the motion of the satellite of the constellation and of the ground stations of interest.

1 | INTRODUCTION

The need for data from space, including Earth observation, dedicated telecommunications, Internet Of Things, and ELINT services, is becoming very demanding since the target of the acquisition can be anywhere on Earth and must be delivered to a specific ground station in short time. This task can be realized by satellite constellations having: (i) a large number of spacecraft, (ii) global character (that is high values of orbit inclinations), and (iii) the possibility to transfer the information to different satellites of the constellation (ISL); see references.[1–4]

The design of the constellation must be optimized with respect to different requirements such as minimizing the revisit time over a target, maximizing the duration of the service over a region, and minimizing the delivery time to a ground station.[5,6]

The number of parameters to be considered in the design and evaluation of such constellations is high, for instance, the requirement of data collection and transmission (from and to specific geographical areas) within a given revisit period implies a number N (hundreds) of satellites—corresponding to a total of $N(N-1)$ possible links for each orbit among the satellites of the constellation—and for each satellite, six orbital parameters shall be chosen for ensuring a desired (minimum) number of passages on a specific target area or ground station.

To cope with such a large number of parameters, a fast tool for constellation design, performance evaluation and networking strategies is needed.

In fact, by some general design principles, the number of orbital parameters can be reduced, and the possible use of algebraic, rather than differential, equations contributes to a significant reduction of the computation time for coverage and ISL analysis.

Namely, to avoid differential action of the J2 perturbation on the orbit planes, the satellites (or families of satellites) composing the constellation must have the same semiaxis a, eccentricity, and inclination i. Eccentricity is generally zero and circular orbits of radius $a = R$ are assumed. The length of R is an input dictated by the payload, that can be slightly modified to produce orbits having closed ground track, that is the satellite ground track repeats after m days corresponding to n orbits. Moreover, the inclination i is chosen according to the maximum latitude of interest.

Then just two orbit parameters remain: the right ascension of the ascending node (RAAN) Ω and the argument of latitude u_0 (measured from the ascending node, that is from the ascending passage at the equator). These free orbit parameters can be selected to optimize the service according to one of the requirements listed before. These considerations drive the design of a large constellation: to test selected configurations and/or to implement iterative schemes to optimize the results a fast algorithm is needed.

The present paper is devoted to obtaining in less than 1 s the performance of any given constellation, even if the number of satellites of the constellation and the duration of the analysis are large (say more than 200 satellites in a period of time of some days).

The algorithm proposed is based on analytical formulas obtained by the use of the stereographic projection onto the equatorial plane of the satellite orbits and the projection of the target and ground station paths.

It is believed that the two-dimensional projection proposed here can offer some advantages with respect to spatial analysis of the satellite orbits and their ground tracks. The advantages are the time reduction of the computation of the station/satellite and satellite/satellite encounter conditions and the clear and easy planar representation of the motion of a constellation of the satellites and of the ground stations of interest.

2 | STEREOGRAPHIC PROJECTIONS

Points on a sphere of radius $\boldsymbol{r}$ are projected into the equatorial plane by the formula:

$$S = \left\{(X,Y,Z)\, s.t.\ X^2+Y^2+Z^2=r^2\right\},$$

$$\varphi : S - \{North\ Pole\} \rightarrow \mathbb{R}^2,$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \begin{bmatrix} \dfrac{r}{r-Z}X \\ \dfrac{r}{r-Z}Y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}.$$

It shall be noted that, because the transformation is singular at the poles, the projection of polar orbits shall be treated differently, as detailed in Section 5. The inverse transformation is

$$\varphi^{-1} :\ \mathbb{R}^2\ \rightarrow S - \{North\ Pole\},$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{x^2+y^2+r^2} \begin{bmatrix} 2r^2x \\ 2r^2y \\ r\left(x^2+y^2-r^2\right) \end{bmatrix}.$$

In the present applications, $\boldsymbol{r}$ is the radius of the Earth R_E when the motion of ground stations and targets at latitude L and geographic longitude λ_g are considered, as well as satellite ground tracks of satellites; $\boldsymbol{r}$ is the radius $\boldsymbol{R}$ of satellite orbits in a constellation having all circular orbits of equal radius and inclination i Figure 1.

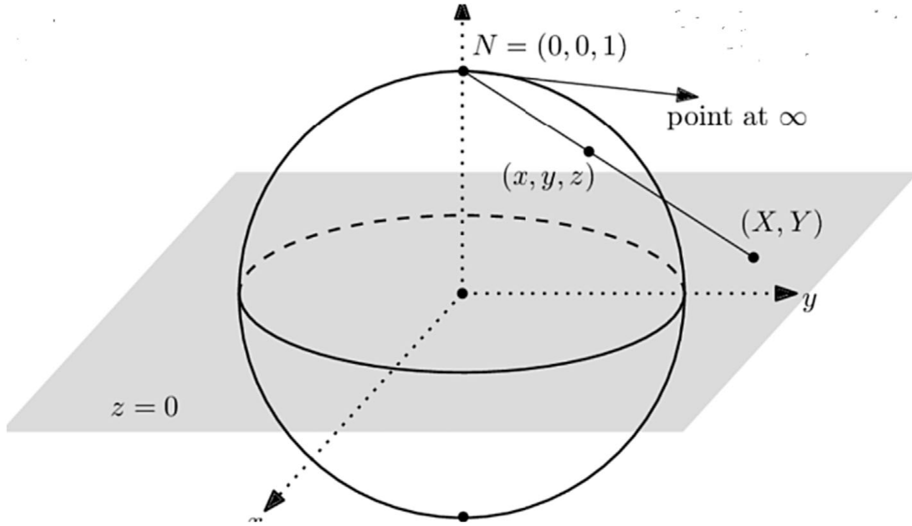

FIGURE 1 Stereographic projection.

2.1 | Ground stations stereographic projection

Any point on Earth at latitude L_0 and geographic longitude λ_g is defined in the Earth-centered inertial (ECI) frame[*,7] by its initial (i.e., at time $t=0$) absolute longitude λ_0, determined by the absolute longitude of the Greenwich meridian at initial time λ_{G0}:

$$\lambda_0 = \lambda_{G0} + \lambda_g.$$

During time we have the variation of (absolute) longitude:

$$\lambda = \lambda(t) = \lambda_0 + \omega_E t,$$

(with ω_E angular velocity of the Earth), while the latitude is constant: $L(t) = L_0$.

The ECI coordinates of the target or ground station are

$$X_E = R_E \cos L \cos \lambda,$$

$$Y_E = R_E \cos L \sin \lambda,$$

$$Z_E = R_E \sin L,$$

then the stereographic projection is

$$\begin{bmatrix} x_E \\ y_E \end{bmatrix} = \frac{R_E}{R_E - Z_E} \begin{bmatrix} X_E \\ Y_E \end{bmatrix} = \begin{bmatrix} \dfrac{\cos L}{1-\sin L} R_E \cos \lambda \\ \dfrac{\cos L}{1-\sin L} R_E \sin \lambda \end{bmatrix}.$$

That is, the target and ground station motion are projected on the equatorial plane into the circle of radius $R_{\pi S} = \frac{\cos L}{1-\sin L} R_E$ followed by constant angular velocity ω_E.

Note that from the definition of $R_{\pi S}$, stations in the North hemisphere ($L > 0$) are circles external to the equator, and stations in the South hemisphere ($L < 0$) are inside the equator.

*A frame with origin in the Earth center, $\hat{c}_1, \hat{c}_2$ axes in the equatorial plane with $\hat{c}_1$ inertially fixed along the intersection between the equatorial and the ecliptic plane and $\hat{c}_3$ axis orthogonal to the equatorial plane and pointing from the South to the North pole.

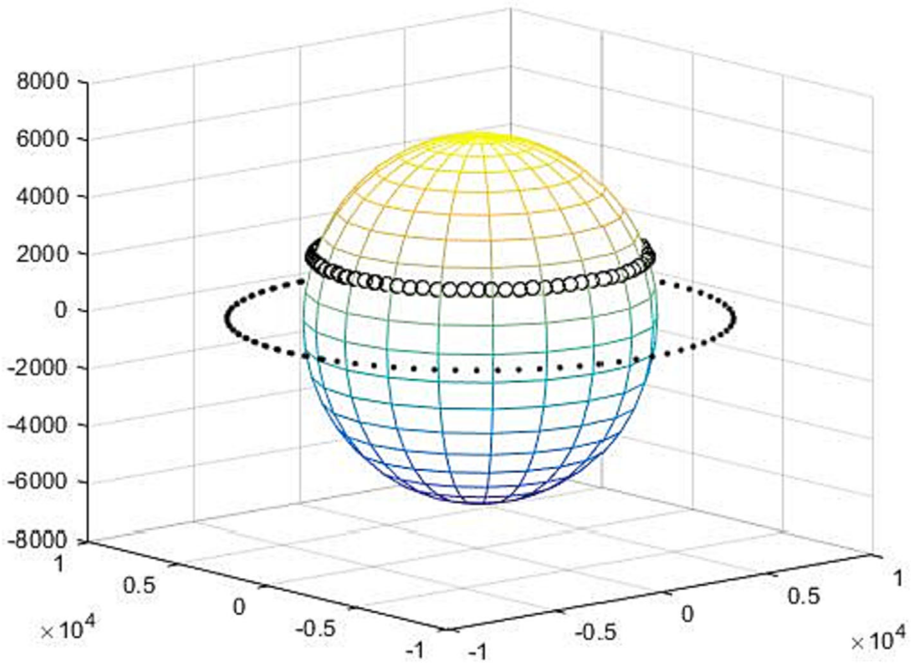

FIGURE 2 Stereographic projection of a ground station @ latitude 30°.

Figure 2 shows the equatorial projection of the motion of a ground station located at the latitude of 30°.

The inverse transformation of the ground station projection map is

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{x^2+y^2+R_E^2} \begin{bmatrix} 2R_E^2 x \\ 2R_E^2 y \\ R_E\left(x^2+y^2-R_E^2\right) \end{bmatrix}.$$

2.2 | Stereographic projection of non-polar circular orbits

Consider now non-polar circular orbits with initial parameters semiaxis, eccentricity, inclination, RAAN, and argument of the node: $a_0 = R, e_0 = 0, i_0, \Omega_0, u_0$. These parameters change in time according to the Kepler law and J2 gravitational (average) perturbation

$$a = a_0,\ e = e_0,\ i = i_0,\ \Omega = \Omega_0 + \omega_{J2} t,\ u = u_0 + (\omega_0 + \omega_{0j2}) t,$$

where $\omega_{J2} = \omega_{J2}(a_0, i_0)$, $\omega_{0j2} = \omega_{0j2}(a_0, i_0)$, $\omega_0 = \sqrt{\frac{\mu}{a^3}}$ are angular velocities of Kepler orbits perturbed by the J2 effect (see, for instance, Prussing and Conway [8]).

The satellite position $\vec{r}$ is defined in the ECI frame and on the orbital frame obtained by the three rotations:

$$\begin{pmatrix} \hat{r} \\ \hat{\theta} \\ \hat{h} \end{pmatrix} = R_u^3 R_i^1 R_\Omega^3 \begin{pmatrix} \hat{c}_1 \\ \hat{c}_2 \\ \hat{c}_3 \end{pmatrix},$$

$$\vec{r} = [R\,0\,0]\ R_u^3 R_i^1 R_\Omega^3 \begin{pmatrix} \hat{c}_1 \\ \hat{c}_2 \\ \hat{c}_3 \end{pmatrix},$$

$$[X\,Y\,Z] = [R\,0\,0]\ R_u^3 R_i^1 R_\Omega^3,$$

$$[X\,Y\,Z] = R\,[\cos u \cos \Omega - \sin u \sin \Omega \cos i, \cos u \sin \Omega + \sin u \cos \Omega \cos i, \sin u \sin i], \tag{1}$$

where R_x^i indicates a rotation around the i-th reference axis of an angle x, $[X\,Y\,Z]$ are the ECI coordinates of the satellite along its circular orbit.

The stereographic projection of the orbit on the equatorial plane is

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \begin{bmatrix} \frac{R}{R-Z}X \\ \frac{R}{R-Z}Y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}.$$

Figure 3 shows the stereographic projection of a circular orbit with parameters $R = 6896$ km, $i = 50°$, $\Omega = 10°$

It is important to note that the stereographic projection of a circular orbit on the equatorial plane is itself a circular orbit with center and radius depending on the orbit inclination i and RAAN Ω.

