The impact of the discrepancies in the yield curve on actuarial forecasting

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Abstract. Both academics and practitioners don't seem to pay too much attention to the databases of zero-coupon rates that they employ in their research and the implementation of models for valuation or risk measuring purposes. However, in this paper we show that significant differences may arise when using alternative popular and usually accepted interest rates databases. In particular, we show that these differences are relevant when calibrating well known interest rates models (Vasicek and Cox,Ingersoll and Ross) and also when valuing some insurance contracts such as temporary life annuities.

Keywords: Yield curve estimation; interest rate databases; Vasicek process; CIR process.

1 Introduction

The term structure of interest rates or yield curves is a basic input for most practitioners and researchers. These zero-coupon interest rates are crucial for a number of purposes in finance, monetary policy, economic theory, and so forth. The final users of yield curves usually download this information from a database provider. Among such users, a few yield curve datasets have become

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popular, some because they are publically available and free and others because they are offered by the primary financial data providers.

It should be pointed out that these popular yield curve datasets may differ considerably. One source of these discrepancies is the model and numerical techniques used to estimate the zero coupon rates. This point has been analyzed in several papers (e.g. [1], and [2]). But another potentially important origin of differences in yield curve estimations is caused by the fact that each provider use as input different prices or yields (transaction prices, quoted prices, or market yields to maturity) as well as different sets of risk-free debt instruments, including/excluding bills, on-the-run bonds, callable bonds, and so forth. However, most papers in the literature and most practitioners appear to be unconcerned about the implications of this key decision that database providers make in estimating their yield curves.

The aim of this study is to examine to what extent the yield curve dataset, that final users obtain from different providers, can affect the interest rate structure model forecasting and, in this case, to analyze their practical effects. The first point will be studied focusing on the sensitivity of the interest rate model parameters on the different dataset. By way of an example we will treat the most popular interest rate structure models, the Cox, Ingersoll and Ross (1981) model (or CIR model) and the Vasicek (1977) process. Additionally, we illustrate the practical implications of the choice of a certain yield curve dataset using an example of an insurance contract.

2 Two alternative popular yield curve datasets

We examine two daily popular yield curve dataset. The first one is simultaneously posted in two different official websites. It appears as Daily Treasury Yield Curve Rate in the US Department of the Treasury website, and also appears as U.S. government securities. Treasury constant maturities included in the Interest Rates H.15 series posted in the Fed website. We refer to this dataset as DoT ⋆. The second considered dataset is posted in the Federal Reserve Board (from herein FRB) ⋆⋆. Other details in particular regarding the collection and validation of the FRB data are provided by Gr̈kaynak et al. (cf. [5]), in the case of DoT by the provider.

There are several sources of discrepancies among these two interest rate datasets. We highlight four of them: the fitting method, the prices/yields that

⋆ This dataset is simultaneously posted in both the Department of Treasury and the Fed websites. See
`http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield`
and
`http://www.federalreserve.gov/releases/h15/data.htm`
respectively.

⋆⋆ This dataset is called the FRB dataset in this paper for convenience. However, the spreadsheet that can be downloaded from the FRB website contains this sentence: *Note: This is not an official Federal Reserve statistical release*. It is daily updated.
`http://www.federalreserve.gov/econresdata/feds/2006/index.htm`

are used as the input in the fitting process, the basket of assets from which the yield curve is estimated, and the reported interest rate.

First, the DoT follows a quasi-cubic Hermite spline function to interpolate among yields-to-maturity of observed Treasury securities. However, the FRB uses a weighted Svensson model (cf. [6]) to fit the yield curve. Different fitting methods imply a different trade-off between having the flexibility to fit complex shapes and the required level of smoothness and convergence over the long end of the yield curve. Second, the DoT employs close of business bid yields-to-maturity. On the contrary, Grkaynak et al. ([5]) note that they use end-of-day prices for their FRB estimates, although they do not specify what type of prices. So, the DoT fits yields-to-maturity whereas FRB uses as dependent variable bond prices. Third, the DoT considers the four most recently auctioned bills (4-, 13-, 26-, and 52-week), the six maturities of on-the-run bonds and notes (2-, 3-, 5-, 7-, 10-, and 30-years), and the composite rate in the 20-year maturity range. The FRB estimates only include second-off-the-run or older bonds, with more than three months to maturity. Further, they exclude quotes of all securities with less than three months to maturity, all Treasury bills, bonds with embedded options, twenty-year bonds since 1996, the on-the-run bond and the first-off-the-run bond for each maturity (2-, 3-, 4-, 5-, 7-, 10-, 20-, and 30-years), and other issues that we judgmentally exclude on an ad hoc basis. The different compositions of the dataset of securities from which the yield curves are estimated have multiple implications. The Treasury market includes traded securities with important liquidity differences. Fourth, the DoT provides par yields whereas the FRB shows spot or zero-coupon rates. As DoT considers on-the-run bills and bonds, the resulting yield curve can be interpreted as a par yield curve instead of the desirable zero-coupon yield curve.

3 The impact within the actuarial context

The discrepancies among databases referring to the same financial instrument, as previously pointed out, suggests to deepen if they have an after-effect on the interest rate structure model forecasting and, in this case, to analyze their practical effects. The first point will be studied focusing on the sensitivity of the interest rate model parameters on the different dataset. By way of an example we will treat the most popular interest rate structure models, the CIR and the Vasicek process.

Within this context, in the following we will check the importance of the choice of different datasets when the parameters of a stochastic term structure models are calibrated. By way of an example we will treat a CIR process and a Vasicek process. In both cases we will indicate by β the long term mean, α the reversion factor and σ the instantaneous volatility. α, β and σ are positive constants.

