Irreversible Thermalization vs Reversible Dynamics Mediated by Phase-Correlations: Wave Turbulence Theory and Experiments in Optical Fibers

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Abstract: We study thermalization of temporally incoherent light waves in a weakly birefringent single mode fiber. We show that the natural process of light thermalization can be arrested by the spontaneous emergence of strongly correlated fluctuations.

Non-integrable Hamiltonian systems of random waves generally undergo thermalization – an irreversible evolution toward thermodynamic equilibrium, characterized by a state of maximum entropy. In the weakly nonlinear regime, this process is accurately described by the wave turbulence (WT) theory [1]. The WT kinetic equation (KE) describes the actual irreversible evolution to the Rayleigh-Jeans equilibrium distribution. It is important to stress that light thermalization has recently been studied theoretically and experimentally in multimode fibers [2-4], thus spurring the emerging field of optical thermodynamics [5-10].

Phase-correlations are known to underlie large-scale coherent structures, such as solitons or condensates, which emerge from strongly nonlinear turbulent regimes. These coherent structures are, by their nature, phase-correlated entities. In contrast, we will focus here on phase-correlations in the weakly nonlinear regime of random dispersive waves.

Phase-correlations – also referred to as 'anomalous correlations' – were central to the original Bardeen–Cooper–Schrieffer theory of superconductivity [11], and have since been extensively studied within the framework of S−theory in parametrically excited dissipative magnetic system [12]. From a different perspective, recent theoretical advances revealed the presence of phase-correlations in conservative (unforced) wave systems [13-14]. In scalar wave systems, the mechanism underlying the emergence of phase-correlations between different frequencies has been recently elucidated, by showing that they may only arise from Hamiltonian terms that break phase invariance, whereas systems that preserve phase invariance preclude such anomalous correlations [15].

In this contribution [16], we investigate theoretically and experimentally a vector system of coherently coupled NLSEs, which describe the polarization evolution of temporally incoherent light waves in optical fibers. Unlike recent 2D spatial studies in multimode optical fibers [2-4], we examine here the 1D temporal dynamics of incoherent waves propagating in a single-mode fiber. The analysis reveals the emergence of two fundamentally distinct turbulent regimes [16].

In a first regime, the waves undergo a slow, irreversible thermalization process, which is accurately described by the wave turbulence kinetic equation and the associated H−theorem of entropy growth. The thermalization process is characterized by the spontaneous repolarization of the temporally incoherent wave. We derive a wave turbulence kinetic equation, which reveals that nonequilibrium repolarization is thermodynamically favorable, as it allows to increase the entropy in the optical field [16]. Because the Rayleigh-Jeans equilibrium state is out of reach in the time domain, the nonequilibrium thermalization process continues indefinitely, making the repolarization effect virtually unlimited through propagation, see Refs.[17-18] for more details.

In contrast to this expected irreversible process, in a second regime we show that the thermalization process can be arrested by the spontaneous emergence of correlated fluctuations between different wave components [16]. It is important to stress that such phase-correlations grow exponentially from fully uncorrelated initial random waves. These anomalous phase-correlations emerge among different wave components in a phase-invariant vector system, hence they are of different nature from those investigated in Ref.[15]. The system then enters a fundamentally different regime that is characterized by a reversible turbulent dynamic, which is governed by the coupled evolution of the usual normal correlator, and the anomalous phase-correlator [16].

We have experimentally investigated the two different turbulent regimes in a weakly birefringent single mode optical fiber. Experimental results provide evidence of both dynamics characterized by an irreversible process of thermalization and the reversible dynamics mediated by phase-correlations among the orthogonal polarization components [16]. In particular, the measurements show that the incoherent wave undergoes a spontaneous repolarization as a result of its thermalization. This latter effect is evidenced by the formation of a spectral tail decaying with the power law $\sim\omega^{-2}$, which is a signature of light thermalization reflecting energy equipartition amongst the modes. The second reversible turbulent regime is evidenced experimentally by means of a direct measurement of the normal correlator and the anomalous phase-correlator, whose evolution has been found in agreement with the theory and simulations.

In conclusion, our study establishes a basis for future advances in understanding the interplay of fundamentally different turbulence regimes. Until now, turbulent regimes dominated by slow, irreversible thermalization and those governed by fast, correlation-driven reversible dynamics have been separately explored, leaving open the fundamental question about the role of anomalous correlators on thermalization, and the reciprocal impact of irreversible processes on the reversible dynamics. A key challenge for future research is therefore the development of a generalized theory that unifies these two aspects – reversible dynamics associated to anomalous correlators and irreversible thermalization – fully accounting for phase-correlations.

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