Namely, let us consider two points on the orbit: the points at maximum and minimum latitude, corresponding to the values $u = \frac{\pi}{2}$, $\frac{3}{2}\pi$ respectively (recall that $u = 0$ corresponds to the point on the equator of the ascending node). The three-dimensional coordinates (ECI coordinates) of these two points are

$$[X_a\,Y_a\,Z_a] = R\,[-\sin\Omega\cos i, \cos\Omega\cos i, \sin i],$$
$$[X_p\,Y_p\,Z_p] = R\,[\sin\Omega\cos i, -\cos\Omega\cos i, -\sin i],$$

having the projections:

$$\begin{bmatrix} x_a \\ y_a \end{bmatrix} = \begin{bmatrix} \frac{R}{R-Z_a}X_a \\ \frac{R}{R-Z_a}Y_a \end{bmatrix}, \begin{bmatrix} x_p \\ y_p \end{bmatrix} = \begin{bmatrix} \frac{R}{R-Z_p}X_p \\ \frac{R}{R-Z_p}Y_p \end{bmatrix}.$$

These stereographic projections correspond to the most distant point (x_a, y_a) from the Earth center O and to the closest point (x_p, y_p). The Euclidean distances

$$d_a = \sqrt{x_a^2 + y_a^2},\ d_p = \sqrt{x_p^2 + y_p^2}$$

are here used to define the parameters a and e of the projected orbit:

$$a = \frac{d_a + d_p}{2} = \frac{R}{|\cos i|},\ e = \frac{d_a - d_p}{d_a + d_p} = \sin i.$$

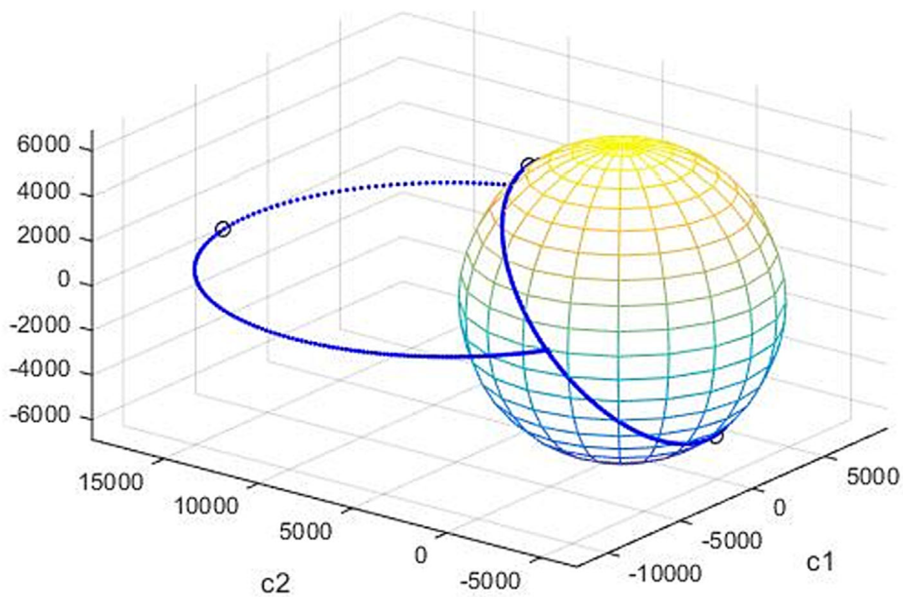

FIGURE 3 Stereographic projection of a circular orbit of inclination of 50°.

In fact, a is the radius of the projected orbit and the product ae is the distance between the center of the Earth O and the center C of the projected orbit. The direction of the center C is determined by the angle $\Omega+\frac{\pi}{2}$ measured from $\hat{c}_1$. The orbit of Figure 3 has the projection of Figure 4, showing the center C of the circle of radius $a=10{,}728$ km and its vector distance from O, having an orientation angle from $\hat{c}_1$ equal to 100° and distance $ae=8218$ km.

Namely, we have the coordinates of the center C

$$C=(x_C,y_C)=\left(ae\cos\left(\Omega+\frac{\pi}{2}\right),ae\sin\left(\Omega+\frac{\pi}{2}\right)\right)=(-ae\sin\Omega,ae\cos\Omega)$$

and the planar coordinates of the projected orbit are as follows:

$$x(\tau)=x_C+a\cos\tau, \tag{2}$$

$$y(\tau)=y_C+a\sin\tau, \tag{3}$$

where

- $a=\frac{R}{|\cos i|}$,
- $ae=R|\tan i|$,
- τ is the phase of the projected circle starting from axis ξ of the (ξ,η) frame parallel to $\hat{c}_1,\hat{c}_2$; see Figure 5.

The initial phase τ_0 in the projected orbit can be derived by the stereographic projection of the initial position of the satellite (X_0,Y_0,Z_0) obtained by Equation 1 with $u=u_0$. Then, the stereographic projection is used to get the initial point on the projected orbit (x_0,y_0), hence, the initial phase τ_0 from

$$\cos\tau_0=\frac{x_0-x_C}{a},$$

$$\sin\tau_0=\frac{y_0-y_C}{a}.$$

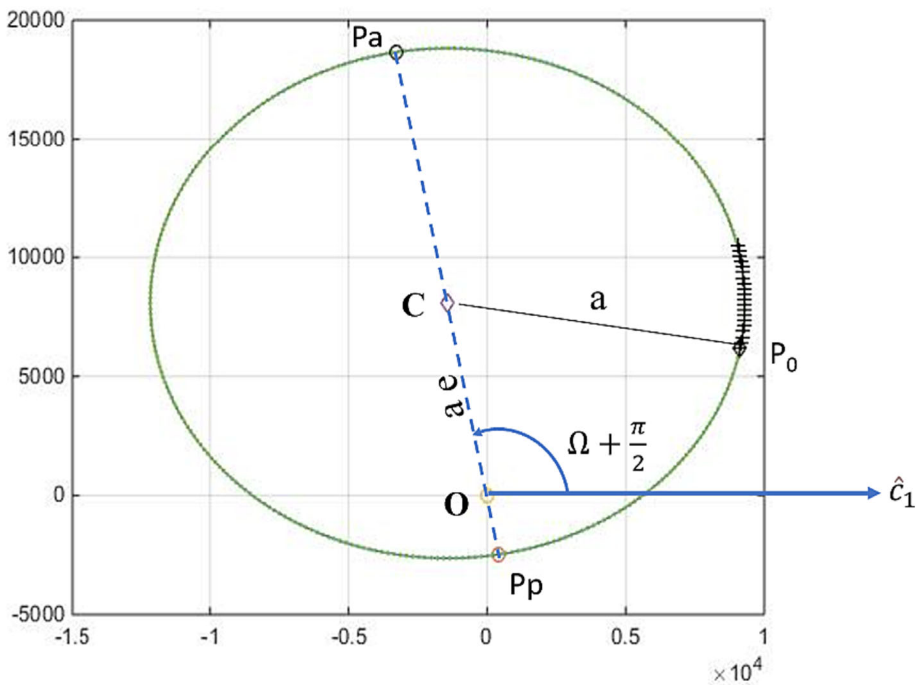

FIGURE 4 The circle on the equatorial projection of the orbit of Figure 3.

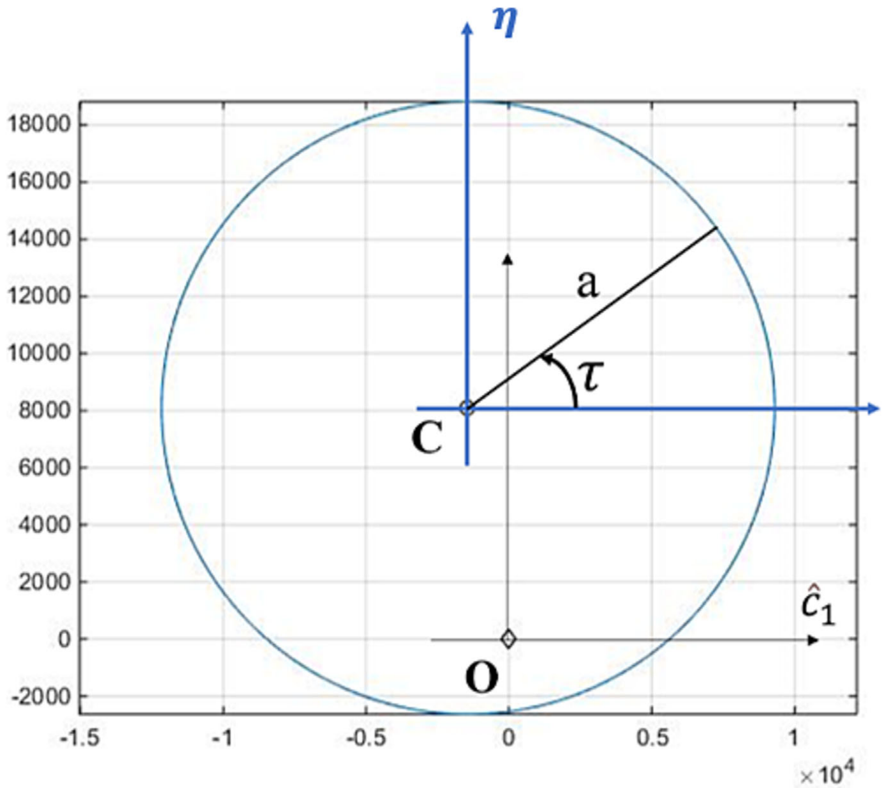

FIGURE 5 The phase angle τ of the projection into the equatorial plane.

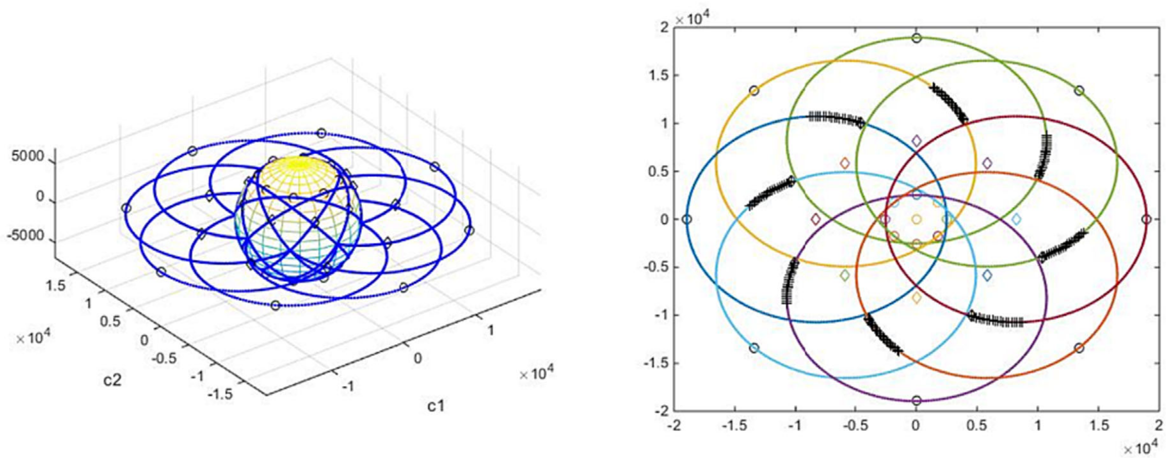

FIGURE 6 Constellation of eight satellites and its planar projection. Black markers in the right figure show the initial point of the satellites on each projected orbit and their motion for some minutes.

The satellites of the constellation differ in RAAN only, then their orbits are projected onto circles of equal radius a rotated around the common point O (center of the Earth) by different angles $\Omega+\frac{\pi}{2}$ with respect to the $\hat{c}_1$ direction. In Figure 6, we see the stereographic projection of a constellation of eight satellites with equally spaced RAAN, inclination of 50°, and initial phases corresponding to maximum and minimum latitude. The 3D orbits of the constellation with the satellite initial positions (two sats for each orbit) are on the left of Figure 6, showing also the equatorial projections with the projected initial positions. On the right side of the figure, we see also Earth's center at the origin of the picture (yellow "dot") and the center points of the projected orbits ("diamonds" of different color). These points have an equal distance ae from the Earth's center and are rotated around the Earth's center according to the equally spaced RAAN angles.

Each of the $N = 8$ satellites of the constellation has two intersections with any other orbit: the number of possible pairs of satellites is given by the sum:

$$(N-1)+(N-2)+\dots+(N-(N-1)) = N^2 - \Sigma_{k=1}^{N-1} k = N^2 - \frac{(N-1)N}{2} = \frac{N^2-N}{2}$$

Since the number of intersections for each pair is equal to 2, the total number of intersections is: $N^2 - N$.

The 56 intersections of the $N = 8$ constellation are clearly visible in the projection of Figure 6. In fact, the use of the stereographic projection simplifies the identification of intersections between the orbit of the satellites, this feature is used, as detailed in Section 3, to rapidly identify ISL among the satellites of the constellation.

3 | INTER SATELLITE LINK

It is supposed that the satellite of the constellation can communicate data with the other satellites within a fixed distance D. When the relative distance of two satellites is less than D, the transmission of data between is possible and the ISL duration can be determined with the following steps:

a. determine the points of intersections among the orbits of the constellation
b. determine the area of possible link ("link area") which is located around these intersections
c. determine the entry times within the link area of each pair of satellites
d. if two satellites are both inside the link area, determine the duration of their link

Step (a)

The determination of the intersection points can be restricted within one orbit period: the J2 perturbation will move rigidly to the intersection points and the duration of the ISL will be obtained just by updating the passage to the node period.

For example, let us consider the two orbits in Figure 7 (orbit 1 and orbit 2) with $\Omega_1 = 10deg$, $\Omega_2 = 20^\circ$.

The projected orbit 1 with planar coordinates (x_1, y_1) has the two intersection points P_1 and P_2 with the projected orbit 2, having planar coordinates (x_2, y_2). Let τ_{121}, τ_{122} be the two phases in the projected orbit 1 corresponding to the points P_1 and P_2, respectively, and τ_{211}, τ_{212} be the two phases in the projected orbit 2 corresponding to the same intersection points P_1 and P_2. In general, the angles $\tau_{k_1 k_2 s}$ denote the phase in orbit k_1 of the intersection point $s = 1,2$ with the orbit k_2.