The calibration of the term structure models CIR and Vasicek has been performed on the two different interest rate datasets collected by the FRB and the DoT. In the time-series analysis, the temporal interval ranges from 4th January 1982 to 1st December 2014.

We will calibrate both the two set of parameters by means of two different methodologies: the time series approach and the panel method. The time-series method is performed on 3-months interest rates; the panel method is performed on 3-months, 6-months, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year interest rates. The results we obtained for the CIR and the Vasicek parameters are reported in Table 1. Here we collected the values got respectively for the speed of adjustment α, the long term mean β and the diffusion coefficient σ of the two considered stochastic processes, having reported the initial position r of the two different datasets we referred to. The first table was determined by the 3-month time series technique while the second one by the panel technique.

As we can plainly observe, the parameters of the CIR and Vasicek processes are sensitive to the two different datasets considered in our analysis, in both the techniques used for calibrating the models themselves. In particular, the sensitivity on the used dataset seems to be stronger concerning the speed of adjustment or reversion factor α and the diffusion coefficient σ, while a certain stability can be observed in the long term means values. This evidence highlights a remarkable difference in the interest rate structure dynamics, more than on its long term stability.

These observations lead to a reflection on the effect the choice of a certain dataset, as previously described, can bear on financial valuations. In order to give an exemplification of this, in the following we will deal with an insurance contract, where the issue takes a central role. In fact, it is not an accident that the European Insurance and Occupational Pensions Authority (EIOPA) provides a monthly basis relevant risk-free interest rate term structures, according to Functional the Solvency II Directive (cf. [4]). In particular the Authority gives suggestions concerning financial instruments used to derive the basic risk-free interest rate term structures.

The applications are performed on a temporary life annuity issued on an insured aged 65, with unitary benefit paid at the beginning of each year if the insured is alive, with a single premium paid at the issue time. The life table is obtained on the basis of a Lee Carter model on the US male dataset (ages 0:110; Years: 1933-2007). We will study the mathematical reserve and the results are collected in Figures 1, 2, 3 and 4. In all the graphs the blue line refers to the DoT data, the red one to FRB data. It is clear the notable impact of the two different datasets in particular when the process is the Vasicek one, while the reserves seem overlapping in the CIR case. In the Vasicek case, the relevant difference is evident in both the techniques used for the estimation procedure. Of course one can guess the amplification the question could have in the case of life annuity contracts characterized by longer duration and high number of contracts.

Further research will be addressed in deepening this topic, considering it a relevant practical aspect of the financial systematic risk, the main risk component in the life contracts valuations.

3 – MONTH TIME SERIES EVALUATION				
	Alpha	Beta	Sigma%	r
VASICEK DoT	0.0840	0.0435	0.5954	0.03
VASICEK FRB	0.0712	0.0440	0.5781	0.021
CIR DoT	0.0884	0.0435	0.6097	0.03
CIR FRB	0.0712	0.0440	0.5429	0.021

Table 1. Time-series technique: Vasicek and CIR parameter estimation

PANEL EVALUATION				
	Alpha	Beta	Sigma%	r
VASICEK DoT	0.1028	0.0523	0.6186	0.03
VASICEK FRB	0.0956	0.0524	0.6050	0.021
CIR DoT	0.1028	0.0523	0.6371	0.03
CIR FRB	0.0957	0.0524	0.6014	0.021

Table 2. Panel technique: Vasicek and CIR parameter estimation

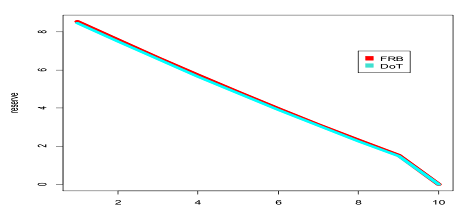

Fig. 1. Comparison between FRB and DoT reserves, pension annuity on policyholder aged x=65 CIR model 3-month time series approach

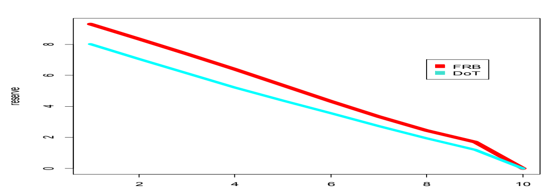

Fig. 2. Comparison between FRB and DoT reserves, pension annuity on policyholder aged x=65 Vasicek model 3-month time series approach

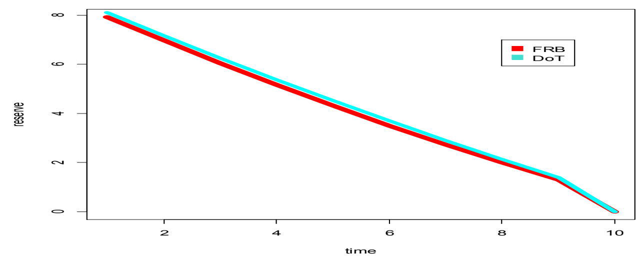

Fig. 3. Comparison between FRB and DoT reserves, pension annuity on policyholder aged x=65 CIR model Panel approach

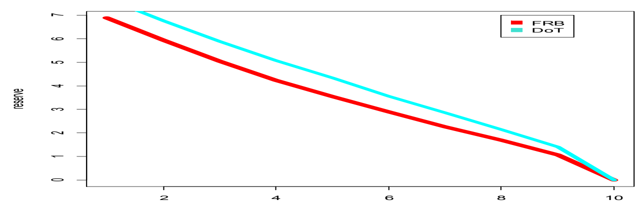

Fig. 4. Comparison between FRB and DoT reserves, pension annuity on policyholder aged x=65 Vasicek model Panel approach

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