The condition on the coordinates of any intersection point P_1 or P_2 between the projections of orbits 1 and 2 is (neglecting index s):

$$x_1(\tau_{12}) = -ae\sin\Omega_1 + a\cos\tau_{12} = x_2(\tau_{21}) = -ae\sin\Omega_2 + a\cos\tau_{21}, \tag{4}$$

$$y_1(\tau_{12}) = ae\cos\Omega_1 + a\sin\tau_{12} = \quad y_2(\tau_{21}) = ae\cos\Omega_2 + a\sin\tau_{21}. \tag{5}$$

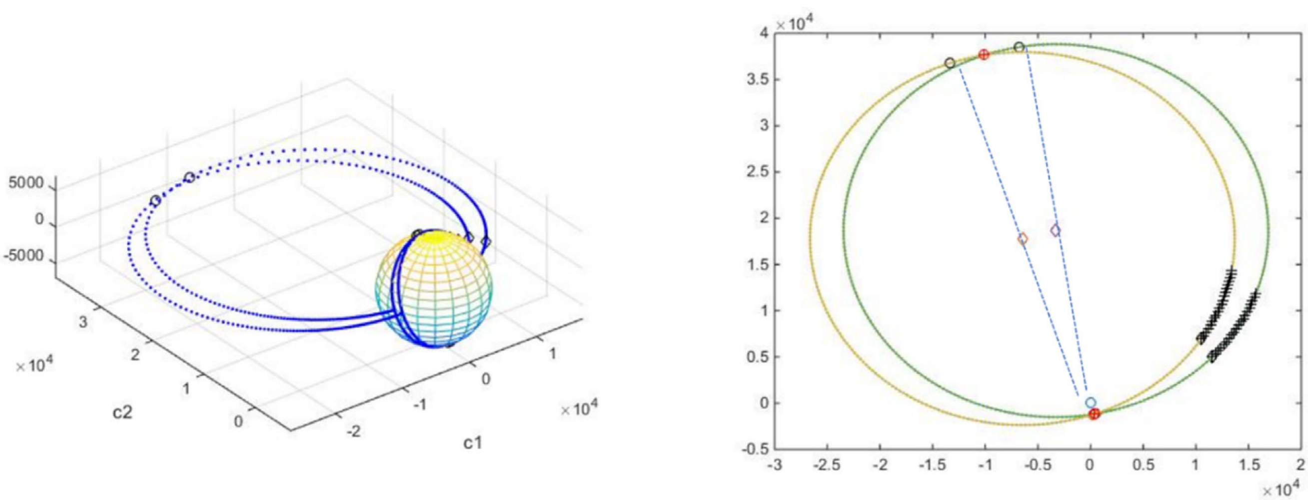

FIGURE 7 Two orbits with different RAAN angles.

In Appendix A, the following two solutions for the angle τ_{21} are derived:

$$\tau_{211} = \text{asin}(-c) - \phi \tag{6}$$

and

$$\tau_{212} = \pi - \text{asin}(-c) - \phi \tag{7}$$

with

$$c = e\sqrt{(1-\cos\,(\Omega_1-\Omega_2))}/\sqrt{2},$$

$$\cos\,\phi = \frac{(\cos\,\Omega_2 - \cos\,\Omega_1)}{\sqrt{1-\cos\,(\Omega_2-\Omega_1)}},$$

$$\sin\,\phi = \frac{(\sin\,\Omega_1 - \sin\,\Omega_2)}{\sqrt{1-\cos\,(\Omega_2-\Omega_1)}}.$$

To obtain the phases on orbit 1, insert the value τ_{211} in Equations (4) and (5) to determine the phase τ_{121} in the projected orbit 1 of the intersection point P_1 then insert the value τ_{212} to determine the phase τ_{122} of the intersection point P_2. In general, we have the phases $\tau_{k_1k_2s}$ for any pair k_1,k_2 of projected orbits $(k_1,k_2=1,..,N,s=1,2)$. It is easy to derive the corresponding phases in the 3D representation. This is done by introducing the coordinates of the intersection points between the projected orbits: $x_{k_1k_2s} = x_{k_1}\left(\tau_{k_1k_2s}\right)$, $y_{k_1k_2s} = y_{k_1}\left(\tau_{k_1k_2s}\right)$ $(s=1,2)$ then, by the inverse of the stereographic projection, the ECI coordinates of the intersection points between orbit k_1 and orbit k_2 are found:

$$X_{k_1k_2s} = \frac{2R^2x_{k_1k_2s}}{x^2_{k_1k_2s}+y^2_{k_1k_2s}+R^2},$$

$$Y_{k_1k_2s} = \frac{2R^2y_{k_1k_2s}}{x^2_{k_1k_2s}+y^2_{k_1k_2s}+R^2},$$

$$Z_{k_1k_2s} = R\,\frac{x^2_{12s}+y^2_{12s}-R^2}{x^2_{k_1k_2s}+y^2_{k_1k_2s}+R^2}.$$

From the three ECI coordinates of the intersection points, the corresponding phases of the 3D representation are derived according to the equations relating orbit parameters to ECI coordinates. For orbit k_1,

$$\cos\,u_{k_1k_2s} = \frac{Y_{k_1k_2s}\sin\,\Omega_{k_1}+X_{k_1k_2s}\cos\,\Omega_{k_1}}{R},$$

$$\sin\,u_{k_1k_2s} = \frac{Z_{k_1k_2s}}{R\,sin(i)},$$

and similar formulas for the phases $u_{k_2k_1s}$ on orbit k_2.

Step (b): "link area"

The region of possible interlink between two satellites is around the intersection points between the two orbits and depends on the maximum distance D of the possible link, see Figure 8.

Considering $D \leq 1000$ km, we are going to approximate the encounter region between the two satellites as the region in the tangent space of the orbits at the intersection point as in Figure 9. If angle Δu between the two satellite orbits is lower than $\frac{\pi}{2}$, the area of possible link between the two satellites is inside the two triangles of Figure 9, since outside this region the distance between the two satellites is bigger than D.

If angle Δu between the two satellite orbits is bigger than $\frac{\pi}{2}$, the area of possible link between the two satellites is inside the two triangles of Figure 10.

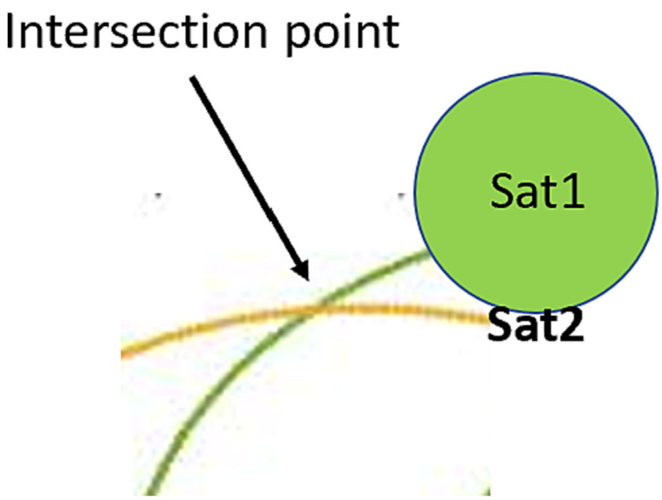

FIGURE 8 ISL distance and Sat1, Sat2 in proximity of the intersection point.

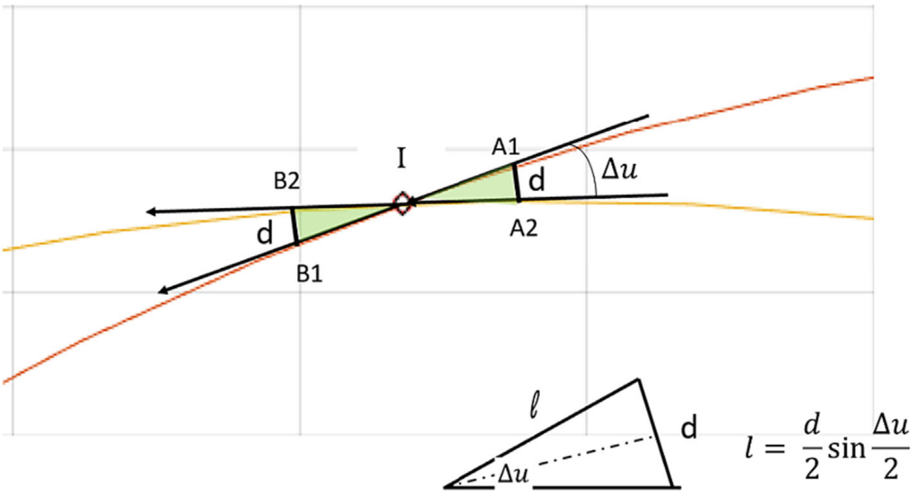

FIGURE 9 The link region for satellites with encounter angle $\Delta u < \frac{\pi}{2}$.

It is important to know that the stereographic projection is a conformal transformation, that is the angles are preserved. It follows that the angle Δu between the two 3D orbits is equal to the angle $\Delta\tau$ between the two projected orbits: $\Delta u = \Delta\tau$. From the geometry in the projected plane, see Figure 11, such an angle $\Delta\tau$ is equal to the difference between the two phases:

$$\Delta\tau = \tau_{k_2k_1s} - \tau_{k_1k_2s}.$$

Step (c): Link area entry and exit times

Let the satellite in orbit k_2 have the initial phase u_{0k_2} and let the phase of one intersection point between this orbit and the orbit k_1 be $u_{k_2k_1s}$. If $u_{0k_2} < u_{k_2k_1s}$, the time needed to Sat k_2 to arrive at the intersection point with the orbit k_1 is

$$t_{intk_2k_1s} = \frac{u_{k_2k_1s} - u_{0k_2}}{n}, \quad n = \sqrt{\frac{\mu}{R^3}}. \tag{8}$$

If $u_{0k_2} > u_{k_2k_1s}$, the time needed to Sat k_2 to arrive at the intersection point with orbit k_1 is

$$t_{intk_2k_1s} = \frac{2\pi + u_{k_2k_1s} - u_{0k_2}}{n}, \quad n = \sqrt{\frac{\mu}{R^3}}. \tag{9}$$

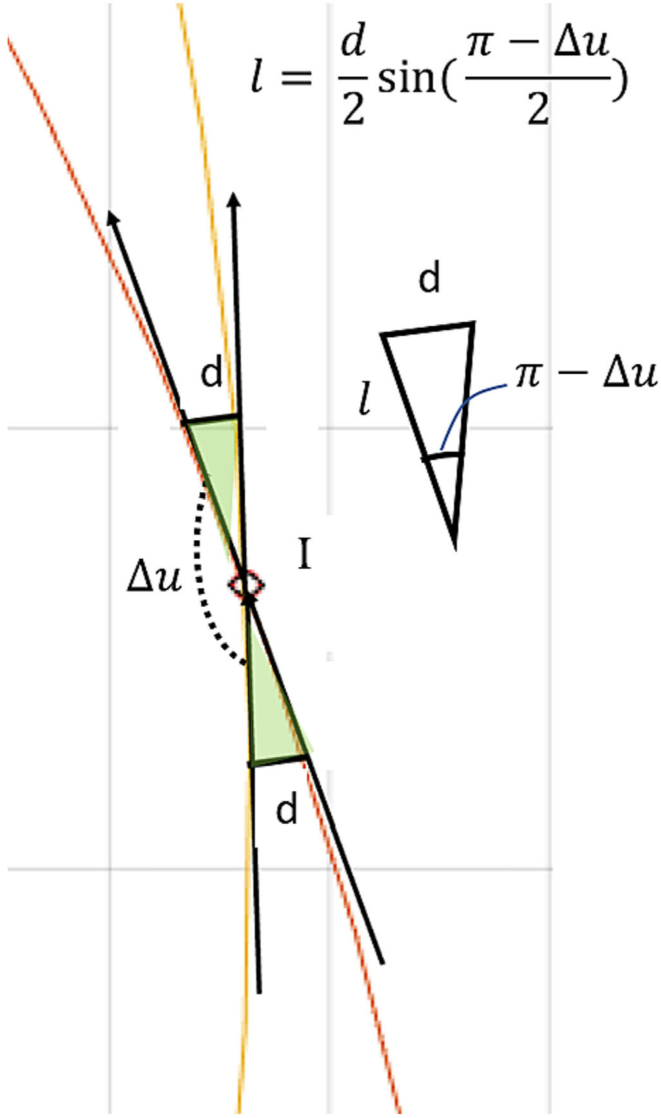

FIGURE 10 The link region for satellites with encounter angle $\Delta u > \frac{\pi}{2}$.

To have the time of arrival to the link region, we have to compute the time to arrive at boundary point A_2 of Figure 9, that is the time needed to move along the distance ℓ between the boundary point A_2 and the intersection point I. Such a time is equal to $\Delta T = \frac{l}{V_0} = \frac{l}{nR}$, then the Sat k_2 arrives to the link region at time

$$t_{inik_2k_1s} = t_{intk_2k_1s} - \Delta T \tag{10}$$

and leaves the region at time $t_{intk_2k_1s} + \Delta T$.

Similar formulas give the arrival time of the Sat k_1, $t_{inik_1k_2s}$, and the exit time $t_{intk_1k_2s} + \Delta T$.

Step (d): Link condition and duration

To have a link between Sat k_1 and Sat k_2, it is necessary that both are within the link region. Suppose that Sat k_2 arrives at time $t_{inik_2k_1s} < t_{inik_1k_2s}$ at point A_2, then Sat k_1 will be distant from the boundary point A_1 for a length equal to $V_0\, dt_{ini}$, where dt_{ini} is the difference in arrival time of the two satellites at the link region and V_0 is the orbital velocity:

$$dt_{ini} = t_{inik_1k_2s} - t_{inik_2k_1s}, \tag{11}$$

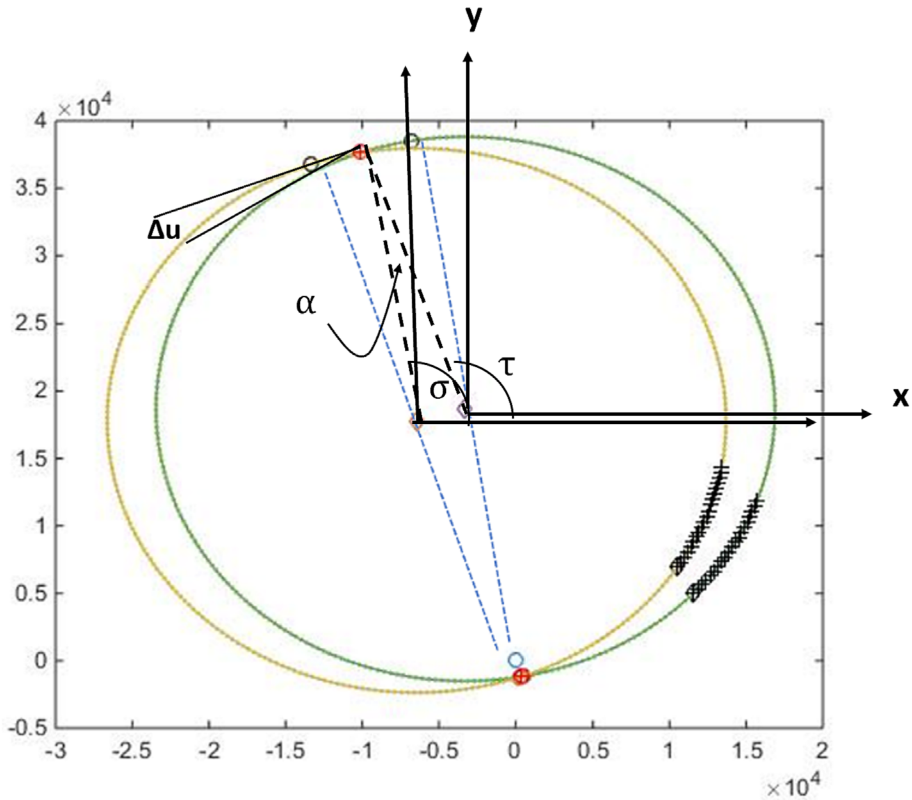

FIGURE 11 The angle Δu generated by the tangent of the two projected orbits on the intersection is equal to the angle α of the picture and this is equal to the difference of phases τ and σ in the two orbits.

It is proved in Appendix A that the minimum distance between the two satellites within the link area is given by the formula:

$$d_{min} = = V_0 \, dt_{ini} \cos \frac{\Delta u}{2}. \tag{12}$$

Then, the condition for the link is

$$d_{\min} < D,$$

and the time duration of the inter satellite link is equal to (see Appendix A):

$$ISL_{time} = \frac{\sqrt{D^2 - d_{min}^2}}{V_0 \sin \frac{\Delta u}{2}}. \tag{13}$$

Note that this is a simple analytic formula depending only on the input data V_0 and D and the encounter angle Δu defined by the stereographic projection.

4 | SATELLITE–GROUND STATION VISIBILITY

Encounters between satellite orbits and stations are determined by the intersections between two circles on the equatorial plane: the circle centered on the Earth performed by the ground station motion and the circle given by the stereographic projection of the satellite ground track (sub-satellite points on the Earth surface).

The subsatellite points are the nadir projection of the points of the orbit on the Earth's surface:

$$[X\,Y\,Z] = R_E[\cos u \cos \Omega - \sin u \sin \Omega \cos i, \cos u \sin \Omega + \sin u \cos \Omega \cos i, \sin u \sin i],$$

with stereographic projection:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \begin{bmatrix} \frac{R_E}{R_E - Z} X \\ \frac{R_E}{R_E - Z} Y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}.$$

Figure 12 represents the stereographic projection of the motion of a ground station at a latitude of 20° and of an orbit with an inclination of 30° and RAAN 20°. The projection of the ground station motion is a circle in the equatorial plane with center equal to Earth's center. This circle intersects the orbit projection in two points.

The steps to identify the visibility between Station and Sat and its duration are similar to the ones performed in the ISL analysis:

a. determine the points of intersections between the ground station and the subsat points belonging to the orbits of the constellation
b. determine the area of possible visibility ("visibility area") which is located around these intersections
c. determine the entry and departure times within the visibility area of each pair ground station/satellite
d. if ground station and satellite are both inside the visibility area, determine the duration of visibility

Step a): Station/subsatellite intersections

The conditions of the intersection of the projected trajectories are

$$x_S = x,$$

$$y_S = y,$$

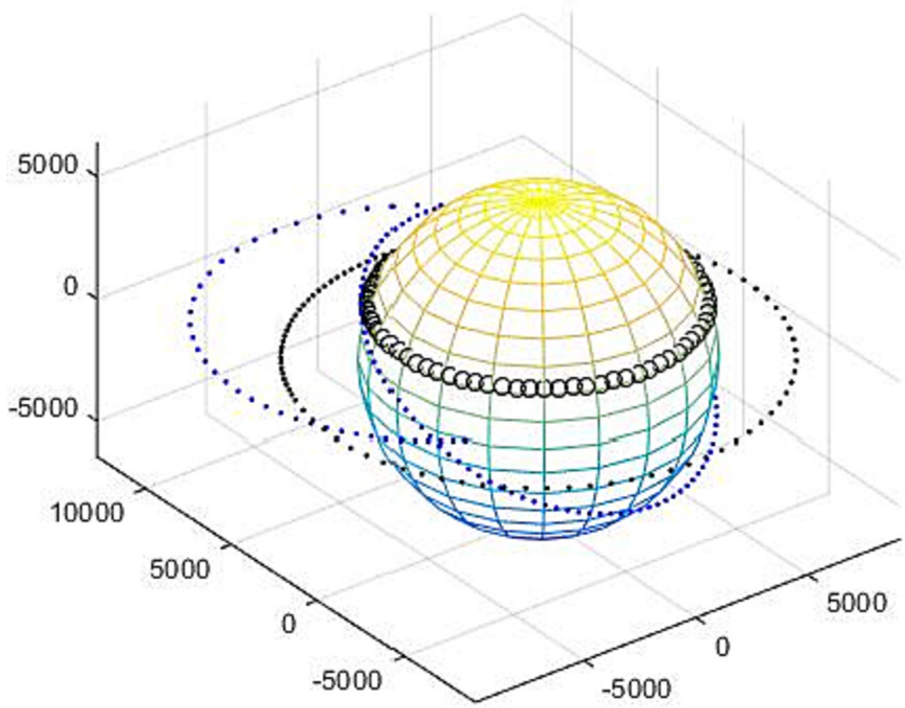

FIGURE 12 Stereographic projection of a ground station and of a circular orbit.

with (x_S, y_S) projected ground station coordinates and (x, y) are the projected subsat coordinates. By definition of the stereographic projection, the above equalities give

$$\frac{R_E \cos L_0}{1-\sin L_0}\cos\lambda = \frac{R_E}{R_E - Z}X, \tag{14}$$

$$\frac{R_E \cos L_0}{1-\sin L_0}\sin\lambda = \frac{R_E}{R_E - Z}Y, \tag{15}$$

where L_0 and λ are the latitude and absolute longitude of the ground station and the coordinates (X, Y, and Z) are the ECI coordinates of the subsatellite points.

Then, squaring and summing Equations (14) and (15), we get:

$$C_{L0} = \frac{cos^2 L_0}{(1-\sin L_0)^2} = \frac{1+\sin L_0}{1-\sin L_0} = \frac{\left(X^2+Y^2\right)}{(R_E - Z)^2} = \frac{cos^2 u + sin^2 u\, cos^2 i}{(1-\sin u \sin i)^2} = \frac{1+\sin u \sin i}{1-\sin u \sin i}, \tag{16}$$

that is,

$$(1-\sin u \sin i)\, C_{L0} = 1 + \sin u \sin i. \tag{17}$$

That is,

u-intersection condition

$$\sin u = \frac{C_{L0}-1}{C_{L0}+1}\frac{1}{\sin i}. \tag{18}$$

This equation has two solutions: u_1 and $u_2 = \pi - u_1$, corresponding to the argument of latitude of the intersection points between the orbit and the ground station. In other words, these are the nodal angles of the satellite when it meets the latitude L_0.

For each of the two solutions $u_k, k = 1,2$, we get the ECI coordinates

$$X(u_k) = R_E(\cos u_k \cos\Omega - \sin u_k \sin\Omega \cos i),$$

$$Y(u_k) = R_E(\cos u_k \sin\Omega + \sin u_k \cos\Omega \cos i),$$

$$Z(u_k) = R_E \sin u_k \sin i.$$

Then, the relationships (14) and (15) define the trigonometric functions cos and sin of the nodal angle for each intersection u_k:

λ-intersection condition

$$\frac{R_E \cos L_0}{1-\sin L_0}\cos\lambda_k = \frac{R_E}{R_E - Z(u_k)}X(u_k) \tag{19}$$

$$\frac{R_E \cos L_0}{1-\sin L_0}\sin\lambda_k = \frac{R_E}{R_E - Z(u_k)}Y(u_k) \tag{20}$$

By the function atan2, the above equations provide the absolute longitudes λ_k $k = 1,2$ of the station/subsat intersection points.

Step (b): area of possible visibility (“visibility area”)

The swath of the satellite sensor (or the elevation angle of the ground station) generates the time and duration of the visibility passage considering the intersection of the swath area with the station point.

Suppose that the satellite has a sensor with aperture angle α corresponding to the minimum elevation angle from the station ε and central angle σ:

$$\sigma = \frac{\pi}{2} - \alpha - \varepsilon.$$

This angle defines the Swath diameter

$$S_w = 2\,\sigma\,R_E$$

and radius $R_{Sw} = S_w/2$.

Let C_1 be the intersection point between the subsat orbit and the ground station. In Figure 13, a small arc of the subsat orbit is shown in blue intersecting on C_1 the ground station latitude, which is the straight line passing through the points $C_3C_1C_4$. A satellite moving from A_1 to C_1 is able to observe objects on the ground within a strip generated by circles with center in orbit blue and radius R_{Sw}. The first possible link is when one of the circles touches the station latitude, that is when the satellite is at distance *c* from the intersection point C_1 and the greatest distance of the Station from the intersection is *d*.

Then the region of possible visibility is the arch $C_3C_1C_4$ of length *2d* for the ground station and the arc $A_1C_1A_2$ of length *2c* for the subsat points.

Step (c) Visibility area: Arrival times

Let us determine the arrival time of the ground station to the ground station/sat intersection points defined by the absolute longitudes λ_1, λ_2 defined in Step (a).

Let $\lambda_1 < \lambda_2$ and be λ_0 the absolute longitude of the ground station at time t_0.

if $\lambda_0 < \lambda_1$ (see Figure 14) then the arrival time of the ground station at the two intersection points are equal to:

$$t_1^* = t_0 + \frac{\lambda_1 - \lambda_0}{\omega_E}\ ,\ t_2^* = t_0 + \frac{\lambda_2 - \lambda_0}{\omega_E}, \tag{21}$$

if $\lambda_1 < \lambda_0 < \lambda_2$, then

$$t_1^* = t_0 + \frac{2\pi + \lambda_1 - \lambda_0}{\omega_E}\ ,\ t_2^* = t_0 + \frac{\lambda_2 - \lambda_0}{\omega_E};$$

if $\lambda_1 < \lambda_2 < \lambda_0$, then

$$t_1^* = t_0 + \frac{2\pi + \lambda_1 - \lambda_0}{\omega_E}\ ,\ t_2^* = t_0 + \frac{2\pi + \lambda_2 - \lambda_0}{\omega_E}.$$

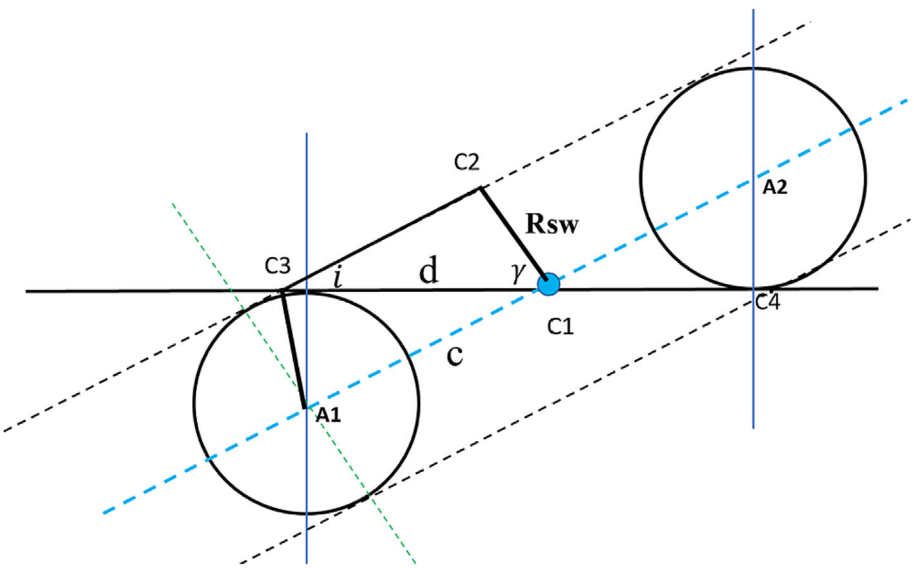

FIGURE 13 Visibility: Arches of possible visibility *d* and *c*.

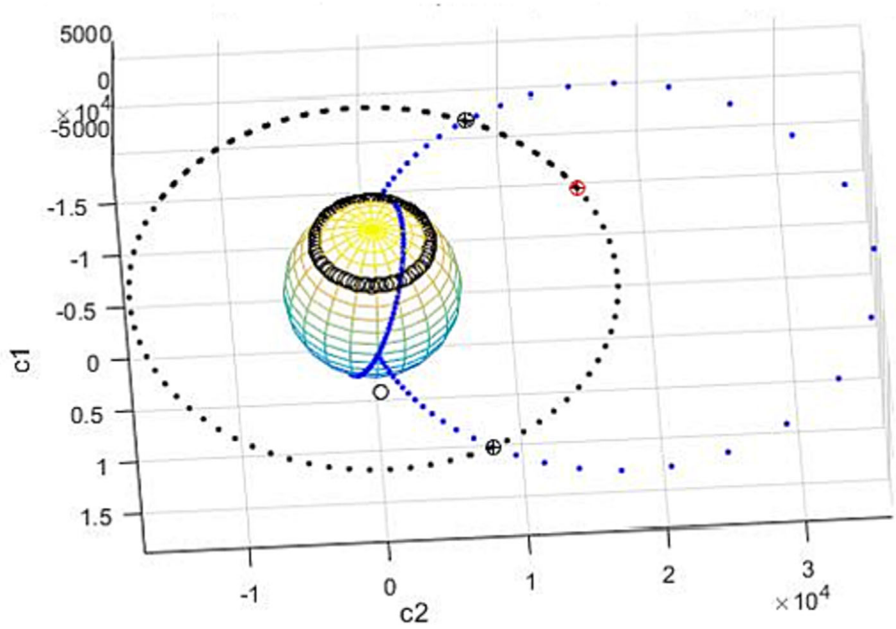

FIGURE 14 Stereographic projection of the ground station path with red marked initial longitude.

The satellite arrival times to the two intersection points are defined by the two arguments of latitude on the orbit u_1 and u_2 Determined in Step a). If u_0 is the argument of the satellite at time t_0, the arrival times of the satellite to the intersection points are:

$$\begin{aligned}
&\text{if } u_0 < u_1 \text{ then } t_1^{**} = t_0 + \frac{u_1 - u_0}{n}, \text{ else } t_1^{**} = t_0 + \frac{2\pi + u_1 - u_0}{n}, \\
&\text{if } u_0 < u_2 \text{ then } t_2^{**} = t_0 + \frac{u_2 - u_0}{n}, \text{ else } t_2^{**} = t_0 + \frac{2\pi + u_2 - u_0}{n}.
\end{aligned} \tag{22}$$

By the arrival times at the intersection point, we can derive the arrival times at the boundary of the visibility area, the points C_3 and A_1 for the ground station and subsat, respectively.

For the ground station, this time is equal to the time t_1^* (or t_2^*) minus the time needed to the ground station to cover the arch d.

Approximating the arches with the straight lines in the tangent plane, we have from the triangle of vertices $C_1C_2C_3$ that

$$d = \frac{R_{Sw}}{\sin i}.$$

So, the time needed to the ground station to cover this distance is

$$\Delta T_{GS} = \frac{d}{\omega_E R_E}.$$

Therefore, the times of arrival of the ground station to the ground station/Sat regions are

$$t_{iniGS1} = t_1^* - \Delta T_{GS}, \; t_{iniGS2} = t_2^* - \Delta T_{GS}.$$

Setting $c = d$, we get the arrival times of the subsat to the ground station/sat regions:

$$t_{iniS1} = t_1^{**} - \Delta T_S, \; t_{iniS2} = t_2^{**} - \Delta T_S$$

with

$$\Delta T_S = \frac{c}{nR}.$$

Step (d) Visibility duration

To have visibility, the ground station must be on the arch C_3C_4 of length 2d before the arrival of the satellite on the arch A_1A_2 of length 2c, that is, a necessary condition for visibility is

$$t_{\text{iniGS}s} < t_{\text{ini}Ss}, s = 1, 2.$$

Since the satellite moves faster than the ground station, the position x of the ground station at the time t_{iniSs} is kept fixed during the subsat motion from A_1 to A_2.

If the position x, counted from C_3, coincides with C_1 then we have the maximum duration of visibility, which is equal to the time needed to the subsat point to run across the entire diameter $2R_{Sw}$, that is,

$$\text{Vis}_{\text{time}} = \frac{2R_{\text{Sw}}}{\text{nR}}.$$

If the position x is different from C_1, the visibility time will be lower and equal to the time needed to run the chord B_1B_2 of Figure 15 having a length $2R_{Sw}\cos\delta(x)$, where the angle $\delta(x)$ is the angle between the line A_1B_2 and the orbit c. In such a case the visibility is equal to the ratio between the chord distance and the Sat velocity, that is,

$$Vis_{time} = \frac{2R_{Sw}\cos\delta(x)}{nR}. \tag{23}$$

Analytic formulas for the ground station position x and the related angle $\delta(x)$ are derived in Appendix A.

5 | POLAR ORBITS

The arguments of Sections 3 and 4 can be applied to polar orbits; note however that if polar orbits pass through the North Pole, which is singular in the stereographic projection, then some peculiarities must be underlined.

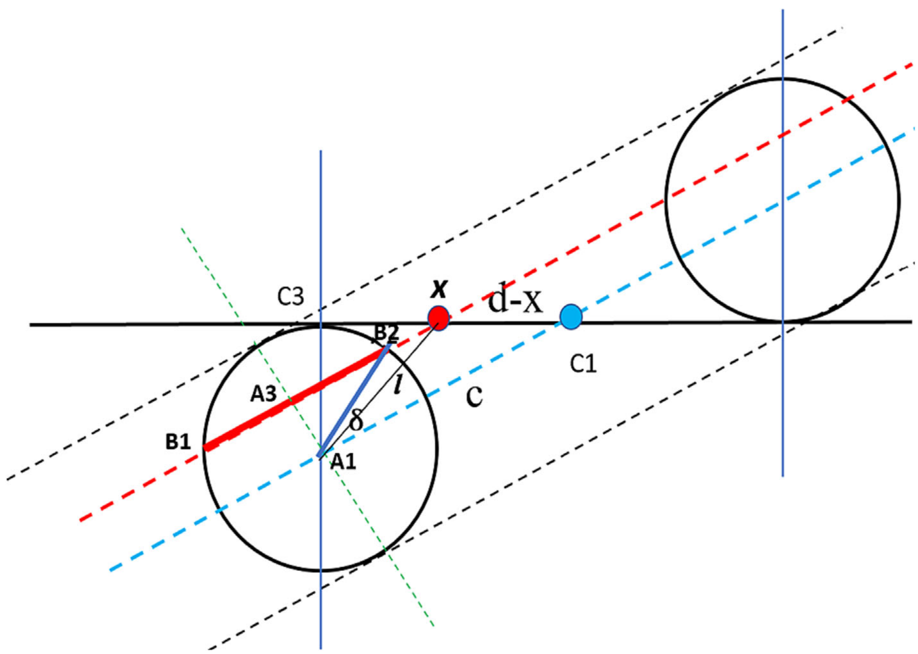

FIGURE 15 Derivation of the angle $\delta(x)$.

Note that ECI coordinates of polar circular orbits with radius R are as follows:

$$X = R\cos u \cos \Omega,$$

$$Y = R\cos u \sin \Omega,$$

$$Z = R\sin u.$$

The stereographic projections of these orbits are as follows:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{R}{R - R\sin u}\begin{bmatrix} R\cos u\cos\Omega \\ R\cos u\sin\Omega \end{bmatrix} = \frac{R\cos u}{1-\sin u}\begin{bmatrix} \cos\Omega \\ \sin\Omega \end{bmatrix}.$$

The subsatellites points of polar orbits have ECI and projected coordinates defined by

$$X = R_E\cos u\cos\Omega,$$

$$Y = R_E\cos u\sin\Omega,$$

$$Z = R_E\sin u,$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{R_E}{R_E - R_E\sin u}\begin{bmatrix} R_E\cos u\cos\Omega \\ R_E\cos u\sin\Omega \end{bmatrix} = \frac{R_E\ \cos u}{1-\sin u}\begin{bmatrix} \cos\Omega \\ \sin\Omega \end{bmatrix}.$$

As the angle u changes in time, the polar orbits define a sheaf of straight lines through the origin (Earth's center) with parametric angle Ω; see Figure 16, for one of these orbits.

In the stereographic projection, the polar orbits intersect at the center O of the equatorial plane, corresponding to the South Pole, and at infinity, corresponding to the North Pole.

For the intersections stations/polar orbits, let us consider an initial date, corresponding to $t_0 = 0$, defining the absolute longitude and latitude of the ground station: λ_0, L_0.

The orbit is defined by parameters $a_0, e_0 = 0, \Omega_0, i_0 = \frac{\pi}{2}$. Intersection between the orbit and the station occurs when the coordinates (x_S, y_S) of the station are equal to the coordinates of the subsatellite points

$$(x,y):$$

$$x_S = x,$$

$$y_S = y,$$

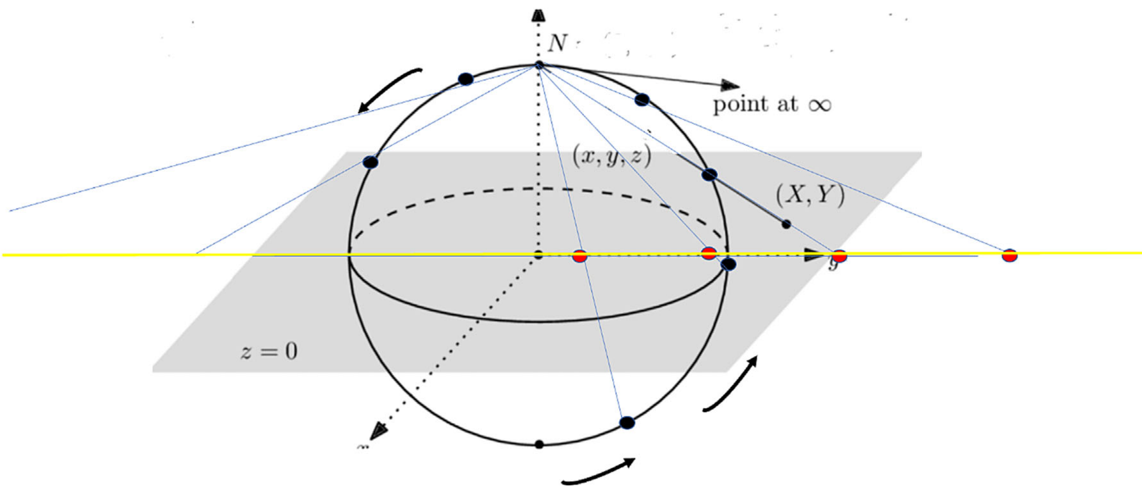

FIGURE 16 The stereographic projection of a polar orbit into the straight line on the equatorial plane.

that is,

$$\frac{R_E \cos L_0}{1-\sin L_0}\cos\lambda = \frac{R_E \cos u}{1-\sin u}\cos\Omega,$$

$$\frac{R_E \cos L_0}{1-\sin L_0}\sin\lambda = \frac{R_E \cos u}{1-\sin u}\sin\Omega.$$

Dividing the second by the first equation we have the *first intersection condition*:

$$\tan\lambda = \tan\Omega.$$

Squaring and summing, we get

$$\frac{\cos {L_0}^2}{(1-\sin L_0)^2} = \frac{\cos u^2}{(1-\sin u)^2}.$$

Since $\frac{\cos x^2}{(1-\sin x)^2} = \frac{1-\sin x^2}{(1-\sin x)^2} = \frac{(1-\sin x)(1-\sin x)}{(1.-\sin x)^2} = \frac{1+\sin x}{1-\sin x}$, we get $\frac{1+\sin L_0}{1-\sin L_0} = \frac{1+\sin u}{1-\sin u}$.

This is equivalent to

$$\sin L_0 = \sin u,$$

and generates the *second intersection condition*

$$u = L_0 + 2k_2\pi.$$

The first condition determines the time t^* when the station reaches the intersection:

$$\lambda = \lambda_0 + \omega_E t^* = \Omega + k_1\pi,$$

that is,

$$t^* = \frac{\Omega - \lambda_0 + k_1\pi}{\omega_E}.$$

The second condition determines the time needed for the satellite to reach the intersection point t^{**}:

$$t^{**} = \frac{u-u_0}{n}.$$

For the ISL between inclined and polar orbit, the same arguments of Section 3 apply, with the observation that the angle between the orbit with inclination i and the polar orbit intersect with angle $\Delta u = \frac{\pi}{2} - i$, and this is independent on the relative RAAN angle.

6 | PRINCIPLES OF CONSTELLATION DESIGN

Basic principles of constellation design are here briefly recalled, more details can be found for instance in.[5,6] A constellation of N satellite in circular Low Earth Orbits is considered, To keep the covering properties of the constellation invariant under the action of the J2 gravitational perturbative term, the satellites of the constellation will have a common value of radius R and inclination i.

The inclination is related to the maximum latitude of interest of the service L_{max}. If $L_{max} < \frac{\pi}{2}$ and ε is the semi-aperture angle of the satellite payload, the inclination is equal to

$$i = L_{max} + \operatorname{asin}\left(\frac{R}{R_E}\sin\varepsilon\right) - \varepsilon.$$

The radius R has to be selected within a range of values according to the payload requirement. Within such a range of values, a further request is to have a periodic ground track, so that the analysis of the covering properties and the planning of the missions can be limited to the fixed and pre-selected period of the ground track. The periodic ground track condition is

$$N_t T_n = m D_n,$$

that is, N_t nodal orbital period is elapsed in exactly m nodal days. The following Table 1 shows the intervals of altitude corresponding to the choices (N_t, m). For instance, $L_{max} = 60^\circ$, $\varepsilon = 20^\circ$ and $(N_t, m) = (44, 3)$ imply that

$$R = 6991\,\text{km}, i = 60.21^\circ$$

The periodic ground track has different symmetry properties depending on: $(N_t + m)$ even or odd, see.[5] This suggests the choice of RAAN and argument of latitude angles.

A simple strategy is to select a reference geographical longitude and consider equally spaced geographical longitudes $\theta_{t,0}^k$ as for instance in Figure 17.

Then, the initial values of the RAAN for each satellite are derived by

$$\Omega_k(t_0) = \lambda_g^{(k)}\left(t_{asc}^{(k)}\right) + \theta_{g0} + t_{asc}^{(k)}(\omega_E - \dot{\Omega}).$$

This configuration will be taken in the next example with $N = 128$.

7 | EXAMPLE

The analytic formulas provided by the stereographic projection are now used to evaluate the performance of the constellation of $N = 128$ satellites in Section 5. Input data are:

1. Initial RAAN and argument of the ascending node of each satellite
2. Latitude and geographic longitude of the target and the ground station
3. Year, month, day, hour, minute, second of the initial time
4. Swath radius of the payload
5. Max distance D for the ISL

An algorithm, based upon the analytic formulas of Sections 3 and 4, generates within 1 second the following output, valid for a duration equal to $Np = 44$ Keplerian periods:

a. Number and time of occurrence of the visibility passages of each satellite over each station/target
b. Duration of each visibility passage (in seconds)
c. Number and time of occurrence of ISL
d. Duration of each ISL (in seconds)

TABLE 1 Range of altitudes depending on the choice of the pair (N_t, m).

Minimum and maximum altitude, H_{min} and H_{max}, depending on (N_t, m)				
N_t	m	r_t	H_{min} (km)	H_{max} (km)
14	1	14	812.4	874.5
43	3	14.333	696.1	761.4
29	2	14.5	639.6	706.5
44	3	14.667	584.1	652.6
15	1	15	476.0	547.9

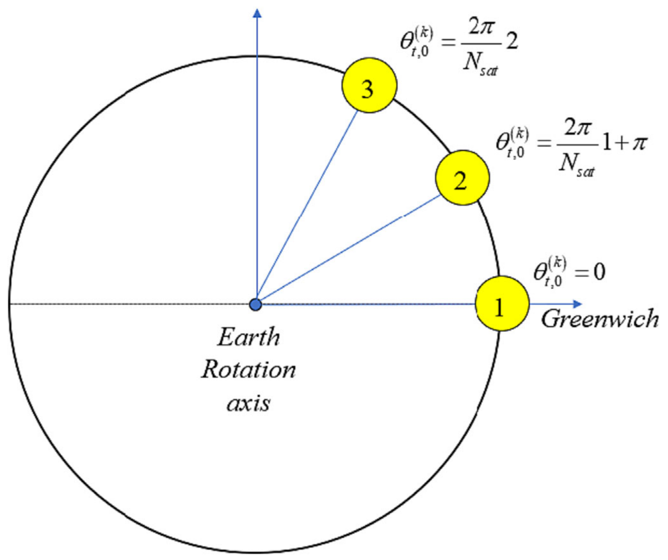

FIGURE 17 A choice of the phases of the satellites of a constellation.

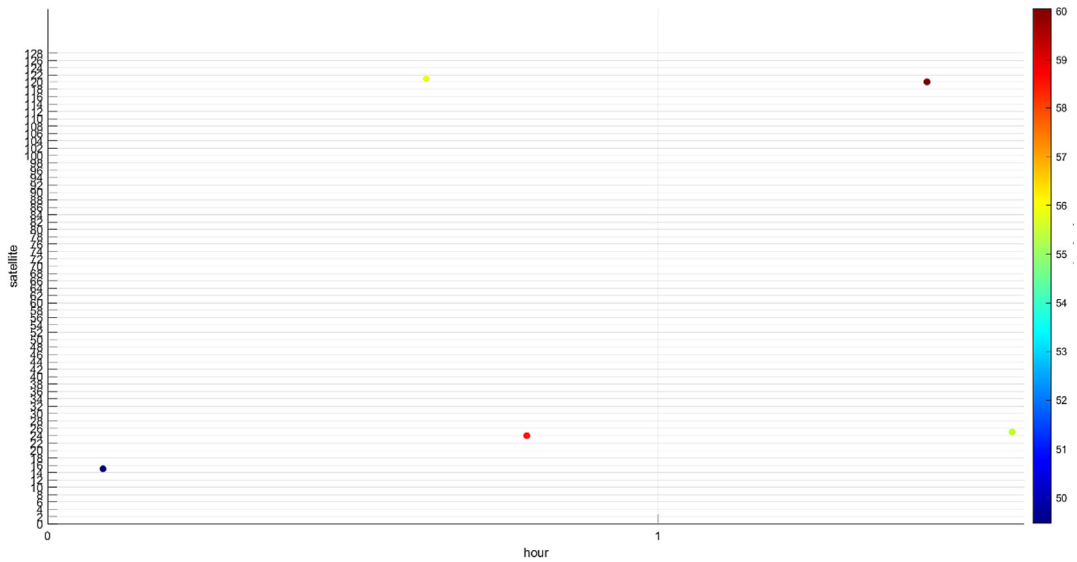

FIGURE 18 The passages over the target within the first orbital period.

The data used for the target are latitude = 50° and geographic longitude 105°. The target has a longitude different by 90° with respect to the longitude of the Ground station (15°). The ground station latitude is equal to 42°. The initial time t_0 corresponds to the first day of 2023 at 00.00 GMT. The payload swath diameter is equal to 1044 km and the ISL maximum distance is equal to $D = 1000$ km.

Figure 18 shows the passages over the target within the first orbital period (1.6 h). Five of the 128 satellites of the constellation have a passage over the target: the first passage occurs 32 s from t_0: the satellite number 15 of the constellation is in link with the target for 50 seconds. The second passage is operated after 37 min by satellite number 121, the duration is equal to 56 s. The third passage is after 47 min, duration of 58 s, by satellite number 24, satellite number 120 reaches the target after 86 min with a passage of duration of 60 s, and finally, satellite number 25 has a passage of 36 s after 95 min. To achieve the transfer of the target data to the ground station within the required time (about 1 hour) the candidate "collector" satellites are Sat15, Sat121, and Sat24.

Figure 19 shows the passages over the ground station within the first orbital period: four satellites pass over the station, satellite numbers 125, 76, 124, and 85. In particular, satellite number 124 meets the station after 64 minutes.

The satellite Sat15 is as collector of the target data and satellite Sat124 is the deliverer.

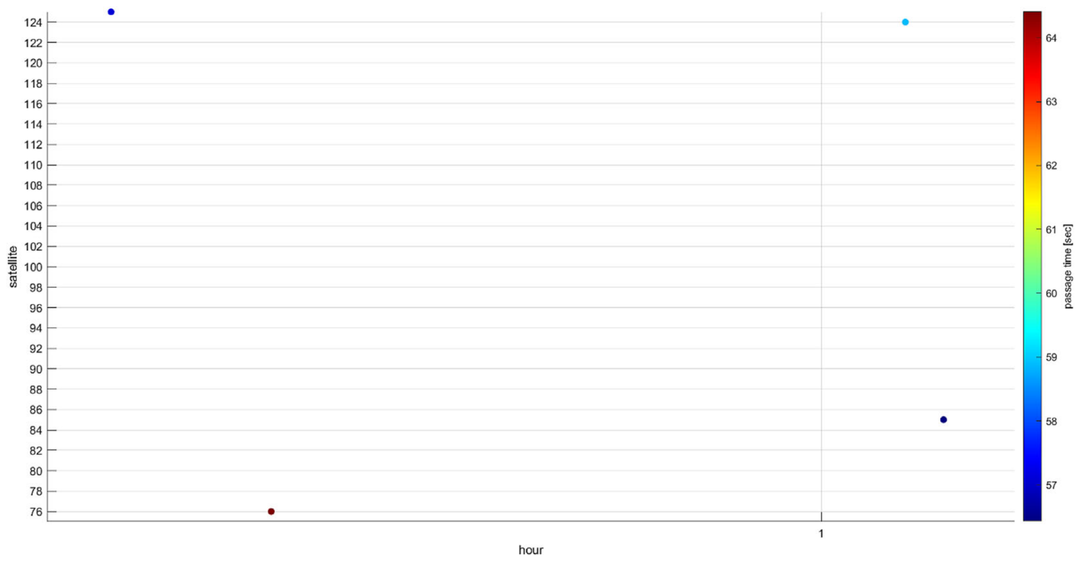

FIGURE 19 The passages over the ground station within the first orbital period.

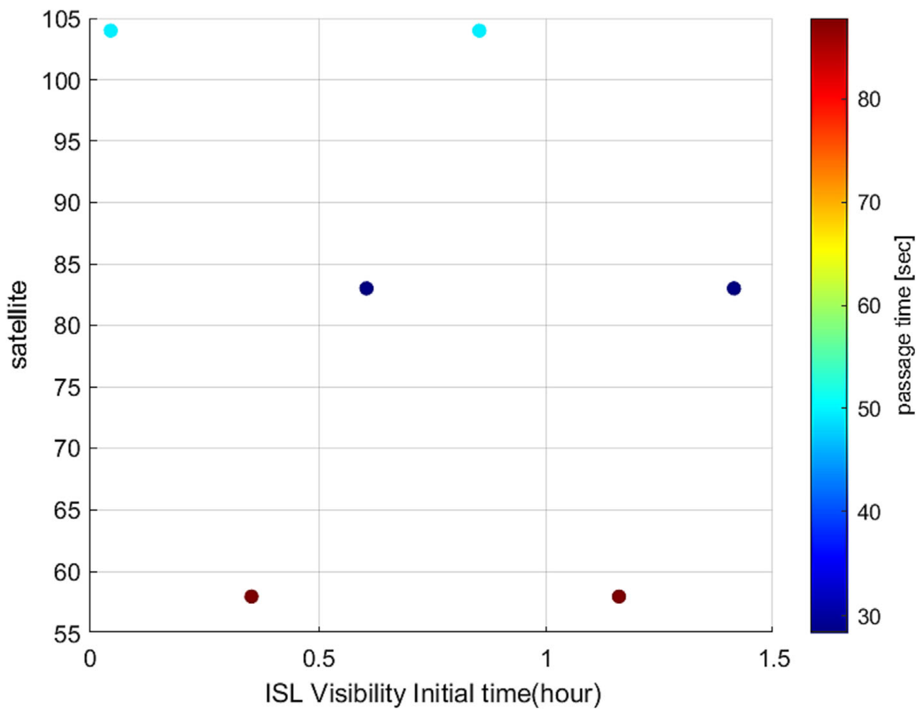

FIGURE 20 Links between Sat15 and the other satellites of the constellation during the first orbital period.

These two satellites have data:

$$\mathrm{Sat15}: \Omega = 161.62\,deg,\ u_0 = 39.37^\circ,$$

$$\mathrm{Sat124}: \Omega = 99.55\ deg,\ u_0 = 165.94^\circ.$$

If the data can be transferred from Sat15 to Sat124 before the delivery to the ground station, the task will be achieved. The analytic algorithm defined in Sections 2 and 3 is used to identify all the possible ISL transfer of data among the satellites of the constellation.

The Sat15 encounters the satellites with numbers 58, 83, and 104, within the times indicated in Figure 20. Figure 21 shows the "tree" of satellite encounters starting from Sat15, and we see that there is the possibility to perform up to three transfers of data from Sat15 to other satellites but this the process runs out of time before possible connections with Sat124.

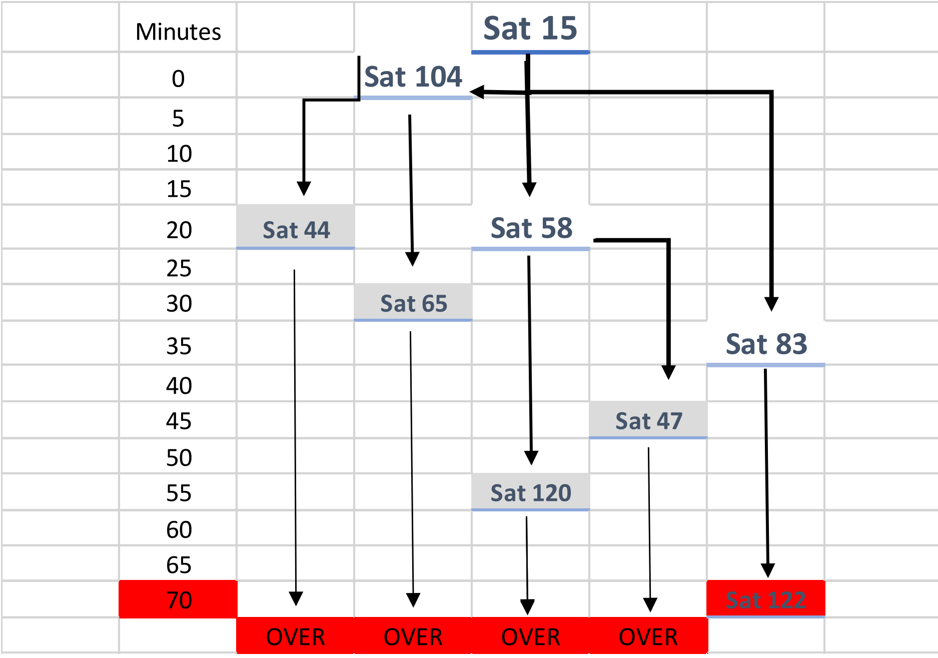

FIGURE 21 The "tree" of satellites that can receive and transmit the data of Sat15 in the first orbital period.

TABLE 2 ISL Sat15—Polar constellation: Times of arrival.

Polar satellites	PSat1 (s)	PSat2 (s)	PSat3 (s)	PSat4 (s)	PSat5 (s)	PSat6 (s)	PSat7 (s)	PSat8 (s)	PSat9 (s)
Sat15@[1]	5726	543	885	1290	2219	299	723	1060	1649
Sat15@[2]	2818	3451	3794	4198	5128	3208	3632	3969	4557

TABLE 3 ISL Sat124 – polar constellation: times of arrival.

Polar satellites	PSat1 (s)	PSat2 (s)	PSat3 (s)	PSat4 (s)	PSat5 (s)	PSat6 (s)	PSat7 (s)	PSat8 (s)	PSat9 (s)
Sat124@[1]	4512	4853	5469	3732	4336	4674	5091	3150	4101
Sat124@[2]	1604	1944	2561	823	1427	1766	2383	242	1193

Then, to accomplish the task of data transmission within 1 h, it is necessary to augment the system by other satellites. Polar or near polar orbits are a good choice: Suppose to select near polar satellites (e.g., $i = 89^\circ$) having equally spaced RAAN. If they have the same initial argument from the node, they arrive at the poles at the same time, and the maximum distance between when they are at the maximum latitude is $R\cos i = 122\,km$

As a consequence, the data delivered by one satellite can be transferred to any other: As a result, the direction of flight of the data to be delivered can be changed at will from 0 to 360° at each North or South Pole arrival.

To test this approach, let us consider nine near polar satellites having a common value of the nodal angle.

Within 1 s, the algorithm generates the following Table 2 and Table 3 reporting the time to arrive at the two intersections of Sat15 and Sat124 with the nine satellites of the polar constellation.

One of the satellites of the polar constellation must receive the data from Sat15 and arrive at Pole to transmit the data to another polar satellite (when needed), which will transmit the data to Sat124. This latter will deliver the data to the ground station at time 3840 s. It follows that all the cases of Table 3 with time to arrival greater than 3840 must be discarded. Moreover, all the combinations where the time to arrival of Sat124 is lower than the time of arrival of Sat15 must be discarded.

This excludes from any combination the polar satellites: PSat1 and PSat9 as well as all the intersections at P2 for Sat15.

The time to arrival of the polar satellites to the Pole T_{Pole} depends on the initial common phase uP of the polar constellation:

$$T_{Pole} = \frac{\left|\frac{\pi}{2} - uP\right|}{n} \text{ if } uP < \frac{\pi}{2} \text{ and } T_{Pole} = \frac{\left|\frac{3}{2}\pi - uP\right|}{n} \text{ if } uP > \frac{\pi}{2}.$$

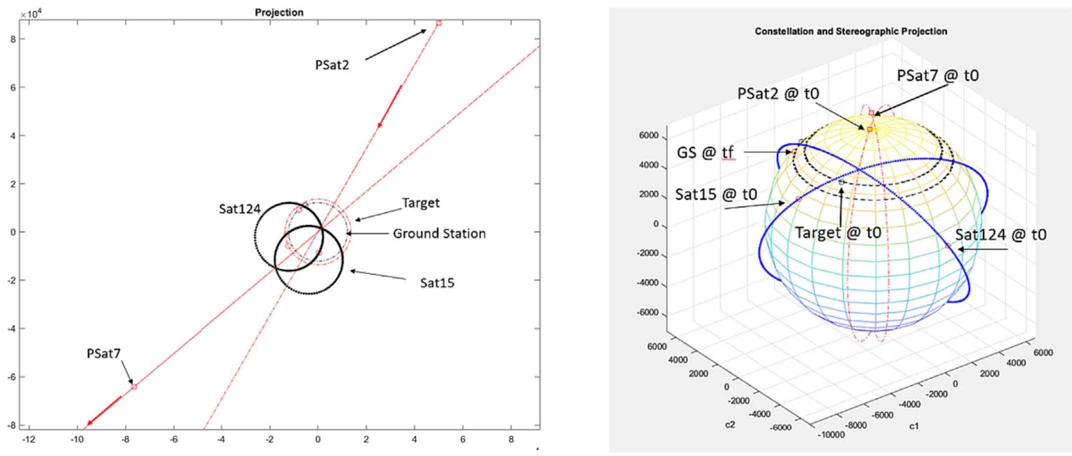

FIGURE 22 The mission in the projected coordinates (left) and in the 3D representation.

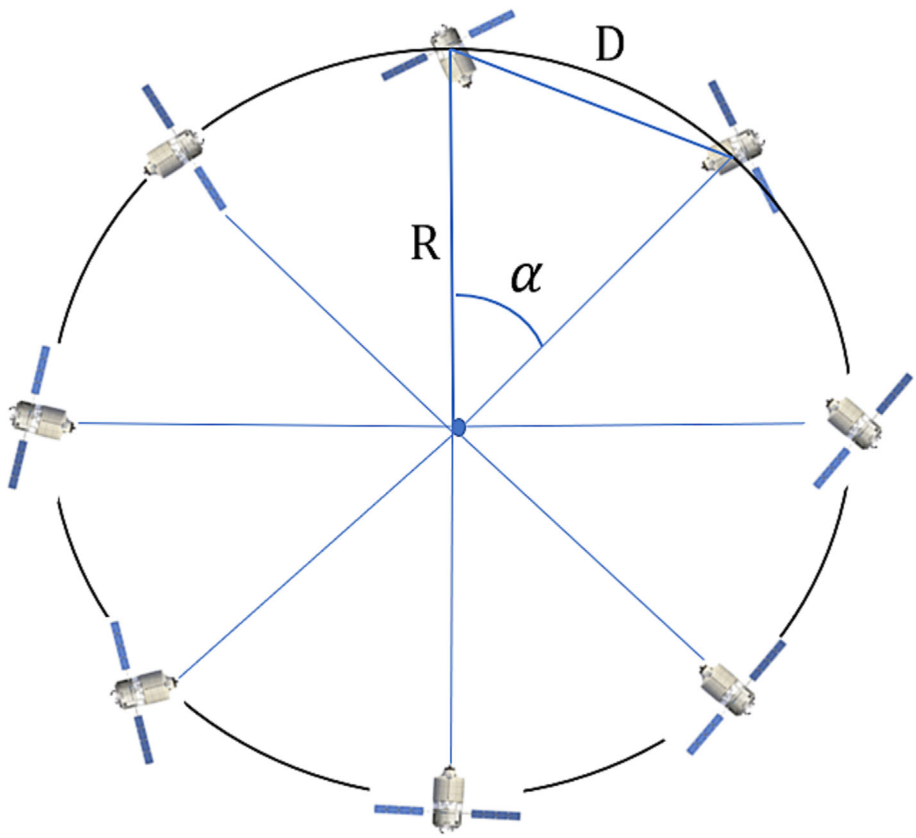

FIGURE 23 The sharing of information within a single orbit.

Inserting a value of *uP*, the algorithm provides in the same run the above data plus the times of arrival of the satellites of the polar constellation to the intersection points with Sat15 and Sat124 and the duration of the ISL (if any) around these points.

It turns out that with the choice $uP = 98^\circ$, PSat2 is able to communicate with Sat15 after 500 s from t_0 and transfer the data to PSat7 at the South Pole around time 2500 s. The satellite PSat7 will transfer the data to Sat124 at around time 2900 s and the latter will transmit the data at the ground station at time 3840 s.

Figure 22 shows the geometry of the mission in the equatorial projection and in the three-dimensional representation.

Although the configuration chosen is successful in the current (and difficult) example, there are too many constraints for the existence of a single phase *uP* accomplishing the mission, and sometimes, the delivery time constraint is violated.

An alternative approach is to reduce the number of planes of the polar constellation and add satellites in the same polar orbit so that they can share the information by an almost immediate sequence of data transfers, see Figure 23.

Of course, this implies a significant increment of the number of polar satellites: to ensure full sharing of data along the polar orbit, the number NP of satellites on each orbit is equal to the nearest integer of $NP = \frac{2\pi}{\alpha}, \alpha = 2\mathrm{asin}\frac{D}{2R}$. With $D = 1000$ km and $R = 6991$ km, we have 48 polar satellites on each orbit. For service at the altitude considered, two polar planes with RAAN difference of 90° and 48 satellites on each of the two orbits. That is 48×2 satellites are added to the previous constellation for a total number of satellites equal to 224. These are enough to accomplish the task of data delivery within 1 h from any site in the world with a latitude lower than 60°.

8 | FINAL COMMENT

In this paper, the stereographic projection onto the equatorial plane of satellite orbits and targets/ground stations motion is used to compute in a very fast way the design and performance of large constellations, providing also a clear two-dimensional picture of the space dynamics of these systems.

The effectiveness of the approach used is shown in the difficult task to deliver in about 1 h the information from a target on the ground to a station having largely different latitudes and longitudes.

It is proved that the output of a constellation of 128 satellites can be generated in less than 1 s and proves that, even with the large number of satellites, the constellation is unable to fulfill the task.

Then, the algorithm is used to augment the constellation with satellites in polar orbits to maximize the use of ISL. The cases where all the satellites of the polar constellation can share the information twice during the orbital period or at any point of the polar orbit are considered.

This research can be continued along two lines: the first line is to use this algorithm to optimize the constellation design using some iterative routine to minimize the number N of satellites of the constellation (in the present example $N = 224$). The second line is to include this algorithm in the optimization of the management of a selected constellation[9–11] to minimize, for instance, the time of transfer of the information to the ground station (at present fixed to about 1 h). In both the lines, the need for a fast computation tool to compare many different possible configurations is even more stringent and the efficiency of the present algorithm appears rather useful.

DATA AVAILABILITY STATEMENT

Research data are not shared.

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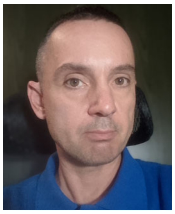

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APPENDIX A: DERIVATION OF SOME ANALYTIC FORMULAS

Derivation of the phases of the intersection points between two orbits of the constellation, (Equations (6) and (7) of Section 3)

Consider two satellites in a circular orbit of equal radius R and inclination i and RAAN Ω_1, Ω_2.

If $a = \frac{R}{|\cos i|}$ and $e = \sin i$, the phases of the satellites in the projected orbits must satisfy at the intersection points the two equations (see Equations (4) and (5) of Section 3):

$$x_1(\tau_{12}) = -ae\sin\Omega_1 + a\cos\tau_{12} = x_2(\tau_{21}) = -ae\sin\Omega_2 + a\cos\tau_{21}$$

$$y_1(\tau_{12}) = ae\cos\Omega_1 + a\sin\tau_{12} = \quad y_2(\tau_{21}) = ae\cos\Omega_2 + a\sin\tau_{21}$$

Let us solve the two equations with respect to the angle τ_{12}:

$$\cos\tau_{12} = e(\sin\Omega_1 - \sin\Omega_2) + \cos\tau_{21} \quad (A.1)$$

$$\sin\tau_{12} = e(\cos\Omega_2 - \cos\Omega_1) + \sin\tau_{21} \quad (A.2)$$

After squaring, the sum gives:

$$1 = 1 + e^2(\sin\,\Omega_1 - \sin\,\Omega_2)^2 + e^2(\cos\,\Omega_2 - \cos\,\Omega_1)^2 + 2e\cos\,\tau_{21}\,(\sin\,\Omega_1 - \sin\,\Omega_2) + \quad +2e\,\sin\,\tau_{21}(\cos\,\Omega_2 - \cos\,\Omega_1)$$

and we look for the zeros τ_{21} of the function:

$$ff = e\,(1 - \cos\,(\Omega_1 - \Omega_2)) + \cos\,\tau_{21}\,(\sin\,\Omega_1 - \sin\,\Omega_2) + \sin\,\tau_{21}(\cos\,\Omega_2 - \cos\,\Omega_1) = 0$$

Write the above equation as:

$$ff = A\sin\,\tau + B\cos\,\tau + C = 0$$

where

$$A = (\cos\,\Omega_2 - \cos\,\Omega_1)$$

$$B = (\sin\,\Omega_1 - \sin\,\Omega_2)$$

$$C = e\,(1 - \cos\,(\Omega_1 - \Omega_2))$$

and consider the equivalent equation

$$\sqrt{A^2 + B^2}\left(\frac{A}{\sqrt{A^2+B^2}}\sin\,\tau + \frac{B}{\sqrt{A^2+B^2}}\cos\,\tau + \frac{C}{\sqrt{A^2+B^2}}\right) = 0$$

where

$$\sqrt{A^2+B^2} = \sqrt{2(1 - \cos\,(\Omega_2 - \Omega_1))}$$

Then the previous equation is equivalent to:

$$a\sin\,\tau + b\cos\,\tau + c = 0 \tag{A.3}$$

where

$$a = \frac{(\cos\,\Omega_2 - \cos\,\Omega_1)}{\sqrt{2(1 - \cos\,(\Omega_2 - \Omega_1))}}$$

$$b = \frac{(\sin\,\Omega_1 - \sin\,\Omega_2)}{\sqrt{2(1 - \cos\,(\Omega_2 - \Omega_1))}}$$

$$c = e\sqrt{(1 - \cos\,(\Omega_1 - \Omega_2))}/\sqrt{2}$$

Since $a,b < 1\,e\,a^2 + b^2 = 1$, let us write $a = \cos\,\phi$, $b = \sin\,\phi$:

$$\cos\,\phi = \frac{(\cos\,\Omega_2 - \cos\,\Omega_1)}{\sqrt{1 - \cos\,(\Omega_2 - \Omega_1)}}$$

$$\sin\,\phi = \frac{(\sin\,\Omega_1 - \sin\,\Omega_2)}{\sqrt{1 - \cos\,(\Omega_2 - \Omega_1)}}$$

Equation (A.3) becomes:

$$\sin\,(\tau + \phi) = -c$$

There are two solutions:

$$\tau+\phi=\text{asin}(-c) \text{ (Equation (6) of Section 3)}$$

and

$$\tau+\phi=\pi-\text{asin}(-c) \text{ (Equation (7) of Section 3)}$$

The two phases τ_{211}, τ_{212} corresponding to the intersections between the projected orbits 1 and 2 are

$$\tau_{211}=\text{asin}\,(-c)-\phi,$$

and

$$\tau_{212}=\pi-\text{asin}\,(-c)-\phi.$$

Insert the value τ_{211} in the Equations (A.1) and (A.2) for $\cos\tau_{121}, \sin\tau_{121}$ to determine the phase of the intersection point P_1 in the projected orbit 1 and the value τ_{212} in the Equations (A.1) and (A.2) for $\cos\tau_{122}, \sin\tau_{122}$ to determine the phase of intersection P_2.

For any of the $\frac{N^2-N}{2}$ pair of projected orbits k_1, k_2, we get the phases of intersection points on the projected orbit k_2:

$$\tau_{k_2k_1 1}=\text{asin}\,(-c)-\phi$$

and

$$\tau_{k_2k_1 2}=\pi-\text{asin}\,(-c)-\phi$$

with

$$c=e\sqrt{\left(1-\cos\,\left(\Omega_{k_2}-\Omega_{k_1}\right)\right)}/\sqrt{2}$$

and

$$\cos\,\phi=\frac{\left(\cos\,\Omega_{k_2}-\cos\,\Omega_{k_1}\right)}{\sqrt{1-\cos\,\left(\Omega_{k_2}-\Omega_{k_1}\right)}},$$

$$\sin\,\phi=\frac{\left(\sin\,\Omega_{k_1}-\sin\,\Omega_{k_2}\right)}{\sqrt{1-\cos\,\left(\Omega_{k_2}-\Omega_{k_1}\right)}}.$$

Finally, the phases of the intersection points ($s=1,2$) on the projection orbit k_1 are obtained by

$$\cos\,\tau_{k_1k_2s}=e\left(\sin\,\Omega_{k_1}-\sin\,\Omega_{k_2}\right)+\cos\,\tau_{k_2k_1s},$$

$$\sin\,\tau_{k_1k_2s}=e\left(\cos\,\Omega_{k_2}-\cos\,\Omega_{k_1}\right)+\sin\,\tau_{k_2k_1s},$$

Derivation of the minimum satellite distance (Equation (12)) and ISL time duration (Equation (13))

The motion of the two satellites around the link region is given respectively by the kinematic equations (see Figure A.1):

$$x_2(t)=x_{20}+\dot{x}_2\,(t-t_0),$$

$$y_2(t)=y_{20}+\dot{y}_2\,(t-t_0).$$

with $x_{20}=-\ell\cos\frac{\Delta u}{2}, y_{20}=-\ell\sin\frac{\Delta u}{2}, \dot{x}_2=V_0\cos\frac{\Delta u}{2}, \dot{y}_2=V_0\sin\frac{\Delta u}{2}$,

$$x_1(t)=x_{10}+\dot{x}_1\,(t-t_0),$$

$$y_1(t)=y_{10}+\dot{y}_1\,(t-t_0),$$

with $x_{10}=-(\ell+V_0dt_{ini})\cos\frac{\Delta u}{2}, y_{10}=(\ell+V_0dt_{ini})\sin\frac{\Delta u}{2}, \dot{x}_1=V_0\cos\frac{\Delta u}{2}, \dot{y}_1=-V_0\sin\frac{\Delta u}{2}$.

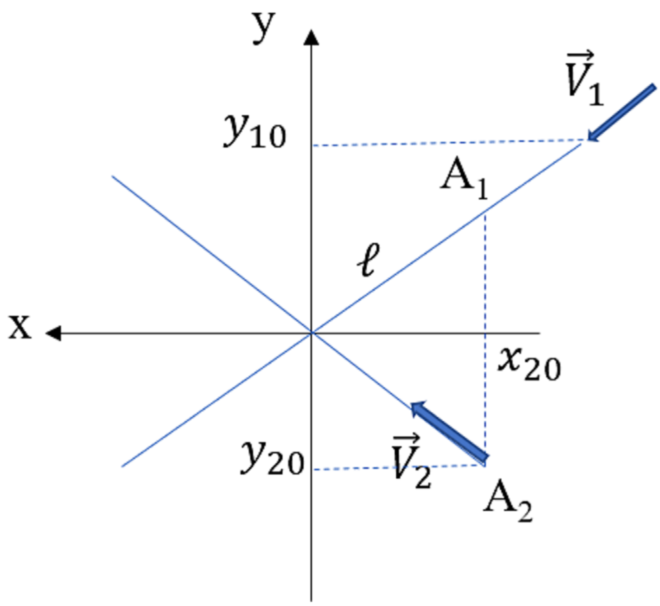

FIGURE A.1 Motion of the satellites near the link region.

These formulas allow to compute the change in time of the satellite distance between them: $d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$ and its minimum found by the equation $\dot{d}=0$, corresponding to the condition $y_1(\bar{t})=y_2(\bar{t})$ for a certain time $\bar{t}$. This time corresponds to the time of minimum distance between the two satellites and it is equal to $\bar{t}=t_0+\frac{y_{10}-y_{20}}{2\dot{y}_2}$, that is,

$$\bar{t}=t_0+\frac{\ell}{V_o}+\frac{\mathrm{dt_{ini}}}{2},$$

In this way, one gets the minimum distance between the satellites, that is, Equation (10) of Section 3:
$d_{min}=|x_1(\bar{t})-x_2(\bar{t})|=V_0\,dt_{ini}\cos\frac{\Delta u}{2}$ (Equation 10 of Section 3)
To have the possibility of link, we have to verify the condition

$$D>V_0\,dt_{ini}\cos\frac{\Delta u}{2}.$$

In such a case, the situation is shown in Figure A.2. The duration of the ISL is the difference $\bar{t}_2-\bar{t}_1$ between the two times where the satellite distance d is equal to D:

$$d(\bar{t}_2)=d(\bar{t}_2)=D.$$

Using the kinematic equations, we get

$$d^2(t)=d_{\min}^2+(y_1-y_2)^2.$$

Then, the condition $d^2=D^2$ implies

$$|y_1-y_2|=\sqrt{D^2-d_{\min}^2}.$$

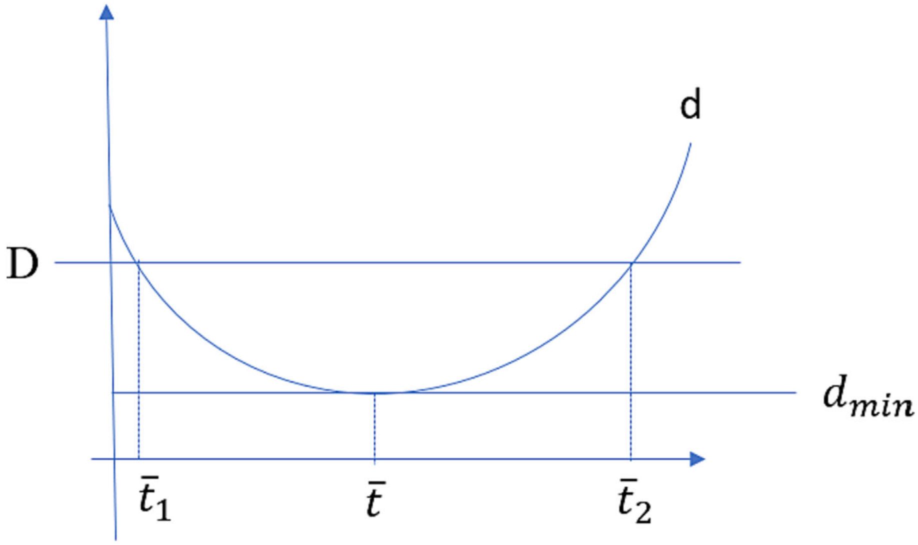

FIGURE A.2 The distance d between the satellites and the ISL distance D.

The modulus has values

$$|y_1 - y_2| = 2\ell \sin\frac{\Delta u}{2} + V_0 dt_{ini} \sin\frac{\Delta u}{2} - 2\,V_0 \sin\frac{\Delta u}{2}\,(t - t_0),$$

$$|y_1 - y_2| = -2\ell \sin\frac{\Delta u}{2} - V_0 dt_{ini} \sin\frac{\Delta u}{2} + 2\,V_0 \sin\frac{\Delta u}{2}\,(t - t_0).$$

Corresponding to the two solutions in time:

$$\bar{t}_{1,2} = t_0 + \frac{2\ell \sin\frac{\Delta u}{2} + V_0 dt_{ini} \sin\frac{\Delta u}{2} \pm \sqrt{D^2 - d_{min}^2}}{2V_0 \sin\frac{\Delta u}{2}}.$$

Then, the ISL duration is equal to

$$ISL_{time} = \bar{t}_2 - \bar{t}_1 = \frac{\sqrt{D^2 - d_{min}^2}}{V_0 \sin\frac{\Delta u}{2}} \text{ (Equation (11) of Section 3)}$$

Derivation of x, $\delta(x)$ of Equation (23), Section 4

The position x of the ground station within the ground station/satellite visibility area and the related angle $\delta(x)$ are hereafter derived.

The position of the ground station at the satellite arrival time t^{**}_{iniS} (see Equation (22) of Section 4) is computed by the angle:

$$\lambda_x = \lambda_0 + t^{**}_{iniS}\,\omega_E.$$

In the case of visibility (see below), we compute

$$d - x = R_E \cos L_0\,|\lambda_k - \lambda_x|.$$

So,

$$x = d - R_E \cos L_0\,|\lambda_k - \lambda_x|.$$

Now, the visibility condition is derived:

If λ_x is such that $R_E \cos L_0 \lceil \lambda_x - \lambda_k \rceil < d$, then we have visibility at intersection point defined by the absolute longitude λ_k.

For the determination of $\delta = \delta(x)$, consider Figure 15 of Section 4, and the triangle of vertices A_1C_3x, its edge ℓ has length

$$\ell^2 = x^2 + R_{sw}^2,$$

with x station position at visibility time. The angle α, within the triangle of edges ℓ, c e $d–x$, see Figure 21, verifies

$$\ell^2 + c^2 - 2\ell c \cos\,\alpha = (d-x)^2 \implies \alpha.$$

Consider now the triangle of Figure 15 with vertices A_1B_2x having edges ℓ, R_{sw}, y. The edge length y can be obtained by the formula:

$$y^2 - 2y\ell\cos\,\alpha + d^2 = R_{sw}{}^2 \implies y, \text{take the lower of the two solutions}.$$

The angle ϵ of the triangle ℓ, R_{sw}, y of Figure 15 at the vertex A_1 is defined by

$$l^2 + R_{sw}{}^2 - 2lR_{sw}\cos\,\epsilon = y^2 \implies \epsilon.$$

Then, finally, we get the angle δ:

$$\delta = \alpha + \varepsilon,$$

Note that the station reaches each intersection point once a day; hence, in the period of ground track repetition, we get the arrival times

$$t^{**}_{\text{iniS}} + m\ \text{Nodal day},$$

where the nodal day takes into account the J_2 effect.